Øving 3.2 Jacob

1.a)

$$v_c^n = \begin{bmatrix} V_c \cos(\beta_{V_c}) \\ V_c \sin(\beta_{V_c}) \\ 0 \end{bmatrix}$$

$$u_c = V_c \cos(\beta_{V_c} - \psi)$$

$$v_c = V_c \sin(\beta_{V_c} - \psi)$$

$$V_c = \sqrt{u_c^2 + v_c^2}$$

irrational constant oceancurrent:

$$\dot{v}_c^n = \dot{R}(\Theta_{nb}) v_c^b + R(\Theta_{nb}) \dot{v}_c^b := 0$$

$$\dot{R}(\Theta_{nb}) = R(\Theta_{nb}) S(\omega_{nb}^b) \quad , \dot{v}_c^b = -S(\omega_{nb}^b) v_c^b$$

$$M_{RB}\dot{\nu} + C_{RB}(\nu)\nu = M_{RB}\dot{\nu}_r + C_{RB}(\nu_r)\nu_r$$

$$\nu_r = \begin{bmatrix} v^b - v_c^b \\ \omega_{nb}^b \end{bmatrix}$$

$$v_c^n = R(\Theta_{nb}) v_c^b$$

$$v_c^b = R(\Theta_{nb})^{-1} v_c^n = \underline{\underline{R(\Theta_{nb})^T v_c^n}}$$

1.c)

$$\gamma_w = \psi - \beta_{V_w} - \pi$$

$$C_Y(\gamma_w) \approx c_Y \sin(\gamma_w)$$

$$C_N(\gamma_w) \approx c_N \sin(2\gamma_w)$$

$$\tau_{wind} = \frac{1}{2}\rho_a V_{rw}^2 \begin{bmatrix} C_X \ldots := 0 \\ C_Y \ldots \\ C_Z \ldots := 0 \\ C_K \ldots := 0 \\ C_M \ldots := 0 \\ C_N \ldots \end{bmatrix}$$

2.a) From 6.9 in Fossen

$$u \approx U, \quad v = r = 0 \Rightarrow C^*_{RB} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & mU \\ 0 & 0 & mx_g U \end{bmatrix}$$

$$C^*_A = U \cdot A^{\{1,2,6\}}(0)\, L^{\{1,2,6\}}$$

$$L^{\{1,2,6\}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad M_A = A^{\{1,2,6\}}(0) = \begin{bmatrix} A_{11}(0) & 0 & 0 \\ 0 & A_{22}(0) & A_{26}(0) \\ 0 & A_{62}(0) & A_{66}(0) \end{bmatrix}$$

$$A \cdot L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{22}(0) \\ 0 & 0 & A_{62}(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -Y_{\dot{v}} \\ 0 & 0 & -N_{\dot{v}} \end{bmatrix}$$

$$\Rightarrow C^*_A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -Y_{\dot{v}} U \\ 0 & 0 & -N_{\dot{v}} \cdot U \end{bmatrix}$$

$\underline{\text{hap}}$

$$C_A(v_r)v_r = \begin{bmatrix} Y_{\dot{v}} v_r r + Y_{\dot{r}} r^2 \\ -X_{\dot{u}} u_r r \\ (X_{\dot{u}} - Y_{\dot{v}}) u_r v_r - Y_{\dot{r}} u_r r \end{bmatrix} \quad v = \begin{bmatrix} u \\ v \\ r \end{bmatrix}$$

$$C^*_A = \begin{bmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} & \frac{\partial f_1}{\partial r} \\ \partial & \frac{\partial f_2}{\partial v} & \frac{\partial f_2}{\partial r} \\ & \frac{\partial f_3}{\partial v} & \frac{\partial f_3}{\partial r} \end{bmatrix} = \begin{bmatrix} 0 & & \\ & 0 & -X_{\dot{u}} u_r \\ & (X_{\dot{u}} - Y_{\dot{v}}) u_r - 0 & Y_{\dot{r}} u_r \end{bmatrix}$$

2b) $M\dot{v}_r + N v_r = b\delta$

$$sMv + Nv = b\delta$$

$$v(sM+N) = b\delta$$

$$v = (sM+N)^{-1} b\delta$$

$$= \begin{bmatrix} v_r \\ r \end{bmatrix} \begin{matrix} = (sM_{11}+N_{11})\cdot b_1\delta + (sM_{12}+N_{12})\, b_2\delta \\ = (sM_{21}+N_{21})\cdot b_1\delta + (sM_{22}+N_{22})\, b_2\delta \end{matrix}$$

$$\frac{r}{\delta} = (sM_{21}+N_{21})\cdot b_1 + (sM_{22}+N_{22})\, b_2$$

$$\frac{\delta}{r} = \frac{1}{\frac{r}{\delta}} = \frac{1}{(sM_{21}+N_{21})b_1 + (sM_{22}+N_{22})b_2}$$

$$(sM+N)^{-1} = \begin{bmatrix} sM_{11}+N_{11} & sM_{12}+N_{12} \\ sM_{21}+N_{21} & sM_{22}+N_{22} \end{bmatrix}^{-1} = H^{-1}$$

$$= \frac{1}{\det(H)} \begin{bmatrix} sM_{22}+N_{22} & -sM_{12}-N_{12} \\ -sM_{21}-N_{21} & sM_{11}+N_{11} \end{bmatrix}$$

$$\Rightarrow \frac{\delta}{r} = \frac{1}{\frac{(-sM_{21}-N_{21})}{\det(H)} b_1 + \frac{(sM_{11}+N_{11})}{\det(H)} b_2}$$

$$M\dot{v} + Nv = b\delta$$

$$\dot{v} = M^{-1}(b\delta - Nv) = M^{-1}b\delta - M^{-1}Nv$$

ss2tf (matlab) $\Rightarrow$ $$H(s) = \frac{0{,}8638\cdot 10^{-4} s + 0{,}0449\cdot 10^{-4}}{s^2 + 0{,}6782 s + 0{,}031}$$

$K_i =$

2a)

C_A^* wrong. $C_A^* = \begin{bmatrix} 0 & \cdot & \cdot \\ \vdots & 0 & -X_i U_r \\ \vdots & (X_i - Y_i) U_r & Y_i U_r \end{bmatrix} \Rightarrow C_{A(2\times 2)}^* = \begin{bmatrix} 0 & -X_i U_r \\ (X_i - Y_i) U_r & Y_i U_r \end{bmatrix}$

$$C_{A(3\times 3)}^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -X_i U_r \\ -X_i U_r & (X_i - Y_i) U_r & Y_i U_r \end{bmatrix}$$

2c) matlab gives

$$H(s) = \frac{10^{-4}(0{,}8638s + 0{,}0615)}{s^2 + 0{,}1509s + 0{,}0008}$$

$$= \frac{K(T_3s+1)}{(T_1s+1)(T_2s+1)} = \frac{K(T_3s+1)}{T_1T_2s^2 + s(T_1+T_2)+1}$$

$$\Rightarrow \frac{\frac{1}{0{,}0615}}{\frac{1}{0{,}0615}} H(s) = \frac{10^{-4}(14{,}046s+1)}{\frac{1}{0{,}0615}(s^2+0{,}1509s+0{,}0008)}$$

$$= \frac{10^{-4}(14{,}046s+1)}{\frac{s^2}{0{,}0615} + \frac{0{,}1509}{0{,}0615}s + \underbrace{\frac{0{,}0008}{0{,}0615}}_{0{,}0130}}$$

$$\frac{\frac{1}{0{,}013}}{\frac{1}{0{,}013}} H(s) = \frac{\frac{10^{-4}}{0{,}013}(14{,}046s+1)}{1250{,}78s^2 + 188{,}745s + 1}$$

$\Rightarrow T_1T_2 = 1250{,}78$

$T_1 + T_2 = 188{,}74 \quad \Rightarrow T_2 = 188{,}74 - T_1$

$\Rightarrow T_1(188{,}74 - T_1) = 1250{,}78$

$T_1 = 6{,}878 \quad , T_2 = 181{,}86$

$T_3 = 14{,}046 \qquad K = \frac{1}{130}$

$T = T_1 + T_2 - T_3 = 174{,}69$

2d) $K_p = \omega_n^2 \frac{T}{K}$, $K_d = \frac{2\zeta\omega_n T - 1}{K}$

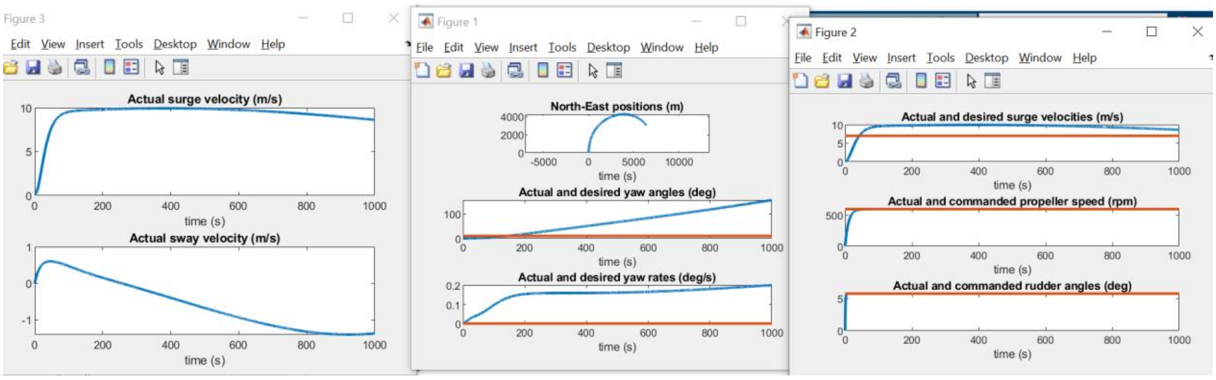
$K_i = \omega_n^3 \frac{T}{10K}$

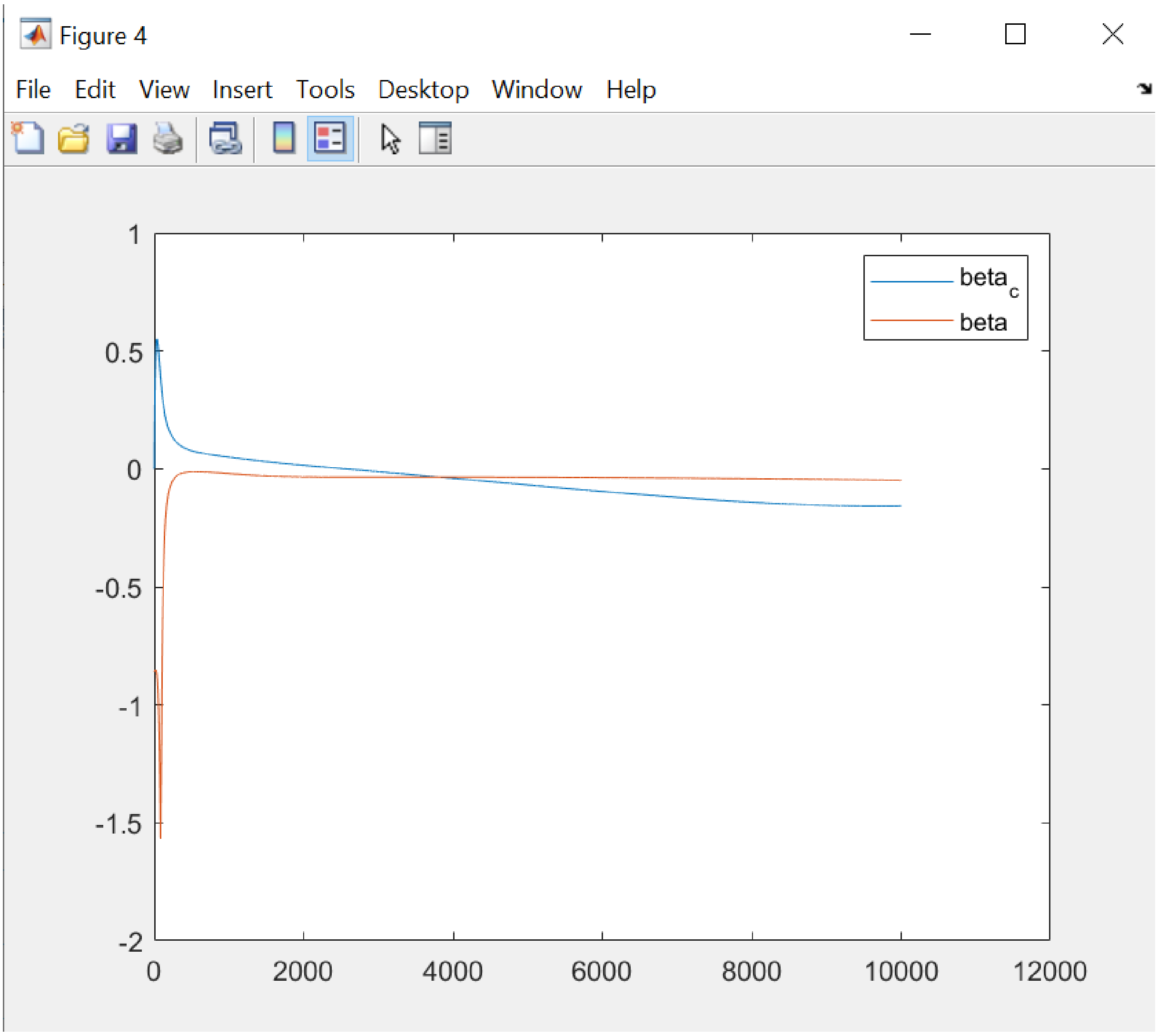
$\omega_b = 0{,}06$, $\zeta = 1$

$\omega_n \overset{(\text{Algo } 15{,}1)}{=} 0{,}0932$

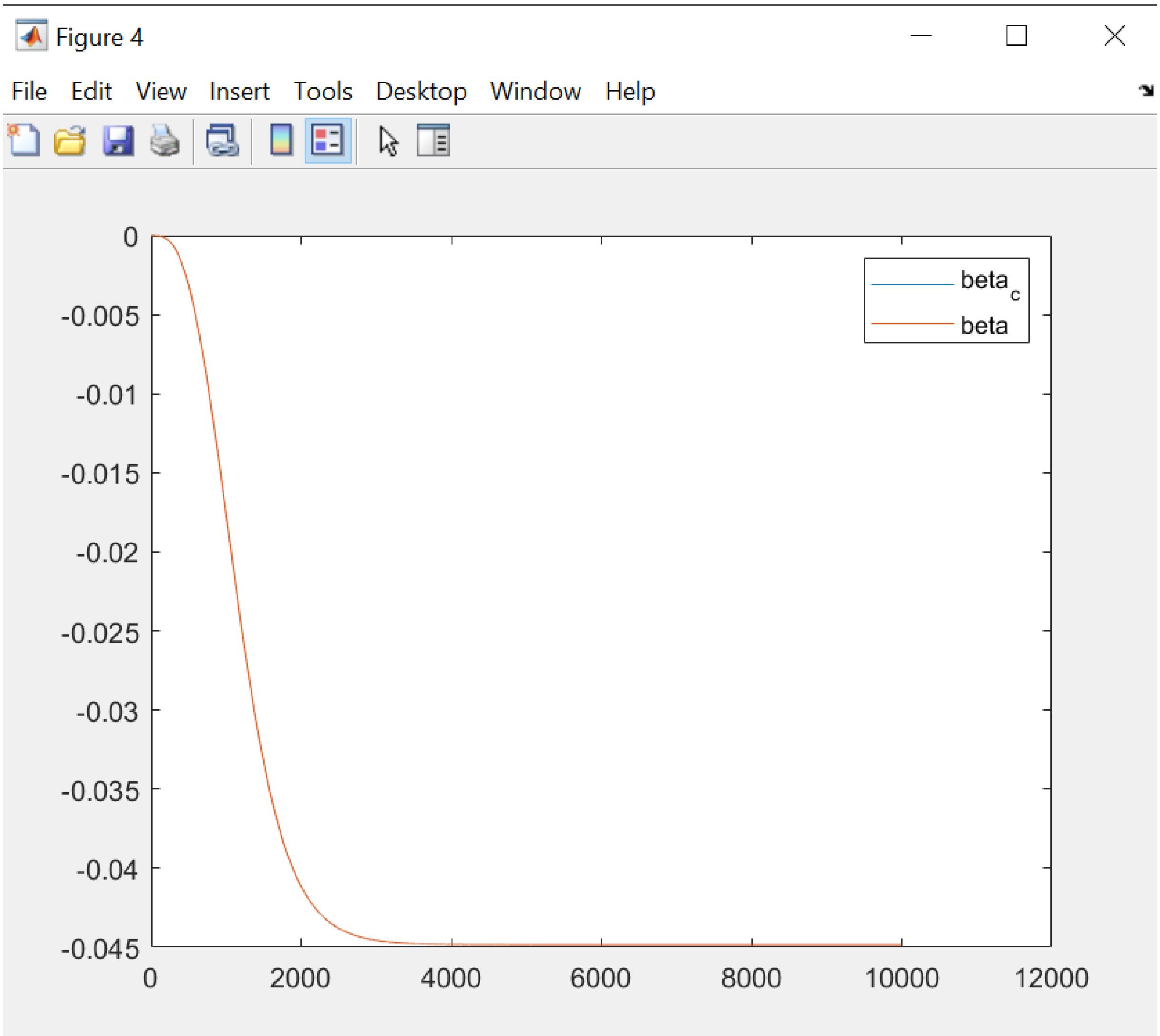
$\underline{K_p = 197{,}4}$ $\underline{K_d = 4103{,}16}$ $\underline{K_i = 1{,}838}$

seems big, but what I got. :) algebra :)





Angles, with current



Angles without current (they are the same)

TASK 2:

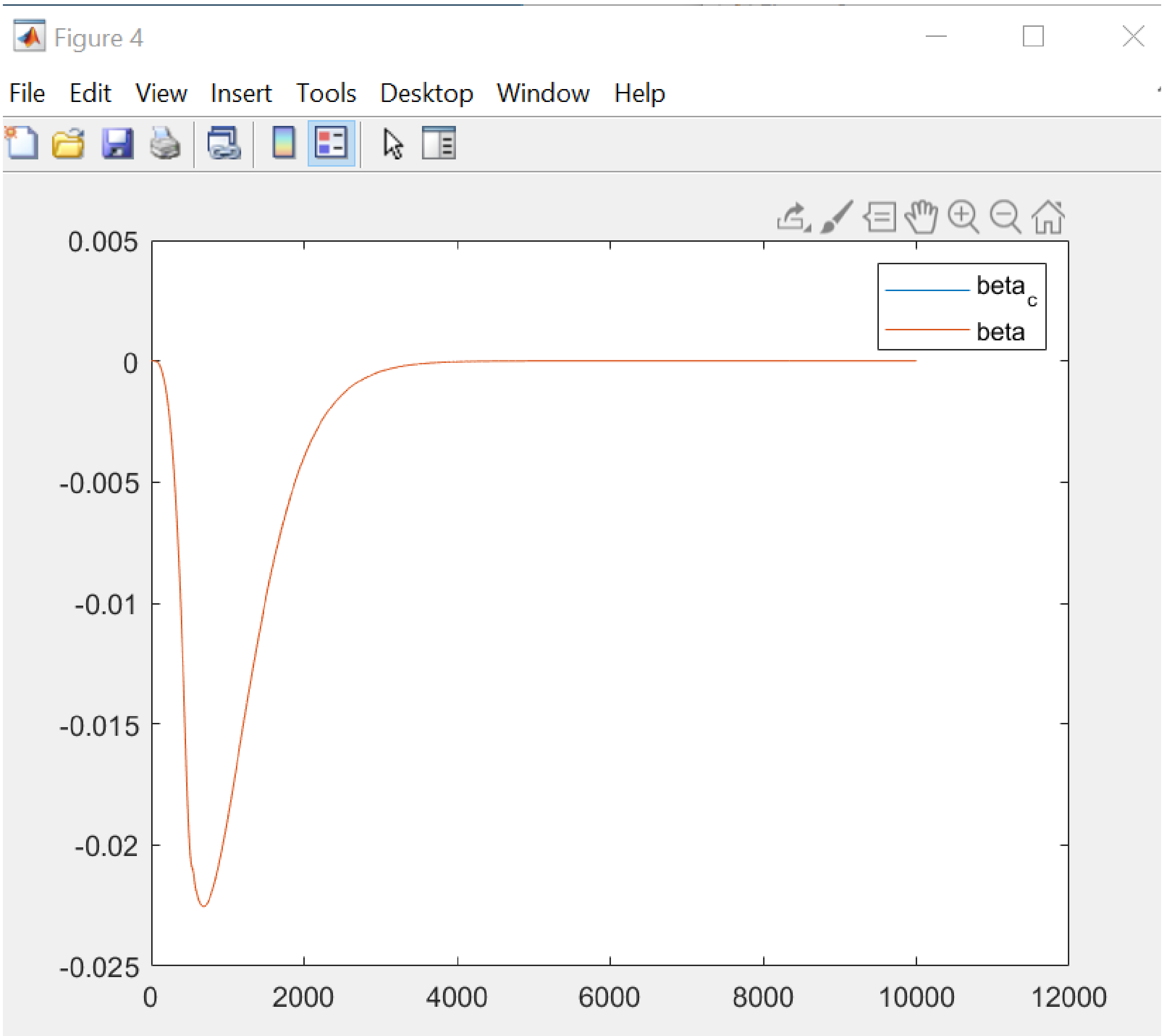


Figure 4
File Edit View Insert Tools Desktop Window Help
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
0
2000
4000
6000
8000
10000
12000
beta$_c$
beta

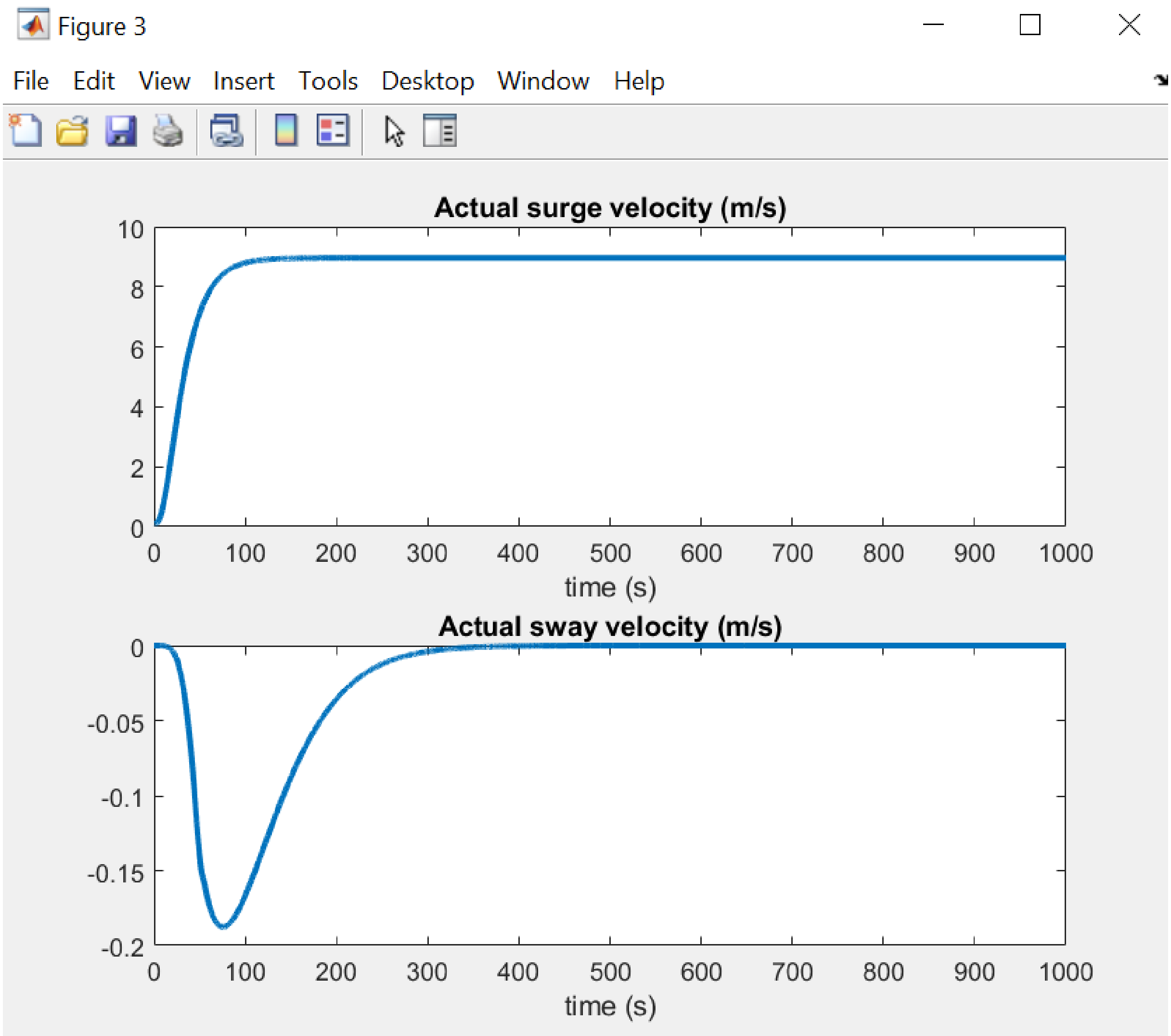


Figure 3
File Edit View Insert Tools Desktop Window Help
Actual surge velocity (m/s)
10
8
6
4
2
0
0 100 200 300 400 500 600 700 800 900 1000
time (s)
Actual sway velocity (m/s)
0
-0.05
-0.1
-0.15
-0.2
0 100 200 300 400 500 600 700 800 900 1000
time (s)

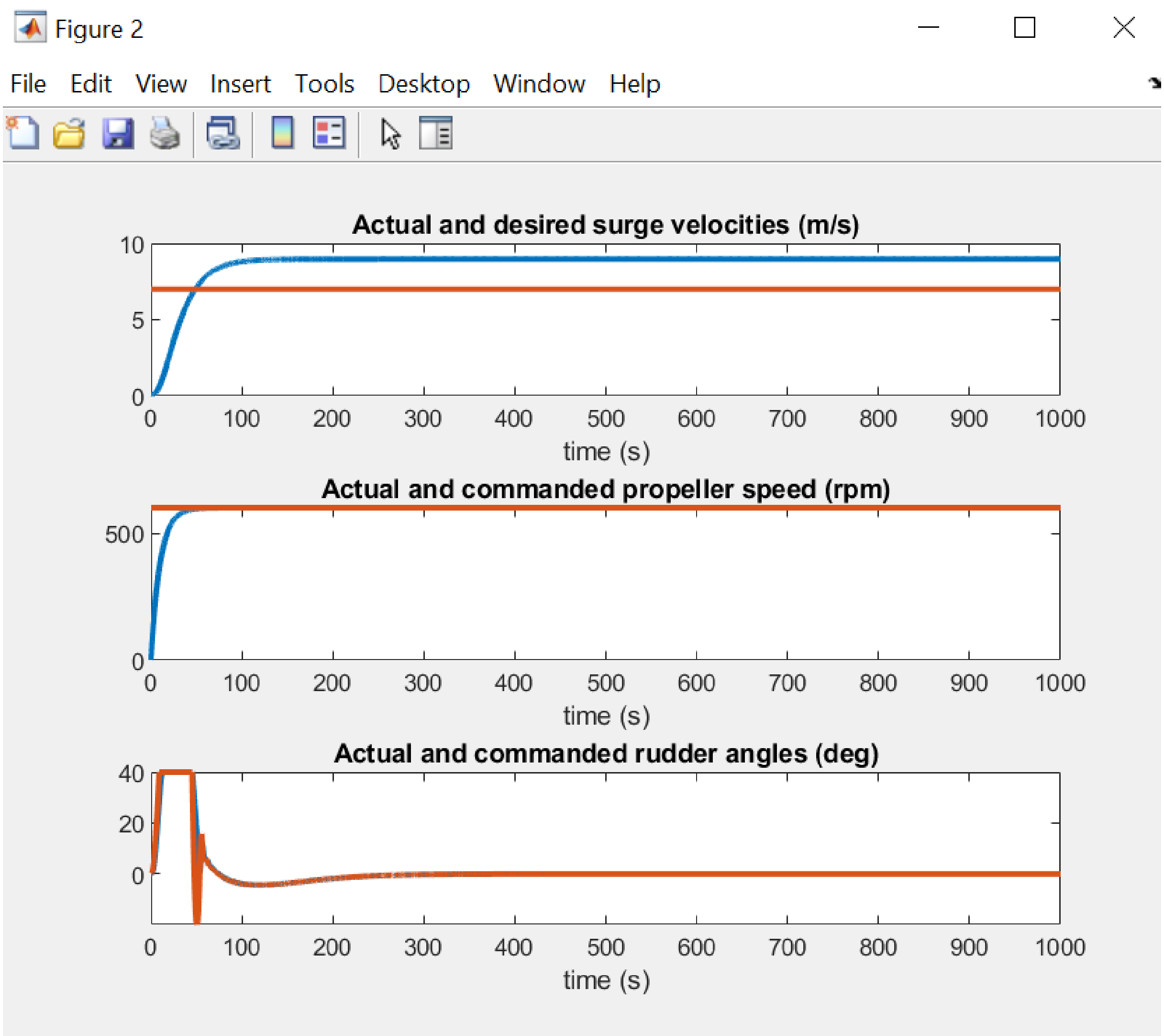
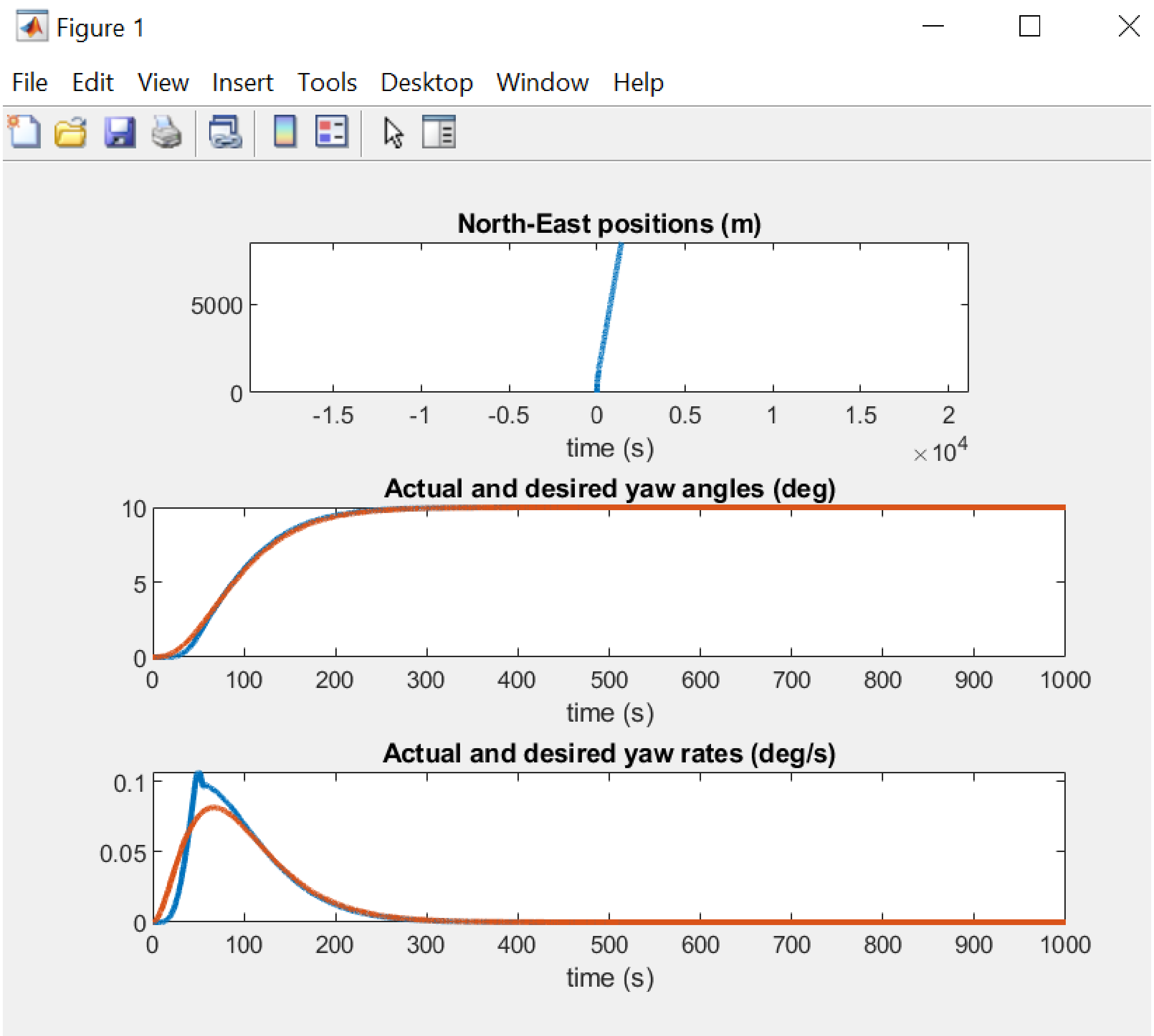


Figure 2
File Edit View Insert Tools Desktop Window Help
Actual and desired surge velocities (m/s)
10
5
0
0 100 200 300 400 500 600 700 800 900 1000
time (s)
Actual and commanded propeller speed (rpm)
500
0
0 100 200 300 400 500 600 700 800 900 1000
time (s)
Actual and commanded rudder angles (deg)
40
20
0
0 100 200 300 400 500 600 700 800 900 1000
time (s)



I did not see much problem with windup, but I added some code commented out just in case.