sigma blowup analysis, large p

Contents

Experiment Overview

In this experiment, I analyze the behavior of the `covdepGE` algorithm when applied to a large dataset. Specifically, I have observed that in the large p regime, the MAPE-fitted σ^2 values blow up along with the μ values, and so, the purpose of this analysis was to more closely examine this blowup.

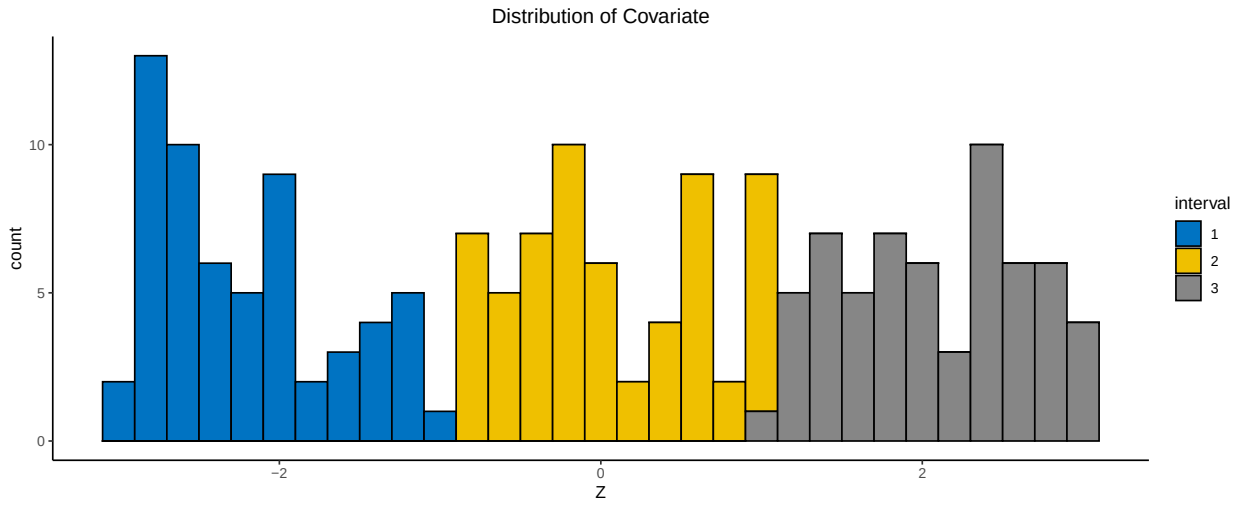
I begin by running a single instance of CAVI using data that will cause the fitted σ^2 values to blow up (*Close Look at Cavi*). In this analysis, the data is generated as described in the section titled *Data Generation*, however, I fix just one variable as the response and then take a closer look at the evolution of the μ and σ^2 parameters in the first few iterations, as well as the terms involved in the calculation of the σ^2 update.

I then perform 10 trials on data generated in the same manner (*Full Precision Trials*). Instead of focusing on one variable as before, I apply the whole algorithm to estimate the precision structure. In each trial, I will record the number of variables for which any of the MAPE fitted σ^2 values exceeded 10. I will also record the number of individuals whose σ^2 values exceeded this mark for each variable, and record the index of these individuals. Finally, I will include a visualization of the condition numbers of the weighted matrices corresponding to the variables for which blowup was observed.

Data Generation

Extraneous Covariate

I generated the covariate, Z, as the union of three almost disjoint intervals of equal measure. That is, $Z = Z_1 \cup Z_2 \cup Z_3$ with $Z_1 = (-3, -1), Z_2 = (a, b) = (-1, 1), Z_3 = (1, 3)$. Within each interval, I generated 60 covariate values from a uniform distribution. For example:



Precision Matrix

All of the individuals in interval 1 had the same precision matrix, $\Omega^{(1)}$:

$$\Omega^{(1)}_{i,j} = \begin{cases} 2 & i = j \\ 1 & (i,j) \in \{(1,2),(2,1),(2,3),(3,2)\} \\ 0 & o.w. \end{cases}$$

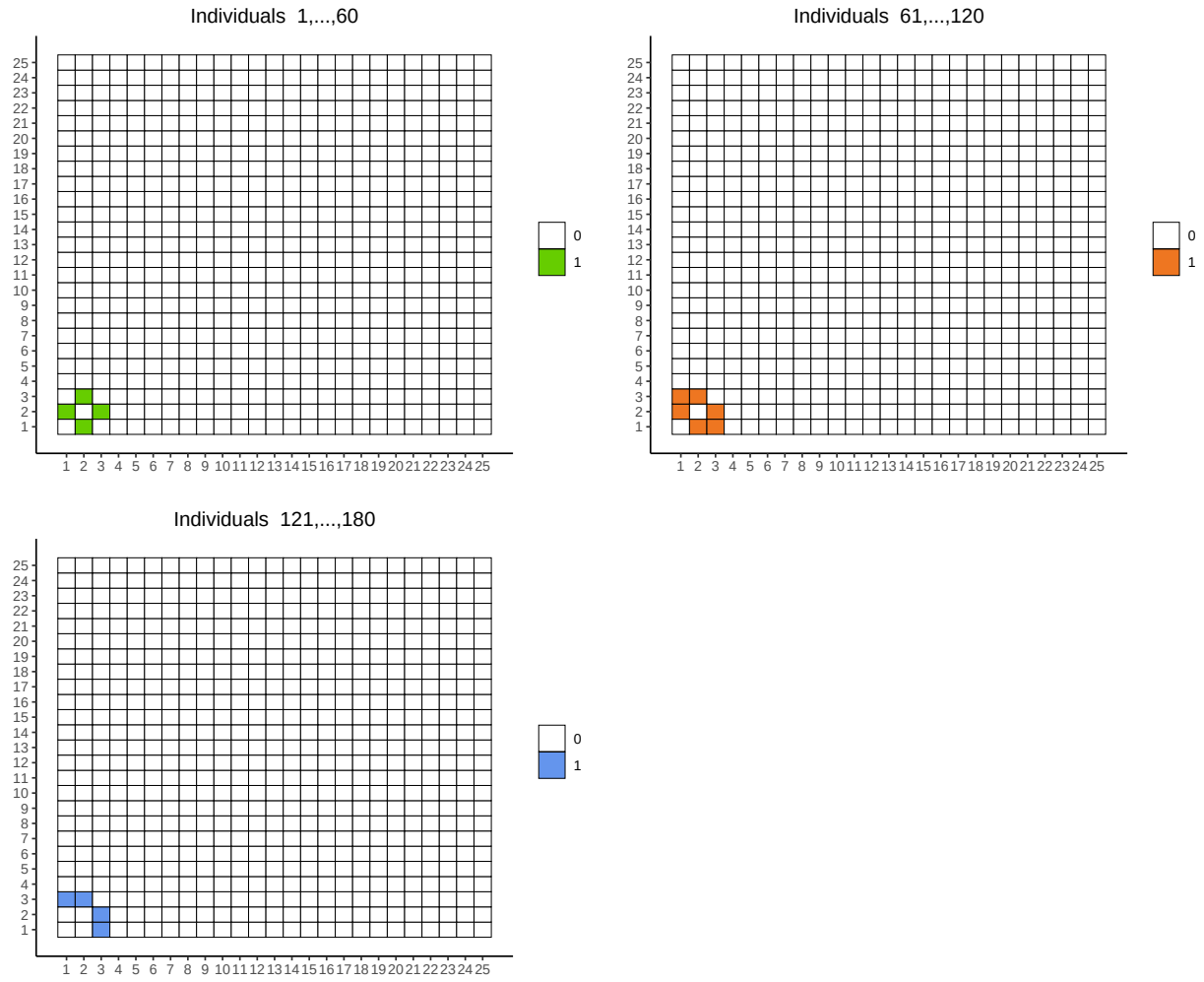
Also, all of the individuals in interval 3 had the same precision matrix, $\Omega^{(3)}$:

$$\Omega^{(3)}_{i,j} = \begin{cases} 2 & i = j \\ 1 & (i,j) \in \{(1,3),(3,1),(2,3),(3,2)\} \\ 0 & o.w. \end{cases}$$

However, the individuals in interval 2 had a precision matrix that was dependent upon Z and (a,b). Let $\beta_0 = -a/(b-a)$ and $\beta_1 = 1/(b-a)$. Then:

$$\Omega^{(2)}_{i,j}(z) = \begin{cases} 2 & i = j \\ 1 & (i,j) \in \{(2,3),(3,2)\} \\ 1-\beta_0-\beta_1 z & (i,j) \in \{(1,2),(2,1)\} \\ \beta_0+\beta_1 z & (i,j) \in \{(1,3),(3,1)\} \\ 0 & o.w. \end{cases}$$

Thus, $\Omega^{(2)}(a) = \Omega^{(1)}$ and $\Omega^{(2)}(b) = \Omega^{(3)}$. That is, an individual on the left or right boundary of Z_2 would have precision matrix $\Omega^{(1)}$ or $\Omega^{(3)}$, respectively. The conditional dependence structures corresponding to each of these precision matrices are visualized below.



Data matrix

Let z_l be the extraneous covariate for the l-th individual. To generate the data matrix for the l-th individual, I took a random sample from $\mathcal{N}(0, \{\Omega_l(z_l)\}^{-1})$, where:

$$\Omega_l(z_l) = \begin{cases} \Omega^{(1)} & z_l \in Z_1 \\ \Omega^{(2)}(z_l) & z_l \in Z_2 \\ \Omega^{(3)} & z_l \in Z_3 \end{cases}$$

Results

Close look at CAVI

```
## Warning in cavi0(y, D, X_mat, mu_mat, alpha_mat, sigmasq, update_sigma, : 908
```

The MAPE update for σ^2 with variable j fixed as the response and with respect to individual l is given by:

$$\sigma^2_{MAPE} = \frac{\|X_j^{w_l} - X_{-j}^{w_l}\rho_j^l\|^2 + \sum_{k=1}^{p-1}[X_{-j}^{w_l}]_k^\top [X_{-j}^{w_l}]_k \left[\alpha_{j,k}^l {s_{j,k}^l}^2 + \alpha_{j,k}^l {\mu_{j,k}^l}^2 - {\alpha_{j,k}^l}^2 {\mu_{j,k}^l}^2\right] + \frac{1}{\sigma_\beta^2}\sum_{k=1}^{p-1}\alpha_{j,k}^l({s_{j,k}^l}^2 + {\mu_{j,k}^l}^2)}{n + \sum_{k=1}^{p-1}\alpha_{j,k}^l}$$

Where $X_{-j}^{w_l}$ denotes the data matrix with the j-th column removed and weighted with respect to individual l, and $\rho_j^l = \mathbb{E}(\beta_j^l) = \alpha_j^l * \mu_j^l$.

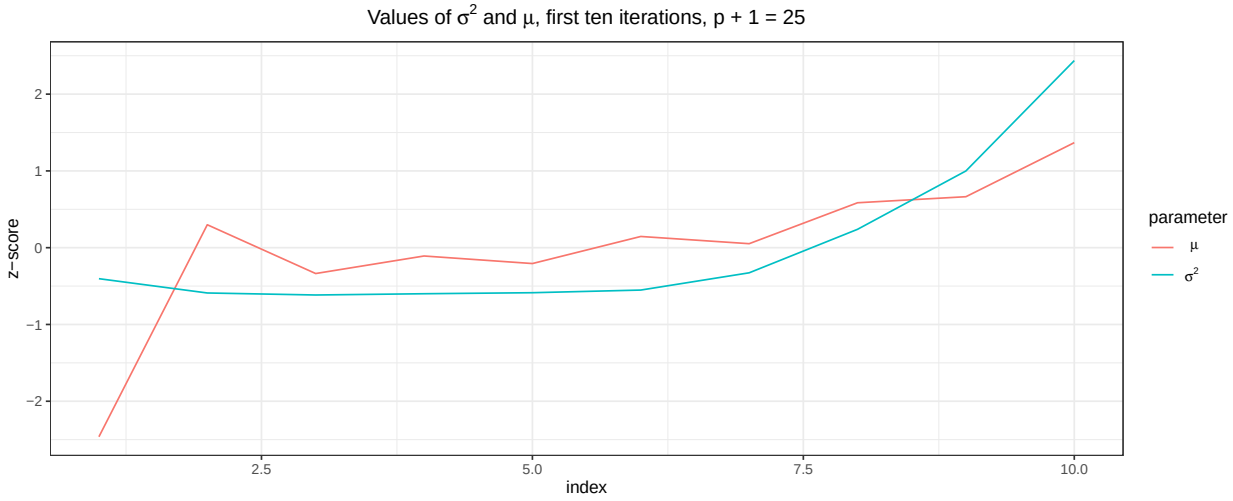
There are three terms in the numerator of the update, which will be referred to later on as:

$$\text{term1} = \|X_j^{w_l} - X_{-j}^{w_l}\rho_j^l\|^2$$

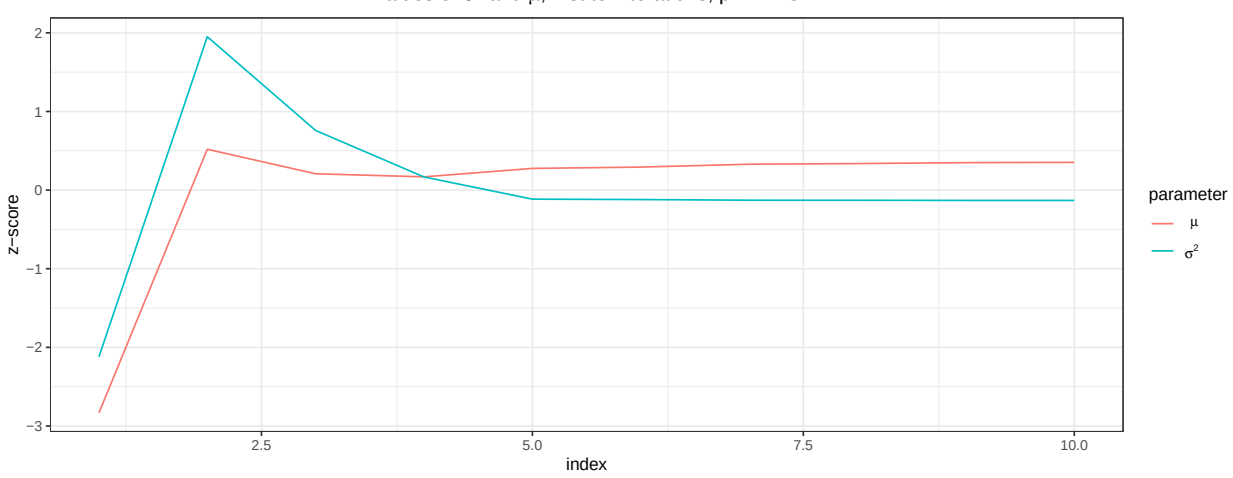
$$\text{term2} = \sum_{k=1}^{p-1}[X_{-j}^{w_l}]_k^\top [X_{-j}^{w_l}]_k \left[\alpha_{j,k}^l {s_{j,k}^l}^2 + \alpha_{j,k}^l {\mu_{j,k}^l}^2 - {\alpha_{j,k}^l}^2 {\mu_{j,k}^l}^2\right]$$

$$\text{term3} = \frac{1}{\sigma_\beta^2}\sum_{k=1}^{p-1}\alpha_{j,k}^l({s_{j,k}^l}^2 + {\mu_{j,k}^l}^2)$$

I first visualize the centered and scaled values of μ and σ^2 through the first ten iterations by averaging the 20 largest values across all individuals. It would appear as though σ^2 begins to blow up before μ does.

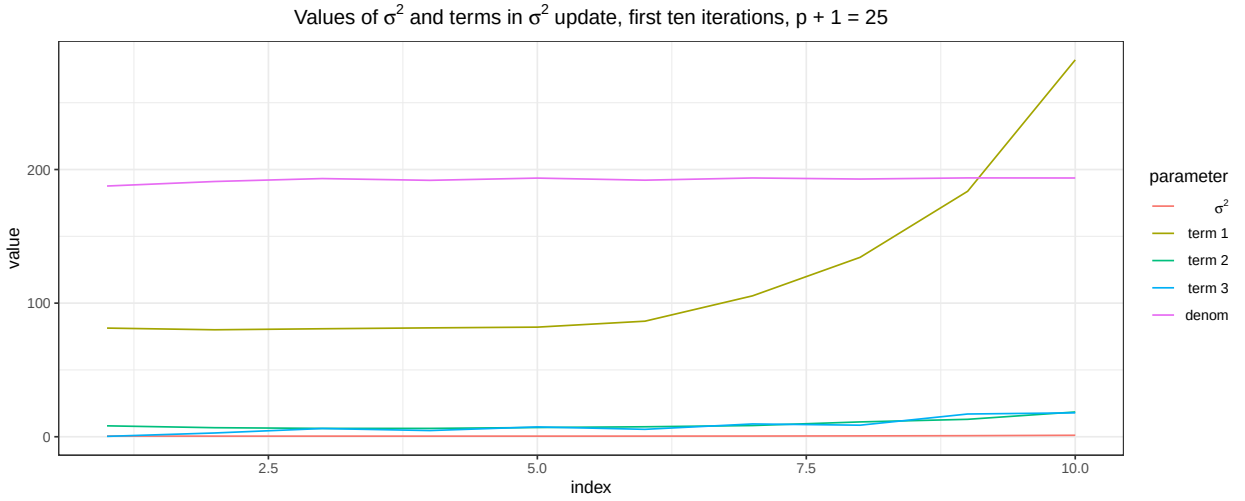


I next create the same visualization, but on a smaller data set that does not result in blow up.

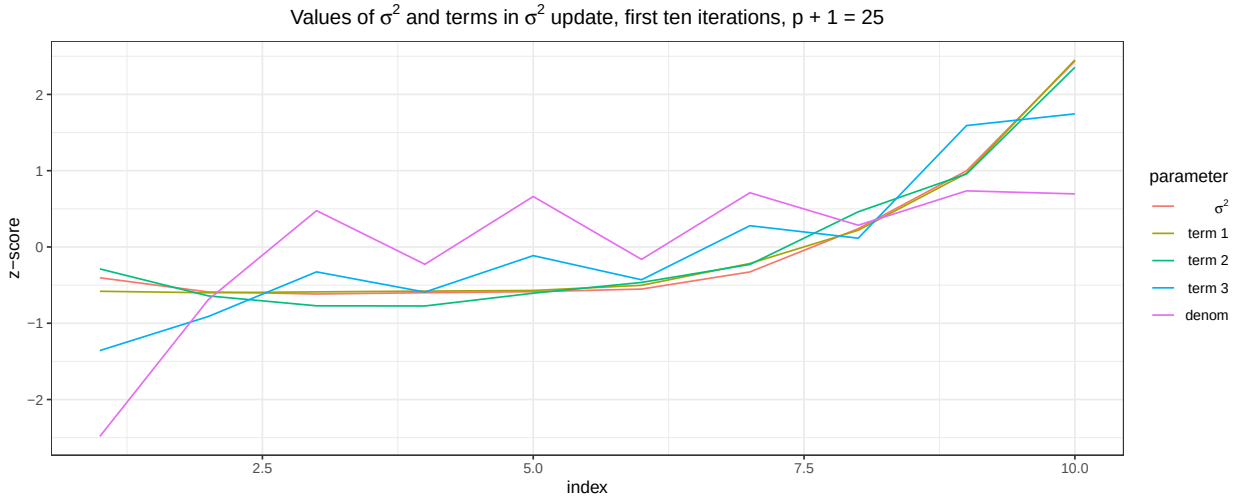


Finally, I examine the evolution of each term in the σ^2 update alongside the value of σ^2. I again reduce the values at each iteration to a scalar by averaging across the 20 largest values.

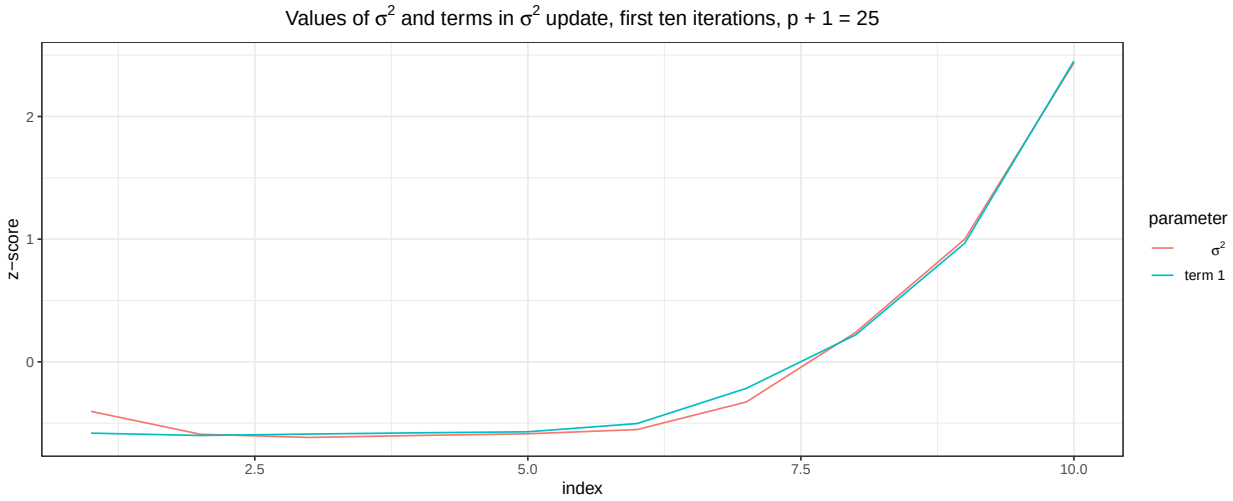
First, I visualize the raw values for each of these quantities through the first ten iterations. It would seem that term 1, the sum of squared weighted residuals, is driving the blowup.



However, after centering and scaling these values, it appears that they grow in conjunction, and so it is not clear whether the true cause of the blowup is the large residual values.



However, in isolating the centered and scaled value of term 1 and σ^2, it is clear that although term 1 may not be the cause of the blow up, it is indeed driving the rapid increase in σ^2.

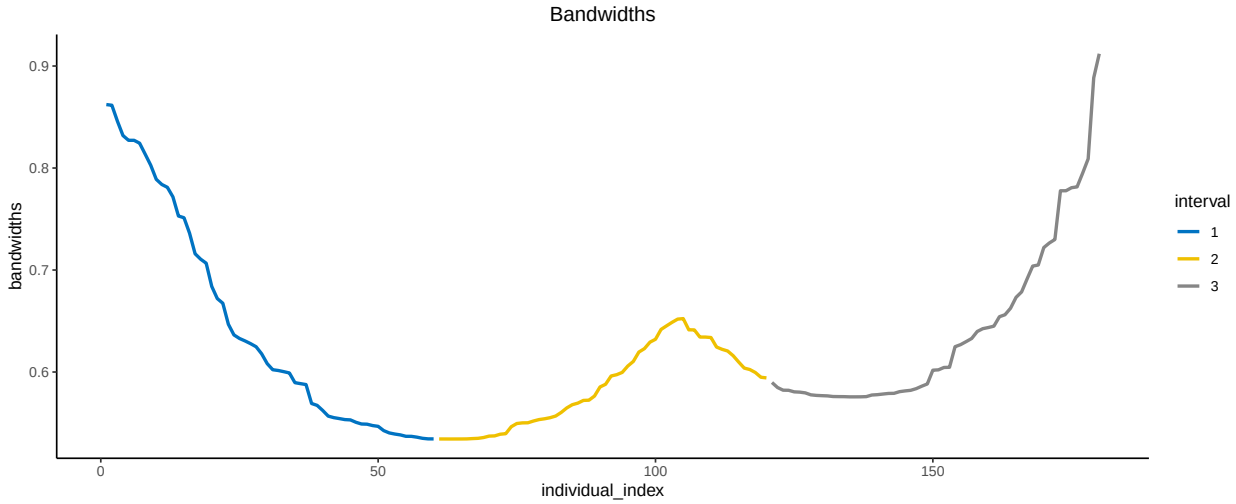


Full Precision Trials

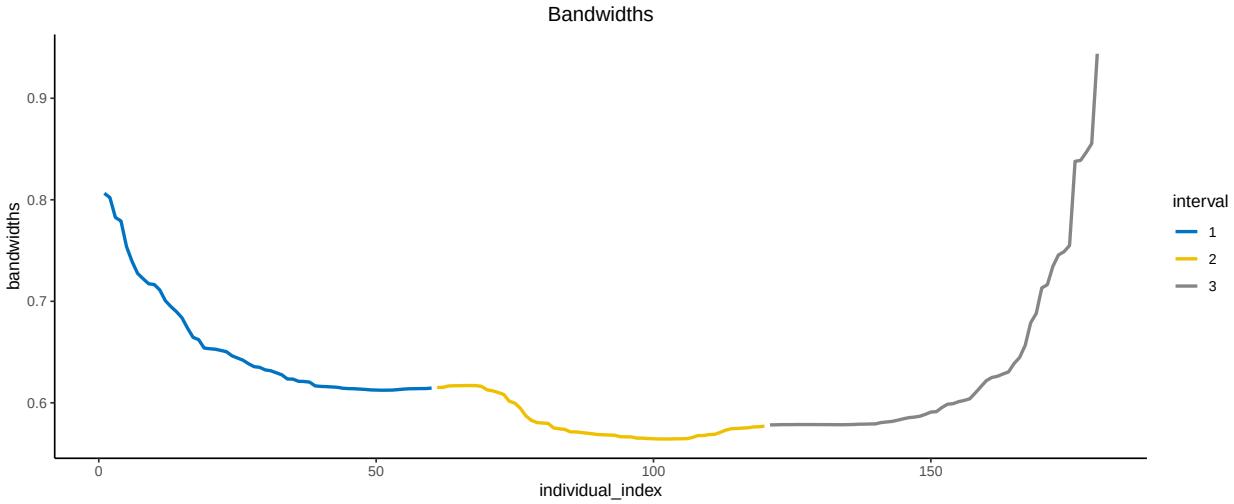
In this section, I run 10 trials on the $p + 1 = 25$ dataset and report the individuals whose fitted σ^2 values blew up by variable. For each of these trials, I visualize the condition numbers of the weighted data matrices for each individual for the variables that had blown up σ^2 values.

```
## Error in unclass(e1) - e2: non-numeric argument to binary operator
```

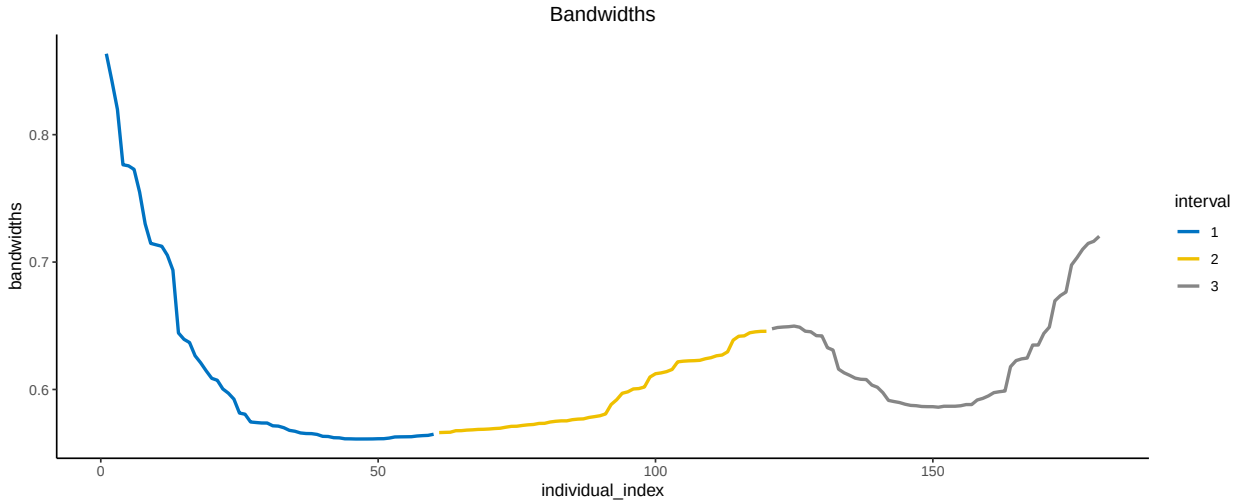
```
## $trial1
## $trial1$summary
##                       Covariate Dependent Graphical Model
##
## Model ELBO: -430143.32             Unique conditional dependence structures: 40
## n: 180, variables: 25                        Hyperparameter grid size: 9 points
## CAVI converged for 19/25 variables
##
## Model fit completed in 5.515 mins
##
## $trial1$sigma_blowup_by_individual
## integer(0)
##
## $trial1$condition_numbers
## named list()
##
## $trial1$bandwidths
```



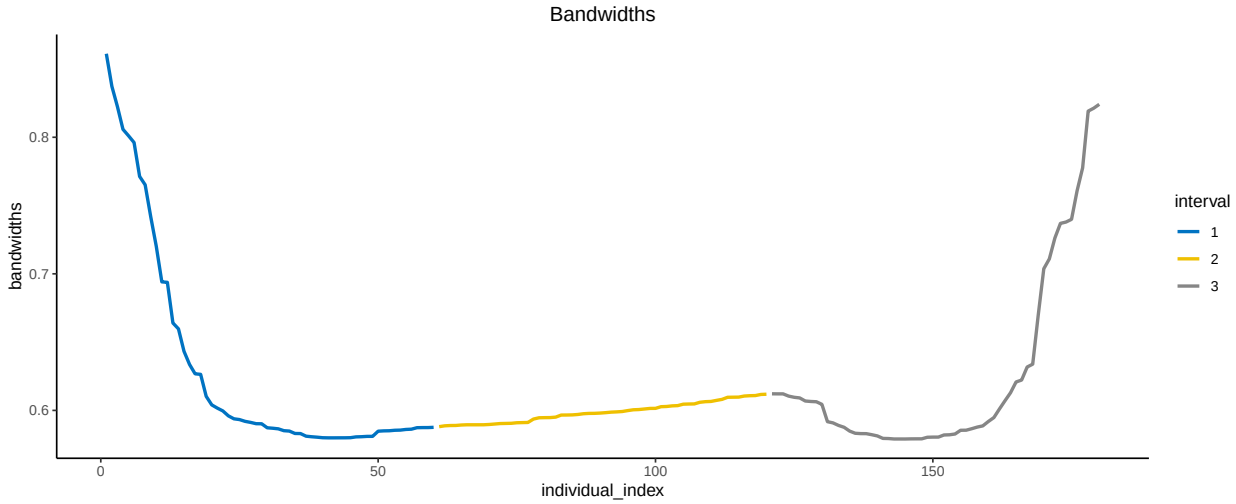

```
##
## $trial1$sigmasq
## NULL
##
##
## $trial2
## $trial2$summary
##                        Covariate Dependent Graphical Model
##
## Model ELBO: -426112.78             Unique conditional dependence structures: 47
## n: 180, variables: 25                       Hyperparameter grid size: 9 points
## CAVI converged for 20/25 variables
##
## Model fit completed in 4.462 mins
##
## $trial2$sigma_blowup_by_individual
## integer(0)
##
## $trial2$condition_numbers
## named list()
##
## $trial2$bandwidths
```




```
##
## $trial2$sigmasq
## NULL
##
##
## $trial3
## $trial3$summary
##                        Covariate Dependent Graphical Model
##
## Model ELBO: -428723.04             Unique conditional dependence structures: 44
## n: 180, variables: 25                       Hyperparameter grid size: 9 points
## CAVI converged for 13/25 variables
##
## Model fit completed in 6.963 mins
##
## $trial3$sigma_blowup_by_individual
## integer(0)
##
## $trial3$condition_numbers
## named list()
##
## $trial3$bandwidths
```

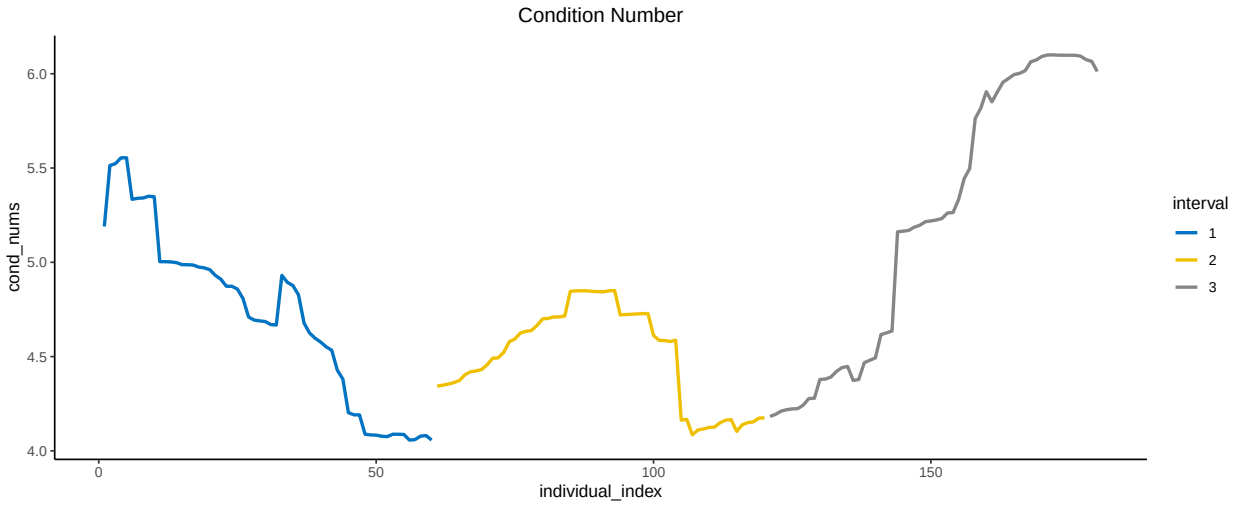



```
##
## $trial3$sigmasq
## NULL
##
##
## $trial4
## $trial4$summary
##                        Covariate Dependent Graphical Model
##
## Model ELBO: -424905.26             Unique conditional dependence structures: 41
## n: 180, variables: 25                        Hyperparameter grid size: 9 points
## CAVI converged for 18/25 variables
##
## Model fit completed in 5.792 mins
##
## $trial4$sigma_blowup_by_individual
## integer(0)
##
## $trial4$condition_numbers
## named list()
##
## $trial4$bandwidths
```

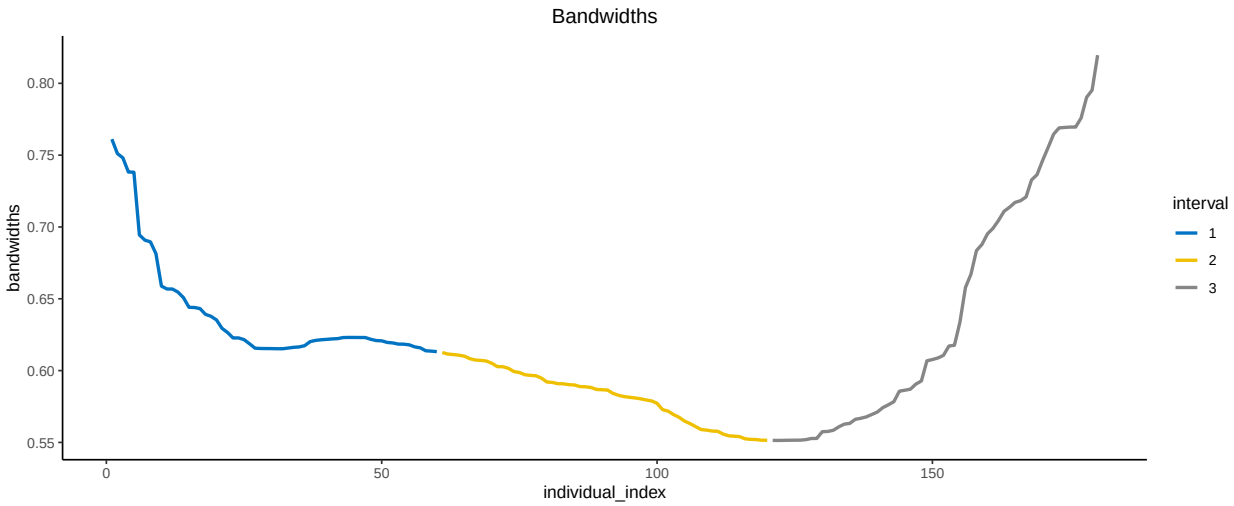



```
##
## $trial4$sigmasq
## NULL
##
##
## $trial5
## $trial5$summary
##                       Covariate Dependent Graphical Model
##
## Model ELBO: NaN                    Unique conditional dependence structures: 34
## n: 180, variables: 25                        Hyperparameter grid size: 9 points
## CAVI converged for 19/25 variables
##
## Model fit completed in 4.157 mins
##
## $trial5$sigma_blowup_by_individual
##       Variable 11
##  [1,]           1
##  [2,]           2
##  [3,]           3
##  [4,]           4
##  [5,]           5
##  [6,]           6
##  [7,]           7
##  [8,]           8
##  [9,]           9
## [10,]          10
## [11,]          11
## [12,]          12
## [13,]          13
## [14,]          14
## [15,]          15
## [16,]          16
## [17,]          17
## [18,]          18
## [19,]          19
## [20,]          20
```

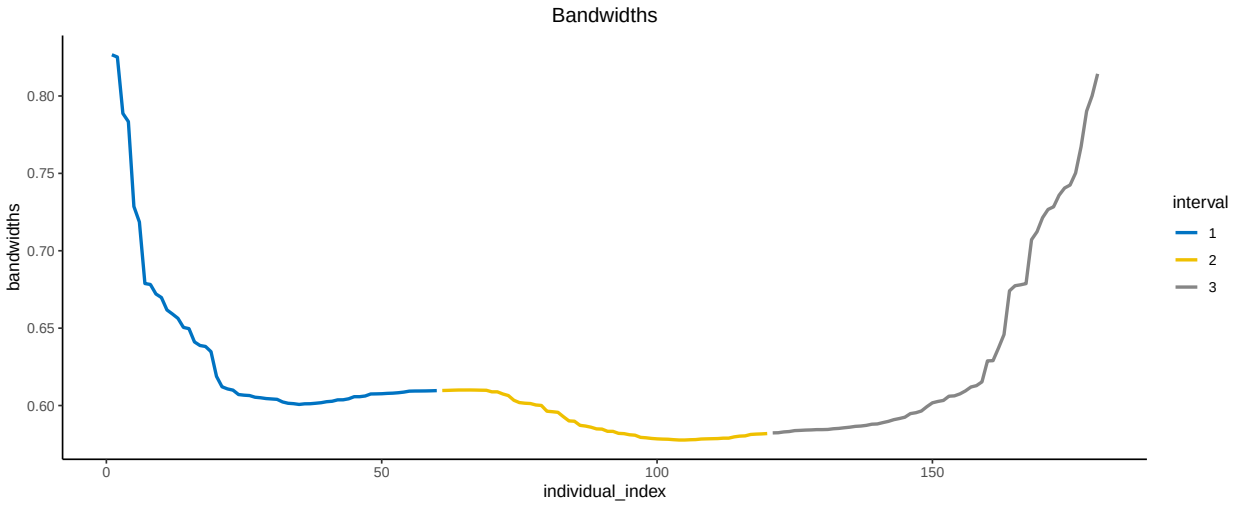
```
## [21,]          21
## [22,]          22
## [23,]          23
## [24,]          24
## [25,]          25
## [26,]          26
## [27,]          27
## [28,]          28
## [29,]          29
## [30,]          30
## [31,]          31
## [32,]          32
##
## $trial5$condition_numbers
## $trial5$condition_numbers$`Variable 11`
```




```
##
##
## $trial5$bandwidths
```

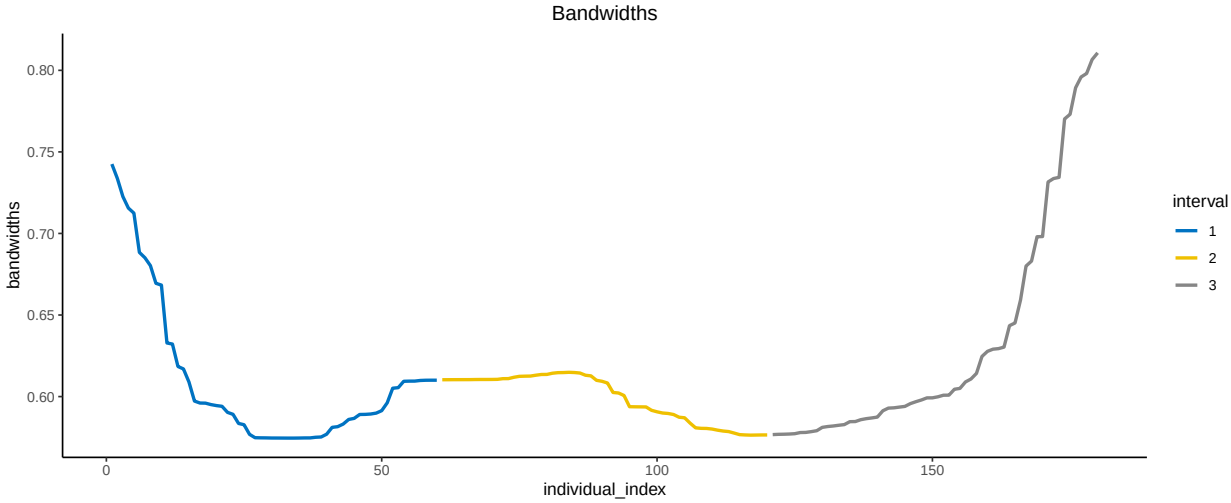


```
##
## $trial5$sigmasq
## NULL
##
##
## $trial6
## $trial6$summary
##                        Covariate Dependent Graphical Model
##
## Model ELBO: -430170.93             Unique conditional dependence structures: 52
## n: 180, variables: 25                       Hyperparameter grid size: 9 points
## CAVI converged for 19/25 variables
##
## Model fit completed in 5.052 mins
##
## $trial6$sigma_blowup_by_individual
## integer(0)
##
## $trial6$condition_numbers
## named list()
##
## $trial6$bandwidths
```

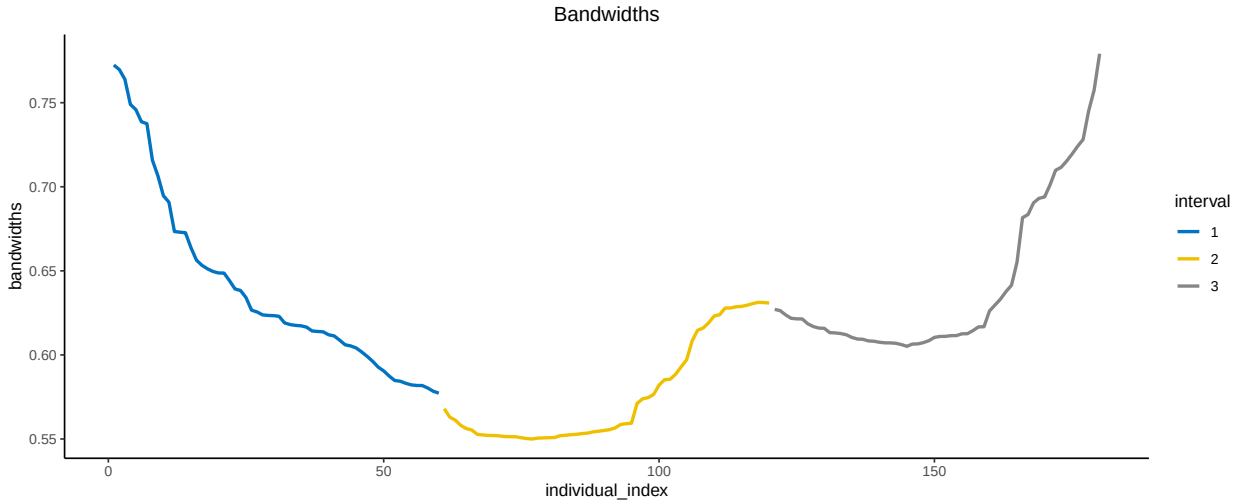



```
##
## $trial6$sigmasq
## NULL
##
##
## $trial7
## $trial7$summary
##                        Covariate Dependent Graphical Model
##
## Model ELBO: -433465.57             Unique conditional dependence structures: 48
## n: 180, variables: 25                       Hyperparameter grid size: 9 points
## CAVI converged for 21/25 variables
##
## Model fit completed in 4.82 mins
```

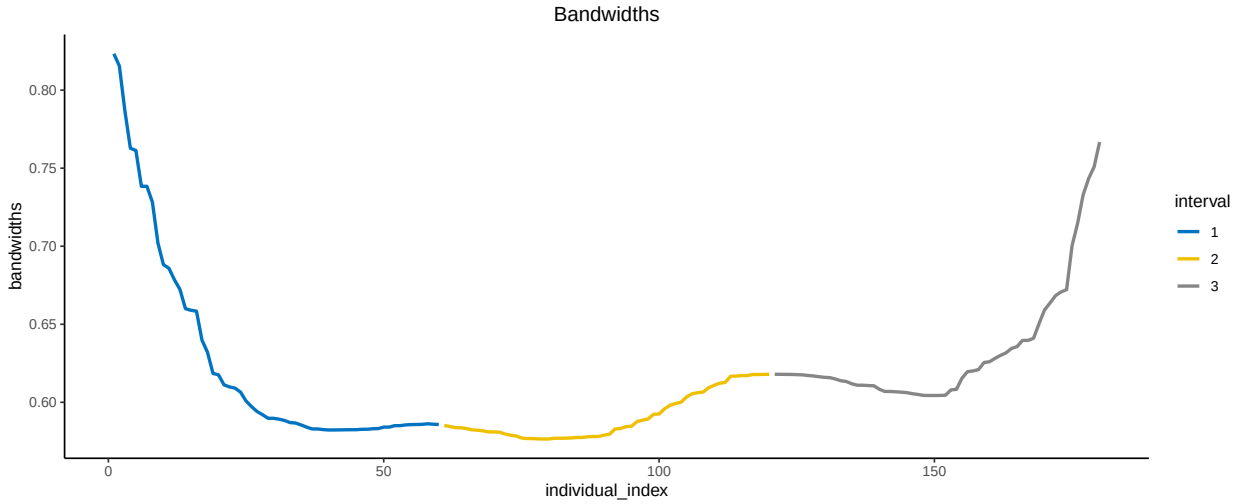
```
##
## $trial7$sigma_blowup_by_individual
## integer(0)
##
## $trial7$condition_numbers
## named list()
##
## $trial7$bandwidths
```



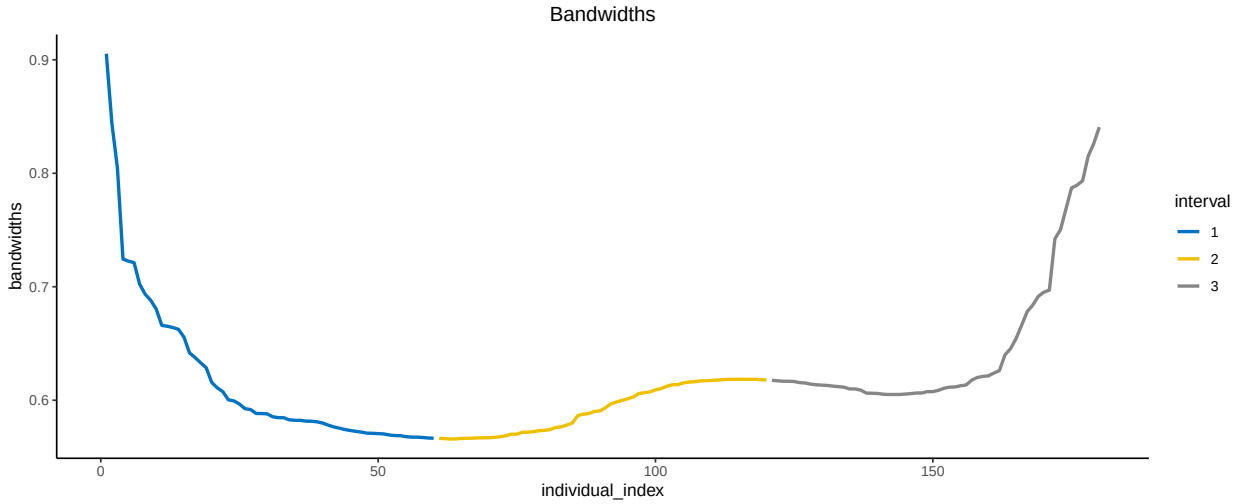

```
##
## $trial7$sigmasq
## NULL
##
##
## $trial8
## $trial8$summary
##                       Covariate Dependent Graphical Model
##
## Model ELBO: -423852.41             Unique conditional dependence structures: 34
## n: 180, variables: 25                        Hyperparameter grid size: 9 points
## CAVI converged for 18/25 variables
##
## Model fit completed in 4.753 mins
##
## $trial8$sigma_blowup_by_individual
## integer(0)
##
## $trial8$condition_numbers
## named list()
##
## $trial8$bandwidths
```




```
##
## $trial8$sigmasq
## NULL
##
##
## $trial9
## $trial9$summary
##                       Covariate Dependent Graphical Model
##
## Model ELBO: -432616.68            Unique conditional dependence structures: 53
## n: 180, variables: 25                      Hyperparameter grid size: 9 points
## CAVI converged for 19/25 variables
##
## Model fit completed in 5.129 mins
##
## $trial9$sigma_blowup_by_individual
## integer(0)
##
## $trial9$condition_numbers
## named list()
##
## $trial9$bandwidths
```




```
##
## $trial9$sigmasq
## NULL
##
##
## $trial10
## $trial10$summary
##                        Covariate Dependent Graphical Model
##
## Model ELBO: -428311.87             Unique conditional dependence structures: 62
## n: 180, variables: 25                        Hyperparameter grid size: 9 points
## CAVI converged for 20/25 variables
##
## Model fit completed in 4.268 mins
##
## $trial10$sigma_blowup_by_individual
## integer(0)
##
## $trial10$condition_numbers
## named list()
##
## $trial10$bandwidths
```




```
##
## $trial10$sigmasq
## NULL
```