Problem 15

In this problem, we have a rotating decoration that initially rotates with an angular velocity of five radians per second. And it has a given radius, a gyration, and a mass. But after a while we come back to it and paperclips are magnetically attached to it, the moment of inertia is changed. So the angular velocity also changes, we're given this angular velocity of 2.1 radians per second. So we're asked given that each paperclip weighs two grams and increases the average radius of gyration of the whole ornament by 0.3 millimeters, how many paper clips were added, we also have to assume that the decoration continues to spin freely, so there's no motor attached to it. And that the decorations or or the paper clips are uniformly distributed around the decoration, and each increase the radius of gyration average by 0.3 millimeters. So this is clearly an angular momentum, conservation of angular momentum question. So we can state that between the two states state one and state two which are drawn in the diagram, we conserve I_G, omega. So I_G omega 1 is equal to I_G 2 omega two, right? We are given the two values for omega, what we're not given is I right, but we're given the radius of gyration in state one, so we can find I in state one. And here were given the rays of variation, initially, plus the increase of radius of duration due to the addition of paperclips. So this will be a function of how many paperclips we have attached to our ornament. So what we're going to do is we're going to apply this equation, so we know that i is equal to k squared. And we can calculate I G one is equal to, m1 k1 squared, and these are properties that are given in the problem, right, then we can see that I G two is equal to m m2k2 squared. Now these are the properties that we do not that are unknown, right, but we can say that m two is equal to m one, which is the mass the initial mass, plus the mass of each paperclip times the number of paper clips 0.002 kilograms, times n, which is the number of paper clips. And we can see the same thing for k, right, k two is equal to k one, the original plus the contribution due to the paper clips 0.0003 meters, times n. And so now we have defined IG one. And we've also defined IG two in terms of m1k1 and the number of paper clips, right. So we know all those parameters, except for the number of paper clips. So this is what we're trying to solve. So we can plug this back into the conservation of angular momentum equation and get the following equation in terms of n. So m one times k one squared times omega one is equal to m one plus 0.002 kilograms times n times k one plus 0.0003 meters times n, all squared times omega two. And as I mentioned, we know all the terms in this equation except for n. And so we can solve this equation. This is a bit more complicated than then a linear quadratic equation because this is indeed a cubic, right? We have this n term multiplied by this n squared term. So we actually get a cubic equation. But we can use a solver to solve and get our answer. So by plugging in all the values, we get that 0.4 kilograms, times 0.12 meters squared times five radians per second, is equal to 0.4 kilograms, plus 0.002 kilograms times n times 0.12 meters, plus 0.0003 meters times n, all squared times 2.1 radians per second. This is our equation and using a solver we get that n is equal to 100. And to so we need 102 paper clips to achieve this angular velocity of 2.1 radians per second. And that is our final answer. We could also use a simplification of this equation to get an estimate of how many paperclips so if we ignore the cubic term, so we simplify this expression here and and we take the cubic term and we assume it's negligible, we actually get n of 104, which is in the same order of magnitude. So this would be an approximation if we don't know how to solve a cubic equation. But ideally, we want to solve the cubic equation and get the exact answer which is 102. paperclips