

Large p small n

Run 1 and Run 2

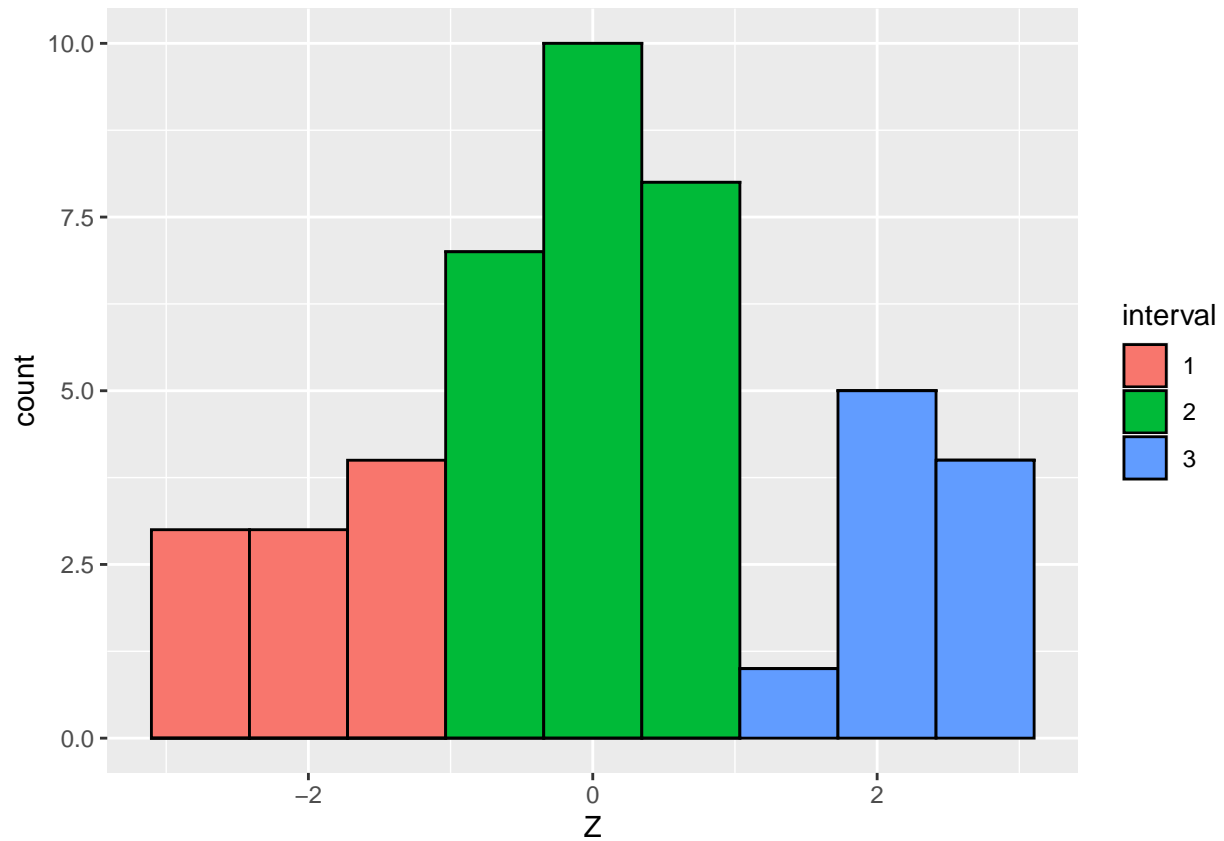
```
library(covdepGE)
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 4.1.3
```

```
# get the data
set.seed(1)
data <- generateData(50, 10, 25, 10)
X <- data$data
Z <- data$covts
interval <- data$interval
prec <- data$true_precision

# get overall and within interval sample sizes
n <- nrow(X)
n1 <- sum(interval == 1)
n2 <- sum(interval == 2)

# visualize the distribution of the extraneous covariate
ggplot(data.frame(Z = Z, interval = as.factor(interval))) +
  geom_histogram(aes(Z, fill = interval), color = "black", bins = n %/% 5)
```



```
# visualize the true precision matrices in each of the intervals
```

```
# interval 1
```

```
cat("\n\nInterval 1: Observations", paste0(range(which(interval == 1)), collapse = ",...,"))
```

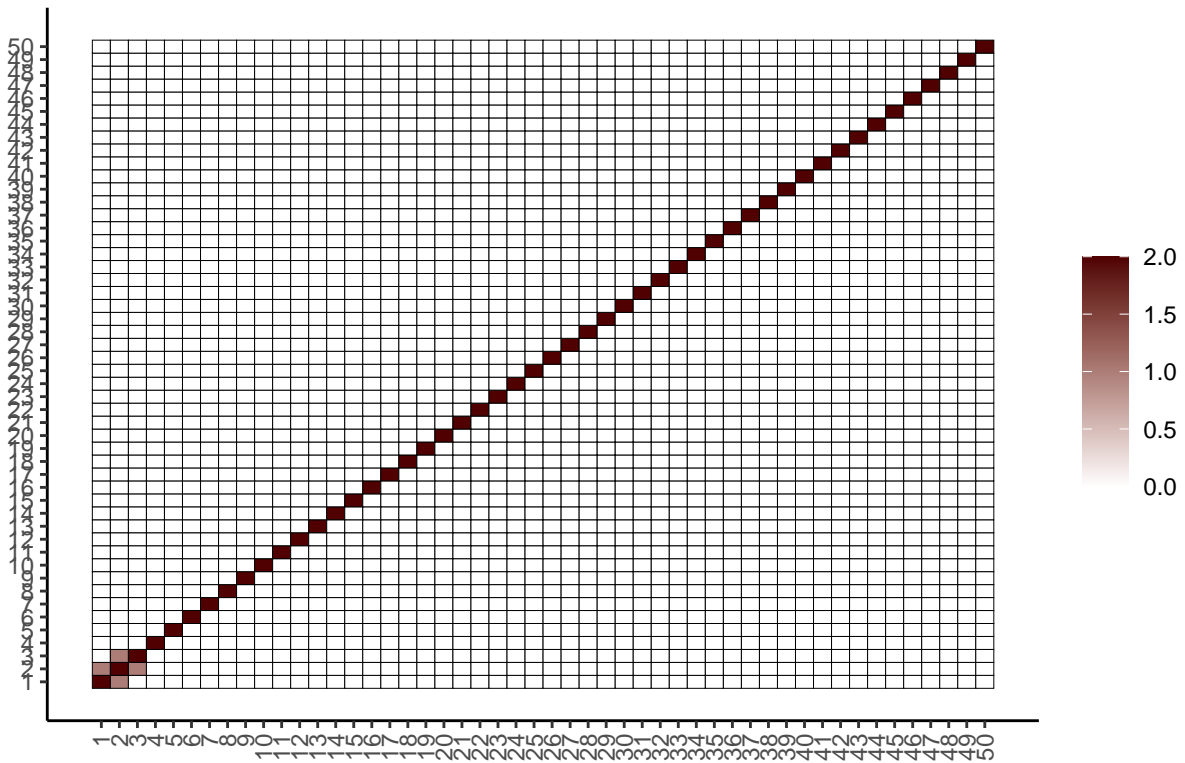
```
##
```

```
##
```

```
## Interval 1: Observations 1,...,10
```

```
matViz(prec[[1]]) + ggtitle("True precision matrix, interval 1") +  
  theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1))
```

True precision matrix, interval 1



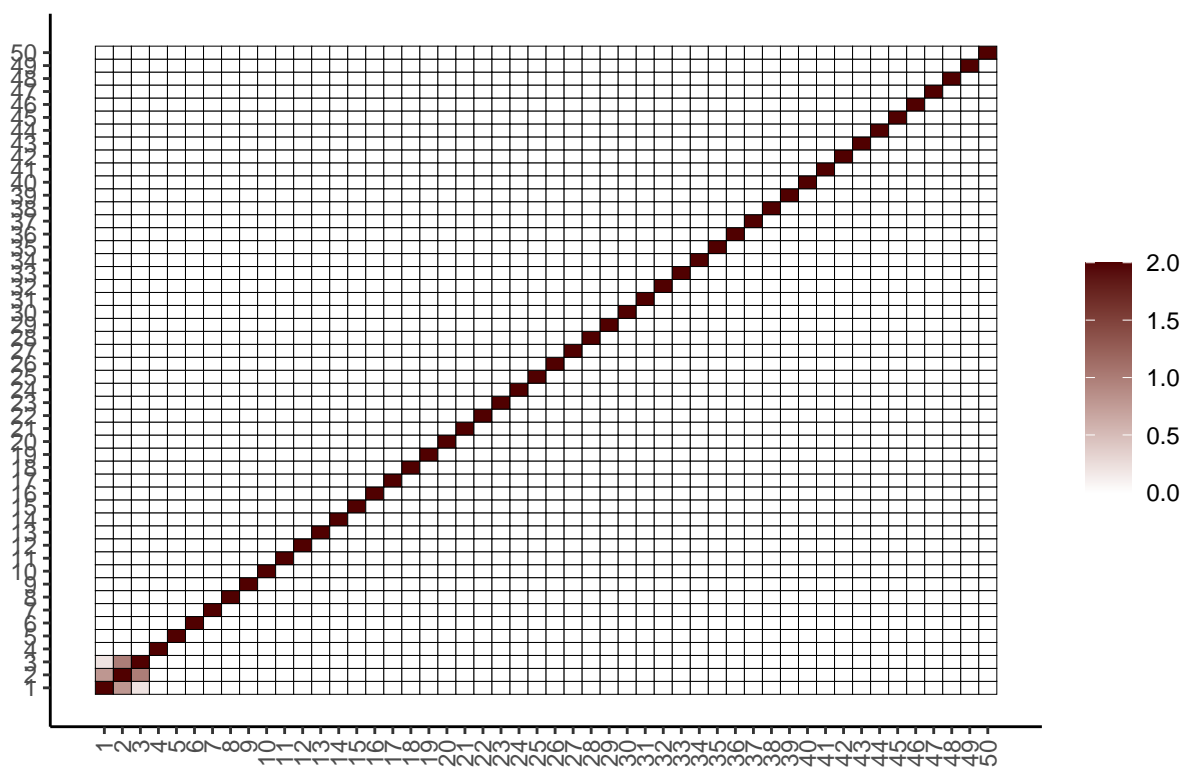
```
# interval 2 (varies continuously with Z)
cat("\n\nInterval 2: Observations", paste0(range(which(interval == 2)), collapse = ",...,"))
```

```
##
##
## Interval 2: Observations 11,...,35
```

```
int2_mats <- prec[interval == 2]
int2_inds <- c(5, n2 %/% 2, n2 - 5)
lapply(int2_inds, function(j) matViz(int2_mats[[j]]) +
  ggtitle(paste("True precision matrix, interval 2, observation", j)) +
  theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1)))
```

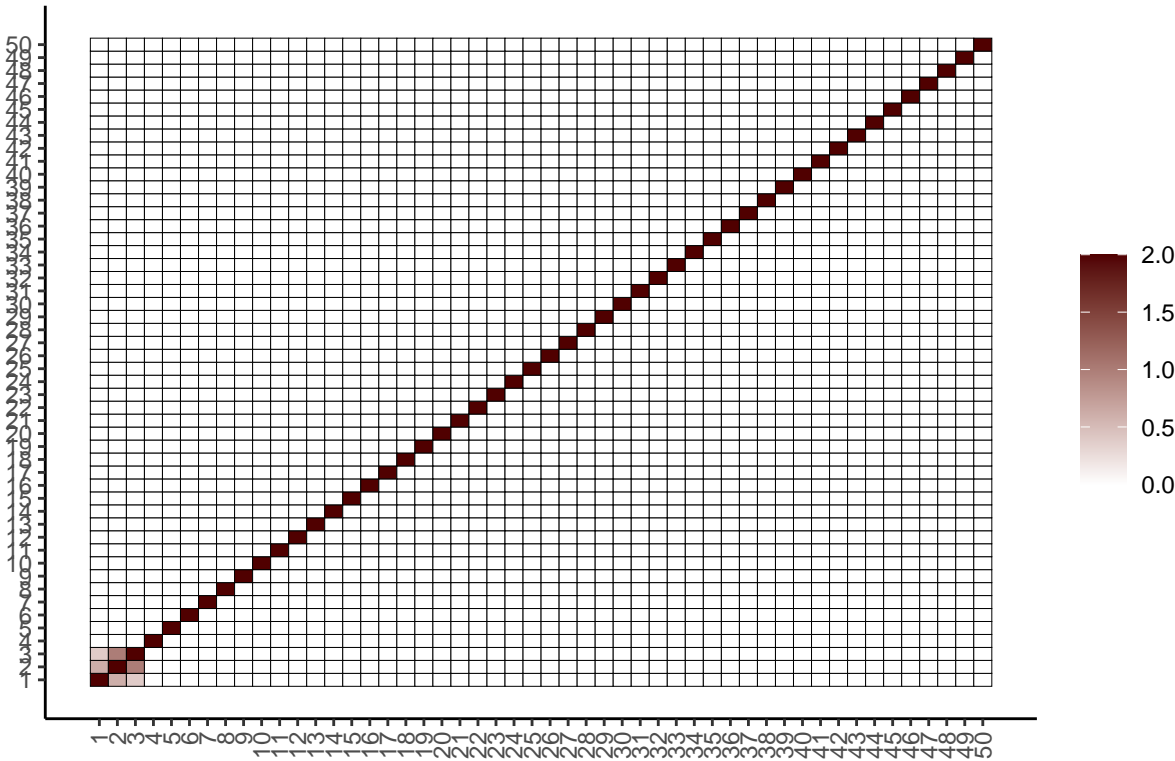
```
## [[1]]
```

True precision matrix, interval 2, observation 5



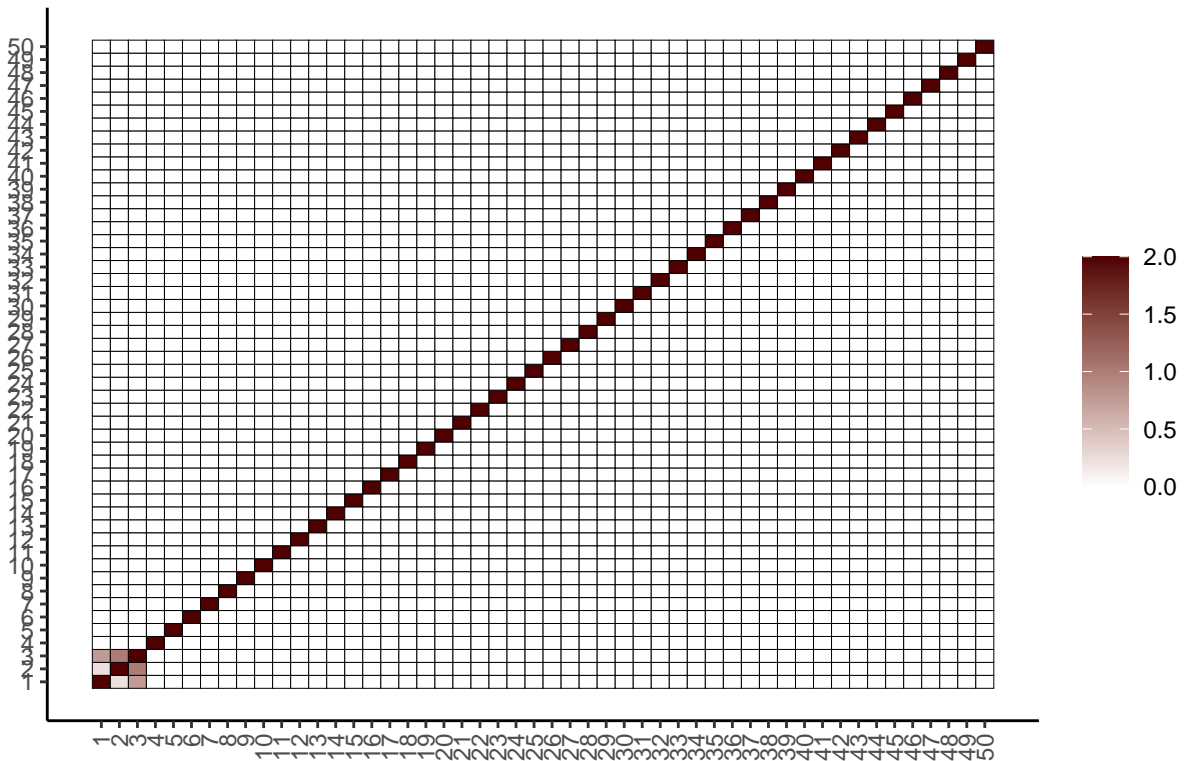
```
##
## [[2]]
```

True precision matrix, interval 2, observation 12



```
##  
## [[3]]
```

True precision matrix, interval 2, observation 20

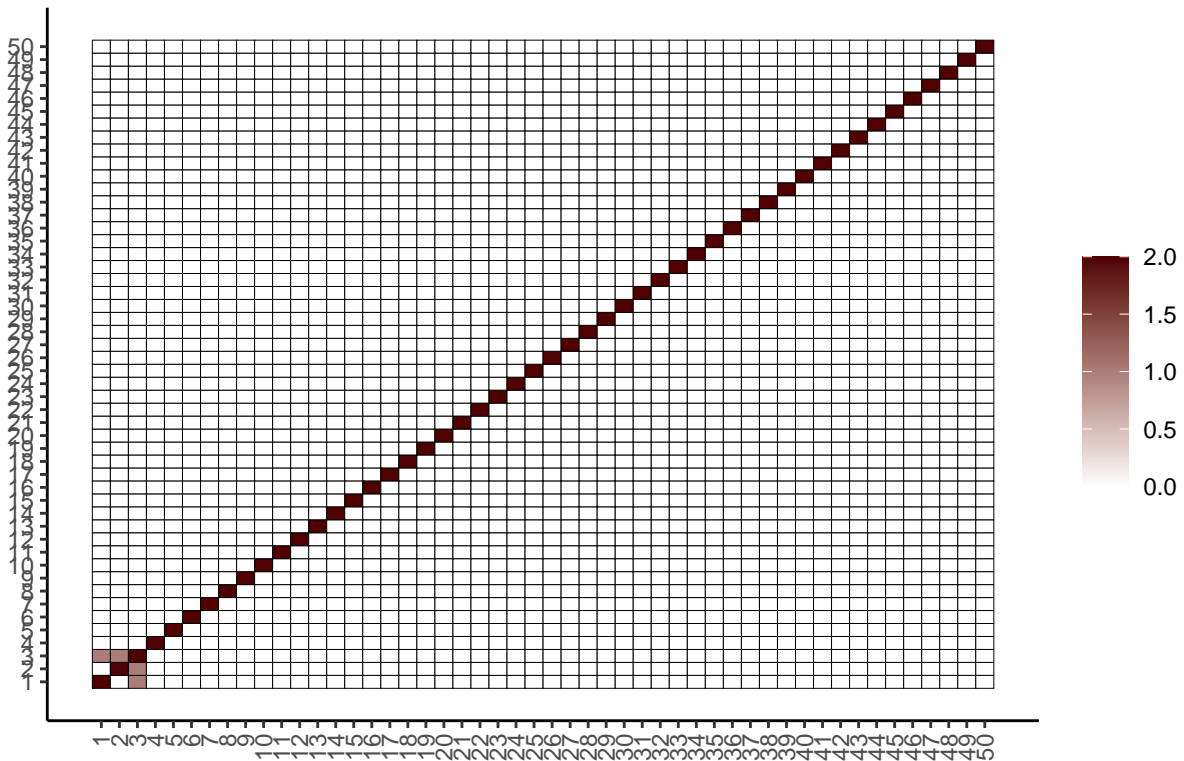


```
# interval 3
cat("\n\nInterval 3: Observations", paste0(range(which(interval == 3)), collapse = ",..."))

##
##
## Interval 3: Observations 36,...,45

matViz(prec[[length(prec)]]) +
  ggtitle("True precision matrix, interval 3") +
  theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1))
```

True precision matrix, interval 3



```
# fit the model and visualize the estimated precision matrices
(out <- covdepGE(X, Z, parallel = T, num_workers = 16))
```

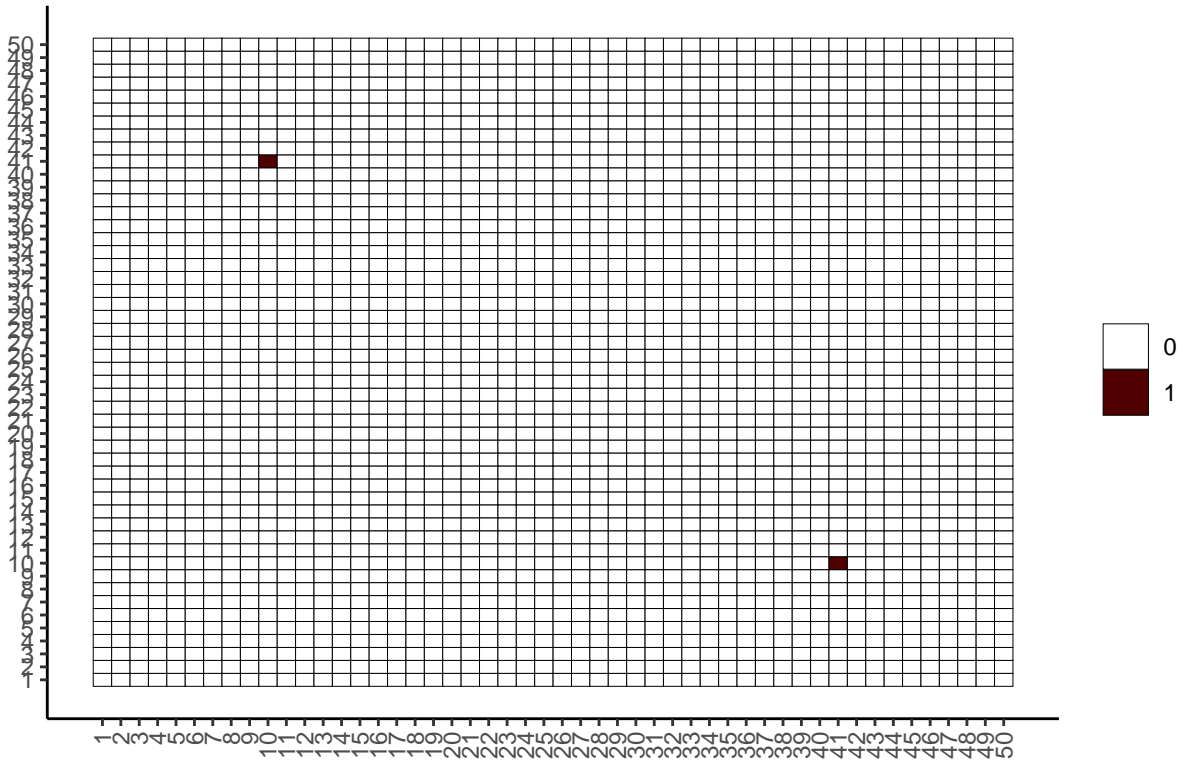
```
## Warning in covdepGE(X, Z, parallel = T, num_workers = 16): No registered workers
## detected; registering doParallel with 16 workers
```

```
##                               Covariate Dependent Graphical Model
##
## ELB0: -108086.7                                # Unique Graphs: 5
## n: 45, variables: 50                        Hyperparameter grid size: 125 points
## Model fit completed in 8.85 secs
```

```
plts <- plot(out)
lapply(plts, function(plt) plt + theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1)))
```

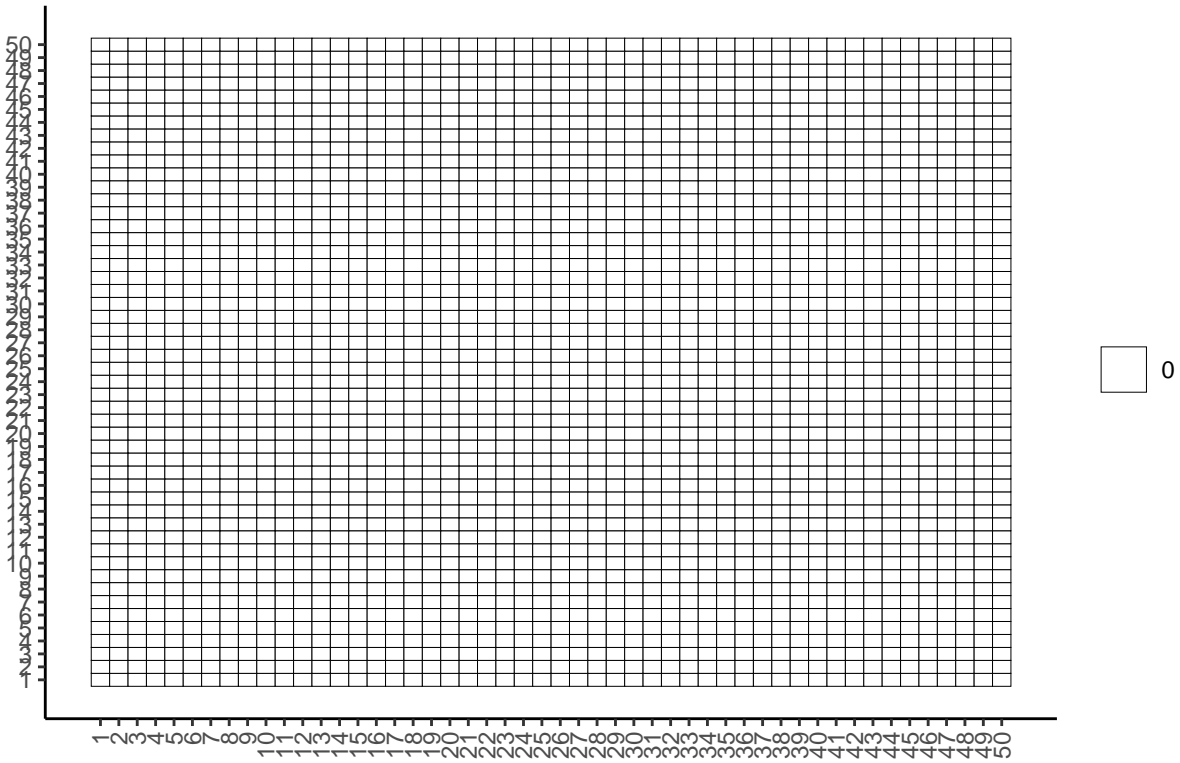
```
## [[1]]
```

Graph 1, observations 1,...,4



```
##  
## [[2]]
```

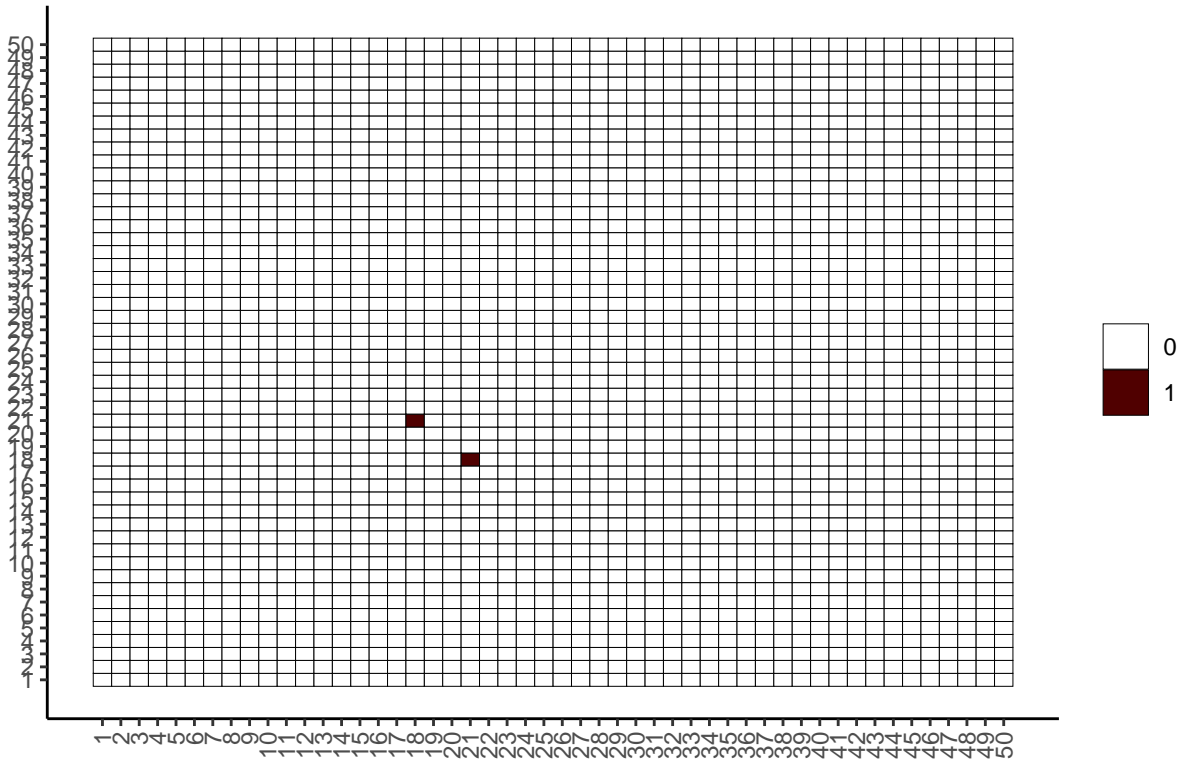

Graph 2, observations 5,...,11,32,...,36



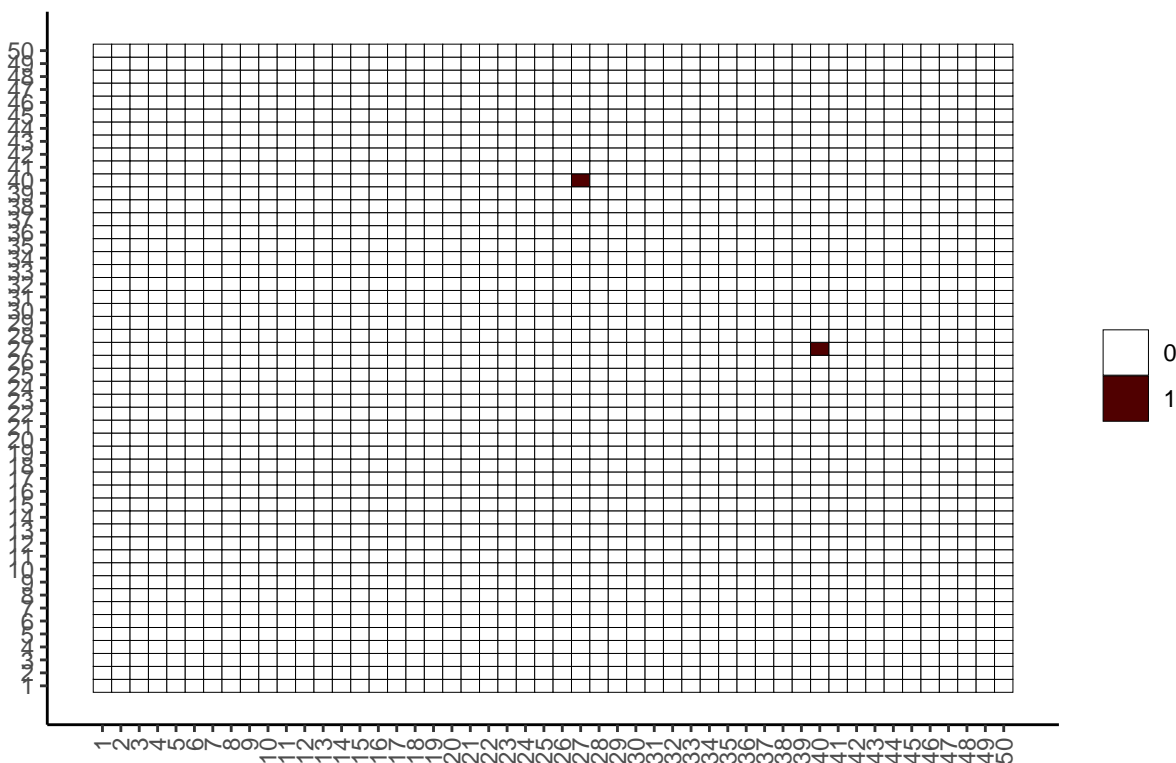
[[3]]

```
##
## [[4]]
```

Graph 4, observations 28,...,31



Graph 5, observations 37,...,45



```
out$weights$bandwidths
```

```
## [1] 1.1504302 0.9943283 0.9521409 0.9037466 0.8339550 0.8110806 0.7969470
## [8] 0.6701070 0.6646864 0.6449568 0.6103186 0.5641069 0.5481653 0.5455225
## [15] 0.5404932 0.5390264 0.5280853 0.5193730 0.5172664 0.5171966 0.5171495
## [22] 0.5170984 0.5195341 0.5204317 0.5207878 0.5344772 0.5452098 0.5540447
## [29] 0.5628004 0.5803914 0.5832557 0.6041737 0.6249997 0.6629558 0.7021263
## [36] 0.7899229 0.8175969 0.7616764 0.7534891 0.7370620 0.7376028 0.7473590
## [43] 0.7726552 0.7794208 0.7980997
```

```
summary(c(sapply(lapply(out$hyperparameters, `[`, "grid"), `[`, "pip")))
```

```
##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
## 0.00001 0.00511 0.01021 0.01082 0.01531 0.08163
```

```
summary(c(sapply(lapply(out$hyperparameters, `[`, "grid"), `[`, "sbsq")))
```

```
##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
## 0.00001 11.70750 23.49777 23.17047 35.38189 47.67528
```

```
summary(c(sapply(lapply(out$hyperparameters, `[`, "grid"), `[`, "ssq")))
```

```
##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
## 0.00001 0.16059 0.37915 0.39858 0.61735 1.31597
```

```
sbsq <- seq(1e-5, 2, length.out = 5)
(out <- covdepGE(X, Z, pip_upper = 0.2, sbsq = sbsq, parallel = T, num_workers = 16))
```

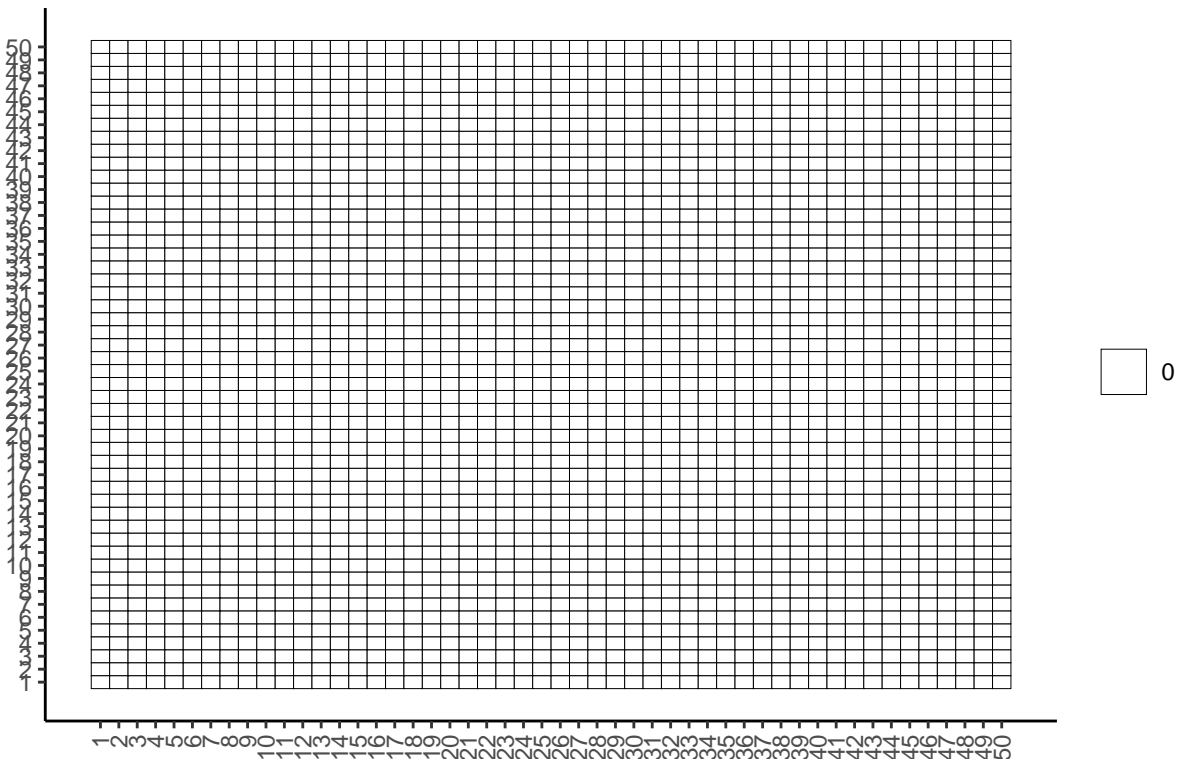
```
## Warning in covdepGE(X, Z, pip_upper = 0.2, sbsq = sbsq, parallel = T,
## num_workers = 16): No registered workers detected; registering doParallel with
## 16 workers
```

```
##                               Covariate Dependent Graphical Model
##
## ELB0: -108327.83                                # Unique Graphs: 3
## n: 45, variables: 50                            Hyperparameter grid size: 125 points
## Model fit completed in 6.515 secs
```

```
plts <- plot(out)
lapply(plts, function(plt) plt + theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1)))
```

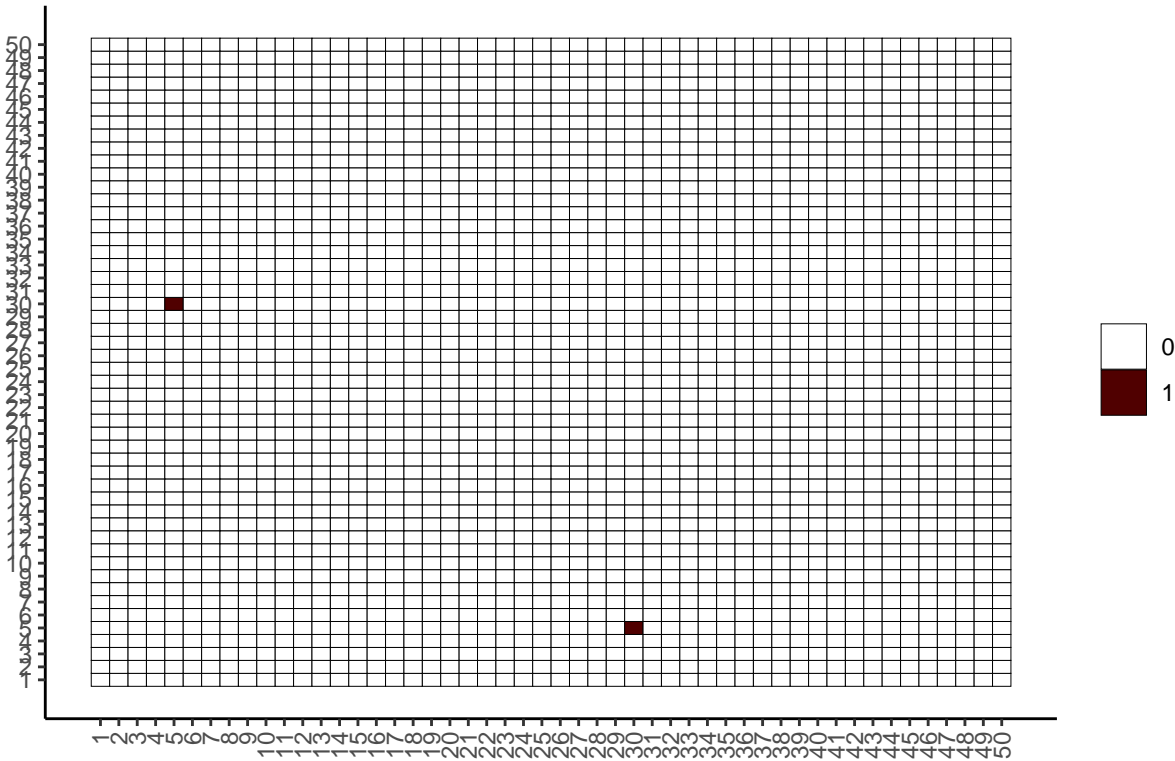
```
## [[1]]
```

Graph 1, observations 1,...,16,26,...,37



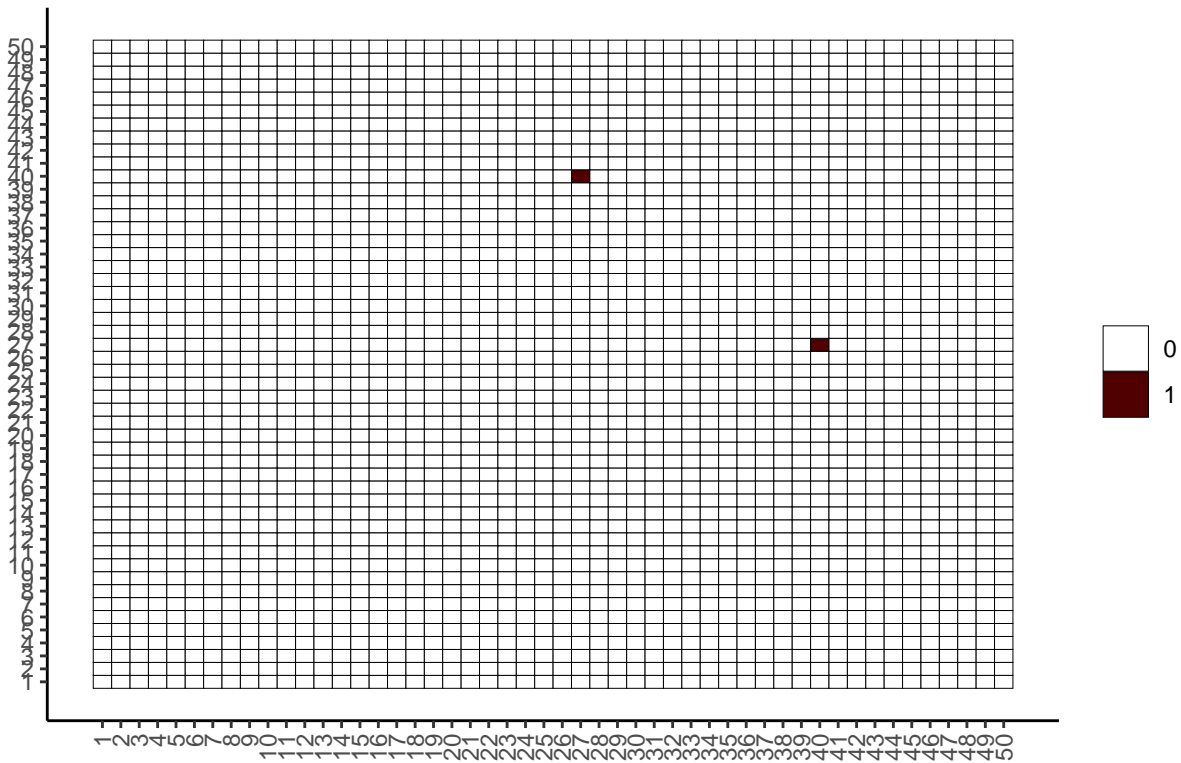
```
##
## [[2]]
```

Graph 2, observations 17,...,25



[[3]]

Graph 3, observations 38,...,45

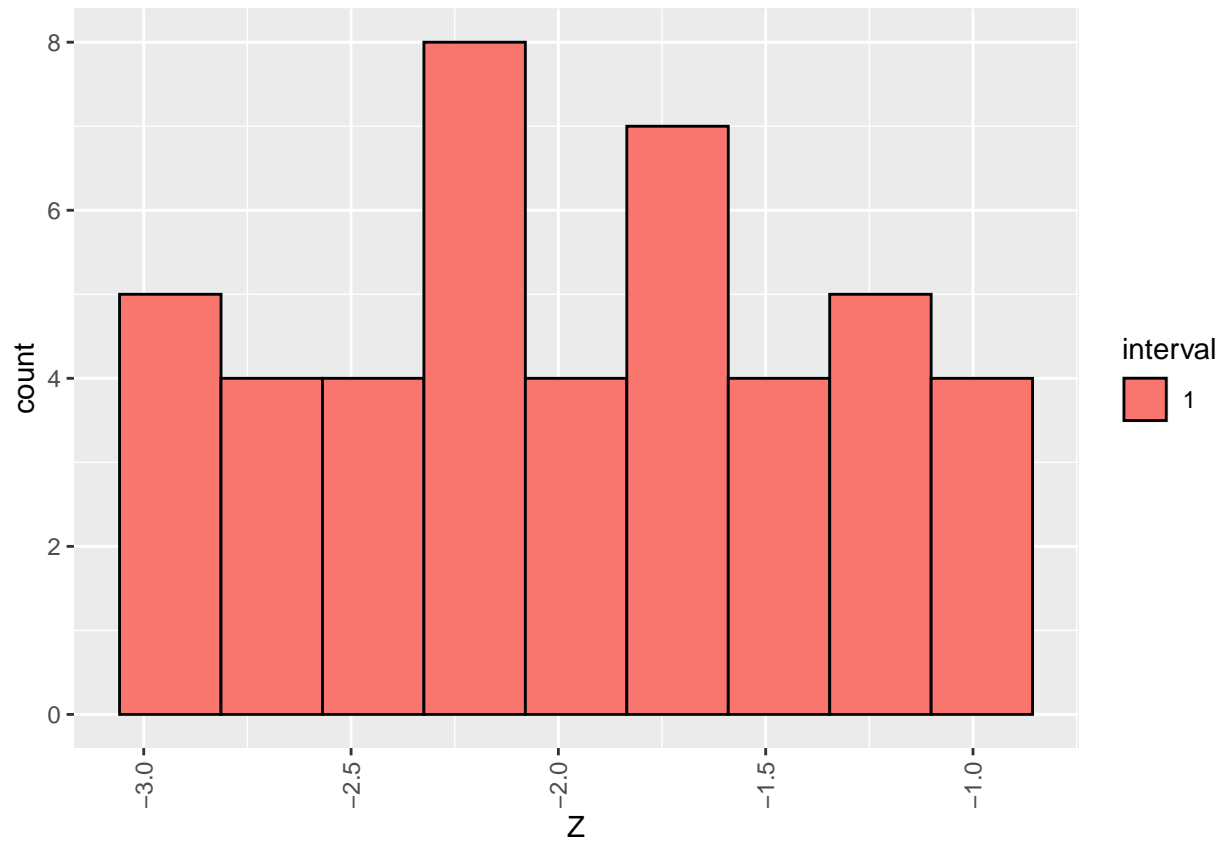


Run 3

```
# only one interval
data <- generateData(50, 45, 0, 0)
X <- data$data
Z <- data$covts
interval <- data$interval
prec <- data$true_precision

# get overall and within interval sample sizes
n <- nrow(X)
n1 <- sum(interval == 1)
n2 <- sum(interval == 2)

# visualize the distribution of the extraneous covariate
ggplot(data.frame(Z = Z, interval = as.factor(interval))) +
  geom_histogram(aes(Z, fill = interval), color = "black", bins = n %/% 5) +
  theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1))
```



```
# visualize the true precision matrices in each of the intervals
```

```
# interval 1
```

```
cat("\n\nInterval 1: Observations", paste0(range(which(interval == 1)), collapse = ",...,"))
```

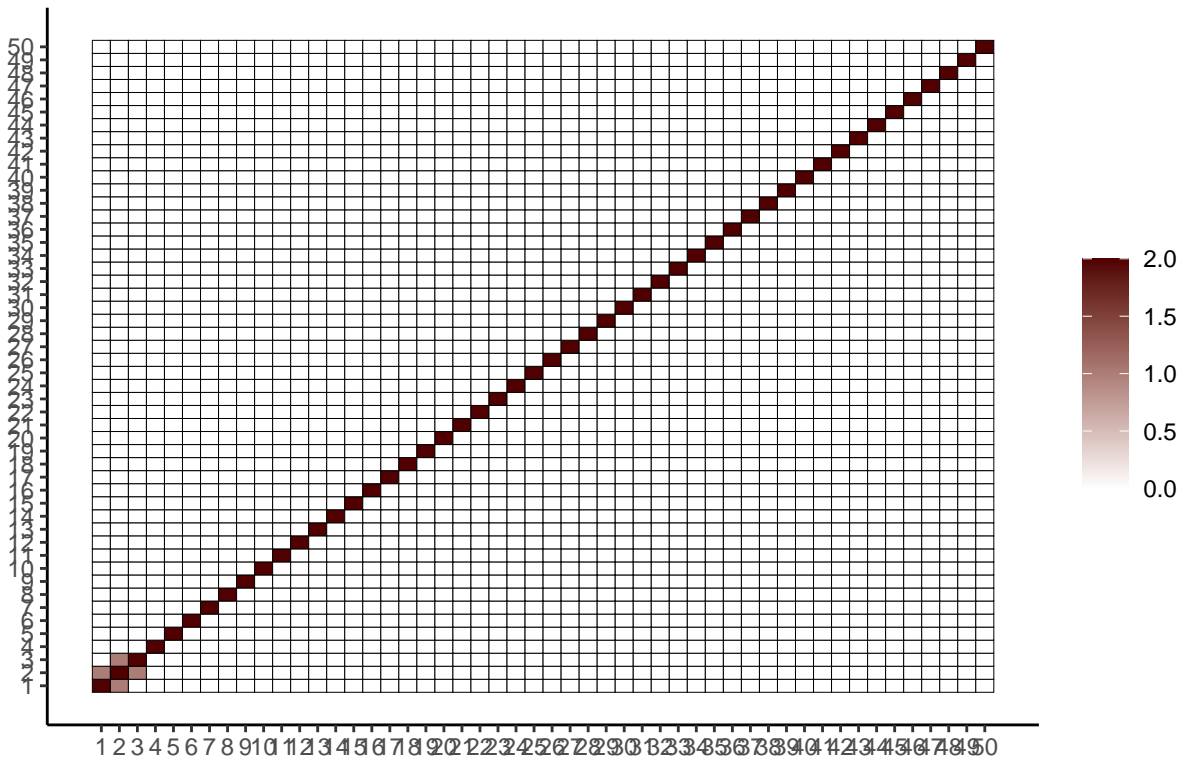
```
##
```

```
##
```

```
## Interval 1: Observations 1,...,45
```

```
matViz(prec[[1]]) + ggtitle("True precision matrix, interval 1")
```


True precision matrix, interval 1



```
# fit the model and visualize the estimated precision matrices
(out <- covdepGE(X, Z, parallel = T, num_workers = 16))
```

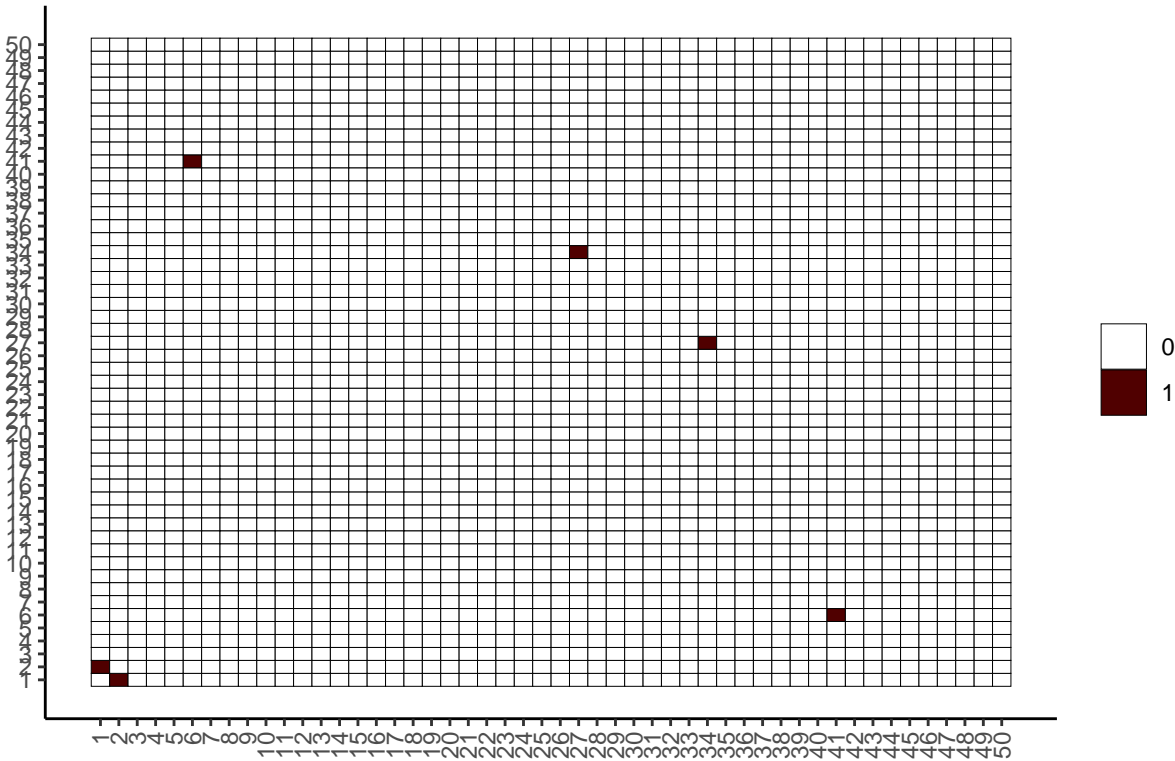
```
## Warning in covdepGE(X, Z, parallel = T, num_workers = 16): No registered workers
## detected; registering doParallel with 16 workers
```

```
##                               Covariate Dependent Graphical Model
##
## ELB0: -108973.85                                # Unique Graphs: 4
## n: 45, variables: 50                            Hyperparameter grid size: 125 points
## Model fit completed in 9.153 secs
```

```
plts <- plot(out)
lapply(plts, function(plt) plt + theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1)))
```

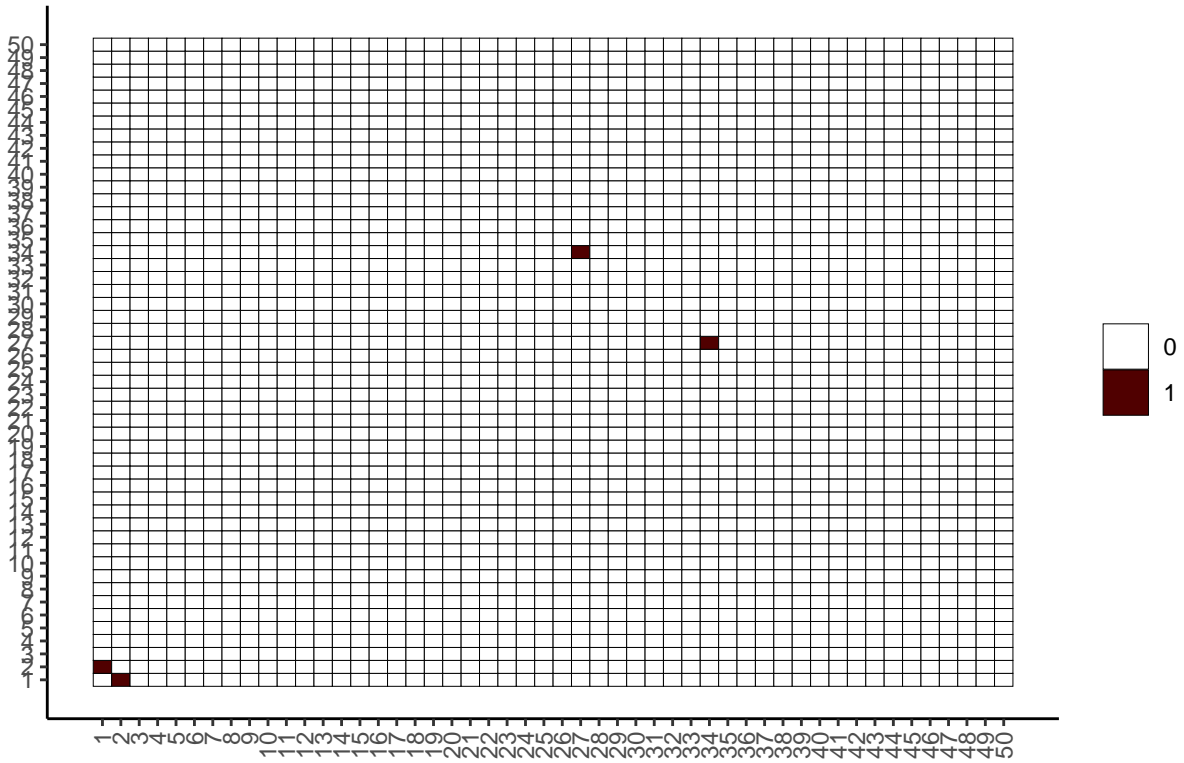
```
## [[1]]
```

Graph 1, observations 1,...,23



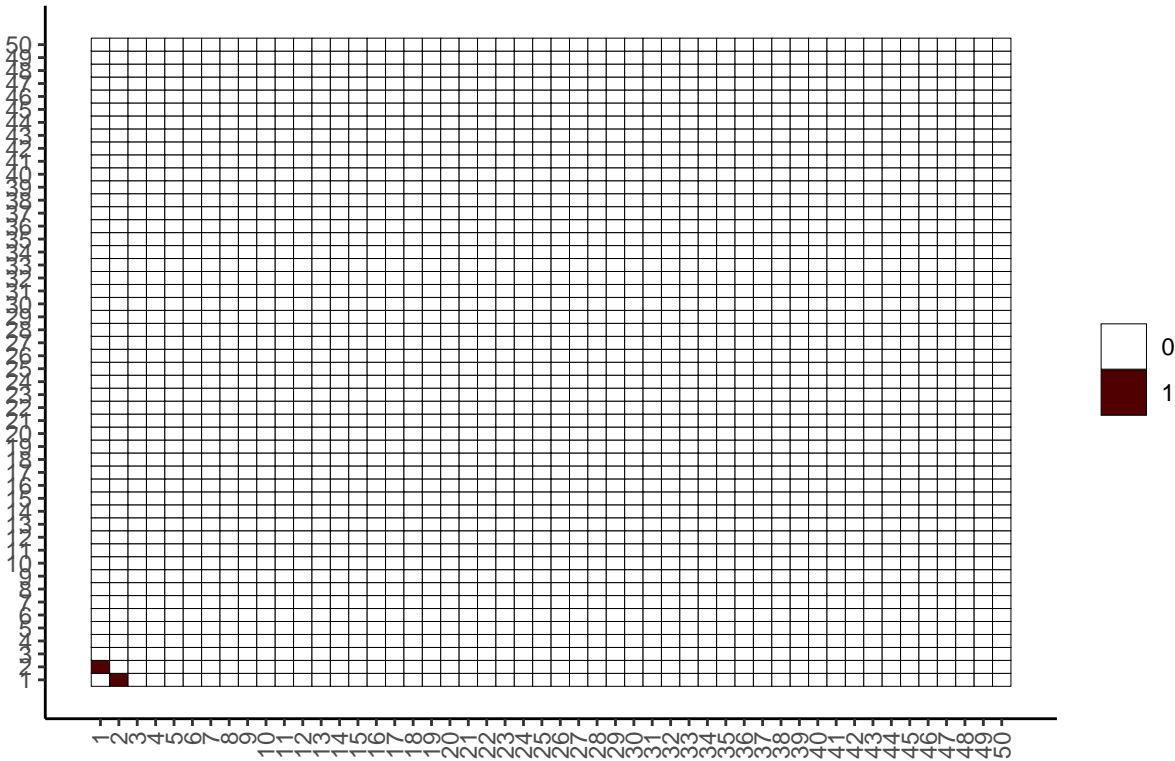
[[2]]

Graph 2, observations 24,25



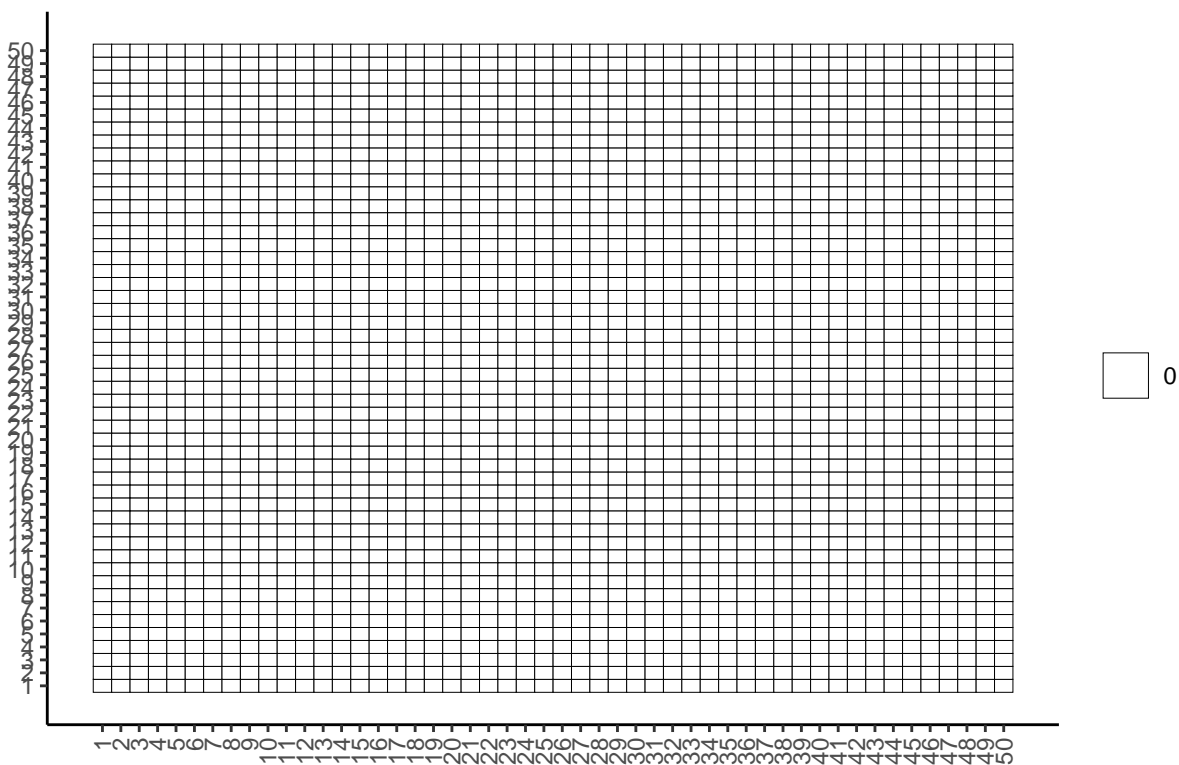
```
##  
## [[3]]
```

Graph 3, observations 26,...,39



[[4]]

Graph 4, observations 40,...,45

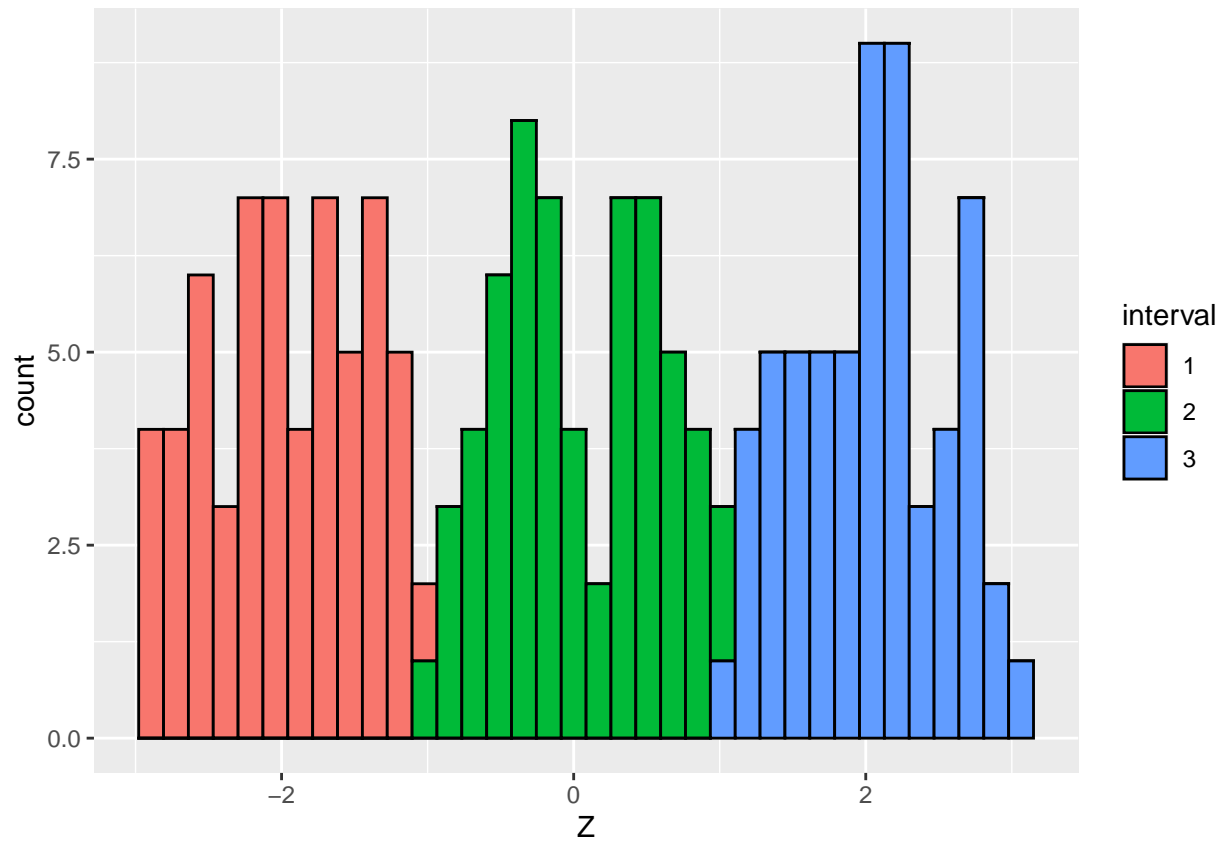


Run 4

```
# get the data
set.seed(1)
data <- generateData(50)
X <- data$data
Z <- data$covts
interval <- data$interval
prec <- data$true_precision

# get overall and within interval sample sizes
n <- nrow(X)
n1 <- sum(interval == 1)
n2 <- sum(interval == 2)

# visualize the distribution of the extraneous covariate
ggplot(data.frame(Z = Z, interval = as.factor(interval))) +
  geom_histogram(aes(Z, fill = interval), color = "black", bins = n %/% 5)
```



```
# visualize the true precision matrices in each of the intervals
```

```
# interval 1
```

```
cat("\n\nInterval 1: Observations", paste0(range(which(interval == 1)), collapse = ",...,"))
```

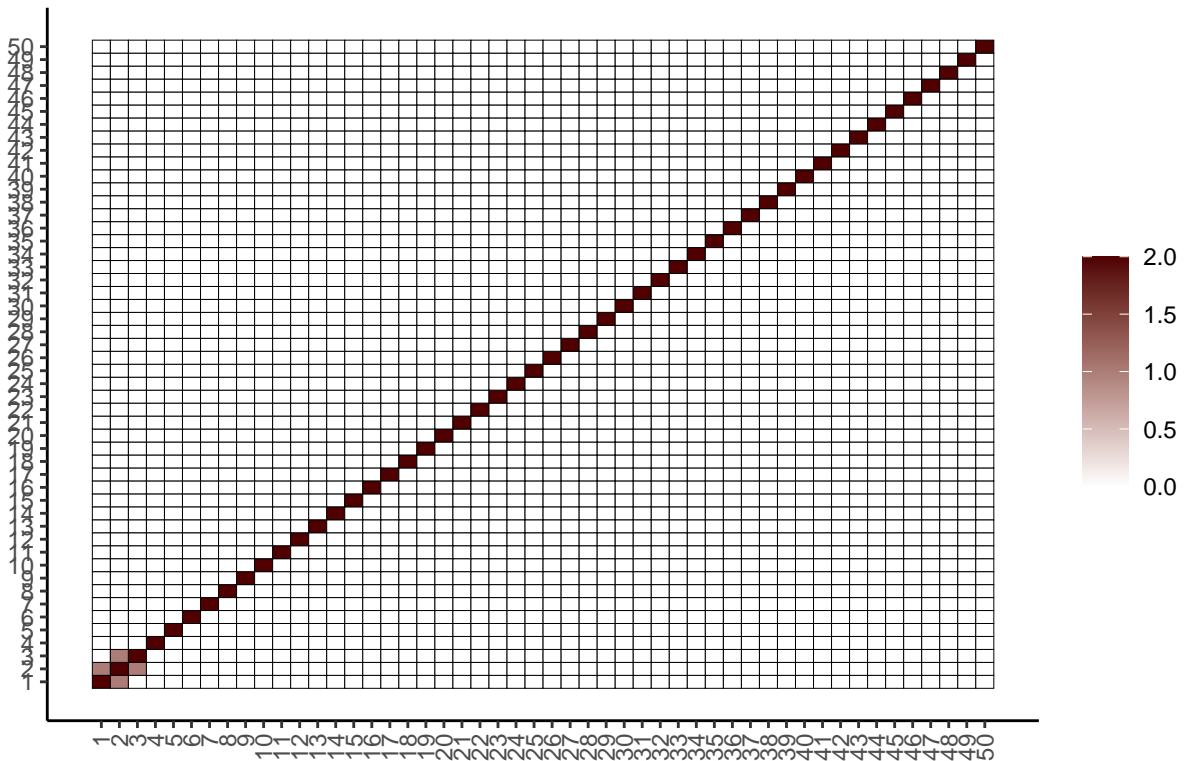
```
##
```

```
##
```

```
## Interval 1: Observations 1,...,60
```

```
matViz(prec[[1]]) + ggtitle("True precision matrix, interval 1") +  
  theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1))
```

True precision matrix, interval 1



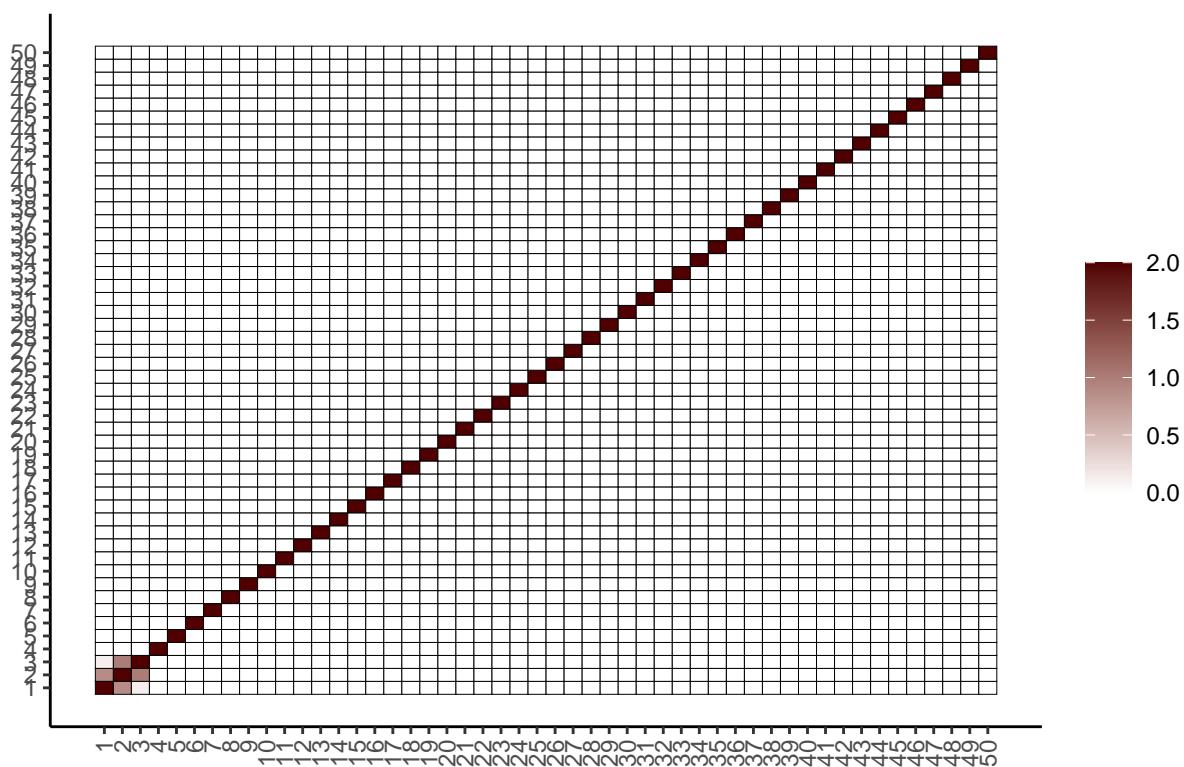
```
# interval 2 (varies continuously with Z)
cat("\n\nInterval 2: Observations", paste0(range(which(interval == 2)), collapse = ",...,"))
```

```
##
##
## Interval 2: Observations 61,...,120
```

```
int2_mats <- prec[interval == 2]
int2_inds <- c(5, n2 %/% 2, n2 - 5)
lapply(int2_inds, function(j) matViz(int2_mats[[j]]) +
  ggtitle(paste("True precision matrix, interval 2, observation", j)) +
  theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1)))
```

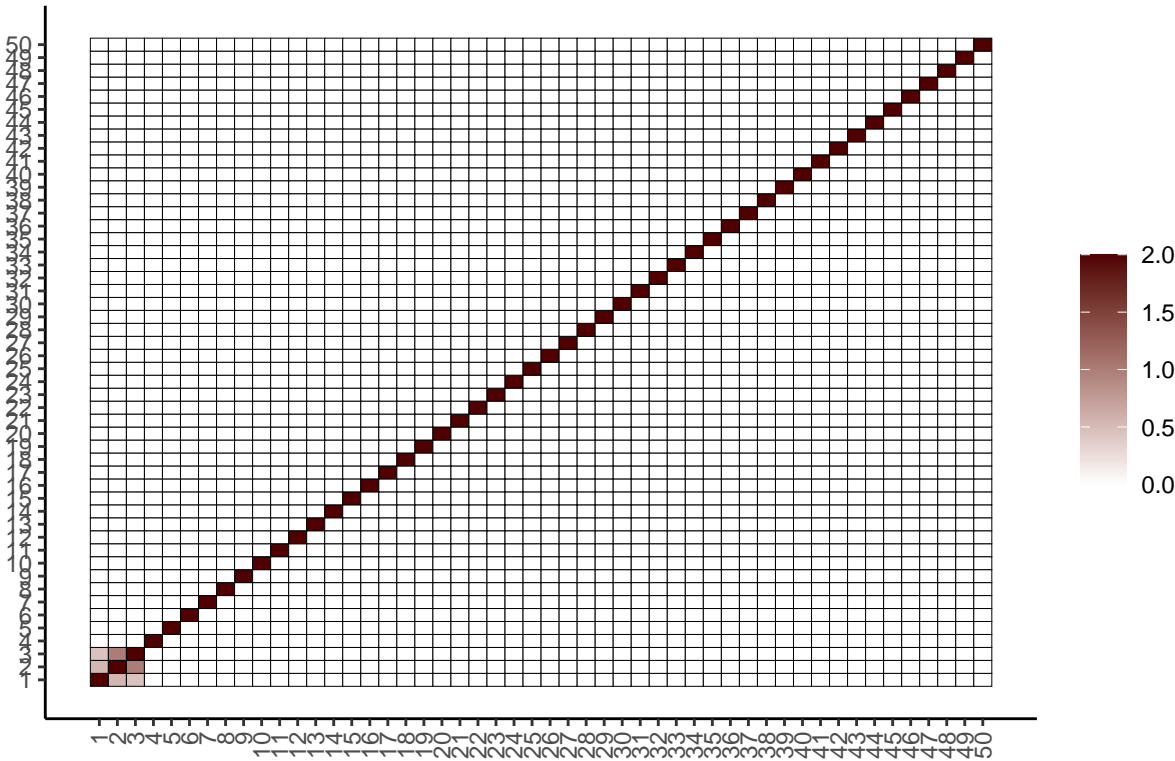
```
## [[1]]
```

True precision matrix, interval 2, observation 5



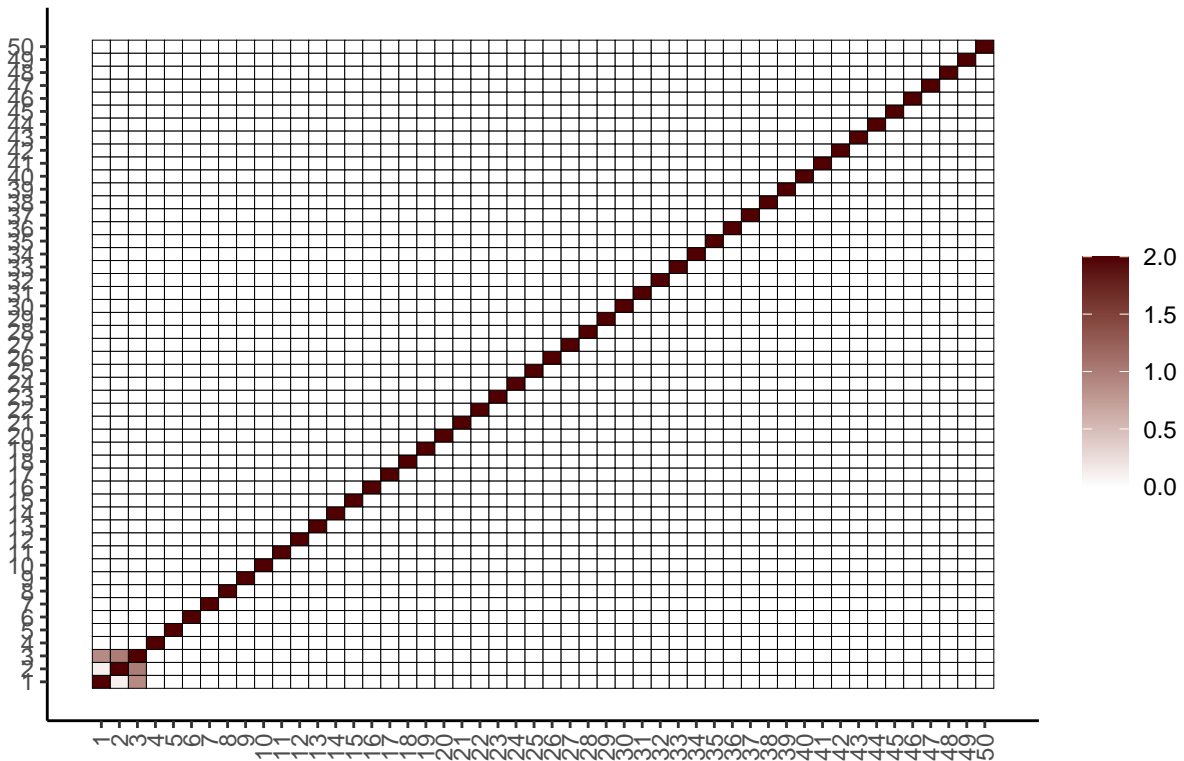
```
##
## [[2]]
```


True precision matrix, interval 2, observation 30



```
##  
## [[3]]
```

True precision matrix, interval 2, observation 55

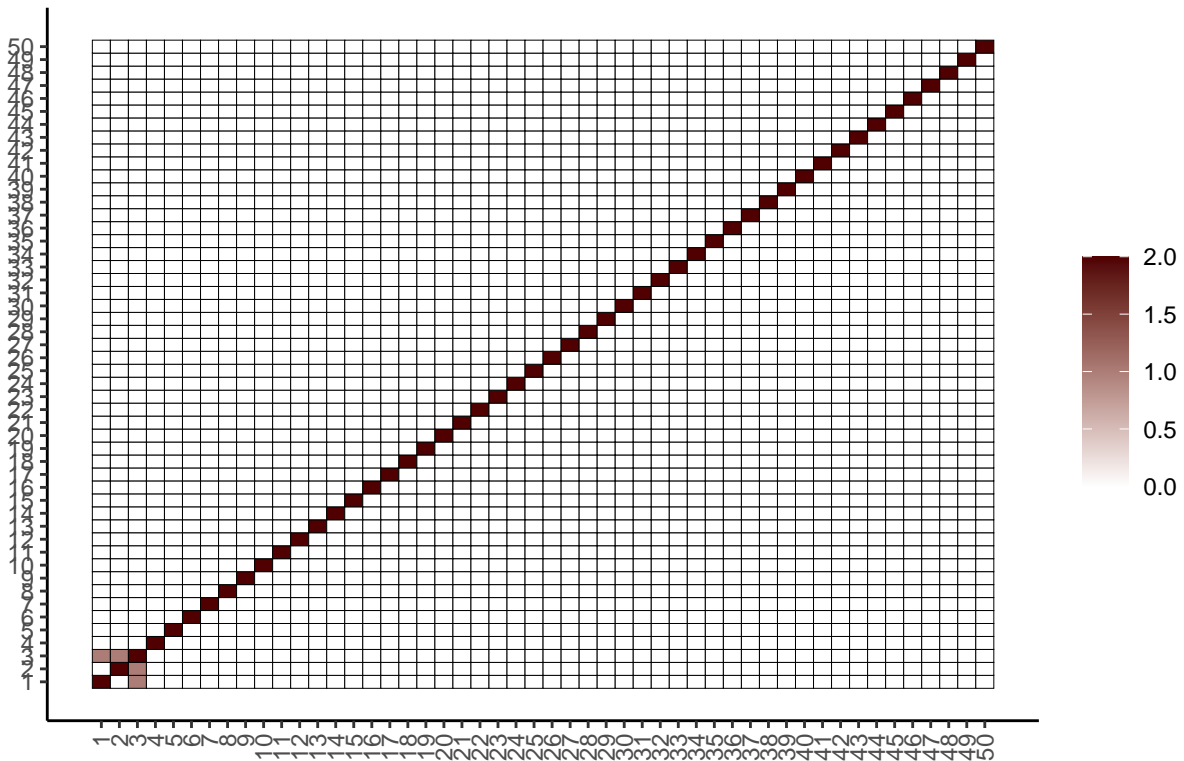


```
# interval 3
cat("\n\nInterval 3: Observations", paste0(range(which(interval == 3)), collapse = ",...,"))

##
##
## Interval 3: Observations 121,...,180

matViz(prec[[length(prec)]]) +
  ggtitle("True precision matrix, interval 3") +
  theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1))
```

True precision matrix, interval 3



```
# fit the model and visualize the estimated precision matrices
(out <- covdepGE(X, Z, parallel = T, num_workers = 16))
```

```
## Warning in covdepGE(X, Z, parallel = T, num_workers = 16): No registered workers
## detected; registering doParallel with 16 workers
```

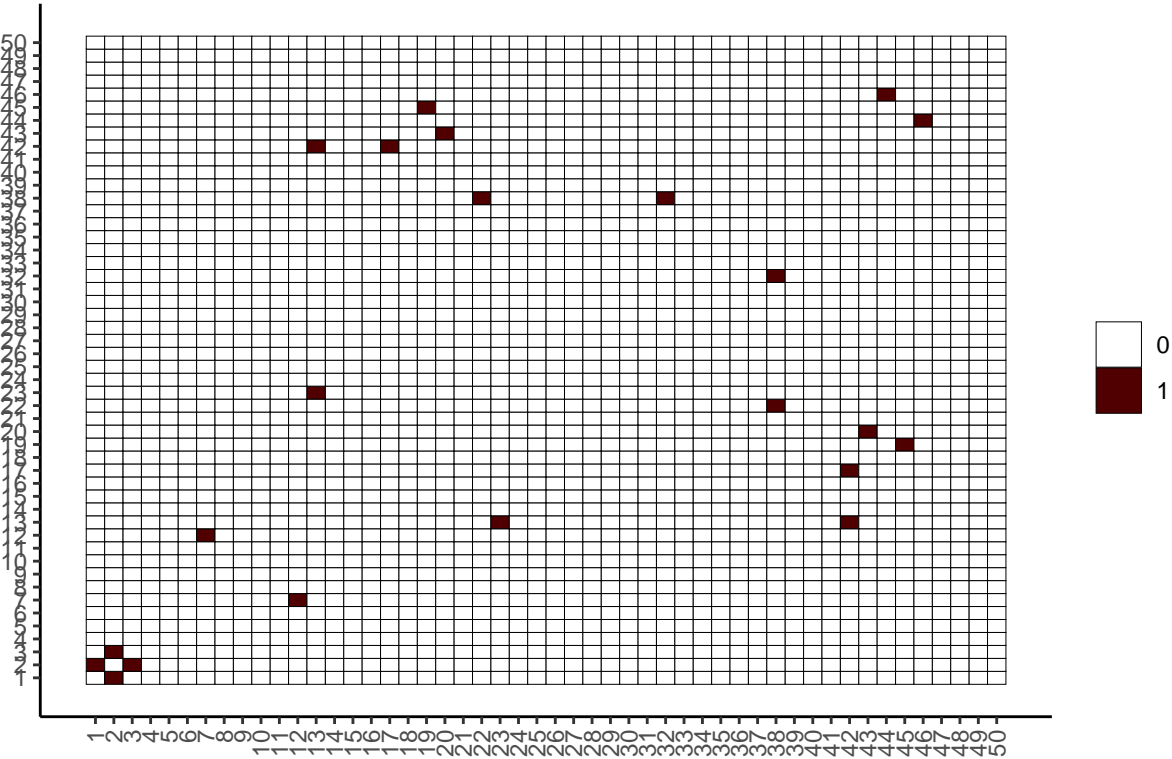
```
##                               Covariate Dependent Graphical Model
##
## ELB0: -1766776.76                                # Unique Graphs: 34
## n: 180, variables: 50                            Hyperparameter grid size: 125 points
## Model fit completed in 1.274 mins
```

```
plts <- plot(out)
lapply(plts, function(plt) plt + theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1)))
```

```
## [[1]]
```

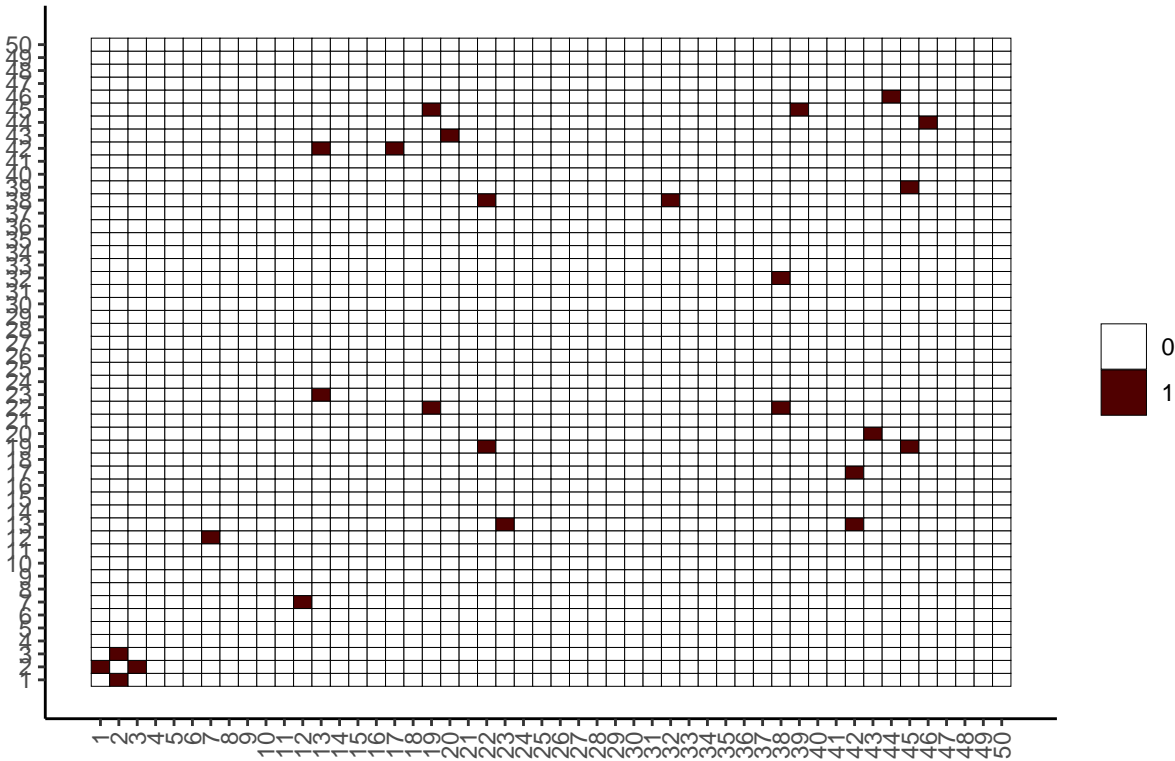
28

Graph 2, observations 5,6,7



[[3]]

Graph 3, observations 8,...,16



[[4]]

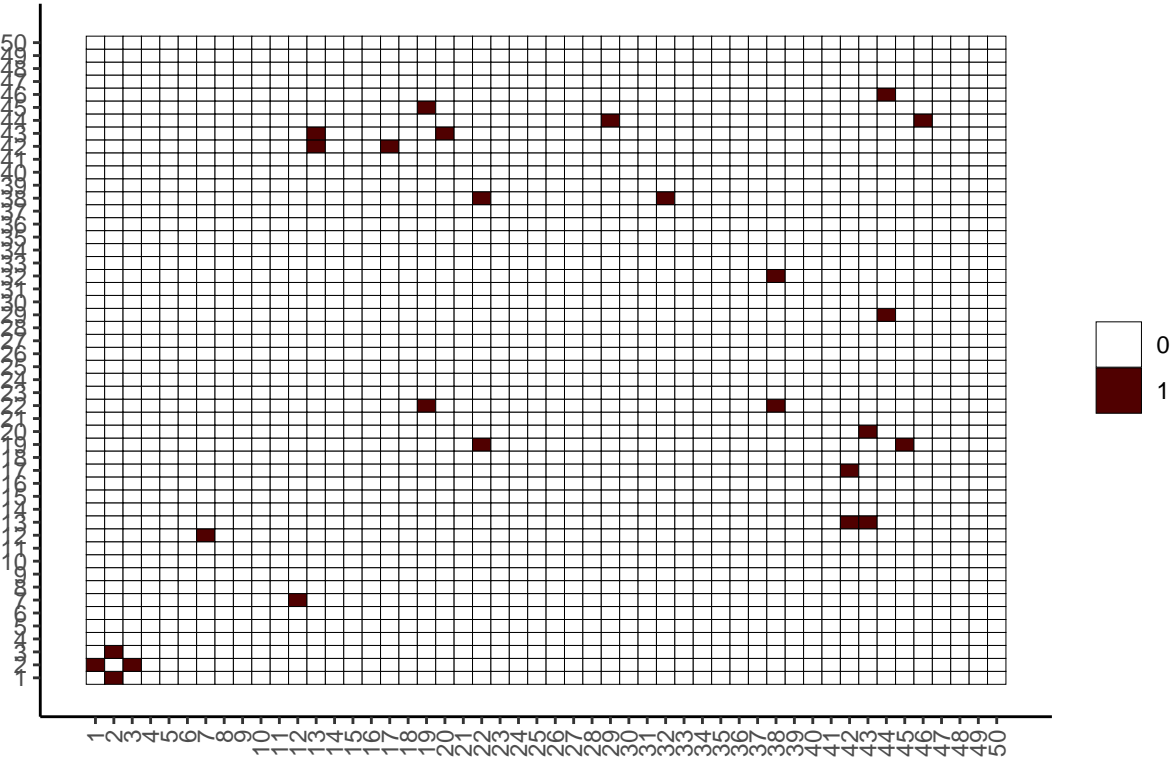
```
##
## [[5]]
```

##

```
## [[6]]
```

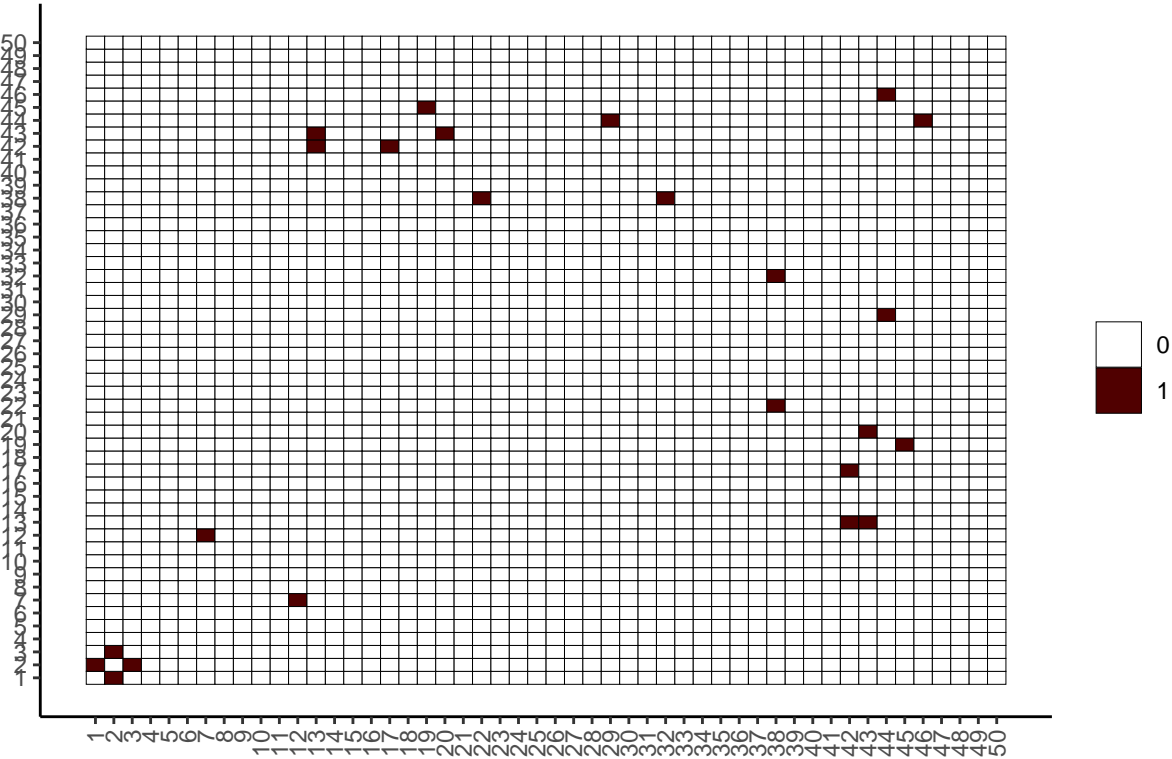

33

Graph 7, observations 25



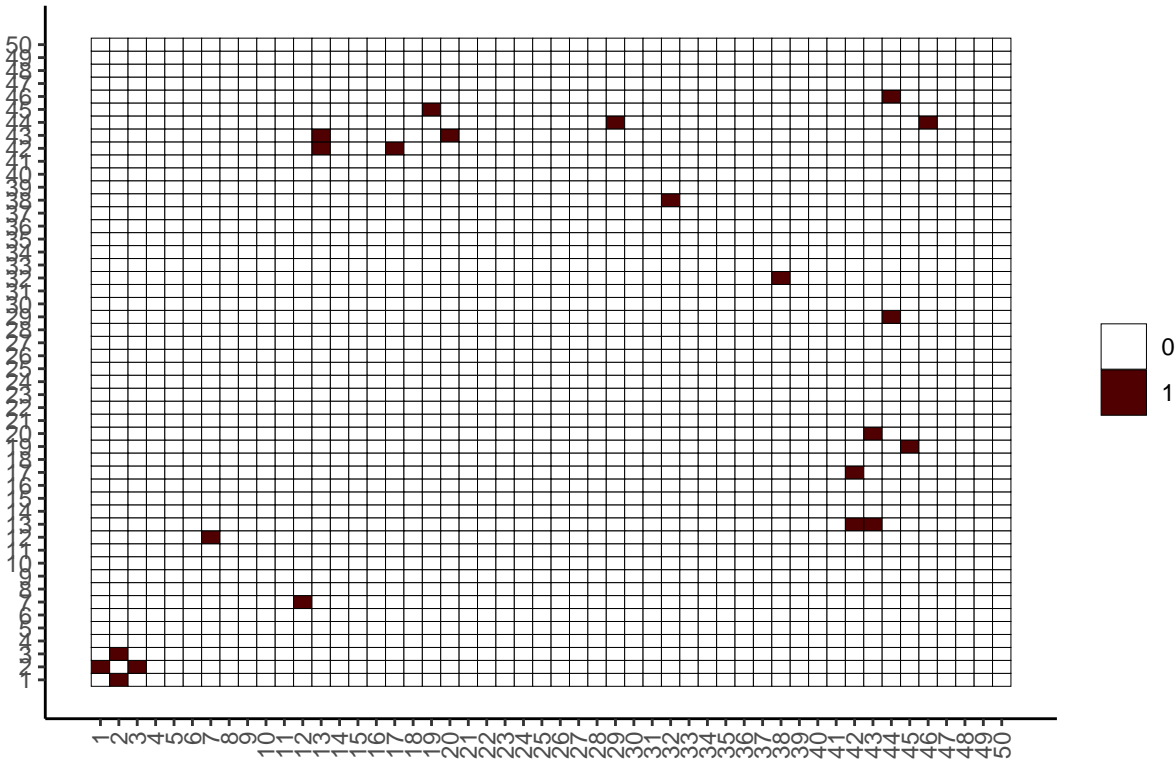
[[8]]

Graph 8, observations 26,27,28



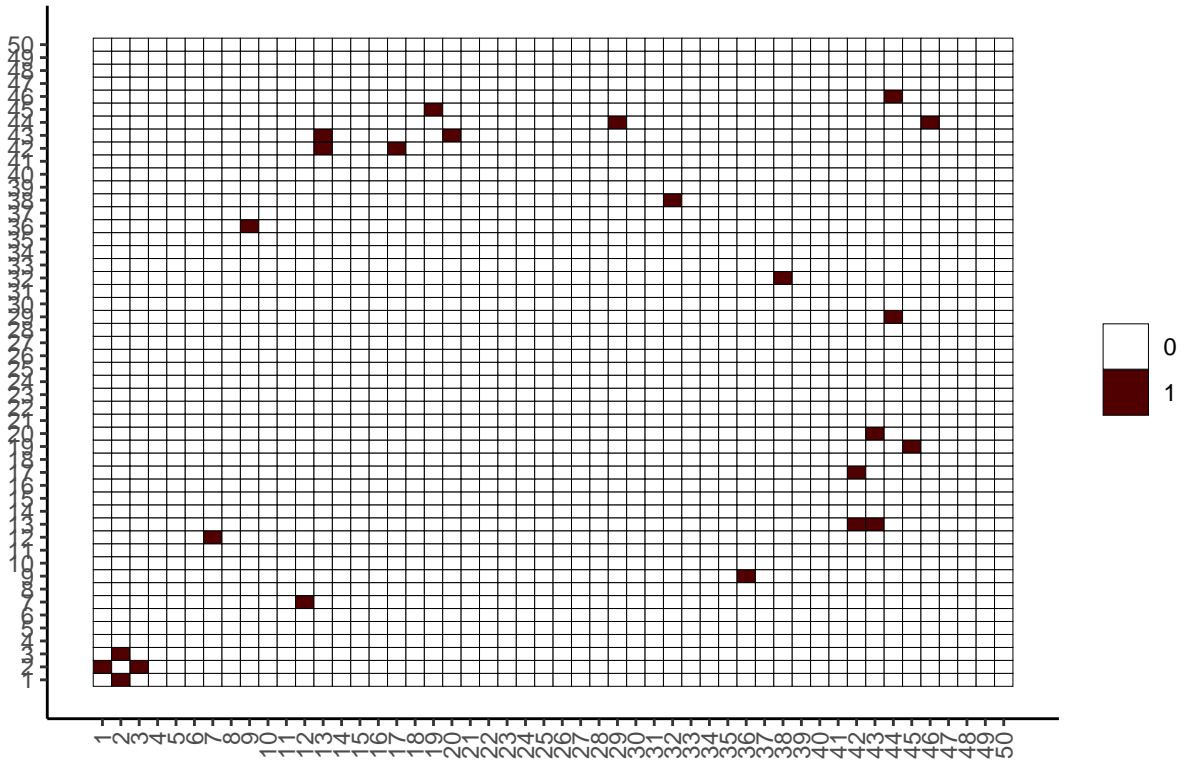
[[9]]

Graph 9, observations 29,...,38



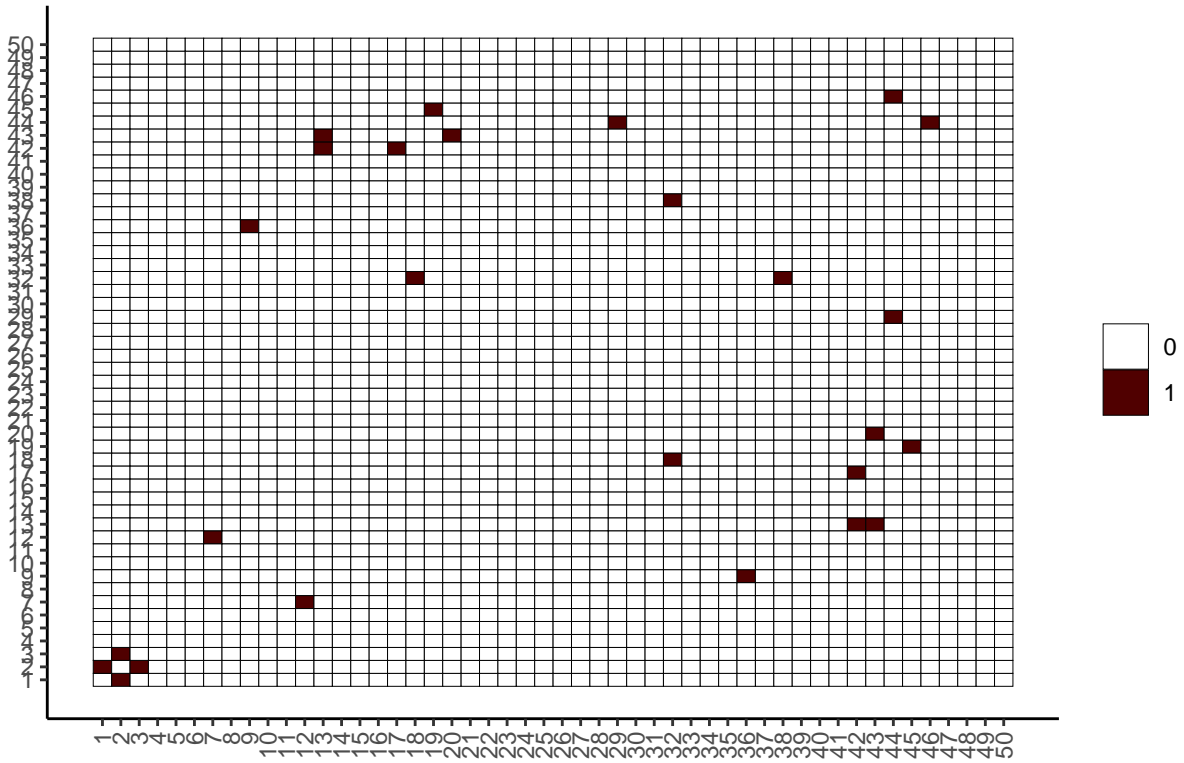
[[10]]

Graph 10, observations 39,40,41



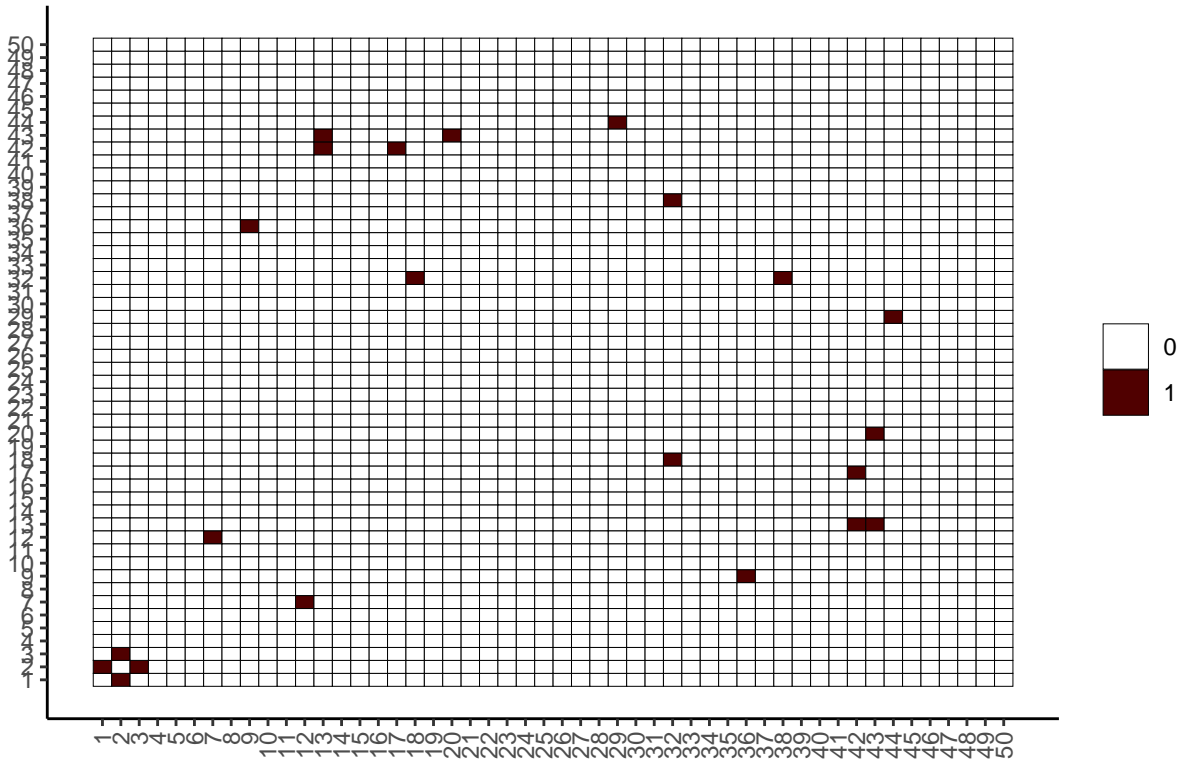
```
##
## [[11]]
```

Graph 11, observations 42,...,46



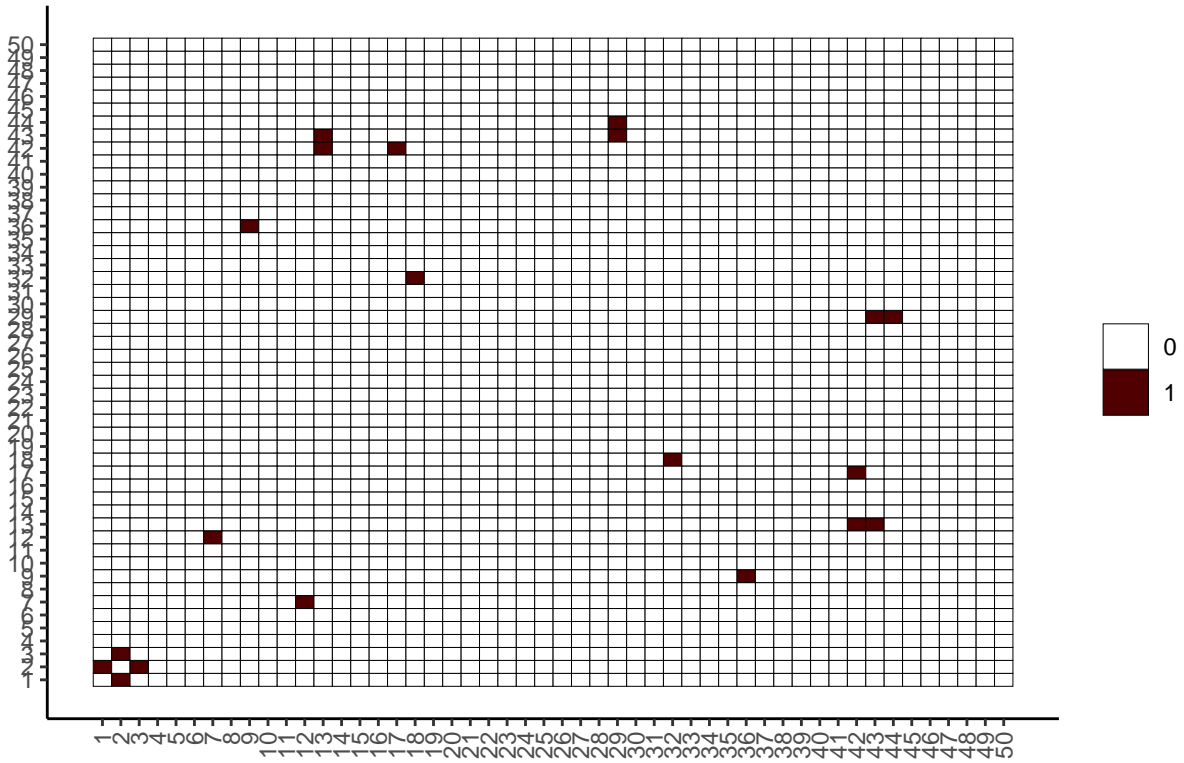
[[12]]

Graph 12, observations 47



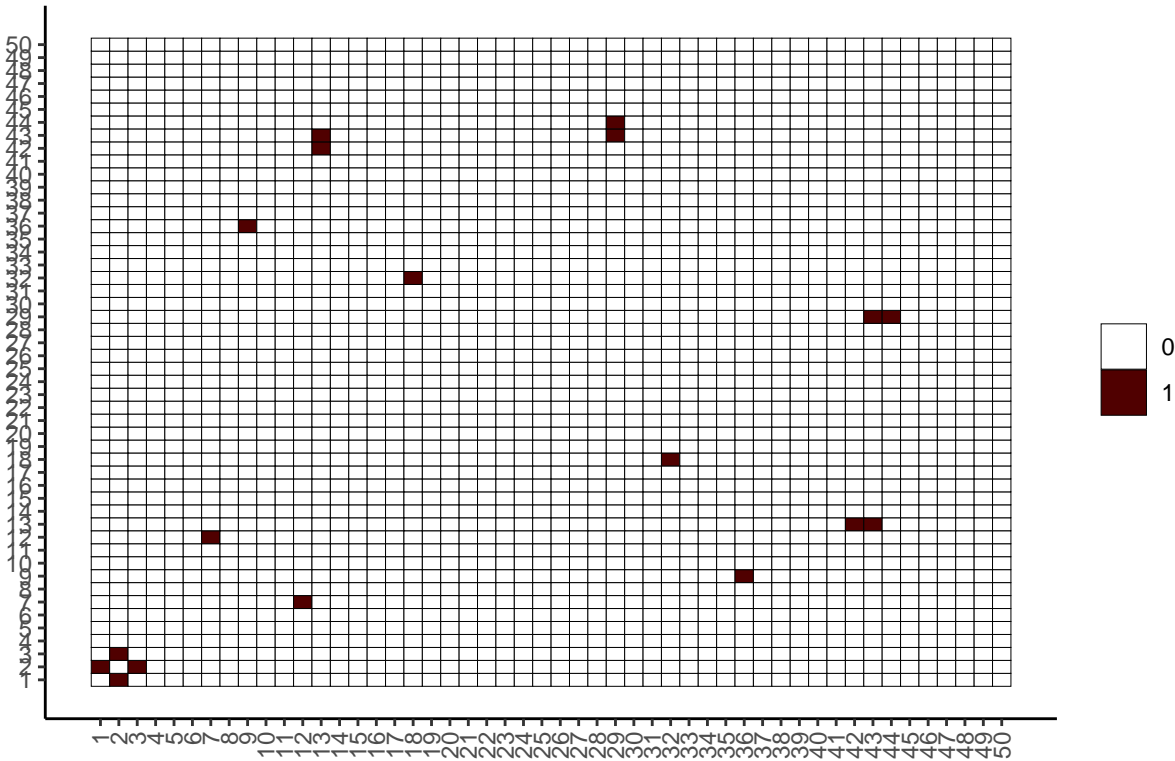
[[13]]

Graph 13, observations 48,...,55



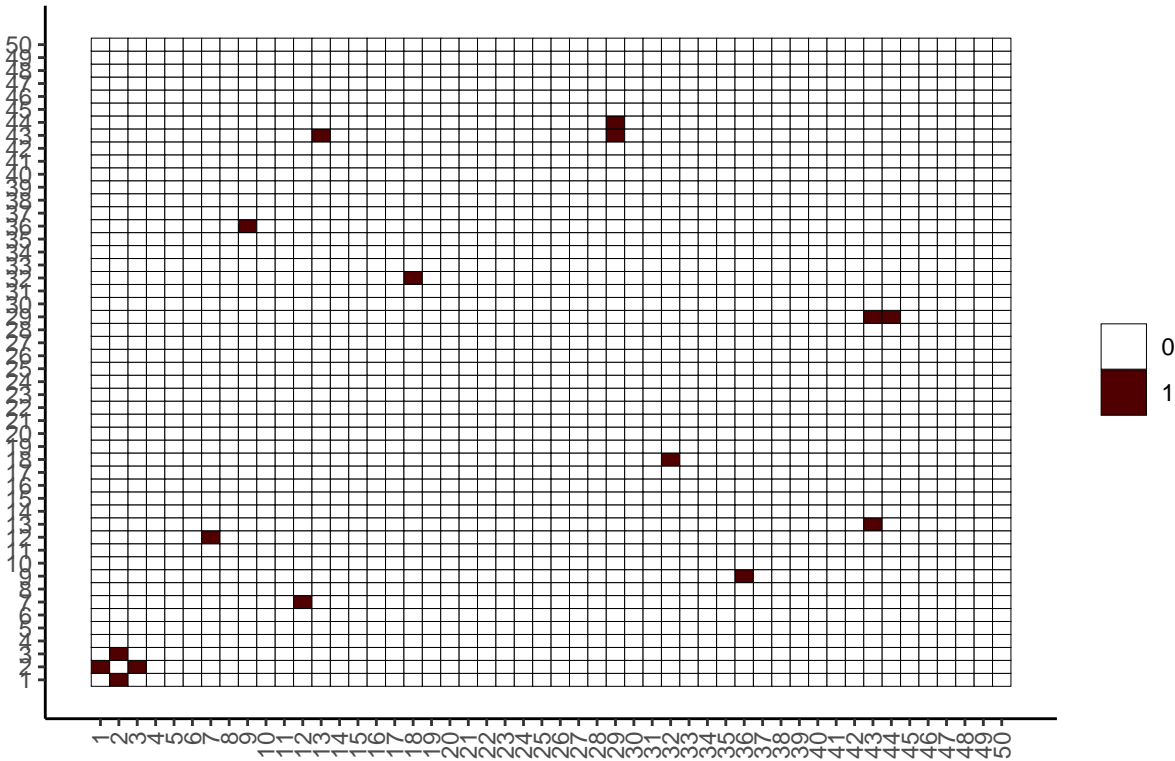
[[14]]

Graph 14, observations 56,57



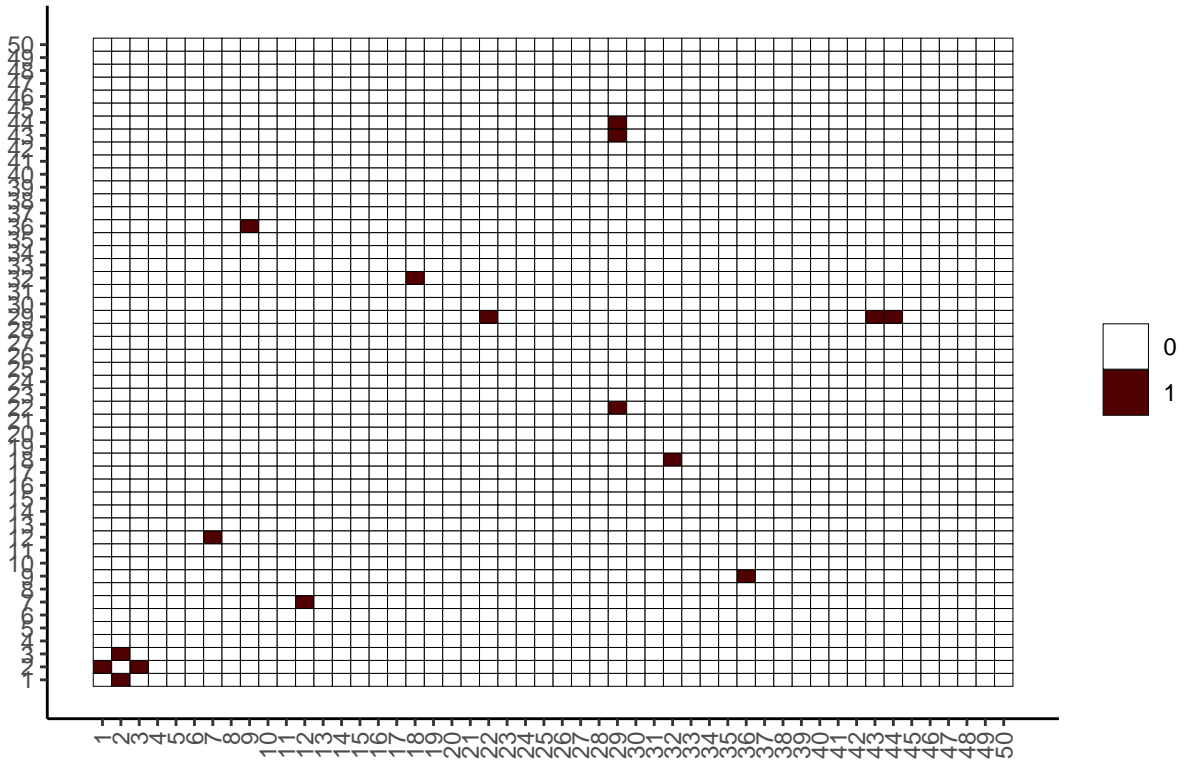
[[15]]

Graph 15, observations 58,59



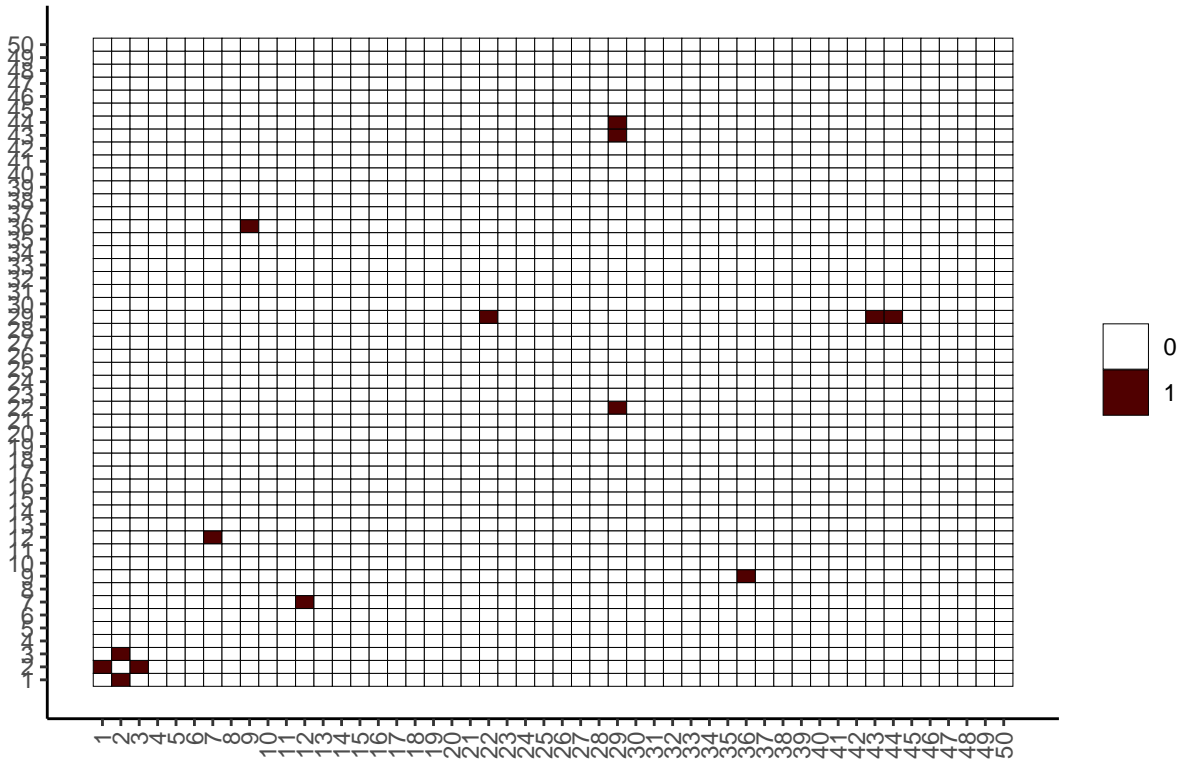
[[16]]

Graph 16, observations 60,...,64



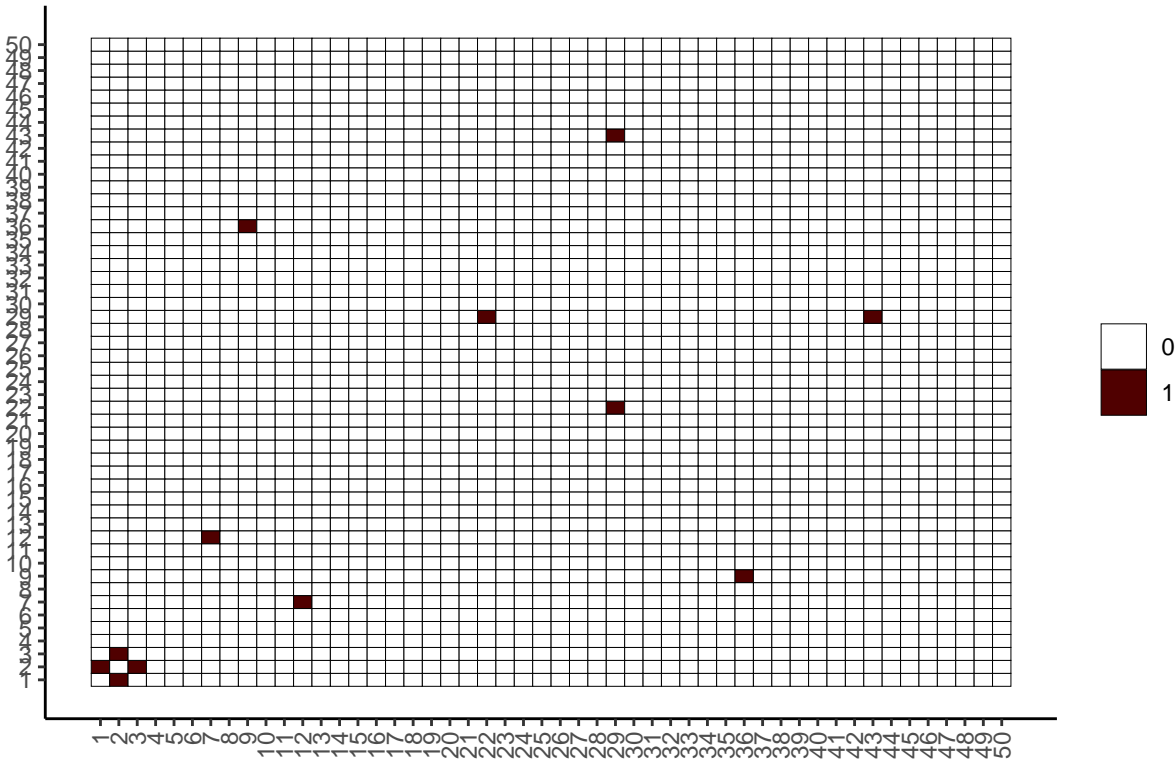
[[17]]

Graph 17, observations 65,...,68



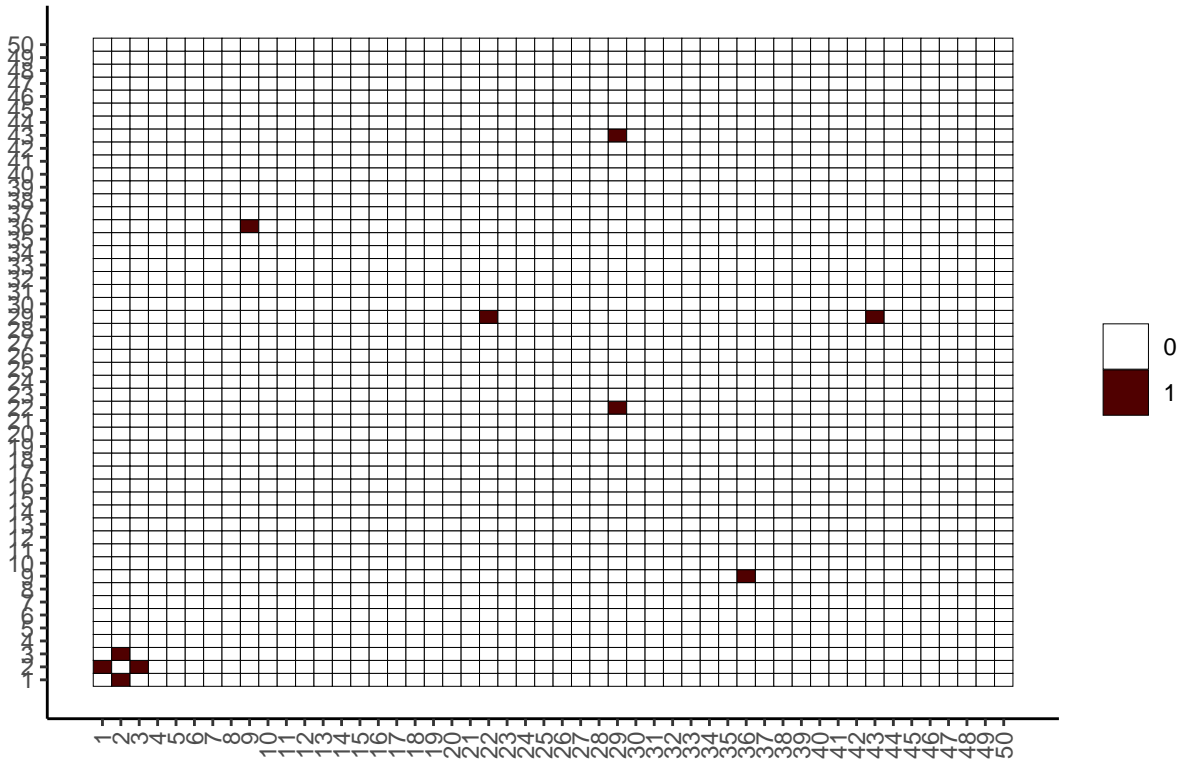
[[18]]

Graph 18, observations 69,70



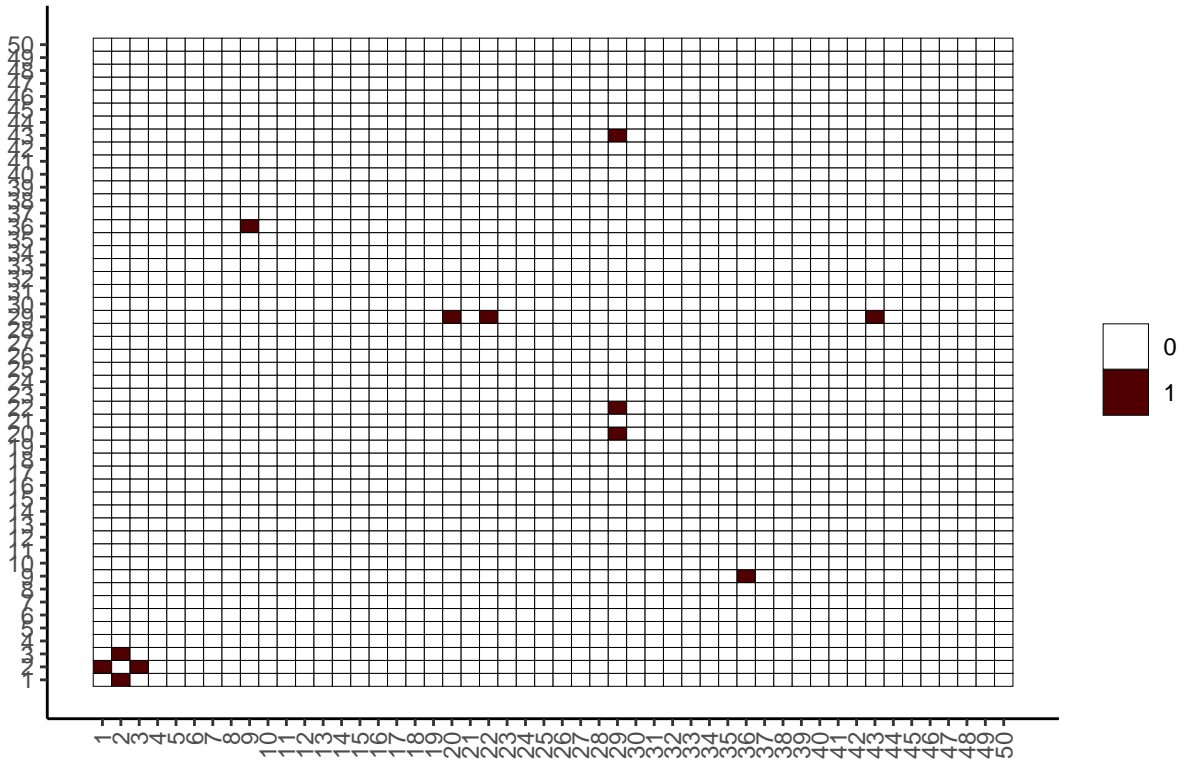
[[19]]

Graph 19, observations 71,...,79



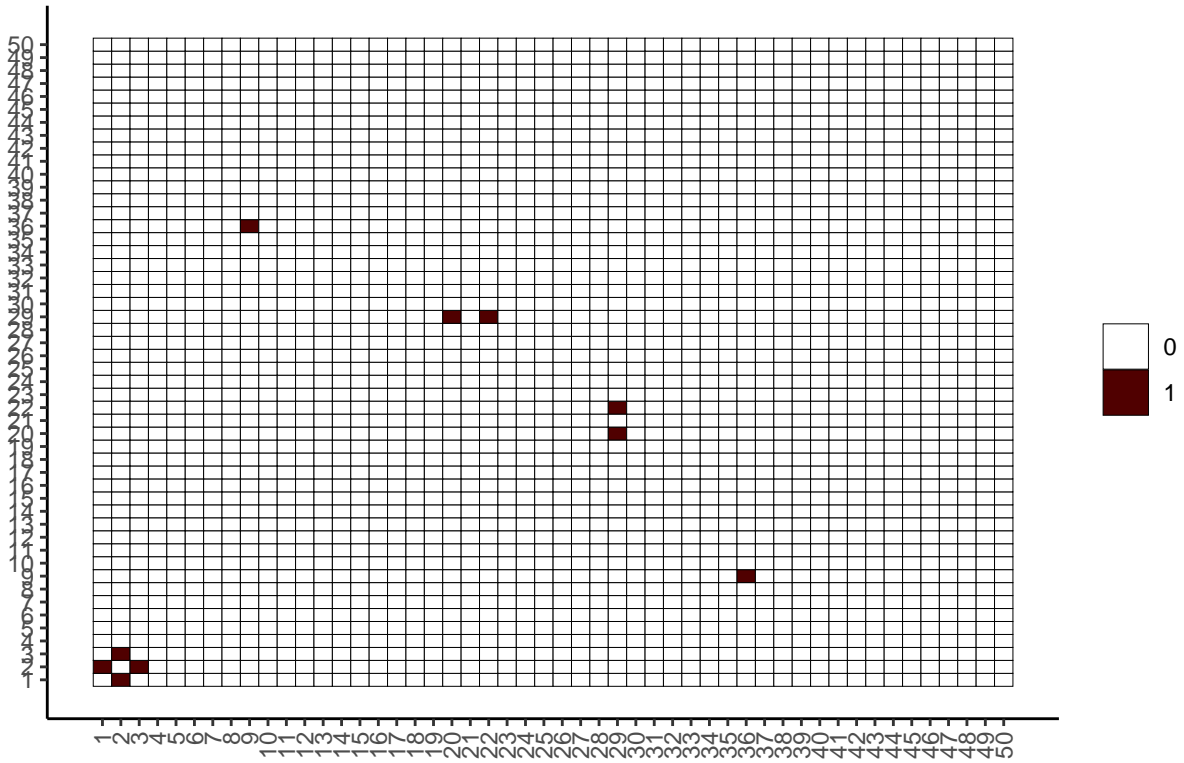
[[20]]

Graph 20, observations 80,81,82



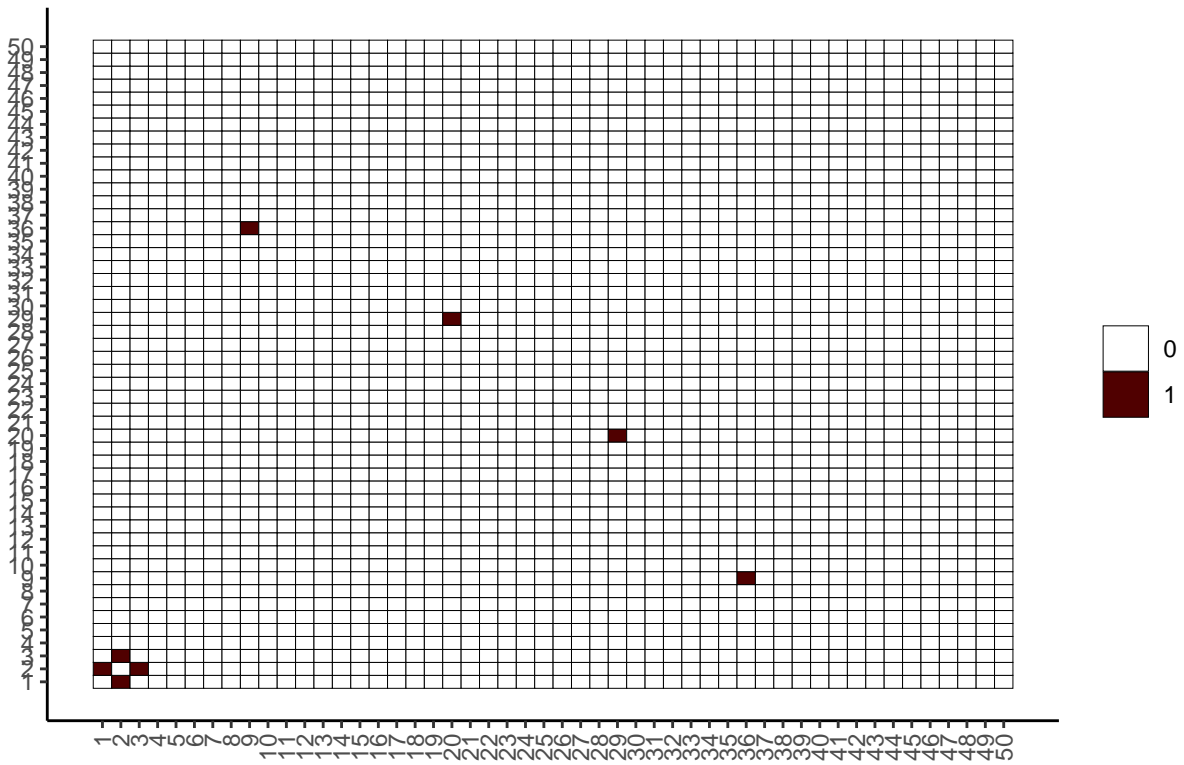
```
##  
## [[21]]
```

Graph 21, observations 83,84



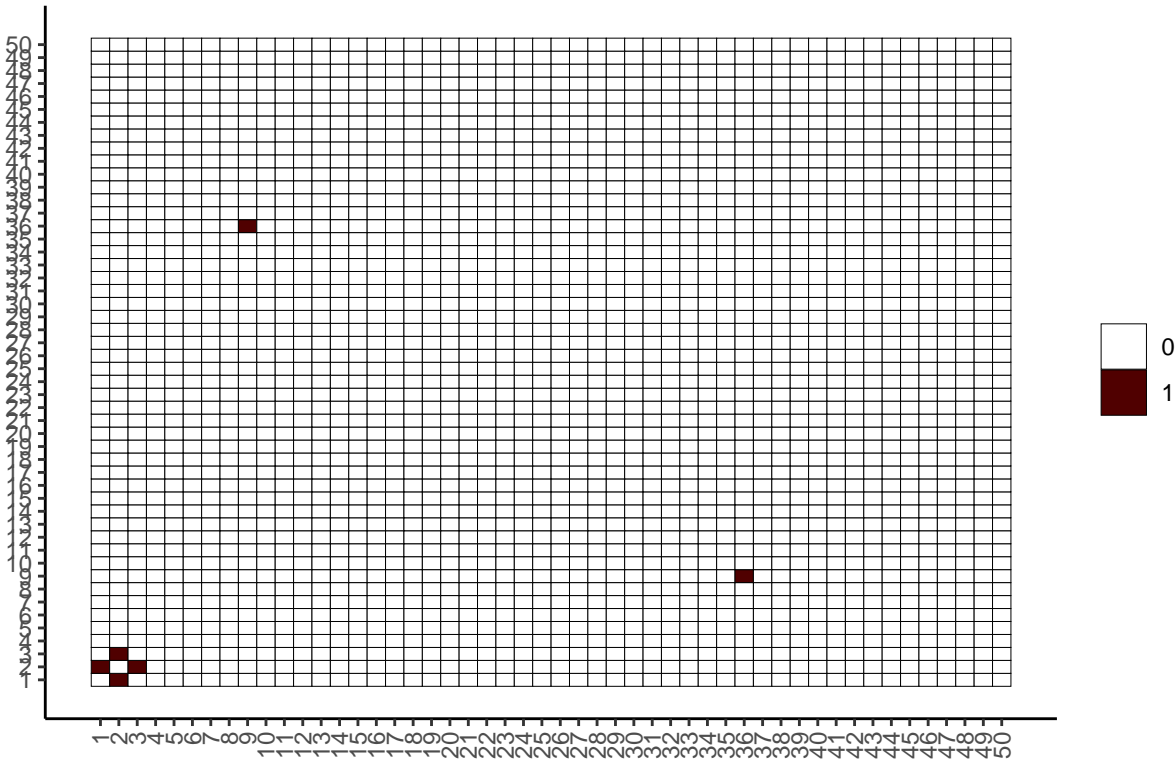
```
##
## [[22]]
```


Graph 22, observations 85,86,87



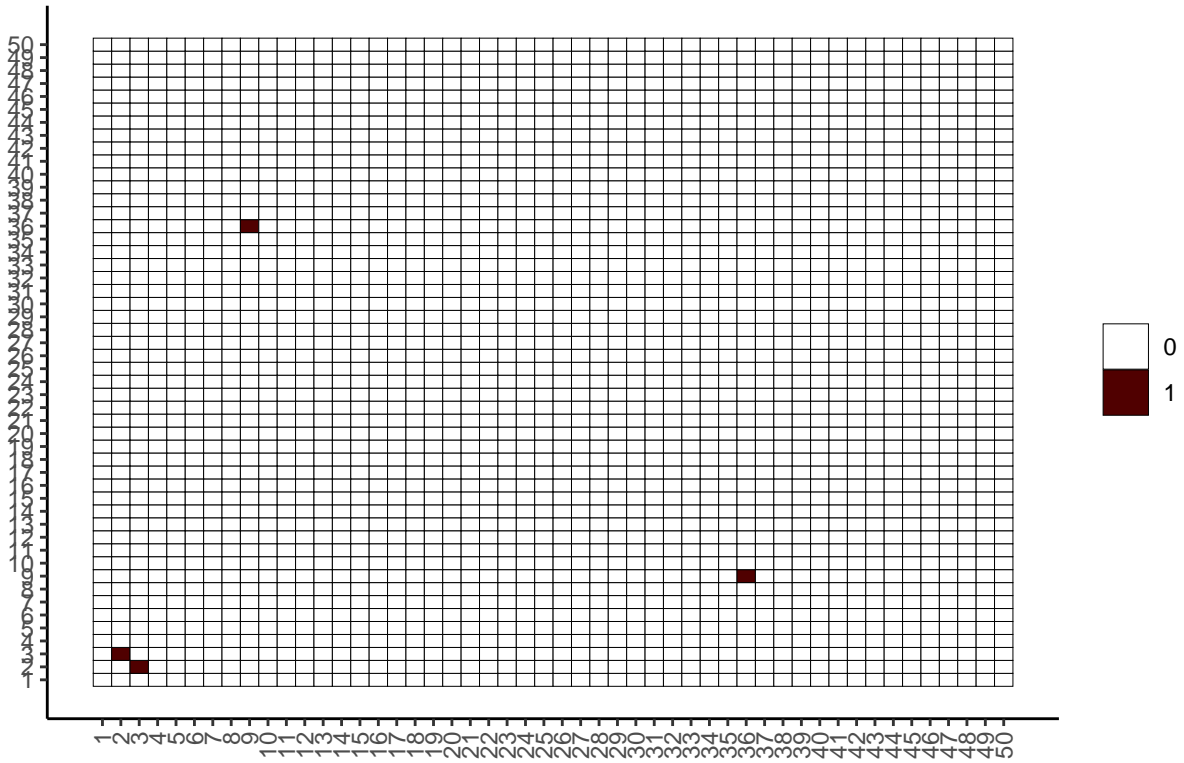
```
##
## [[23]]
```

Graph 23, observations 88,89



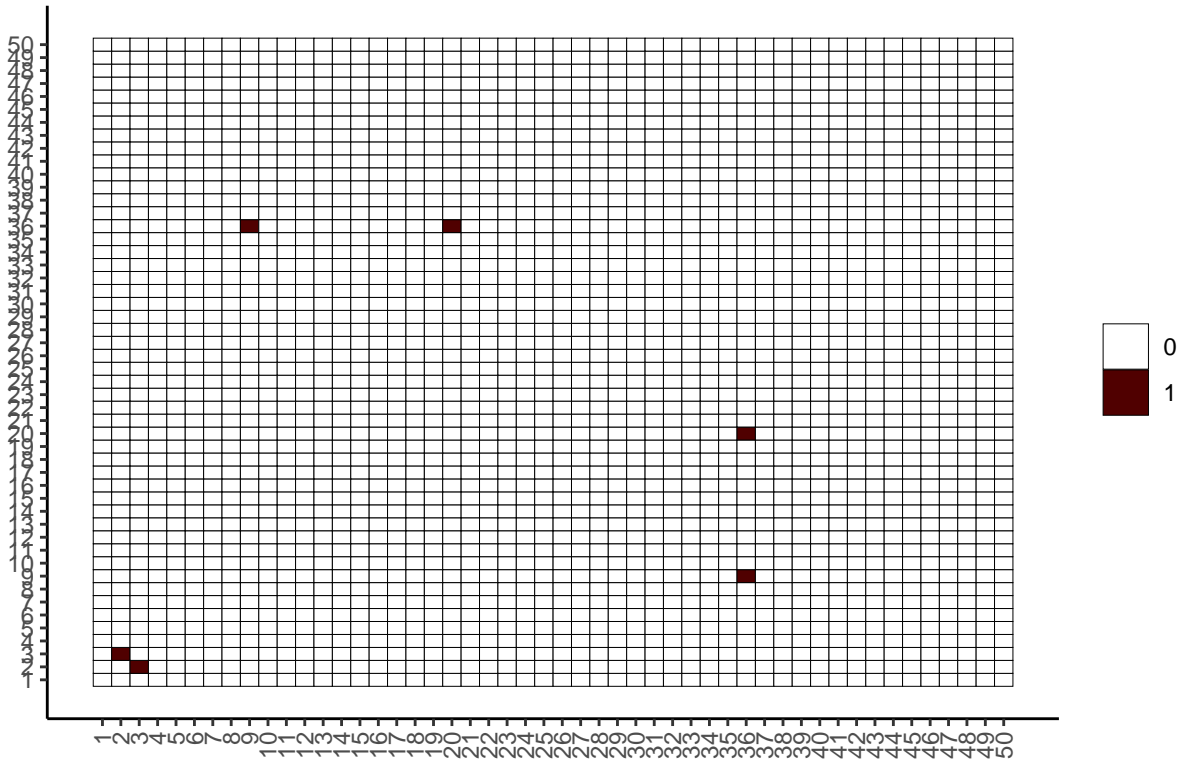
[[24]]

Graph 24, observations 90,...,93



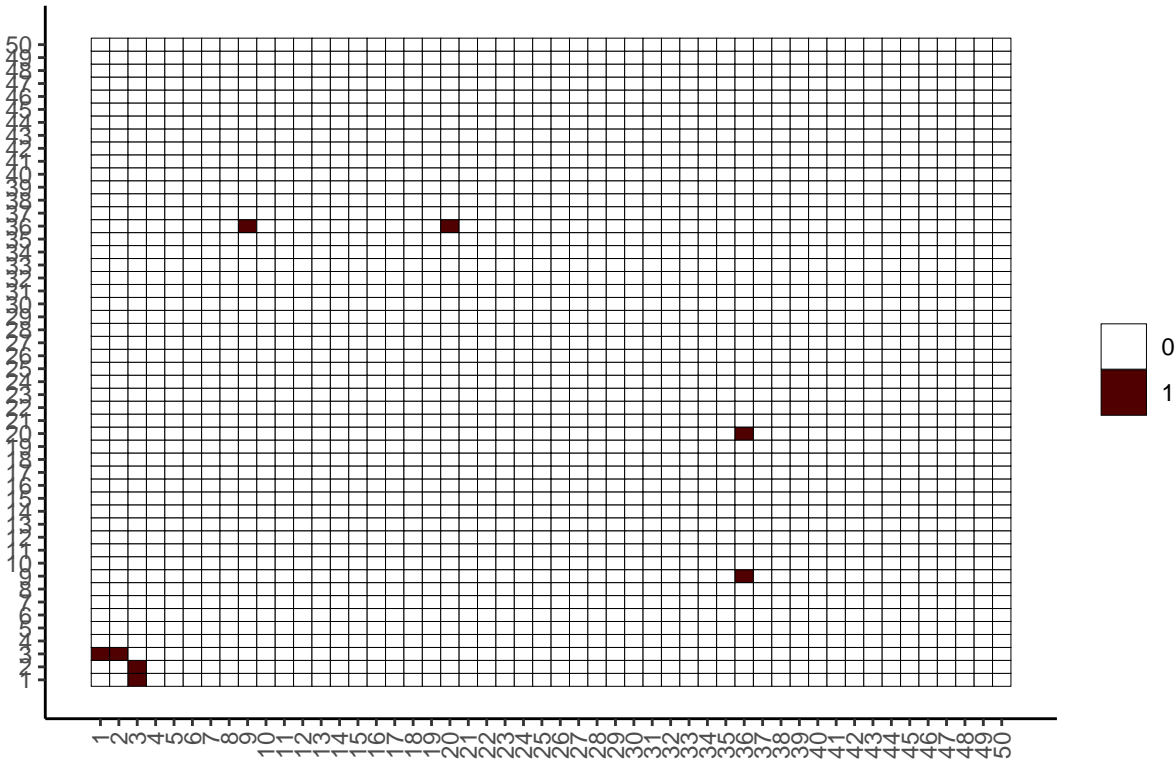
[[25]]

Graph 25, observations 94



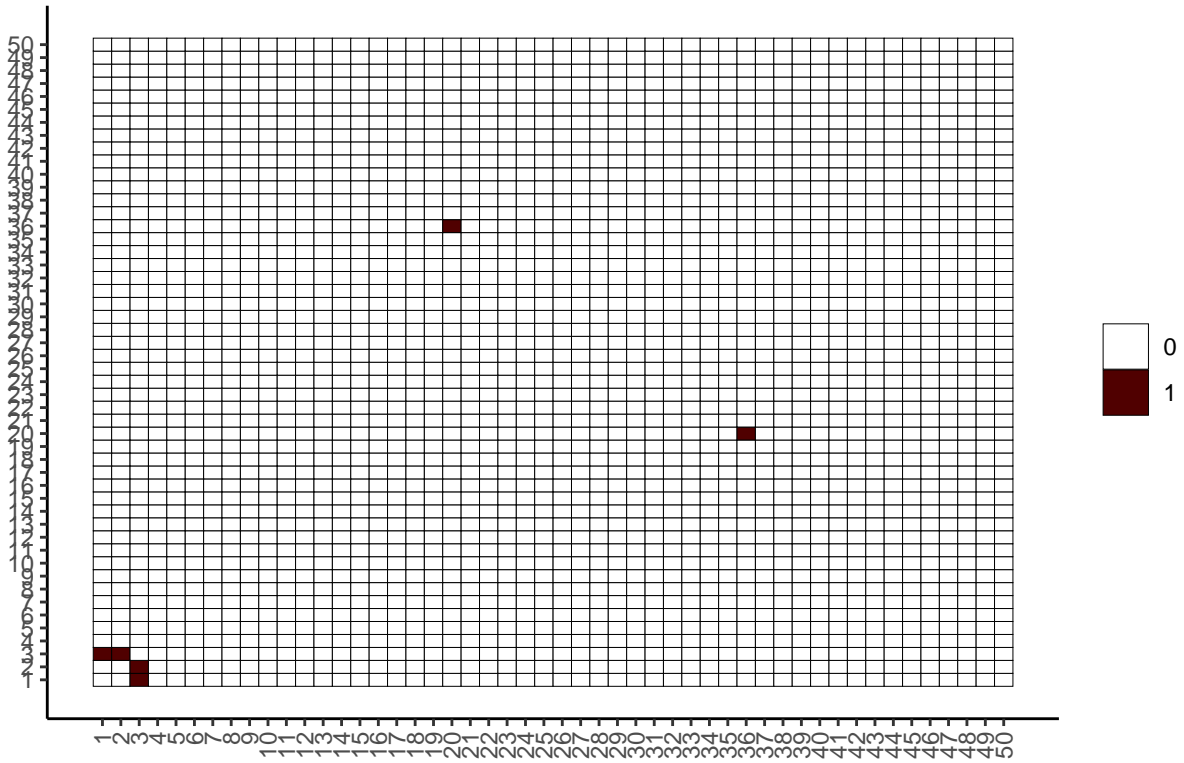
```
##  
## [[26]]
```

Graph 26, observations 95,...,100



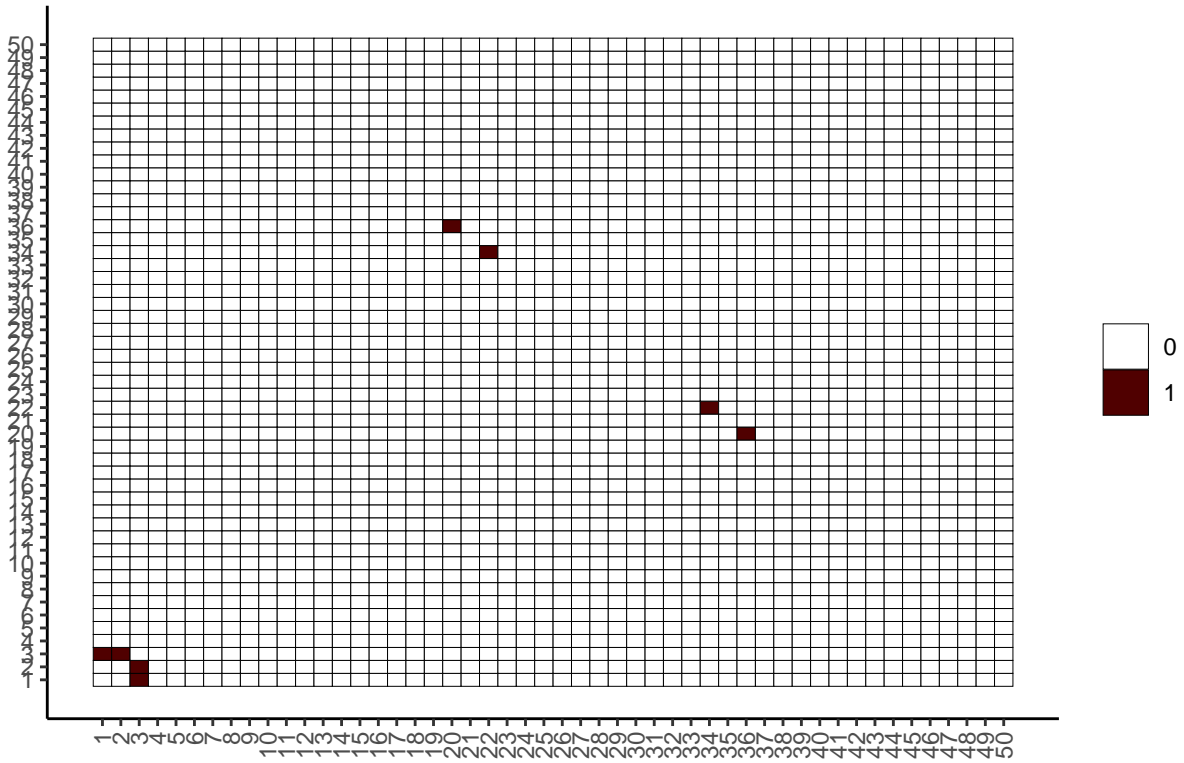
[[27]]

Graph 27, observations 101,...,122



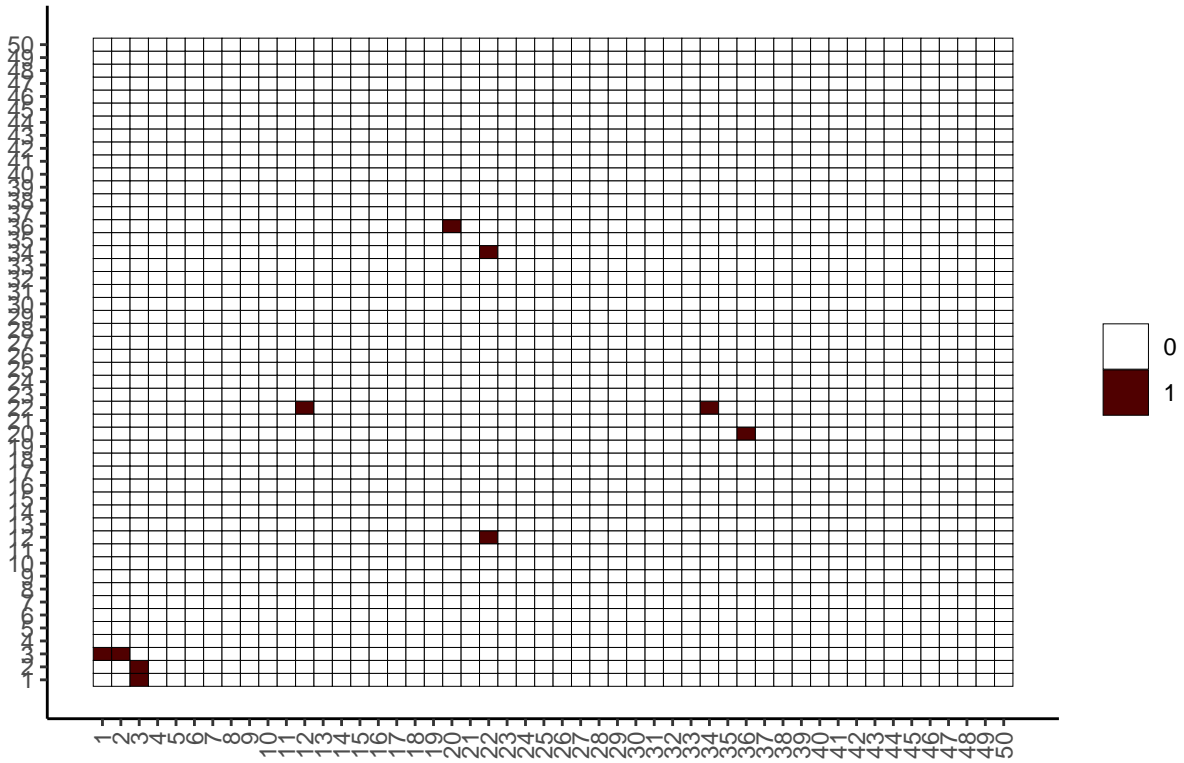
[[28]]

Graph 28, observations 123,124



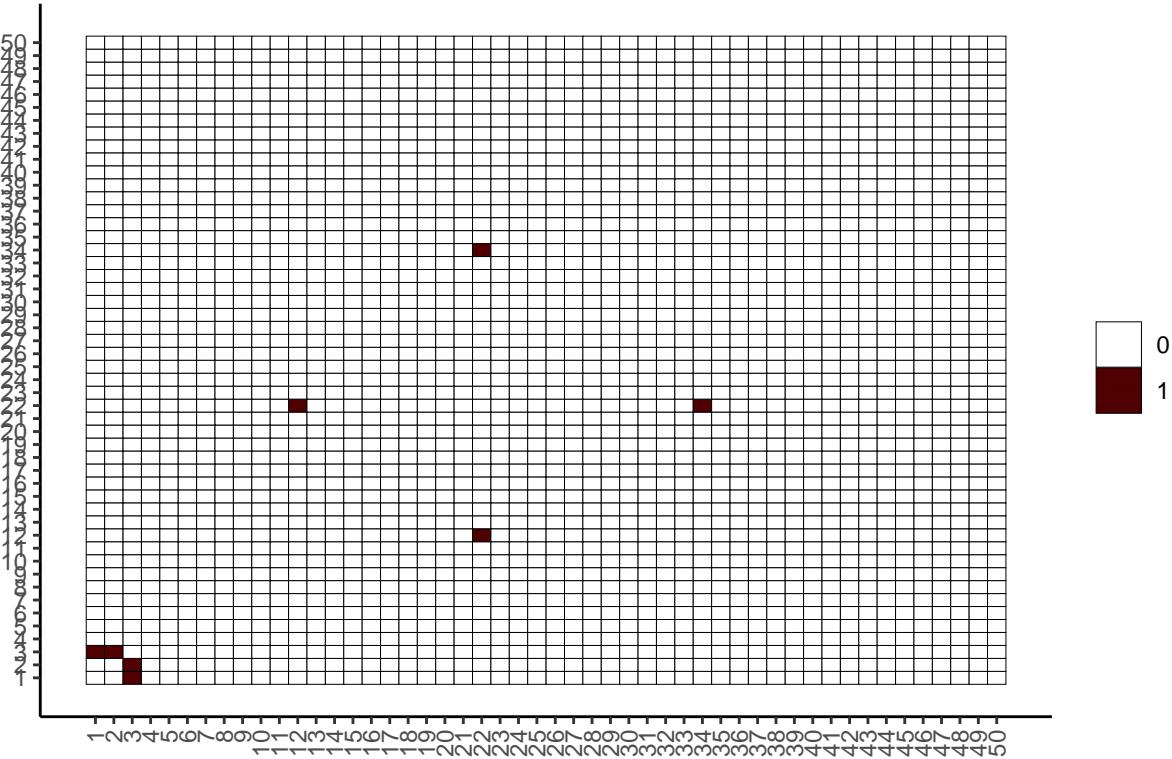
[[29]]

Graph 29, observations 125,...,128



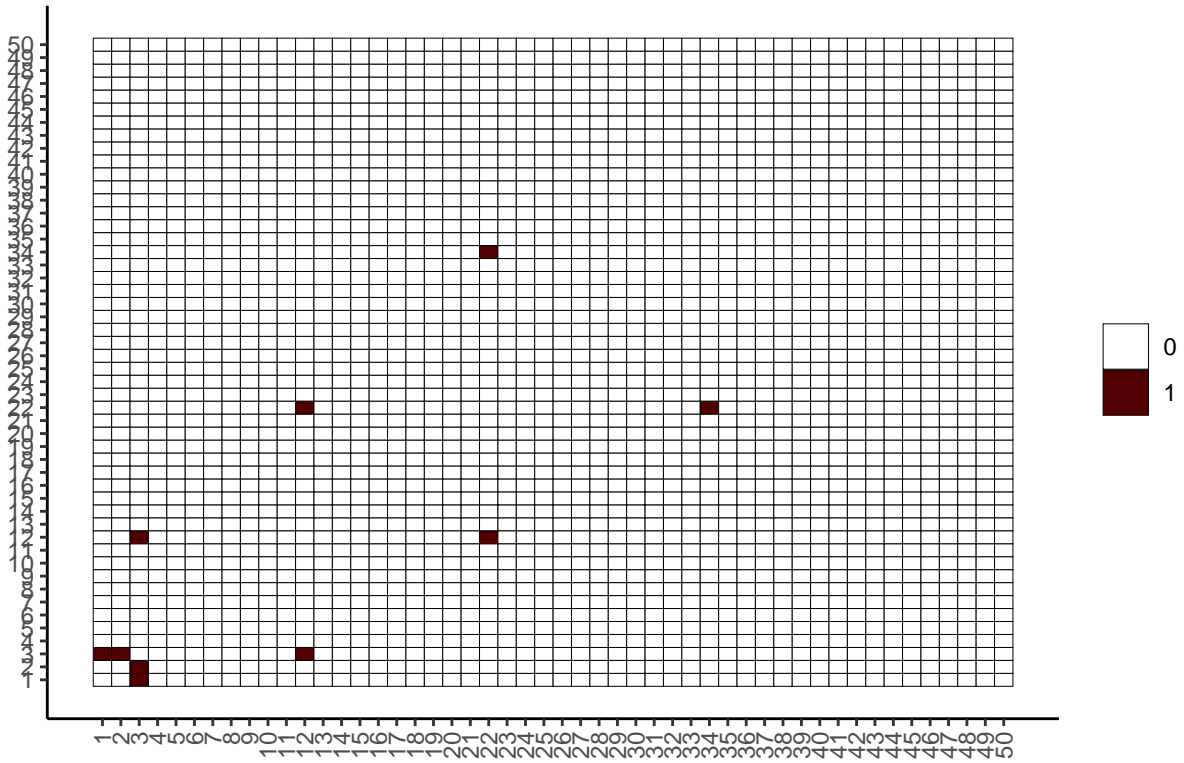
[[30]]

Graph 30, observations 129,...,132,140,...,152



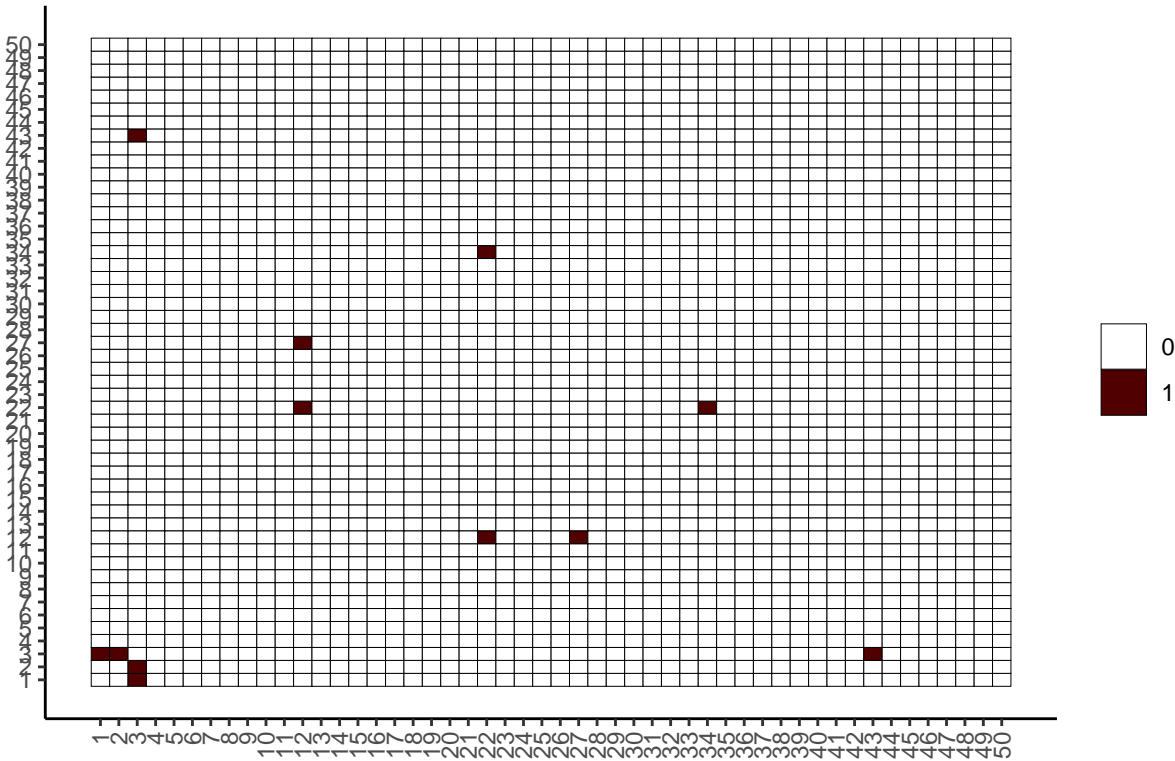
[[31]]

Graph 31, observations 133,...,139



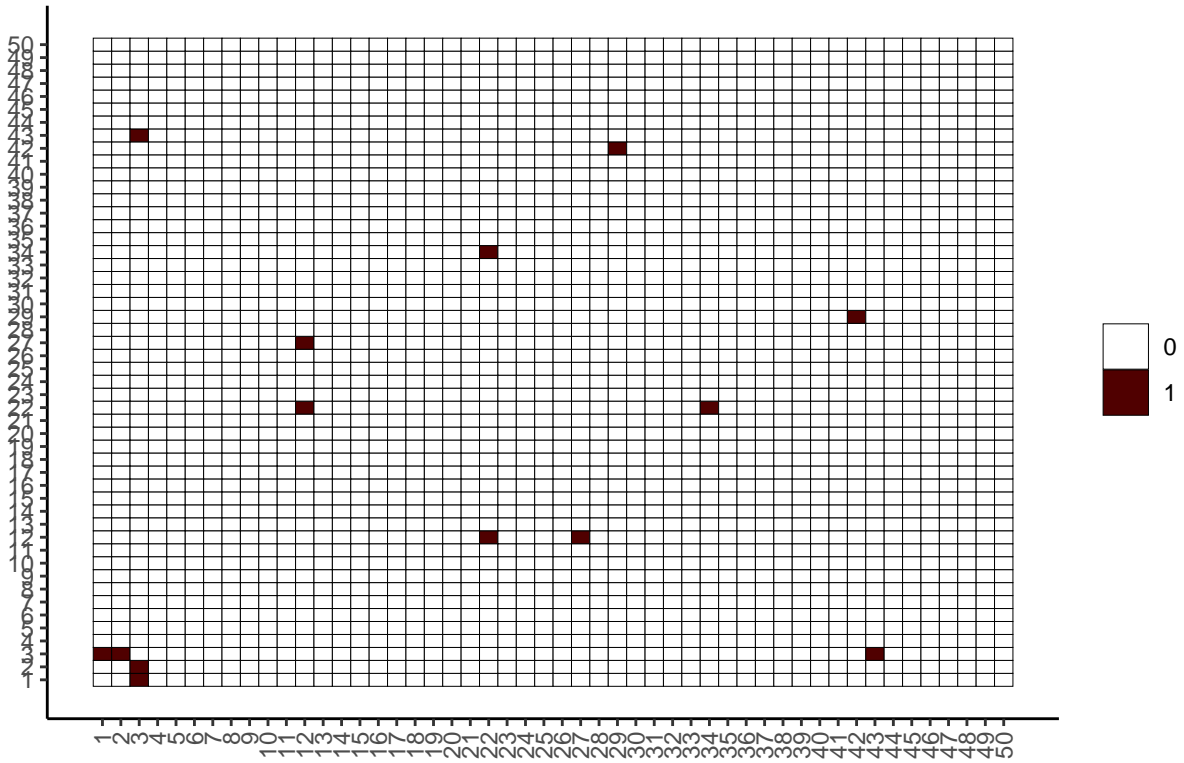
[[32]]

Graph 32, observations 153,...,169,174,...,178



[[33]]

Graph 33, observations 170,...,173



[[34]]

This figure is a heatmap visualization of the 2008 US Presidential election results by county. The x-axis represents the year (2008) and the y-axis represents the county. The color scale indicates the winning candidate: 0 (white) and 1 (dark red). The heatmap shows a distribution of election results across the United States, with a concentration of '1' (dark red) in the southern and central regions, and '0' (white) in the northern and western regions.