Dominant Currency Paradigm

A New Model for Small Open Economies*

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Abstract

Most trade is invoiced in very few currencies. Despite this, the Mundell-Fleming benchmark and its variants focus on pricing in the producer's currency or in local currency. We model instead a 'dominant currency paradigm' for small open economies characterized by three features: pricing in a dominant currency; pricing complementarities, and imported input use in production. Under this paradigm: (a) the terms-of-trade is stable; (b) dominant currency exchange rate pass-through into export and import prices is high regardless of destination or origin of goods; (c) exchange rate pass-through of non-dominant currencies is small; (d) expenditure switching occurs mostly via imports, driven by the dollar exchange rate while exports respond weakly, if at all; (e) strengthening of the dominant currency relative to non-dominant ones can negatively impact global trade; (f) optimal monetary policy targets deviations from the law of one price arising from dominant currency fluctuations, in addition to the inflation and output gap. Using data from Colombia we document strong support for the dominant currency paradigm.

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1 Introduction

Nominal exchange rates have always been at the center of fierce economic and political debates on spillovers, currency wars, and competitiveness. It is easy to understand why: in the presence of price rigidities, nominal exchange rate fluctuations are associated with fluctuations in relative prices and therefore have consequences for real variables such as the trade balance, consumption, and output.

The relationship between nominal exchange rate fluctuations and other nominal and real variables depends critically on the currency in which prices are rigid. The first generation of New Keynesian (NK) models, the leading paradigm in international macroeconomics, assumes prices are sticky in the currency of the producing country. Under this 'producer currency pricing' paradigm (PCP), the law of one price holds and a nominal depreciation raises the price of imports relative to exports (the terms-of-trade) thus improving competitiveness. This paradigm was developed in the seminal contributions of Mundell (1963) and Fleming (1962), Svensson and van Wijnbergen (1989), and Obstfeld and Rogoff (1995).

There is, however, pervasive evidence that the law of one price fails to hold. Out of this observation grew a second pricing paradigm. In the original works of Betts and Devereux (2000) and Devereux and Engel (2003), prices are instead assumed to be sticky in the currency of the destination market. Under this 'local currency pricing' paradigm (LCP), a nominal depreciation lowers the price of imports relative to exports, a decline in the terms-of-trade, thus worsening competitiveness. Both paradigms have been extensively studied in the literature and are surveyed in Corsetti et al. (2010).

Recent empirical work using granular data on international prices questions the validity of both approaches. Firstly, there is very little evidence that the best description of pricing in international markets follows either PCP or LCP. Instead, the vast majority of trade is invoiced in a small number of 'dominant currencies', with the U.S. dollar playing an outsized role. This is documented in Goldberg and Tille (2008) and in Gopinath (2015). Moreover, these prices are found to be rigid for significant durations in their currency of invoicing, as documented by Gopinath and Rigobon (2008) and Fitzgerald and Haller (2012). Secondly, exporters price in markets characterized by strategic complementarities in pricing that give rise to variations in the elasticity of demand and desired mark-ups.[1] Thirdly, most exporting firms employ imported inputs in production reducing the value added content of exports.[2] The workhorse NK models in the literature instead assume constant

[1]Burstein and Gopinath (2014) survey the evidence on variable mark-ups.

[2]The fact that most exporters are also importers is now well documented in the literature. See Bernard et al. (2009), Kugler and Verhoogen (2009), Manova and Zhang (2009) among others. This is also reflected in the fact that value added exports are significantly lower than gross exports, particularly for manufacturing, as documented in the works of Johnson (2014) and

demand elasticity and/or abstract from intermediate inputs.

Based on these observations, this paper proposes an alternative: the 'dominant currency paradigm' (DCP). Under DCP, firms set export prices in a dominant currency (most often the dollar) and change them infrequently. They face strategic complementarities in pricing, so that desired markups vary over time and across destination markets. Finally, there is roundabout production, with domestic and foreign inputs employed in production. With these assumptions, the model departs fundamentally from the canonical NK small open economy model *à la* Galí and Monacelli (2005).

We emphasize the following main results. First, at both short and medium horizons the terms-of-trade is stable, playing little to no role in expenditure switching. Second, the dominant currency exchange rate pass-through into export and import prices is high, regardless of the destination or origin of goods. Third, the exchange rate pass-through of non-dominant currencies is negligible. Fourth, while depreciations have a limited expansionary impact on exports, expenditure switching still occurs through imports, arising from fluctuations in the relative price of imported to domestic goods. In turn, these are driven by movements in a country's exchange rate relative to the dominant currency, regardless of the country of origin of the imported goods. Fifth, a strengthening of the dominant currency relative to non-dominant ones can negatively impact global trade. Sixth, optimal monetary policy targets deviations from the law of one price arising from fluctuations in the dominant currency, in addition to the inflation and output gap.

Using customs data for a representative small open economy, Colombia, we document strong support for the predictions of the model.

Sections 2 and 3 present the baseline model and describe in detail its predictions for the terms-of-trade, exchange rate pass-through, and the impact of monetary policy shocks across pricing regimes. In contrast to the PCP and LCP paradigms, DCP is associated with stable terms-of-trade. This stability, however, differs from predictions of models with flexible prices and strategic complementarities in pricing such as Atkeson and Burstein (2008). Unlike these models, the terms-of-trade stability is associated with volatile movements of the relative price of imported to domestic goods for non-dominant (currency) countries that will be the focus of our analysis. Furthermore, this volatility is driven by fluctuations in the value of the country's currency relative to the dominant currency, regardless of the country of origin of the imported goods. Consequently, when a country's currency depreciates relative to the dominant currency, all else equal, it reduces its demand for imports from *all* countries.

Johnson and Noguera (2012). Amiti et al. (2014) present empirical evidence of the influence of strategic complementarities in pricing and of imported inputs on pricing decisions of Belgian firms.

In the case of exports, in contrast to PCP, which associates exchange rate depreciations with increases in quantities exported, DCP predicts a negligible impact on goods exported to the dominant-currency destination. For exporting firms whose dominant currency prices are unchanged there is no increase in exports. For those firms changing prices the rise in marginal cost following the rise in the price of imported inputs and the complementarities in pricing dampen their incentive to reduce prices, leaving exports mostly unchanged. The impact on exports to non-dominant currency destinations depends on the fluctuations of the exchange rate of the destination country currency with the dominant currency. If the exchange rate is stable then DCP predicts a weak impact on exports to non-dollar destinations. On the other hand, if the destination country currency weakens (strengthens) relative to the dominant currency it can lead to a decline (increase) in exports.

Taken together, we find that the inflation-output trade off in response to a monetary policy shock (under an inflation targeting monetary rule) worsens under DCP relative to PCP. That is, a monetary rate cut raises inflation by much more than it increases output, as compared to PCP.

Fluctuations in the value of dominant currencies can also have implications for cyclical fluctuations in global trade (the sum of exports and imports). Under DCP, a strengthening of dominant currencies relative to non-dominant ones is associated with a decline in imports across the periphery without a commensurate increase in exports, thus negatively impacting global trade. In contrast, in the case of PCP, the rise in export competitiveness for the periphery generates an increase in exports. Moreover, the increase in exports dampens the decline in imports as production relies on imported intermediate inputs. In the case of LCP, both the import and export response is muted so the impact on global trade is weak, but remains positive.

Section 4 then proceeds to test the novel empirical predictions of our model for a small open economy, Colombia, that is representative of emerging markets in its heavy reliance on dollar invoicing, with 98.3% (98.4%) of its exports (manufacturing exports) invoiced in dollars.

We document that, as predicted by DCP, the pass-through into import and export (Colombian) peso prices measured as the elasticity relative to the peso-dollar exchange rate starts out high for import prices *and* export prices and then gradually declines over time. This is true regardless of the origin of imports or destination of exports. In the case of export prices to dollar destinations, the contemporaneous pass-through estimate is 84% while the cumulative pass-through slowly decreases after two years to 56%. In the case of import prices from dollar origins, the pass-through is very high, around 100%, and the cumulative effect after two years declines to 81%. For exports (imports) to (from) non-dollar destinations, the estimated pass-through starts at around 86% (87%) and decreases to 47% (49%) after two years.

Secondly, we find that, conditional on the peso-dollar exchange rate, the bilateral exchange rate is quantitatively insignificant as an explanatory factor in bilateral transactions with non-dollar economies. Unconditionally, the pass-through of the bilateral exchange rate into peso export prices to non-dollar destinations is 70% at the annual horizon. However, when we control for the peso-dollar exchange rate the coefficient on the bilateral exchange rate drops to 9% while the coefficient on the peso-dollar exchange rate is 70%. These predictions are also consistent with DCP.

Thirdly, we also find that, following a weaker peso/dollar exchange rate, the pass-through to export quantities to dollar destinations is mainly insignificantly different from zero while there is a pronounced decline in quantities imported from both dollar and non-dollar countries. Exports to non-dollar destinations also decline. Further, when quantities respond, the relevant exchange rate is the peso/dollar exchange rates as opposed to the bilateral exchange rate for both export and import quantities.

Lastly, while Colombia's overall terms-of-trade is very volatile and strongly correlated with the exchange rate, when we strip out commodity prices we find the terms-of-trade to be highly stable—a feature consistent with the predictions of DCP.

To further compare the different pricing paradigms we simulate in Section 5 a model economy that is subject to commodity price shocks, productivity shocks, and third country exchange rate shocks, and test its ability to match the data. As the model nests DCP, PCP and LCP we can evaluate the success of the various paradigms. Using a combination of calibration and estimation we document that the data strongly rejects the PCP and LCP paradigms in favor of DCP.

The data also favors a model with strategic complementarities in pricing and imported input use. For example, under our benchmark DCP specification we obtain, in line with the data, the export pass-through at four quarters to both dollar and non-dollar destinations to be 65%. Instead when we shut down strategic complementarities and imported input use the predicted pass-through declines by a half to 30%.

Section 6 derives optimal monetary policy for a small open economy with dominant currency pricing under parameter restrictions similar to Galí and Monacelli (2005). The second-order approximation to the welfare loss function under dominant currency pricing differs from that under PCP: in addition to inflation and the output gap, it includes a term that captures misalignment due to the failure of the law of one price for Home goods driven by dominant currency fluctuations. The terms-of-trade is also independent of monetary policy, under common parameter restrictions, in contrast to PCP where it is influenced by monetary policy. This gives rise to a breakdown of "divine coincidence": it is no longer possible to attain simultaneously zero inflation and a zero output gap.[3] Optimal monetary policy calls for domestic producer price inflation targeting while the output gap fluctuates with the terms of trade. A final section concludes.

Related Literature: Our paper is related to a relatively small literature that models dollar pricing. These include Corsetti and Pesenti (2005), Goldberg and Tille (2008), Goldberg and Tille (2009), Devereux et al. (2007), Cook and Devereux (2006) and Canzoneri et al. (2013). All of these models, with the exception of Canzoneri et al. (2013), are effectively static with one period ahead price stickiness. Unlike Canzoneri et al. (2013) we explore a three region world, which is crucial to analyze differences between dominant and non-dominant currencies. Goldberg and Tille (2009) explore three regions but in a static environment. In addition, the dollar pricing literature assumes constant desired mark-ups and production functions that use only labor.

Our contribution to this literature is three-fold. Firstly, we develop a quantitative new Keynesian small open economy model that combines dynamic dominant currency pricing, variable mark-ups and imported input use in production. All of these features are important ingredients required to match facts on pricing in international trade. The model also provides a counterpart for the empirical pass-through regressions employed in the data. Secondly, we empirically evaluate the dominant currency paradigm employing data from Colombia using novel tests that the model generates. Lastly, we derive the target criteria for optimal monetary policy for a small open economy under dominant currency pricing.

The evidence on asymmetric responses of the volume of exports and imports is consistent with that documented by Alessandria et al. (2013) for exports and Gopinath and Neiman (2014) for imports.[4] Boz et al. (2017) extend and affirm our findings for global trade using bilateral export and import price indices for 2,500 country pairs.

[3]The new Keynesian literature has emphasized a number of important deviations from the divine coincidence in the open economy, even in the absence of cost-push shocks. See Monacelli (2013) for a discussion. However, the breakdown of divine coincidence under DCP occurs even under conditions such that the divine coincidence would obtain under PCP.

[4]The typical explanations for the sluggish export response has to do with quantity frictions arising from say sunk costs or search costs, while the relative price of exports to destination market prices are assumed to move strongly with the exchange rate. DCP, consistent with the data predicts that such relative prices are stable and therefore does not require quantity frictions in the short-term to generate slow adjustments in exports.

2 Model

We model a small open economy, H (for Home) that trades goods and assets with a rest of the world that we divide into two regions: U (for the dominant currency country) and R (for the Rest). The nominal exchange rate between country $i \in \{U, R\}$ and Home is denoted $\mathcal{E}_{i,t}$, expressed as Home currency per unit of foreign currency, so that an increase in $\mathcal{E}_{i,t}$ represents a depreciation of the Home currency against that of country i. Under the small open economy assumption, we assume that prices and quantities in U and R are exogenous from the perspective of H.

As in the canonical small open economy framework of Galí (2008) firms adjust prices infrequently, *à la* Calvo. We however depart from Galí (2008) along the following dimensions: Firstly, we nest three different pricing paradigms: local currency pricing and dominant currency pricing alongside producer currency pricing. Secondly, the production function uses not just labor but also intermediate inputs produced domestically and abroad. Thirdly, we allow for strategic complementarity in pricing that gives rise to variable mark-ups, as opposed to constant mark-ups. Fourthly, international asset markets are incomplete with only riskless bonds being traded, as opposed to the assumption of complete markets. We describe the details below.

2.1 Households

Home is populated with a continuum of symmetric households of measure one. In each period household h consumes a bundle of traded goods $C_t(h)$. Each household also sets a wage rate $W_t(h)$ and supplies an individual variety of labor $N_t(h)$ in order to satisfy demand at this wage rate. Households own all domestic firms. To simplify exposition we omit the indexation of households when possible. The per-period utility function is separable in consumption and labor and given by,

$$U(C_t, N_t) = \frac{1}{1-\sigma_c} C_t^{1-\sigma_c} - \frac{\kappa}{1+\varphi} N_t^{1+\varphi} \tag{1}$$

where $\sigma_c > 0$ is the household's coefficient of relative risk aversion, $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply and κ scales the disutility of labor.

The consumption aggregator C is implicitly defined by a Kimball (1995) homothetic demand aggregator:

$$\sum_i \frac{1}{|\Omega_i|} \int_{\omega \in \Omega_i} \gamma_i \Upsilon \left(\frac{|\Omega_i| C_{iH}(\omega)}{\gamma_i C} \right) d\omega = 1. \tag{2}$$

In eq. (2) $C_{iH}(\omega)$ represents the consumption by households in country H of variety ω produced by country i where $i \in \{H, U, R\}$. γ_i is a parameter that captures home bias in H with $\sum_i \gamma_i = 1$, and

$|\Omega_i|$ is the measure of varieties produced in region i. The function Υ satisfies the constraints $\Upsilon(1) = 1$, $\Upsilon'(.) > 0$ and $\Upsilon''(.) < 0$. This demand structure gives rise to strategic complementarities in pricing and variable mark-ups. It captures the classic Dornbusch (1987) and Krugman (1987) channel of variable mark-ups that gives rise to pricing to market as described below.

Home households solve the following optimization problem,

$$\max_{C_t, W_t, B_{U,t+1}, B_{t+1}(s')} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to the per-period budget constraint expressed in home currency,

$$P_t C_t + \mathcal{E}_{U,t}(1+i_{U,t-1})B_{U,t} + B_t = W_t(h)N_t(h) + \Pi_t + \mathcal{E}_{U,t}B_{U,t+1} + \sum_{s' \in \mathcal{S}} Q_t(s')B_{t+1}(s') + \mathcal{E}_{U,t}\zeta_t \quad (3)$$

where P_t is the price index for the domestic consumption aggregator C_t. Π_t represents domestic profits that are transfered to households who own the domestic firms. Households also trade a risk-free international bond denominated in dollars that pays a nominal interest rate $i_{U,t}$ and $B_{U,t+1}$ denotes the dollar debt holdings of this bond at time t. Households also have access to a full set of domestic state contingent securities (in H currency) that are traded domestically and in zero net supply. Denoting $\mathcal{S}$ the set of possible states of the world, $Q_t(s)$ is the period-t price of the security that pays one unit of home currency in period $t+1$ and state $s \in \mathcal{S}$, and $B_{t+1}(s)$ are the corresponding holdings. Finally, ζ_t represents an exogenous dollar income shock to the domestic budget constraint. This is a simple way to capture shocks such as commodity price movements for small commodity exporters.

The optimality conditions of the household's problem yield the following demand system:

$$C_{iH,t}(\omega) = \gamma_i \psi \left(D_t \frac{P_{iH,t}(\omega)}{P_t} \right) C_t, \quad (4)$$

where $\psi(.) \equiv \Upsilon'^{-1}(.) > 0$ so that $\psi'(.) < 0$, $D_t \equiv \sum_i \int_{\Omega_i} \Upsilon' \left(\frac{|\Omega_i| C_{iH,t}(\omega)}{\gamma_i C_t} \right) \frac{C_{iH,t}(\omega)}{C_t} d\omega$ and $P_{iH,t}(\omega)$ denotes the home price of variety ω produced in country i and sold in H. Define the elasticity of demand $\sigma_{iH,t}(\omega) \equiv -\frac{\partial \log C_{iH,t}(\omega)}{\partial \log Z_{iH,t}(\omega)}$, where $Z_{iH,t}(\omega) \equiv D_t \frac{P_{iH,t}(\omega)}{P_t}$. The log of the optimal flexible price mark-up is $\mu_{iH,t}(\omega) \equiv \log \left(\frac{\sigma_{iH,t}}{\sigma_{iH,t}-1} \right)$. It is time-varying and we denote $\Gamma_{iH,t}(\omega) \equiv \frac{\partial \mu_{iH,t}}{\partial \log Z_{iH,t}(\omega)}$ the elasticity of that markup.

The price index P_t satisfies,

$$P_t C_t = \sum_i \int_{\Omega_i} P_{iH,t}(\omega) C_{iH,t}(\omega) d\omega$$

Inter-temporal optimality conditions for U bonds and H bonds are given by the usual Euler equation:

$$C_t^{-\sigma_c} = \beta(1+i_{U,t})\mathbb{E}_t C_{t+1}^{-\sigma_c} \frac{P_t}{P_{t+1}} \frac{\mathcal{E}_{U,t+1}}{\mathcal{E}_{U,t}} \tag{5}$$

$$C_t^{-\sigma_c} = \beta(1+i_t)\mathbb{E}_t C_{t+1}^{-\sigma_c} \frac{P_t}{P_{t+1}} \tag{6}$$

where $(1+i_t) = (\sum_{s' \in \mathcal{S}} Q_t(s'))^{-1}$ is the inverse of the price of a risk-free nominal H currency bond at time t that delivers one unit of H currency in every state of the world in period $t+1$.

Households are subject to a Calvo friction when setting wages in H currency: in any given period, they may adjust their wage with probability $1-\delta_w$, and maintain the previous-period nominal wage otherwise. As we will see, they face a downward sloping demand for the specific variety of labor they supply given by, $N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\vartheta} N_t$, where $\vartheta > 1$ is the constant elasticity of labor demand and W_t is the aggregate wage rate. The standard optimality condition for wage setting is thus given by:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \delta_w^{s-t} \Theta_{t,s} N_s W_s^{\vartheta(1+\varphi)} \left[\frac{\vartheta}{\vartheta-1} \kappa P_s C_s^{\sigma} N_s^{\varphi} - \frac{\bar{W}_t(h)^{1+\vartheta\varphi}}{W_s^{\vartheta\varphi}} \right] = 0, \tag{7}$$

where $\Theta_{t,s} \equiv \beta^{s-t} \frac{C_s^{-\sigma_c}}{C_t^{-\sigma_c}} \frac{P_t}{P_s}$ is the stochastic discount factor between periods t and $s \geq t$ used to discount profits and $\bar{W}_t(h)$ is the optimal reset wage in period t. This implies that $\bar{W}_t(h)$ is preset as a constant markup over the expected weighted-average of future marginal rates of substitution between labor and consumption and aggregate wage rates, during the duration of the wage. This is a standard result in the New Keynesian literature, as derived, for example, in Galí (2008).

2.2 Producers

Each home producer manufactures a unique variety ω that is sold both domestically and internationally. The output of the firm is used both for final consumption and as an intermediate input for production. The production function uses a combination of labor L_t and intermediate inputs X_t, with a Cobb Douglas production function:

$$Y_t = e^{a_t} L_t^{1-\alpha} X_t^{\alpha} \tag{8}$$

where α is the constant share of intermediates in production and a_t is a productivity shock. The intermediate input aggregator X_t takes the same form as the consumption aggregator in eq. (2):

$$\sum_i \frac{1}{|\Omega_i|} \int_{\omega \in \Omega_i} \gamma_i \Upsilon \left(\frac{|\Omega_i| X_{iH,t}(\omega)}{\gamma_i X_t} \right) d\omega = 1, \tag{9}$$

where $X_{iH,t}(\omega)$ represents the demand by firms in country H for variety ω produced in country i as intermediate input. The labor input L_t is a CES aggregator of the individual varieties supplied by each household,

$$L_t = \left[\int_0^1 L_t(h)^{(\vartheta-1)/\vartheta} \mathrm{d}h\right]^{\vartheta/(\vartheta-1)}$$

with $\vartheta > 1$.

Similarly, a good produced in H can be used for consumption or as an intermediate input in each country i. We assume that the foreign demand for domestic individual varieties (both for consumption and as intermediate input) takes a form similar to that in eq. (4).

Markets are assumed to be segmented so firms can set different prices by destination market and invoicing currency. Denote $P^j_{Hi,t}(\omega)$ the price of a domestic variety ω sold in market i and invoiced in currency j. The per-period profits of the domestic firm producing variety ω are then given by:

$$\Pi_t(\omega) = \sum_{i,j} \mathcal{E}_{j,t} P^j_{Hi,t}(\omega) Y^j_{Hi,t}(\omega) - \mathcal{MC}_t\, Y_t(\omega) \tag{10}$$

with the convention that $\mathcal{E}_{H,t} \equiv 1$. In that expression, $Y^j_{Hi,t}(\omega) = C^j_{Hi,t}(\omega) + X^j_{Hi,t}(\omega)$ is the demand for domestic variety ω in country i invoiced in currency j, both used for consumption and as an input in production, while $Y_t(\omega) = \sum_{i,j} Y^j_{Hi,t}(\omega)$ is the total demand across destination markets and invoicing currencies. $\mathcal{MC}_t$ denotes the nominal marginal cost of domestic firms in domestic currency. Given eq. (8), it is given by:

$$\mathcal{MC}_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \cdot \frac{W_t^{1-\alpha} P_t^\alpha}{e^{a_t}}. \tag{11}$$

The optimality conditions for hiring labor are given by,

$$(1-\alpha)\frac{Y_t}{L_t} = \frac{W_t}{\mathcal{MC}_t}, \qquad L_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\vartheta} L_t, \tag{12}$$

with

$$W_t = \left[\int W_t(h)^{1-\vartheta} dh\right]^{\frac{1}{1-\vartheta}},$$

while the demand for intermediate inputs is determined by,

$$\alpha\frac{Y_t}{X_t} = \frac{P_t}{\mathcal{MC}_t}, \qquad X_{iH,t}(\omega) = \gamma_i \psi\left(D_t \frac{P_{iH,t}(\omega)}{P_t}\right) X_t. \tag{13}$$

2.2.1 Pricing

Firms choose prices at which to sell in H and in international markets U and R, with prices reset infrequently. As in Galí (2008) we consider a Calvo pricing environment where firms are randomly chosen to reset prices with probability $1-\delta_p$. A core focus of this paper is on the implications of various pricing choices by firms. We assume that firms set their prices either in the producer currency, in the destination currency, or in the dominant currency.

Without lack of generality, we define U's currency to be the dominant currency. Denote θ_{ij}^k as the fraction of exports from region i to region j that are priced in currency k, with $\sum_k \theta_{ij}^k = 1$ for any $\{i,j\} \in \{H,U,R\}^2$. The benchmark of producer currency pricing (PCP) corresponds to the case where $\theta_{i,j}^i = 1$ for every $i \neq j$. The case of local currency pricing (LCP) corresponds to $\theta_{ij}^j = 1$ for every $i \neq j$. Under the dominant currency paradigm (DCP), $\theta_{ij}^U = 1$ for every $i \neq j$. Lastly, we assume that all domestic prices are sticky in the home currency, an assumption consistent with a large body of evidence: $\theta_{ii}^i = 1$ for every i.

Consider the pricing problem of a domestic firm selling in country i and invoicing in currency j, and denote $\bar{P}_{Hi,t}^j(\omega)$ its *reset* price. This reset price satisfies the following optimality condition:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \delta_p^{s-t} \Theta_{t,s} Y_{Hi,s|t}^j(\omega)(\sigma_{Hi,s}^j(\omega)-1)\left(\mathcal{E}_{j,s}\bar{P}_{Hi,t}^j(\omega) - \frac{\sigma_{Hi,s}^j(\omega)}{\sigma_{Hi,s}^j(\omega)-1}\mathcal{MC}_s\right) = 0 \tag{14}$$

with the convention that $\mathcal{E}_{H,t} \equiv 1$. In this expression, $Y_{Hi,s|t}^j(\omega)$ is the quantity sold in country i invoiced in currency j at time s by a firm that resets prices at time $t \leq s$ and $\sigma_{Hi,s}^j(\omega)$ is the elasticity of demand. This expression implies that $\bar{P}_{Hi,t}^j(\omega)$ is preset as a markup over expected future marginal costs expressed in currency j, $\mathcal{MC}_s(\omega)/\mathcal{E}_{j,s}$, during the duration of the price. Observe that because of strategic complementarities, the markup over expected future marginal costs is not constant.

2.3 Interest Rates

2.3.1 Home interest rate i_t

The domestic risk-free interest rate is set by H's monetary authority and follows an inflation targeting Taylor rule with inertia:

$$i_t - \bar{i} = \rho_m(i_{t-1} - \bar{i}) + (1-\rho_m)\phi_M \pi_t + \varepsilon_{i,t} \tag{15}$$

In eq. (15), ϕ_M captures the sensitivity of policy rates to domestic price inflation $\pi_t = \Delta \ln P_t$, while ρ_m captures the inertia in setting rates. $\varepsilon_{i,t}$ evolves according to an $AR(1)$ process, $\varepsilon_{i,t} =$

$\rho_{\varepsilon_i}\varepsilon_{i,t-1} + \varepsilon_{m,t}$, while $\bar{i}$ denotes the target nominal interest rate. In a zero inflation steady state equilibrium, we assume that this target nominal rate equals the exogenous international borrowing rate i^*: $\bar{i} = i^*$.

2.3.2 Dollar interest rate $i_{U,t}$

As in Schmitt-Grohe and Uribe (2003), we assume that the spread between the dollar interest rate at which H borrows internationally $i_{U,t}$ and the exogenous international interest rate i^* is an increasing function of the deviation of the aggregate level of debt from the steady state level of debt:

$$i_{U,t} = i^* + \psi(e^{B_{U,t+1}-\bar{B}} - 1). \tag{16}$$

$\psi > 0$ measures the responsiveness of the dollar rate to the country's net foreign position $B_{U,t+1}$ and $\bar{B}$ is the steady state (exogenous) dollar denominated debt.[5] Because of the dependence on aggregate debt individual households do not internalize the effect of their borrowing choices on the interest rate.

2.3.3 Relation between $\mathcal{E}_{U,t}$ and $\mathcal{E}_{R,t}$

We capture the relation between $\mathcal{E}_{U,t}$ and $\mathcal{E}_{R,t}$ using the following reduced form relation between the two real exchange rates, that we later discipline with data:

$$\ln \mathcal{E}_{R,t} + \ln P^R_{R,t} - \ln P_t = \eta \left(\ln \mathcal{E}_{U,t} + \ln P^U_{U,t} - \ln P_t\right) + \epsilon_{R,t} \tag{17}$$

In eq. (17), $P^R_{R,t}$ and $P^U_{U,t}$ are the consumer price level in R and U in their respective currencies, $\epsilon_{R,t}$ captures idiosyncratic fluctuations in the U-R exchange rate while η captures the comovement between the two real exchange rates.

This specification generates exogenous fluctuations in the bilateral exchange rate between U and R, that will allow us in Section 5 to explore separately how fluctuations in $\mathcal{E}_{U,t}$ and $\mathcal{E}_{R,t}$ impact prices and quantities in H, under different pricing paradigms.[6]

2.4 Equilibrium and Some Analytics

Given the preceding assumptions, the monopolistically competitive equilibrium of the small open economy is defined as follows.

[5]This is a standard assumption in the SOE literature to induce stationarity of $B_{U,t}$ in a log-linearized environment.

[6]An alternative set-up would be to allow for the SOE to borrow internationally in both U and R currencies. Then (even if interest rates in U and R do not change) shocks that drive a wedge in the UIP conditions (commonly used to capture risk-premia shocks) for each of the two currencies will generate fluctuations in $\mathcal{E}_{U,t}/\mathcal{E}_{R,t}$.

Definition 1 (Equilibrium) *A monopolistically competitive equilibrium of the small open economy H consists of:*

a) Households maximizing utility over consumption, labor supply and portfolio choice, and firms maximizing profits over labor demand, intermediate inputs and prices in each market.

b) Market clearing: $L_t = N_t$, $B_t^h = 0$, $Y_{Hi,t} = C_{Hi,t} + X_{Hi,t}$.

c) Real exchange rates of R and U related according to eq. (17).

d) Exogenous shocks to domestic monetary policy, $\epsilon_{M,t}$, the budget constraint, ζ_t, productivity a_t, and the real exchange rate $\epsilon_{R,t}$ that follow $AR(1)$ processes.

■

We solve the model by log-linearizing around a symmetric zero inflation steady state. Before proceeding to the models dynamics in the general case, we provide some insights into its inner workings. This in turn generates testable predictions that we take to the data in Section 4. In Section 3 we adopt a specific functional form for the demand aggregator Υ and provide an expression for the elasticity of the mark-up defined previously, $\Gamma_{ij,t}$. Importantly, approximating up to the first order around a symmetric point, the pricing equations only depend on the constant $\Gamma_{ij,t} = \Gamma$ evaluated at the steady state.

2.4.1 Exchange Rate Pass-through

We first discuss exchange rate pass-through ($ERPT$), that is, the impact of a nominal exchange rate movement on prices for the two extremes of flexible prices and fully rigid preset prices. In the following expressions, p, w and e denote $\ln P$, $\ln W$ and $\ln \mathcal{E}$ respectively. We keep all foreign prices and quantities fixed at exogenous values. All proofs are relegated to the appendix.

Proposition 1 (Flexible prices) *When prices are fully flexible ($\delta_p = 0$) exchange rate pass-through into* *export prices ($p_{Hi,t}$) and import prices ($p_{iH,t}$) expressed in H currency are given by:*

$$\begin{aligned}
\Delta p_{Hi,t} &= \frac{1}{1+\Gamma}\left[\frac{\alpha\gamma_i}{1-\alpha\gamma_H}+\Gamma\right]\Delta e_{i,t} \\
&+\frac{1}{1+\Gamma}\frac{\alpha\gamma_j}{1-\alpha\gamma_H}\Delta e_{j,t} \\
&+\frac{1}{1+\Gamma}\frac{1-\alpha}{1-\alpha\gamma_H}\Delta w_t-\frac{1}{1+\Gamma}\frac{1}{1-\alpha\gamma_H}\Delta a_t \qquad (18)
\end{aligned}$$

$$\begin{aligned}
\Delta p_{iH,t} &= \frac{1}{1+\Gamma}\left[1+\Gamma\frac{\gamma_i}{1-\alpha\gamma_H}\right]\Delta e_{i,t} \\
&+\frac{\Gamma}{1+\Gamma}\frac{\gamma_j}{1-\alpha\gamma_H}\Delta e_{j,t} \\
&+\frac{\Gamma}{1+\Gamma}\frac{\gamma_H(1-\alpha)}{1-\alpha\gamma_H}\Delta w_t-\frac{\Gamma}{1+\Gamma}\frac{\gamma_H}{1-\alpha\gamma_H}\Delta a_t \qquad (19)
\end{aligned}$$

where $j \neq i$, for $i, j \in \{U, R\}^2$. ■

Consider first export prices, Eq. (18). When prices are fully flexible the export price is determined by the marginal cost of H firms and their desired mark-up.

The marginal cost of H firms depends on wages, the price of intermediate inputs, and productivity. The price of intermediate inputs in H depends in turn on the cost of production in each country expressed in H currency and the preference shares γ_i in the aggregator eq. (9). Because of the roundabout nature of production, the impact of wages on marginal cost $(1-\alpha)/(1-\alpha\gamma_H)$ exceeds its direct share $(1-\alpha)$ in the production function, and is increasing in γ_H, the preference for home goods. If there is full home-bias ($\gamma_H = 1$) the impact of wages on marginal costs is one to one.

Secondly, exchange rate fluctuations directly affect the cost of imported inputs and therefore affect the marginal cost of producing H goods. This cost is increasing in the share of these inputs γ_i, $i \neq H$. What this implies is that third currency exchange rates matter for bilateral export prices in addition to bilateral exchange rates.

Lastly, the desired mark-up depends on the degree of strategic complementarity, controlled by Γ, the elasticity of the mark-up to prices. When $\Gamma > 0$, firms wish to keep their prices stable relative to their competitors' in destination markets. This is captured by the term $\Gamma/(1+\Gamma)\Delta e_{i,t}$ in equation (18).

If domestic wages are rigid ($\Delta w_t = 0$), productivity is unchanged ($\Delta a_t = 0$), and $\eta = 1$ in eq. (17), we obtain the following expression for the export price exchange rate pass-through:

$$ERPT^x \equiv \frac{\Delta p_{Hi,t}}{\Delta e_{i,t}} = 1-\frac{1-\alpha}{(1+\Gamma)(1-\alpha\gamma_H)} \qquad (20)$$

In the case with no intermediate inputs used in production, $\alpha = 0$, and constant mark-ups $\Gamma = 0$ as in Galí and Monacelli (2005), $ERPT^x$ is equal to zero or equivalently the pass-through into destination currency prices is 100%, the full pass-through benchmark in the literature: firms set their local price as a constant markup above a fixed wage, regardless of the exchange rate.[7] When intermediate inputs are used in production but there is full home bias so that $\gamma_H = 1$ and $\Gamma = 0$, then again $ERPT^x = 0$, since in that case, marginal cost depends only on local wages and productivity.

When $\gamma_H < 1$ or $\Gamma > 0$, we obtain $ERPT^x > 0$ or equivalently an imperfect pass-through into destination currency prices. With less than full home bias, $\gamma_H < 1$ the cost of imported inputs and domestic marginal costs increase with a depreciation of the domestic currency, pushing up local currency prices. The lower the home bias in intermediate inputs the higher is $ERPT^x$. Similarly, with strategic complementarities, $\Gamma > 0$, domestic firms increase their markup when the domestic currency depreciates. The stronger the strategic complementarities, the higher is $EPRT^x$.

Consider next import prices, eq. (19). Import prices of foreign goods in domestic currency depend on the foreign cost of production, foreign firms' desired mark-up and the exchange rate of the foreign currency. It follows that variation in import prices are driven by fluctuations in desired mark-up and the bilateral exchange rate. In turn, with strategic complementarities, the desired mark-up varies with the local competitors' price.

By analogy with eq. (20), we can define an import price exchange rate pass-through under the same assumptions:

$$ERPT^m \equiv \frac{\Delta p_{iH,t}}{\Delta e_{i,t}} = \frac{1}{1+\Gamma} + \frac{\Gamma}{1+\Gamma}\frac{1-\gamma_H}{1-\alpha\gamma_H} \tag{21}$$

According to eq. (21), when $\Gamma = 0$, the pass through into home currency prices is 1 (100%): foreign firms set a constant price in foreign currency, converted into H currency at the prevailing exchange rate. By contrast, with strategic complementarities, $\Gamma > 0$, foreign firms set prices that depend on their local competitors' marginal costs and the pass-through is incomplete: $ERPT^m < 1$. The first term captures the direct impact of strategic complementarities in pricing, that is holding fixed competitors prices a higher Γ dampens pass-through. The second term captures the indirect effect that works in the opposite direction because the exchange rate change is associated with higher marginal costs for H firms through the imported input channel. This causes H firms to raise prices too and that in turn leads foreign firms to raise theirs. This effect is increasing in Γ and in the share

[7] Equation (20) can be compared to the analysis in Burstein and Gopinath (2014) where the pass-through is in terms of destination currency prices from exchange rate changes expressed as destination currency per unit of home currency, equal in our notations to $1 - ERPT^x = \frac{1}{1+\Gamma}\frac{1-\alpha}{1-\alpha\gamma_H}$. This collapses to the formula in Burstein and Gopinath (2014) when $\gamma_H = 0$, that is when only imported intermediate inputs are used in production.

of imported inputs in production $(1-\gamma_H)$.

The next proposition considers the case of fully rigid prices ($\delta_p = 1$).

Proposition 2 (Fully rigid prices) *When prices are fully rigid and pre-determined in their currency of invoicing, pass-through into export and import prices expressed in H currency for $i \in \{U, R\}$ are given by,*

$$\Delta p_{Hi,t} = \theta^U_{Hi} \Delta e_{U,t} + \mathbb{I}_{i=R} \cdot \theta^R_{Hi} \Delta e_R \tag{22}$$

$$\Delta p_{iH,t} = \theta^U_{iH} \Delta e_{U,t} + \mathbb{I}_{i=R} \cdot \theta^R_{iH} \Delta e_R \tag{23}$$

where $\mathbb{I}_{i=R}$ takes the value 1 when $i = R$ and 0 otherwise.

- *In the case of PCP, $\theta^H_{Hi} = 1$ and $\theta^i_{iH} = 1$ for $i \in \{U, R\}$*

$$\begin{aligned} \Delta p_{Hi,t} &= 0 \cdot \Delta e_{i,t} + 0 \cdot \Delta e_{j \neq i,t}, \qquad \Delta p_{iH,t} = 1 \cdot \Delta e_{i,t} + 0 \cdot \Delta e_{j \neq i,t}, \qquad \forall i \\ tot_{iH,t} &= \Delta p_{iH,t} - \Delta p_{Hi,t} = 1 \cdot \Delta e_{i,t} \qquad \forall i \end{aligned}$$

- *In the case of LCP, $\theta^i_{Hi} = 1$ and $\theta^H_{iH} = 1$ for $i \in \{U, R\}$.*

$$\begin{aligned} \Delta p_{Hi,t} &= 1 \cdot \Delta e_{i,t} + 0 \cdot \Delta e_{j \neq i,t} \qquad \Delta p_{iH,t} = 0 \cdot \Delta e_{i,t} + 0 \cdot \Delta e_{j \neq i,t} \qquad \forall i \\ tot_{iH,t} &= \Delta p_{iH,t} - \Delta p_{Hi,t} = -1 \cdot \Delta e_{i,t} \qquad \forall i \end{aligned}$$

- *In the case of DCP, $\theta^U_{Hi} = 1$ and $\theta^U_{iH} = 1$ for $i \in \{U, R\}$*

$$\begin{aligned} \Delta p_{Hi,t} &= 1 \cdot \Delta e_{U,t} + 0 \cdot \Delta e_{i \neq U,t} \qquad \Delta p_{iH,t} = 1 \cdot \Delta e_{U,t} + 0 \cdot \Delta e_{i \neq U,t} \qquad \forall i \\ tot_{iH,t} &= \Delta p_{iH,t} - \Delta p_{Hi,t} = 0 \qquad \forall i \end{aligned}$$

where tot_{iH} is the terms-of-trade between regions H and i ■

This proposition highlights that in the event of dominant currency pricing and extreme price stickiness the only relevant exchange rate is the dollar exchange rate $e_{U,t}$, regardless of destination or origin country. Moreover, because export and import prices load perfectly on the dollar exchange rate, the terms-of-trade is constant. This contrasts with the predictions under PCP and LCP where one of the export or import prices loads on the bilateral exchange rate $e_{i,t}$, and therefore movements in the terms-of-trade load fully on the bilateral exchange rate: under PCP a depreciation of the nominal exchange rate worsens the terms-of-trade. The reverse occurs under LCP. We test empirically these propositions in the data in section 4.

2.4.2 Price dynamics: the general case

Define the (log) export price index to country i for goods invoiced in currency j, $p^j_{Hi,t}$, and the (log) import price index from country i for goods invoiced in currency j, $p^j_{iH,t}$, with $\pi^j_{Hi,t}$ and $\pi^j_{iH,t}$ the corresponding destination/source and currency specific inflation rates. Log-linearizing the equilibrium reset price equation (14) around a steady state with zero inflation and following standard steps (see the appendix for derivations) we arrive at the following destination/source and currency specific export and import price index inflation:

$$\pi^j_{Hi,t} = \frac{\lambda_p}{1+\Gamma}\left[\left(mc^j_{H,t} - p^j_{Hi,t}\right) + \Gamma\left(p^j_{i,t} - p^j_{Hi,t}\right) + \mu\right] + \beta\mathbb{E}_t\pi^j_{Hi,t+1} \tag{24}$$

$$\pi^j_{iH,t} = \frac{\lambda_p}{1+\Gamma}\left[\left(mc^j_{i,t} - p^j_{iH,t}\right) + \Gamma\left(p^j_{H,t} - p^j_{iH,t}\right) + \mu\right] + \beta\mathbb{E}_t\pi^j_{iH,t+1} \tag{25}$$

where $\lambda_p = (1-\delta_p)(1-\beta\delta_p)/\delta_p$, $mc^j_{i,t}$ is the (log) nominal marginal cost of firms in country i, expressed in currency j (e.g. $mc^j_{H,t} = \ln(\mathcal{MC}_t/\mathcal{E}_{j,t})$), $p^j_{i,t}$ is the (log) of the aggregate price level of country i in currency j, μ is the log of the steady state desired gross markup, and Γ is the steady-state elasticity of that markup.

Eq. (24) reveals that the destination/ currency specific export price index inflation rate $\pi^j_{Hi,t}$ varies with (a) the destination/currency specific (log) markup $p^j_{Hi,t} - mc^j_{H,t}$, (b) the ratio of export prices to the destination price index, expressed in the same currency, $p^j_{Hi,t} - p^j_{i,t}$ and (c) expected future export price inflation. Strategic complementarities, $\Gamma > 0$, dampen the impact of movements in real marginal cost or markups on export price inflation. At the same time a higher Γ raises the sensitivity of export price inflation to the ratio of export prices to the destination price index (expressed in the same currency) since firms pay more attention to the price of their competitors. A similar interpretation applies to the source/currency specific import price index inflation rate $\pi^j_{iH,t}$ in equ. (25).

Because marginal costs rely on imported inputs, cost-shocks in U and R directly impact pricing decisions of H firms. This is in contrast to standard NK open economy models where foreign shocks have no direct impact on marginal costs and only impact it indirectly through risk-sharing and its effect on consumption and therefore on wages.

3 Impulse Response to a Monetary Policy Shock

As the previous discussion reveals, there are starkly different implications for exchange rate pass-through, the terms-of-trade and the volume of trade under the different currency pricing regimes. In this section we present numerical impulse responses to a monetary policy shock to contrast the responses under different pricing regimes.

Preference Aggregator: To start with, we specify a functional form for the demand function Υ. We adopt the Klenow and Willis (2016) formulation that gives rise to the following demand for individual varieties:

$$Y_{iH,t}(\omega) \equiv C_{iH,t}(\omega) + X_{iH,t}(\omega) = \gamma_i \left(1 + \epsilon \ln \frac{\sigma - 1}{\sigma} - \epsilon \ln Z_{iH,t}\right)^{\sigma/\epsilon} (C_t + X_t)$$

where $Z \equiv \frac{P_{iH}(\omega)}{P} D$ as previously defined and σ and ϵ are two parameters that determine the elasticity of demand and its variability as follows:

$$\sigma_{iH,t} = \frac{\sigma}{\left(1 + \epsilon \ln \frac{\sigma-1}{\sigma} - \epsilon \ln Z_{iH,t}\right)} \qquad \Gamma_{iH,t} = \frac{\epsilon}{\left(\sigma - 1 - \epsilon \ln \frac{\sigma-1}{\sigma} + \epsilon \ln Z_{iH,t}\right)}.$$

In a symmetric steady state $Z_{iH,t} = (\sigma - 1)/\sigma$, the elasticity of demand is σ and the elasticity of the mark-up $\Gamma \equiv \frac{\epsilon}{\sigma-1}$.

Parameter Values: Table 1 lists parameter values employed in the simulation. The time period is a quarter. Several parameters take values standard in the literature (see e.g. Galí, 2008). Following Christiano et al. (2011) we set the wage stickiness parameter $\delta^w = 0.85$ corresponding roughly to a year and a half average duration of wages. The steady state elasticity of substitution σ is assumed in the model to be the same across varieties within a region and also across regions. Accordingly, we calibrate to an average of these elasticities measured in the literature. Specifically, Broda and Weinstein (2006) obtain a median elasticity estimate of 2.9 for substitution across imported varieties, while Feenstra et al. (2010) estimate a value close to 1 for the elasticity of substitution across domestic and foreign varieties. Thus, we set $\sigma = 2$.

To parameterize ϵ we rely on estimates from the micro pass-through literature that converges on very similar values for Γ despite the differences in data and methodology. Following Amiti et al. (2016), Amiti et al. (2014), Gopinath and Itskhoki (2010) we set $\Gamma = 1$. Because in steady state $\Gamma = \frac{\epsilon}{\sigma-1}$ this implies $\epsilon = 1$.

The home bias shares are set to $\{\gamma_H, \gamma_U, \gamma_R\} = \{3/5, 1/5, 1/5\}$. This implies steady state spending on imported goods in the consumption bundle and intermediate input bundle equal to forty percent. Lastly, we set $\eta = 1$, so both currencies depreciate identically in response to a monetary policy shock in H. In Section 5 we estimate η and home bias parameters directly from the data for Colombia.

Figures 1 and 2 plot the impulse response to a negative 25 basis point exogenous cut in interest rates. In each sub-figure we contrast the response under the regimes of DCP, PCP, and LCP.

Table 1: Parameter Values

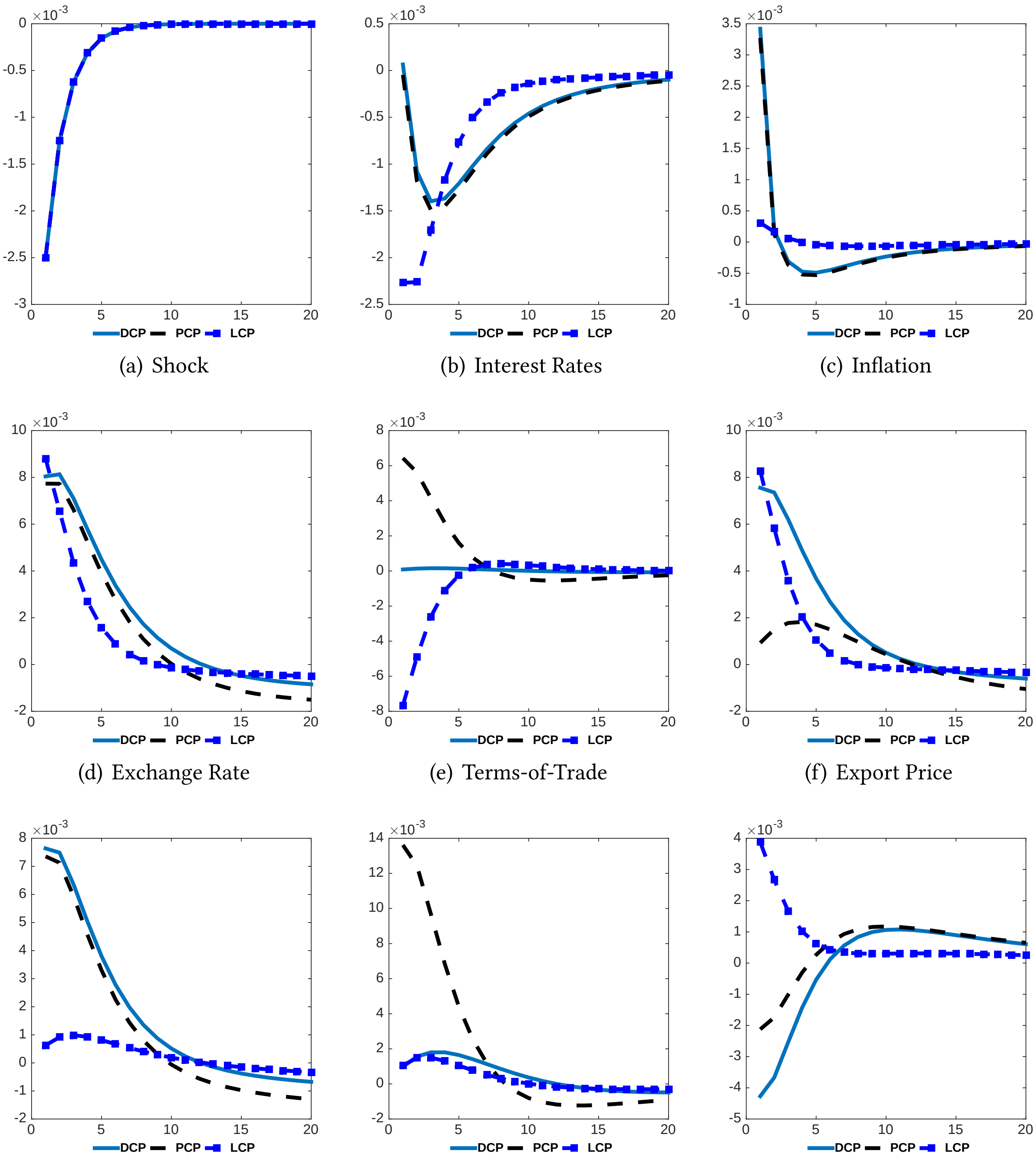
	Parameter	Value
Household Preferences		
Discount factor	β	0.99
Risk aversion	σ_c	2.00
Frisch elasticity of N	φ^{-1}	0.50
Disutility of labor	κ	1.00
Production		
Intermediate share	α	2/3
Demand		
Elasticity	σ	2.00
Super-elasticity	ϵ	1.00
Rigidities		
Wage	δ_w	0.85
Price	δ_p	0.75
Monetary Rule		
Inertia	ρ_m	0.50
Inflation sensitivity	ϕ_M	1.50
Shock persistence	ρ_{ε_i}	0.50

Note: other parameter values as reported in the text.

ER and Inflation: Following the monetary shock, domestic interest rates decline (Figure 1(b)) but less than one-to-one as the exchange rate $\mathcal{E}_U$ and $\mathcal{E}_R$ depreciates by around 0.8% (Figure 1(d)) raising inflationary pressures on the economy (Figure 1(c)). This in turn dampens the fall in nominal interest rates via the monetary rule. As seen in Figure 1(c) the increase in inflation in the case of DCP and PCP far exceeds that of LCP since exchange rate movements have a smaller impact on the domestic prices of imported goods when import prices are sticky in local currency (i.e. LCP).

Terms-of-Trade: The exchange rate depreciation is associated with almost a one to one depreciation of the terms-of-trade in the case of PCP and a one to one appreciation in the case of LCP (Figure 1(e)). Distinctively, in the case of DCP the terms-of-trade depreciates negligibly and remains stable because both export and import prices are stable in the dominant currency in that case.

Exports and Imports: With stable export and import prices in the dominant currency under DCP, the H currency price of exports and imports rise with the exchange rate depreciation as depicted in Figures 1(f)-1(g). This in turn generates a significant decline in trade weighted imports (0.43%), despite the expansionary effect of monetary policy, and only a modest increase in trade weighted exports (0.1%) (Figures 1(h)-1(i)). This contrasts with the PCP benchmark that generates a large increase



(a) Shock (b) Interest Rates (c) Inflation

(d) Exchange Rate (e) Terms-of-Trade (f) Export Price

(g) Import Price (h) Export Quantity (i) Import Quantity

Figure 1: Impulse Response to a Domestic Monetary policy shock. Note: TW refers to Trade Weighted.

in exports and with the LCP benchmark that generates an increase in imports (from the demand expansion). The decline in imports in the case of PCP is lower than that under DCP because of export expansion under PCP and the use of imported inputs.

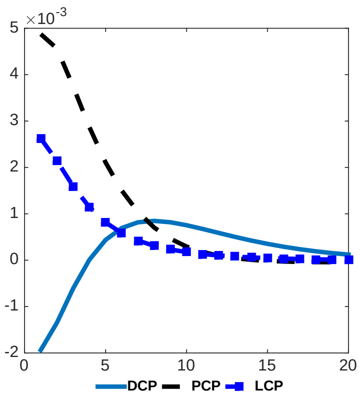
World Trade: An implication of these diverging patterns is that a strengthening of the dominant currency may be associated with a decline in trade (defined as the sum of export and import quantities) as shown in Figure 2(a), in contrast to the case of PCP and LCP. In the case of DCP trade declines by 0.2% as imports fall without a commensurate increase in exports. In the case of PCP trade expands by 0.47% as the increase in exports outweighs the decrease in imports and the latter is dampened because of the induced demand for imported inputs arising from the export expansion. In the case of LCP trade increases by 0.27% mainly because of the increase in imports.

Output: As depicted in Figure 2(b) the expansionary impact on output is muted under DCP relative to PCP, with the lowest impact under LCP. Under DCP there is an expenditure switching effect from imports towards domestic output that is absent under LCP, while DCP misses out on the expansionary impact on exports under PCP. Comparing Figures 2(b) and 1(c), the inflation-output trade off in response to expansionary monetary policy worsens under DCP relative to both PCP and LCP (where output does not expand much, but inflation increases the least). In the case of DCP inflation rises by 0.35% on impact and output by 0.67%, a ratio of 0.52. In the case of PCP that ratio is almost halved to $0.35/1.2 = 0.3$. The ratio is lowest for LCP at 0.1.

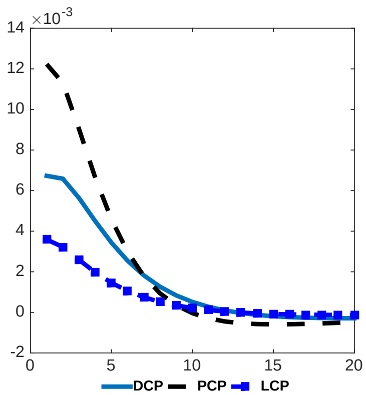
Consumption: Consumption increases by most under LCP as compared to PCP and DCP. This follows partly because real interest rates decline by the most under LCP on impact (-0.24%), as compared to PCP (-0.03%) and DCP (-0.01%) (Figures 2(c)).

Mark-up, Pricing-to-market: The stability of prices in the dominant currency alongside the rigidity of wages in home currency generates an increase in mark-ups in the case of DCP as depicted in Figure 2(d). While this is similar to the case of LCP where mark-ups also rise, there is a more modest increase in mark-ups in the case of DCP because of the increase in marginal costs arising from the higher price of imported inputs, an effect absent in the case of LCP. In contrast, mark-ups decline in the case of PCP as marginal costs increase alongside a stable price in home currency.

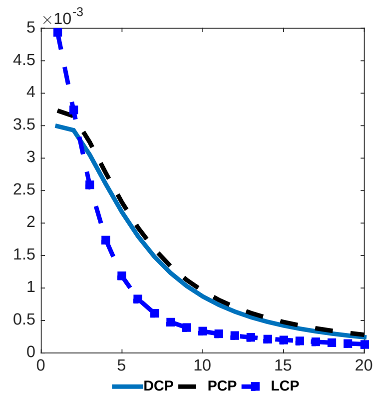
Lastly, figure 2(e) plots the differences in (log) prices at which goods are sold at home relative to



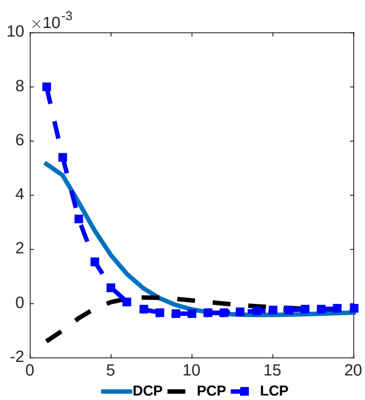
(a) Trade



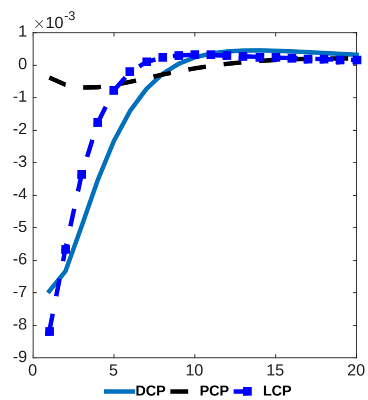
(b) Output



(c) Consumption



(d) Mark-up



(e) Pricing to Market

Figure 2: Impulse Response to a Domestic Monetary policy shock (continued)

exported (trade-weighted). As is evident there is a large decline in the relative price of goods sold at home in the case of LCP and DCP. This is far more muted in the case of PCP where it arises entirely through the variable mark-up channel.

4 Empirical Evidence

To test the implications of the model we use unique customs data from Colombia on exports and imports at the firm level. After describing our data sources we present empirical pass-through results for import and export prices and quantities, which we later compare to the model's predictions in Section 5.

4.1 Data Sources

The data on international trade are from the customs agency (DIAN), and the department of statistics (DANE), and include information on the universe of Colombian importers and exporters. We have access to the data through the Banco de la República. The data include the trading firm's tax identification number, the 10-digit product code (according to the Nandina classification system, based on the Harmonized System), the FOB value (in U.S. dollars) and volume (net kilograms) of exports (imports), and the country of destination (origin), among other details.[8] The data are available on a monthly basis, and for our analysis we aggregate exports and imports at the annual or quarterly level. These data are available for the period between 2000 and 2015.

Further, starting in 2007, our exports data include information on the invoicing currency of each transaction. In Table 2 we present the distribution of currencies, broken down by destination groups. It is evident that the vast majority of Colombian exports are priced in dollars. Even for exports to the euro zone, the overwhelming invoicing currency is the dollar. Although some transactions are negotiated in euros, Colombian pesos, or Venezuelan bolívares among other currencies, the U.S. dollar accounts for over 98% of all exports. Moreover, the distribution is very similar if we look at the value of exports negotiated in each currency instead of the number of transactions. In this regard the Colombian economy is representative of a large number of economies that rely extensively on dollar invoicing.

We obtain data on exchange rates from the International Monetary Fund. The Colombian ex-

[8]In the case of imports, there are cases where the imported good was produced in one country but actually arrived to Colombia from a third country. This case is most commonly seen for goods produced in China arriving to Colombia from either the United States or Panama. To avoid introducing unnecessary noise in our empirical work, we only keep in our regressions those observations where the country of origin and purchase are the same.

Table 2: Currency Distribution, by Destination

Destination	Currency	All Exports	Manufactures
US	US Dollar	99.71%	99.93%
	Euro	0.02%	0.03%
	Colombian Peso	0.27%	0.03%
Dollar economies	US Dollar	99.73%	99.91%
	Euro	0.03%	0.04%
	Colombian Peso	0.23%	0.03%
CAN	US Dollar	99.75%	99.90%
	Euro	0.07%	0.07%
	Colombian Peso	0.18%	0.03%
Latin America	US Dollar	99.18%	99.34%
	Euro	0.13%	0.13%
	Colombian Peso	0.22%	0.03%
	Bolívar (Ven)	0.44%	0.45%
	Mexican Peso	0.02%	0.02%
	Colón (CR)	0.01%	0.01%
European Union	US Dollar	90.73%	86.19%
	Euro	8.64%	13.28%
	Colombian Peso	0.31%	0.21%
	Sterling Pound	0.28%	0.26%
Euro zone	US Dollar	88.78%	84.48%
	Euro	10.80%	15.22%
	Colombian Peso	0.39%	0.25%
	Sterling Pound	0.01%	0.01%
All destinations	US Dollar	98.28%	98.39%
	Euro	0.72%	0.70%
	Colombian Peso	0.67%	0.52%
	Venezuelan Bolívar	0.27%	0.33%
	Sterling Pound	0.02%	0.01%

Source: Authors' calculations based on data from DIAN/DANE.
Notes: (1) Exports of coke, refined petroleum products, and nuclear fuel (ISIC 23), and basic metals (ISIC 27) excluded from "Manufactures". (2) Distribution calculated for number of invoices in each currency.

change rate (peso) is a commodity currency, and fluctuations in the peso are strongly negatively correlated with fluctuations in commodity prices.[9] Figure 3 displays the relation between the Colombian peso (solid black line) and the overall (log) terms-of-trade (dashed blue line), defined as the log difference between import and export prices. The correlation between the two series is 0.62, and the regression coefficient is 1.15 with an R^2 of 0.38. This terms-of-trade is driven primarily by com-

[9] The Colombian peso officially switched to a floating status in 1999. Commodity prices can be considered as exogenous to the economy: while mining output makes up 58.4% of total exports for Colombia, it is small relative to world commodity markets. For example, Colombia's oil production was 1.1% of world oil production in 2014.

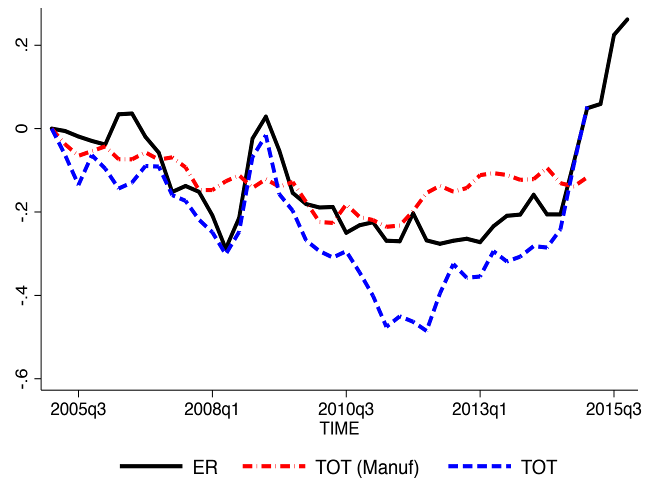


Figure 3: Exchange Rate and Terms-of-Trade

modity prices. If we focus instead on the non-commodity terms-of-trade (dots-and-dash red line) we find that the terms-of-trade is far more stable with a regression coefficient of 0.33 and R^2 of 0.36, consistent with the predictions of the model under DCP.[10]

4.2 Results

We use these data to test the main implications of the model. In all of our empirical analysis, we focus on manufactured goods, excluding products in the petrochemicals and basic metals industries and we follow the ISIC Rev. 3.1 classification to define which products are manufactures. As a robustness check we also use the subsample of differentiated products only (instead of the full set of manufactures presented) constructed using the classification of goods by Rauch (1999).[11] We define prices and quantities at the 10-digit product, country, year (or quarter) level. Prices are given by the FOB value per net kilogram, and quantities are given by total net kilograms. Exchange rates are the annual (or quarterly) average.

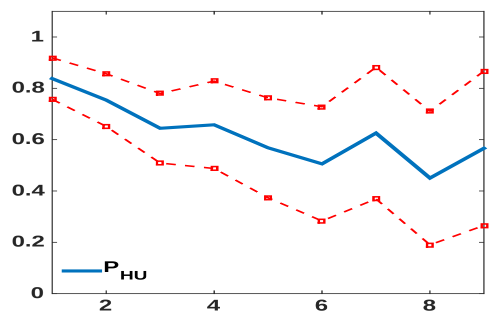
Exchange rate pass-through: We estimate the pass-through of exchange rates into import and export prices using the dynamic lag regression described in Burstein and Gopinath (2014):

[10]The non-commodity terms-of-trade is constructed by excluding 'traditional' exports/imports such as oil, coal, metals, coffee, bananas or flowers. Although it does not consist exclusively of manufactured goods, these represent more than 90 percent of the basket.

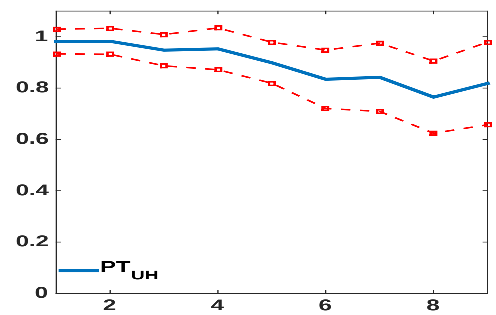
[11]In our reported estimates, we follow Rauch's conservative classification, although the results are virtually unchanged if we use the liberal definition instead.

$$\Delta x_t = \alpha + \sum_{s=0}^{8} \beta_s \Delta e_{t-s} + \mathbf{Z}_t + \epsilon_t, \tag{26}$$

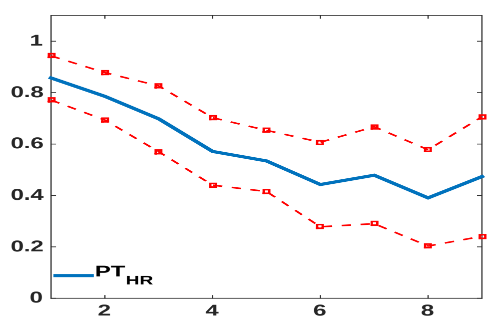
where Δx_t is the quarterly log change in export/import prices expressed in pesos. Δe_{t-s} is the quarterly log change in the nominal exchange rate of the *peso relative to the dollar* regardless of origin or destination country. We include the contemporaneous effect and eight lags. $\mathbf{Z}_t$ is a control vector that includes fixed effects by firm-industry-country and quarter dummies to account for seasonality.[12] The cumulative estimates, $\sum_{s=0}^{k} \beta_s$, and two standard error bands (where the standard errors are clustered at the level of quarter-year) are plotted as the blue solid line and the dashed with squares red line in Figure 4(a) for export prices from Colombia to dollar destinations and Figure 4(b) for import prices from dollar destinations. For non-dollar countries the figures are similarly reported in Figures 4(c) and 4(d).



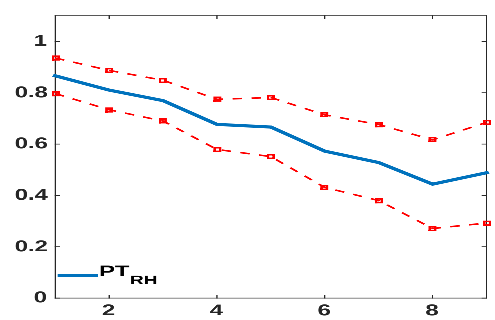
(a) *Export prices (dollar destination)*



(b) *Import prices (dollar origin)*



(c) *Export prices (non-dollar destination)*



(d) *Import prices (non-dollar origin)*

Figure 4: ERPT - Export and Import Prices

A striking feature of the pass-through estimates is that all pass-throughs start out high at close to

[12]We also estimate the regression controlling for contemporaneous and eight lags of quarterly log changes in the producer price index in Colombia and in the origin/destination country and our estimates are practically unchanged.

one and decline over time. This is the case for both export and import prices and for dollar and non-dollar destinations/origins and follows the prediction of DCP where if prices are set in the dominant currency, in this case the dollar, the pass-through of peso/dollar exchange rates into export and import prices in pesos is almost one to one initially and then declines over time. In the case of export prices to dollar destinations the contemporaneous estimate is 0.84 and then the cumulative pass-through slowly decreases after two years to 0.56. In the case of import prices from dollar origins pass-through is very high, around 1 and the cumulative effect declines to 0.81. For non-dollar destinations the estimated pass-through starts at around 0.86 and decreases to 0.47 after two years.

The second set of regressions we estimate tests the importance of non-dominant currencies in pass-through. We report here the results from annual regressions of the log change in export/import prices on the log change in the bilateral exchange rates and then we add in the peso/dollar exchange rate and the peso/euro exchange rate. Specifically,

$$\Delta x_t = \alpha + \tilde{\beta}_U \Delta e_{R,t} + \tilde{\beta}_R \Delta e_{U,t} + \mathbf{Z}_t + \epsilon_t, \tag{27}$$

where $\mathbf{Z}_t$ includes log changes in the producer price index in Colombia and in the origin/destination country and we cluster the standard errors by year.

The estimates are reported in Tables 3-6 respectively for the various specifications. As is clearly evident from non-dollar destinations the introduction of the peso/dollar exchange rate knocks down the coefficient on the bilateral exchange rate in all specifications. This finding once again is consistent with DCP.

Table 3: ERPT into Colombian Export Prices (Dollarized Economies, U)

	(1) Δp_{COP}	(2) Δp_{COP}	(3) Δp_{COP}	(4) Δp_{COP}	(5) Δp_{COP}	(6) Δp_{COP}
$\Delta e_{COP/USD}$	0.699***	0.677***	0.830***	0.863***	0.798***	0.821***
	(0.0324)	(0.0630)	(0.0341)	(0.0410)	(0.0440)	(0.0595)
$\Delta e_{COP/Euro}$		0.0366		-0.0460		-0.0323
		(0.0667)		(0.0288)		(0.0447)
ΔPPI			-0.0611	-0.0547	0.116	0.120
			(0.141)	(0.113)	(0.143)	(0.126)
ΔPPI^*			0.218***	0.227***	0.193***	0.199***
			(0.0490)	(0.0468)	(0.0495)	(0.0505)
Observations	169,749	169,749	159,002	159,002	98,820	98,820
R-squared	0.289	0.289	0.290	0.290	0.304	0.304
Sample	M	M	M	M	D	D

Notes: All regressions include Firm-Industry-Country fixed effects. Standard errors clustered at the year level. The sample includes all manufactured (M) products excluding petrochemicals and metal industries in columns (1)-(4) and only differentiated (D) products in columns (5)-(6). The export destinations are the Dollarized economies: USA, Panama, Puerto Rico, Ecuador, and El Salvador. '***', '**', and '*' indicate significance at the 1, 5, and 10 percent level, respectively.

Table 4: ERPT into Colombian Export Prices (Non-Dollarized Economies, R)

	(1) Δp_{COP}	(2) Δp_{COP}	(3) Δp_{COP}	(4) Δp_{COP}	(5) Δp_{COP}	(6) Δp_{COP}	(7) Δp_{COP}
$\Delta e_{COP/LCU}$	0.697***	0.0896*	0.0801**	0.559***	0.110*	0.143**	0.122
	(0.115)	(0.0464)	(0.0333)	(0.155)	(0.0542)	(0.0453)	(0.0906)
$\Delta e_{COP/USD}$		0.660***	0.652***		0.626***	0.681***	0.671***
		(0.0473)	(0.0750)		(0.0533)	(0.0603)	(0.0928)
$\Delta e_{COP/Euro}$			0.0422			-0.0701	-0.0438
			(0.0842)			(0.0590)	(0.0762)
ΔPPI				1.100**	0.280	0.208	0.161
				(0.362)	(0.162)	(0.172)	(0.202)
ΔPPI^*				-0.355	0.0647	0.117	0.183
				(0.277)	(0.161)	(0.174)	(0.187)
Observations	204,664	204,664	184,825	137,151	137,151	118,198	72,408
R-squared	0.306	0.308	0.300	0.310	0.312	0.303	0.320
Sample	M	M	M	M	M	M	D

Notes: All regressions include Firm-Industry-Country fixed effects. Standard errors clustered at the year level. The sample includes all manufactured (M) products excluding petrochemicals and metal industries in columns (1)-(6) and only differentiated (D) products in column (7). The export destinations include all countries except the Dollarized economies (USA, Panama, Puerto Rico, Ecuador, and El Salvador), economies with currencies pegged to the dollar, and Venezuela. Columns (3) and (6) exclude euro destinations. '***', '**', and '*' indicate significance at the 1, 5, and 10 percent level, respectively.

Table 5: ERPT into Colombian Import Prices (Dollarized, U)

	(1) Δp_{COP}	(2) Δp_{COP}	(3) Δp_{COP}	(4) Δp_{COP}	(5) Δp_{COP}	(6) Δp_{COP}
$\Delta e_{COP/USD}$	0.976***	0.975***	1.003***	1.034***	0.969***	0.970***
	(0.0173)	(0.0369)	(0.0278)	(0.0435)	(0.0328)	(0.0375)
$\Delta e_{COP/Euro}$		0.00159		-0.0404		-0.00132
		(0.0563)		(0.0534)		(0.0603)
ΔPPI			0.147	0.151	0.253**	0.253**
			(0.0963)	(0.102)	(0.0988)	(0.0983)
ΔPPI^*			0.0947**	0.113***	-0.0127	-0.0121
			(0.0359)	(0.0327)	(0.0530)	(0.0396)
Observations	508,559	508,559	508,247	508,247	264,495	264,495
R-squared	0.226	0.226	0.226	0.226	0.252	0.252
Sample	M	M	M	M	D	D

Notes: All regressions include Firm-Industry-Country fixed effects. Standard errors clustered at the year level. The sample includes all manufactured (M) products excluding petrochemicals and metal industries in columns (1)-(4) and only differentiated (D) products in columns (5)-(6). The imports originate from the Dollarized economies: USA, Panama, Puerto Rico, Ecuador, and El Salvador. '***', '**', and '*' indicate significance at the 1, 5, and 10 percent level, respectively.

Table 6: ERPT into Colombian Import Prices (Non-Dollarized, R)

	(1) Δp_{COP}	(2) Δp_{COP}	(3) Δp_{COP}	(4) Δp_{COP}	(5) Δp_{COP}	(6) Δp_{COP}	(7) Δp_{COP}
$\Delta e_{COP/LCU}$	0.742***	0.301***	0.289***	0.461***	0.257**	0.282***	0.289**
	(0.126)	(0.0791)	(0.0861)	(0.132)	(0.0829)	(0.0873)	(0.0923)
$\Delta e_{COP/USD}$		0.540***	0.484***		0.547***	0.628***	0.624***
		(0.0662)	(0.119)		(0.0460)	(0.0646)	(0.0760)
$\Delta e_{COP/Euro}$			0.182			-0.0365	-0.0360
			(0.167)			(0.0974)	(0.108)
ΔPPI				1.623**	0.696**	0.834***	0.739***
				(0.664)	(0.229)	(0.137)	(0.119)
ΔPPI^*				-0.631**	0.185	0.276***	0.244*
				(0.211)	(0.121)	(0.0774)	(0.120)
Observations	824,364	824,364	600,041	582,201	582,201	368,247	182,233
R-squared	0.287	0.290	0.316	0.268	0.271	0.294	0.306
Sample	M	M	M	M	M	M	D

Notes: All regressions include Firm-Industry-Country fixed effects. Standard errors clustered at the year level. The sample includes all manufactured (M) products excluding petrochemicals and metal industries in columns (1)-(6) and only differentiated (D) products in column (7). The imports originate from al countries except for the Dollarized economies (USA, Panama, Puerto Rico, Ecuador, and El Salvador), economies with currencies pegged to the dollar, and Venezuela. Columns (3) and (6) exclude euro destinations. '***', '**', and '*' indicate significance at the 1, 5, and 10 percent level, respectively.

Quantities: An important prediction of DCP that differs substantially from PCP and LCP is the differential quantity responses of imports and exports. Using a first order approximation we have for export and import quantities respectively,

$$\frac{\Delta y_{Hi}}{\Delta e_U} = -\sigma \left(\frac{\Delta p_{Hi}}{\Delta e_U} - \frac{\Delta e_i}{\Delta e_U} \right) \tag{28}$$

$$\frac{\Delta y_{iH}}{\Delta e_U} = -\sigma \left(\frac{\Delta p_{iH}}{\Delta e_U} - \frac{\Delta p}{\Delta e_U} \right) + \frac{\Delta y_d}{\Delta e_U} \tag{29}$$

where $y_d = \log(C + X)$ is (log) domestic demand and all prices are in H currency. We have suppressed terms that are held fixed because of the SOE assumption. Consider first the case of imports and exports from and to U. In this case $\frac{\Delta e_i}{\Delta e_U} = 1$ and $\frac{\Delta p_{Hi}}{\Delta e_U}$ is also close to 1. Consequently, from Eq. 28, the impact on exports is close to 0. In the case of imports, controlling for demand and home competitors prices, Eq. (29) states that quantities are almost as sensitive as the elasticity of demand σ, given that $\frac{\Delta p_{iH}}{\Delta e_U}$ is close to 1.

This would also be the case for imports from R. Importantly the relevant exchange rate here again is the exchange rate of H relative to the dominant currency U with the bilateral exchange rate playing a minor role. In the case of exports to R quantity responses are less straightforward as Eq. (28) indicates that it depends on the co-movement between e_U and e_R. If this co-movement is lower than 1, then a weakening of H's currency relative to the dominant currency can lead to a decline in exports to R destination, because of the depreciation of R's currency relative to U's.

Tables 7-10 report the results from the quantity regressions in the data. A few things stand out. Starting with the dollarized economies, the pass-through to export quantities to U is insignificantly different from zero in all specifications except one where exports decline. On the other hand, for imports from U there is a pronounced decline in quantities imported across all specifications. In the case of the nondollarized economies, the decline in imports from R is also significantly negative and, importantly, the relevant exchange rate is the peso/dollar exchange rates as opposed to the bilateral exchange rate. For exports we again have that the relevant exchange rate is the peso/dollar exchange rate. We however observe exports declining following a weakening of the peso relative to the dollar which as we pointed out previously, is possible when the co-movement of the destination currency with the dollar is sufficiently weak.

Table 7: ERPT into Colombian Export Quantities (Dollarized, U)

	(1) Δq	(2) Δq	(3) Δq	(4) Δq
$\Delta e_{COP/USD}$	-0.608*	-0.466	-0.421	-0.0447
	(0.277)	(0.344)	(0.331)	(0.372)
$\Delta e_{COP/Euro}$		-0.203		-0.536
		(0.386)		(0.428)
ΔPPI	1.172	1.207	0.576	0.662
	(0.940)	(1.008)	(1.069)	(1.296)
ΔPPI^{*}	0.454	0.487*	0.803**	0.897***
	(0.259)	(0.247)	(0.311)	(0.265)
ΔGDP^{*}	0.289	0.325	-0.00557	0.0573
	(1.304)	(1.318)	(1.548)	(1.508)
Observations	159,002	159,002	98,820	98,820
R-squared	0.225	0.225	0.232	0.232
Sample	M	M	D	D

Notes: All regressions include Firm-Industry-Country fixed effects. Standard errors clustered at the year level. The sample includes all manufactured (M) products excluding petrochemicals and metal industries in columns (1)-(2) and only differentiated products in columns (3)-(4). The export destinations are the Dollarized economies: USA, Panama, Puerto Rico, Ecuador, and El Salvador. '***', '**', and '*' indicate significance at the 1, 5, and 10 percent level, respectively.

Table 8: ERPT into Colombian Import Quantities (Dollarized, U)

	(1) Δq	(2) Δq	(3) Δq	(4) Δq
$\Delta e_{COP/USD}$	-1.104***	-0.939**	-1.123***	-0.950*
	(0.255)	(0.397)	(0.296)	(0.455)
$\Delta e_{COP/Euro}$		-0.233		-0.243
		(0.414)		(0.462)
ΔPPI	1.500	1.584	1.369	1.459
	(1.068)	(1.075)	(1.174)	(1.174)
ΔPPI^{*}	-0.128	-0.0972	0.0418	0.0739
	(0.317)	(0.327)	(0.364)	(0.363)
ΔGDP	3.538	3.916	2.699	3.096
	(2.750)	(2.798)	(3.199)	(3.250)
Observations	508,263	508,263	264,501	264,501
R-squared	0.184	0.184	0.206	0.206
Sample	M	M	D	D

Notes: All regressions include Firm-Industry-Country fixed effects. Standard errors clustered at the year level. The sample includes all manufactured (M) products excluding petrochemicals and metal industries in columns (1)-(2) and only differentiated (D) products in columns (3)-(4). The imports originate from the Dollarized economies: USA, Panama, Puerto Rico, Ecuador, and El Salvador. '***', '**', and '*' indicate significance at the 1, 5, and 10 percent level, respectively.

Table 9: ERPT into Colombian Export Quantities (Non-Dollarized, R)

	(1)	(2)	(3)	(4)	(5)	(6)
	Δq	Δq	Δq	Δq	Δq	Δq
$\Delta e_{COP/LCU}$	-0.872***	-0.113	-0.251	-1.136***	-0.283	-0.416
	(0.254)	(0.245)	(0.278)	(0.306)	(0.295)	(0.294)
$\Delta e_{COP/USD}$		-1.057***	-0.972**		-1.156***	-0.966**
		(0.271)	(0.327)		(0.277)	(0.325)
$\Delta e_{COP/Euro}$			0.0359			-0.0352
			(0.321)			(0.323)
ΔPPI	1.869	2.852**	2.986**	1.927	2.990**	2.978**
	(1.420)	(1.222)	(1.108)	(1.533)	(1.275)	(1.208)
ΔPPI^{*}	0.051	-0.328	-0.463	-0.396	-0.792	-0.861*
	(0.469)	(0.393)	(0.297)	(0.544)	(0.495)	(0.388)
ΔGDP^{*}	2.995***	1.676	1.753	3.479***	2.049	2.195
	(0.882)	(1.194)	(1.153)	(0.989)	(1.349)	(1.248)
Observations	137,151	137,151	118,198	83,948	83,948	72,408
R-squared	0.253	0.254	0.249	0.261	0.262	0.256
Sample	M	M	M	D	D	D

Notes: All regressions include Firm-Industry-Country fixed effects. Standard errors clustered at the year level. The sample includes all manufactured (M) products excluding petrochemicals and metal industries in columns (1)-(3) and only differentiated (D) products in columns (4)-(6). The export destinations include all countries except the Dollarized economies (USA, Panama, Puerto Rico, Ecuador, and El Salvador), economies with currencies pegged to the dollar, and Venezuela. Columns (3) and (6) exclude euro destinations. '***', '**', and '*' indicate significance at the 1, 5, and 10 percent level, respectively.

Table 10: ERPT into Colombian Import Quantities (Non-Dollarized, R)

	(1)	(2)	(3)	(4)	(5)	(6)
	Δq	Δq	Δq	Δq	Δq	Δq
$\Delta e_{COP/LCU}$	-0.569**	-0.174	-0.297	-0.597**	-0.183	-0.259
	(0.216)	(0.125)	(0.246)	(0.234)	(0.142)	(0.243)
$\Delta e_{COP/USD}$		-0.881***	-0.942***		-0.908***	-0.983**
		(0.188)	(0.270)		(0.234)	(0.315)
$\Delta e_{COP/Euro}$			-0.0828			-0.0901
			(0.353)			(0.363)
ΔPPI	0.587	1.738*	2.130*	0.605	1.785*	2.146*
	(1.120)	(0.829)	(0.983)	(1.112)	(0.844)	(0.968)
ΔPPI^{*}	0.0695	-0.794**	-1.164***	0.103	-0.780**	-1.057**
	(0.398)	(0.260)	(0.364)	(0.397)	(0.286)	(0.342)
ΔGDP	6.306***	4.561**	4.982**	6.614***	4.813**	4.894**
	(1.593)	(2.026)	(2.177)	(1.586)	(2.035)	(2.171)
Observations	582,306	582,306	368,351	292,551	292,551	182,298
R-squared	0.209	0.210	0.220	0.232	0.234	0.247
Sample	M	M	M	D	D	D

Notes: All regressions include Firm-Industry-Country fixed effects. Standard errors clustered at the year level. The sample includes all manufactured (M) products excluding petrochemicals and metal industries in columns (1)-(3) and only differentiated (D) products in columns (4)-(6). The imports originate from al countries except for the Dollarized economies (USA, Panama, Puerto Rico, Ecuador, and El Salvador), economies with currencies pegged to the dollar, and Venezuela. Columns (3) and (6) exclude euro destinations. '***', '**', and '*' indicate significance at the 1, 5, and 10 percent level, respectively.

Table 11: Parameter Values

	Parameter	Value
Measured		
Export Invoicing Shares		
to U	θ^U_{HU}	1.00
to R	$\theta^U_{HR}, \theta^R_{HR}$	0.93,0.07
Shocks		
commodity prices	σ_ζ, ρ_ζ	0.09, 0.74
Estimated		
Home bias	γ_H	0.88
from U	γ_U	0.06
from R	γ_R	0.06
Exports		
to U	D_U	-2.38
to R	D_R	-0.87
Oil endowment	$\bar{\zeta}$	0.27
Import Invoicing Shares		
from U	θ^U_{UH}	1.00
from R	$\theta^U_{RH}, \theta^R_{RH}$	0.93, 0.07
e_R process	$\eta, \rho_{\epsilon_r}, \sigma_{\varepsilon_r}$	0.74, 0.82,0.016
a process	$\sigma_a, \rho_a, \rho_{a,\zeta}$	0.13,0.49,-0.26

Note: other parameter values as reported in the text.

5 Discerning Pricing Paradigms

The empirical evidence points strongly to DCP. To further test the different pricing paradigms along the lines suggested in Section 2.4 we simulate the model economy subject to three shocks: commodity price shocks, productivity shocks, shocks to the exchange rate between U and R (eq. (17)). We use a combination of calibration and estimation to parameterize the model, reported in Table 11 while other parameter values are as reported in Table 1.

The export invoicing shares are measured in the data directly. In addition, we specify the following processes for the three shocks (commodity price shock, productivity shock and exchange rate shock) as follows:

$$\zeta_t - \bar{\zeta} = \rho_\zeta(\zeta_{t-1} - \bar{\zeta}) + \varepsilon_{\zeta,t} \tag{30}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{31}$$

$$\epsilon_{R,t} = \rho_{\epsilon_r}\epsilon_{R,t-1} + \varepsilon_{R,t} \tag{32}$$

where $\bar{\zeta}$ is the steady state value of the commodity price, and $\varepsilon_{i,t}$ are serially independently distributed innovations. We allow the productivity and commodity price innovations to be correlated, and denote $\rho_{a,\zeta} = \text{corr}(\varepsilon_{a,t}, \varepsilon_{\zeta,t})$.

We calibrate the process for commodity price shocks in equation (30) to match the autocorrelation and standard deviation of HP-filtered commodity prices.[13] The values for $\bar{\zeta}$, D_U, D_R, γ_H, are chosen such that in steady state the model matches the Colombian data for the share of oil exports in total exports of 58%, a 10% share of oil exports over GDP, and the share of manufacturing exports going to the U.S. of 18%.

We estimate the remaining parameters using a minimum distance estimator that minimizes the sum of squared deviations from moments in the data. Specifically, we minimize,

$$\mathbf{m}(\vec{\tau})\Omega^{-1}\mathbf{m^T}(\vec{\tau})$$

where $\vec{\tau} = \{\theta^U_{UH}, \theta^U_{RH}, \theta^R_{RH}, \eta, \sigma_r, \rho_{\epsilon_r}, \sigma_a, \rho_a, \rho_{a,\zeta}\}$ is a vector of nine parameters. We allow for common shocks to a and ζ by allowing for a non-zero correlation $\rho_{a,\zeta}$. To estimate these parameters we use the following eleven moments $\mathbf{m}(\vec{\tau})$ that theory suggests are informative. We estimate all parameters jointly and consequently all moments matter for all parameter values. The most informative moment for each parameter is described next.

- Import Invoicing Shares: To estimate the import invoicing shares,
  - θ^U_{UH}: We use the contemporaneous estimate β_0 from regression eq. (26) for import prices from dollar countries.
  - θ^R_{RH} and θ^U_{RH}: We use the coefficients from regressing the quarterly change in import prices from non-dollar destinations on the peso/dollar and peso/origin country exchange rates. $\Delta p_{RH,t} = \beta_U \cdot \Delta e_{U,t} + \beta_R \cdot \Delta e_{R,t} + \epsilon_t$
- Relation between e_R and e_U: To estimate η and σ_{ε_r} we construct the real exchange rate for Colombia relative to the U.S. and the (export share weighted) real exchange rate for Colombia relative to its other trading partners. We use these series to estimate the two equations (17)

[13] Specifically, we use the IMF's price index for all primary commodities, at the quarterly frequency, from 2000Q1 to 2016Q2. We HP filter the log of the index and compute the autocorrelation and the standard deviation of the cyclical component.

and (32) which we rewrite here:

$$\begin{aligned}\ln \mathcal{E}_{R,t} + \ln P^R_{R,t} - \ln P_t &= \eta \left(\ln \mathcal{E}_{U,t} + \ln P^U_{U,t} - \ln P_t\right) + \epsilon_{R,t} \\ \epsilon_{R,t} &= \rho_{\epsilon_r} \epsilon_{R,t-1} + \varepsilon_{R,t}\end{aligned}$$

We use the empirical estimate for $\hat{\eta}$, $\hat{\rho}_{\epsilon_r}$ and the standard deviation of $\varepsilon_{R,t}$ to obtain η, ρ_{ϵ_r}, σ_{ε_r}.

- Process for a: We match moments for the standard deviation (0.023) and autocorrelation (0.62) of manufacturing value added. To ascertain the correlation $\rho_{a,\zeta}$ we match the time zero pass-through into export prices to dollar destinations.

- Additional Moments: We match the time zero coefficient on pass-through from $\mathcal{E}_U$ into export and import prices for R goods.

The weighting matrix Ω^{-1} is a diagonal matrix where the entries are the inverse of the variance of the data moments. The estimated values from this minimization are reported in Table 11 and the moment match between the model and data are reported in Table 12. As Table 11 reports the data strongly points towards DCP with almost all of the import invoicing share in dollars.

Table 12: Moment Matching

	Data	Model
$\beta^U_{0,UH}$	0.98	0.97
$\beta^U_{0,RH}$	0.89	0.80
$\beta^H_{0,RH}$	0.18	0.13
$\hat{\eta}$	0.54	0.54
$\hat{\sigma}_{\varepsilon_r}$	0.018	0.017
$\hat{\rho}_{\epsilon_r}$	0.78	0.78
$\hat{\rho}_{a,\zeta}$	0.84	0.87
$\hat{\sigma}_a$	0.023	0.026
$\hat{\rho}_a$	0.64	0.64
$\beta^U_{0,HR}$	0.86	0.81
$\beta^U_{0,RH}$	0.87	0.90

With these parameters we simulate the model and plot the pass-through estimates from the estimated model, the DCP model, the PCP and LCP models against the estimates from the data. In the case of the latter three we force the invoicing shares to take the extreme values of each of the paradigms, keeping all other values unchanged.

Price PT: Figure 5 reports the values for price pass-through for dollar destinations and Figure 6 for non-dollar destinations. The red circles marked on the graphs represent pass-through values that were used in moment matching. The pass-through at other lags were not used in estimating parameters. As is evident the estimated model replicates the pass-through estimates at various lags for export prices to U and R and for import prices from U quite closely. The match is less good for import prices from R but we still obtain that pass-through starts high and declines gradually. Regardless, the estimated model and DCP perform much better than the other paradigms. The PCP paradigm gets the pass-through into export prices wrong because it implies low pass-through initially, with prices sticky in the exporting currency and then it gradually increases over time. The LCP paradigm gets import pass-through wrong as it assumes prices are sticky in the destination currency. So pass-through into import prices is initially low and then it increases over time. In the case of non-dollar trading partners we similarly observe that the DCP models performance is far better than the PCP and LCP case.

Relevance of bilateral exchange rates: The estimated model and DCP both match the fact in the data that while bilateral exchange rates show up as large and significant when it is the only exchange rate control in the regression (for non-dollar destinations and origins), they drop significantly as a predictor of prices when the dollar exchange rate is also included in the regression. This is reported in Table 13. On the other hand PCP and LCP do not match this fact.

Quantity PT: Table 14 reports quantity pass-through estimates from the (estimated) model generated data that replicates the empirical regressions reported in Tables 7-10. The estimated model generates a weak expansion in exports to U destinations following a depreciation and a more pronounced contraction in imports from both U and R consistent with the empirical evidence in Tables 7-9. Exports to R are negatively impacted by depreciations relative to the dollar. Here again the dollar exchange rate is a major predictor of quantities for non-dollar regions.

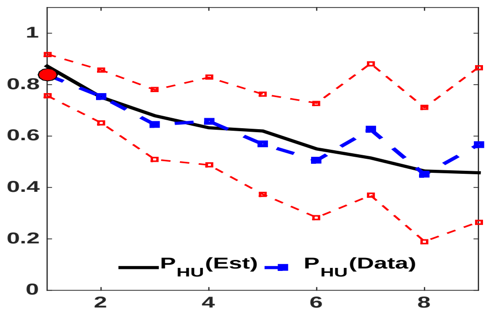
Importance of non-zero α and Γ: Figure 7 contrasts the pass-through estimates when Γ and α are set to 0 relative to the benchmark of $\Gamma = 1$ and $\alpha = 2/3$ (solid line). Export price pass-through into H prices declines by a half at the one year horizon when Γ and α are both set equal to 0 (line with solid circles), compared to the data and the benchmark model predictions. In the case of import pass-through the difference is smaller (as to be expected given that the marginal cost of foreign firms are taken as exogenous), but in all cases the models match with the data is the best under the benchmark specification.

Table 13: ERPT (Non-Dollarized Economies, R)

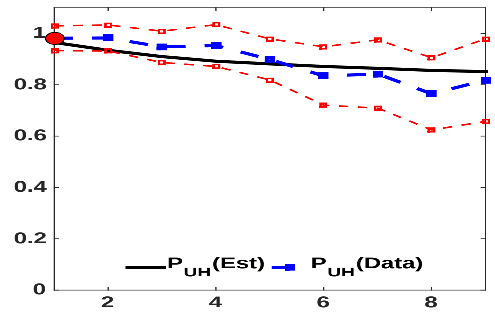
	(1) Δp_{HR}	(2) Δp_{HR}	(3) Δp_{RH}	(4) Δp_{RH}
		Data		
Δe_R	0.697***	0.0896*	0.742***	0.301***
	(0.115)	(0.0464)	(0.126)	(0.0791)
Δe_U		0.660***		0.540***
		(0.0473)		(0.0662)
		Estimated		
Δe_R	0.72	0.28	0.68	0.22
Δe_U		0.66		0.70
		DCP		
Δe_R	0.71	0.23	0.67	0.17
Δe_U		0.71		0.75
		PCP		
Δe_R	0.49	0.26	0.92	0.88
Δe_U		0.36		0.06
		LCP		
Δe_R	0.98	0.93	0.44	0.19
Δe_U		0.08		0.39

Table 14: ERPT Quantities

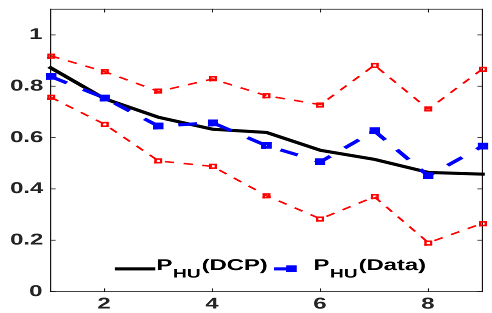
	(1) Δy_{HU}	(2) Δy_{UH}	(3) Δy_{HR}	(4) Δy_{RH}
Δe_U	0.26	-1.60	-1.33	-1.19
Δe_R	-0.18	0.28	1.43	-0.11



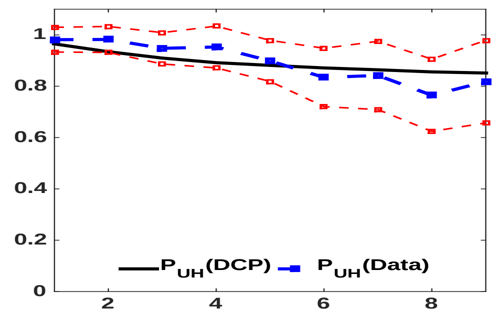
(a) *Estimated*



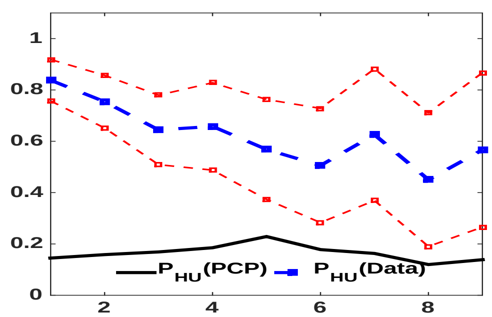
(b) *Estimated*



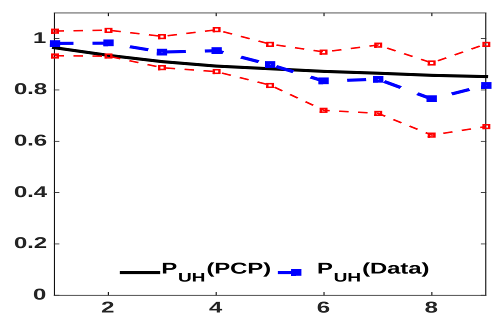
(c) *DCP*



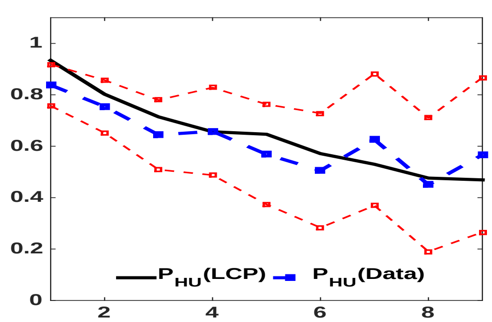
(d) *DCP*



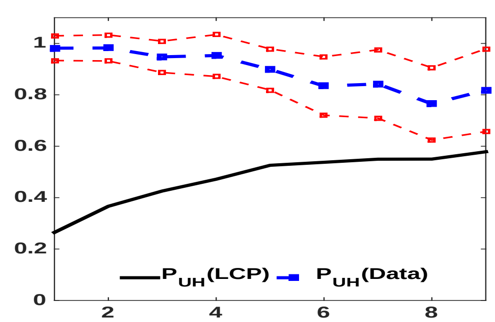
(e) *PCP*



(f) *PCP*

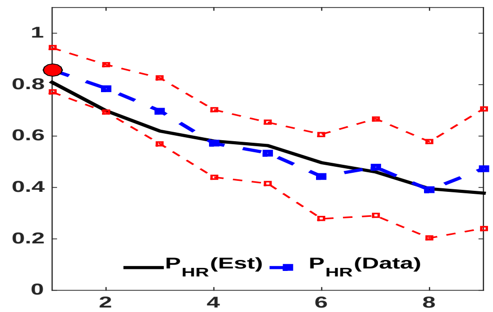


(g) *LCP*

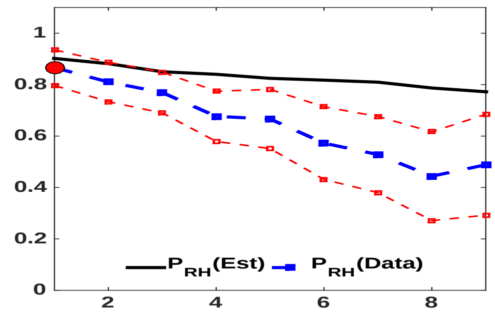


(h) *LCP*

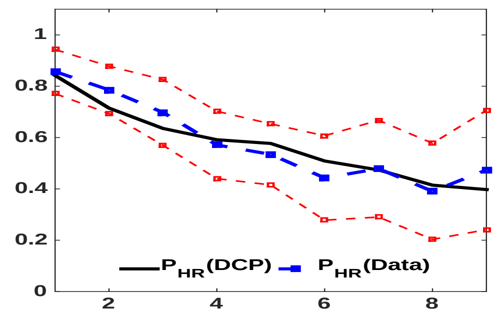
Figure 5: ERPT - Export and Import Prices, U



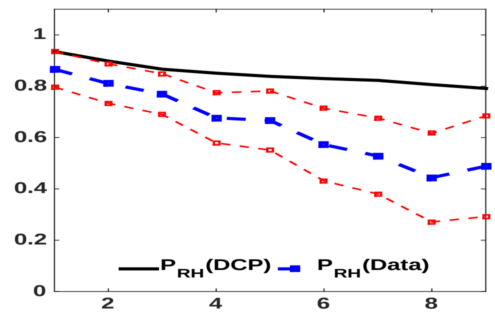
(a) *Estimated*



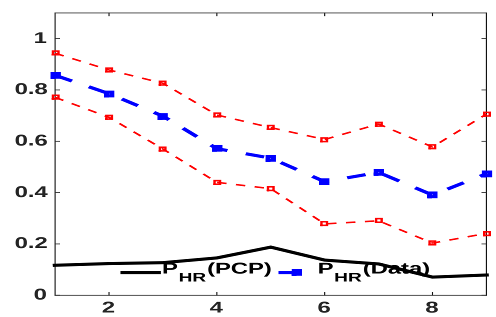
(b) *Estimated*



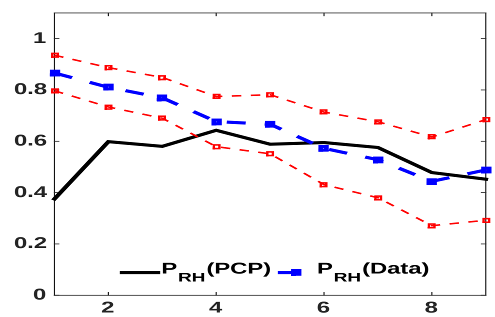
(c) *DCP*



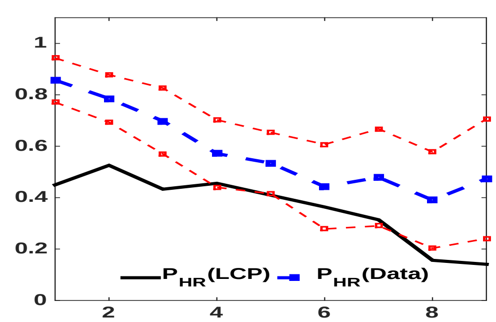
(d) *DCP*



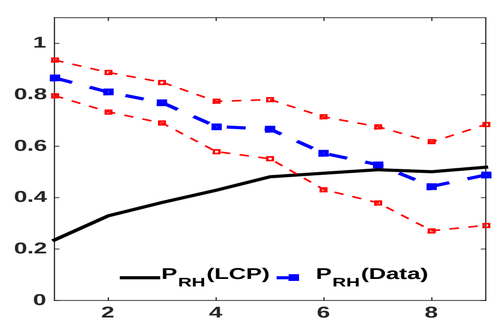
(e) *PCP*



(f) *PCP*

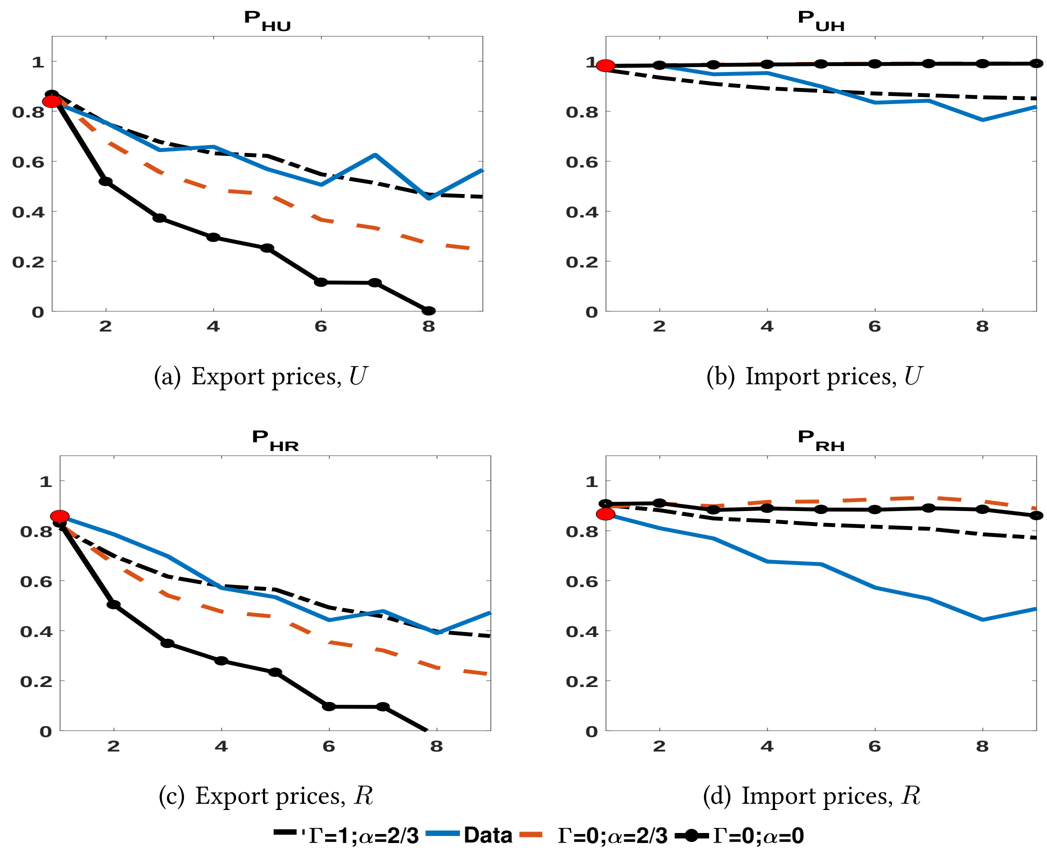


(g) *LCP*



(h) *LCP*

Figure 6: ERPT - Export and Import Prices, R



(a) Export prices, U

(b) Import prices, U

(c) Export prices, R

(d) Import prices, R

Figure 7: ERPT, Varying α and Γ

6 Optimal Monetary Policy

This section derives the linear quadratic representation of the optimal monetary policy problem for a small open economy with dominant currency pricing, following Woodford (2003). We consider a special environment relative to the benchmark case, for which we obtain an explicit representation. Specifically we restrict demand to be C.E.S., the production function only uses labor and international asset markets are complete. With these restrictions we analytically derive and compare our loss function and optimal policy to those for producer currency pricing derived in Galí and Monacelli (2005), where these assumptions apply. We continue to allow for three regions, H, U and R. The details of the derivation are provided in the appendix.

6.1 Canonical Representation

We begin by deriving a canonical representation of the small open economy under DCP with three equations: a New Keynesian Phillip's Curve, a Dynamic IS Curve, and an additional equation that characterizes the behavior of the deviations of the law of one price arising from dominant currency fluctuations. Together with a monetary policy rule, such as Eq. (15), this fully characterizes the behavior of the domestic economy.

Proposition 3 *When $\varepsilon = \alpha = \varphi = 0$, $\sigma_c = 1$, and international asset markets are complete,*

- *The evolution of inflation, output gap and law of one price departures, in deviations from the flexible price allocation (tilde notation), are given by the following relation:*

$$\pi_{HH,t} = \frac{\lambda_p}{\gamma}\left[\tilde{y}_t - (1-\gamma)\tilde{s}_t\right] + \beta \mathbb{E}_t \pi_{HH,t+1} \tag{33}$$

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - (i_t - \mathbb{E}_t \pi_{HH,t+1} - r_t^n) + (1-\gamma)\mathbb{E}_t(\Delta \tilde{m}_{t+1}) \tag{34}$$

$$\tilde{m}_t = \frac{1}{\gamma}(\tilde{y}_t - \tilde{s}_t) \tag{35}$$

  where $\tilde{m}_t = \tilde{e}_{U,t} + \tilde{p}^U_{HU,t} - \tilde{p}_{HH,t} = \tilde{e}_{U,t} + \tilde{p}^U_{HR,t} - \tilde{p}_{HH,t}$ captures deviations from the law of one price against the dominant currency. $r_t^n = \log \beta + \mathbb{E}_t \Delta a_{t+1}$ is the natural real rate, γ measures home-bias and $\lambda_p = (1-\delta_p)(1-\beta\delta_p)/\delta_p$. The tilde notation refers to deviations from the flexible price allocation.

- *The terms-of-trade $\tilde{s}_t$ evolves independently of monetary policy.*

■

Eq. (33) is the New Keynesian Philip's Curve (NKPC) and eq. (34) is the dynamic IS curve both for the DCP environment while Eq. (35) characterizes deviations from the law of one price against the dominant currency. These equations make clear that, in the case of DCP, there is no "divine coincidence": it is not possible to attain simultaneously zero inflation and a zero output gap. Contrast this with PCP, where, under the same parameter restrictions, the relation between PPI inflation and the output gap is governed by

$$\pi_{HH,t} = \lambda_p \tilde{y}_t + \beta \mathbb{E}_t \pi_{HH,t+1}.$$

Under PCP, the monetary authority can accomplish both zero inflation and a zero output gap. In the case of DCP even if the output gap is closed, inflation fluctuates with the terms of trade (Eq. (33)).[14] This is because, different from PCP, the terms-of-trade evolves exogenously and independently from monetary policy under DCP, given the parameter restrictions. The exogeneity of the import price in the dominant currency follows from the small open economy assumption. The exogeneity of the export price in the dominant currency follows from the assumptions of complete markets, and $\varphi = 0$. Under these assumptions, the wage expressed in the dominant currency is equal to the nominal level of consumption in U (and the nominal level of consumption in R expressed in U currency). It follows that export prices in U's currency are exogenous from policy. With both export and import prices exogenous, so is the terms-of-trade.

To derive optimal monetary policy we start with characterizing the welfare loss function.

6.2 Welfare Loss Function

We characterize the second-order approximation to the welfare function of the domestic planner for the small open economy in the presence of a tax $\tau = 1/\sigma$ on labor income , where σ is the elasticity of substitution across varieties produced within a country.

Proposition 4 *When $\varepsilon = \alpha = \varphi = 0$, $\sigma_c = 1$, international asset markets are complete and $\tau = 1/\sigma$, the welfare loss function for the small open economy under dominant currency pricing approximated up to the second order is given by,*

$$\mathbb{W}^{DCP} \approx \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \tilde{y}_t^2 + \gamma \frac{\sigma}{2\lambda_p} \pi_{HH,t}^2 + \frac{\gamma(1-\gamma)}{2} \tilde{m}_t^2 \right] + t.i.p \tag{36}$$

■

[14]It is well known that, in an open economy, the divine coincidence obtains under PCP only under narrow parameter assumptions. See Monacelli (2013). Our discussion establishes that, even under these parameter restrictions, the divine coincidence fails under DCP.

This can be contrasted with the welfare loss function under PCP as derived in Galí and Monacelli (2005):[15]

$$\mathbb{W}^{PCP} \approx \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_H \left[\frac{1}{2} \tilde{y}_t^2 + \frac{\sigma}{2\lambda_p} \pi_{HH,t}^2 \right] + t.i.p \tag{37}$$

Both loss functions involve the variance of inflation and the output-gap. In addition, under DCP there is an additional misalignment term that arises from the failure of the law of one price of H's good in the domestic and export markets. Under PCP, $m_t = 0$. In the case of DCP fluctuations in m_t lead to fluctuations in the real exchange rate as long as there is home-bias in consumption.[16] All else equal, the real exchange rate appreciates (depreciates) when m_t declines (increases). Fluctuations in the real exchange rate impact home consumption through the complete markets (international risk-sharing) condition. This source of fluctuation in consumption generates losses relative to the flexible price allocation.

The misaligment term, $\tilde{m}_t$, is similar to that in Engel (2011) who derives the *global* welfare loss function under LCP. There are however important differences between the DCP and LCP environments. Firstly, under DCP, despite the fact that in our environment H sells to multiple locations, there is only one misaligment term and the only policy relevant exchange rate is the dominant currency exchange rate, regardless of the share of exports to U. Unlike DCP, with LCP it is the bilateral exchange rates with the trading partner that impacts the misaligment between H good prices at H and in the destination market. Secondly, in the case of DCP it is the terms-of-trade that cannot be influenced by monetary policy, while under LCP it is the relative price of imports to home produced goods that is independent of monetary policy, under the same set of parameter restrictions.[17]

6.3 Monetary policy trade-offs

We now characterize optimal monetary policy. The main findings are as follows: Unlike the case of PCP there is no "divine coincidence" that makes it possible to attain zero PPI inflation and output gap. Optimal monetary policy targets deviations from the law of one price arising from dominant currency fluctuations, in addition to the inflation and output gap. In the absence of cost-push shocks optimal monetary policy calls for domestic producer price inflation targeting while the output gap fluctuates with the terms of trade.

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[15] Specifically, this is the result in Galí and Monacelli (2005) with $\varphi = 0$ imposed. Importantly though the optimal policy trade-off in Galí and Monacelli (2005) does not depend on φ.

[16] The real exchange rate is $q_t = \gamma(s_t + m_t)$.

[17] Monacelli (2005) derives optimal monetary policy for a small open economy with incomplete pass-through from import prices to consumer prices, PCP at-the-dock, and for an ad-hoc loss function.

Discretion: The monetary authority minimizes eq. (36) subject to the implementation constraints eq. (33) and eq. (35), taking future inflation as given. Optimal discretionary policy is characterized by the trade-off,

$$\tilde{y}_t + (1-\gamma)\tilde{m}_t = -\sigma\pi_{HH,t} \tag{38}$$

By contrast, in the case of PCP, the minimization of eq. (37) subject to the single implementation constraint under PCP, $\pi_{HH,t} = \lambda_p \tilde{y}_t + \beta \mathbb{E}_t \pi_{HH,t+1}$, generates the optimal trade-off,

$$\tilde{y}_t = -\sigma\pi_{HH,t}. \tag{39}$$

In the case of PCP the monetary authority leans against inflation pressures by reducing the output gap. Instead, in the case of DCP the monetary authority can lean against inflation by lowering the output gap $\tilde{y}_t$ and/or the misalignment term m_t.

The role of m_t can be understood as follows. Inflation depends on the real marginal cost,

$$\tilde{mc}_t - \tilde{p}_{HH,t} = \tilde{w}_t - \tilde{p}_{HH,t} = \tilde{p}_t - \tilde{p}_{HH,t} + \tilde{c}_t = (1-\gamma)\tilde{s}_t + (1-\gamma)\tilde{m}_t + \tilde{c}_t \tag{40}$$

where the third equality follows from the labor supply decision of households, $\tilde{w}_t = \tilde{p}_t + \tilde{c}_t$ and the fourth equality from the definition of the price index, $\tilde{p}_t - \tilde{p}_{HH,t} = (1-\gamma)(\tilde{s}_t + \tilde{m}_t)$. All else equal an increase in $\tilde{m}_t$ raises the price of the consumption basket relative to the price of H's goods at home. This is because for a given terms of trade $\tilde{s}_t$ an increase in $\tilde{m}_t$ (given $p_{HH,t}$) is associated with an increase in the price of imports that in turn raises the price of the consumption basket and the extent to which it does so depends on the level of openness of the economy $(1-\gamma)$. The increase in $p_t - p_{HH,t}$, in turn raises the real marginal cost of H firms and therefore increases inflationary pressure in H good prices.[18]

Replacing the optimal trade-off (eq. (38)) in the implementation constraints we have that optimal policy under DCP gives rise to,

$$\begin{cases} \pi_{HH,t} = 0 & (41) \\ \tilde{y}_t = (1-\gamma)\tilde{s}_t & (42) \end{cases}$$

That is, optimal policy calls for *producer price inflation targeting*. This is the same as under PCP, but unlike PCP where inflation targeting goes along with a zero output gap, in the case of DCP the

[18] A first order approximation of the market clearing condition, combined with the complete markets condition gives, $\tilde{c}_t = \tilde{y}_t - (1-\gamma)\tilde{s}_t$. Replacing this in eq. (40) we have that the real marginal cost is given by $\tilde{mc}_t - \tilde{p}_{HH,t} = \tilde{y}_t + (1-\gamma)\tilde{m}_t$, the terms in the square brackets of the Philip's curve.

output gap fluctuates with the terms of trade gap and this is increasing in the level of openness of the economy.

Consider the case when the only shocks are shocks to domestic productivity a_t. Specifically consider a positive shock to productivity. Under the flexible price allocation prices at home decline relative to imported goods and the terms of trade deteriorates. Under DCP we obtain that the dollar exchange rate moves one to one with productivity[19] $e_{U,t} = a_t$. This generates the same movement in the relative price of imports as under flexible prices. However it does not generate the desired movement in the terms of trade ($\tilde{s}_t < 0$) giving rise to a negative output gap.[20]

As we demonstrate next there are no gains to commitment in monetary policy (without cost-push shocks), despite the absence of divine coincidence.

Commitment: As derived in Appendix A.3.10, optimal policy under commitment at time $t = 0$ is given by,

$$\left\{\begin{aligned} \tilde{y}_t &= (1-\gamma)\tilde{s}_t - \gamma\sigma(p_{HH,t} - p_{HH,-1}) \\ (p_{HH,t} - p_{HH,-1}) &= \frac{(p_{HH,t-1} - p_{HH,-1}) + \beta\mathbb{E}_t(p_{HH,t+1} - p_{HH,-1})}{1 + \lambda_p\sigma + \beta} \end{aligned}\right.$$

where $p_{HH,-1}$ is the -given- initial price level from period $t = -1$.

There are two things to note. Firstly, the optimal price level under commitment follows the same path as the PCP case. The desired output gap however fluctuates with the terms of trade under DCP, unlike the case of PCP. Secondly, the solution under discretion and commitment are the same, in the absence of cost-push shocks. This is because under discretion optimal policy calls for zero inflation in each period. There are therefore no gains to being able to smooth inflation over time as is possible under commitment.

7 Conclusion

This paper presents a new pricing paradigm for small open economies, the dominant currency paradigm. DCP is characterized by three main features: pricing in a dominant currency, strategic complementarities in pricing and imported input use in production. We use these elements to develop a new model for small open economies, and we use it to understand the consequences of shocks that gen-

[19]To see this, note that $\tilde{y}_t + (1-\gamma)\tilde{m}_t = 0$ when $\pi_{HH,t} = 0$. Because $\tilde{y}_t = (1-\gamma)\tilde{s}_t$ we have $\tilde{m}_t = m_t = -\tilde{s}_t$ (because in the flexible price allocation $m_t = 0$.) We have $m_t + s_t^n = e_{U,t} - \tilde{s}_t$. Because $s_t^n = a_t$ we have $e_{U,t} = a_t$.

[20]Additionally, because import prices and world prices in dollars are unchanged due to SOE assumption, the real exchange rate behaves as in the flex price allocation, yielding $\tilde{c}_t = \tilde{q}_t = 0$.

erate fluctuations in the exchange rate on small open economies.

In particular, we find that the model predicts stability in the terms-of-trade while, at the same time, the price of imported goods relative to domestic goods remains volatile. Moreover, this volatility is driven by fluctuations in the exchange rate with respect to the dominant currency. Hence, following a depreciation of the exchange rate, imports from all origins will decrease. In contrast, DCP predicts that exports to dominant-currency destinations will not be responsive to currency movements, while the impact on exports to other destinations will depend on the co-movement of the exchange rate of the destination country with the dominant currency.

Taken together, these findings imply that a weakening of emerging market currencies relative to the dominant (dollar) currency following, say, a monetary policy easing in the former or a decline in commodity prices, will be associated with a decline in world trade (exports plus imports) relative to PCP or LCP.

We demonstrate that these DCP predictions when compared to the data (from Colombia) outperform the dominant paradigms of producer and local currency pricing in the literature. Lastly, optimal monetary policy for a small open economy deviates from the Mundell-Fleming benchmarks and involves targeting, alongside output and inflation, deviations from the law of one price against the dominant currency. Optimal policy (in the absence of cost-push shocks) calls for domestic PPI inflation targeting while the output gap fluctuates with the terms of trade.

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ONLINE APPENDIX: NOT FOR PUBLICATION

A Appendix

A.1 Proof of Proposition 1

The proof follows from a first order approximation of the first order condition for flexible prices:

$$\Delta p_{Hi,t} = \frac{1}{1+\Gamma}\Delta mc_t + \frac{\Gamma}{1+\Gamma}\left(\Delta p^i_{i,t} + \Delta e_{i,t}\right)$$

$$\Delta mc_t = (1-\alpha)\Delta w_t + \alpha \Delta p_t - \Delta a_t$$

$$\begin{aligned}
\Delta p_t &= \gamma_H \Delta p_{HH,t} + \gamma_U \Delta p_{UH,t} + \gamma_R \Delta p_{RH,t} \\
&= \gamma_H \Delta mc_t + \sum_{i\in U,R} \gamma_i \left(\Delta mc^i_{i,t} + \Delta e_{i,t}\right) \\
\Delta mc_t &= \frac{1-\alpha}{1-\alpha\gamma_H}\Delta w_t + \frac{\alpha}{1-\alpha\gamma_H}\sum_{i\in U,R}\gamma_i\left(\Delta mc^i_{i,t} + \Delta e_{i,t}\right) - \frac{1}{1-\alpha\gamma_H}\Delta a_t
\end{aligned}$$

The final expression follows when setting $\Delta mc^i_{i,t} = 0$.

On the import side,

$$\Delta p_{iH,t} = \frac{1}{1+\Gamma}\left(\Delta mc^i_{i,t} + \Delta e_{i,t}\right) + \frac{\Gamma}{1+\Gamma}\left(\Delta p_t\right)$$

Through simple substitution for Δp and Δmc we arrive at equation (19).

A.2 Derivation of Equation 24

The first order approximation to the optimal reset price for exports from H to i denominated in currency j $\bar{p}^j_{Hi,t}$ is given by,

$$\bar{p}^j_{Hi,t} = \frac{1-\beta\delta_p}{1+\Gamma}\left(mc^j_{H,t} + \Gamma p^j_{i,t} + \mu\right) + \beta\delta_p \mathbb{E}_t \bar{p}^j_{Hi,t+1}$$

where $mc^j_{H,t}$ is the (log) nominal marginal cost for H firms expressed in the currency of country j, and $p^j_{i,t}$ is the price index in country i expressed in currency j, μ is the (log) of the steady state markup.

From Calvo pricing we have,

$$\begin{aligned}
\pi^j_{Hi,t} &= (1-\delta_p)\left[\bar{p}^j_{Hi,t} - p^j_{Hi,t-1}\right] \\
&= (1-\delta_p)\left[\frac{1-\beta\delta_p}{1+\Gamma}\left(mc^j_{H,t} + \Gamma p^j_{i,t} + \mu\right) + \beta\delta_p \mathbb{E}_t \bar{p}^j_{Hi,t+1} - p^j_{Hi,t-1}\right] \\
&= (1-\delta_p)\left[\frac{1-\beta\delta_p}{1+\Gamma}\left(mc^j_{H,t} + \Gamma p^j_{i,t} + \mu\right) + \beta\delta_p\left(\mathbb{E}_t \bar{p}^j_{Hi,t+1} - p^j_{Hi,t}\right) + \beta\delta_p p^j_{Hi,t} - p^j_{Hi,t-1}\right] \\
&= (1-\delta_p)\left[\frac{1-\beta\delta_p}{1+\Gamma}\left(mc^j_{H,t} + \Gamma p^j_{i,t} + \mu\right) + \beta\delta_p\left(\frac{\mathbb{E}_t p^j_{Hi,t+1} - p^j_{Hi,t}}{1-\delta_p}\right) + \beta\delta_p p^j_{Hi,t} - p^j_{Hi,t-1}\right] \\
&= (1-\delta_p)\left[\frac{1-\beta\delta_p}{1+\Gamma}\left(mc^j_{H,t} + \Gamma p^j_{i,t} + \mu\right) + \beta\delta_p p^j_{Hi,t} - p^j_{Hi,t-1}\right] + \beta\delta_p \mathbb{E}_t \pi^j_{Hi,t+1} \\
&= (1-\delta_p)\left[\frac{1-\beta\delta_p}{1+\Gamma}\left(\left(mc^j_{H,t} - p^j_{Hi,t}\right) + \Gamma\left(p^j_{i,t} - p^j_{Hi,t}\right) + \mu\right) + \pi^j_{Hi,t}\right] + \beta\delta_p \mathbb{E}_t \pi^j_{Hi,t+1}
\end{aligned}$$

Re-grouping we arrive at equation 24.

A.3 Proof of Proposition 4

A.3.1 Functional forms

The restrictions $\varepsilon = \alpha = \varphi = 0$, $\sigma_c = 1$ imply,

$$U = \ln(C) - N, \qquad C = C_{HH}^{\gamma} C_{UH}^{\gamma_U} C_{RH}^{\gamma_R}, \qquad C_{ij} = \left(\int_0^1 C_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \qquad Y = e^a N$$

The share spent on domestic goods is given the home bias parameter γ common to every country; it ensures that the amount spent on domestic goods has positive measure. Moreover, the remaining share $1-\gamma$ is divided between the continuum of small open economies which compose the world. Because all countries other than H belong to either region U or R, we can aggregate the expenditure on a given region according to their relative sizes $\frac{\gamma_U}{1-\gamma}$ and $\frac{\gamma_R}{1-\gamma}$, yielding the above C.

A.3.2 Utility

$$U = \ln(C) - N \tag{A.1}$$

A second order approximation where hat notation represents deviations from the steady state gives,

$$U - \bar{U} \quad \approx \quad \hat{c}_t - \bar{N}\hat{n}_t - \frac{1}{2}\bar{N}\hat{n}_t^2$$

where $\bar{U} = \ln(\bar{C}) - \bar{N}$.

A.3.3 Labor demand

Lemma 2 *Under dollar pricing, each firm ω in region H sets the same export price in dollars in every country it exports to, conditional on being able to reset prices. That is, $\bar{P}_{Hj,t}^{U}$ is the same for all $j \in U \cup R$ and for all t. This also implies that $P_{HU,t}^{U} = P_{HR,t}^{U}$.*

This follows straightforwardly from section 2.4.2 when $\Gamma = 0$ (that is, $\epsilon = 0$). Dollar export prices are a function of marginal costs expressed in dollars and this does not vary by destination. Further with $\Gamma = 0$ optimal mark-ups are independent of destination.

Labor. Aggregate labor hired by firms is given by

$$N_t = \int_0^1 N_t(\omega) d\omega = \int_0^1 \frac{Y_t(\omega)}{e^{a_t}} d\omega.$$

The output of firm ω satisfies

$$\begin{aligned}
Y_t(\omega) =& Y_{HH,t}(\omega) + Y_{HU,t}(\omega) + Y_{HR,t}(\omega) \\
=& \left(\frac{P_{HH,t}(\omega)}{P_{HH,t}} \right)^{-\sigma} Y_{HH,t} + \left(\frac{P_{HU,t}(\omega)}{P_{HU,t}} \right)^{-\sigma} Y_{HU,t} + \left(\frac{P_{HR,t}(\omega)}{P_{HR,t}} \right)^{-\sigma} Y_{HR,t} \\
=& \left(\frac{P_{HH,t}(\omega)}{P_{HH,t}} \right)^{-\sigma} Y_{HH,t} + \left(\frac{P_{HU,t}^{U}(\omega)}{P_{HU,t}^{U}} \right)^{-\sigma} Y_{HU,t} + \left(\frac{P_{HU,t}^{U}(\omega)}{P_{HU,t}^{U}} \right)^{-\sigma} Y_{HR,t} \\
=& \left(\frac{P_{HH,t}(\omega)}{P_{HH,t}} \right)^{-\sigma} Y_{HH,t} + \left(\frac{P_{HU,t}^{U}(\omega)}{P_{HU,t}^{U}} \right)^{-\sigma} \left(Y_{HU,t} + Y_{HR,t} \right)
\end{aligned}$$

Aggregating over all firms,

$$N_t e^{a_t} = Y_{HH,t} \underbrace{\int_0^1 \left(\frac{P_{HH,t}(\omega)}{P_{HH,t}} \right)^{-\sigma} d\omega}_{V_t} + \underbrace{\left(Y_{HU,t} + Y_{HR,t} \right)}_{Y^*_{H,t}} \underbrace{\int_0^1 \left(\frac{P^U_{HU,t}(\omega)}{P^U_{HU,t}} \right)^{-\sigma} d\omega}_{V^*_t} \tag{A.2}$$

Now we approximate eq. (A.2) to second order. The left-hand side (LHS) becomes

$$e^{\bar{n}+\bar{a}} \left[1 + \hat{n}_t + \hat{a}_t + \frac{1}{2}(\hat{n}_t + \hat{a}_t)^2 \right]$$

and the right-hand side (RHS) can be approximated up to second order by

$$e^{\bar{y}_H + \bar{v}} \left[1 + \hat{y}_{HH,t} + \hat{v}_t + \frac{1}{2} \left(\hat{y}_{HH,t} + \hat{v}_t \right)^2 \right] + e^{\bar{y}^*_H + \bar{v}^*} \left[1 + \hat{y}^*_{H,t} + \hat{v}_t{}^* + \frac{1}{2} \left(\hat{y}^*_{H,t} + \hat{v}_t{}^* \right)^2 \right]$$

In a symmetric steady state, $v = v^* = 0$, all prices are symmetric, initial $NFA = 0$ and aggregate quantities are equal. Consequently, Cobb-Douglas consumption bundles entail:

$$\frac{e^{\bar{y}_H}}{e^{\bar{n}+\bar{a}}} = \gamma \quad \text{and} \quad \frac{e^{\bar{y}^*_H}}{e^{\bar{n}+\bar{a}}} = (1-\gamma)$$

Then, LHS=RHS becomes

$$\hat{n}_t + \hat{a}_t = \gamma \left[\hat{y}_{HH,t} + \hat{v}_t + \frac{1}{2} \left(\hat{y}_{HH,t} + \hat{v}_t \right)^2 \right] + (1-\gamma) \left[\hat{y}^*_{H,t} + \hat{v}_t{}^* + \frac{1}{2} \left(\hat{y}^*_{H,t} + \hat{v}_t{}^* \right)^2 \right] - \frac{1}{2}(\hat{n}_t + \hat{a}_t)^2$$

The relation $Y_t = Y_{HH,t} + Y^*_{H,t}$ can be written as $\frac{Y_t - Y}{\bar{Y}} = \gamma \frac{(Y_{HH,t} - \bar{Y}_{HH})}{\bar{Y}_{HH}} + (1-\gamma) \frac{(Y^*_{H,t} - \bar{Y}^*_H)}{\bar{Y}^*_H}$. This can then be approximated up to second order as

$$\hat{y}_t + \frac{\hat{y}_t^2}{2} = \gamma \left(\hat{y}_{HH,t} + \frac{\hat{y}^2_{HH,t}}{2} \right) + (1-\gamma) \left(\hat{y}^*_{H,t} + \frac{\hat{y}^{2*}_{H,t}}{2} \right)$$

Because v_t equals zero up to first order, $\left(\hat{y}_{HH,t} + \hat{v}_t \right)^2$ equals $\hat{y}^2_{HH,t}$ up to second order (analogously for v^*_t). Hence we can rewrite the labor demand equation as:

$$\hat{n}_t + \hat{a}_t = \hat{y}_t + \gamma \hat{v}_t + (1-\gamma)\hat{v}_t{}^* + \frac{\hat{y}_t^2}{2} - \frac{1}{2}(\hat{n}_t + \hat{a}_t)^2$$

Since $\hat{n}_t + \hat{a}_t$ equals $\hat{y}_t$ plus second or higher order terms, $(\hat{n}_t + \hat{a}_t)^2$ equals $\hat{y}_t^2$ up to second order so that the last two terms of the above equation cancel out. Finally, one can follow the standard steps in order to express price dispersion as

$$\begin{aligned} v_t &\approx \frac{\sigma}{2} \int_0^1 \left(p_{HH,t}(\omega) - p_{HH,t} \right)^2 d\omega \\ &\approx \frac{\sigma}{2} var_\omega \{ p_{HH,t}(\omega) \} \end{aligned}$$

$$\begin{aligned} v^*_t &\approx \frac{\sigma}{2} \int_0^1 \left(p^U_{HU,t}(\omega) - p^U_{HU,t} \right)^2 d\omega \\ &\approx \frac{\sigma}{2} var_\omega \{ p^U_{HU,t}(\omega) \} \end{aligned}$$

Substitute this into previous approximation and up to the second order we derive

$$\hat{n}_t + \hat{a}_t = \hat{y}_t + \gamma \frac{\sigma}{2} var_\omega \{ p_{HH,t}(\omega) \} + (1-\gamma) \frac{\sigma}{2} var_\omega \{ p^U_{HU,t}(\omega) \} \tag{A.3}$$

A.3.4 Market clearing condition

By goods market clearing[21], we have

$$Y_t = \gamma \left(\frac{P_t}{P_{HH,t}} \right) C_t + \gamma_U \left(\frac{P_t^U}{P_{HU,t}^U} \right) C_t^U + \gamma_R \left(\frac{P_t^R \mathcal{E}_{R,t}}{P_{HR,t}^U \mathcal{E}_{U,t}} \right) C_t^R$$

Complete markets gives us (under the assumption of exante symmetry and $NFA = 0$),

$$\mathcal{E}_{U,t} P_t^U C_t^U = P_t C_t, \qquad \mathcal{E}_{R,t} P_t^R C_t^R = P_t C_t \tag{A.4}$$

In addition we define a misaligment term ala Engel to capture the failure of the law of one price across destinations,

$$M_t^U = M_t^R = M_t = \frac{\mathcal{E}_{U,t} P_{HU,t}^U}{P_{HH,t}} = \frac{\mathcal{E}_{U,t} P_{HR,t}^U}{P_{HH,t}} \tag{A.5}$$

Under dollar pricing there is only one misalignment that arises from fluctuations in the dollar exchange rate regardless of destination market (this holds due to Lemma 3). Combining goods market clearing and the complete markets condition:

$$Y_t = \frac{P_t C_t}{P_{HH,t}} \left[\gamma + \frac{\gamma_U}{M_t} + \frac{\gamma_R}{M_t} \right]$$

Define the trade weighted TOT,

$$S = S_U^{\frac{\gamma_U}{1-\gamma}} S_R^{\frac{\gamma_R}{1-\gamma}}$$

where (note that exports and imports are invoiced in currency U under DCP)

$$S_U = \frac{P_{UH}^U}{P_{HU}^U}, \qquad S_R = \frac{P_{RH}^U}{P_{HR}^U}$$

With this, we can express,

$$\frac{P_t}{P_{HH,t}} = S_t^{1-\gamma} M_t^{1-\gamma}$$

$$Y_t = \gamma S_t^{1-\gamma} M_t^{1-\gamma} C_t + (1-\gamma) M_t^{-\gamma} S_t^{1-\gamma} C_t$$

Second Order Approximation to Market Clearing

$$LHS \approx \bar{Y} \left(1 + \hat{y}_t + \frac{1}{2} \hat{y_t}^2 \right)$$

$$\begin{aligned} RHS &\approx \gamma \bar{C} \left[1 + (1-\gamma)\hat{s}_t + (1-\gamma)\hat{m}_t + \hat{c}_t + \frac{1}{2} \left((1-\gamma)\hat{s}_t + (1-\gamma)\hat{m}_t + \hat{c}_t \right)^2 \right] \\ &+ (1-\gamma)\bar{C} \left[1 + (1-\gamma)\hat{s}_t + (-\gamma)\hat{m}_t + \hat{c}_t + \frac{1}{2} \left((1-\gamma)\hat{s}_t + (-\gamma)\hat{m}_t + \hat{c}_t \right)^2 \right] \end{aligned}$$

Recalling that $\bar{Y} = \bar{C}$ due to symmetric steady state:

$$\begin{aligned} RHS &\approx \bar{Y} \left[1 + (1-\gamma)\hat{s}_t + \hat{c}_t \right] + \gamma \bar{Y} \left[\frac{1}{2} \left((1-\gamma)\hat{s}_t + (1-\gamma)\hat{m}_t + \hat{c}_t \right)^2 \right] \\ &+ (1-\gamma)\bar{Y} \left[\frac{1}{2} \left((1-\gamma)\hat{s}_t + (-\gamma)\hat{m}_t + \hat{c}_t \right)^2 \right] \end{aligned}$$

[21]Once again, this condition arises because elasticity of substitution between foreign goods is unit and because U and R are composed of a continuum of SOE with relative masses $\gamma_U/(1-\gamma)$ and $\gamma_R/(1-\gamma)$ respectively and where every country has home bias in preferences according to γ.

$LHS = RHS$ entails:

$$
\begin{aligned}
\hat{y_t} + \frac{1}{2}\hat{y_t}^2 &= (1-\gamma)\hat{s}_t + \hat{c}_t + \gamma\left[\frac{1}{2}\left((1-\gamma)\hat{s}_t + (1-\gamma)\hat{m}_t + \hat{c}_t\right)^2\right] \\
&+ (1-\gamma)\left[\frac{1}{2}\left((1-\gamma)\hat{s_t} + (-\gamma)\hat{m}_t + \hat{c}_t\right)^2\right]
\end{aligned}
$$

$$
\begin{aligned}
\hat{y_t} &= (1-\gamma)\hat{s_t} + \hat{c} + \gamma\left[\frac{1}{2}\left((1-\gamma)\hat{s_t} + (1-\gamma)\hat{m}_t + \hat{c}_t\right)^2\right] \\
&+ (1-\gamma)\left[\frac{1}{2}\left((1-\gamma)\hat{s_t} + (-\gamma)\hat{m}_t + \hat{c}_t\right)^2\right] - \frac{1}{2}\hat{y_t}^2
\end{aligned}
$$

Now note that because $\hat{y_t} = [(1-\gamma)\hat{s_t} + \hat{c_t}] + S.O.T.$, $\hat{y_t}^2 = [(1-\gamma)\hat{s_t} + \hat{c_t}]^2 + h.o.t.$ and we ignore terms with third order or higher. Thus:

$$
\begin{aligned}
\hat{y_t} &= (1-\gamma)\hat{s}_t + \hat{c}_t + \frac{\gamma}{2}\left[(1-\gamma)\hat{s}_t + (1-\gamma)\hat{m}_t + \hat{c}_t\right]^2 \\
&+ \frac{(1-\gamma)}{2}\left[(1-\gamma)\hat{s}_t + (-\gamma)\hat{m}_t + \hat{c}\right]^2 - \frac{1}{2}\left[(1-\gamma)\hat{s}_t + \hat{c}_t\right]^2
\end{aligned}
$$

Simplifying the above equation yields:

$$
\hat{y_t} = (1-\gamma)\hat{s_t} + \hat{c}_t + \frac{\gamma(1-\gamma)}{2}\hat{m_t}^2
$$

A.3.5 Complete Markets

$$
c_t = c_{U,t} + q_{U,t}, \qquad c_t = c_{R,t} + q_{R,t}
$$

where q is the log of the real exchange rate. Summing the two equations using weights, $\frac{\gamma_U}{1-\gamma}$ and $\frac{\gamma_R}{1-\gamma}$, we arrive at,

$$
c_t = c_t^* + q_t
$$

where $c_t^* = \frac{\gamma_U}{1-\gamma}c_{U,t} + \frac{\gamma_R}{1-\gamma}c_{R,t}$ and $q_t = \frac{\gamma_U}{1-\gamma_H}q_{U,t} + \frac{\gamma_R}{1-\gamma_H}q_{R,t}$. The consumer price index,

$$
p_t = (1-\gamma_H)s_t + (1-\gamma_H)m_t + p_{HH,t}
$$

$$
q_t = e_{U,t} + p_t^* - p_t
$$

where $p_t^* = \frac{\gamma_U}{1-\gamma_H}p_{U,t} + \frac{\gamma_U}{1-\gamma_H}p_{R,t}^U$. Note that we are expressing the R price level in dollars and because of the exogeneity of the exchange rate between R and U this can be treated as exogenous. Next we use the definition of the TOT to arrive at,[22]

$$
s_{U,t} = e_{U,t} + p_{U,t} - p_{HH,t} - m_t \qquad s_{R,t} = e_{U,t} + p_{R,t}^U - p_{HH,t} - m_t
$$

$$
s_t = e_{U,t} + p_t^* - m_t - p_{HH,t}
$$

$$
\begin{aligned}
q_t &= e_{U,t} + p_t^* - p_t \\
&= s_t + m_t + p_{HH,t} - p_t \\
&= \gamma(s_t + m_t)
\end{aligned}
$$

[22] To simplify notation we use $p_{U,t}$ ($p_{R,t}^U$) to represent the CPI and export price for U (R). With two large regions U and R these will not be the same but the difference will involve terms independent of monetary policy in H and consequently we abuse notation without any costs.

Substituting in the complete markets condition we obtain,

$$c_t = c_t^* + \gamma(s_t + m_t)$$

Note that this is exact in logs and not an approximation.

A.3.6 Terms-of-Trade

Lemma 3 *When $\varphi = 0$ and assets markets are complete then the terms-of-trade evolves independently of policy.*

Proof: It follows from the small open economy assumption that prices of imported goods in the dominant currency, $\hat{p}_{UH}^U(\omega)$ and $\hat{p}_{RH}^U(\omega)$ evolves exogenously. We only need to ensure that H export prices in dominant currency evolves exogenous from monetary policy. When $\varphi = 0$ this is indeed the case because H firms marginal cost in dollars is exogenous. Specifically, from the labor supply decision with $\varphi = 0$ we have, $C_t = W_t/P_t$. This combined with complete markets, $P_t C_t = \mathcal{E}_{U,t} P_t^U C_t^U$, leads to an expression for marginal cost in dollars,

$$\frac{MC_{H,t}}{\mathcal{E}_{U,t}} = \frac{W_t}{\mathcal{E}_{U,t} e^{a,t}} = \frac{P_t^U C_t^U}{e^{a,t}}$$

that evolves exogenously.

A.3.7 Natural/Flexible price allocation

When prices are flexible there is no misalignment and the law of one price holds, $P_{HH,t}^n = \mathcal{E}_{U,t} P_{HU,t}^U = \mathcal{E}_{R,t} P_{HU,t}^U$, where $P_{HH,t}^n$ refers to the 'natural' price in the flexible price allocation. The pricing decision is given by:

$$P_{HH,t}^n = \frac{\sigma}{\sigma - 1} \frac{(1-\tau) W_t^n}{e^{a_t}} = \frac{\sigma}{\sigma - 1} \frac{(1-\tau) P_t^n C_t^n}{e^{a_t}}$$

using the complete markets condition and where τ is a static tax on hiring labor.

$$\frac{(1-\tau) P_t^n C_t^n}{e^{a_t} P_{HH,t}^n} = \frac{(1-\tau)(S_t^n)^{1-\gamma_H} C_t^n}{e^{a_t}} = \frac{(1-\tau) Y_t^n}{e^{a_t}}$$

where for the last equality we used the market clearing condition $Y_t^n = (S_t^n)^{1-\gamma_H} C_t^n$. Combining both equations and using $Y_t^n = N_t^n e^{a_t}$ yield:

$$(1-\tau) N_t^n = \frac{\sigma - 1}{\sigma}$$

Hence $N_t^n = 1$ when $(1-\tau) = \frac{\sigma-1}{\sigma}$. That is, when the tax rate offsets the mark-up distortion $N_t = 1$ at all points in time (in the absence of mark-up shocks).

Remark 4 *To arrive at a second-order representation of the loss function that requires only first order approximations to the equilibrium solution to solve for optimal policy we approximate around a steady state where the tax $(1-\tau) = \frac{\sigma-1}{\sigma}$ and $\bar{N} = 1$.*

We therefore approximate around a constrained efficient steady state where a tax that only gets rid of the mark-up distortion is used. As derived in Galí and Monacelli (2005) an efficient steady state has a tax that addresses both the mark-up distortion and the monopoly power of the firm in international markets. Specifically in the efficient steady state $(1-\tau) = \frac{1}{\gamma} \frac{\sigma-1}{\sigma}$ and $\bar{N} = \gamma$, that is employment is lower than in the constrained efficient steady state. In the case of PCP it is important to approximate around the efficient steady state otherwise policy can generate first order gains by manipulating the terms-of-trade. In the case of dollar pricing this is not the case. The terms-of-trade is independent of policy (under the parameter restrictions) and consequently it cannot be manipulated.

To derive a linear-quadratic representation of the problem we approximate around the constrained efficient steady state that only gets rid of the monopoly mark-up. By doing so we are left with only second-order terms in the loss function that depend on policy.

A.3.8 Second order welfare loss function

$$\mathbb{W} \quad \approx \quad \mathbb{E}_0 \sum_0^{\infty} \beta^t \left(\hat{c}_t - \bar{N}\hat{n}_t - \frac{1}{2}\bar{N}\hat{n}_t^2 \right)$$

Define deviations from the natural (flexible price) allocation as, $\tilde{x}_t \equiv \hat{x}_t - \hat{x}_t^n$. We then have $\hat{x}_t = \tilde{x}_t + \hat{x}_t^n$. Recalling that $\bar{N}$ =1 since we are approximating around the inefficient steady state, we can can then write the welfare function in deviation from the natural allocation:

$$\mathbb{W} \quad \approx \quad \mathbb{E}_0 \sum_0^{\infty} \beta^t \left(\tilde{c}_t - \tilde{n}_t - \frac{1}{2}\tilde{n}_t^2 \right) + t.i.p.$$

Since $m_t^n = 0 = \bar{m}_t \Rightarrow \tilde{m}_t = \hat{m}_t$, the market clearing condition can be stated as:

$$\tilde{y}_t + \hat{y}_t^n \quad = \quad [(1-\gamma)(\tilde{s}_t + \hat{s}_t^n) + \tilde{c}_t + \hat{c}_t^n] + \frac{\gamma(1-\gamma)}{2}\tilde{m}_t^2$$

Combining it with $\hat{y}_t^n = \hat{a}_t$ simplifies to:

$$\tilde{y}_t \quad = \quad (1-\gamma)\tilde{s}_t + \tilde{c}_t + \frac{\gamma(1-\gamma)}{2}\tilde{m}_t^2$$

Labor demand:

$$\tilde{n}_t + \cancelto{0}{\hat{n}_t^n} + \cancel{\hat{a}_t} = \tilde{y}_t + \cancel{\hat{y}_t^n} + \gamma\frac{\sigma}{2}var_w(p_{HH,t}^H) + (1-\gamma)\frac{\sigma}{2}var_w(p_{HU,t}^U)$$

Combing the previous two expressions and using the result in chapter 6 of Woodford (2003) for the evolution of price dispersion,

$$\sum_0^{\infty} \beta^t var_\omega p_{HH,t} = \frac{1}{\lambda_p}\sum_0^{\infty} \beta^t \pi_{HH,t}^2 \tag{A.6}$$

$$\sum_0^{\infty} \beta^t var_\omega p_{HU,t}^U = \frac{1}{\lambda_p}\sum_0^{\infty} \beta^t \left(\pi_{HU,t}^U\right)^2$$

we arrive at the second order loss function in Proposition 4 (expressed in deviation from the natural allocation). Note that $\tilde{n}_t^2 = \tilde{y}_t^2 + h.o.t.$ where $h.o.t.$ are third or higher order terms and that $\tilde{s}_t$ and $\pi_{HU,t}^2$ show up in the terms independent of policy.

Constraints: Because there are only quadratic terms in the second order approximation of the welfare loss function, we need to write the constraints only up to first order. The first constraint is the NKPC:

$$\pi_{HH,t}^H = \lambda_p \left(\tilde{mc}_{H,t}^H - \tilde{p}_{HH,t}^H \right) + \beta\mathbb{E}_t\pi_{HH,t+1}^H$$

where

$$\tilde{mc}_{H,t}^H - \tilde{p}_{HH,t}^H = \tilde{p}_t - \tilde{p}_{HH,t} + \tilde{c}_t = (1-\gamma)(\tilde{s}_t + \tilde{m}_t) + \tilde{c}_t$$

$$\tilde{c}_t = \tilde{y}_t - (1-\gamma)\tilde{s}_t$$

Combining the two equations and replacing in the pricing equation for $p_{HH,t}$ we arrive at the first constraint as stated in Preposition 4. The second constraint restricts the movement between $\tilde{y}_t$ and $\tilde{m}_t$. Combining $\tilde{y}_t = (1-\gamma)\tilde{s}_t + \tilde{c}_t$ (market clearing condition) and $\tilde{c}_t = \gamma(\tilde{s}_t + \tilde{m}_t)$ (complete markets condition) we have:

$$\tilde{s}_t = \tilde{y}_t - \gamma\tilde{m}_t$$

A.3.9 PCP

In the case of PCP the market clearing condition is linear in logs and consequently the derivation is much simpler. Specifically we have from the market clearing condition,

$$\tilde{y}_t = (1-\gamma)\tilde{s}_t + \tilde{c}_t$$

And the complete markets condition

$$\tilde{c} = \gamma \tilde{s}_t$$

Combining the two we have

$$\tilde{c}_t = \gamma \tilde{y}_t$$

Linearizing around the *efficient* steady state where $\bar{N} = \gamma$ and substituting,

$$\tilde{n}_t = \tilde{y}_t + \frac{\sigma}{2} var_w(p^H_{HH,t}) \tag{A.7}$$

we have

$$U - \bar{U} \quad \approx \quad -\gamma \left(\frac{\sigma}{2} var_w(p^H_{HH,t}) + \frac{1}{2}\tilde{y}_t^2 \right)$$

Using equation (A.6) yields the loss function in Proposition 4, with the only constraint being the NKPC where we substitute in $\left(\tilde{mc}^H_{H,t} - \tilde{p}^H_{HH,t}\right) = \tilde{c}_t + \left(\tilde{p}_t - \tilde{p}^H_{HH,t}\right) = \tilde{c}_t + (1-\gamma)\tilde{s}_t = \tilde{y}_t$.

A.3.10 Optimal policy under commitment

DCP: Firstly, it is helpful to use the second constraint to obtain $\tilde{m}_t$ in terms of $\tilde{y}_t$ and $\tilde{s}_t$ and rewrite the problem as:

$$\min \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\tilde{y}_t^2 + \gamma \frac{\sigma}{2\lambda_p}\pi^2_{HH,t} + \frac{\gamma(1-\gamma)}{2}\left(\frac{\tilde{y}_t - \tilde{s}_t}{\gamma}\right)^2 \right] + t.i.p \tag{A.8}$$

s.t.

$$\pi_{HH,t} \quad = \quad \frac{\lambda_p}{\gamma}\left[\tilde{y}_t - (1-\gamma)\tilde{s}_t\right] + \beta \mathbb{E}_t \pi_{HH,t+1}$$

where now the central bank internalizes the effect of its policy on inflation expectations. We can then set the Lagrangian with multiplier θ_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\tilde{y}_t^2 + \gamma \frac{\sigma}{2\lambda_p}\pi^2_{HH,t} + \frac{\gamma(1-\gamma)}{2}\left(\frac{\tilde{y}_t - \tilde{s}_t}{\gamma}\right)^2 + \theta_t \left(\pi_{HH,t} - \frac{\lambda_p}{\gamma}\left[\tilde{y}_t - (1-\gamma)\tilde{s}_t\right] - \beta\pi_{HH,t+1} \right) \right] \tag{A.9}$$

The FOCs w.r.t. $\tilde{y}_t$ and $\pi_{HH,t}$ are:

$$\tilde{y}_t + (1-\gamma)\left(\frac{\tilde{y}_t - \tilde{s}_t}{\gamma}\right) - \frac{\lambda_p \theta_t}{\gamma} = 0 \Rightarrow \tilde{y}_t - (1-\gamma)\tilde{s}_t - \lambda_p\theta_t = 0$$

$$\frac{\gamma\sigma}{\lambda_p}\pi_{HH,t} + \theta_t - \theta_{t-1} = 0$$

Combining the two optimality conditions yield:

$$[\tilde{y}_t - (1-\gamma)\tilde{s}_t] = [\tilde{y}_{t-1} - (1-\gamma)\tilde{s}_{t-1}] - \gamma\sigma\pi_{HH,t}$$

$$[\tilde{y}_0 - (1-\gamma)\tilde{s}_0] = -\gamma\sigma\pi_{HH,0}$$

This allows us to write optimal policy in terms of levels targeting:

$$\tilde{y}_t = (1-\gamma)\tilde{s}_t - \gamma\sigma(p_{HH,t} - p_{HH,-1})$$

Although we find a similar result to the standard PCP case in which output gaps depend upon the price level rather than inflation, now this commitment depends on the terms of trade which evolves exogenously. In a sense, the central bank conditions his response on the level of the terms of trade.

Moreover, the price level follows a second order difference equation:

$$(p_{HH,t} - p_{HH,-1}) = \frac{(p_{HH,t-1} - p_{HH,-1}) + \beta\mathbb{E}_t(p_{HH,t+1} - p_{HH,-1})}{1 + \lambda_p\sigma + \beta}$$

Interestingly, the optimal price level under commitment follows the same path as the PCP case. Consequently, while the desired output gap fluctuates with the terms of trade under DCP, the price level will follow a similar behavior to the standard case.

PCP: With commitment, the optimality conditions are:

$$\tilde{y}_t - \lambda_p \theta_t = 0$$

$$\frac{\sigma}{\lambda_p}\pi_{HH,t} + \theta_t - \theta_{t-1} = 0$$

This yields:

$$\tilde{y}_t = \tilde{y}_{t-1} - \sigma\pi_{HH,t}$$
$$\tilde{y}_0 = -\sigma\pi_{HH,0}$$

This allows us to write optimal policy in terms of levels targeting:

$$\tilde{y}_t = -\sigma(p_{HH,t} - p_{HH,-1})$$

Combining with the NKPC gives the following price level targeting equation:

$$(p_{HH,t} - p_{HH,-1}) = \frac{(p_{HH,t-1} - p_{HH,-1}) + \beta\mathbb{E}_t(p_{HH,t+1} - p_{HH,-1})}{1 + \lambda_p\sigma + \beta}$$

Hence we recover the same price level formula as in Galí and Monacelli (2005) but with our parameter restriction.

A.4 Dynamic IS Curve

The IS curve under DCP and simplified parametrization is:

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} - (i_t - \mathbb{E}_t\pi_{HH,t+1} - \rho) + (1-\gamma)\mathbb{E}_t(\Delta\tilde{m}_{t+1}) + \mathbb{E}_t\Delta a_{t+1}$$

Comparing with equation 38 of Gali's chapter 7, one can see that the only difference is the misalignment term that now appears in the IS curve. This occurs because now the equation linking PPI inflation (above) and CPI inflation (in original Euler equation) depends on misalignments in addition to the terms of trade. Finally, the IS curve can also be written as a function of the natural interest rate (as in Gali's equation 39):

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} - (i_t - \mathbb{E}_t\pi_{HH,t+1} - r_t^n) + (1-\gamma)\mathbb{E}_t(\Delta\tilde{m}_{t+1})$$

A.4.1 Derivation

Euler condition:

$$C_t^{-\sigma_c} = \beta(1+i_{U,t})\mathbb{E}_t C_{t+1}^{-\sigma_c}\frac{P_t}{P_{t+1}}\frac{\mathcal{E}_{U,t+1}}{\mathcal{E}_{U,t}}$$
$$C_t^{-\sigma_c} = \beta(1+i_t)\mathbb{E}_t C_{t+1}^{-\sigma_c}\frac{P_t}{P_{t+1}}$$

where $(1+i_t) = (\sum_{s'\in\mathcal{S}} Q_t(s'))^{-1}$. Log-linearizing the first equation gives:

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}(i_t - \mathbb{E}_t\pi_{t+1} - \rho)$$

where $\rho = \log\beta$ and the approximation $(1+i_t)\approx i_t$ was used. This is the standard IS curve in Gali. It is easy to see that log-linearizing the Euler equation of the dollar bond gives $c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}(i_{U,t} - \mathbb{E}_t\pi_{t+1} - \rho + \mathbb{E}_t e_{U,t+1})$. Combining both yields the UIP condition:

$$i_t = i_{U,t} + \mathbb{E}_t e_{U,t+1}$$

Imposing $\sigma_c = 1$ and writing the log-linearized Euler equation in deviations of steady state gives:

$$\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - (i_t - \mathbb{E}_t\pi_{t+1} - \rho)$$

Combining with the market clearing condition up to first order yields:

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - (i_t - \mathbb{E}_t\pi_{t+1} - \rho) - (1-\gamma)\mathbb{E}_t(\Delta\hat{s}_{t+1})$$

Moreover, we have that

$$p_t - p_{HH,t} = (1-\gamma)(s_t + m_t) \Rightarrow \pi_t = \pi_{HH,t} + (1-\gamma)(\Delta s_t + \Delta m_t)$$

Since naturally $\Delta s_t = \Delta\hat{s}_t$, IS curve becomes:

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - (i_t - \mathbb{E}_t\pi_{HH,t+1} - \rho) + (1-\gamma)\mathbb{E}_t(\Delta m_{t+1})$$

Using the fact that $\hat{y}_t = \tilde{y}_t + \hat{y}_t^n = \tilde{y}_t + \hat{a}_t$ we can write the IS curve in terms of flexible price deviation

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} - (i_t - \mathbb{E}_t\pi_{HH,t+1} - \rho) + (1-\gamma)\mathbb{E}_t(\Delta\tilde{m}_{t+1}) + \mathbb{E}_t\Delta a_{t+1}$$

Defining $r_t^n = \rho + \mathbb{E}_t\Delta a_{t+1}$ which is the same definition of equation 39 of chapter 7 of Gali for the simplified parametrization, we can also write the IS curve as:

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} - (i_t - \mathbb{E}_t\pi_{HH,t+1} - r_t^n) + (1-\gamma)\mathbb{E}_t(\Delta\tilde{m}_{t+1})$$