Chapter 4: Elasticity

Introductory Microeconomics

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Learning goals for Week 2

By the end of this week, you should be able to:

- Define and compute price elasticity of demand and supply (point and arc).
- Relate price elasticity of demand to total expenditure/revenue.
- Analyze excise taxes: tax incidence and efficiency with different elasticities.
- Compute income and cross-price elasticities; classify goods and pairs.

Price elasticity of demand: definition

Idea: Elasticity is a unit-free measure of responsiveness.

Point elasticity (calculus):

$$\varepsilon_d \equiv \frac{\mathrm{d}Q}{\mathrm{d}P} \cdot \frac{P}{Q} \quad \text{(usually negative; use } |\varepsilon_d| \text{ when discussing magnitude)}$$

Arc (midpoint) elasticity between points A and B:

$$\varepsilon_d^{\text{arc}} \equiv \frac{\Delta Q}{\Delta P} \cdot \frac{\overline{P}}{\overline{Q}} \quad \text{where} \quad \overline{P} = \frac{P_A + P_B}{2}, \ \overline{Q} = \frac{Q_A + Q_B}{2}$$

Categories (by magnitude):

- Elastic: $|\varepsilon_d| > 1$ Inelastic: $|\varepsilon_d| < 1$ Unit elastic: $|\varepsilon_d| = 1$

PED: quick numerical example

Consider a move from A to B: $P_A = 10,\ Q_A = 100;\ P_B = 12,\ Q_B = 88.$

Arc elasticity:

$$\Delta Q = -12,\ \Delta P = 2,\ \overline{P} = 11,\ \overline{Q} = 94$$

$$\varepsilon_d^{\text{arc}} = \frac{-12}{2} \cdot \frac{11}{94} \approx -0.70$$

Interpretation: A 1% increase in price reduces quantity by about 0.70% along this segment (inelastic).

Determinants of demand elasticity

- Availability of close substitutes (more substitutes $\Rightarrow$ more elastic).
- Budget share (larger share $\Rightarrow$ more elastic).
- Time horizon (longer horizon $\Rightarrow$ more elastic).
- Definition of the market (narrower category $\Rightarrow$ more elastic).
- Necessities vs. luxuries (luxuries typically more elastic).

Total expenditure and price elasticity of demand

Total expenditure (revenue for sellers): $TE \equiv P \times Q$.

When price changes:

- If demand is **elastic** ($|\varepsilon_d| > 1$): $P \uparrow \Rightarrow Q \downarrow$ a lot $\Rightarrow TE \downarrow$.
- If demand is **inelastic** ($|\varepsilon_d| < 1$): $P \uparrow \Rightarrow Q \downarrow$ a little $\Rightarrow TE \uparrow$.
- If demand is **unit elastic** ($|\varepsilon_d| = 1$): TE unchanged for small changes.

Slope vs. elasticity: Slope depends on units, elasticity does not.

Price elasticity of supply: definition

Point elasticity (calculus):

$$\varepsilon_s \equiv \frac{\mathrm{d}Q_s}{\mathrm{d}P} \cdot \frac{P}{Q_s} \;\; \geq 0$$

Arc (midpoint) elasticity:

$$\varepsilon_s^{\text{arc}} \equiv \frac{\Delta Q_s}{\Delta P} \cdot \frac{\overline{P}}{\overline{Q_s}}$$

Determinants:

- Flexibility of inputs and capacity utilization.
- Time horizon (supply more elastic in the long run).
- Storability and ease of inventory adjustment.

PES: quick numerical example

Move from A to B: $P_A = 20,\ Q_{s,A} = 50;\ P_B = 22,\ Q_{s,B} = 65.$

$$\Delta Q_s = 15,\ \Delta P = 2,\ \overline{P} = 21,\ \overline{Q_s} = 57.5$$

$$\varepsilon_s^{\text{arc}} = \frac{15}{2} \cdot \frac{21}{57.5} \approx 2.74$$

Interpretation: Supply is elastic over this segment.

Excise tax basics

Specific tax t per unit creates a wedge between price buyers pay P_b and price sellers receive P_s:

$$P_b = P_s + t$$

Market clears where $Q_d(P_b) = Q_s(P_s)$ with the wedge t.

Tax incidence (who bears the burden):

- The side of the market that is **more inelastic** bears **more** of the tax.
- If demand perfectly inelastic: buyers bear all. If supply perfectly inelastic: sellers bear all.

Incidence: algebra with linear curves (back-of-the-envelope)

Suppose $Q_d = a - bP_b$, $Q_s = c + dP_s$, tax t.

Solve $a - bP_b = c + dP_s$ with $P_b = P_s + t$:

$$a - b(P_s + t) = c + dP_s \;\Rightarrow\; (b + d)P_s = a - c - bt$$

$$P_s^\star = \frac{a - c - bt}{b + d}, \quad P_b^\star = P_s^\star + t$$

Burden shares (per unit):

$$\Delta P_b = \frac{d}{b + d}\, t, \qquad \Delta P_s = \frac{b}{b + d}\, t \text{ (received price falls)}$$

Larger b (flatter demand $\Rightarrow$ more elastic) $\Rightarrow$ *smaller* buyer burden. Larger d (more elastic supply) $\Rightarrow$ *larger* buyer burden.

Deadweight loss (efficiency) and elasticity

- A tax reduces traded quantity below the efficient level $\Rightarrow$ deadweight loss (DWL).
- DWL is larger when either demand or supply is more elastic (quantity responds more).
- For small taxes, $DWL \propto \frac{1}{2}\, t^2 \times$ slope-adjusted responsiveness.

Graph intuition: wedge and triangle area *(see blackboard)*.

Tax example: quick numbers

Let $Q_d = 100 - 2P_b,\ Q_s = 10 + P_s,\ t = 9$.

$$(2+1)P_s = 100 - 10 - 2(9) = 72 \Rightarrow P_s^\star = 24,\ \ P_b^\star = 33$$

$$Q^\star = Q_s(24) = 10 + 24 = 34$$

Burden per unit: buyers pay $+9$ more than sellers receive, but relative to no-tax prices:

$$\Delta P_b = \frac{d}{b+d}t = \frac{1}{3}\cdot 9 = 3, \quad \Delta P_s = \frac{2}{3}\cdot 9 = 6 \text{ (received price falls by 6)}$$

Sellers bear more because supply is more inelastic than demand at these slopes.

Income elasticity of demand

Definition:

$$\varepsilon_I \equiv \frac{\mathrm{d}Q}{\mathrm{d}Y} \cdot \frac{Y}{Q} \quad \text{or} \quad \varepsilon_I^{\text{arc}} = \frac{\Delta Q}{\Delta Y} \cdot \frac{\overline{Y}}{\overline{Q}}$$

Classification:

- Normal good: $\varepsilon_I > 0$ (luxury if $\varepsilon_I > 1$; necessity if $0 < \varepsilon_I < 1$).
- Inferior good: $\varepsilon_I < 0$.

Example (arc): Y rises from 50 to 55, Q rises 100 to 108.

$$\Delta Q = 8,\ \Delta Y = 5,\ \overline{Y} = 52.5,\ \overline{Q} = 104$$

$$\varepsilon_I^{\text{arc}} = \frac{8}{5} \cdot \frac{52.5}{104} \approx 0.81 \text{ (normal necessity)}$$

Cross-price elasticity of demand

Definition for good x with respect to price of good y:

$$\varepsilon_{xy} \equiv \frac{\mathrm{d}Q_x}{\mathrm{d}P_y} \cdot \frac{P_y}{Q_x} \quad \text{or} \quad \varepsilon_{xy}^{\text{arc}} = \frac{\Delta Q_x}{\Delta P_y} \cdot \frac{\overline{P_y}}{\overline{Q_x}}$$

Classification:

- Substitutes: $\varepsilon_{xy} > 0$.
- Complements: $\varepsilon_{xy} < 0$.
- Unrelated: $\varepsilon_{xy} \approx 0$.

Example (arc): P_y increases $10 \to 11$, Q_x increases $200 \to 206$.

$$\Delta Q_x = 6,\ \Delta P_y = 1,\ \overline{P_y} = 10.5,\ \overline{Q_x} = 203$$

$$\varepsilon_{xy}^{\text{arc}} = \frac{6}{1} \cdot \frac{10.5}{203} \approx 0.31 \text{ (substitutes)}$$

Practice: compute and classify

1. Demand move: $(P, Q) : (8, 120) \to (10, 96)$. Compute $\varepsilon_d^{\text{arc}}$ and classify.
2. Supply move: $(P, Q_s) : (15, 60) \to (18, 81)$. Compute $\varepsilon_s^{\text{arc}}$ and classify.
3. Income rises $Y : 40 \to 44$, $Q : 50 \to 60$. Compute $\varepsilon_I^{\text{arc}}$; luxury or necessity?
4. $P_y : 20 \to 22$, $Q_x : 300 \to 282$. Compute $\varepsilon_{xy}^{\text{arc}}$; substitutes or complements?

Graphical intuition and DWL triangles: see blackboard.

Key takeaways

- Elasticities are unit-free responsiveness measures; use point or midpoint formulas as appropriate.
- With inelastic demand, price increases raise TE; with elastic demand, they reduce TE.
- Tax burden falls more on the less elastic side; DWL grows with elasticity and with t^2.
- Income elasticity classifies goods (normal/inferior; necessity/luxury). Cross-price elasticity classifies pairs (substitutes/complements).