Important limits

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Polynomials and Rational Functions

- $\lim_{x\to\infty} x^n = \infty$
- $\lim_{x\to-\infty} x^n = \begin{cases} \infty & \text{if } n \text{ is even} \\ -\infty & \text{if } n \text{ is odd} \end{cases}$
- If $r = \frac{p}{q} > 0$, $\lim_{x\to\infty} \frac{1}{x^r} = 0$
- If $r = \frac{p}{q} > 0$ and x^r is defined when $x < 0$, then $\lim_{x\to-\infty} \frac{1}{x^r} = 0$
- If n is an even positive integer, then $\lim_{x\to 0} \frac{1}{x^n} = \infty$
- If n is an odd positive integer, then:
  - $\lim_{x\to 0^+} \frac{1}{x^n} = \infty$;
  - $\lim_{x\to 0^-} \frac{1}{x^n} = -\infty$.

Exponentials and Logarithms

- $\lim_{x\to\infty} a^x = \begin{cases} \infty & \text{if } a > 1 \\ 0 & \text{if } 0 < a < 1 \end{cases}$
- $\lim_{x\to-\infty} a^x = \begin{cases} 0 & \text{if } a > 1 \\ \infty & \text{if } 0 < a < 1 \end{cases}$
- In particular, $\lim_{x\to\infty} e^x = \infty$ and $\lim_{x\to-\infty} e^x = 0$.
- $\lim_{x\to\infty} \log_b(x) = \begin{cases} \infty & \text{if } b > 1 \\ -\infty & \text{if } 0 < b < 1 \end{cases}$
- $\lim_{x\to 0^+} \log_b(x) = \begin{cases} -\infty & \text{if } b > 1 \\ \infty & \text{if } 0 < b < 1 \end{cases}$
- In particular, $\lim_{x\to 0^+} \ln(x) = -\infty$ and $\lim_{x\to\infty} \ln(x) = \infty$

Trigonometric Functions and their inverses

- $\lim_{x\to\infty} \sin(x)$ DNE, $\lim_{x\to-\infty} \sin(x)$ DNE
- $\lim_{x\to\infty} \cos(x)$ DNE $\lim_{x\to-\infty} \cos(x)$ DNE
- If k is an odd integer, $\lim_{x\to\left(\frac{\pi k}{2}\right)^+} \tan(x) = -\infty$, $\lim_{x\to\left(\frac{\pi k}{2}\right)^-} \tan(x) = \infty$
- $\lim_{x\to\infty} \arctan(x) = \frac{\pi}{2}$, $\lim_{x\to-\infty} \arctan(x) = -\frac{\pi}{2}$.

Other remarkable limits

- $\lim_{h\to 0}(1+h)^{\frac{1}{h}} = e$, $\lim_{x\to\infty}(1+\frac{1}{x})^x = e$
- $\lim_{h\to 0} \frac{e^h-1}{h} = 1$
- $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$
- $\lim_{x\to 0} \frac{\cos(x)-1}{x} = 0$