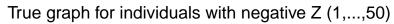
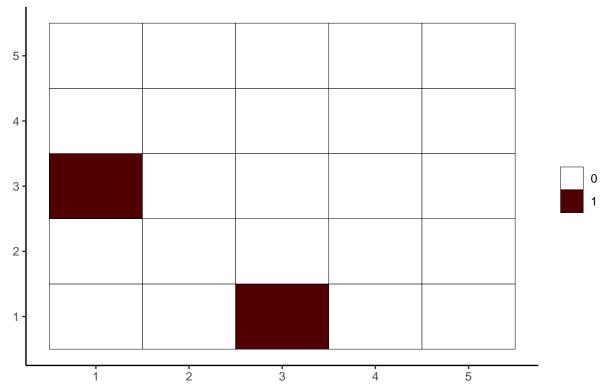
Implementation-details

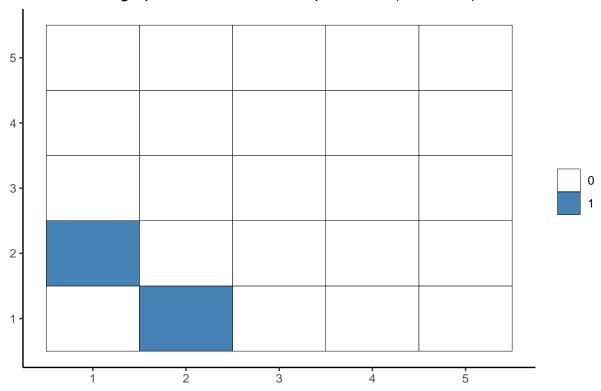
```
# install the package if necessary
if (!("covdepGE" %in% installed.packages())){
  devtools::install_github("JacobHelwig/covdepGE")
library(covdepGE)
?covdepGE
## starting httpd help server ... done
set.seed(1)
n <- 100
p < -4
# generate the extraneous covariate
Z_{neg} \leftarrow sort(runif(n / 2) * -1)
Z_pos <- sort(runif(n / 2))</pre>
Z <- c(Z_neg, Z_pos)</pre>
summary(Z)
##
       Min. 1st Qu.
                       Median
                                   Mean 3rd Qu.
                                                       Max.
## -0.99191 -0.55799 0.02277 -0.01475 0.45622 0.96062
\# create true covariance structure for 2 groups: positive Z and negative Z
true_graph_pos <- true_graph_neg <- matrix(0, p + 1, p + 1)</pre>
true_graph_pos[1, 2] <- true_graph_pos[2, 1] <- 1</pre>
true_graph_neg[1, 3] <- true_graph_neg[3, 1] <- 1</pre>
# visualize the true covariance structures
(gg_adjMat(true_graph_neg) +
    ggplot2::ggtitle("True graph for individuals with negative Z (1,...,50)"))
```



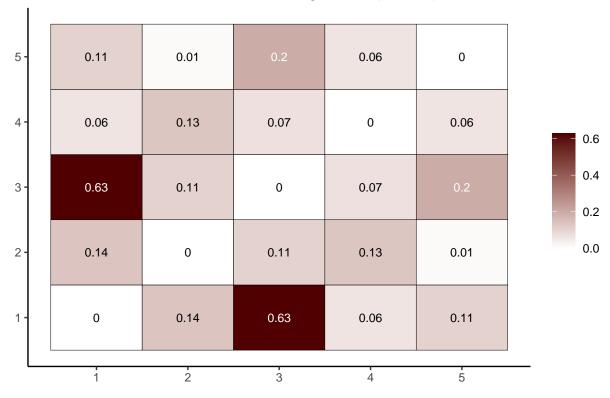


```
(gg_adjMat(true_graph_pos, color1 = "steelblue") +
    ggplot2::ggtitle("True graph for individuals with positive Z (51,...,100)"))
```

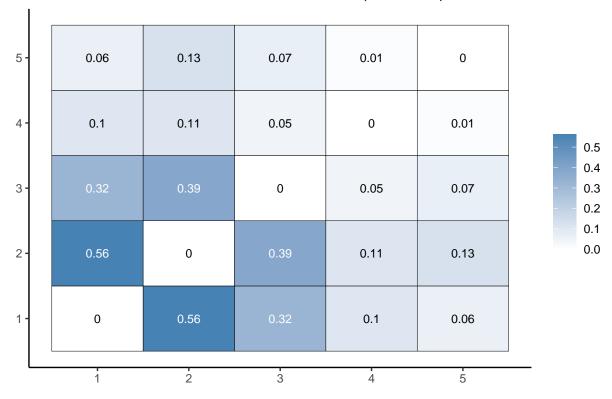
True graph for individuals with positive Z (51,...,100)



Correlation Matrix for Negative Z (1,...,50)



Correlation Matrix for Positive Z (51,...,100)



```
# use varbvs to get the hyperparameter sigma
sigmasq <- rep(NA, p + 1)
for (j in 1:(p + 1)){
   sigmasq[j] <- mean(varbvs::varbvs(data_mat[ , -j], Z, data_mat[ , j], verbose = F)$sigma)
}
sigmasq</pre>
```

[1] 0.7875369 1.2113484 0.8469619 0.7154704 1.0762645

mean(sigmasq)

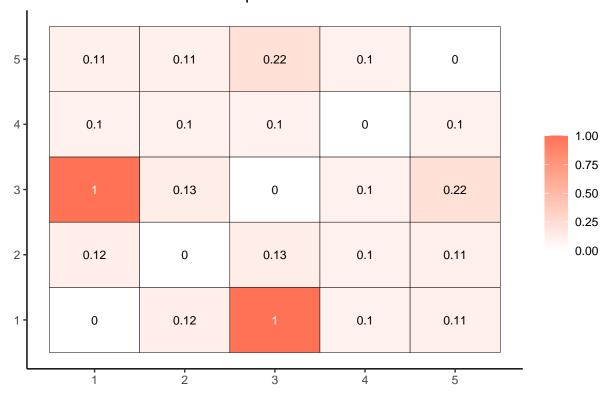
[1] 0.9275164

```
# estimate the covariance structure
out <- covdepGE(</pre>
                data mat,
                Z, # extraneous covariates
                kde = T, # whether KDE should be used to calculate bandwidths
                sigmasq = mean(sigmasq), # hyperparameter residual variance
                var_min = 1e-4, # smallest sigmabeta_sq grid value
                var_max = 1, # largest sigmabeta_sq grid value
                n_sigma = 10, # length of the sigmabeta_sq grid
                pi_vec = seq(0.1, 0.3, 0.05), # prior inclusion probability grid
                norm = Inf, # norm to calculate the weights with
                scale = T, # whether the extraneous covariates should be scaled
                tolerance = 1e-15, # variational parameter exit condition 1
                max_iter = 1e5, # variational parameter exit condition 2
                edge_threshold = 0.75, # minimum inclusion probability
                sym_method = "min", # how to symmetrize the alpha matrices
                print_time = T,
                warnings = T # whether warnings should be displayed
## Warning in covdepGE(data_mat, Z, kde = T, sigmasq = mean(sigmasq), var_min =
## 1e-04, : Response 1: 3/50 candidate models did not converge in 1e+05 iterations
## Warning in covdepGE(data_mat, Z, kde = T, sigmasq = mean(sigmasq), var_min =
## 1e-04, : Response 2: 17/50 candidate models did not converge in 1e+05 iterations
## Warning in covdepGE(data_mat, Z, kde = T, sigmasq = mean(sigmasq), var_min =
## 1e-04, : Response 2: final model did not converge in 1e+05 iterations
## Warning in covdepGE(data_mat, Z, kde = T, sigmasq = mean(sigmasq), var_min =
## 1e-04, : Response 3: 17/50 candidate models did not converge in 1e+05 iterations
## Warning in covdepGE(data_mat, Z, kde = T, sigmasq = mean(sigmasq), var_min =
## 1e-04, : Response 3: final model did not converge in 1e+05 iterations
## Warning in covdepGE(data_mat, Z, kde = T, sigmasq = mean(sigmasq), var_min =
## 1e-04, : For 1/5 responses, the selected value of sigmabeta sq was on the grid
## boundary. See return value ELBO for details
## Warning in covdepGE(data_mat, Z, kde = T, sigmasq = mean(sigmasq), var_min =
## 1e-04, : For 5/5 responses, the selected value of pi was on the grid boundary.
## See return value ELBO for details
## Time difference of 17.38476 mins
# grid search results
out$ELBO
## $'Response 1'
## $'Response 1'$'sigma^2_beta'
## [1] 0.129155
```

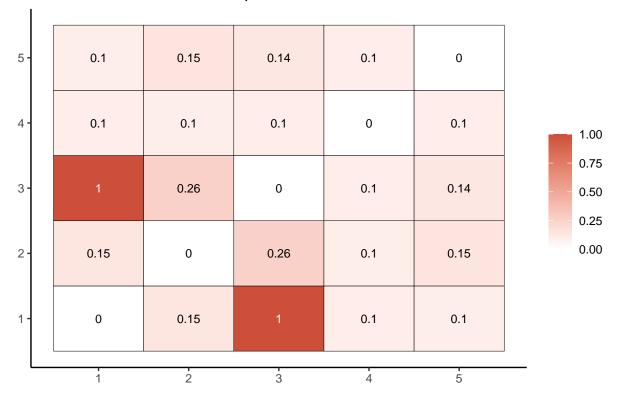
```
##
## $'Response 1'$pi
## [1] 0.3
##
## $'Response 1'$ELBO
## [1] -4206.716
##
##
## $'Response 2'
## $'Response 2'$'sigma^2_beta'
## [1] 0.129155
## $'Response 2'$pi
## [1] 0.3
##
## $'Response 2'$ELBO
## [1] -5978.314
##
##
## $'Response 3'
## $'Response 3'$'sigma^2_beta'
## [1] 0.129155
##
## $'Response 3'$pi
## [1] 0.3
## $'Response 3'$ELBO
## [1] -4871.425
##
##
## $'Response 4'
## $'Response 4'$'sigma^2_beta'
## [1] 1e-04
##
## $'Response 4'$pi
## [1] 0.1
##
## $'Response 4'$ELBO
## [1] -3968.535
##
##
## $'Response 5'
## $'Response 5'$'sigma^2_beta'
## [1] 0.002154435
## $'Response 5'$pi
## [1] 0.3
##
## $'Response 5'$ELBO
## [1] -6046.502
{\it\#\ individual-specific\ bandwidths\ calculated\ using\ KDE}
out$bandwidths
```

```
[1] 0.8971926 0.8185741 0.8045396 0.7710095 0.7598766 0.7309506 0.6970164
##
##
     [8] 0.6926737 0.6766866 0.6741037 0.6708585 0.6682201 0.6647626 0.6508583
    [15] 0.6483599 0.6467354 0.6412826 0.6402868 0.6376642 0.6368427 0.6360554
##
    [22] 0.6357334 0.6349457 0.6351369 0.6367247 0.6386644 0.6416763 0.6469182
    [29] 0.6476813 0.6498723 0.6508363 0.6656945 0.6719934 0.6725067 0.6729458
    [36] 0.6735493 0.6755900 0.6839099 0.7029164 0.7033434 0.7157298 0.7170107
##
    [43] 0.7178789 0.7208353 0.7225332 0.7291790 0.7304082 0.7305625 0.7270552
    [50] 0.7256076 0.7091386 0.7056539 0.7014279 0.6964823 0.6889947 0.6815783
##
    [57] 0.6619228 0.6512742 0.6499261 0.6497488 0.6466575 0.6392574 0.6356924
    [64] \ \ 0.6345264 \ \ 0.6337265 \ \ 0.6335805 \ \ 0.6330526 \ \ 0.6323850 \ \ 0.6305934 \ \ 0.6306508
##
   [71] 0.6307874 0.6308795 0.6321204 0.6323676 0.6338561 0.6342392 0.6362190
   [78] 0.6363787 0.6364964 0.6424749 0.6597773 0.6688075 0.6710351 0.6740264
##
   [85] 0.6889789 0.6894551 0.7067720 0.7109540 0.7162684 0.7170686 0.7269144
  [92] 0.7349568 0.7544093 0.7718605 0.7745784 0.7845610 0.7854562 0.8013423
##
   [99] 0.8245164 0.8909799
```

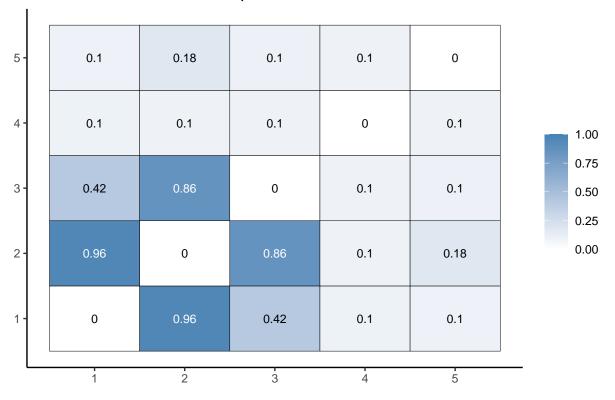
```
# analyze results
?gg_adjMat
gg_adjMat(out, 1, color1 = "coral1")
```



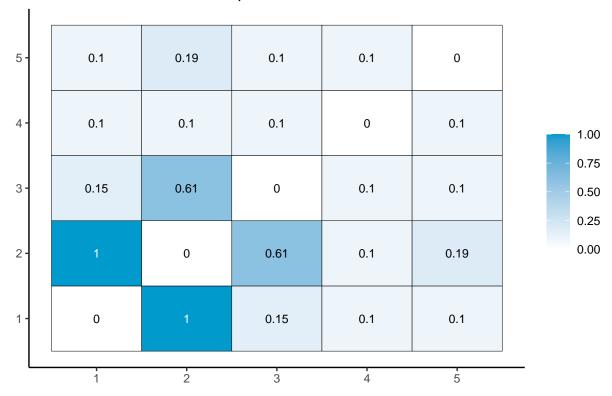
```
gg_adjMat(out, 50, color1 = "tomato3")
```



gg_adjMat(out, 60, color1 = "steelblue")

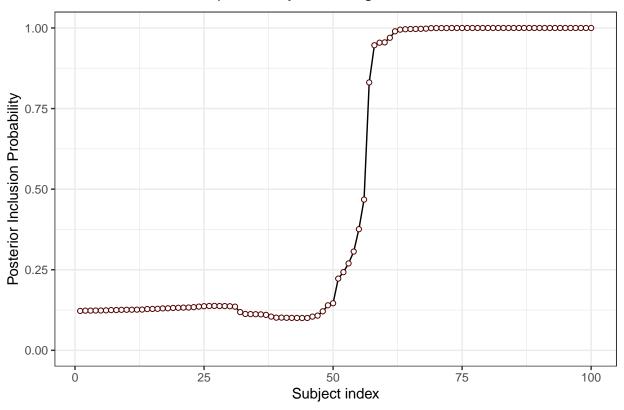


gg_adjMat(out, 100, color1 = "deepskyblue3")



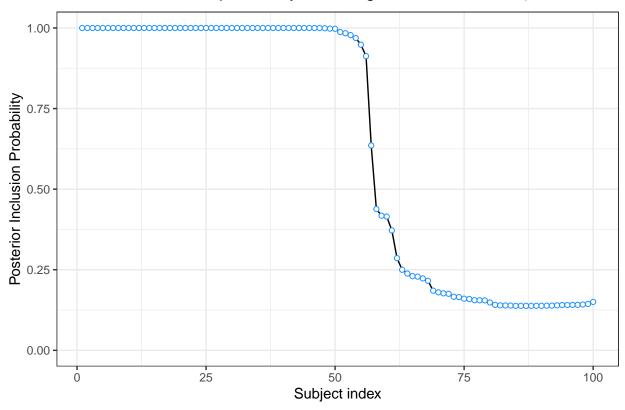
?gg_inclusionCurve
gg_inclusionCurve(out, 1, 2)

Inclusion probability of an edge between x_1 and x_2



gg_inclusionCurve(out, 1, 3, point_color = "dodgerblue")

Inclusion probability of an edge between x₁ and x₃



```
# find sensitivity, specificity, and accuracy
# true positives
TP_neg <- sum(sapply(out$graphs[1:(n / 2)],</pre>
                      function(graph) sum(graph == 1 & true_graph_neg == 1)))
TP_pos \leftarrow sum(sapply(out\$graphs[(n / 2 + 1):n],
                      function(graph) sum(graph == 1 & true_graph_pos == 1)))
TP <- TP_neg + TP_pos
# total positives
num_pos <- sum(true_graph_pos) * n / 2 + sum(true_graph_neg) * n / 2</pre>
# true negatives
TN_neg <- sum(sapply(out$graphs[1:(n / 2)],</pre>
                      function(graph) sum(graph == 0 & true_graph_neg == 0)))
TN_pos <- sum(sapply(out$graphs[(n / 2 + 1):n],</pre>
                      function(graph) sum(graph == 0 & true_graph_pos == 0)))
TN <- TN_neg + TN_pos
# total negatives
num_neg <- length(true_graph_pos) * n - num_pos</pre>
(sensitivity <- TP / num_pos)</pre>
```

[1] 0.94

```
(specificity <- TN / num_neg)
## [1] 0.9686957

(accuracy <- (TN + TP) / (num_pos + num_neg))</pre>
```

[1] 0.9664