Problem **9**

In this problem, we have a cylinder that is rolling and it hits an incline, it then proceeds to roll up the incline along the incline for 10 meters until it stops, we're asked to find what is the initial angular velocity required to achieve this distance. So we know that this is a work energy problem where the kinetic energy at the initial state is converted into potential energy at the final state. So let's first start by defining this the first state state 1. And then the final state is when this cylinder over here is aligned with the of the top here, it's traveled distance out. So this is step 2. So this is going to be our center of gravity. And we're going to start by setting a datum for the potential energy at the center of gravity of the state 1, so we can cancel the potential energy at state 1, then we set an initial omega. And we know that at this final state, there is no rotation because it reaches a complete stop. So we do not have any kinetic energy or omega at that state. So let's start with our energy equation, T1 plus v1 is going to be equal to T2 plus V2. Now we can start eliminating on a terms, because we know that the initial potential energy is going to be zero, since we've set our datum to the center of gravity of the initial state, and we also know that at the final state, the kinetic energy will be zero, because it will reaches a complete rest, right, so we can condense this equation to the initial kinetic energy being equal to the final potential energy change, the change in potential energy or the final potential energy with a datum set at the center of gravity of the initial state. So let's go ahead and calculate these terms. So let's start with T1. So T1 is the kinetic energy at state 1. Now, this, we can get an equation for this, since we know that T1 is equal to one half v g squared, plus one half I G omega squared, right, so we have two components to this kinetic energy, we have the first component with which deals with the velocity of the center of gravity. So that's the translational component. And then this component here, which deals with the rotation about the center of gravity, and this is the rotational kinetic energy, and the addition of the two gives us the total kinetic energy. And it's really important to note that this g and this G here, these are because this is all about the center of gravity, G, right. So let's calculate V G, and let's calculate IG. So, we know V G, is going to point along this direction. So that is V G, right? And VG is going to be omega cross r, right. And that is starting from the instantaneous center of zero velocity, which we're rolling wheel that does not slip is located at the bottom here. So r is going to point up in this direction. So this is our G with respect to A let's call this point A. So, we know that V G is going to be equal to omega cross r of G with respect to a now, we can we can see that all these vectors are perpendicular right and Omega points out of out of the page. So, we can get rid of the vectors and just have a simple multiplication. So V G is equal to omega R of G with respect to a and that is just the magnitude right? And that is just going to be equal to omega r, right, because this is the radius r. So V G is omega r. Now, we come up with the first expression, so we can replace this VG here with omega r right? So we have this term in terms of omega and then this radius which is known. Next up we need to find IG. So IG is inertia about G. So since this is a cylinder, IG is equal to one half m r squared, right? So what is this? Well, we we have the mass we have of the radius, we can directly solve for this quantity and plug it into here. And then we have omega, which is what we're trying to solve for. So we leave the equation in terms of this. Right. So now we can take these quantities and plug them into this equation to get a final equation for the kinetic energy at state 1. This is going to be equal to one half m omega squared R squared plus one half times one half m r squared times omega squared. And you can see that we can simplify this because we have an omega squared omega squared R squared, R squared, and M M. So this condenses to the following: three quarters M omega squared R squared. This is the kinetic energy just based on omega, right. Next up, we need to solve for this term, the potential energy at the final state based on this datum over here, it's important to note that we've cancelled this because we've set the datum over here. So that's where we need to start with this datum, we need to keep the same datum and find the new height of the center of gravity. And this is going to be the H in mgh, right, the change of height based on the datum in meters, multiplied times the mass and G, right. So v2 is equal to m G H. And this is going to be equal to m g, and the height here is just going to be based on geometry, right? So the height here is based on this similar triangle over here. So this on the right here is h, and the hypotenuse here is our L. Right? That's the distance traveled. So L is our H is just going to be L sine of theta, right? Because we use a sign relationship with this angle and this hypotenuse to find h over there. We have theta we have L, we have m, and we know g. So now we can put these together into this equation over here and solve for our angular velocity. So we have three quarters, m omega squared R squared r is equal to m g l sine theta. Okay. And we can now take this solve for omega, omega is going to be equal to the square root of 15 kilograms, times 9.81 meters per second squared, times 10 meters, and sine of 40 degrees divided by three quarters, times 15 kilograms, times one meter. And this is all in the square root. And we get that the final omega is equal to 9.17 radians per second. And this is in the k hat direction. So this is k the points into or out of the page. And this is our final answer.