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Design of a Linkage System to Write in Cursive

This paper presents a design methodology for a system of linkages that can trace planar Bezier curves that represent cursive handwriting of the alphabet and Chinese characters. This paper shows that the standard degree n *Bezier curve can be reparameterized so that it takes the form of a trigonometric curve that can be drawn by a one degree-of-freedom coupled serial chain consisting of* 2n *links. A series of cubic Bezier curves that define a handwritten name yields a series of six-link coupled serial chains that trace these curves. We then show how to simplify this system using cubic trigonometric Bezier curves to obtain a series of four-link serial chains that approximate the system of Bezier curves. The result is a methodology for the design of a mechanical system that draws complex plane curves such as the cursive alphabet and Chinese characters.*
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1 Introduction

Kempe's universality theorem [1] states that for every real algebraic curve, there exists a linkage that draws the curve, which means every Bezier curve has an associated linkage constructed using Kempe's approach [2]. However, this construction yields a complex linkage even for the case of cubic curves [3].

In this paper, we show that a degree n Bezier curve can be drawn by a coupled serial chain with $2n$ links. A coupled serial chain is a serial chain constructed so each of the joints is driven by the base joint such that the system has one degree-of-freedom [4,5]. Therefore, a cubic Bezier curve is drawn by a six-link coupled serial chain.

Finally, we use the shape parameter in cubic trigonometric Bezier curves to simplify the system by eliminating two links to obtain four-link serial chains that draw the desired curves. We apply this to the design of linkage systems that generate cursive alphabet and script Chinese characters.

2 Literature Review

Recent research on Kempe's universality theorem has yielded drawing linkages for rational plane and space curves [6–8] by factoring a suitable motion polynomial in the Clifford algebra of dual quaternions [9]. Bezier curves formed from dual quaternions with trigonometric weighting functions [10–12] have been used for robot path planning.

Ge et al. [13] modified the polynomial formulation of Bezier curves using trigonometric weighting functions, which they called harmonic rational Bezier curves, in order to eliminate higher harmonics. Sanchez-Reyes [14] and Han [15] reformulated harmonic Bezier curves as trigonometric Bezier curves to draw plane curves. More recently, Juhasz and Roth [16] showed how to use trigonometric spline curves for interpolation.

Our use of cubic trigonometric Bezier curves follows the formulation of Han [17], who used four control points to define a curve that has the same properties as cubic polynomial Bezier curves [18]. Therefore, in this paper, we define a system of plane curves using cubic Bezier curve segments and then use the results of Liu and McCarthy [19] to design a mechanical system that draws the curves.

Our examples consist of cursive alphabet [20] and Chinese characters [21] that are drawn with a set of continuous strokes that form a series plane curves. We design planar systems of coupled serial chains that draw these curves using a single input.

The contribution of this research is an extension of computer-aided mechanism design techniques described by Sonawale and McCarthy [22] and Ge et al. [23] to achieve devices that draw complex curves, which has a range of applications such as physical models for profile curves that define tolerance zones [24], kinematic models for protein movement [25], and mechanical devices that support handwriting rehabilitation [26,27] and handwriting recognition [28].

3 Degree *n* Bezier Curve

In this section, we show that a degree n Bezier curve can be reparameterized to obtain a curve defined by component functions that are Fourier series, known as a trigonometric curve [15]. The equation for degree n Bezier curve is given by [29]

$$\text{Bezier}(n,t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i \mathbf{P}_i, \quad 0 \leqslant t \leqslant 1 \tag{1}$$

We introduce the reparameterization $t = \phi(\theta)$ given by

$$t = \phi(\theta) = \sin\theta, \quad 0 \leqslant \theta \leqslant \pi/2 \tag{2}$$

Note that $\phi(\theta)$ ranges from $0 \leqslant t \leqslant 1$ as required by Eq. (1). This allows us to write the Bezier curve as

$$\text{Bezier}(n,\phi(\theta)) = \sum_{i=0}^{n} \binom{n}{i} (1-\sin\theta)^{n-i} (\sin\theta)^i \mathbf{P}_i, \quad 0 \leqslant \theta \leqslant \pi/2 \tag{3}$$

The powers $\sin^{n-i}\theta$ and $\sin^i\theta$ can be reduced to sine and cosine of multiples k of the angle θ [30]. For example, the formulas for $\sin^2\theta$, $\cos^2\theta$, $\sin^3\theta$, and $\cos^3\theta$ are given by

$$\begin{aligned} \sin^2\theta &= \frac{1}{2}(1-\cos 2\theta), \quad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta) \\ \sin^3\theta &= \frac{1}{4}(3\sin\theta - \sin 3\theta), \quad \cos^3\theta = \frac{1}{4}(3\cos\theta + \cos 3\theta) \end{aligned} \tag{4}$$

Therefore, the component functions of a degree n Bezier curve are defined by a finite Fourier series in θ, which means it is a trigonometric curve [15]. Liu and McCarthy [19] showed that a trigonometric curve $\mathbf{p}(\theta)$ defined by Fourier series is given by

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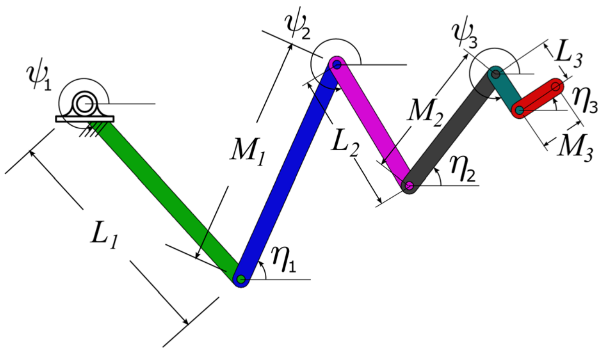

Fig. 1 The six-link coupled serial chain is constructed so that each sequence of two links, L_k and M_k, rotates in opposite directions at a multiple k of the base joint rate. The phase angles, ψ_k and η_k, define the initial configuration of the trigonometric Bezier curve.

$$\mathbf{p}(\theta) = \sum_{k=0}^{n} \mathbf{a}_k \cos k\theta + \sum_{k=0}^{n} \mathbf{b}_k \sin k\theta \tag{5}$$

where $\mathbf{a}_k = (a_{xk}, a_{yk})$ and $\mathbf{b}_k = (b_{xk}, b_{yk})$, $k = 0, \ldots, n$ are the real coefficients and can be drawn by a coupled serial chain consisting of $2n$ links.

For the particular case of a cubic Bezier curve for which $n = 3$, we obtain

$$\begin{aligned} &\text{CubicBezier}(\phi(\theta)) \\ &\quad = \sum_{i=0}^{3} \binom{3}{i} (1-\sin\theta)^{3-i} (\sin\theta)^i \, \mathbf{P}_i, \quad 0 \leqslant \theta \leqslant \pi/2 \end{aligned} \tag{6}$$

Expand the powers of $\sin\theta$ to obtain

$$\begin{aligned} &\text{CubicBezier}(\phi(\theta)) \\ &\quad = (2.5\mathbf{P}_0 - 3\mathbf{P}_1 + 1.5\mathbf{P}_2) + (-1.5\mathbf{P}_0 + 3\mathbf{P}_1 - 1.5\mathbf{P}_2)\cos 2\theta \\ &\qquad + (-3.75\mathbf{P}_0 + 5.25\mathbf{P}_1 - 2.25\mathbf{P}_2 + 0.75\mathbf{P}_3)\sin\theta \\ &\qquad + (0.25\mathbf{P}_0 - 0.75\mathbf{P}_1 + 0.75\mathbf{P}_2 - 0.25\mathbf{P}_3)\sin 3\theta \end{aligned} \tag{7}$$

This curve is drawn by the coupled serial chain constructed with six-links, see Fig. 1. The links L_1, L_2, and L_3 rotate counterclockwise, while the links M_1, M_2, and M_3 rotate clockwise, with the joint angles θ, 2θ, and 3θ, respectively. The initial configuration of the chain is defined by the phase angles ψ_1, ψ_2, and ψ_3 and η_1, η_2, and η_3. The dimensions of the eight six-link chains that draw "Yang" are listed in Table 1.

4 Cubic Trigonometric Bezier Curve

A cubic trigonometric Bezier curve has weighting functions that are cubic polynomials of sine and cosine functions [17]. For

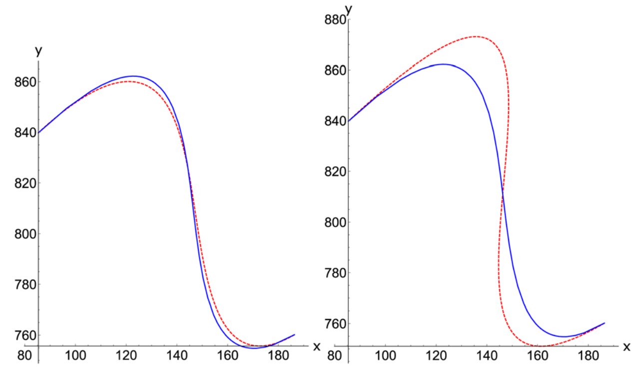

Fig. 2 Cubic trigonometric Bezier curve with $\lambda = -0.5$ in dashed compared to the cubic Bezier curve in solid and cubic trigonometric Bezier curve with $\lambda = 0.5$ in dashed compared to the cubic Bezier curve in solid

the same four control points, it is the parameterized curve given by

$$\begin{aligned} &\text{CubicTrigBezier}(\theta, \lambda) \\ &\quad = (1-\sin\theta)^2(1-\lambda\sin\theta)\mathbf{P}_0 + \sin\theta(1-\sin\theta) \\ &\qquad \times (2+\lambda(1-\sin\theta))\mathbf{P}_1 + \cos\theta(1-\cos\theta) \\ &\qquad \times (2+\lambda(1-\cos\theta))\mathbf{P}_2 + (1-\cos\theta)^2 \\ &\qquad \times (1-\lambda\cos\theta)\mathbf{P}_3, \quad 0 \leqslant \theta \leqslant \pi/2 \end{aligned} \tag{8}$$

The parameter λ is called the shape parameter. For example, consider the cubic Bezier curve defined by the control points

$$\begin{aligned} \mathbf{P}_0 &= \begin{bmatrix} 85.37 \\ 839.79 \end{bmatrix}, \quad \mathbf{P}_1 = \begin{bmatrix} 180.60 \\ 929.26 \end{bmatrix}, \\ \mathbf{P}_2 &= \begin{bmatrix} 116.57 \\ 719.11 \end{bmatrix}, \quad \mathbf{P}_3 = \begin{bmatrix} 186.35 \\ 760.16 \end{bmatrix} \end{aligned} \tag{9}$$

Figure 2 shows the comparison of the cubic Bezier curves with cubic trigonometric Bezier curves that have $\lambda = -0.5$ and $\lambda = 0.5$, respectively.

5 Compute the Coupled Serial Chain

Once each of the Bezier curves is written in the form of a trigonometric curve, we can use the results of Liu and McCarthy [19] to design the associated coupled serial chain. Expanding the powers of sine and writing the trigonometric Bezier curve in the form

$$\begin{aligned} \text{CubicTrigBezier}(\theta, \lambda) &= \mathbf{a}_0 + \mathbf{a}_1\cos\theta + \mathbf{a}_2\cos 2\theta + \mathbf{a}_3\cos 3\theta \\ &\quad + \mathbf{b}_0 + \mathbf{b}_1\sin\theta + \mathbf{b}_2\sin 2\theta + \mathbf{b}_3\sin 3\theta \end{aligned} \tag{10}$$

Table 1 Six-bar serial chain ground pivots, link lengths (mm), and phase angles to draw Yang

Curve j	$\mathbf{G}_j$	L_1^j	ψ_1^j	M_1^j	η_1^j	L_2^j	ψ_2^j	M_2^j	η_2^j	L_3^j	ψ_3^j	M_3^j	η_3^j
1	(−153.51, 390.36)	424.26	−0.64	424.26	2.50	254.49	1.08	254.49	1.08	77.99	2.65	77.99	−0.49
2	(−140.11, 845.14)	302.04	−1.70	302.04	1.45	168.67	−0.25	168.67	−0.25	67.36	0.69	67.36	−2.45
3	(−1.99, 1752.64)	897.66	−2.92	897.66	0.22	513.13	−1.35	513.13	−1.35	125.61	0.43	125.61	−2.71
4	(219.63, 178.15)	513.37	−0.13	513.37	3.01	308.40	1.40	308.40	1.40	104.75	2.81	104.75	−0.33
5	(299.55, 1232.29)	379.33	−3.05	379.33	0.02	223.21	−1.52	223.21	−1.52	78.73	0.61	78.73	−3.12
6	(211.30, 268.16)	456.62	−0.35	456.62	2.79	179.71	1.30	257.70	1.30	75.92	2.82	75.92	−0.32
7	(495.31, 459.73)	282.42	0.15	282.42	−3.00	74.43	1.66	179.71	1.66	69.87	3.03	69.87	−0.11
8	(157.96, 1455.96)	679.62	−2.75	679.62	0.39	361.73	−1.13	361.73	−1.13	86.67	0.86	86.67	−2.28

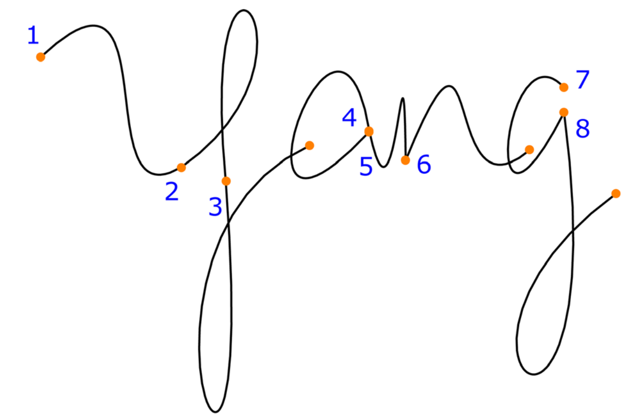

Fig. 3 The cursive letters that define Yang are defined by eight cubic Bezier curves listed in Table 2

where the coefficient vectors $\mathbf{a}_k$ and $\mathbf{b}_k$, $i = 0, 1, 2, 3$, are given by

$$\begin{aligned}
\mathbf{a}_0 &= \left(\frac{3}{2}+\lambda\right)\mathbf{P}_0 - (1+\lambda)\mathbf{P}_1 - (1+\lambda)\mathbf{P}_2 + \left(\frac{3}{2}+\lambda\right)\mathbf{P}_3 \\
\mathbf{a}_1 &= \left(2+\frac{7}{4}\lambda\right)\mathbf{P}_2 - \left(2+\frac{7}{4}\lambda\right)\mathbf{P}_3 \\
\mathbf{a}_2 &= \left(-\frac{1}{2}-\lambda\right)\mathbf{P}_0 + (1+\lambda)\mathbf{P}_1 - (1+\lambda)\mathbf{P}_2 + \left(\frac{1}{2}+\lambda\right)\mathbf{P}_3 \\
\mathbf{a}_3 &= \frac{1}{4}\lambda\mathbf{P}_2 - \frac{1}{4}\lambda\mathbf{P}_3 \quad \text{and} \\
\mathbf{b}_0 &= 0 \\
\mathbf{b}_1 &= \left(-2-\frac{7}{4}\lambda\right)\mathbf{P}_0 + \left(2+\frac{7}{4}\lambda\right)\mathbf{P}_1 \\
\mathbf{b}_2 &= 0 \\
\mathbf{b}_3 &= \frac{1}{4}\lambda\mathbf{P}_0 - \frac{1}{4}\lambda\mathbf{P}_1
\end{aligned} \tag{11}$$

Liu and McCarthy [19] showed how for each value k the coefficients $\mathbf{a}_k$ and $\mathbf{b}_k$ are used to compute the magnitudes, L_k and M_k, and phase angles ψ_k and η_k of vectors defining the links of the coupled serial chain

$$\begin{aligned}
L_k &= \frac{1}{2}\sqrt{(a_{xk}+b_{yk})^2 + (a_{yk}-b_{xk})^2} \\
M_k &= \frac{1}{2}\sqrt{(a_{xk}-b_{yk})^2 + (a_{yk}+b_{xk})^2}, \quad k = 0, 1, 2, 3
\end{aligned} \tag{12}$$

and

$$\psi_k = \arctan\frac{a_{yk}-b_{xk}}{a_{xk}+b_{yk}}, \quad \eta_k = \arctan\frac{a_{yk}+b_{xk}}{a_{xk}-b_{yk}}, \quad k = 0, 1, 2, 3 \tag{13}$$

Table 3 Serial chain ground pivot positions and link lengths for Yang (unit: mm)

Curve j	$\mathbf{G}_j$	L_1^j	M_1^j	L_2^j	M_2^j	L_3^j	M_3^j
1	(110.16, 751.07)	138.90	169.68	103.63	103.63	0.21	0.17
2	(108.55, 449.37)	125.10	355.82	93.15	93.15	0.44	0.16
3	(376.38, 1190.91)	522.31	207.94	169.45	169.45	0.26	0.64
4	(460.24, 766.96)	103.86	296.30	140.99	140.99	0.37	0.13
5	(316.65, 740.14)	147.30	177.45	107.94	107.94	0.22	0.18
6	(401.51, 681.93)	137.06	209.34	105.53	105.53	0.26	0.17
7	(570.09, 887.60)	85.04	203.83	92.53	92.53	0.25	0.10
8	(602.07, 1197.44)	496.26	166.32	122.50	122.50	0.21	0.61

The result is the equation of the cubic trigonometric curve $\mathbf{s}(\theta, \lambda)$ given by

$$\mathbf{s}(\theta, \lambda) = \sum_{k=0}^{3} L_k\mathbf{u}_k + \sum_{k=0}^{3} M_k\mathbf{v}_k \tag{14}$$

where $\mathbf{u}_k$ and $\mathbf{v}_k$ are the unit vectors

$$\mathbf{u}_k = \begin{bmatrix}\cos(k\theta-\psi_k)\\ \sin(k\theta-\psi_k)\end{bmatrix}, \quad \mathbf{v}_k = \begin{bmatrix}\cos(-k\theta-\eta_k)\\ \sin(-k\theta-\eta_k)\end{bmatrix}, \quad k = 0, 1, 2, 3 \tag{15}$$

The ground pivot, $\mathbf{G}$, for the serial chain is given by

$$\mathbf{G} = \mathbf{s}(0, \lambda) = L_0\mathbf{u}_0 + M_0\mathbf{v}_0 \tag{16}$$

This shows that the cubic trigonometric Bezier curve is drawn by a six-link serial chain with three links rotating counterclockwise and three links rotating clockwise. In Sec. 6, we show how to design the coupled serial chains that draw a sequence of cubic Bezier curves.

6 Linkage System to Draw Cursive Letters

In this section, we design the system of eight coupled serial chains that draw the eight cubic Bezier curves shown in Fig. 3, which spells the name Yang in cursive letters. The control points for each of these eight curves are listed in Table 2. The link lengths of the mth coupled serial chain are obtained using Eq. (12) with $\lambda = 0.01$ for each of the eight curve segments. These lengths are listed in Table 3.

Table 3 shows that the last two links are less than 1% of the lengths of the other links in the serial chain. Therefore, we evaluate the effect of using only the first four-links to draw the Bezier curves by selecting $N = 1000$ points on the two curves and computing the maximum difference Δ between corresponding points. In order to scale this measurement, use the distance $d = |\mathbf{P}_0 - \mathbf{P}_3|$, to obtain $\varepsilon = \Delta/d$.

Table 2 Control points for the eight Bezier curves that define Yang (unit: mm)

Curve j	$\mathbf{P}_0^j$	$\mathbf{P}_1^j$	$\mathbf{P}_2^j$	$\mathbf{P}_3^j$
1	(85.37, 839.79)	(180.60, 929.26)	(116.57, 719.11)	(186.35, 760.16)
2	(186.35, 760.16)	(300.52, 838.94)	(197.06, 974.37)	(218.36, 750.31)
3	(218.36, 750.31)	(245.45, 394.84)	(125.64, 707.59)	(279.11, 775.76)
4	(321.80, 786.43)	(298.81, 927.62)	(207.71, 663.29)	(321.80, 785.61)
5	(321.80, 786.43)	(341.53, 691.99)	(346.43, 894.79)	(348.06, 765.09)
6	(348.06, 765.09)	(405.41, 906.64)	(371.58, 716.91)	(437.54, 772.47)
7	(462.17, 817.62)	(419.19, 865.06)	(398.30, 673.90)	(462.17, 799.56)
8	(462.17, 799.56)	(502.42, 493.34)	(339.86, 624.71)	(499.93, 741.28)

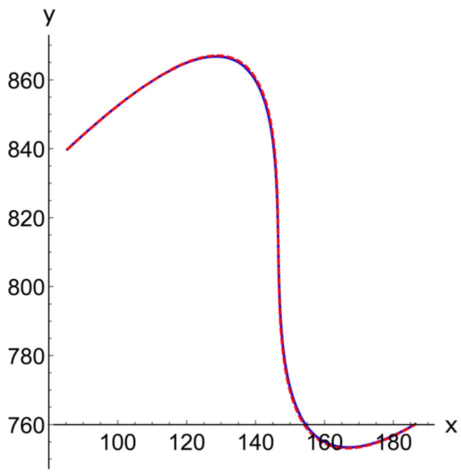

Fig. 4 Comparison of curve 1 in Yang drawn by a six-link chain (solid) and by a four-link chain (dashed)

Table 4 Gap between consecutive curves using four-link serial chain (unit: mm)

Gap	d_1^2	d_2^3	d_4^5	d_5^6	d_7^8
Value	0.45	0.76	0.85	0.24	0.50

Consider the first curve in collection that has the control points

$$\mathbf{P}_0 = \begin{bmatrix} 85.37 \\ 839.78 \end{bmatrix}, \quad \mathbf{P}_1 = \begin{bmatrix} 180.60 \\ 929.26 \end{bmatrix}, \quad \mathbf{P}_2 = \begin{bmatrix} 116.57 \\ 719.11 \end{bmatrix}, \quad \mathbf{P}_3 = \begin{bmatrix} 186.34 \\ 760.16 \end{bmatrix} \tag{17}$$

This yields a coupled serial chain with the six-link lengths

$$L_1^1 = 138.90, \quad M_1^1 = 169.68, \quad L_2^1 = 103.63, \quad M_2^1 = 103.63, \quad L_3^1 = 0.21, \quad M_3^1 = 0.17 \tag{18}$$

If we eliminate the last two links, L_3 and M_3, we obtain the difference between the resulting curves as

$$\varepsilon = \frac{\Delta}{d} = 0.002 \tag{19}$$

which means the maximum variation is less than 1% of the size of the curve. Figure 4 shows a comparison of these two curves.

The elimination of short links does not change the shape of a curve but it does introduce a gap between curves that is approximately the size of the last two links. In Sec. 6.1, we close this gap and show its effect on the resulting curves.

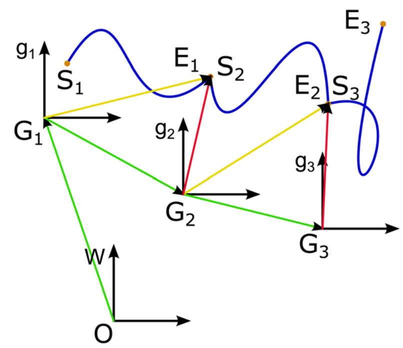

Fig. 5 An example showing the vectors used to calculate the ground pivots $\mathbf{G}_j$

6.1 Correction for Four-Link Serial Chains. Neglecting the last two links of the six-link serial chains used to draw Yang introduces a gap of length d_m^{m+1} between the end point of curve m and the start point of curve $m+1$. The size of this gap is bounded by the size of the neglected links

$$|d_m^{m+1}| \leqslant |L_3^m| + |M_3^m| \tag{20}$$

These are listed in Table 4. We eliminate this gap by adjusting the ground pivot location of each of the four-link drawing linkages.

For convenience, we introduce local coordinates for each of the m serial chains so that $\mathbf{G}_j = (0,\ 0)$, $j = 1, \ldots, m$. This yields the local coordinates of these curves

$$\mathbf{h}_j(\theta, \lambda) = L_1^j \mathbf{u}_1^j + M_1^j \mathbf{v}_1^j + L_2^j \mathbf{u}_2^j + M_2^j \mathbf{v}_2^j, \quad j = 1, \ldots, m \tag{21}$$

The start and end points, $\mathbf{S}_j$ and $\mathbf{E}_j$, of these curves are given by

$$\mathbf{S}_j = \mathbf{h}_j(0, \lambda), \text{ and } \mathbf{E}_j = \mathbf{h}_j(\pi/2, \lambda), \quad j = 1, \ldots, m \tag{22}$$

We can compute the new ground pivots for each of the serial chains relative to the ground pivot of the first chain, $\mathbf{G}_{1j}$, by requiring that the end point $\mathbf{E}_j$ of curve j be the start point $\mathbf{S}_{j+1}$ of the curve $j+1$, see Fig. 5. This yields

$$\mathbf{G}_{1j} = \sum_{i=1}^{j-1} (\mathbf{E}_i - \mathbf{S}_{i+1}), \quad j = 2, \ldots, m \tag{23}$$

Assemble the new system of curves by positioning the ground pivot of the first serial chain at the original ground pivot location to obtain

$$\bar{\mathbf{s}}_j(\theta, \lambda) = \mathbf{G}_1 + \mathbf{G}_{1j} + \mathbf{h}_j(\theta, \lambda), \quad j = 1, \ldots, m \tag{24}$$

where we consider $\mathbf{G}_{11} = 0$.

Figure 6 shows the curves $\mathbf{s}_j(\theta,\ \lambda)$ drawn by the six-link serial chains and the curves $\bar{\mathbf{s}}_j(\theta, \lambda)$ drawn by four-link serial chains. Figure 7 provides a comparison of the two curves.

Table 5 Four-link serial chain ground pivots, link lengths (mm), and phase angles to draw Yang

Curve j	$\mathbf{G}_j$	L_1^j	ψ_1^j	M_1^j	η_1^j	L_2^j	ψ_2^j	M_2^j	η_2^j
1	(110.16, 751.07)	138.90	−1.43	169.68	2.81	103.63	0.98	103.63	0.98
2	(108.26, 449.70)	125.10	1.09	355.82	1.86	93.15	−0.87	93.15	−0.87
3	(375.42, 1190.88)	522.31	−2.96	207.94	−0.20	169.45	−1.11	169.45	−1.11
4	(460.24, 766.96)	103.87	−1.30	296.30	−2.62	140.99	1.24	140.99	1.24
5	(316.70, 739.29)	147.30	2.29	177.45	1.02	107.94	−1.53	107.94	−1.53
6	(401.35, 681.18)	137.07	−0.98	209.34	3.13	105.53	1.18	105.53	1.18
7	(570.09, 887.60)	85.04	−1.77	203.83	−2.15	92.53	1.46	92.53	1.46
8	(601.78, 1197.03)	496.27	−2.82	166.32	−0.48	122.50	−0.72	122.50	−0.72

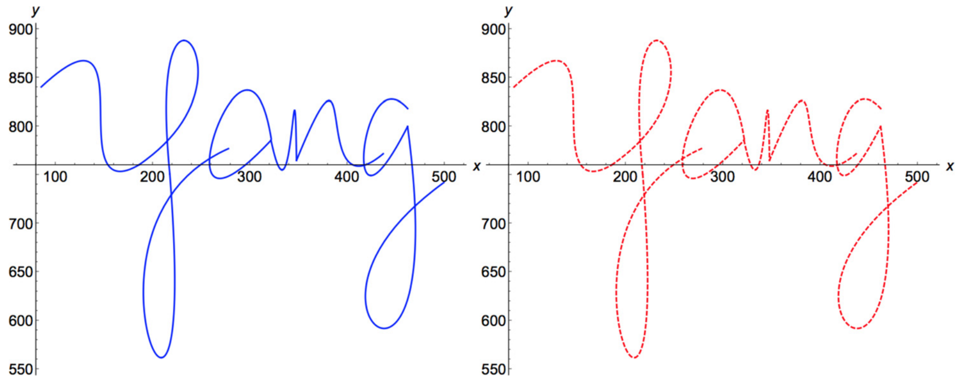

Fig. 6 The curves generated by six-link serial chains and the curves generated by corrected four-link serial chains

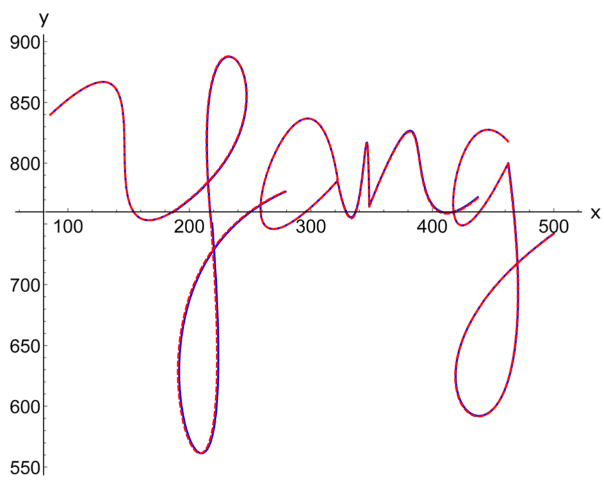

Fig. 7 The comparison of curves generated by six-link and corrected four-link serial chains

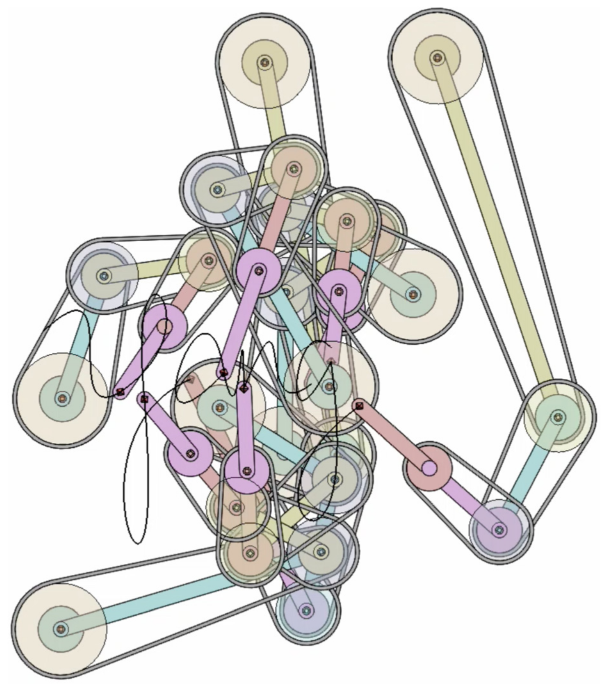

Fig. 9 The linkage system that can draw Yang in cursive. These chains are driven by the same input.

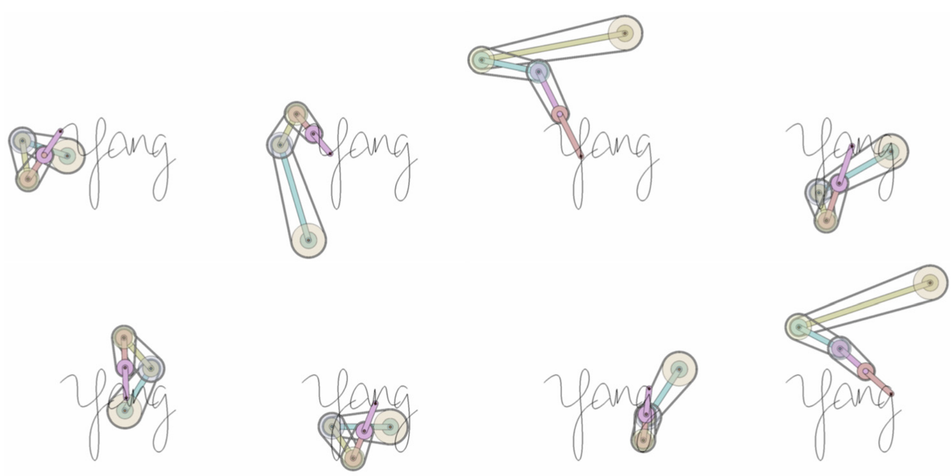

Fig. 8 The eight four-link coupled serial chains that draw the Bezier curves defining the cursive Yang

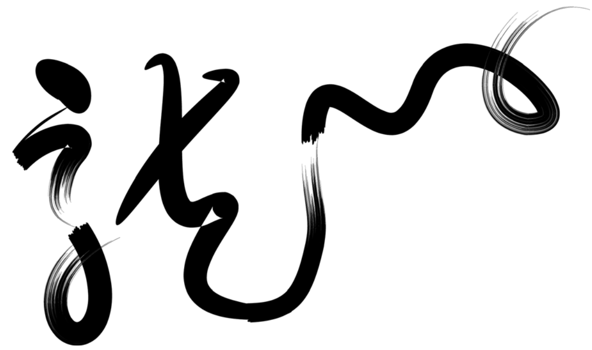

Fig. 10 Script form of the Chinese character long or dragon

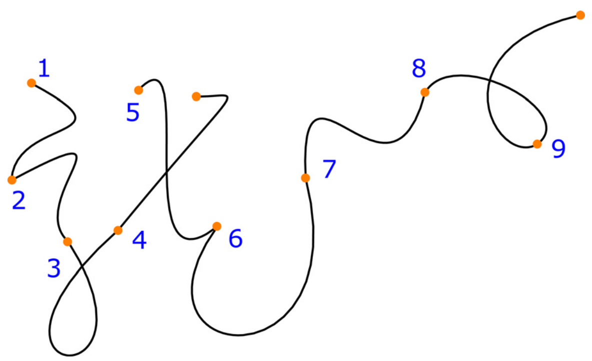

Fig. 11 Nine Bezier curves used to define the script form of the Chinese character long. The control points are listed in Table 6.

Thus, the drawing linkage for the name Yang consists of eight four-link coupled serial chains (see Table 5) that are shown separately in Fig. 8 and together in Fig. 9.

7 Linkage System to Draw Script Chinese Characters

In this section, we design the drawing linkage for the script form of the Chinese character "long," which means dragon. Figure 10 is a copy of the Chinese script for long drawn using the drawing software INKSCAPE. From this drawing, we obtain nine cubic Bezier curves that form the character, as shown in Fig. 11. The control points for these nine cubic Bezier curves are listed in Table 6.

For this example, we found that the shape parameter can be used either to match the cubic Bezier curves or to reduce the size of the end-links to simplify the coupled serial chain. Figure 12 shows that by specifying $\lambda = -0.5$, we can make the maximum deviation $\varepsilon \leq 0.01$ for all the curves. On the other hand, if we adjust the desired shape of this Chinese character so that $\lambda = 0.01$, we find that the last two links are of lengths less than 1% of the other four-links. This provides a simpler drawing linkage.

Therefore, we select $\lambda = 0.01$ and, as was done in the previous example, we drop these last two links of the resulting serial chain. The character long is drawn using nine four-link coupled serial chains. The dimension of these chains is listed in Table 7 and illustrated in Figs. 13 and 14.

8 Practical Considerations

This paper shows that a cubic Bezier curve can be drawn by a six-link serial chain. The designer can use the shape parameter of cubic trigonometric Bezier curves to adjust the curve so that the two end-links of the six-link serial chain are small enough to be neglected resulting in a four-link drawing linkage.

This means that an assembly of m cubic Bezier curves can be drawn mechanically by m coupled six-link serial chains connected

Table 6 Control points for the nine Bezier curves that define the Chinese character long (unit: mm)

Curve j	$\mathbf{P}_0^j$	$\mathbf{P}_1^j$	$\mathbf{P}_2^j$	$\mathbf{P}_3^j$
1	(175.31, 724.27)	(224.14, 698.20)	(168.03, 709.63)	(166.37, 680.51)
2	(166.37, 680.51)	(226.98, 713.13)	(171.78, 671.48)	(191.97, 652.84)
3	(191.97, 652.84)	(239.31, 579.43)	(135.07, 587.58)	(214.57, 657.92)
4	(214.57, 657.92)	(273.38, 729.78)	(271.95, 717.82)	(250.07, 718.32)
5	(223.72, 721.20)	(252.27, 750.96)	(217.29, 626.42)	(259.36, 659.51)
6	(259.36, 659.51)	(213.07, 595.59)	(323.64, 585.64)	(299.35, 681.46)
7	(299.35, 681.46)	(295.66, 750.18)	(337.36, 658.35)	(353.31, 720.09)
8	(353.31, 720.09)	(366.15, 742.17)	(424.29, 711.36)	(403.75, 696.70)
9	(403.75, 696.70)	(385.52, 686.45)	(356.55, 739.57)	(423.29, 755.08)

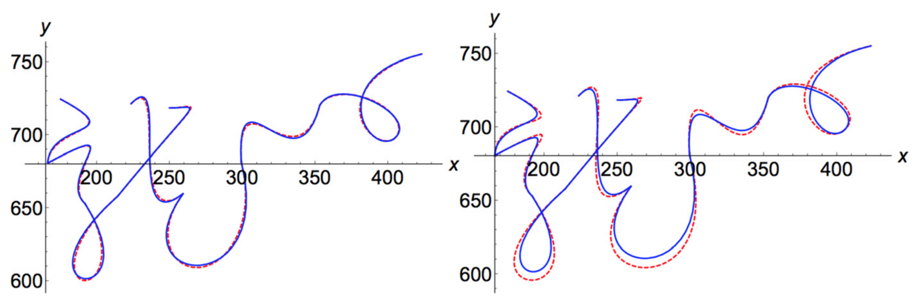

Fig. 12 The cubic trigonometric Bezier curves with $\lambda = -0.5$ (dashed) fit the cubic Bezier curves (solid), and the cubic trigonometric Bezier curves with $\lambda = 0.01$ (dashed) vary from the cubic Bezier curves (solid)

Table 7 The ground pivots, link lengths (mm), and phase angles of the four-bar serial chains to draw the script Chinese character

Curve j	$\mathbf{G}_j$	L_1^j	ψ_1^j	M_1^j	η_1^j	L_2^j	ψ_2^j	M_2^j	η_2^j
1	(119.85, 699.30)	31.64	−2.46	83.46	1.23	31.07	−0.58	31.07	−0.58
2	(138.17, 615.00)	44.15	−1.28	96.07	2.16	37.13	0.39	37.13	0.39
3	(235.22, 800.41)	194.63	−2.49	23.99	−1.83	58.47	−0.05	58.47	−0.05
4	(150.23, 616.07)	111.90	−0.56	77.48	2.28	23.56	1.14	23.56	1.14
5	(255.19, 693.70)	63.41	−1.77	72.61	−3.08	54.22	1.055	54.22	1.05
6	(301.55, 831.49)	63.98	−2.25	168.72	−1.02	46.86	2.91	46.86	2.91
7	(346.22, 693.40)	79.14	−0.83	107.94	−2.48	56.70	1.70	56.70	1.70
8	(344.94, 670.83)	43.03	0.04	27.79	1.63	19.09	2.61	19.09	2.61
9	(499.26, 751.36)	77.71	3.11	66.37	−2.60	22.96	−0.55	22.96	−0.55

Fig. 13 The nine four-link coupled serial chains that draw the Bezier curves defining Chinese character long

to a single actuator, and in some cases, by m four-link serial chains. This is a significant reduction in complexity of drawing linkages relative to current implementations of Kempe's universality theorem [3,6–8].

However, the manufacture of an assembly of coupled serial chains shown in Figs. 9 and 14 requires further study to evaluate accuracy and interference.

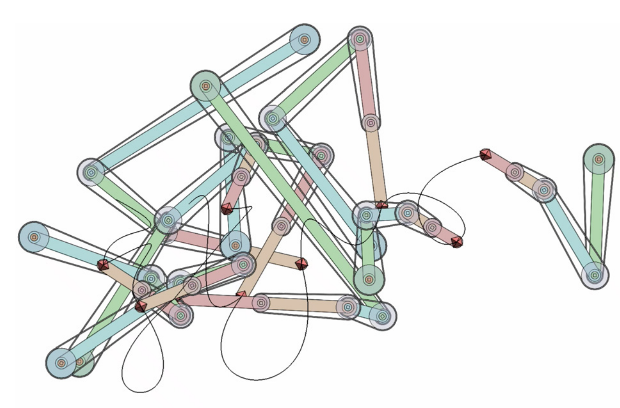

Fig. 14 The linkage system that draws the script form of the Chinese character long

9 Conclusion

This paper shows that a collection of planar Bezier curves obtained from cursive handwriting can be used to design a system of coupled serial chains that draw these curves. Furthermore, by using trigonometric Bezier curves, the shape function can be used to adjust the original curves and simplify these serial chains.

Two examples demonstrate this methodology. The first yields a linkage system that draws the cursive letters spelling Yang using eight four-link coupled serial chains driven by one actuator. The second draws a script version of the Chinese character long, or dragon, using nine interconnected four-link coupled serial chains.

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