Problem **20-R-IM-DK-16**

In this problem, we have a one meter by one meter plate that is being hit by a beanbag with mass of two kilograms, and we are asked to determine what is the angular velocity of the plate right after the beanbags strikes, and the beanbag strikes with a given velocity. And we're also given the mass of the plates and the radius of gyration. And we're also given the coefficient of restitution. So clearly, this is an angular momentum problem, because we're trying to find the angular velocity. And then we're also given a coefficient of restitution, which involves a momentum question. So let's get started. So we have this beanbag with a velocity v, in that direction, and when it strikes, it's going to transfer some of its energy to the plate and get the plate to start rotating with an Omega in this direction. And this beanbag is hitting the plate right at the center of gravity, G. Okay, so that simplifies our calculations. Alright, so what we're going to start with is the sum of angular momentum. So sum of Ha one is equal to the sum of Ha two. Okay. So, about a, okay, which is this point over here. Okay, and one is before the beanbag strikes, or two is after the beanbag strikes. Okay. So at the beginning, we're at point one, this whole system is stationary and nothing is moving. The only thing that's moving is the beanbag with a velocity v. So the only component to H a one is that from linear velocity of B, which is the beanbag. Okay, so we'll call this B as in the beanbag, and this is p as in the plate. Okay. So we have let me go on a new line here. I over two, m BV b. Okay, and this is again, BB one, because that's the velocity before hitting, okay. And this is because our h b is equal to h g, plus r of g with respect to A of m, then VB. Okay, now you can clearly see that H and g is going to be zero. So the only term we're left with is this RF g with respect to A and M BBB. So we have m BBB and RG with respect to a is L over two because it strikes right at the midpoint, which is this distance here is L over two. Okay? Then this is equal to, and on the right side, we're gonna have more components. Okay. So, again, this beanbag doesn't stop after it hits, right, we have a coefficient of restitution, and it doesn't move with the same velocity. Okay. Again, because we have this coefficient of restitution. So we're going to have the same component that we had here, but with a different velocity. And then we're going to have the components that arise from the rotation and translation of this plate. Okay, so let's start with the beanbag. So we have the same exact formula L over two times MV, but here we're going to have the B, two. Okay, which is different from VB one, as I mentioned, on so this is just a different linear velocity. Okay, then we're going to add all the other components, okay. So we have two components for the plate, which is the component due to translation of the plate plus the component due to rotation of the plate. So the translation one is the same exact one as the other ones, L over two mass of the plate times the velocity of the plate at two because, well, the first one is zero. Okay, so this is the translation of this center of gravity. And again, the distance from a To the center of gravity is L over two. So that's why we have this L over two here, okay? Plus we have this rotation component, so plus IGP m omega two, okay, and it's omega two is what we're trying to sell for, because they're trying to sell for the Omega after the hit. So at that time, point two. Um, so this is our conservation of angular momentum. Okay. And we can start plugging numbers in to see what unknowns we need to solve for. So I over two, we know we have one meter over two times massive B, which is two kilograms. So the beanbag times the velocity of B one, which is 20 meters per second. This we have, then we have again, one meter divided by two times the mass of the beanbag, which is again two kilograms times v v two, which we don't know. Okay, plus one meter over two times NP, which is the mass of the plate, which is 20 kilograms, times v2, which we also don't know. And then we have IGP, which we can write down as 1/12 L squared, so 20 kilograms, lb one meter squared times omega two. So just as a summary, this here is 112 L squared. Okay. So now, we can see from this equation that we have three unknowns, VB two, v two and omega two. And again, B is rebag. p is the plate, so we don't know, what are the linear velocities of the plate and the beanbag after the impact, okay. And if we know these, then we can solve for omega two directly. But we can simplify a few things. So we can relate omega two to the P two, right? Because omega two is related to VB two due to this play being pinned at the top, right. So we know that A is the instantaneous center of zero velocity. So given an omega, and given this radius, which we know we can find the velocity v g, which is also VP to which all right down here, the P two. Okay, and this is that G. And so we can actually find that, given omega, we don't know omega, but we can find it in terms of omega. So we know that V p two is going to be equal to L over two omega two, which is also equal to one meter, or two, omega two, which is equal to omega two over two. Okay. And this is because remember, V. So this is from I'll do it in brackets. V equals to omega cross r. and M, we know omega, it's in the k hat direction. And we know our it's in the negative j hat direction. And if you take the cross product of the two, you get V, which is in the positive x i hat direction. And it's going to have since they're perpendicular, so since this radius here is perpendicular to the velocity coming straight out of the page, the Omega coming through the page, that omega the velocity will be towards the right, and it will be just the for multiplication of these two properties based so that's why we have our crossbar to multiply by omega. Okay. And then we can rearrange to solve and we can plug in value sorry to solve for VB two in terms of omega two. So this here, we can directly plug into here and we've eliminated one unknown because this is in terms of omega two, and we need to find what VB two is. And as I mentioned before, for this, we need to use the coefficient of restitution, okay, so the coefficient of restitution e is equal to V p two minus v b two. So this is the after the impact divided by v v one minus V p one. Okay. And we can already cancel out this term over here. Because we know that V p one is zero, there's no velocity. And we can plug in the other values. So I can plug in coefficient of restitution is equal to 0.4 is equal to or V p two, we've just solved in terms of omega. So that's what I'm going to use this equation here. Omega two over two, let me write that nicer. And then we subtract VB two, which we're trying to solve for, so we just leave it in terms of the v two, and we divide by 20 meters per second, okay, because that is VB one, and then we subtract zero, which doesn't do anything. Okay. So when we rearrange here, we get the following from eight is equal to omega two or two, minus v v two. And when we solve for VB two, we get the following omega two over two, minus eight. Okay, and all of these units are in meters per second, the full all the terms are in meters per second. Okay, so now we have the v two in terms of omega two, we can plug this into here, and this whole equation becomes in terms of omega two, and we can directly solve. Okay, so let's plug in everything into this equation. This star equation over here. So star, we get the following 20 is equal to and I'm going to leave out the unit's here, but because I multiplied everything together, the B two plus five, omega two plus 20 over 12 omega two, which is equal to b, b to us, 20 over three, omega two, and then so in here, I just plugged in this equation here, omega two over two into the P two. And then I've also multiplied all these things together. Okay. Now let's plug in this part of the equation here into the B two. Okay, so we have 20 is equal to omega two over two minus eight, plus 20 over three omega two. So I've essentially plugged in this equation into there. And now we can directly solve for omega two and we get that omega two is equal to 3.91 radians per second. And this is the final answer.