AEM 3101 Project - Mars Entry

Joseph Mueller

Fall 2019

---

1 Overview

Your goal in this project is to study the kinematic trajectory and the heat profile of an Orion capsule entering the Martian atmosphere. The equations that govern the atmospheric entry trajectory and heat transfer are provided in this document. You will need to find and implement a model of the Martian atmosphere.

2 Dynamic Model

Here we describe the dynamics that govern the orbit of an object about a central body.

Two forces will govern the trajectory: weight and drag.

2.1 Assumptions

We make several assumptions.

Point Mass Model The capsule is assumed to be a point mass. Thus, for the trjajectory, we model only the translation of the capsule through space – we ignore the attitude of the capsule and the corresponding rotational dynamics.

Uniform gravitational field This is consistent with a uniform mass distribution of the central body (which in this case is Mars). This effectively means that the force of gravity depends only on the distance from the central body, regardless of latitude and longitude.

No gravitational disturbaces We assume that the orbiting body experiences gravitation only from the central body. We neglect all other bodies.

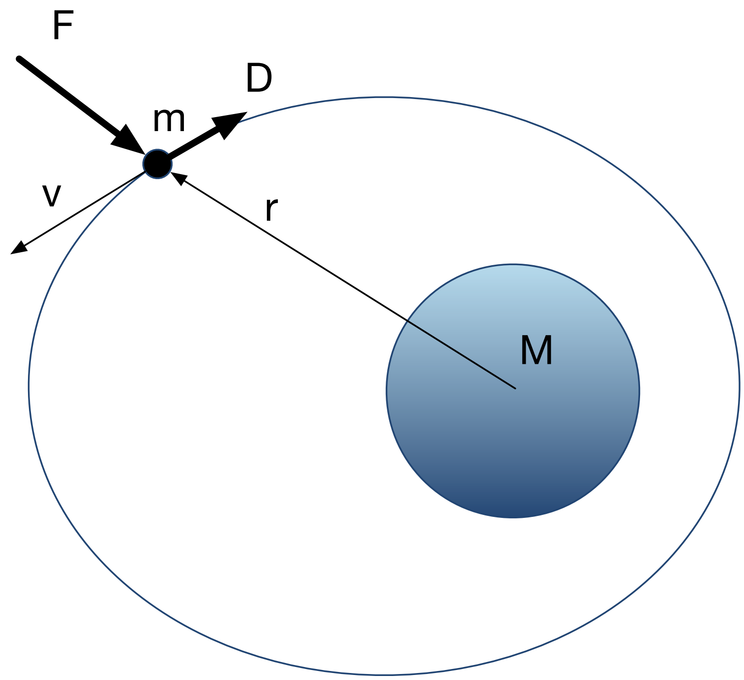

Figure 1: Free body diagram of an object in orbit.

Atmospheric Drag We will include a model of atmospheric drag. This will be the only disturbance force we consider. We will use an atmospheric density model for Mars that depends only on altitude.

A free body diagram of the object in its orbit is shown in the figure below. The orbit is described by the position ($\boldsymbol{r}$) and velocity ($\boldsymbol{v}$) of the mass m. Only two forces act on the body: gravitational force $\boldsymbol{F}$ and drag force $\boldsymbol{D}$. These are described in the next section.

2.2 Atmospheric Density

The atmospheric density of Mars varies with temperature and pressure, both of which change based on the altitude, latitude, and the location of the sun. For this project, assume that the temperature, pressure and atmospheric density vary only with altitude.

You can find a simple model of the atmospheric density as a function of altitude at this NASA site: https://www.grc.nasa.gov/www/k-12/airplane/atmosmrm.html. However, it is only valid for the lower atmosphere. Assume that it is valid up to 65 km. For altitudes above 65 km, use the following model:

$$\rho(h) = 0.88325 \exp\left(a_0 + a_1 \log(h) + a_2(\log(h))^2 + a_3(\log(h))^3\right)$$

where h is in units of kilometers, and:

$$\begin{aligned} a_0 &= 49.8118119899434 \\ a_1 &= -5.9123700325916 \\ a_2 &= -3.5638800977374 \\ a_3 &= 0.380908561109888 \end{aligned}$$

2.3 Drag Force

The drag force is:

$$D = \frac{1}{2}\rho C_D A v^2$$

where ρ is the atmospheric density, C_D is the drag coefficient (depends on the shape of the object), A is the cross-sectional area, and v is the magnitude of velocity.

The drag vector is aligned opposite the direction of the velocity. We therefore use the above formulation for the magnitude of the drag force, and multiply this by a unit vector that points opposite the velocity vector.

$$\boldsymbol{D} = -\left(\frac{1}{2}\rho C_D A v^2\right)\frac{\boldsymbol{v}}{v}$$

Let's define the ballistic coefficient B as:

$$B = m/(C_D A)$$

The acceleration due to drag is then:

$$\boldsymbol{a_D} = \boldsymbol{D}/m = -\frac{1}{2}\rho B^{-1} v \boldsymbol{v}$$

IMPORTANT: Be careful to use consistent units! The density ρ should be defined in units of kg/m^3. The ballistic coefficient should be in units of kg/m^2. The position r and velocity v should always be in units of km and km/s, respectively. As a result, you will have to do a units converstion in the above equation for acceleration so that you get it in units of km/s/s. That units conversion is **not** shown in the equation.

2.4 Gravigational Force

The magnitude of the force of gravity between two objects of mass M and m is:

$$F = \frac{GMm}{r^2}$$

where M is the mass of a planet (Earth for example), m is the mass of our projectile, r is the distance between the masses, and $G = 6.67408E - 11$ m^3 kg^{-1} s^{-2} is the universal gravitational constant.

Let $\mu = GM$ be the gravitational constant of the planet. The force of gravity is now written as:

$$F = m\frac{\mu}{r^2}$$

The gravitational force vector is aligned in a direction that is opposite the position vector.

$$\boldsymbol{F} = -m\frac{\mu \boldsymbol{r}}{r^3}$$

The values of μ for several solar system bodies are given in the table below.

Gravitational Parameters and Equatorial Radii for Solar System Bodies		
Body	μ (km^3/s^2)	Eq. Radius (km)
Sun	1.32712440018E11	695508
Mercury	2.2032E4	2439.7
Venus	3.24859E5	6051.8
Earth	3.986004418E5	6378.14
Moon	4.9048695E3	1738.1
Mars	4.282837E4	3396.2
Jupiter	1.26686534E8	71492
Saturn	3.7931187E7	60268
Uranus	5.793939E6	25559
Neptune	6.836529E6	24764
Pluto	8.71E2	1195

2.5 Heating Rate

We will use the body-averaged heating rate for this analysis. The equation is:

$$q_{avg} = \frac{1}{4}\rho v^3 C_F$$

where ρ is the atmospheric density, v is the velocity, and C_F is the body-averaged skin friction coefficient. The skin friction coefficient depends on the thickness of the boundary layer, which varies with Mach number and Reynolds number. We will use the following approximation for C_F:

$$C_F = \frac{0.65 + 0.339\left[(2/\pi)\tan^{-1}(10 - M) + 1\right]}{\sqrt{R_E}}$$

where M is the Mach number and R_E is the Reynold's number. The above expression causes the ratio $C_F/R_E^{1/2}$ to asymptoticallyh approach 0.65 at Mach 20, and 1.328 at low speed, which is valid for a flat plate.

The Mach number is the ratio of your velocity to the speed of sound, s, which depends on temperature. And because we use a model where the temperature varies only with altitude, it follows that the speed of sound also varies with altitude.

$$M = v/s$$

Your Reynold's number should be:

$$R_E = \frac{vD}{\nu}$$

where D is the diameter of your capsule and ν is the kinematic viscosity of the Martian atmosphere.

Most of the Martian atmosphere is carbon dioxide. Therefore, you can look up the kinematic viscosity and speed of sound for carbon dioxide. Note that they will change with temperature.

In summary, you should create the following functions:

- One that computes the body-averaged heating rate given your speed v, the atmospheric density ρ, and the body-averaged skin friction coefficient C_F.
- One that computes the body-averaged skin friction coefficient given the Mach number M and Reynolds number R_E.
- One that computes the Reynolds number given your speed v, your diameter D, and the kinematic viscosity ν (at your current altitude h).
- One that computes the Mach number given your current speed v and the speed of sound s (at your current altitude h).
- One that returns the speed of sound of carbon dioxide given the temperature T.
- One that returns the kinematic viscosity of carbon dioxide given the temperature T.
- One that returns the temperature given your current altitude h. Use the simple Mars atmospheric model (see link above) for temperature between the altitudes of 0 and 65 km. Assume that it reaches a minimum temperature of -167.7 deg Celsius at 65 km, and stays at that temperature for all altitudes $h > 65$ km.

2.6 Flight Path Angle

The flight path angle is measured from the horizontal plane to the velocity vector. To compute the flight path angle, you can use the following approach:

1. Define the local vertical direction (up) as the unit vector that points along the position vector. $\hat{e}_{up} = \boldsymbol{r}/r$.
2. Define the velocity unit vector: $\hat{e}_{vel} = \boldsymbol{v}/v$.

3. Compute the dot product between the velocity unit vector and the local vertical: $d = \hat{e}_{up} \cdot \hat{e}_{vel}$
4. Compute the angle between the local vertical and the velocity direction: $\alpha = \cos^{-1}(d)$
5. Compute the flight path angle: $\gamma = \pi/2 - \alpha$

2.7 States and State Derivatives

The states are:

- x x position
- y y position
- z z position
- v_x x velocity
- v_y y velocity
- v_z z velocity

The time derivative of each position is just the corresponding velocity:

$$\dot{x} = v_x \tag{1}$$
$$\dot{y} = v_y \tag{2}$$
$$\dot{z} = v_z \tag{3}$$

The time derivative of the velocity vector is the acceleration: $\dot{\boldsymbol{v}} = \boldsymbol{a}$. The acceleration is the total force divided by the mass:

$$\boldsymbol{a} = \frac{1}{m}(\boldsymbol{F} + \boldsymbol{D}) \tag{4}$$
$$= -\frac{\mu \boldsymbol{r}}{r^3} - \frac{1}{2}\rho B^{-1} v \boldsymbol{v} \tag{5}$$

AGAIN... be careful to use consistent units for this equation. The acceleration should be in units of km/s/s.

Given the acceleration vector, the three components of the velocity state derivatives are, of course:

$$\dot{v}_x = a_x \tag{6}$$
$$\dot{v}_y = a_y \tag{7}$$
$$\dot{v}_z = a_z \tag{8}$$

3 Analysis

1. Calculate the circular orbit velocity for a parking orbit about Mars at a prescribed altitude. Use a starting altitude of 200 km.
2. Run a simulation that lasts for 10 orbit periods and plot the altituede over time. You should see that the altitude decays (drops), but very slowly. For example, using a ballistic coefficient of 50, your altitude should still be above 199.75 km after 10 orbits.
3. Compute a delta-v that will induce an atmospheric entry trajectory. Your delta-v should be oriented opposite the current velocity vector. This will take some trial and error. I suggest you start by disabling your drag model. Run a simulation for 1-2 orbit periods with an initial delta-v and plot the altitude over time. You'd like the minimum altitude to be well above zero. Once you have this where you want it, go back and run it again with your drag model on.
4. Numerically integrate the equations of motion as the capsule descends through the atmosphere.
5. Approximate the heating rate during reentry using the formula provided.
6. Terminate the simulation at an altitude of 5 km.
7. What is the maximum heating rate and at what altitude does it occur?
8. Record the final velocity, flight path angle, and duration.
9. Repeat the above process for several different initial delta-v's (from step 2) and comment on how the final conditions vary with that initial condition.