pi stability analysis, large p

Contents

Experiment Overview

In this experiment, I compared the results for three methods of hyperparameter selection: `M1`, `M2`, and `M3`. 50 trials were performed. `M1` proceeded as follows: In each trial, for each variable, π was selected by maximizing ELBO over the following grid:

$$\underset{\sim}{\pi} = \{0.001, 0.113, 0.226, 0.338, 0.450\}$$

σ^2 and σ^2_β were fit to the data for each variable and individual using MAPE.

The optimal π was then stabilized in the following manner: upon concluding the first grid search, another grid search was performed using the values of μ, α, σ^2, and σ^2_β corresponding to the optimal π as initial values. If the optimal π remained unchanged from the first grid search, then the grid search was concluded. However, if the optimal π changed, then the grid search was repeated until the optimal π stabilized.

`M2` proceeded exactly as `M1`, except instead of fitting σ^2 to the data using MAPE, σ^2 was fixed for all individuals as the sample variance of the variable being treated as the response.

`M3` was a pure grid search in which each of the hyperparameters was optimized by maximizing ELBO over a 3-D grid of 125 points $\sigma^2 \times \underset{\sim}{\pi} \times \underset{\sim}{\sigma^2_\beta}$. Since none of the hyperparameters were being fit to the data, the grid search was not iterated until stability as in `M1` and `M2`. $\underset{\sim}{\pi}$ was chosen as above, while $\underset{\sim}{\sigma^2_\beta}$ and $\underset{\sim}{\sigma^2}$ were:

$$\underset{\sim}{\sigma^2_\beta} = \{0.001, 0.005, 0.022, 0.106, 0.5\} \quad \underset{\sim}{\sigma^2} = \{0.2, 0.4, 0.6, 0.8, 1\}$$

The first 30 trials were performed on a relatively small dataset, with $p + 1 = 5$. The last 20 trials were performed on a larger dataset, with $p + 1 = 25$. At the end of each set of trials, the performance of all methods were compared in terms of sensitivity, specificity, accuracy, and time to fit.

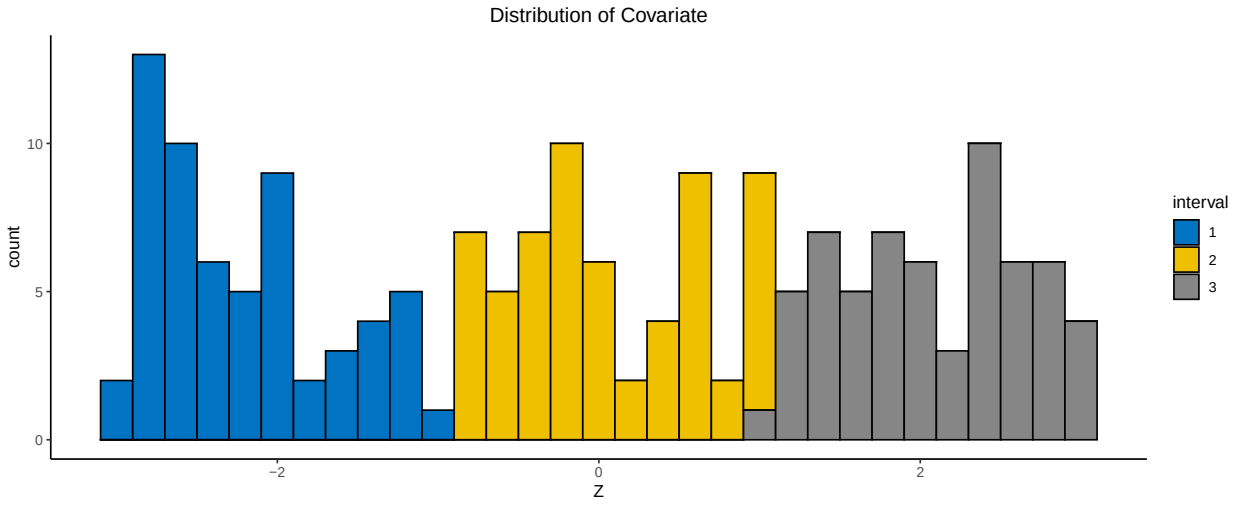
Note that I observed the fitted σ^2 blow up, resulting in erroneous results. This happened most often in the large p case. Thus, if any errors resulted for any of the 3 methods, for example, due to blowup of fitted σ^2 values, the trial was discarded.

Under each method, the CAVI updates were performed for 100 iterations before exiting, assuming that the tolerance criteria was not met prior to this.

Data Generation

Extraneous Covariate

I generated the covariate, Z, as the union of three almost disjoint intervals of equal measure. That is, $Z = Z_1 \cup Z_2 \cup Z_3$ with $Z_1 = (-3, -1), Z_2 = (a, b) = (-1, 1), Z_3 = (1, 3)$. Within each interval, I generated 60 covariate values from a uniform distribution. For example:



Precision Matrix

All of the individuals in interval 1 had the same precision matrix, $\Omega^{(1)}$:

$$\Omega^{(1)}_{i,j} = \begin{cases} 2 & i = j \\ 1 & (i,j) \in \{(1,2),(2,1),(2,3),(3,2)\} \\ 0 & o.w. \end{cases}$$

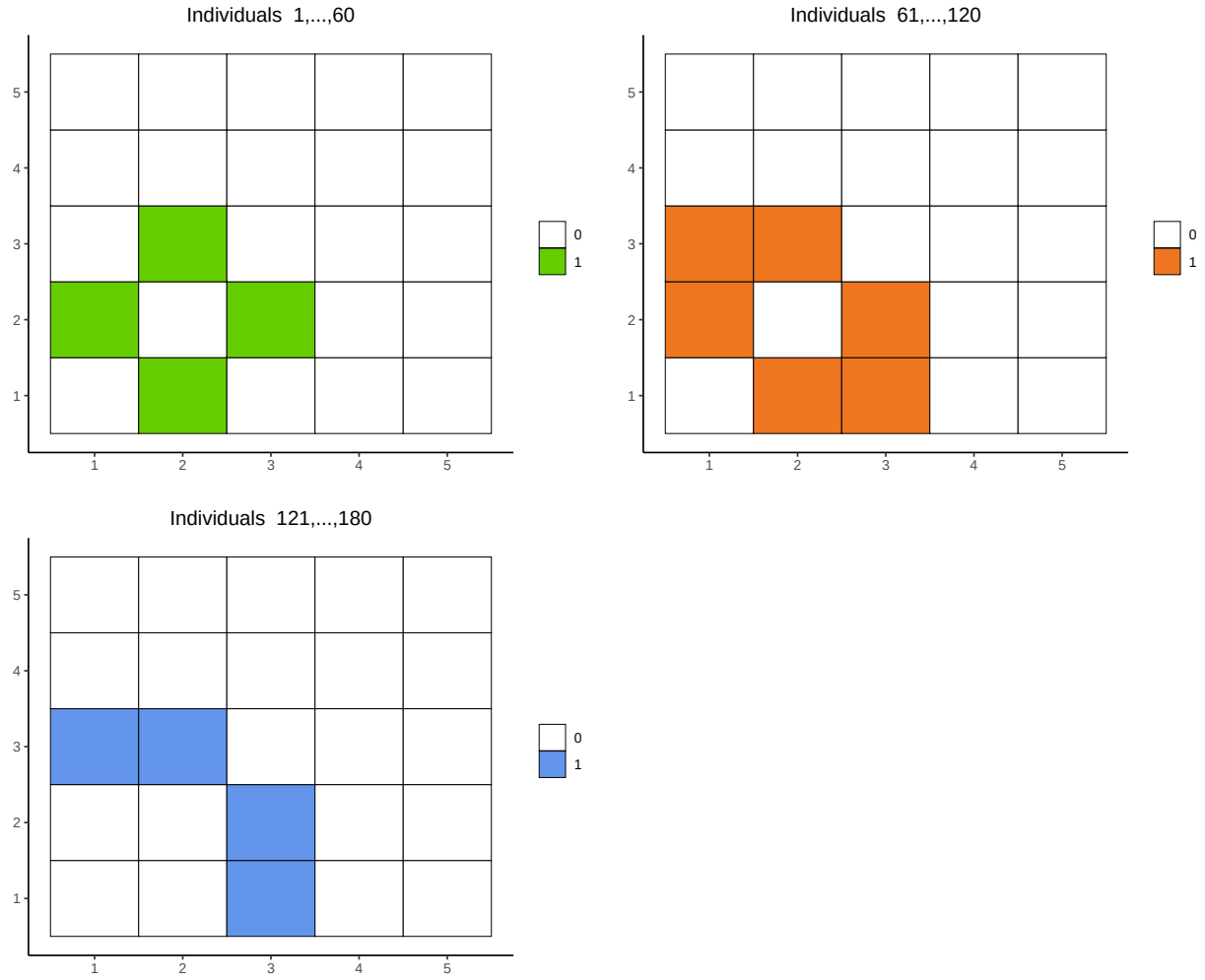
Also, all of the individuals in interval 3 had the same precision matrix, $\Omega^{(3)}$:

$$\Omega^{(3)}_{i,j} = \begin{cases} 2 & i = j \\ 1 & (i,j) \in \{(1,3),(3,1),(2,3),(3,2)\} \\ 0 & o.w. \end{cases}$$

However, the individuals in interval 2 had a precision matrix that was dependent upon Z and (a, b). Let $\beta_0 = -a/(b-a)$ and $\beta_1 = 1/(b-a)$. Then:

$$\Omega^{(2)}_{i,j}(z) = \begin{cases} 2 & i = j \\ 1 & (i,j) \in \{(2,3),(3,2)\} \\ 1 - \beta_0 - \beta_1 z & (i,j) \in \{(1,2),(2,1)\} \\ \beta_0 + \beta_1 z & (i,j) \in \{(1,3),(3,1)\} \\ 0 & o.w. \end{cases}$$

Thus, $\Omega^{(2)}(a) = \Omega^{(1)}$ and $\Omega^{(2)}(b) = \Omega^{(3)}$. That is, an individual on the left or right boundary of Z_2 would have precision matrix $\Omega^{(1)}$ or $\Omega^{(3)}$, respectively. The conditional dependence structures corresponding to each of these precision matrices are visualized below.



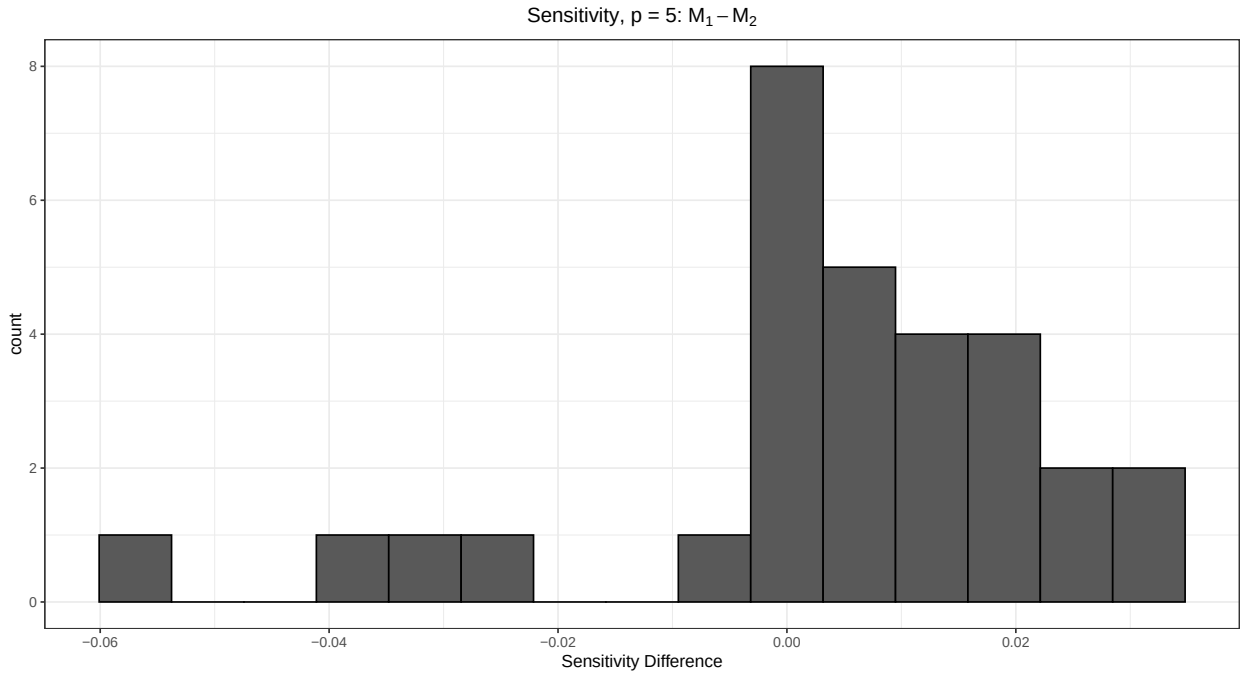
Data matrix

Let z_l be the extraneous covariate for the l-th individual. To generate the data matrix for the l-th individual, I took a random sample from $\mathcal{N}(0, \{\Omega_l(z_l)\}^{-1})$, where:

$$\Omega_l(z_l) = \begin{cases} \Omega^{(1)} & z_l \in Z_1 \\ \Omega^{(2)}(z_l) & z_l \in Z_2 \\ \Omega^{(3)} & z_l \in Z_3 \end{cases}$$

Results

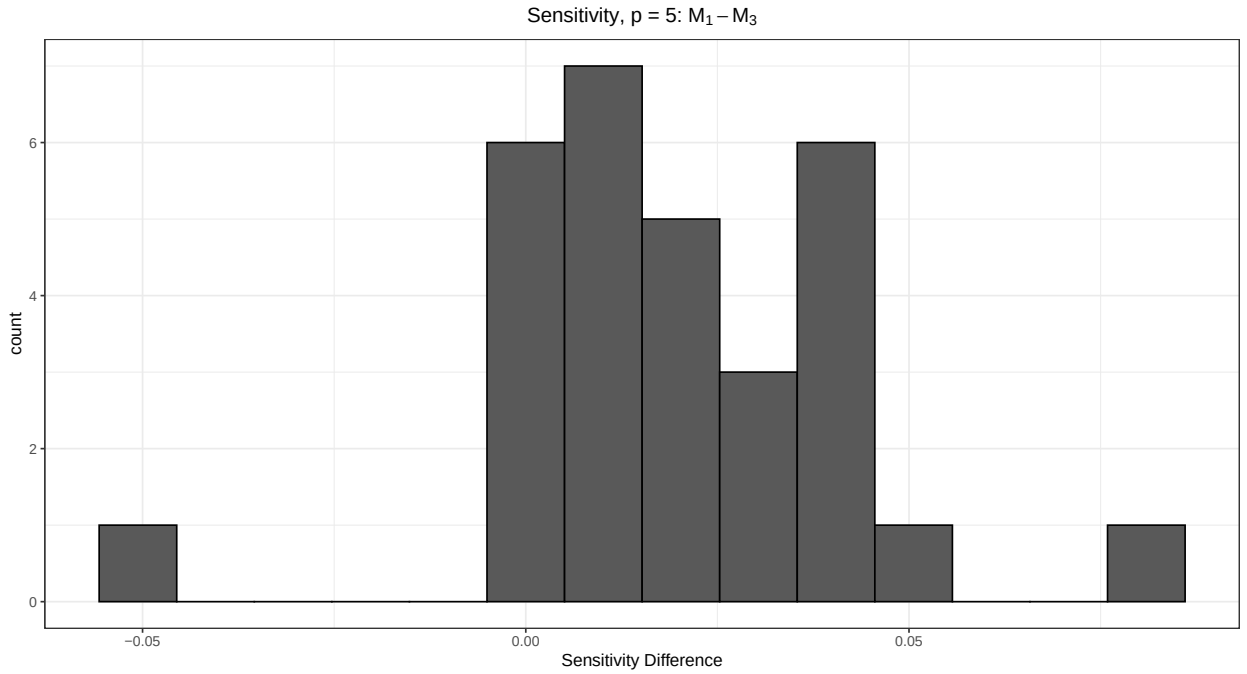
```
n <- 180
n_3 <- n^(-1/3)

# sensitivity comparison (5)

# m1 - m2
data.frame(X = m1_sens5 - m2_sens5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 5: $M_1 - M_2$")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_sens5 - m2_sens5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m1_sens5 - m2_sens5)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## -0.054762  0.000000  0.004762  0.003810  0.017857  0.030952
```

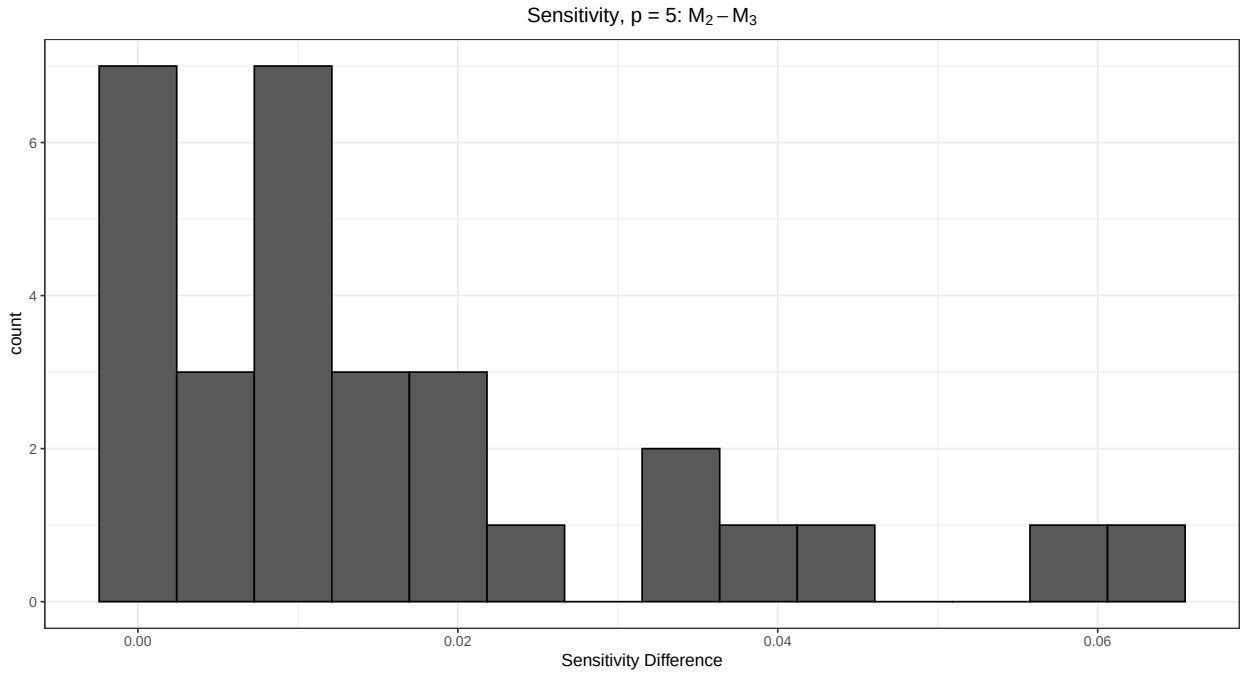
```
# m1 - m3
data.frame(X = m1_sens5 - m3_sens5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 5: $M_1 - M_3$")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_sens5 - m3_sens5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m1_sens5 - m3_sens5)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## -0.047619  0.008333  0.016667  0.020556  0.036905  0.083333
```

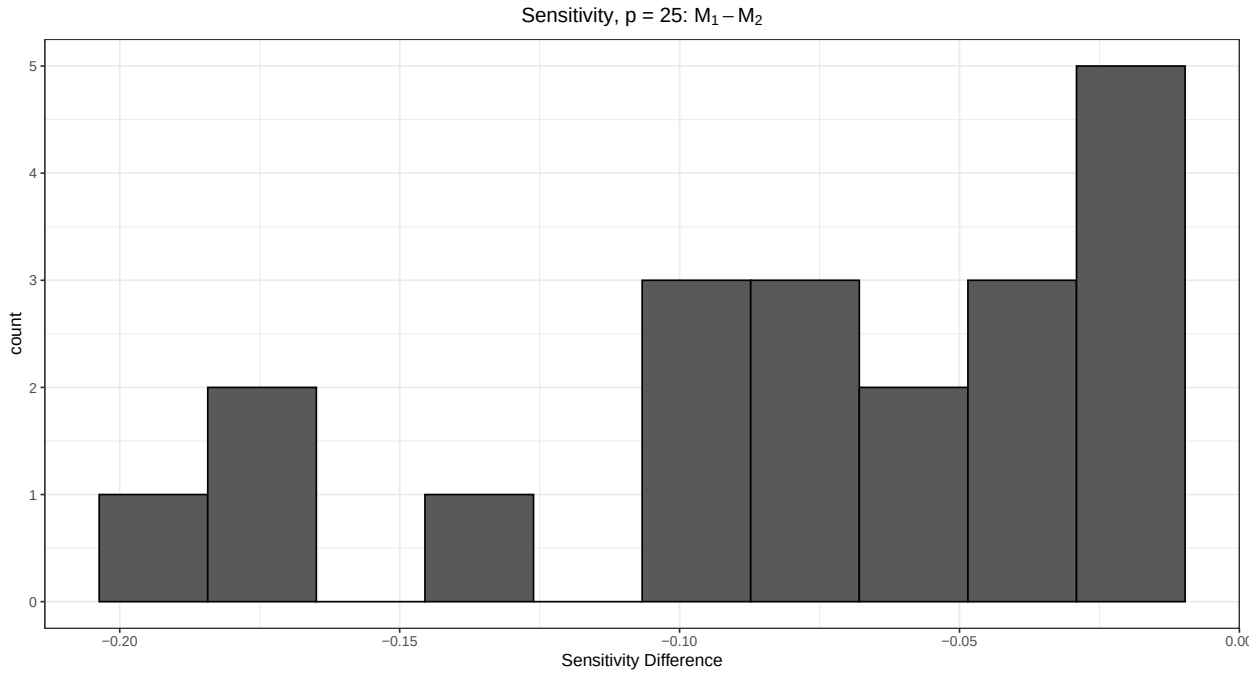
```
# m2 - m3
data.frame(X = m2_sens5 - m3_sens5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 5: $M_2 - M_3$")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m2_sens5 - m3_sens5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m2_sens5 - m3_sens5)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## 0.000000 0.007143 0.011905 0.016746 0.020833 0.061905
```

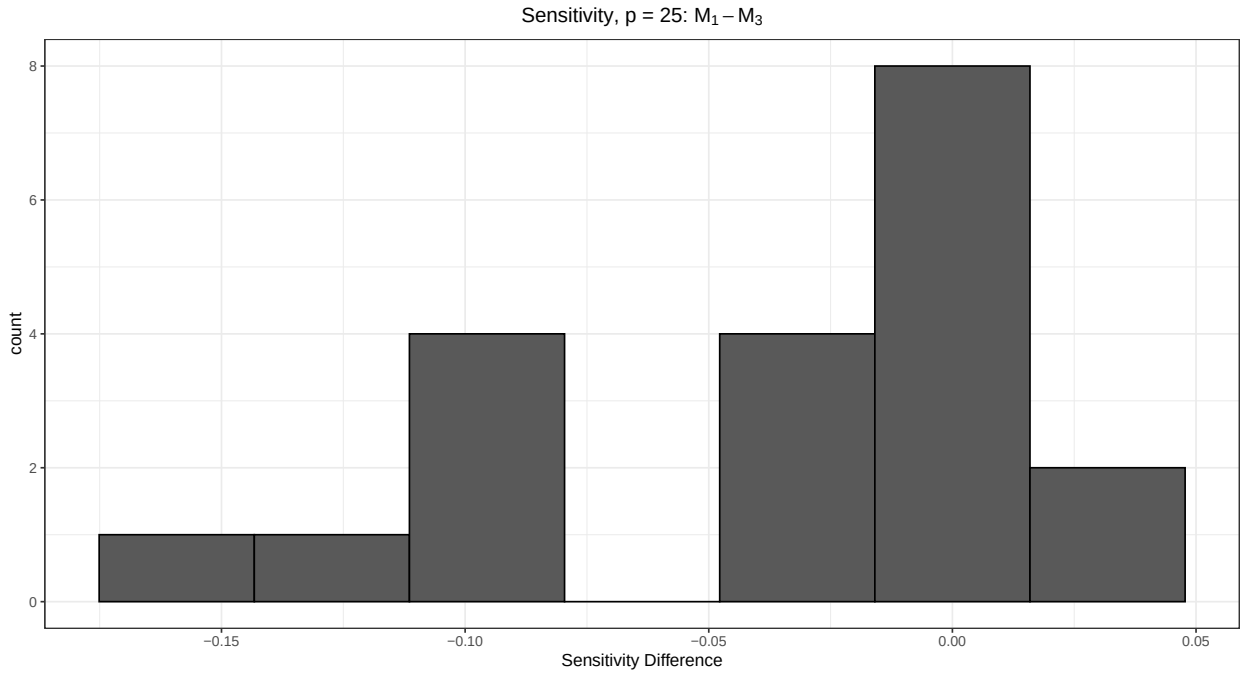
```
# sensitivity comparison (25)

# m1 - m2
data.frame(X = m1_sens25 - m2_sens25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 25: $M_1 - M_2$")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_sens25 - m2_sens25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m1_sens25 - m2_sens25)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -0.20000 -0.09048 -0.06667 -0.07548 -0.03571 -0.01429
```

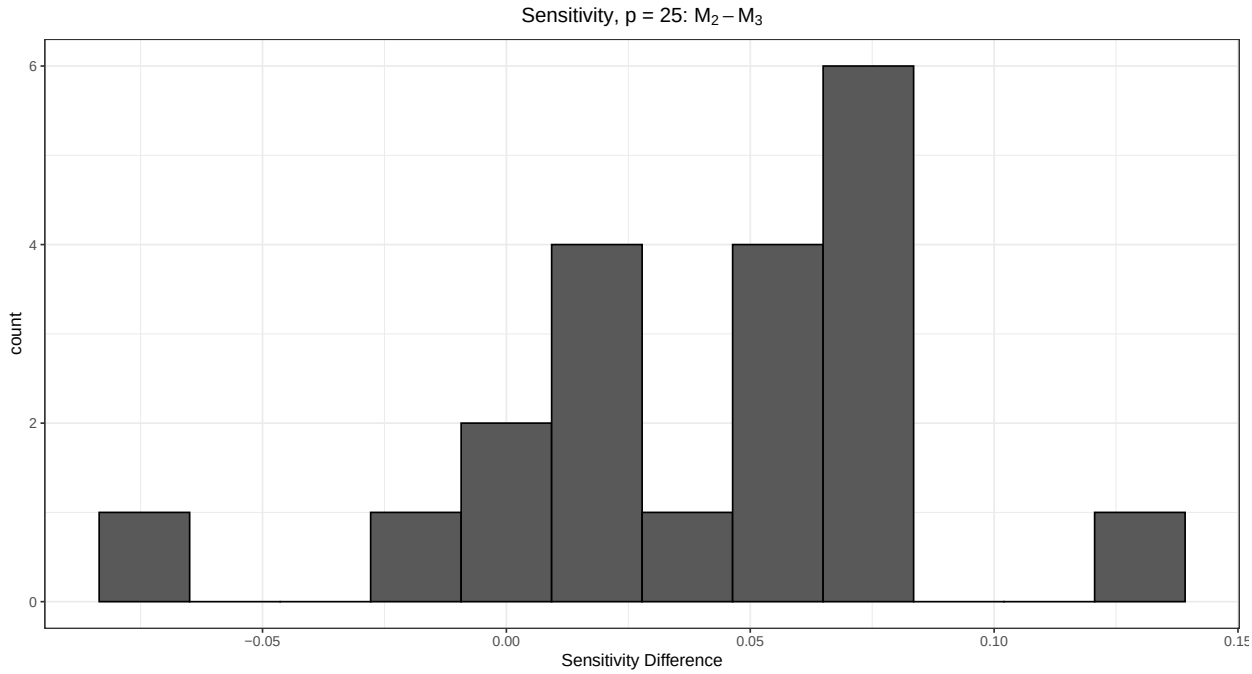
```
# m1 - m3
data.frame(X = m1_sens25 - m3_sens25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 25: $M_1 - M_3$")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_sens25 - m3_sens25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m1_sens25 - m3_sens25)
```

```
##       Min.    1st Qu.     Median       Mean    3rd Qu.       Max.
## -0.1523809 -0.0892857 -0.0154762 -0.0378571  0.0005952  0.0261905
```

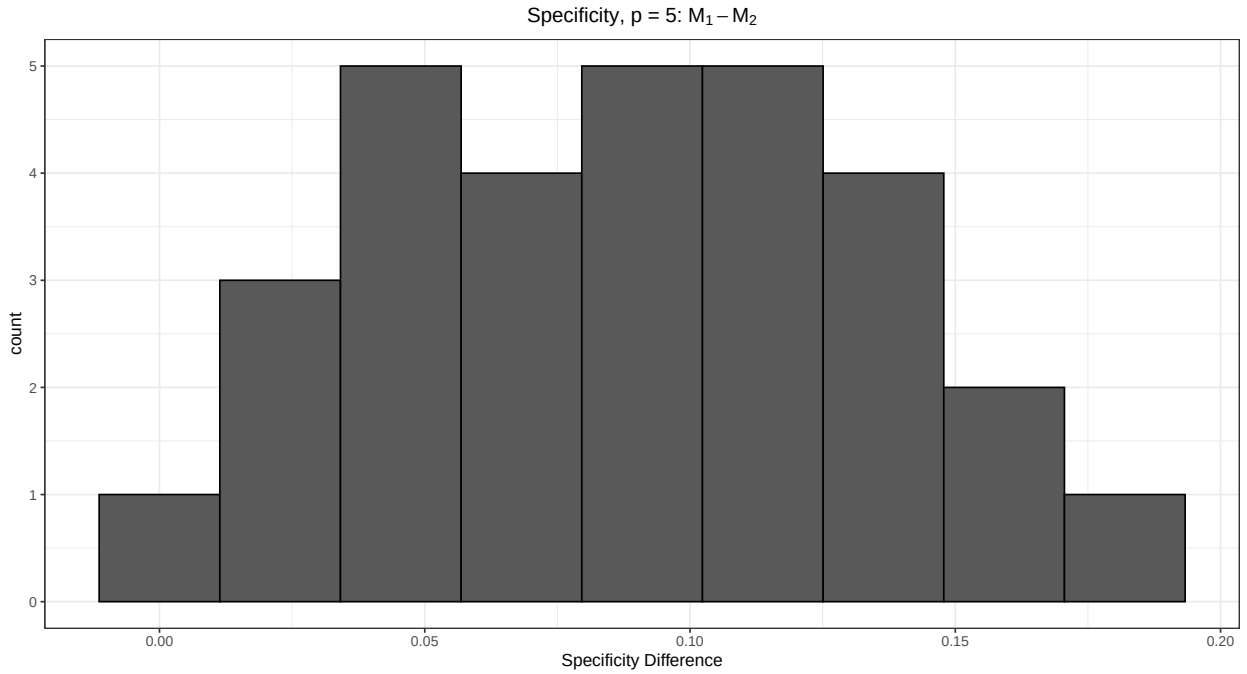
```
# m2 - m3
data.frame(X = m2_sens25 - m3_sens25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 25: $M_2 - M_3$")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m2_sens25 - m3_sens25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m2_sens25 - m3_sens25)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -0.08333  0.01667  0.04762  0.03762  0.06905  0.12381
```

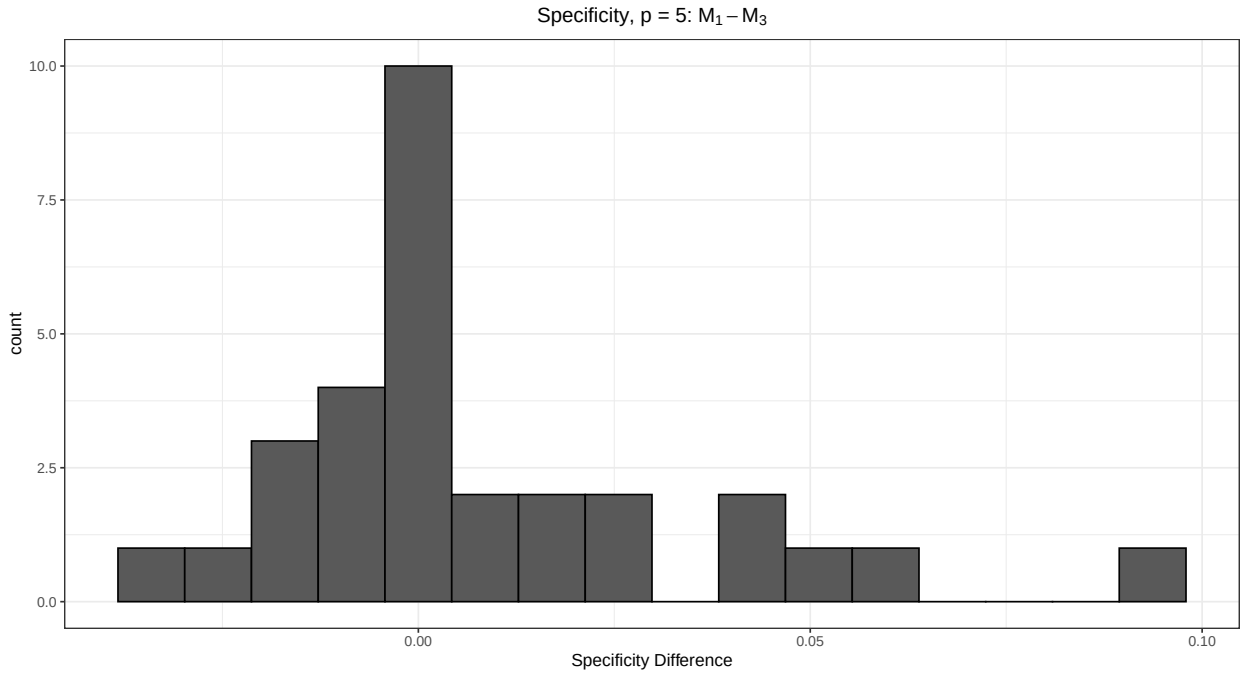
```
# specificity comparison (5)

# m1 - m2
data.frame(X = m1_spec5 - m2_spec5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 5: $M_1 - M_2$")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_spec5 - m2_spec5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m1_spec5 - m2_spec5)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## 0.0005464 0.0498634 0.0907104 0.0882878 0.1140710 0.1868853
```

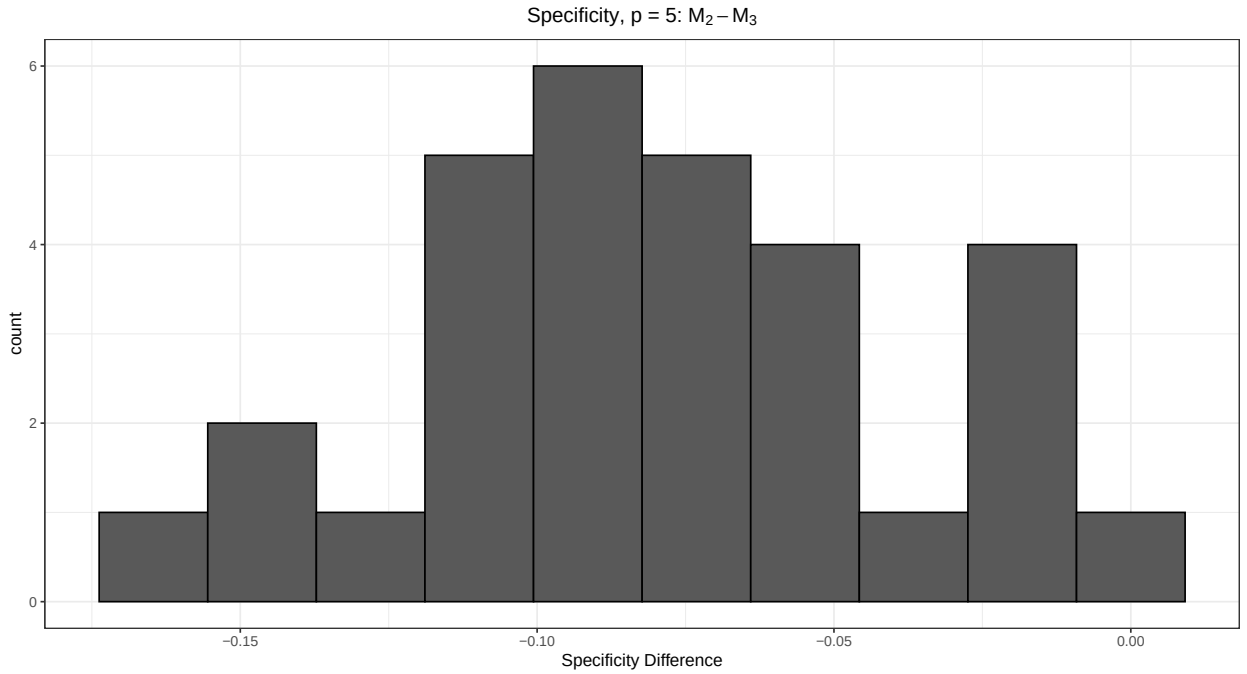
```
# m1 - m3
data.frame(X = m1_spec5 - m3_spec5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 5: $M_1 - M_3$")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_spec5 - m3_spec5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m1_spec5 - m3_spec5)
```

```
##       Min.    1st Qu.     Median       Mean    3rd Qu.       Max.
## -0.032787 -0.005874  0.000000  0.007978  0.018169  0.093443
```

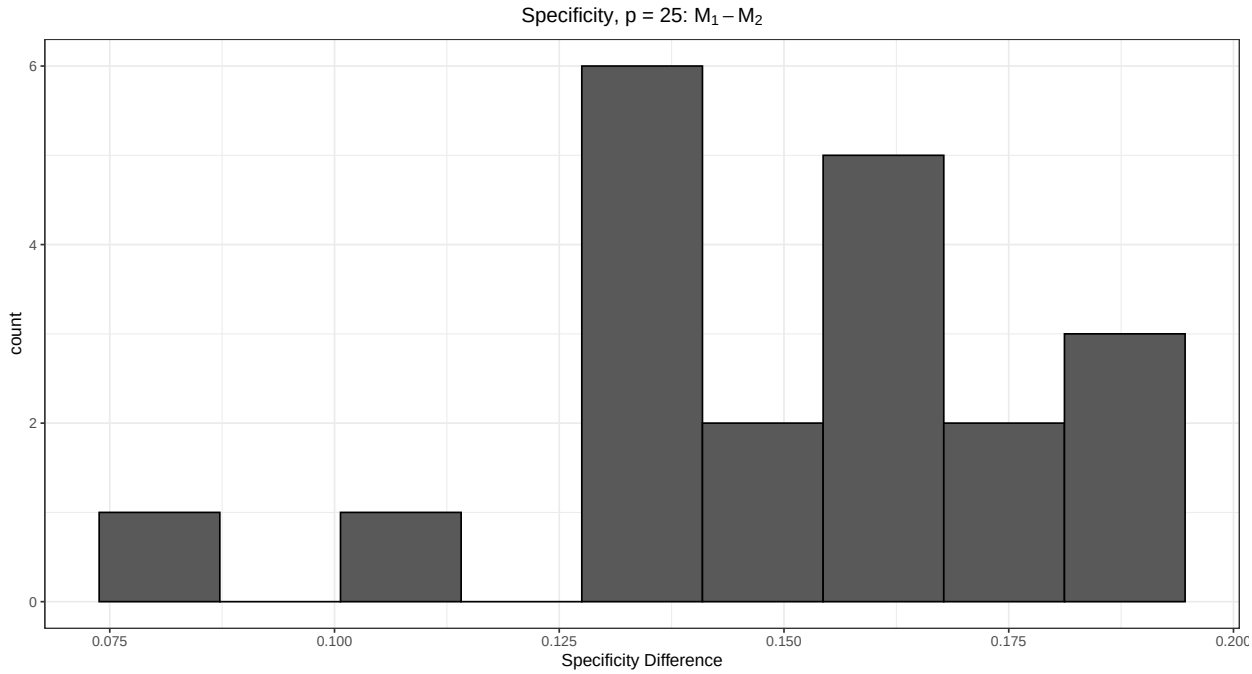
```
# m2 - m3
data.frame(X = m2_spec5 - m3_spec5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 5: $M_2 - M_3$")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m2_spec5 - m3_spec5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m2_spec5 - m3_spec5)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## -0.171038 -0.105328 -0.084426 -0.080310 -0.053689 -0.006011
```

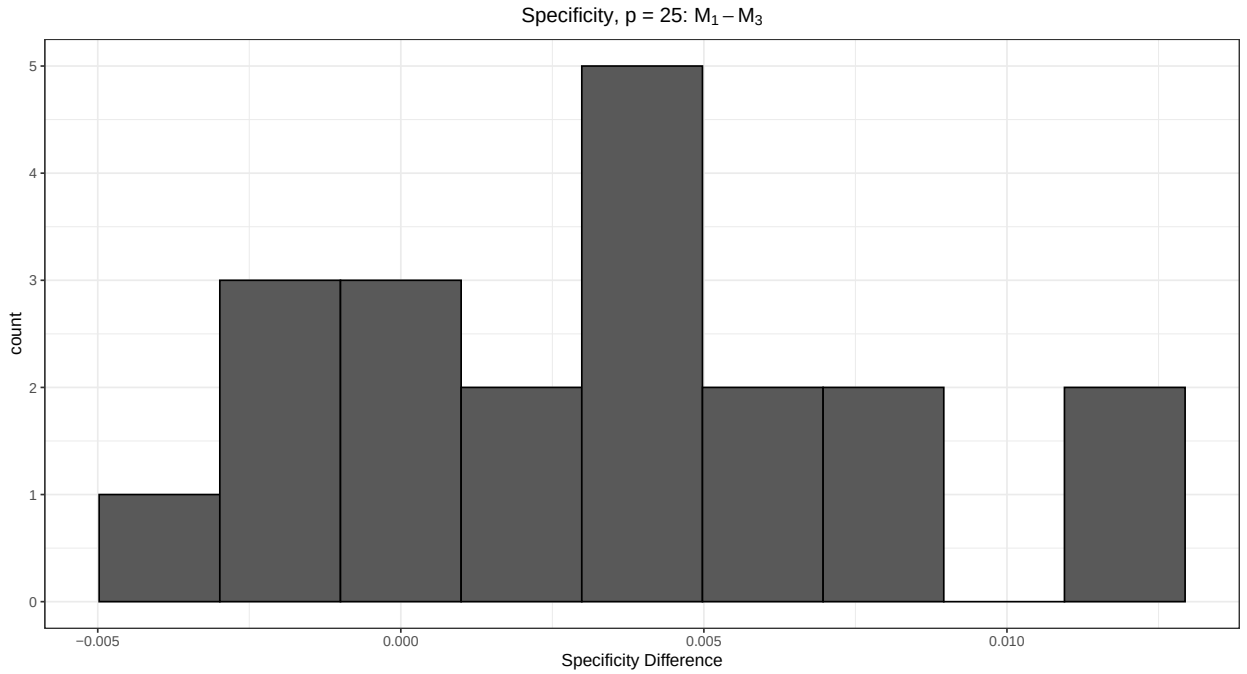
```
# specificity comparison (25)

# m1 - m2
data.frame(X = m1_spec25 - m2_spec25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 25: $M_1 - M_2$")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_spec25 - m2_spec25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m1_spec25 - m2_spec25)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.08601 0.13079 0.15419 0.14986 0.16868 0.18812
```

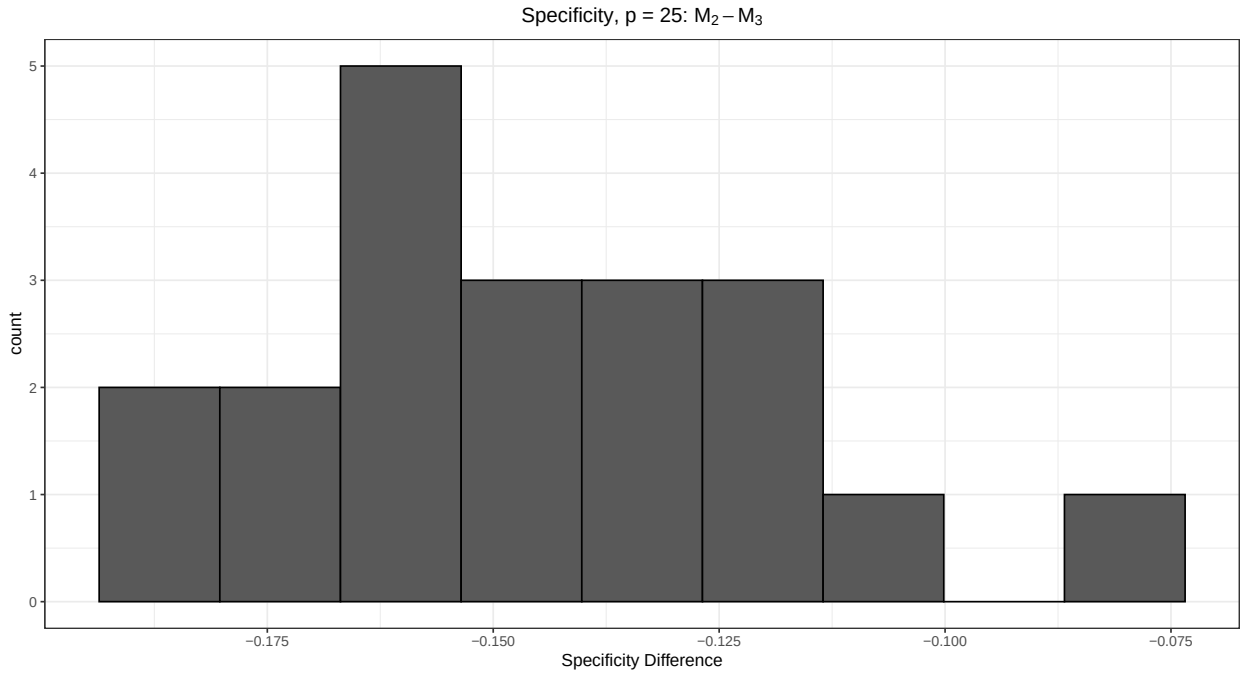
```
# m1 - m3
data.frame(X = m1_spec25 - m3_spec25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 25: $M_1 - M_3$")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_spec25 - m3_spec25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m1_spec25 - m3_spec25)
```

```
##       Min.    1st Qu.     Median       Mean    3rd Qu.       Max.
## -0.0032957  0.0003717  0.0038062  0.0033763  0.0059914  0.0120724
```

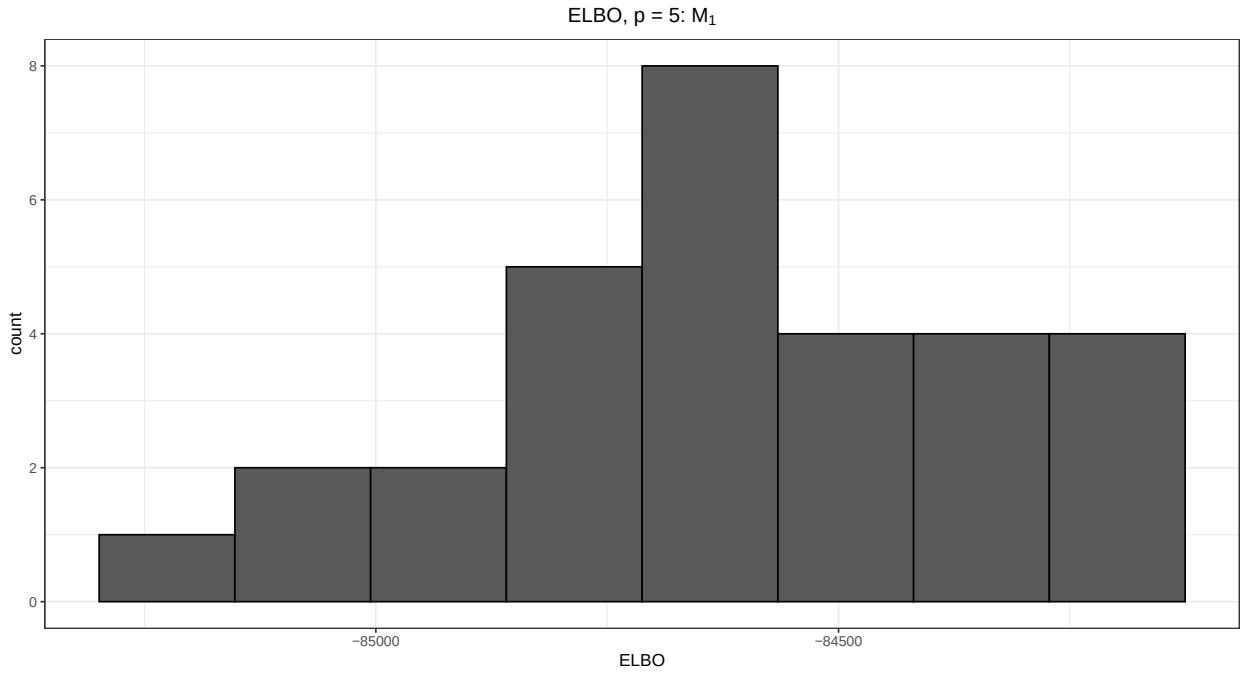
```
# m2 - m3
data.frame(X = m2_spec25 - m3_spec25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 25: $M_2 - M_3$")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m2_spec25 - m3_spec25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m2_spec25 - m3_spec25)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -0.18791 -0.16494 -0.14841 -0.14648 -0.12724 -0.08109
```

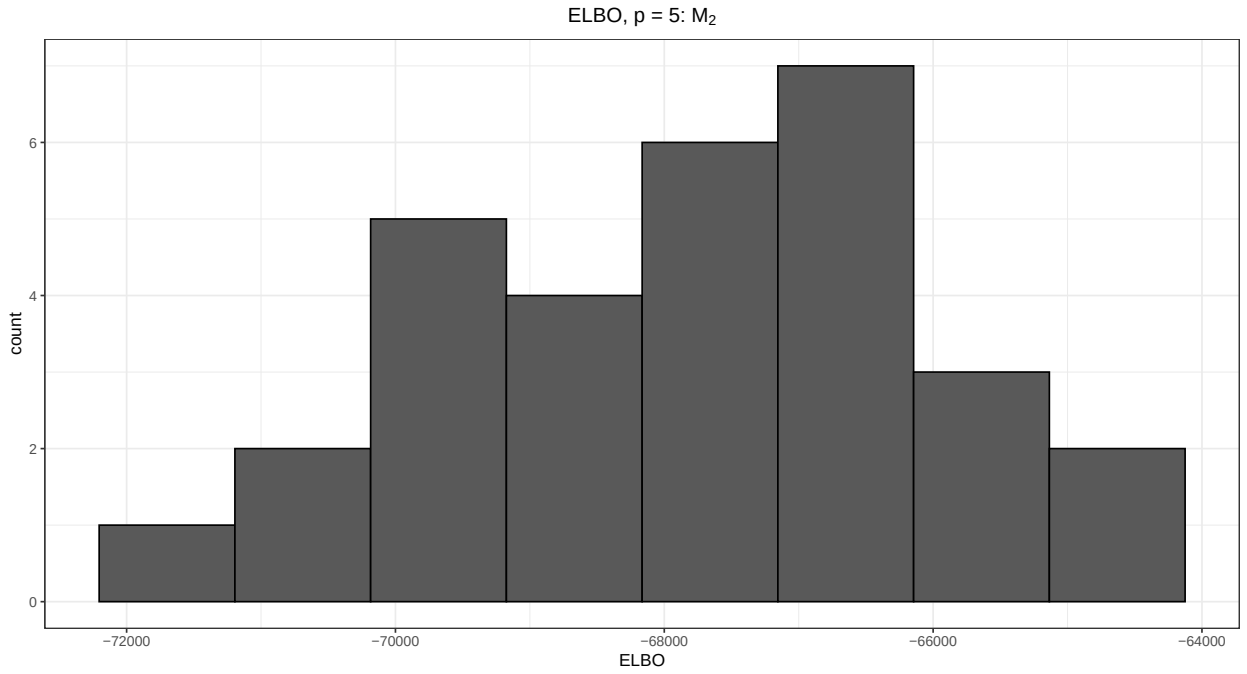
```
# distribution of ELBO

# p = 5

# m1
data.frame(X = m1_ELBO5) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO, p = 5: $M_1$")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m1_ELBO5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m1_ELBO5)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  -85164  -84793  -84608  -84623  -84379  -84210
```

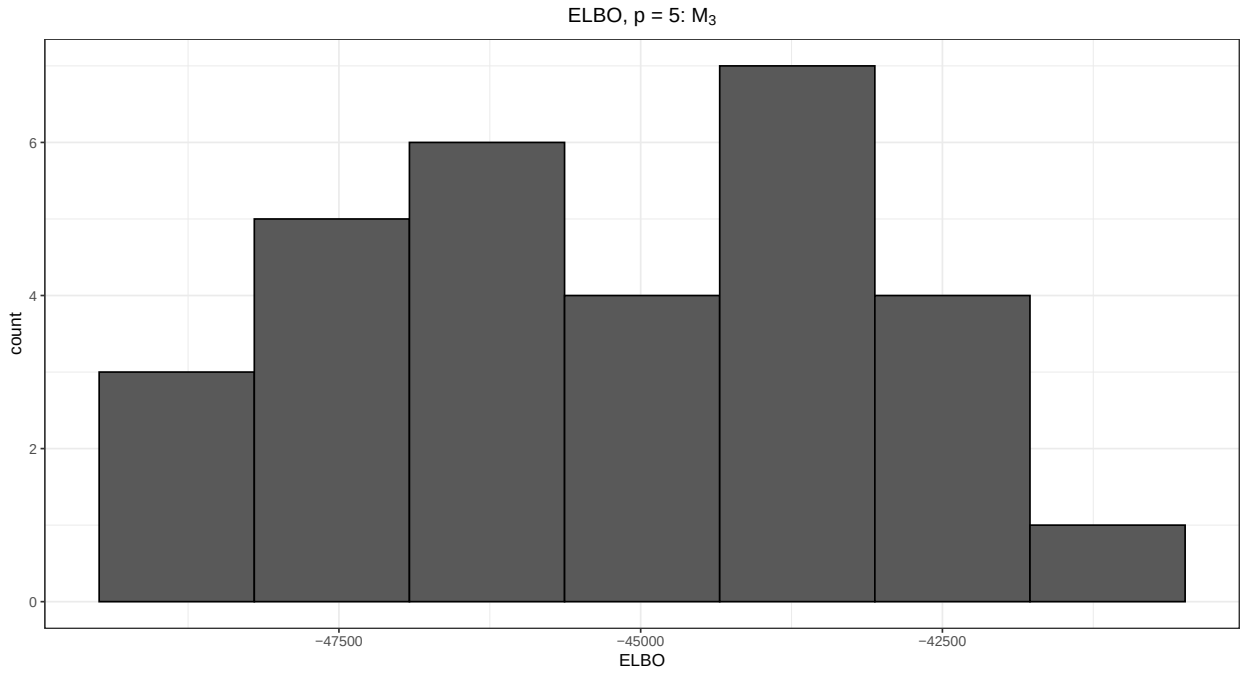
```
# m2
data.frame(X = m2_ELBO5) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO, p = 5: $M_2$")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m2_ELBO5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m2_ELBO5)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  -71496  -69328  -67904  -67857  -66477  -64592
```

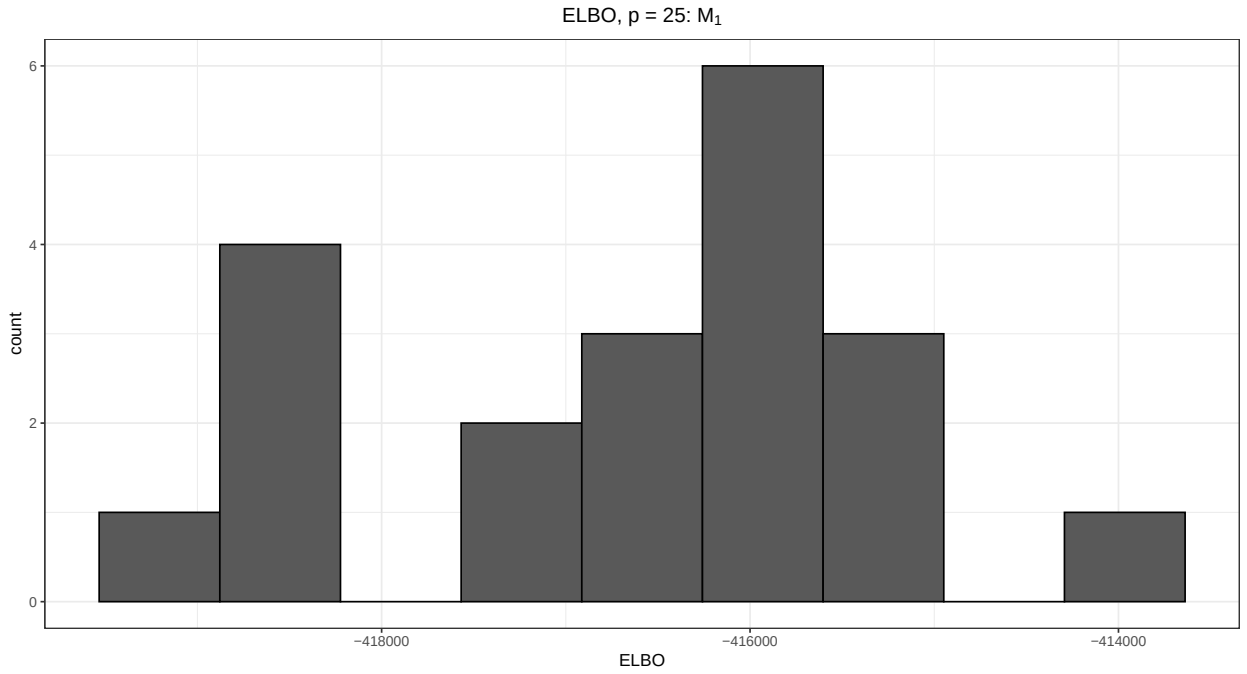
```
# m3
data.frame(X = m3_ELBO5) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO, p = 5: $M_3$")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m3_ELBO5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m3_ELBO5)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  -49121  -46917  -45228  -45239  -43288  -40581
```

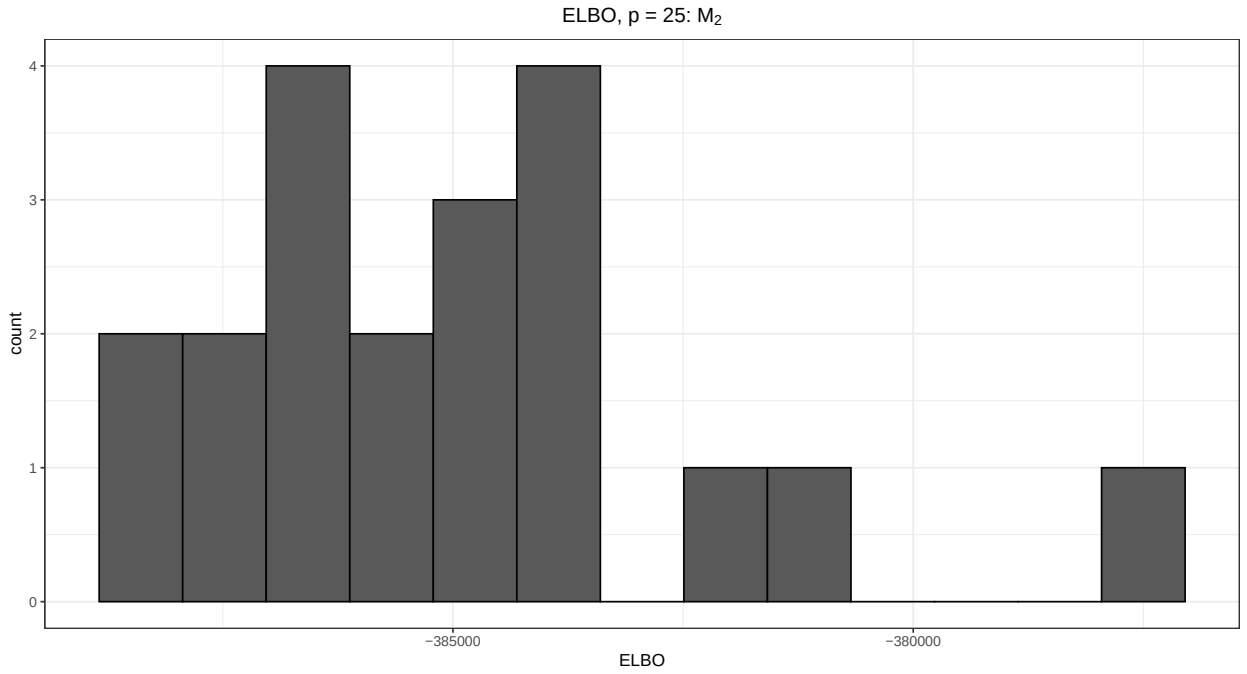
```
# p = 25

# m1
data.frame(X = m1_ELBO25) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO, p = 25: $M_1$")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m1_ELBO25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m1_ELBO25)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -419360 -417484 -416282 -416620 -415635 -414012
```

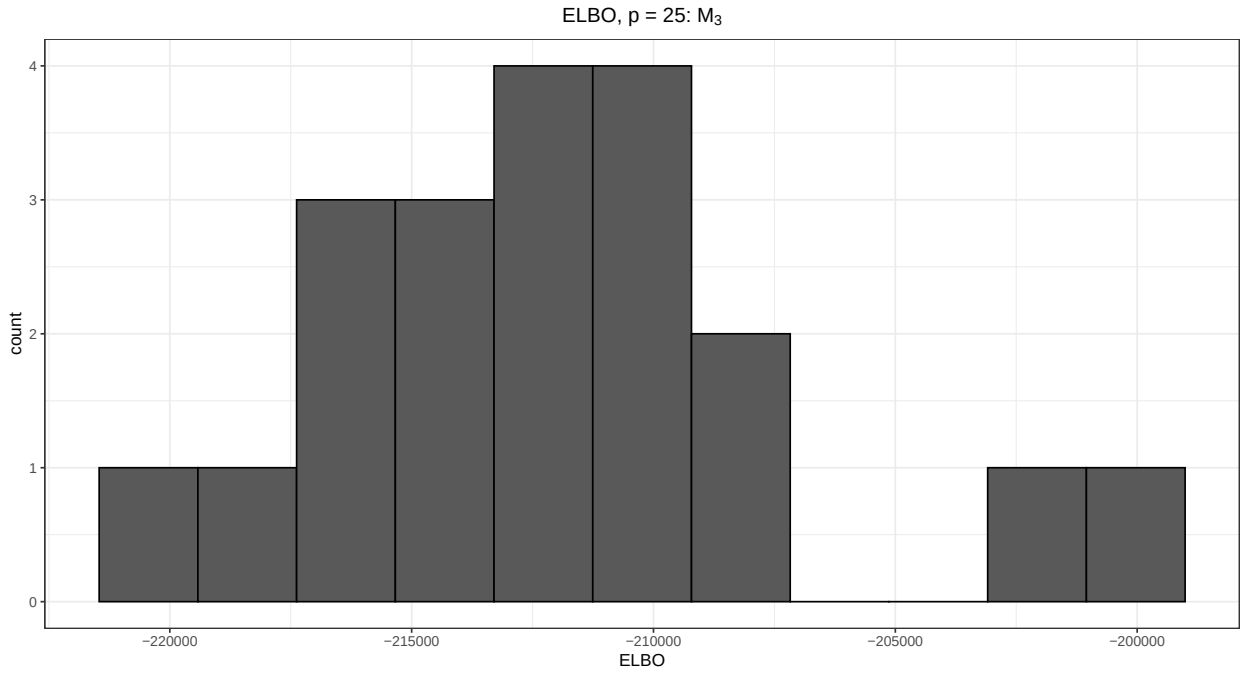
```
# m2
data.frame(X = m2_ELBO25) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO, p = 25: $M_2$")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m2_ELBO25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m2_ELBO25)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -388137 -386381 -385196 -384894 -383819 -377801
```

```
# m3
data.frame(X = m3_ELBO25) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO, p = 25: $M_3$")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m3_ELBO25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
summary(m3_ELBO25)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -220222 -215170 -212452 -211904 -209408 -199046
```

```
Sys.time() - start
```

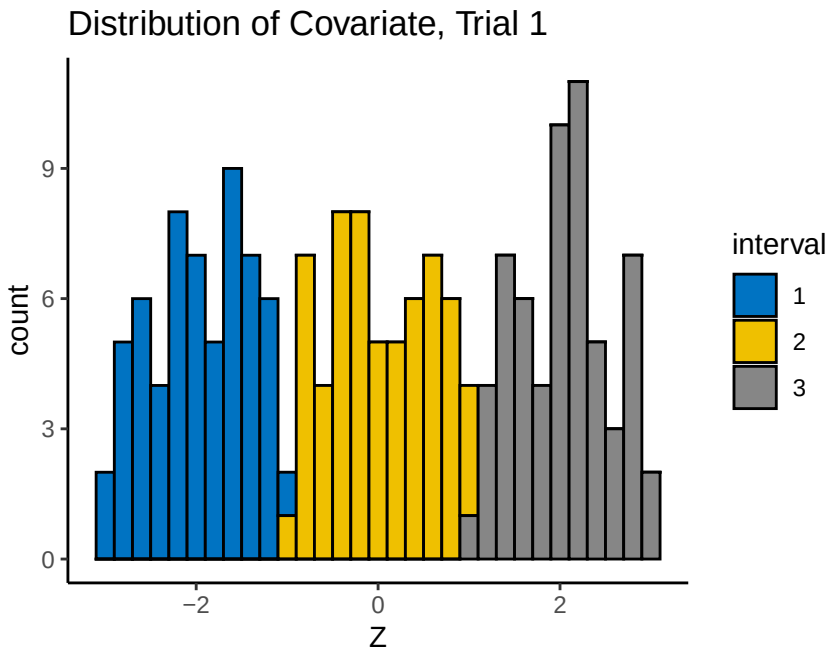
```
## Time difference of 1.000593 hours
```

```
# find errors
errs <- sapply(results, `[[`, "error")
err_log <- !sapply(errs, is.null)
errs[err_log]
```

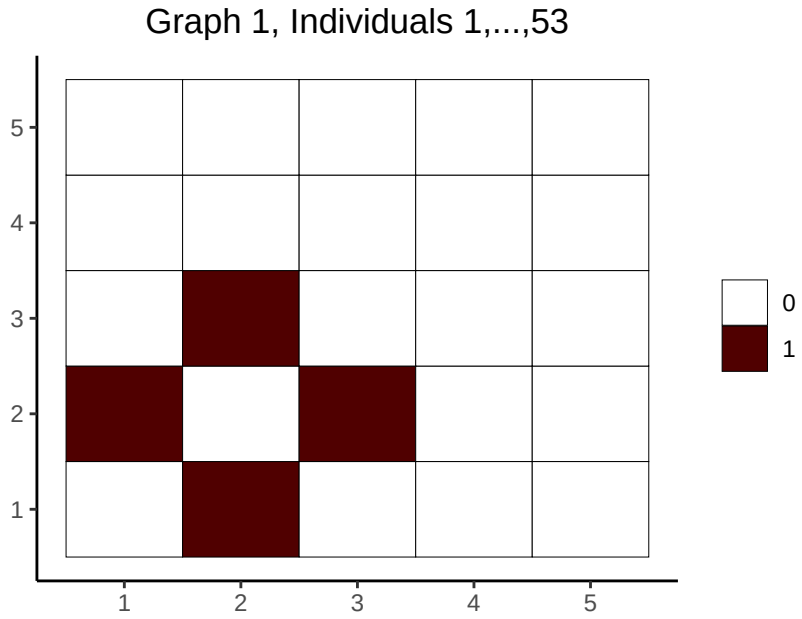
```
## named list()
```

```
# display a single non-large example
results[!err_log][1]
```

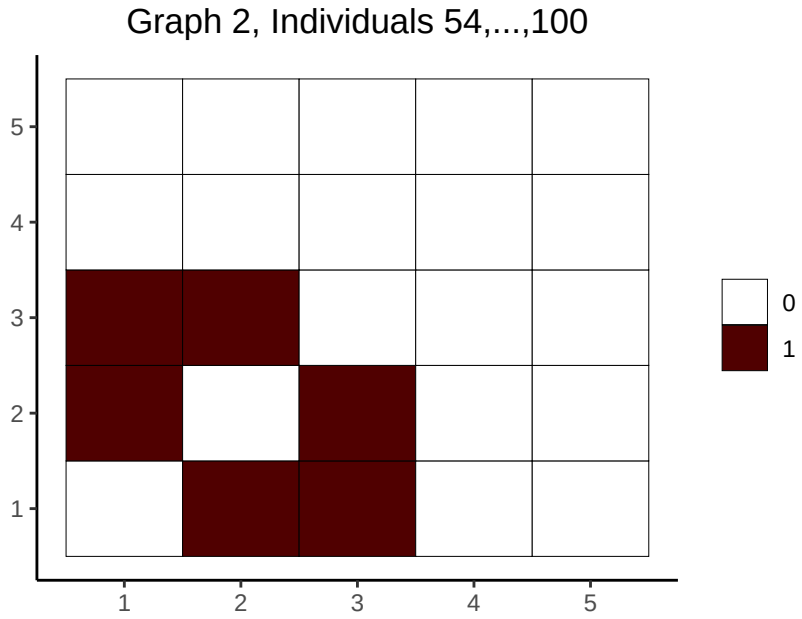
```
## $trial1
## $trial1$covariates
```



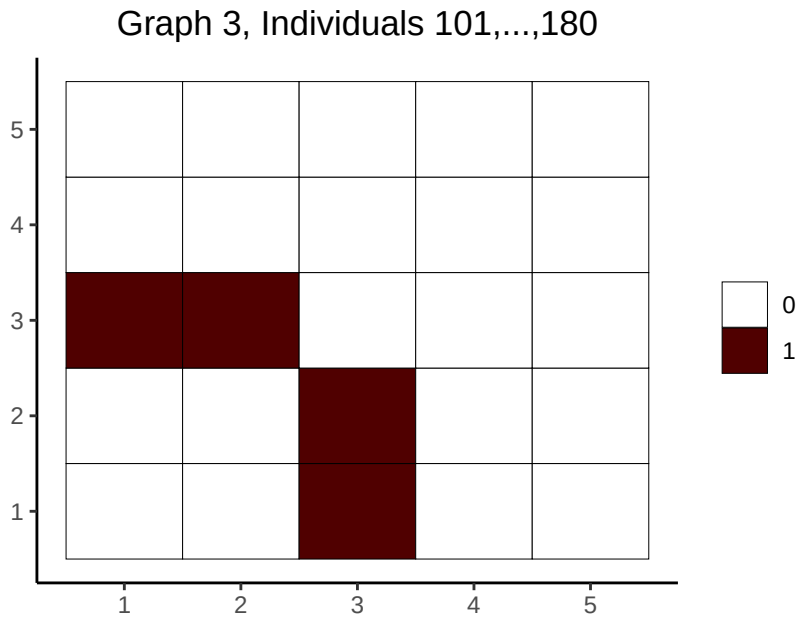

```
##
## $trial1$M1
## $trial1$M1$summary
##                         Covariate Dependent Graphical Model
##
## Model ELBO: -84663.66               Unique conditional dependence structures: 3
## n: 180, variables: 5                          Hyperparameter grid size: 5 points
## CAVI converged for 3/5 variables
##
## Model fit completed in 3.032 secs
##
## $trial1$M1$unique_graphs
## $trial1$M1$unique_graphs[[1]]
```



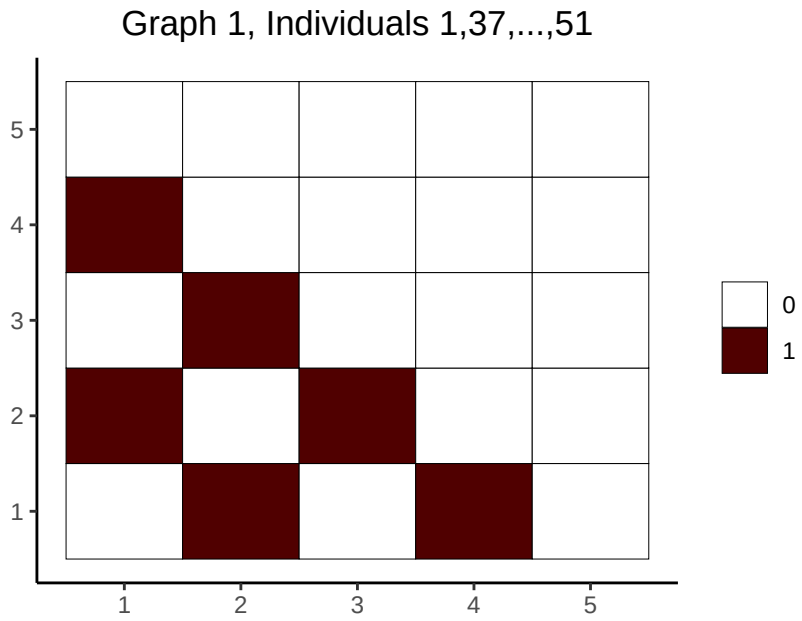

```
##
## $trial1$M1$unique_graphs[[2]]
```



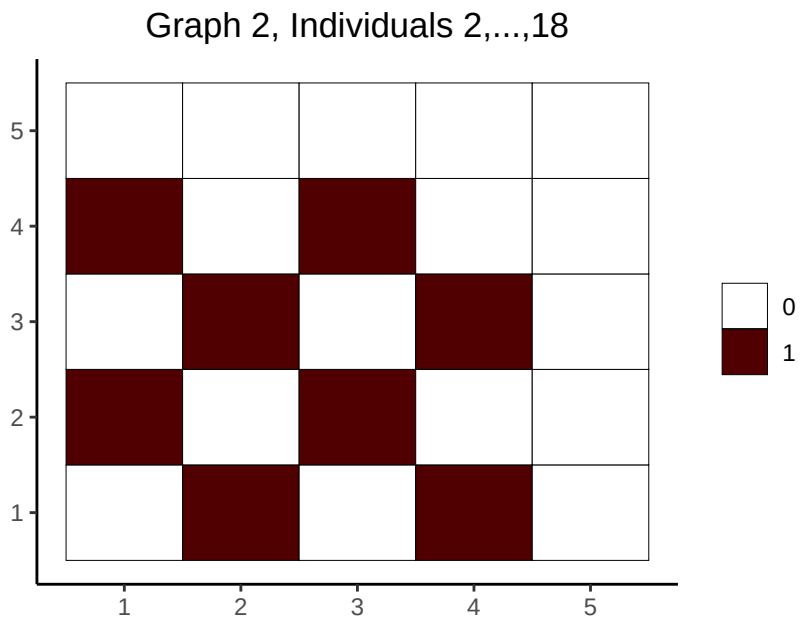

```
##
## $trial1$M1$unique_graphs[[3]]
```



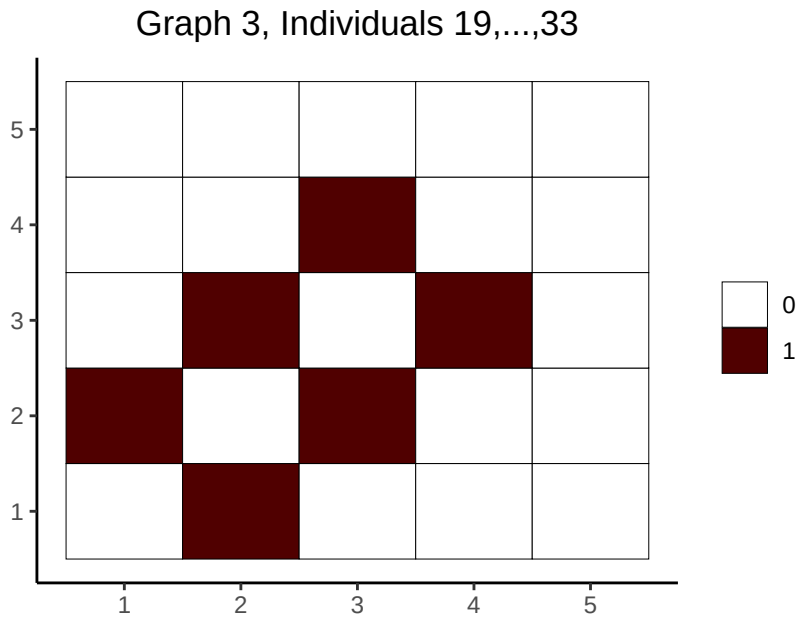

```
##
##
## $trial1$M1$sensitivity
## [1] 0.952381
##
## $trial1$M1$specificity
## [1] 0.9961749
##
## $trial1$M1$accuracy
## [1] 0.988
##
## $trial1$M1$ELBO
## [1] -84663.66
##
## $trial1$M1$time
## [1] 3.032029
##
##
## $trial1$M2
## $trial1$M2$summary
##                       Covariate Dependent Graphical Model
##
## Model ELBO: -67017.2               Unique conditional dependence structures: 13
## n: 180, variables: 5                          Hyperparameter grid size: 5 points
## CAVI converged for 3/5 variables
##
## Model fit completed in 1.106 secs
##
## $trial1$M2$unique_graphs
## $trial1$M2$unique_graphs[[1]]
```



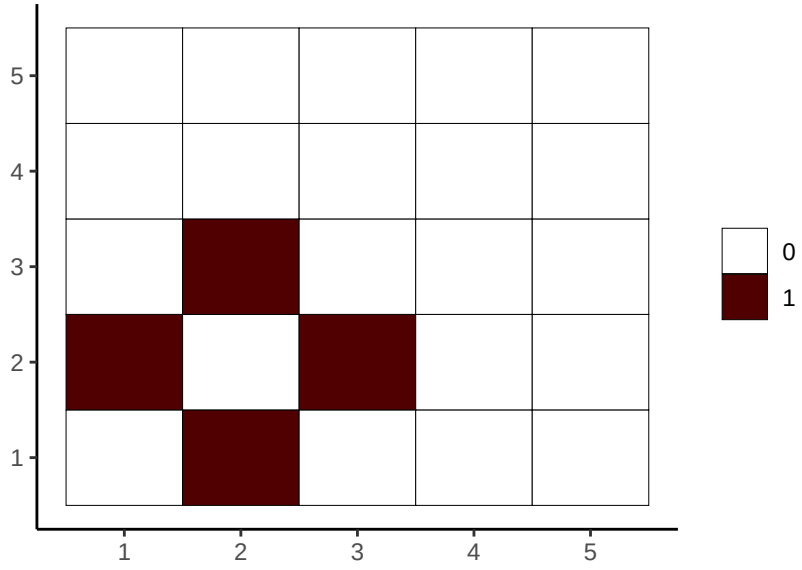

```
##
## $trial1$M2$unique_graphs[[2]]
```



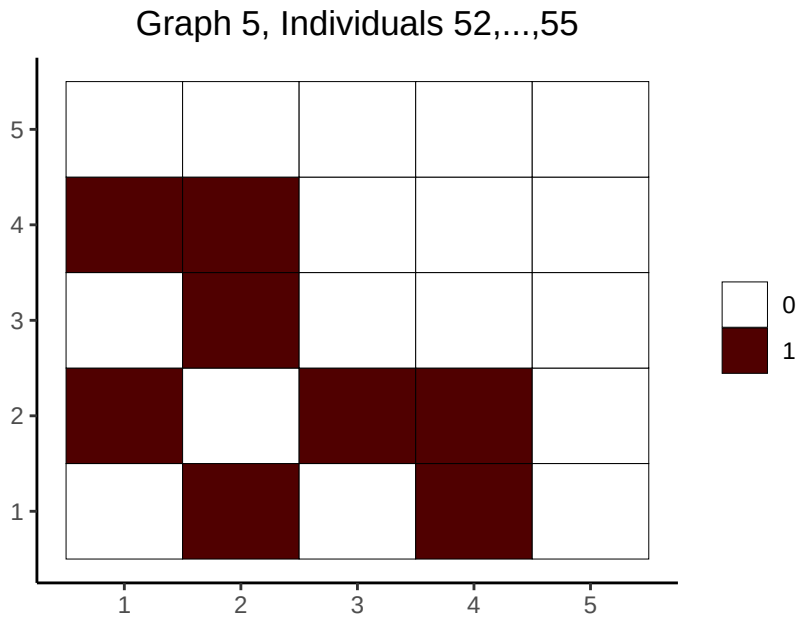

```
##
## $trial1$M2$unique_graphs[[3]]
```



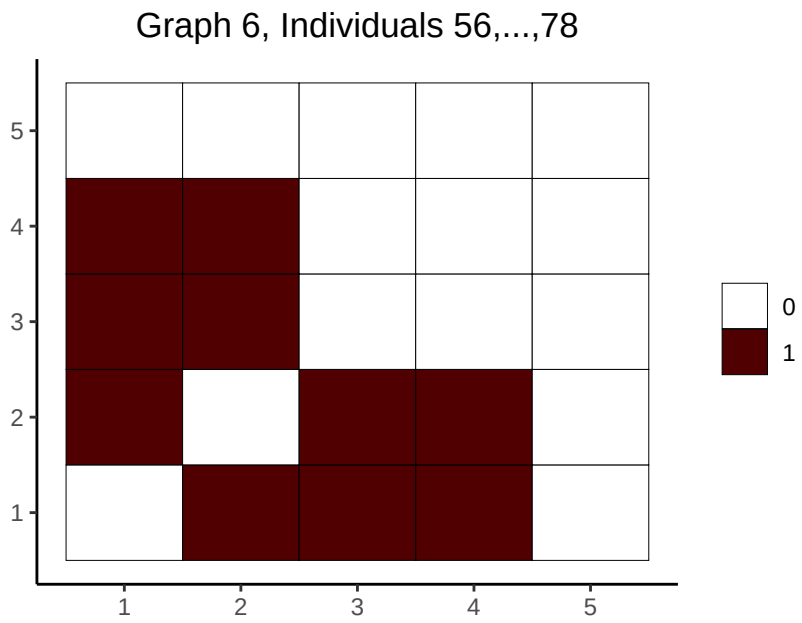

```
##
## $trial1$M2$unique_graphs[[4]]
```



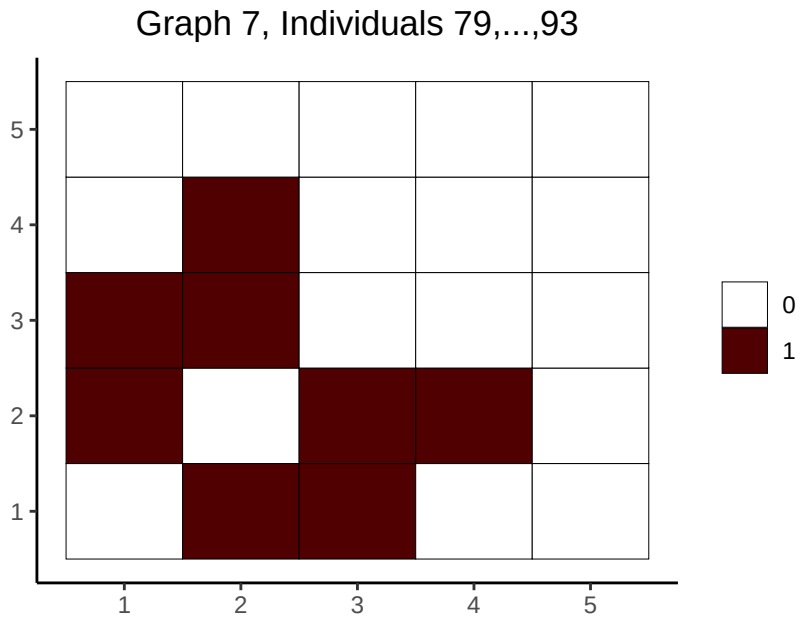

```
##
## $trial1$M2$unique_graphs[[5]]
```



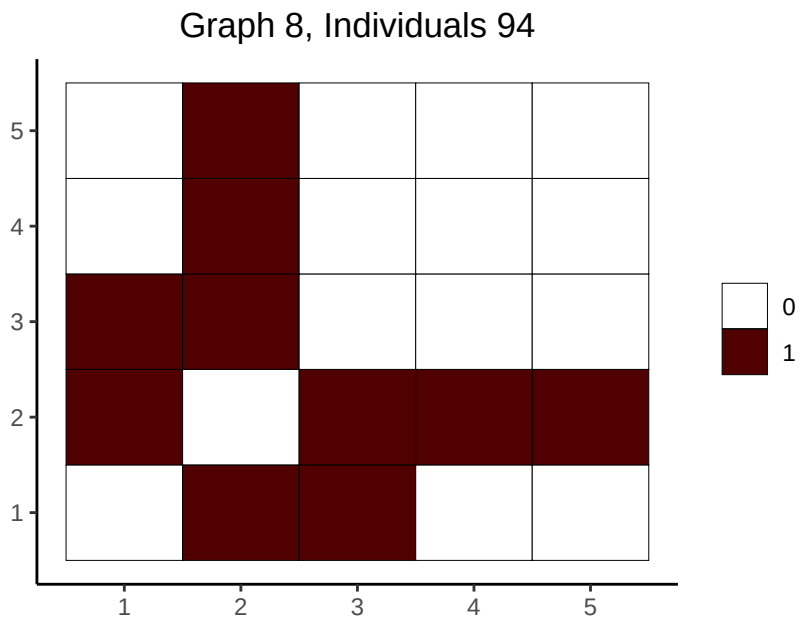

```
##
## $trial1$M2$unique_graphs[[6]]
```



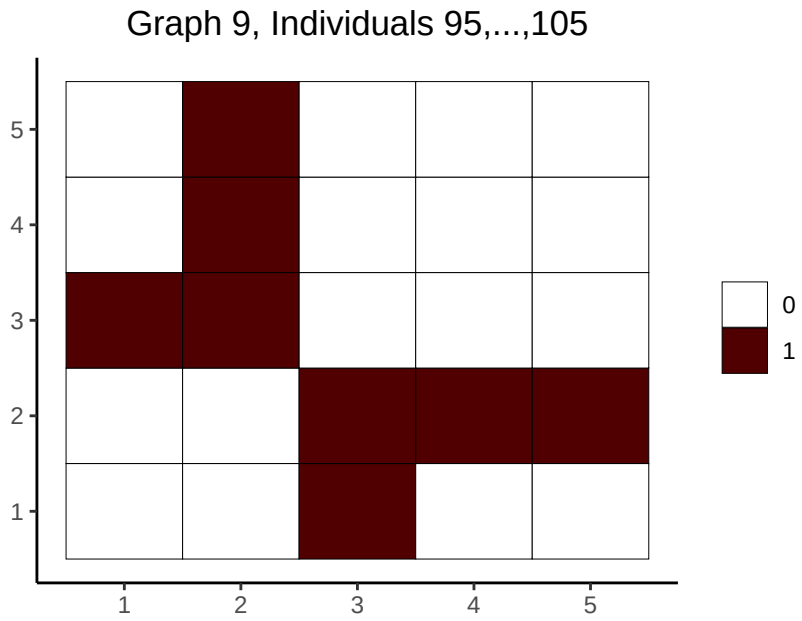

```
##
## $trial1$M2$unique_graphs[[7]]
```



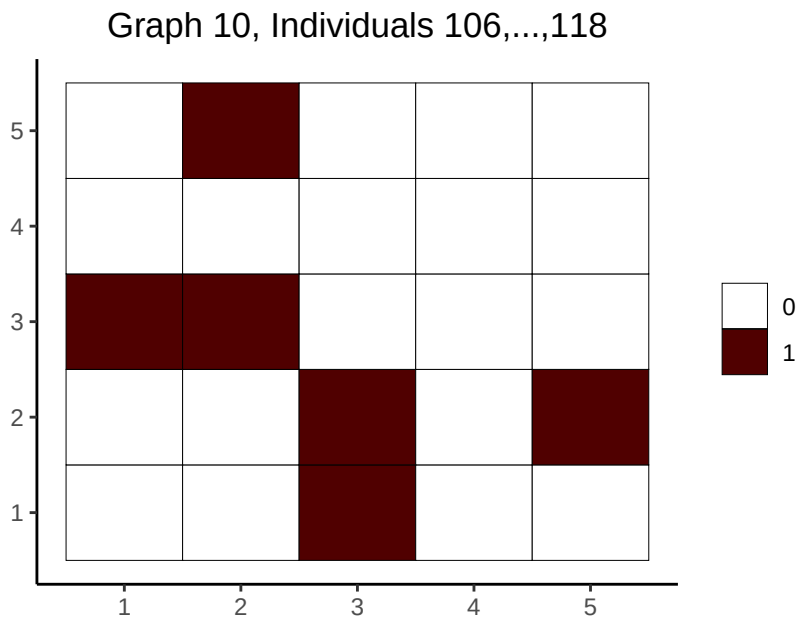

```
##
## $trial1$M2$unique_graphs[[8]]
```



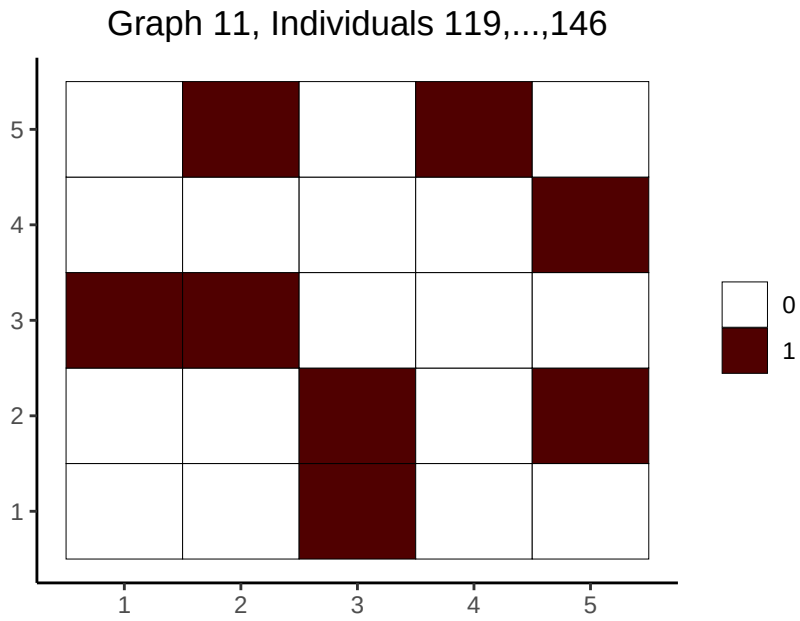

```
##
## $trial1$M2$unique_graphs[[9]]
```



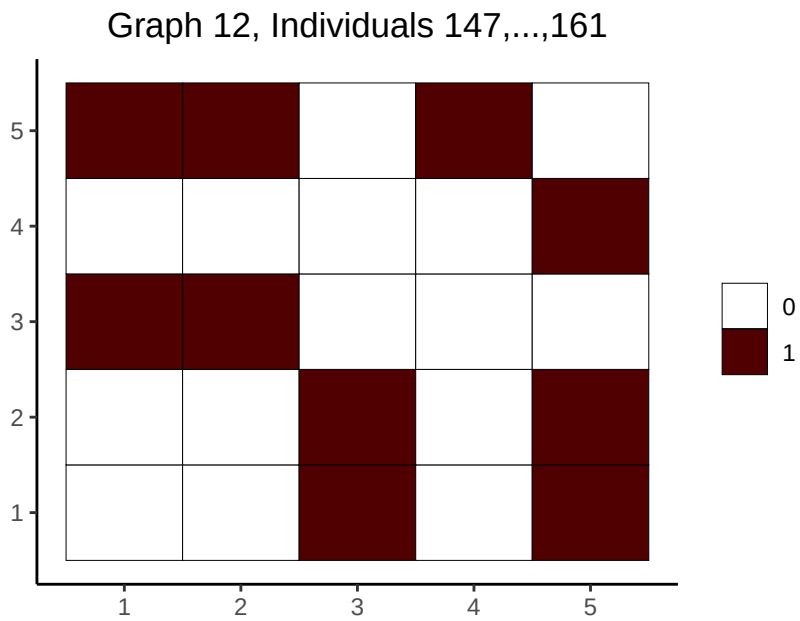

```
##
## $trial1$M2$unique_graphs[[10]]
```



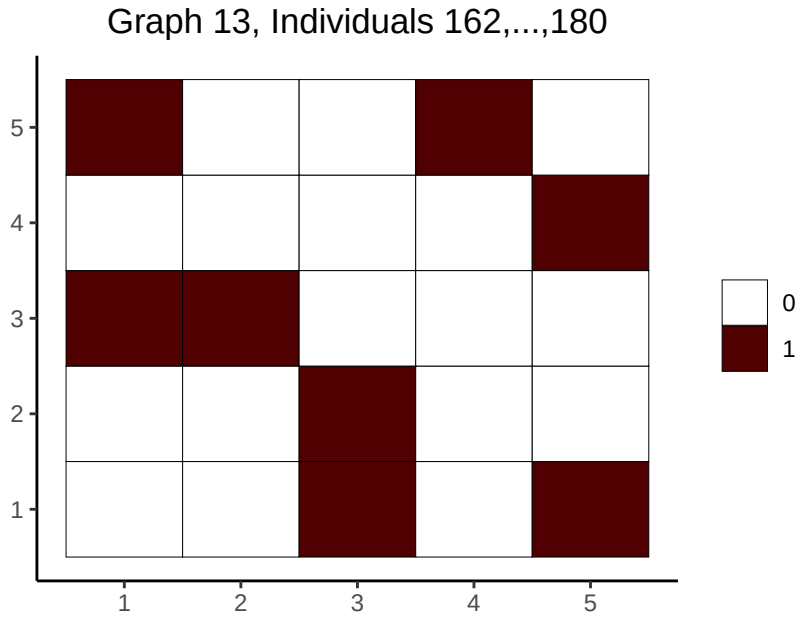

```
##
## $trial1$M2$unique_graphs[[11]]
```



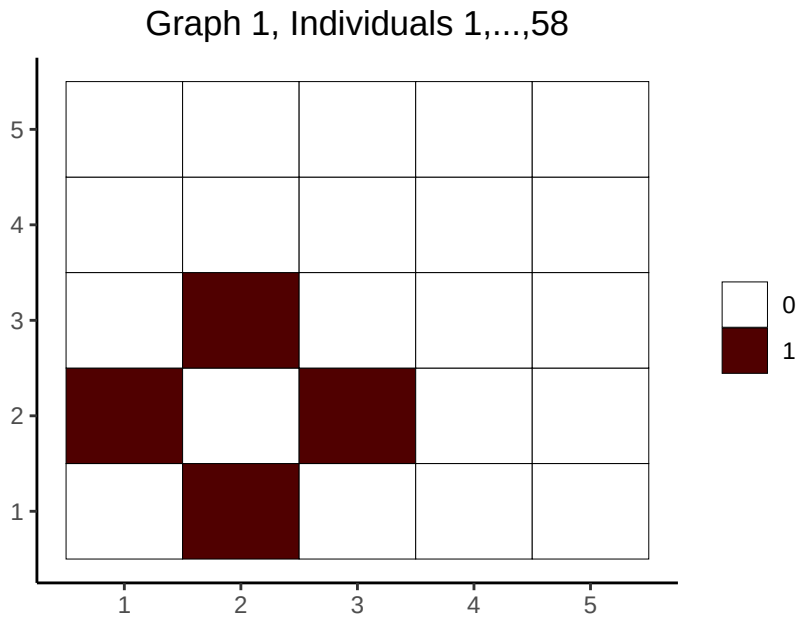

```
##
## $trial1$M2$unique_graphs[[12]]
```



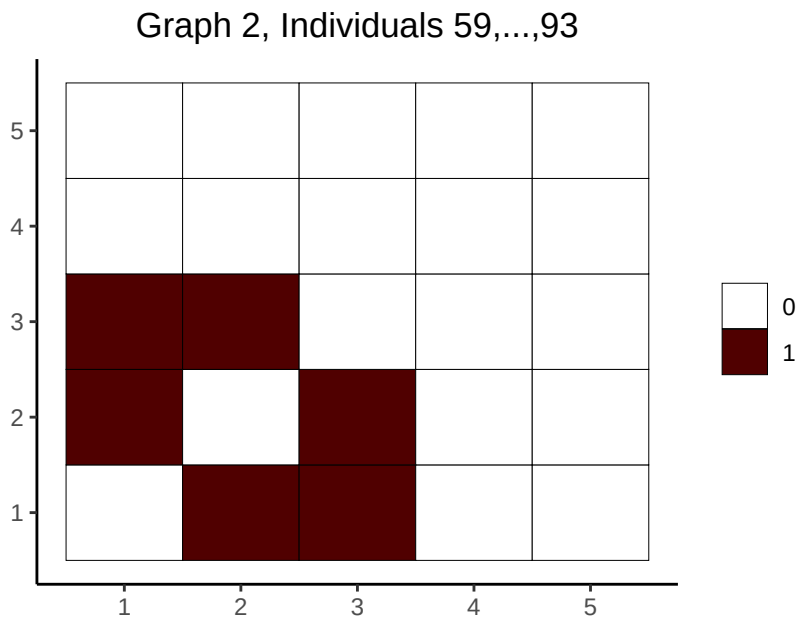

```
##
## $trial1$M2$unique_graphs[[13]]
```



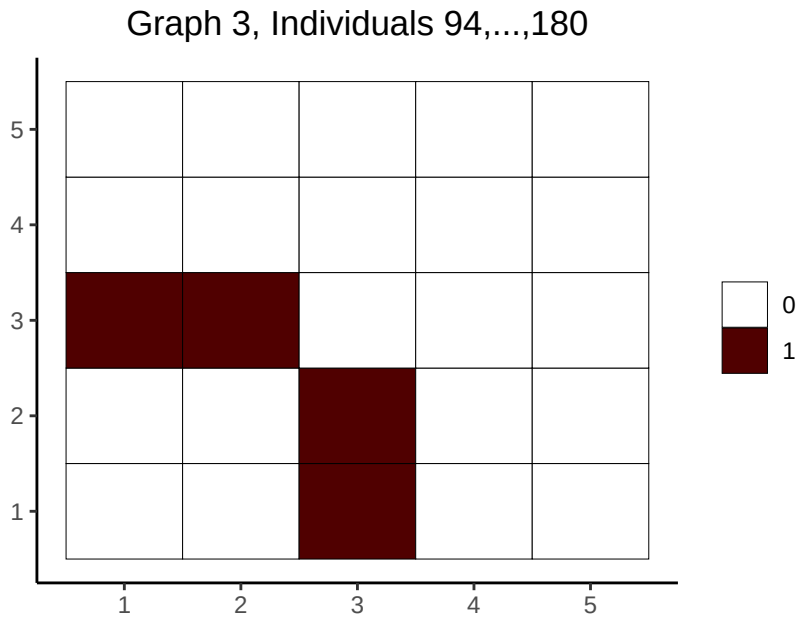

```
##
##
## $trial1$M2$sensitivity
## [1] 0.9380952
##
## $trial1$M2$specificity
## [1] 0.8278689
##
## $trial1$M2$accuracy
## [1] 0.8484444
##
## $trial1$M2$ELBO
## [1] -67017.2
##
## $trial1$M2$time
## [1] 1.105693
##
##
## $trial1$M3
## $trial1$M3$summary
##                       Covariate Dependent Graphical Model
##
## Model ELBO: -49121.09               Unique conditional dependence structures: 3
## n: 180, variables: 5                       Hyperparameter grid size: 125 points
## CAVI converged for 4/5 variables
##
## Model fit completed in 6.251 secs
##
## $trial1$M3$unique_graphs
## $trial1$M3$unique_graphs[[1]]
```




```
##
## $trial1$M3$unique_graphs[[2]]
```



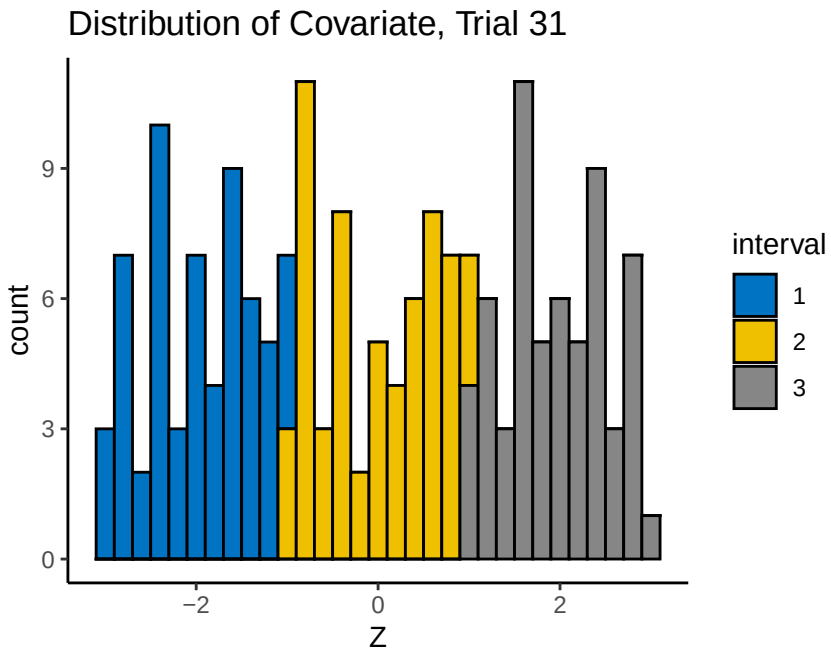

```
##
## $trial1$M3$unique_graphs[[3]]
```



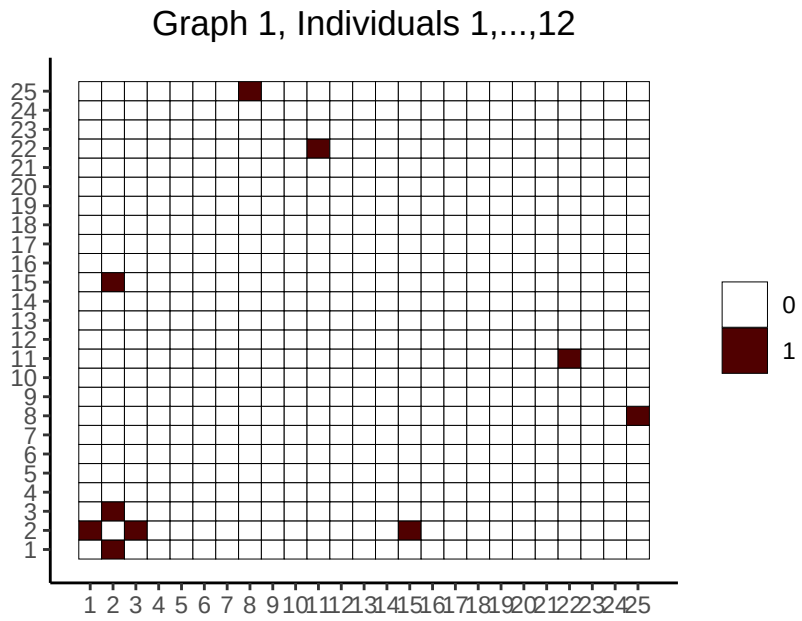

```
##
##
## $trial1$M3$sensitivity
## [1] 0.9357143
##
## $trial1$M3$specificity
## [1] 0.9989071
##
## $trial1$M3$accuracy
## [1] 0.9871111
##
## $trial1$M3$ELBO
## [1] -49121.09
##
## $trial1$M3$time
## [1] 6.25109
##
##
## $trial1$error
## NULL
```

```
# display a single large example
results[!err_log & 1:n_trials > (n_trials - n_large)][1]
```

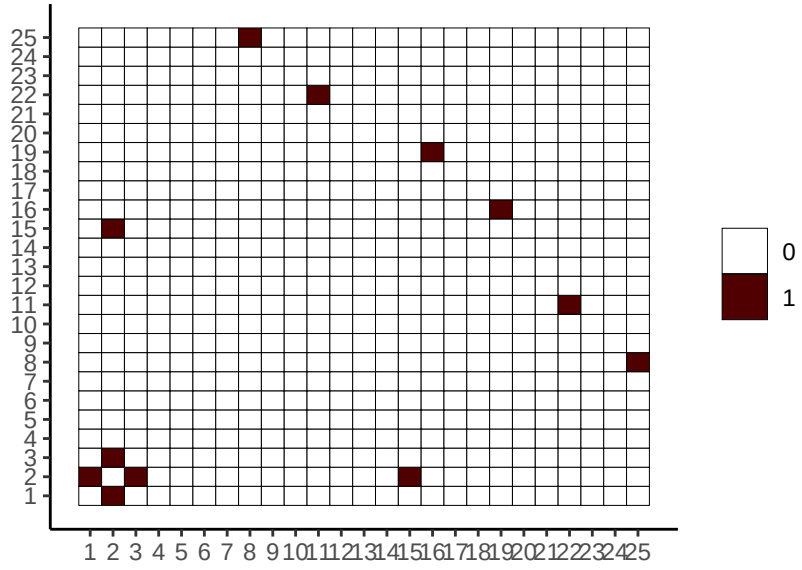
```
## $trial31
## $trial31$covariates
```



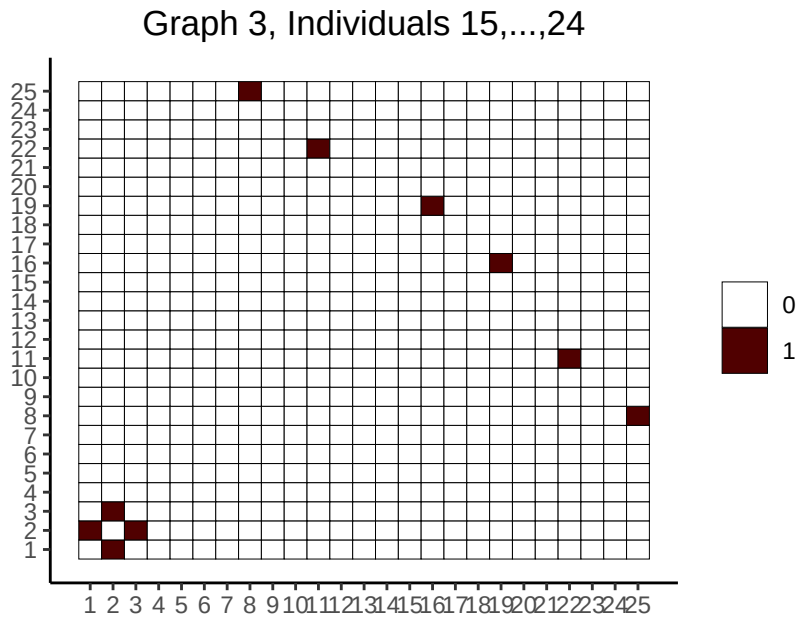

```
##
## $trial31$M1
## $trial31$M1$summary
##                        Covariate Dependent Graphical Model
##
## Model ELBO: -417210.07             Unique conditional dependence structures: 11
## n: 180, variables: 25                        Hyperparameter grid size: 5 points
## CAVI converged for 18/25 variables
##
## Model fit completed in 24.872 secs
##
## $trial31$M1$unique_graphs
## $trial31$M1$unique_graphs[[1]]
```



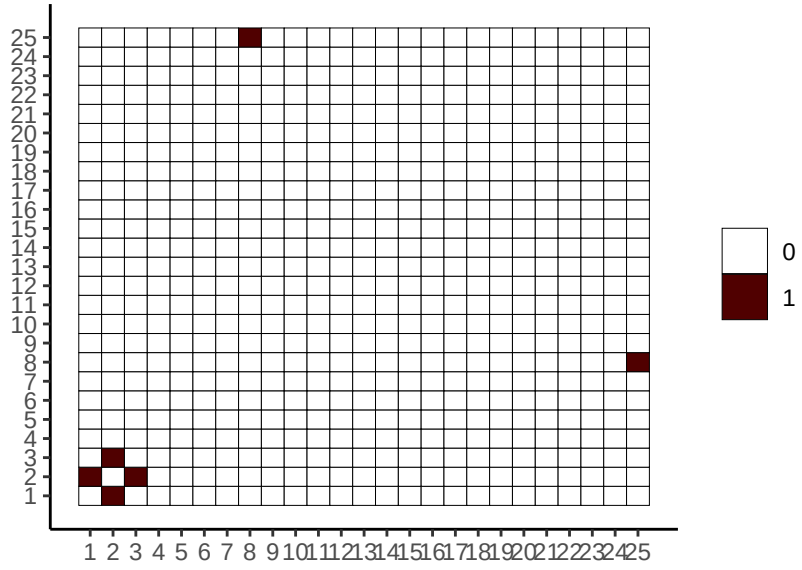

```
##
## $trial31$M1$unique_graphs[[2]]
```




```
##
## $trial31$M1$unique_graphs[[3]]
```

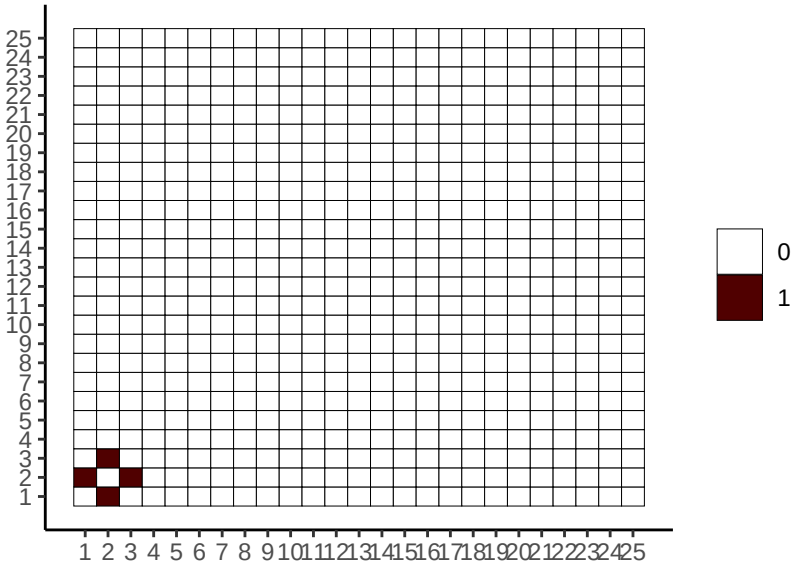



```
##
## $trial31$M1$unique_graphs[[4]]
```



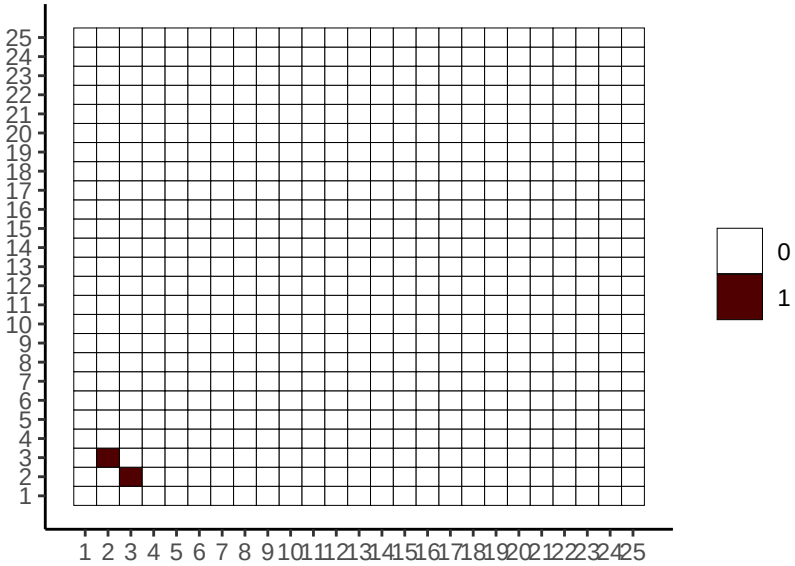

```
##
## $trial31$M1$unique_graphs[[5]]
```

Graph 5, Individuals 34,...,99

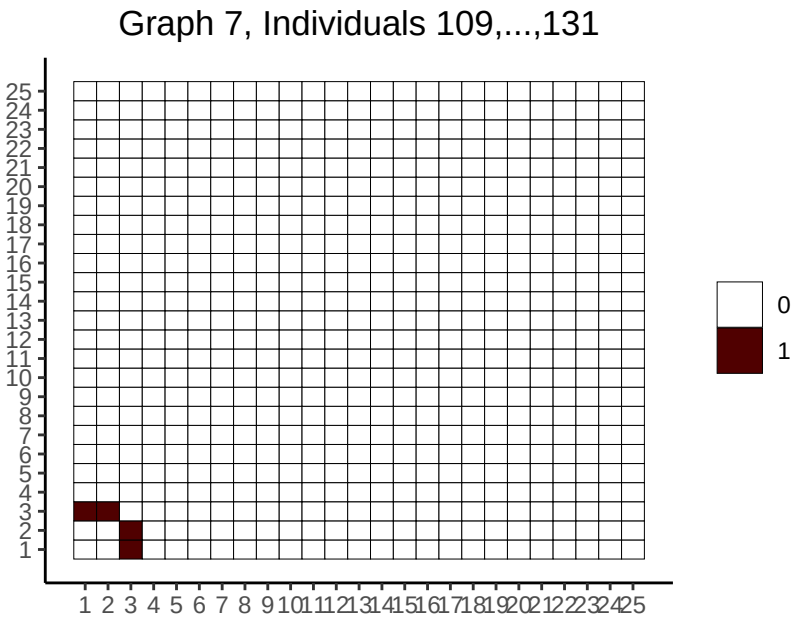



```
##
## $trial31$M1$unique_graphs[[6]]
```

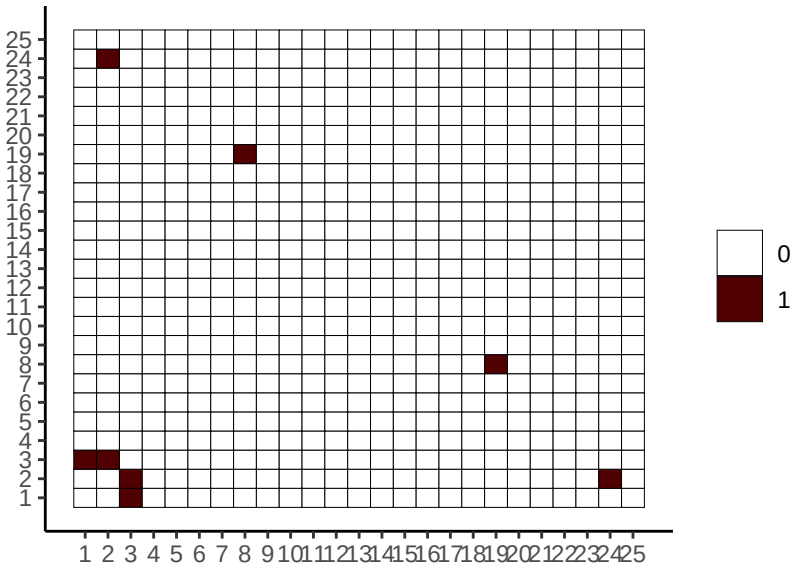
Graph 6, Individuals 100,...,108



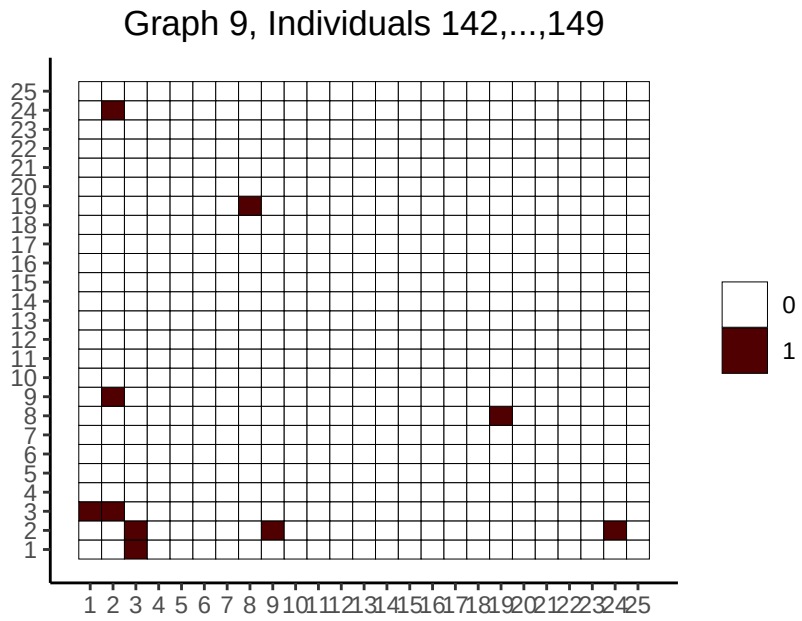

```
##
## $trial31$M1$unique_graphs[[7]]
```



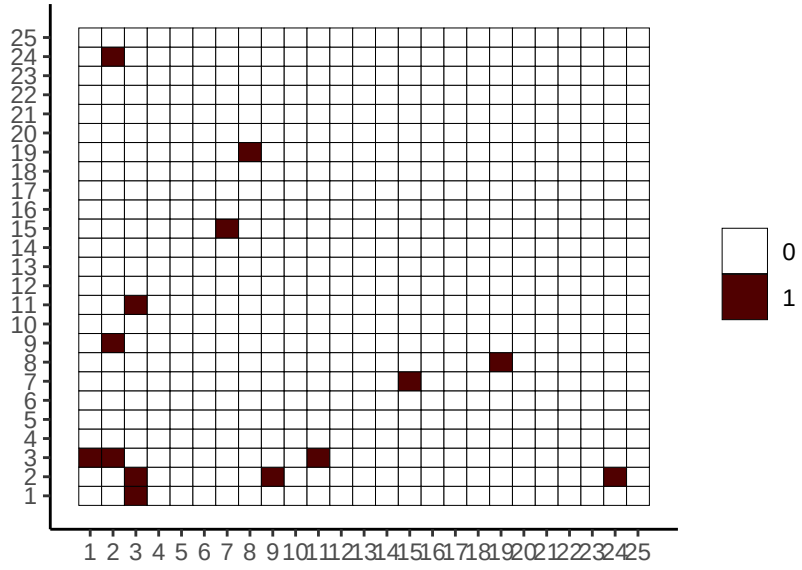

```
##
## $trial31$M1$unique_graphs[[8]]
```



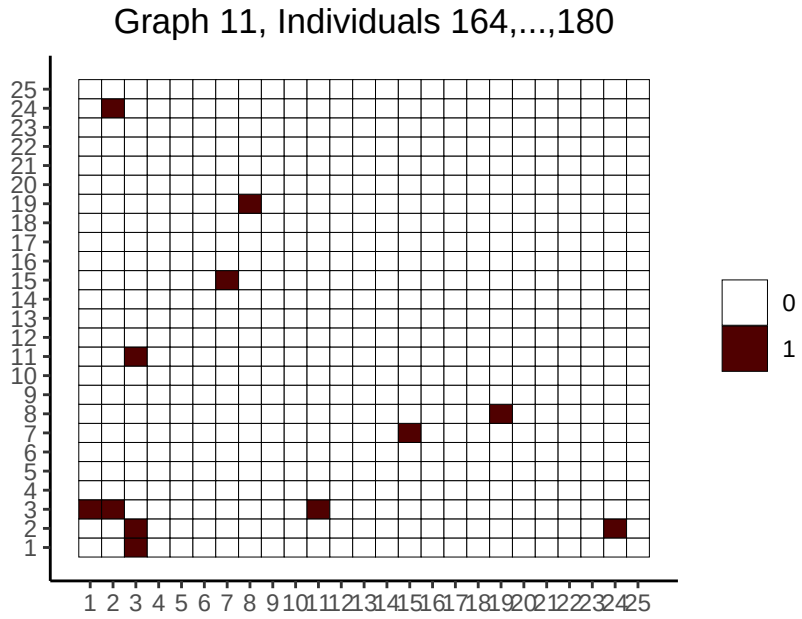

```
##
## $trial31$M1$unique_graphs[[9]]
```



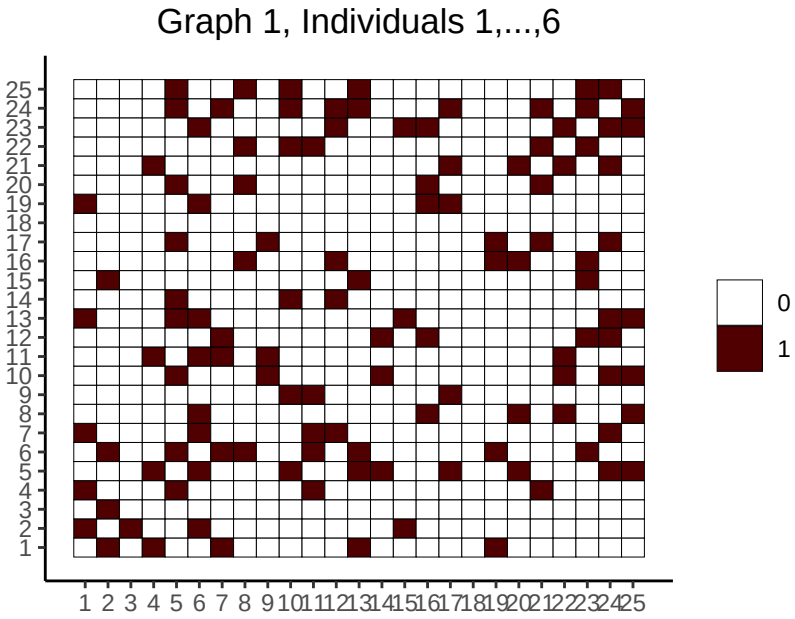

```
##
## $trial31$M1$unique_graphs[[10]]
```



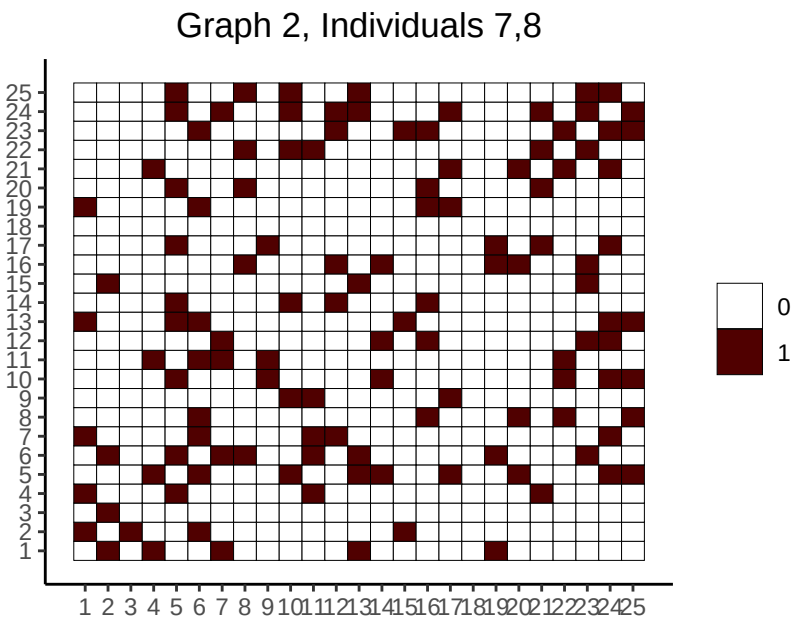

```
##
## $trial31$M1$unique_graphs[[11]]
```



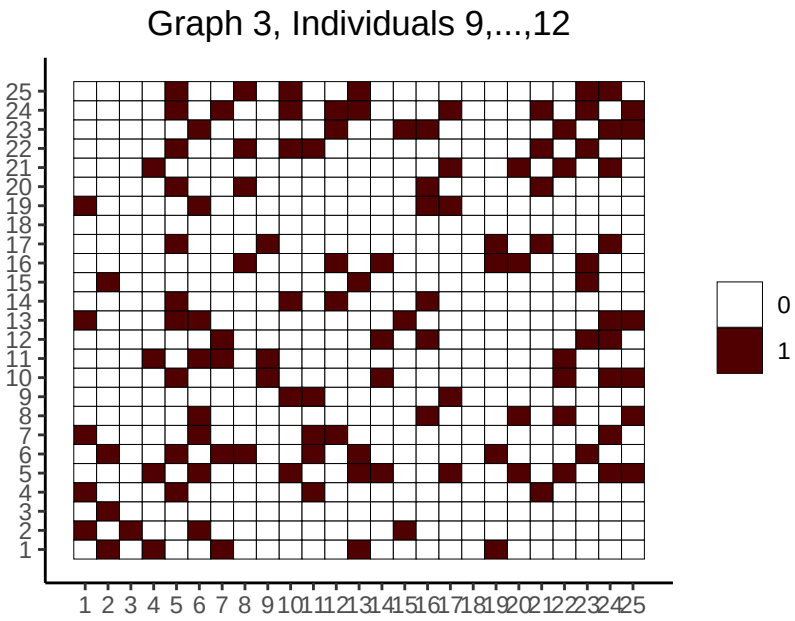

```
##
##
## $trial31$M1$sensitivity
## [1] 0.8357143
##
## $trial31$M1$specificity
## [1] 0.9952534
##
## $trial31$M1$accuracy
## [1] 0.9940622
##
## $trial31$M1$ELBO
## [1] -417210.1
##
## $trial31$M1$time
## [1] 24.87153
##
##
## $trial31$M2
## $trial31$M2$summary
##                       Covariate Dependent Graphical Model
##
## Model ELBO: -385317.49            Unique conditional dependence structures: 102
## n: 180, variables: 25                          Hyperparameter grid size: 5 points
## CAVI converged for 6/25 variables
##
## Model fit completed in 32.559 secs
##
## $trial31$M2$unique_graphs
## $trial31$M2$unique_graphs[[1]]
```



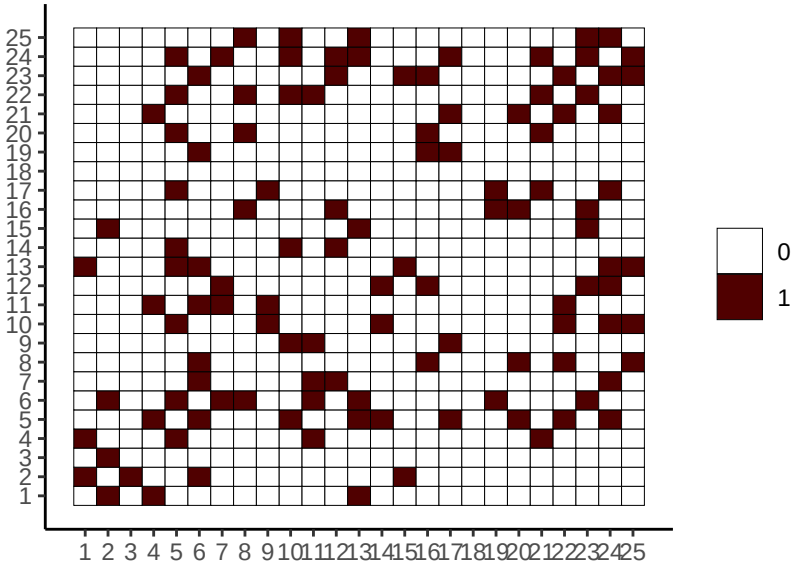

```
##
## $trial31$M2$unique_graphs[[2]]
```



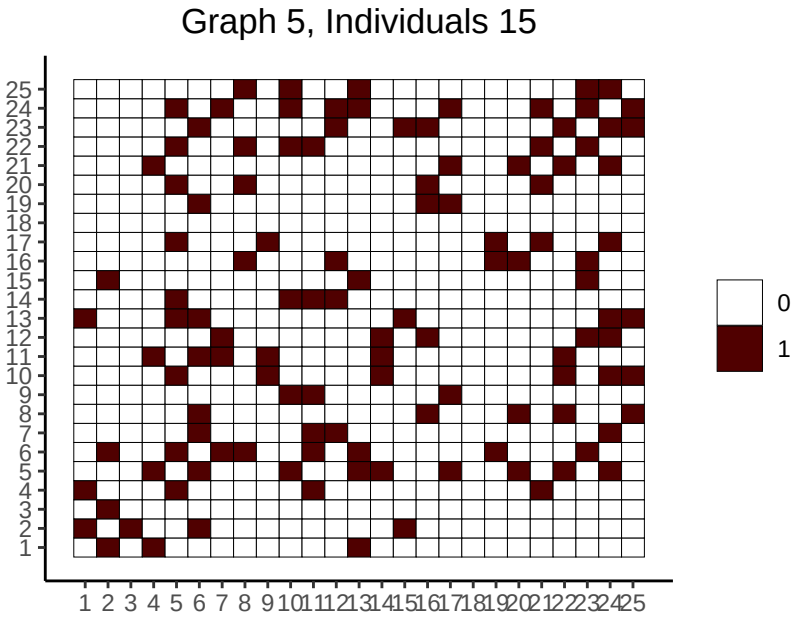

```
##
## $trial31$M2$unique_graphs[[3]]
```



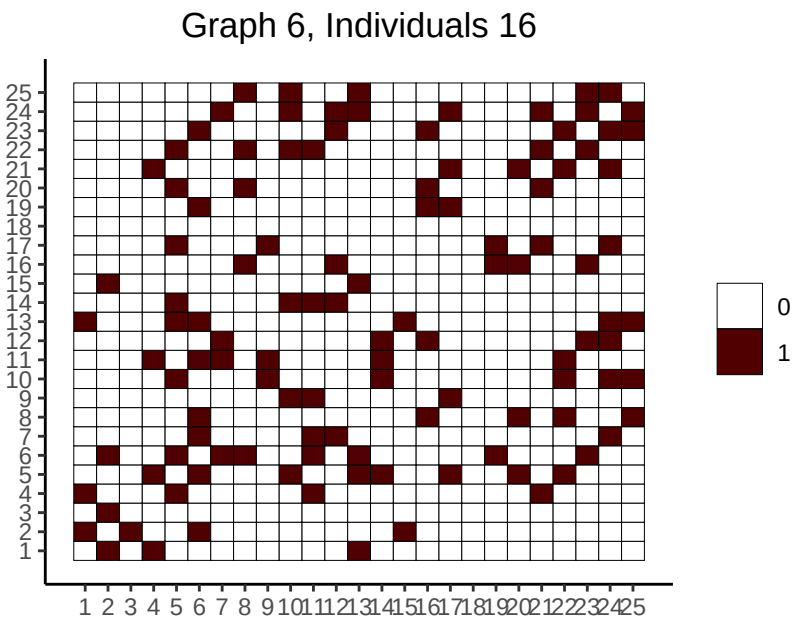

```
##
## $trial31$M2$unique_graphs[[4]]
```



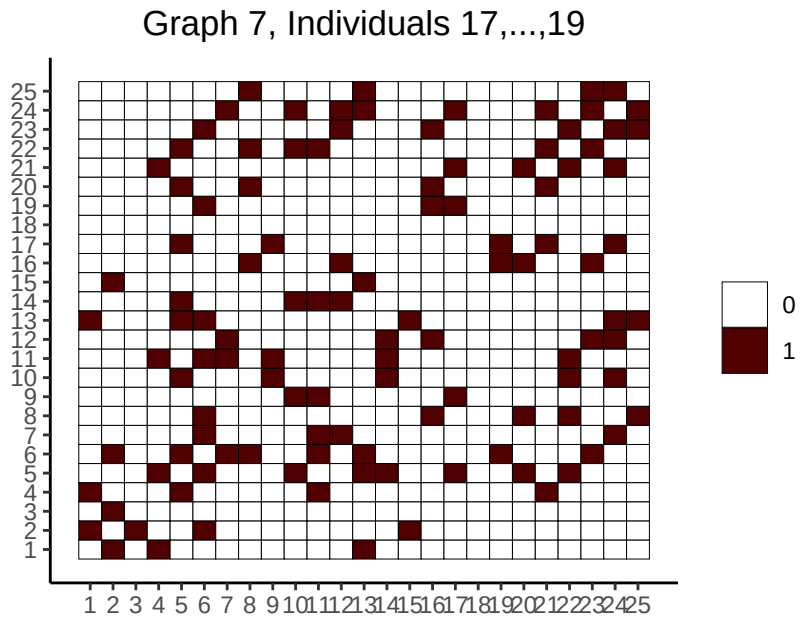

```
##
## $trial31$M2$unique_graphs[[5]]
```



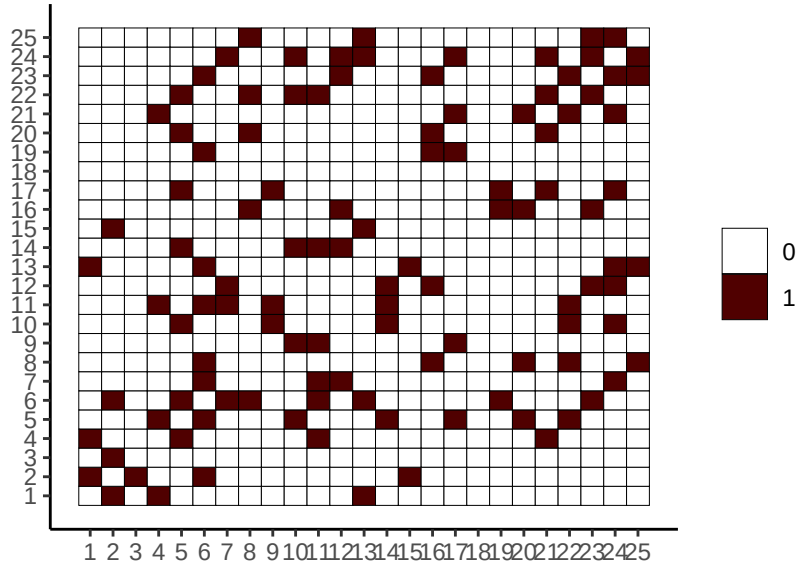

```
##
## $trial31$M2$unique_graphs[[6]]
```



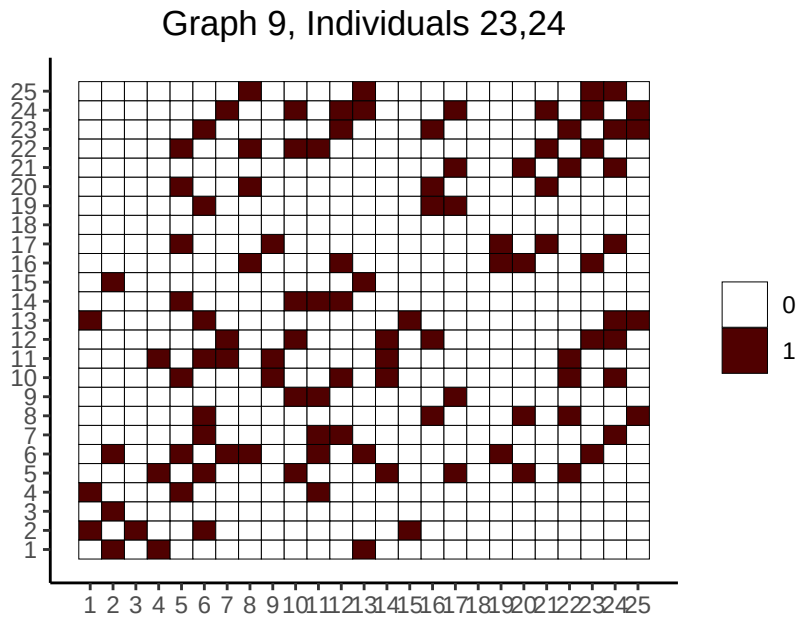

```
##
## $trial31$M2$unique_graphs[[7]]
```



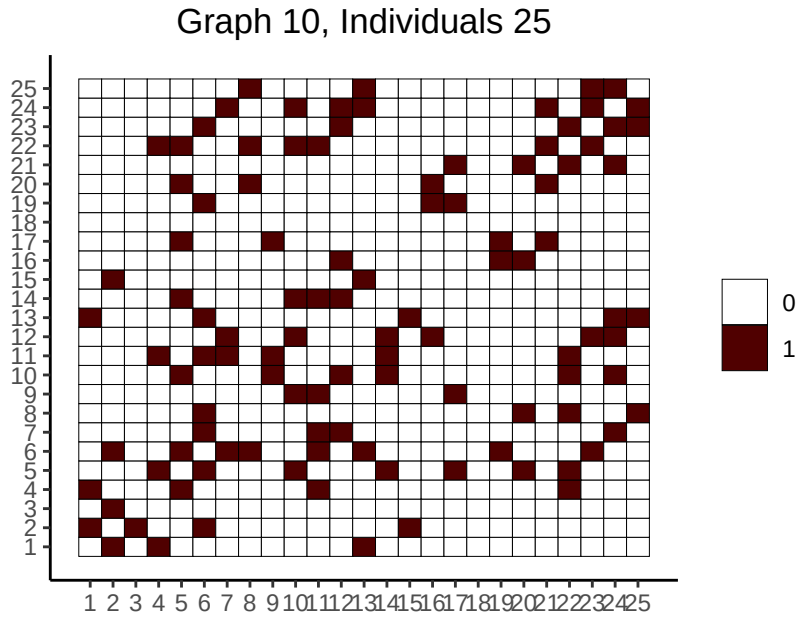

```
##
## $trial31$M2$unique_graphs[[8]]
```



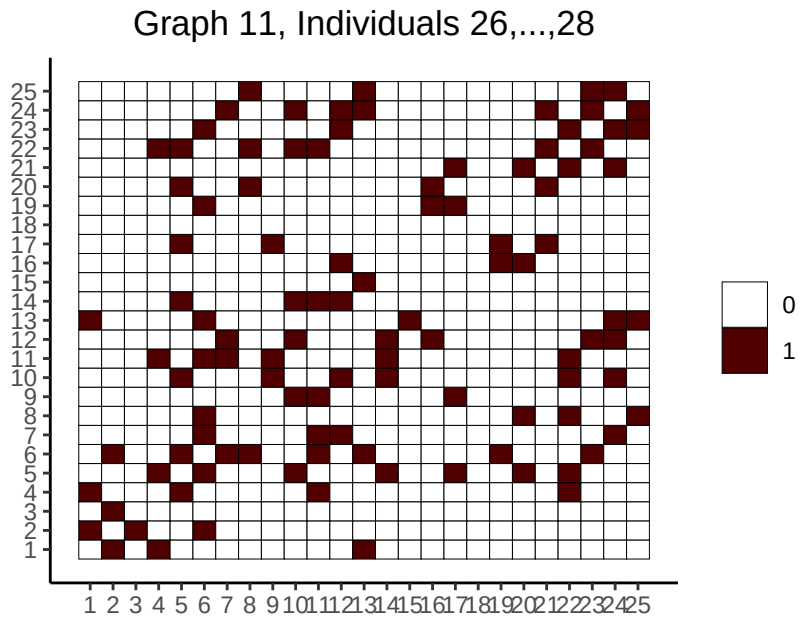

```
##
## $trial31$M2$unique_graphs[[9]]
```



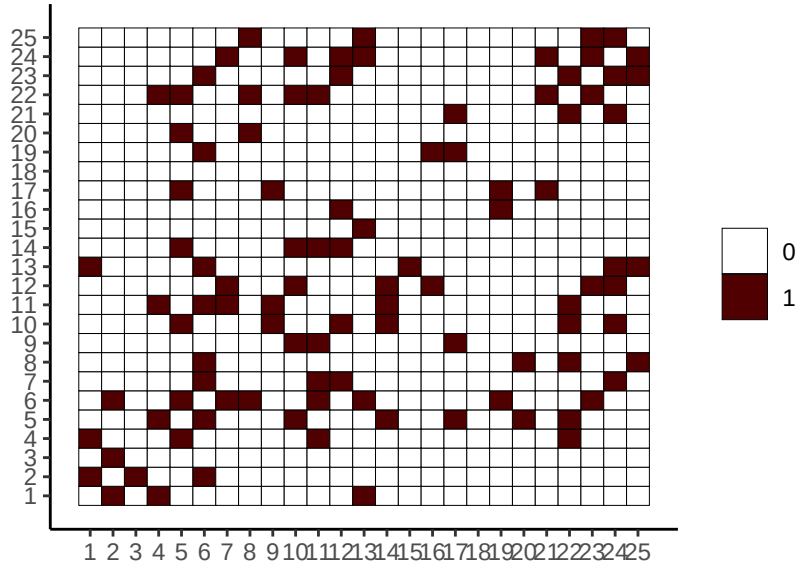

```
##
## $trial31$M2$unique_graphs[[10]]
```



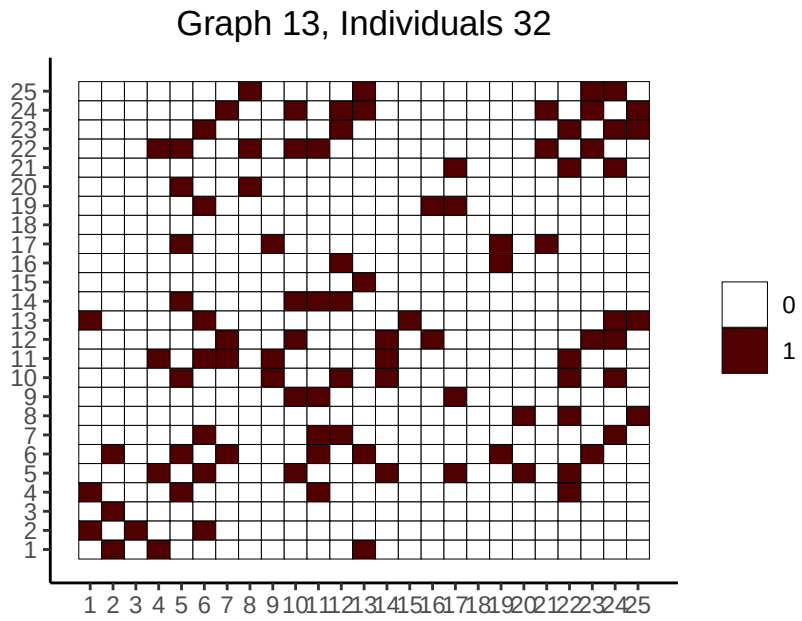

```
##
## $trial31$M2$unique_graphs[[11]]
```



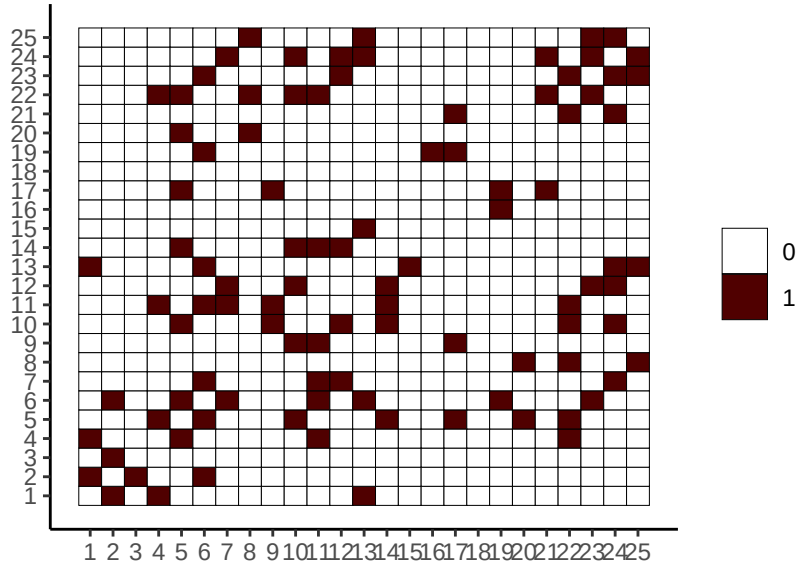

```
##
## $trial31$M2$unique_graphs[[12]]
```



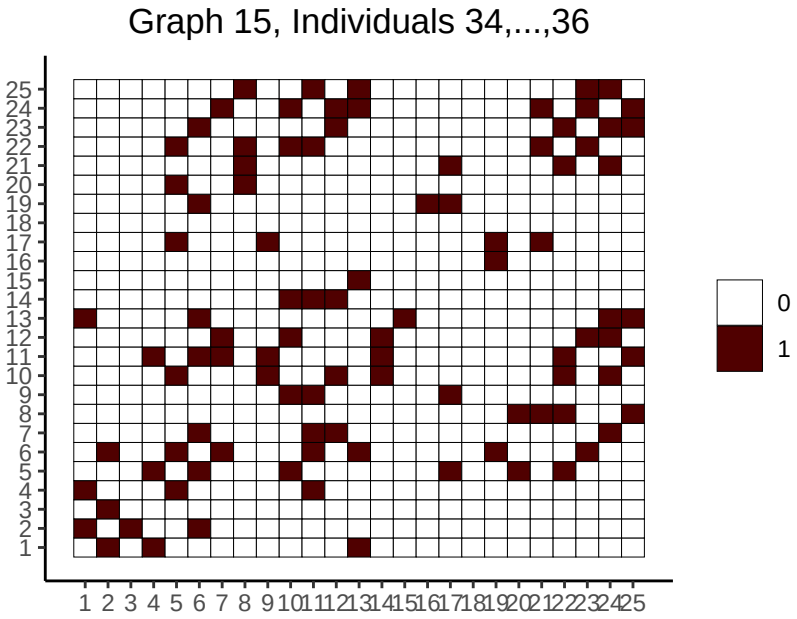

```
##
## $trial31$M2$unique_graphs[[13]]
```



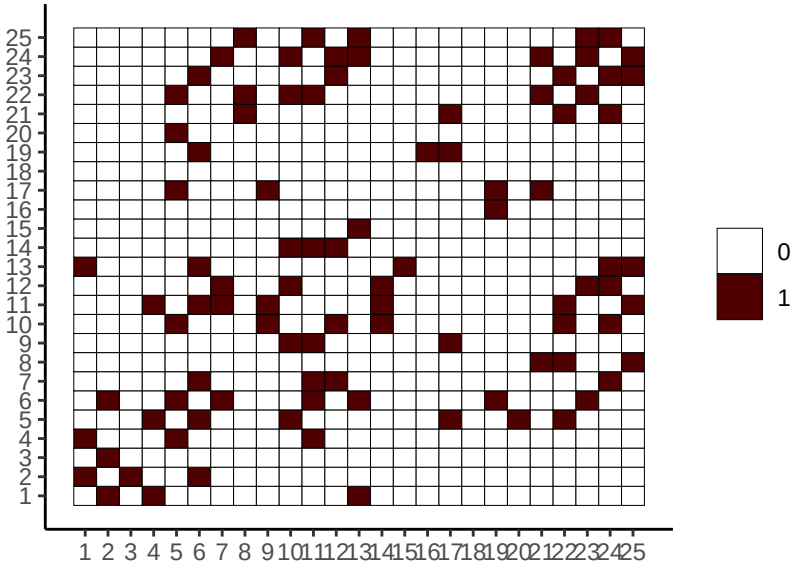

```
##
## $trial31$M2$unique_graphs[[14]]
```



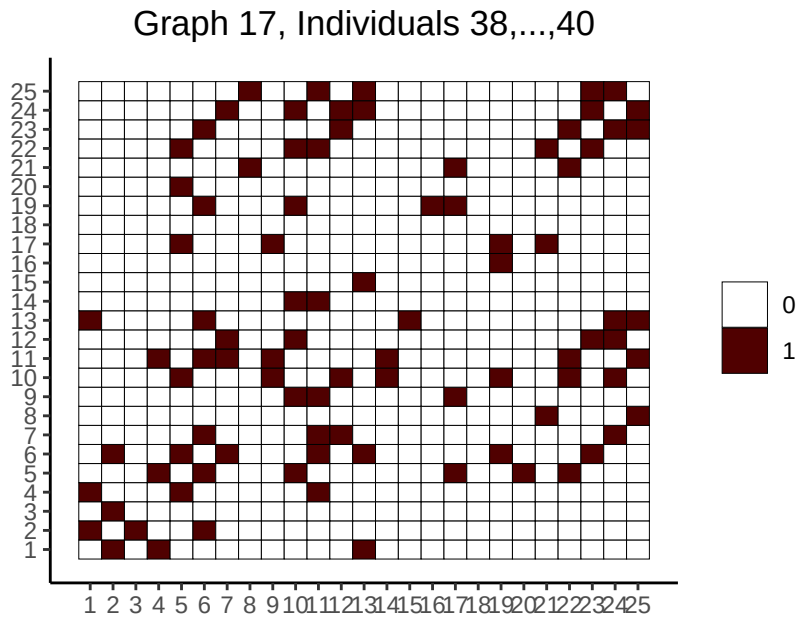

```
##
## $trial31$M2$unique_graphs[[15]]
```



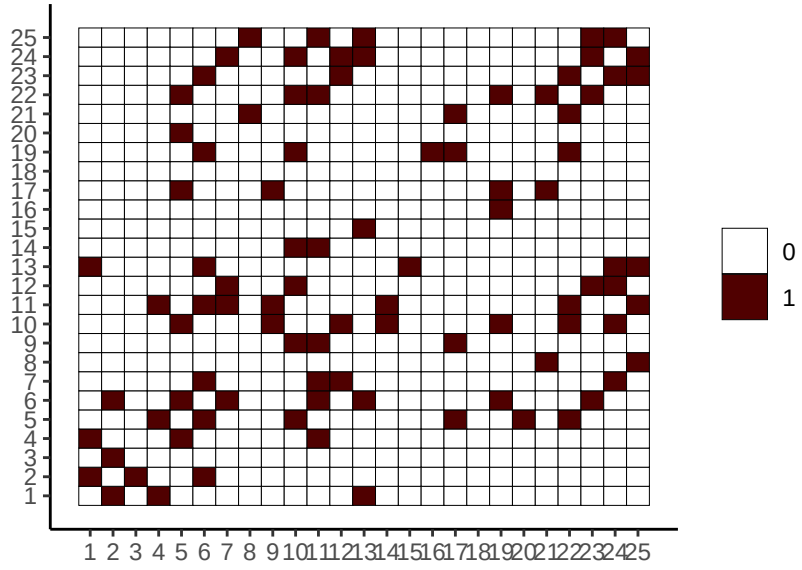

```
##
## $trial31$M2$unique_graphs[[16]]
```



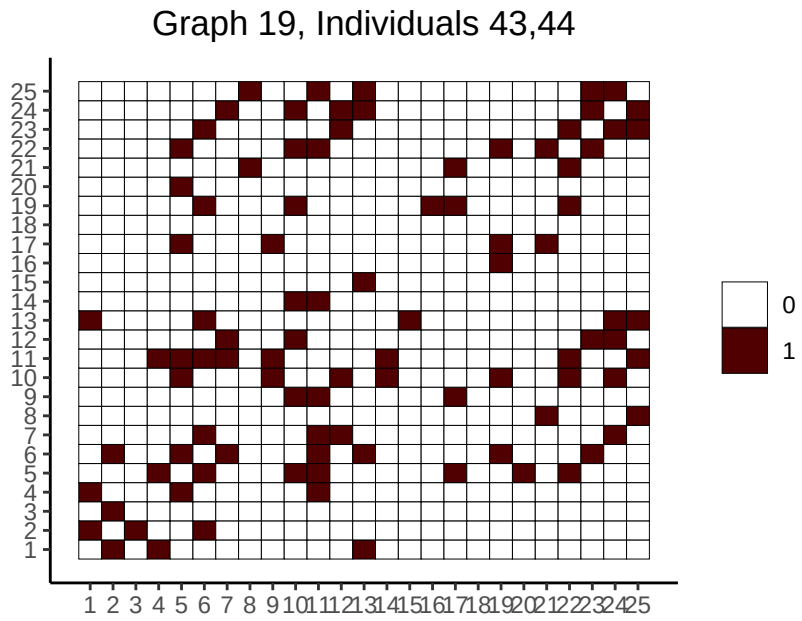

```
##
## $trial31$M2$unique_graphs[[17]]
```



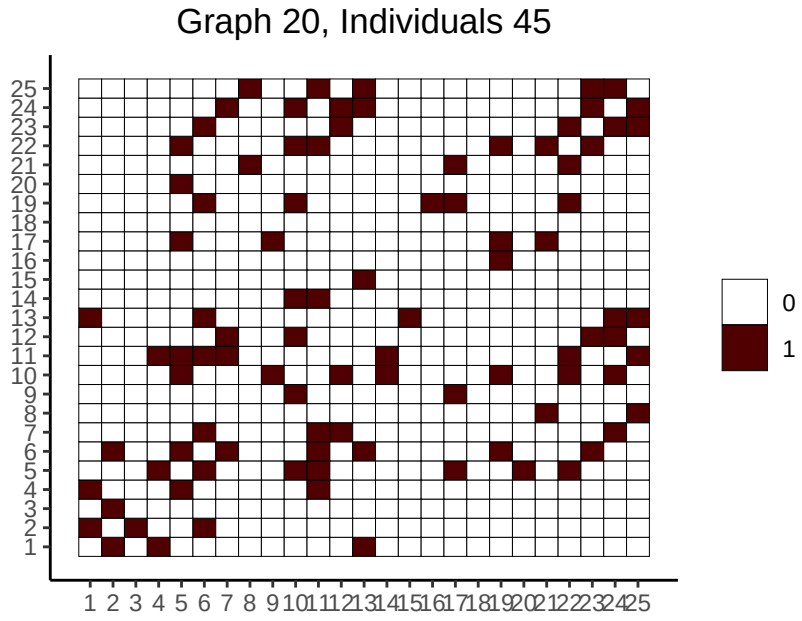

```
##
## $trial31$M2$unique_graphs[[18]]
```



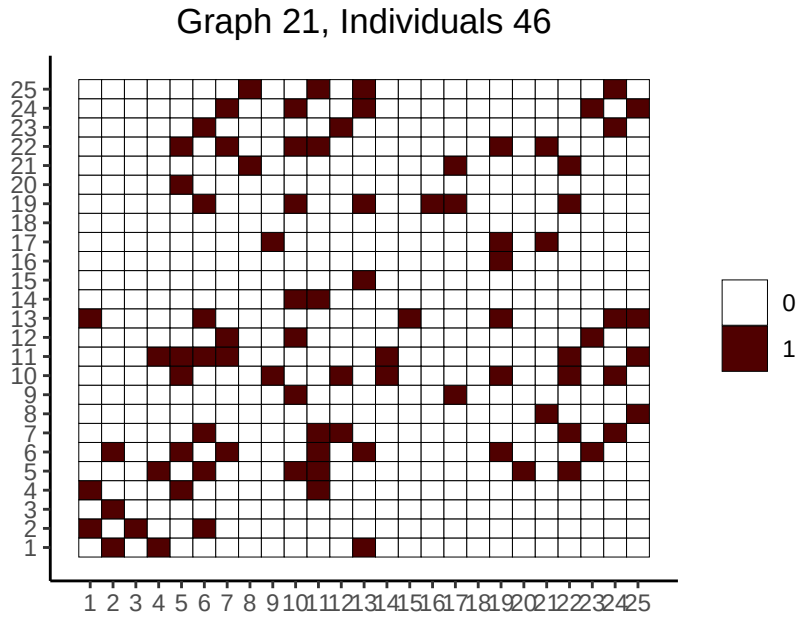

```
##
## $trial31$M2$unique_graphs[[19]]
```



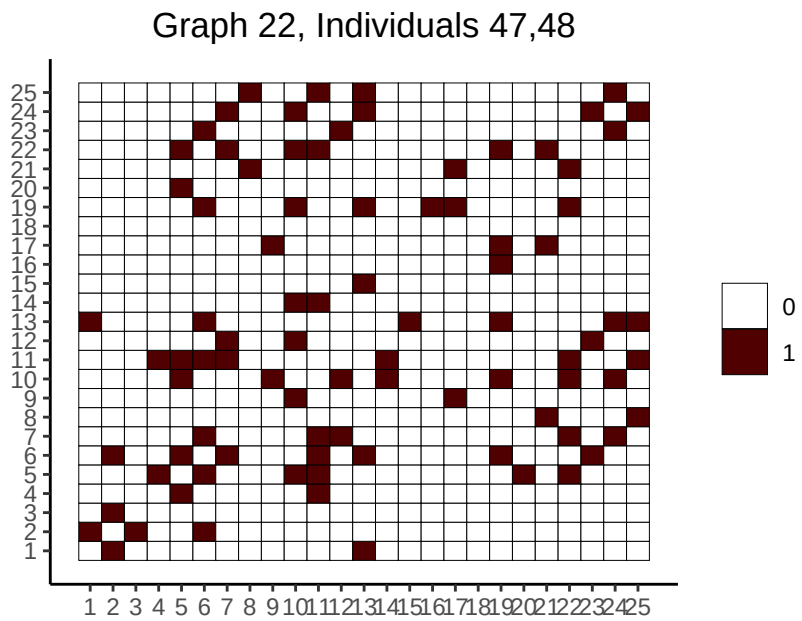

```
##
## $trial31$M2$unique_graphs[[20]]
```



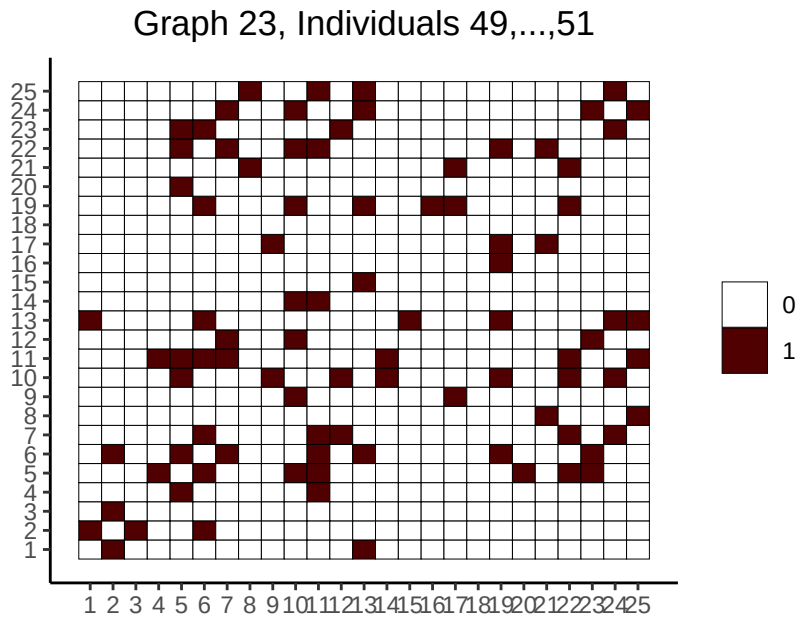

```
##
## $trial31$M2$unique_graphs[[21]]
```



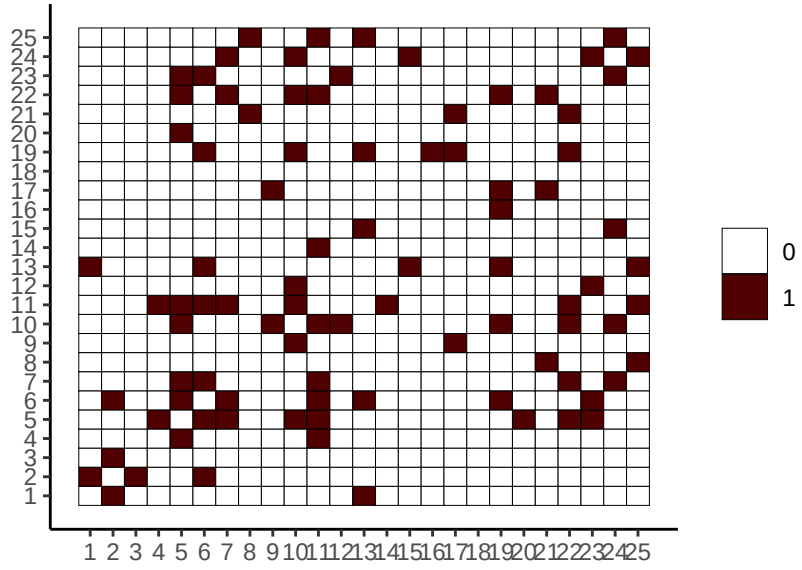

```
##
## $trial31$M2$unique_graphs[[22]]
```



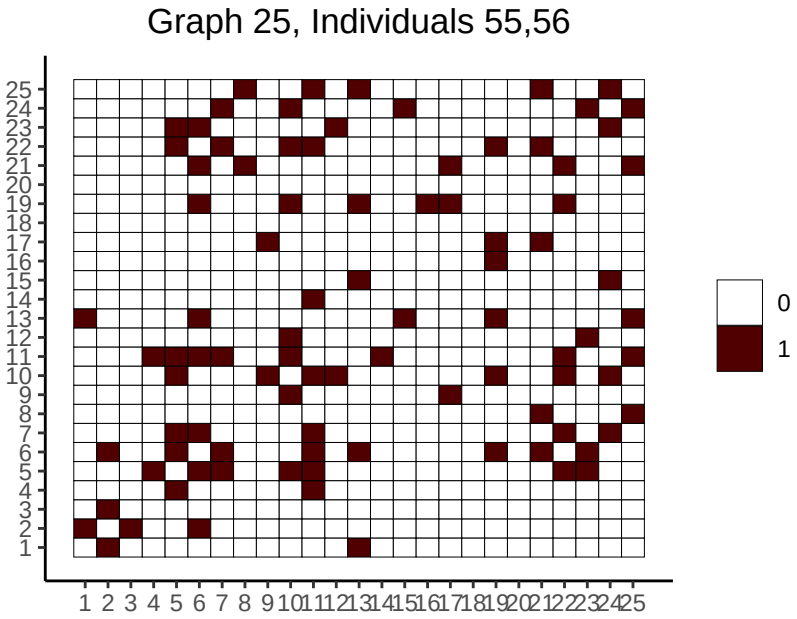

```
##
## $trial31$M2$unique_graphs[[23]]
```



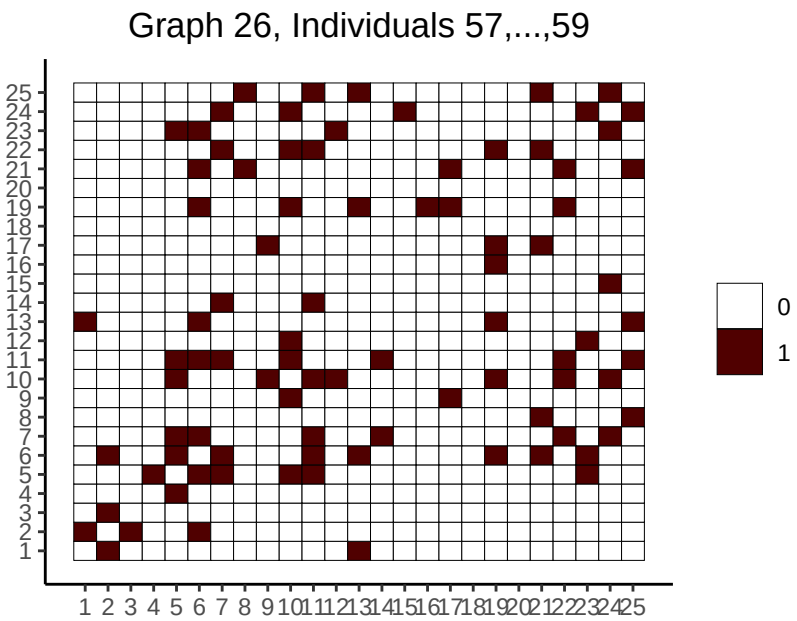

```
##
## $trial31$M2$unique_graphs[[24]]
```



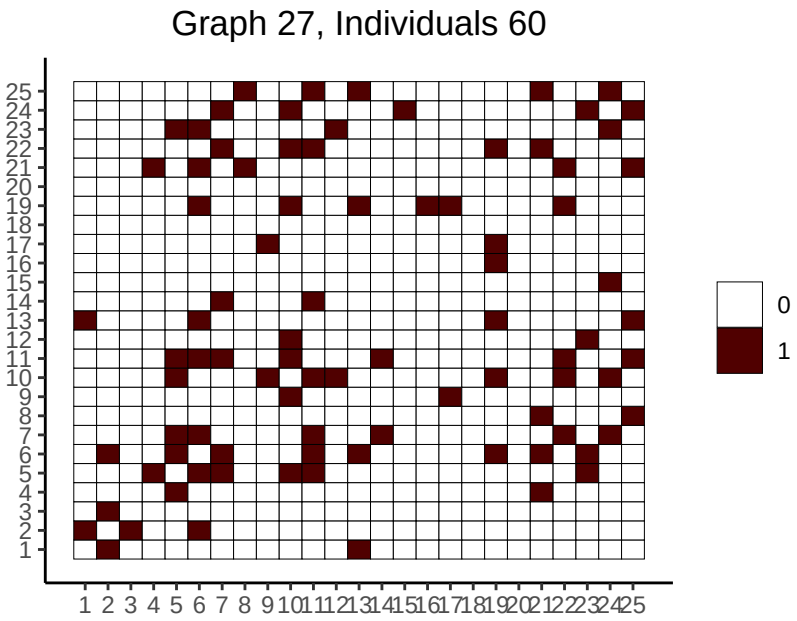

```
##
## $trial31$M2$unique_graphs[[25]]
```



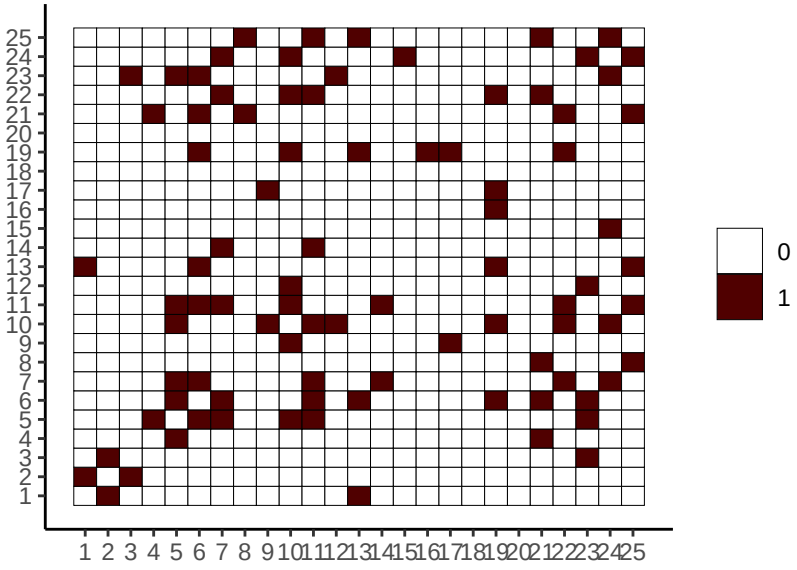

```
##
## $trial31$M2$unique_graphs[[26]]
```



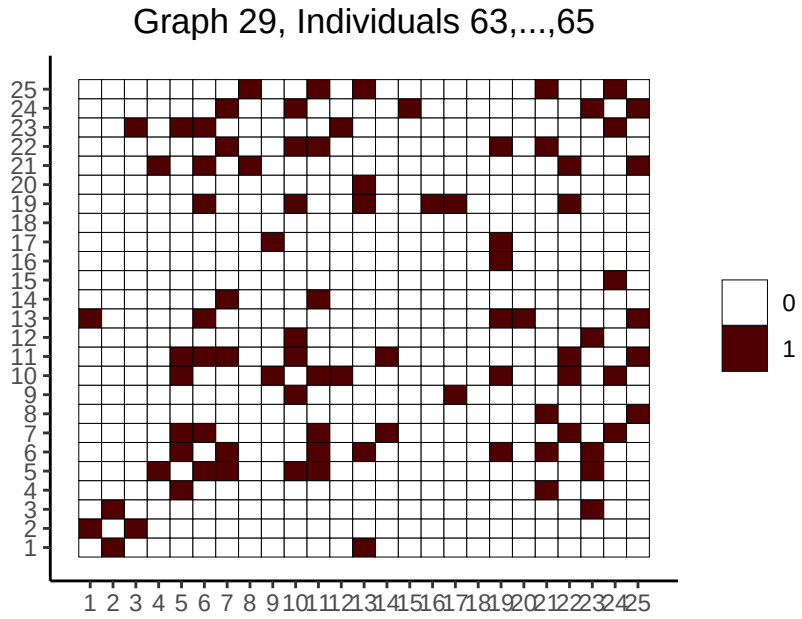

```
##
## $trial31$M2$unique_graphs[[27]]
```



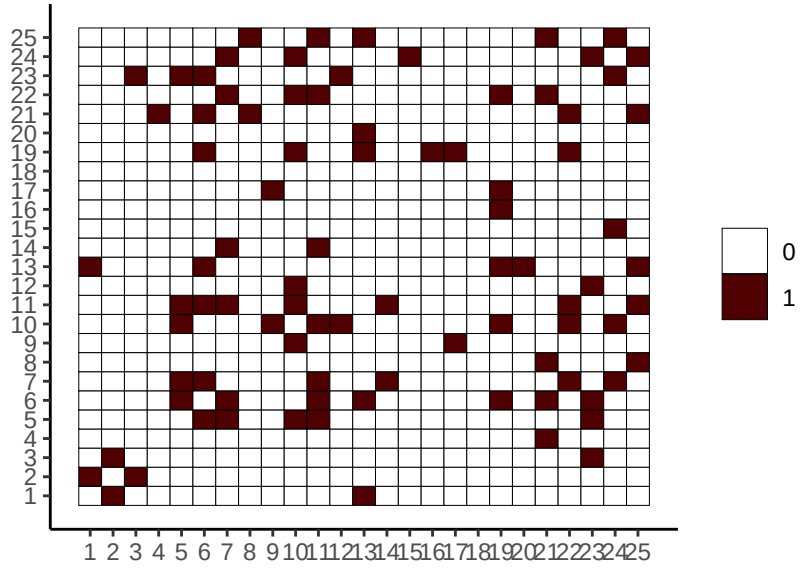

```
##
## $trial31$M2$unique_graphs[[28]]
```



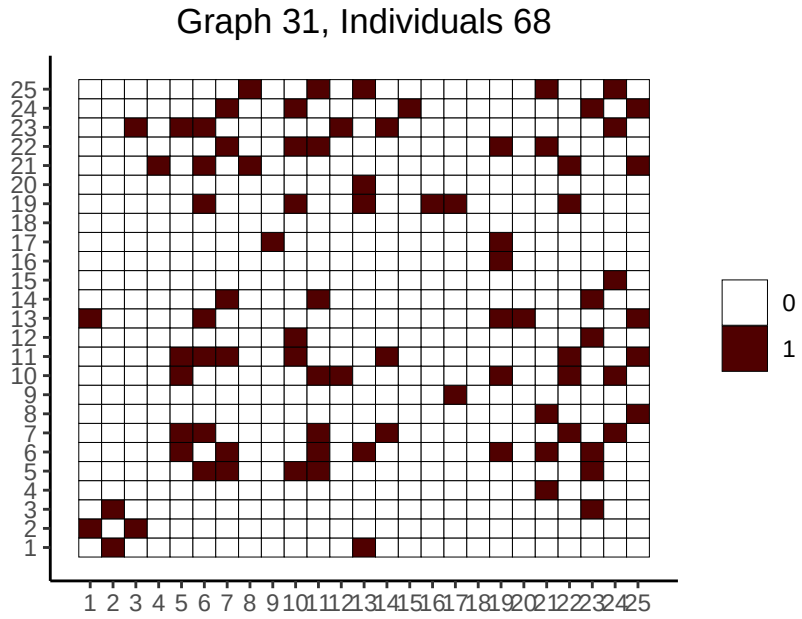

```
##
## $trial31$M2$unique_graphs[[29]]
```



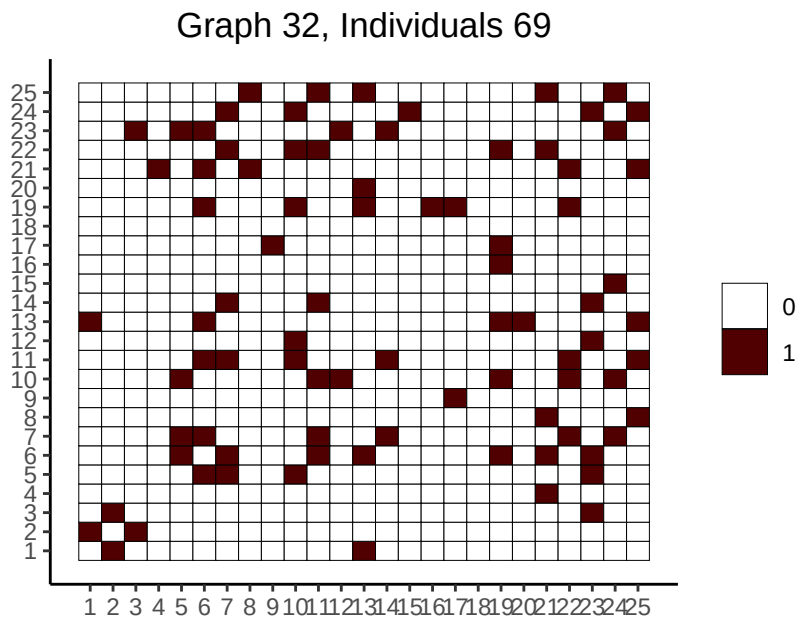

```
##
## $trial31$M2$unique_graphs[[30]]
```



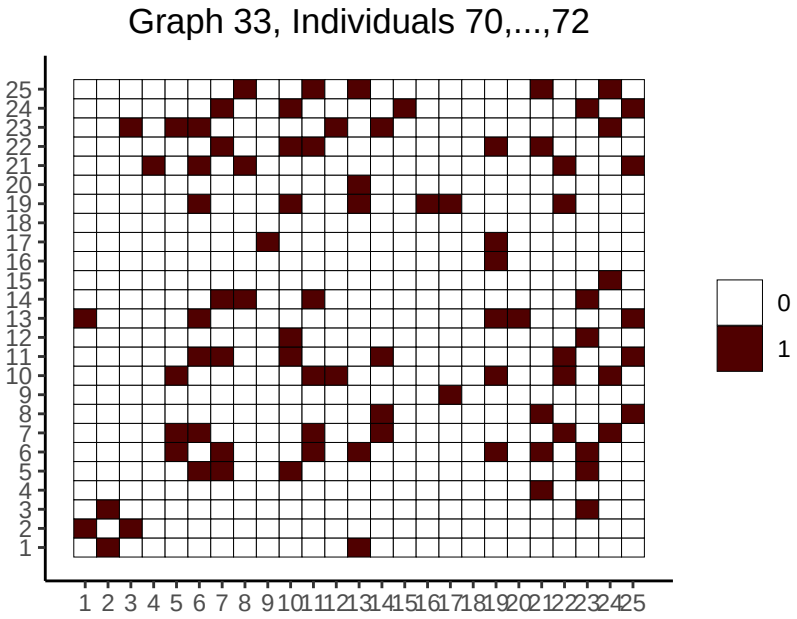

```
##
## $trial31$M2$unique_graphs[[31]]
```



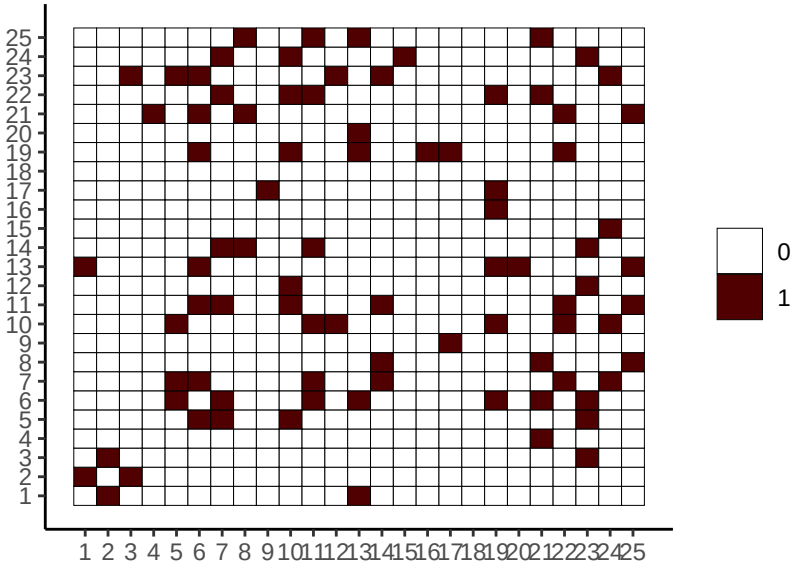

```
##
## $trial31$M2$unique_graphs[[32]]
```



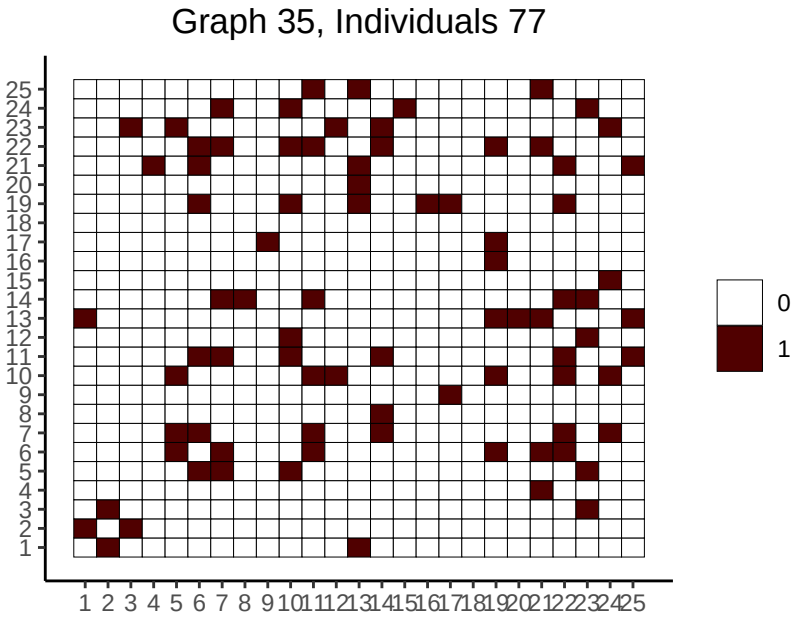

```
##
## $trial31$M2$unique_graphs[[33]]
```



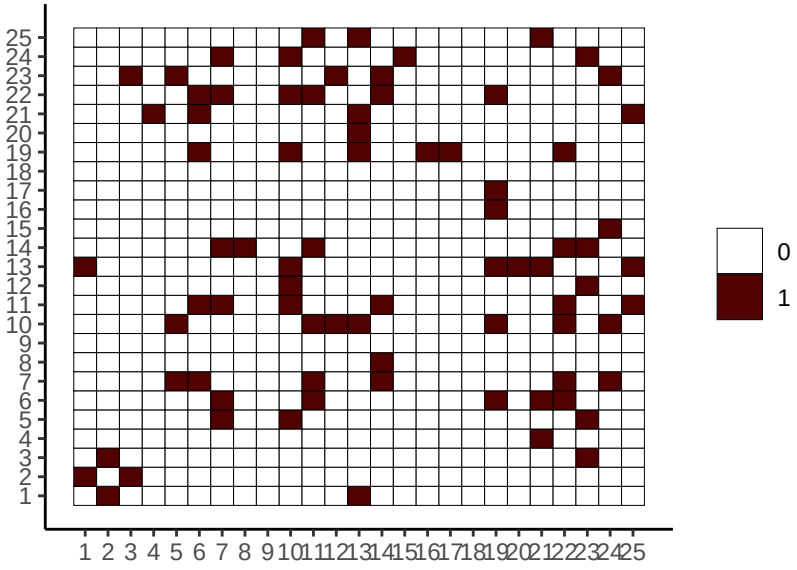

```
##
## $trial31$M2$unique_graphs[[34]]
```



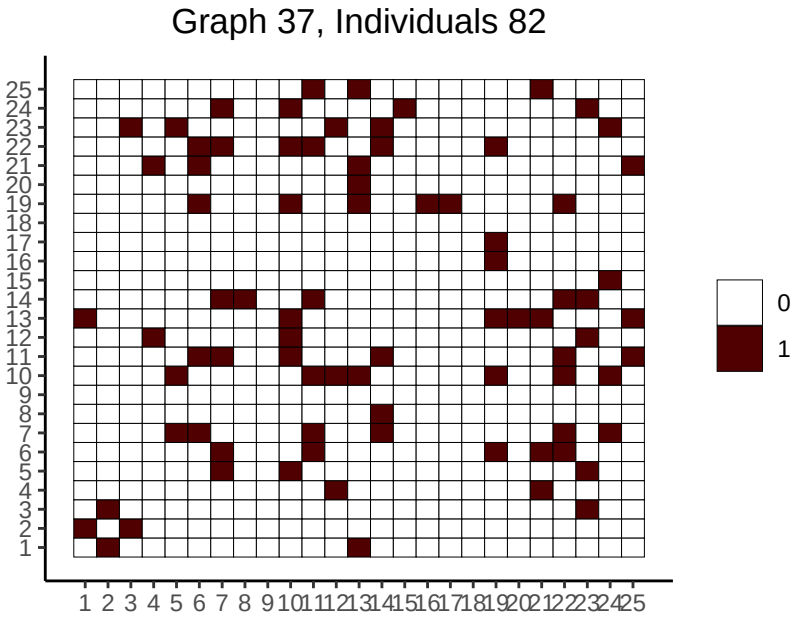

```
##
## $trial31$M2$unique_graphs[[35]]
```



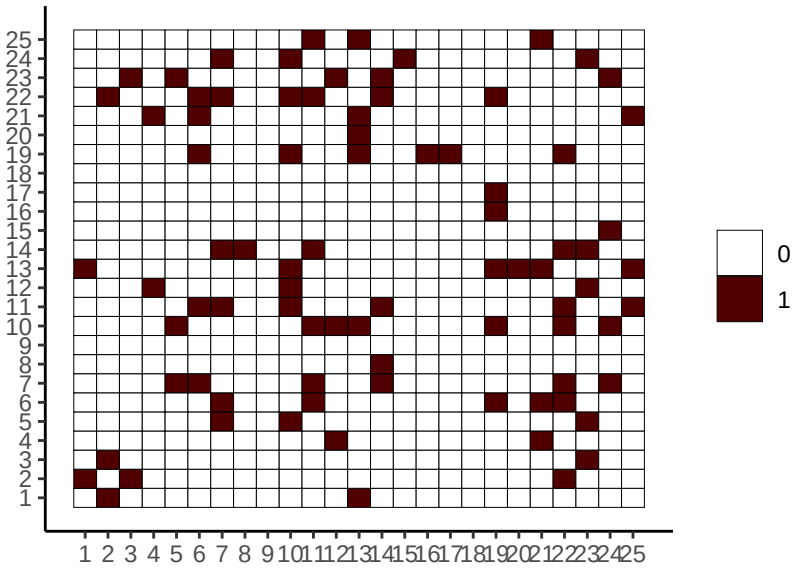

```
##
## $trial31$M2$unique_graphs[[36]]
```



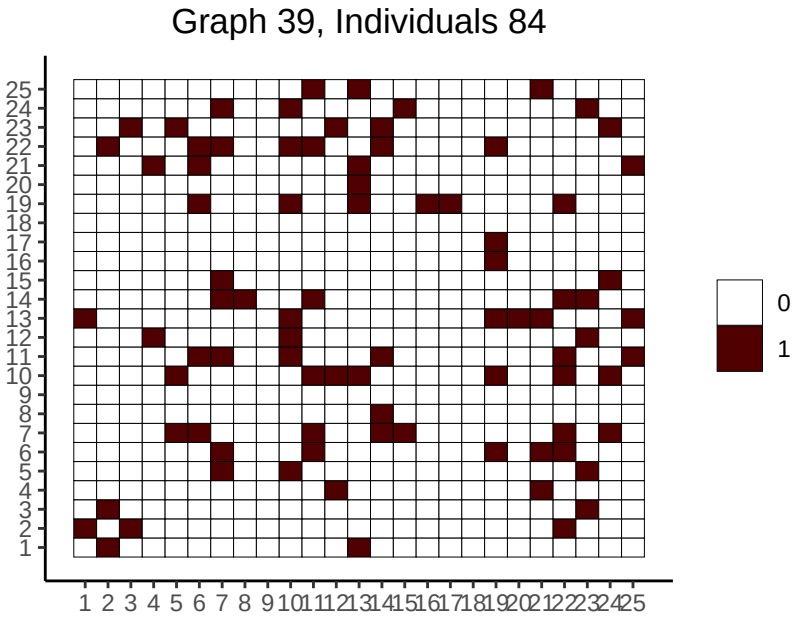

```
##
## $trial31$M2$unique_graphs[[37]]
```



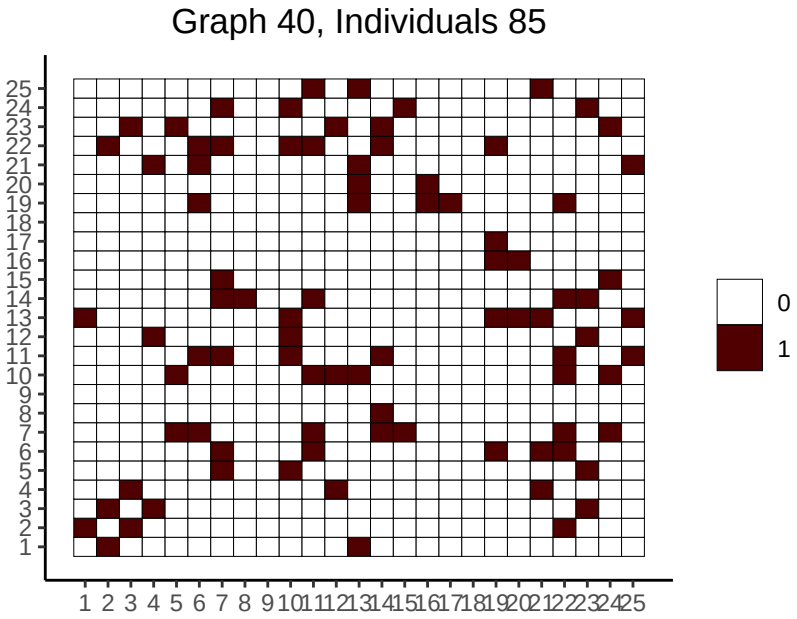

```
##
## $trial31$M2$unique_graphs[[38]]
```



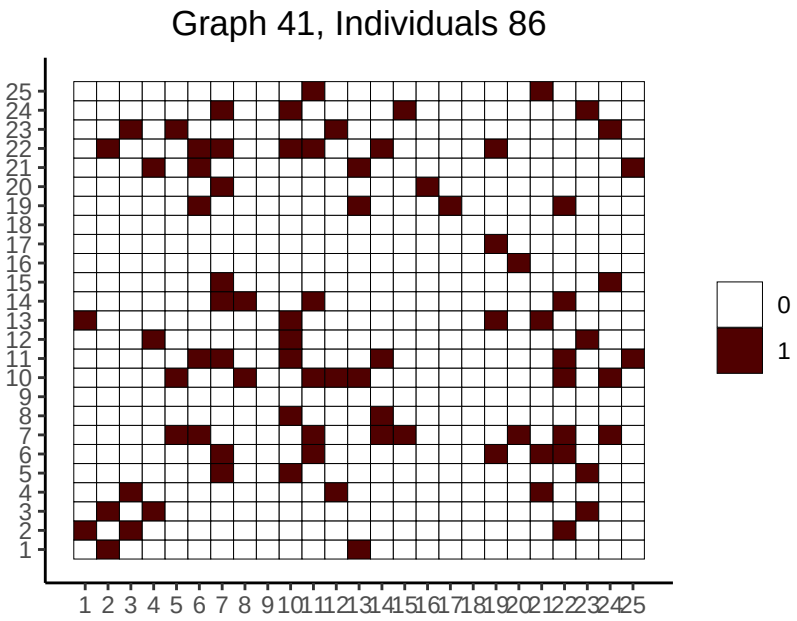

```
##
## $trial31$M2$unique_graphs[[39]]
```



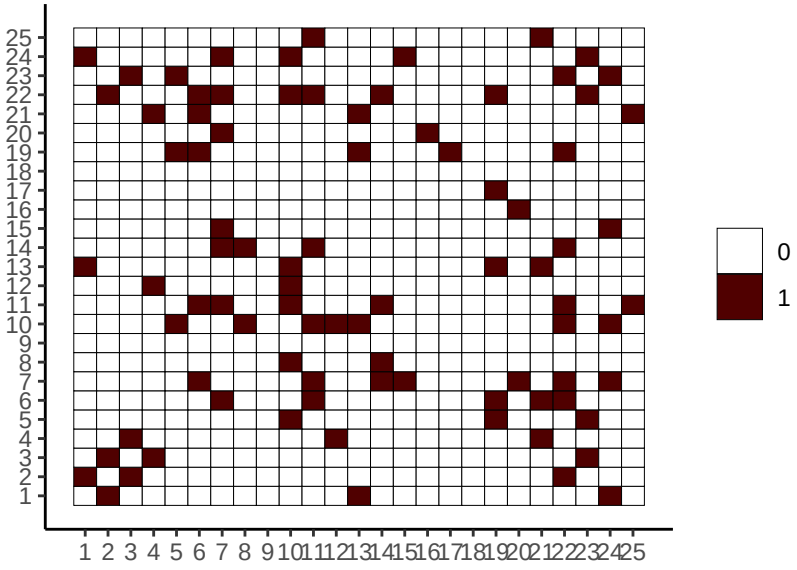

```
##
## $trial31$M2$unique_graphs[[40]]
```



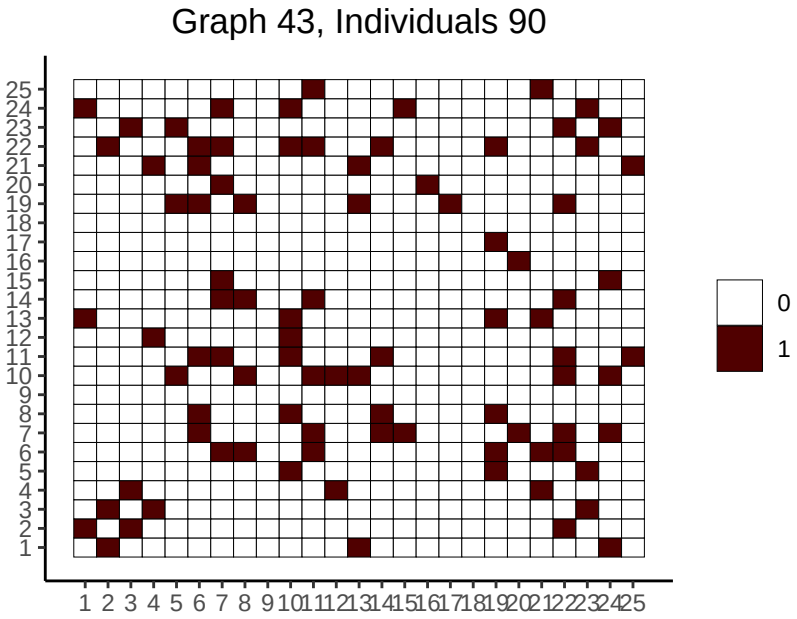

```
##
## $trial31$M2$unique_graphs[[41]]
```



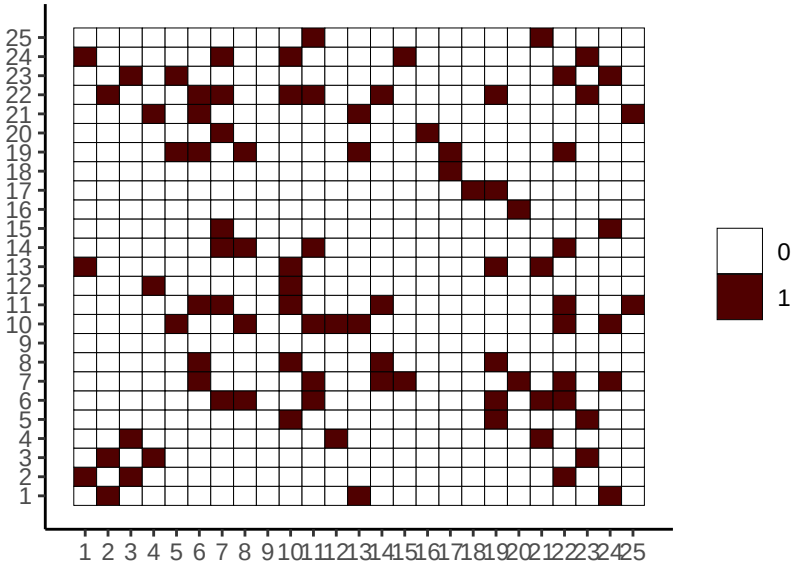

```
##
## $trial31$M2$unique_graphs[[42]]
```



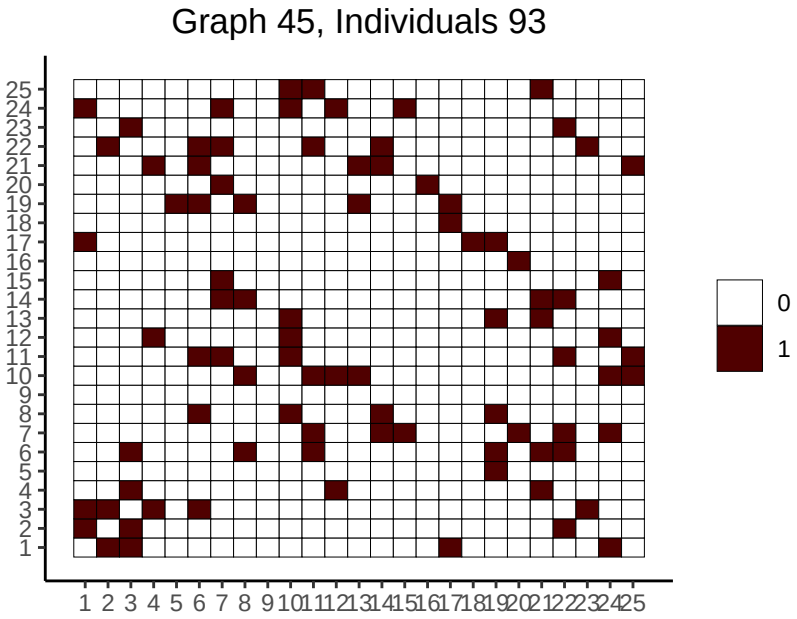

```
##
## $trial31$M2$unique_graphs[[43]]
```



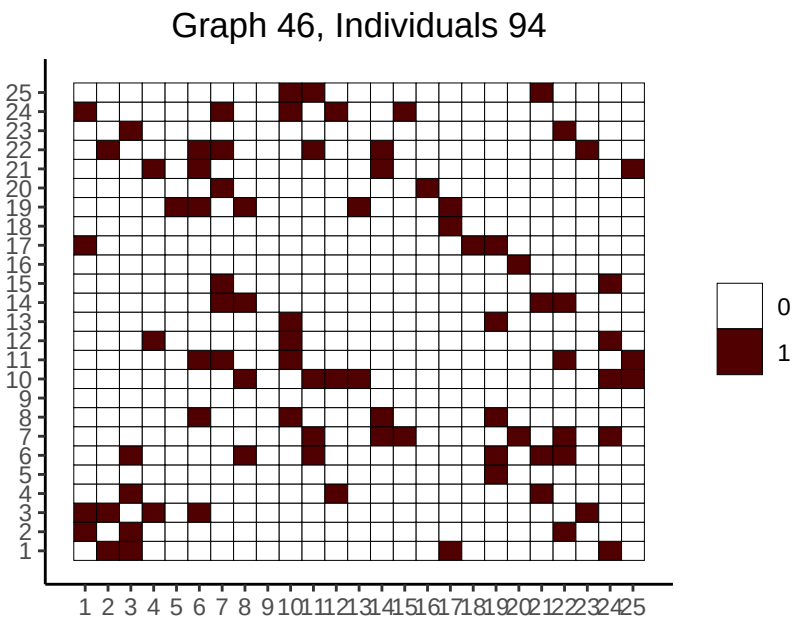

```
##
## $trial31$M2$unique_graphs[[44]]
```



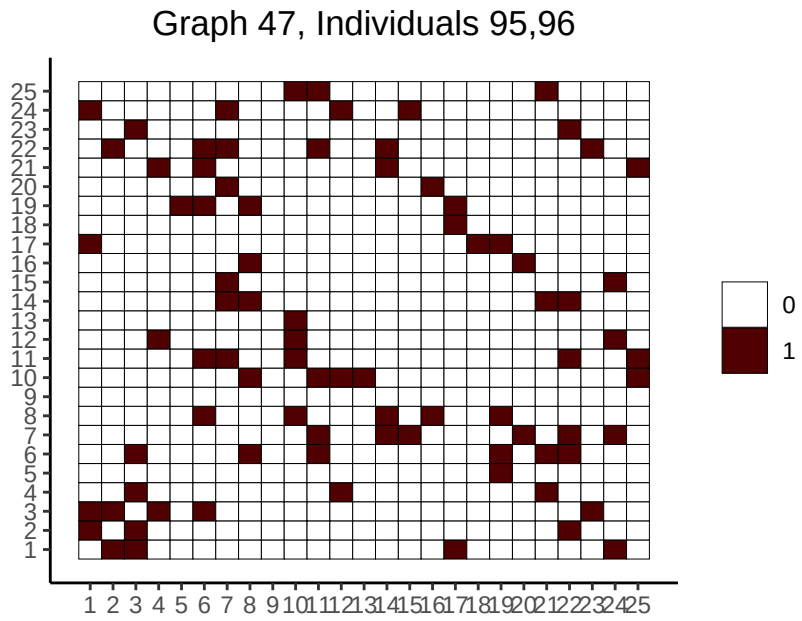

```
##
## $trial31$M2$unique_graphs[[45]]
```



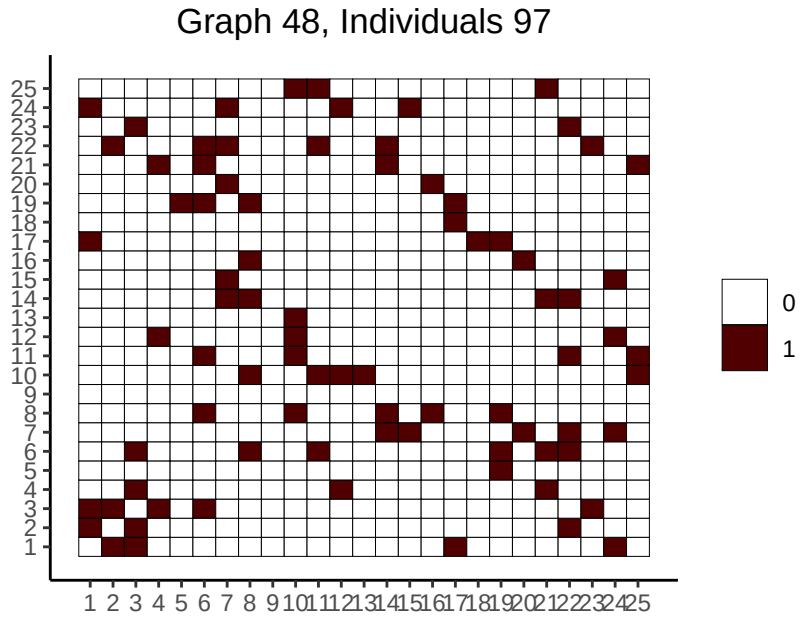

```
##
## $trial31$M2$unique_graphs[[46]]
```



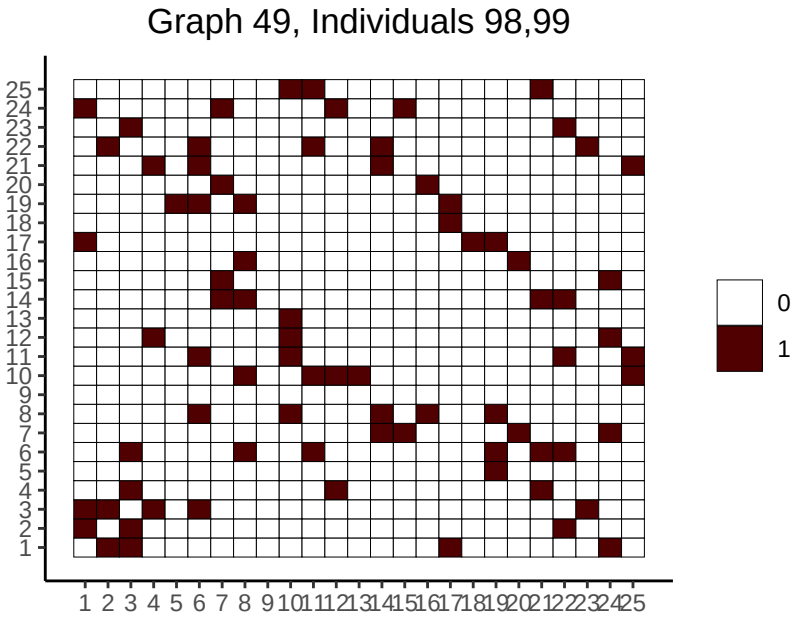

```
##
## $trial31$M2$unique_graphs[[47]]
```



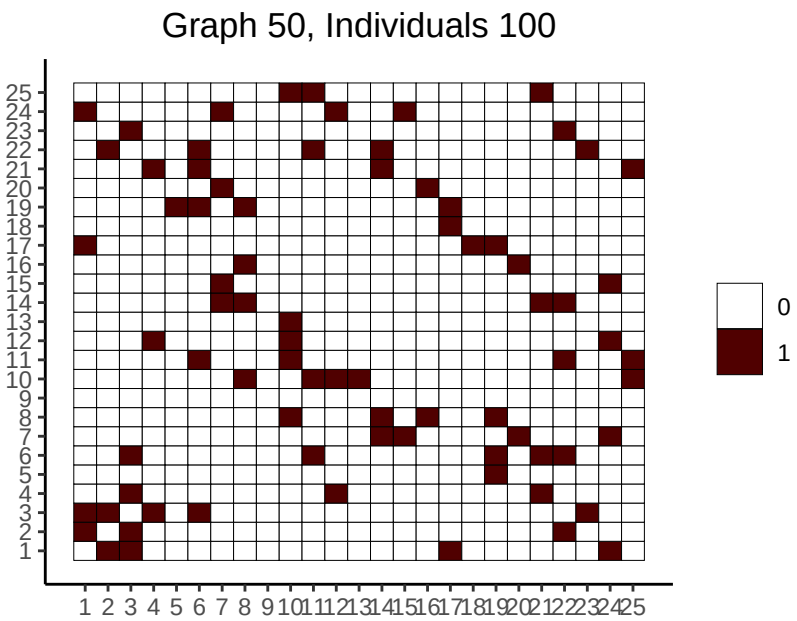

```
##
## $trial31$M2$unique_graphs[[48]]
```



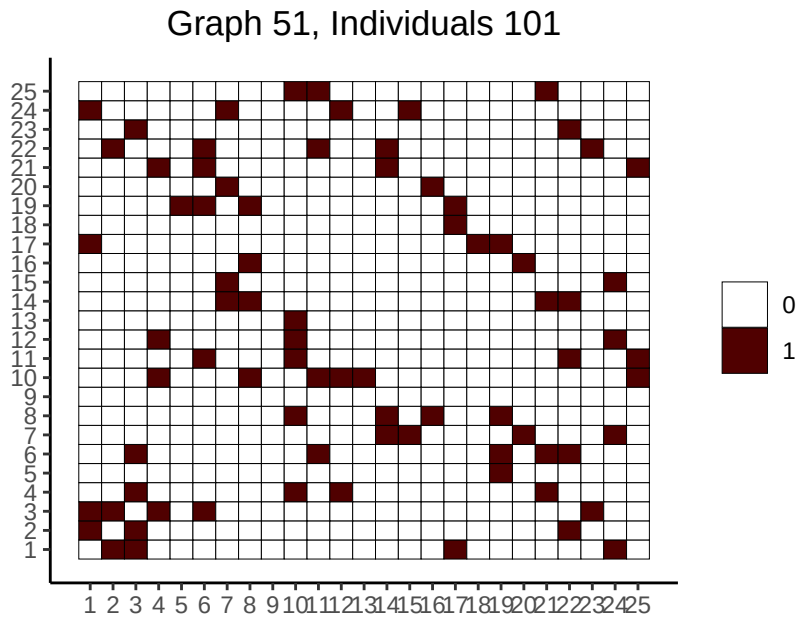

```
##
## $trial31$M2$unique_graphs[[49]]
```



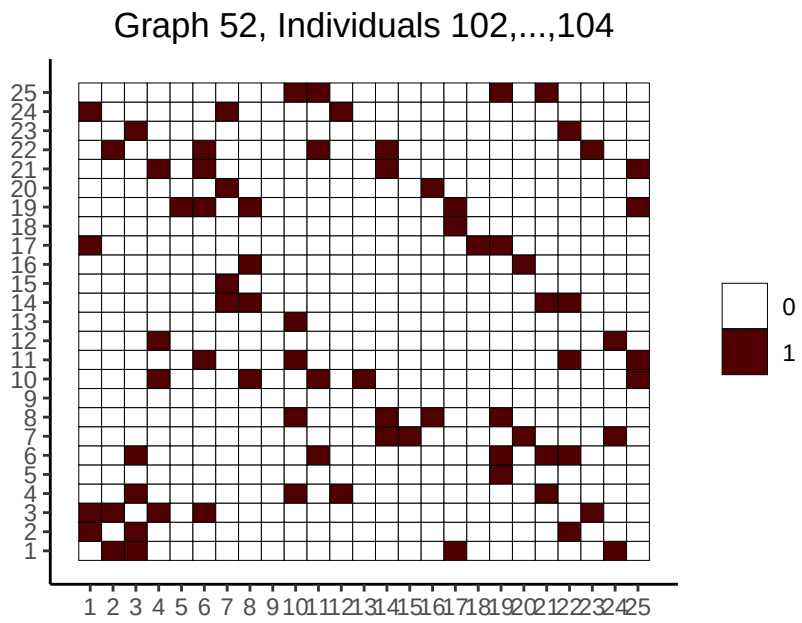

```
##
## $trial31$M2$unique_graphs[[50]]
```



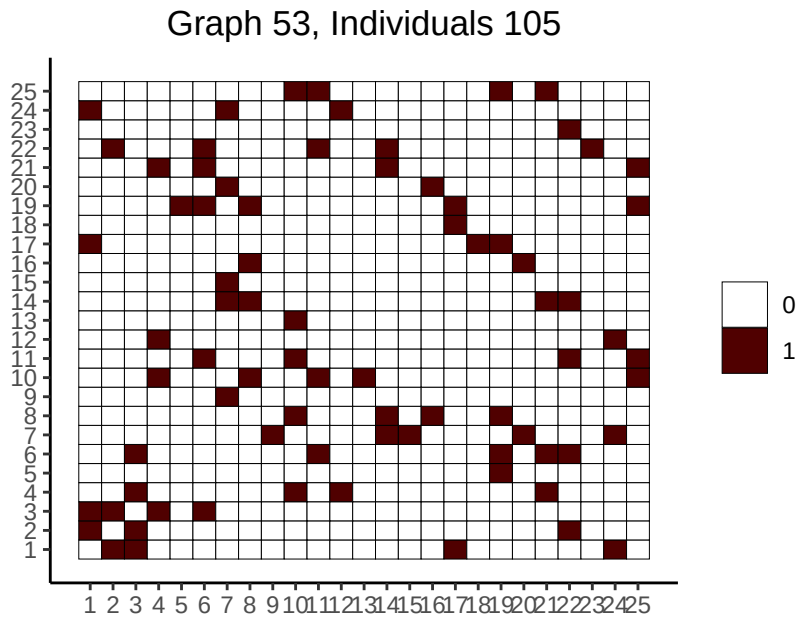

```
##
## $trial31$M2$unique_graphs[[51]]
```



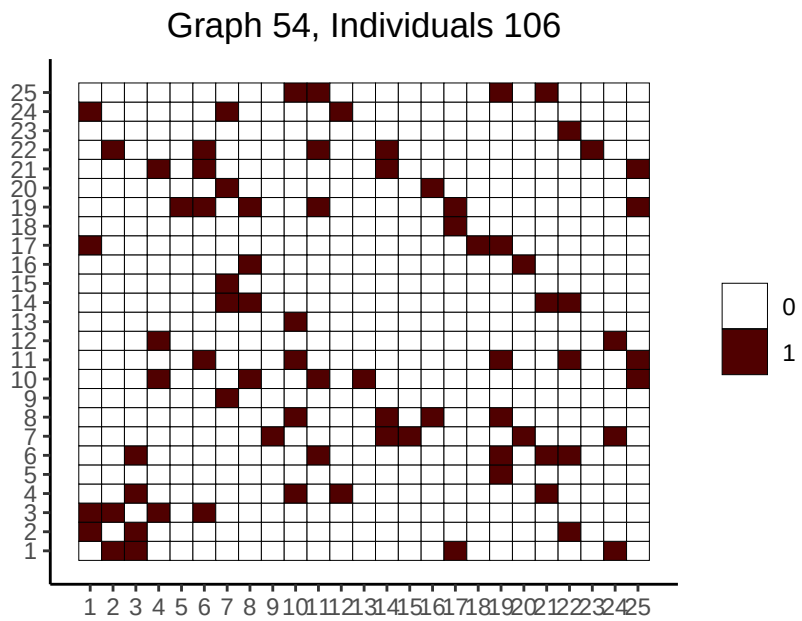

```
##
## $trial31$M2$unique_graphs[[52]]
```



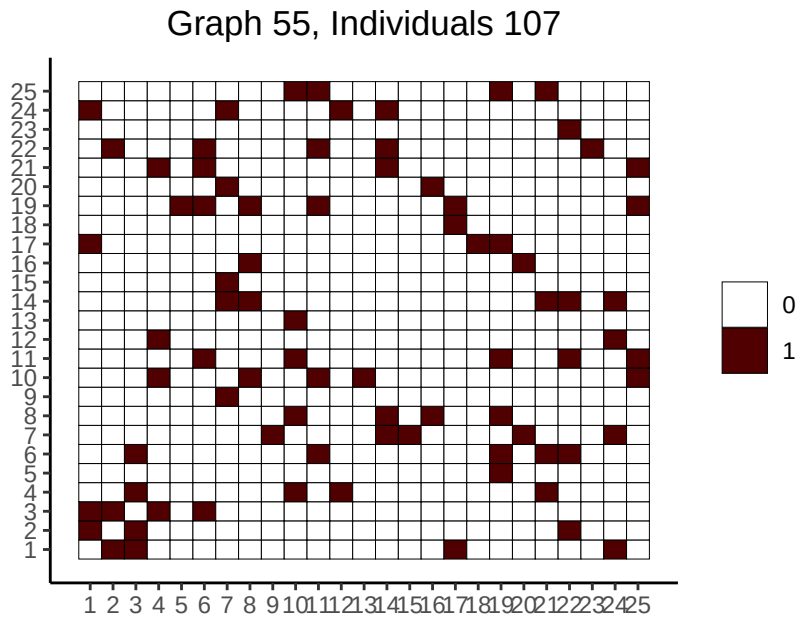

```
##
## $trial31$M2$unique_graphs[[53]]
```



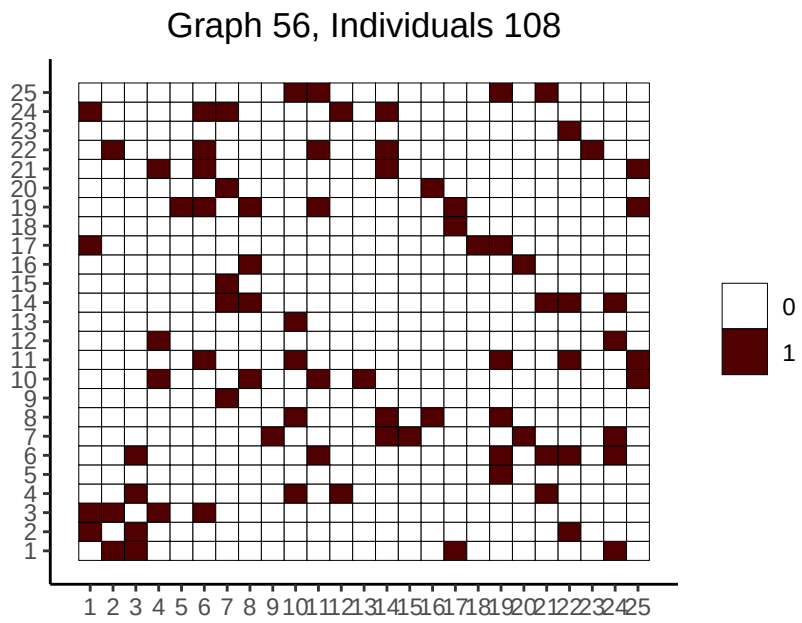

```
##
## $trial31$M2$unique_graphs[[54]]
```



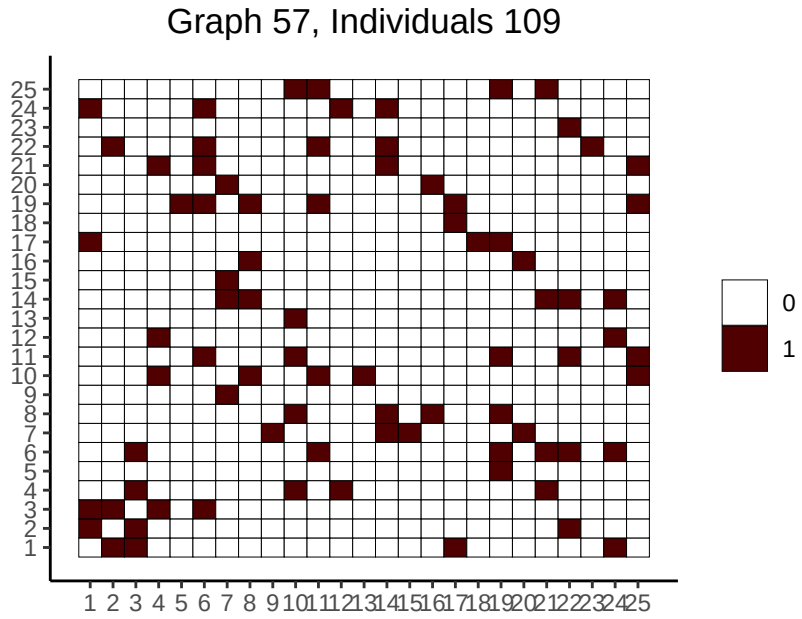

```
##
## $trial31$M2$unique_graphs[[55]]
```



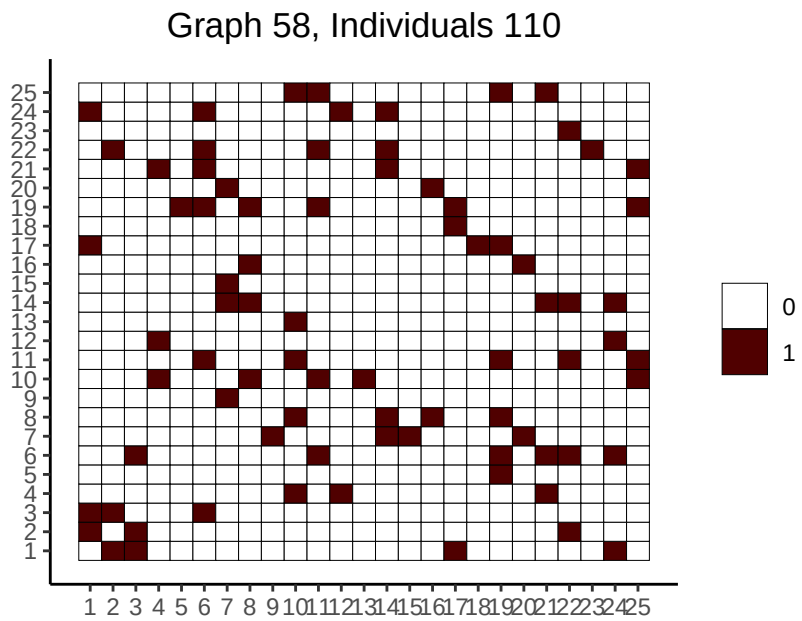

```
##
## $trial31$M2$unique_graphs[[56]]
```



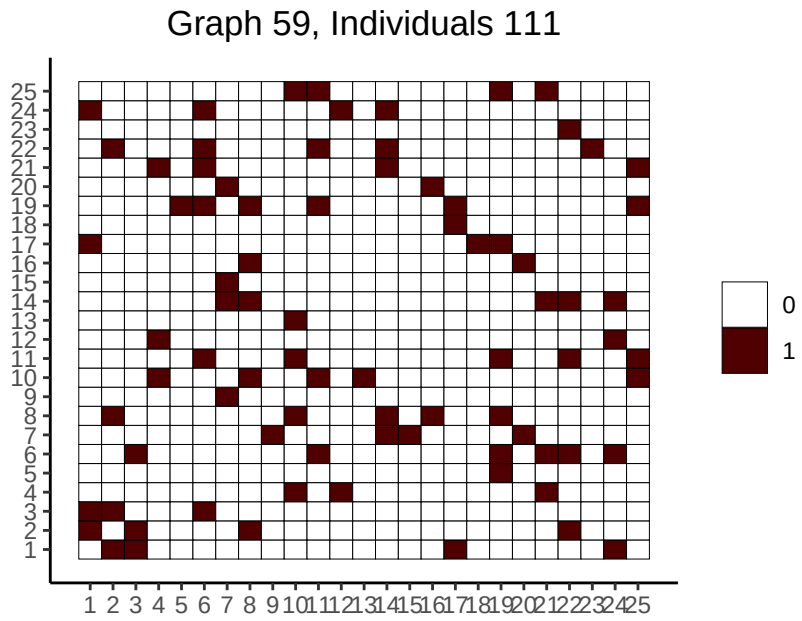

```
##
## $trial31$M2$unique_graphs[[57]]
```



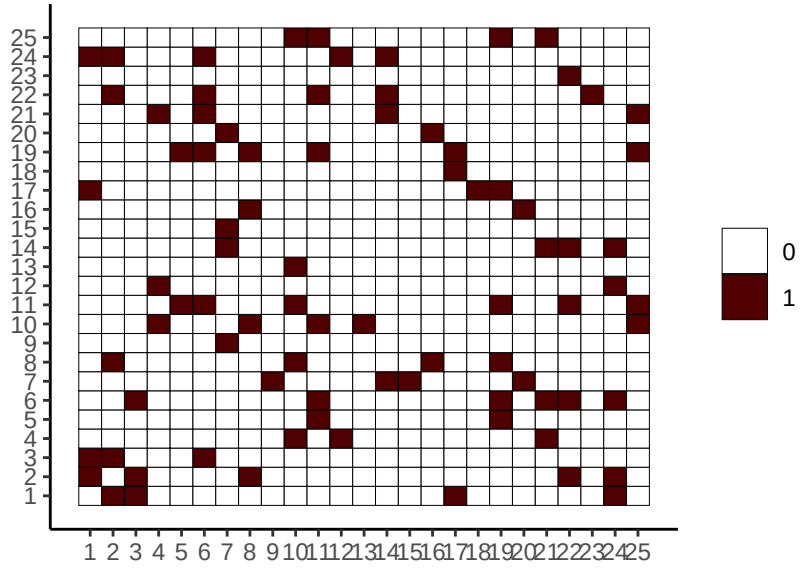

```
##
## $trial31$M2$unique_graphs[[58]]
```




```
##
## $trial31$M2$unique_graphs[[59]]
```

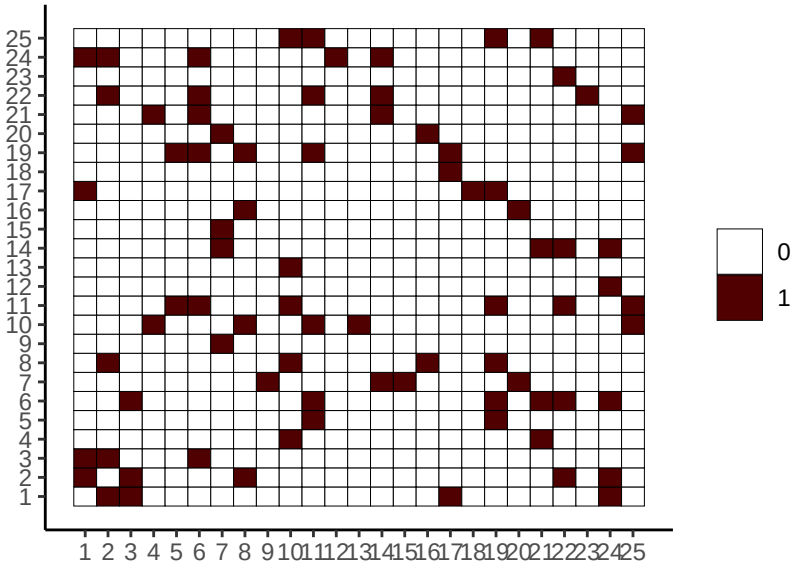



```
##
## $trial31$M2$unique_graphs[[60]]
```



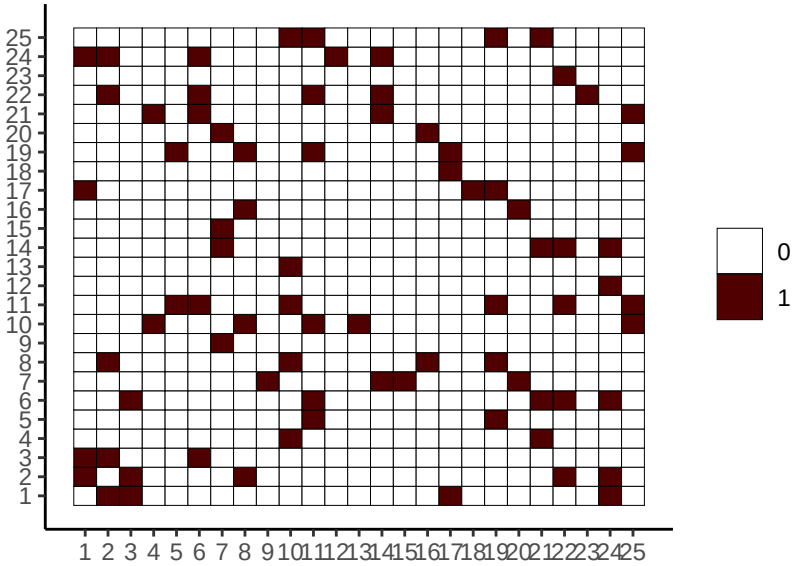

```
##
## $trial31$M2$unique_graphs[[61]]
```

Graph 61, Individuals 114

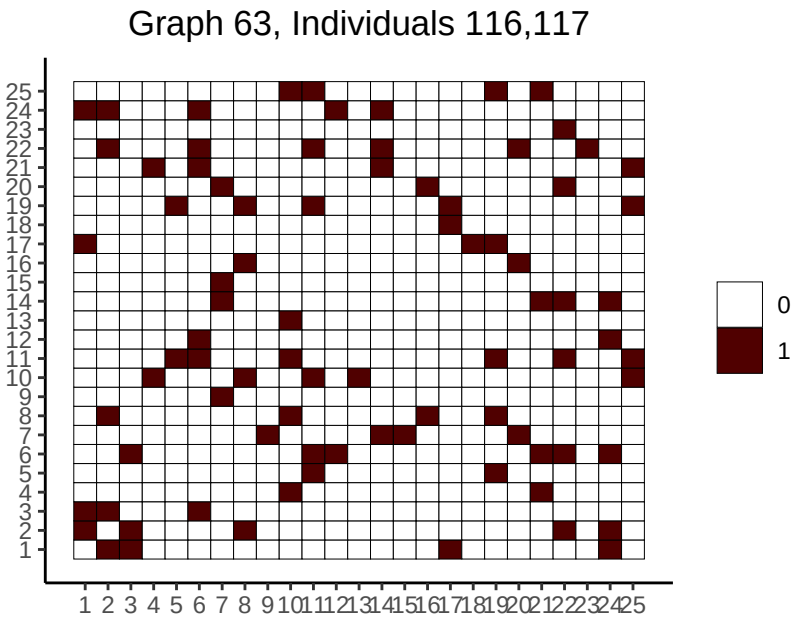



```
##
## $trial31$M2$unique_graphs[[62]]
```

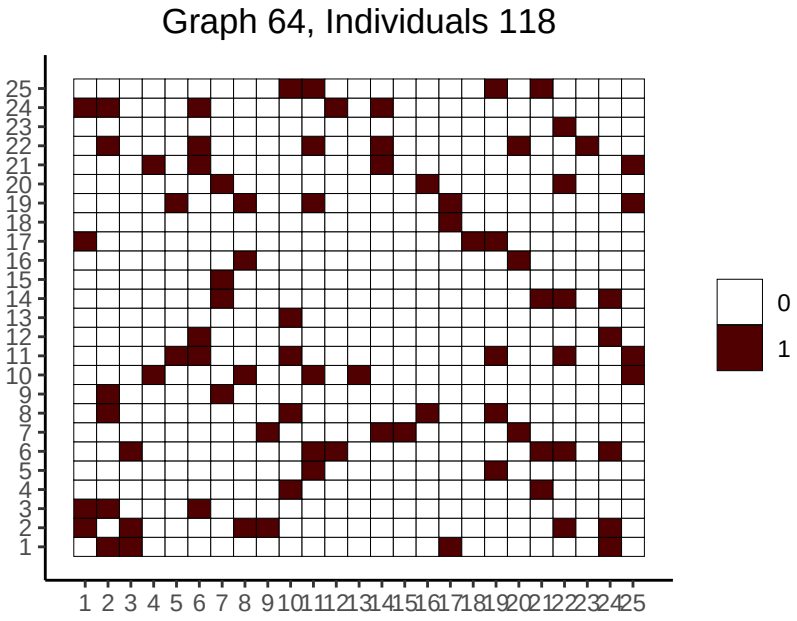
Graph 62, Individuals 115



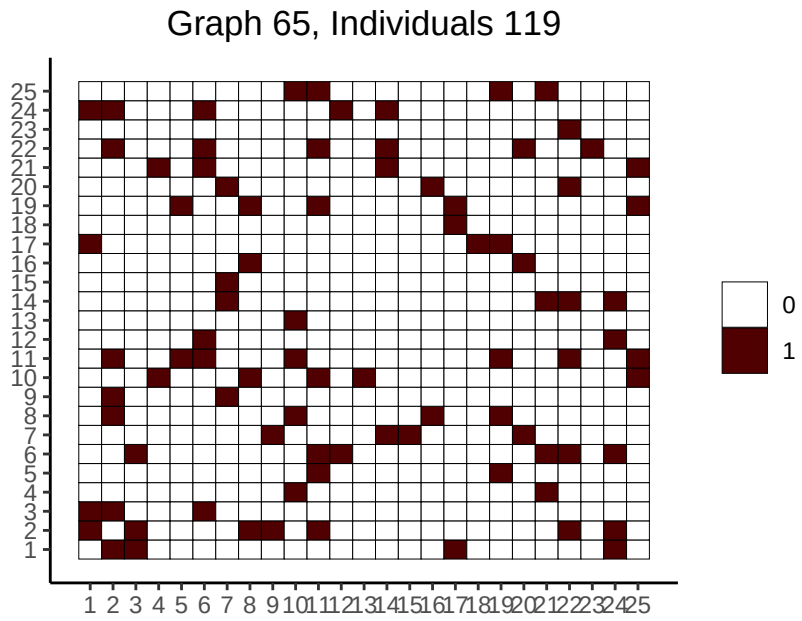

```
##
## $trial31$M2$unique_graphs[[63]]
```



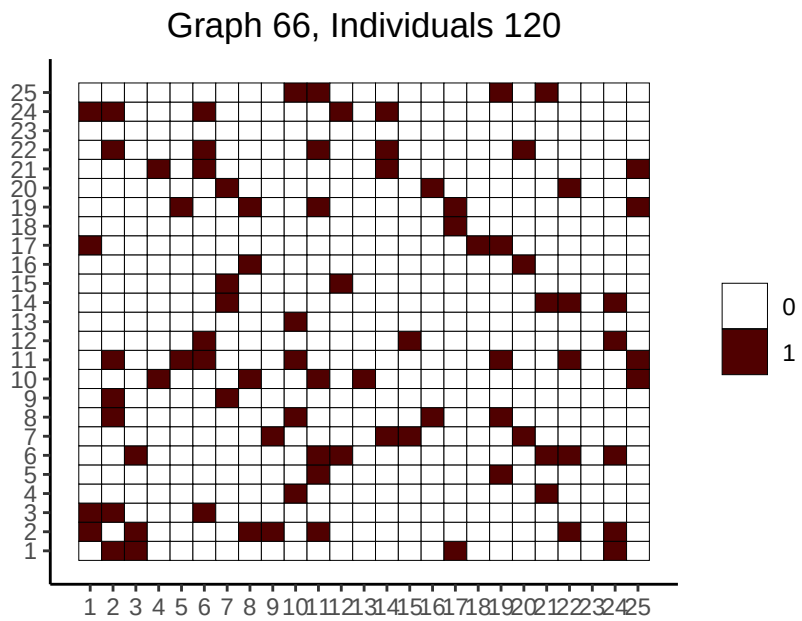

```
##
## $trial31$M2$unique_graphs[[64]]
```



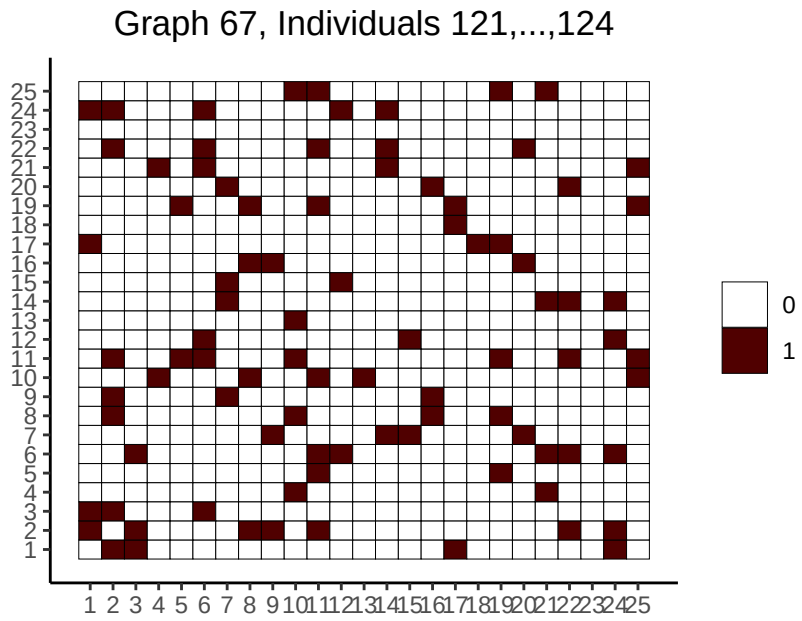

```
##
## $trial31$M2$unique_graphs[[65]]
```



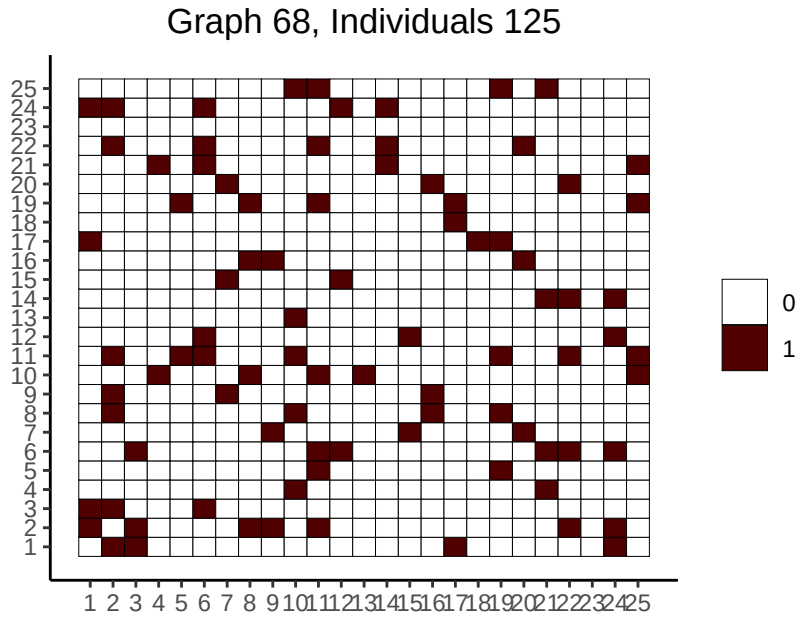

```
##
## $trial31$M2$unique_graphs[[66]]
```



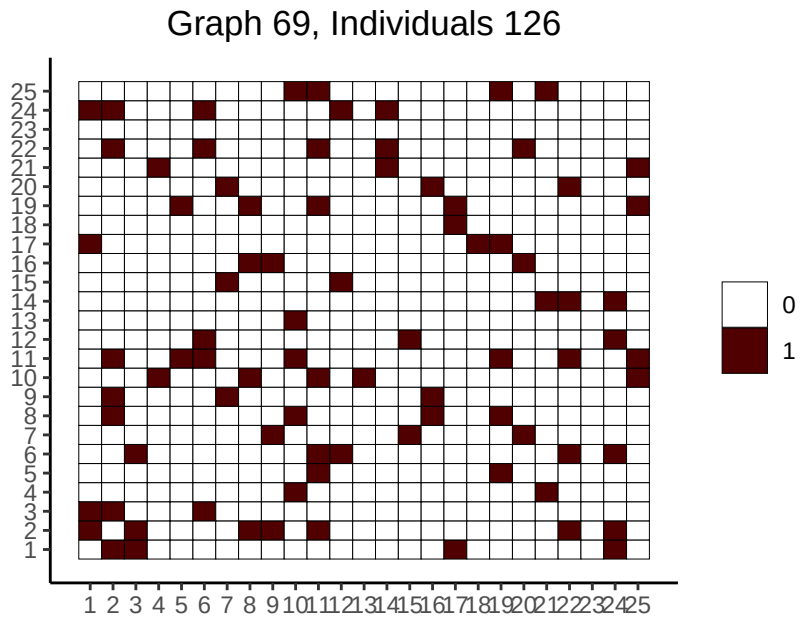

```
##
## $trial31$M2$unique_graphs[[67]]
```



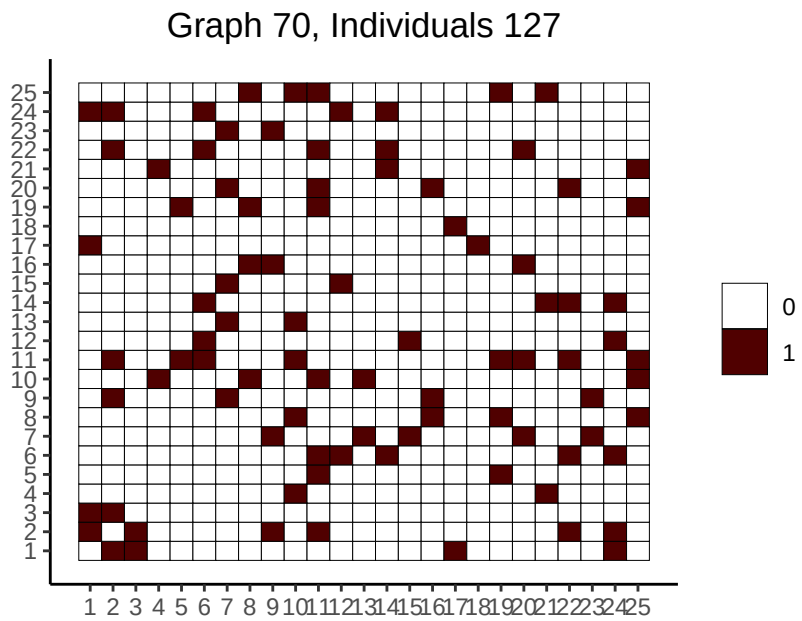

```
##
## $trial31$M2$unique_graphs[[68]]
```



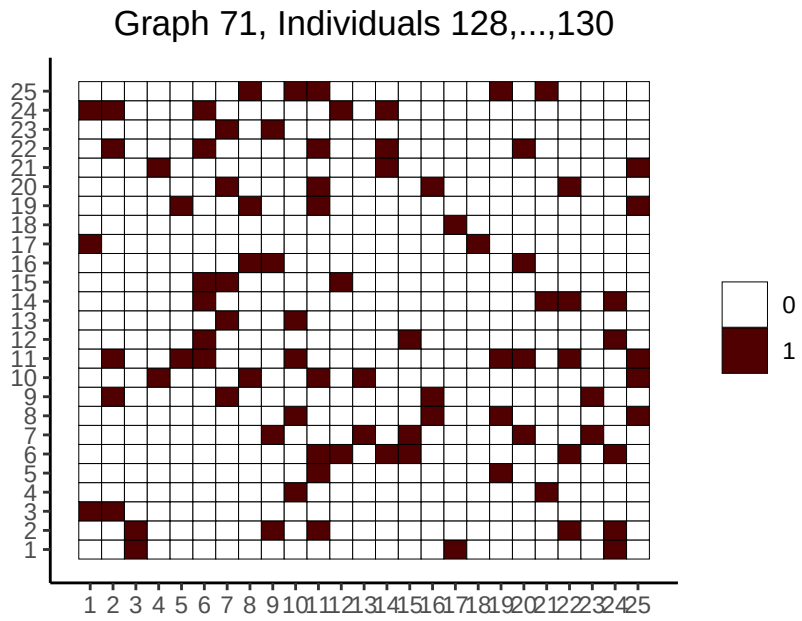

```
##
## $trial31$M2$unique_graphs[[69]]
```



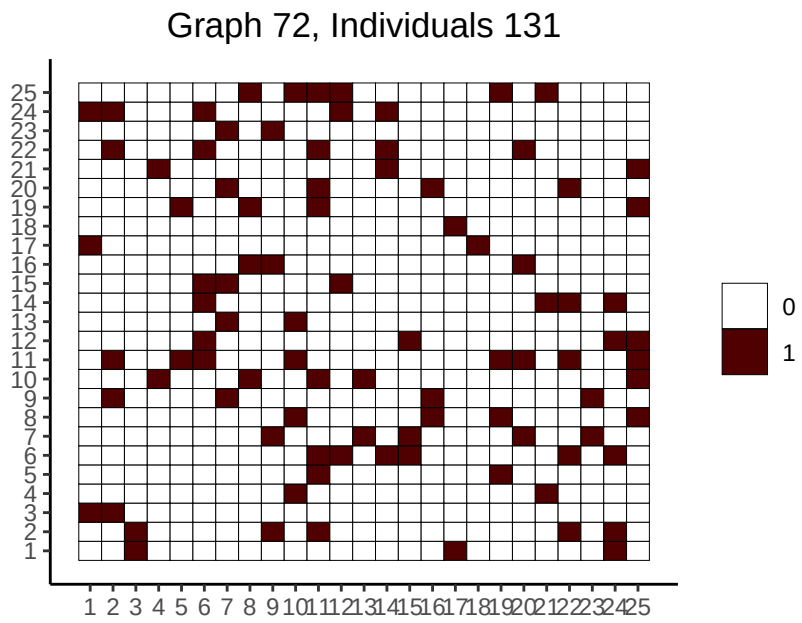

```
##
## $trial31$M2$unique_graphs[[70]]
```



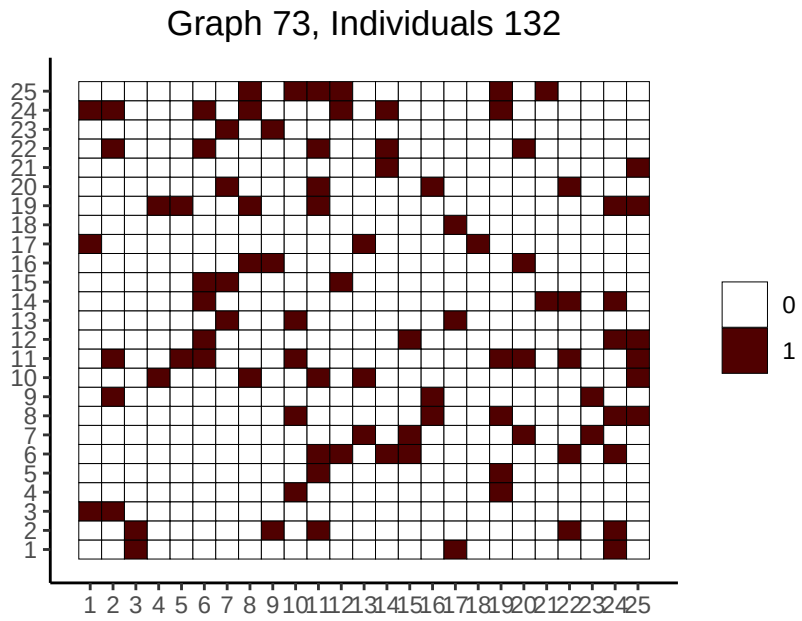

```
##
## $trial31$M2$unique_graphs[[71]]
```



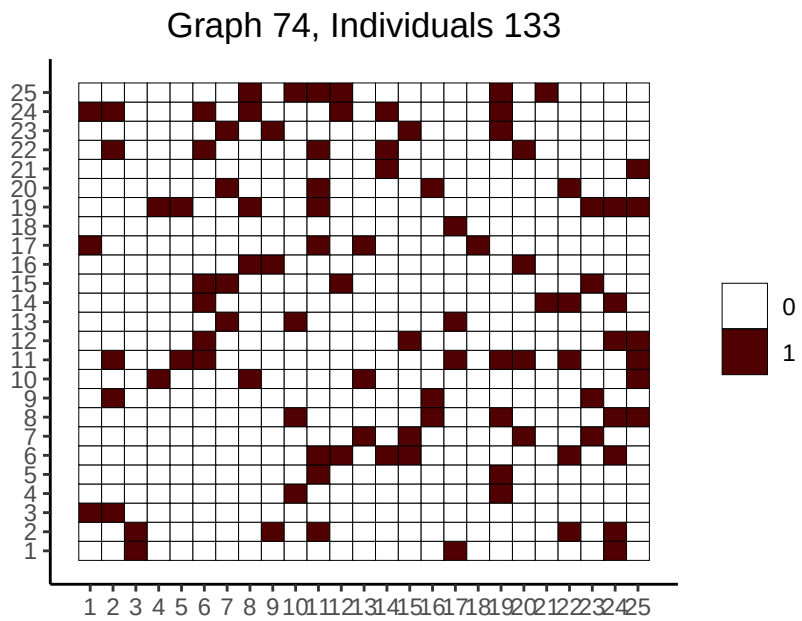

```
##
## $trial31$M2$unique_graphs[[72]]
```



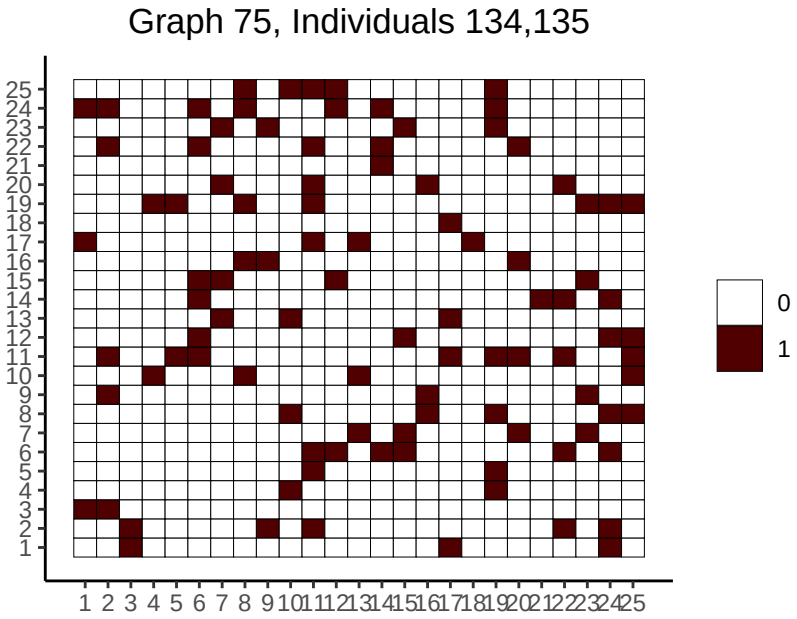

```
##
## $trial31$M2$unique_graphs[[73]]
```



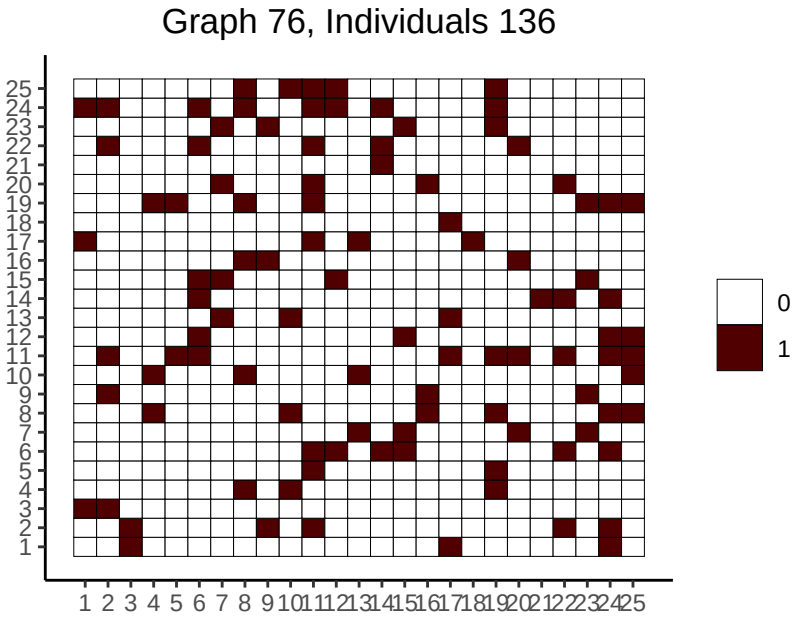

```
##
## $trial31$M2$unique_graphs[[74]]
```



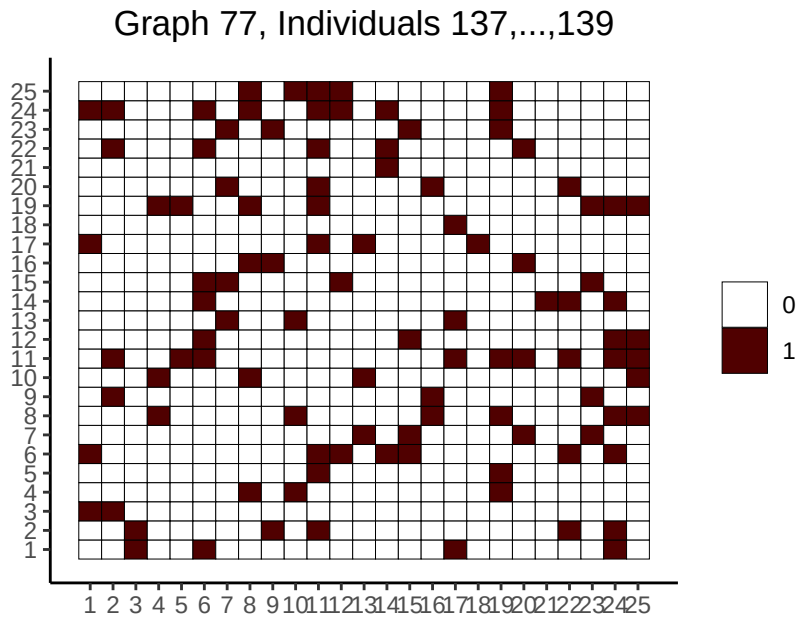

```
##
## $trial31$M2$unique_graphs[[75]]
```



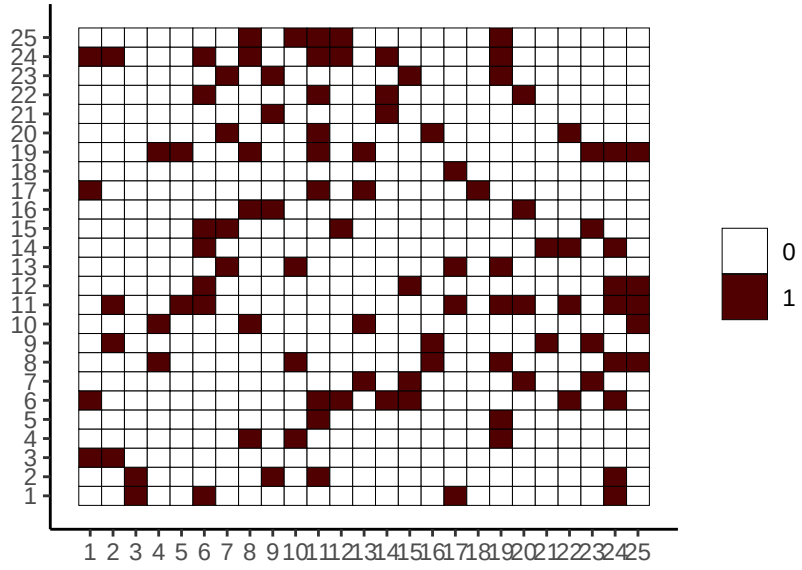

```
##
## $trial31$M2$unique_graphs[[76]]
```



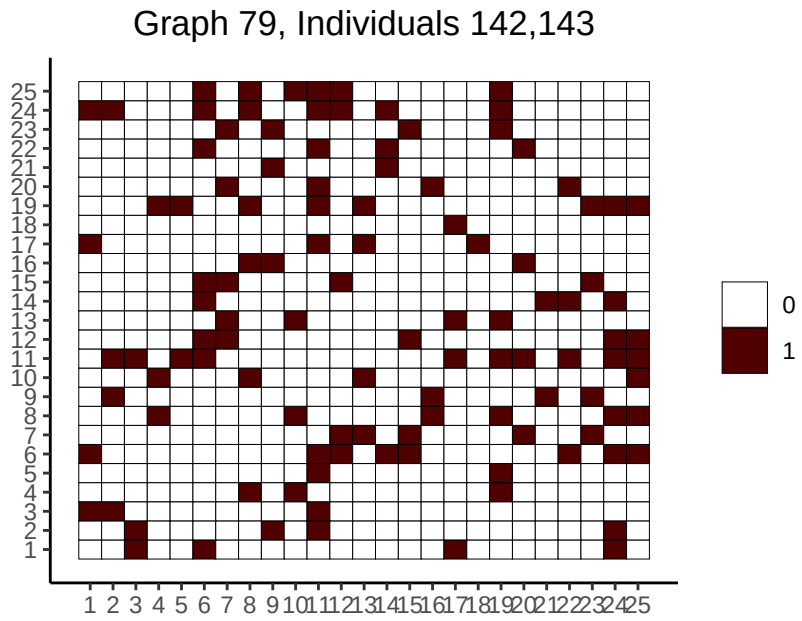

```
##
## $trial31$M2$unique_graphs[[77]]
```



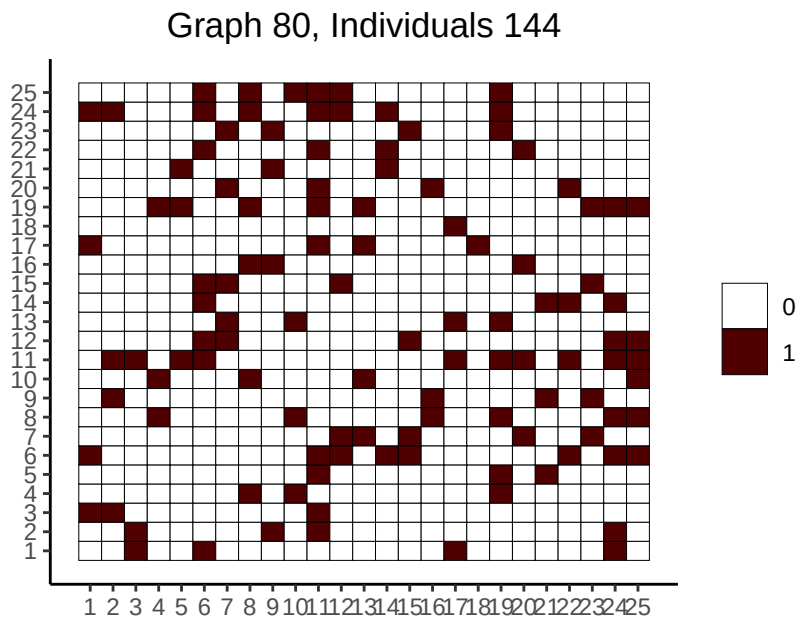

```
##
## $trial31$M2$unique_graphs[[78]]
```



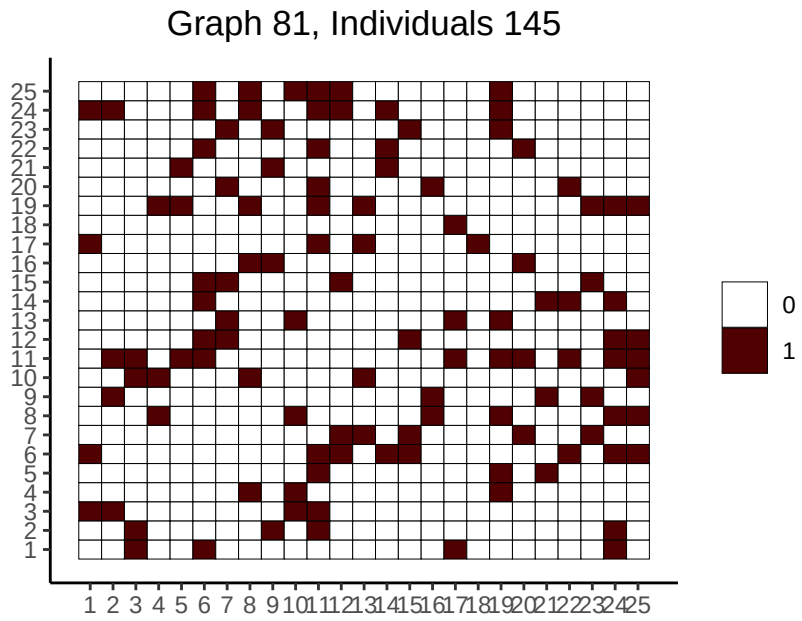

```
##
## $trial31$M2$unique_graphs[[79]]
```



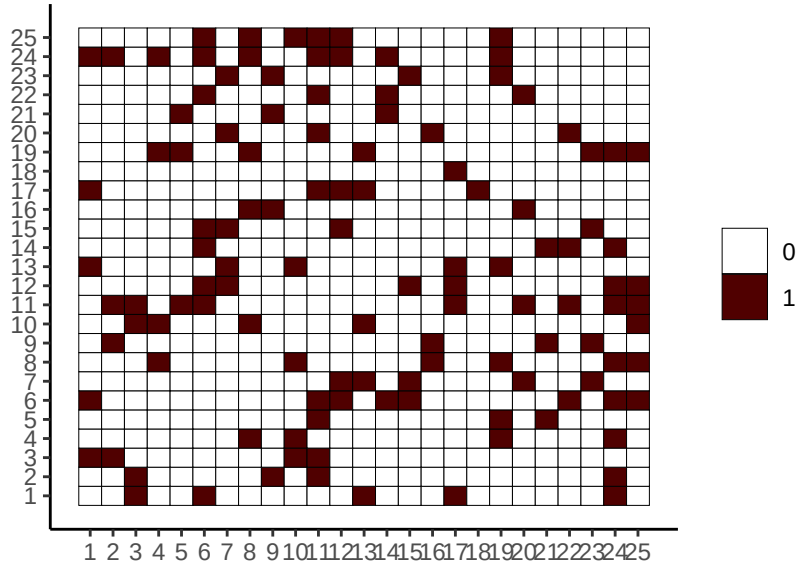

```
##
## $trial31$M2$unique_graphs[[80]]
```



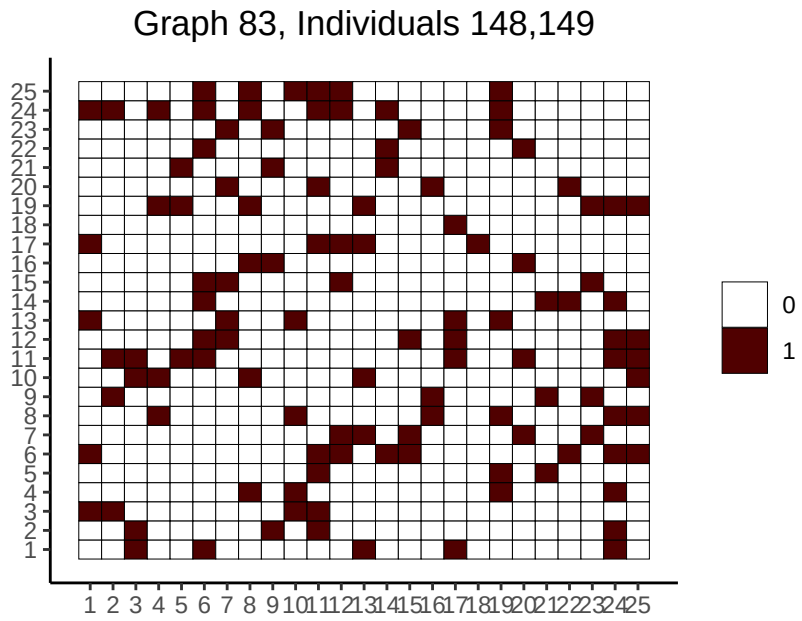

```
##
## $trial31$M2$unique_graphs[[81]]
```



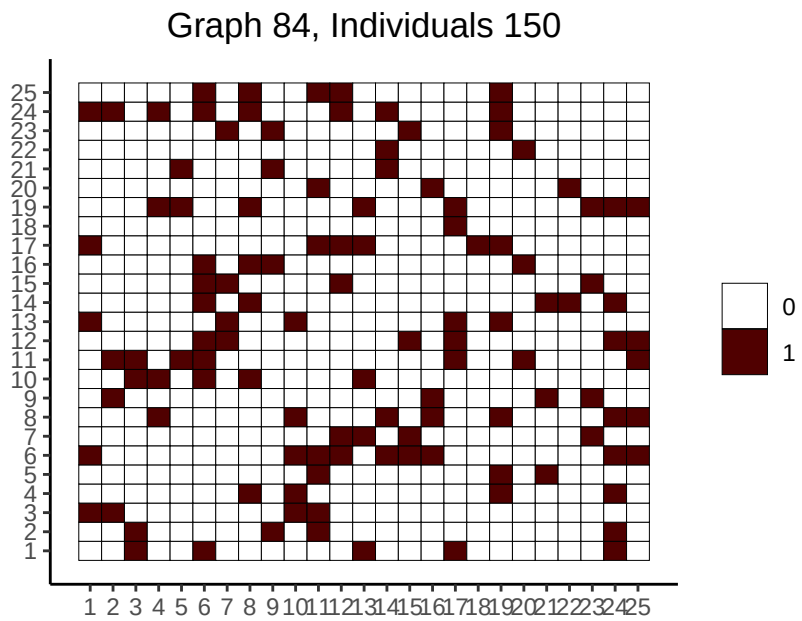

```
##
## $trial31$M2$unique_graphs[[82]]
```



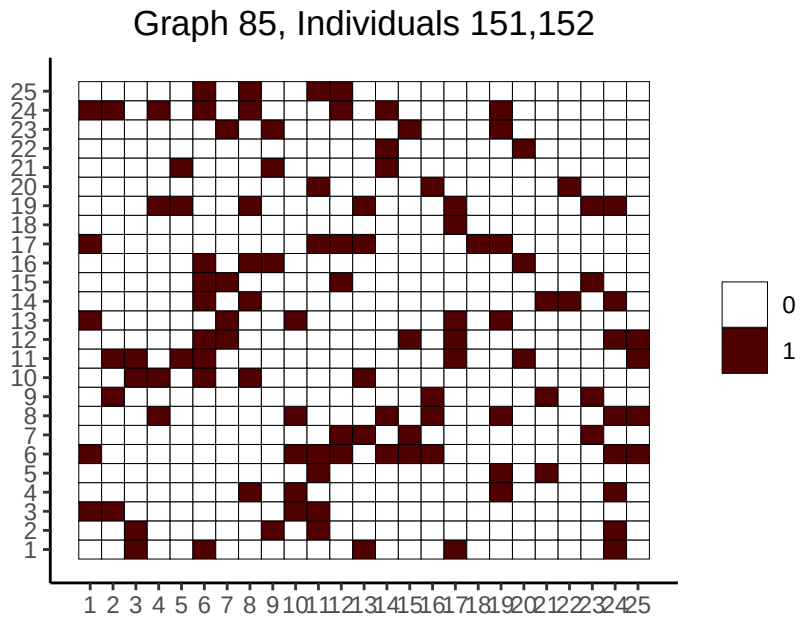

```
##
## $trial31$M2$unique_graphs[[83]]
```



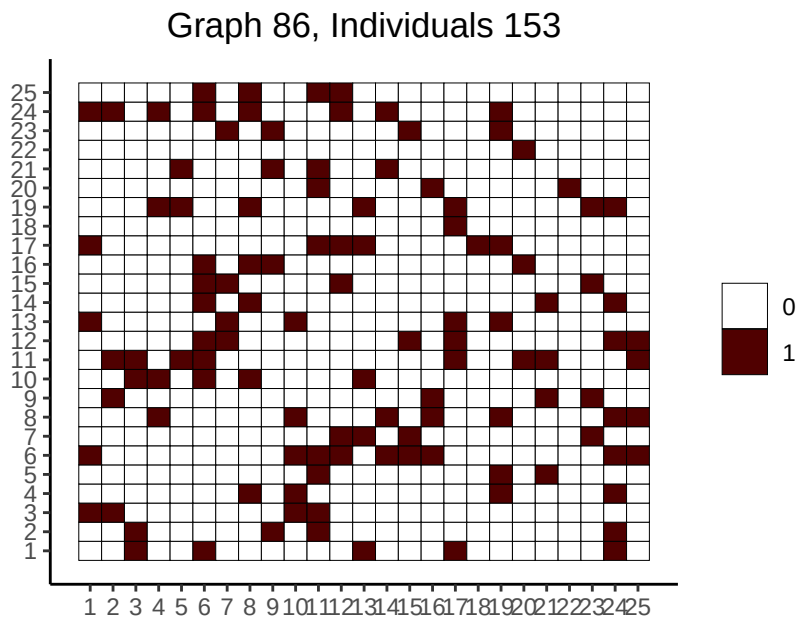

```
##
## $trial31$M2$unique_graphs[[84]]
```



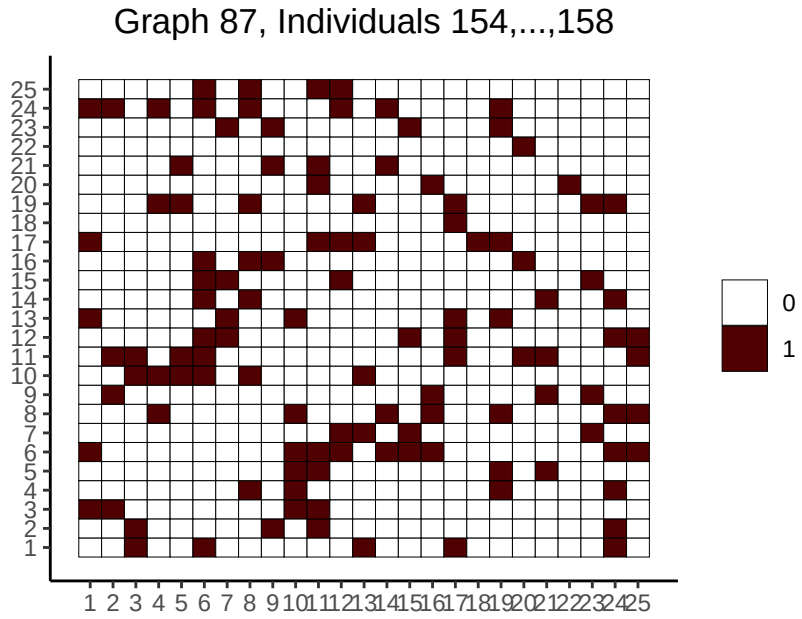

```
##
## $trial31$M2$unique_graphs[[85]]
```



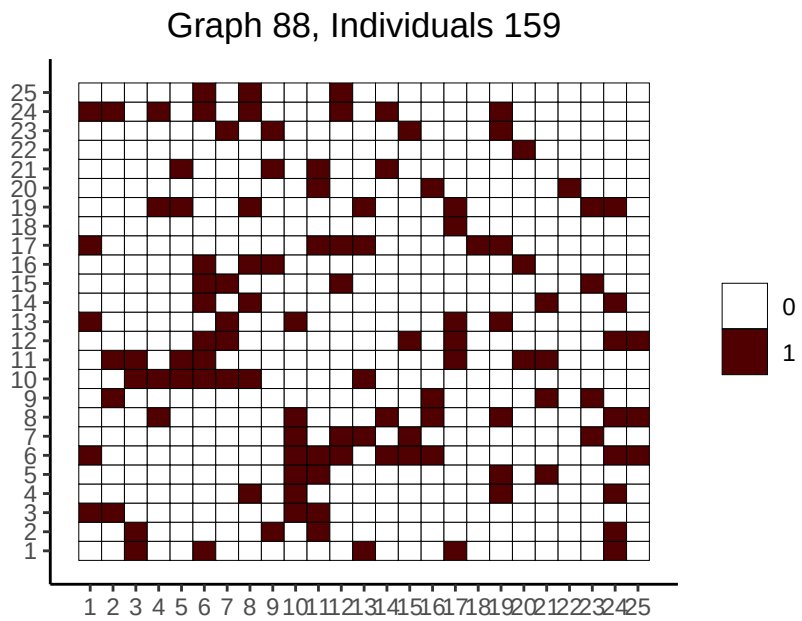

```
##
## $trial31$M2$unique_graphs[[86]]
```



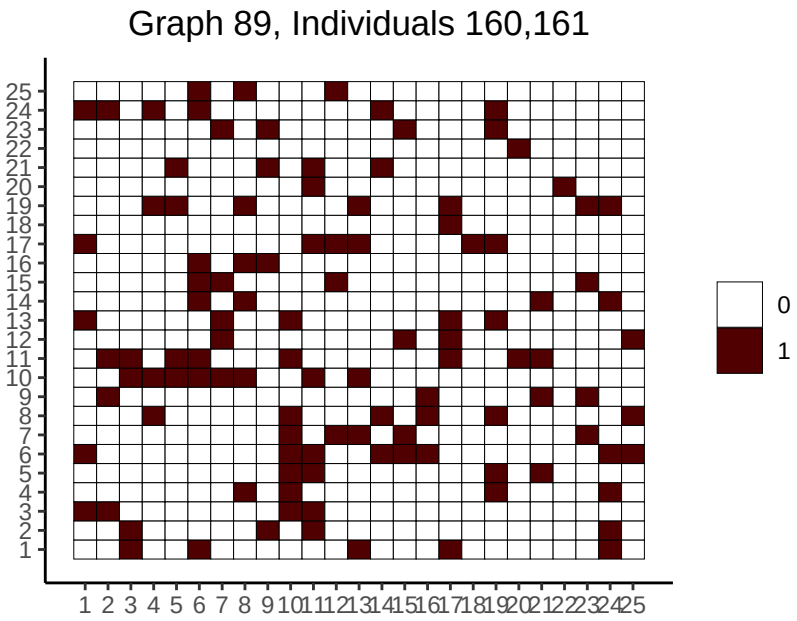

```
##
## $trial31$M2$unique_graphs[[87]]
```



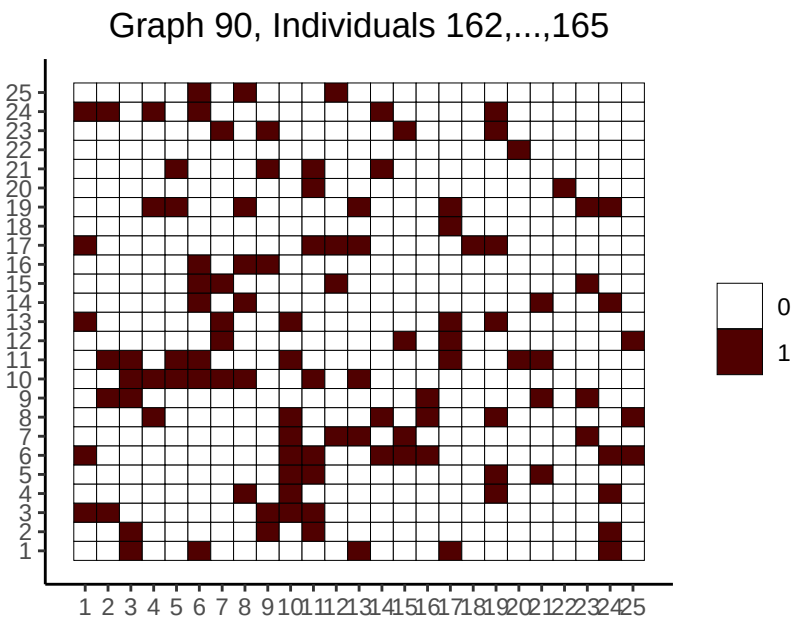

```
##
## $trial31$M2$unique_graphs[[88]]
```




```
##
## $trial31$M2$unique_graphs[[89]]
```

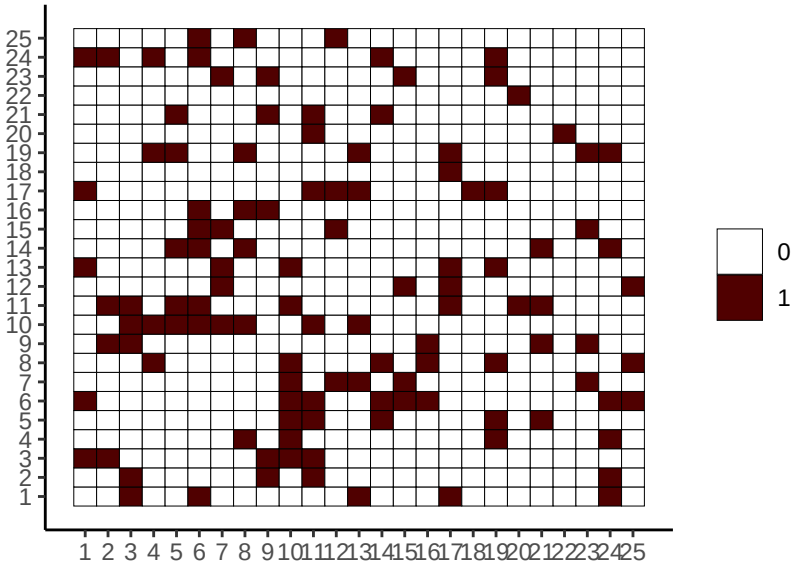



```
##
## $trial31$M2$unique_graphs[[90]]
```



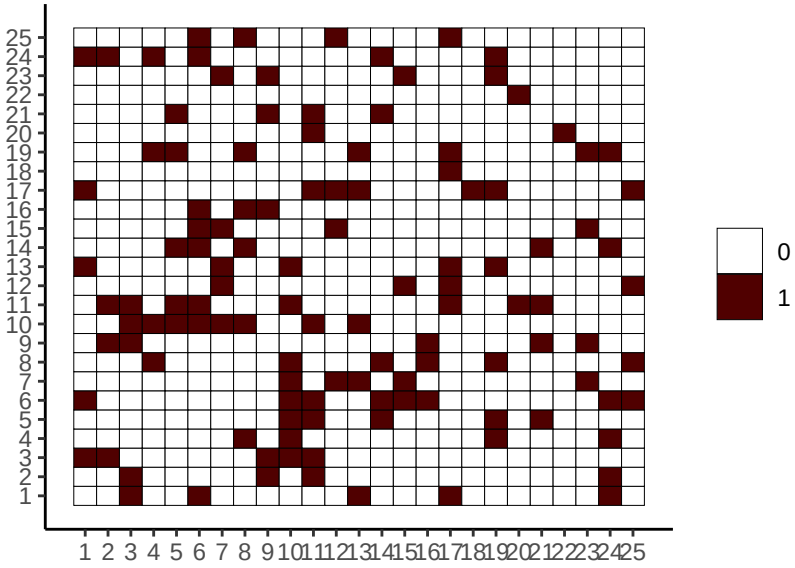

```
##
## $trial31$M2$unique_graphs[[91]]
```

Graph 91, Individuals 166

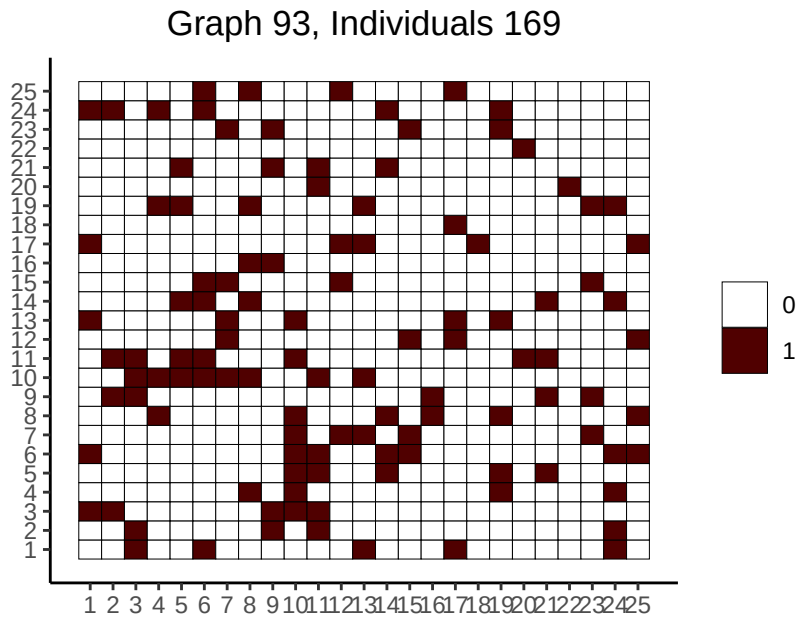



```
##
## $trial31$M2$unique_graphs[[92]]
```

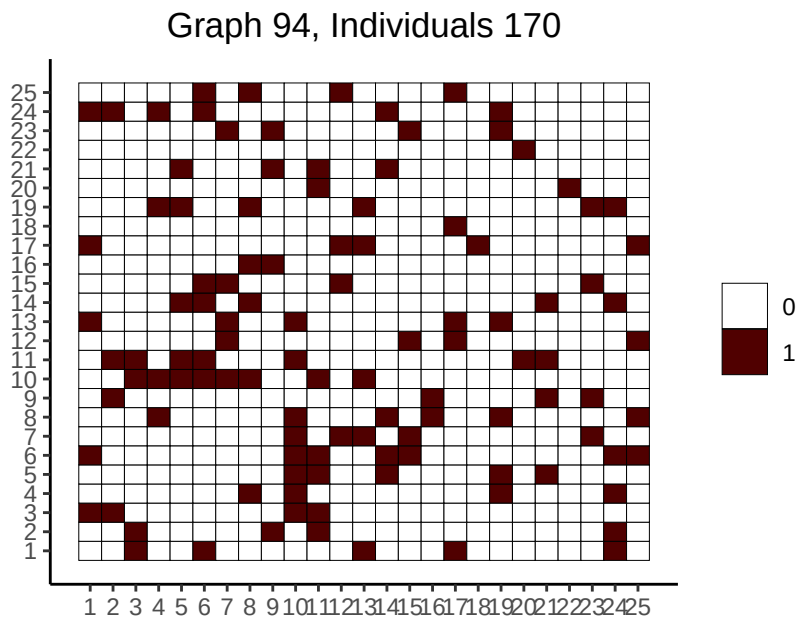
Graph 92, Individuals 167,168



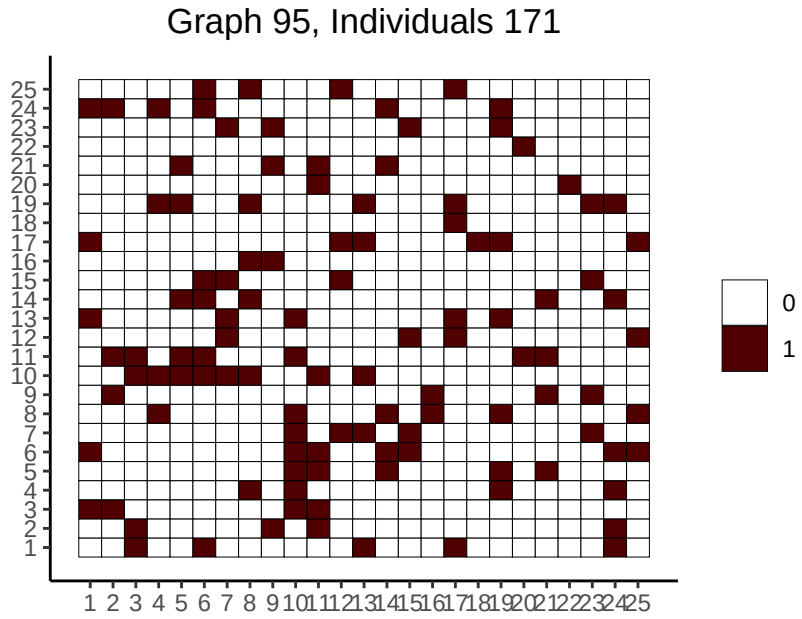

```
##
## $trial31$M2$unique_graphs[[93]]
```



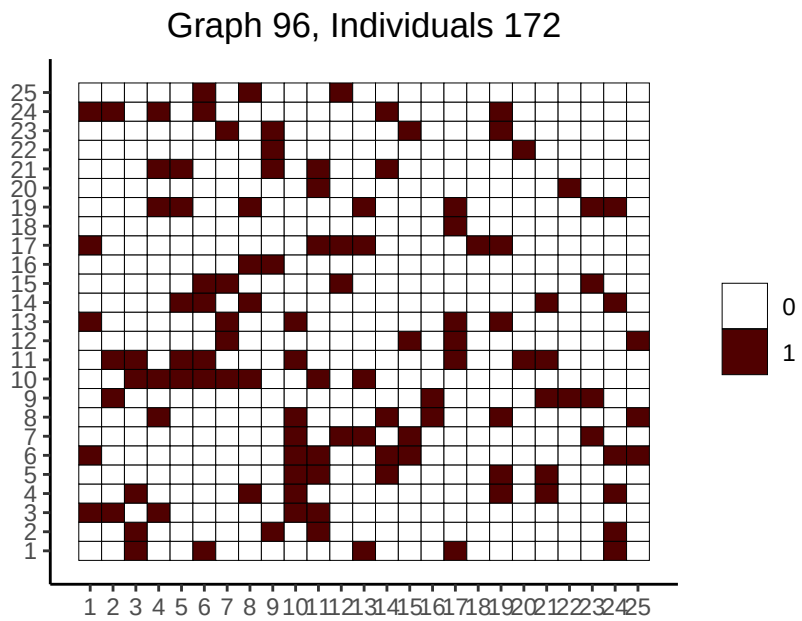

```
##
## $trial31$M2$unique_graphs[[94]]
```



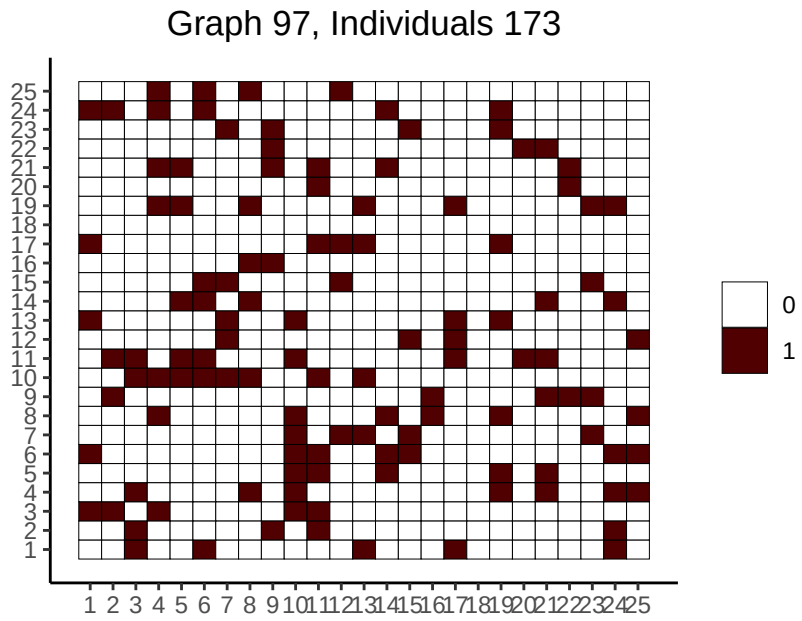

```
##
## $trial31$M2$unique_graphs[[95]]
```



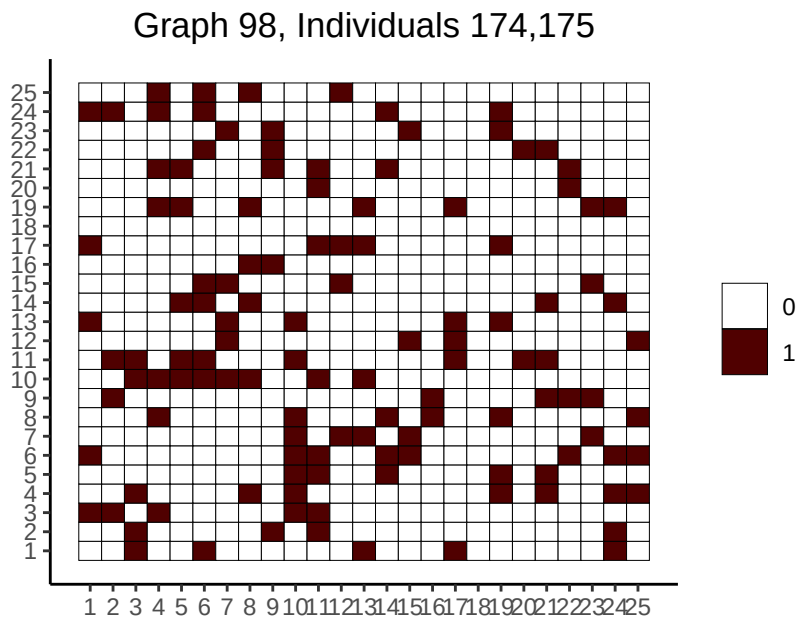

```
##
## $trial31$M2$unique_graphs[[96]]
```



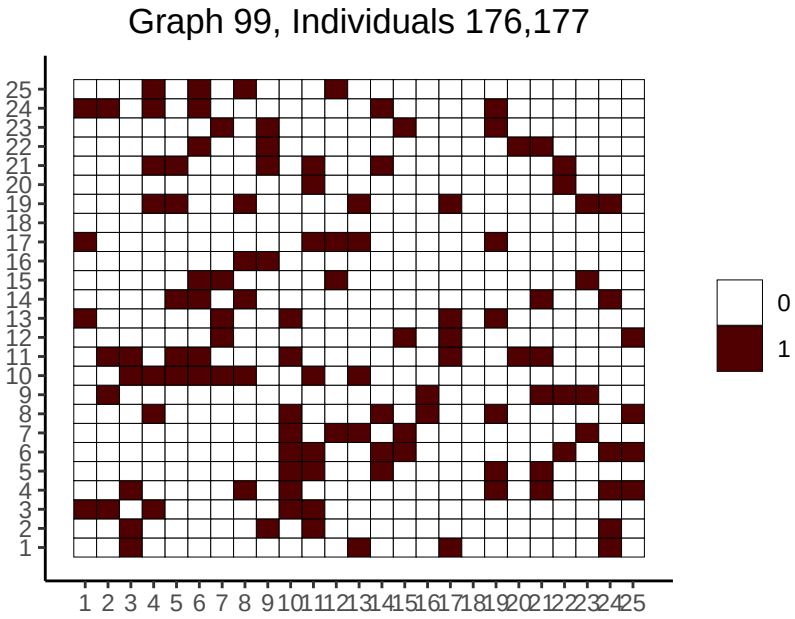

```
##
## $trial31$M2$unique_graphs[[97]]
```



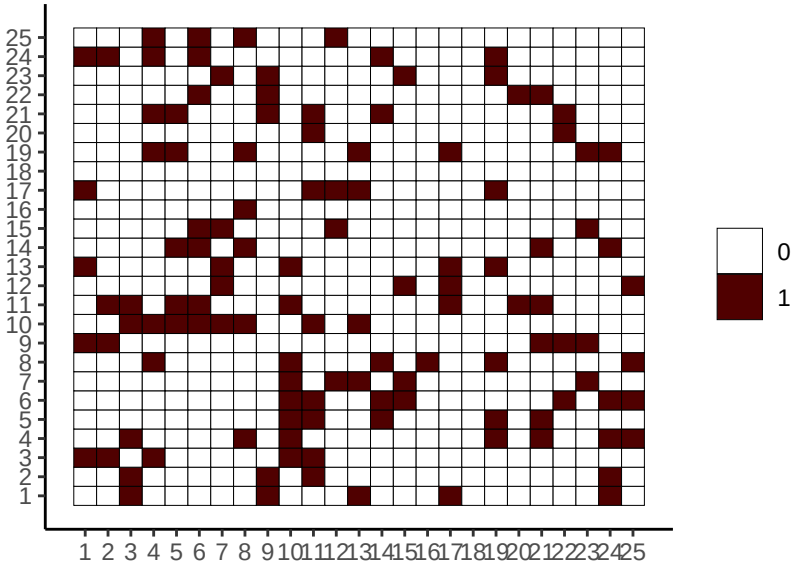

```
##
## $trial31$M2$unique_graphs[[98]]
```



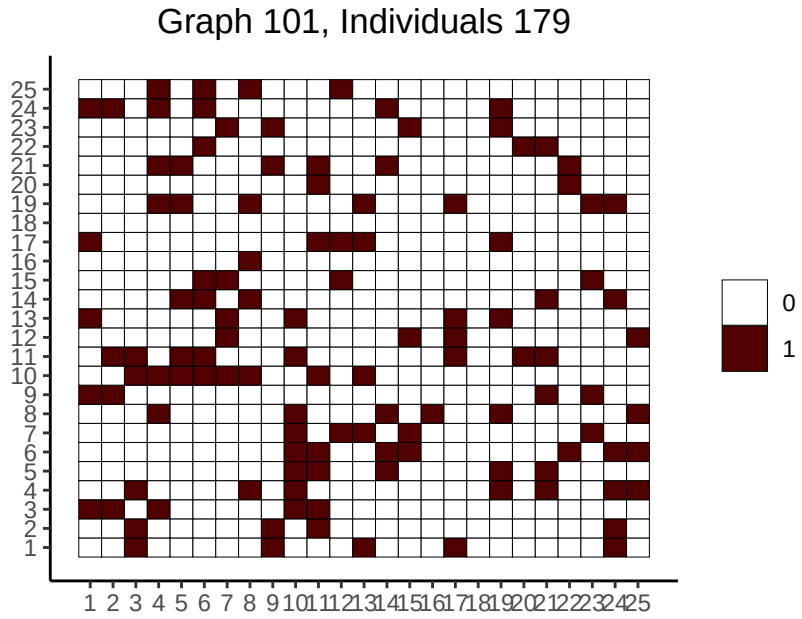

```
##
## $trial31$M2$unique_graphs[[99]]
```



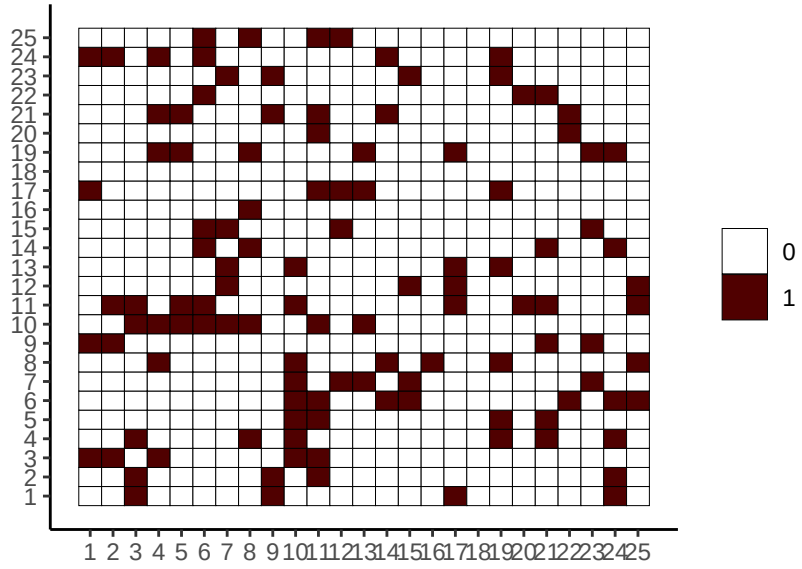

```
##
## $trial31$M2$unique_graphs[[100]]
```




```
##
## $trial31$M2$unique_graphs[[101]]
```

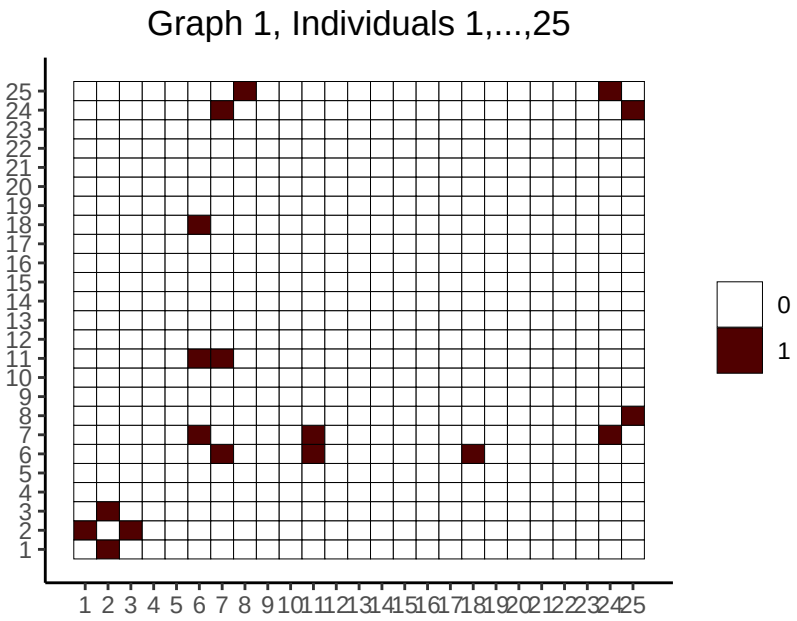



```
##
## $trial31$M2$unique_graphs[[102]]
```

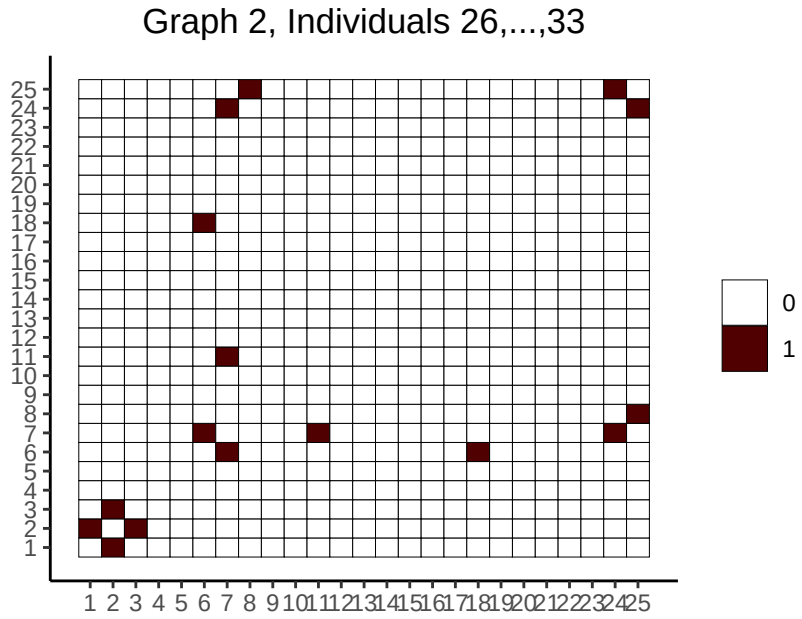



```
##
##
## $trial31$M2$sensitivity
## [1] 0.9238095
##
## $trial31$M2$specificity
```

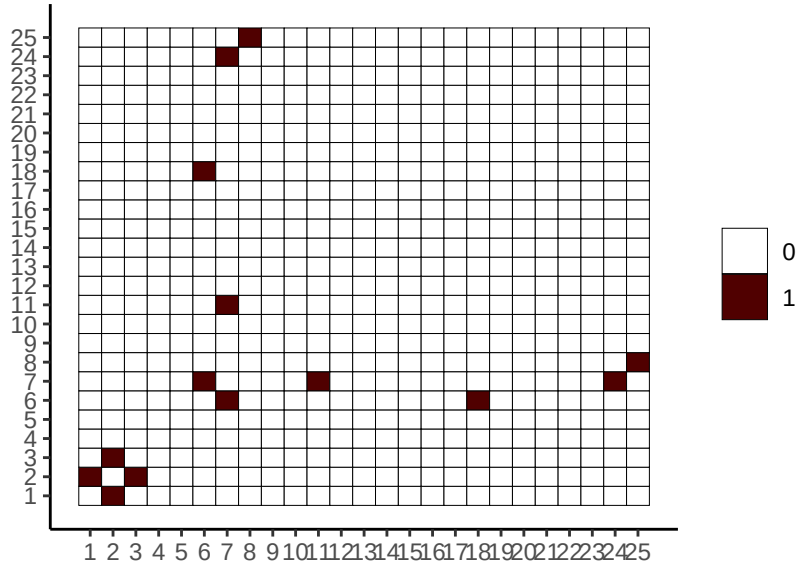
```
## [1] 0.8457997
##
## $trial31$M2$accuracy
## [1] 0.8463822
##
## $trial31$M2$ELBO
## [1] -385317.5
##
## $trial31$M2$time
## [1] 32.55868
##
##
## $trial31$M3
## $trial31$M3$summary
##                        Covariate Dependent Graphical Model
##
## Model ELBO: -214790.25             Unique conditional dependence structures: 17
## n: 180, variables: 25                      Hyperparameter grid size: 125 points
## CAVI converged for 25/25 variables
##
## Model fit completed in 1.986 mins
##
## $trial31$M3$unique_graphs
## $trial31$M3$unique_graphs[[1]]
```



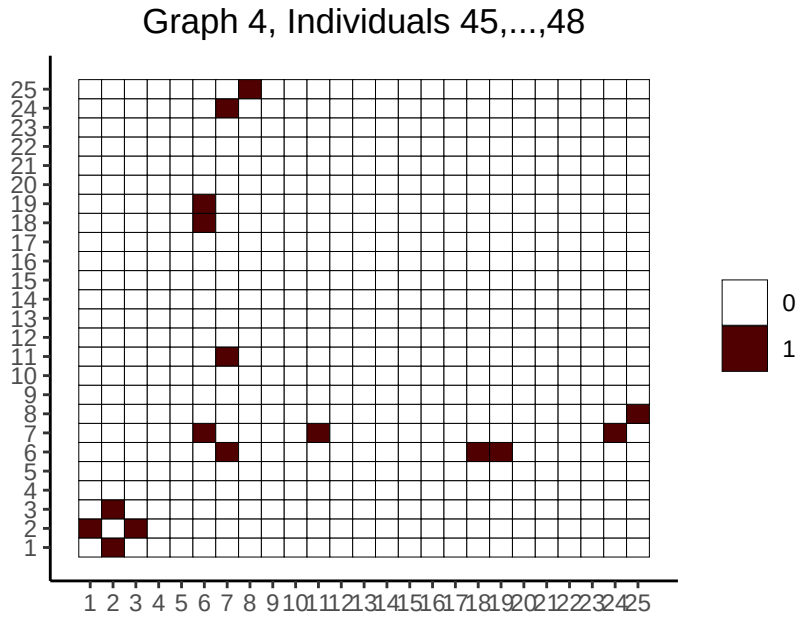

```
##
## $trial31$M3$unique_graphs[[2]]
```



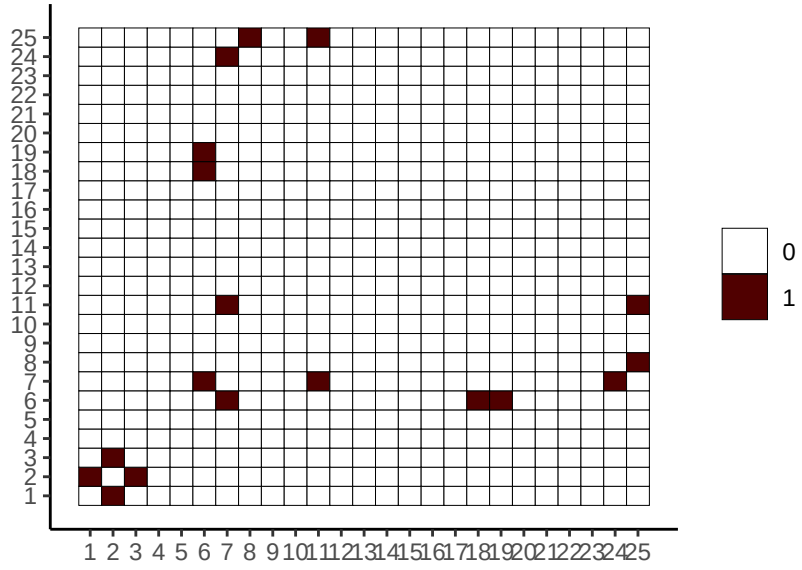

```
##
## $trial31$M3$unique_graphs[[3]]
```



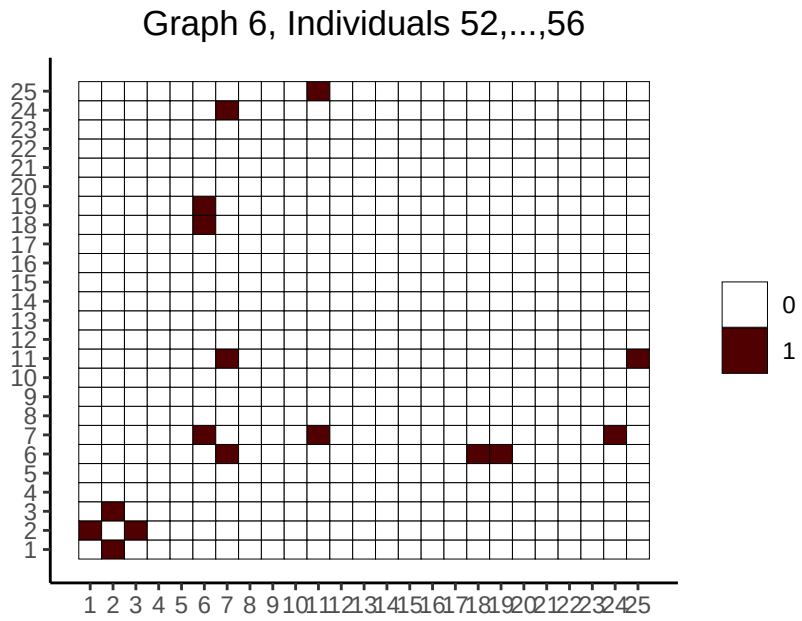

```
##
## $trial31$M3$unique_graphs[[4]]
```



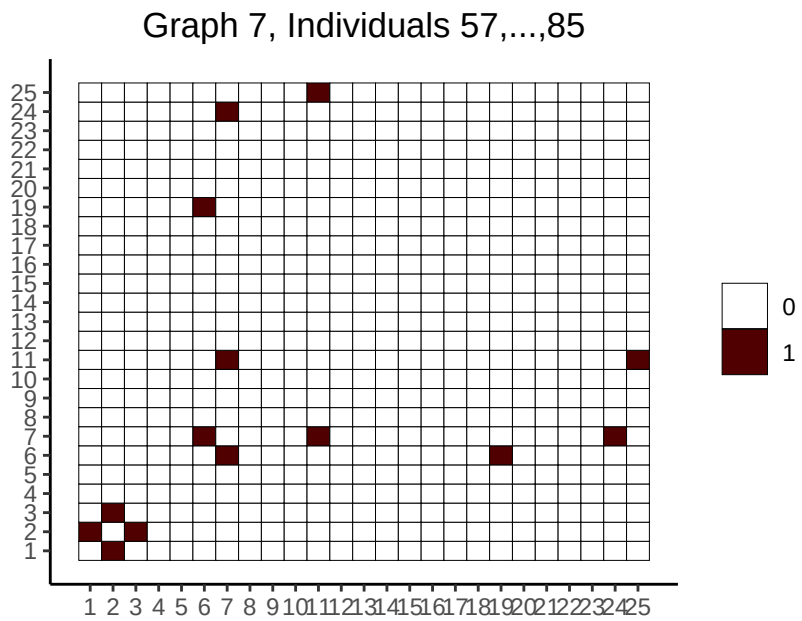

```
##
## $trial31$M3$unique_graphs[[5]]
```



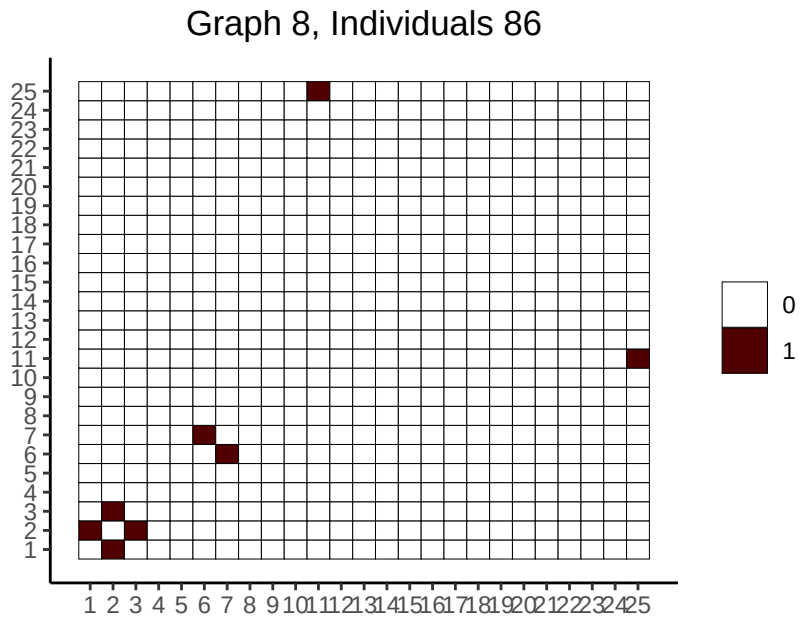

```
##
## $trial31$M3$unique_graphs[[6]]
```



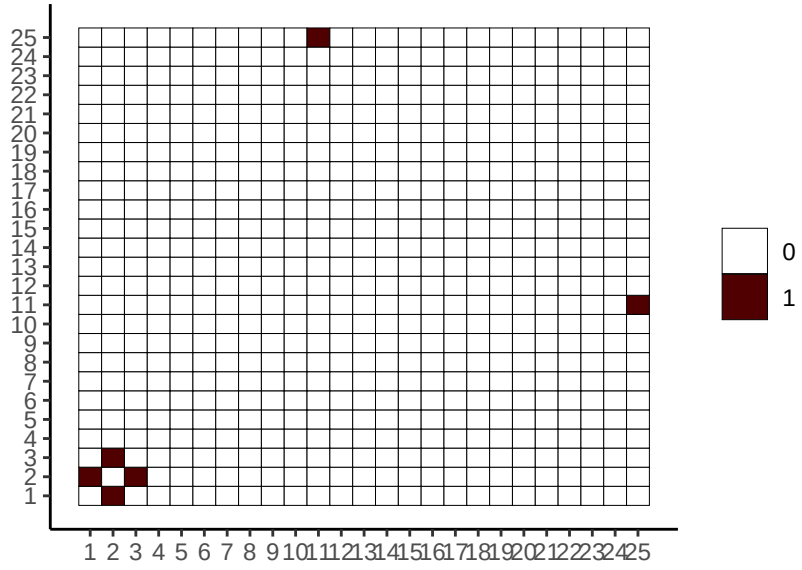

```
##
## $trial31$M3$unique_graphs[[7]]
```



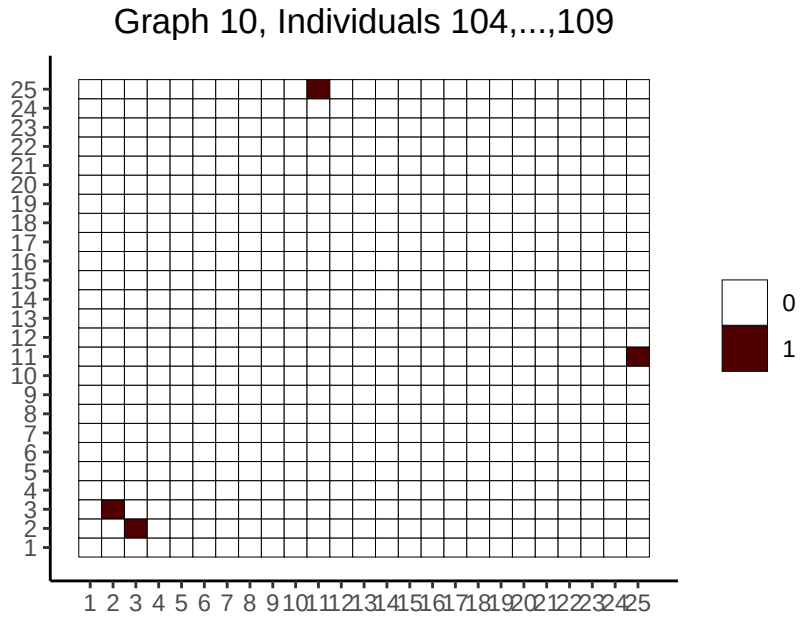

```
##
## $trial31$M3$unique_graphs[[8]]
```



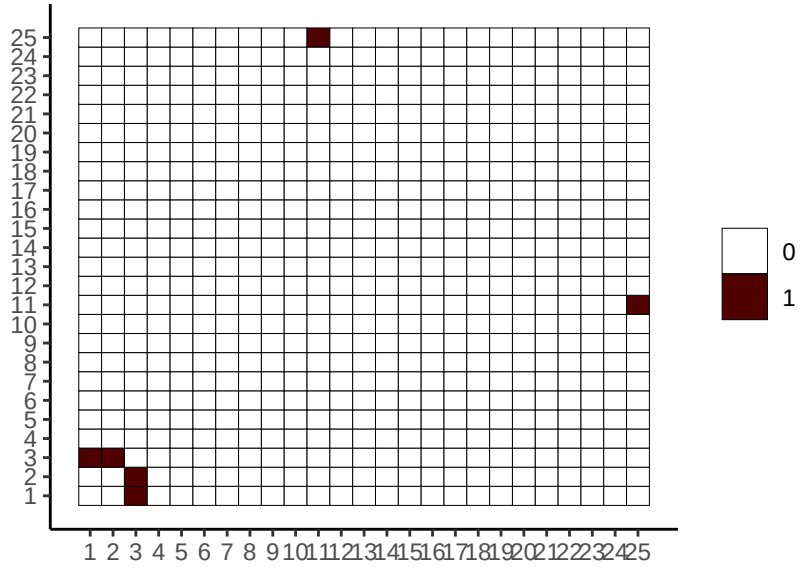

```
##
## $trial31$M3$unique_graphs[[9]]
```



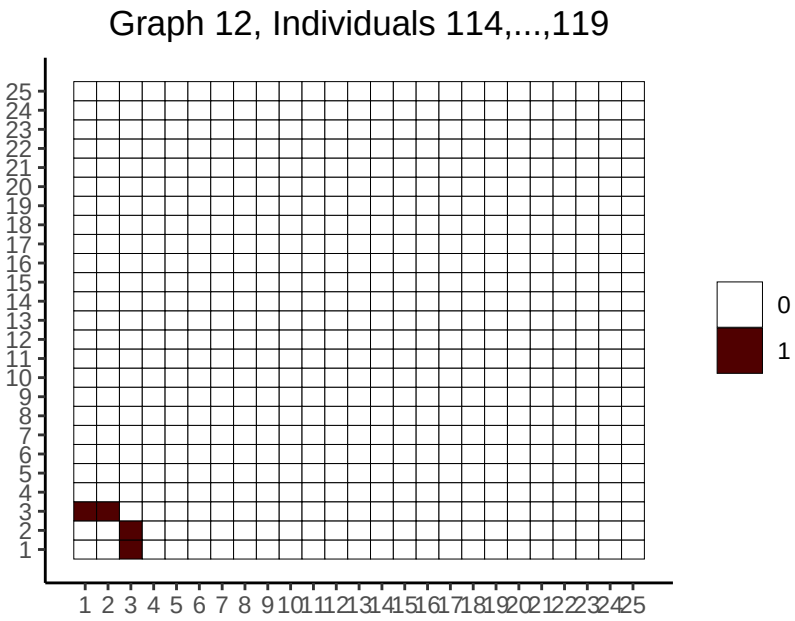

```
##
## $trial31$M3$unique_graphs[[10]]
```



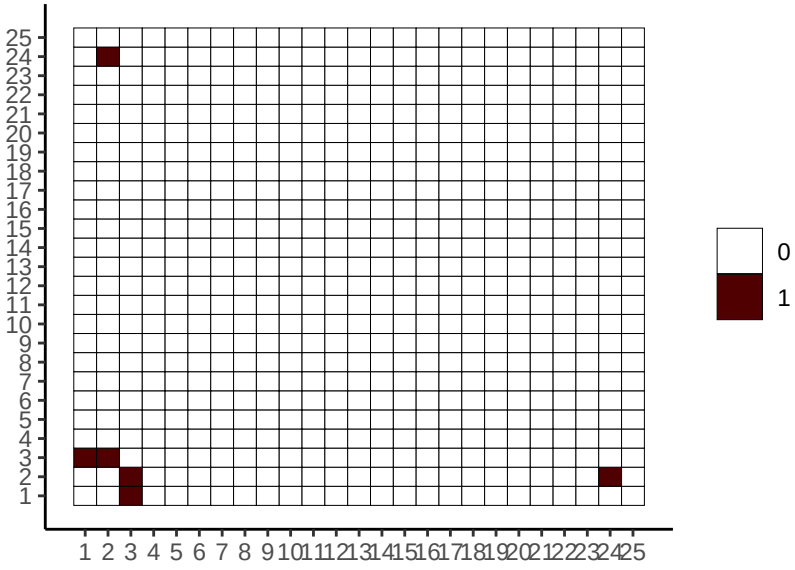

```
##
## $trial31$M3$unique_graphs[[11]]
```



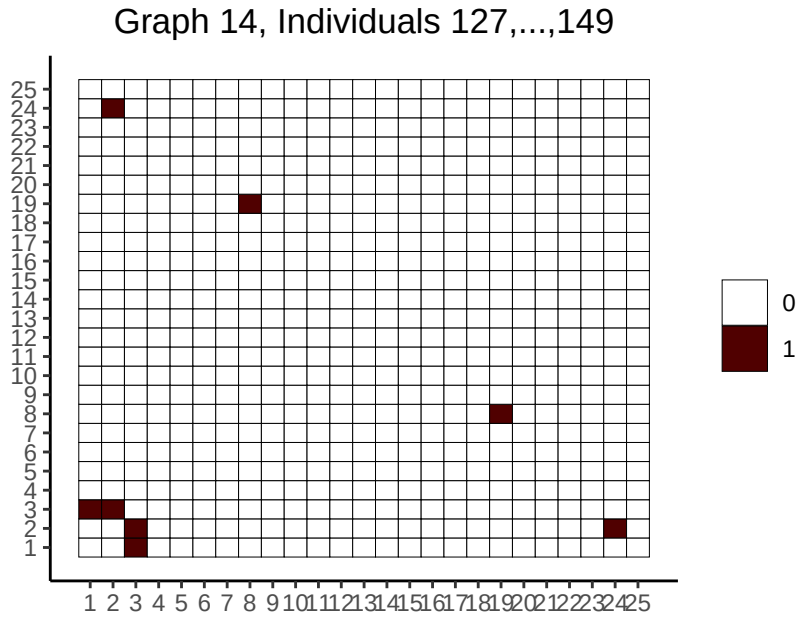

```
##
## $trial31$M3$unique_graphs[[12]]
```



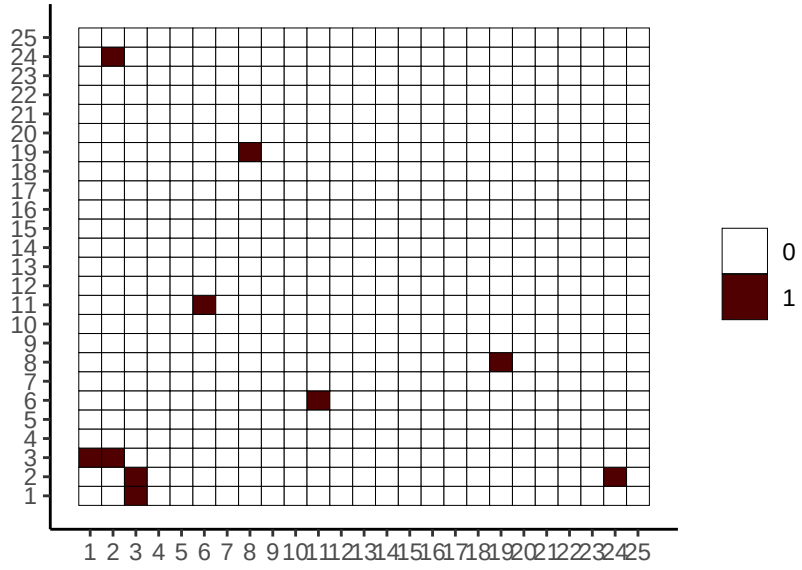

```
##
## $trial31$M3$unique_graphs[[13]]
```



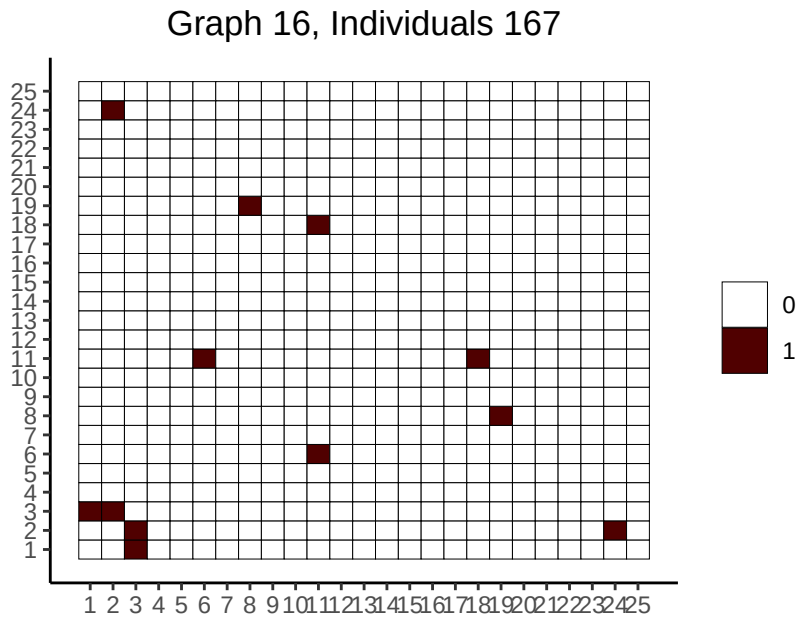

```
##
## $trial31$M3$unique_graphs[[14]]
```



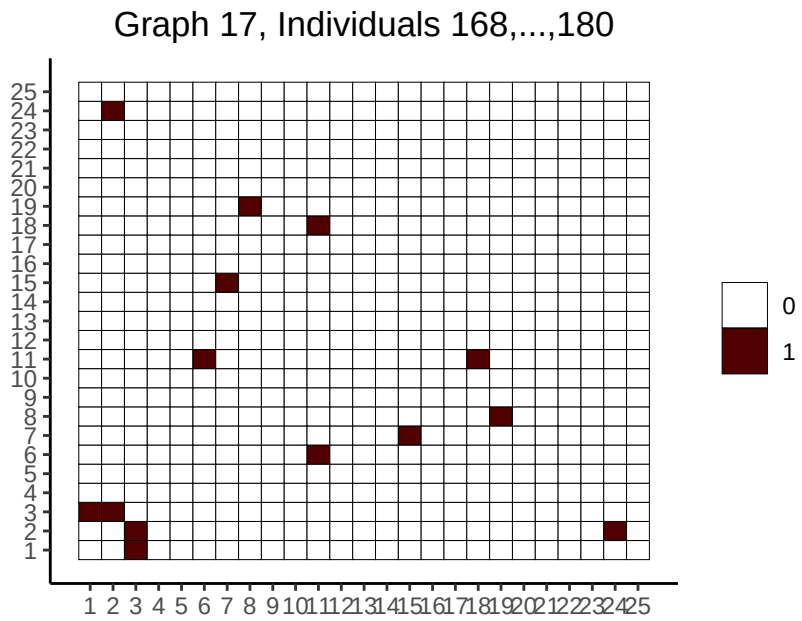

```
##
## $trial31$M3$unique_graphs[[15]]
```




```
##
## $trial31$M3$unique_graphs[[16]]
```




```
##
## $trial31$M3$unique_graphs[[17]]
```




```
##
##
## $trial31$M3$sensitivity
## [1] 0.8428571
##
## $trial31$M3$specificity
```

```
## [1] 0.9874619
##
## $trial31$M3$accuracy
## [1] 0.9863822
##
## $trial31$M3$ELBO
## [1] -214790.2
##
## $trial31$M3$time
## [1] 119.1823
##
##
## $trial31$error
## NULL
```