

## sigma bounding experiment

```
load("~/TAMU/Research/An approximate Bayesian approach to covariate dependent/covdepGE/dev/analyses_demo")
library(covdepGE)
dat <- results$trial86$data
X <- dat$data
Z <- dat$covts
```

### Model 1

In this model, I do not bound any of the hyperparameters. Both  $\sigma^2$  and  $\mu$  exhibit instability, while  $\sigma_\beta^2$  remains stable.

```
# fit the model without restricting the size of ssq
out1 <- covdepGE(X, Z, bound_ssq = F, bound_sbsq = F, max_iter = 100,
                 parallel = T, num_workers = 14)
```

```
## Warning in covdepGE(X, Z, bound_ssq = F, bound_sbsq = F, max_iter = 100, : No
## registered workers detected; registering doParallel with 14 workers
```

```
## Warning in FUN(X[[i]], ...): Variable 1: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 2: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 3: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 4: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 5: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 6: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 7: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 8: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 10: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 13: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 14: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 18: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 20: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 22: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 23: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 24: final CAVI did not converge in 100
## iterations
```

```
out1
```

```
##                      Covariate Dependent Graphical Model
##
## Model ELBO: -441698.65          Unique conditional dependence structures: 60
## n: 180, variables: 25          Hyperparameter grid size: 9 points
## CAVI converged for 9/25 variables
##
## Model fit completed in 47.627 secs
```

$\sigma^2$

```
# analyze ssq
ssq1 <- out1$hyperparameters$sigmasq
summary(as.numeric(ssq1))
```

```
##      Min.   1st Qu.   Median     Mean   3rd Qu.     Max.
## 0.000e+00 0.000e+00 0.000e+00 1.428e+71 1.000e+00 7.075e+73
```

```
# analyze the ssq were in excess of 1.25
summary(as.numeric(ssq1[ssq1 < 1.25]))
```

```
##      Min. 1st Qu.  Median     Mean 3rd Qu.     Max.
## 0.3235  0.4201  0.4625  0.4681  0.5110  1.2267
```

```
sum(ssq1 < 1.25)
```

```
## [1] 4344
```

```
summary(as.numeric(ssq1[ssq1 > 1.25]))
```

```
##      Min.   1st Qu.   Median     Mean   3rd Qu.    Max.
## 2.447e+15 5.327e+28 7.697e+55 4.118e+72 8.688e+66 7.075e+73
```

```
sum(ssq1 > 1.25)
```

```
## [1] 156
```

```
# which of the individuals had at least 1 ssq exceed 1.25?
which(rowSums(ssq1 > 1.25) > 0)
```

```
## [1] 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170
## [20] 171 172 173 174 175 176 177 178 179 180
```

```
# which of the variables had at least 1 ssq exceed 1.25?
which(colSums(ssq1 > 1.25) > 0)
```

```
## Variable 1 Variable 2 Variable 4 Variable 5 Variable 8 Variable 10
##          1          2          4          5          8          10
## Variable 14
##          14
```

$\mu$  and  $\sigma_\beta^2$

```
# analyze mu and sbsq
summary(abs(unlist(out1$mu_matrices)))
```

```
##      Min.   1st Qu.   Median     Mean   3rd Qu.    Max.
## 0.000e+00 0.000e+00 0.000e+00 3.317e+33 0.000e+00 2.824e+36
```

```
summary(as.numeric(out1$hyperparameters$sigmabeta_sq))
```

```
##      Min.   1st Qu.   Median     Mean   3rd Qu.    Max.
## 0.0004494 0.0338762 0.0602351 0.0780340 0.0913357 0.4948567
```

## Model 2

In this model, I bound  $\sigma^2$  using the weighted least squares estimate,  $\hat{\sigma}^2$ . Let  $\sigma_{l,j}^2$  denote the error term variance for the regression weighted with respect to individual  $l$  and with variable  $j$  fixed as the response. Then, if the MAPE-fitted  $\sigma_{l,j}^2$  exceeds two times  $\hat{\sigma}_{l,j}^2$ ,  $\sigma_{l,j}^2$  is fixed as  $\hat{\sigma}_{l,j}^2$  and is not updated any further.

The results show that although  $\sigma^2$  is stabilized,  $\sigma_\beta^2$  is now unstable, as is  $\mu$ . Examining the MAPE update for  $\sigma_\beta^2$ , it seems this instability comes from the instability in  $\mu$  appearing in the numerator of the update.

$$\sigma_{\beta}^2_{MAPE} = \frac{\sum_{k=1}^{p-1} \alpha_{j,k}^l (s_{j,k}^l{}^2 + \mu_{j,k}^l{}^2)}{\sigma^2 \sum_{k=1}^{p-1} \alpha_{j,k}^l}$$

Note also that the same variables and individuals suffer from this instability as before.

```
# fit a model restricting ssq but not sbsq
out2 <- covdepGE(X, Z, bound_ssq = T, ssq_bound_mult = 2, bound_sbsq = F,
                max_iter = 100, parallel = T, num_workers = 15, R = T)

## Warning in covdepGE(X, Z, bound_ssq = T, ssq_bound_mult = 2, bound_sbsq = F, :
## No registered workers detected; registering doParallel with 15 workers

## Warning in FUN(X[[i]], ...): Variable 2: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 3: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 4: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 5: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 6: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 7: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 8: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 10: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 13: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 14: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 18: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 20: final CAVI did not converge in 100
## iterations
```

```

## Warning in FUN(X[[i]], ...): Variable 22: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 23: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 24: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 2: Detected sigmasq instability for
## individuals 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172,
## 173, 174, 175, 176, 177, 178, 179, 180; using weighted OLS sigmasq

## Warning in FUN(X[[i]], ...): Variable 3: Detected sigmasq instability for
## individuals 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169,
## 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180; using weighted OLS
## sigmasq

## Warning in FUN(X[[i]], ...): Variable 4: Detected sigmasq instability for
## individuals 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173,
## 174, 175, 176, 177, 178, 179, 180; using weighted OLS sigmasq

## Warning in FUN(X[[i]], ...): Variable 5: Detected sigmasq instability for
## individuals 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164,
## 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180;
## using weighted OLS sigmasq

## Warning in FUN(X[[i]], ...): Variable 8: Detected sigmasq instability for
## individuals 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166,
## 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180; using
## weighted OLS sigmasq

## Warning in FUN(X[[i]], ...): Variable 10: Detected sigmasq instability for
## individuals 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166,
## 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180; using
## weighted OLS sigmasq

## Warning in FUN(X[[i]], ...): Variable 14: Detected sigmasq instability for
## individuals 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178,
## 179, 180; using weighted OLS sigmasq

```

```
out2
```

```

##                               Covariate Dependent Graphical Model
##
## Model ELB0: -1.74962117351457e+125 Unique conditional dependence structures: 58
## n: 180, variables: 25                               Hyperparameter grid size: 9 points
## CAVI converged for 10/25 variables
##
## Model fit completed in 6.872 mins

```

$\mu$  and  $\sigma^2$

```
# analyze mu and ssq
summary(as.numeric(out2$hyperparameters$sigma_sq))
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.2540 0.4149 0.4600 0.4599 0.5093 0.6865
```

```
summary(abs(unlist(out2$mu_matrices)))
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.000e+00 0.000e+00 0.000e+00 2.375e+57 0.000e+00 2.965e+60
```

$\sigma_\beta^2$

```
# analyze sbsq
sbsq2 <- out2$hyperparameters$sigmabeta_sq
summary(as.numeric(sbsq2))
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.000e+00 0.000e+00 0.000e+00 9.789e+117 0.000e+00 6.970e+120
```

```
# analyze the sbsq in excess of 0.5
summary(as.numeric(sbsq2[sbsq2 < 0.5]))
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.0004446 0.0333437 0.0582631 0.0778529 0.0907554 0.4948567
```

```
sum(sbsq2 < 0.5)
```

```
## [1] 4312
```

```
summary(as.numeric(sbsq2[sbsq2 > 0.5]))
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 2.628e+12 3.934e+67 5.851e+102 2.343e+119 2.762e+110 6.970e+120
```

```
sum(sbsq2 > 0.5)
```

```
## [1] 188
```

```
# which of the individuals had at least 1 sbsq exceed 0.5?
which(rowSums(sbsq2 > 0.5) > 0)
```

```
## [1] 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170
## [20] 171 172 173 174 175 176 177 178 179 180
```

```
# which of the variables had at least 1 ssq exceed 0.5?
which(colSums(sbsq2 > 0.5) > 0)
```

```
## Variable 1 Variable 2 Variable 3 Variable 4 Variable 5 Variable 8
##          1          2          3          4          5          8
## Variable 10 Variable 14
##          10          14
```

## Model 3

Finally, I bound both  $\sigma^2$  and  $\sigma_{\beta}^2$ . If  $\sigma_{l,j}^2$  exceeds  $2\hat{\sigma}_{l,j}^2$ , I again fix its value as  $\hat{\sigma}_{l,j}^2$  and stop updating it. Additionally, I also stop updating  $\sigma_{\beta l,j}^2$  and fix its value as:

$$\sigma_{\beta l,j}^2 = \text{median}\{\sigma_{\beta k,j}^2 : \sigma_{k,j}^2 \text{ is still being updated}\}$$

The results show that although both  $\sigma^2$  and  $\sigma_{\beta}^2$  are now stable,  $\mu$  remains unstable. Note that the same individuals and variables suffer from instability as in the first 2 models.

```
# fit a model restricting both ssq and sbsq
out3 <- covdepGE(X, Z, bound_ssq = T, ssq_bound_mult = 3, bound_sbsq = T,
                 sbsq_bound_mult = 2, max_iter = 100, parallel = T,
                 num_workers = 15, R = T)
```

```
## Warning in covdepGE(X, Z, bound_ssq = T, ssq_bound_mult = 3, bound_sbsq = T, :
## No registered workers detected; registering doParallel with 15 workers
```

```
## Warning in FUN(X[[i]], ...): Variable 3: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 4: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 5: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 6: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 7: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 8: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 13: final CAVI did not converge in 100
## iterations
```

```
## Warning in FUN(X[[i]], ...): Variable 14: final CAVI did not converge in 100
## iterations
```

```

## Warning in FUN(X[[i]], ...): Variable 18: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 20: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 22: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 23: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 24: final CAVI did not converge in 100
## iterations

## Warning in FUN(X[[i]], ...): Variable 4: Detected sigmasq instability for
## individuals 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21,
## 22, 23, 24, 25, 26, 27, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150,
## 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166,
## 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180; using
## weighted OLS sigmasq

## Warning in FUN(X[[i]], ...): Variable 5: Detected sigmasq instability for
## individuals 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146,
## 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162,
## 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178,
## 179, 180; using weighted OLS sigmasq

## Warning in FUN(X[[i]], ...): Variable 8: Detected sigmasq instability for
## individuals 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160,
## 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176,
## 177, 178, 179, 180; using weighted OLS sigmasq

## Warning in FUN(X[[i]], ...): Variable 14: Detected sigmasq instability for
## individuals 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156,
## 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172,
## 173, 174, 175, 176, 177, 178, 179, 180; using weighted OLS sigmasq

```

out3

```

##                               Covariate Dependent Graphical Model
##
## Model ELBO: -4.75836543854953e+83 Unique conditional dependence structures: 66
## n: 180, variables: 25                               Hyperparameter grid size: 9 points
## CAVI converged for 12/25 variables
##
## Model fit completed in 7.232 mins

```

$\sigma^2$  and  $\sigma_\beta^2$



```
summary(as.numeric(out3$hyperparameters$sigma_sq))
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.2540 0.4124 0.4576 0.4569 0.5015 1.0983
```

```
summary(as.numeric(out3$hyperparameters$sigma_beta_sq))
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.0003743 0.0260191 0.0499239 0.0729439 0.0865847 0.4948567
```

$\mu$

```
mu3 <- abs(unlist(out3$mu_matrices))
summary(mu3)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.000e+00 0.000e+00 0.000e+00 4.069e+36 0.000e+00 6.226e+39
```

```
# analyze the mu that are in excess of 1
summary(mu3[mu3 < 1])
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.0000009 0.0219567 0.0498451 0.0650426 0.0910465 0.9605224
```

```
sum(mu3 < 1)
```

```
## [1] 101731
```

```
summary(mu3[mu3 > 1])
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 1.000e+00 9.753e+13 9.675e+22 7.010e+37 6.949e+26 6.226e+39
```

```
sum(mu3 > 1)
```

```
## [1] 6269
```

```
# which of the variables have at least one mu in excess of 1?
which(sapply(out3$mu_matrices, function(mat) any(abs(mat) > 1)))
```

```
## [1] 1 2 4 5 8 10 14
```

```
# which of the individuals have at least one mu in excess of 1?
which(rowSums(sapply(out3$mu_matrices, function(mat) rowSums(abs(mat) > 1) > 0)) > 0)
```

```
## [1] 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152
## [20] 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171
## [39] 172 173 174 175 176 177 178 179 180
```

## Model 4

Model 4 performs a 3-D grid search over the hyperparameter space.

```
hp <- expand.grid(ssq = exp(seq(log(0.0001), log(2), length.out = 10)),
                 sbsq = exp(seq(log(0.0001), log(2), length.out = 10)),
                 pi = out3$hyperparameters$pi_grid)
out4 <- covdepGE(X, Z, sigmasq_vec = hp$ssq, sigmabetasq_vec = hp$sbsq, pi_vec = hp$pi,
                max_iter = 100, parallel = T, num_workers = 15)
```

```
## Warning in covdepGE(X, Z, sigmasq_vec = hp$ssq, sigmabetasq_vec = hp$sbsq, : No
## registered workers detected; registering doParallel with 15 workers
```

```
## Warning in FUN(X[[i]], ...): Variable 3: final CAVI did not converge in 100
## iterations
```

```
out4
```

```
##                      Covariate Dependent Graphical Model
##
## Model ELBO: -109120.69          Unique conditional dependence structures: 3
## n: 180, variables: 25          Hyperparameter grid size: 900 points
## CAVI converged for 24/25 variables
##
## Model fit completed in 16.043 mins
```

```
out4$hyperparameters$sigmasq[1, ]
```

```
## Variable 1 Variable 2 Variable 3 Variable 4 Variable 5 Variable 6
##          2          2          2          2          2          2
## Variable 7 Variable 8 Variable 9 Variable 10 Variable 11 Variable 12
##          2          2          2          2          2          2
## Variable 13 Variable 14 Variable 15 Variable 16 Variable 17 Variable 18
##          2          2          2          2          2          2
## Variable 19 Variable 20 Variable 21 Variable 22 Variable 23 Variable 24
##          2          2          2          2          2          2
## Variable 25
##          2
```

```
out4$hyperparameters$sigmabeta_sq[1, ]
```

```
## Variable 1 Variable 2 Variable 3 Variable 4 Variable 5 Variable 6
## 0.02451665 0.07368063 0.02451665 0.00010000 0.00010000 0.00010000
## Variable 7 Variable 8 Variable 9 Variable 10 Variable 11 Variable 12
## 0.00010000 0.00010000 0.00010000 0.00010000 0.00010000 0.00010000
## Variable 13 Variable 14 Variable 15 Variable 16 Variable 17 Variable 18
## 0.00010000 0.00010000 0.00010000 0.00010000 0.00010000 0.00010000
## Variable 19 Variable 20 Variable 21 Variable 22 Variable 23 Variable 24
## 0.00010000 0.00010000 0.00010000 0.00010000 0.00010000 0.00010000
## Variable 25
## 0.00010000
```

```
out4$hyperparameters$pi
```

```
## Variable 1 Variable 2 Variable 3 Variable 4 Variable 5 Variable 6
##      0.05      0.10      0.20      0.05      0.05      0.05
## Variable 7 Variable 8 Variable 9 Variable 10 Variable 11 Variable 12
##      0.05      0.05      0.05      0.05      0.05      0.05
## Variable 13 Variable 14 Variable 15 Variable 16 Variable 17 Variable 18
##      0.05      0.05      0.05      0.05      0.05      0.05
## Variable 19 Variable 20 Variable 21 Variable 22 Variable 23 Variable 24
##      0.05      0.05      0.05      0.05      0.05      0.05
## Variable 25
##      0.05
```

*mu*

```
summary(abs(unlist(out4$mu_matrices)))
```

```
##      Min.   1st Qu.   Median     Mean   3rd Qu.     Max.
## 0.0000000 0.0003356 0.0007480 0.0105911 0.0013778 0.5934859
```