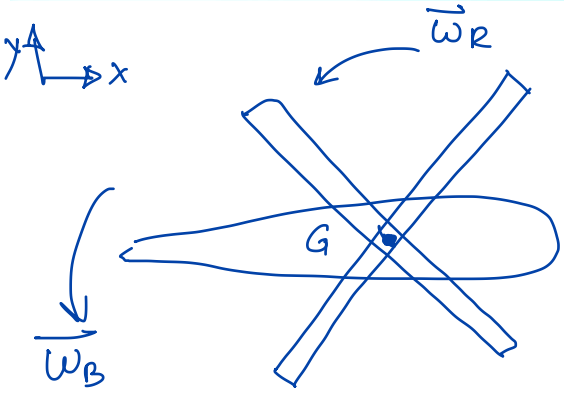
A helicopter tail rotor is used to prevent unwanted rotation of the body of the helicopter when the main rotor changes speed. Assume that the four main rotor blades are each long thin rods with length of 5 m and mass of 30 kg. Assume that the helicopter body has a mass of 750 kg and a mass moment of inertia of 1300 kg-m2 at its centre of gravity, located vertically in line with the main rotor. The distance between the main and tail rotors is 6 m.

If the tail rotor is NOT functioning, find the final angular velocity of the helicopter body when the main rotor changes speed from 200 rpm to 300 rpm. (Hint: consider the horizontal x-y plane).

$\vec{\omega}_{B2}$

Conservation of angular mom. of system



$$\sum_{sys} \vec{K}_{G1} = \sum_{sys} \vec{K}_{G2}$$

$I_B = 1300\ kg\text{-}m^2$

$I_R = 4\left(\frac{1}{3}(30kg)(5m)^2\right) = 1000\ kg\text{-}m^2$

state 1 $\omega_{R1} = 200\ rev/min \cdot \frac{1\ min}{60\ sec} \cdot \frac{2\pi}{rev} = 20.93\ rad/s$

$\omega_{B1} = 0$

$$\sum_{sys} \vec{K}_{G1} = \vec{K}_{GR1} + \cancel{\vec{K}_{GB1}}^{\,0} = I_R \vec{\omega}_{R1}$$

$$= 20930\ kg\text{-}m^2/s\ \hat{k}$$

state 2 $\omega_{R2} = 300\ rev/min \cdot \frac{1}{60} \cdot 2\pi = 31.4\ rad/s$

$\omega_{B2} \neq 0$

$$\sum_{sys} \vec{K}_{G2} = \vec{K}_{GR2} + \vec{K}_{GB2} \qquad \vec{\omega}_{B2} = \omega_{B2}\hat{k}$$

$$= I_R \vec{\omega}_{R2} + I_B \vec{\omega}_{B2}$$

$$= 31400 + 1300\,\omega_{B2}\ \hat{k}$$

Cons. of mom.

$20930 = 31400 + 1300\,\omega_{B2} \Rightarrow \omega_{B2} = -8.05\ rad/s$

$\boxed{\vec{\omega}_{B2} = -8.05\ rad/s\ \hat{k}}$