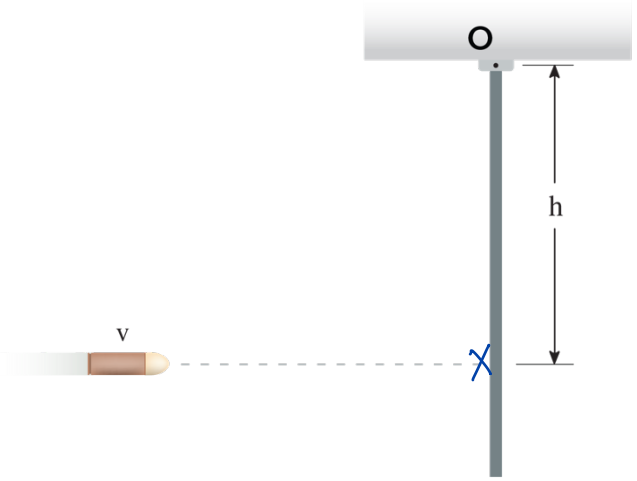
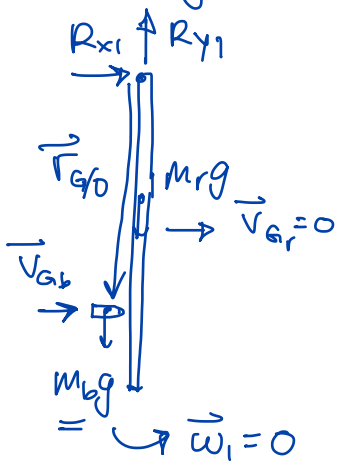
Consider the bullet shown with mass m_b = 0.025 kg and velocity just prior to impact with the rod of v_1 = 400 m/s. It impacts a uniform thin rod with mass m_r = 3.3 kg, and length L = 0.8 m. If this is a perfectly plastic impact, what is the angular velocity of the rod, ω_2, immediately after the impact? The distance h = 0.6 m.



Find $\vec{\omega}_2$

Plastic impact — bullet embedded into rod, they move as one object.
Energy NOT conserved across the impact.

State 1 just before impact



$$\sum_{sys} \vec{K}_{O_1} = \sum_{sys} \vec{K}_{O_2}$$

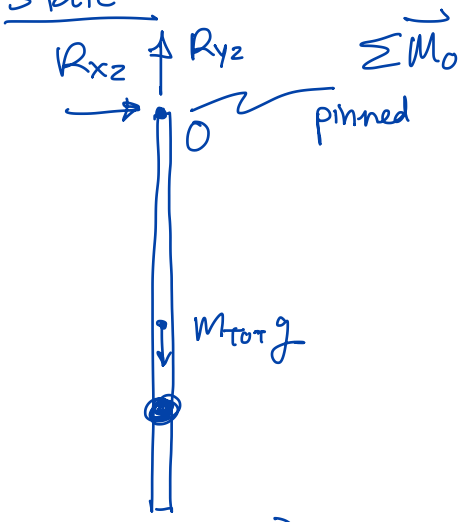
$$\sum_{sys} \vec{K}_{O_1} = \vec{K}_{O_{1b}} + \vec{K}_{O_{1r}}$$ ($\vec{K}_{O_{1r}}$: 0 not moving)

$$\vec{K}_{O_{1b}} = \vec{I}_{G_b}\vec{\omega}_b + \vec{r}_{G/O} \times m\vec{v}_{G_b}$$ ($\vec{I}_{G_b}\vec{\omega}_b$: 0 pt. mass)

$$\vec{r}_{G/O} = h(-\hat{j}) \qquad \vec{v}_{G_b} = v\hat{i}$$

$$\vec{K}_{O_{1b}} = m_b h v \hat{k} = \sum_{sys} \vec{K}_{O_1}$$

State 2



$\vec{\omega}_2 \neq 0$

$$\sum_{sys} \vec{K}_{O_2} = \vec{K}_{O_{2b}} + \vec{K}_{O_{2r}}$$

$$\vec{K}_{O_{2b}} = I_{O_b}\vec{\omega}_2 = (\vec{I}_{G_b}^{\,0} + m_b h^2)\vec{\omega}_2 = m_b h^2 \omega_2 \hat{k}$$

$$\vec{K}_{O_{2r}} = I_{O_r}\vec{\omega}_2 = \frac{1}{3} m_r L^2 \vec{\omega}_2$$

$$\sum_{sys} \vec{K}_{O_1} = \sum_{sys} \vec{K}_{O_2}$$

$$m_b h v = m_b h^2 \omega_2 + \frac{1}{3} m_r L^2 \omega_2$$

$$\Rightarrow \omega_2 = \frac{m_b h v}{m_b h^2 + \frac{1}{3} m_r L^2}$$

$$= \frac{(0.025\,kg)(0.6m)(400\,m/s)}{(0.025\,kg)(0.6m)^2 + \frac{1}{3}(3.3\,kg)(0.8m)^2}$$

$$\omega_2 = 8.415\,rad/s$$

$$\boxed{\vec{\omega}_2 = 8.415\,rad/s\,\hat{k}}$$