Problem **20-R-IM-DK-14**

In this problem, a person is flying down vertically, and then after a height h, they attach to a hook and they start to purely rotate about point P, we are asked to find, what is the what after 90 degrees of rotation? What is the angular velocity of this person rotating this way? Okay. So this question is actually a combination of questions, impact and momentum and also work energy. Okay. So, let's start. So initially, the person is traveling down vertically down with an initial velocity v one. And after a while, after they travel a height h, on their velocity increases, they're still horizontal. Okay, so they're just gonna end up here, horizontal, but the new velocity, so with that, we can use work energy to find the final velocity before the person hooks on. Now after the person hooks onto p, they purely start to rotate. This is where angular momentum will come in. And then after this, so after the impact, then we have we have conservation of angular momentum, then we can use work energy again to determine the the new position or the energy at the new position, or the velocity at the new position here after this 90 degree rotation when the person ends up fully vertical. Okay, so first of all, let's do the easy part, let's calculate the velocity of when the person becomes horizontal just before connecting to the hook. Okay, so we have the so again, conservation of energy says that potential and kinetic energy at one are going to be equal to kinetic energy and potential energy at two. And in this case, at one, so one is going to be over here. And then two, I'm going to draw it in, in red, the person is going to be holding on partially, the person is going to be holding on here, and the legs are going to be out but they're not holding on yet. Okay? And on the velocity here, it will be of the two, so this is two. Okay. And so actually, let me draw the arms that are not connected yet. So the arms are still there. Okay, that's stage two. So initially, we're going to set our datum to be at the height of point p, so this is going to be your datum. And initially, we're going to have both kinetic and potential energy because we have a velocity and we're also at a height. Okay? So we're gonna have one half V one squared plus g h one, okay? And this is going to be equal to just kinetic energy because there's no more height because we're at the datum now. Okay, so height is zero, so that term cancels out. So we have one half V two squared. Okay, so we can actually plug all those these values in because we have v one we have H, mass we have so we can actually solve for V to found so let me plug in these values. So we have one half, the mass is 60 kilograms, times v one, which is three meters per second squared, plus mgh, which is 60 kilograms times 9.81 meters per second squared, times 1.5 meters, which is the height is going to be equal to one half times 60 kilograms, times v two squared. So when we solve this quadratic equation, we get that v two is equal to 6.199 meters per second. Okay, so now we have the velocity right before impact or right before the person hooks onto the hook at P. Okay, so now we You have to get from state two to state three, okay, and state three is essentially the same. But in this state, we have the person actually latching on to this hook here, and the person is still going to be horizontal, but this is going to be stage three, and the person is actually holding on to point P. Okay. And, in this case, so to get from two to three, we're going to have conservation of angular momentum. Okay, so let's write here from one to two. And then down here, we're going to have two to three. Okay. And conservation of angular momentum states. of angular momentum of a P of two is going to be equal to the sum of angular momentum about P of three. Okay, and now we can actually we have to determine what HP two and HP three are. So, initially, HP two is just before latching on an HP three is after latching on. Okay, so right after the person starts holding on to the book, okay, so let's start with hp two, hp. Two, too, is equal to, oh, we're gonna have two terms. So first, we are going to have a linear velocity term. So this person is traveling straight down with v2. So that's why we're going to have our g with respect to P times two, v two, sorry, okay. So this is just linear momentum times this radius, and g with respect to P is just the radius between here and the center of gravity of the person. Okay. And so this is the first term, and then we're gonna have a rotational term, which is i g omega two, but this one is going to be zero, we can already cancel it right? On because the person is just linearly translating, there is no omega to see, let me put this over on the Omega. Okay, so there actually is no term for rotation. After, so in state three, we are going to have rotation, so h p three is going to be equal to, again a linear term and a rotational term, but this time rotation we will have rotation, because the person is holding on and rotating about point P. Okay, so the first term is the linear one. So m r, and then times the velocity. So the and this is just RG, the velocity here is going to be a function of omega, right? So this is going to be V three, but we know that V three is going to be equal to omega times are, okay. So that in later and then we're going to have a rotational term, so we have M I times omega. Okay. But this this year is going to be this weekend, so we can use plug in with the radius of gyration. You can find I and then this whole equation will be in terms of omega. So let me make it all in terms of omega and RG squared, omega three, I'm sorry, this is omega three plus MRG squared omega three. Sorry, kg not RG kg squared omega three. Okay. So here this year, let me highlight it this year is I because I just MK g squared, and then here, one of the RS times omega three is going to be our V three. Okay. So that's how we get that equation. And again, this is all in terms of omega three. Okay, this is we're given kg and we know this. So all all we care about in this equation is omega three. And we've solved in terms of that. And then here in this equation here, we know the radius, and we know B two, because we just calculated it. So when we equate these two terms, we can actually solve for omega three. So let's do that. So we have 0.9 times meters, sorry, times 60 kilograms, times 6.199 meters per second. So this is this first term over here, or sorry, this first term over here, plus zero, which is this term over here, is equal to, and then here, I will pull out omega three of this equation. So we can have one term of omega three, so we have times 60 kilograms, times RG, which is 0.25 meters squared, plus 60 kilograms, times 0.9 meters squared. And then we have omega three outside, so we can quickly solve for omega three. Okay? So when we solve for omega three, we get the following. Omega three is equal to 6.395 radians per second. Okay, so this is omega three, this is the angular velocity right after from the person hooks on to P. But we're actually asked to find what is the angular velocity after a rotation of 90 degrees. Okay, so that's going to be different than the angular velocity up here. Okay. But um, with this, we can use work energy, again, to simply solve for that, okay, so this state three, travels to state four, and state four is when the person is vertical. Okay? So what's going to happen is, there's a difference in potential energy because this G travels downwards. And therefore, there's going to be a difference in kinetic energy. Okay, so let's do that. So this is to get from three to four. Okay. And we're going to use again, work energy, so t three, plus v three is equal to T four plus a V four. And we're simply going to have zero potential energy at the beginning. So in state three, because the person is horizontal, right at the datum, so this term here cancels out. And we're gonna have all the other terms. So let's start with T three, we have both on a rotational velocity and a linear velocity, and angular velocity and linear, so there's gonna be two terms. Okay. So these two terms are going to couple them together, and again, pull out the Omega term. So this is what we're going to have, we're going to have one half, I also pulled out the one half m RG squared plus m kg squared times omega squared. Okay, and this is going to be the first term, then we have zero v three, as I mentioned, and this is going to be equal to t four plus v four. So again, I'm going to pull out the one half because I want a couple of the omegas together. And I'm going to have my mass, which is the same times RG because the person is rotating in a perfect circle, so RG stays the same. So the distance between here in G and it is extreme here in G is exactly the same times r plus sorry, m times k g squared times omega four squared, okay? Because now we have a different omega, right? It's not omega three anymore. And then we're gonna have plus mgh. And here this H here will be negative, right? Because it's below the datum. This G is traveling below the data on so we're going to have to plug in a negative number. Okay, so let's plug in actual values and solve for omega four. Okay, because we have everything in this equation except And sorry, let me plug in here omega three, except for omega four over here. Okay, so let's solve. So we have one half times 60 kilograms times 0.25 meters squared plus 60 kilograms, times 0.9 meters squared times omega three, which is 6.395 meters per second, or radians for a second, sorry. And this is all going to be equal to, and I'm going to go on a new line. Actually, let me pull this to this side over here. Let's put it back. is going to be equal to, so I'm going to be rewriting the equal sign to one half times 60 kilograms, times 0.25 meters squared plus 60 kilograms times 0.9 meters squared times omega four plus 60 kilograms times 9.81 meters per second squared times h, which is on zero point, negative 0.9 meters. Let me make some space. Negative 0.9 meters. Okay. And if we rearrange and solve for omega four, we get that omega four is equal to 7.819 radians per second. And this is our final answer.