

# pi stability analysis, large p

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## Experiment Overview

In this experiment, I compared the results for three methods of hyperparameter selection: M1, M2, and M3. 50 trials were performed. M1 proceeded as follows: In each trial, for each variable,  $\pi$  was selected by maximizing ELBO over the following grid:

$$\pi_{\sim} = \{0.001, 0.113, 0.226, 0.338, 0.450\}$$

$\sigma^2$  and  $\sigma_{\beta}^2$  were fit to the data for each variable and individual using MAPE.

The optimal  $\pi$  was then stabilized in the following manner: upon concluding the first grid search, another grid search was performed using the values of  $\mu, \alpha, \sigma^2$ , and  $\sigma_{\beta}^2$  corresponding to the optimal  $\pi$  as initial values. If the optimal  $\pi$  remained unchanged from the first grid search, then the grid search was concluded. However, if the optimal  $\pi$  changed, then the grid search was repeated until the optimal  $\pi$  stabilized.

M2 proceeded exactly as M1, except instead of fitting  $\sigma^2$  to the data using MAPE,  $\sigma^2$  was fixed for all individuals as the sample variance of the variable being treated as the response.

M3 was a pure grid search in which each of the hyperparameters was optimized by maximizing ELBO over a 3-D grid of 125 points  $\sigma^2 \times \pi_{\sim} \times \sigma_{\beta}^2$ . Since none of the hyperparameters were being fit to the data, the grid search was not iterated until stability as in M1 and M2.  $\pi_{\sim}$  was chosen as above, while  $\sigma_{\beta}^2$  and  $\sigma^2$  were:

$$\sigma_{\beta}^2_{\sim} = \{0.001, 0.005, 0.022, 0.106, 0.5\} \quad \sigma^2_{\sim} = \{0.2, 0.4, 0.6, 0.8, 1\}$$

The first 30 trials were performed on a relatively small dataset, with  $p + 1 = 5$ . The last 20 trials were performed on a larger dataset, with  $p + 1 = 25$ . At the end of each set of trials, the performance of all methods were compared in terms of sensitivity, specificity, accuracy, and time to fit.

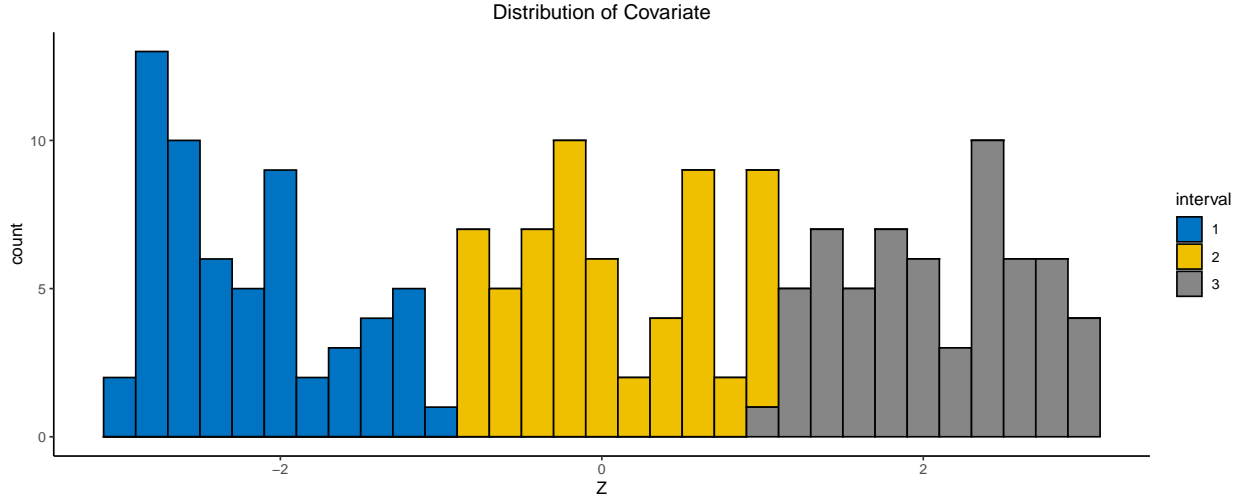
Note that I observed the fitted  $\sigma^2$  blow up, resulting in erroneous results. This happened most often in the large  $p$  case. Thus, if any errors resulted for any of the 3 methods, for example, due to blowup of fitted  $\sigma^2$  values, the trial was discarded.

Under each method, the CAVI updates were performed for 100 iterations before exiting, assuming that the tolerance criteria was not met prior to this.

## Data Generation

### Extraneous Covariate

I generated the covariate,  $Z$ , as the union of three almost disjoint intervals of equal measure. That is,  $Z = Z_1 \cup Z_2 \cup Z_3$  with  $Z_1 = (-3, -1)$ ,  $Z_2 = (a, b) = (-1, 1)$ ,  $Z_3 = (1, 3)$ . Within each interval, I generated 60 covariate values from a uniform distribution. For example:



### Precision Matrix

All of the individuals in interval 1 had the same precision matrix,  $\Omega^{(1)}$ :

$$\Omega_{i,j}^{(1)} = \begin{cases} 2 & i = j \\ 1 & (i,j) \in \{(1,2), (2,1), (2,3), (3,2)\} \\ 0 & o.w. \end{cases}$$

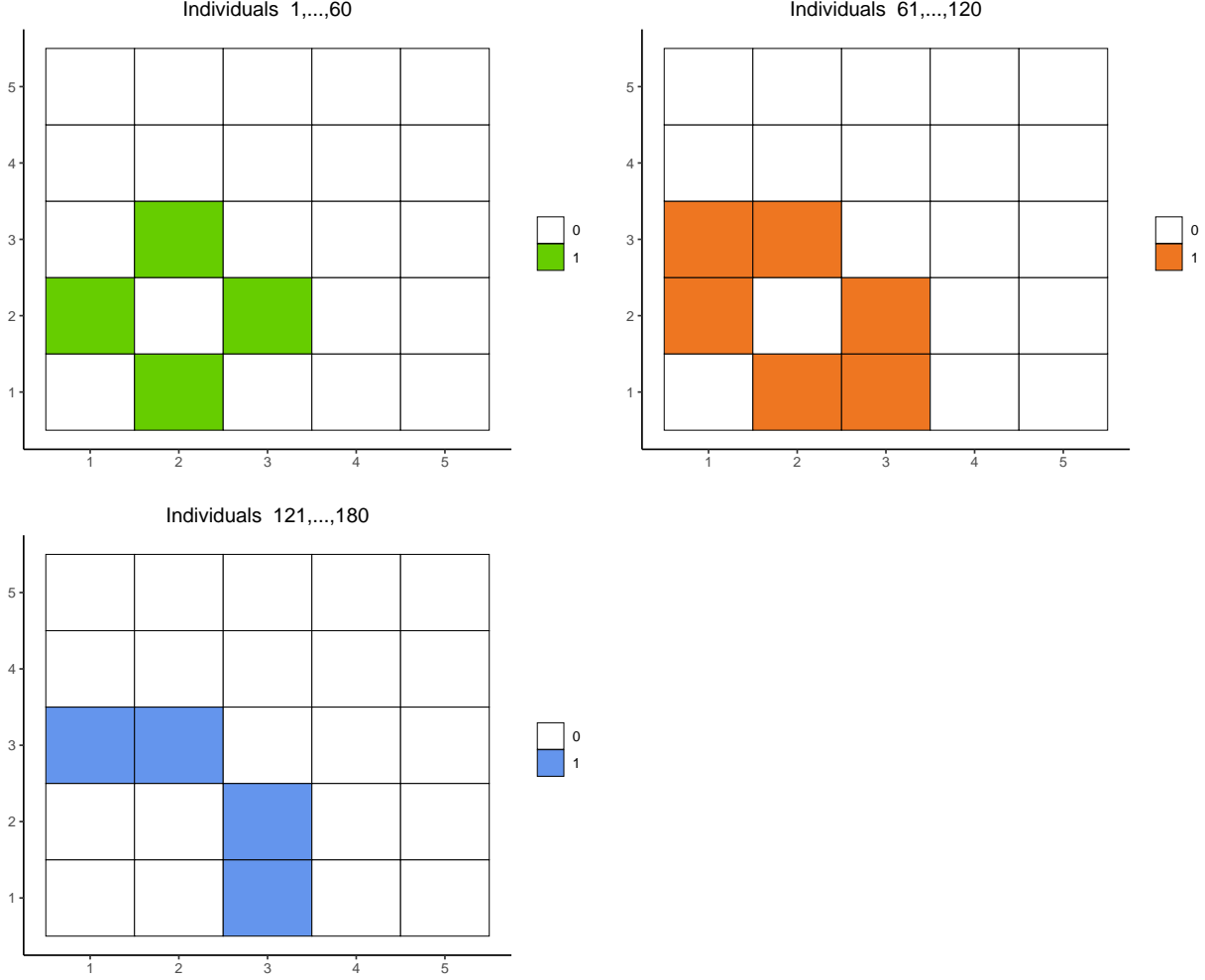
Also, all of the individuals in interval 3 had the same precision matrix,  $\Omega^{(3)}$ :

$$\Omega_{i,j}^{(3)} = \begin{cases} 2 & i = j \\ 1 & (i,j) \in \{(1,3), (3,1), (2,3), (3,2)\} \\ 0 & o.w. \end{cases}$$

However, the individuals in interval 2 had a precision matrix that was dependent upon  $Z$  and  $(a, b)$ . Let  $\beta_0 = -a/(b-a)$  and  $\beta_1 = 1/(b-a)$ . Then:

$$\Omega_{i,j}^{(2)}(z) = \begin{cases} 2 & i = j \\ 1 & (i,j) \in \{(2,3), (3,2)\} \\ 1 - \beta_0 - \beta_1 z & (i,j) \in \{(1,2), (2,1)\} \\ \beta_0 + \beta_1 z & (i,j) \in \{(1,3), (3,1)\} \\ 0 & o.w. \end{cases}$$

Thus,  $\Omega^{(2)}(a) = \Omega^{(1)}$  and  $\Omega^{(2)}(b) = \Omega^{(3)}$ . That is, an individual on the left or right boundary of  $Z_2$  would have precision matrix  $\Omega^{(1)}$  or  $\Omega^{(3)}$ , respectively. The conditional dependence structures corresponding to each of these precision matrices are visualized below.



## Data matrix

Let  $z_l$  be the extraneous covariate for the  $l$ -th individual. To generate the data matrix for the  $l$ -th individual, I took a random sample from  $\mathcal{N}(0, \{\Omega_l(z_l)\}^{-1})$ , where:

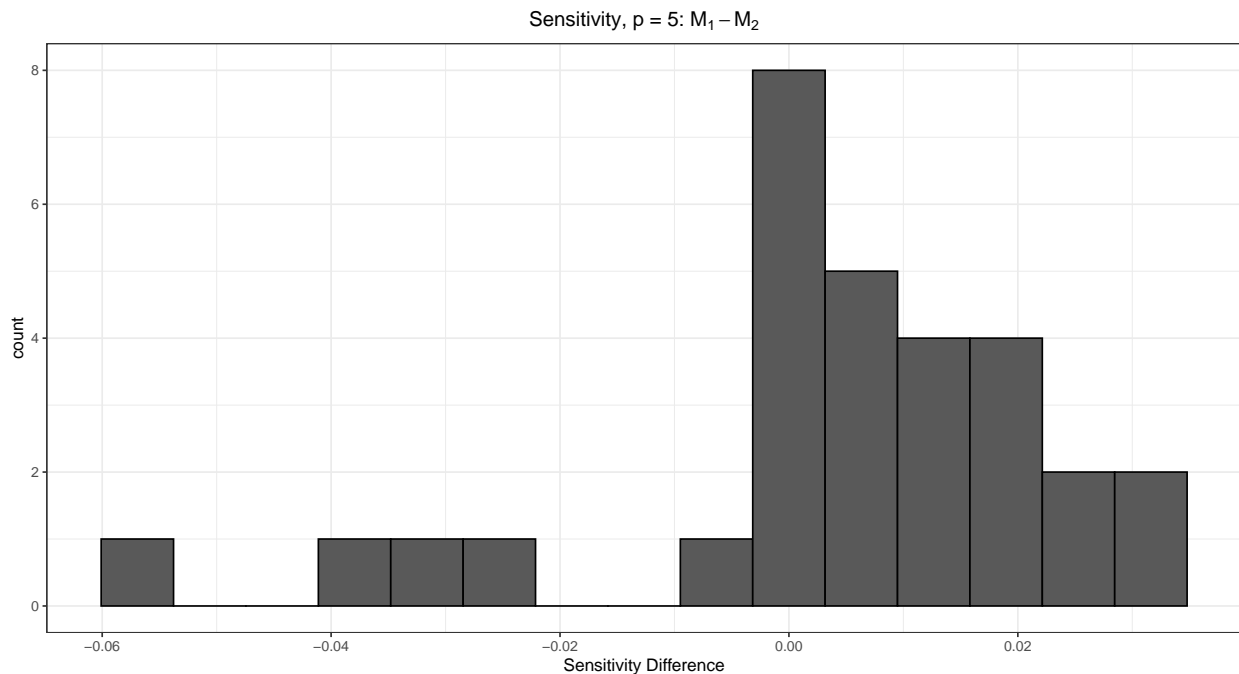
$$\Omega_l(z_l) = \begin{cases} \Omega^{(1)} & z_l \in Z_1 \\ \Omega^{(2)}(z_l) & z_l \in Z_2 \\ \Omega^{(3)} & z_l \in Z_3 \end{cases}$$

## Results

```
n <- 180
n_3 <- n^(-1/3)

# sensitivity comparison (5)

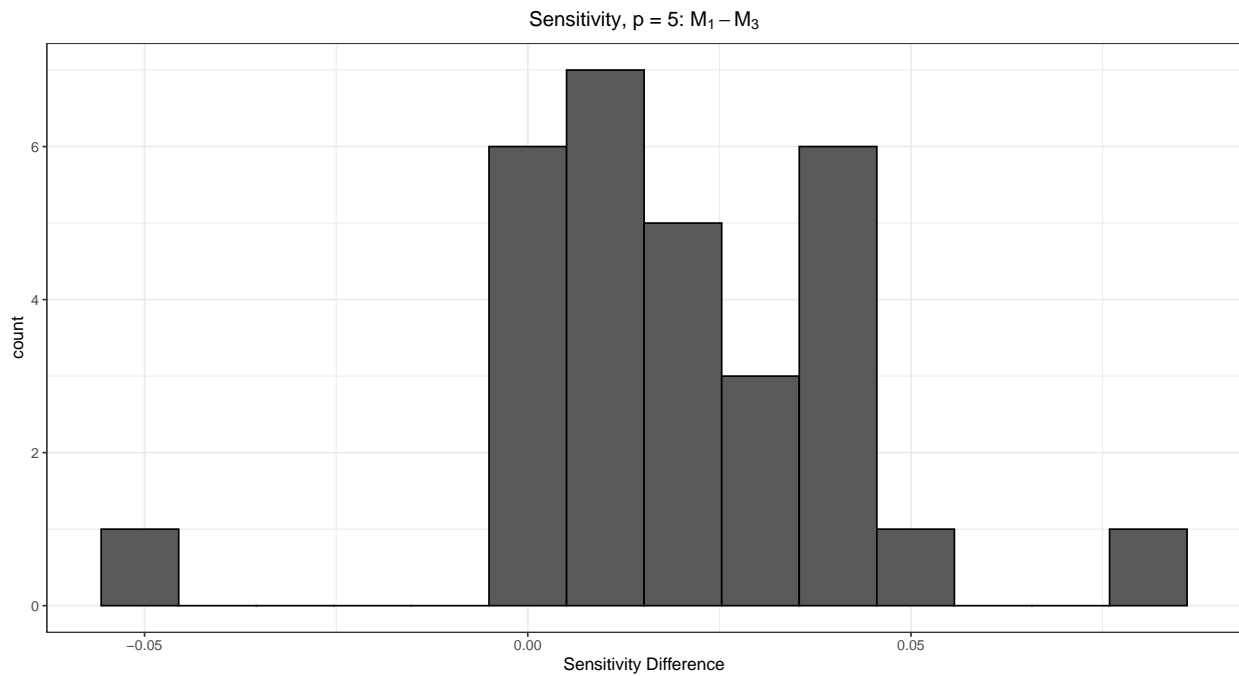
# m1 - m2
data.frame(X = m1_sens5 - m2_sens5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 5:  $M_1 - M_2$ ")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_sens5 - m2_sens5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



```
summary(m1_sens5 - m2_sens5)
```

```
##      Min.   1st Qu.   Median     Mean   3rd Qu.     Max.
## -0.054762  0.000000  0.004762  0.003810  0.017857  0.030952
```

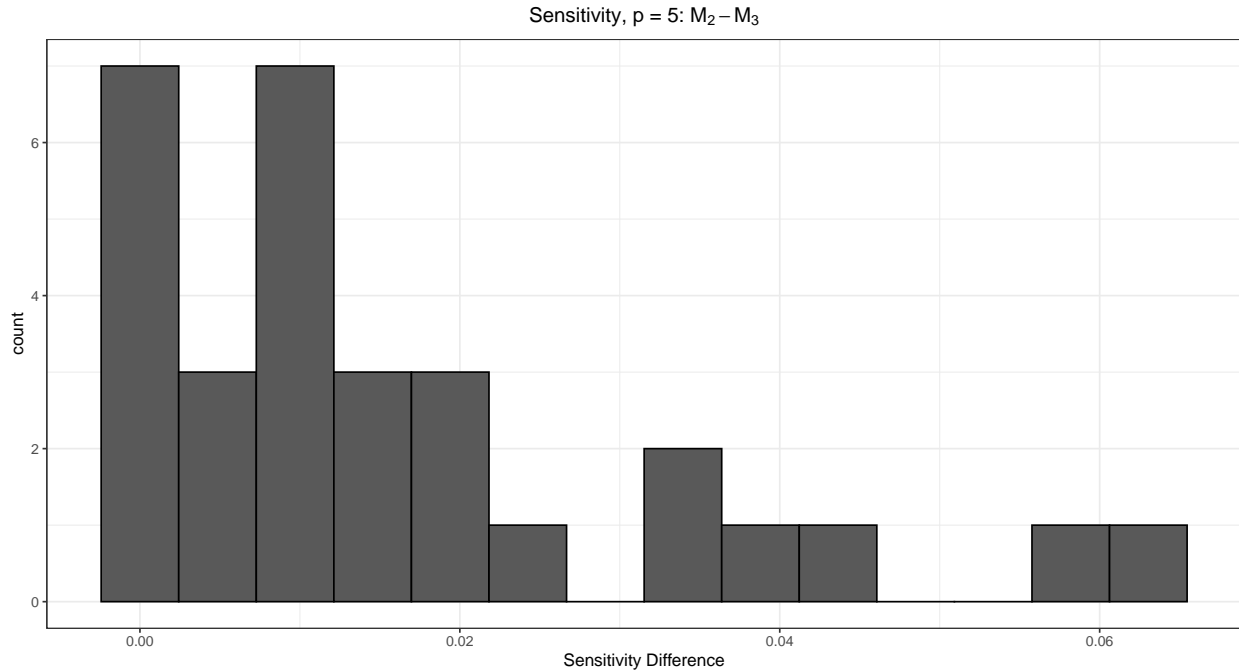
```
# m1 - m3
data.frame(X = m1_sens5 - m3_sens5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 5:  $M_1 - M_3$ ")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_sens5 - m3_sens5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



```
summary(m1_sens5 - m3_sens5)
```

```
##      Min.   1st Qu.   Median     Mean   3rd Qu.    Max.
## -0.047619  0.008333  0.016667  0.020556  0.036905  0.083333
```

```
# m2 - m3
data.frame(X = m2_sens5 - m3_sens5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 5:  $M_2 - M_3$ ")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m2_sens5 - m3_sens5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



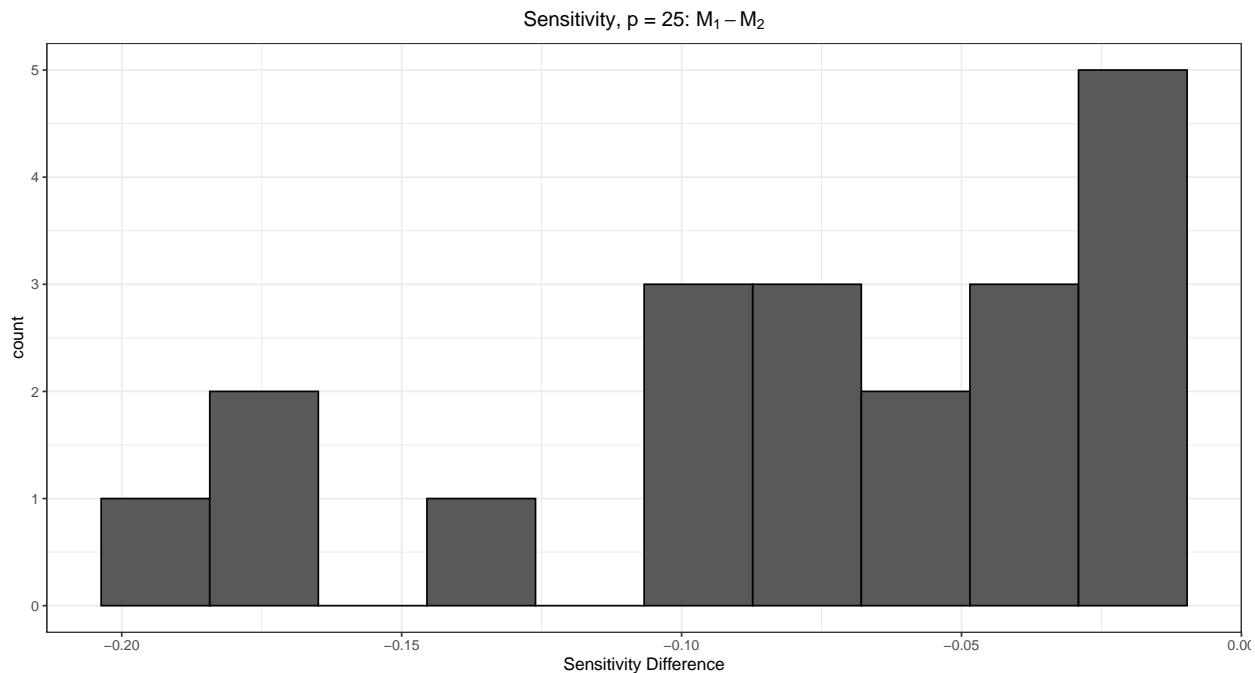
```
summary(m2_sens5 - m3_sens5)
```

```
##      Min.   1st Qu.   Median     Mean  3rd Qu.     Max.
## 0.000000 0.007143 0.011905 0.016746 0.020833 0.061905
```

```
# sensitivity comparison (25)
```

```
# m1 - m2
```

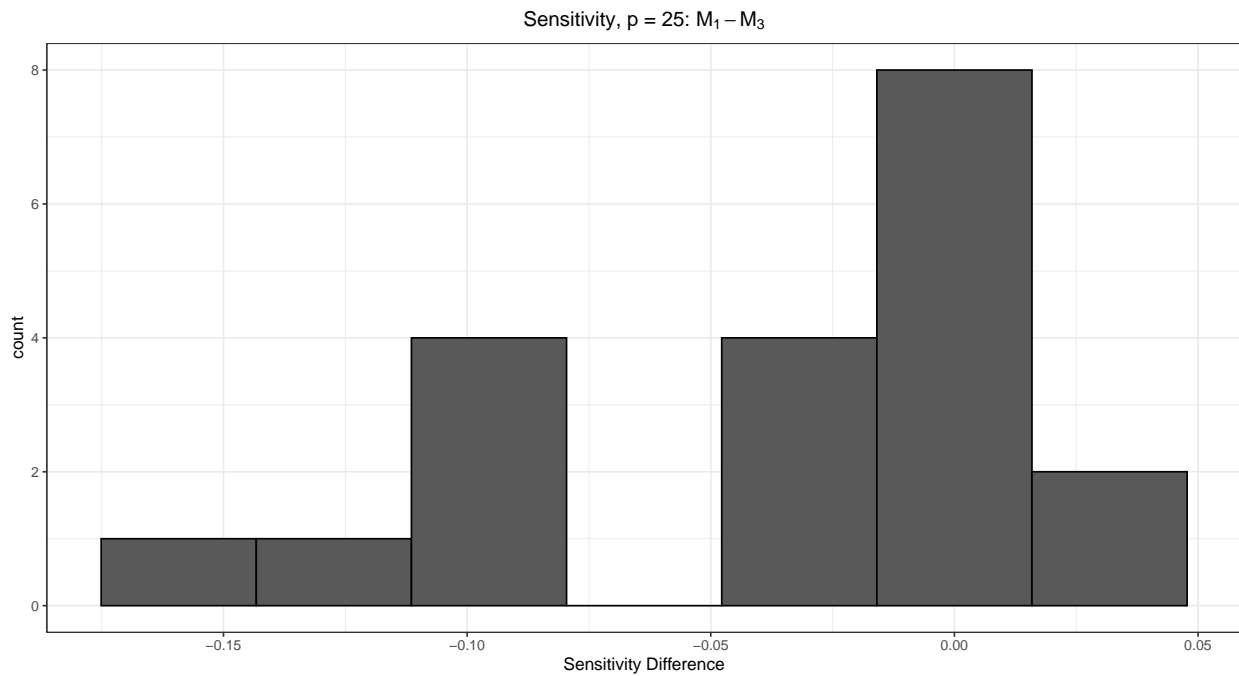
```
data.frame(X = m1_sens25 - m2_sens25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 25:  $M_1 - M_2$ ")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_sens25 - m2_sens25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



```
summary(m1_sens25 - m2_sens25)
```

```
##      Min.   1st Qu.   Median     Mean  3rd Qu.     Max.
## -0.20000 -0.09048 -0.06667 -0.07548 -0.03571 -0.01429
```

```
# m1 - m3
data.frame(X = m1_sens25 - m3_sens25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 25: $M_1 - M_3$")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_sens25 - m3_sens25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```

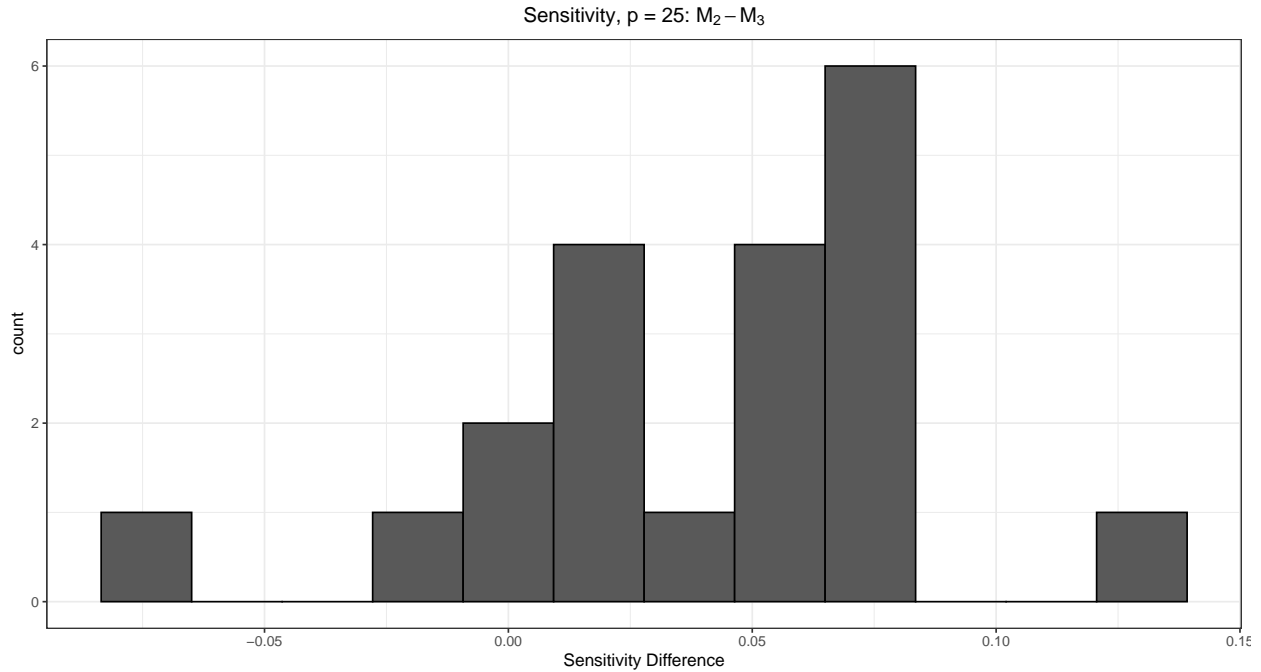


```
summary(m1_sens25 - m3_sens25)
```

```
##      Min.    1st Qu.    Median      Mean   3rd Qu.      Max.
## -0.1523809 -0.0892857 -0.0154762 -0.0378571  0.0005952  0.0261905
```

```
# m2 - m3
data.frame(X = m2_sens25 - m3_sens25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Sensitivity, p = 25:  $M_2 - M_3$ ")) +
  xlab("Sensitivity Difference") +
  geom_histogram(binwidth = 2 * IQR(m2_sens25 - m3_sens25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```





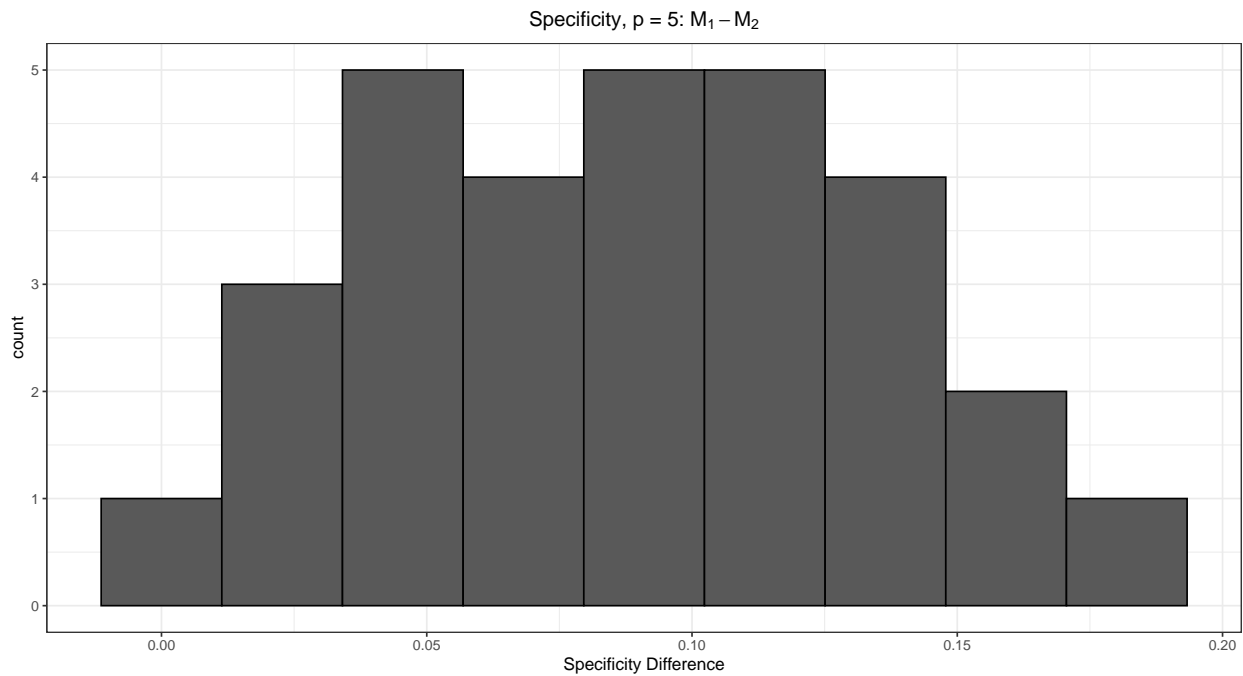
```
summary(m2_sens25 - m3_sens25)
```

```
##      Min.   1st Qu.   Median     Mean  3rd Qu.    Max.
## -0.08333  0.01667  0.04762  0.03762  0.06905  0.12381
```

```
# specificity comparison (5)
```

```
# m1 - m2
```

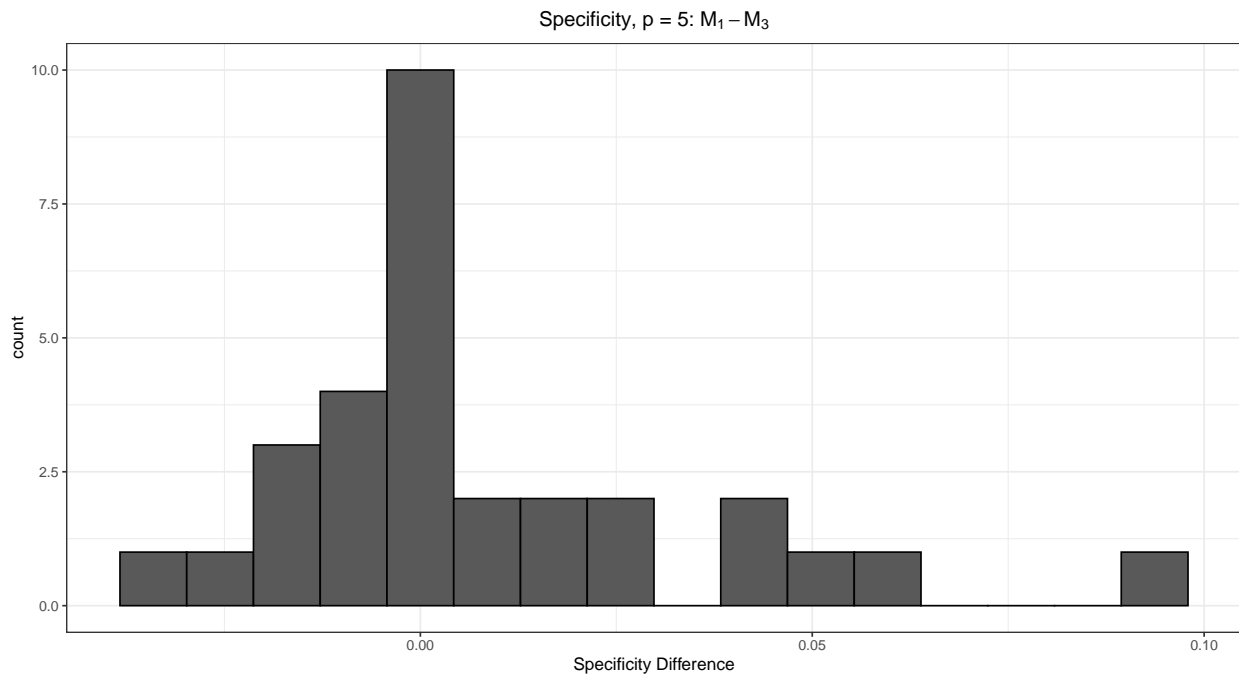
```
data.frame(X = m1_spec5 - m2_spec5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 5: $M_1 - M_2$")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_spec5 - m2_spec5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



```
summary(m1_spec5 - m2_spec5)
```

```
##      Min.   1st Qu.   Median     Mean   3rd Qu.     Max.
## 0.0005464 0.0498634 0.0907104 0.0882878 0.1140710 0.1868853
```

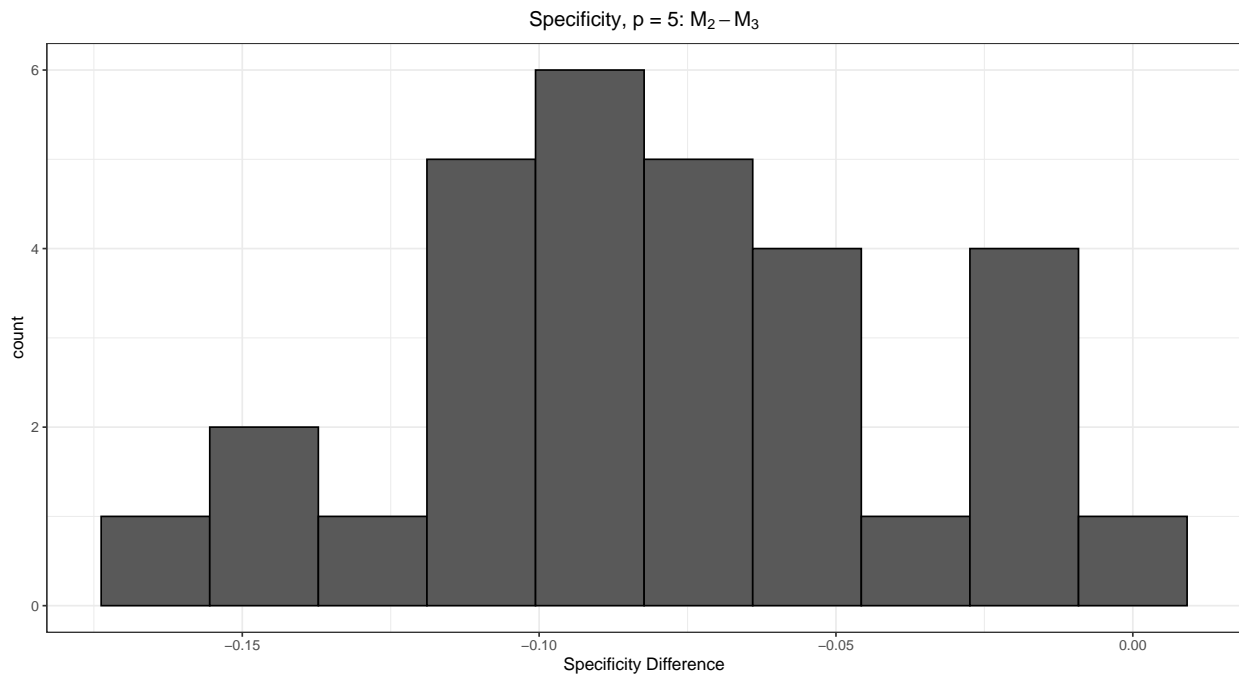
```
# m1 - m3
data.frame(X = m1_spec5 - m3_spec5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 5: $M_1 - M_3$")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_spec5 - m3_spec5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



```
summary(m1_spec5 - m3_spec5)
```

```
##      Min.   1st Qu.   Median     Mean   3rd Qu.    Max.
## -0.032787 -0.005874  0.000000  0.007978  0.018169  0.093443
```

```
# m2 - m3
data.frame(X = m2_spec5 - m3_spec5) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 5: $M_2 - M_3$")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m2_spec5 - m3_spec5) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



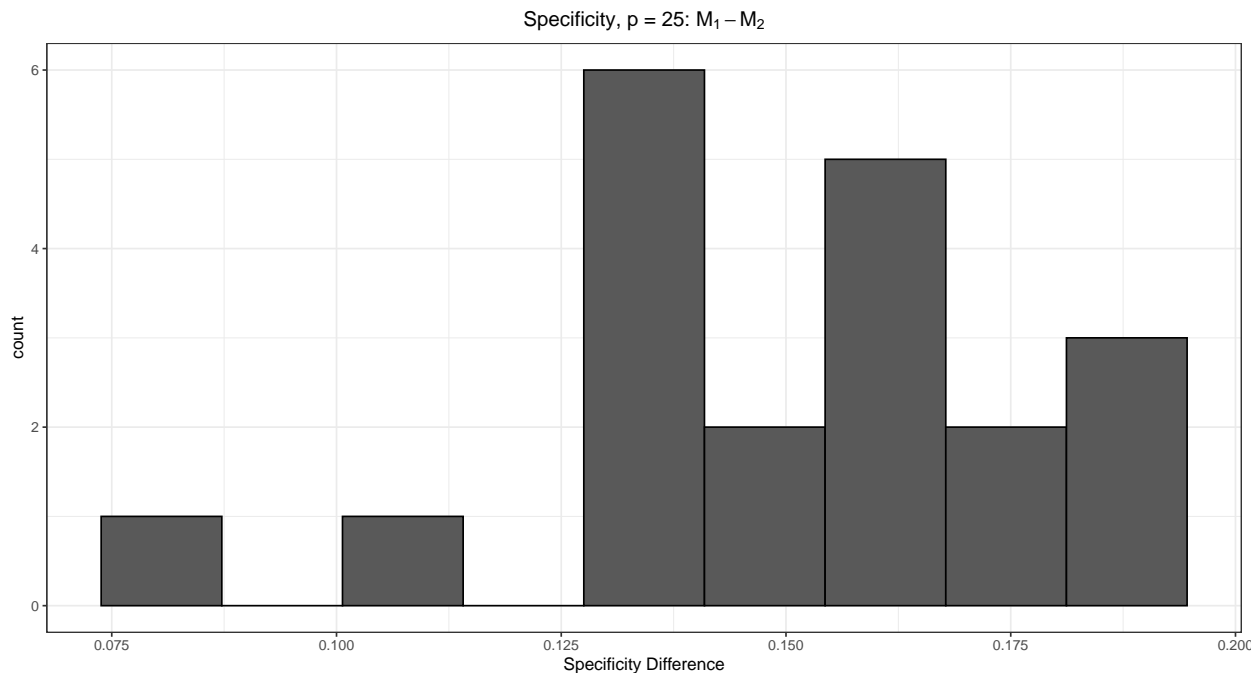
```
summary(m2_spec5 - m3_spec5)
```

```
##      Min.   1st Qu.   Median     Mean   3rd Qu.     Max.
## -0.171038 -0.105328 -0.084426 -0.080310 -0.053689 -0.006011
```

```
# specificity comparison (25)
```

```
# m1 - m2
```

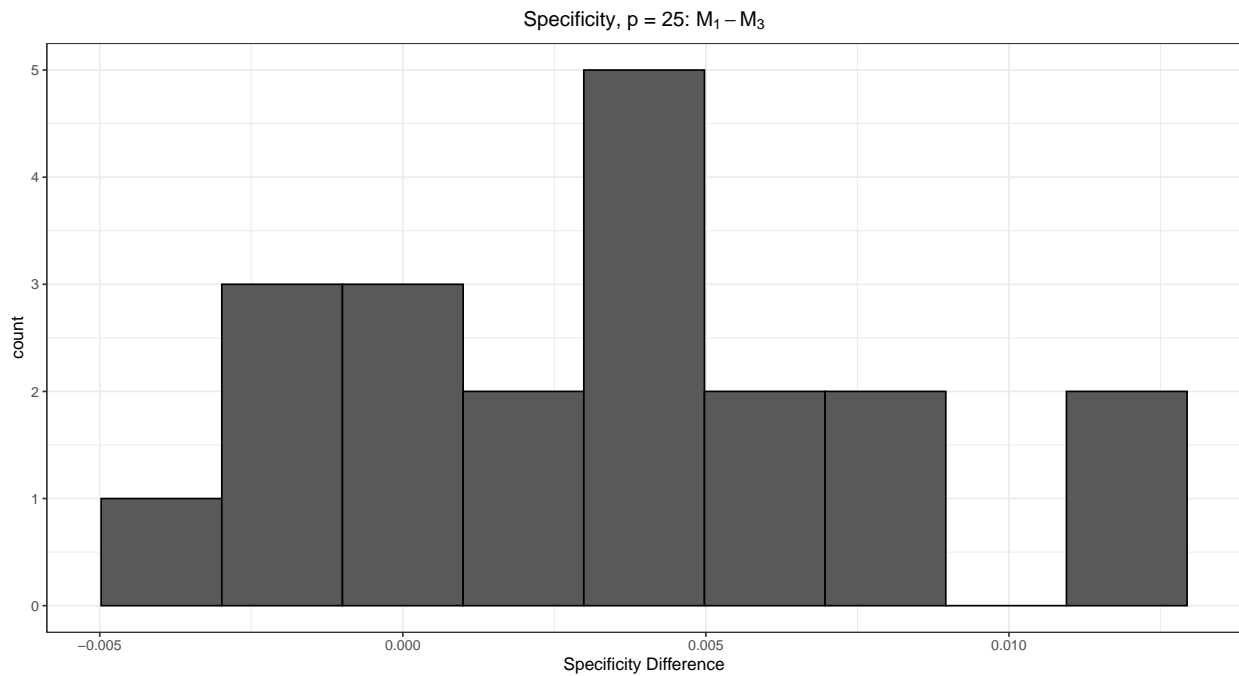
```
data.frame(X = m1_spec25 - m2_spec25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 25:  $M_1 - M_2$ ")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_spec25 - m2_spec25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



```
summary(m1_spec25 - m2_spec25)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.08601 0.13079 0.15419 0.14986 0.16868 0.18812
```

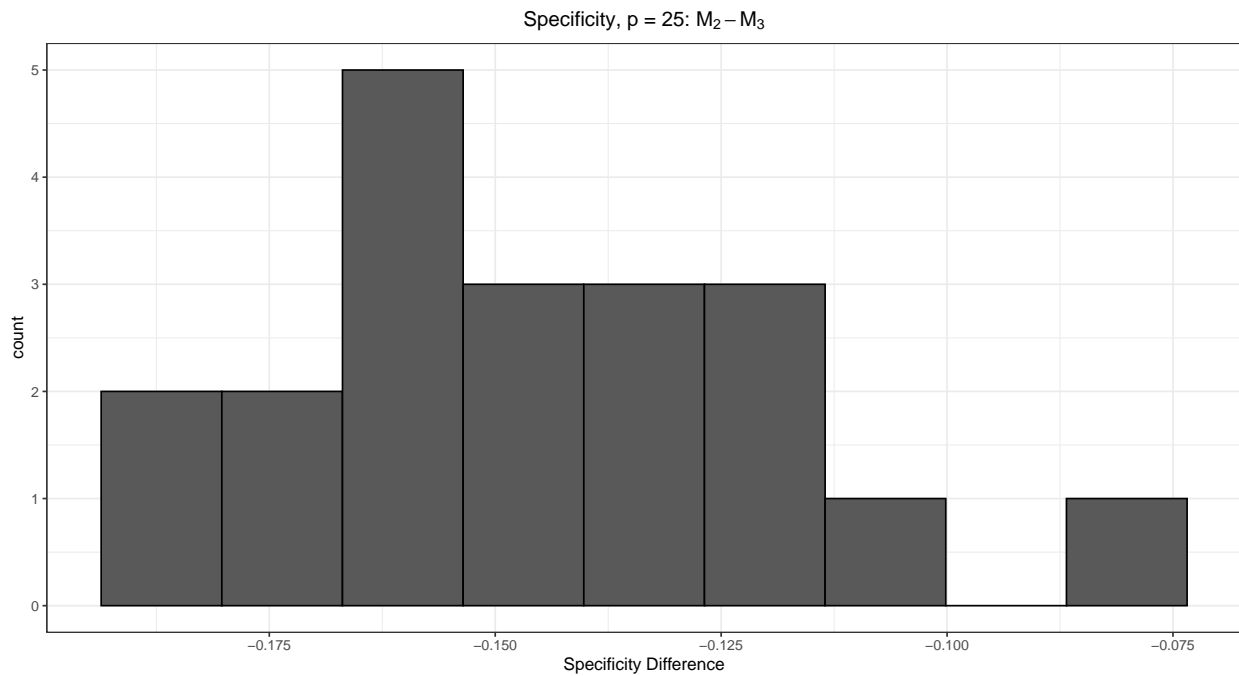
```
# m1 - m3
data.frame(X = m1_spec25 - m3_spec25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 25: $M_1 - M_3$")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m1_spec25 - m3_spec25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



```
summary(m1_spec25 - m3_spec25)
```

```
##      Min.    1st Qu.    Median      Mean   3rd Qu.      Max.
## -0.0032957 0.0003717 0.0038062 0.0033763 0.0059914 0.0120724
```

```
# m2 - m3
data.frame(X = m2_spec25 - m3_spec25) %>% ggplot(aes(X)) +
  ggtitle(TeX("Specificity, p = 25: $M_2 - M_3$")) +
  xlab("Specificity Difference") +
  geom_histogram(binwidth = 2 * IQR(m2_spec25 - m3_spec25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



```
summary(m2_spec25 - m3_spec25)
```

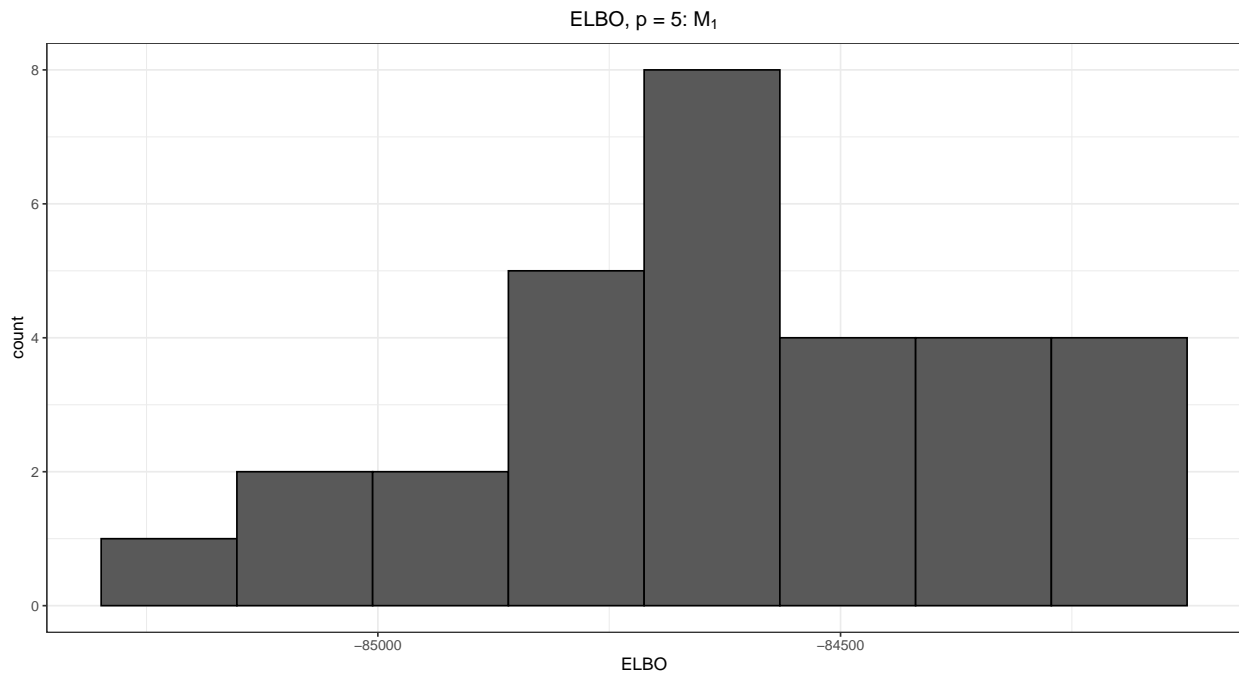
```
##      Min.   1st Qu.   Median     Mean  3rd Qu.     Max.
## -0.18791 -0.16494 -0.14841 -0.14648 -0.12724 -0.08109
```

```
# distribution of ELBO
```

```
# p = 5
```

```
# m1
```

```
data.frame(X = m1_ELB05) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO, p = 5:  $M_1$ ")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m1_ELB05) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```

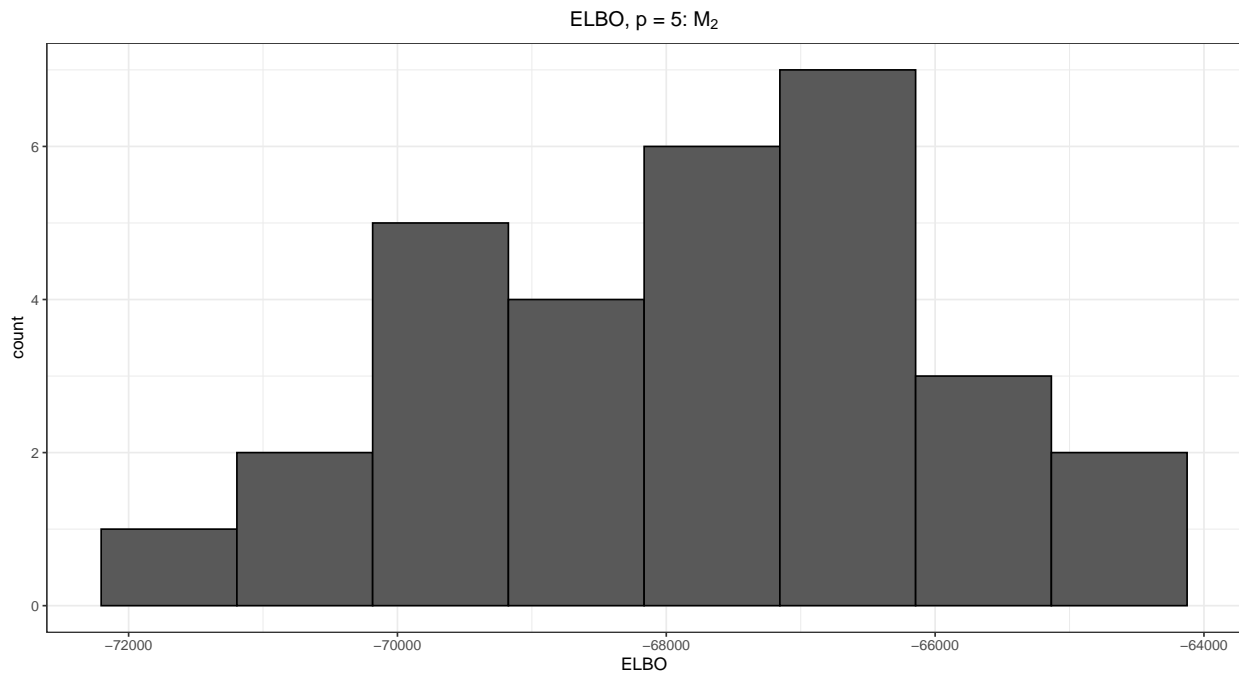


```
summary(m1_ELB05)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -85164  -84793  -84608  -84623  -84379  -84210
```

```
# m2
data.frame(X = m2_ELB05) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO, p = 5:  $M_2$ ")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m2_ELB05) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```

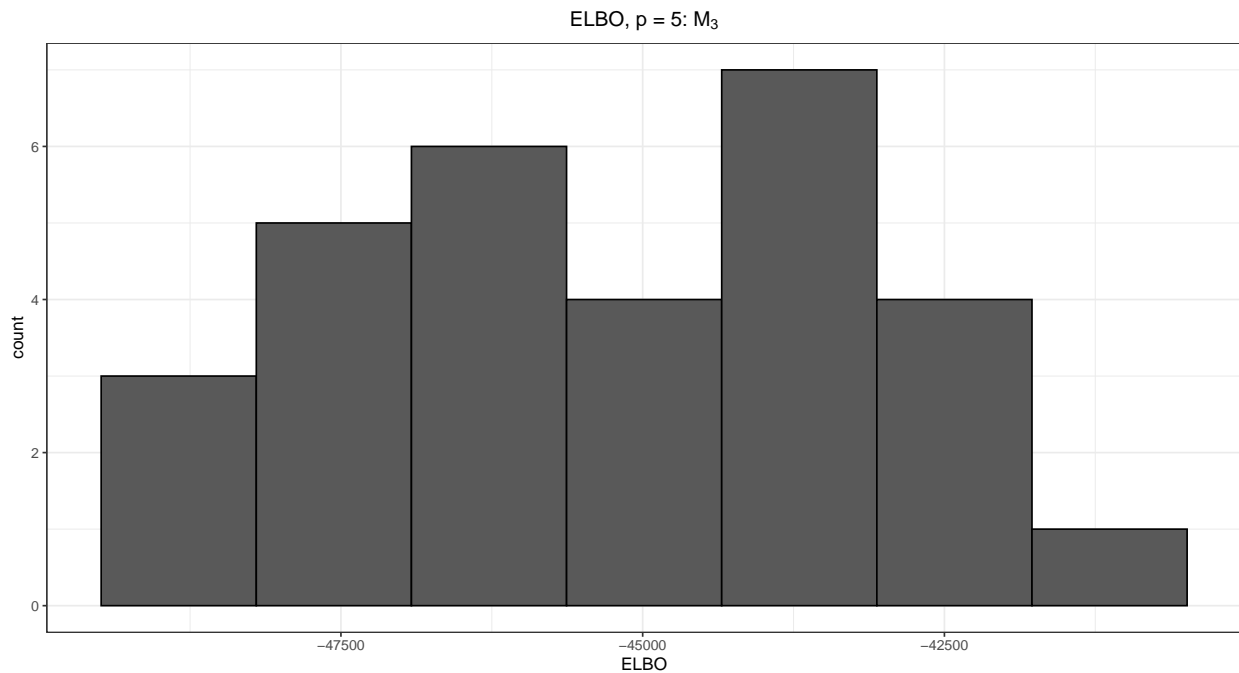




```
summary(m2_ELB05)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -71496 -69328  -67904  -67857  -66477  -64592
```

```
# m3
data.frame(X = m3_ELB05) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO, p = 5:  $M_3$ ")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m3_ELB05) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



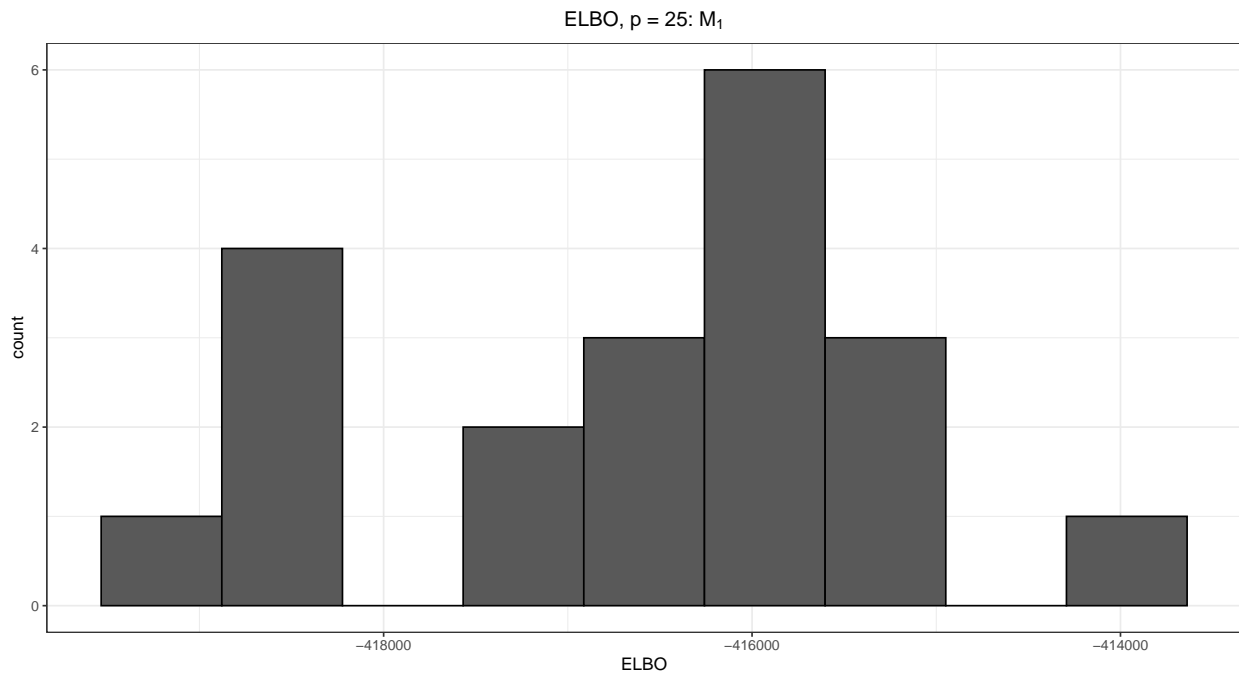
```
summary(m3_ELB05)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -49121 -46917  -45228  -45239  -43288  -40581
```

```
#  $p = 25$ 
```

```
#  $m_1$ 
```

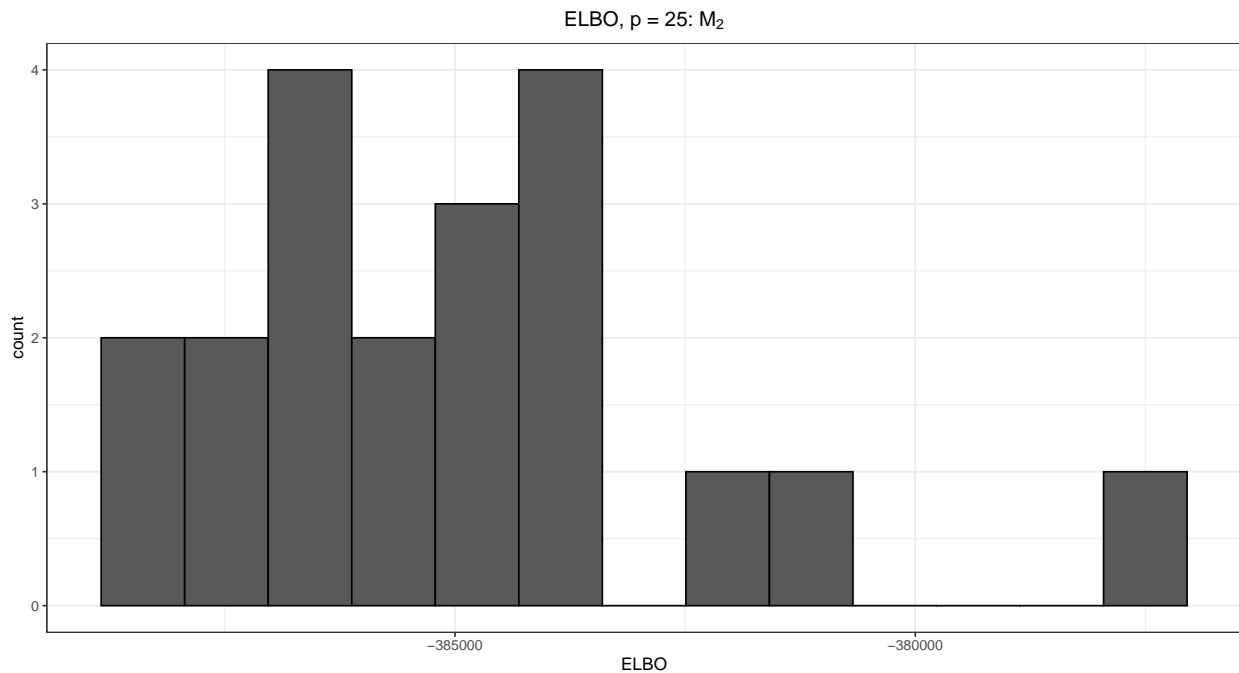
```
data.frame(X = m1_ELB025) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO,  $p = 25: M_1$ ")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m1_ELB025) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



```
summary(m1_ELBO25)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -419360 -417484 -416282 -416620 -415635 -414012
```

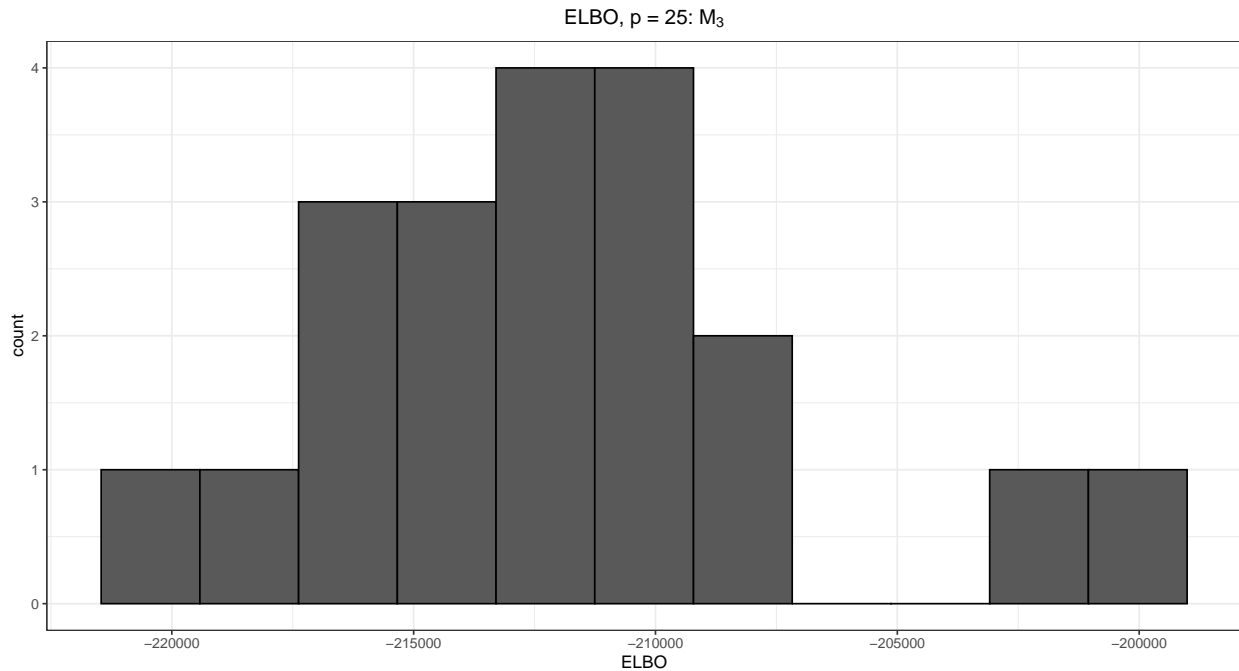
```
# m2
data.frame(X = m2_ELBO25) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO, p = 25: $M_2$")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m2_ELBO25) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



```
summary(m2_ELB025)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -388137 -386381 -385196 -384894 -383819 -377801
```

```
# m3
data.frame(X = m3_ELB025) %>% ggplot(aes(X)) +
  ggtitle(TeX("ELBO, p = 25:  $M_3$ ")) +
  xlab("ELBO") +
  geom_histogram(binwidth = 2 * IQR(m3_ELB025) * n_3, color = "black") +
  theme_bw() +
  theme(plot.title = element_text(hjust = 0.5))
```



```
summary(m3_ELB025)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -220222 -215170 -212452 -211904 -209408 -199046
```

```
Sys.time() - start
```

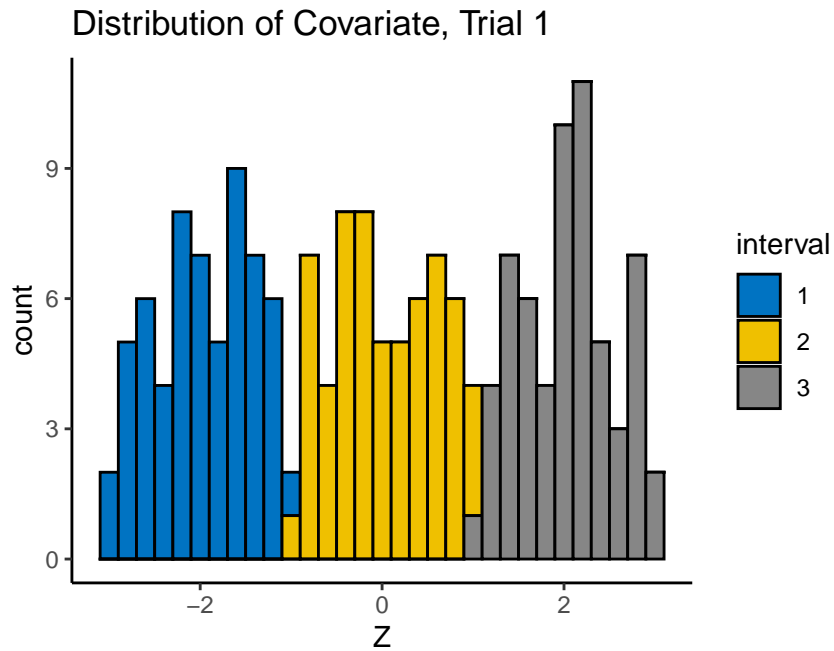
```
## Time difference of 1.000593 hours
```

```
# find errors
errs <- sapply(results, `[`, "error")
err_log <- !sapply(errs, is.null)
errs[err_log]
```

```
## named list()
```

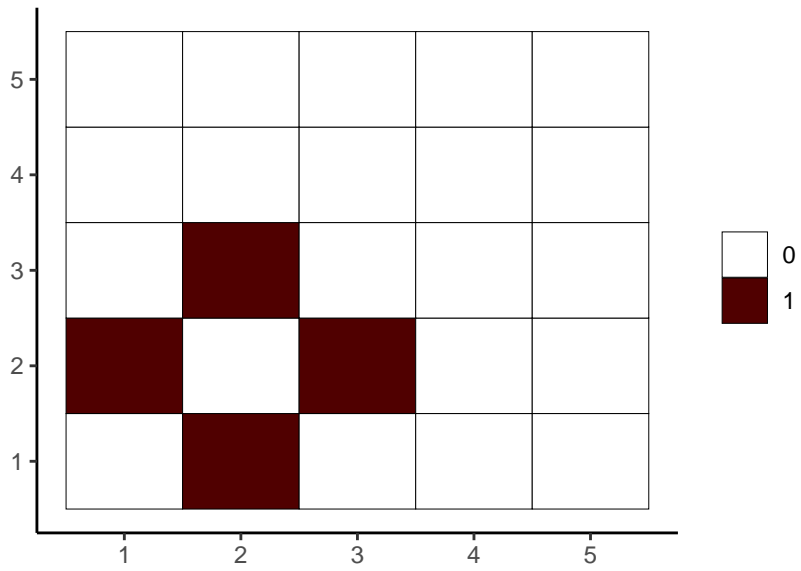
```
# display a single non-large example
results[!err_log][1]
```

```
## $trial1
## $trial1$covariates
```



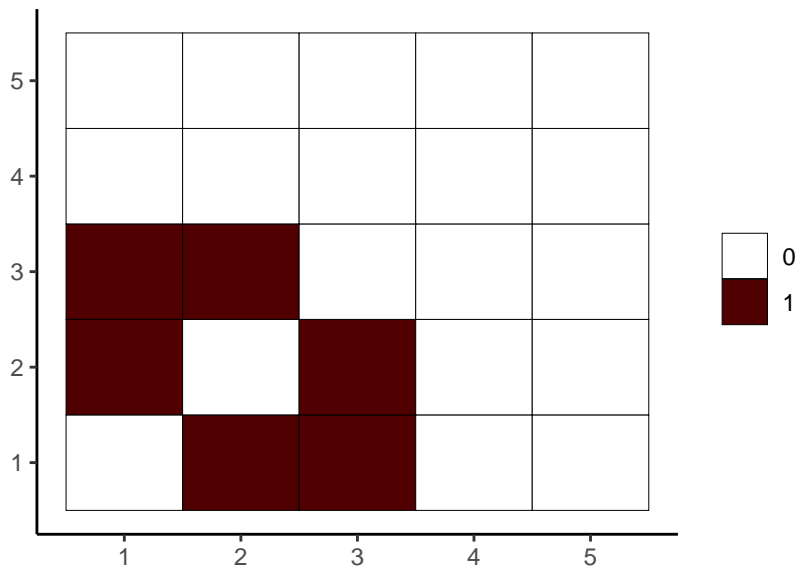
```
##
## $trial1$M1
## $trial1$M1$summary
##               Covariate Dependent Graphical Model
##
## Model ELBO: -84663.66           Unique conditional dependence structures: 3
## n: 180, variables: 5             Hyperparameter grid size: 5 points
## CAVI converged for 3/5 variables
##
## Model fit completed in 3.032 secs
##
## $trial1$M1$unique_graphs
## $trial1$M1$unique_graphs[[1]]
```

Graph 1, Individuals 1,...,53

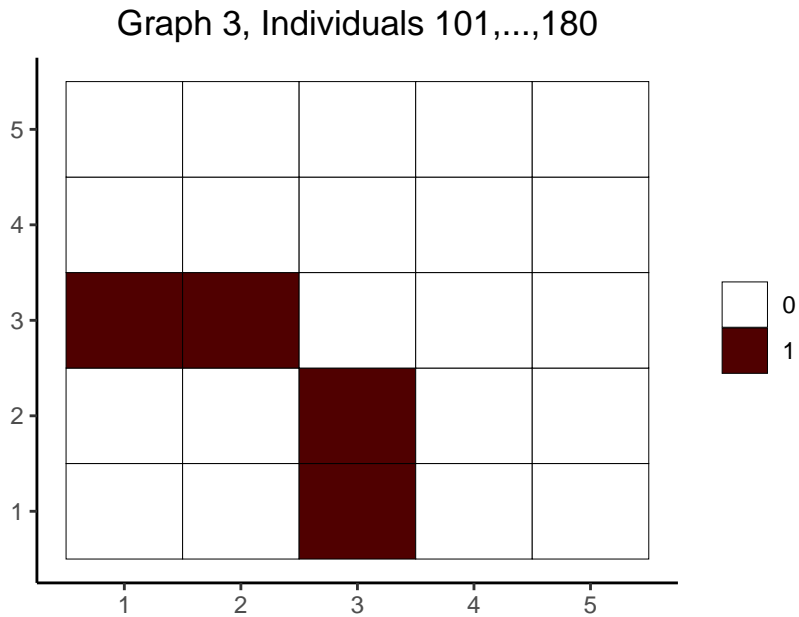


```
##
## $trial1$M1$unique_graphs[[2]]
```

Graph 2, Individuals 54,...,100



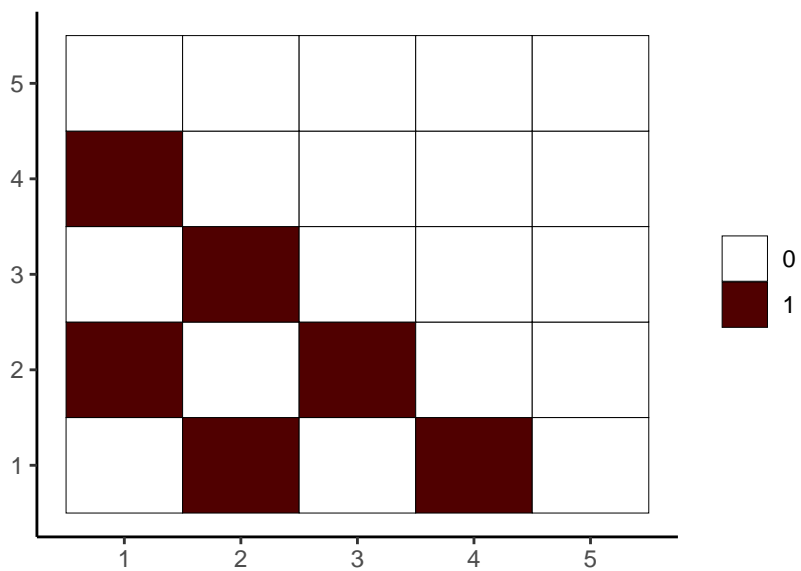
```
##
## $trial1$M1$unique_graphs[[3]]
```



```
##
##
## $trial1$M1$sensitivity
## [1] 0.952381
##
## $trial1$M1$specificity
## [1] 0.9961749
##
## $trial1$M1$accuracy
## [1] 0.988
##
## $trial1$M1$ELBO
## [1] -84663.66
##
## $trial1$M1$time
## [1] 3.032029
##
##
## $trial1$M2
## $trial1$M2$summary
##               Covariate Dependent Graphical Model
##
## Model ELBO: -67017.2           Unique conditional dependence structures: 13
## n: 180, variables: 5           Hyperparameter grid size: 5 points
## CAVI converged for 3/5 variables
##
## Model fit completed in 1.106 secs
##
## $trial1$M2$unique_graphs
## $trial1$M2$unique_graphs[[1]]
```

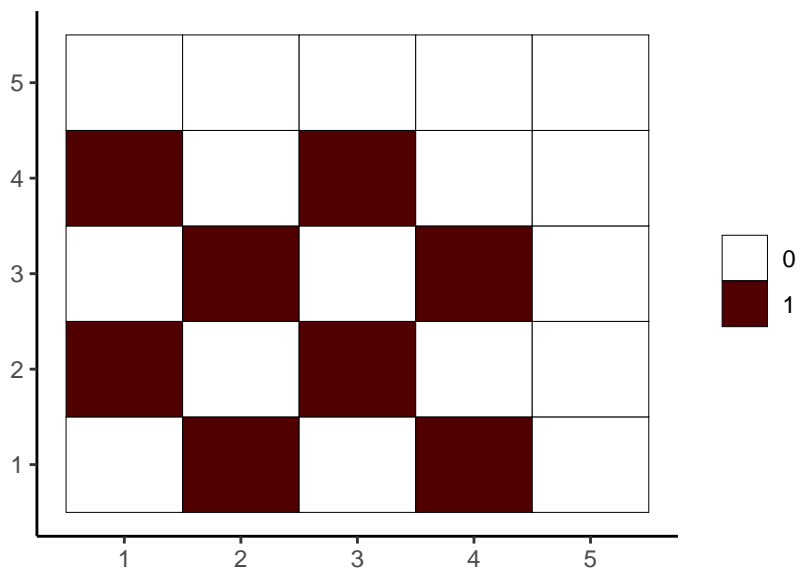


Graph 1, Individuals 1,37,...,51



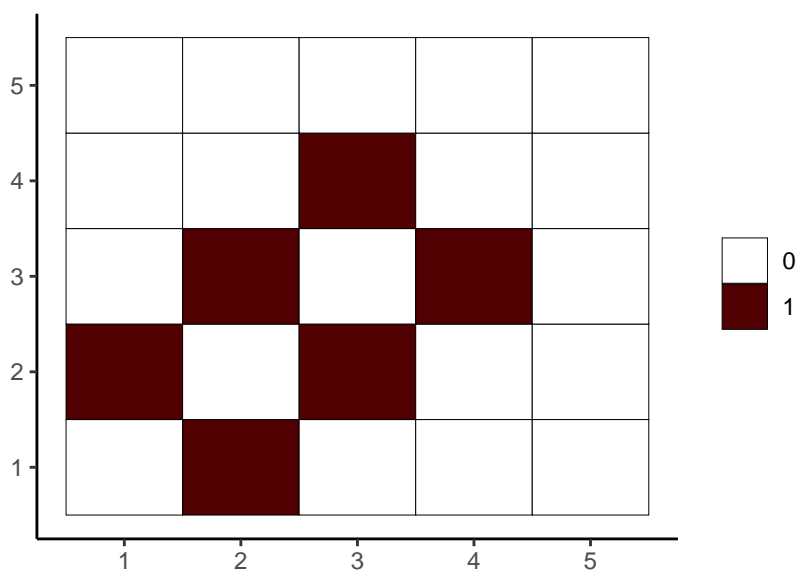
```
##
## $trial1$M2$unique_graphs[[2]]
```

Graph 2, Individuals 2,...,18



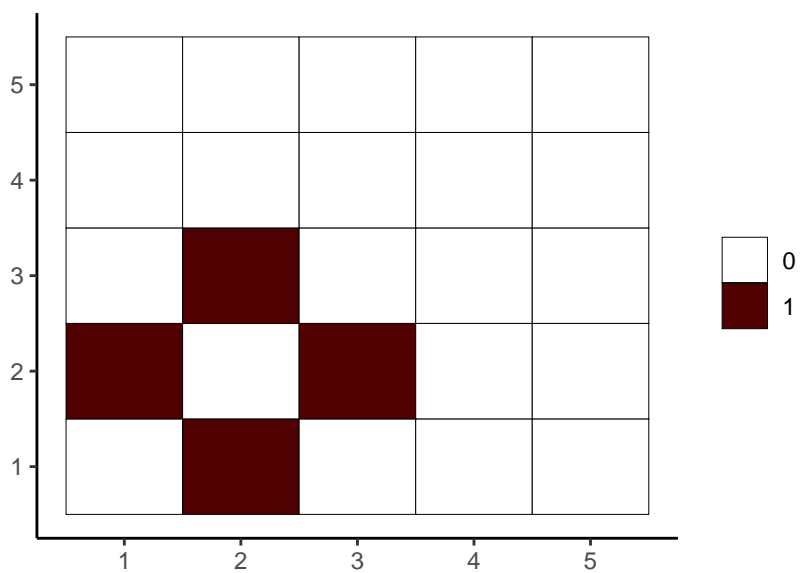
```
##
## $trial1$M2$unique_graphs[[3]]
```

Graph 3, Individuals 19,...,33



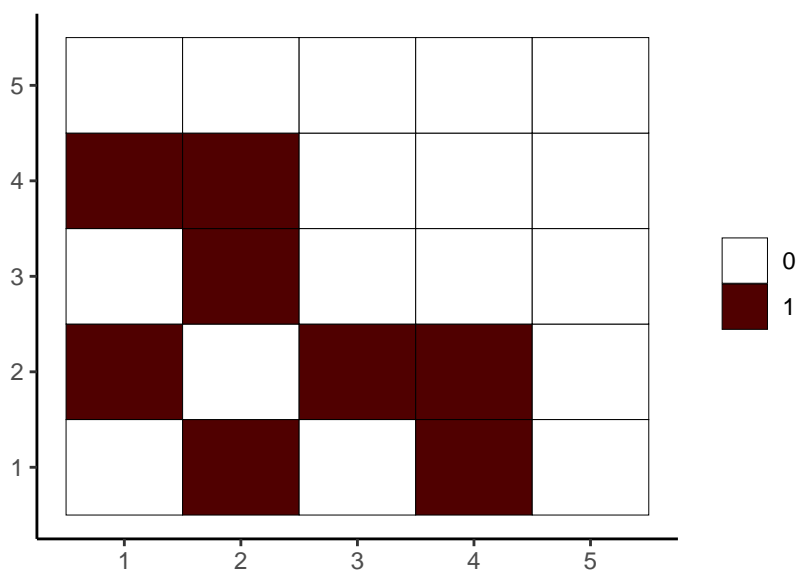
```
##
## $trial1$M2$unique_graphs[[4]]
```

Graph 4, Individuals 34,...,36



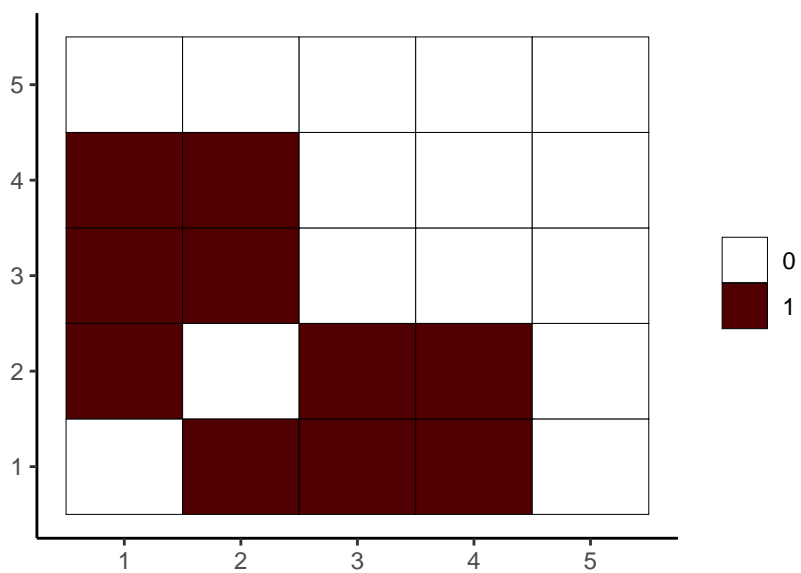
```
##
## $trial1$M2$unique_graphs[[5]]
```

Graph 5, Individuals 52,...,55



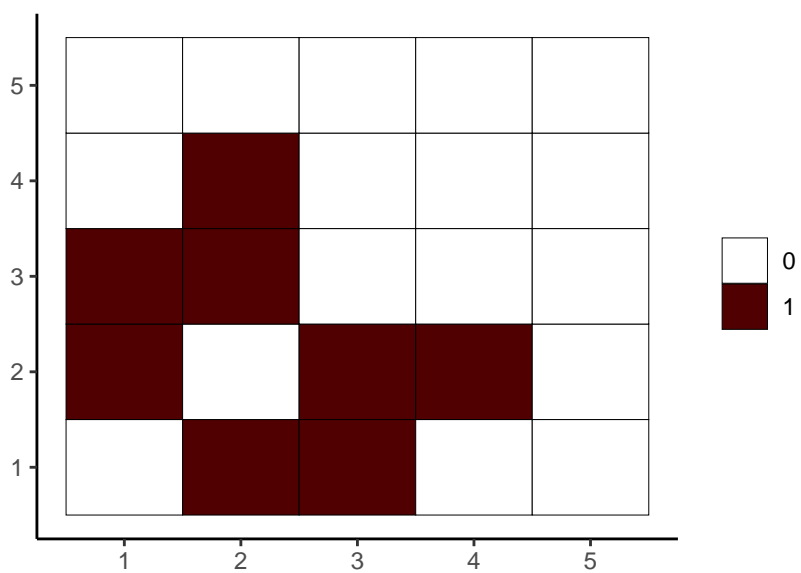
```
##
## $trial1$M2$unique_graphs[[6]]
```

Graph 6, Individuals 56,...,78



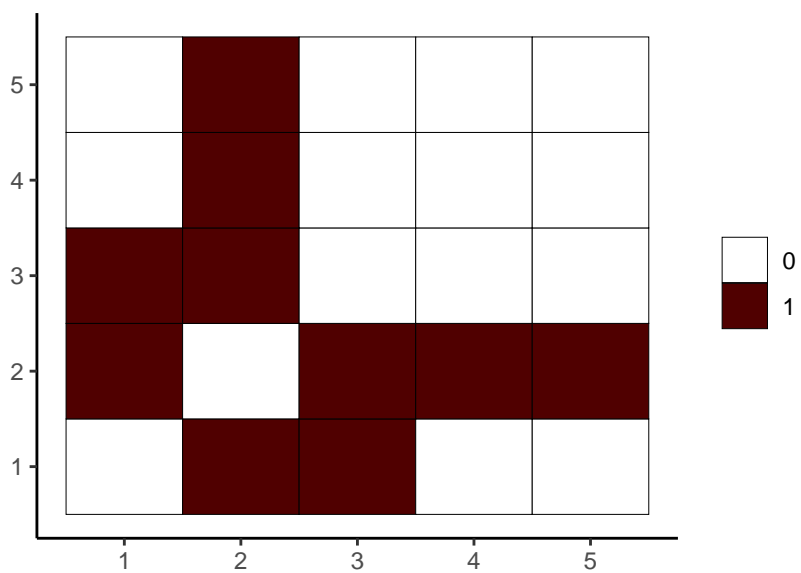
```
##
## $trial1$M2$unique_graphs[[7]]
```

Graph 7, Individuals 79,...,93



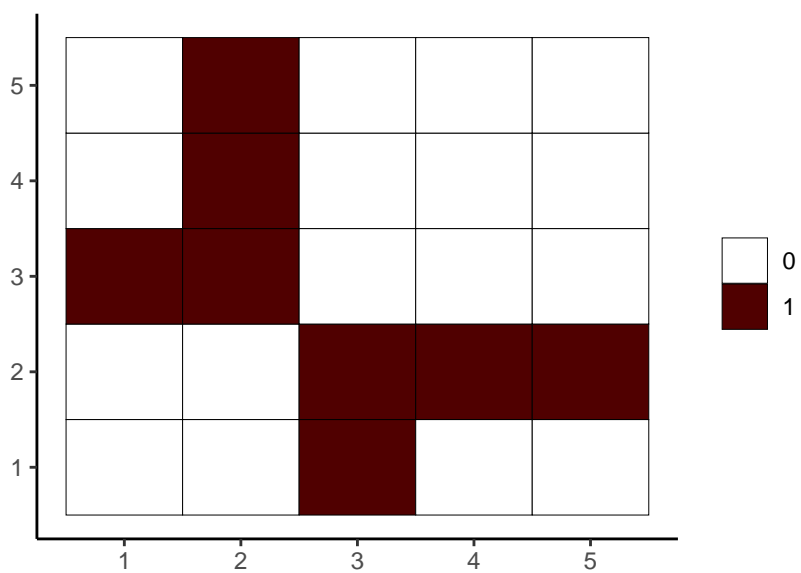
```
##  
## $trial1$M2$unique_graphs[[8]]
```

Graph 8, Individuals 94



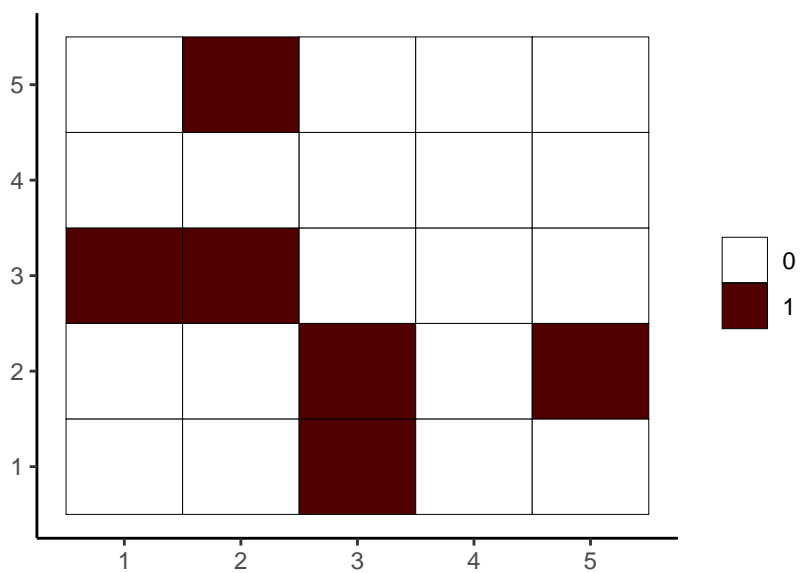
```
##  
## $trial1$M2$unique_graphs[[9]]
```

Graph 9, Individuals 95,...,105

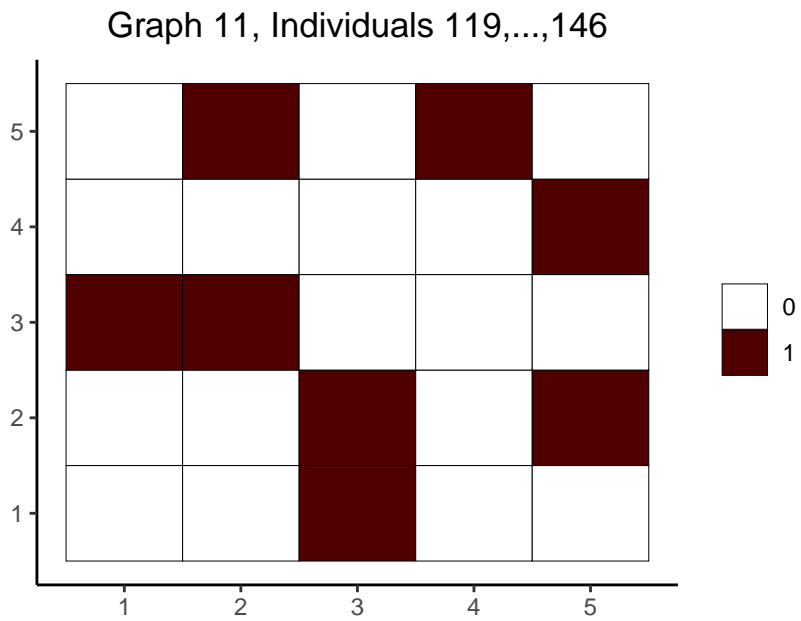


```
##
## $trial1$M2$unique_graphs[[10]]
```

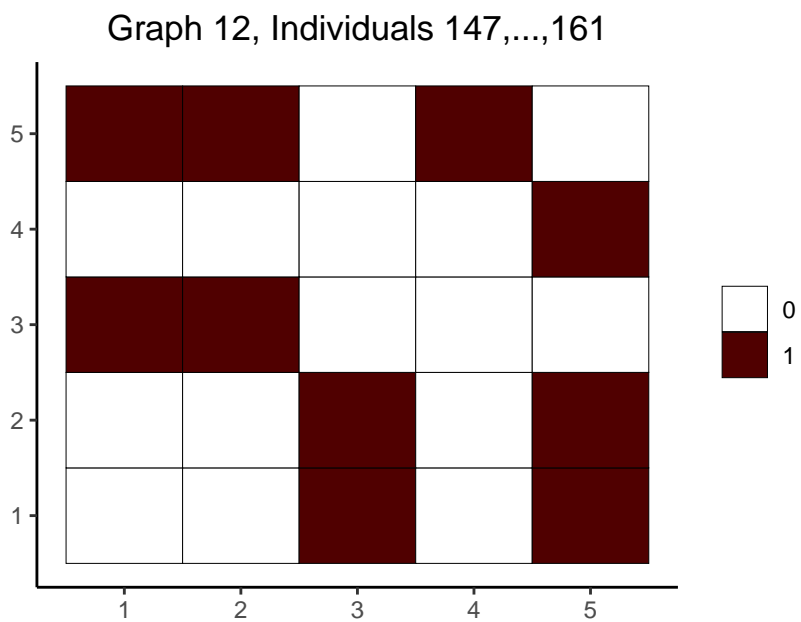
Graph 10, Individuals 106,...,118



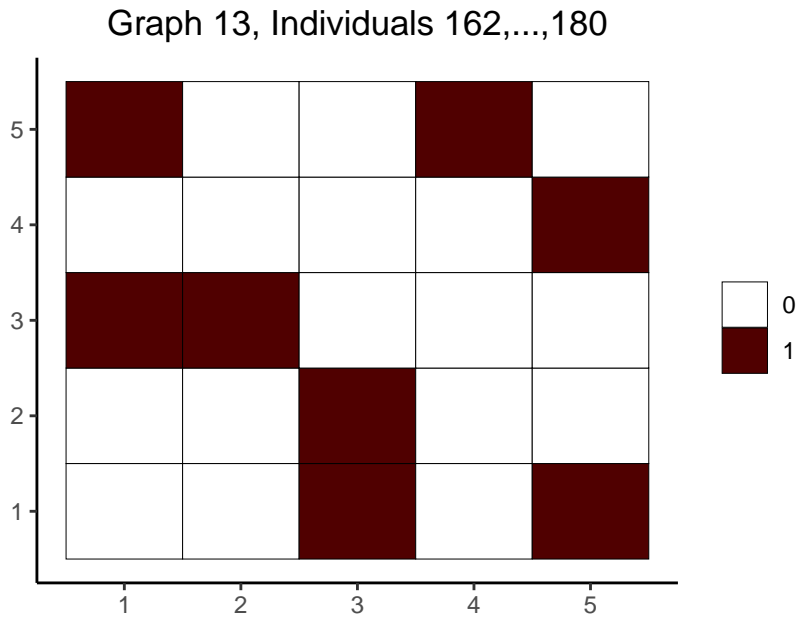
```
##
## $trial1$M2$unique_graphs[[11]]
```



```
##  
## $trial1$M2$unique_graphs[[12]]
```

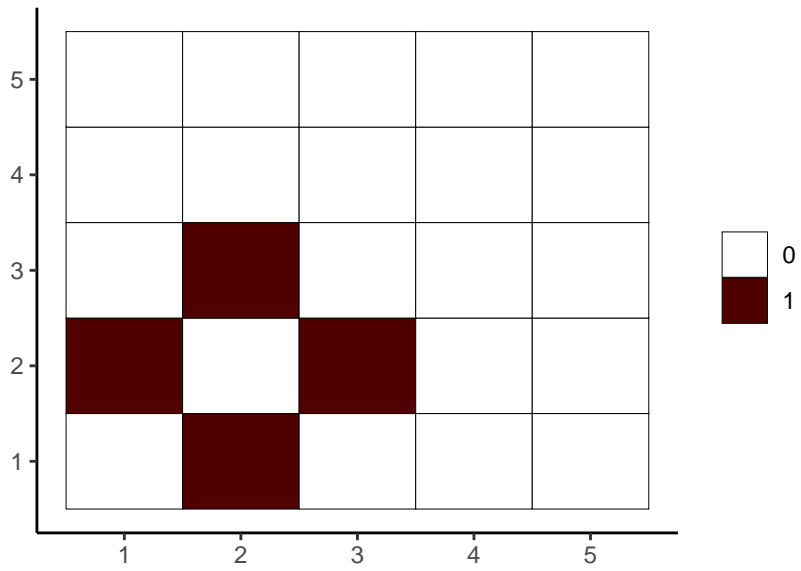


```
##  
## $trial1$M2$unique_graphs[[13]]
```



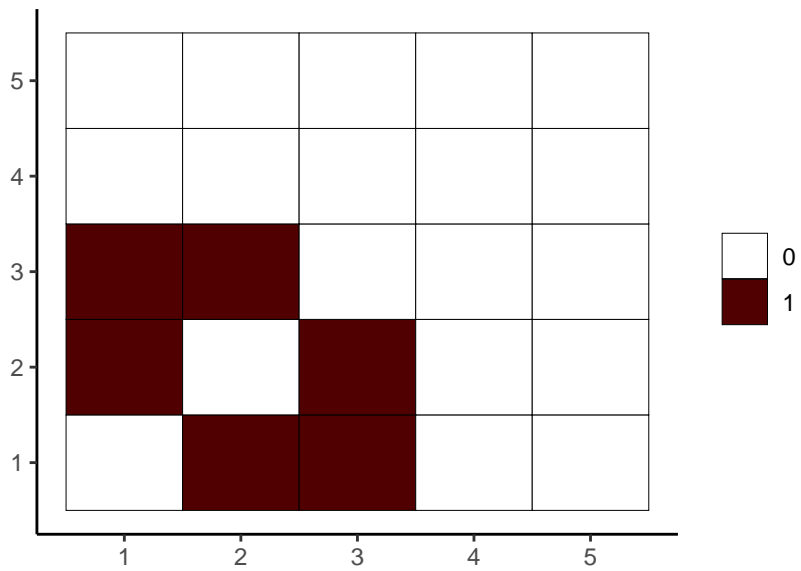
```
##
##
## $trial1$M2$sensitivity
## [1] 0.9380952
##
## $trial1$M2$specificity
## [1] 0.8278689
##
## $trial1$M2$accuracy
## [1] 0.8484444
##
## $trial1$M2$ELBO
## [1] -67017.2
##
## $trial1$M2$time
## [1] 1.105693
##
##
## $trial1$M3
## $trial1$M3$summary
##               Covariate Dependent Graphical Model
##
## Model ELBO: -49121.09           Unique conditional dependence structures: 3
## n: 180, variables: 5           Hyperparameter grid size: 125 points
## CAVI converged for 4/5 variables
##
## Model fit completed in 6.251 secs
##
## $trial1$M3$unique_graphs
## $trial1$M3$unique_graphs[[1]]
```

Graph 1, Individuals 1,...,58



```
##
## $trial1$M3$unique_graphs[[2]]
```

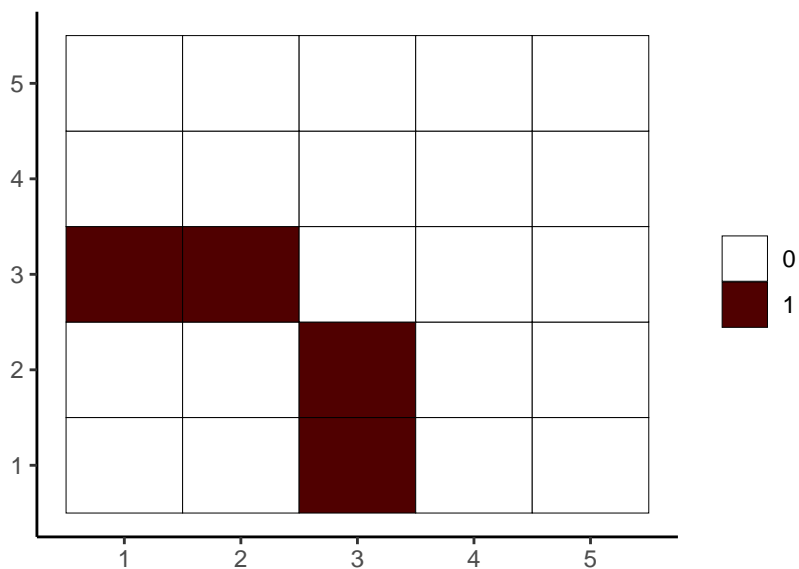
Graph 2, Individuals 59,...,93



```
##
## $trial1$M3$unique_graphs[[3]]
```



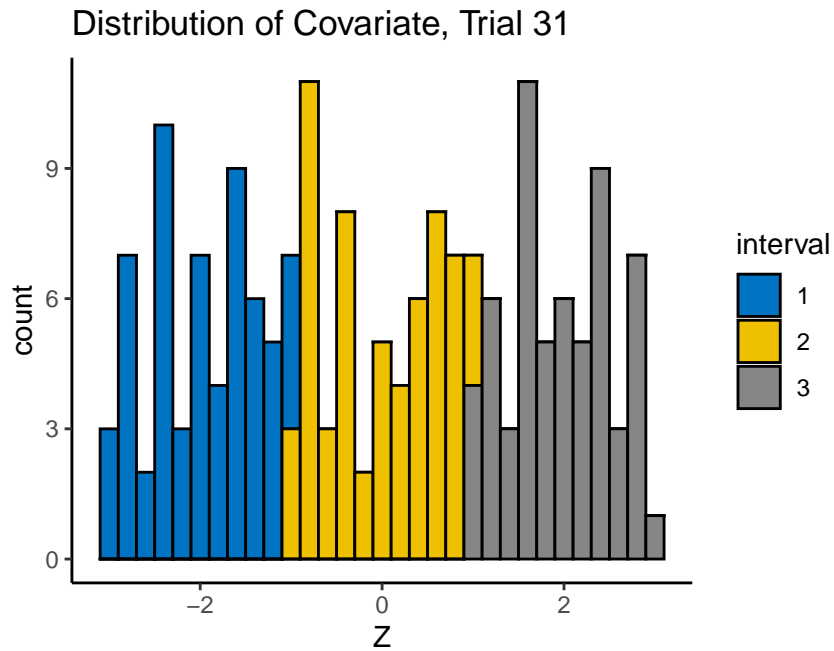
Graph 3, Individuals 94,...,180



```
##
##
## $trial1$M3$sensitivity
## [1] 0.9357143
##
## $trial1$M3$specificity
## [1] 0.9989071
##
## $trial1$M3$accuracy
## [1] 0.9871111
##
## $trial1$M3$ELBO
## [1] -49121.09
##
## $trial1$M3$time
## [1] 6.25109
##
##
## $trial1$error
## NULL
```

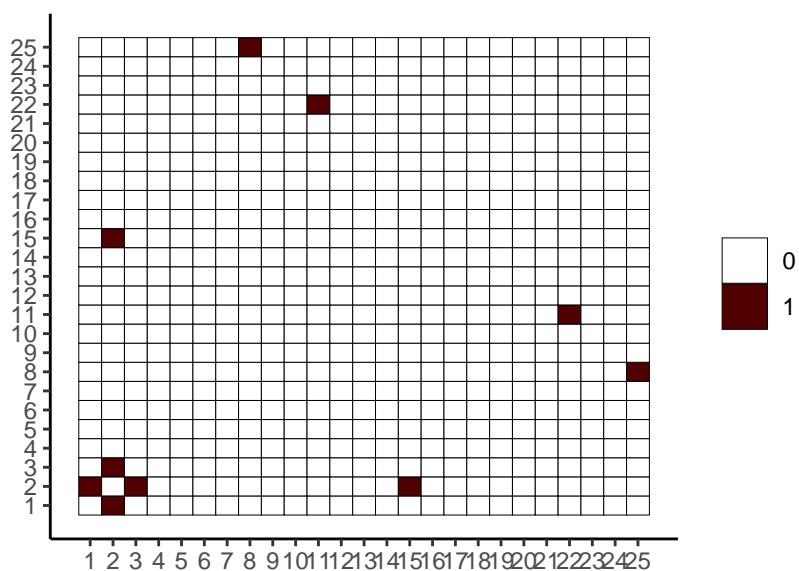
```
# display a single large example
results[!err_log & 1:n_trials > (n_trials - n_large)][1]
```

```
## $trial31
## $trial31$covariates
```



```
##
## $trial31$M1
## $trial31$M1$summary
##           Covariate Dependent Graphical Model
##
## Model ELBO: -417210.07           Unique conditional dependence structures: 11
## n: 180, variables: 25             Hyperparameter grid size: 5 points
## CAVI converged for 18/25 variables
##
## Model fit completed in 24.872 secs
##
## $trial31$M1$unique_graphs
## $trial31$M1$unique_graphs[[1]]
```

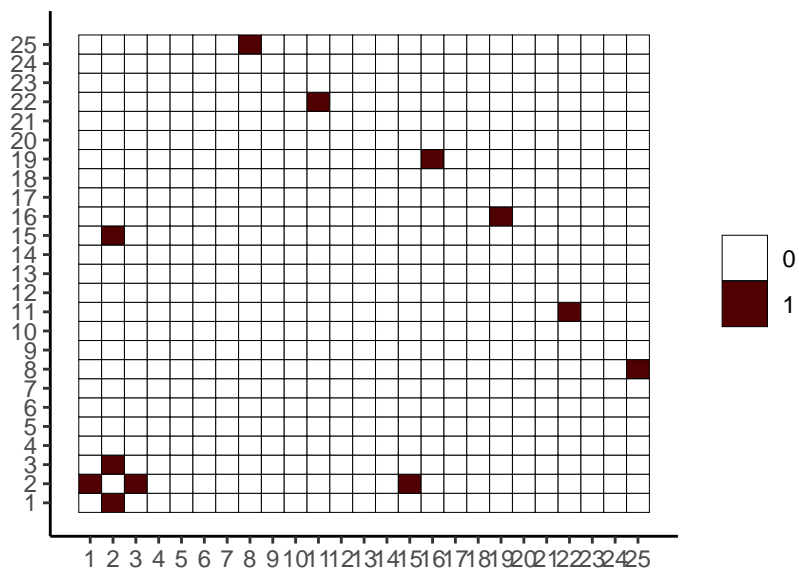
Graph 1, Individuals 1,...,12



##

## \$trial31\$M1\$unique\_graphs[[2]]

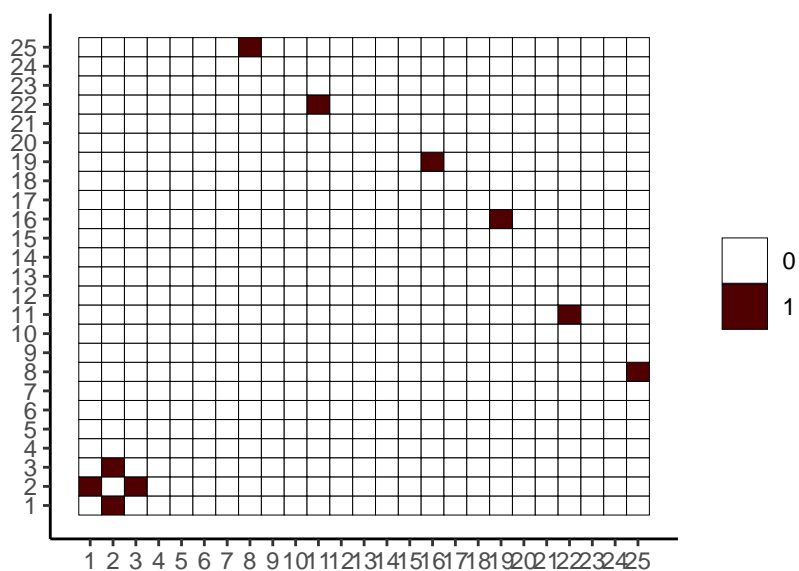
Graph 2, Individuals 13,14



##

## \$trial31\$M1\$unique\_graphs[[3]]

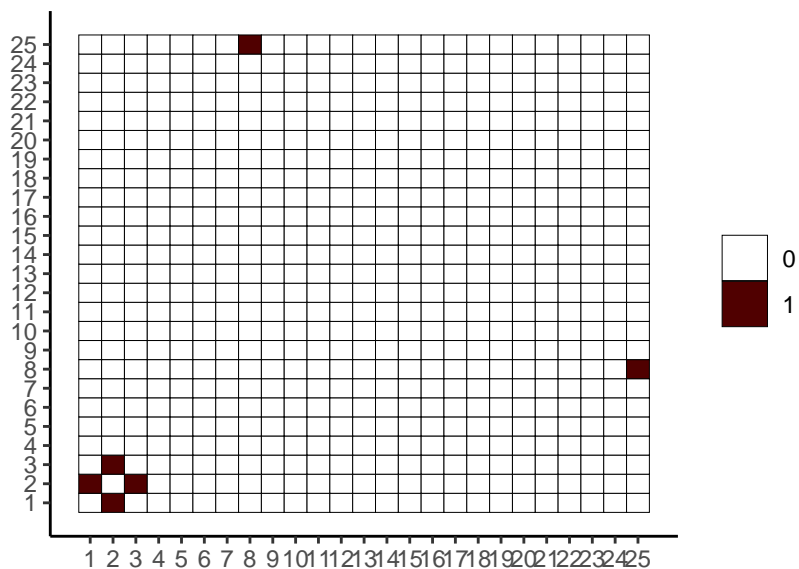
Graph 3, Individuals 15,...,24



##

## \$trial31\$M1\$unique\_graphs[[4]]

Graph 4, Individuals 25,...,33



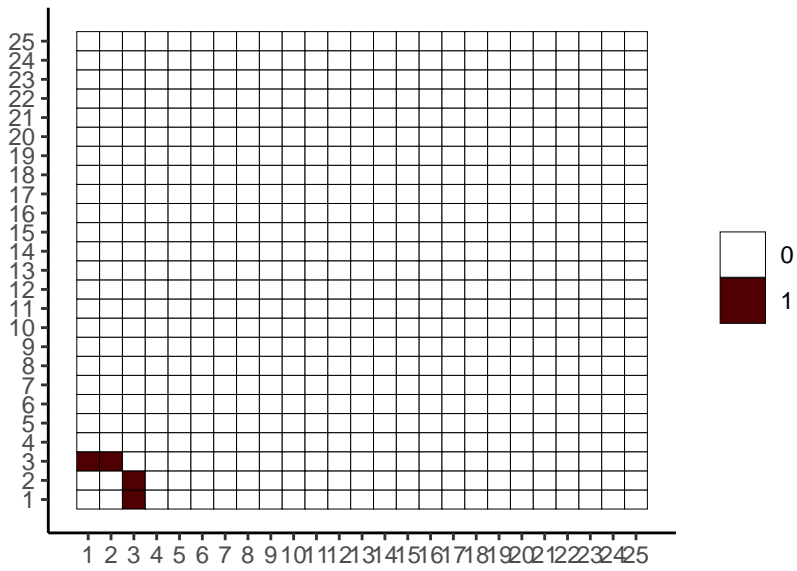
##

## \$trial31\$M1\$unique\_graphs[[5]]

```
##
## $trial31$M1$unique_graphs[[6]]
```

```
##
## $trial31$M1$unique_graphs[[7]]
```

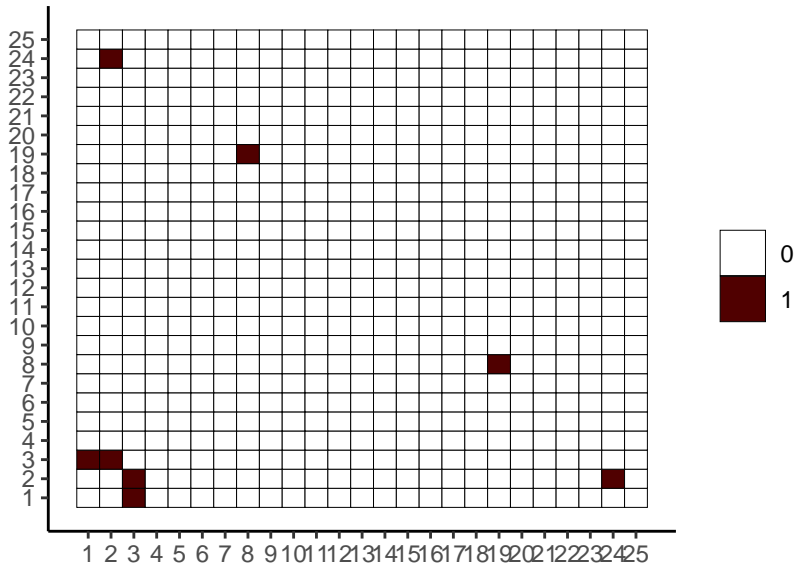
Graph 7, Individuals 109,...,131



##

## \$trial31\$M1\$unique\_graphs[[8]]

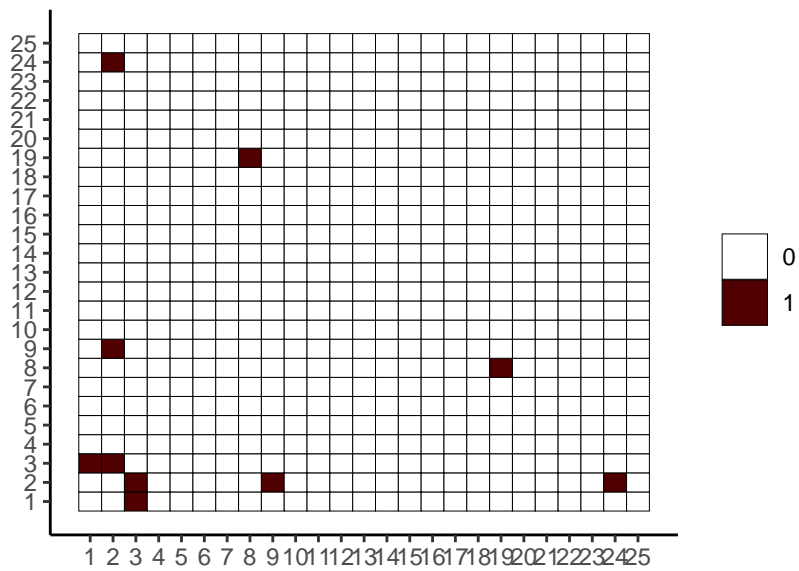
Graph 8, Individuals 132,...,141



##

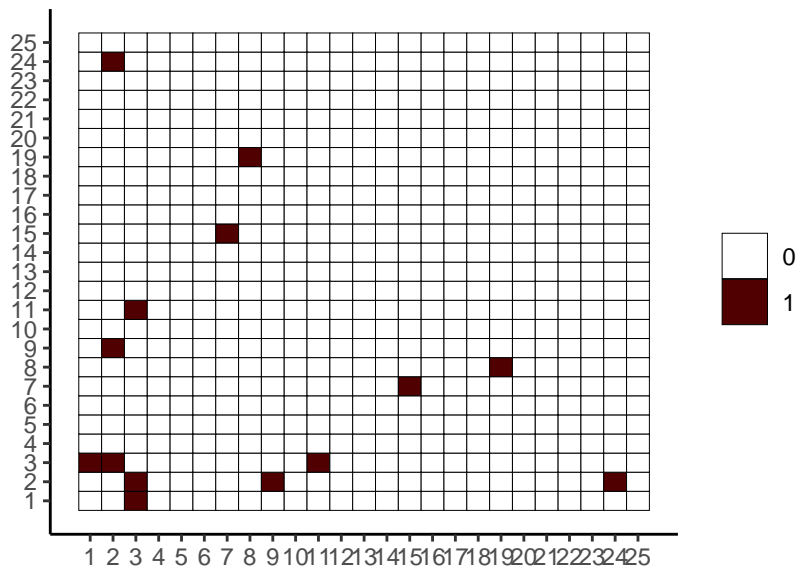
## \$trial31\$M1\$unique\_graphs[[9]]

Graph 9, Individuals 142,...,149



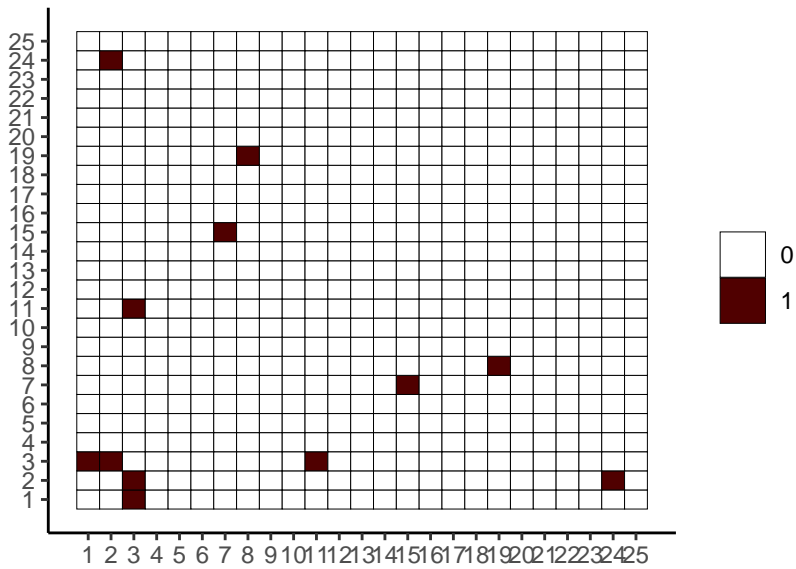
```
##
## $trial31$M1$unique_graphs[[10]]
```

Graph 10, Individuals 150,...,163



```
##
## $trial31$M1$unique_graphs[[11]]
```

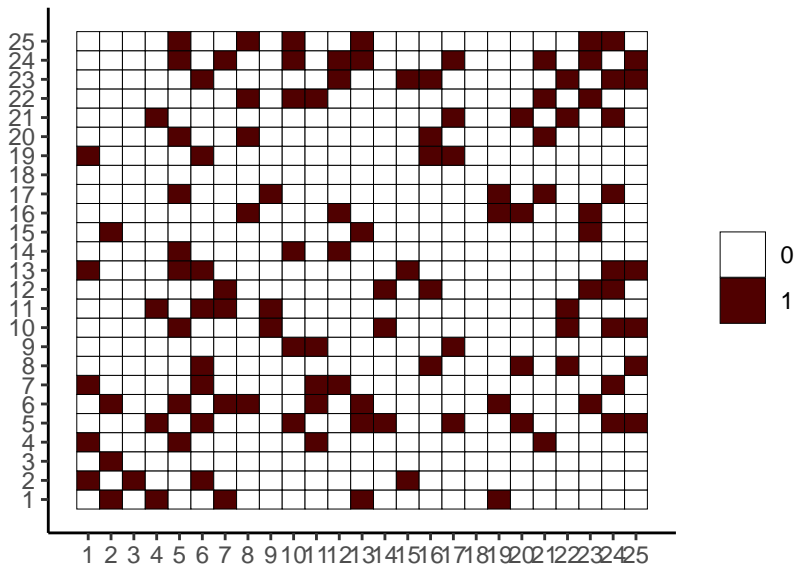
Graph 11, Individuals 164,...,180



```
##
##
## $trial31$M1$sensitivity
## [1] 0.8357143
##
## $trial31$M1$specificity
## [1] 0.9952534
##
## $trial31$M1$accuracy
## [1] 0.9940622
##
## $trial31$M1$ELBO
## [1] -417210.1
##
## $trial31$M1$time
## [1] 24.87153
##
##
## $trial31$M2
## $trial31$M2$summary
##               Covariate Dependent Graphical Model
##
## Model ELBO: -385317.49           Unique conditional dependence structures: 102
## n: 180, variables: 25           Hyperparameter grid size: 5 points
## CAVI converged for 6/25 variables
##
## Model fit completed in 32.559 secs
##
## $trial31$M2$unique_graphs
## $trial31$M2$unique_graphs[[1]]
```

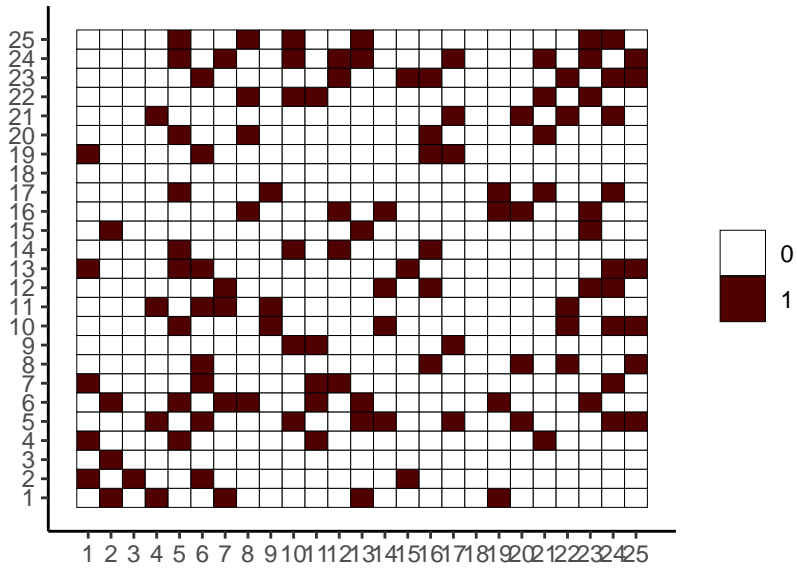


Graph 1, Individuals 1,...,6



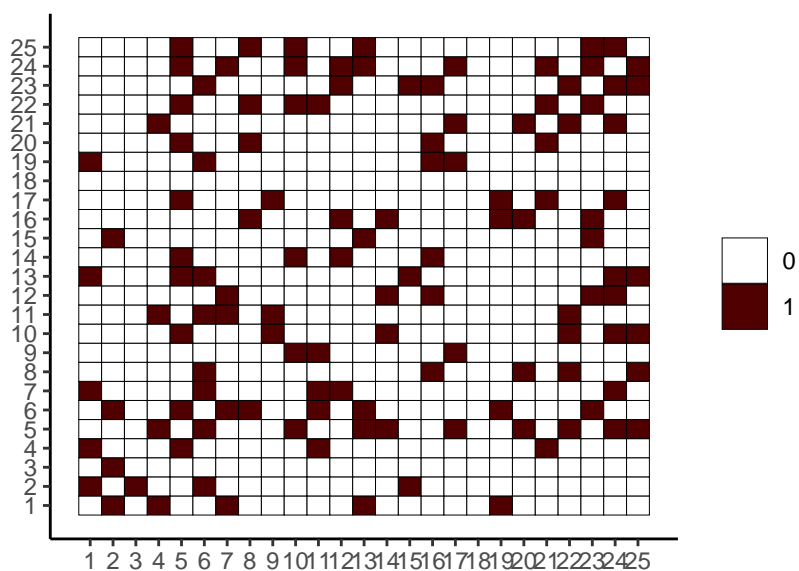
```
##
## $trial31$M2$unique_graphs[[2]]
```

Graph 2, Individuals 7,8



```
##
## $trial31$M2$unique_graphs[[3]]
```

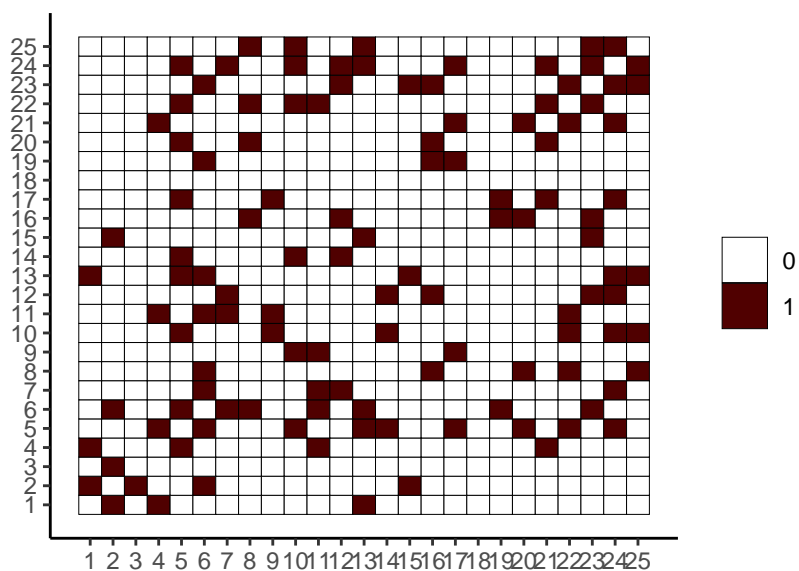
Graph 3, Individuals 9,...,12



##

## \$trial31\$M2\$unique\_graphs[[4]]

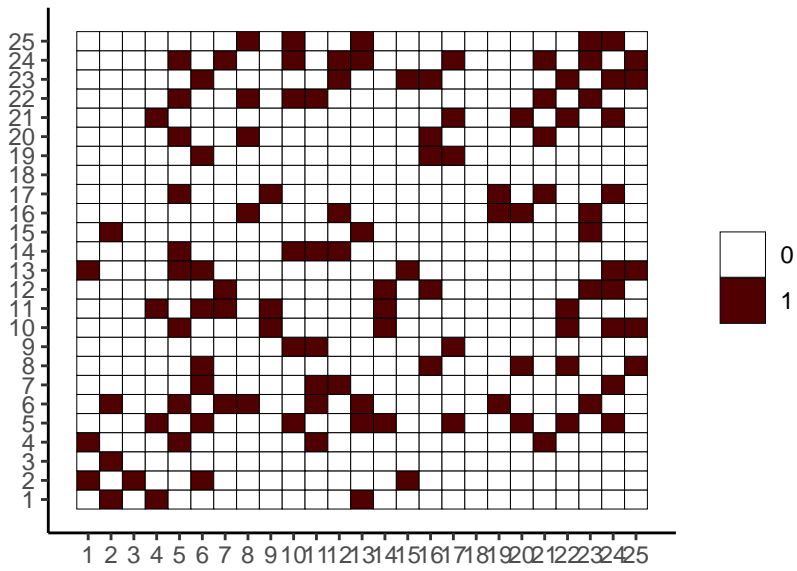
Graph 4, Individuals 13,14



##

## \$trial31\$M2\$unique\_graphs[[5]]

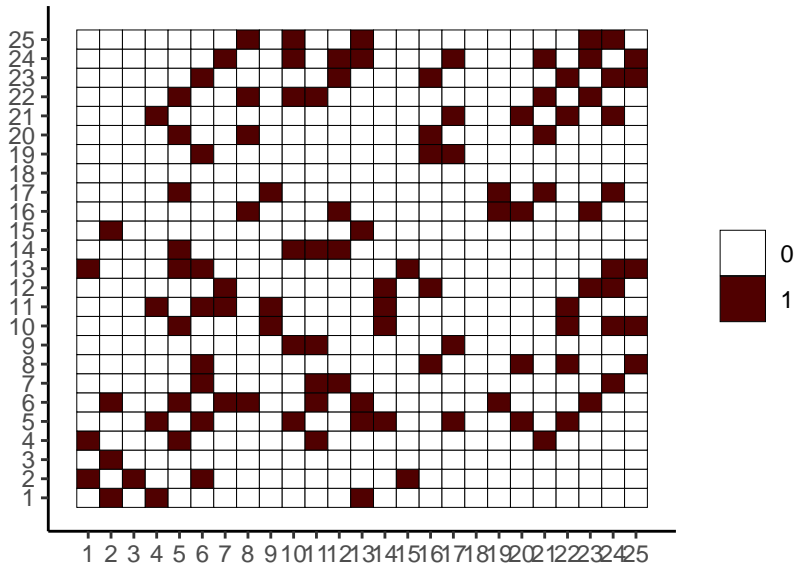
Graph 5, Individuals 15



##

## \$trial31\$M2\$unique\_graphs[[6]]

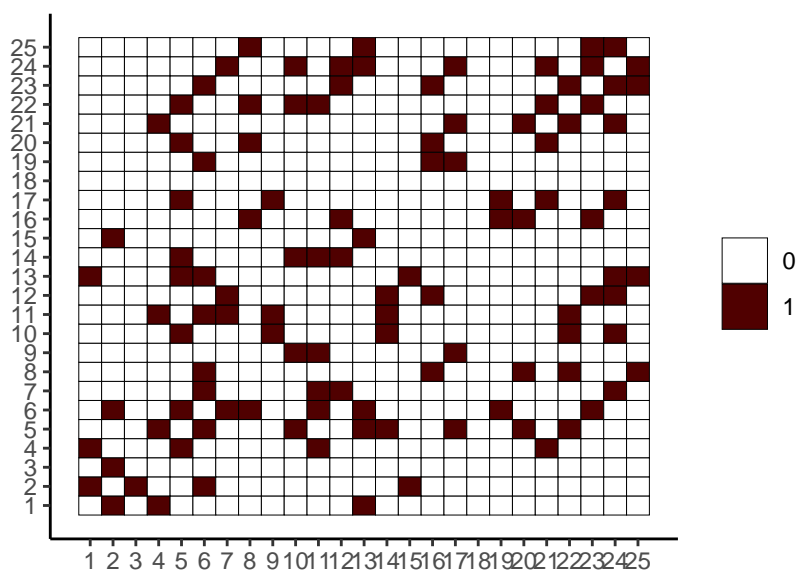
Graph 6, Individuals 16



##

## \$trial31\$M2\$unique\_graphs[[7]]

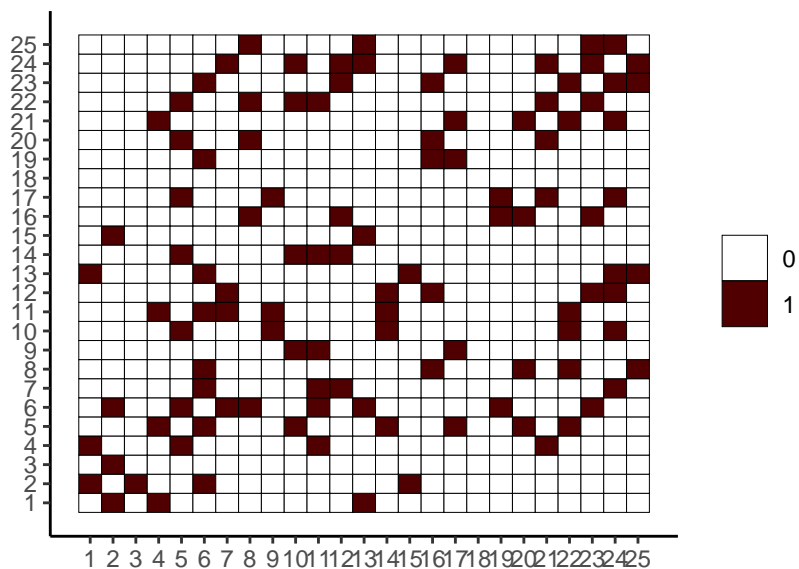
Graph 7, Individuals 17,...,19



##

## \$trial31\$M2\$unique\_graphs[[8]]

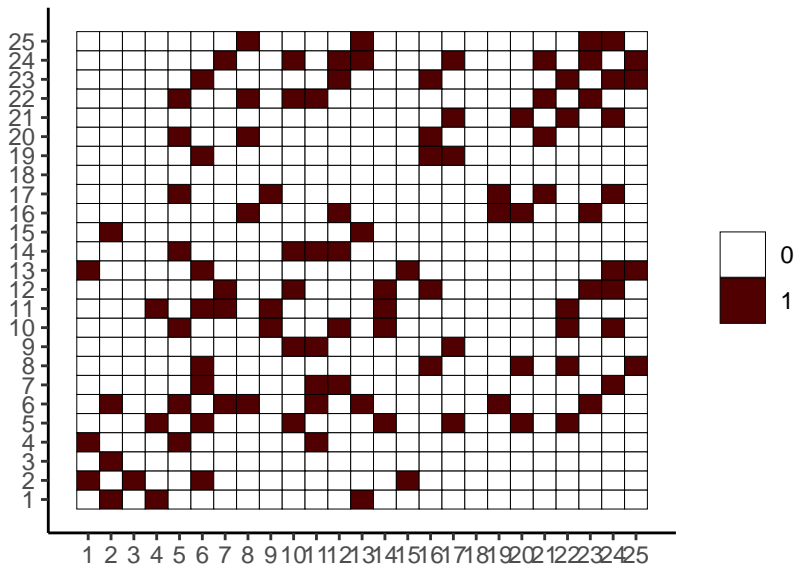
Graph 8, Individuals 20,...,22



##

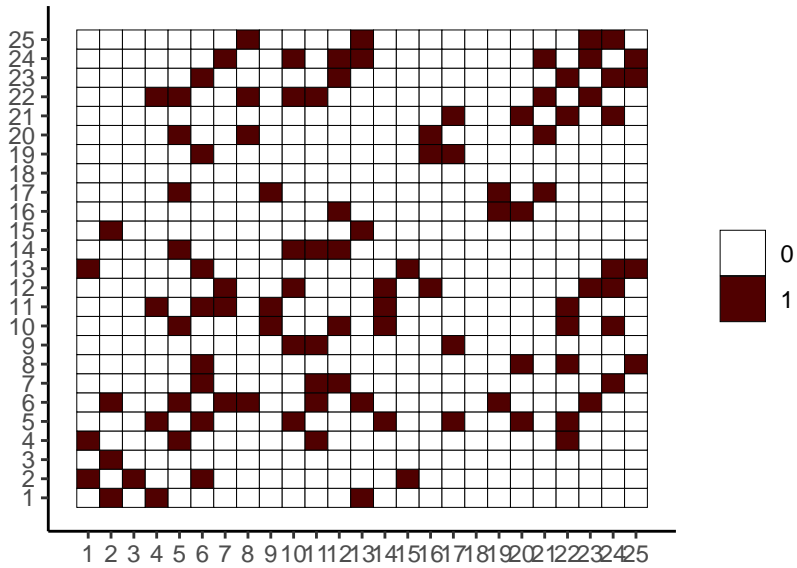
## \$trial31\$M2\$unique\_graphs[[9]]

Graph 9, Individuals 23,24



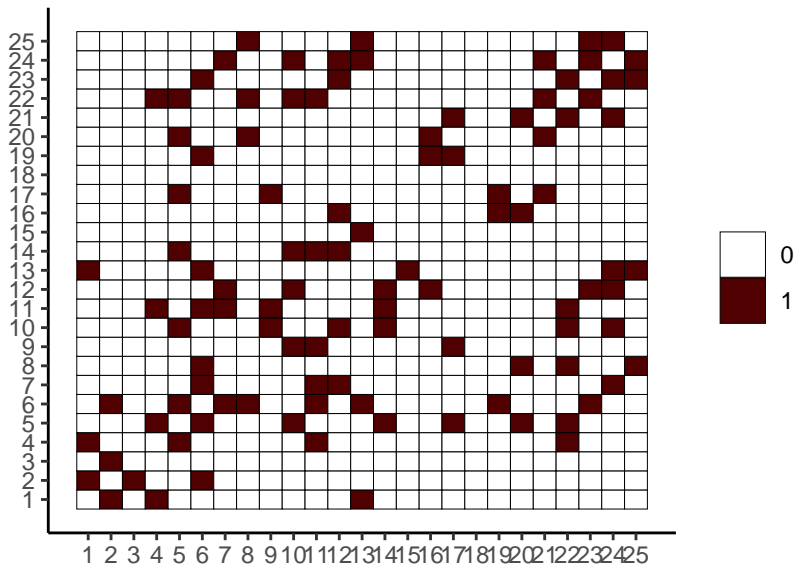
```
##
## $trial31$M2$unique_graphs[[10]]
```

Graph 10, Individuals 25



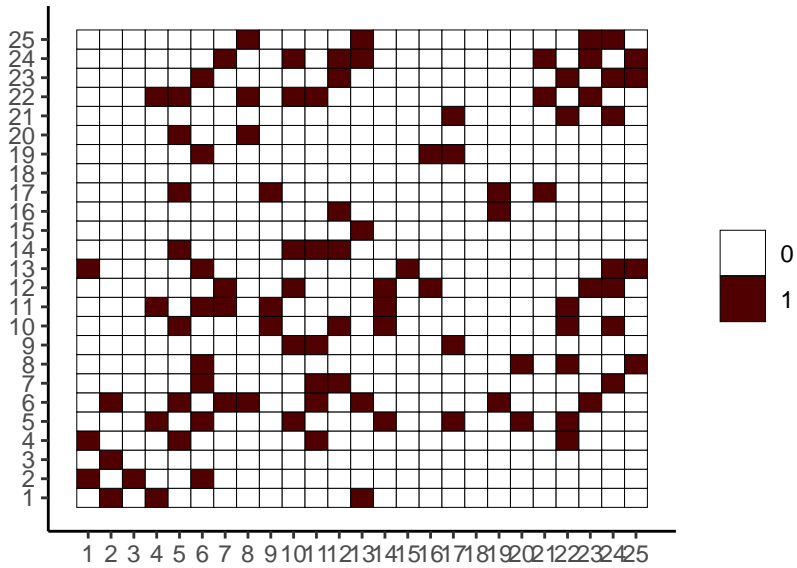
```
##
## $trial31$M2$unique_graphs[[11]]
```

Graph 11, Individuals 26,...,28



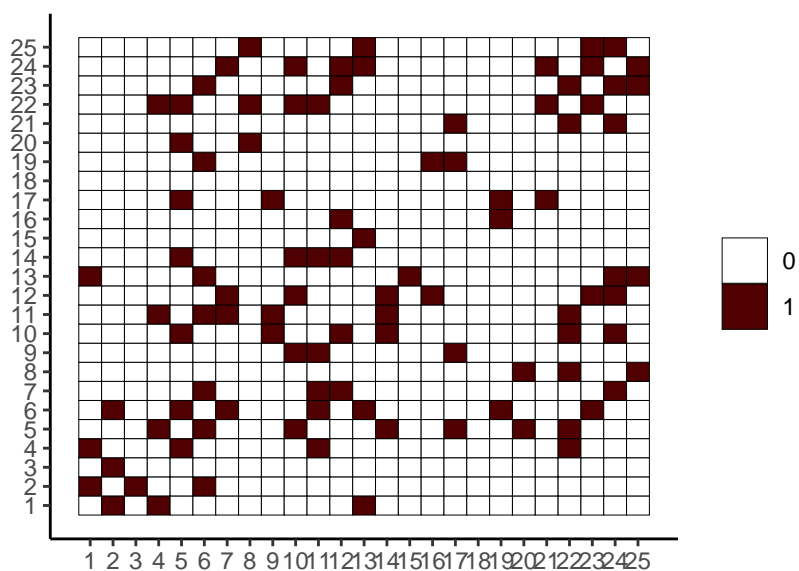
```
##
## $trial31$M2$unique_graphs[[12]]
```

Graph 12, Individuals 29,...,31



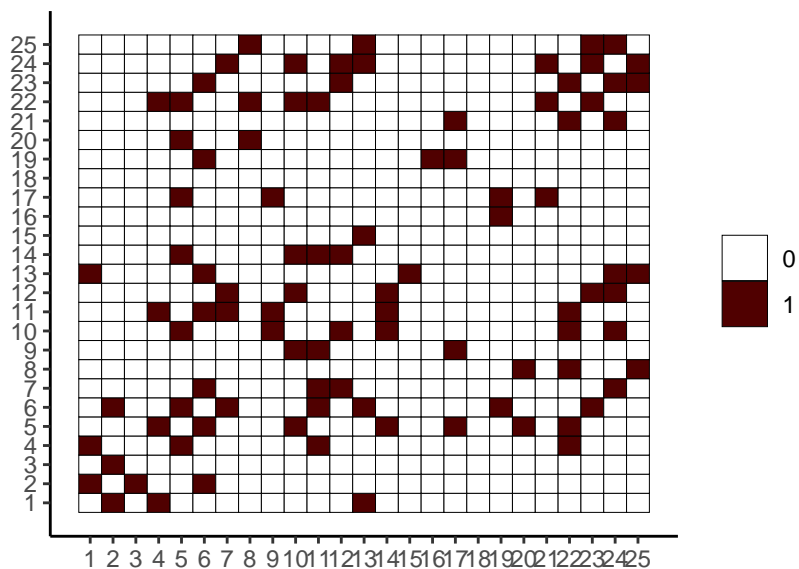
```
##
## $trial31$M2$unique_graphs[[13]]
```

Graph 13, Individuals 32



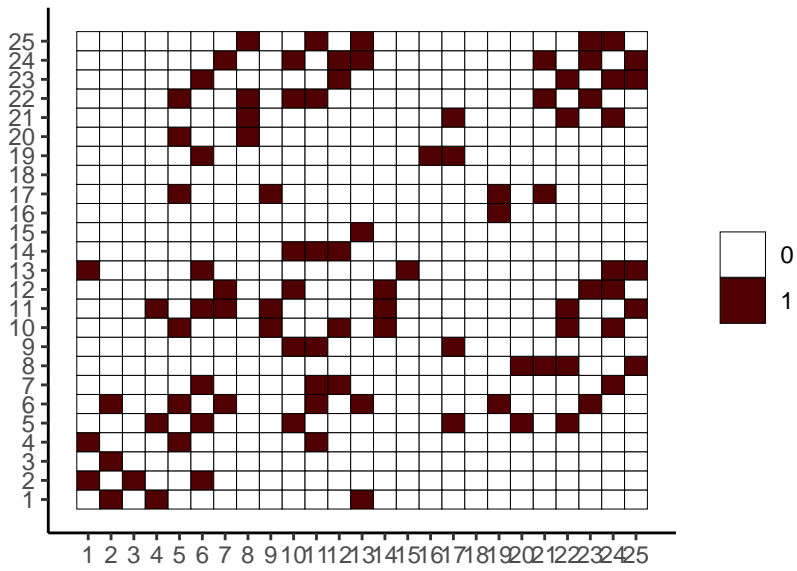
```
##
## $trial31$M2$unique_graphs[[14]]
```

Graph 14, Individuals 33



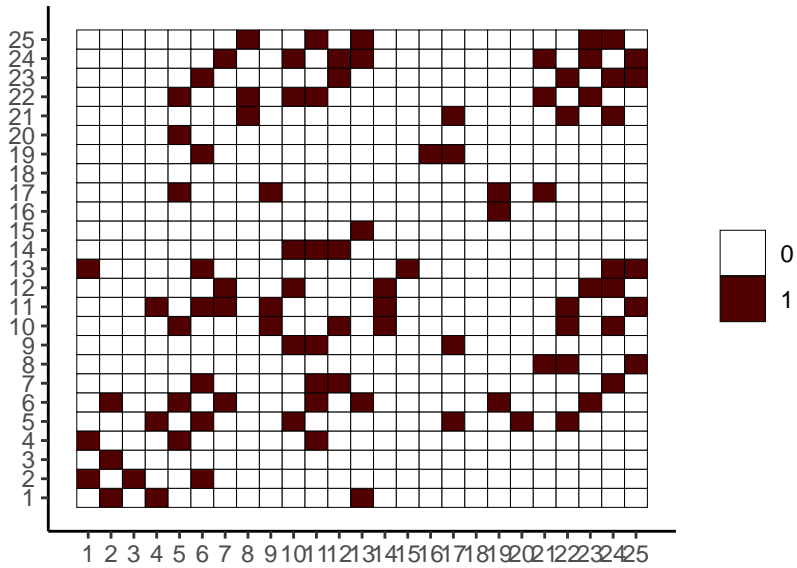
```
##
## $trial31$M2$unique_graphs[[15]]
```

Graph 15, Individuals 34,...,36



```
##
## $trial31$M2$unique_graphs[[16]]
```

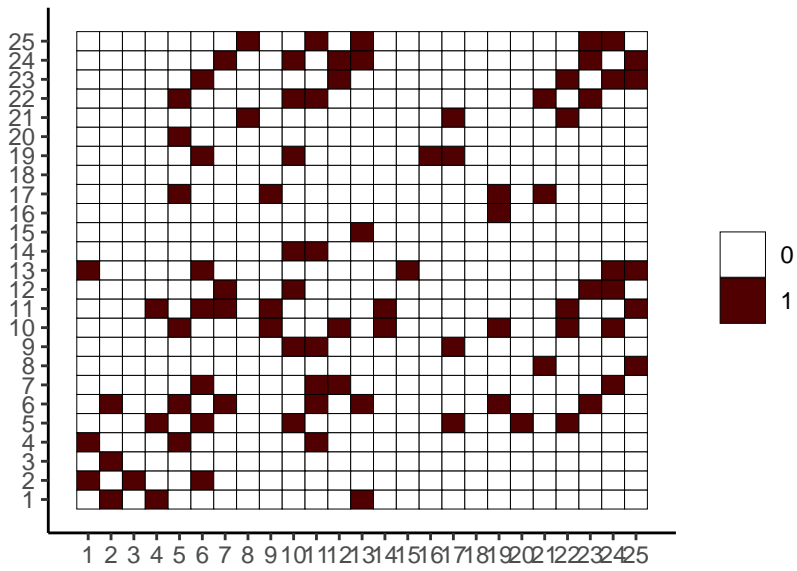
Graph 16, Individuals 37



```
##
## $trial31$M2$unique_graphs[[17]]
```

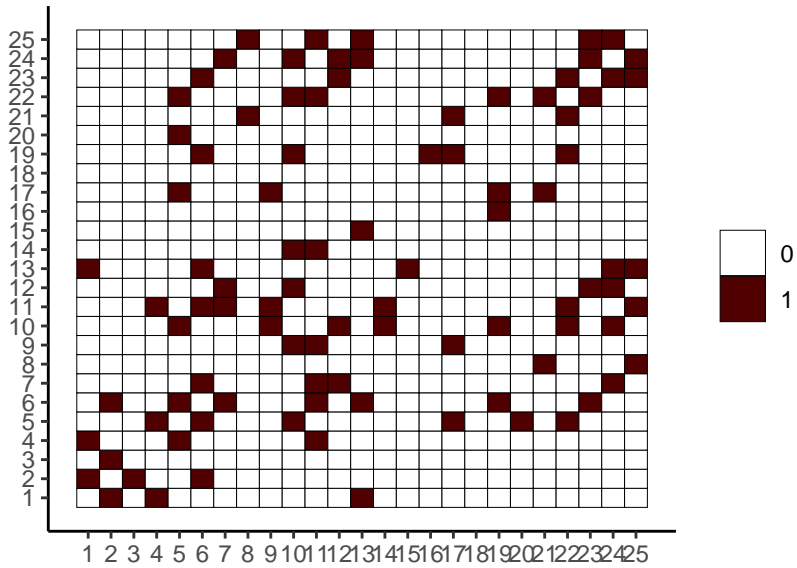


Graph 17, Individuals 38,...,40



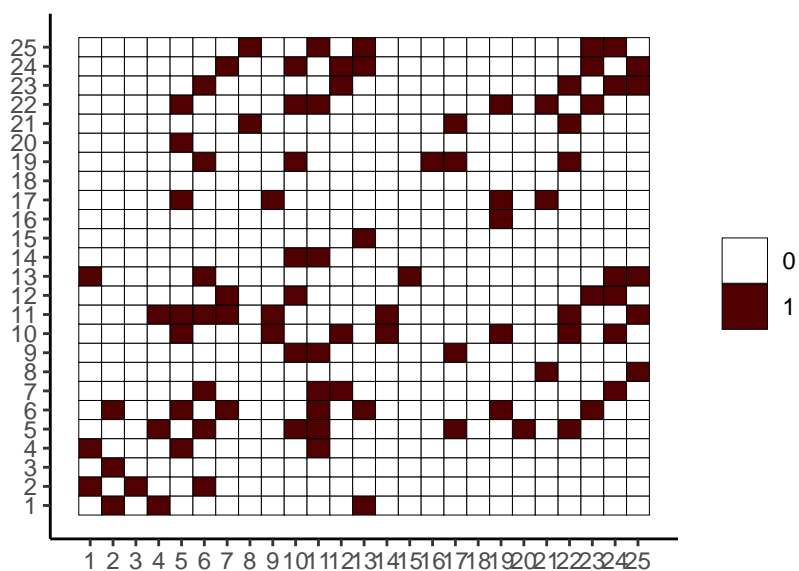
```
##
## $trial31$M2$unique_graphs[[18]]
```

Graph 18, Individuals 41,42



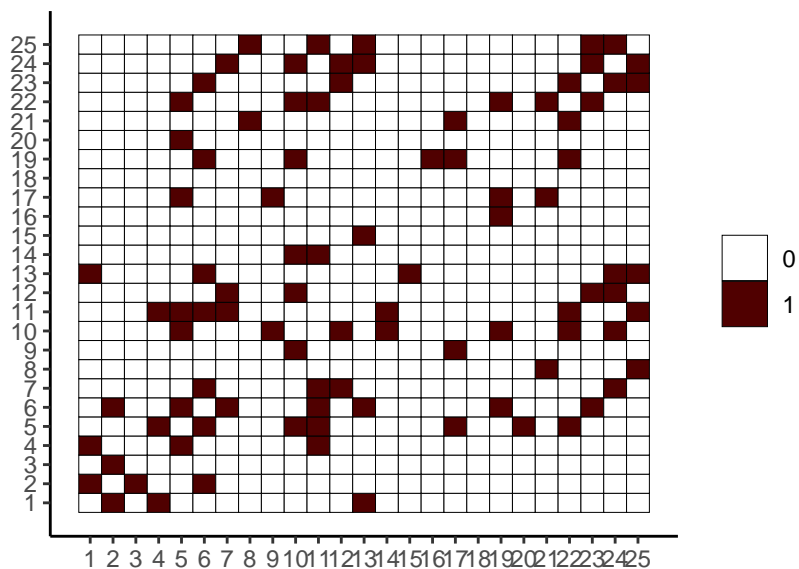
```
##
## $trial31$M2$unique_graphs[[19]]
```

Graph 19, Individuals 43,44



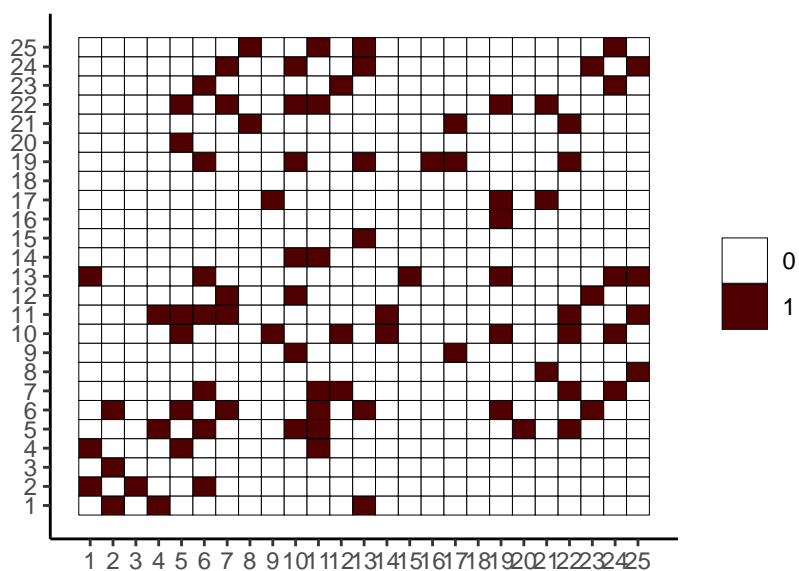
```
##
## $trial31$M2$unique_graphs[[20]]
```

Graph 20, Individuals 45



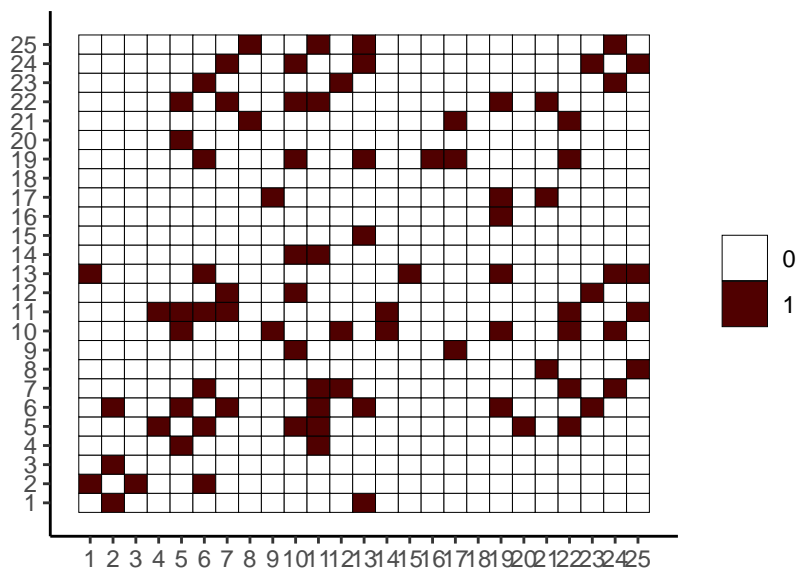
```
##
## $trial31$M2$unique_graphs[[21]]
```

Graph 21, Individuals 46



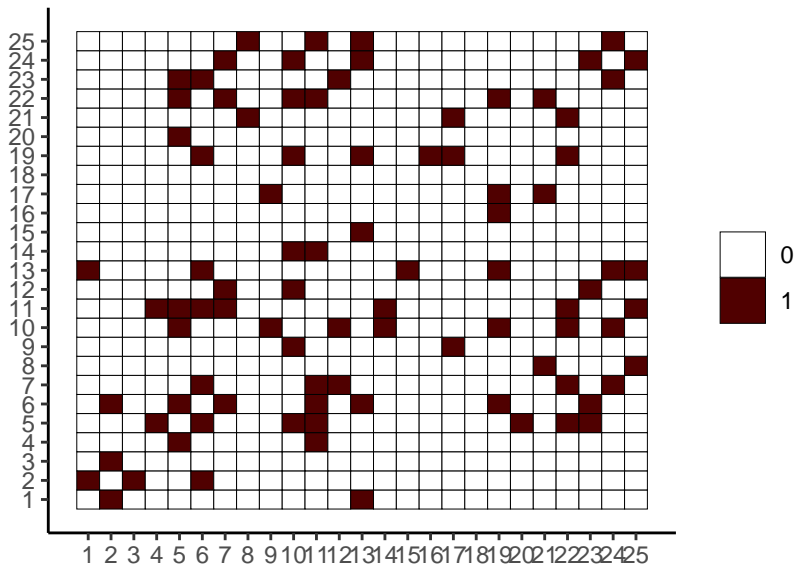
```
##
## $trial31$M2$unique_graphs[[22]]
```

Graph 22, Individuals 47,48



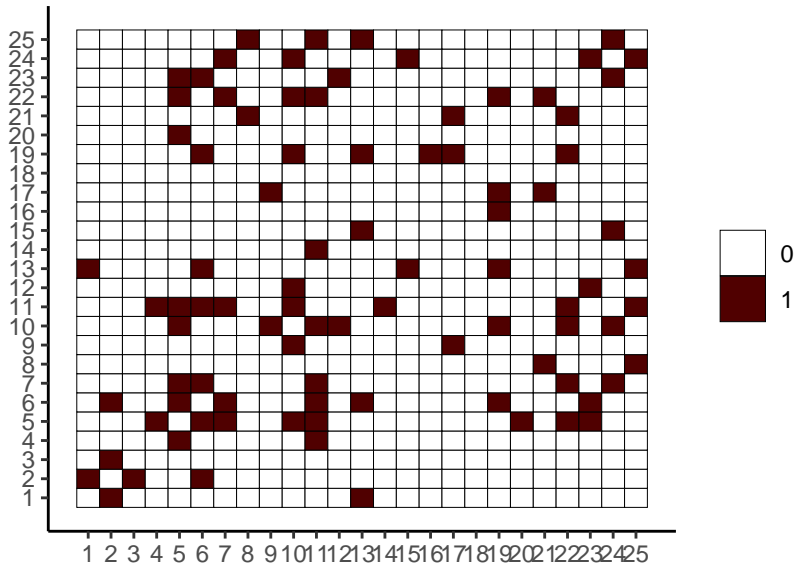
```
##
## $trial31$M2$unique_graphs[[23]]
```

Graph 23, Individuals 49,...,51



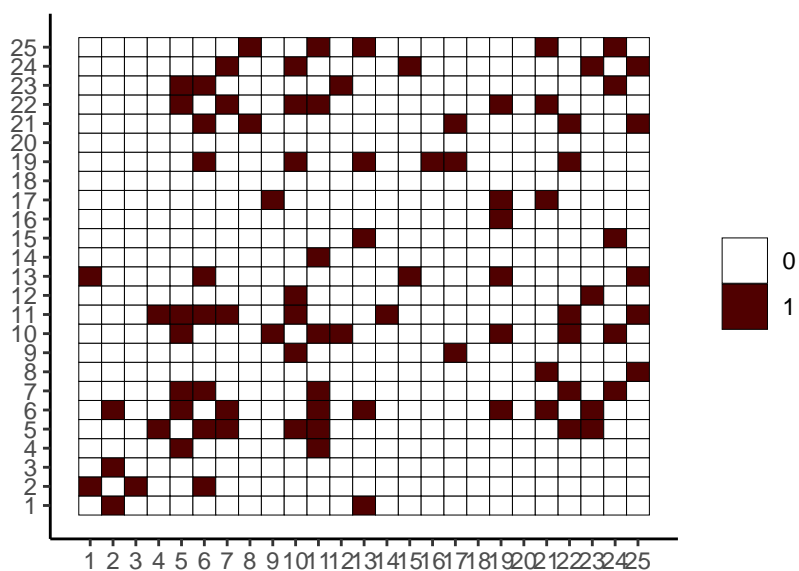
```
##
## $trial31$M2$unique_graphs[[24]]
```

Graph 24, Individuals 52,...,54



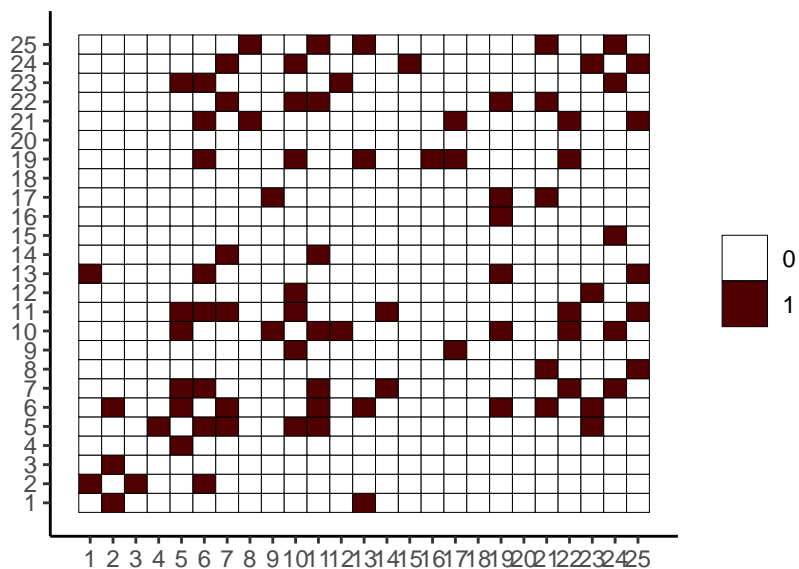
```
##
## $trial31$M2$unique_graphs[[25]]
```

Graph 25, Individuals 55,56



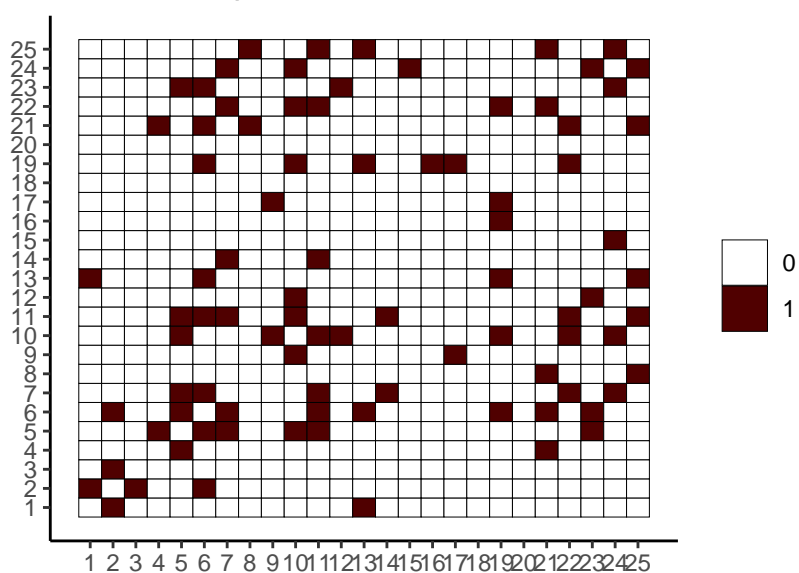
```
##
## $trial31$M2$unique_graphs[[26]]
```

Graph 26, Individuals 57,...,59



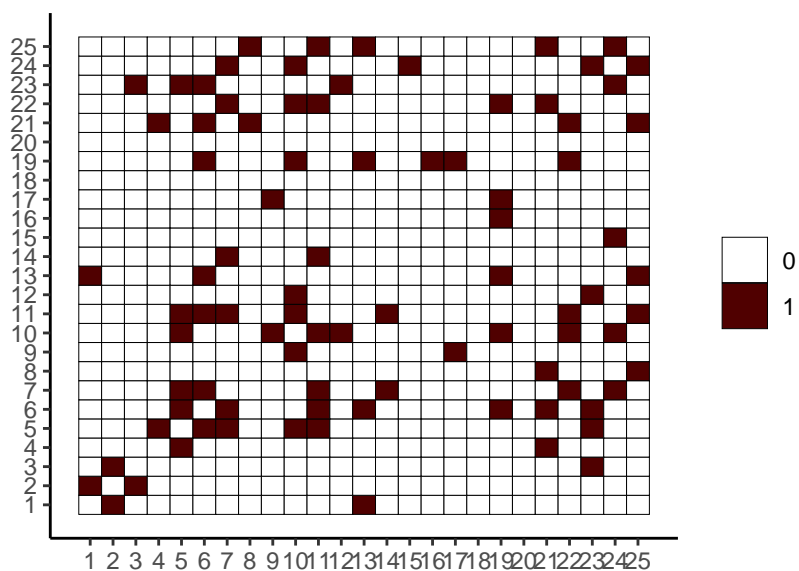
```
##
## $trial31$M2$unique_graphs[[27]]
```

Graph 27, Individuals 60



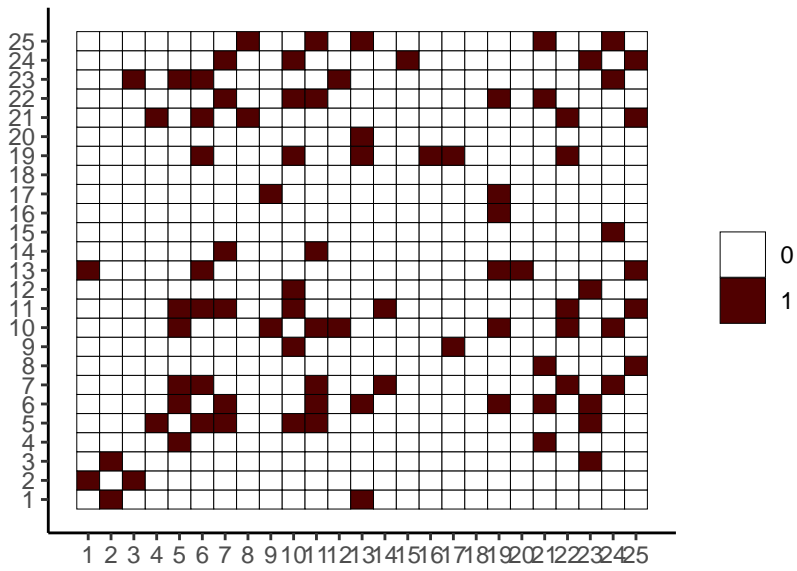
```
##
## $trial31$M2$unique_graphs[[28]]
```

Graph 28, Individuals 61,62



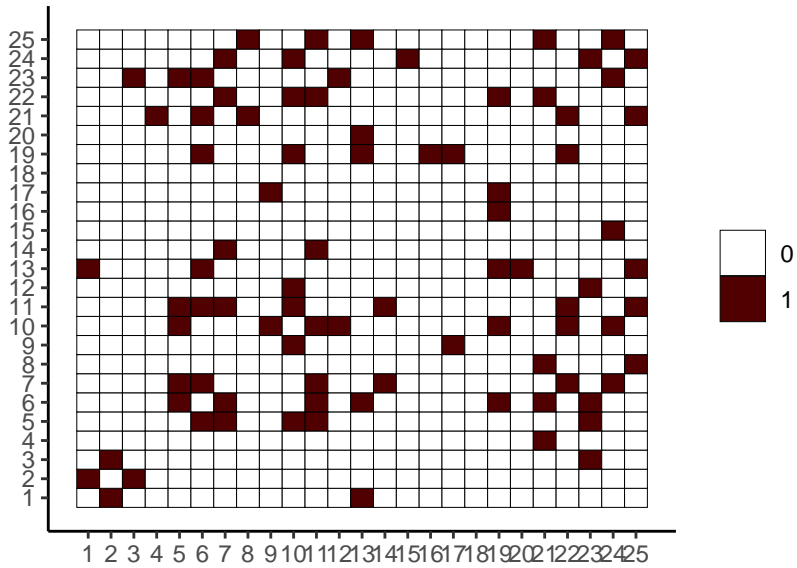
```
##
## $trial31$M2$unique_graphs[[29]]
```

Graph 29, Individuals 63,...,65



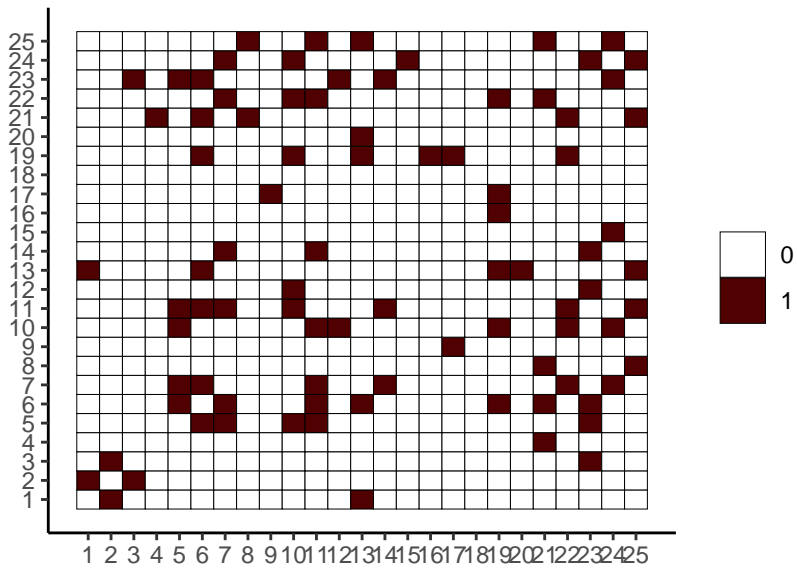
```
##
## $trial31$M2$unique_graphs[[30]]
```

Graph 30, Individuals 66,67



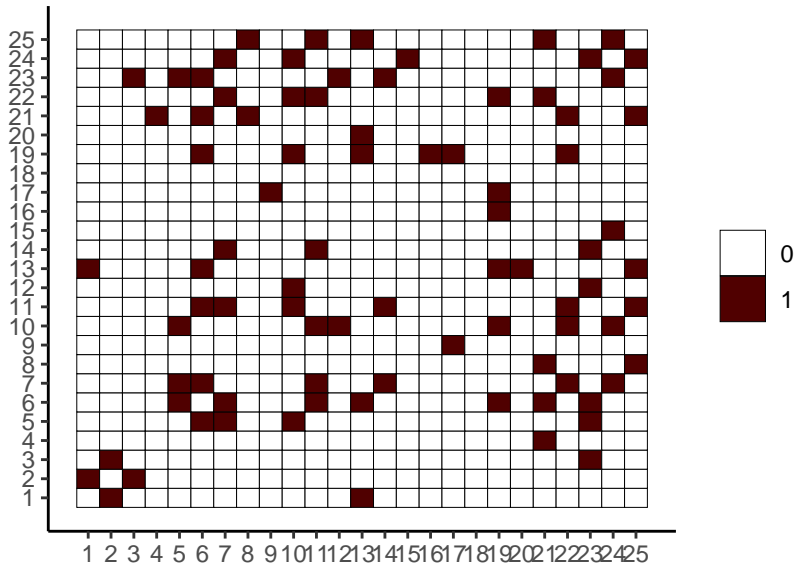
```
##
## $trial31$M2$unique_graphs[[31]]
```

Graph 31, Individuals 68



```
##
## $trial31$M2$unique_graphs[[32]]
```

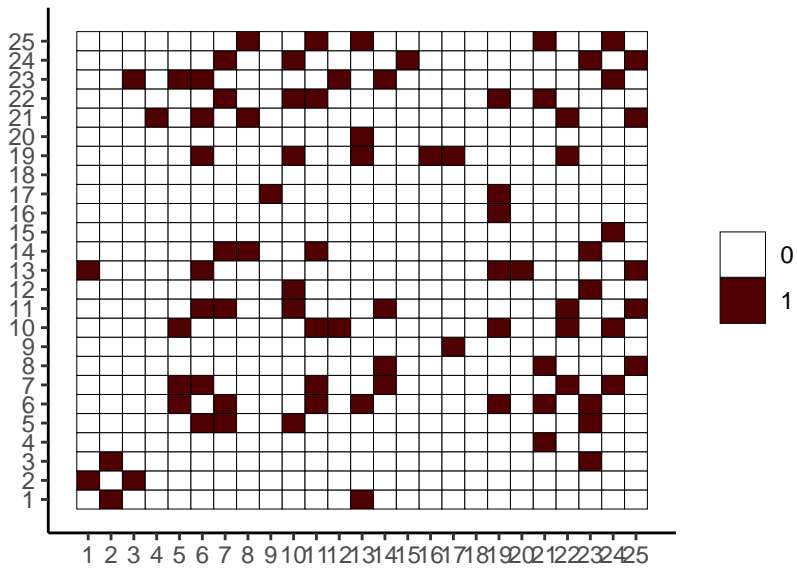
Graph 32, Individuals 69



```
##
## $trial31$M2$unique_graphs[[33]]
```

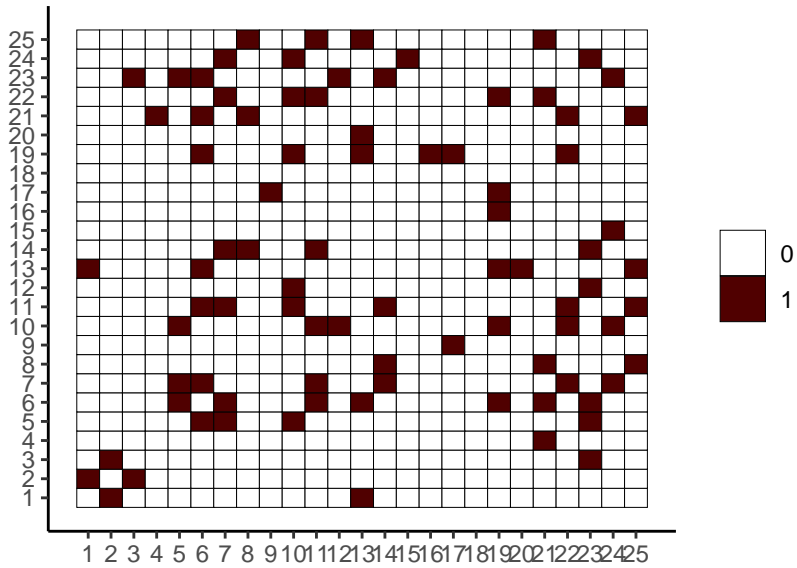


Graph 33, Individuals 70,...,72



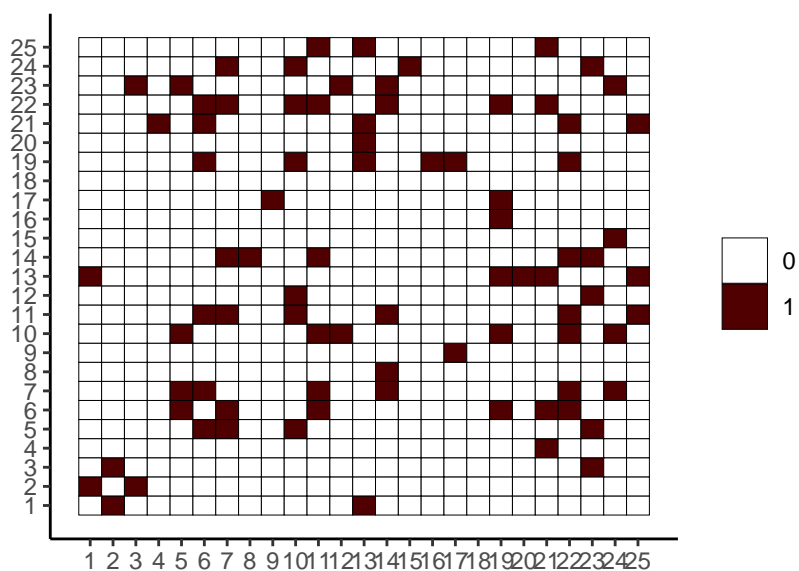
```
##
## $trial31$M2$unique_graphs[[34]]
```

Graph 34, Individuals 73,...,76



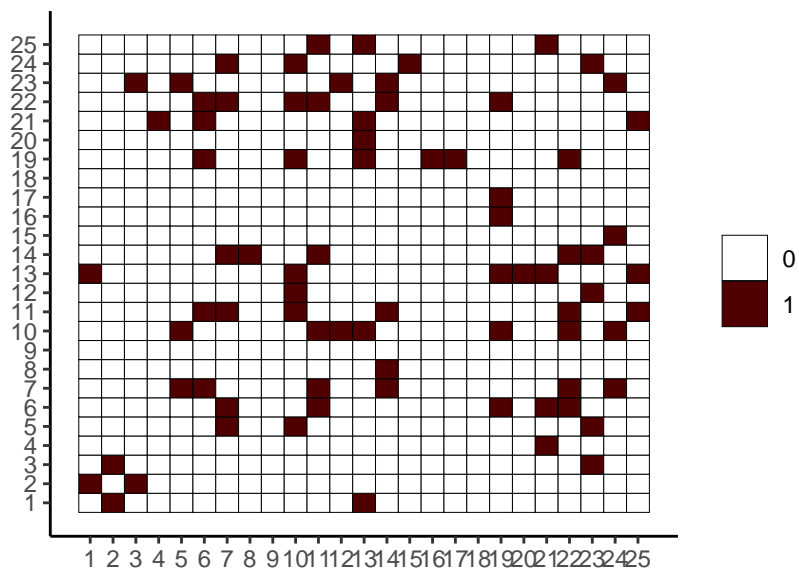
```
##
## $trial31$M2$unique_graphs[[35]]
```

Graph 35, Individuals 77



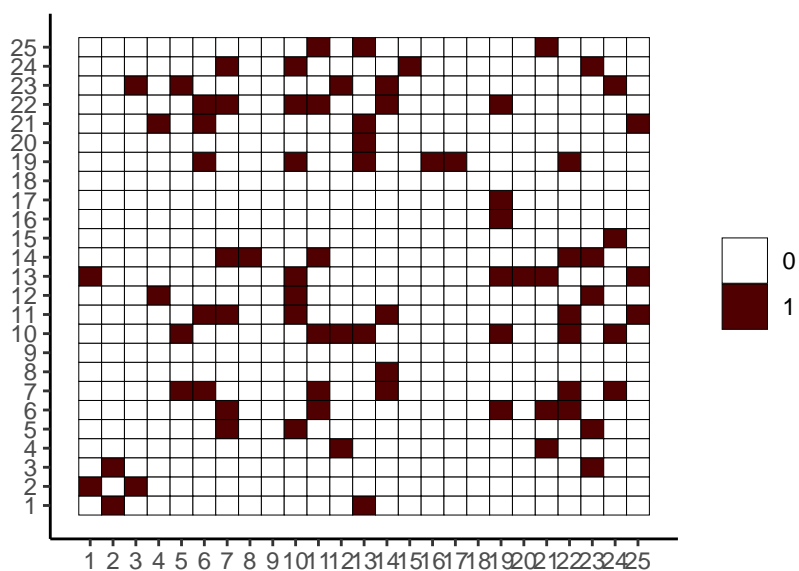
```
##
## $trial31$M2$unique_graphs[[36]]
```

Graph 36, Individuals 78,...,81



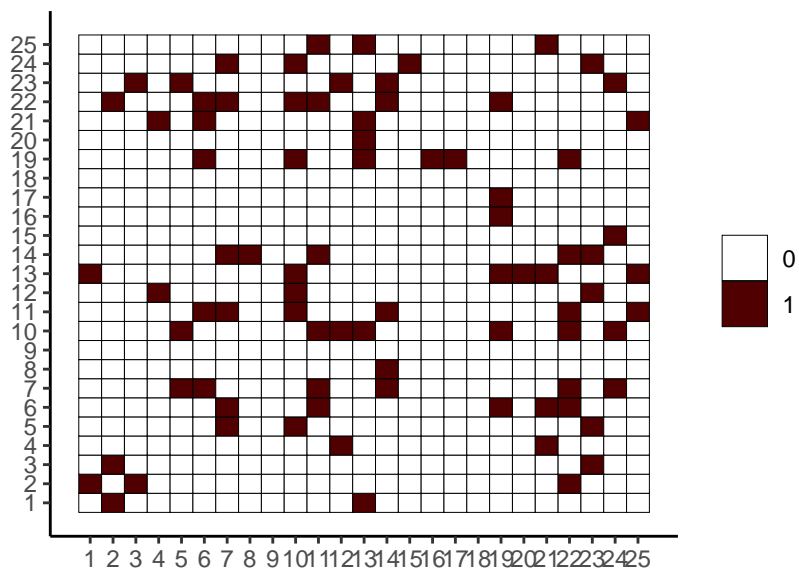
```
##
## $trial31$M2$unique_graphs[[37]]
```

Graph 37, Individuals 82



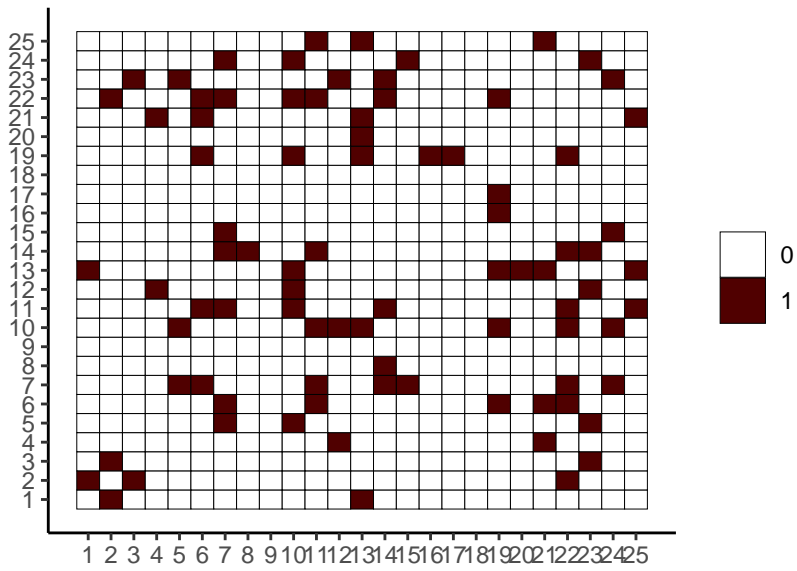
```
##
## $trial31$M2$unique_graphs[[38]]
```

Graph 38, Individuals 83



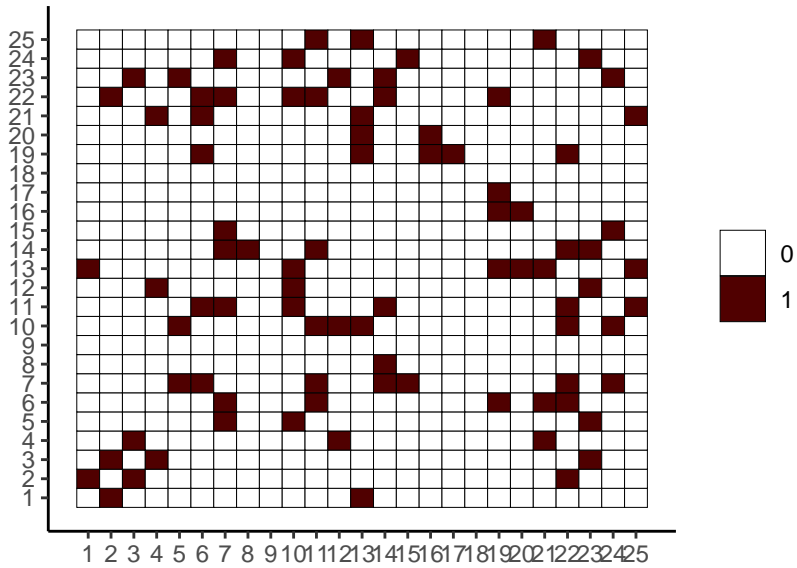
```
##
## $trial31$M2$unique_graphs[[39]]
```

Graph 39, Individuals 84



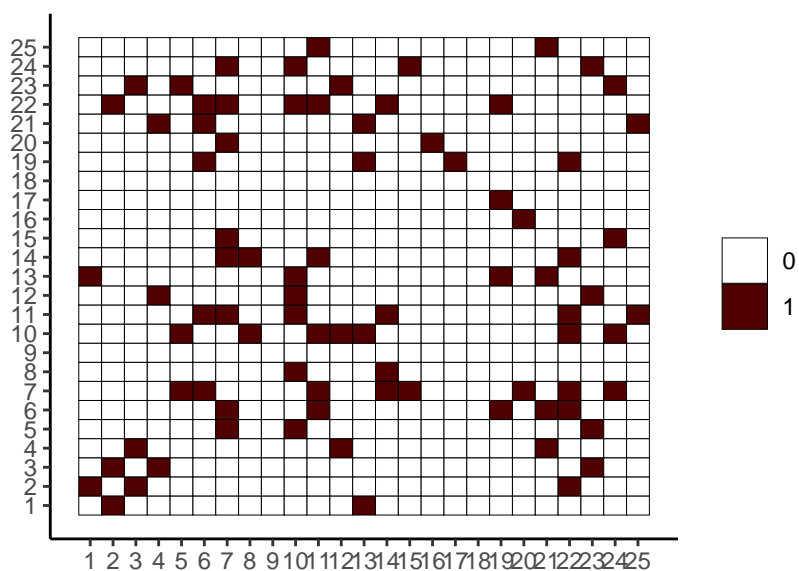
```
##
## $trial31$M2$unique_graphs[[40]]
```

Graph 40, Individuals 85



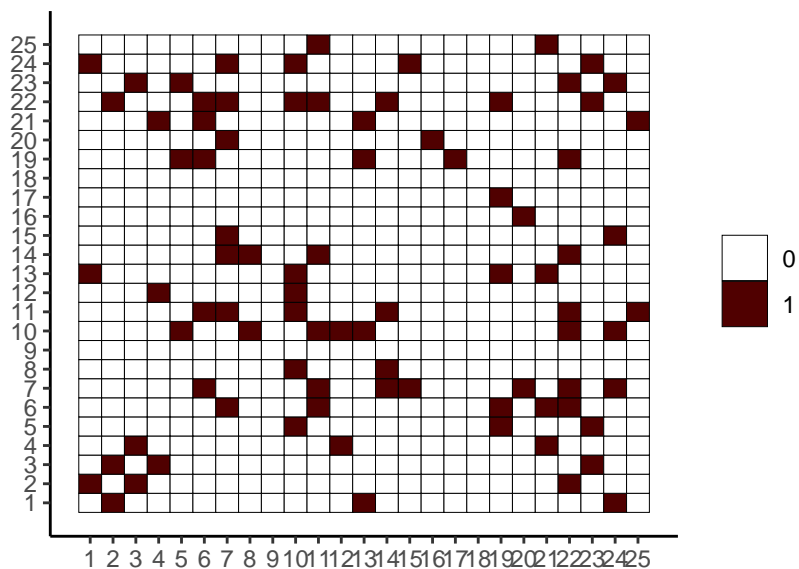
```
##
## $trial31$M2$unique_graphs[[41]]
```

Graph 41, Individuals 86



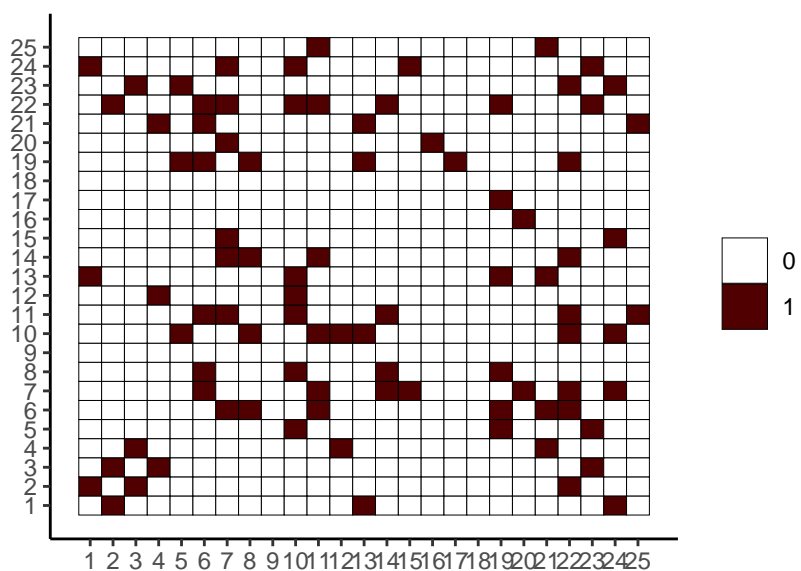
```
##
## $trial31$M2$unique_graphs[[42]]
```

Graph 42, Individuals 87,...,89



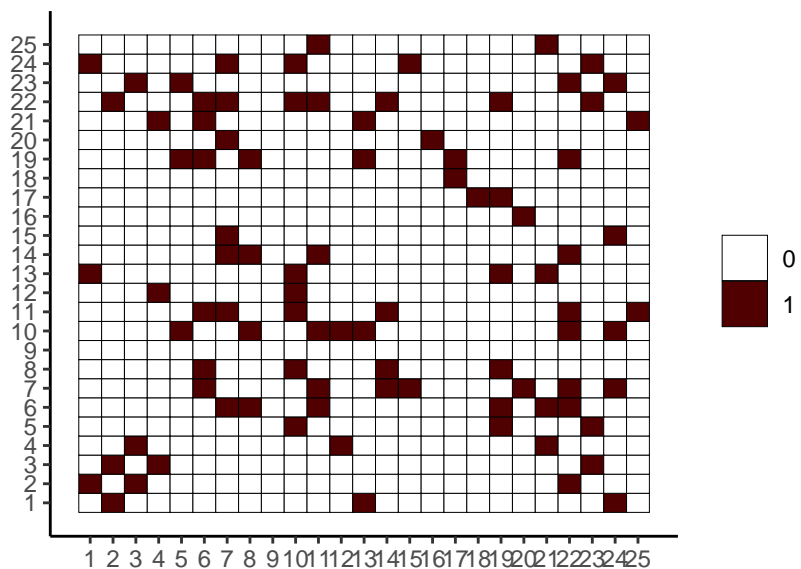
```
##
## $trial31$M2$unique_graphs[[43]]
```

Graph 43, Individuals 90



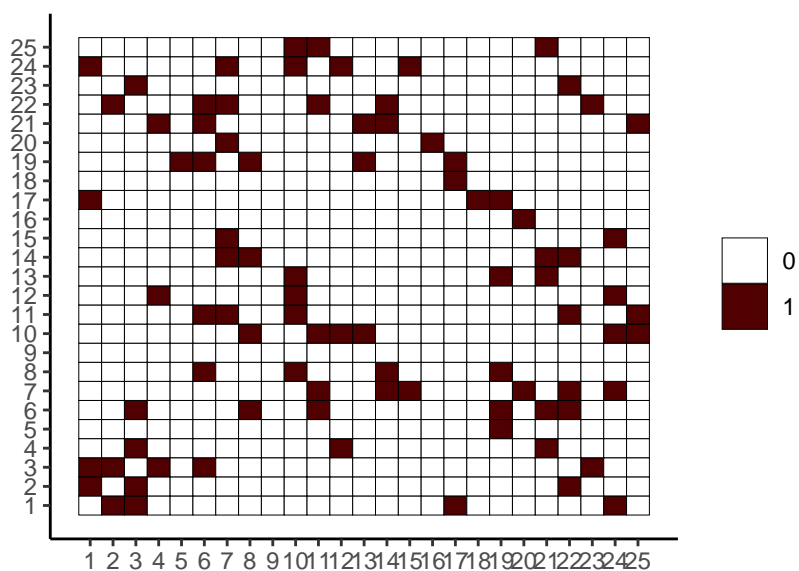
```
##
## $trial31$M2$unique_graphs[[44]]
```

Graph 44, Individuals 91,92



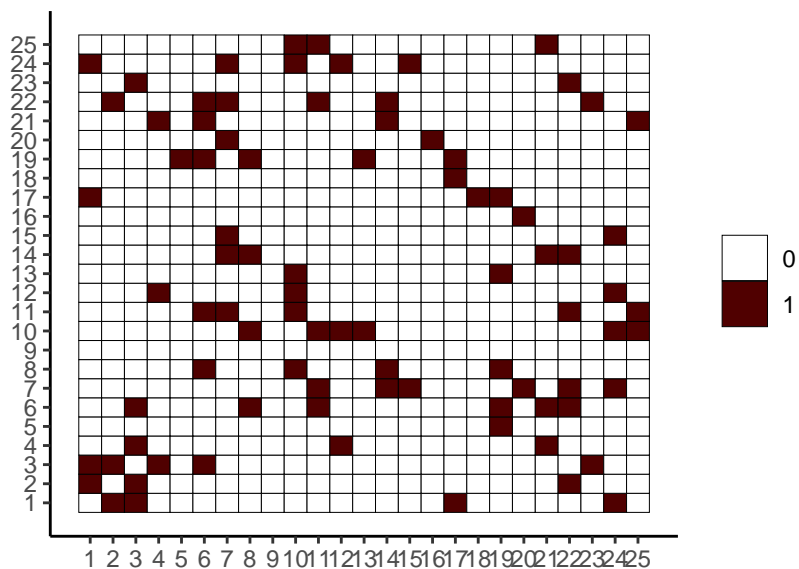
```
##
## $trial31$M2$unique_graphs[[45]]
```

Graph 45, Individuals 93



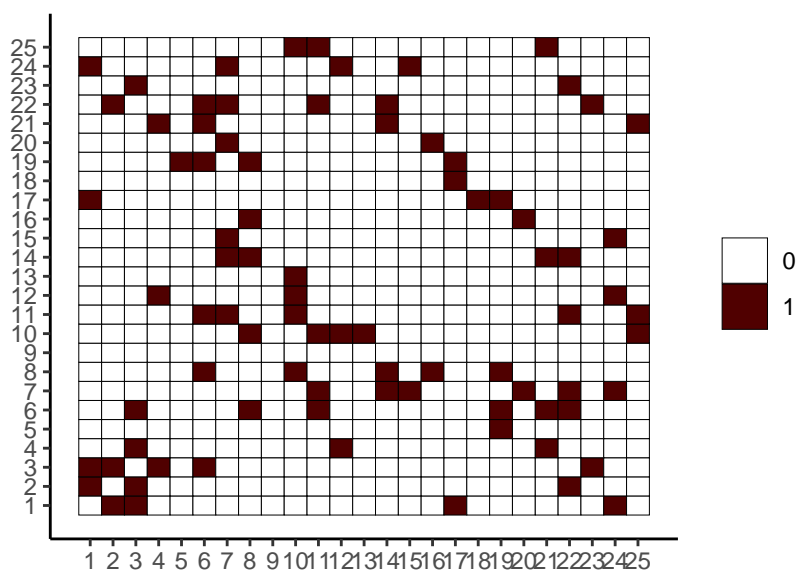
```
##
## $trial31$M2$unique_graphs[[46]]
```

Graph 46, Individuals 94



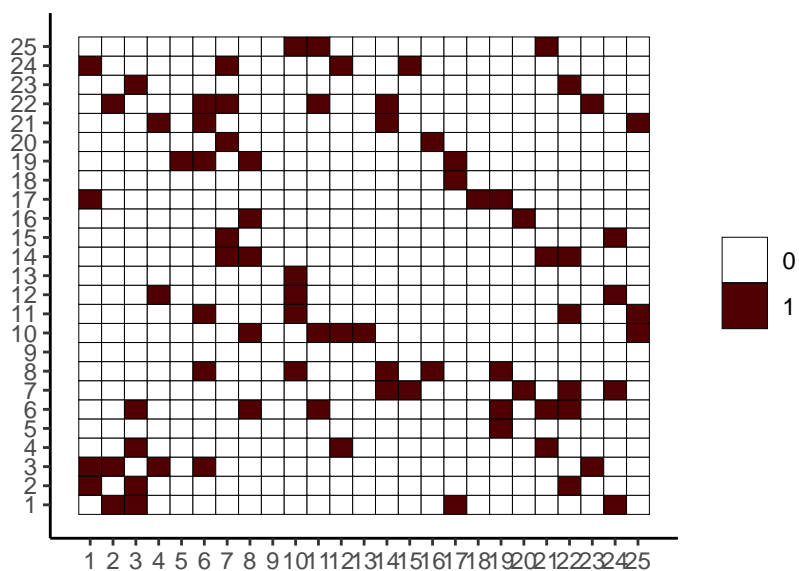
```
##
## $trial31$M2$unique_graphs[[47]]
```

Graph 47, Individuals 95,96



```
##
## $trial31$M2$unique_graphs[[48]]
```

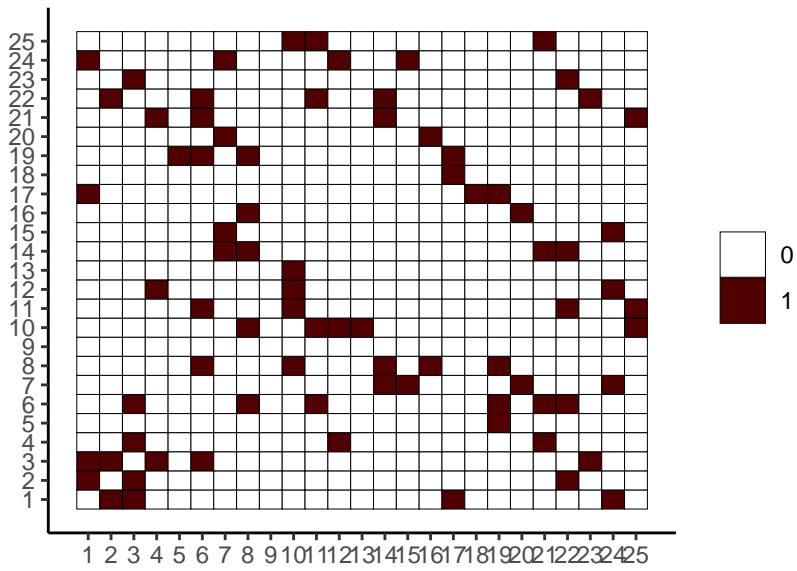
Graph 48, Individuals 97



```
##
## $trial31$M2$unique_graphs[[49]]
```

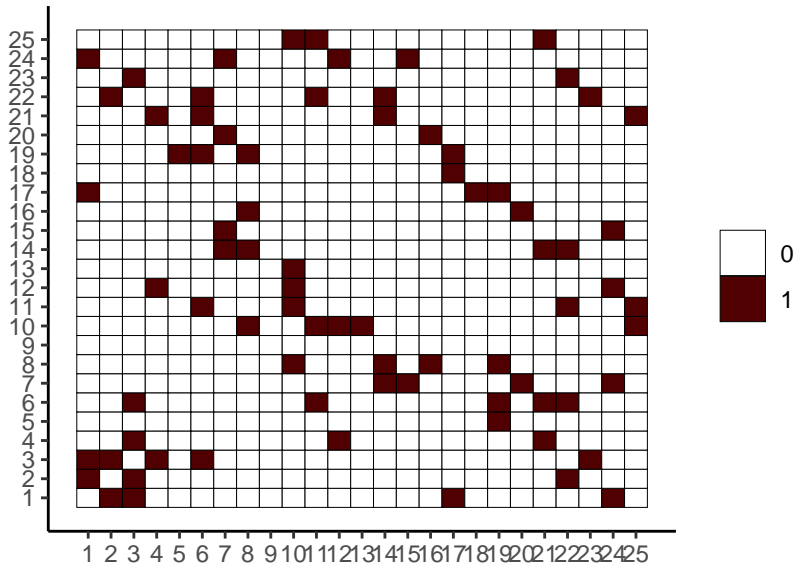


Graph 49, Individuals 98,99



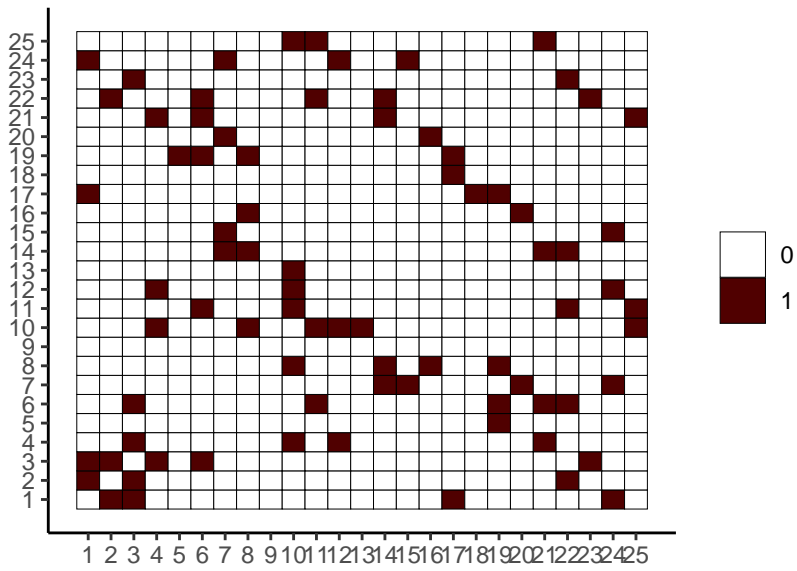
```
##
## $trial31$M2$unique_graphs[[50]]
```

Graph 50, Individuals 100



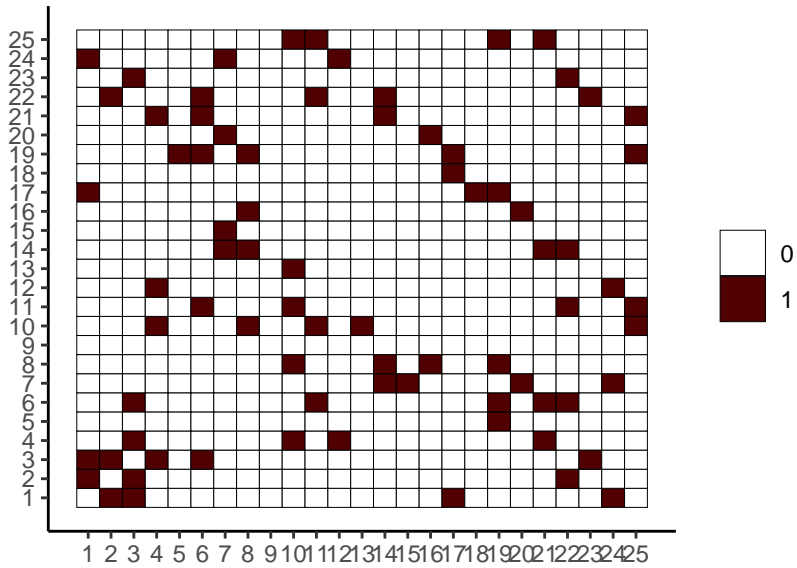
```
##
## $trial31$M2$unique_graphs[[51]]
```

Graph 51, Individuals 101



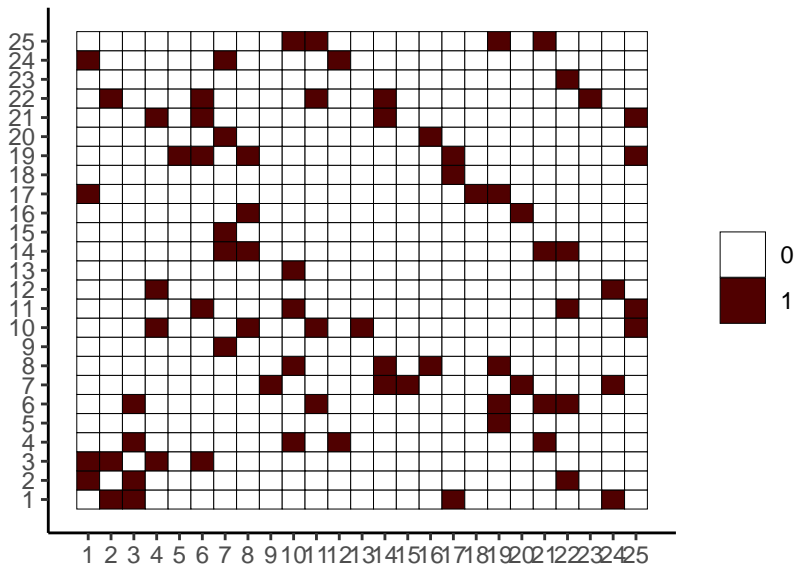
```
##
## $trial31$M2$unique_graphs[[52]]
```

Graph 52, Individuals 102,...,104



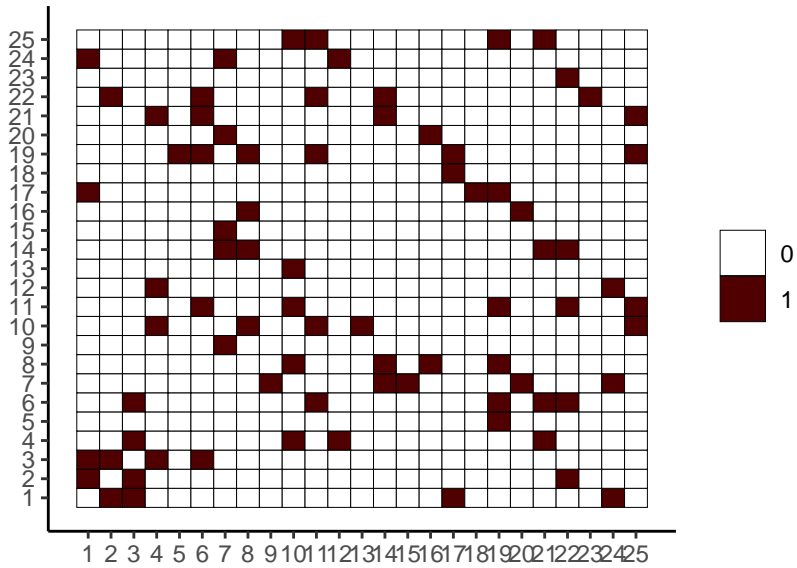
```
##
## $trial31$M2$unique_graphs[[53]]
```

Graph 53, Individuals 105



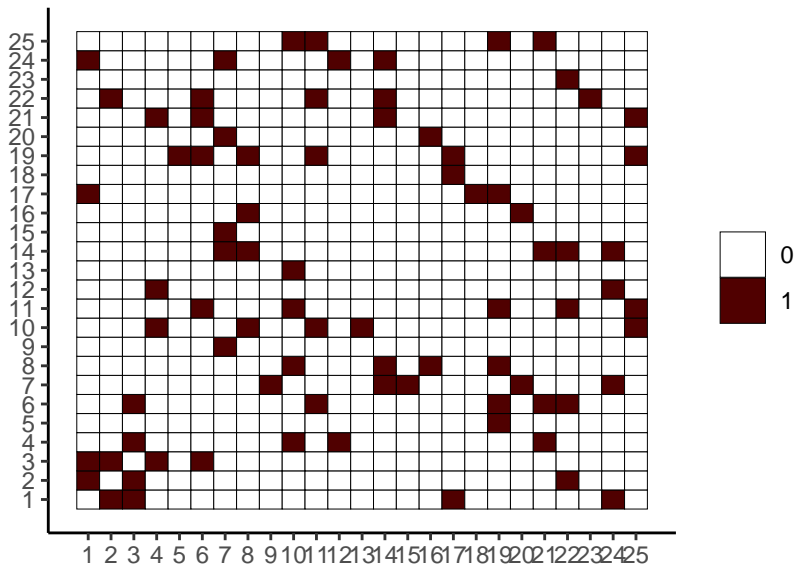
```
##
## $trial31$M2$unique_graphs[[54]]
```

Graph 54, Individuals 106



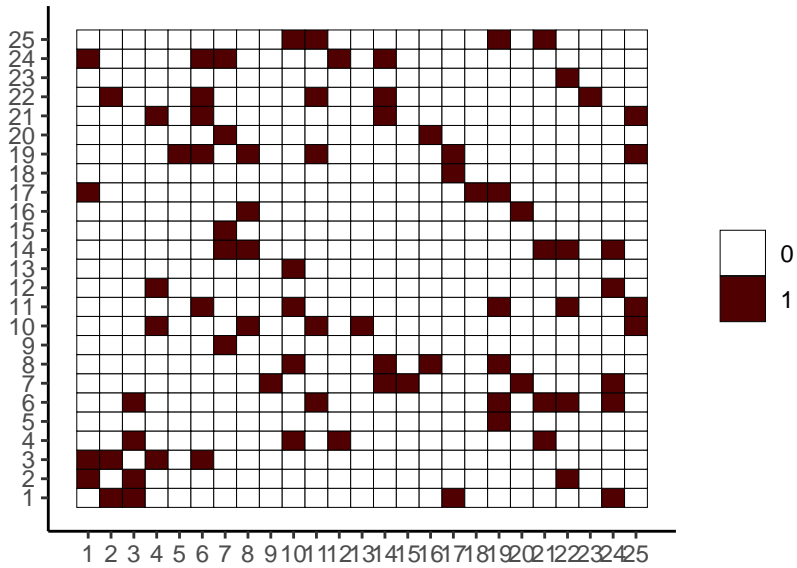
```
##
## $trial31$M2$unique_graphs[[55]]
```

Graph 55, Individuals 107



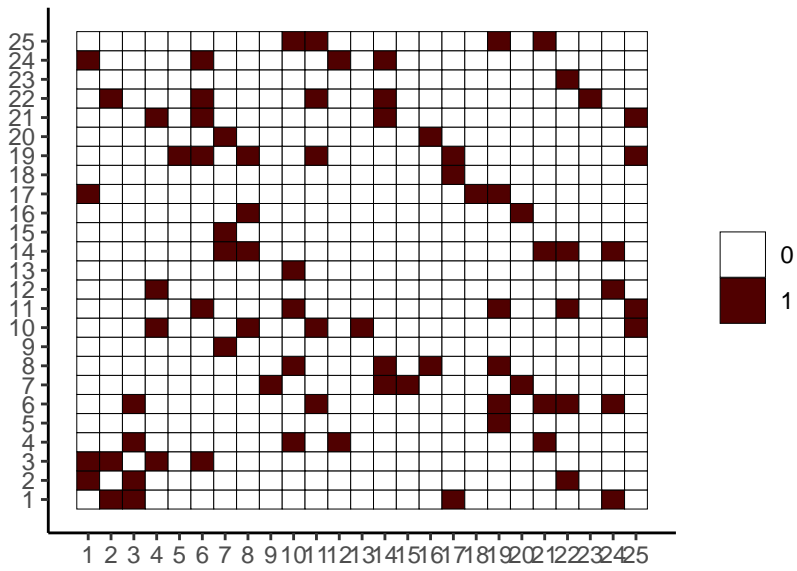
```
##
## $trial31$M2$unique_graphs[[56]]
```

Graph 56, Individuals 108



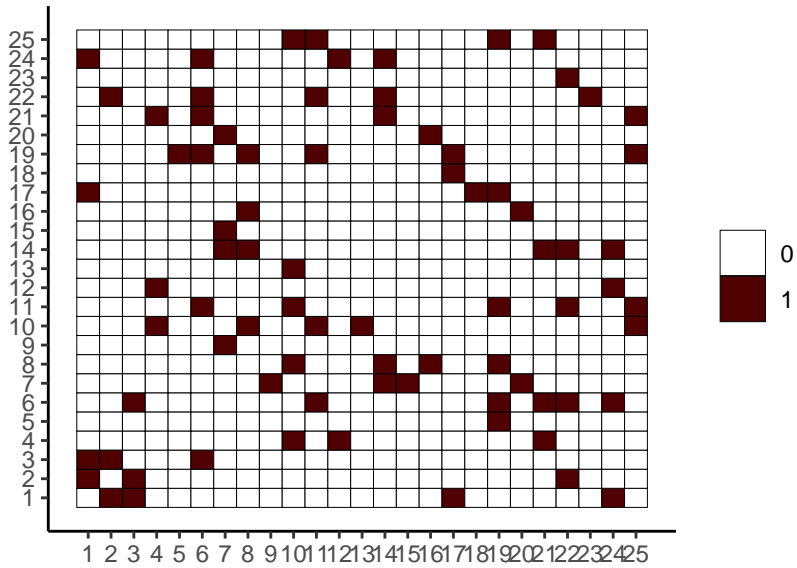
```
##
## $trial31$M2$unique_graphs[[57]]
```

Graph 57, Individuals 109



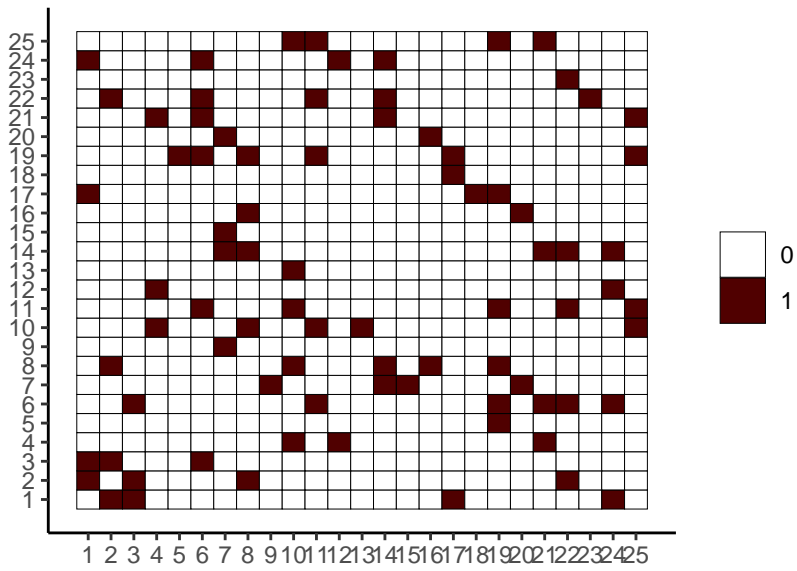
```
##
## $trial31$M2$unique_graphs[[58]]
```

Graph 58, Individuals 110



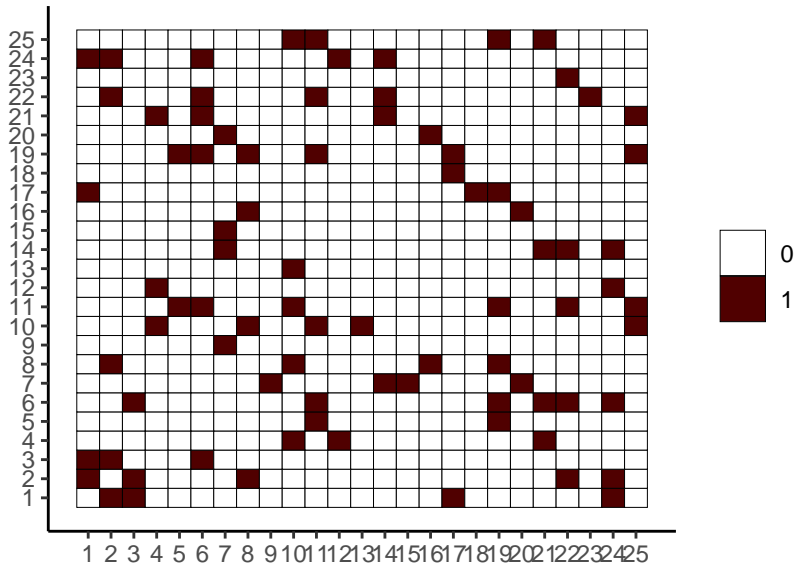
```
##
## $trial31$M2$unique_graphs[[59]]
```

Graph 59, Individuals 111



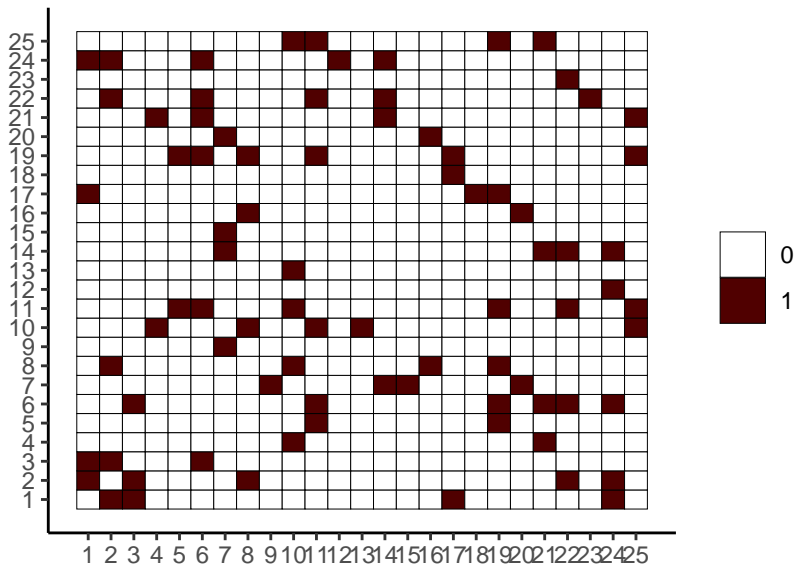
```
##
## $trial31$M2$unique_graphs[[60]]
```

Graph 60, Individuals 112,113



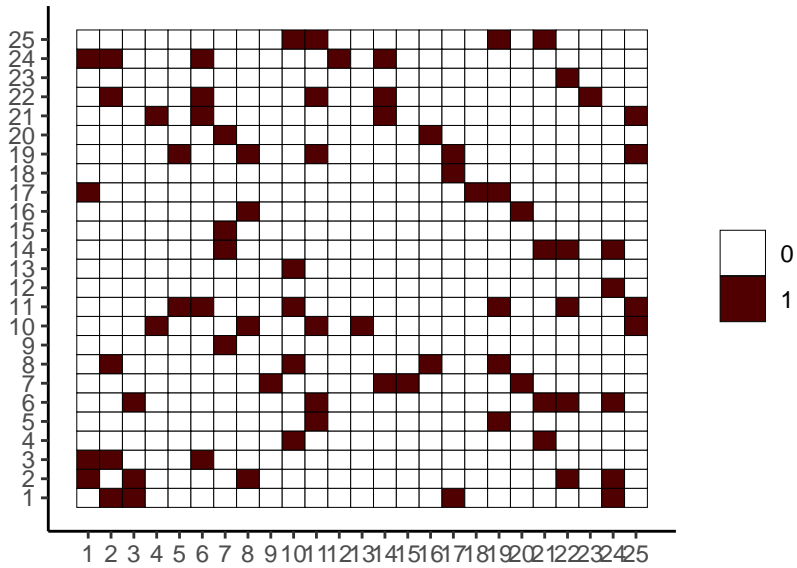
```
##
## $trial31$M2$unique_graphs[[61]]
```

Graph 61, Individuals 114



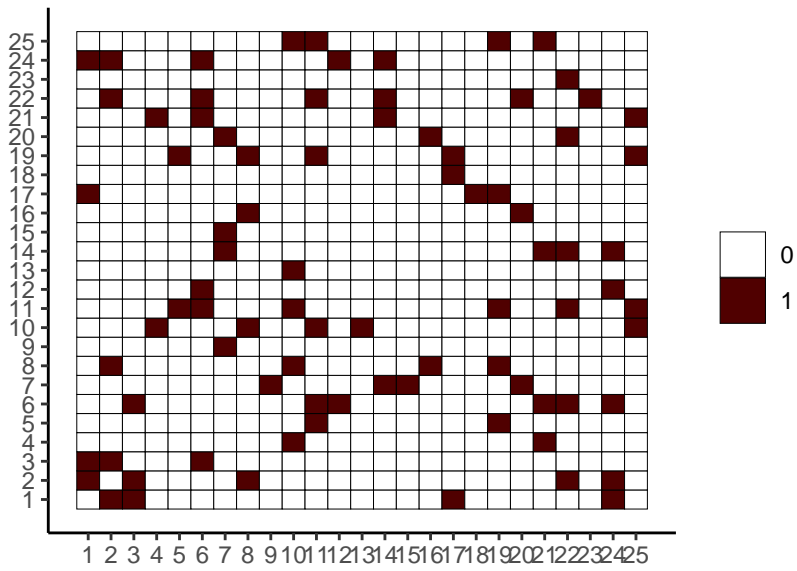
```
##
## $trial31$M2$unique_graphs[[62]]
```

Graph 62, Individuals 115



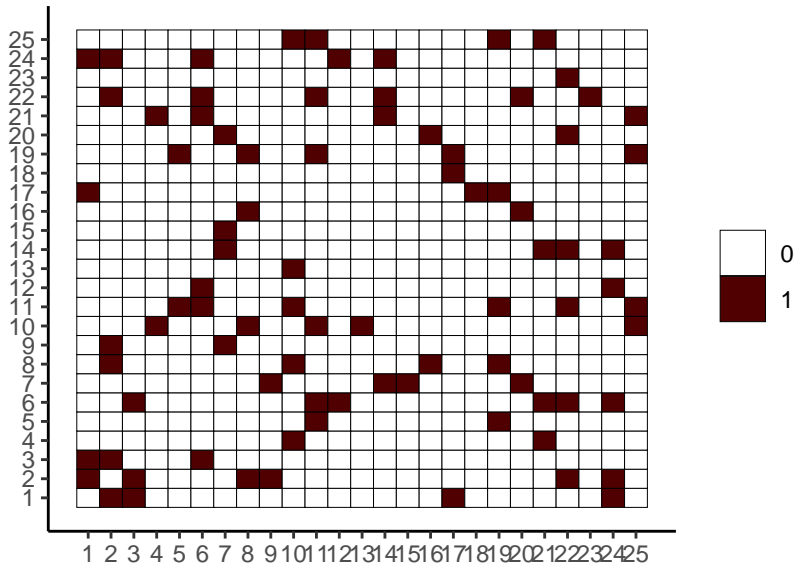
```
##
## $trial31$M2$unique_graphs[[63]]
```

Graph 63, Individuals 116,117



```
##
## $trial31$M2$unique_graphs[[64]]
```

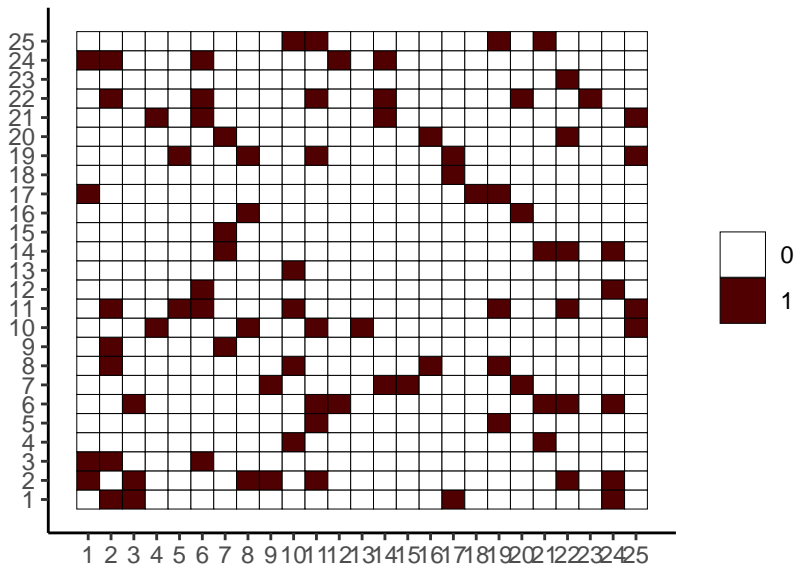
Graph 64, Individuals 118



```
##
## $trial31$M2$unique_graphs[[65]]
```

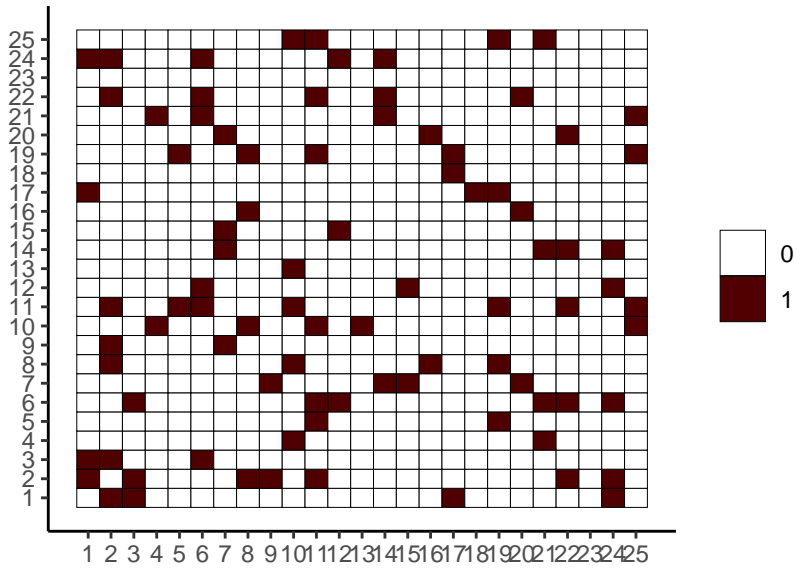


Graph 65, Individuals 119



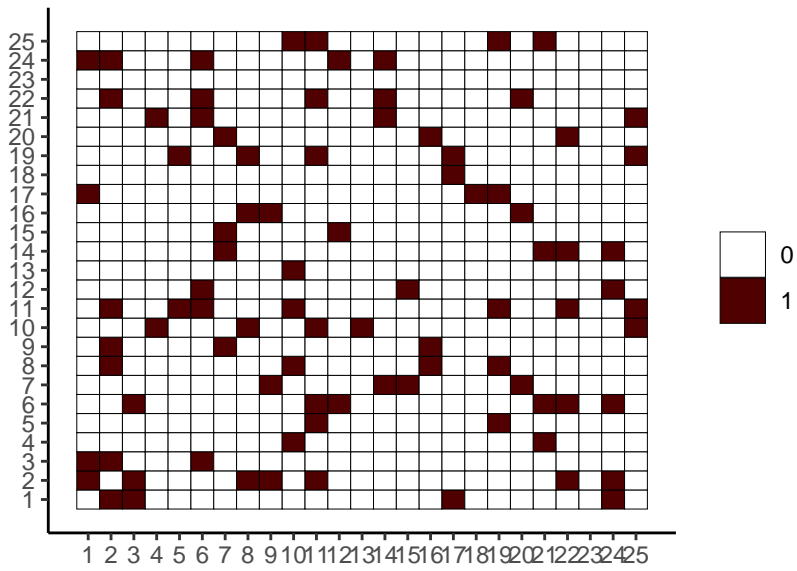
```
##
## $trial31$M2$unique_graphs[[66]]
```

Graph 66, Individuals 120



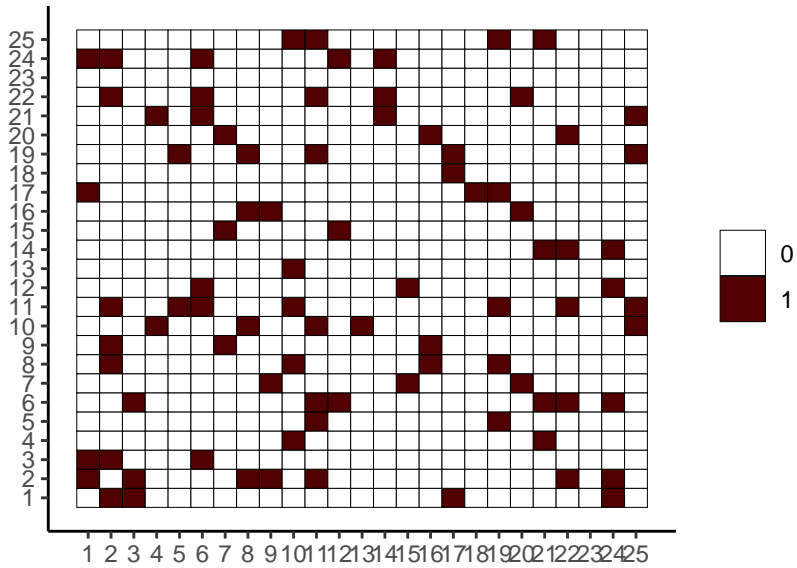
```
##
## $trial31$M2$unique_graphs[[67]]
```

Graph 67, Individuals 121,...,124



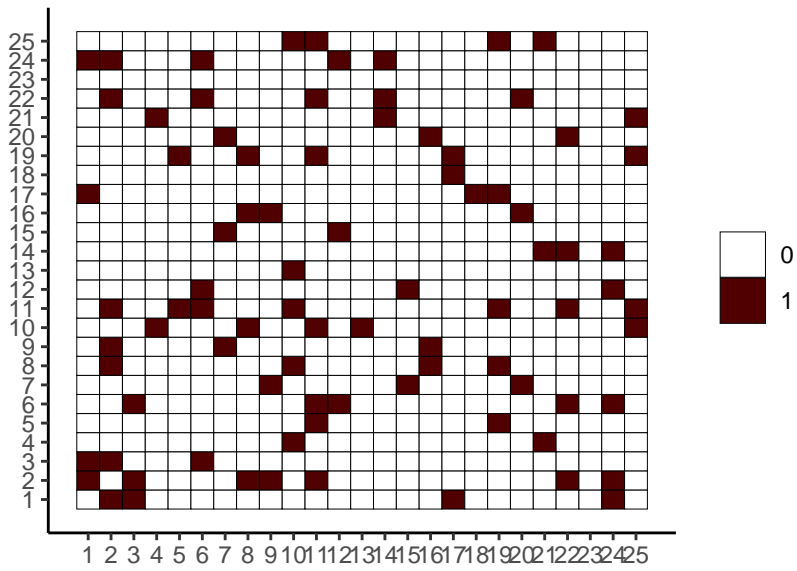
```
##
## $trial31$M2$unique_graphs[[68]]
```

Graph 68, Individuals 125



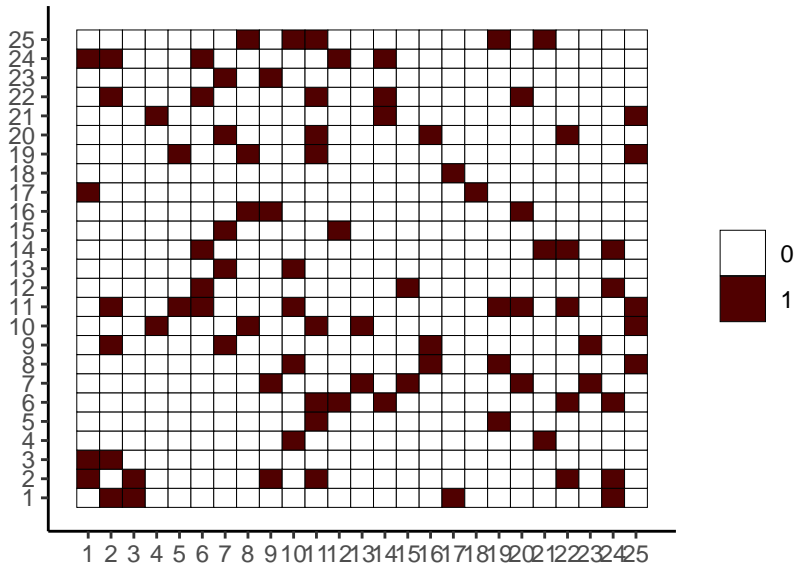
```
##
## $trial31$M2$unique_graphs[[69]]
```

Graph 69, Individuals 126



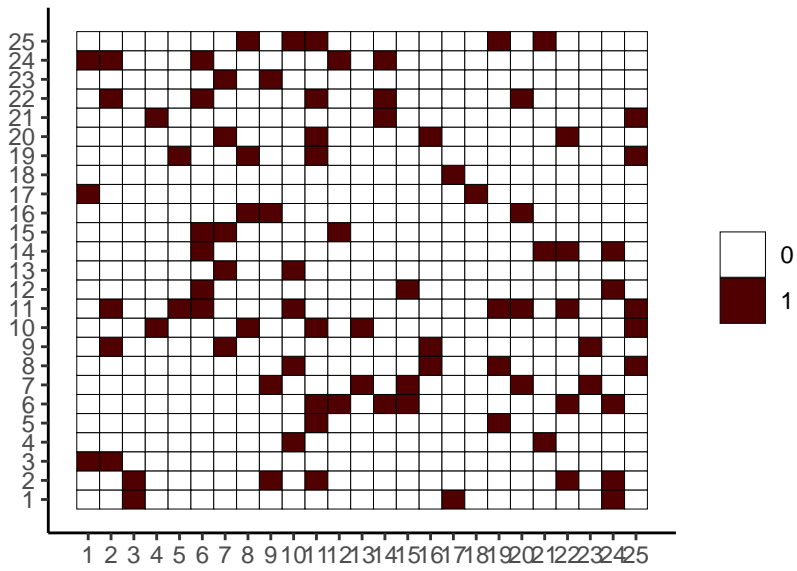
```
##
## $trial31$M2$unique_graphs[[70]]
```

Graph 70, Individuals 127



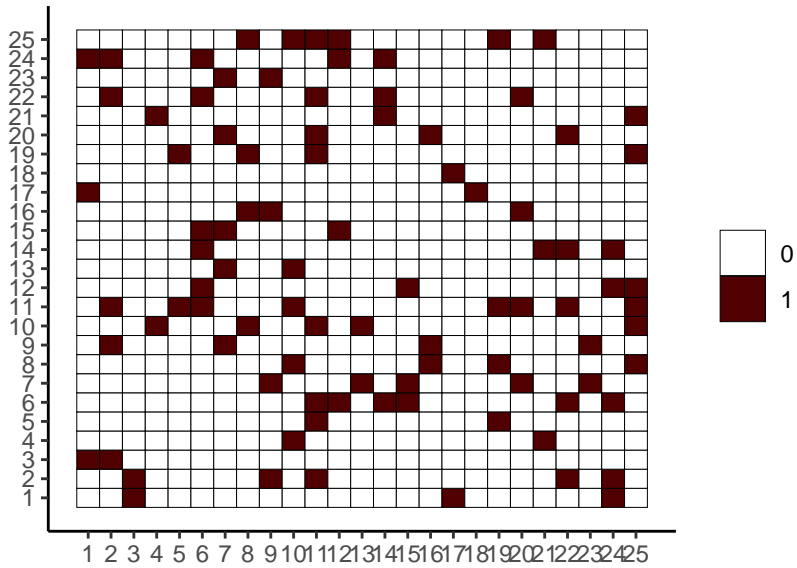
```
##
## $trial31$M2$unique_graphs[[71]]
```

Graph 71, Individuals 128,...,130



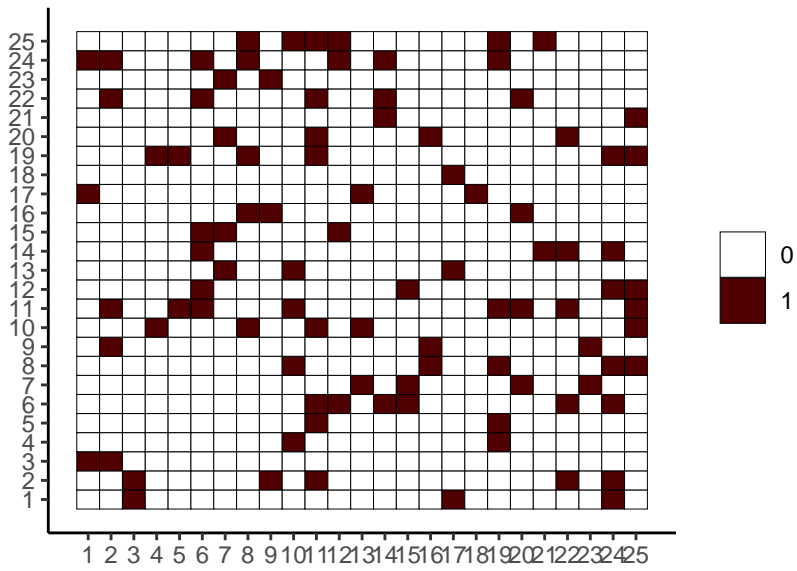
```
##
## $trial31$M2$unique_graphs[[72]]
```

Graph 72, Individuals 131



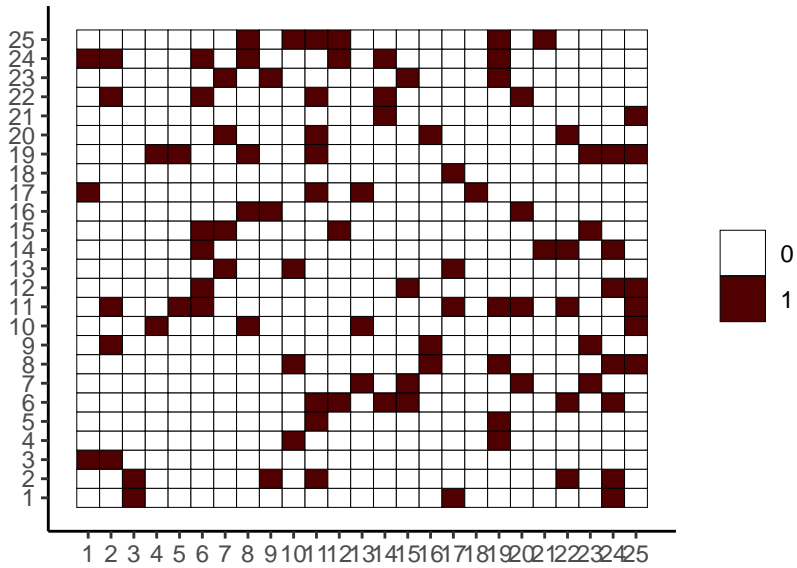
```
##
## $trial31$M2$unique_graphs[[73]]
```

Graph 73, Individuals 132



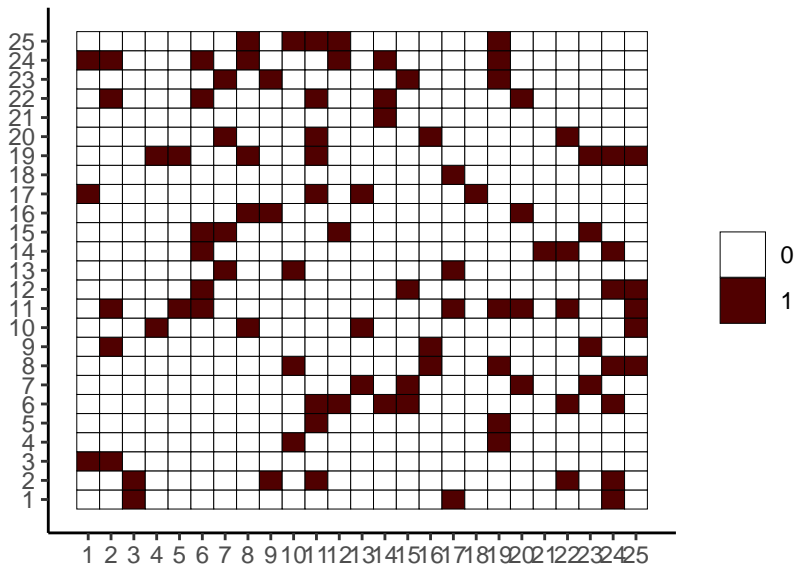
```
##
## $trial31$M2$unique_graphs[[74]]
```

Graph 74, Individuals 133



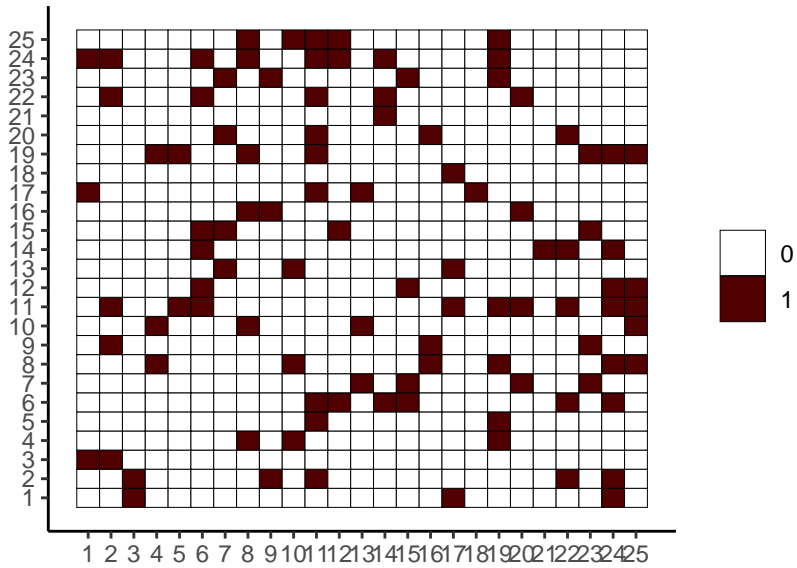
```
##
## $trial31$M2$unique_graphs[[75]]
```

Graph 75, Individuals 134,135



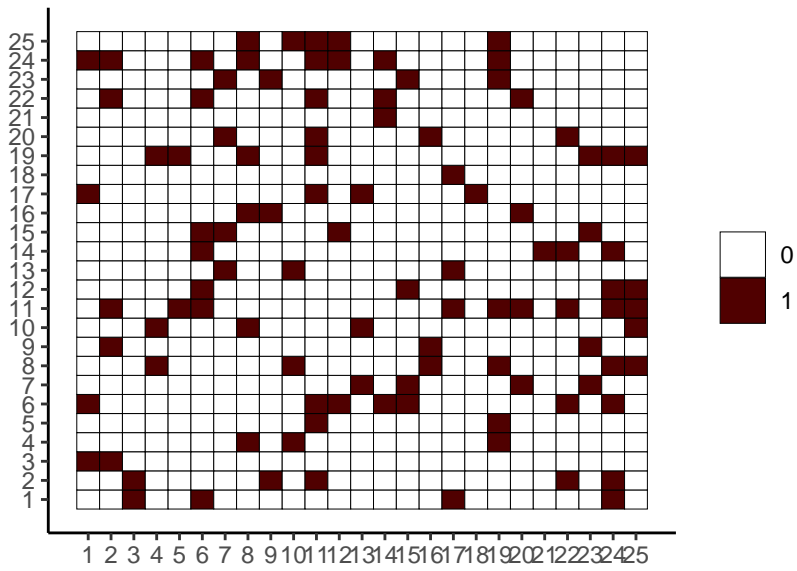
```
##
## $trial31$M2$unique_graphs[[76]]
```

Graph 76, Individuals 136



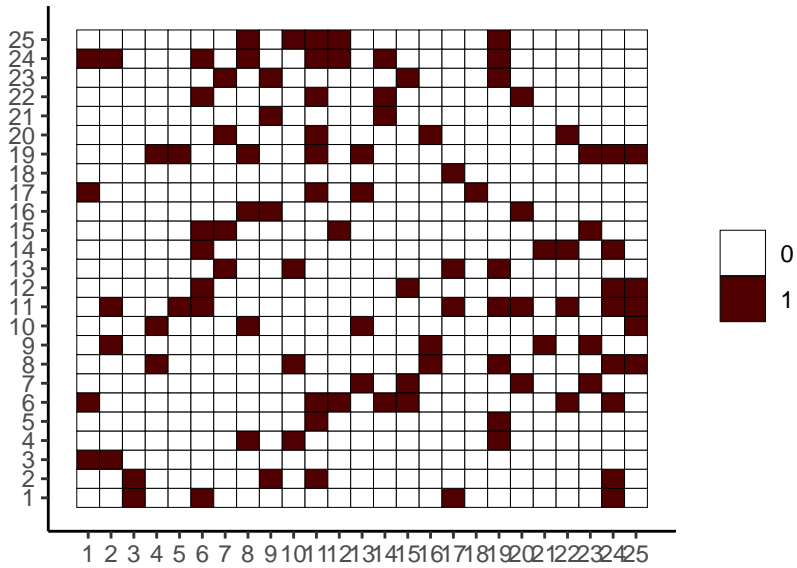
```
##
## $trial31$M2$unique_graphs[[77]]
```

Graph 77, Individuals 137,...,139



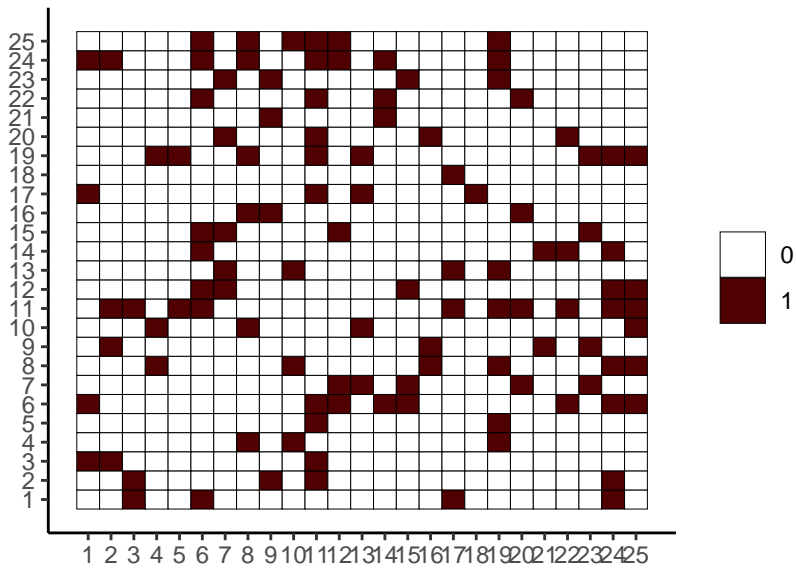
```
##
## $trial31$M2$unique_graphs[[78]]
```

Graph 78, Individuals 140,141



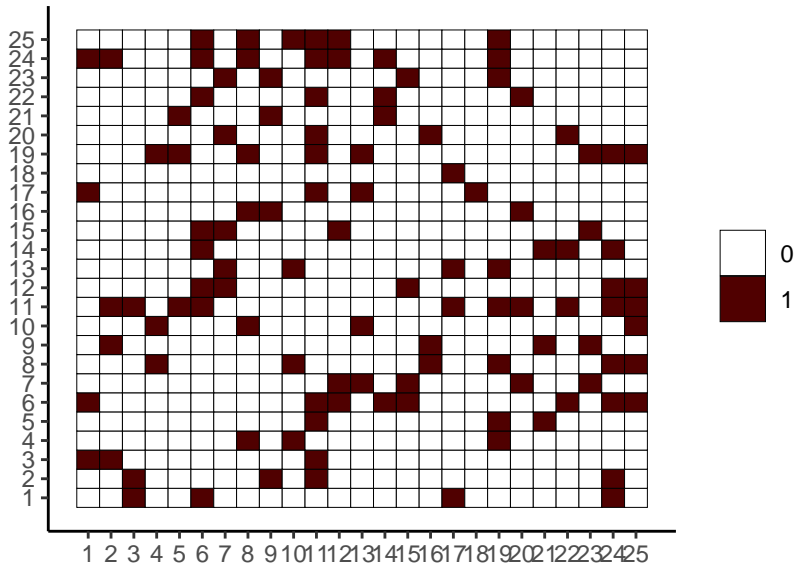
```
##
## $trial31$M2$unique_graphs[[79]]
```

Graph 79, Individuals 142,143



```
##
## $trial31$M2$unique_graphs[[80]]
```

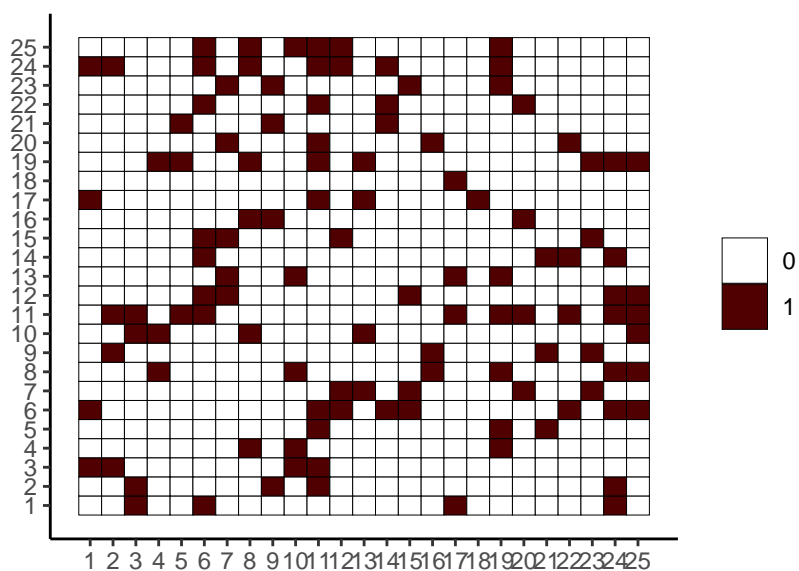
Graph 80, Individuals 144



```
##
## $trial31$M2$unique_graphs[[81]]
```

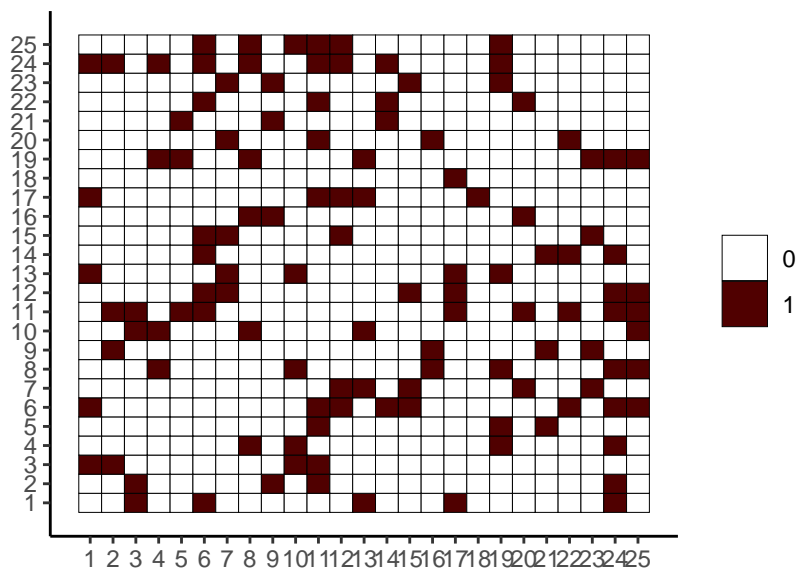


Graph 81, Individuals 145



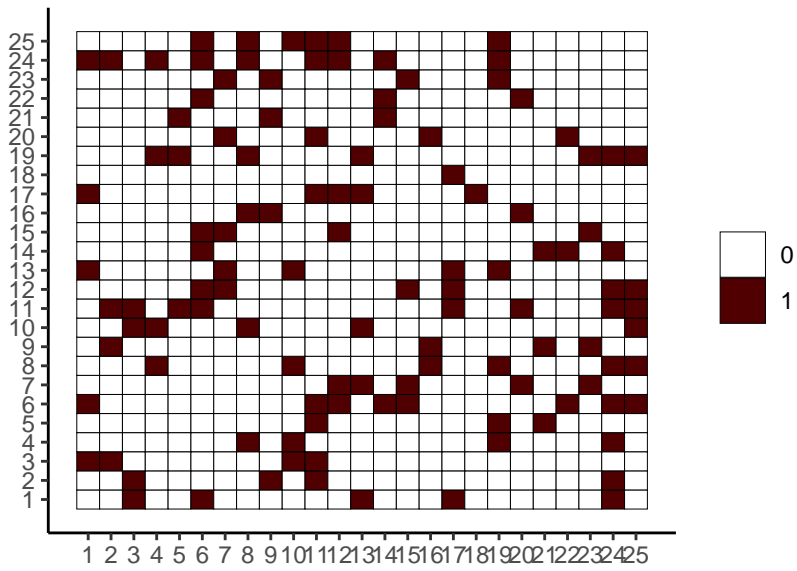
```
##
## $trial31$M2$unique_graphs[[82]]
```

Graph 82, Individuals 146,147



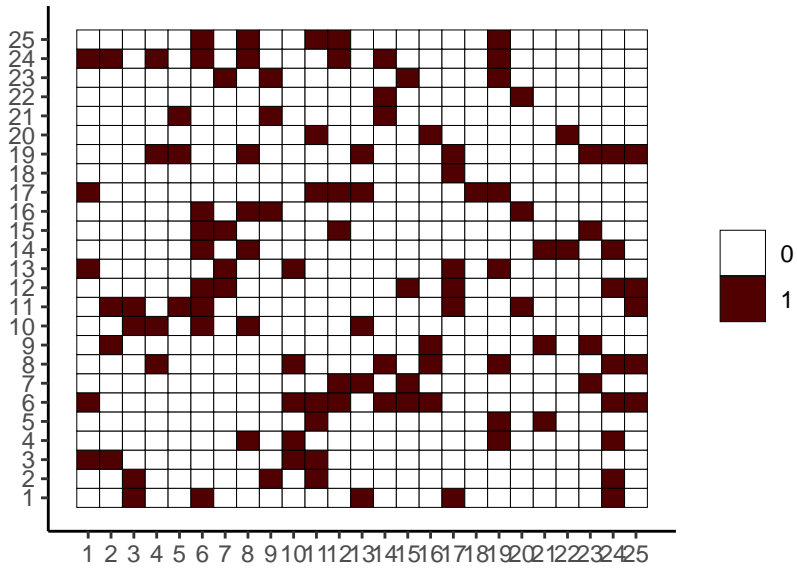
```
##
## $trial31$M2$unique_graphs[[83]]
```

Graph 83, Individuals 148,149



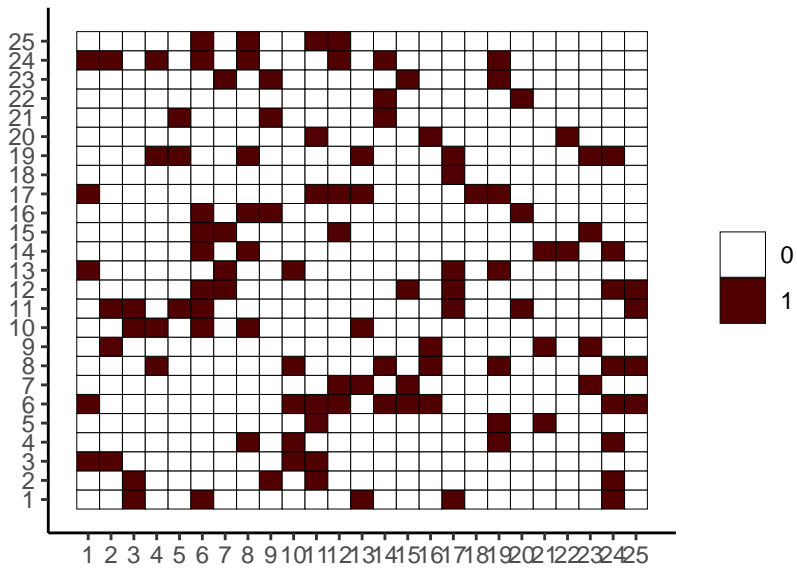
```
##
## $trial31$M2$unique_graphs[[84]]
```

Graph 84, Individuals 150



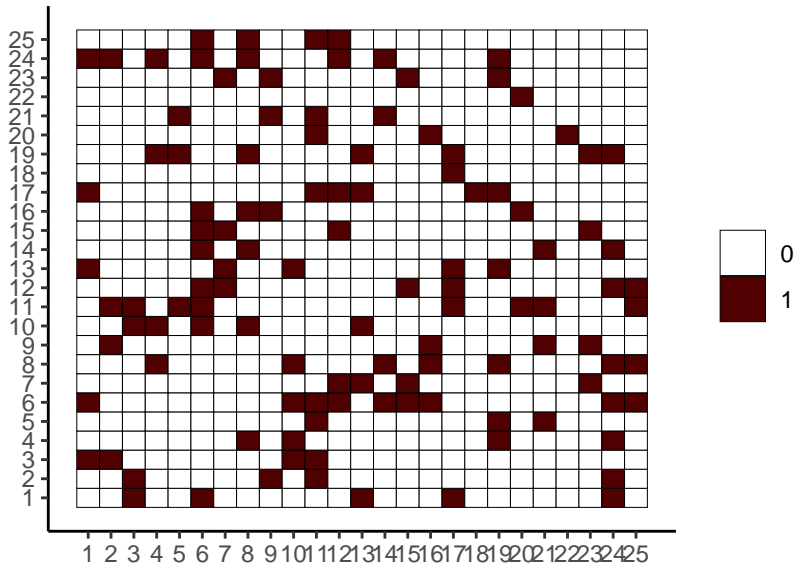
```
##
## $trial31$M2$unique_graphs[[85]]
```

Graph 85, Individuals 151,152



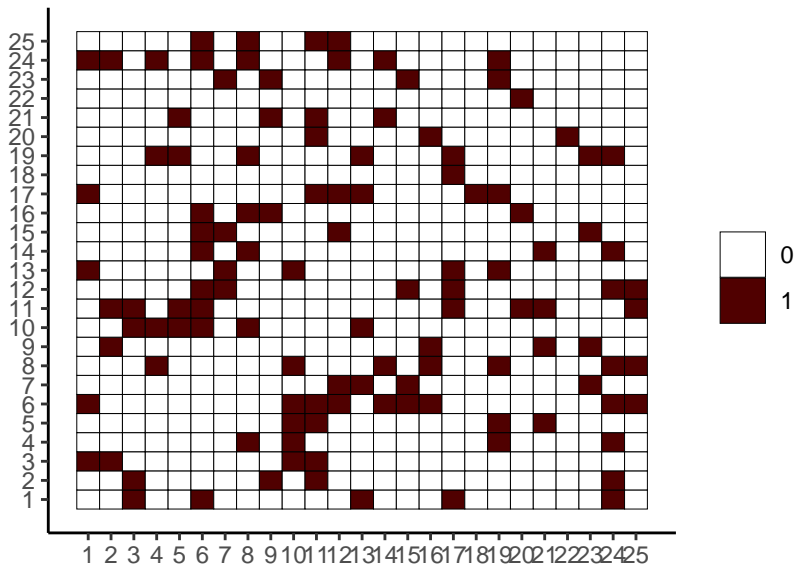
```
##
## $trial31$M2$unique_graphs[[86]]
```

Graph 86, Individuals 153



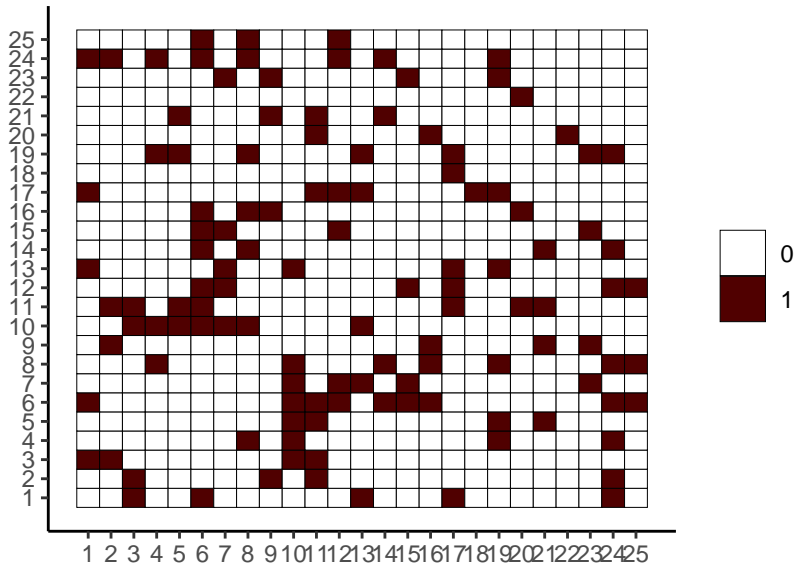
```
##
## $trial31$M2$unique_graphs[[87]]
```

Graph 87, Individuals 154,...,158



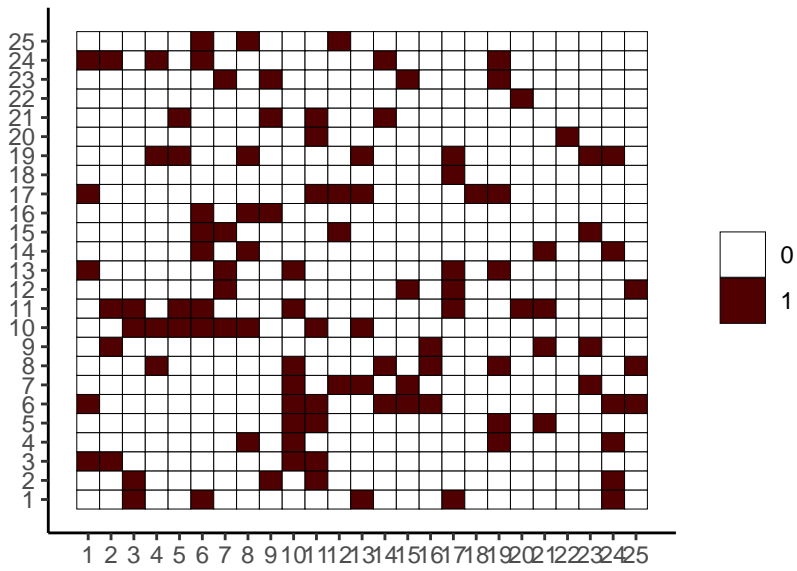
```
##
## $trial31$M2$unique_graphs[[88]]
```

Graph 88, Individuals 159



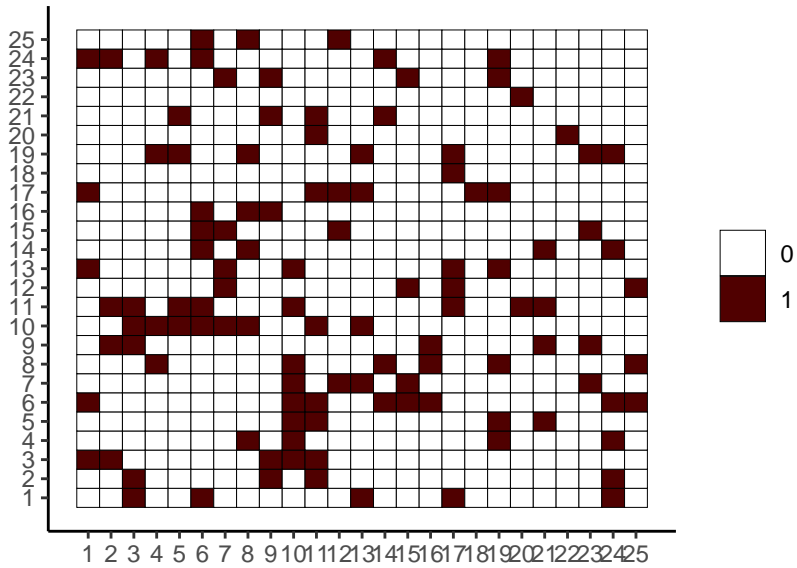
```
##
## $trial31$M2$unique_graphs[[89]]
```

Graph 89, Individuals 160,161



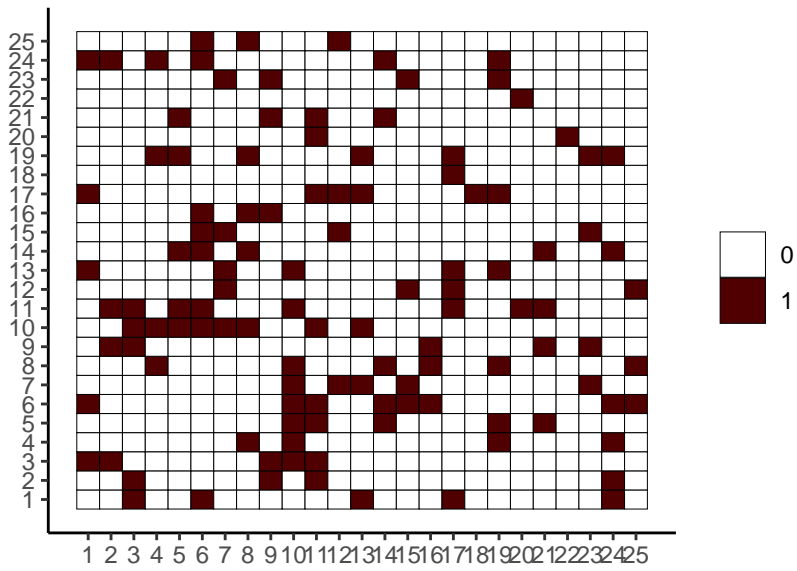
```
##
## $trial31$M2$unique_graphs[[90]]
```

Graph 90, Individuals 162,...,165



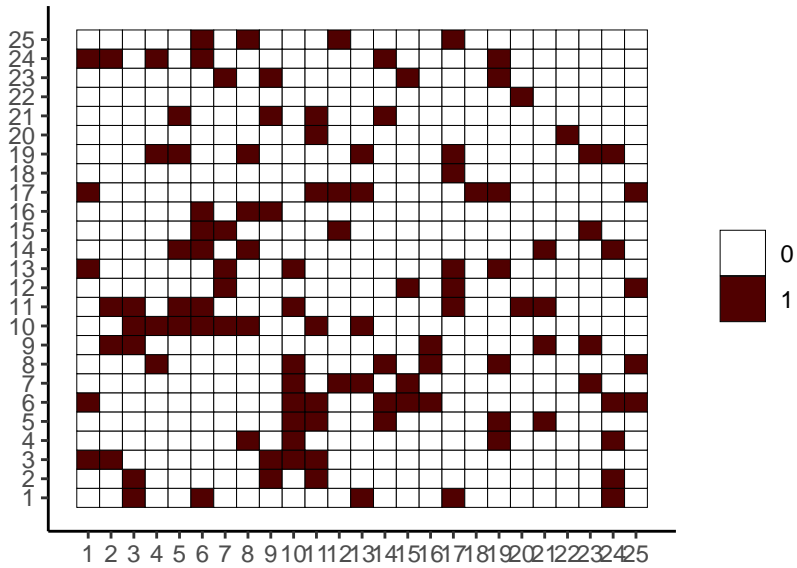
```
##
## $trial31$M2$unique_graphs[[91]]
```

Graph 91, Individuals 166



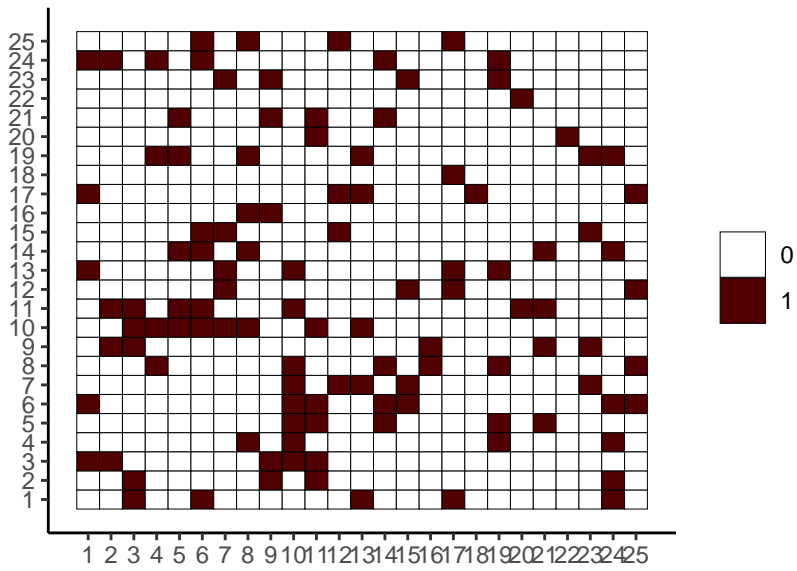
```
##
## $trial31$M2$unique_graphs[[92]]
```

Graph 92, Individuals 167,168



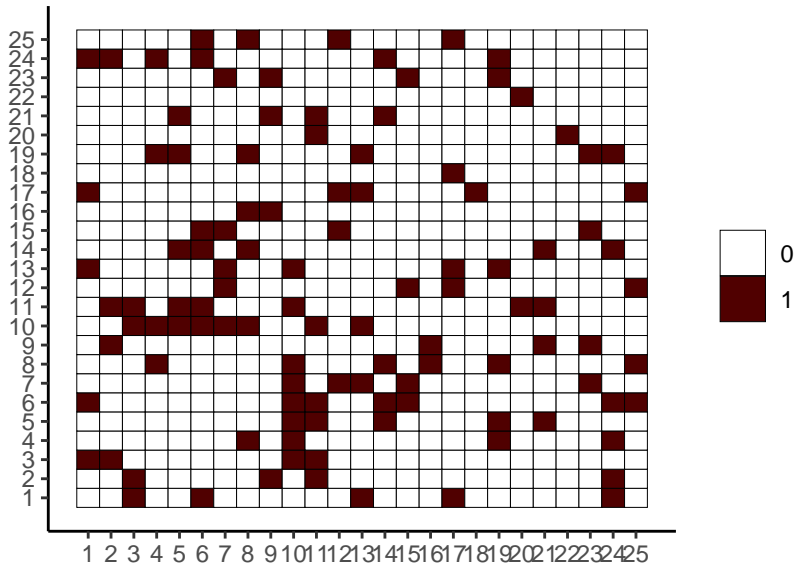
```
##
## $trial31$M2$unique_graphs[[93]]
```

Graph 93, Individuals 169



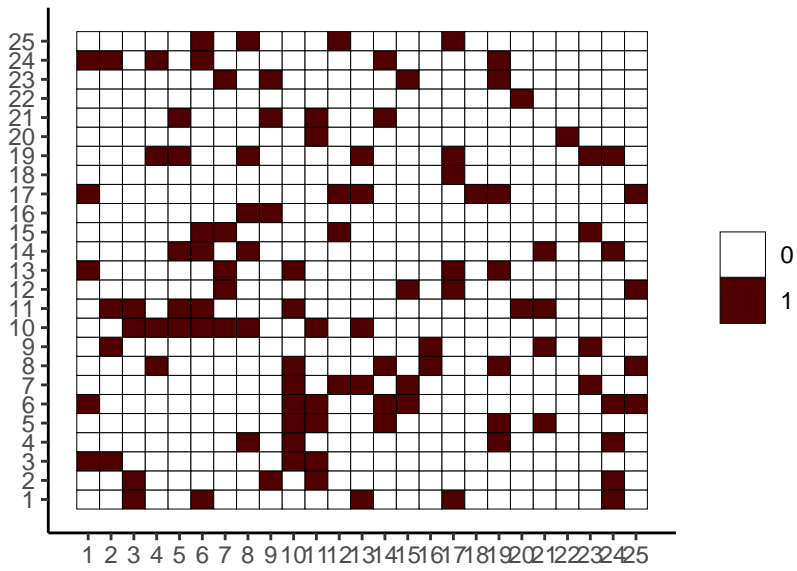
```
##
## $trial31$M2$unique_graphs[[94]]
```

Graph 94, Individuals 170



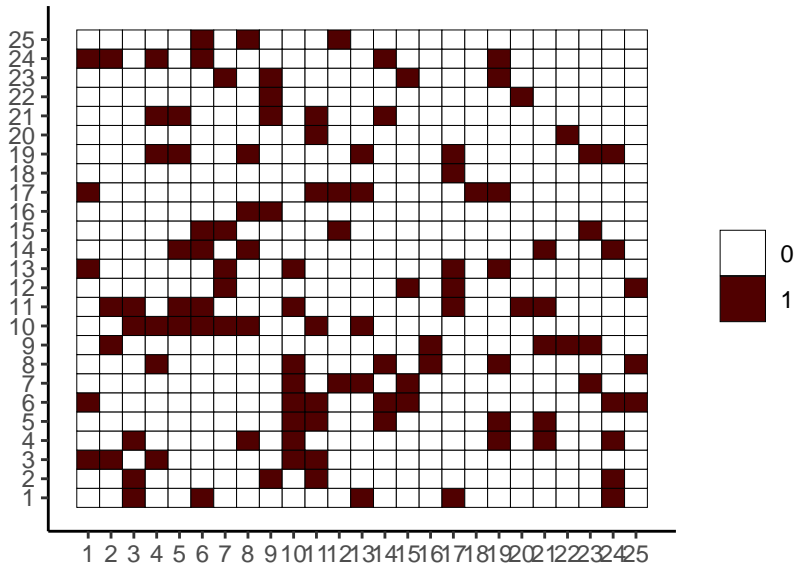
```
##
## $trial31$M2$unique_graphs[[95]]
```

Graph 95, Individuals 171



```
##
## $trial31$M2$unique_graphs[[96]]
```

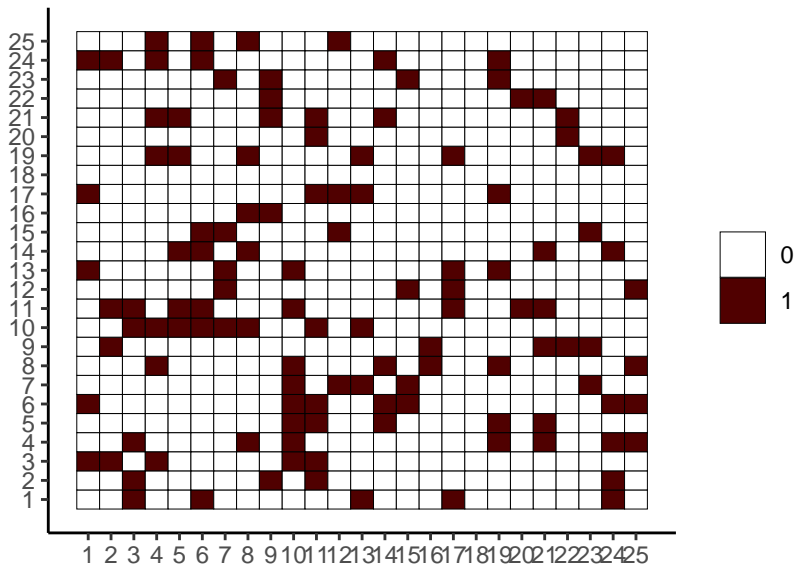
Graph 96, Individuals 172



```
##
## $trial31$M2$unique_graphs[[97]]
```

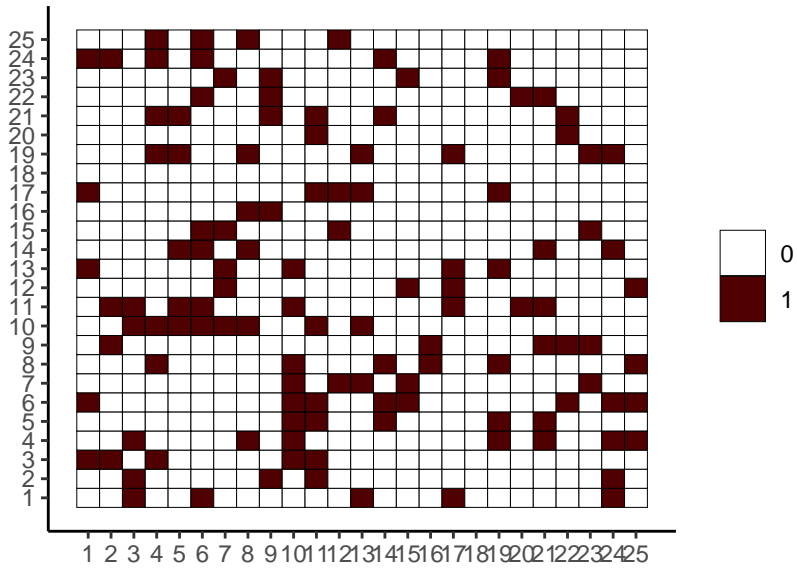


Graph 97, Individuals 173



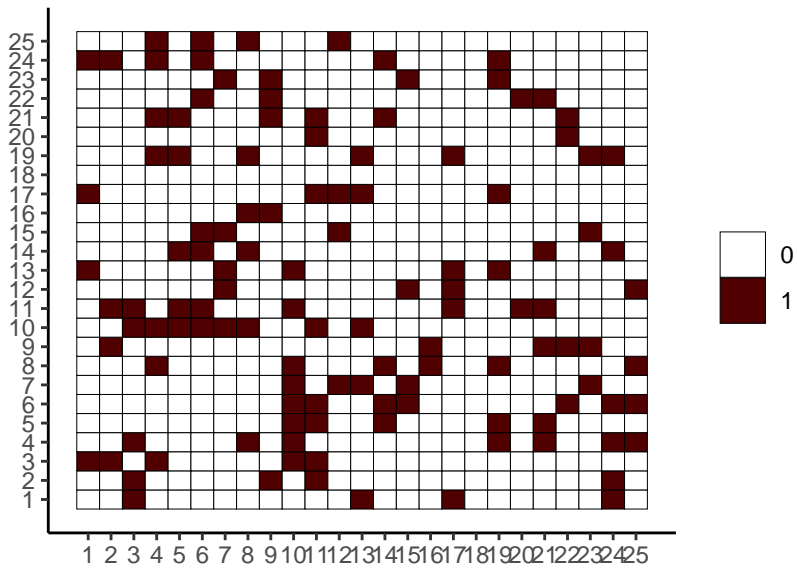
```
##
## $trial31$M2$unique_graphs[[98]]
```

Graph 98, Individuals 174,175



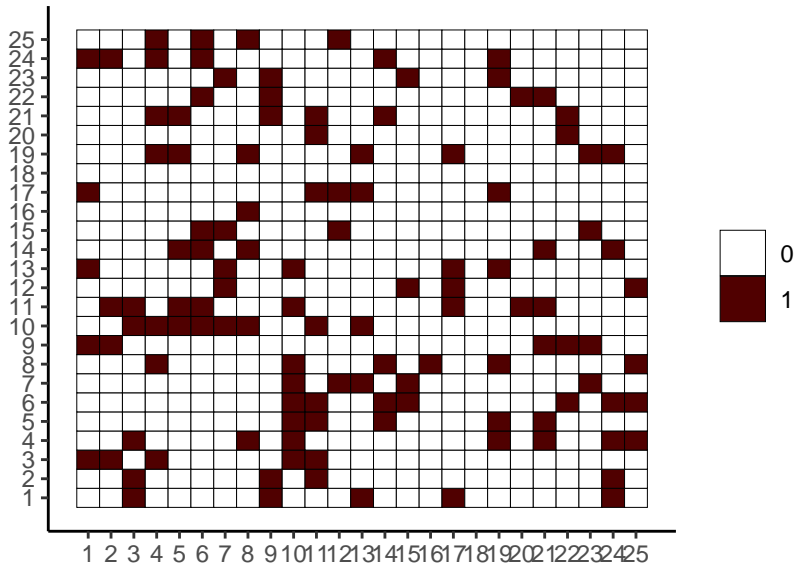
```
##
## $trial31$M2$unique_graphs[[99]]
```

Graph 99, Individuals 176,177



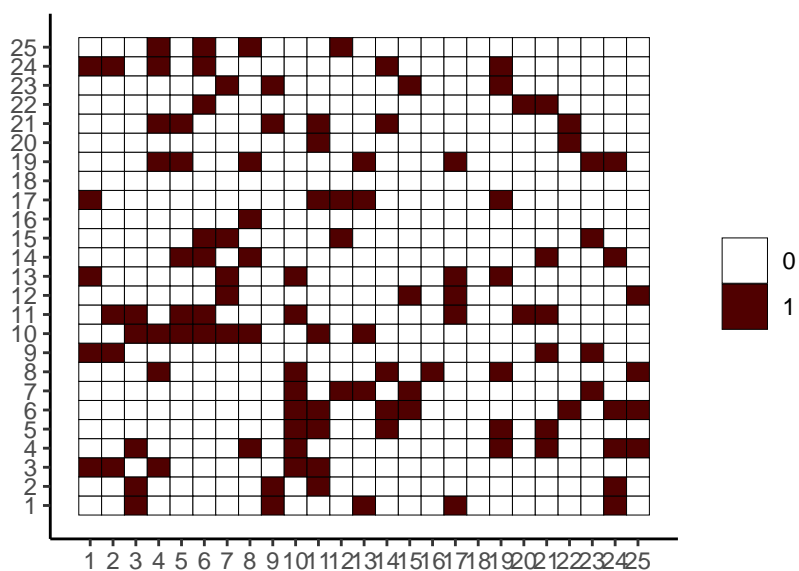
```
##
## $trial31$M2$unique_graphs[[100]]
```

Graph 100, Individuals 178



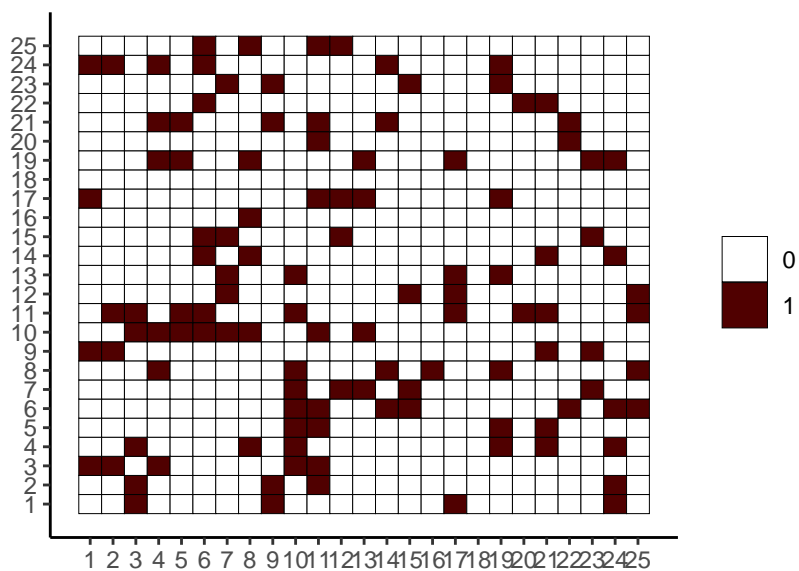
```
##
## $trial31$M2$unique_graphs[[101]]
```

Graph 101, Individuals 179



```
##
## $trial31$M2$unique_graphs[[102]]
```

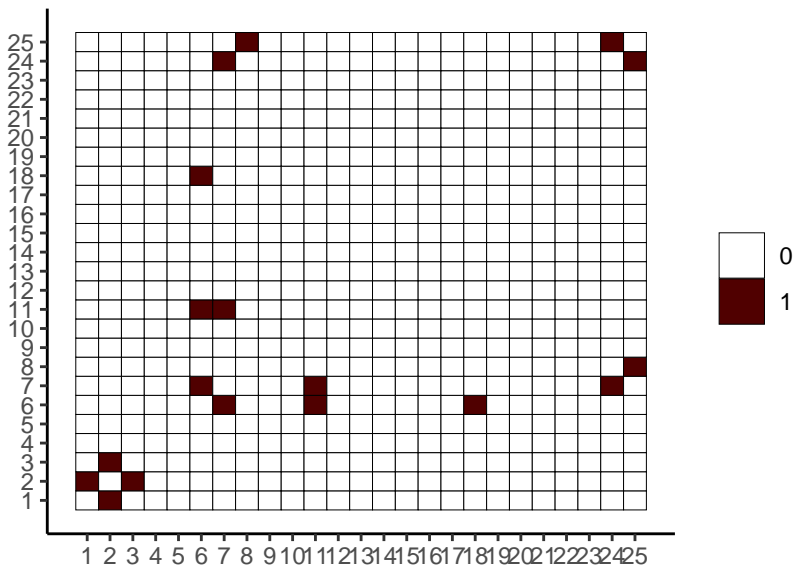
Graph 102, Individuals 180



```
##
##
## $trial31$M2$sensitivity
## [1] 0.9238095
##
## $trial31$M2$specificity
```

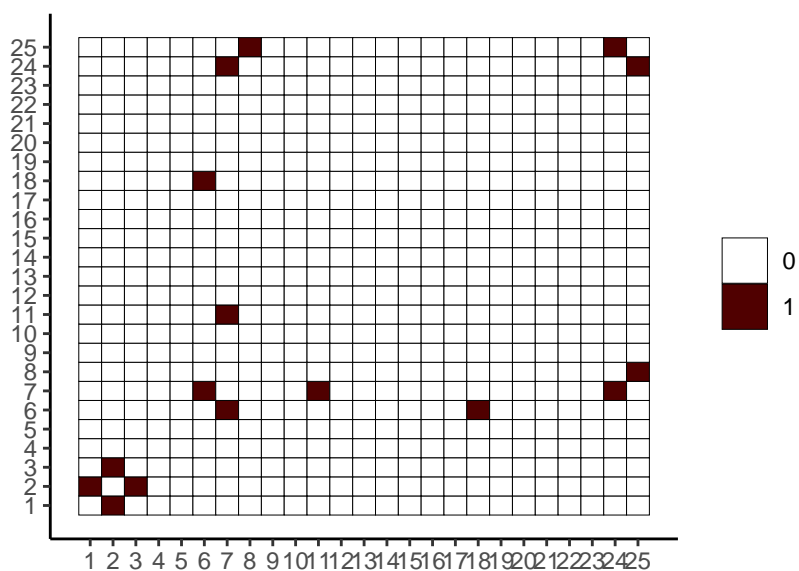
```
## [1] 0.8457997
##
## $trial31$M2$accuracy
## [1] 0.8463822
##
## $trial31$M2$ELBO
## [1] -385317.5
##
## $trial31$M2$time
## [1] 32.55868
##
##
## $trial31$M3
## $trial31$M3$summary
##          Covariate Dependent Graphical Model
##
## Model ELBO: -214790.25          Unique conditional dependence structures: 17
## n: 180, variables: 25          Hyperparameter grid size: 125 points
## CAVI converged for 25/25 variables
##
## Model fit completed in 1.986 mins
##
## $trial31$M3$unique_graphs
## $trial31$M3$unique_graphs[[1]]
```

Graph 1, Individuals 1,...,25



```
##
## $trial31$M3$unique_graphs[[2]]
```

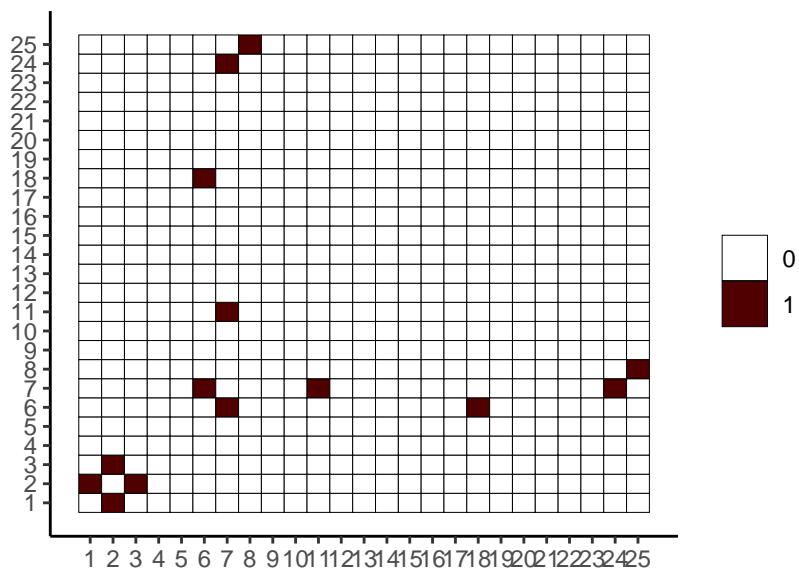
Graph 2, Individuals 26,...,33



##

## \$trial31\$M3\$unique\_graphs[[3]]

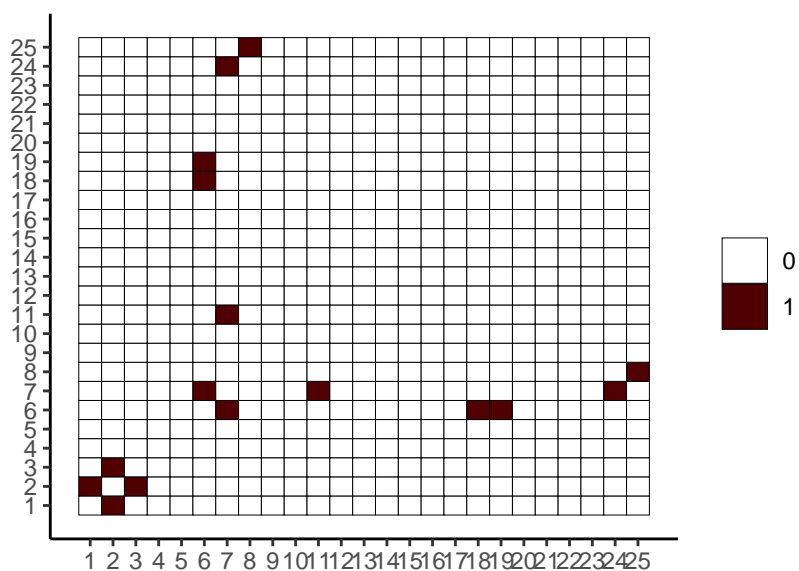
Graph 3, Individuals 34,...,44



##

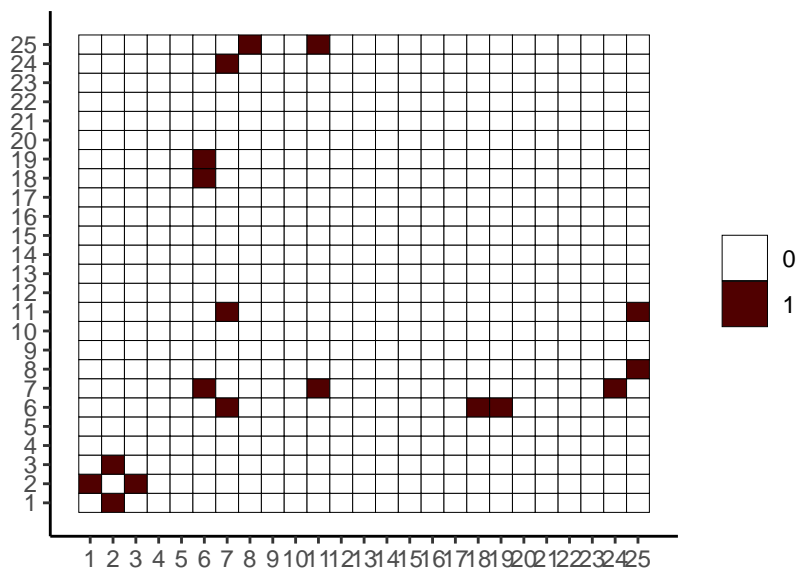
## \$trial31\$M3\$unique\_graphs[[4]]

Graph 4, Individuals 45,...,48



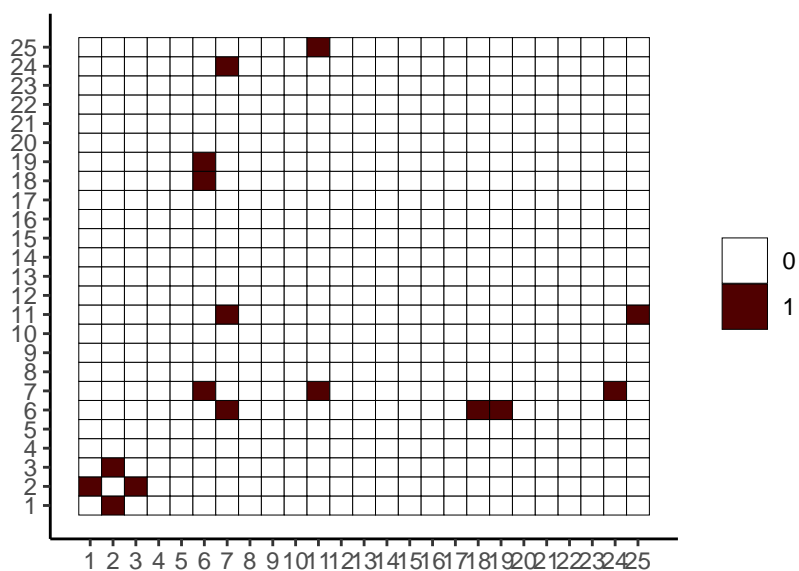
```
##
## $trial31$M3$unique_graphs[[5]]
```

Graph 5, Individuals 49,...,51



```
##
## $trial31$M3$unique_graphs[[6]]
```

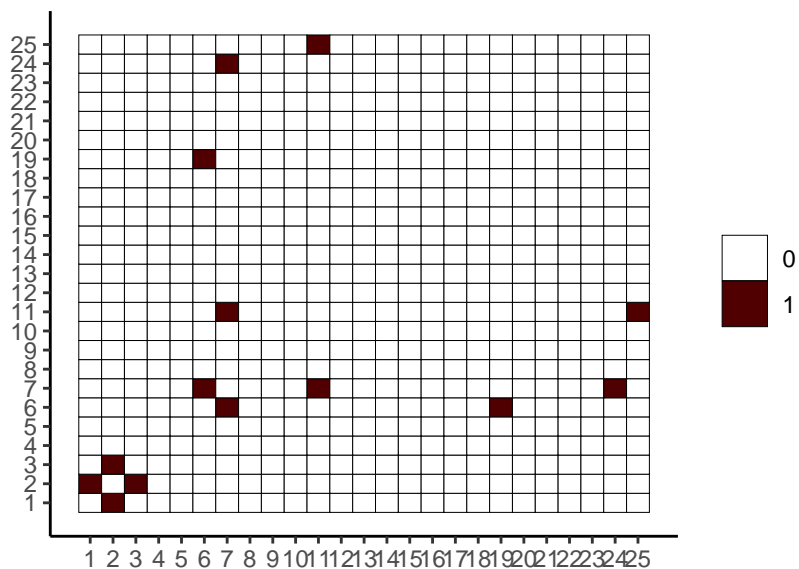
Graph 6, Individuals 52,...,56



##

## \$trial31\$M3\$unique\_graphs[[7]]

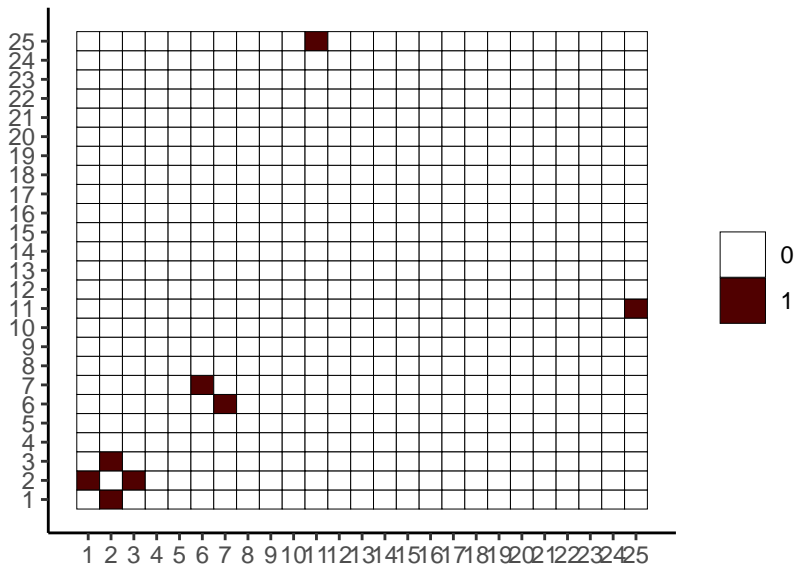
Graph 7, Individuals 57,...,85



##

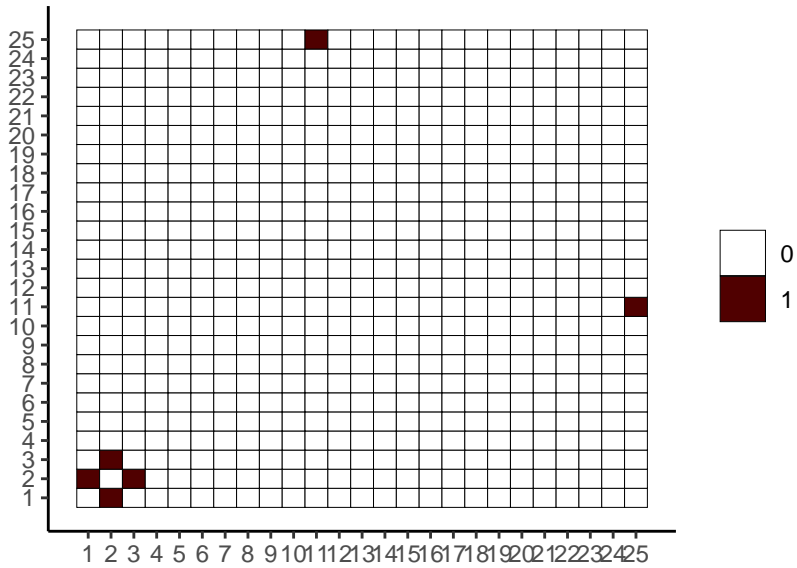
## \$trial31\$M3\$unique\_graphs[[8]]

Graph 8, Individuals 86



```
##
## $trial31$M3$unique_graphs[[9]]
```

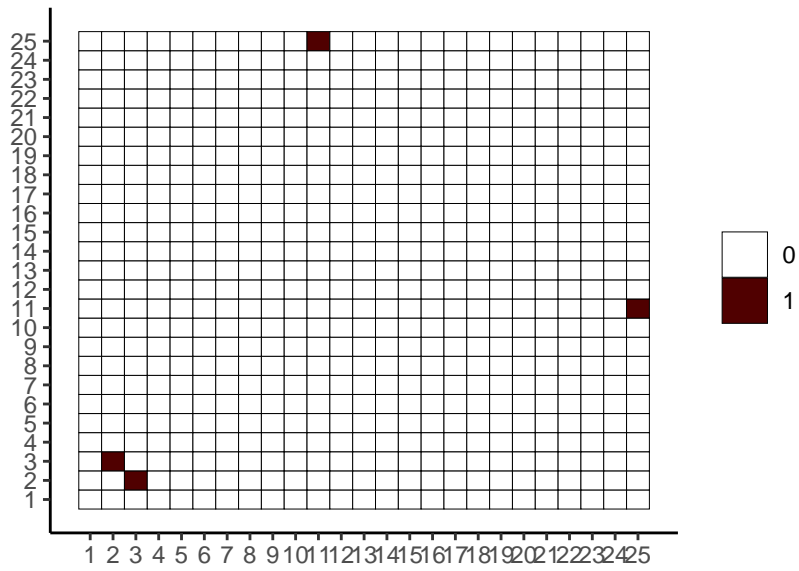
Graph 9, Individuals 87,...,103



```
##
## $trial31$M3$unique_graphs[[10]]
```

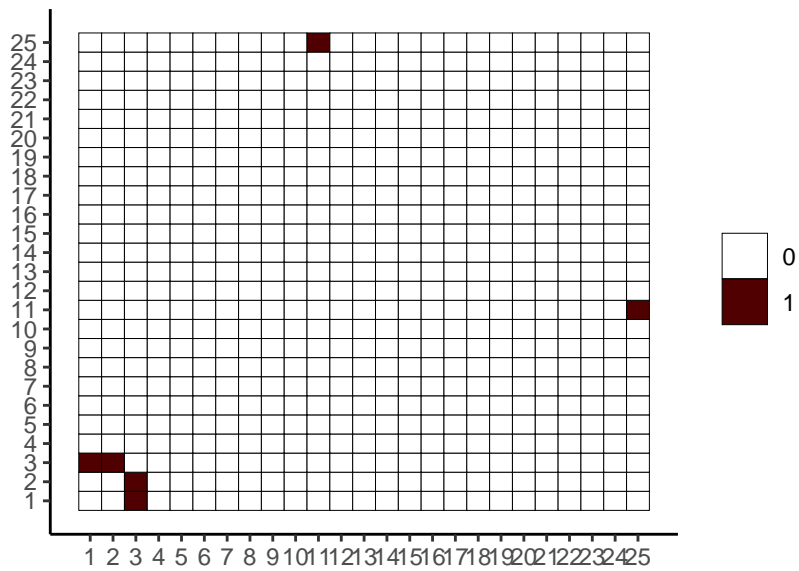


Graph 10, Individuals 104,...,109



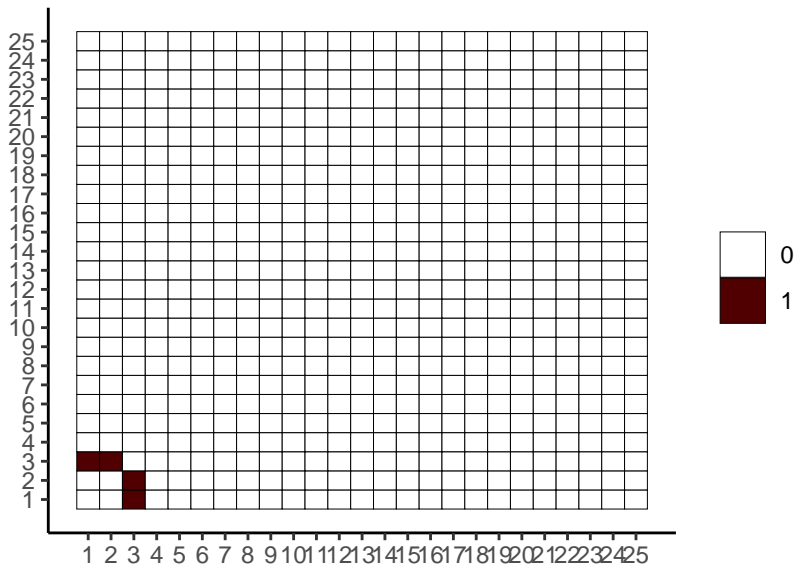
```
##
## $trial31$M3$unique_graphs[[11]]
```

Graph 11, Individuals 110,...,113



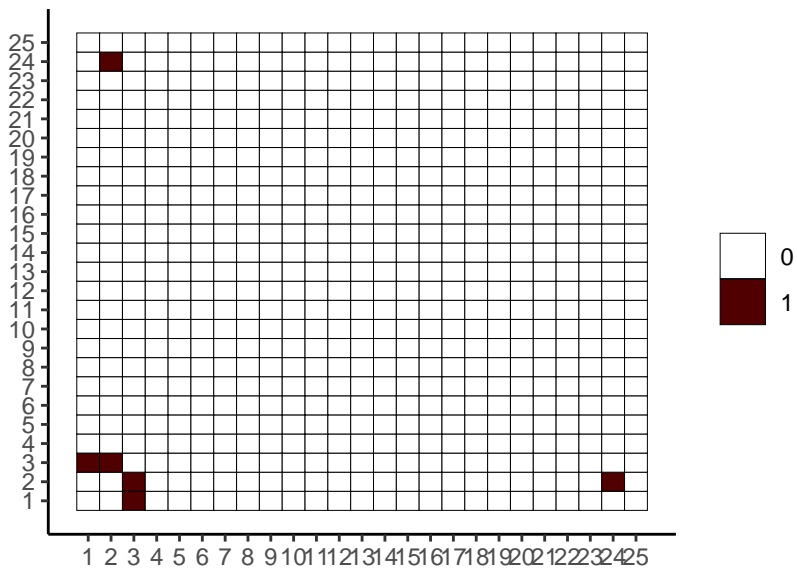
```
##
## $trial31$M3$unique_graphs[[12]]
```

Graph 12, Individuals 114,...,119



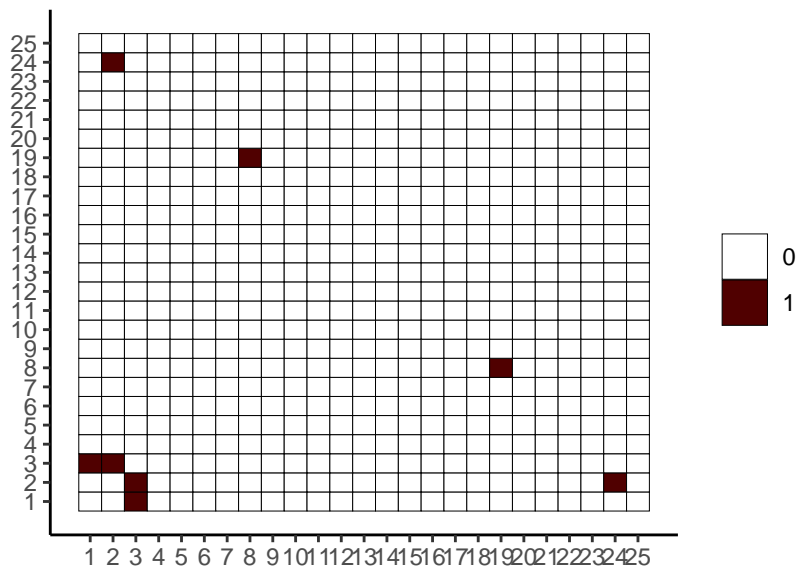
```
##
## $trial31$M3$unique_graphs[[13]]
```

Graph 13, Individuals 120,...,126



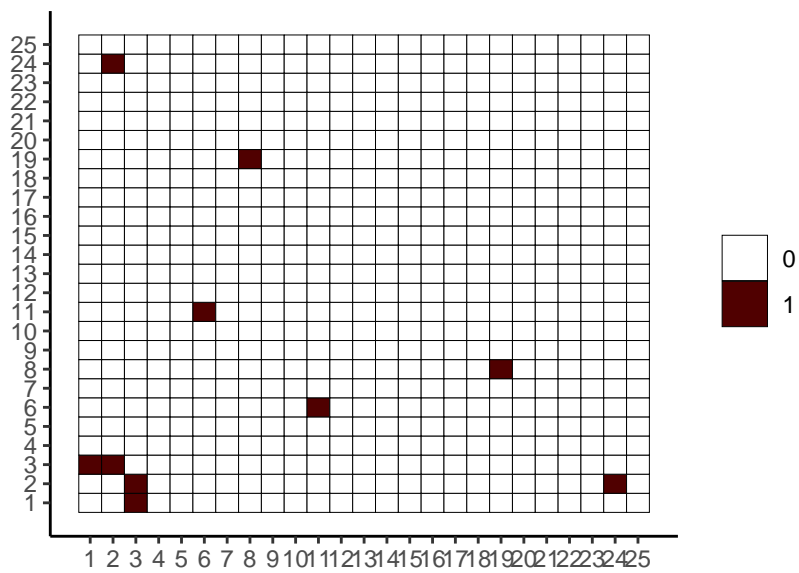
```
##
## $trial31$M3$unique_graphs[[14]]
```

Graph 14, Individuals 127,...,149



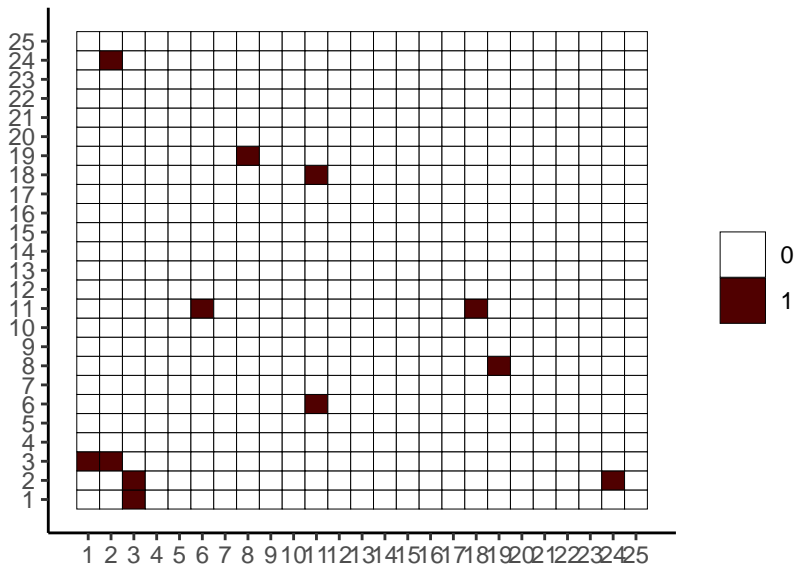
```
##
## $trial31$M3$unique_graphs[[15]]
```

Graph 15, Individuals 150,...,166



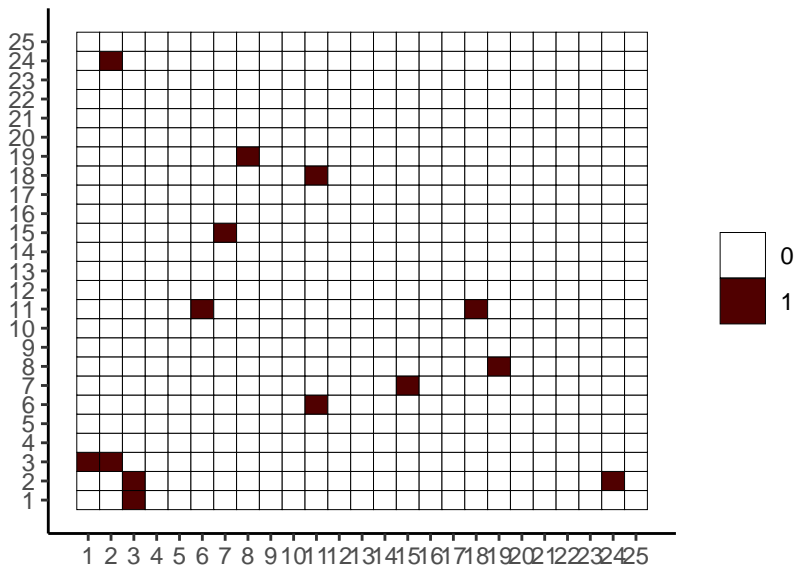
```
##
## $trial31$M3$unique_graphs[[16]]
```

Graph 16, Individuals 167



```
##
## $trial31$M3$unique_graphs[[17]]
```

Graph 17, Individuals 168,...,180



```
##
##
## $trial31$M3$sensitivity
## [1] 0.8428571
##
## $trial31$M3$specificity
```

```
## [1] 0.9874619
##
## $trial31$M3$accuracy
## [1] 0.9863822
##
## $trial31$M3$ELBO
## [1] -214790.2
##
## $trial31$M3$time
## [1] 119.1823
##
##
## $trial31$error
## NULL
```