About the 1^{st} Quiz

- It will cover:
  - Sections 1.1 - 1.4.
  - Section 1.5, <u>EXCLUDING</u> inverse trigonometric functions ($\sin^{-1}, \cos^{-1}, \tan^{-1}$)
  - Appendix D

- It will be open-book, i.e.,

  textbook, lecture notes & personal notes ✓

  calculators, phones, other aids ✗

- It will be 20-min long and at the beginning of class on Wed. <u>BE ON TIME</u>.

Domain and Range of common functions			
Name	**Formula/How it looks like**	**Domain**	**Range**
Constant Function	$f(x) = b$, for some fixed number $b \in \mathbb{R}$.	$\mathbb{R}$	$\{b\}$
Linear Function	$f(x) = ax+b$, for fixed numbers $a, b \in \mathbb{R}$, with $a \neq 0$.	$\mathbb{R}$	$\mathbb{R}$
Power Function	$f(x) = x^n$, for some $n \in \mathbb{N}$, $n \geq 2$.	$\mathbb{R}$	$\mathbb{R}$ if n is odd, $\mathbb{R}_{\geq 0} = [0, +\infty)$ if n is even.
Polynomial Function	$f(x) = a_n x^n + \ldots + a_1 x + a_0$, with $a_0, a_1, \ldots, a_n$ fixed numbers and $a_n \neq 0$.	$\mathbb{R}$	It depends!
Rational Function	$f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials.	$x \in \mathbb{R}$ such that $Q(x) \neq 0$.	It depends!
Exponential	$f(x) = e^x$	$\mathbb{R}$	$\mathbb{R}_+ = (0, +\infty)$
Logarithm	$f(x) = \ln(x)$	$\mathbb{R}_+ = (0, +\infty)$	$\mathbb{R}$
Sine	$f(x) = \sin(x)$	$\mathbb{R}$	$[-1, 1]$
Cosine	$f(x) = \cos(x)$	$\mathbb{R}$	$[-1, 1]$
Tangent	$f(x) = \tan(x)$	All real numbers but $\frac{k\pi}{2}$, where k is an odd integer. (This is where $\cos(x) = 0$.)	$\mathbb{R}$
Secant	$f(x) = \sec(x) = \frac{1}{\cos(x)}$	All real numbers but $\frac{k\pi}{2}$, where k is an odd integer. (This is where $\cos(x) = 0$.)	$(-\infty, -1] \cup [1, +\infty)$
Cosecant	$f(x) = \csc(x) = \frac{1}{\sin(x)}$	All real numbers but those of the form $k\pi$, for any integer k. (This is where $\sin(x) = 0$.)	$(-\infty, -1] \cup [1, +\infty)$
Cotangent	$f(x) = \cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$	All real numbers but those of the form $k\pi$, for any integer k. (This is where $\sin(x) = 0$.)	$\mathbb{R}$
Arcsine or Inverse Sine	$f(x) = \arcsin(x)$ or $f(x) = \sin^{-1}(x)$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Arc-cosine or Inverse Cosine	$f(x) = \arccos(x)$ or $f(x) = \cos^{-1}(x)$	$[-1, 1]$	$[0, \pi]$
Arctangent or Inverse Tangent	$f(x) = \arctan(x)$ or $f(x) = \tan^{-1}(x)$	$\mathbb{R}$	$\left(-\frac{\pi}{2}, +\frac{\pi}{2}\right)$

Lecture 3 - Trigonometry (Sep 17)

• Trigonometric functions and their inverses

Trigonometric functions are functions which take an angle as an input and spit out a number that is related to that angle in some way.

This lecture is on Appendix D & Section 1.5

We will review some things you already know about (tri)angles and add a few more layers to them.

Let's start off with a note on how we consider angles in this course. Namely, we are going to use radians to measure angles, instead of degrees.

Radians and degrees are connected by the conversion formula

$$\boxed{2\pi \text{ radians} = 360 \text{ degrees}}$$

From this we get that for example,

Get familiar with these!

π rad $= 180$ deg $\qquad \frac{\pi}{2} \left(= \frac{2\pi}{4}\right)$ rad $= 90$ deg

$\frac{\pi}{3}$ rad $= 60$ deg $\qquad \frac{\pi}{4}$ rad $= 45$ deg $\qquad \frac{\pi}{6}$ rad $= 30$ deg

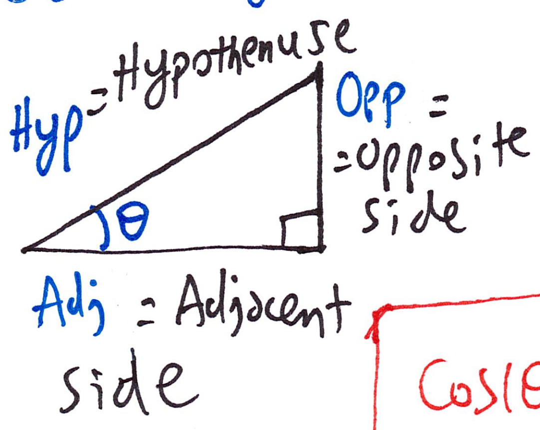
We will omit "rad" from now on when talking about angles. So, for example, we simply write

$$\frac{\pi}{4} \text{ to mean } \frac{\pi}{4}\text{ rad} = 45 \text{ deg}$$

when talking about the measure of an angle α.

"Trigonometry" means "measure of triangles", so trigonometric functions should have something to do with, guess what, triangles. Well, they do!

Let's then fix an angle θ ("theta") $(0 < \theta < \frac{\pi}{2})$ and let's consider a right-angle triangle with one of the non-right angles being θ:



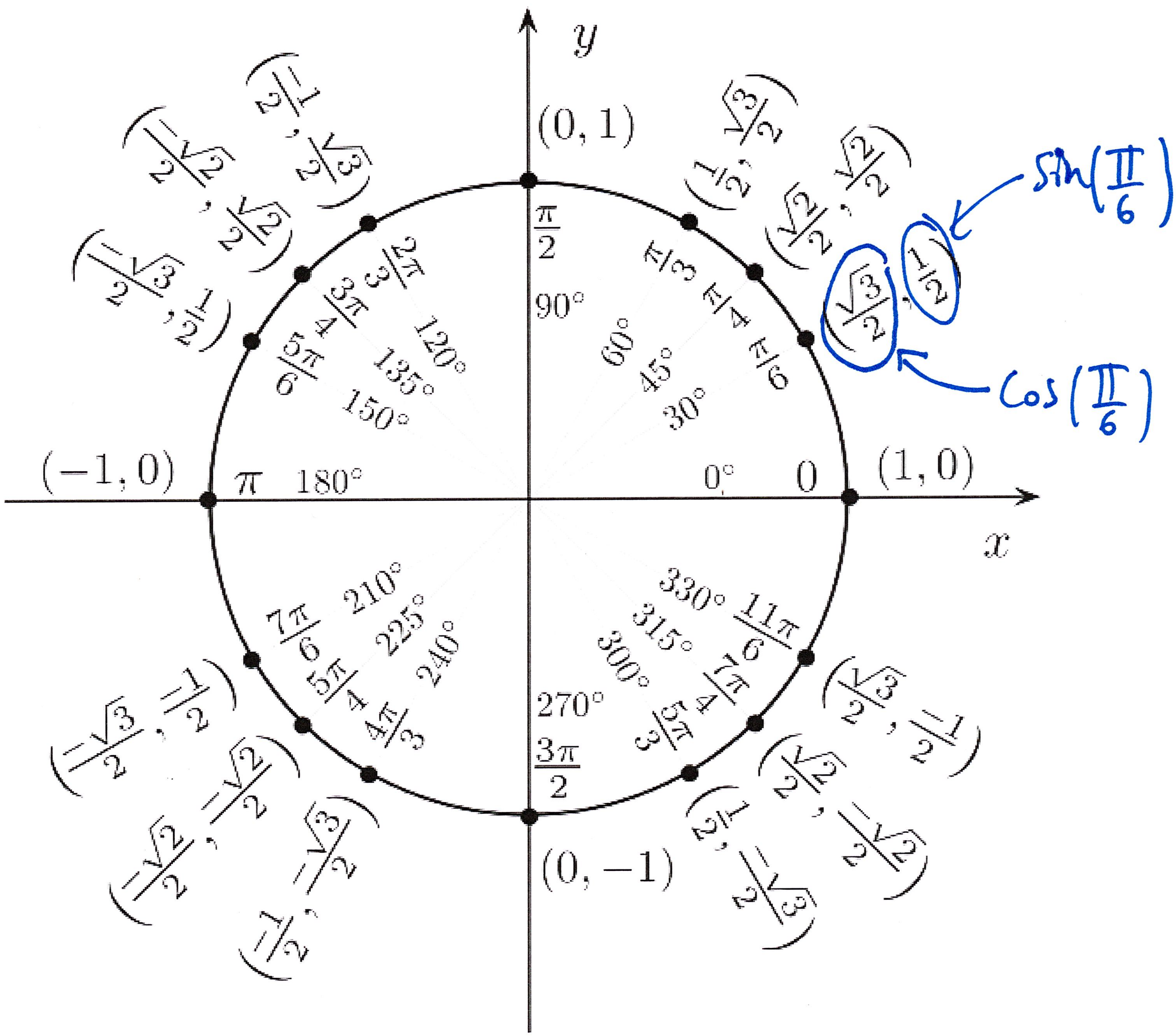
Then we define:

$$\boxed{\sin(\theta) := \frac{\text{Opp}}{\text{Hyp}}}$$ (sine of θ)

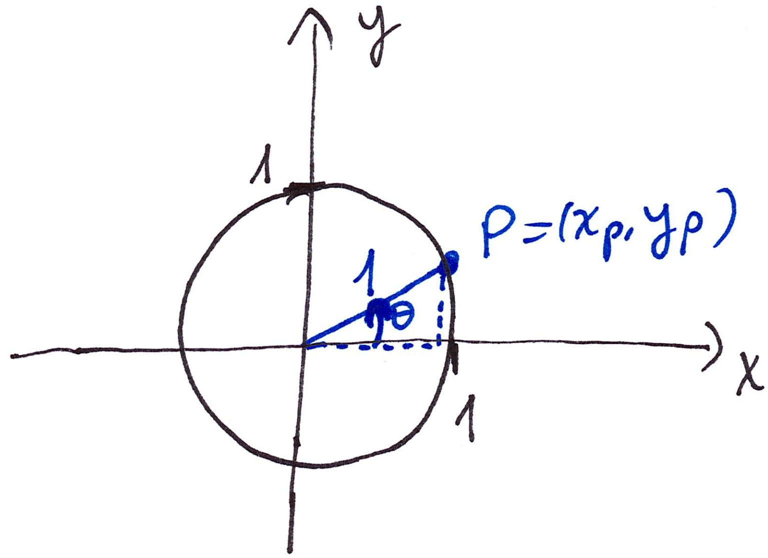
$$\boxed{\cos(\theta) = \frac{\text{Adj}}{\text{Hyp}}}$$ (cosine of θ)

$$\boxed{\tan(\theta) = \frac{\text{Opp}}{\text{Adj}}}$$ (tangent of θ)

Here are some examples of sine and cosine for some special angles. Learn these ones!

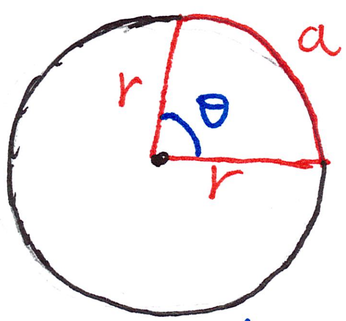


From this circle, we gathered that in a situation like:



the point P has cordinates $P=(x_p, y_p)$ with
$x_p = \cos(\theta)$
$y_p = \sin(\theta)$

Here it's important that we consider a circle of radius 1. We can use angles to compute arc-lengths.



Circular sector of radius r and angle θ

If a is the length of the arc to the left, we have the formulas

$$\boxed{\theta = \frac{a}{r}} \qquad \boxed{a = r\theta}$$

θ has to be in radians for this to work.

Ex: Find the radius r of a circular sector with angle $\frac{3\pi}{4}$ and arc length 6 cm.

Sol: $a = 6\,\text{cm}$, $\theta = \frac{3\pi}{4}$. We want to find r. From $a = r\theta$, we get
$$r = \frac{a}{\theta} = \frac{6\,\text{cm}}{\frac{3\pi}{4}} = \cancel{6}^{\,2} \cdot \frac{4}{\cancel{3}\pi} = \frac{8}{\pi}\,\text{cm}.$$

There is a 2500-year-old piece of Math which says

$$(*) \quad \boxed{Opp^2 + Adj^2 = Hyp^2}$$

<u>Pythagoras' theorem.</u>

If we divide this identity by Hyp^2, we get

$$\frac{Opp^2}{Hyp^2} + \frac{Adj^2}{Hyp^2} = \frac{\cancel{Hyp^2}\,1}{\cancel{Hyp^2}\,1}$$

$$\underbrace{\left(\frac{Opp}{Hyp}\right)}_{=\sin(\theta)}^2 + \underbrace{\left(\frac{Adj}{Hyp}\right)}_{=\cos(\theta)}^2 = 1$$

$$(\sin(\theta))^2 + (\cos(\theta))^2 = 1$$

We usually write $\sin^2(\theta)$ for $\sin(\theta)^2$ and $\cos^2(\theta)$ for $\cos(\theta)^2$. Hence, from Pythagora's theorem we get that

$$\boxed{\sin^2(\theta) + \cos^2(\theta) = 1} \quad (**)$$ (Pythagora's identity for angles)

for every angle θ. From $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, we can obtain other numbers associated to θ:

$\sec(\theta) = \dfrac{1}{\cos(\theta)}$ <u>secant of θ</u} $\qquad$ $\csc(\theta) = \dfrac{1}{\sin(\theta)}$ <u>cosecant of θ</u>

$\cot(\theta) = \dfrac{1}{\tan(\theta)}$ <u>cotangent of θ</u>

If we divide by $\cos^2(\theta)$ $(**)$, we get

$$\underbrace{\frac{\sin^2(\theta)}{\cos^2(\theta)}}_{\tan^2(\theta)} + \underbrace{\frac{\cos^2(\theta)}{\cos^2(\theta)}}_{1} = \underbrace{\frac{1}{\cos^2(\theta)}}_{\sec^2(\theta)}$$

So we obtain the identity:

$$\boxed{1 + \tan^2(\theta) = \sec^2(\theta)}$$

Similarly, we have $\boxed{1 + \cot^2(\theta) = \csc^2(\theta)}$

Now, all these quantities we have seen associated with an angle θ we can actually consider as **functions**.

We can do this by **thinking to a number $x \in \mathbb{R}$ as an angle of x-many radians.**

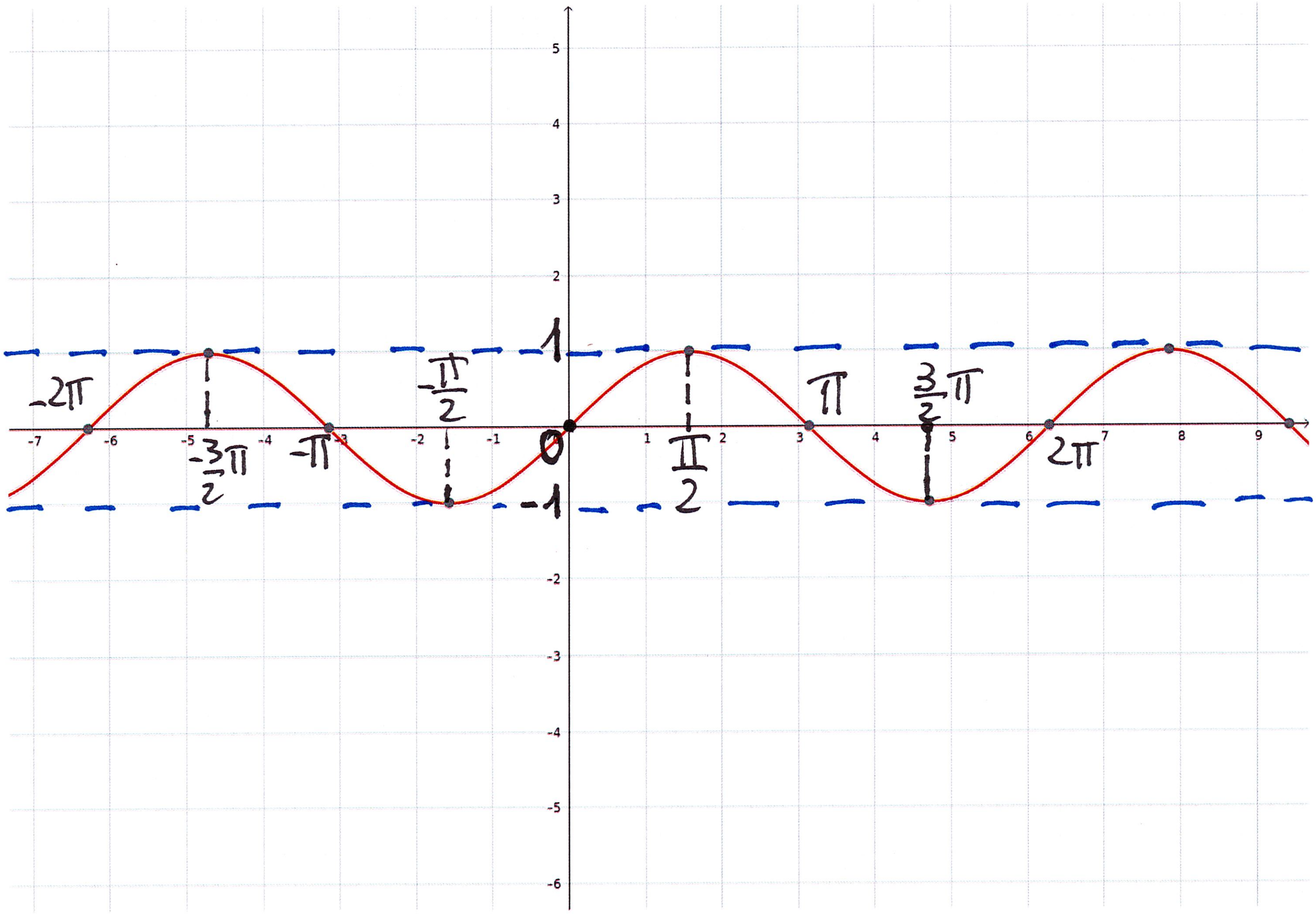
(Remember, we <u>always</u> use radians to measure angles)

Let's see how these functions look like for:

- sin (page 8)
- cos (page 9)
- tan (page 10)

The function sin.

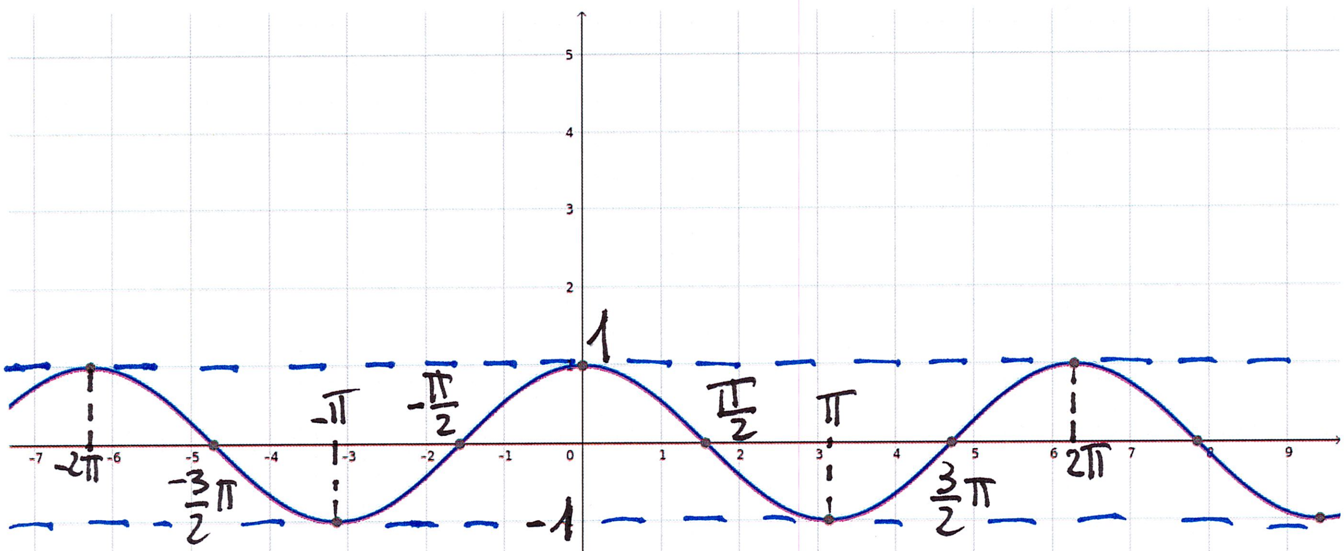
Domain of sin $= \mathbb{R}$ Range of sin $= [-1, 1]$



- Sin is an odd function: $-\sin(x) = \sin(-x)$ (graph symmetric about the origin)
- $\sin(x + 2\pi) = \sin(x)$ (For ex., the graph is the same for $x \in [-2\pi, 0]$ and for $x \in [0, 2\pi]$)
- $\sin(x + \pi) = -\sin(x)$

The function cos

Domain of cos $= \mathbb{R}$ Range of cos $= [-1,1]$



Check out the "circle with angles" on page 3 too

Compare the graphs of $\sin(x)$ & $\cos(x)$ and notice:

$\sin(x+\frac{\pi}{2}) = \cos(x)$ $\cos(x+\frac{\pi}{2}) = -\sin(x)$

$\sin(x-\frac{\pi}{2}) = -\cos(x)$ $\cos(x-\frac{\pi}{2}) = \sin(x)$

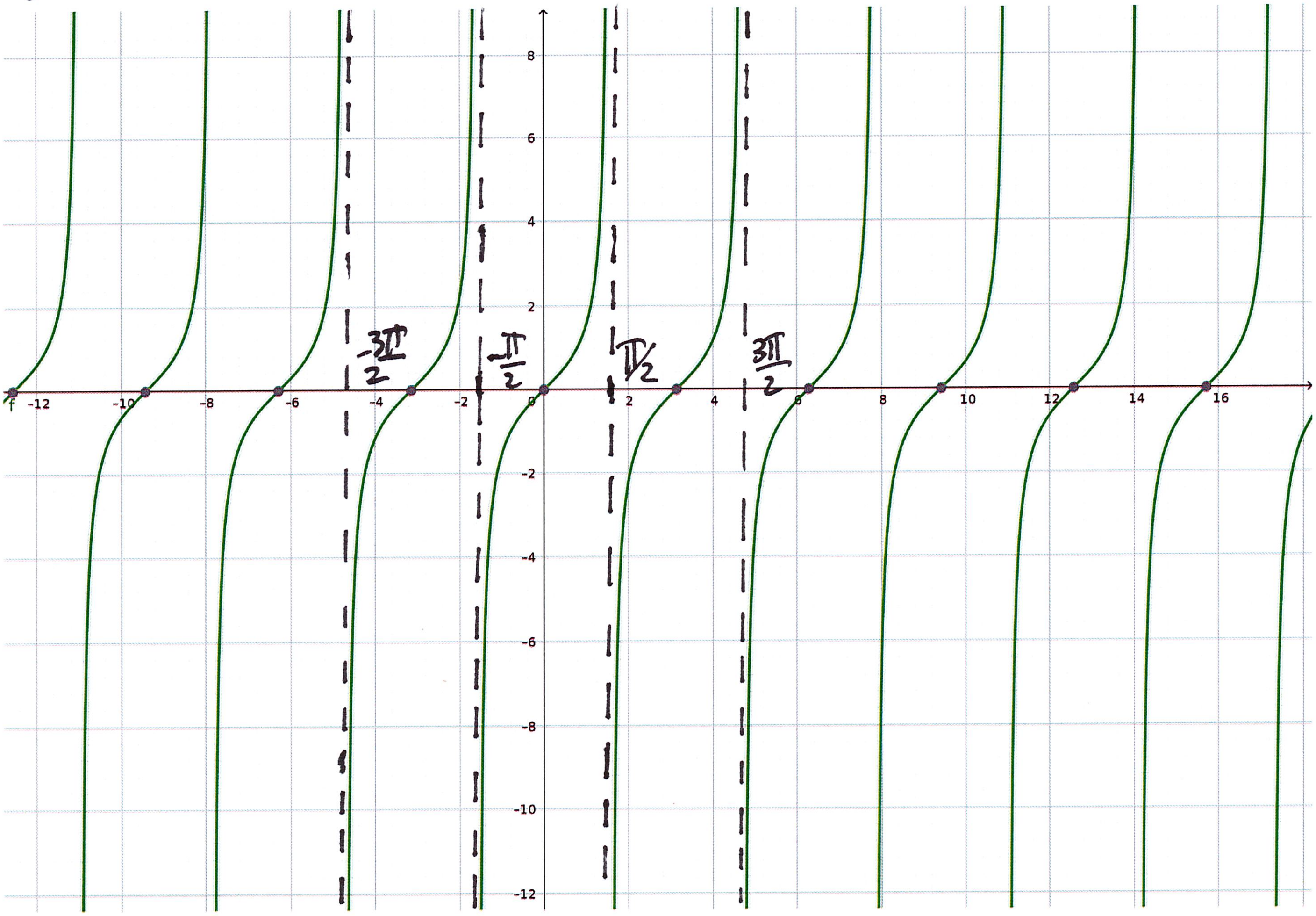
- cos is an even function: $\cos(-x) = \cos(x)$ (graph is symmetric wrt y-axis)
- $\cos(x+2\pi) = \cos(x)$
- $\cos(x+\pi) = -\cos(x)$

• The function $\tan = \frac{\sin}{\cos}$

Domain of $\tan = \mathbb{R} \setminus \{ k\frac{\pi}{2} : k$ is an odd integer$\}$

Range of $\tan = \mathbb{R}$

Remember: $\cos(x) = 0$ if $x = k\frac{\pi}{2}$ w/ k odd!



• tan is an odd function : $\tan(-x) = -\tan(x)$

• $\tan(x+\pi) = \tan(x)$ (Compare this with what happens with sin & cos)

• $\tan\left(x+\frac{\pi}{2}\right) = -\frac{1}{\tan(x)} = -\cot(x)$

• $\tan\left(x-\frac{\pi}{2}\right) = \frac{1}{\tan(x)} = \cot(x)$

(You can check the graphs of the other trig. functions – $\csc(x)$, $\sec(x)$, $\cot(x)$ – on page A31 of the book)

There's a bunch of formulas which are helpful when doing computations with sin, cos, tan. You'll be using these many times, over and over again, during this term. In other words, you should learn them. Really.

↳ Addition formulas

$$\boxed{\begin{aligned} \sin(x \pm y) &= \sin(x)\cos(y) \pm \cos(x)\sin(y) \\ \cos(x \pm y) &= \cos(x)\cos(y) \mp \sin(x)\sin(y) \end{aligned}}$$

These formulas are true for <u>every</u> pair of $x, y \in \mathbb{R}$. In particular, we can take $y = x$ in the formula for $\sin(x+y)$ and for $\cos(x+y)$ and we get more formulas. (Check this)

↳ Double-angle formulas

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) \quad \text{(version 1)}$$

$$\overset{!}{=} 1 - 2\sin^2(x) \quad \text{(version 2)}$$

$$\overset{!}{=} 2\cos^2(x) - 1 \quad \text{(version 3)}$$

$\cos(2x) = 1 - 2\sin^2(x) \Rightarrow \sin^2(x) = \frac{1-\cos(2x)}{2}$ - Similarly for $\cos^2(x)$

Exercise: Use the identity $\sin^2(x) + \cos^2(x) = 1$ to derive version 2 and version 3 from version 1 above.

We almost forgot about our friend tan, didn't we? She gets her formula too.

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

Exercise: Use the addition formulas to derive this one for tan. At some point, you might want to use that $1 = \frac{\cos(x)\cos(y)}{\cos(x)\cos(y)}$ ....

We can use these identities / formulas to solve equations involving cos, sin, tan.

Ex: Solve $\cos(2x) + 3\sin(x) - 2 = 0$, for $x \in [0, 2\pi]$

Idea: try to only have "sin" or "cos".

$$\cos(2x) = \begin{cases} \to \cos^2(x) - \sin^2(x) & \times \text{ (both cos \& sin)} \\ \to 2\cos^2(x) - 1 & \times \text{ (we don't "get rid" of cos)} \\ \to 1 - 2\sin^2(x) & \checkmark \end{cases}$$

$$\cos(2x) + 3\sin(x) - 2 = 0 \iff 1 - 2\sin^2(x) + 3\sin(x) - 2 = 0$$

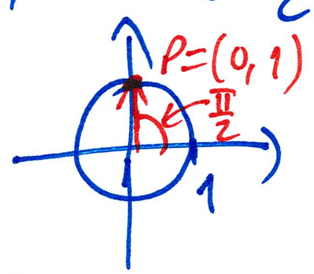
$$\iff 2\sin^2(x) - 3\sin(x) + 1 = 0$$

Notice: This eq. is $2t^2 - 3t + 1 = 0$ where $t \overset{s}{=} \sin(x)$. We solve it as a "normal" eq. in t and remember that $t = \sin(x)$.

$$2t^2 - 3t + 1 = 0 \qquad t_{1,2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4} = \begin{cases} t_1 = 1 \\ t_2 = \frac{1}{2} \end{cases}$$

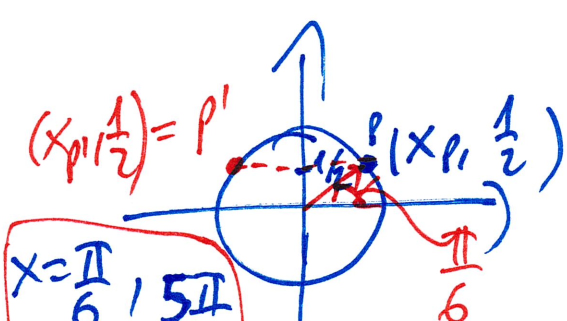
w/ $x \in [0, 2\pi)$

- $t_1 = 1$: $\sin(x) = 1 \Rightarrow \boxed{x = \frac{\pi}{2}}$



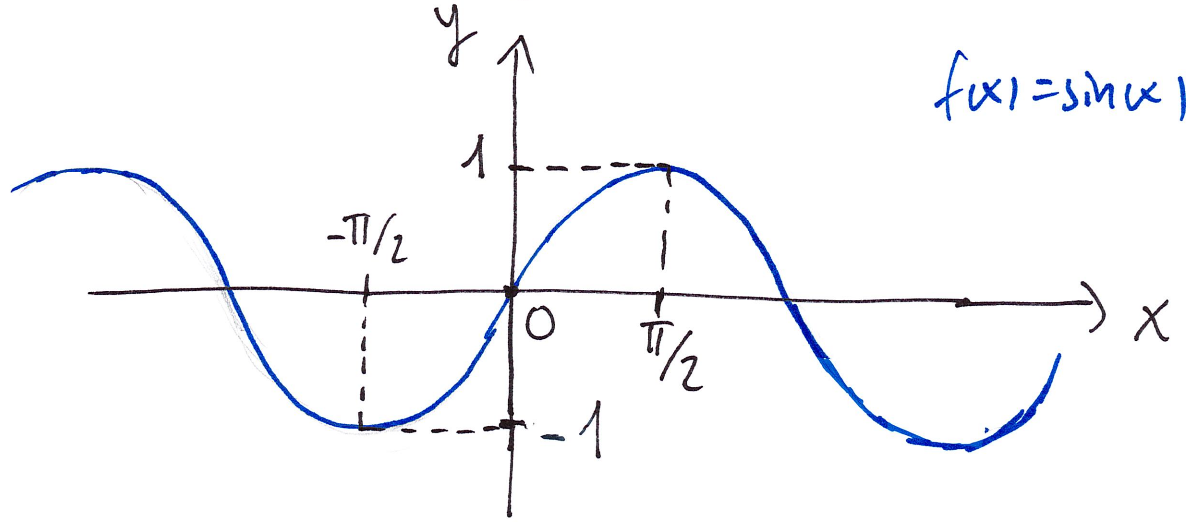
- $t_2 = \frac{1}{2}$: $\sin(x) = \frac{1}{2}$

$x \in [0, 2\pi) \Rightarrow \boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$



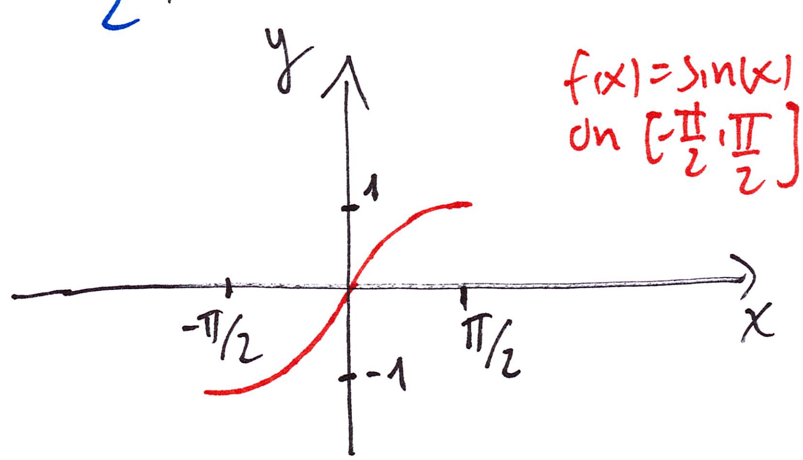
• Inverse trigonometric functions

Let's take another glimpse at $\sin(x)$, shall we?



If you ask me, this looks very much **NOT one-to-one**. Remember what we did with $f(x) = x^2$? Also that one wasn't one-to-one. Yet, we found an inverse for it ($f^{-1}(x) = \sqrt{x}$) by **changing its domain** so that it became 1-to-1.

On $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, sin looks like



This looks pretty much **one-to-one**.

On $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $\sin(x)$ is one-to-one, so we can invert it!

The inverse function to $\sin(x)$ is denoted $\left(\frac{1}{\sin(x)} \neq\right) \sin^{-1}(x)$ or $\arcsin(x)$.

Its domain is $[-1,1]$ (= range of $\sin(x)$) and its range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

So, for $x \in [-1,1]$, we have that:

- $\sin^{-1}(x)$ is a number in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ which we can think of as an angle θ with $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$;
- this angle $\theta = \sin^{-1}(x)$ is the only angle $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ such that $\sin(\theta) = x$.

The formulas $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$ for $f = \sin$ read:

$\sin^{-1}(\sin(x)) = x$ for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

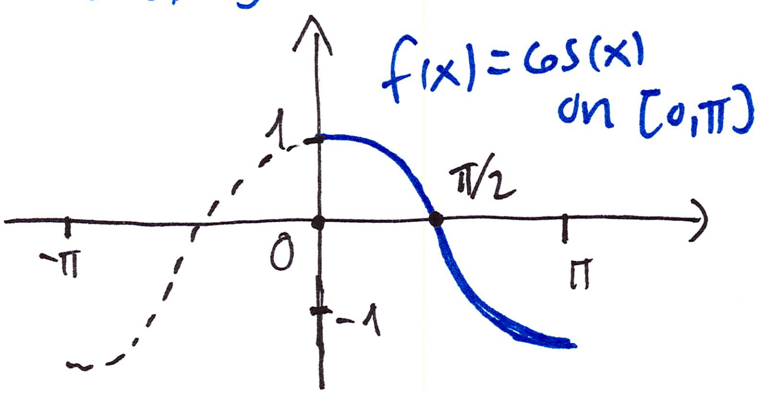
$\sin(\sin^{-1}(x)) = x$ for $x \in [-1,1]$

For ex, $\sin^{-1}(1) = \frac{\pi}{2}$ because $\alpha = \frac{\pi}{2}$ is the only angle $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ such that $\sin(\alpha) = 1$.

We can do the same operation we did with sin also with cos and tan: we can shrink the domains of cos and tan to make them one-to-one.

Cos

On $[0, \pi]$



The inverse function to cos is $\left(\frac{1}{\cos(x)} \neq\right) \cos^{-1}(x)$ or $\arccos(x)$

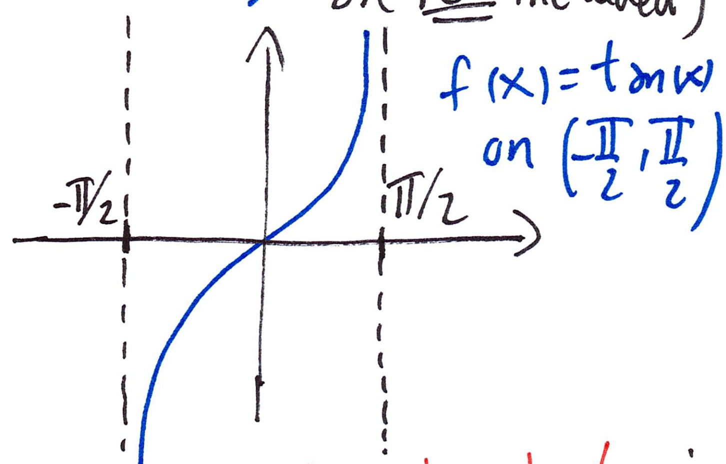
- Domain of $\cos^{-1}$: $[-1, 1]$
- Range of $\cos^{-1}$: $[0, \pi]$
- $\cos^{-1}(\cos(x)) = x, \quad x \in [0, \pi]$
- $\cos(\cos^{-1}(x)) = x, \quad x \in [-1, 1]$

Ex: $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

because $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Tan

On $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (Note the round brackets: $\pm\frac{\pi}{2}$ are not included)



The inverse function to tan is $\left(\frac{1}{\tan(x)} \neq\right) \tan^{-1}(x)$ or $\arctan(x)$

- Domain of $\tan^{-1}(x)$: $\mathbb{R}$ $(=(-\infty, +\infty))$
- Range of $\tan^{-1}(x)$: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- $\tan^{-1}(\tan(x)) = x, \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- $\tan(\tan^{-1}(x)) = x, \quad x \in \mathbb{R}$

Ex: $\tan^{-1}(1) = \frac{\pi}{4}$, because

$$\tan\left(\frac{\pi}{4}\right) = \frac{\sin(\pi/4)}{\cos(\pi/4)} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1.$$

Note: Computations that involve combinations of $\sin, \cos, \tan, \sin^{-1}, \cos^{-1}, \tan^{-1}$ can be done "using trigonometric identities" or "using the triangle". Let's understand what this means with an example.

$\boxed{\text{Ex:}}$ Find $\sin\left(2\sin^{-1}\left(\frac{3}{5}\right)\right)$.

Sol: Write $\alpha = \sin^{-1}\left(\frac{3}{5}\right)$. We know two things from this:

(1) $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (because α is $\sin^{-1}$ of something and $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ = range of $\sin^{-1}$)

(2) $\sin(\alpha) = \frac{3}{5}$, because $\alpha = \sin^{-1}\left(\frac{3}{5}\right)$.

We need to compute $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$

- "Using trig identities". We know $\sin(\alpha) = \frac{3}{5}$ by (2) above. We also know $\cos^2(\alpha) + \sin^2(\alpha) = 1$.

From this, we get $\cos^2(\alpha) = 1 - \sin^2(\alpha) \Rightarrow$

$$\Rightarrow \cos(\alpha) = \pm\sqrt{1-\sin^2(\alpha)} = \pm\sqrt{1-\left(\frac{3}{5}\right)^2} = \pm\sqrt{1-\frac{9}{25}} =$$

$= \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$. From (1), we know that $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow \cos(\alpha) = \frac{4}{5}$ ($\cos(x) \geq 0$ when $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$).

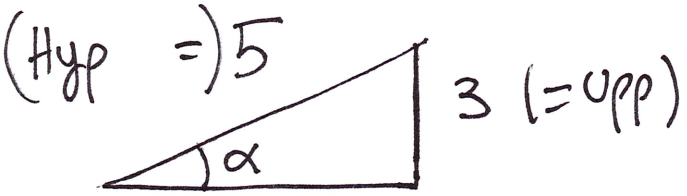
So: $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$.

• "Using the triangle". Remember

$$\sin(\alpha) = \frac{Opp}{Hyp}$$

when we consider a right-angle triangle with one of the non-right angles being θ.

In our case, because $\sin(\alpha) = \frac{3}{5}$, we have:



We know that

$$\cos(\alpha) = \frac{Adj}{Hyp}$$

But Adj is just the (length of) the missing side of the triangle! We can find this with Pythagoras's thm:

$$Adj^2 + 3^2 = 5^2 \Rightarrow Adj^2 = 16 \Rightarrow Adj = 4$$

(because a side-length can never be < 0)

So $\cos(\alpha) = \frac{4}{5}$ and we get again

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}.$$

$\boxed{\text{Ex:}}$ Compute $\tan(\cos^{-1}(x))$ (for $x \in [-1,1]$).

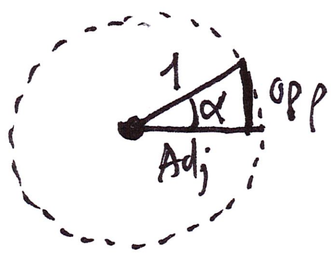
"Using the triangle". We know that, if

$$\alpha = \cos^{-1}(x),$$

then: (1) $\alpha \in [0, \pi]$ ($0 \leq \alpha \leq \pi$)

(2) $\cos(\alpha) = x$.

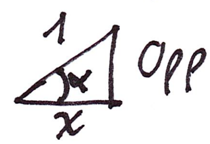
We want to draw a right-angle triangle with one of the non-right angles being α and use that $\cos(\alpha) = \frac{\text{Adj}}{\text{Hyp}}$ and $\cos(\alpha) = x$. We can choose to draw the triangle so that $\text{Hyp} = 1$:



If $\text{Hyp} = 1$, then

$$x = \cos(\alpha) = \frac{\text{Adj}}{\text{Hyp}} = \frac{\text{Adj}}{1} = \text{Adj},$$

So $\text{Adj} = x$:



Then, using Pythagoras's thm: $\text{Opp}^2 + x^2 = 1 \Rightarrow$

$\Rightarrow \text{Opp}^2 = 1 - x^2 \Rightarrow \text{Opp} = \sqrt{1-x^2}$. We have:

$\sin(\alpha) = \frac{\text{Opp}}{\text{Hyp}} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$. Then:

(Try to do this "using trig. identities")

$$\boxed{\tan(\cos^{-1}(x))} = \tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \boxed{\frac{\sqrt{1-x^2}}{x}}$$

Let's do an exercise that puts together several things we have seen in the various lectures so far.

Ex: Consider the function

$$f(x) = e^{\sin^{-1}(x)}$$

defined for $x \in [-1,1]$.

(a) Find the range of f.

(b) Find a formula for f^{-1}.

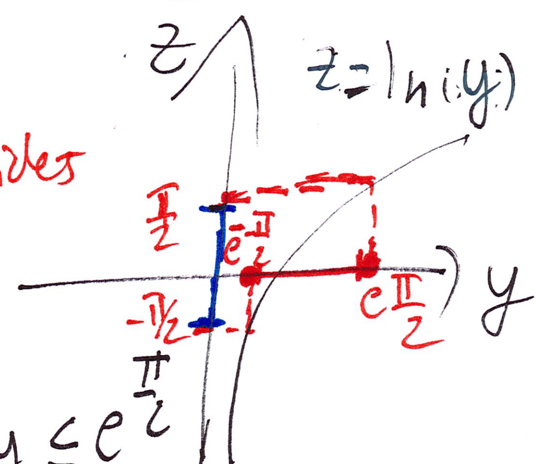
We can solve (a) & (b) together at the same time. The recipe for both (a) and (b) starts by writing $y = e^{\sin^{-1}(x)}$ and then solving for x in terms of y. Keeping track of the conditions we need to impose on y at each step will allow us to find the range.

- Apply $\ln$ to "get rid of e". To do this, we need to be sure that y is in the domain of $\ln = (0, +\infty)$. So, we need $\boxed{y > 0}$. So, for $y > 0$, $\ln(y) = \ln\left(e^{\sin^{-1}(x)}\right) = \sin^{-1}(x)$

• Apply the inverse of $\sin^{-1}$ to "get rid of $\sin^{-1}$". the inverse of $\sin^{-1}$ is.... sin, but <u>ONLY</u> on $[-\frac{\pi}{2}, \frac{\pi}{2})$. So, in order to apply sin, we need to have

$$\ln(y) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right) \iff$$

$$\iff -\frac{\pi}{2} \le \ln(y) \le \frac{\pi}{2}$$



Now:

(i) $\ln(y) = -\frac{\pi}{2} \iff y = e^{-\frac{\pi}{2}}$

↑Apply e to both sides

(ii) $\ln(y) = \frac{\pi}{2} \iff y = e^{\frac{\pi}{2}}$

$$-\frac{\pi}{2} \le \ln(y) \le \frac{\pi}{2} \iff e^{-\frac{\pi}{2}} \le y \le e^{\frac{\pi}{2}}$$

(This is analogous to trying to solve something like $4 \le x^2 \le 5$)

So, when $\boxed{y \in [e^{-\frac{\pi}{2}}, e^{\frac{\pi}{2}}]}$,

$$\ln(y) = \sin^{-1}(x) \iff x = \sin(\ln(y))$$

Apply sin

↑formula for f^{-1} (exchange x and y and get $y = \sin(\ln(x))$)

Range $= [e^{-\frac{\pi}{2}}, e^{\frac{\pi}{2}}]$

(If something is between $e^{-\frac{\pi}{2}}$ and $e^{\frac{\pi}{2}}$, it is >0 already).

For next class:

- prepare for the Quiz, that is:
  (1) Understand and know the definitions of what is covered in the quiz.
  (2) Do the suggested ex. for the parts covered in the quiz.
  (3) Look back at what we have seen in class, including examples we did together.
- Start doing the suggested exercises on Section 1.5 about $\sin^{-1}, \cos^{-1}, \tan^{-1}$
- During next class, we will cover part of the material in sections 2.2, 2.3 & 2.5 of the book, so you can check those out if you want a preview of our next lecture.