Improving Liver Allocation Using Optimized Neighborhoods

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Background. Geographic disparities persist in access to liver transplantation. Candidates with similar urgency experience varying opportunities for transplants across the United States. Policymakers are poised to act and 1 proposal entails reorganizing the current Organ Procurement and Transplant Network (OPTN) of 11 regions into 8 districts. However, redistricting has the shortcomings that Organ Procurement Organizations (OPOs) are disconnected from their immediate neighbors by district borders and that it is not easily responsive to uncertainty resulting from variability in donor and listing rates. **Methods.** We introduce the notion of an OPO's neighborhood—a collection of donor service areas (DSA) surrounding the OPO that acts as the OPO's region in the current local-regional-national framework. Districts and concentric circles are special cases. We design 58 neighborhoods for the DSAs with several attractive properties and optimize them to balance supplies and demands using 10 years of Organ Procurement and Transplant Network data. We conduct a simulation experiment comparing current allocation, redistricting, and neighborhoods under current sharing policies with respect to the following metrics: total mortalities, DSA-average model for end-stage liver disease (MELD) at transplant, DSA-average MELD standard deviation, and average organ transport distance. Liver-simulated allocation model cannot accommodate neighborhoods, so we programmed a discrete-event simulator, LivSim, to approximate liver-simulated allocation model. **Results.** We exhibited a neighborhood solution. Compared with the current allocation, simulation results showed that neighborhoods reduce the DSA-average MELD standard deviation by 29% and save about 65 lives annually. Compared with redistricting, the neighborhoods had smaller average transport distances that were more uniform across DSAs, saved about 20 additional lives, and reduced DSA-average MELD standard deviation by an additional 17%. **Conclusions.** Alternatives to redistricting with desirable properties and performance are possible and should be considered.

(*Transplantation* 2017;101: 350–359)

Liver transplantation is the only restorative therapy for irreversible and progressive liver failure.[1,2] The longevity and quality of life of the thousands of Americans listed with end-stage liver disease are significantly influenced by the performance of the national organ procurement and transplantation system. For the 14 637 patients waitlisted in the United States in 2014, the vast majority of organs for transplantation were obtained from deceased donors (6449 [96%] of 6729 transplants).[3-5] Liver transplantation is thus marred by the shortage of available donated livers with 1767 patients dying while waiting for a transplant in 2013 and an additional 1223 patients removed from the waitlist because they become too sick while waiting.[4] Regrettably, the current structure of the liver allocation system allows geographic disparities in access to a transplant to exist among those with similar medical urgency.[6-8]

The United Network for Organ Sharing (UNOS) is responsible for overseeing the national network for organ procurement and organ allocation, and for promoting organ donation. Within UNOS, the Liver and Intestinal Committee[9] is actively seeking to resolve the disparity issue. The current proposal being put to public comment in August 2016 entails redistricting the nation into 8 districts in order to promote fairer distribution of transplanted organs.[10-14] Mehrotra

Received 12 August 2016. Revision received 22 September 2016.

Accepted 23 September 2016.

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This work is funded by National Science Foundation award CMMI-1131568 and Agency for Healthcare Research and Quality award 1R36HS024840-01.

The authors declare no conflicts of interest.

The content of this work does not necessarily reflect the views or policies of the Department of Health and Human Services, nor does mention of trade names, commercial products, or organizations imply endorsement by the US Government. Data for this study were provided by the Minneapolis Medical Research Foundation as the contractor for the Scientific Registry of Transplant Recipients and by the United Network for Organ Sharing. Two anonymous reviewers provided helpful comments. The authors assume responsibility for the integrity of this work and all views expressed herein.

V.K. implemented research design, conducted experiments and analyses, and drafted the article. S.M. developed research design, supervised experiments and analyses, and edited the article.

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Supplemental digital content (SDC) is available for this article. Direct URL citations appear in the printed text, and links to the digital files are provided in the HTML text of this article on the journal's Web site (www.transplantjournal.com).

ISSN: 0041-1337/17/10102-350

DOI: 10.1097/TP.0000000000001505

et al[15,16] affirmed the significance of the redistricting plan and its methodology but also argued for further independent testing and exploration of alternatives. This article responds to invitations to provide such an alternative.[17]

CURRENT LIVER ALLOCATION

Liver allocation in the United States is overseen through 2 separate congressionally mandated contracts: the Organ Procurement and Transplantation Network (OPTN) currently held by UNOS; and the Scientific Registry of Transplant Recipients currently held by the Minneapolis Medical Research Foundation. Input is solicited from the transplantation community with oversight provided by the Division of Transplantation within the Health Resources and Services Administration of the US Department of Health and Human Services.[18-20] The current geographic structure for the OPTN divides the US into 11 regions, each of which is a grouping of several neighboring states. These regions are further subdivided into 58 Donor Service Areas (DSAs) total with the DSAs not necessarily having boundaries that correspond with state borders. Each DSA has a designated Organ Procurement Organization (OPO) that facilitates local procurement and allocation procedures. Allocation of livers is based primarily on a 3-tier geographic system—local/regional/national (local refers to the DSA of the procuring OPO).[21]

The OPTN follows certain policies in its operations. These policies mainly prioritize which candidates on the waitlist are offered an organ for transplantation in the 3-tier system. The recent liver allocation policies and their history are summarized in Elwir and Lake and Trotter.[22,23] Current policy adheres to the principles of transplanting "the sickest first" and that "organs and tissues ought to be distributed on the basis of objective priority criteria and not on the basis of accidents of geography" as promulgated by the Health Resources and Services Administration, and refined by the Institute of Medicine, into the US Department of Health and Human Services Final Rule.[18] Compliance with the regulations is ongoing and has resulted in several incremental changes to liver allocation policy (eg, Status 1, model for end-stage liver disease [MELD], Share 15, Share 35).[4,24,25] The MELD score, a predictor of 3-month mortality without liver transplantation, currently serves as the key metric for assessing medical urgency[26-29]; however, there is no similarly established standard for assessing geographic disparity in organ transplantation although several possibilities have been proposed.[30] MELD scores range from 6 to 40 points (more points implying greater urgency) and are based on laboratory values (INR, bilirubin, creatinine, and as of January 2016, sodium[31-33]). However, point assignment is not purely model-based, as candidates may receive additional "exception points" that augment their MELD score based on circumstantial criteria (of which hepatocellular carcinoma [HCC] is a prominent example[34]).

Table 1 provides an overview of current liver allocation. The geographic structure of the OPTN and the sharing policies together comprise how deceased donor-livers are allocated to recipients.

TABLE 1.

Overview of deceased donor liver allocation policy for adult[a] recipients

Deceased donor liver prioritization[b]
Local and regional status 1A or 1B candidates
Regional and local candidates with MELD 35-40 (ie, local 40, regional 40, local 39, …)
Local candidates with MELD $\geq$ 15
Regional candidates with MELD $\geq$ 15
National status 1A or 1B candidates
National candidates with MELD $\geq$ 15
Local candidates with MELD < 15
Regional candidates with MELD <15
National candidates with MELD <15

[a] Pediatric candidates are prioritized differently.
[b] Overview of prioritization. Rules shown have minor modifications based on ABO blood type, donor-recipient compatibility, and multiple transplant recipients.

POLICY INITIATIVES

Despite existing organ sharing policies, there continues to be discrepancies across the 58 DSAs in a number of metrics such as the mean MELD score at transplant (>10 points), transplant rates (>20-fold),[6] placement on waitlist (>14-fold),[7] and deaths due to end-stage liver disease (>19-fold).[35] The redistricting plan under consideration involves regrouping the DSAs into 8 districts instead of the current 11 UNOS regions.[10-14] The proposal has evolved over the past 4 years and was simulated under different sharing policies and compared with various alternatives. The redistricting plan is based on an optimization model that solves for a new grouping of DSAs into districts where MELD at transplant across the DSAs in the district is as equal as possible.[10] The plan is projected to reduce total mortalities while slightly increasing organ transport distances and times.[14]

The methodology for redistricting demonstrates the value of optimization techniques, but some of its limitations ought to be addressed.[10] We confine the critique to the proposed geographic structure (ie, regrouping the 58 DSAs into 8 instead of 11 districts) rather than any specific sharing policy that was tested. Figure 1 demonstrates the chief structural shortcoming of the redistricting solution and the concept of districting in general. We use Tennessee as an example. According to the redistricting plan, the OPO serving Western Tennessee will share organs with parts of Arkansas and Missouri rather than with Eastern Tennessee during regional allocation. Eastern Tennessee will instead share with parts of Illinois, Indiana, Kentucky, Ohio, and Wisconsin.[16] Because redistricting partitions the country into geographically disjoint subsets, organs procured near district boundaries may be transported to recipients farther away, whereas candidates with greater medical urgency who are also closer to the procuring DSA, but are out-of-district, will not receive the organ. Unfortunately, this lack of connectivity among neighboring DSAs will be symptomatic of any redistricting plan. Concentric circles (where candidates within a specified physical radius of the donor hospital or procuring OPO are given additional priority) or momentarily granting out-of-district candidates that are closer to the donor hospital additional MELD points have been suggested to remedy this deficiency. Additionally, the example also demonstrates that districts are imbalanced in the numbers of OPOs within the district and in the population sizes necessary for supporting donor pools.

Interestingly, Tennessee in 1992 implemented a statewide sharing variance for kidney allocation (which proceeds similarly

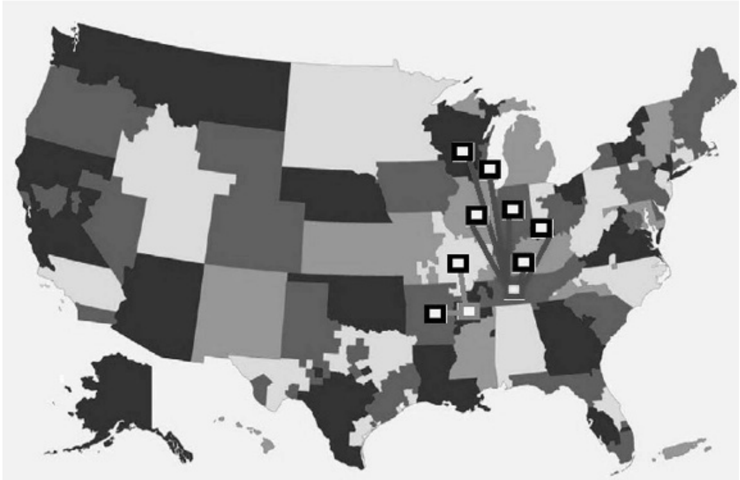

FIGURE 1. Redistricting limits interconnectivity among neighboring DSAs. The OPO serving primarily Western Tennessee also shares with OPOs in Arkansas and Eastern Missouri during regional allocation but not with the OPO serving primarily Eastern Tennessee in the current 8-district redistricting plan. The OPO in Eastern Tennessee will potentially send organs as far as Wisconsin before Western Tennessee in the redistricting plan.

to liver allocation, but more closely follows a local-regional-national setup without MELD-scoring). The policy variance reduced geographic disparities in kidney transplant rates and ischemic times by allowing the OPOs in Tennessee to preempt regional allocation and share with each other before sharing with OPOs out-of-state.[36] This historical incident further motivates the value of the notion of a DSA's neighborhood discussed in the following section.

A second major structural deficiency of any districting solution is its inability to locally respond to changes in policies and/or practices that may occur within an ever evolving transplant system. The OPTN is a dynamic system in which the behaviors of OPOs and transplant centers change over time. For example, the number of newly listed candidates needing a liver for transplant in each DSA from 2005 to 2015 fluctuated by approximately 15% year-to-year on average.[3] Any revision to the districts in response to these inevitable organ supply-demand imbalances will simultaneously affect multiple DSAs.

We present an approach that retains attractive features of both redistricting and concentric circles and is also amenable to the current operation of the OPTN. The framework is based on mathematical theory in operations research that surmounts some of the aforementioned limitations of redistricting.

MATERIALS AND METHODS

DSA/OPO Neighborhoods

The core concept of our proposal is to define regions as a specified set of neighboring DSAs for each DSA, instead of DSAs in a fixed district. A DSA's set of neighbors is called that DSA's *neighborhood*. Liver allocation may proceed just as before, except that during regional allocation, organs are shared with the procuring OPO's set of neighboring DSAs; thus maintaining the current local-regional-national hierarchy.

Figure 2 depicts an example of a neighborhood for Western Tennessee. In contrast to Figure 1, where Eastern and Western Tennessee are separated during regional allocation under redistricting, Figure 2 shows that the DSAs in Western Tennessee's neighborhood include its geographically immediate neighbors among others. The figure illustrates a neighborhood for a single DSA; each of the 58 DSAs has its own neighborhood. OPOs in each of these respective neighborhoods can be made to share with their geographically immediate neighbors among others during regional allocation. This requirement forces neighborhoods of adjacent DSAs to "overlap," that is, 2 adjacent DSAs will have some neighbors in common. This feature has an underpinning in the operations research literature on the theory of the design of manufacturing systems that are resilient to demand and supply uncertainty.[37-40] This literature discusses manufacturing systems and networks abstractly, but when translated into the context of the OPTN, it recommends the following: increasing a DSA's connectivity and creating overlapping neighborhoods promotes resilience in responding to demand and supply uncertainty; and balancing supply and demand across neighborhoods ensures greater equity. Interconnectivity is achieved by having each DSA's neighborhood contain other nearby DSAs. Supply-demand balancing is discussed in the following subsection.

The neighborhoods concept provides additional rigor to and generalizes the concept of concentric circles, as geographically immediate neighbors of a procuring OPO are within its neighborhood during regional allocation. Regions in the current OPTN and districts in the redistricting proposal are special types of neighborhoods that do not overlap; hence, the neighborhoods framework also generalizes districting.

Constructing the Neighborhoods

Selecting which DSAs belong in each DSA's neighborhood using multiple years of supply and demand data requires solving an optimization model. The neighborhood of a DSA identified from the optimization model forms the DSA's region in a local-regional-national policy. An explicit mathematical formulation of the optimization model is included in the **Technical Appendix, SDC**, http://links.lww.com/TP/B357.

Much like the geographic structure used in districting,[10] the neighborhoods can be constructed so that each of them has attractive properties. Table 2 summarizes the most important properties of the neighborhoods that were included in the modeling framework. They include that a neighborhood

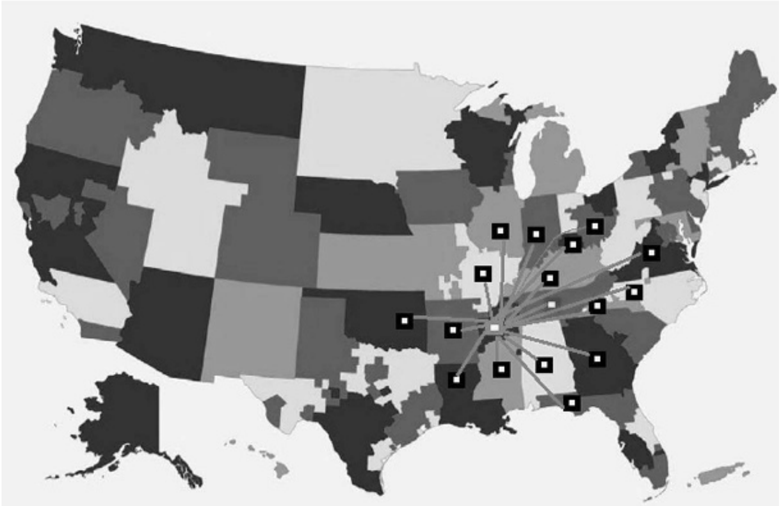

FIGURE 2. An example of a DSA's neighborhood. The OPO serving primarily Western Tennessee now shares with neighboring DSAs including Eastern Tennessee during regional allocation under the neighborhoods framework in contrast to redistricting in Figure 1.

for each DSA contains its geographically immediate neighbors; has relationships that are reciprocal or symmetric (ie, DSA *A* is in DSA *B*'s neighborhood if and only if DSA *B* is in DSA *A's* neighborhood); has a minimum and a maximum number of DSAs inside of it; attains a minimum population size; includes a minimum number of transplant centers; has bounded average organ travel distance/time; and is geographically contiguous. In the spirit of concentric circles, once geographically immediate neighbors are included in a specific DSA's neighborhood, the model will then consider including the geographically-immediate neighbors of those immediate neighbors, and so on; however, not all such neighbors are necessarily included and the model does not discriminate among neighbors at each stage. These properties promote resilience in the solution.

In addressing disparity, we adopt a Rawlsian[41] principle of justice in ensuring that the worst-off neighborhood is as close to the best-off neighborhood as possible. The model's objective is to balance the ratio of supply and demand across neighborhoods (not for a specific DSA). Additionally, the model departs from that used by Gentry et al[10] in 4 important ways. First, the model uses 10 years of historical supply and demand data to mitigate the uncertainty of annual changes in donor and listing rates. Second, demand in a specific year is measured by the number of waitlist additions to the liver transplantation list during that year. Other definitions for demand are possible. Third, the objective of the optimization model in Gentry et al[10] minimizes the sum of absolute deviations in the number of donors from demand across districts; it is preferable to minimize the ratios of supply and demand to avoid penalizing DSAs based on the number of donors and candidates that they have.[15] Thus, for each neighborhood, the model considers the ratio of the supply and demand and its deviation from the ratio of the expected value of the nationally aggregated supply and demand. Fourth, it minimizes the expected value of the maximum of these deviations, where the expectation is taken over the years 2005 to 2014.

We used Julia 0.3.10 and a commercial solver Gurobi 6.5 to solve the optimization model.[42,43] Hawaii and Puerto Rico were excluded from the model and their neighborhoods were defined to include the 4 closest DSAs in California and Florida respectively. Data on the numbers of transplant centers (as of 2015), population sizes (as of 2013), historical transplant volumes, and the numbers of organs recovered for transplant and the number of waitlist additions from 2005 to 2014 were obtained from UNOS and the Scientific Registry of Transplant Recipient. Transport distances (miles) were calculated using latitudes and longitudes of donor hospitals and transplant centers via the method of geodesics in SAS 9.4[44]

TABLE 2.

Properties ensured in neighborhood solutions

- √→***Formed using 10-year historical data:*** The DSA-neighborhoods are formed using a 10-y period (2005-2014) and hence incorporate uncertainty in organ availability and needs
- √→***Immediate Neighbors*:** DSAs have their geographically immediate neighbors in their neighborhood
- √→***Population:*** Each DSA's neighborhood has a minimum population
- √→***Symmetry in DSA Relationships*:** DSA *A* has DSA *B* in its neighborhood if and only if DSA *B* has DSA *A* in its neighborhood[a]
- √→***Density:*** Each DSA has a minimum and a maximum number of neighbors
- √→***Contiguity:*** Each DSA's neighborhood is geographically contiguous[a]
- √→***Compactness:*** The average transport distance/time for a DSA's neighborhood is bounded[a]
- √→***Transplant Centers:*** Each DSA's neighborhood has a minimum number of transplant centers[a]
- √→***Possibility to generate a spectrum of solutions, instead of 1:*** By parameter specifications, it is possible to generate many alternative solutions with different properties (eg, average distance, mortality, disparity, etc.) to facilitate decision making.

[a] Property also possessed by redistricting.[10]

Simulating Neighborhood Solutions

We test the performance of a neighborhoods solution from the optimization model in a simulation environment. Unfortunately, to our knowledge, the architecture of the simulation tool used in the transplantation community, the liver simulated allocation model (LSAM v Aug 2014), does not allow for neighborhoods.[45] We therefore programmed a discrete-event liver allocation simulator in Python 3, hereafter referred to as LivSim.

LivSim approximates LSAM from information available in publicly released sources. LivSim begins with an initial waitlist and takes 3 input streams: additions to the liver transplant waitlist, status updates of waitlist candidates, and arrivals of donors. LivSim then processes each of these events. When candidates arrive to a particular DSA, they are assigned a MELD score, ABO blood type, Status 1 exception (yes or no), and HCC exception (yes or no). During a status update, LivSim updates the candidate's MELD score and potentially removes the candidate from the waitlist or indicates their death. After a donor arrives, the liver is assigned an ABO blood type and is offered to ABO blood type-compatible candidates in accordance with the sharing policies and geographic structure in place. The current version of LivSim uses a reduced form of LSAM's organ acceptance model to calculate whether a candidate accepts a liver for transplant. The acceptance model uses LSAM's coefficients for whether the potential recipient is Status 1, the potential recipient's waiting time, whether the potential recipient is listed in the DSA of the procuring OPO, and donor blood type and assumes that all other patient attributes are held at the baseline. These 4 sets of coefficients included are also the 4 most significant predictors in LSAM's acceptance model. After LivSim processes these streams, it will calculate the posttransplant deaths and the average organ transport distance. Organ transport distances are calculated by assuming that any organ traveling between any 2 DSAs travel (including within a DSA) the historical average amount of distance; distances are not calculated using donor hospitals and transplant centers. The current version of LivSim operates at the DSA level and does not incorporate re-lists, re-transplants, and multiple transplants; it also assumes candidates will remain active on the waitlist once they are assigned a MELD score.

The input files generated by LSAM input generator modules for waitlist, patient listing, patient status updates, and posttransplant survival are used. LivSim incorporates Status 1, HCC exceptions, Share 15, and Share 35 sharing policies in addition to MELD scoring with and without sodium.

We calibrate LivSim against LSAM by comparing results generated by both simulators on the same input data for the current geographic structure and sharing policies. 5-year (January 2010- December 2014) patient listing data and status updates were generated by the LSAM Candidate Generator and organ donor data were generated by the LSAM Donor Generator.

Simulation Experiment for Comparing Geographic Structures

The simulation experiments using LivSim compare the performance of the geographic structures under the current allocation system, redistricting, the specific neighborhoods solution obtained, and national allocation. For each system, we assume that the Status 1, Share 15, and Share 35 policies are in place; the experiment only varies the geographic structure employed.

Performance Measures

We measure disparity by DSA mean transplant MELD standard deviation. DSA mean transplant MELD aims to measure the overall medical urgency of patients being transplanted and its standard deviation across DSAs measures geographic disparity in access to transplant. Additional important performance measures are waitlist and posttransplant mortalities, waitlist removals, and average organ transport distance. Average organ transport distances have implications for costs. The simulation experiment is conducted on the same input data used in the calibration with a 5-year run-length (January 2010 to December 2014). Differences in the performance measures relative to current allocation and between redistricting and neighborhoods were computed and significance was assessed using 2-tailed z-tests on differences between replication means. We performed 5 replications (25 replication-years) and modeled 2 cases:

(1) MELD without sodium: No candidate was excluded, even those with MELD exception points. HCC exceptions were included, but the recent cap and delay policy was not incorporated.[46]
(2) MELD with sodium: No candidate was excluded and it was assumed that no exception points for non-HCC candidates were awarded. HCC exceptions were included, but the recent cap and delay policy was not incorporated.[46]

RESULTS

Calibration Results

Table 3 presents the results of the calibration of LivSim against LSAM for current allocation. LivSim's results for all performance metrics except for average organ transport distance are within 10% relative error of LSAM. LivSim overestimates average organ transport distances because it uses right-skewed DSA-to-DSA historical averages of distances rather than donor-hospital-to-transplant-center distances. Actual distances are likely to be less than those reported.

Neighborhood Solution Found

We found a neighborhood solution where each neighborhood had at least 9 transplant centers and a population of 25 million; and the volume-weighted organ transport distance was less than 400 miles. Each DSA had at least 5 neighbors including itself and no more than 20 neighbors including itself. Table 4 provides a listing of the DSAs in each DSA's neighborhood. Bounds on distance, transplant centers,

TABLE 3.
Calibration results of LivSim vs LSAM

	LSAM	LivSim
Category	Current allocation	Current allocation
Annualized waitlist and posttransplant deaths	2301.9	2181.9
Annualized waitlist deaths	1230.8	1149.8
Annualized posttransplant deaths	1071.1	1032.1
Annualized waitlist removals	3453.4	3091.0
DSA mean transplant MELD	23.32	24.30
DSA mean transplant MELD standard deviation	2.00	2.03
Average organ transport distance (miles)	257	332

Input data generated by LSAM candidate and donor generators for 2010-2014.

population, and number of DSAs in the neighborhood may be adjusted.

Simulation Experiment Results

Tables 5 and 6 present the 5-year comparative performances of the current allocation, redistricting, the neighborhood solution found, and national allocation for the cases of using MELD without sodium and with sodium respectively. We emphasize results for the latter case since it is more representative of the most recent liver allocation policy. All estimates are differences relative to current allocation.

For standard deviation in DSA mean transplant MELD, in the case of MELD without sodium, redistricting and neighborhoods both achieve significant reductions in the standard deviation of mean transplant MELD of 0.48 and 0.50 points, respectively, (a 24% and 25% reduction with respect to current allocation) when compared to current allocation ($P < 0.05$). In the case of MELD with sodium, both achieve significant reductions of 0.50 and 0.59 points, respectively (a 25% and 29% reduction with respect to current allocation) ($P < 0.05$). Compared to redistricting in this case, neighborhoods significantly reduced disparity by an additional 17% ($P < 0.05$).

Experiment results for either case demonstrate that both redistricting and neighborhoods achieve significant annual reductions in the total of posttransplant and waitlist mortalities compared to current allocation ($P < 0.05$), with neighborhoods saving an additional 20-25 lives annually compared to redistricting in both cases ($P < 0.05$). MELD with sodium scoring improves mortality reductions for all structures. Both redistricting and neighborhoods reduce waitlist removals by 40 to 55 each year compared to current allocation, but the finding was not significant ($P > 0.05$); additionally, no significant difference was found between redistricting and neighborhoods in this regard.

Redistricting will increase DSA mean transplant MELD by approximately 0.6 points in either case ($P < 0.05$), and neighborhoods will do so by 0.8 points in the case of MELD without sodium and 0.9 points in the case of MELD with sodium ($P < 0.05$) when compared with current allocation. The differences between redistricting and neighborhoods are also statistically significant ($P < 0.05$). Both structures will increase average organ transport distances compared with current allocation; redistricting by approximately 36 miles per organ, and neighborhoods by approximately 24 miles, or 33% less in the case of MELD without sodium while MELD with sodium scoring shows an increase of 43 and 36 miles respectively. All differences in transportation distances between redistricting and neighborhoods with current allocation and among each other were significant ($P < 0.05$).

Further analysis of the simulation results show that the benefits of neighborhoods are also borne more uniformly. For both the cases of MELD without sodium and MELD with sodium, respectively, we calculated the DSA ranges for waitlist mortalities; total miles procured organs are transported; and MELD at transplant. The DSA ranges are defined as the difference of the minimum number of waitlist mortalities (resp. total miles transported, average MELD at transplant) across DSAs from the maximum number of waitlist mortalities (resp. total miles transported, average MELD at transplant) across DSAs averaged over all replications. For MELD without sodium and with sodium, DSA ranges for mortalities decreased by 13% and 15% respectively for neighborhoods relative to current allocation. This decrease is 11% and 12% respectively for redistricting. Ranges in total miles transported fell by 12% in both cases for neighborhoods. However, they rose by 3% and 2% respectively for redistricting. The ranges for MELD at transplant fell by 12% and 17%, respectively, for neighborhoods. This reduction is 8% and 9% respectively for redistricting. All changes were statistically significant ($P < 0.001$) relative to current allocation. Differences between redistricting and neighborhoods were significant for ranges of MELD at transplant in the case of MELD with sodium ($P = 0.004$) and ranges of total miles transported for both cases ($P < 0.001$).

DISCUSSION

Optimally designed districts and neighborhoods both further the goals of transplanting the sickest first, reducing total mortalities, and promoting fairness in transplant access when compared with the current allocation system. However, as the particular neighborhood solution demonstrates, interventions that exceed the redistricting structure in these aims are possible. Moreover, the neighborhood solution, while exhibiting smaller increases in average organ transport distances, exceeds redistricting in improvements on average DSA transplant MELD (especially in the MELD with sodium case where exceptions were not granted to most candidates); total mortalities; and DSA transplant MELD standard deviation.

These advantages stem from the structural design. A neighborhood for a given DSA expands the DSA's regional allocation and thereby results in more organs being directed to sicker candidates by sharing policies (eg, Share 35). The inclusion of geographically immediate neighbors of that DSA helps forestall rising organ transport distances. The neighborhoods are optimally constructed so that available organs for transplantation relative to demand are as equal as possible—thereby reducing geographic disparity. We emphasize that the framework is not specifying the number of organs an OPO will send to its region but merely ensuring similar opportunities to access organs from regional allocation. Also, the shapes of each neighborhood (with respect to compactness in historical-transplant volumes) may be further constrained with guidance from policymakers (eg, use forecasted transplant volumes, contain a limited number of US states, have maximum geographic areas, and so on).

The underlying optimization model confers the neighborhoods with additional resilience against uncertainty. Several trends (eg, acute alcoholic hepatitis transplantation, healthcare reform, ex vivo liver perfusion, varying organ refusal rates, evolving community demographics, etc.) can cause unforeseen changes in donor organ supply and demand. Historical variability in procurement rates and listing rates are incorporated so that a particular neighborhood solution remains the same as long as the ratios of donors to candidates at individual DSAs remain close to their 2005 to 2014 historical averages. An advantage of using a stochastic rather than a deterministic optimization methodology is that data forecasting procurement and listing rates may be used in lieu of historical data to construct the neighborhoods in future work—and such solutions would remain similarly stable

TABLE 4.

Example neighborhoods solution[a]

Procuring OPO	DSAs belonging to procuring OPO's neighborhood																		
ALOB-OP1 Alabama Organ Center	ALOB	AROR	FLUF	GALL	KYDA	LAOP	MOMA	MSOP	NCNC	TNDS	TNMS	VATB							
AROR-OP1 Arkansas Reg. Organ Recovery Agency	ALOB	AROR	AZOB	CORS	IAOP	ILIP	INOP	KYDA	LAOP	MOMA	MSOP	MWOB	NEOR	NMOP	OKOP	TNDS	TNMS	TXGC	TXSA	TXSB
AZOB-OP1 Donor Network of Arizona	AROR	AZOB	CADN	CAGS	CAOP	CASD	CORS	NMOP	NVLV	OKOP	ORUO	TXGC	TXSA	TXSB	UTOP	WALC				
CADN-OP1 Donor Network West	AZOB	CADN	CAGS	CAOP	HIOP	NVLV	ORUO	WALC												
CAGS-OP1 Sierra Donor Services	AZOB	CADN	CAGS	CAOP	CASD	HIOP	NVLV	ORUO	UTOP	WALC										
CAOP-OP1 OneLegacy	AZOB	CADN	CAGS	CAOP	CASD	HIOP	NVLV	ORUO	UTOP											
CASD-IO1 Lifesharing - A Donate Life Org.	AZOB	CAGS	CAOP	CASD	HIO	NVLV	UTOP													
CORS-OP1 Donor Alliance	AROR	AZOB	CORS	IAOP	MNOP	MOMA	MWOB	NEOR	NMOP	NVLV	OKOP	ORUO	TXGC	TXSA	TXSB	UTOP	WALC	WIUW		
CTOP-OP1 LifeChoice Donor Services	CTOP	MAOB	NJTO	NYAP	NYFL	NYRT	NYWN	OHLP	PADV	PATF										
DCTC-OP1 Washington Reg Transplant Community	DCTC	MDPC	NCNC	NJTO	NYRT	PADV	PATF	TNDS	VATB											
FLFH-IO1 TransLife	FLFH	FLMP	FLUF	FLWC	GALL	PRLL														
FLMP-OP1 Life Alliance Organ Recovery Agency	FLFH	FLMP	FLUF	FLWC	GALL	PRLL														
FLUF-IO1 LifeQuest Organ Recovery Services	ALOB	FLFH	FLMP	FLUF	FLWC	GALL	PRLL	SCOP	TNMS											
FLWC-OP1 LifeLink of Florida	FLFH	FLMP	FLUF	FLWC	GALL	PRLL														
GALL-OP1 LifeLink of Georgia	ALOB	FLFH	FLMP	FLUF	FLWC	GALL	KYDA	MDPC	MSOP	NCCM	NCNC	PATF	SCOP	TNDS	TNMS	VATB				
HIOP-OP1 Legacy of Life Hawaii	CADN	CAGS	CAOP	CASD	HIOP															
IAOP-OP1 Iowa Donor Network	AROR	CORS	IAOP	ILIP	INOP	MNOP	MOMA	MWOB	NEOR	OKOP	WALC	WIDN	WIUW							
ILIP-OP1 Gift of Hope	AROR	IAOP	ILIP	INOP	MIOP	MNOP	MOMA	NEOR	OHLC	OHOV	TNMS	WIDN	WIUW							
INOP-OP1 Indiana Donor Network	AROR	IAOP	ILIP	INOP	KYDA	MIOP	MOMA	MWOB	OHLB	OHLC	OHLP	OHOV	TNMS	WIDN	WIUW					
KYDA-OP1 KY Organ Donor Affiliates	ALOB	AROR	GALL	INOP	KYDA	MDPC	MOMA	NCCM	NCNC	NYFL	NYWN	OHLB	OHLC	OHLP	OHOV	PATF	TNDS	TNMS	VATB	
LAOP-OP1 Louisiana Organ Procurement Agency	ALOB	AROR	LAOP	MOMA	MSOP	MWOB	NMOP	OKOP	TNDS	TNMS	TXGC	TXSA	TXSB							
MAOB-OP1 New England Organ Bank	CTOP	MAOB	NJTO	NYAP	NYFL	NYRT	NYWN	OHLP	PADV	PATF										
MDPC-OP1 The Living Legacy Foundation of MD	DCTC	GALL	KYDA	MDPC	NCNC	NJTO	NYAP	NYFL	NYRT	NYWN	OHLB	OHLP	PADV	PATF	TNDS	VATB				
MIOP-OP1 Gift of Life Michigan	ILIP	INOP	MIOP	OHLB	OHLC	OHLP	OHOV	WIDN	WIUW											
MNOP-OP1 LifeSource Upper Midwest OPO	CORS	IAOP	ILIP	MNOP	MOMA	MWOB	NEOR	OKOP	ORUO	WALC	WIDN	WIUW								
MOMA-OP1 Mid-America Transplant Svcs	ALOB	AROR	CORS	IAOP	ILIP	INOP	KYDA	LAOP	MNOP	MOMA	MWOB	OHLC	OHLP	OHOV	OKOP	TNMS	WIDN	WIUW		
MSOP-OP1 Mississippi Organ Recovery Agency	ALOB	AROR	GALL	LAOP	MSOP	NCNC	OKOP	PATF	TNDS	TNMS	TXGC	VATB								
MWOB-OP1 Midwest Transplant Network	AROR	CORS	IAOP	INOP	LAOP	MNOP	MOMA	MWOB	NEOR	OKOP	TXGC	TXSB	WALC	WIUW						
NCCM-IO1 LifeShare of the Carolinas	GALL	KYDA	NCCM	NCNC	PATF	SCOP	TNDS	TNMS	VATB											
NCNC-OP1 Carolina Donor Services	ALOB	DCTC	GALL	KYDA	MDPC	MSOP	NCCM	NCNC	OHLB	PATF	SCOP	TNDS	TNMS	VATB						
NEOR-OP1 Nebraska Organ Recovery System	AROR	CORS	IAOP	ILIP	MNOP	MWOB	NEOR	OKOP	WALC	WIUW										
NJTO-OP1 NJ Organ and Tissue Sharing Network	CTOP	DCTC	MAOB	MDPC	NJTO	NYAP	NYFL	NYRT	NYWN	OHLP	PADV	PATF								
NMOP-OP1 New Mexico Donor Services	AROR	AZOB	CORS	LAOP	NMOP	NVLV	OKOP	TXGC	TXSA	TXSB	WALC									
NVLV-OP1 Nevada Donor Network	AZOB	CADN	CAGS	CAOP	CASD	CORS	NMOP	NVLV	ORUO	UTOP	WALC									
NYAP-OP1 Ctr for Donation and Transplant	CTOP	MAOB	MDPC	NJTO	NYAP	NYFL	NYRT	NYWN	OHLB	OHLP	PADV	PATF	TNDS	VATB						
NYFL-IO1 Finger Lakes Donor Recovery Network	CTOP	KYDA	MAOB	MDPC	NJTO	NYAP	NYFL	NYRT	NYWN	OHLB	OHLP	PADV	PATF	TNDS	VATB					
NYRT-OP1 LiveOnNY	CTOP	DCTC	MAOB	MDPC	NJTO	NYAP	NYFL	NYRT	NYWN	PADV	PATF	VATB								
NYWN-OP1 Upstate NY Transplant Svcs	CTOP	KYDA	MAOB	MDPC	NJTO	NYAP	NYFL	NYRT	NYWN	OHLB	OHLP	PADV	PATF	TNDS	VATB					
OHLB-OP1 LifeBanc	INOP	KYDA	MDPC	MIOP	NCNC	NYAP	NYFL	NYWN	OHLB	OHLC	OHLP	OHOV	PADV	PATF	TNDS	VATB				

OHLC-OP1 Life Connection of Ohio	ILIP INOP KYDA MIOP MOMA OHLB OHLC OHLP OHOV WIUW
OHLP-OP1 Lifeline of Ohio	CTOP INOP KYDA MAOB MDPC MIOP MOMA NJTO NYAP NYFL NYWN OHLB OHLC OHLP OHOV PADV PATF TNDS TNMS VATB
OHOV-OP1 LifeCenter Organ Donor Network	ILIP INOP KYDA MIOP MOMA OHLB OHLC OHLP OHOV TNMS
OKOP-OP1 LifeShare Transplant Donor Svcs of OK	AROR AZOB CORS IAOP LAOP MNOP MOMA MSOP MWOB NEOR NMOP OKOP ORUO TNMS TXGC TXSA TXSB UTOP WALC
ORUO-IO1 Pacific NW Transplant Bank	AZOB CADN CAGS CAOP CORS MNOP NVLV OKOP ORUO UTOP WALC WIUW
PADV-OP1 Gift of Life Donor Program	CTOP DCTC MAOB MDPC NJTO NYAP NYFL NYRT NYWN OHLB OHLP PADV PATF TNDS VATB
PATF-OP1 Center for Organ Recovery and Educ.	CTOP DCTC GALL KYDA MAOB MDPC MSOP NCCM NCNC NJTO NYAP NYFL NYRT NYWN OHLB OHLP PADV PATF TNDS VATB
PRLL-OP1 LifeLink of Puerto Rico	FLFH FLMP FLUF FLWC PRLL
SCOP-OP1 LifePoint, Inc.	FLUF GALL NCCM NCNC SCOP VATB
TNDS-OP1 Tennessee Donor Svcs	ALOB AROR DCTC GALL KYDA LAOP MDPC MSOP NCCM NCNC NYAP NYFL NYWN OHLB OHLP PADV PATF TNDS TNMS VATB
TNMS-OP1 Mid-South Transplant Foundation	ALOB AROR FLUF GALL ILIP INOP KYDA LAOP MOMA MSOP NCCM NCNC OHLP OHOV OKOP TNDS TNMS VATB
TXGC-OP1 LifeGift Organ Donation Ctr	AROR AZOB CORS LAOP MSOP MWOB NMOP OKOP TXGC TXSA TXSB
TXSA-OP1 Texas Organ Sharing Alliance	AROR AZOB CORS LAOP NMOP OKOP TXGC TXSA TXSB
TXSB-OP1 Southwest Transplant Alliance	AROR AZOB CORS LAOP MWOB NMOP OKOP TXGC TXSA TXSB
UTOP-OP1 Intermountain Donor Services	AZOB CAGS CAOP CASD CORS NVLV OKOP ORUO UTOP WALC
VATB-OP1 LifeNet Health	ALOB DCTC GALL KYDA MDPC MSOP NCCM NCNC NYAP NYFL NYRT NYWN OHLB OHLP PADV PATF SCOP TNDS TNMS VATB
WALC-OP1 LifeCenter Northwest	AZOB CADN CAGS CORS IAOP MNOP MWOB NEOR NMOP NVLV OKOP ORUO UTOP WALC WIUW
WIDN-OP1 Wisconsin Donor Network	IAOP ILIP INOP MIOP MNOP MOMA WIDN WIUW
WIUW-IO1 UW Health Organ and Tissue Donation	CORS IAOP ILIP INOP MIOP MNOP MOMA MWOB NEOR OHLC ORUO WALC WIDN WIUW

[a] Neighborhood solution found with requirements that neighborhoods have average volume-weighted organ transport distance less than 400 miles; and have at least 9 transplant centers and population of 25 million. Each DSA has at least 5 neighbors including itself and no more than 20 neighbors including itself. Bounds on distance, transplant centers, and number of DSAs in the neighborhood may be adjusted.

during the forecast period. Should supply and demand change significantly from the forecasted values, the model can be adaptively used to specify a new set of neighbors without redesigning the borders.

We emphasize that the results herein demonstrate the promise of the neighborhood framework more so than that of the particular geographic structure obtained. This framework is quite general and alternative solutions meeting different requirements of stakeholders may arise from it in future refinements. It is advisable for the transplantation community to compare the advantages and disadvantages of the neighborhood and redistricting frameworks relative to current allocation before enacting policy changes. Notably, neighborhoods are perhaps also amenable to current OPTN practice. From the perspective of an individual OPO, their activities remain much the same in relation to sharing, albeit with different neighbors. The only major operational change for the DSAs would be where the sharing of organs might occur during regional allocation. Moreover, a remarkable advantage of the neighborhoods is that should the neighborhoods ever be modified in the future, it can be done so 1 OPO at a time and without disrupting the ongoing practices of other OPOs, whereas changing the districts in redistricting affects the operations of every OPO in the district. However, whether such modifications can be made systematically and regularly is influenced by the culture of the OPTN and the ease of securing reform.[47] Inertia in this regard only underscores the importance of having the best possible solution at the outset.

LIMITATIONS

This study has some limitations. Foremost, the results asserting the improved performance of neighborhoods were based on our own simulator that only approximates LSAM. Differences in mortality and disparity estimates were due to omission of re-transplants/re-lists and the use of less patient and donor characteristics in the current version of LivSim. Transport distances provided by LivSim are conservative overestimates, so actual distances will be smaller. However, we treated all geographic structures consistently in the simulation experiment. Lastly, the simulation experiment results are based on sharing policies at the time of writing. Modifications of the sharing policies affect the performance of the geographic structures tested and the validity of comparisons among them.

FUTURE WORK

This work focused on the geographic aspects of the OPTN's design in a local-regional-national framework under current policy at a systems-level. It did not consider changes in transplant centers' and patients' behaviors in response to the neighborhoods design. Incorporating behavioral models into the optimization framework to obtain a more refined solution requires further investigation. We believe, however, that the flexibility provided by the neighborhood structure will be an important factor in developing such a model. This flexibility can also be used for developing an allocation design that judiciously directs low-quality livers to DSAs for rapid placement to reduce discards. Sharing policies are also the counterparts of the geographic structures and help perfect liver allocation. For example, boosting local priority by assigning

TABLE 5.

5-y Comparative performance of allocation systems (MELD without sodium case)[a]

	Current	Redistricting[b]	Neighborhoods[b]	Difference (neighborhoods-redistricting)	National[b]
Annualized waitlist and posttransplant deaths	—	−23.1 (0.050)	−48.2 (< 0.001)	−25.1 (0.038)	−237.6 (< 0.001)
Annualized waitlist deaths	—	−32.6 (0.005)	−45.2 (< 0.001)	−12.6 (0.160)	−155.3 (< 0.001)
Annualized posttransplant deaths	—	+9.5 (0.060)	−3.0 (0.314)	−12.4 (0.021)	−82.3 (< 0.001)
Annualized waitlist removals	—	−53.2 (0.113)	−46.7 (0.144)	+6.5 (0.441)	−143.2 (< 0.001)
DSA mean transplant MELD	—	+0.6 (< 0.001)	+0.8 (< 0.001)	+0.2 (< 0.001)	+1.6 (< 0.001)
DSA mean transplant MELD standard deviation	—	−0.48 (< 0.001)	−0.50 (< 0.001)	−0.02 (0.274)	−0.8 (< 0.001)
Average organ transport distance (miles)	—	+35.5 (< 0.001)	+24.3 (< 0.001)	−11.3 (< 0.001)	> +300 (< 0.001)

[a] *P* values in parentheses.
[b] All results obtained from LivSim for 2010-2014 and relative to current allocation. Input data generated by LSAM Candidate and Donor generators.

TABLE 6.

5-y Comparative performance of allocation systems (MELD with sodium case)[a]

	Current	Redistricting[b]	Neighborhoods[b]	Difference (neighborhoods-redistricting)	National[b]
Annualized waitlist and posttransplant deaths	—	−45.8 (< 0.001)	−64.2 (< 0.001)	−18.4 (0.014)	−272.4 (< 0.001)
Annualized waitlist deaths	—	−41.0 (< 0.001)	−56.2 (< 0.001)	−15.1 (0.004)	−142.7 (< 0.001)
Annualized posttransplant deaths	—	−4.8 (0.218)	−8.1 (0.093)	−3.3 (0.293)	−129.7 (< 0.001)
Annualized waitlist removals	—	−41.3 (0.174)	−46.8 (0.144)	−5.5 (0.450)	−116.2 (0.004)
DSA mean transplant MELD	—	+0.6 (< 0.001)	+0.9 (< 0.001)	0.3 (P < 0.001)	+1.7 (< 0.001)
DSA mean transplant MELD standard deviation	—	−0.50 (< 0.001)	−0.59 (< 0.001)	−0.09 (P < 0.001)	−0.9 (< 0.001)
Average organ transport distance (miles)	—	+43.4 (< 0.001)	+36.1 (< 0.001)	−7.3 (P < 0.001)	> +300 (< 0.001)

[a] *P* values in parentheses.
[b] All results obtained from LivSim for 2010-2014 and relative to current allocation. Input data generated by LSAM Candidate and Donor generators.

temporary MELD exception points is a possibility. Such policies should be studied in conjunction with the geographic structures— and ideally, optimized with them simultaneously.

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