# Wikipedia Illustration Task Force
## Visualizing Kempe's Universality Theorem

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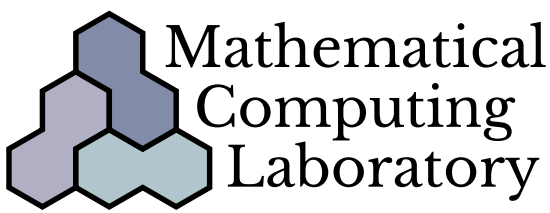
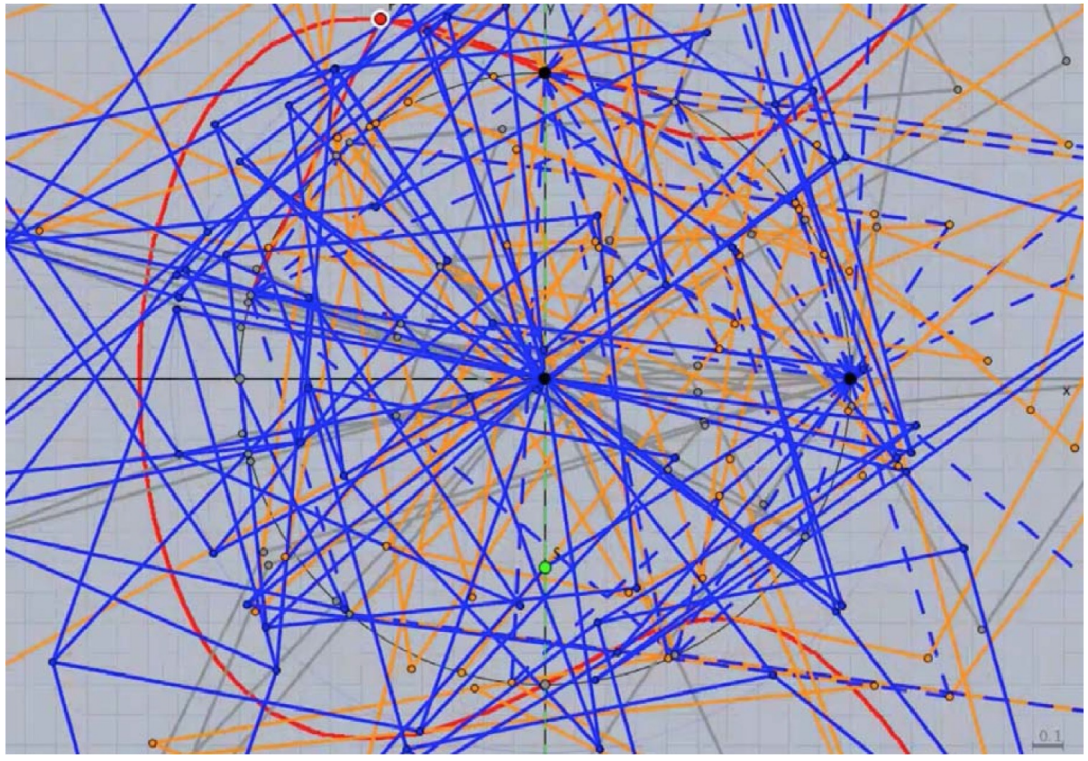
October 25, 2017




# Kempe's Universality Theorem

## Theorem

*For an arbitrary algebraic plane curve a linkage can be constructed that draws the curve.*



# Trigonometric Plane Curves

## Definition

A trigonometric plane curve, $P = (x(\theta), y(\theta))$, is a parameterized curve with coordinate functions that are finite Fourier series.

## Example

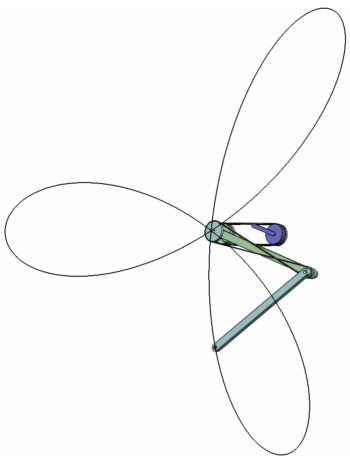
$$P_T = \begin{Bmatrix} -\cos 2\theta - \cos 4\theta \\ \sin 2\theta - \sin 4\theta \end{Bmatrix}$$

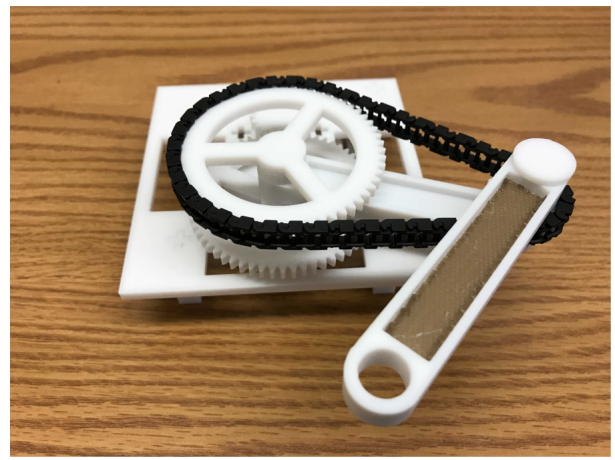
# Mechanisms to Draw Trigonometric Plane Curves

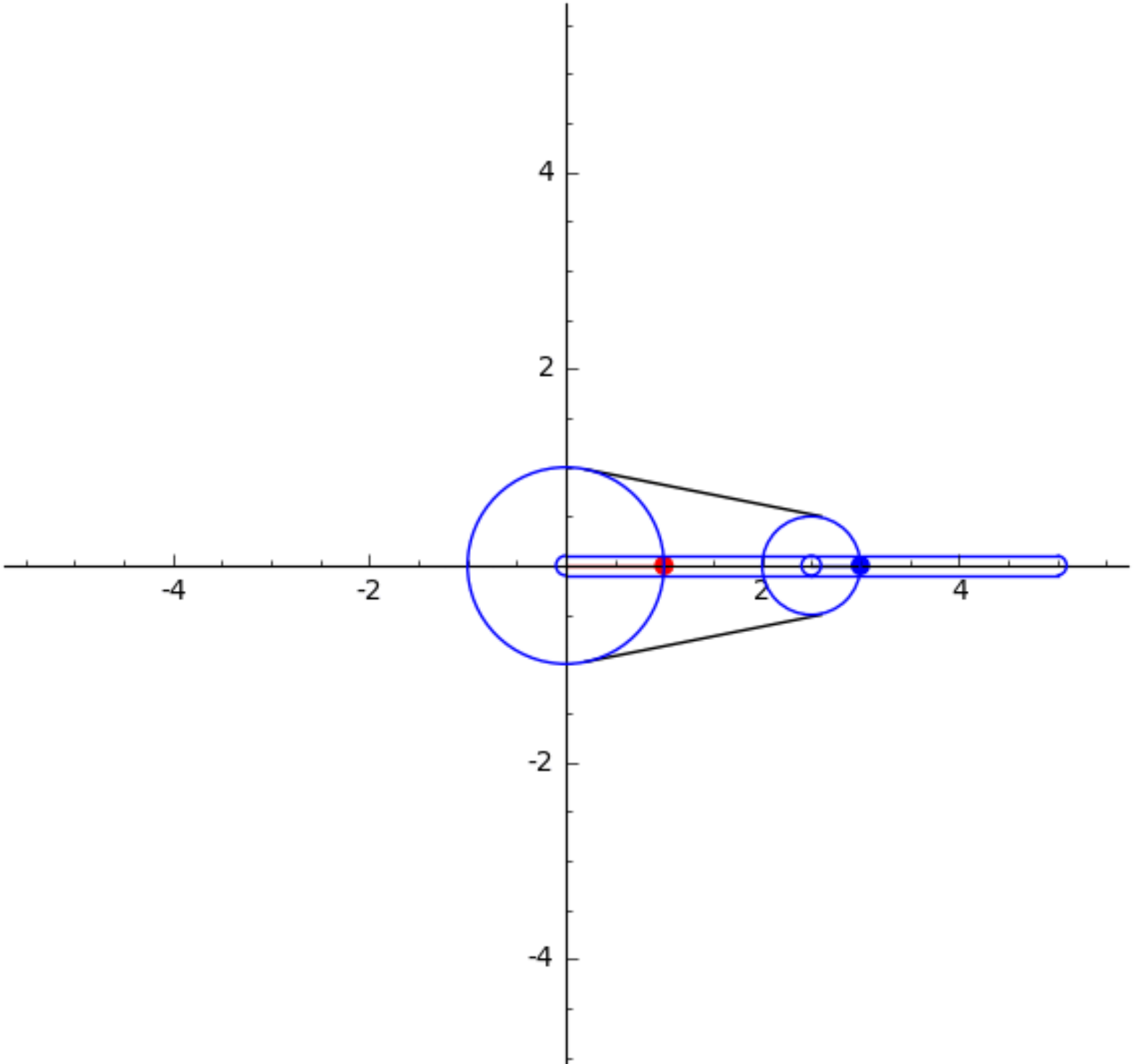
## Definition

A single-coupled serial chain is a mechanism that can be realized by coupling successive joint rotations of a serial chain linkage, using gears or cable-pulley drives.

- all links in a single-coupled serial chain are connected to the same input angle $\theta$
- cable drives at increasing speeds by pulleys of decreasing radius



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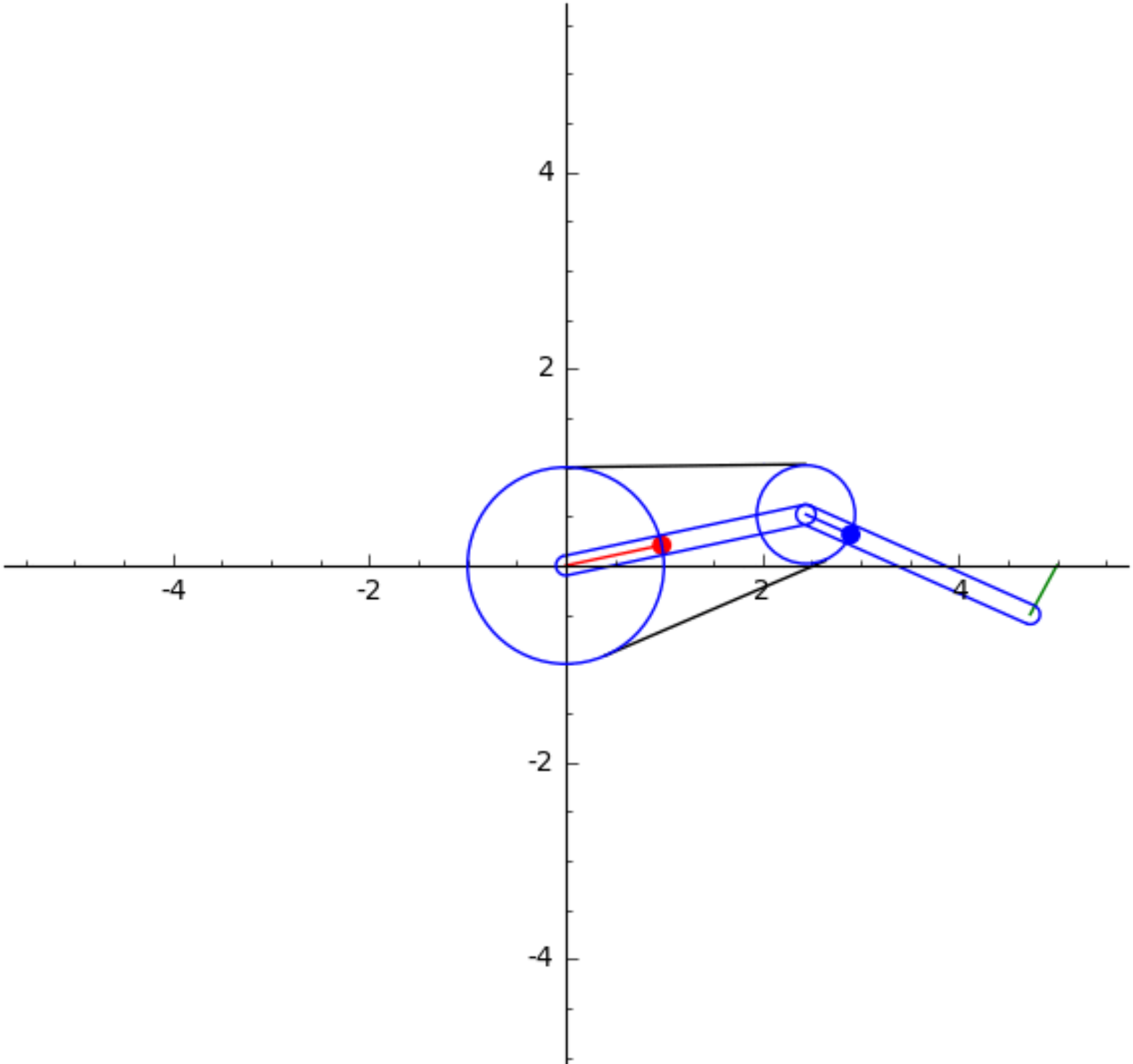
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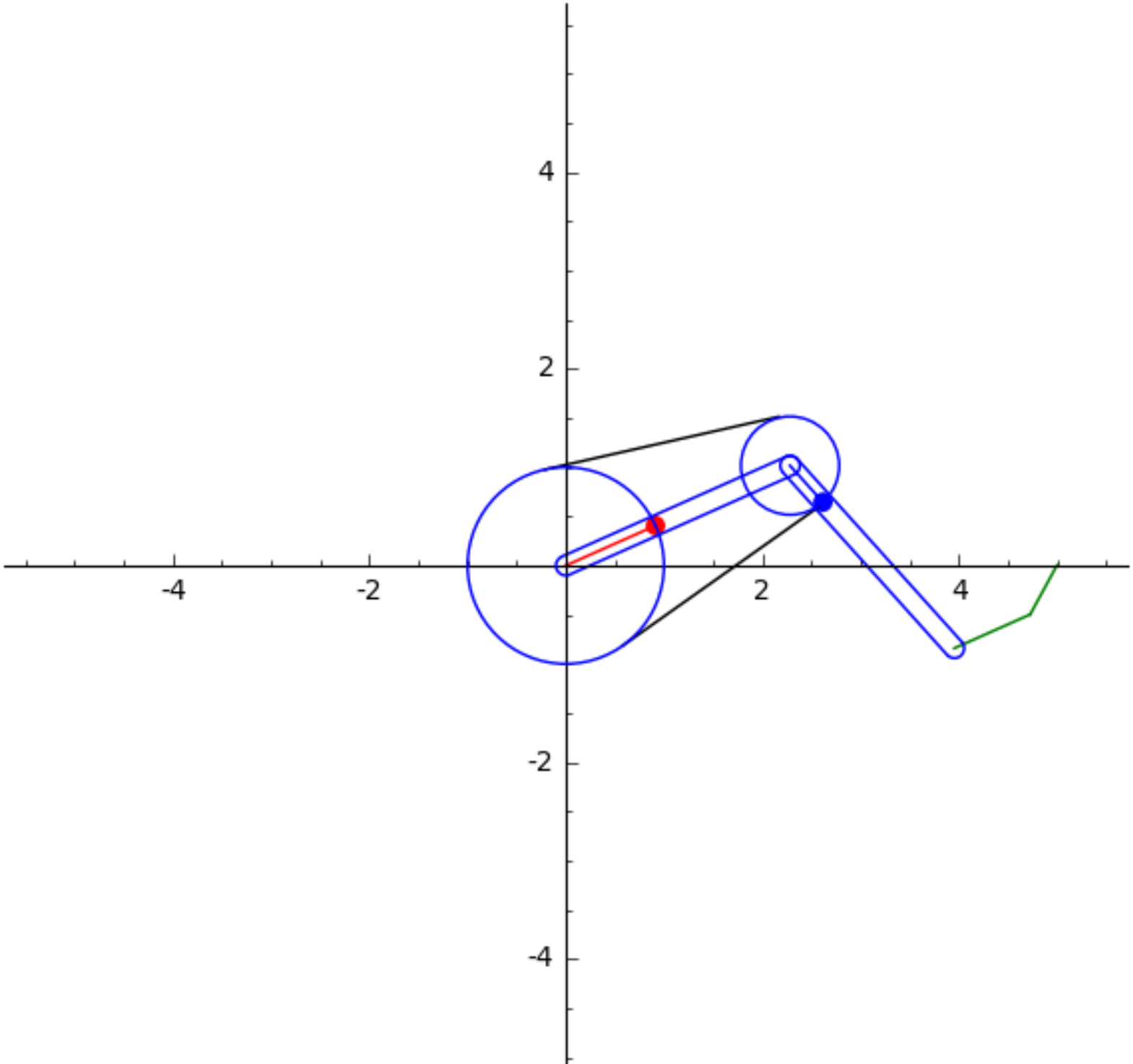
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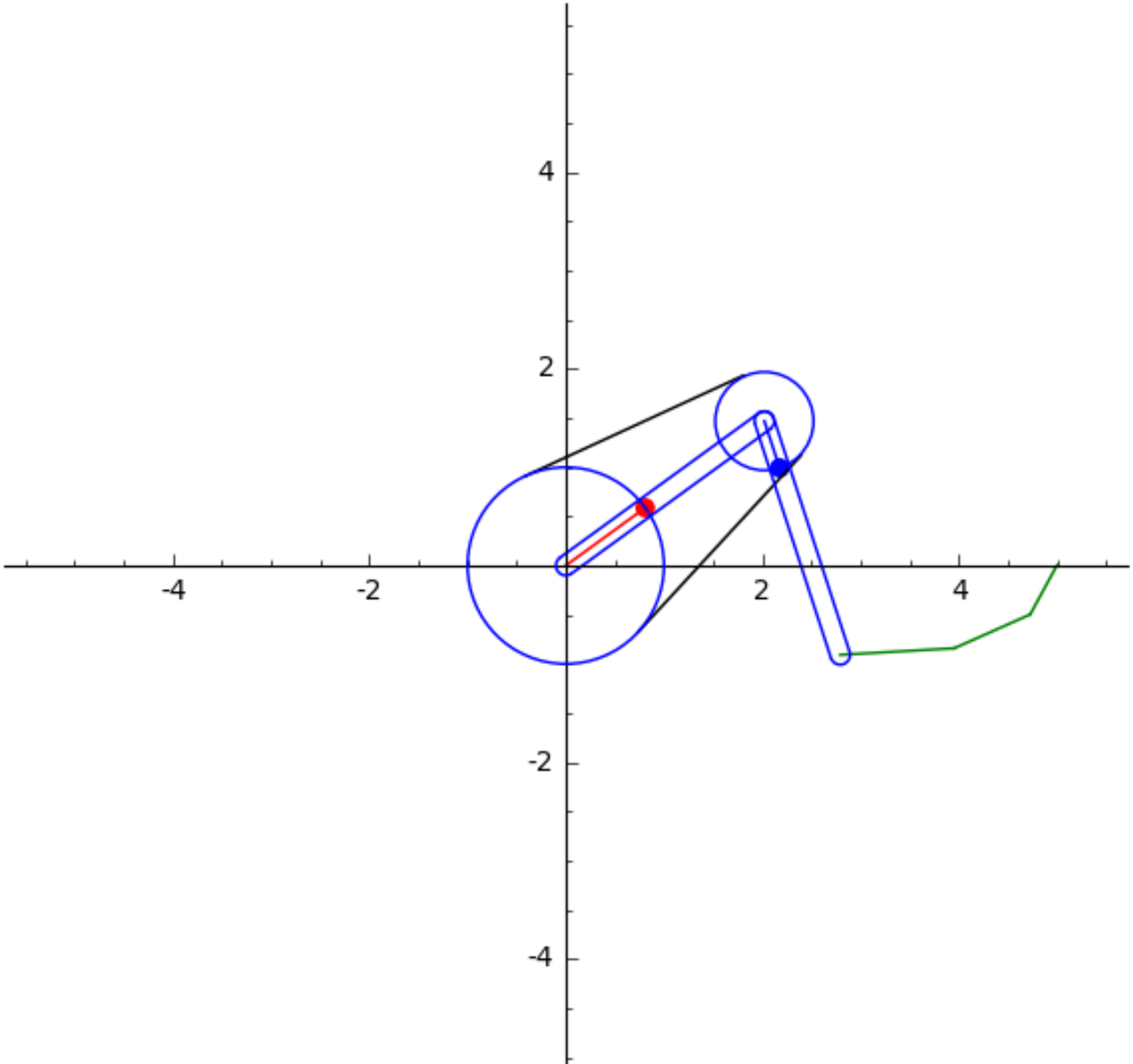
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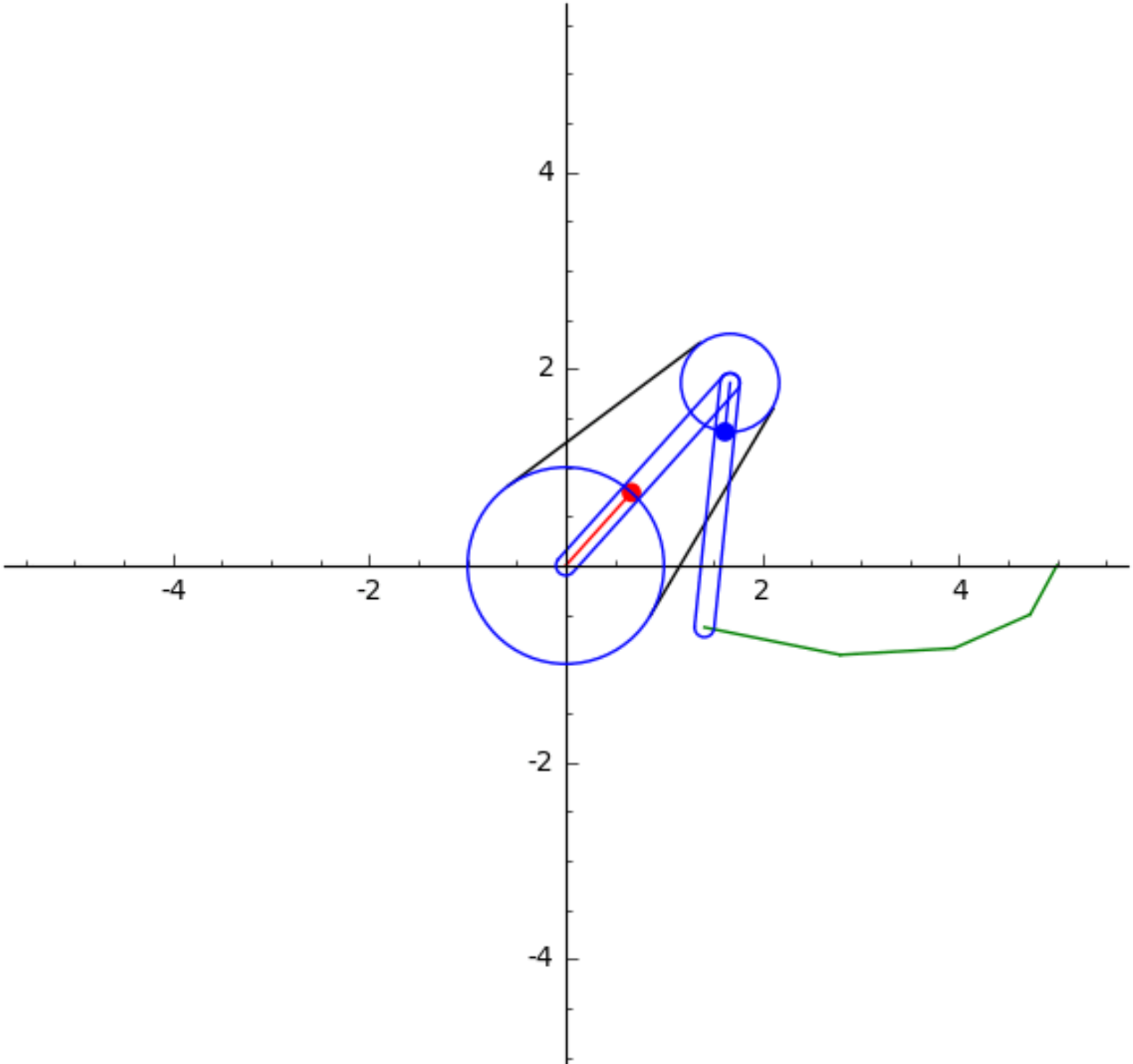
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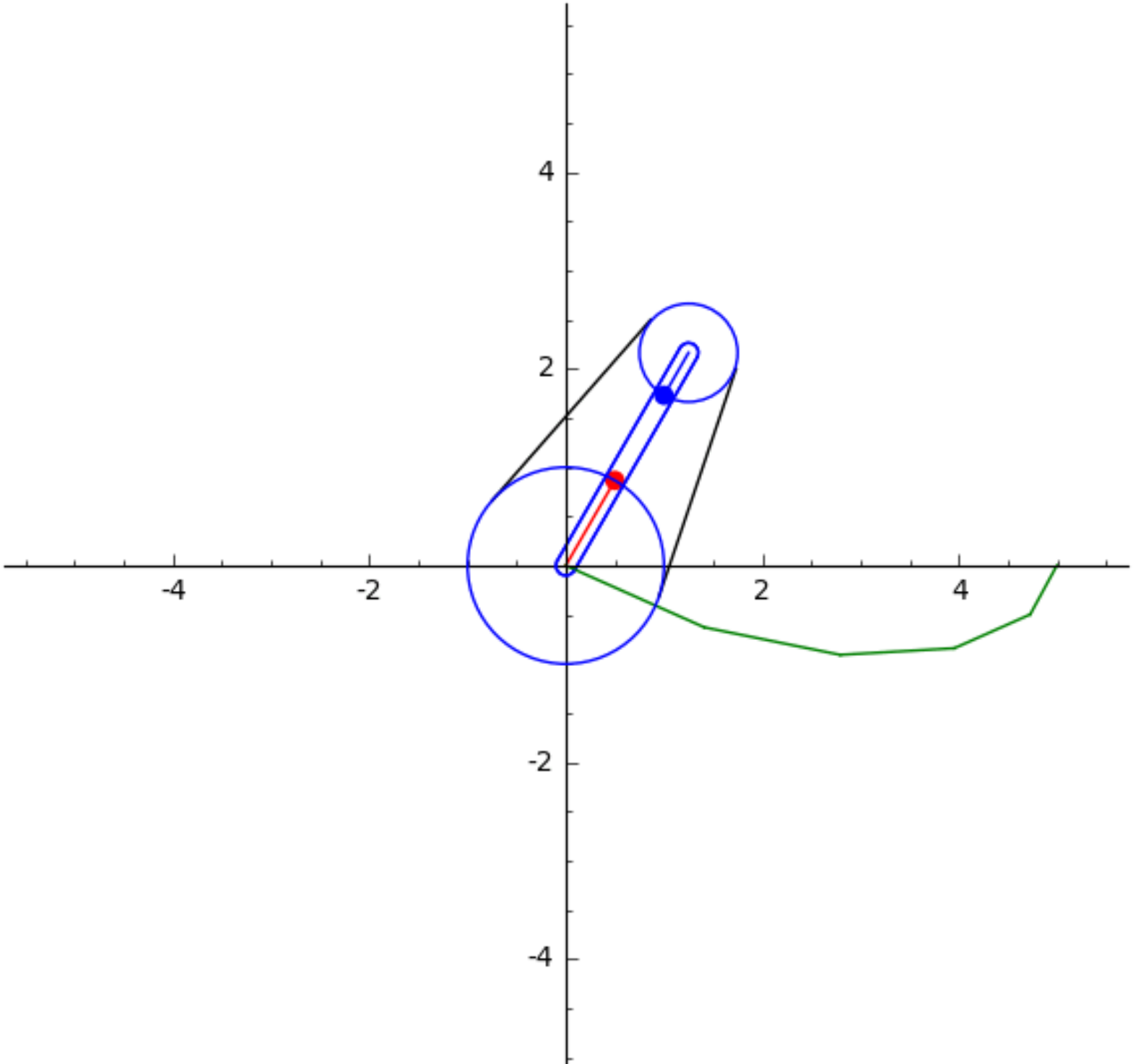
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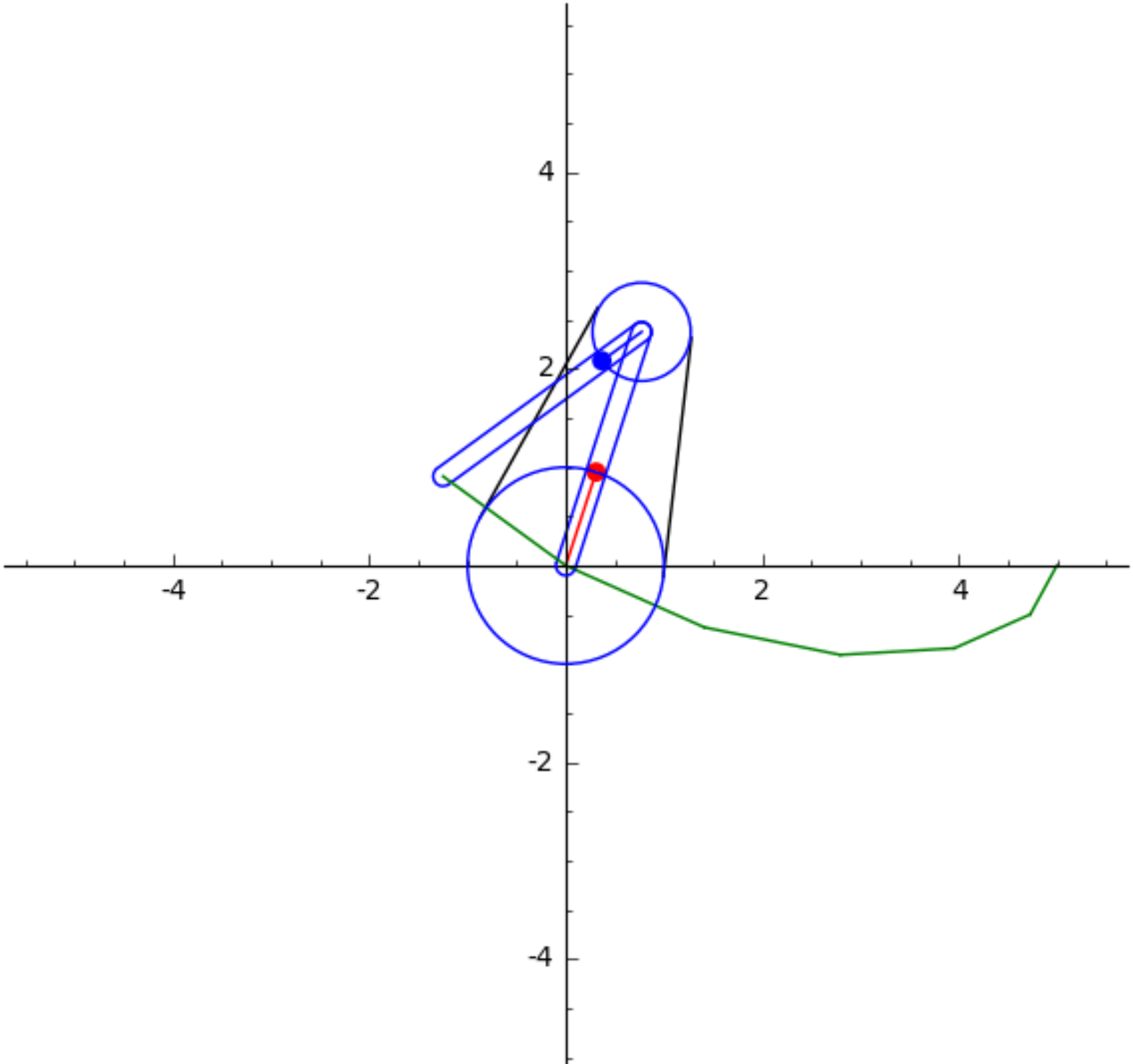
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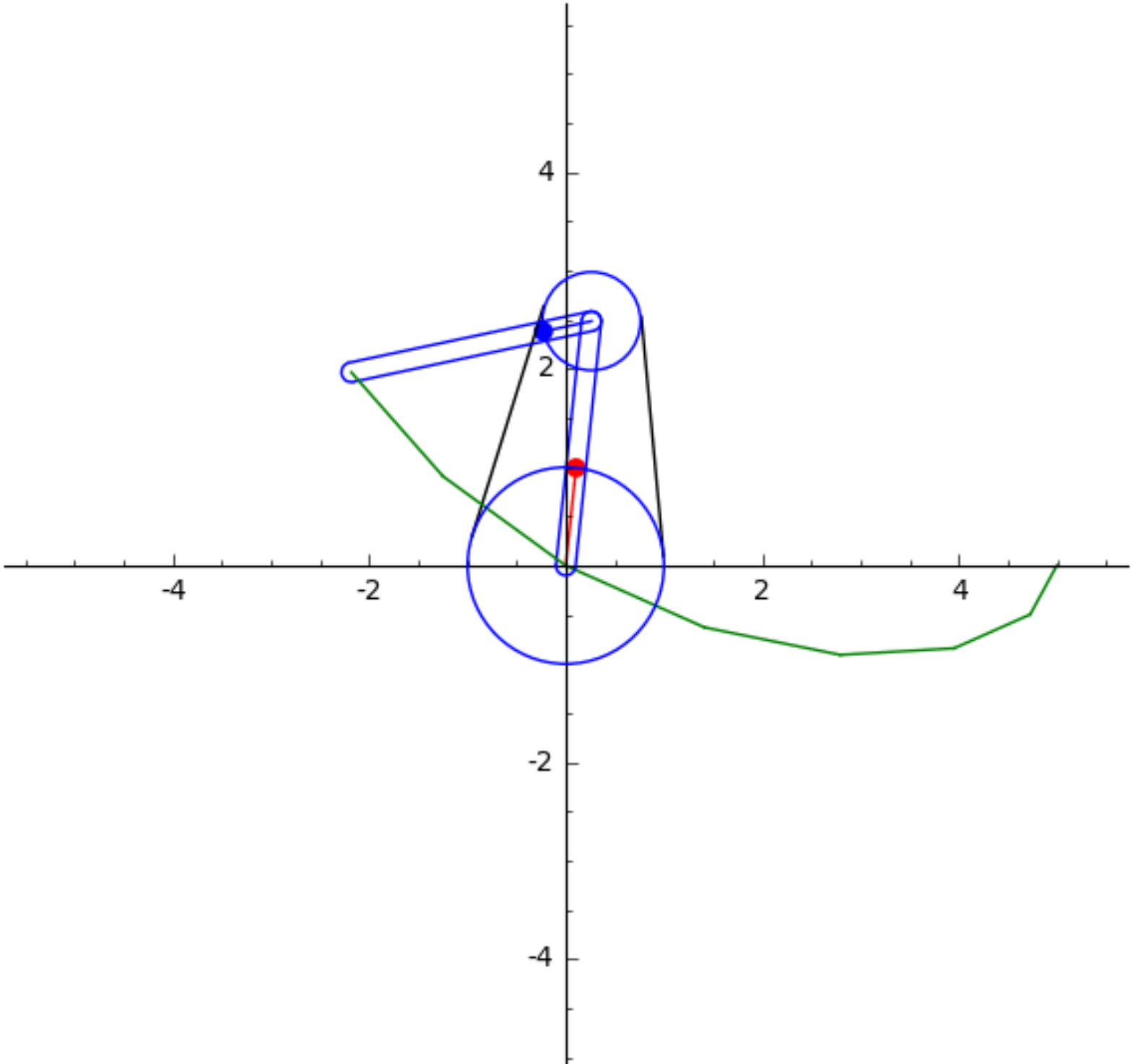
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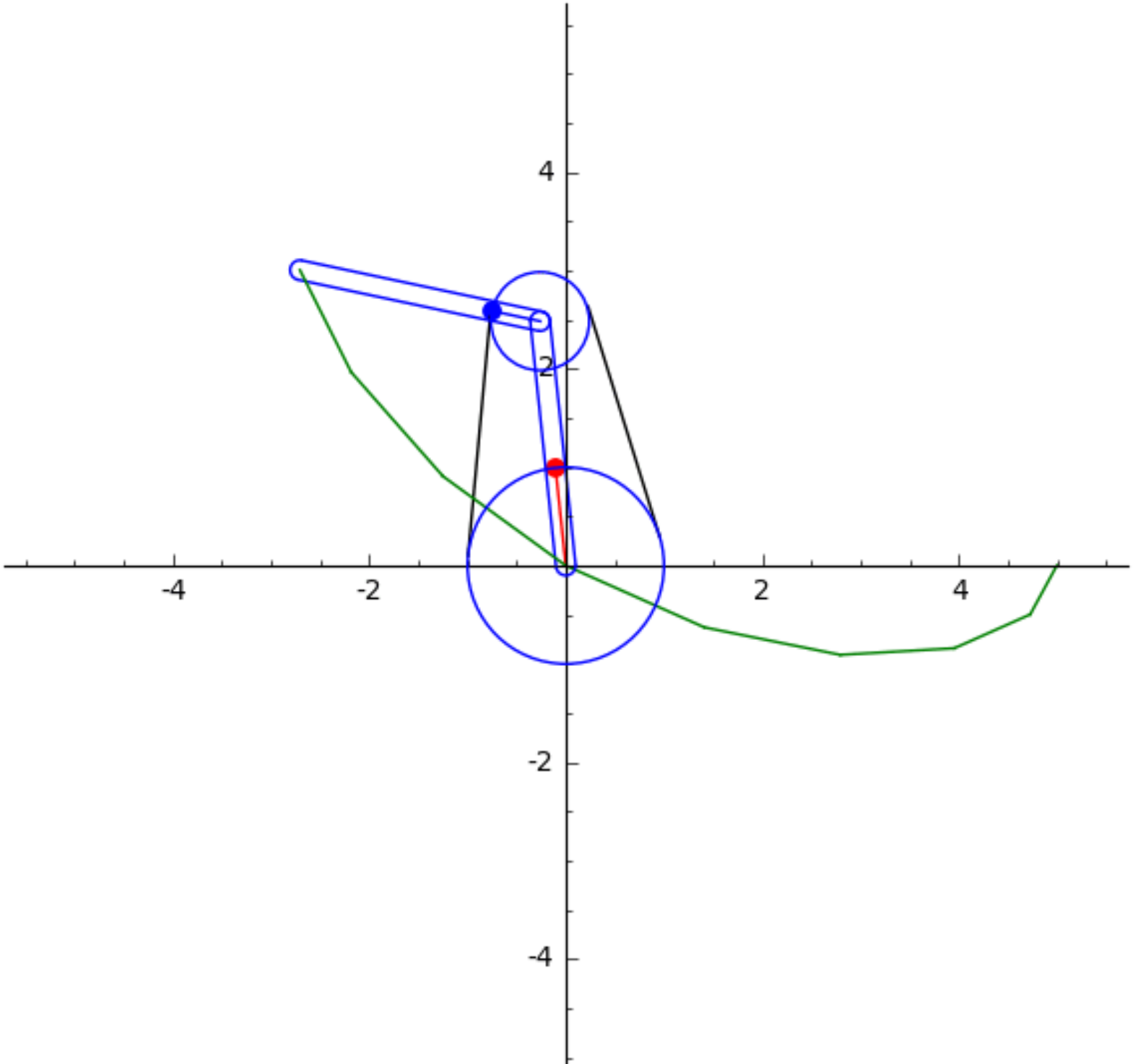
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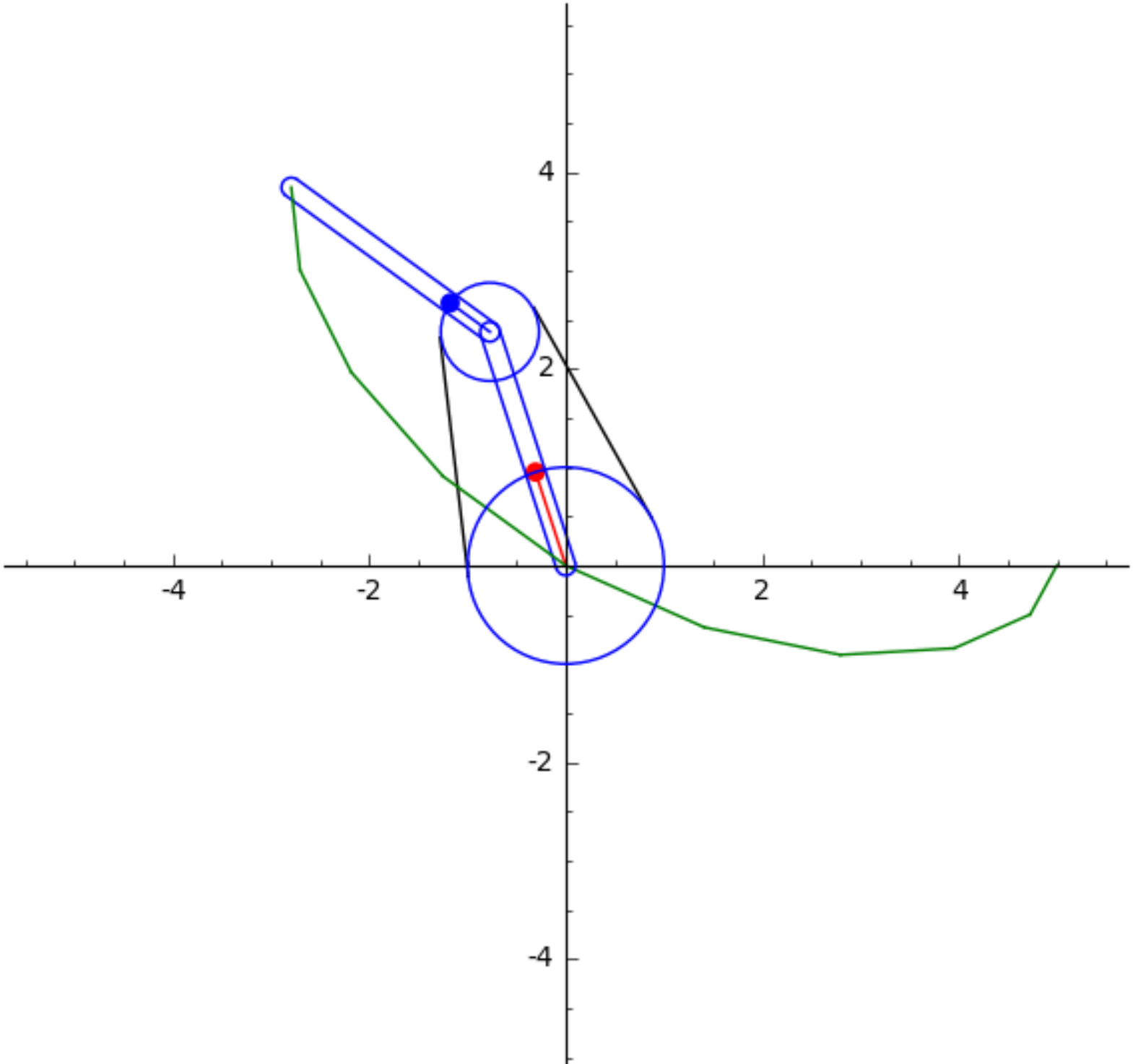
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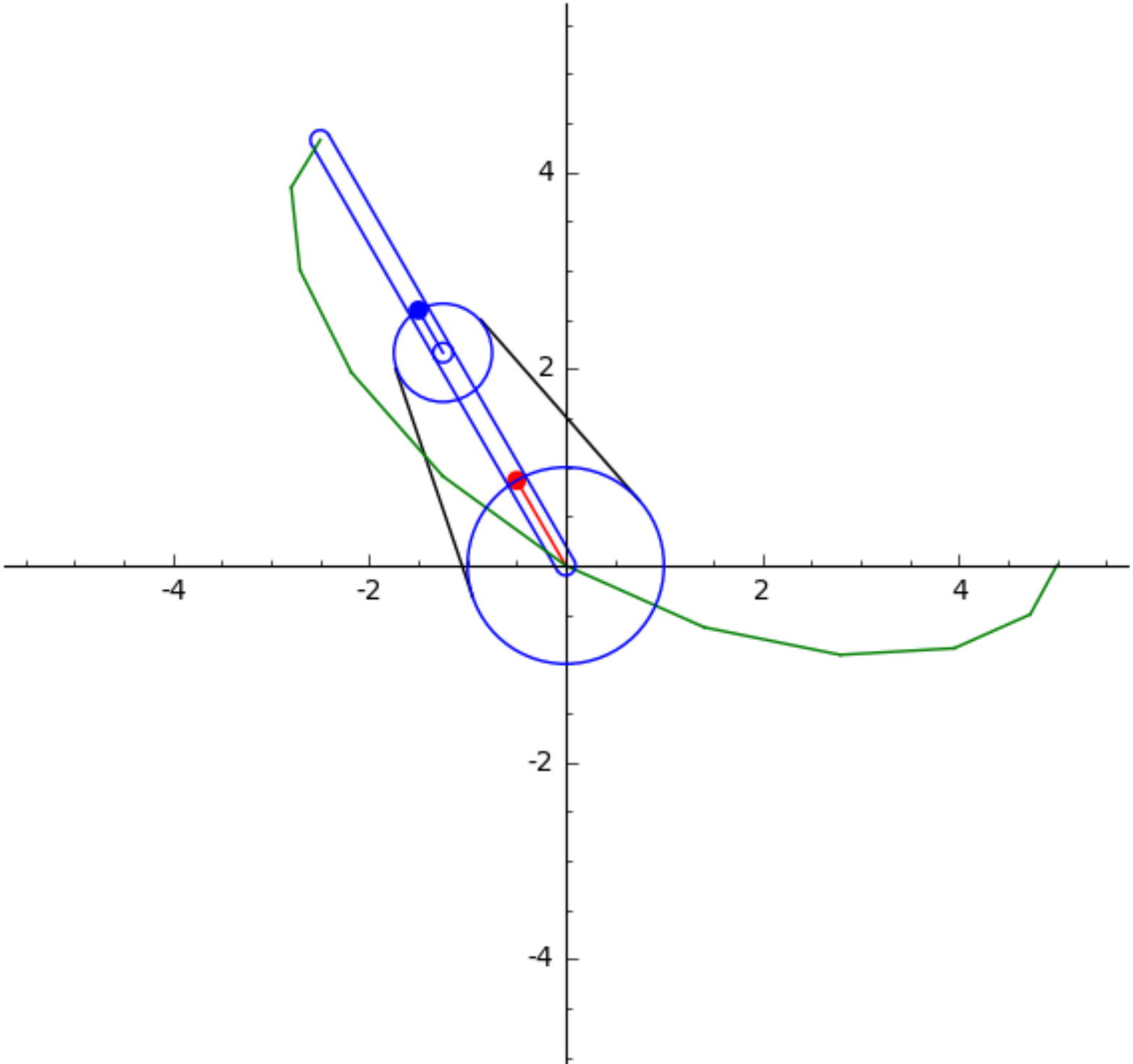
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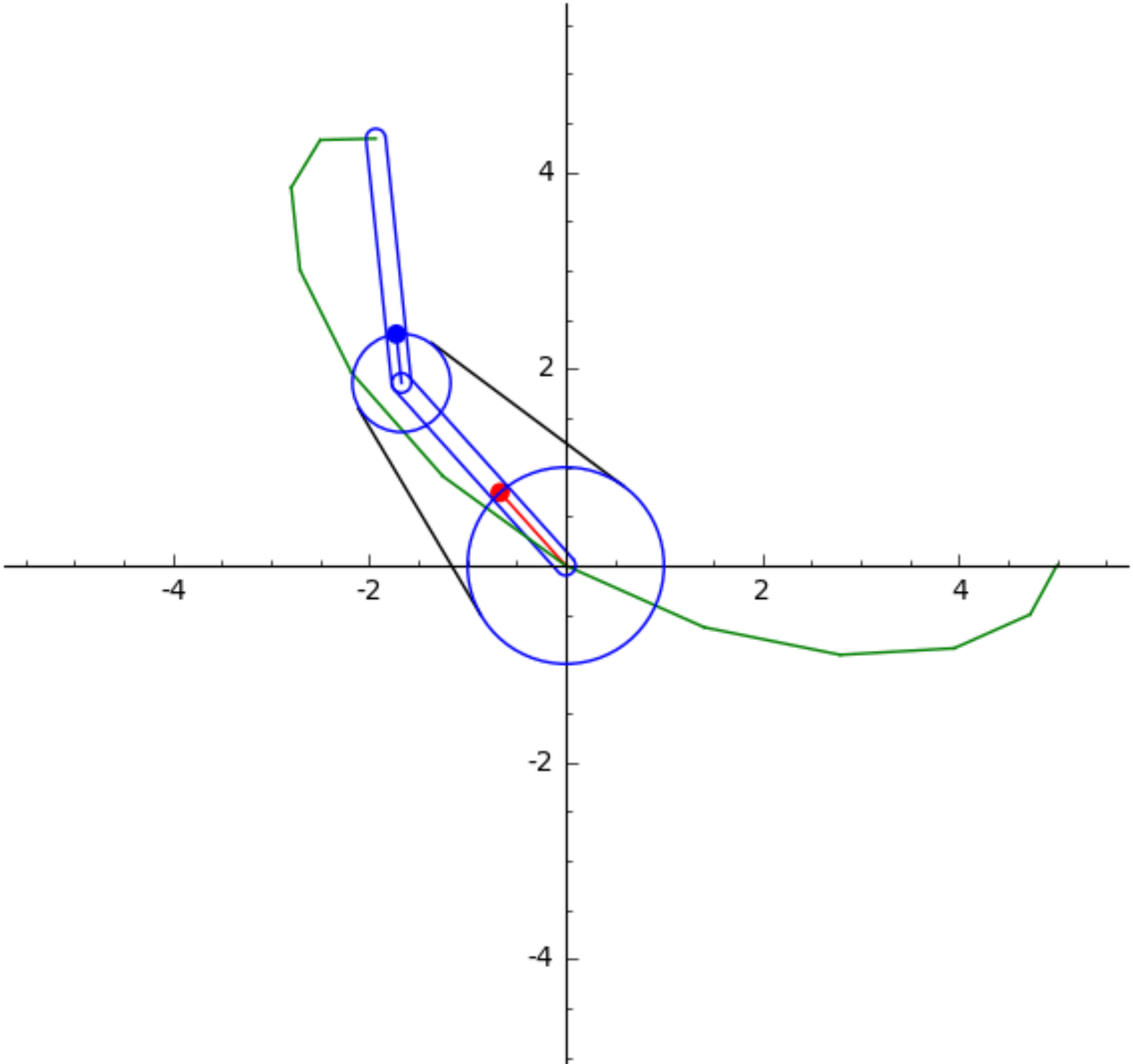
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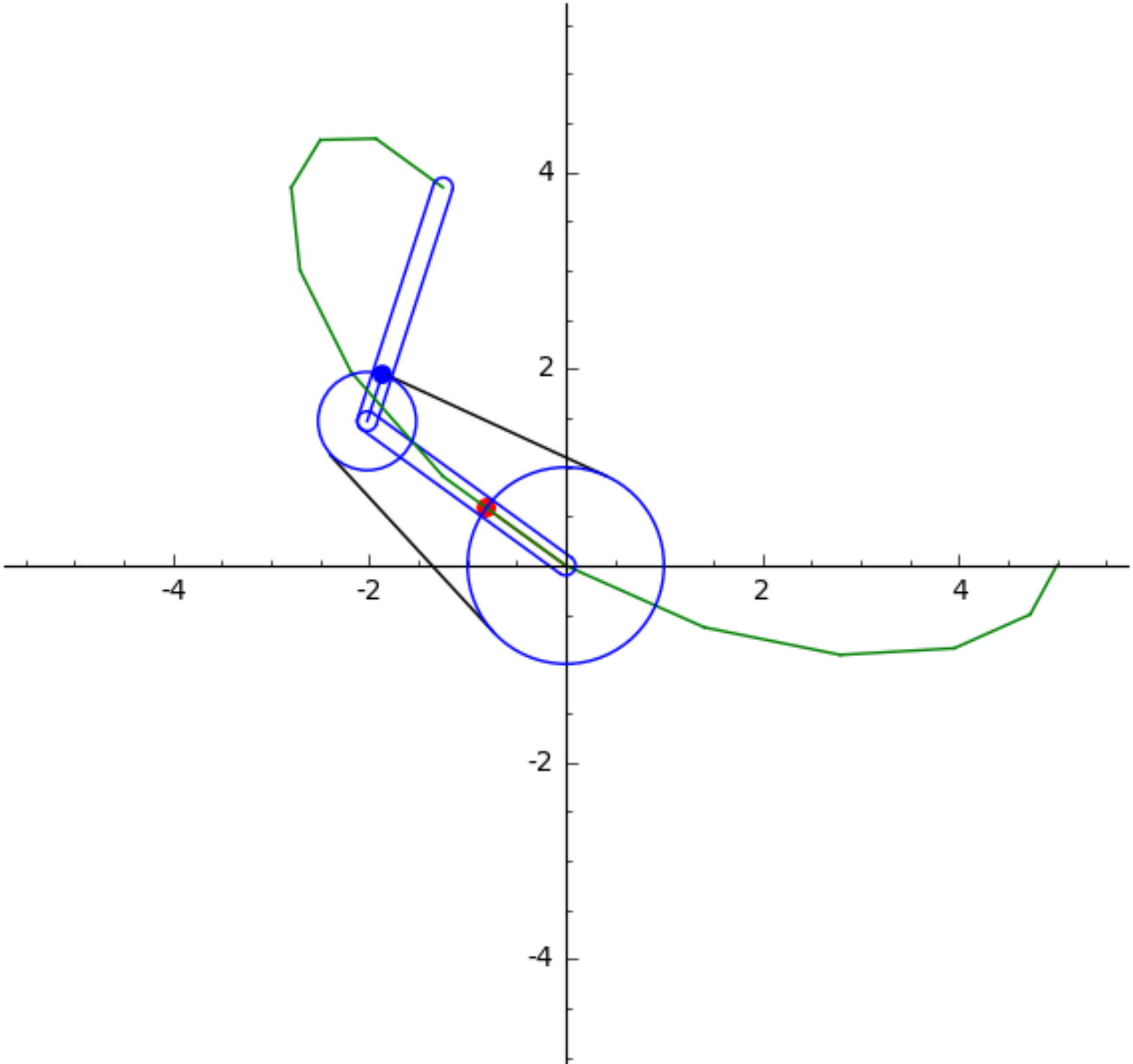
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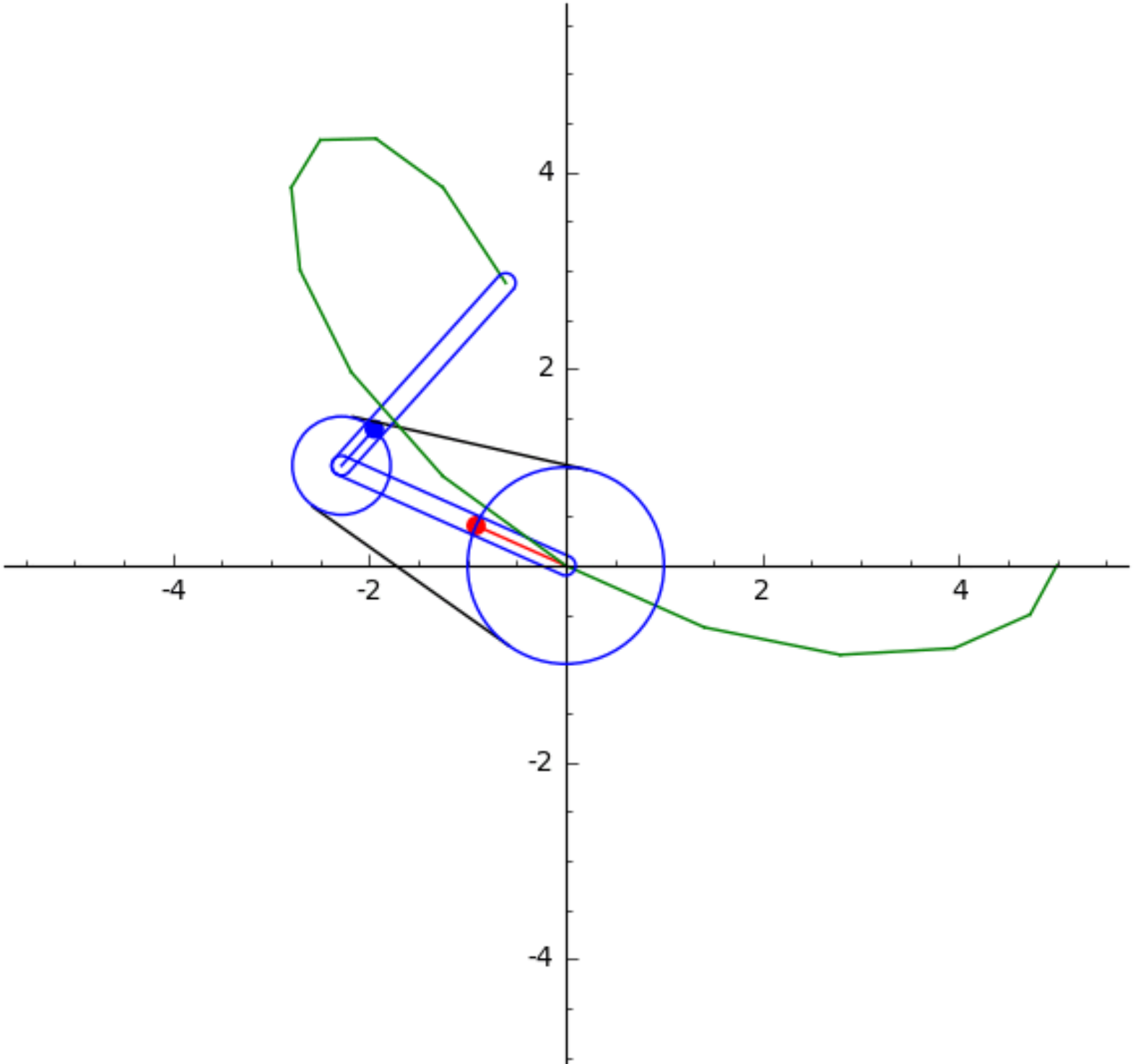
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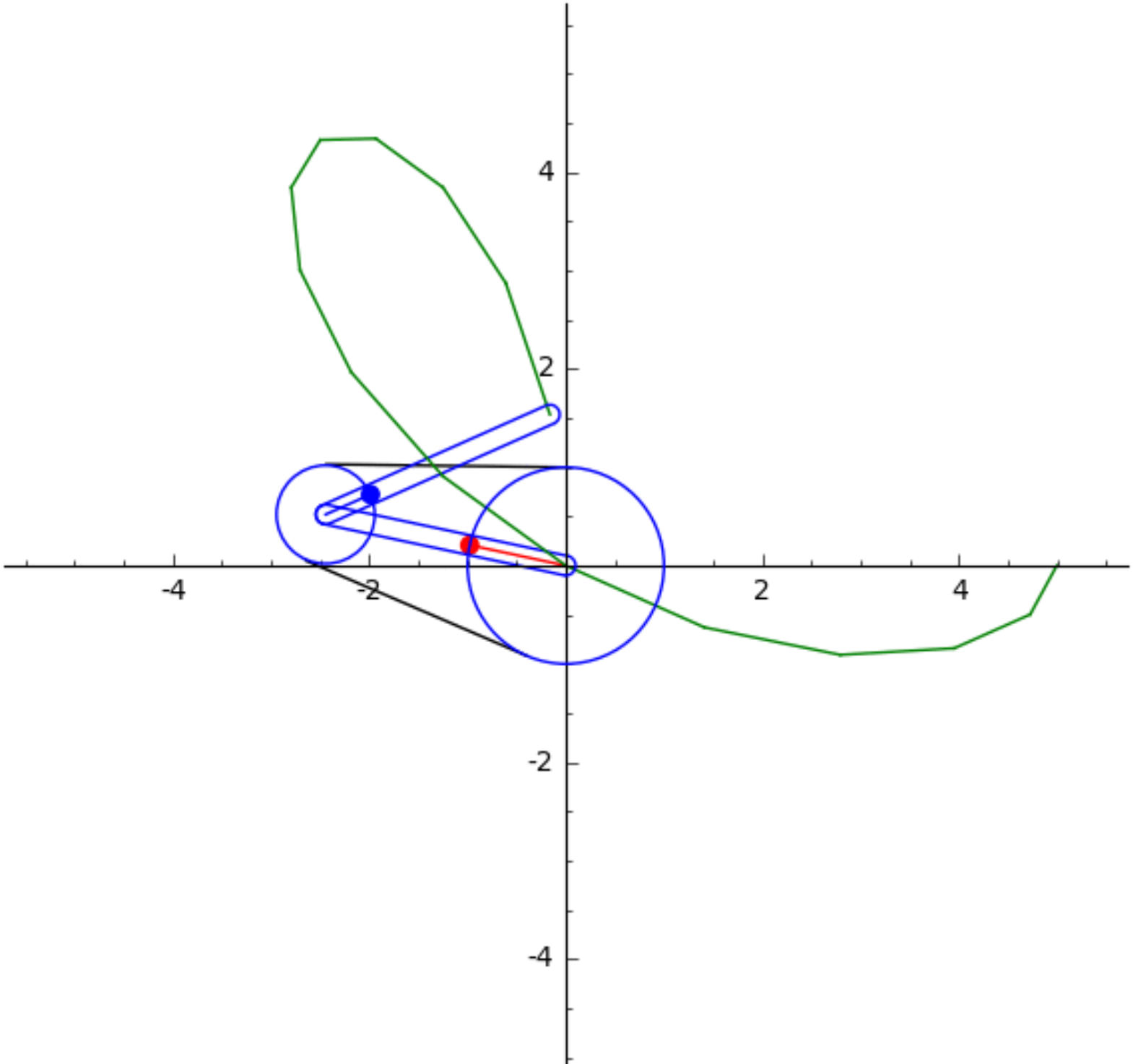
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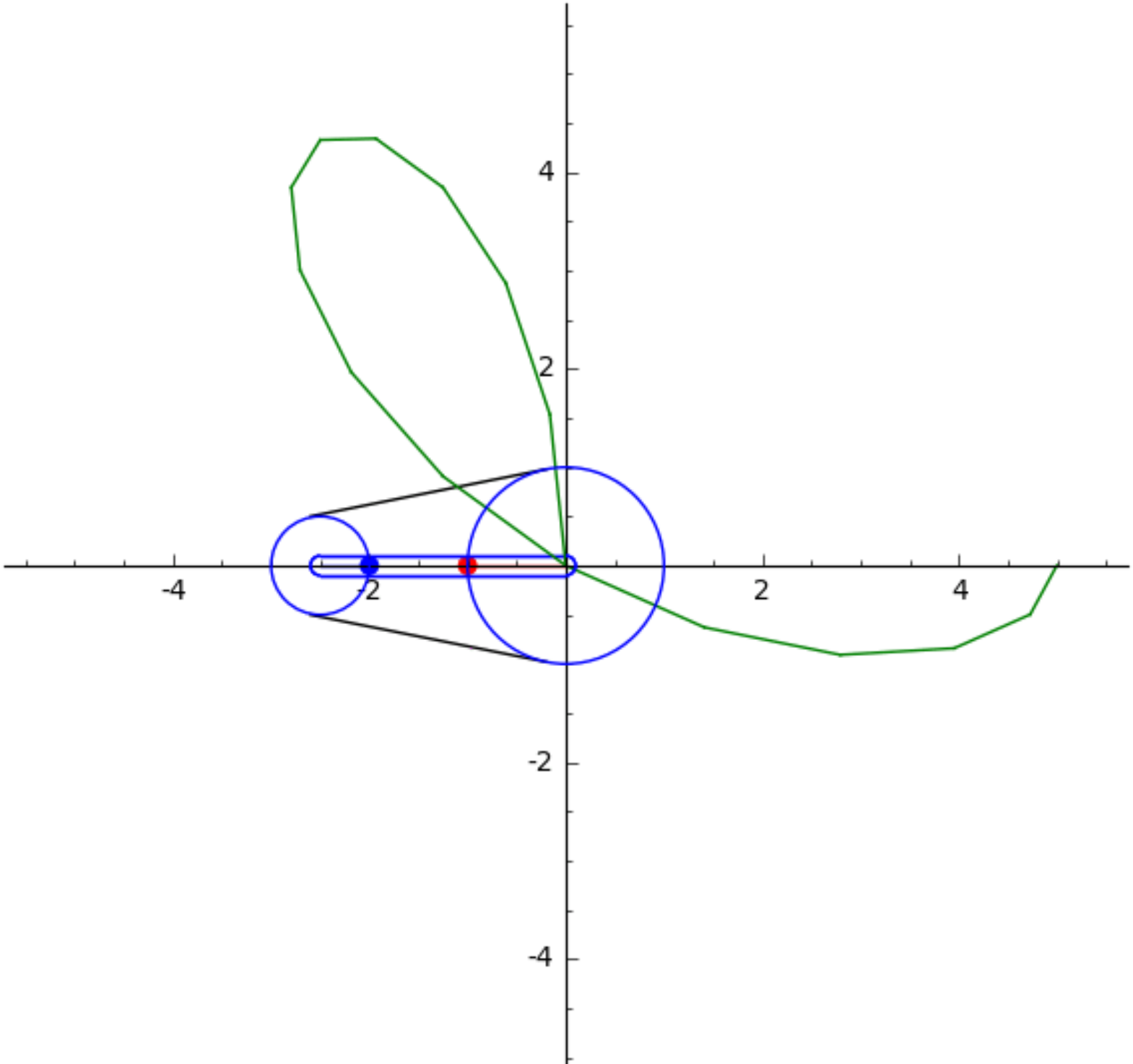
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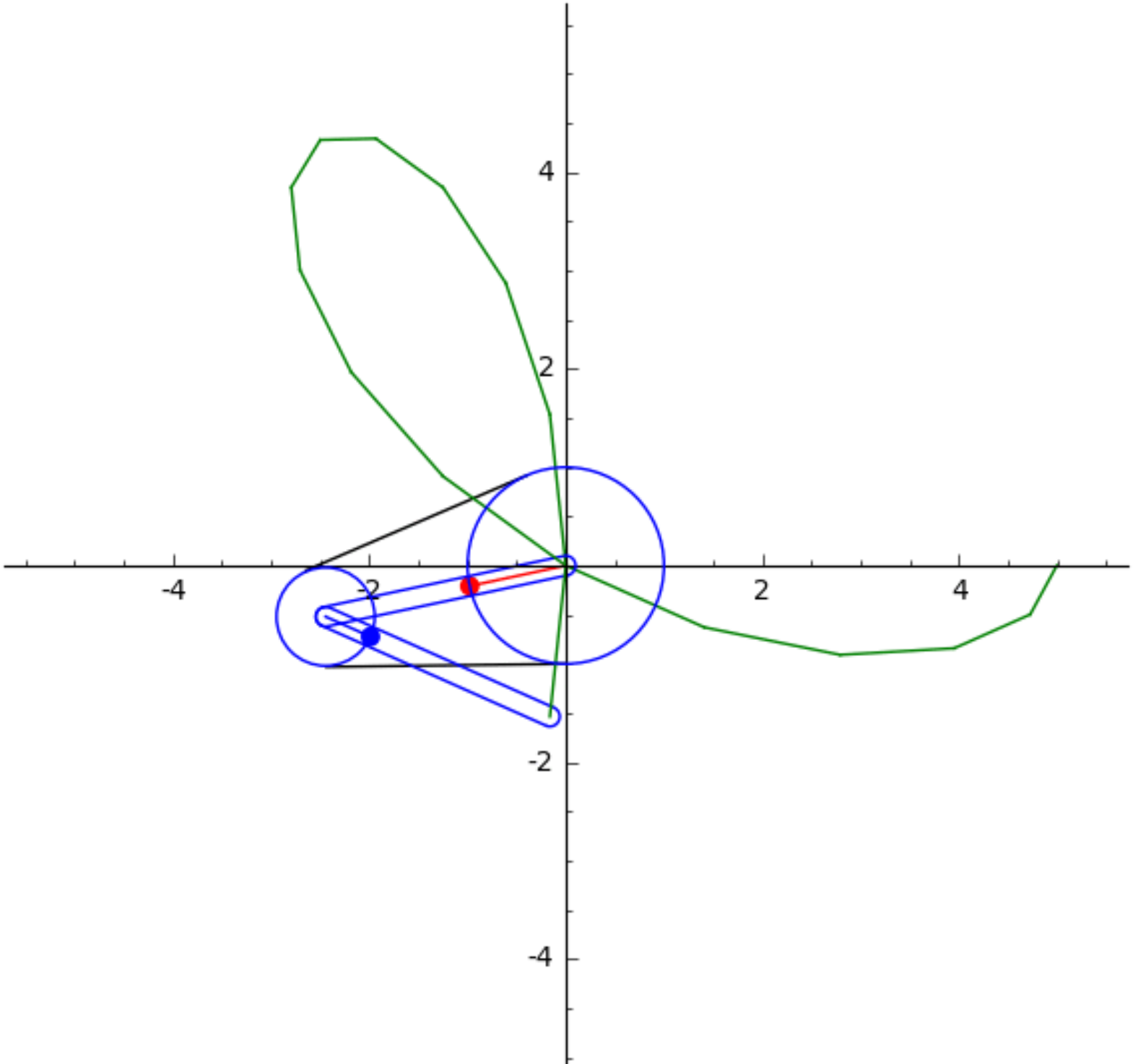
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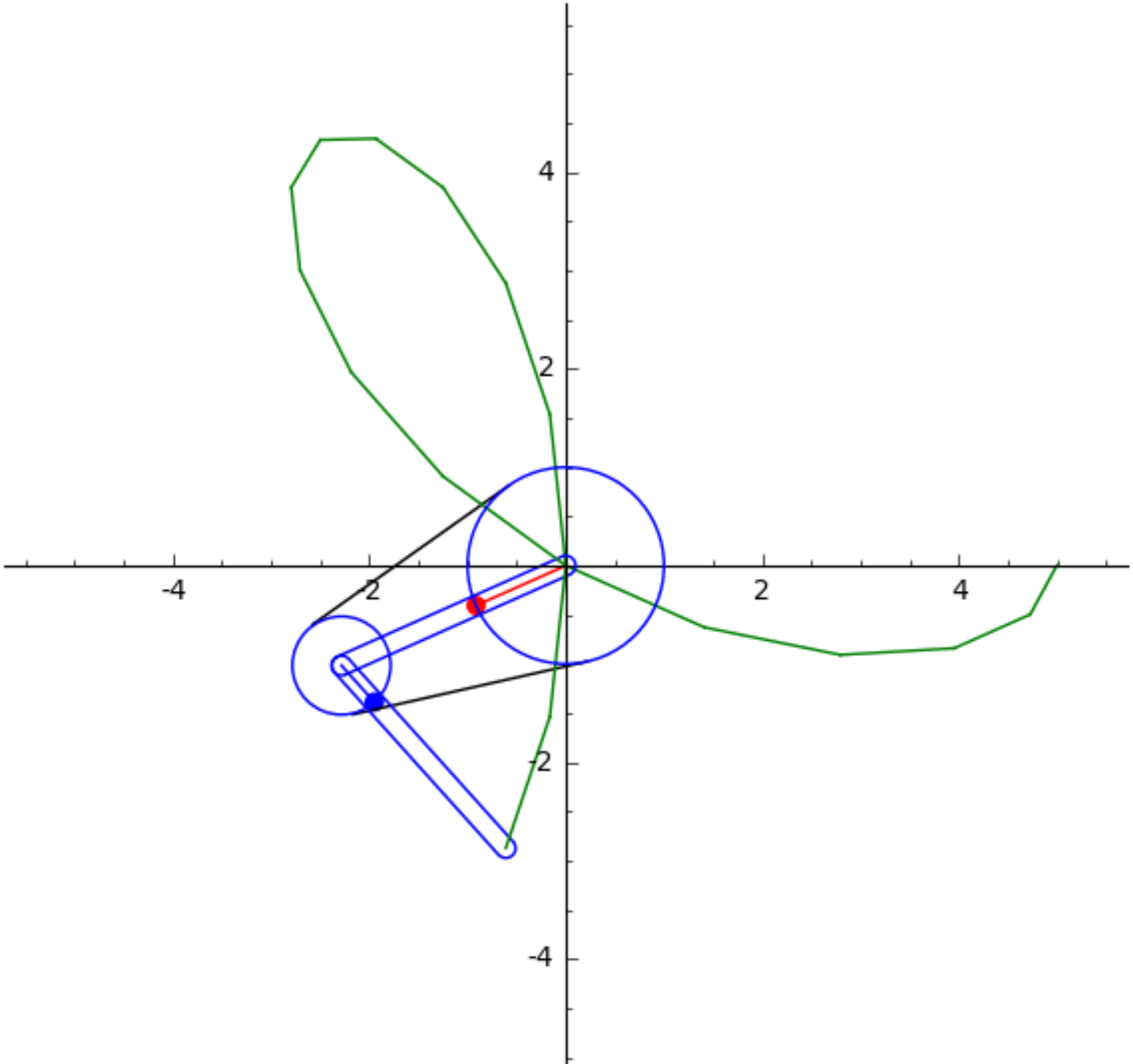
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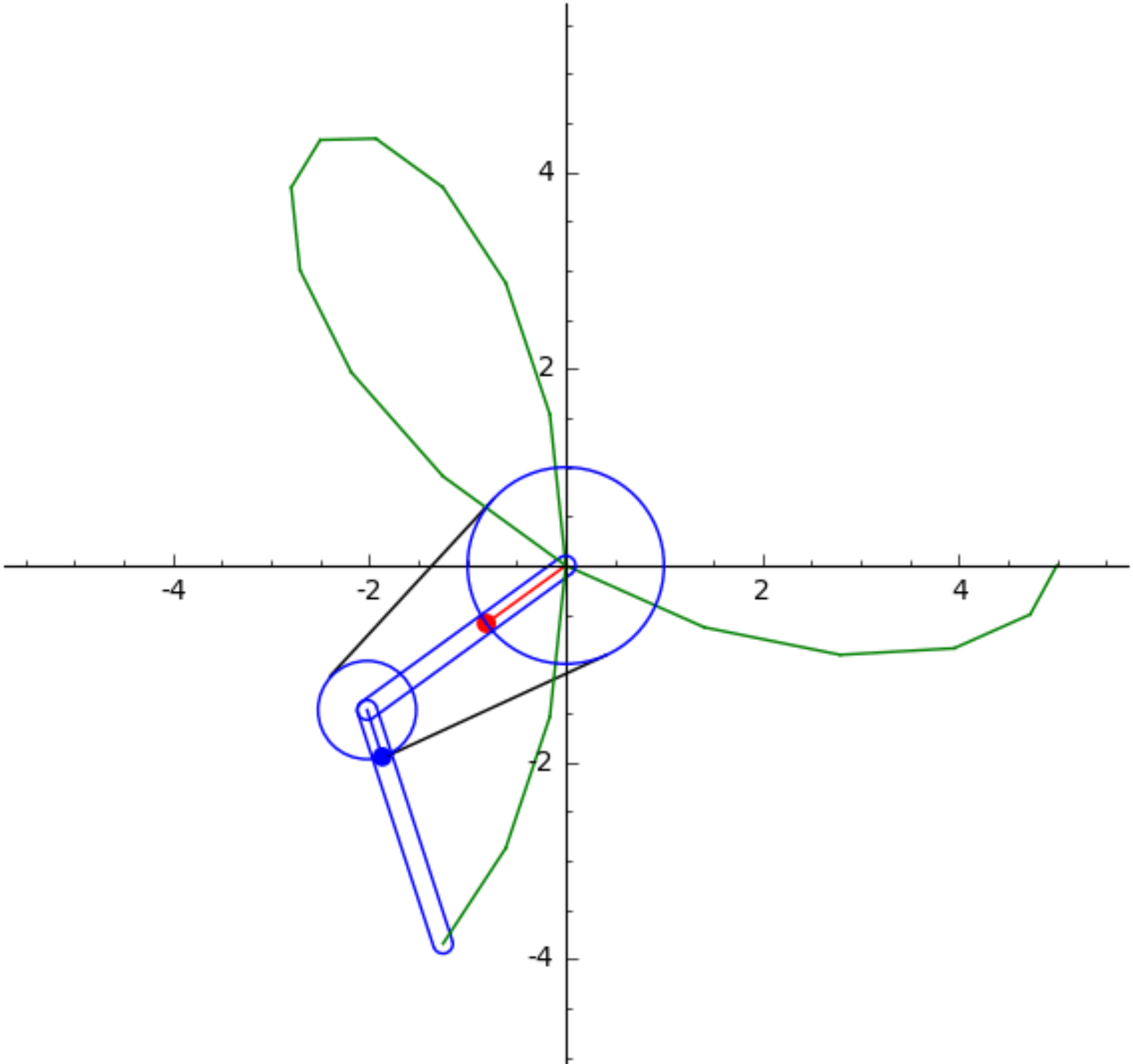
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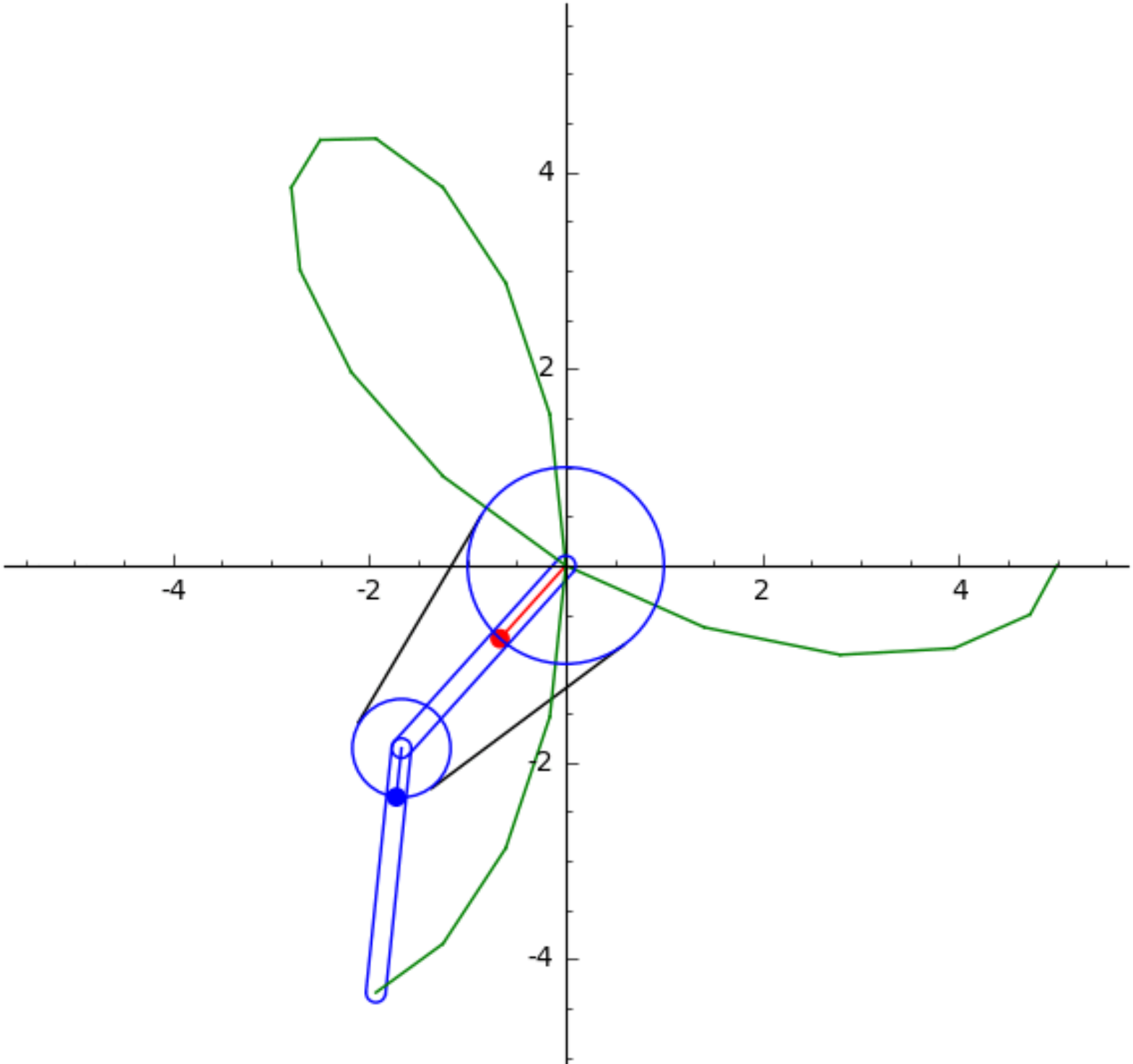
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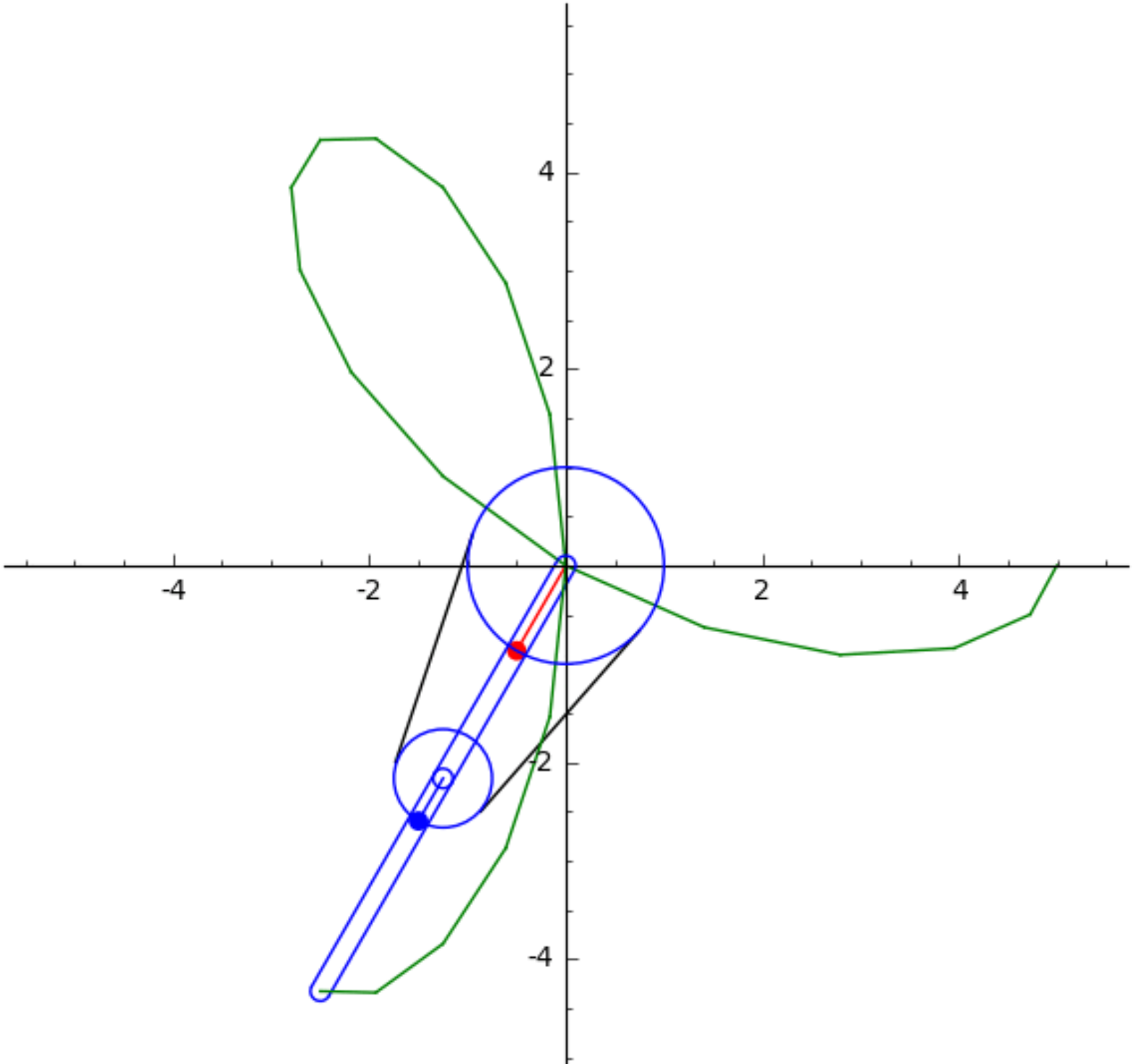
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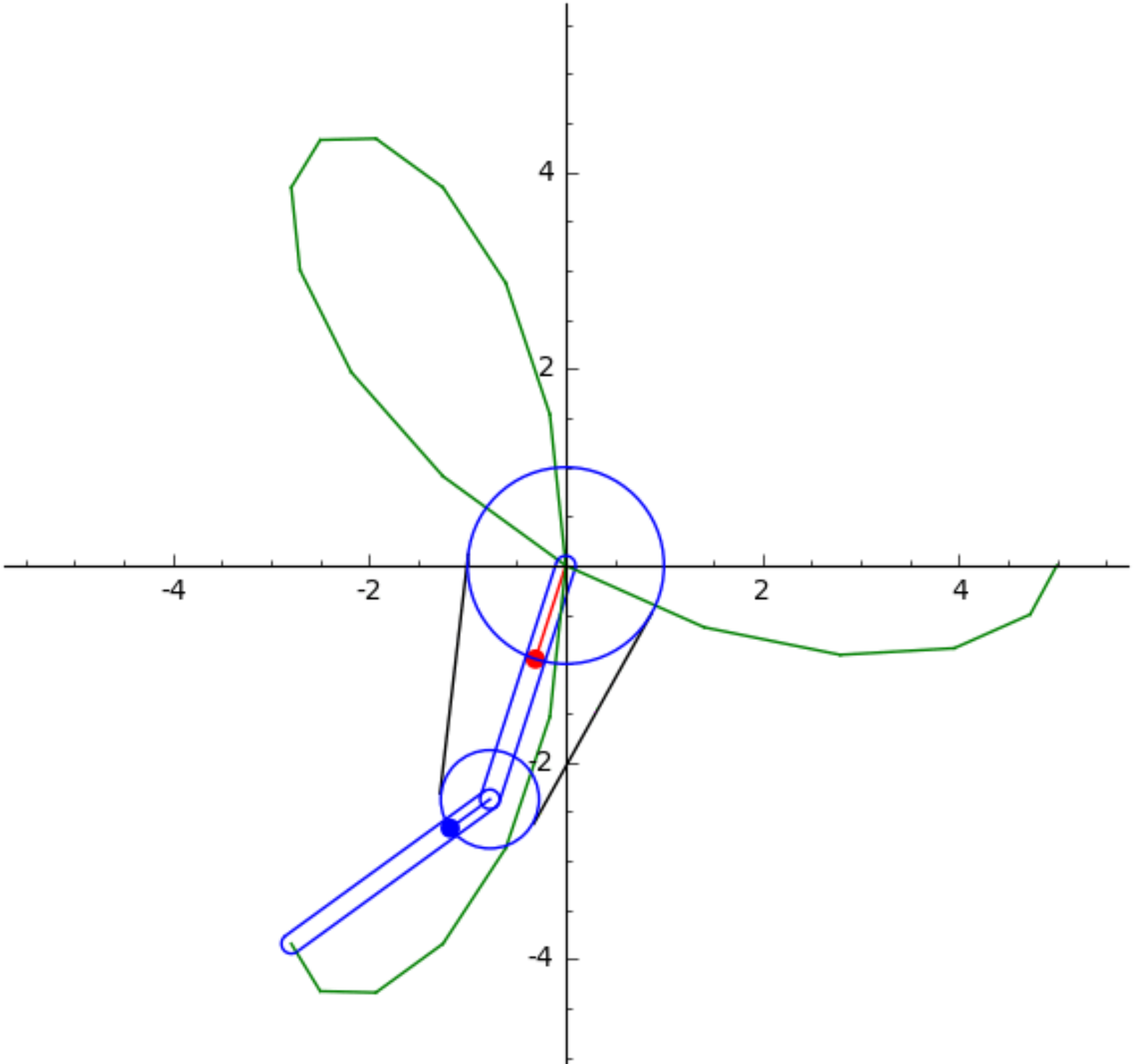
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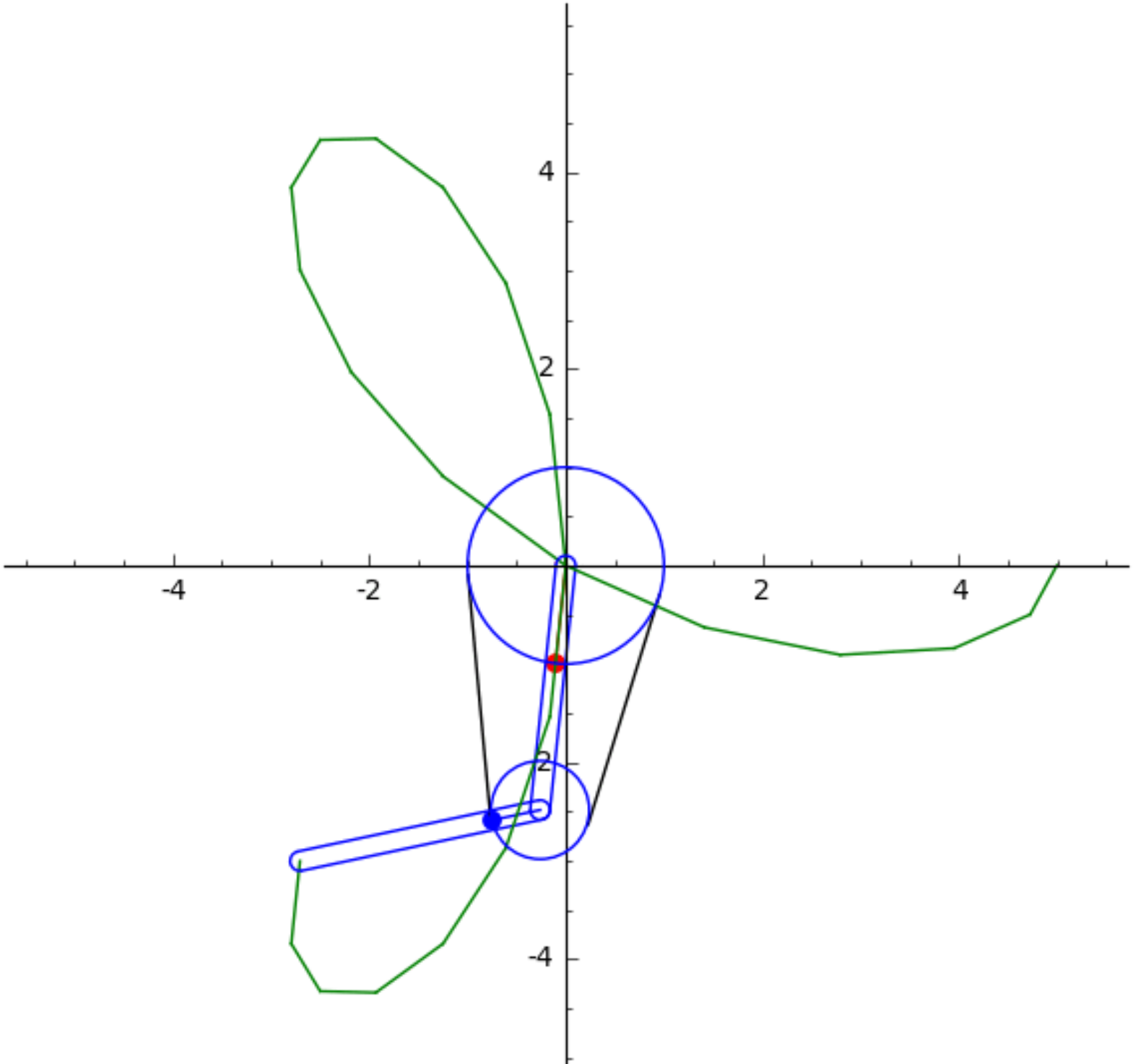
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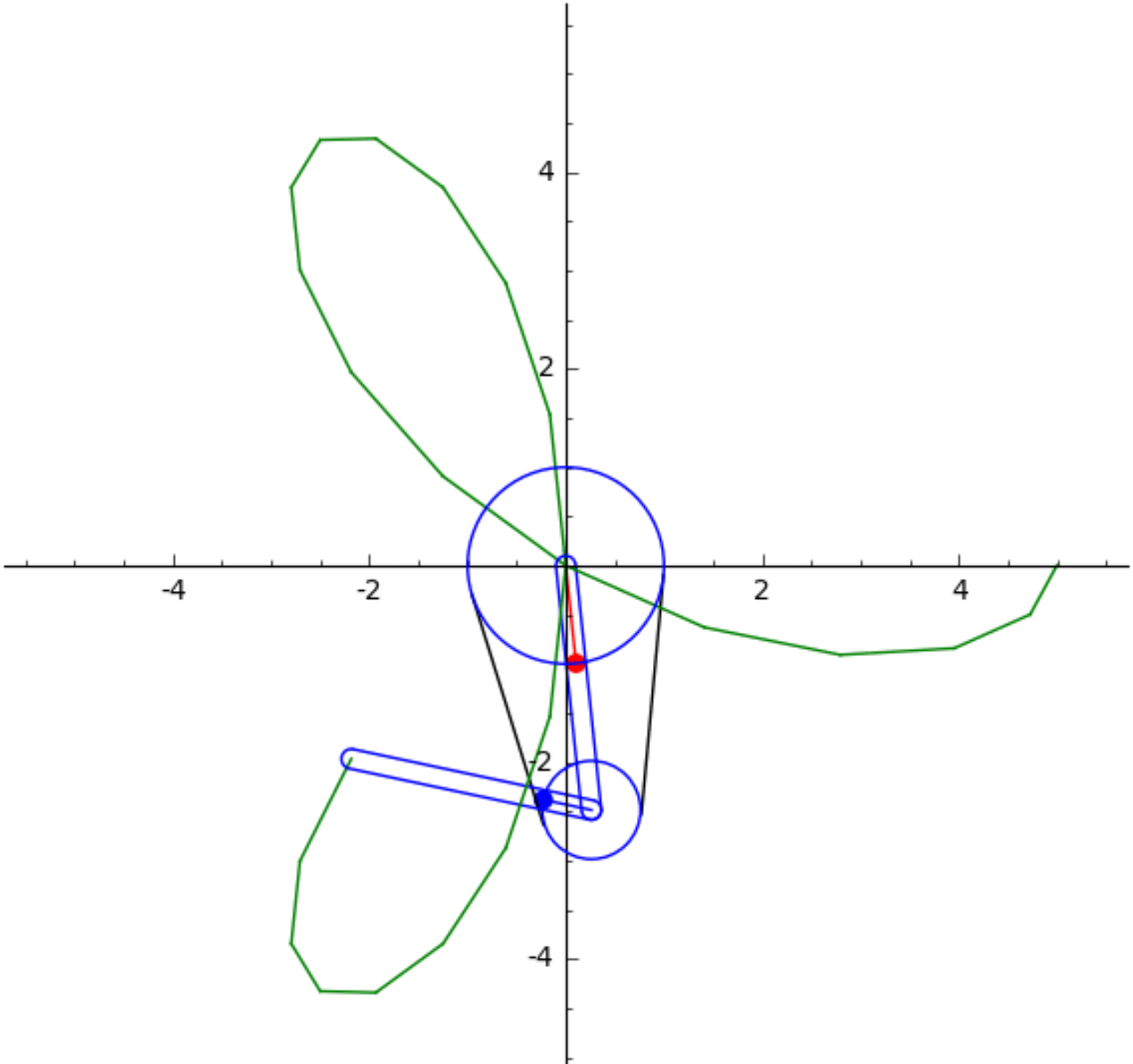
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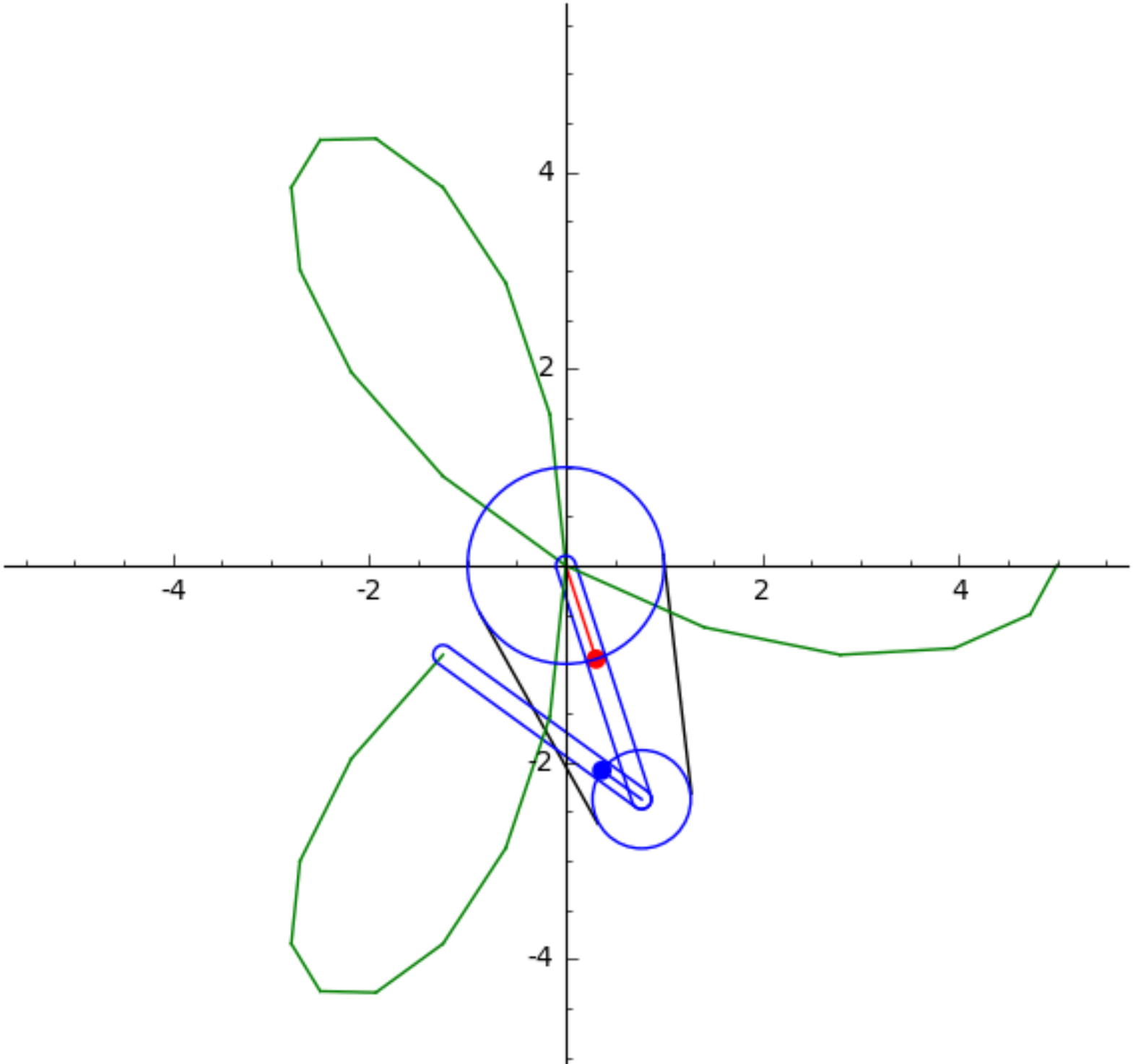
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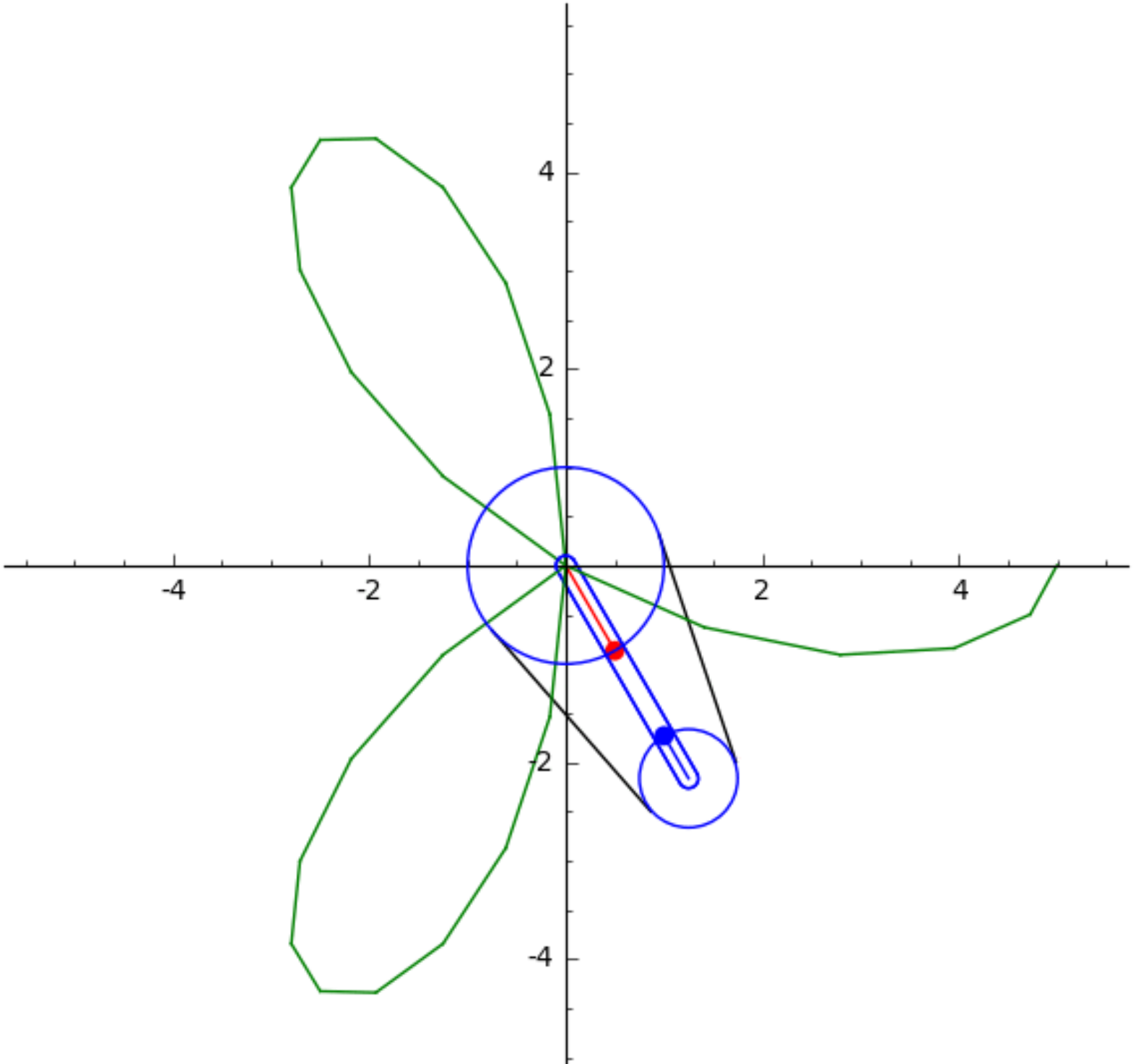
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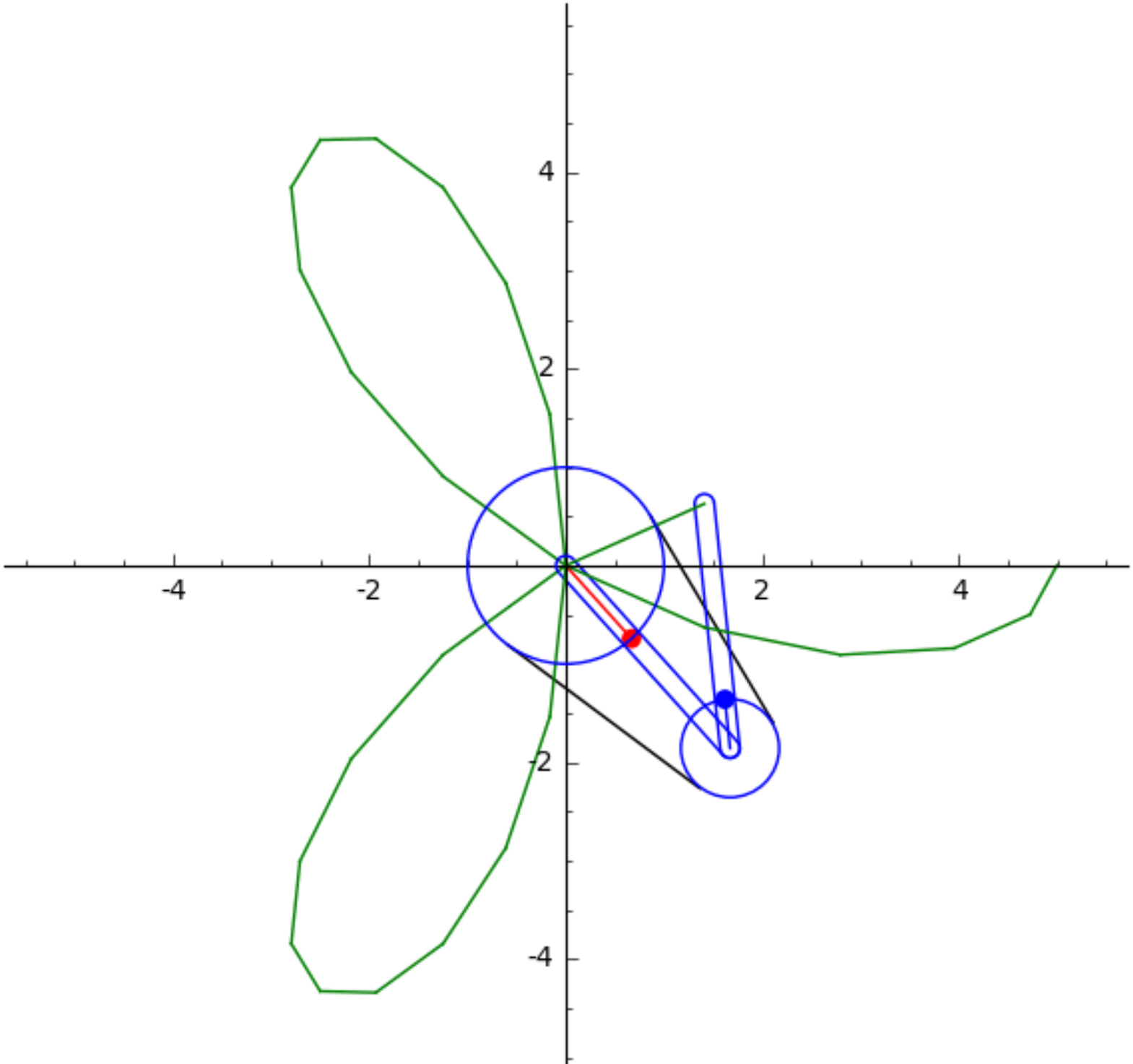
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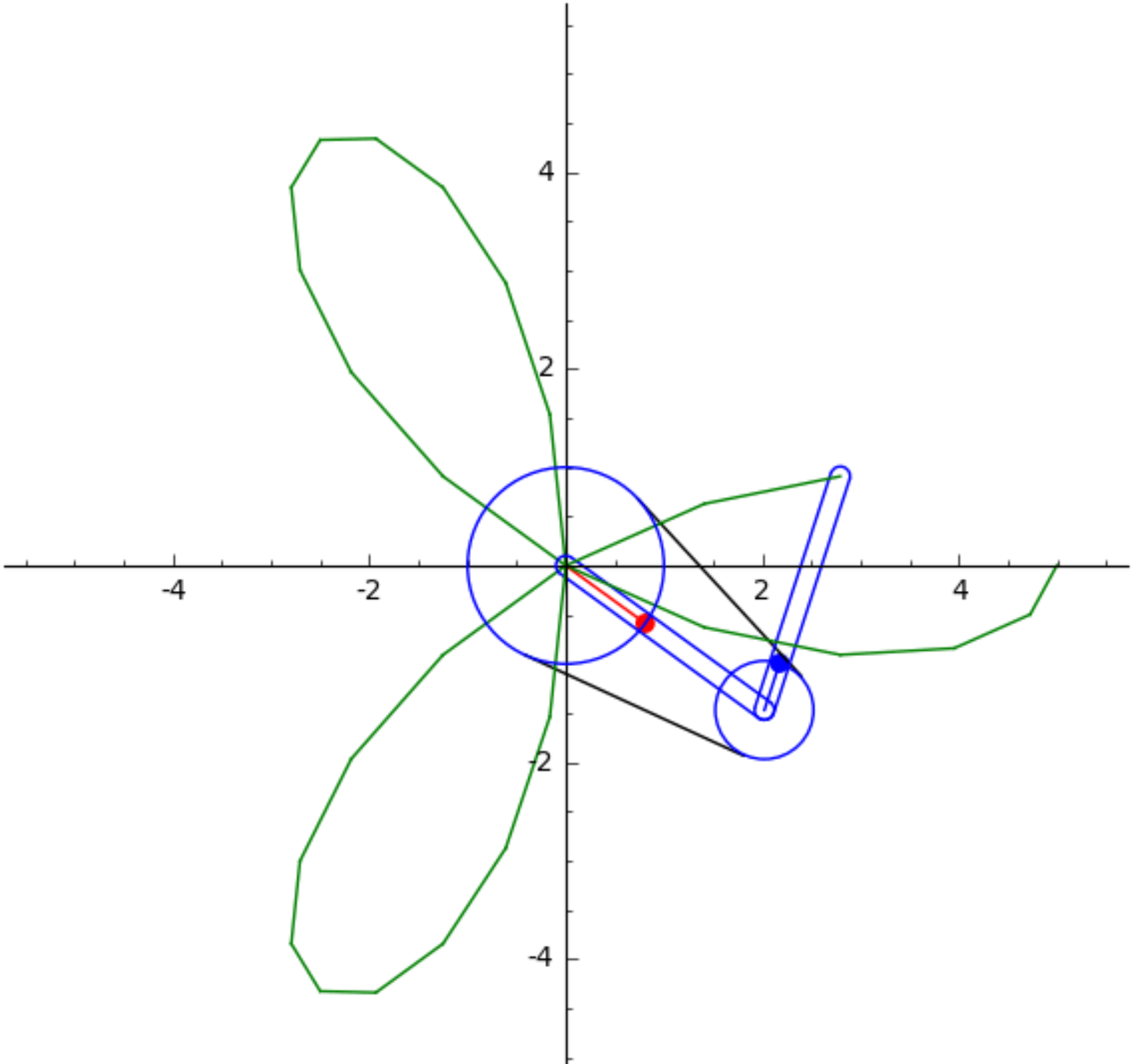
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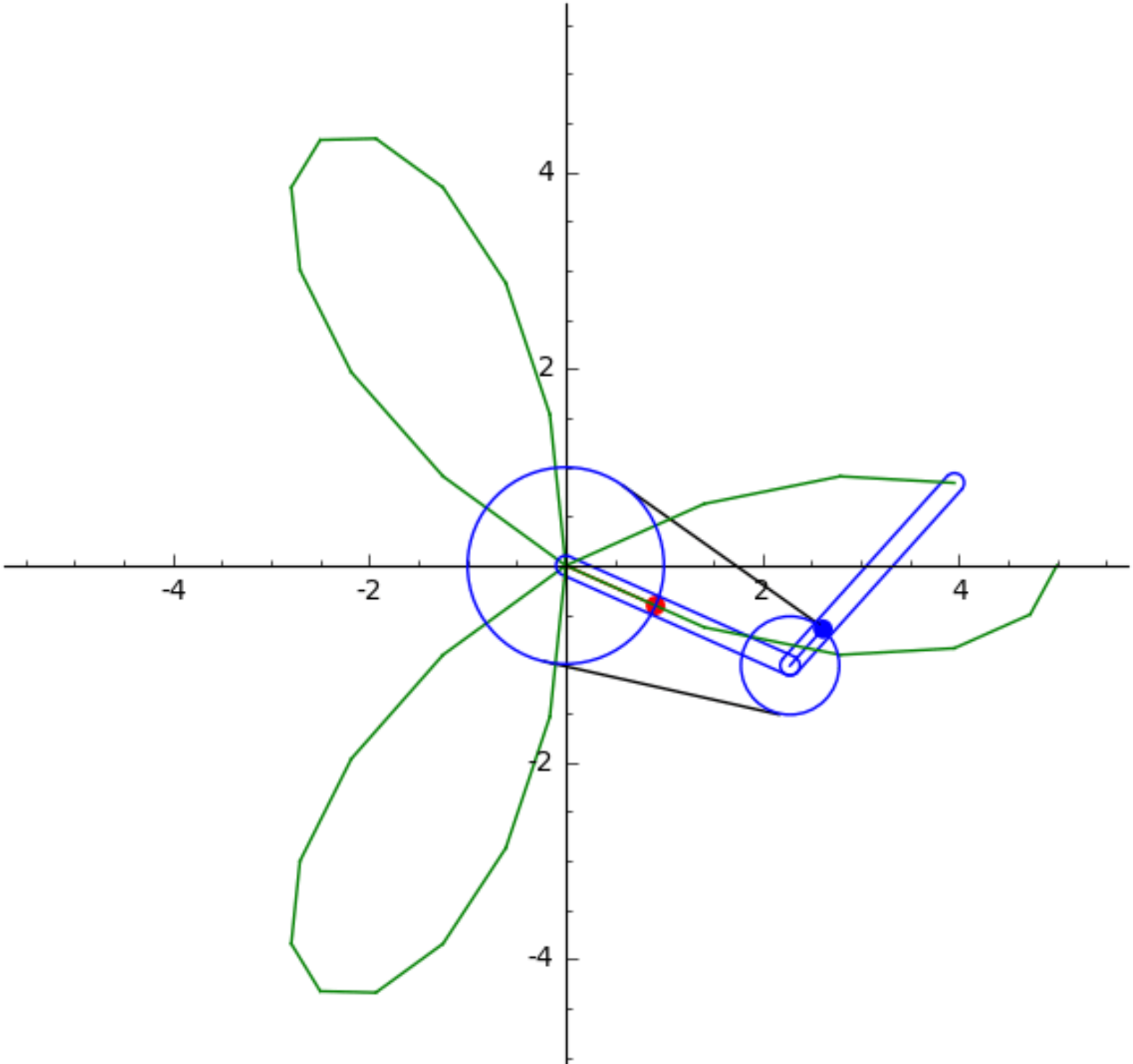
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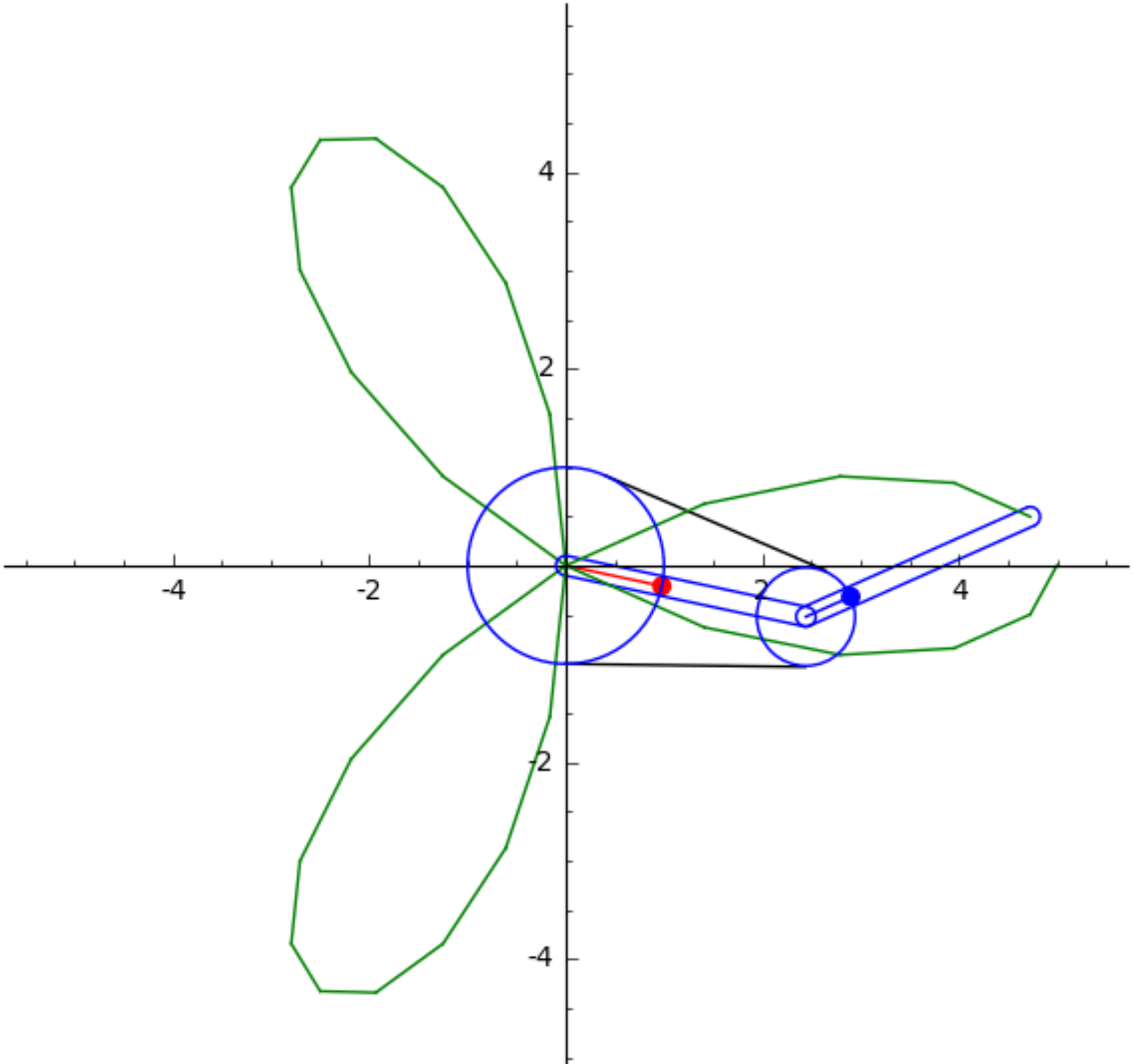
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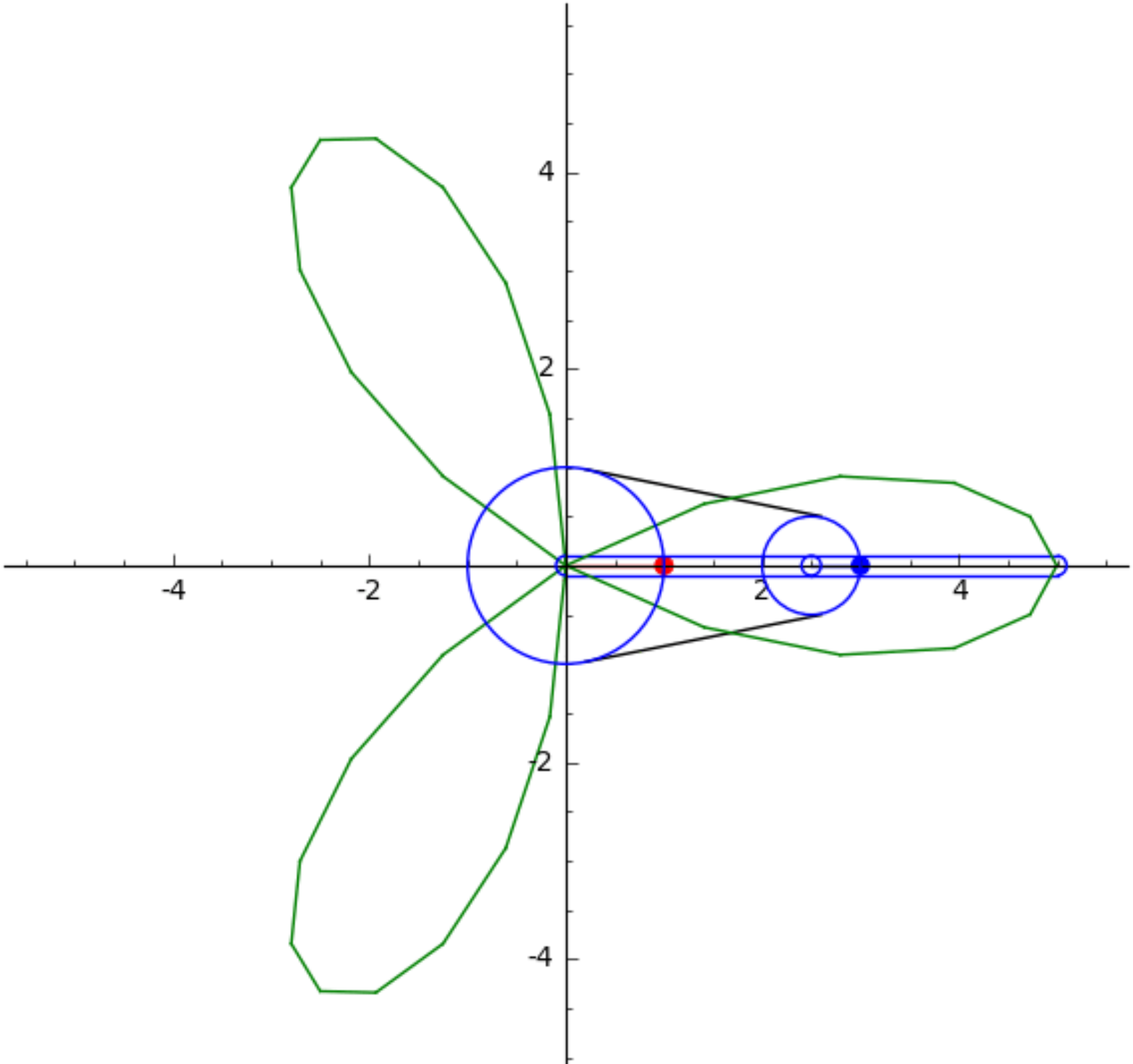
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# References


- Liu Y, Michael McCarthy JJ. *Design of Mechanisms to Draw Trigonometric Plane Curves.* ASME. J. Mechanisms Robotics. 2017;9(2):024503-024503-8. doi:10.1115/1.4035882.
- SageMath, the Sage Mathematics Software System (Version 7.5.1), The Sage Developers, 2017, http://www.sagemath.org.
- Wikipedia contributors. *"Kempe's universality theorem."* Wikipedia, The Free Encyclopedia. Wikipedia, The Free Encyclopedia, 6 Sep. 2017. Web. 25 Oct. 2017.