Chapter 4: Ceilings, Floors, and Quotas

Introductory Microeconomics

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Today

- 5.1 Government-Controlled Prices
- 5.2 Rent Controls: A Case Study of Price Ceilings
- 5.3 An Introduction to Market Efficiency

Learning goals

1. Describe how legislated price ceilings and price floors affect equilibrium price and quantity.
2. Compare the short-run and long-run effects of rent controls.
3. Describe the relationship between economic surplus and market efficiency.
4. Explain why price controls and output quotas tend to be inefficient.

Price ceilings (legal maximum price)

Definition: A price ceiling sets a legal upper bound on price.

Binding condition:

$$P_c < P^* \quad \Rightarrow \quad \text{binding (effective)}$$

Market effects when binding:

- Price cannot rise to P^*; transaction price at or below P_c.
- Quantity supplied falls and quantity demanded rises:

$$Q_s(P_c) < Q^* < Q_d(P_c)$$

- *Shortage* of size $Q_d(P_c) - Q_s(P_c)$.
- Non-price rationing emerges: queues, waitlists, first-come-first-served, discrimination, black markets.

Board sketch: Standard S-D with P_c below P^*; label shortage.

Price floors (legal minimum price)

Definition: A price floor sets a legal lower bound on price.

Binding condition:

$$P_f > P^* \quad \Rightarrow \quad \text{binding (effective)}$$

Market effects when binding:

- Price cannot fall to P^*; transaction price at or above P_f.
- Quantity demanded falls and quantity supplied rises:

$$Q_d(P_f) < Q^* < Q_s(P_f)$$

- *Surplus* of size $Q_s(P_f) - Q_d(P_f)$.
- Disposal/stockpiles or government purchase; potential waste/misalallocation.

Board sketch: Standard S-D with P_f above P^*; label surplus.

Binding vs. non-binding; incidence and elasticities

- **Non-binding:** If $P_c \geq P^*$ or $P_f \leq P^*$, no effect on P, Q.
- **Who is constrained?** For ceilings, sellers cannot raise price; for floors, buyers cannot negotiate down.
- **Elasticities matter:** The size of shortages/surpluses depends on the price responsiveness of S and D.
- **Quality margins:** When price cannot adjust, quality and non-price attributes adjust (maintenance, service, perks).

Rent control basics

Policy: A price ceiling on rent P_{rent}.

If binding:

$$P_{\text{rent}} < P^*_{\text{rent}} \quad \Rightarrow \quad Q_d > Q_s \text{ (housing shortage)}$$

Rationing: Waitlists, key money, discrimination, informal side payments. **Quality:** Landlords reduce maintenance/amenities when price is constrained.

Short run vs. long run under rent control

Short run (SR):

- Housing supply is relatively inelastic (fixed stock).
- Shortage exists but is limited by low supply responsiveness.
- Small reductions in new listings/maintenance.

Long run (LR):

- Supply becomes more elastic: construction discouraged, conversions to condos/other uses, exit from rental market.
- Demand more elastic: more households want controlled units at low rent.
- *Larger shortage*, increased misallocation (units not going to highest-valuation renters).
- Quality deterioration accumulates; under-maintenance and slower upgrading.

Distributional notes (who gains/loses?)

- **Winners:** Incumbent tenants who secure units at controlled rents.
- **Losers:** Prospective tenants rationed out; some landlords; tenants facing reduced quality/search costs.
- **Misallocation:** Units may be occupied by lower-valuation users while higher-valuation users are rationed out.

Efficiency preview: These features generate deadweight loss (foregone mutually beneficial trades).

Consumer, producer, and total surplus

Consumer Surplus (CS): Willingness to pay minus price, summed over buyers.
Producer Surplus (PS): Price minus marginal cost (or minimum acceptable price), summed over sellers.

Total Surplus (TS):

$$TS = CS + PS$$

Key result in competitive markets:

$$(P^*, Q^*) \text{ maximizes } TS \quad \text{(no externalities, no market power)}$$

Board sketch: CS and PS areas under D and above S up to Q^*.

Why price controls reduce efficiency

When a binding ceiling/floor is imposed:

- Quantity traded falls below the efficient level Q^*.
- Mutually beneficial trades between Q and Q^* are *not realized*.
- **Deadweight loss (DWL):** Lost TS due to underproduction or overproduction.

$$\text{DWL} = TS^* - TS^{\text{policy}}$$

Board sketch: Triangle between S and D over the range of foregone trades.

Output quotas and inefficiency

Quota (quantity control): Legal limit $Q_{\max}$ on output or transactions.

If binding:

$$Q_{\max} < Q^* \quad \Rightarrow \quad \text{underproduction and DWL}$$

Quota wedge and license value:

- Creates a price gap (wedge) between buyers and sellers:

$$\text{Quota rent} = P_{\text{buyers}} - P_{\text{sellers}}$$

- Value accrues to holders of production/transaction licenses.

Board sketch: Vertical quota line at $Q_{\max}$; wedge between D and S prices; DWL triangle.

Checklist: linking back to learning goals

1. **Ceilings/floors** shift the transaction price away from P^* and reduce traded quantity when binding.
2. **Rent control SR vs. LR:** Shortage grows over time as elasticities rise; quality and allocation issues intensify.
3. **Efficiency:** $TS = CS + PS$ maximized at Q^*; controls move the market away from efficiency.
4. **Inefficiency from controls/quotas:** They create DWL by preventing mutually beneficial trades.

Worked linear example

Choose:

$$Q_d = 120 - 2P, \qquad Q_s = -20 + 2P$$

Equilibrium:

$$120 - 2P^* = -20 + 2P^* \;\Rightarrow\; 140 = 4P^* \;\Rightarrow\; P^* = 35, \quad Q^* = 120 - 2 \cdot 35 = 50.$$

Binding price ceiling example

Let a ceiling $P_c = 30$ **(binding since** $30 < 35$**).**

$$Q_d(P_c) = 120 - 2(30) = 60, \qquad Q_s(P_c) = -20 + 2(30) = 40.$$

Shortage:

$$\text{Shortage} = Q_d(P_c) - Q_s(P_c) = 60 - 40 = 20.$$

Board sketch: Draw horizontal line at $P = 30$, mark $Q_d = 60$, $Q_s = 40$, label shortage 20.

Binding price floor example

Let a floor $P_f = 40$ **(binding since** $40 > 35$**).**

$$Q_d(P_f) = 120 - 2(40) = 40, \qquad Q_s(P_f) = -20 + 2(40) = 60.$$

Surplus:

$$\text{Surplus} = Q_s(P_f) - Q_d(P_f) = 60 - 40 = 20.$$

Board sketch: Draw horizontal line at $P = 40$, mark $Q_d = 40$, $Q_s = 60$, label surplus 20.

Quota example and DWL

Let a binding quota $Q_{\max} = 40$ (since $40 < 50$).

$$P_{\text{buyers}} = P_d(40) = \frac{120 - 40}{2} = 40, \qquad P_{\text{sellers}} = P_s(40) = \frac{40 + 20}{2} = 30.$$

Quota rent per unit:

$$\text{Quota rent} = P_{\text{buyers}} - P_{\text{sellers}} = 40 - 30 = 10.$$

Deadweight loss:

$$\text{DWL} = \tfrac{1}{2} \times (\text{wedge height } = 10) \times (\text{reduction in } Q = 50 - 40 = 10) = 50.$$

Board sketch: Vertical line at $Q = 40$; draw wedge from 30 to 40; shade the DWL triangle to $Q^* = 50$.

Worked example

Demand and supply:

$$Q_d = 120 - 2P, \qquad Q_s = -20 + 2P$$

Equilibrium:

$$120 - 2P^* = -20 + 2P^* \Rightarrow 140 = 4P^* \Rightarrow P^* = 35, \quad Q^* = 50$$

Useful numbers from the graph:

$$P_{\text{choke}} = 60 \quad (\text{where } Q_d = 0), \qquad P_s(0) = 10 \quad (\text{where } Q_s = 0).$$

CS and PS at equilibrium

Consumer surplus (triangle):

$$\text{height} = 60 - 35 = 25, \quad \text{base} = Q^* = 50$$

$$CS^* = \tfrac{1}{2} \times 50 \times 25 = 625$$

Producer surplus (triangle):

$$\text{height} = 35 - 10 = 25, \quad \text{base} = 50$$

$$PS^* = \tfrac{1}{2} \times 50 \times 25 = 625$$

$$TS^* = CS^* + PS^* = 1250$$

Board tip: At equilibrium the two triangles are the same size here.

Binding price ceiling $P_c = 30$

Quantity traded:

$$Q_{\text{trade}} = Q_s(30) = -20 + 2 \cdot 30 = 40$$

Consumer surplus (trapezoid above price):

$$\text{left gap} = 60 - 30 = 30, \quad \text{right gap} = (\text{demand at } Q = 40) - 30 = 40 - 30 = 10$$

$$CS(P_c) = \tfrac{1}{2} \times (30 + 10) \times 40 = \tfrac{1}{2} \times 40 \times 40 = 800$$

Producer surplus (trapezoid below price):

$$\text{left gap} = 30 - 10 = 20, \quad \text{right gap} = 30 - (\text{supply at } Q = 40) = 30 - 30 = 0$$

$$PS(P_c) = \tfrac{1}{2} \times (20 + 0) \times 40 = 400$$

$$TS(P_c) = 800 + 400 = 1200, \qquad \text{DWL} = 1250 - 1200 = 50$$

Board sketch: Shade a buyer trapezoid and a seller trapezoid up to $Q = 40$.

Binding price floor $P_f = 40$

Quantity traded:

$$Q_{\text{trade}} = Q_d(40) = 120 - 2 \cdot 40 = 40$$

Consumer surplus (trapezoid above price):

$$\text{left gap} = 60 - 40 = 20, \quad \text{right gap} = (\text{demand at } Q = 40) - 40 = 40 - 40 = 0$$

$$CS(P_f) = \tfrac{1}{2} \times (20 + 0) \times 40 = 400$$

Producer surplus (trapezoid below price):

$$\text{left gap} = 40 - 10 = 30, \quad \text{right gap} = 40 - (\text{supply at } Q = 40) = 40 - 30 = 10$$

$$PS(P_f) = \tfrac{1}{2} \times (30 + 10) \times 40 = \tfrac{1}{2} \times 40 \times 40 = 800$$

$$TS(P_f) = 400 + 800 = 1200, \qquad \text{DWL} = 1250 - 1200 = 50$$

Note: Same total surplus as the ceiling here because $Q_{\text{trade}} = 40$ in both cases.

Quota $Q_{\max} = 40$: CS, PS, and license value

Find prices at $Q = 40$ by solving the lines:

$$\text{Demand: } 120 - 2P = 40 \Rightarrow P_{\text{buyers}} = 40, \qquad \text{Supply: } -20 + 2P = 40 \Rightarrow P_{\text{sellers}} = 30$$

Consumer surplus (triangle at buyers' price):

$$\text{height} = 60 - 40 = 20, \quad \text{base} = 40 \Rightarrow CS = \tfrac{1}{2} \times 40 \times 20 = 400$$

Producer surplus (triangle at sellers' price):

$$\text{height} = 30 - 10 = 20, \quad \text{base} = 40 \Rightarrow PS = \tfrac{1}{2} \times 40 \times 20 = 400$$

Quota rent (rectangle):

$$\text{height} = 40 - 30 = 10, \quad \text{width} = 40 \Rightarrow \text{Rent} = 10 \times 40 = 400$$

$$TS(Q_{\max}) = 400 + 400 + 400 = 1200, \qquad \text{DWL} = 1250 - 1200 = 50$$

Board sketch: Vertical line at $Q = 40$; draw the 10-by-40 rent rectangle between the two prices.