Problem 7

In this problem, we have a wheel of radius 0.5 meters, that is rolling down a hill with an initial velocity of six meters per second, the wheel travels a vertical distance of four meters. And we're asked to find the final velocity of the center of mass of the wheel, we're assuming that there's no friction, there's no energy loss due to friction. And we can assume the wheel to be a disc of uniform density. So this is a work energy problem where potential energy is converted into kinetic energy. And we're asked to find the final velocity. So this is determined from the final kinetic energy. So we first have to come up with the expressions for the kinetic and potential energy. So the potential energy V, about the center of gravity is going to be equal to m g y, where y is going to be the distance from our datum to the center of gravity. So this is y, and this is our datum. So this vertical distance is going to be again, the distance from here to our center of gravity, G. And that's at the center of the wheel. That is a simple expression for the potential energy. Now let's look at the kinetic energy, which is a bit more involved. So T, the kinetic energy is going to be equal to one half the G squared plus one half by omega squared. Okay, so we have a component due to the translation of the center of gravity and a component due to the rotation, right? So we know VG, we need to find an expression for i omega squared in terms of VG, which we can do based on the geometry of the wheel. So if we look at the wheel, we know that the point of contact is going to be our instantaneous center of zero velocity, right. And if we know omega, so let's assume that the wheel is rolling with omega in this direction, we know that given a radius, we can get a velocity of the center of gravity that points in the following direction v g. So we know based on v is equal to omega cross r, because this is our radius, that since these are all perpendicular, and omega points out of the page, we know that we can just directly multiply so we can actually actually find the relationship that omega is equal to V G divided by R. Okay. The other thing is in the question, it says we can assume that the wheel is a disc. So I four disc is one half m r squared. And this is just because it's a disk. So if we plug this into this equation, we get that one half m v g squared plus one half i which is one half m r squared times omega squared, which is the G over R all squared is reduces to the following. Once equals two three fourths the G squared okay. Now, we can we now have expressions for the potential energy and the kinetic energy of the two states. So now we apply conservation of energy which is as follows. At the initial state V naught plus t naught, we have the same energy at the final state which is Vf plus TF Okay, so, this is the initial state or zero and when the wheel rolls down to this point over here, this is the final state okay. So, initially, we have the following potential energy and G H right because H is the distance between the datum and the center of gravity. Plus we know an initial velocity of the center of gravity. So, three fourth m V G naught squared, this is going to be equal to the final velocity, which is what we're trying to find three fourths,m V g f squared plus the potential energy sorry, this is the kinetic energy goes here, potential energy is m g r. And this is because at the final state, a center of gravity will be at a height r with respect to our datum, right? That's why we have Mgr. Now we take this equation, we know this term. We know this term, and we know this term so we can finally solve for V G F. As that is our final answer. So V g, f is equal to the square root and I'm going to start plugging in values. So 9.81 meters per second squared times four meters minus 0.5 meters. So with this, I have condensed these two terms into one where this is h and this is R, plus three fourths times V G naught squared, which is six meters per second squared. And then we take the, we multiply all of this by four thirds. And then we take the square root and you get the final answer, which is nine meters per second. And this is our final answer.