

Overall, informal outline

- SMBH mass measurements and the various techniques → why do we care about mass measurements in the first place?
 - Help inform assembly history and timeline of galaxy formation, understand star formation through feedback with the central black hole and quenching, understand current population of SMBHs and their demographics, understand and predict the gravitational wave signal from binary SMBHs, etc
 - Trace build up of mass in SMBHs over cosmic time/growth channels of galaxies over time
 - These are tough questions - MASSIVE survey was designed to mitigate these challenges and understand massive galaxy formation and the supermassive black holes at the centers of these large ellipticals
- Data Processing – what data do we use? How do we go from photometry and spectra to inputs?
 - Need to mention: exquisite IFU and photometry needed; IFU data used for full spectrum fitting for the velocity profiles of these galaxies expressed as Gauss-Hermite moments
 - Photometry needed to model the mass distribution of the stars
 - Show some typical velocity profiles, and maybe some unique ones that make the modeling challenging
- Modeling
 - Intro to axisymmetric and triaxial Schwarzschild models; TriOS code; give a sense of how complex these models are
 - Superposition of orbits is constructed to mimic the input light profile and kinematics from densely sampled orbits in phase space
- Turning galaxy models into parameters
 - Briefly run through nested sampling routine and how we extract posterior information on these parameters
 - Maybe introduce here that for axisymmetric models, more often than not our models prefer edge-on inclinations and this was a sign that maybe these models are too flexible for their own good, leading to potential biases in the parameters
- Model Flexibility and Mock Recovery
 - We've been testing this idea at Berkeley on the more complicated triaxial models.
 - Axisymmetric: does seem that edge-on is preferred when the models are good enough, the covariance with the other parameters doesn't seem to be too strong
 - Triaxial: model flexibility is a less strong and less obvious effect, and in our tests, we don't seem to see a need to penalize our models by their intrinsic flexibilities; our models seem to do a decent job at recovering the inputs, especially at the level our measurement uncertainties limit us to
- Where we are now and what's to come

Stellar Dynamical Mass Measurements of MASSIVE Elliptical Galaxies

More details in:
[Liepold+20](#),
[Quenneville+21,+22](#),
[Pilawa+22](#),
[Pilawa+23 \(coming soon!\)](#)

Jacob Pilawa (UCB),

February 24, 2023 @ TAMU

Emily Liepold (UCB), Matthew Quenneville (UCB), Chung-Pei Ma (UCB)

+ Silvana Delgado Andrade (TAMU), Jonelle Walsh (TAMU)

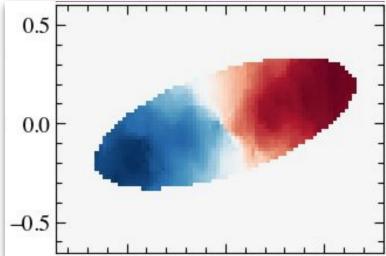
+ Jenny Greene (Princeton), John Blakeslee (NOIRLab)

A roadmap for the talk:

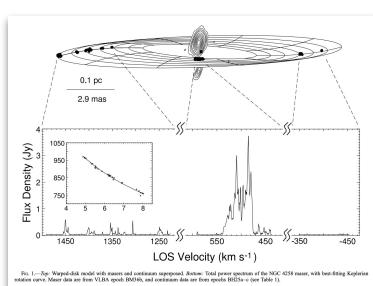
- 
1. Supermassive black holes in the largest galaxies
 2. Triaxial Schwarzschild modeling of massive ellipticals
 3. Robustness and Recovery Tests of Dynamical Modeling Techniques

main techniques for z~0 “direct” BH discovery:

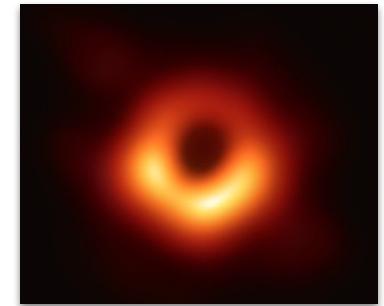
gas disk dynamics



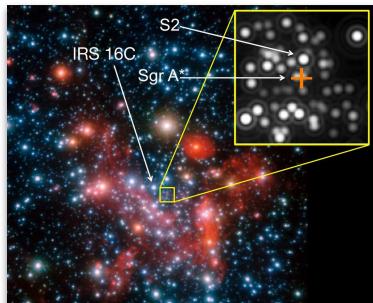
maser disks



EHT

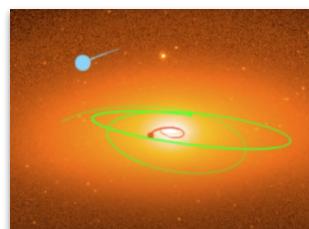


galactic center



stellar dynamics

integrated
light

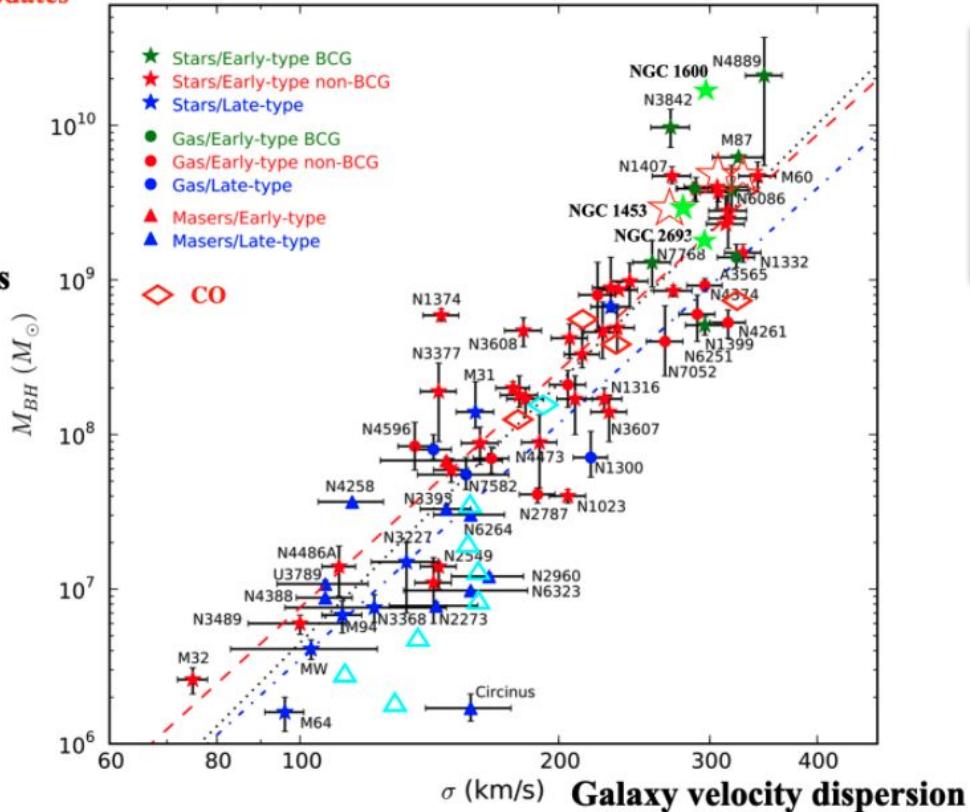


...

see: van den Bosch+08 (stellar),
Farrington+01, Walsh+10 (gas dynamics),
Miyoshi+95, Herrnstein+99 (masers).
slide inspiration: J. Walsh (TAMU)

BLACK HOLES + GALAXY EVOLUTION

McConnell & Ma (2013)
+ some updates



$$r_{\text{SOI}} \approx \frac{GM}{\sigma^2} \approx 10^{\prime}\text{s to } 100^{\prime}\text{s of pc}$$

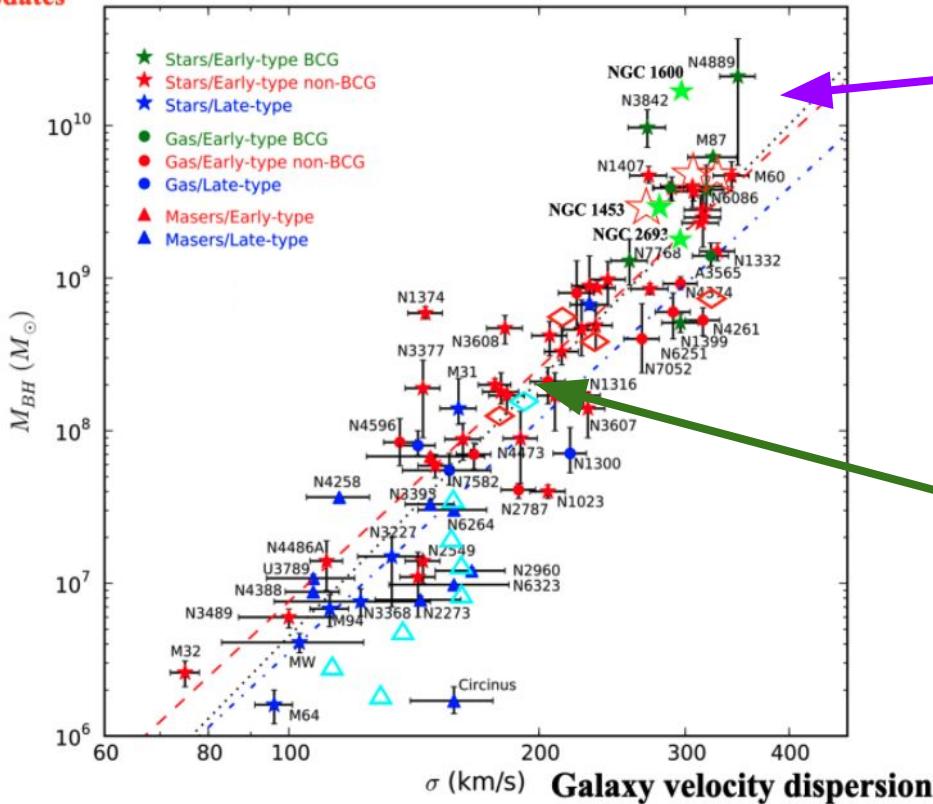
$r_{\text{galaxy}} \approx 10^{\prime}\text{s}$ of kpc

how do galaxies and SMBHs evolve together?

A CLUE: the most MASSIVE galaxies

McConnell & Ma (2013)

+ some updates



growth by gas-poor mergers

→ BH increases

→ σ saturates

→ kinematically misaligned,
slow rotation ($V/\sigma \sim 0.2$)

→ indicative of triaxiality?

growth by gas
accretion/gas-rich mergers

→ BH increases

→ fast rotators ($V/\sigma > 0.3$)

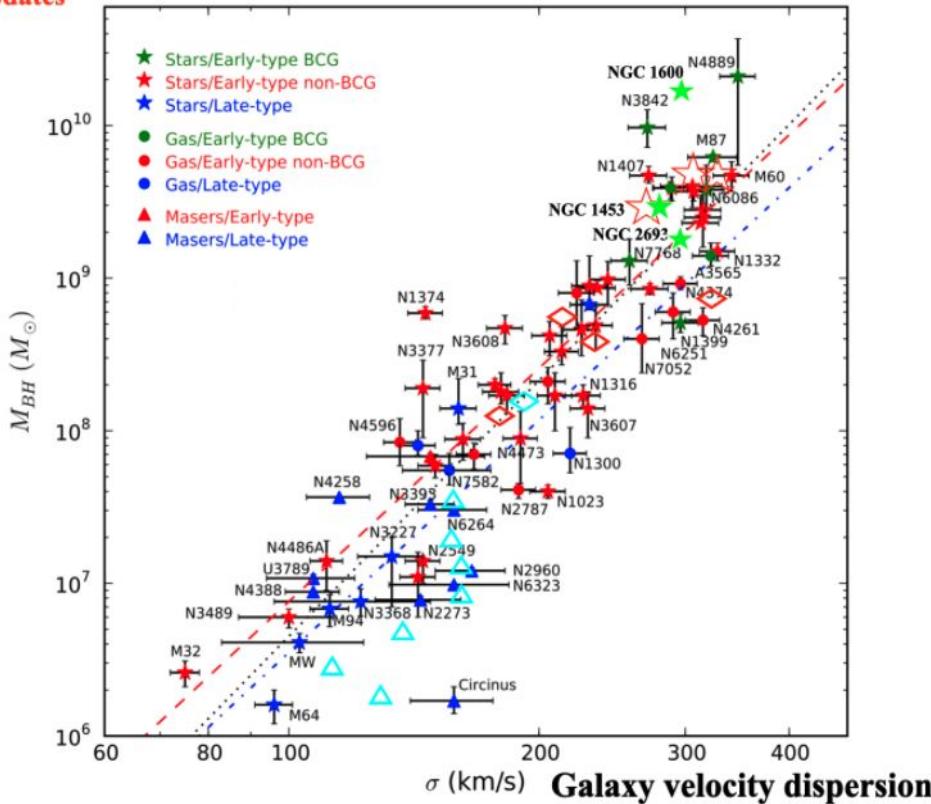
→ kinematically aligned

→ axisymmetric shapes

A CLUE: the most MASSIVE galaxies

*our interpretation depends
on M_{BH} 's at the most
massive end*

McConnell & Ma (2013)
+ some updates



1. Estimating BH Masses
2. Cross-checking of methods (e.g., ALMA gas dynamics [e.g., Cohn+2021, Kabasares+2022])
3. Calibration of reverberation mapped AGN
4. $z \sim 0$ mass function \rightarrow BBH merger rates
5. Comparison to simulations of AGN feedback modes and mechanisms

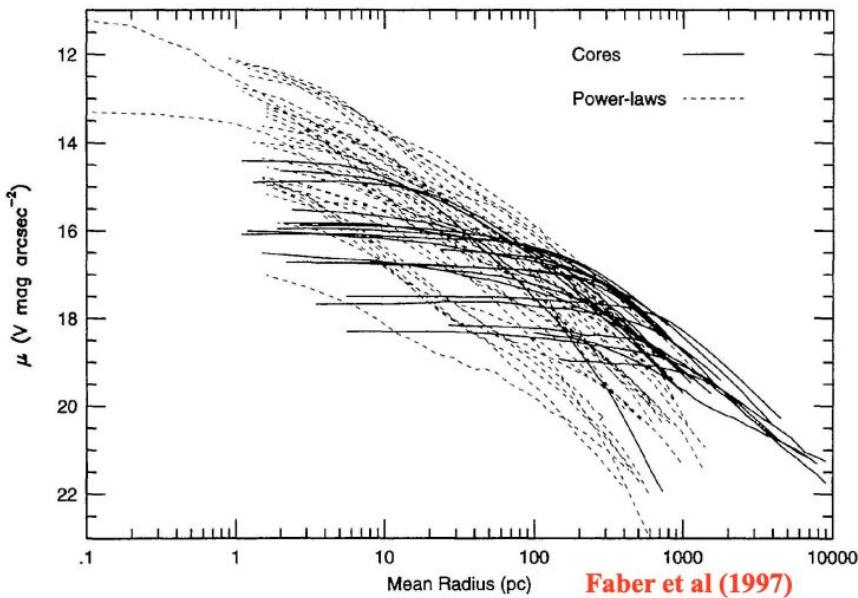
... but it's challenging...

[i.e., Yu+2019 (reverberation mapping), Thater+21 (gas dynamics)],

[i.e., Shannon+2015, Arzoumanian+2019], [i.e., Li+2019, Habouzit+2020]

why are massive ellipticals a challenge?

they're both rare, and have
extremely faint/flat cores



sphere of influence
is **tiny**

$$r = \frac{GM_{BH}}{\sigma^2} \approx 50\text{pc} \frac{M_{BH}}{10^9 M_\odot} \left(\frac{300 \text{ km s}^{-1}}{\sigma} \right)^2$$

a few 0.1's arcsec at ~100Mpc

→ long exposures on 8-10m telescopes!

The MASSIVE Survey

McDonald Observatory



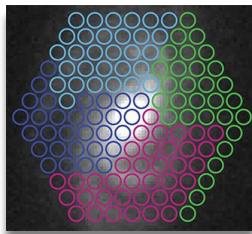
Gemini North



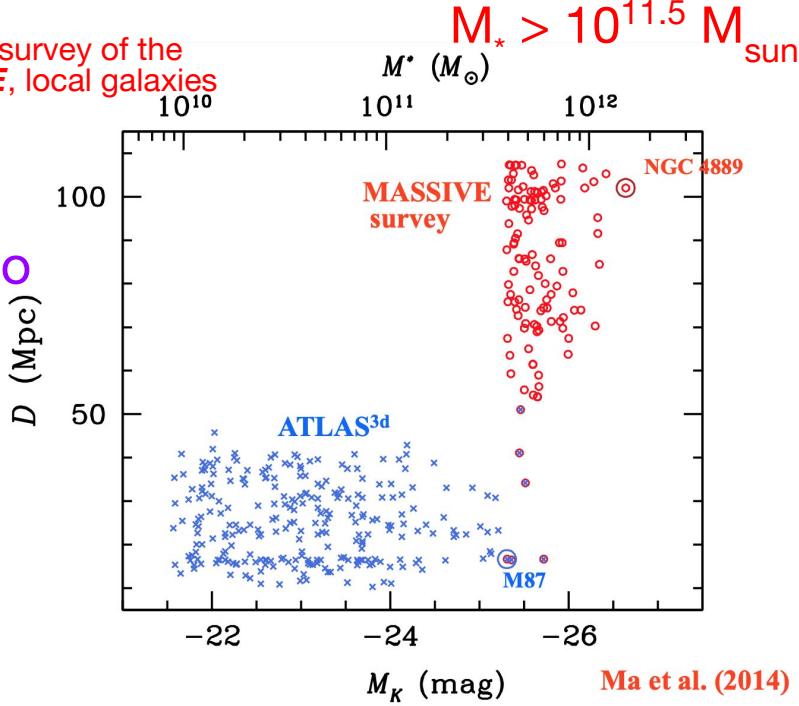
Photometry: CFHT, HST, UKIRT, PanSTARRS

a volume-limited survey of the
~100 most **MASSIVE**, local galaxies

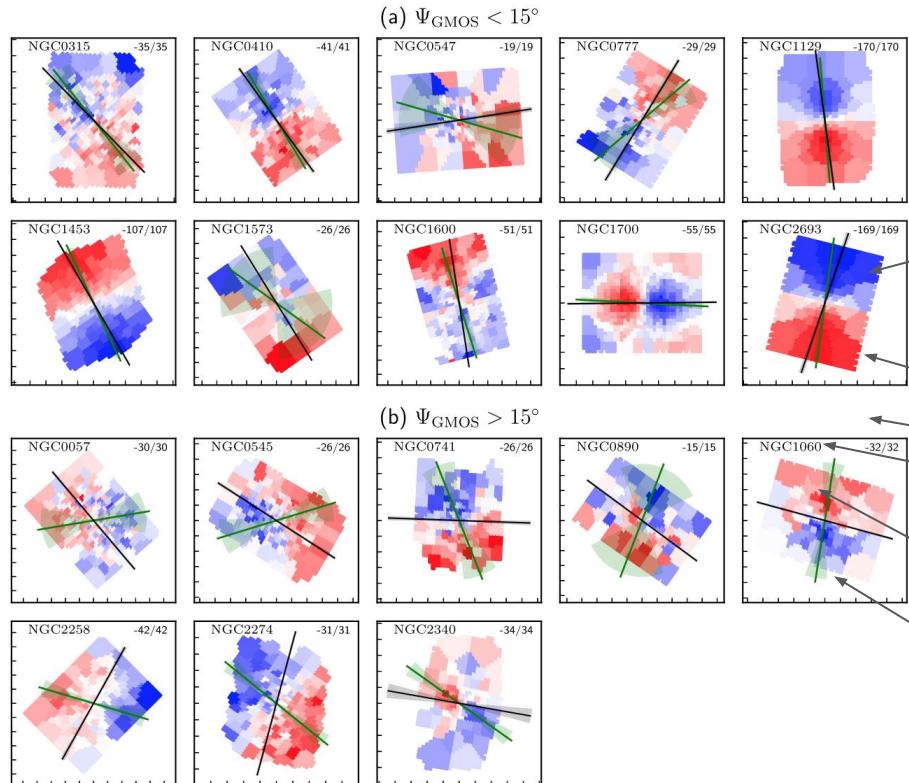
Wide-field IFU
($107'' \times 107''$, out to
 $\sim 2 R_{\text{eff}}$)



High-resolution,
high SNR IFU
($\sim 0.3''$ to $\sim 5''$ at
 $\text{SNR} \sim 125$)



The MASSIVE Survey: Stellar Kinematics



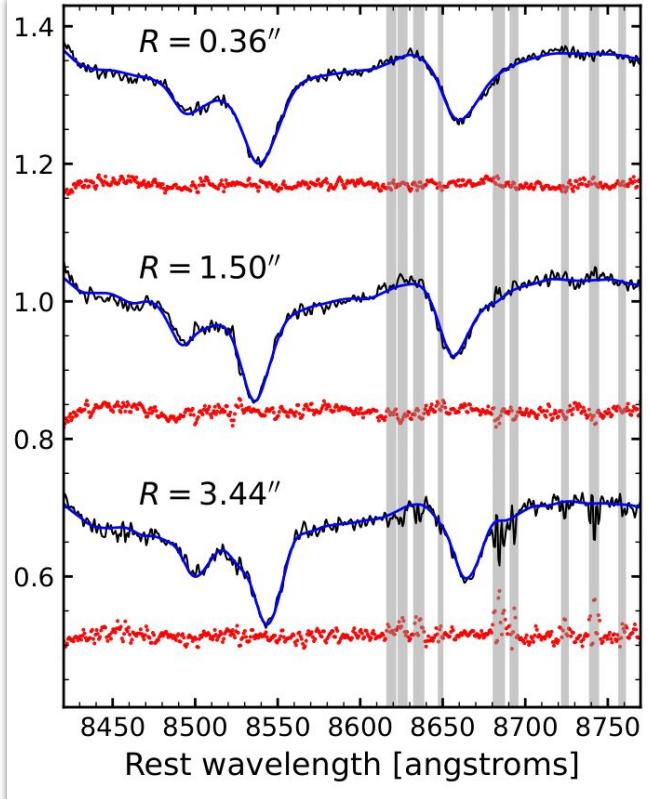
MASSIVE Survey Related Papers

- | | | |
|------------------------------|---------------|------------------|
| I Survey paper | Ma + | (2014) ApJ |
| II Stellar pop gradient | Greene + | (2015) ApJ |
| III Molecular gas #1 | Davis + | (2016) MNRAS |
| IV X-ray properties | Goulding + | (2016) ApJ |
| NGC1600 black hole | Thomas + | (2016) Nature |
| V Stellar kinematics | Veale + | (2017) MNRAS |
| VI Ionized gas | Pandya + | (2017) ApJ |
| VII λ & environment | Veale + | (2017) MNRAS |
| VIII σ radial profile | Veale + | (2018) MNRAS |
| IX WFC3 photometry | Goulaud + | (2018) ApJ |
| X Kinematic alignment | Ene + | (2018) MNRAS |
| XI Molecular gas #2 | Davis + | (2019) MNRAS |
| XII Stellar pop vs env | Greene + | (2019) ApJ |
| XIII Core kinematics #1 | Ene + | (2019) ApJ |
| XIV Core kinematics #2 | Ene + | (2020) ApJ |
| NGC1453 black hole | Liepold + | (2020) ApJ |
| SBF distances | Jensen + | (2021) ApJ |
| SBF H_0 | Blakeslee + | (2021) ApJ |
| Axisymmetric orbit code | Quenneville + | (2021) ApJ |
| Triaxial orbit code | Quenneville + | (2022) ApJ |
| XVI Stellar pop & IMF | Gu + | (2022) ApJ |
| NGC2693 black hole | Pilawa+ | (2022) ApJ |
| XVIII CFHT imaging | Quenneville + | (2022) submitted |
| Globular clusters | Hartmann + | (2023) submitted |

The MASSIVE Survey: Stellar Kinematics

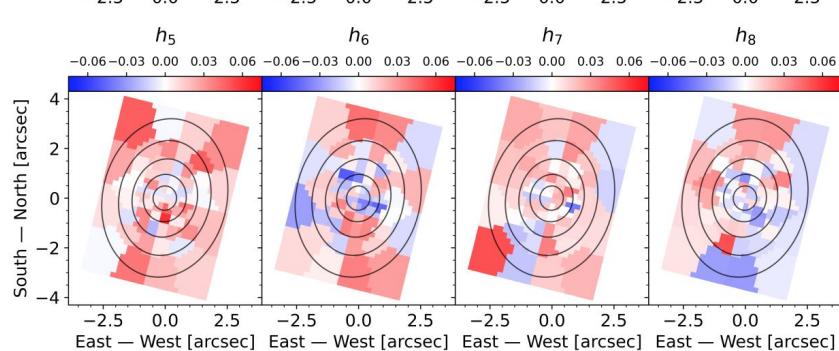
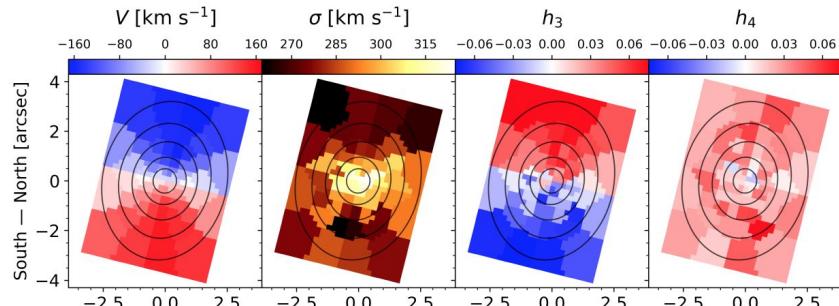
Observables:

3 sample spectra from GMOS



measure **spatially resolved** line-of-sight velocity distributions of stars

$$f(v) \propto \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(v-V)^2}{\sigma^2}} \left[1 + \sum_{m=3}^n h_m H_m \left(\frac{v-V}{\sigma} \right) \right]$$



A roadmap for the talk:

1. Supermassive black holes in the largest galaxies
2. **Triaxial Schwarzschild modeling of massive ellipticals**
3. Robustness and Recovery Tests of Dynamical Modeling Techniques



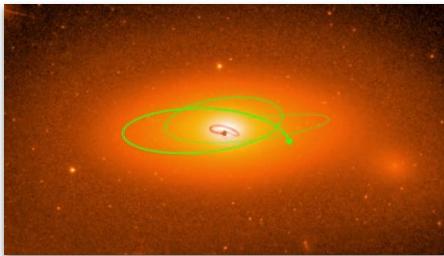
dynamical modeling

a new code for
Schwarzschild modeling:

TriOS: Triaxial Orbit Superposition

(van den Bosch+08,
Quenneville+21 updates)

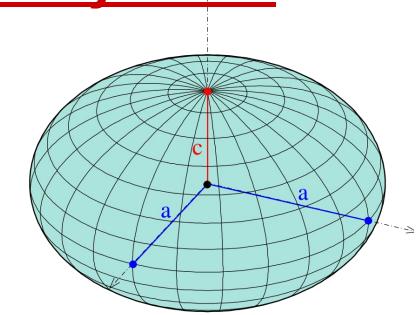
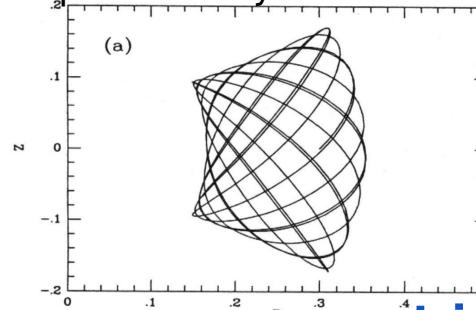
inputs: stellar kinematics, surface
brightness; galaxy model parameters



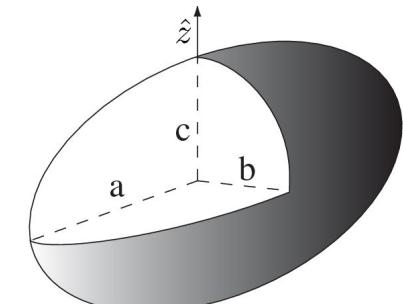
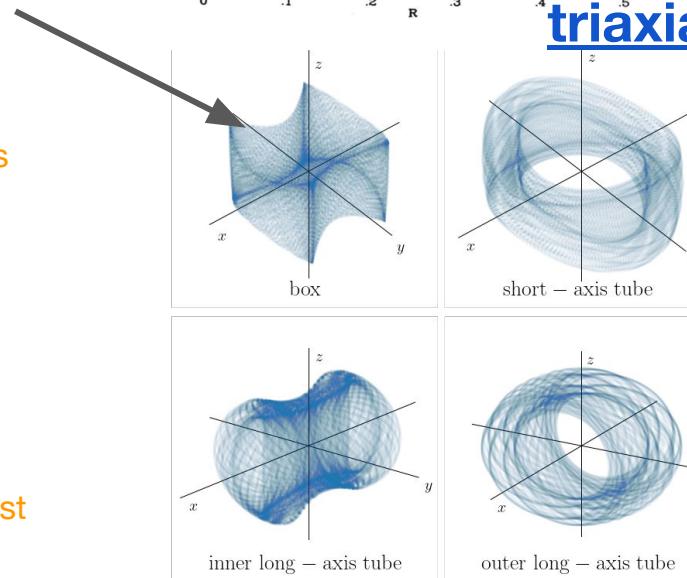
outputs: set of stellar orbits which best
reproduces the input kinematics

axisymmetric systems

loop orbits only



triaxial systems



dynamical modeling

1. Choose a trial potential:

$$\text{Galaxy} = (\text{DM}) + (\text{STARS}) + (\text{BH}) + \dots$$

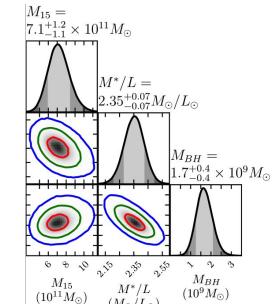
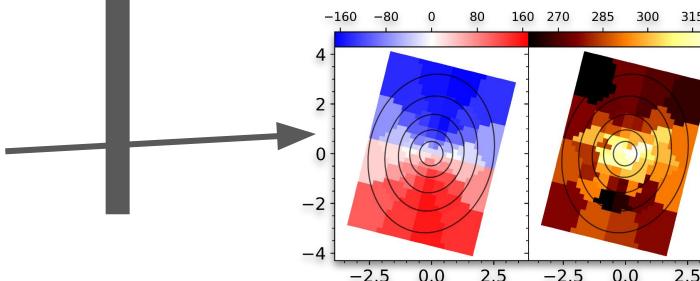
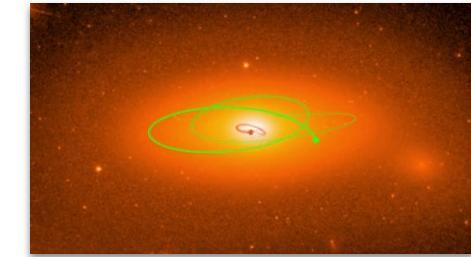
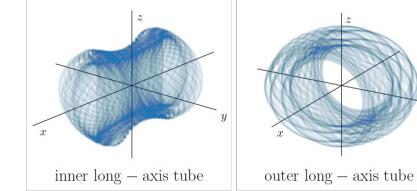
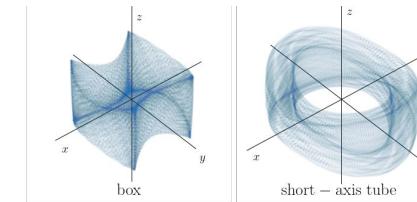
2. Generate stellar orbits in trial potential

3. Determine which orbits most accurately reproduce **kinematics** + **photometry** for a *single* trial potential

4. Find which assumed potential fits **kinematics** + **photometry** best **across trial potentials (BH, ML, Shapes)**

triaxial: BH, M/L, DM Halo, 3 shape parameters

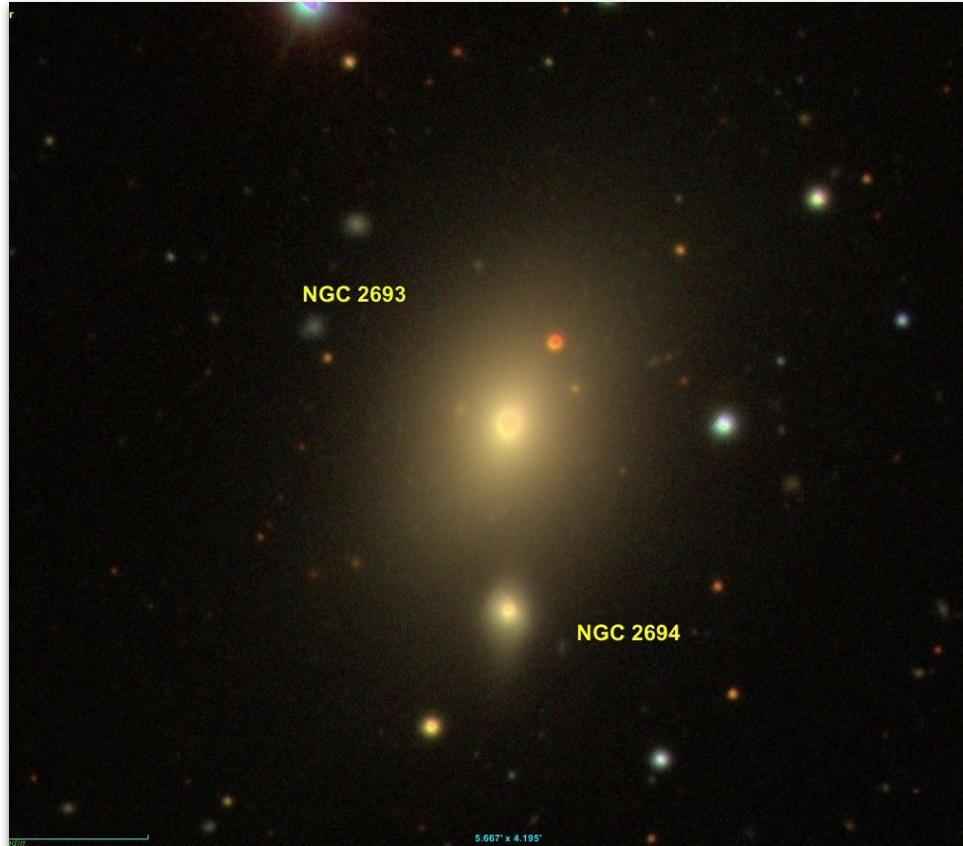
axisymmetric: BH, M/L, DM Halo, inclination



stellar kinematics of NGC 2693

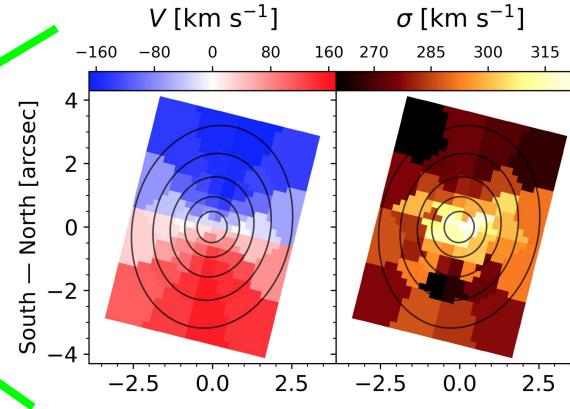
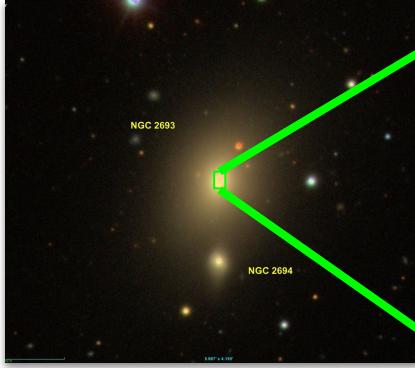
Liepold+20
Quenneville+21
Pilawa+22

NGC 2693



stellar kinematics of NGC 2693

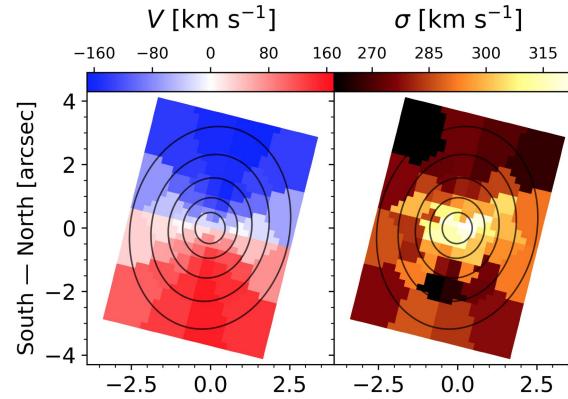
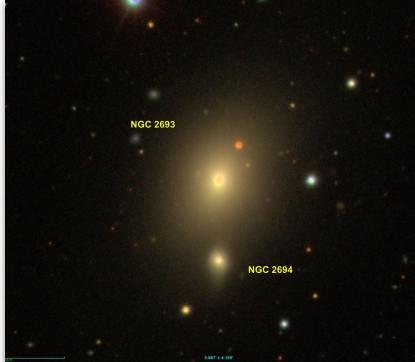
NGC 2693



- regular, fast rotator ($V \sim 150 \text{ km/s}$, $\sigma \sim 320 \text{ km/s}$)

stellar kinematics of NGC 2693

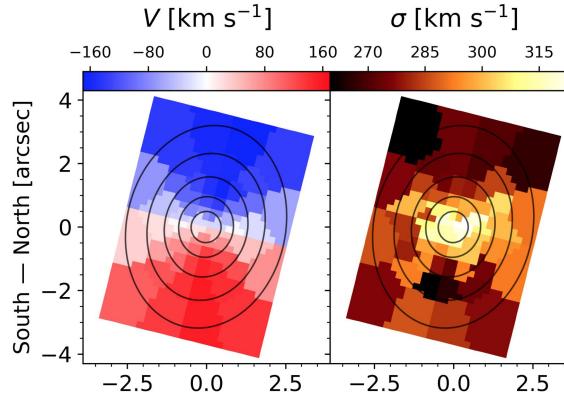
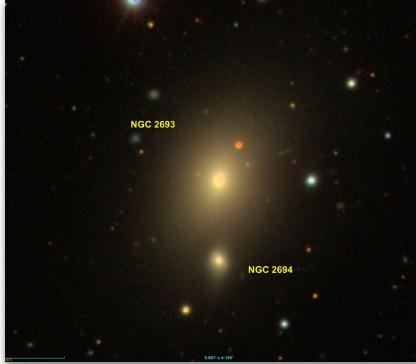
NGC 2693



- regular, fast rotator ($V \sim 150 \text{ km/s}$, $\sigma \sim 320 \text{ km/s}$)
- mostly regular, elliptical isophotes

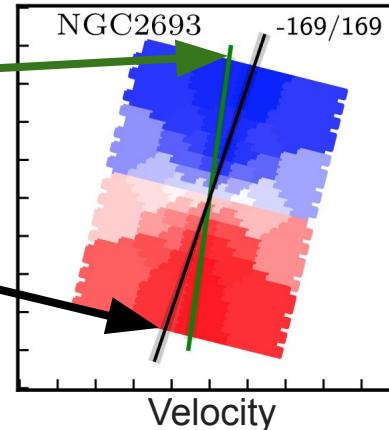
stellar kinematics of NGC 2693

NGC 2693



average orientation of stellar motion

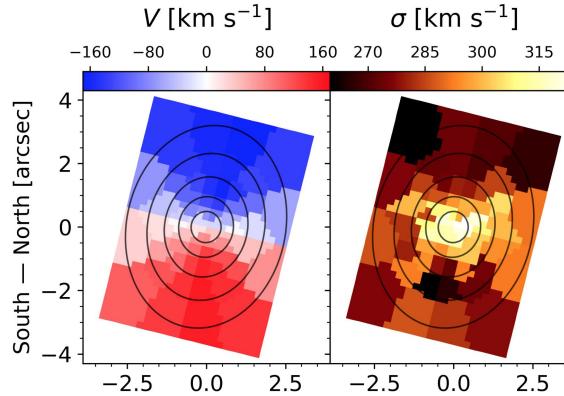
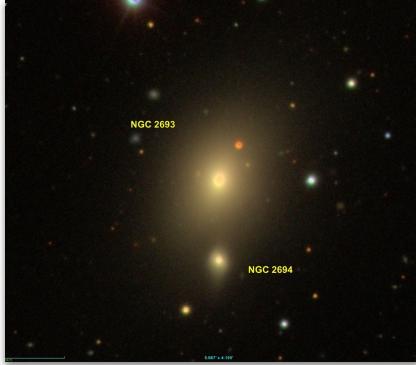
photometric major axis



- regular, fast rotator ($V \sim 150 \text{ km/s}$, $\sigma \sim 320 \text{ km/s}$)
- mostly regular, elliptical isophotes
- **kinematic** and **photometric** major axes are **nearly** aligned ($\Delta\Psi \sim 5^\circ$)

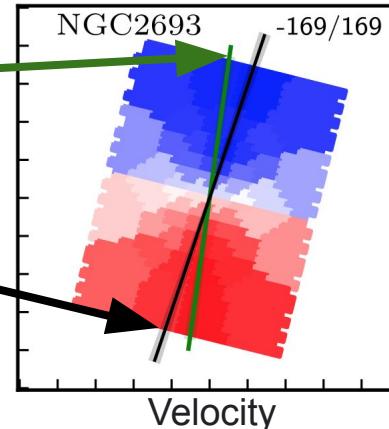
stellar kinematics of NGC 2693

NGC 2693



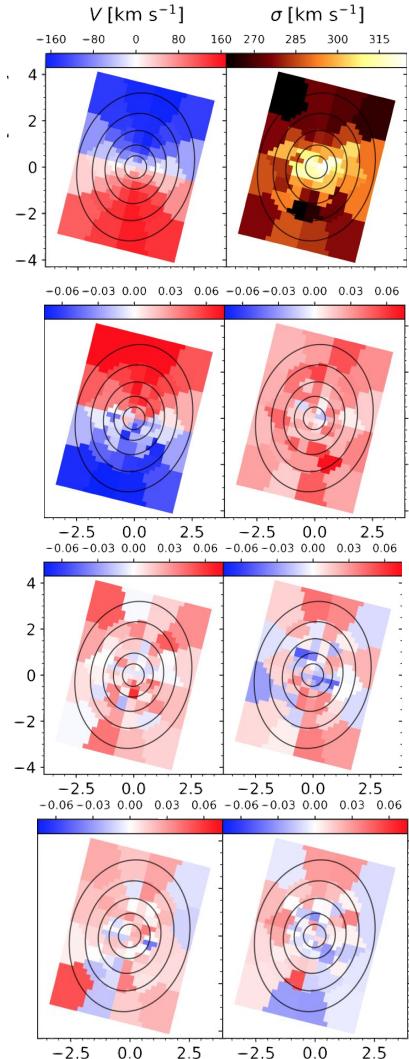
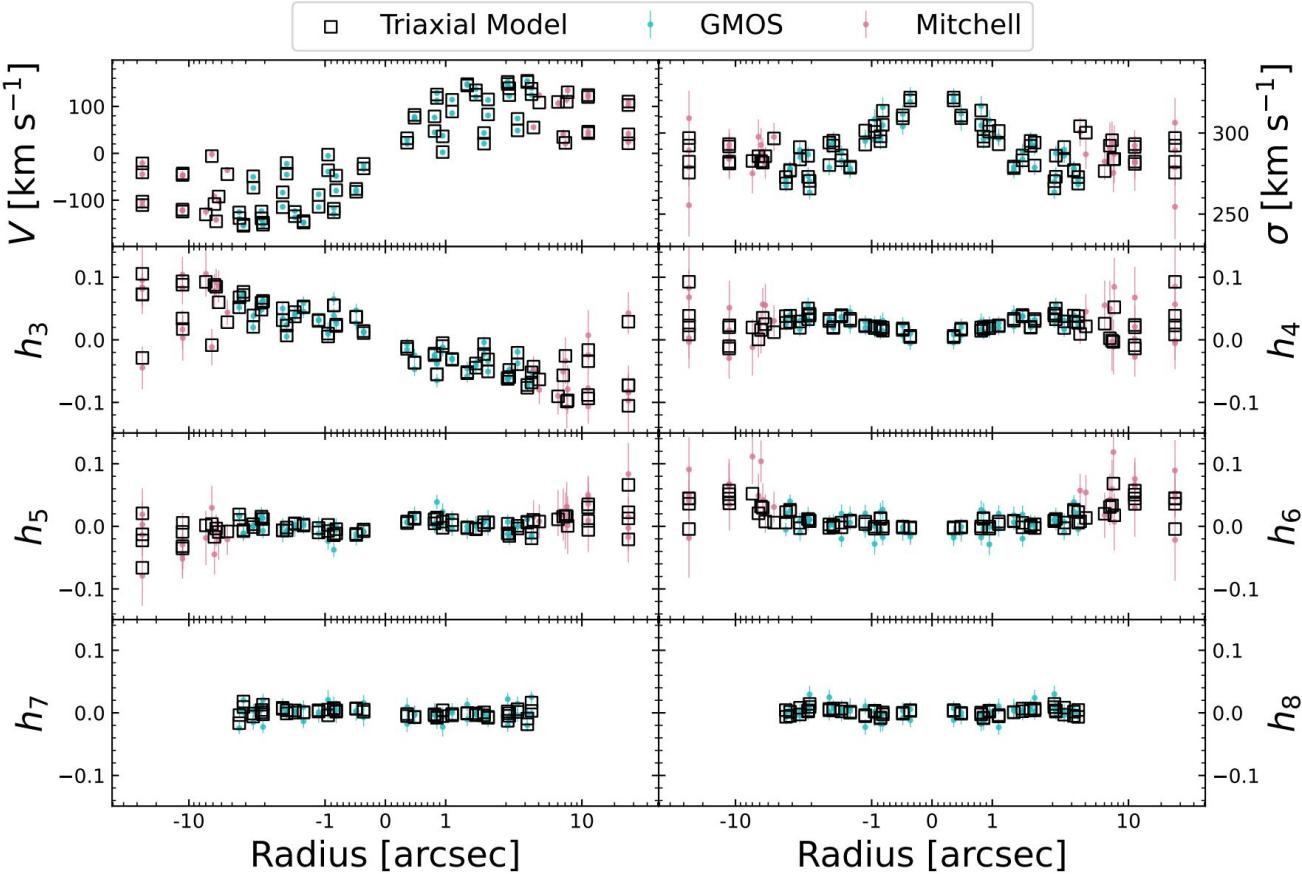
average orientation of stellar motion

photometric major axis



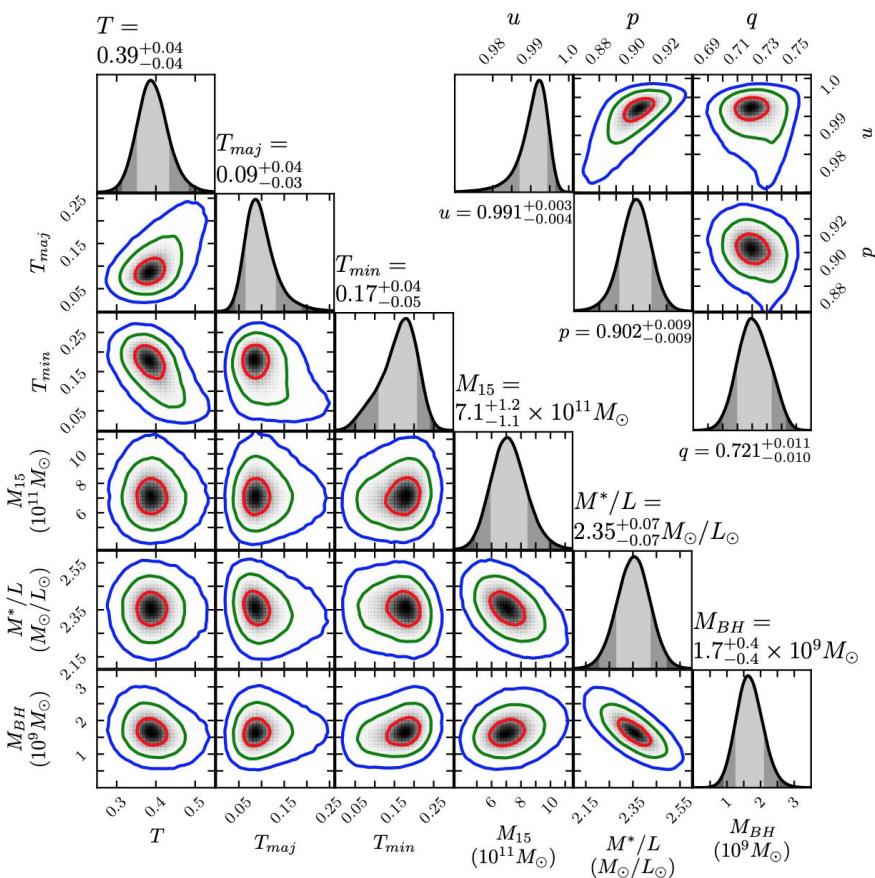
- regular, fast rotator ($V \sim 150 \text{ km/s}$, $\sigma \sim 320 \text{ km/s}$)
- mostly regular, elliptical isophotes
- **kinematic and photometric major axes are *nearly* aligned ($\Delta\Psi \sim 5^\circ$)**
properties ***nearly*** consistent with **axisymmetric** intrinsic shapes, so let's test!

stellar dynamical modeling: applications to NGC 2693



stellar dynamical modeling: applications to NGC 2693

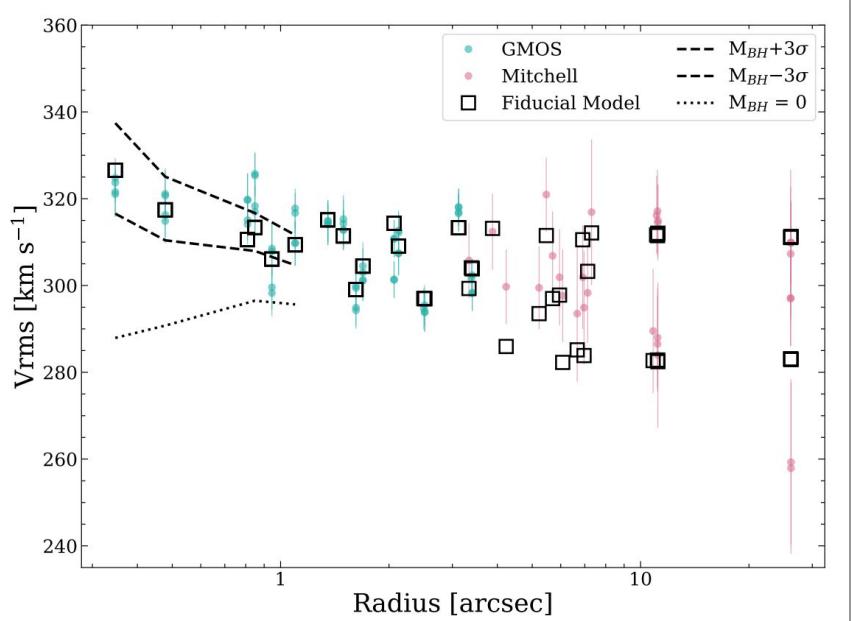
- first simultaneous measurements of DM mass, BH mass, **and intrinsic shapes**
- N2693 has a moderately triaxial intrinsic shape,
 - Intermediate-to-major axis ratio: $p = b/a \sim 0.9$
 - Minor-to-major axis ratio: $q = c/a \sim 0.7$
- NGC 2693 $M_{\text{BH}} = (1.7 \pm 0.4) \times 10^9 M_{\text{sun}}$



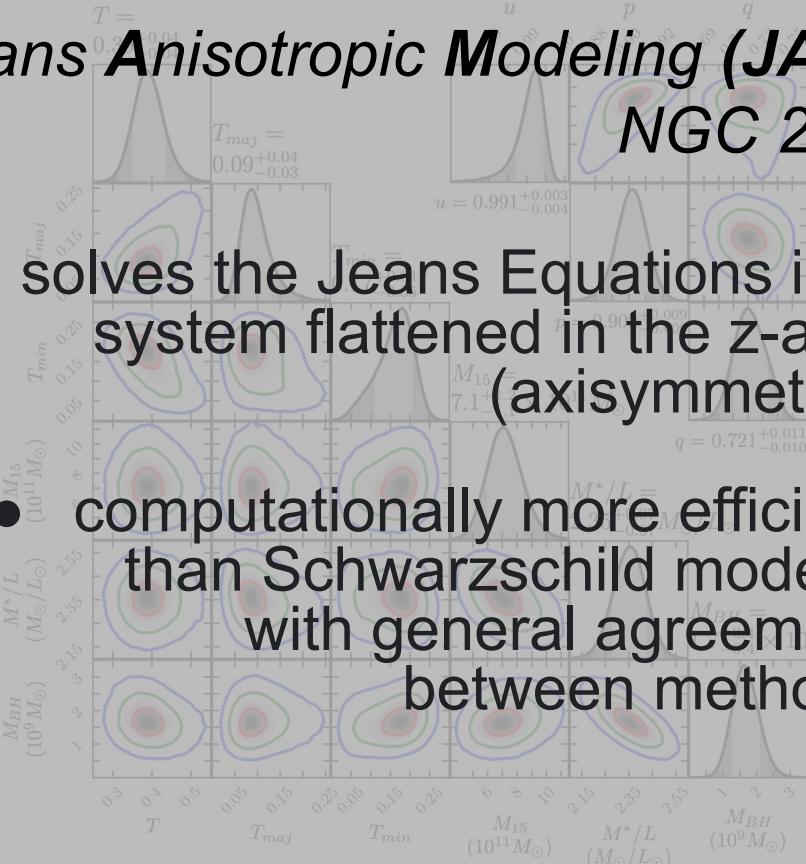
TRIAXIAL

NGC 2693 BH: $(1.7 \pm 0.4) \times 10^9 M_{\text{sun}} \rightarrow (2.4 \pm 0.6) \times 10^9 M_{\text{sun}}$

Jeans Anisotropic Modeling (JAM): NGC 2693



- solves the Jeans Equations in a system flattened in the z-axis (axisymmetric)
- computationally more efficient than Schwarzschild models, with general agreement between methods



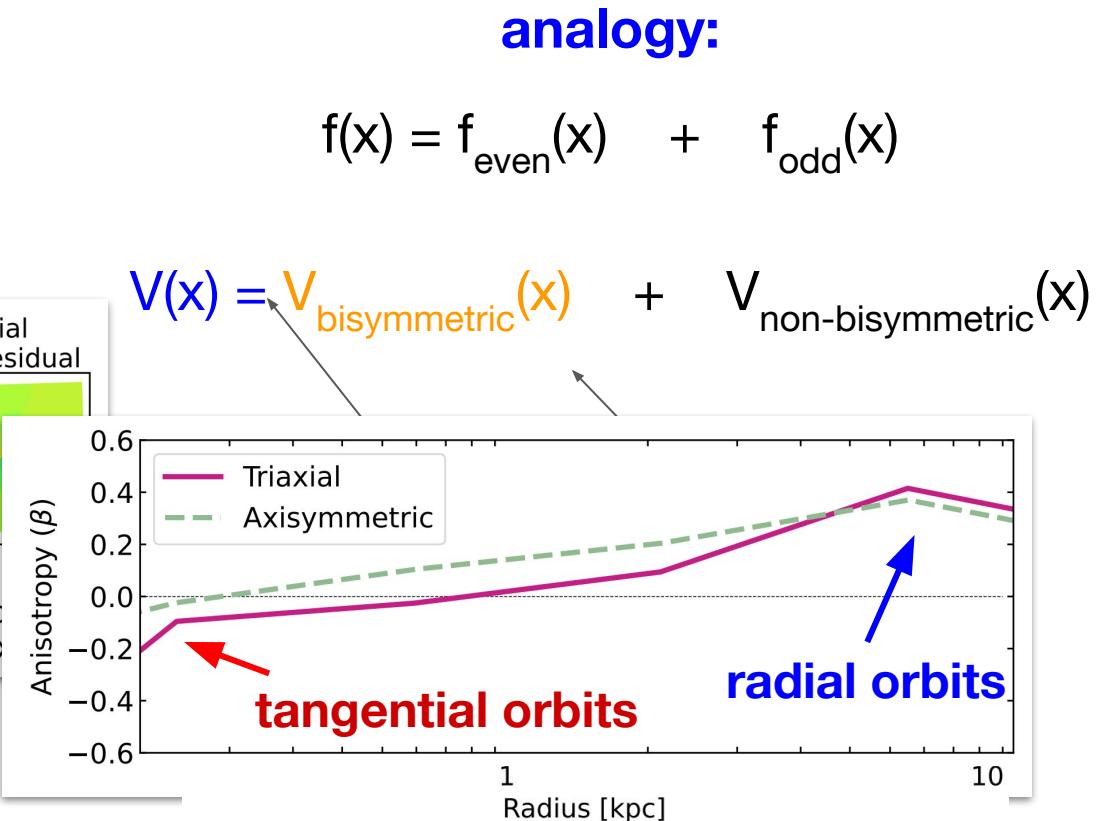
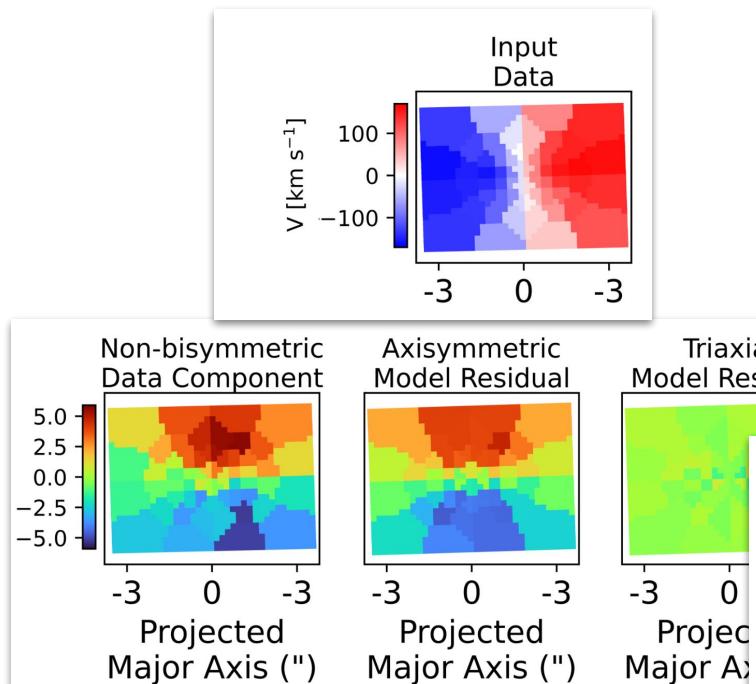
- $\text{NGC 2693 } M_{\text{BH}} = (1.7 \pm 0.4) \times 10^9 M_{\text{sun}}$

JAM

AXISYMMETRIC Schwarzschild

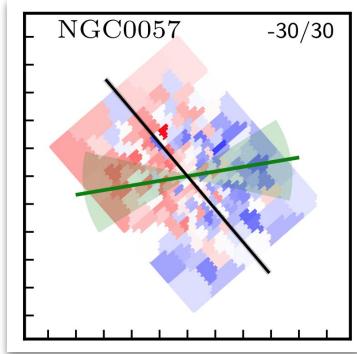
$$(2.9 \pm 0.3) \times 10^9 M_{\text{sun}} \rightarrow (2.4 \pm 0.6) \times 10^9 M_{\text{sun}}$$

what happened to the axisymmetric models?



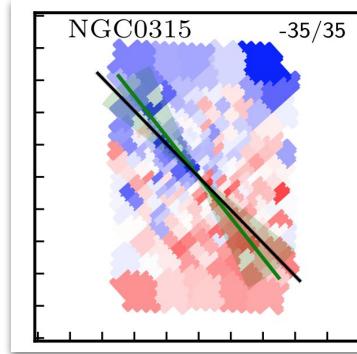
stellar dynamical modeling: ongoing work

NGC 57



- Slow rotation (~ 25 km/s), **kinematically** and **photometrically** misaligned
- Most isolated MASSIVE galaxy

NGC 315

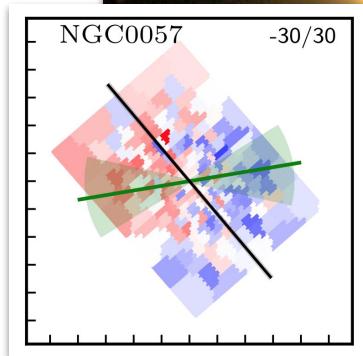


- Slow rotation (~ 30 km/s), **kinematically** and **photometrically** aligned
- Has ALMA CO M_{BH} measurement [Boizelle, Walsh+2021]

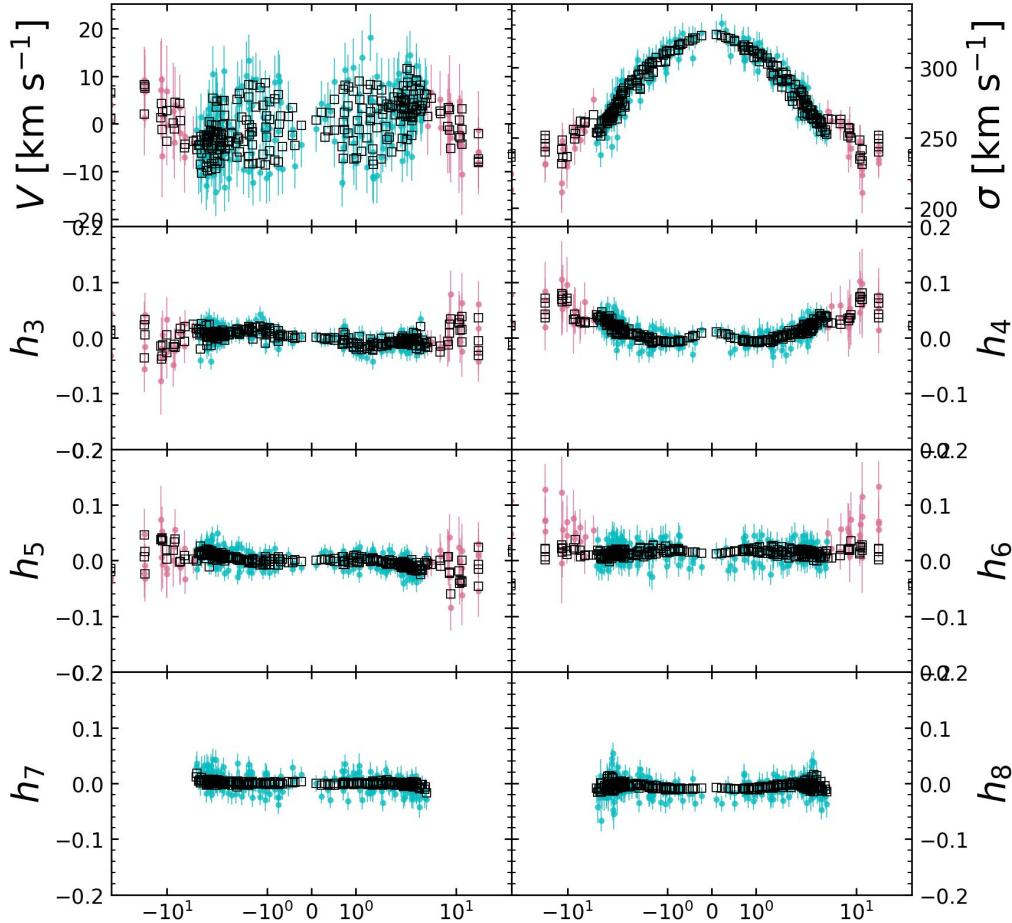
stellar dynamical modeling: ongoing work

□ Triaxial Model ♦ Mitchell ♦ GMOS

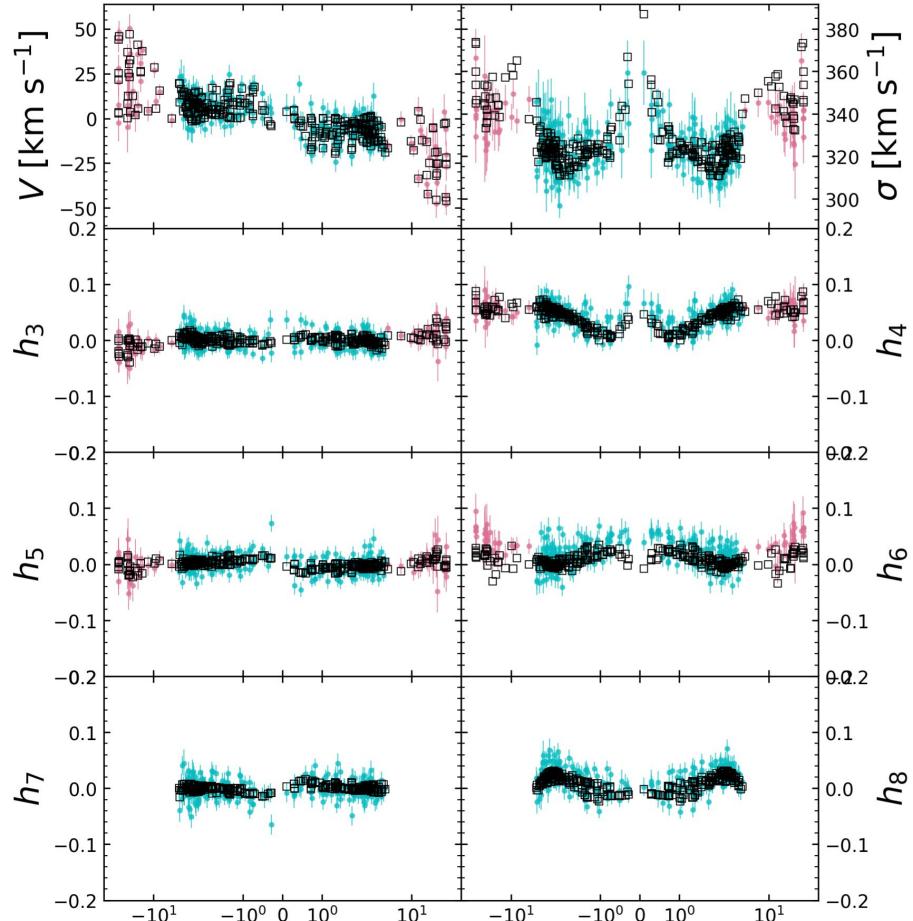
NGC 57



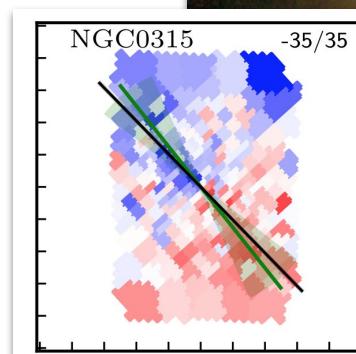
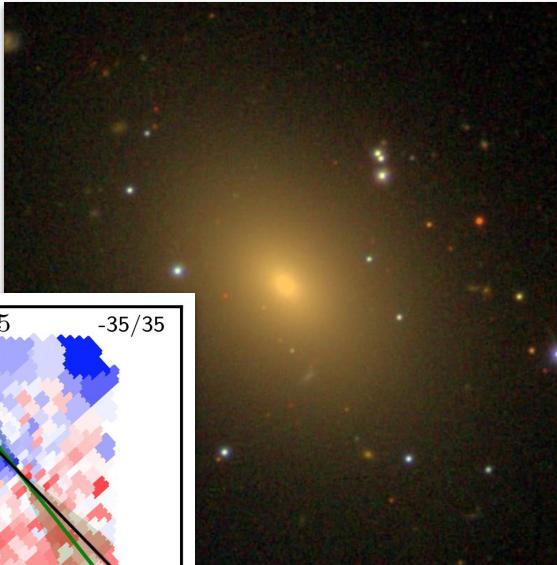
- Slow rotation (~ 25 km/s),
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stellar dynamical modeling: ongoing work



NGC 315



- Slow rotation (~ 30 km/s), **kinematically and photometrically** aligned
- Has ALMA CO M_{BH} measurement

A roadmap for the talk:

1. Supermassive black holes in the largest galaxies
2. Triaxial Schwarzschild modeling of massive ellipticals
3. **Robustness and Recovery Tests of Dynamical Models**

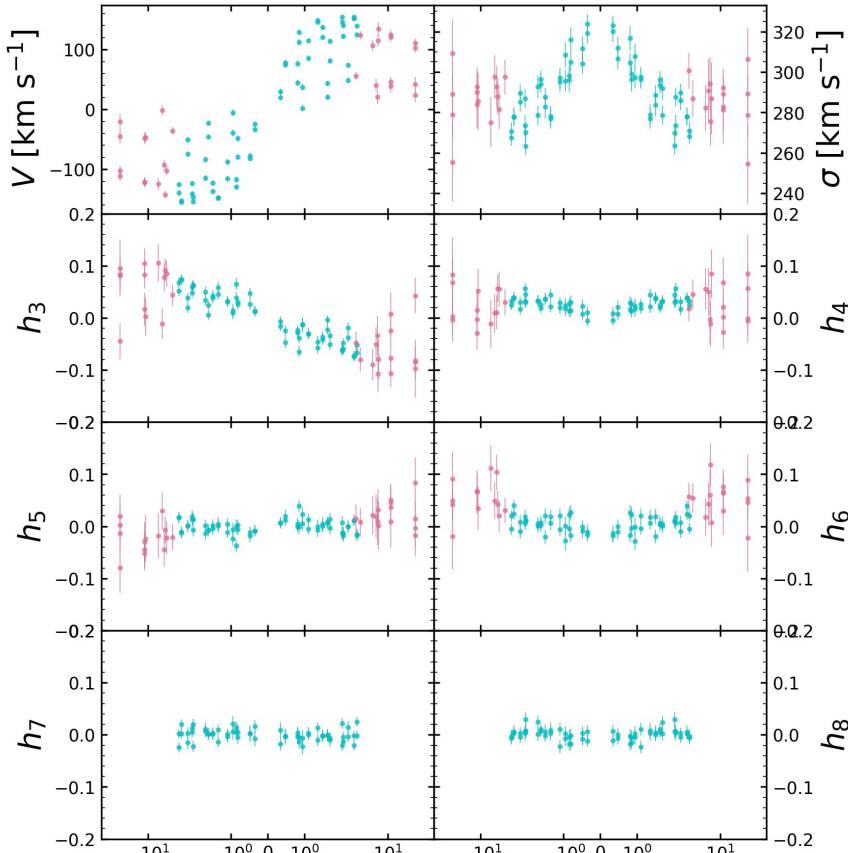


Taking a step back... how robust are Schwarzschild orbit models?

how well can we
recover a series of
known inputs?

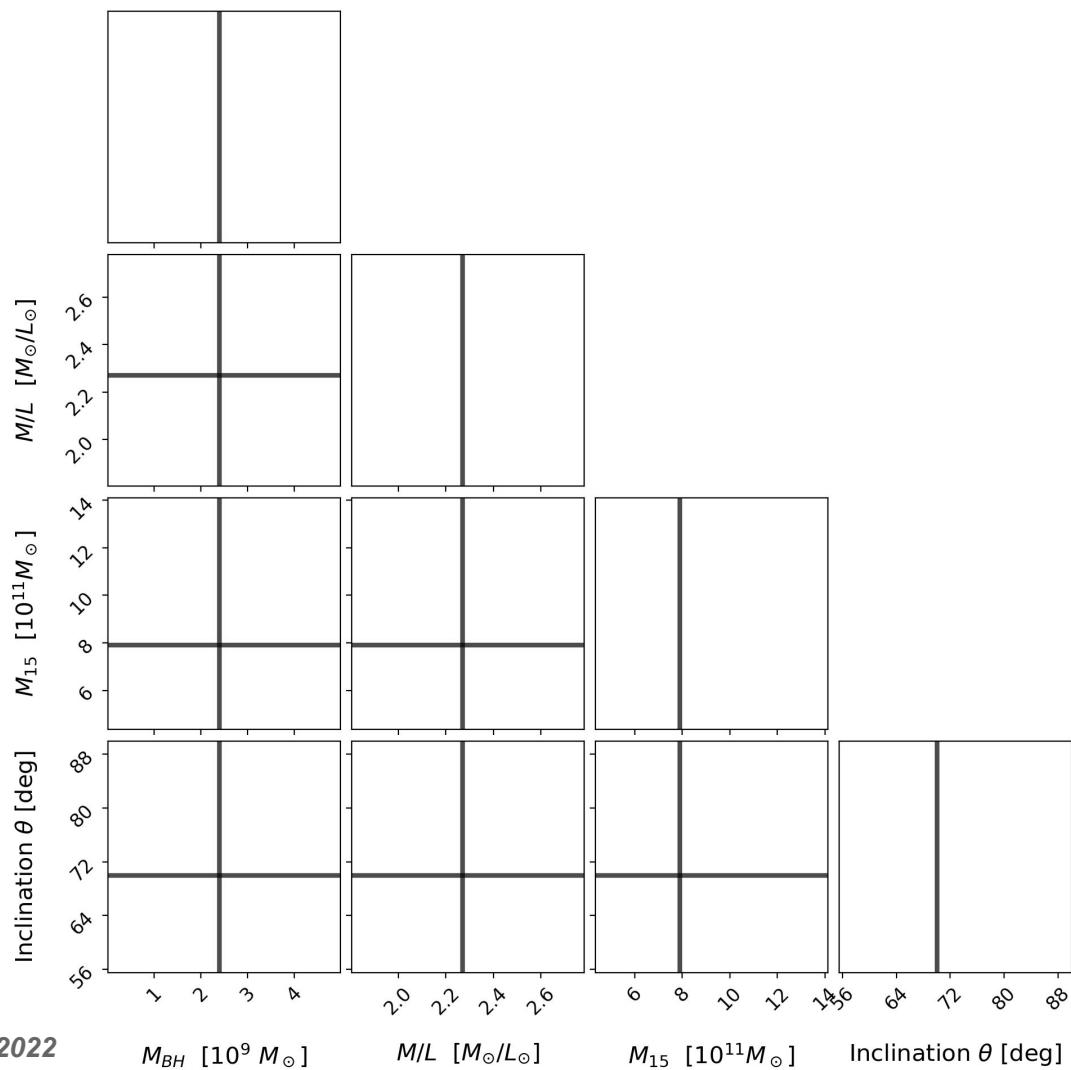
← *TriOS*

input: mock kinematics from a
galaxy with known parameters



Schwarzschild Mock Recovery: Axisymmetric Models

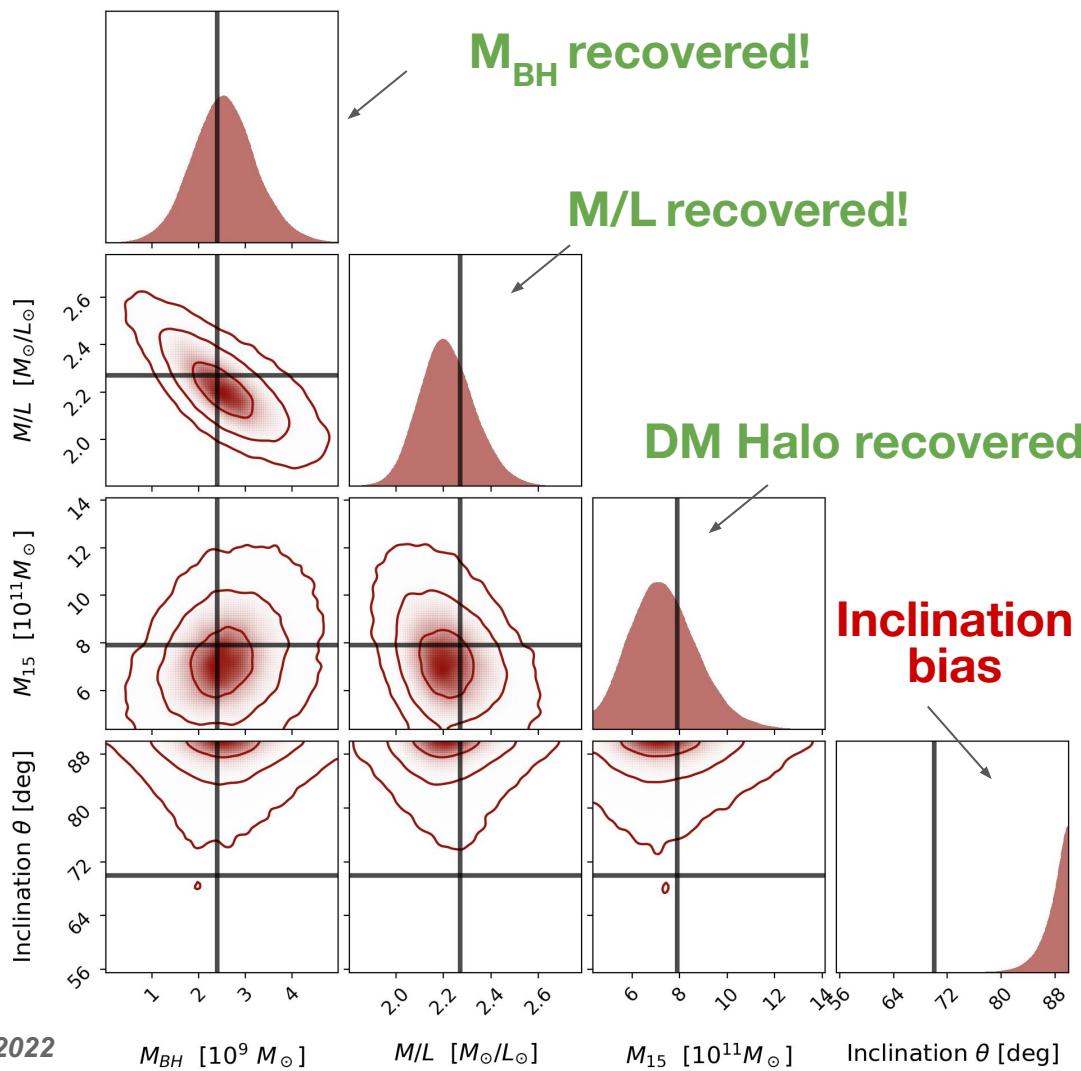
black:
input model kinematics
(from an $i = 70^\circ$ mock
galaxy model)



Schwarzschild Mock Recovery: Axisymmetric Models

black:
input model kinematics
(from an $i = 70^\circ$ mock
galaxy model)

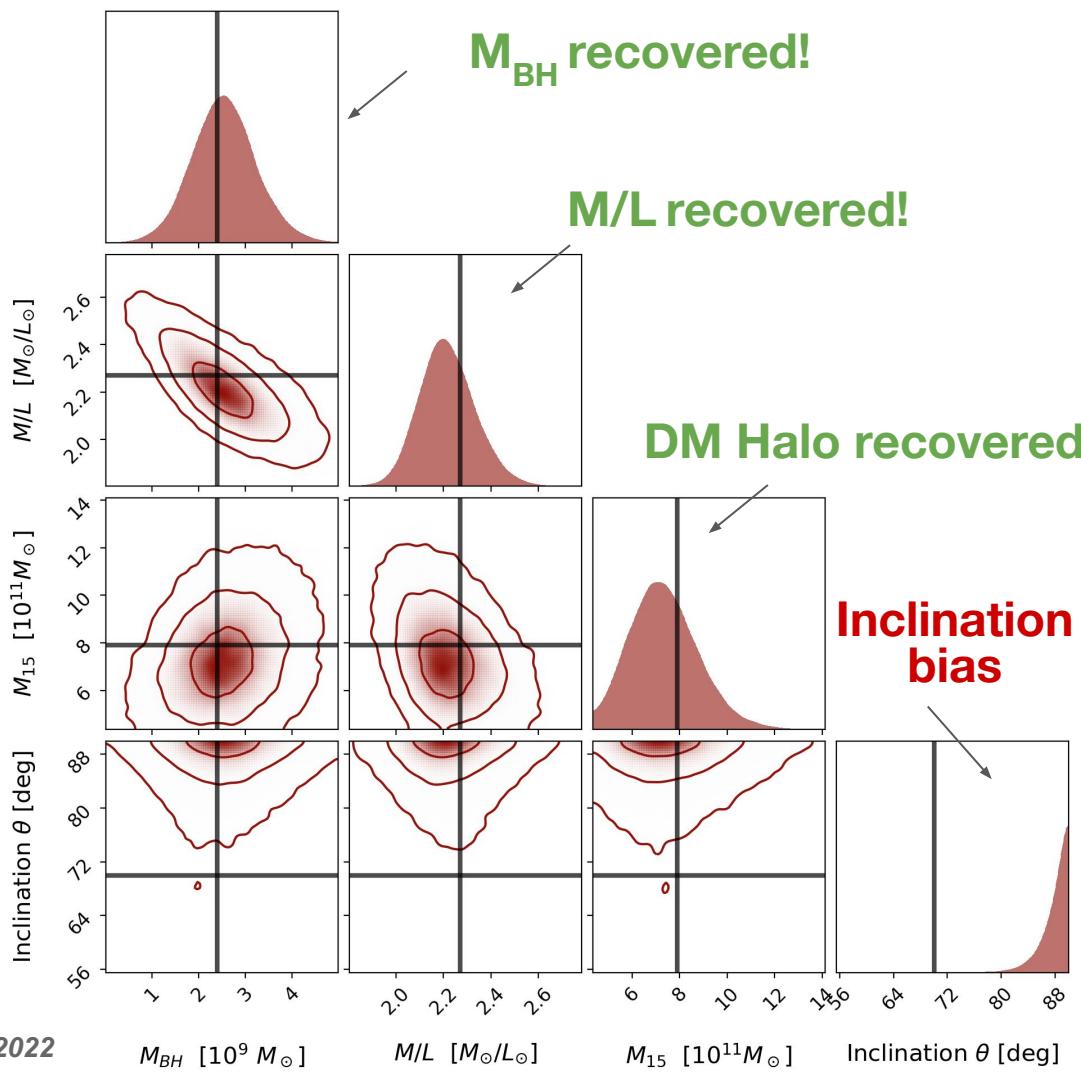
red: recovered posteriors
on parameters from
Schwarzschild models



Schwarzschild Mock Recovery: Axisymmetric Models

best case scenario:
large range of inclinations
are consistent with the data

typical case:
axisymmetric schwarzschild models exhibit a bias toward edge-on inclinations



Schwarzschild Model

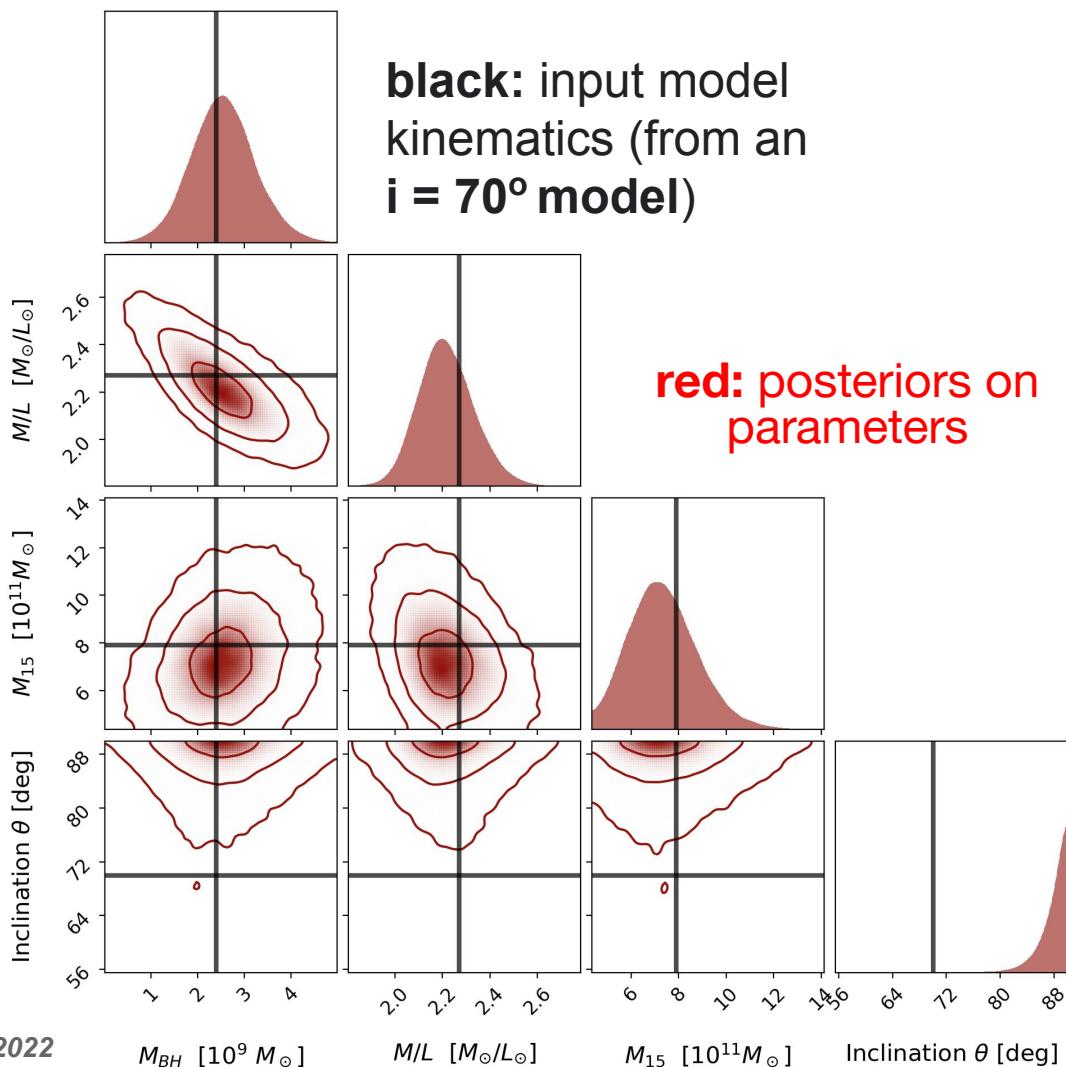
Flexibility:

Axisymmetric Models

edge on:
prograde and retrograde
orbits are maximally
different

face on:
prograde and retrograde
orbits are identical

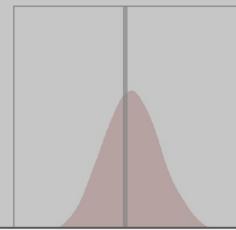
→ edge-on models have a
larger set of unique **basis
functions** for the
superposition of orbits!
→ larger “**flexibility**” in
these models



Schwarzschild Mock Recovery: Axisymmetric Models

best case
large range of
parameters are consistent

typical
axisymmetric
schwarzschild
models have
bias toward
edge-on
inclinations

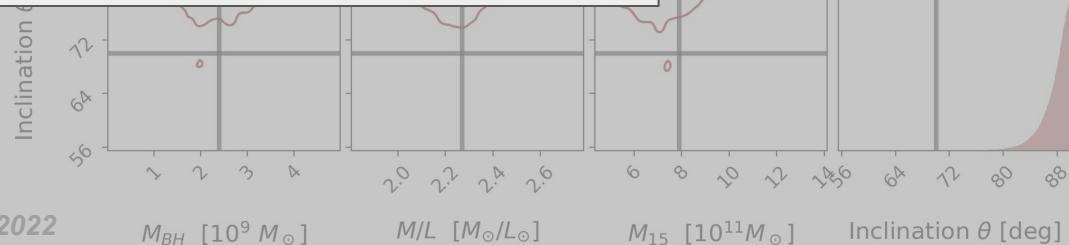


black: input model
kinematics (from an
 $i = 70^\circ$ model)

red: posteriors on
parameters

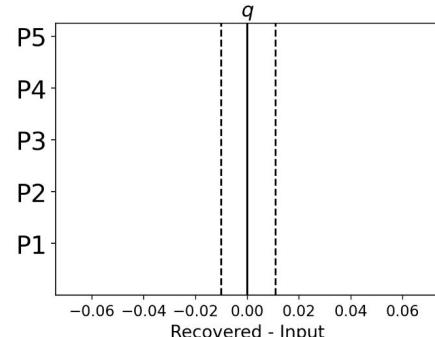
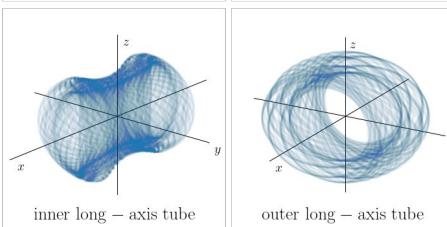
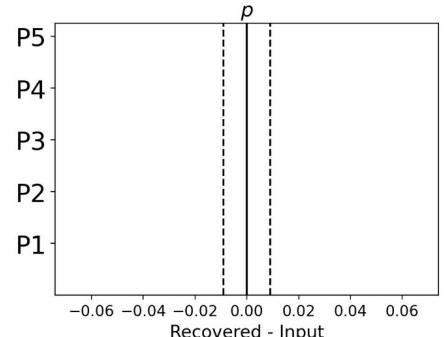
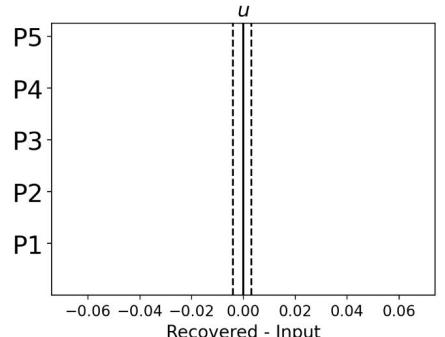
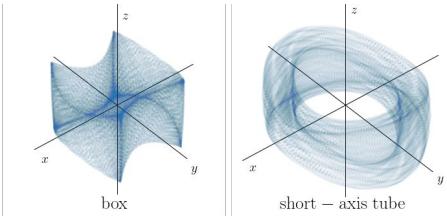
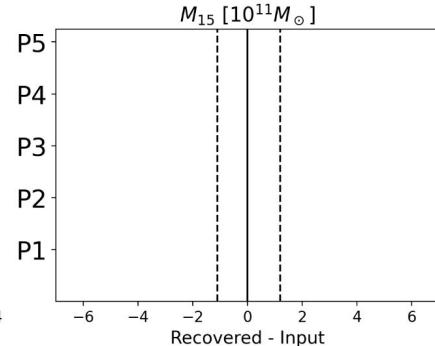
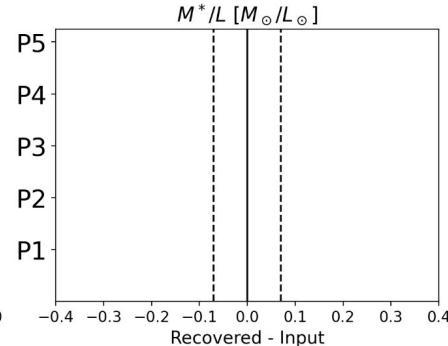
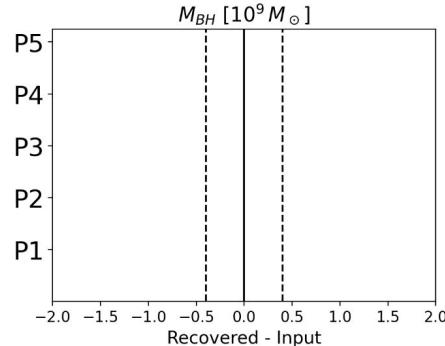
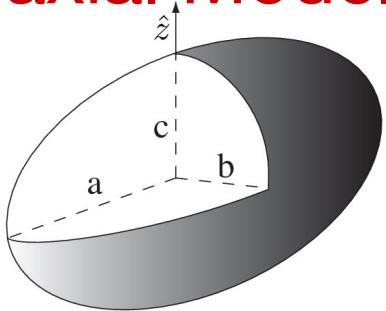
- (a) how well can we recover a series of known inputs from our **triaxial** models?

- (b) are there analogous biases in our triaxial modeling scheme?



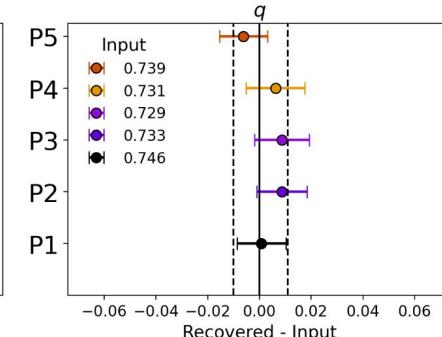
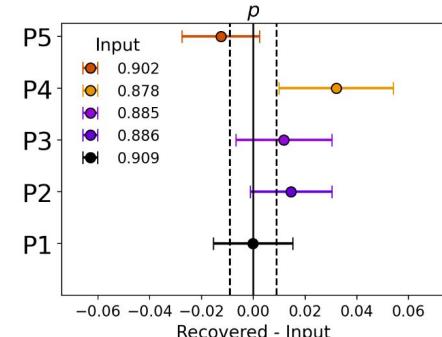
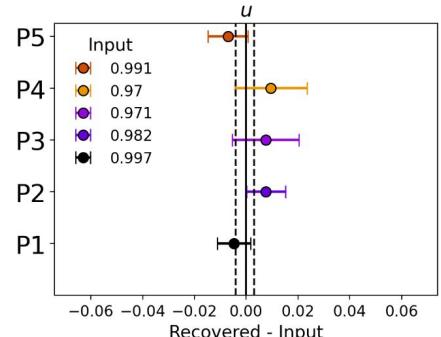
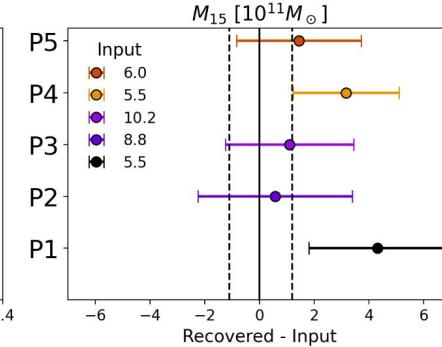
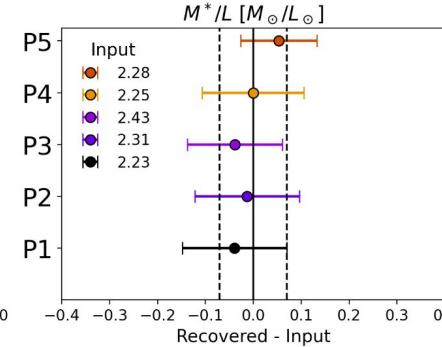
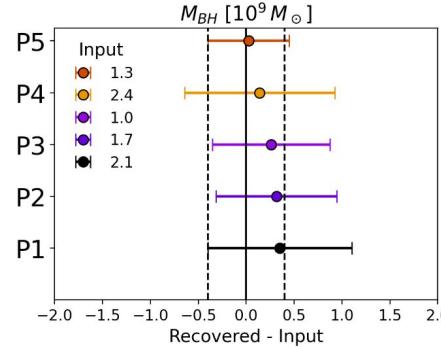
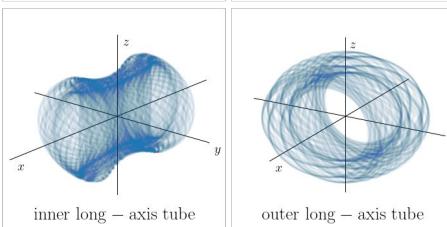
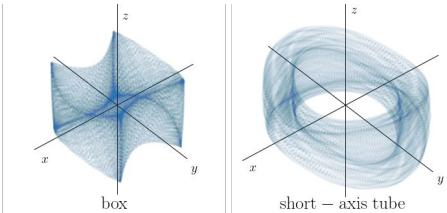
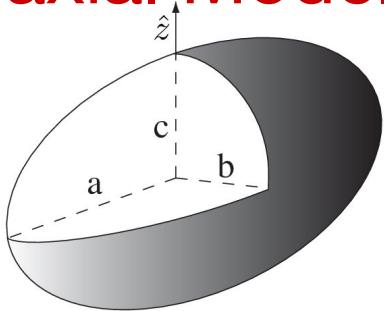
Schwarzschild Model

Flexibility: Triaxial Models

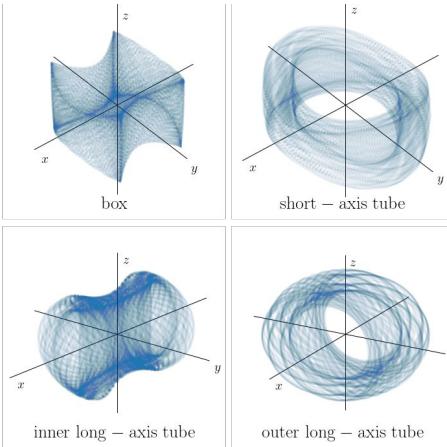
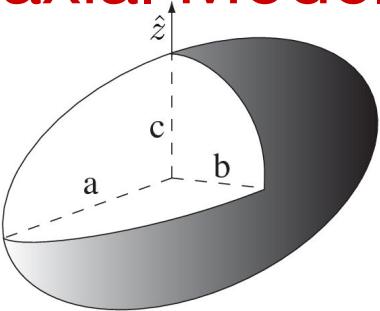


Schwarzschild Model

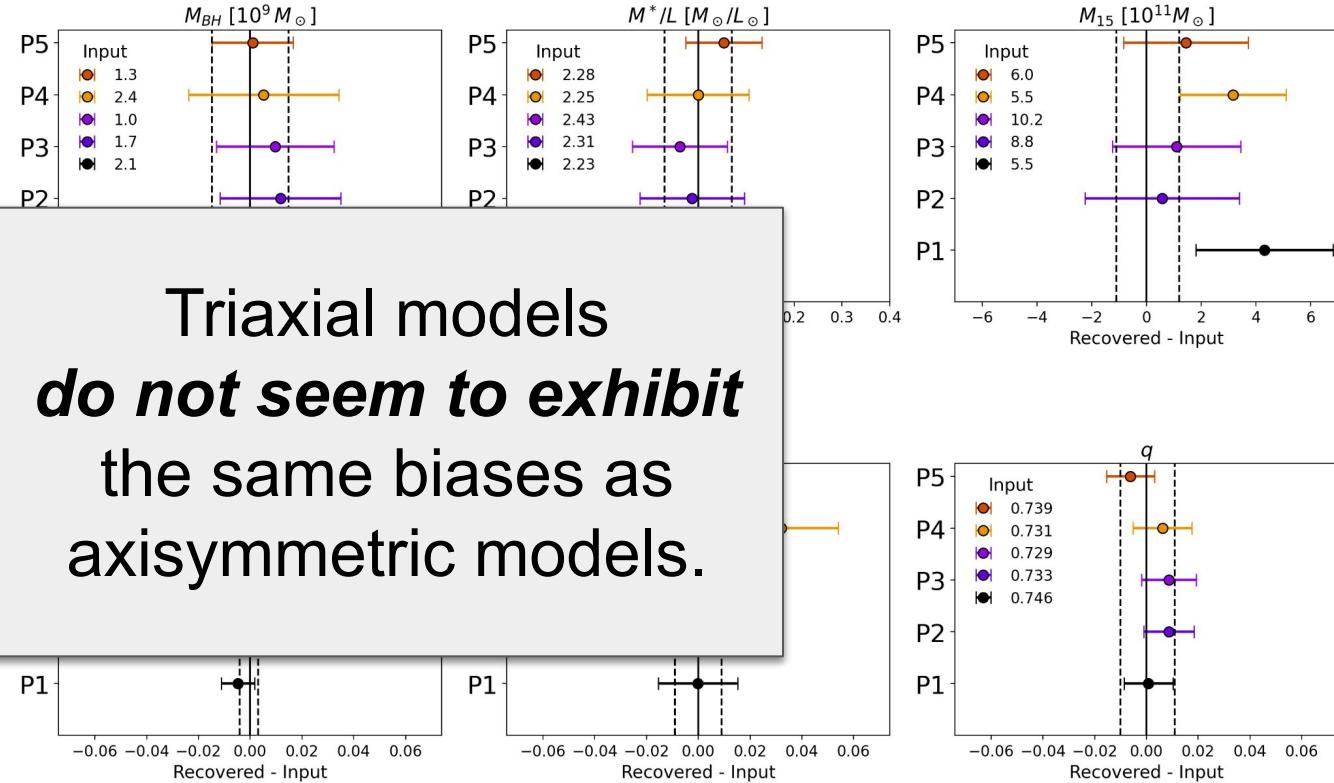
Flexibility: Triaxial Models



Schwarzschild Model Flexibility: Triaxial Models

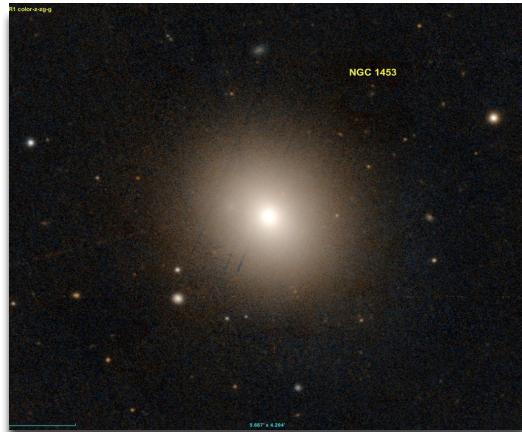


Triaxial models
do not seem to exhibit
the same biases as
axisymmetric models.



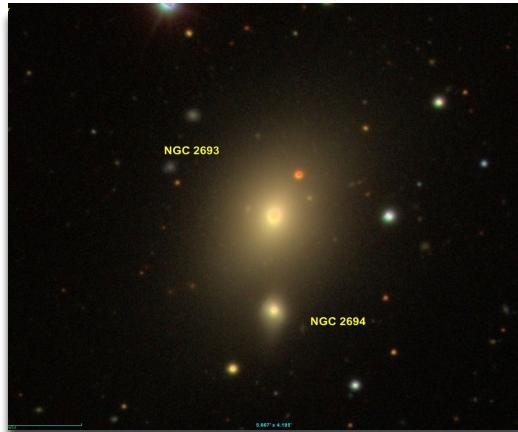
Summary: Recent Dynamical Modeling Efforts

NGC 1453



Quenneville+2021

NGC 2693



Pilawa+2022

M87



Liepold+2023

- $M_{BH} = (2.9 \pm 0.4) \times 10^9 M_{\text{sun}}$
- $p = b/a = (0.933 \pm 0.015)$
- $q = c/a = (0.779 \pm 0.012)$

- $M_{BH} = (1.7 \pm 0.4) \times 10^9 M_{\text{sun}}$
- $p = b/a = (0.902 \pm 0.009)$
- $q = c/a = (0.721 \pm 0.010)$

- $(5.37^{+0.37}_{-0.25} \pm 0.22) \times 10^9 M_{\odot}$
- $p = b/a = (0.845 \pm 0.004)$
- $q = c/a = (0.772 \pm 0.004)$

Thank you!

Summary:

discovery and dynamical modeling of new supermassive black holes:
NGC 2693: $(1.7 \pm 0.4) \times 10^9 M_{\text{Sun}}$
NGC 1453: $(1.7 \pm 0.4) \times 10^9 M_{\text{Sun}}$

schwarzschild models are **extremely** powerful, and we can robustly recover known inputs

we are ready for **more dynamical modeling** more complicated kinematic structure

first simultaneous measurements of **BH + DM halo + galaxy shape**; axisymmetric + triaxial + JAM

computationally cheap ways to estimate **model flexibility** and its effects (or lack thereof) on recovered parameters

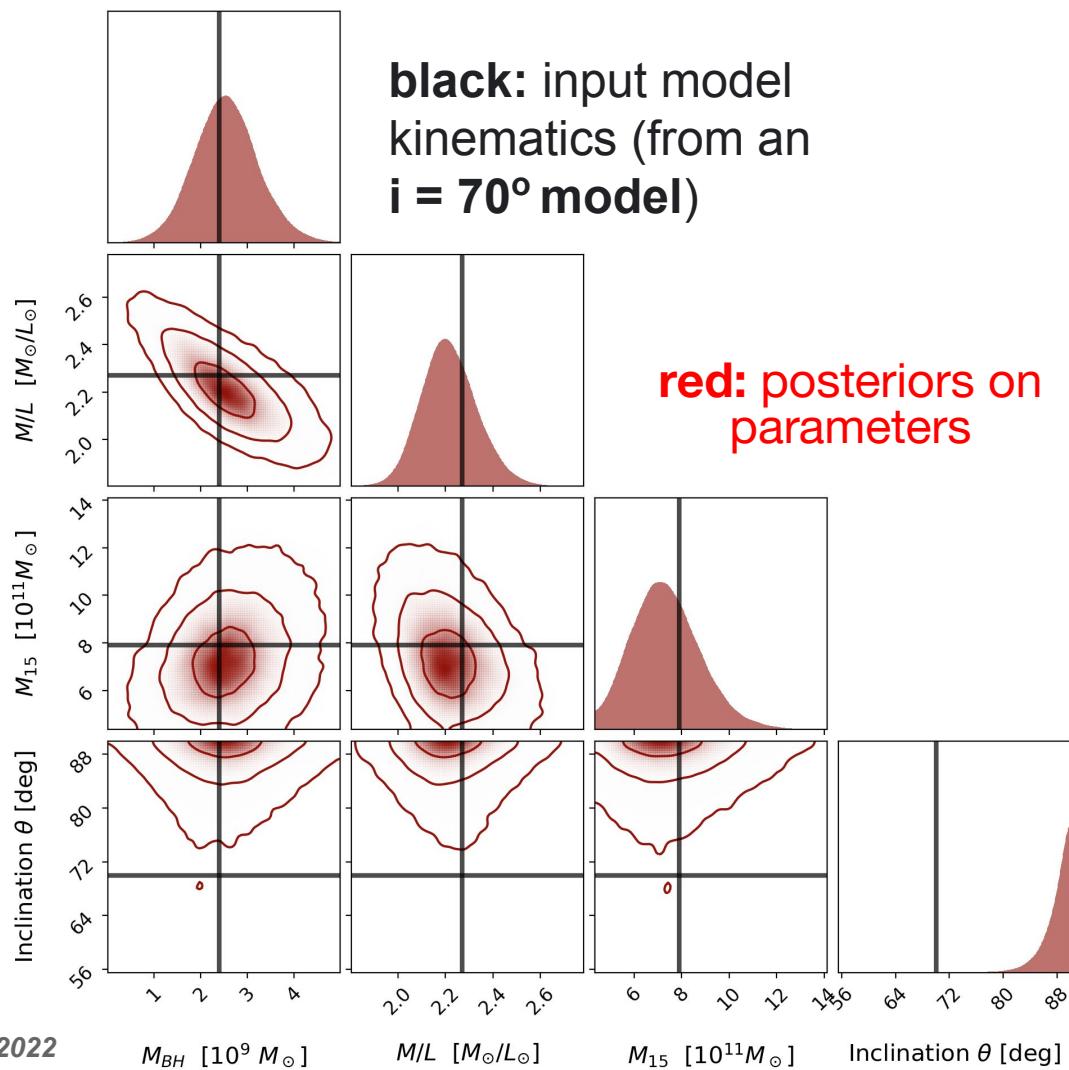
~20 MASSIVE galaxies are ready for modeling, none of which are simple fast rotators

Thank you!

Extra Slides

Schwarzschild Model Flexibility: Axisymmetric Models

can we fix this?



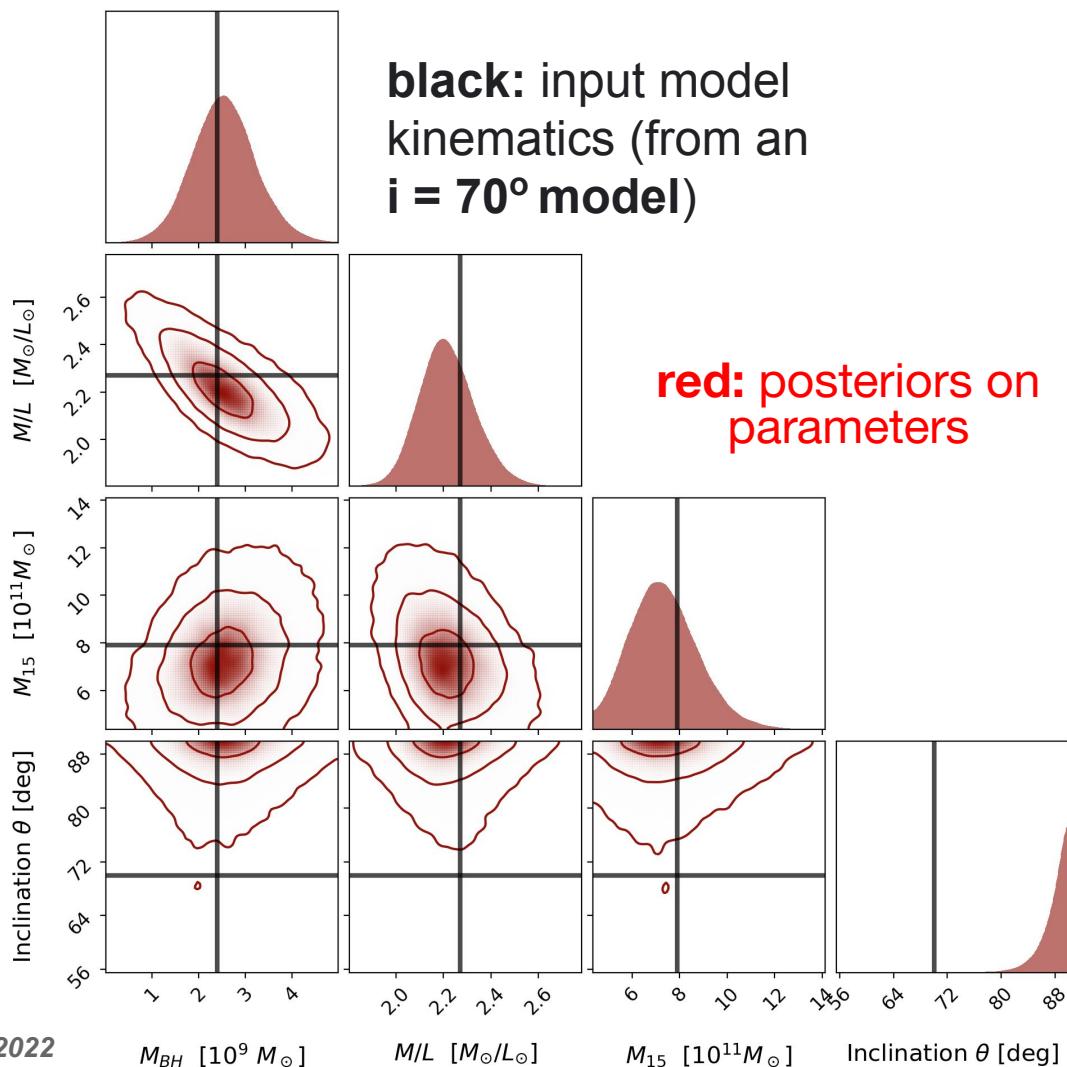
Schwarzschild Model

Flexibility:

Axisymmetric Models

can we fix this?

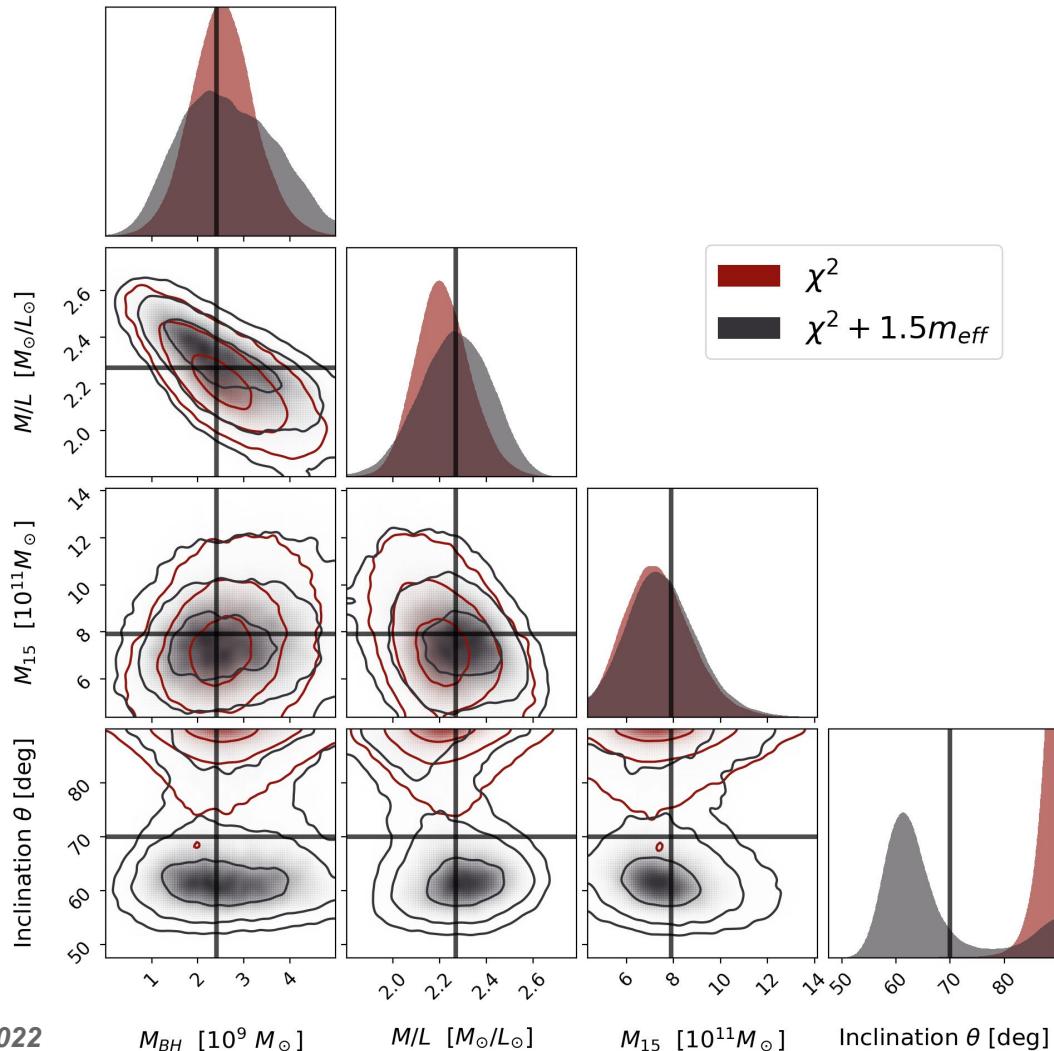
- one approach (Ye 1998):
generalized d.o.f. m_{eff}



Schwarzschild Model Flexibility: Axisymmetric Models

can we fix this?

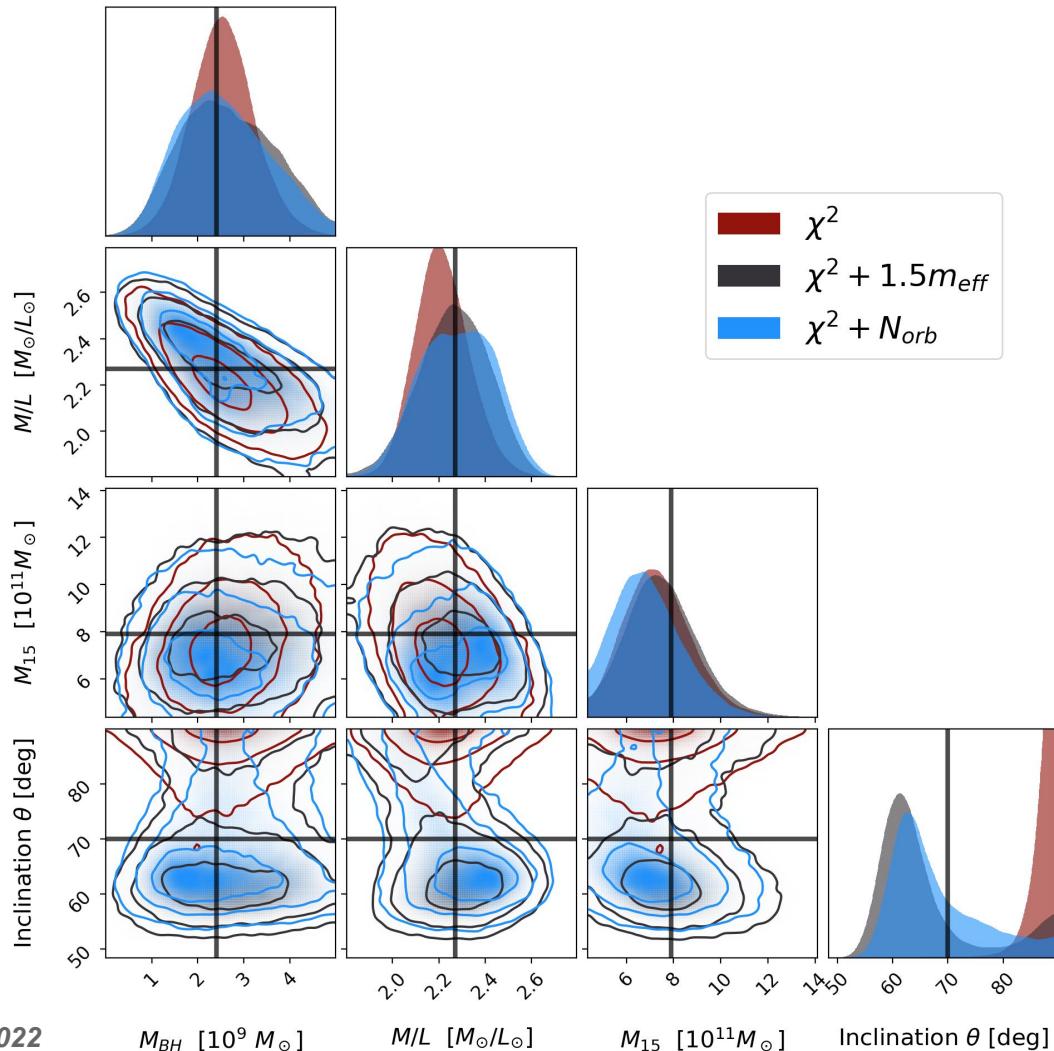
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Schwarzschild Model Flexibility: Axisymmetric Models

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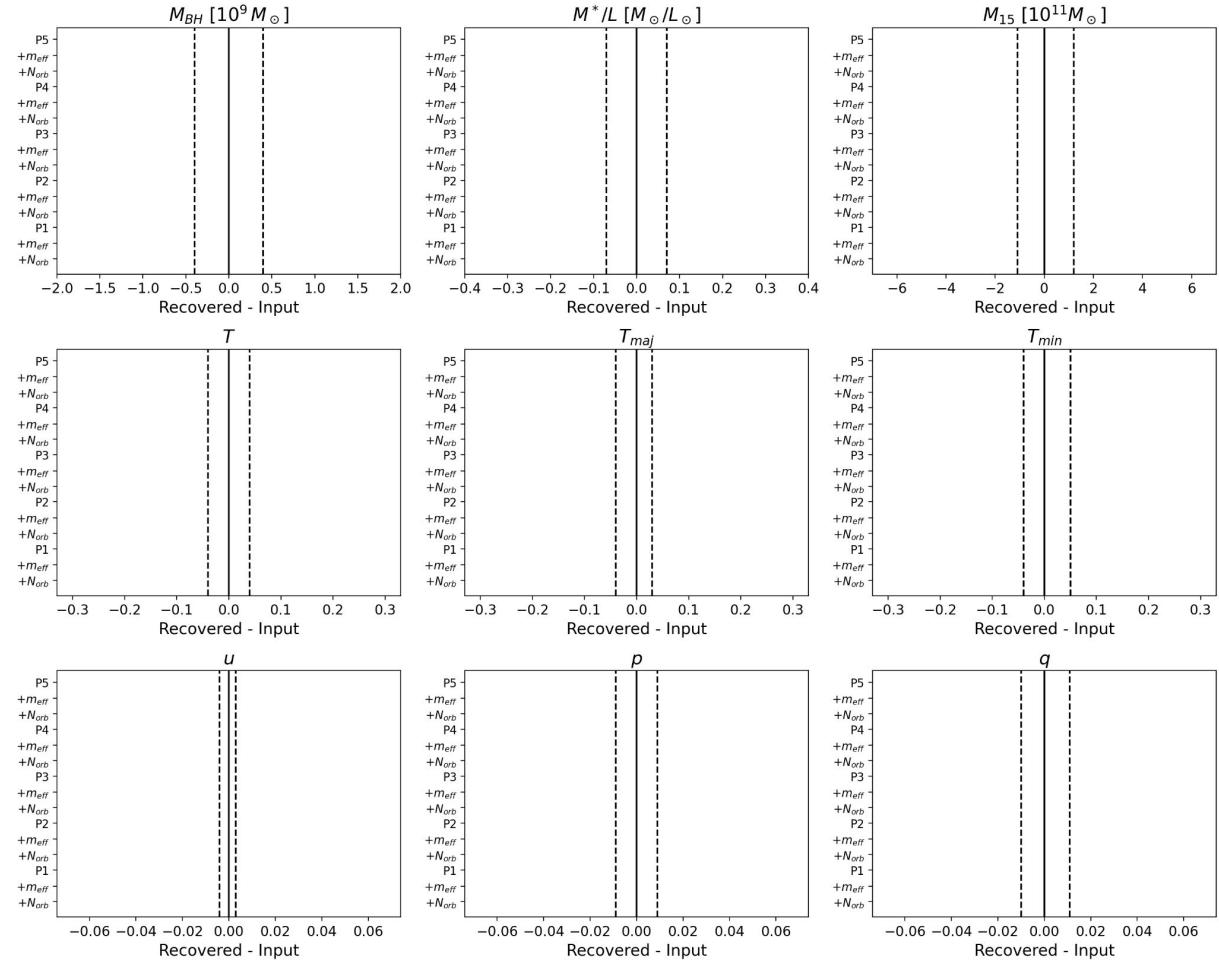
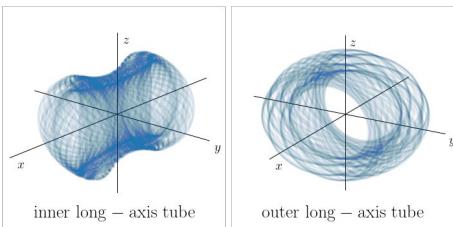
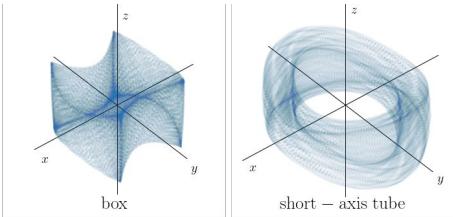
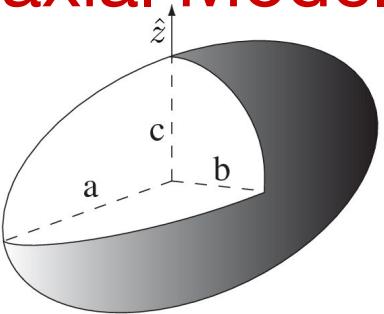
- one approach (Ye 1998): **generalized d.o.f. m_{eff}**
- alternative, **faster** approach:
 - penalize the model's χ^2 by “activated” orbits N_{orb} (orbits with non-zero weight in superposition)



Schwarzschild Model

Flexibility:

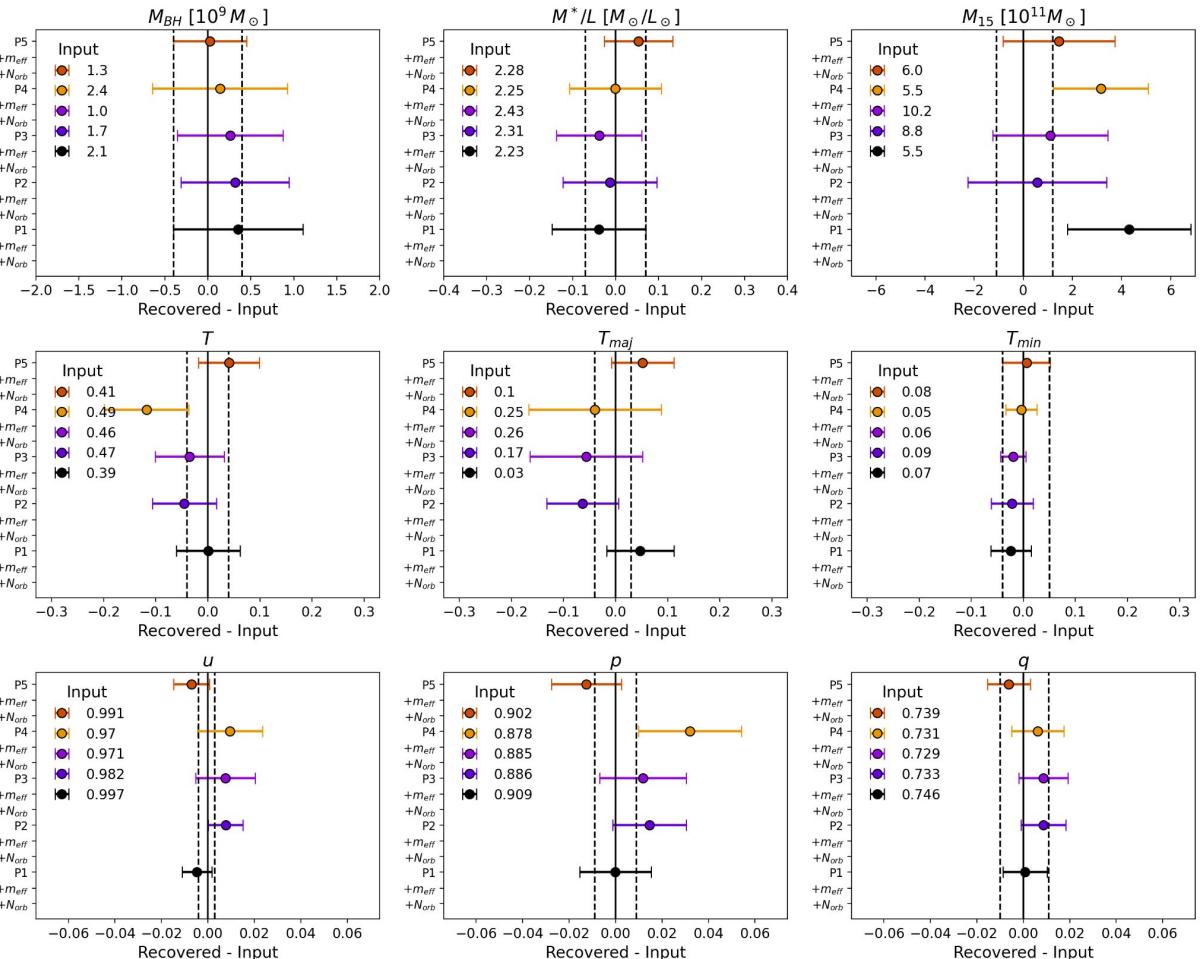
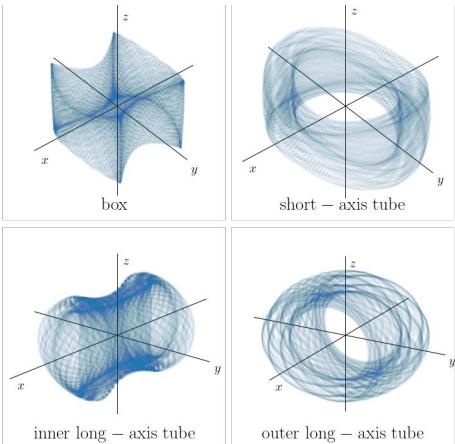
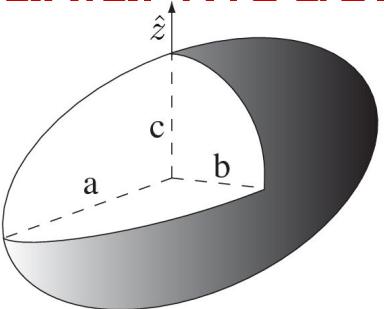
Triaxial Models



Schwarzschild Model

Flexibility:

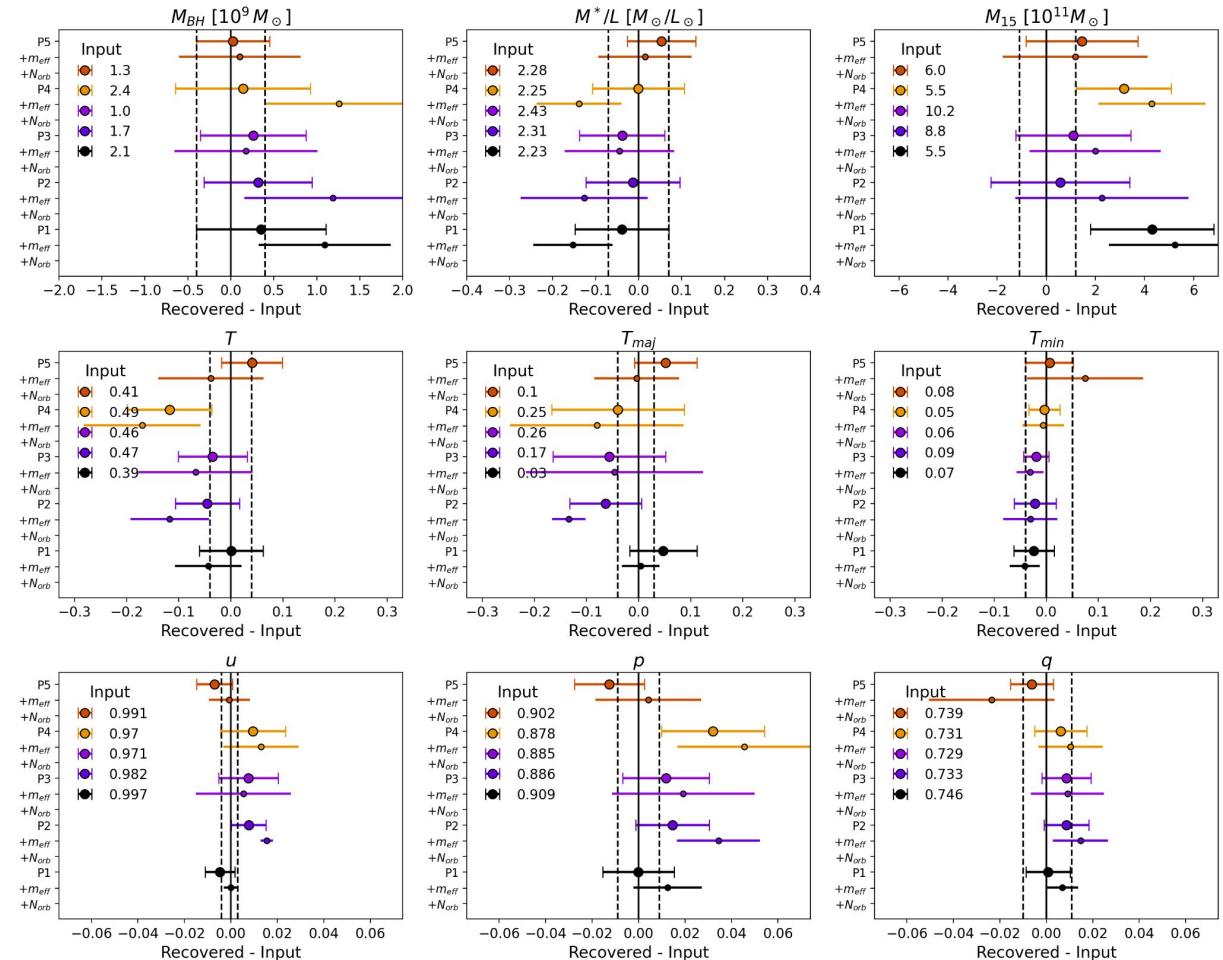
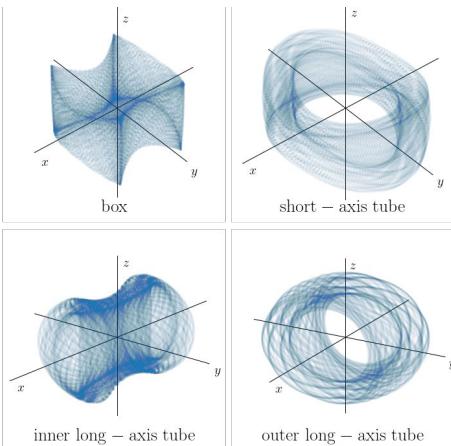
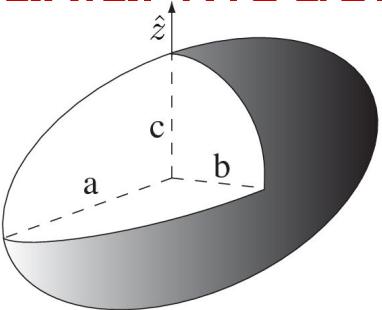
Triaxial Models



Schwarzschild Model

Flexibility:

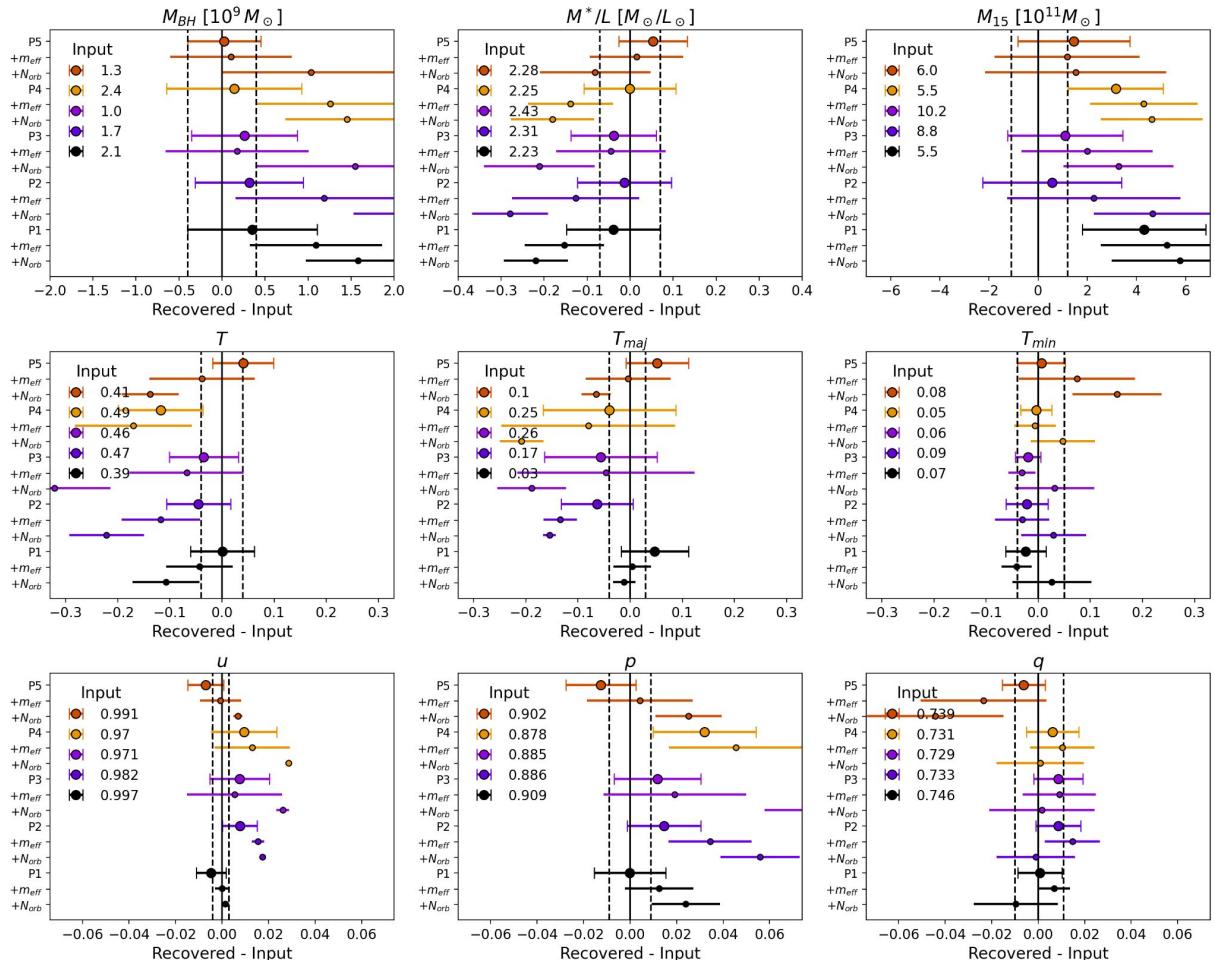
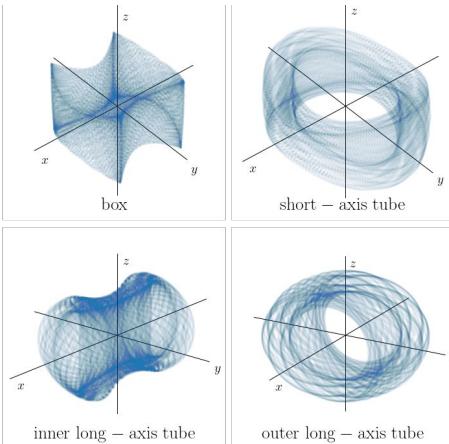
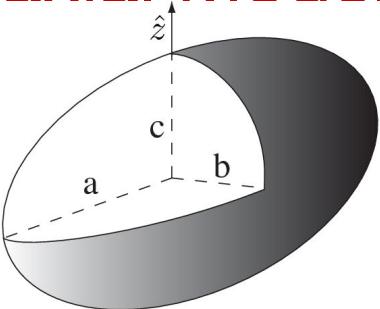
Triaxial Models



Schwarzschild Model

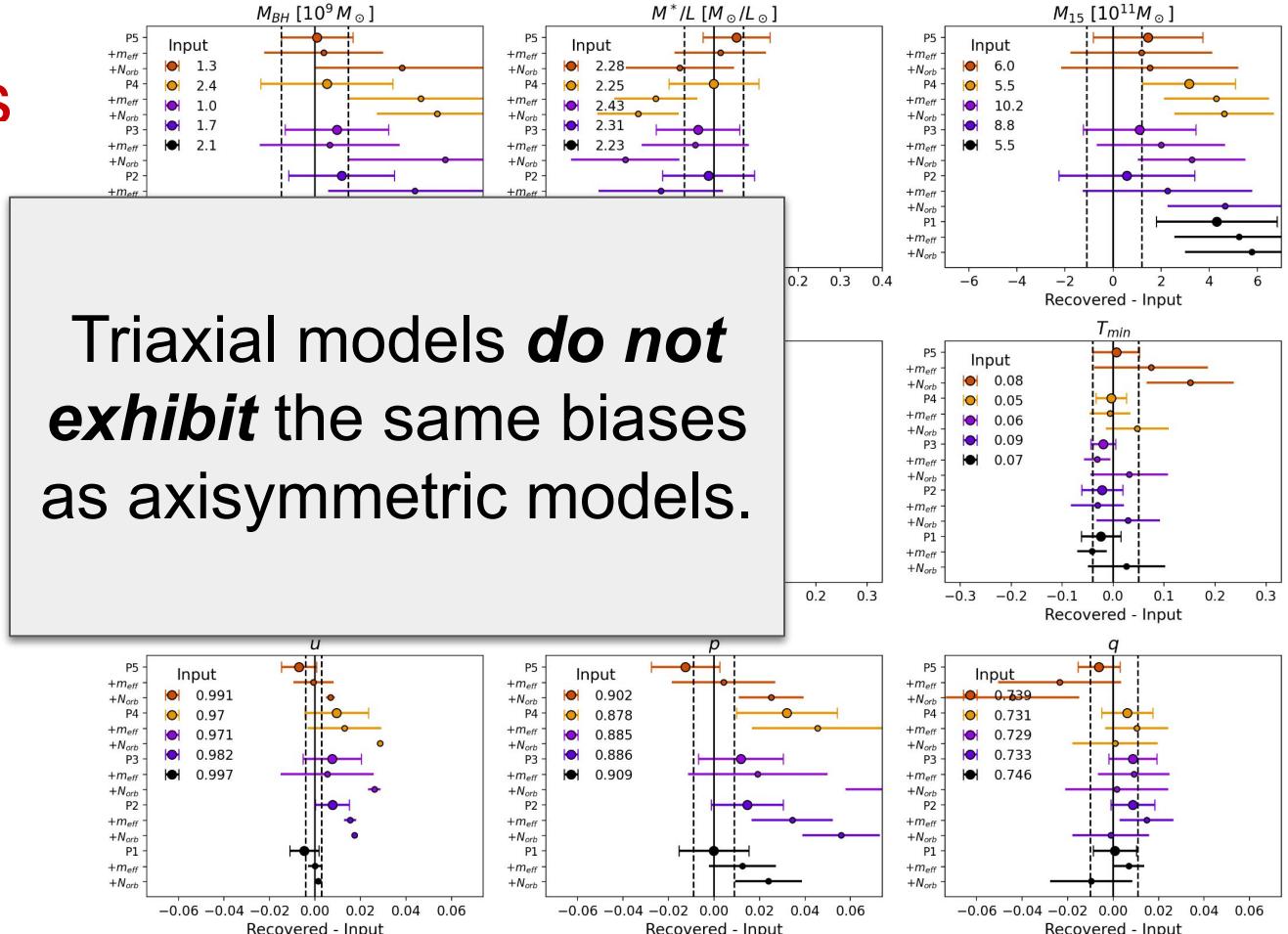
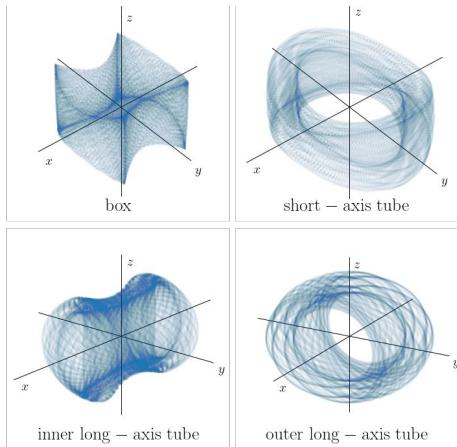
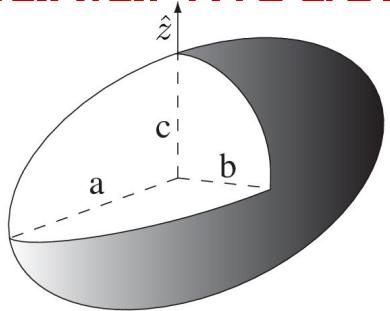
Flexibility:

Triaxial Models



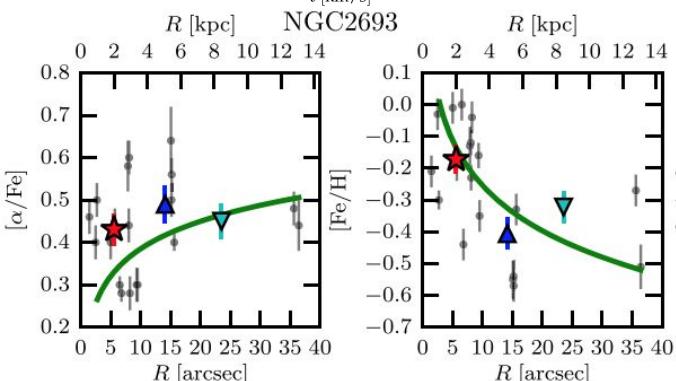
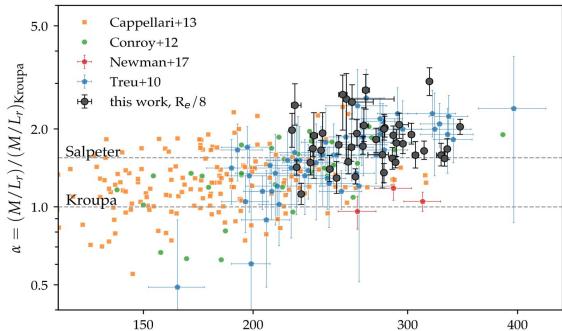
Schwarzschild Model

Flexibility: Triaxial Models



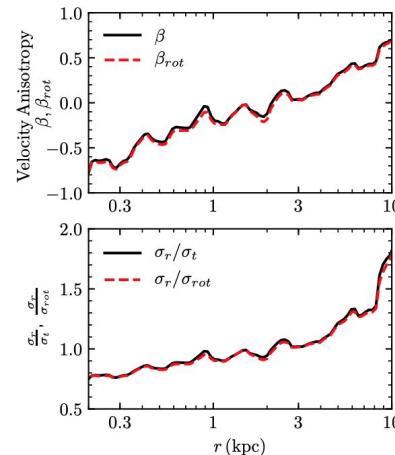
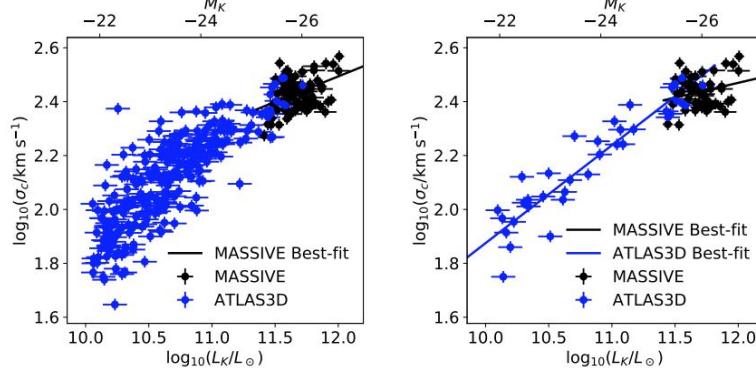
The MASSIVE Survey: Recent Studies

vibrations in the IMF in massive ellipticals (Gu+2022)



chemical abundance gradients/formation history (Greene+2019)

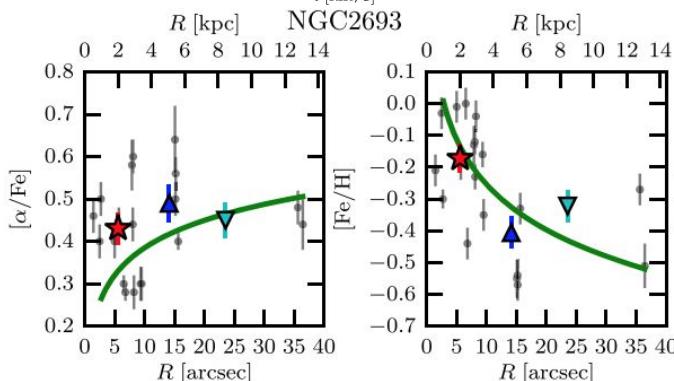
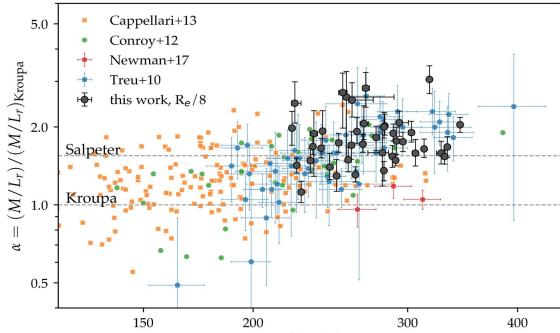
local scaling relations (Quenneville+2022)



dynamical modeling to constrain SMBH masses, shapes (Liepold+2020)

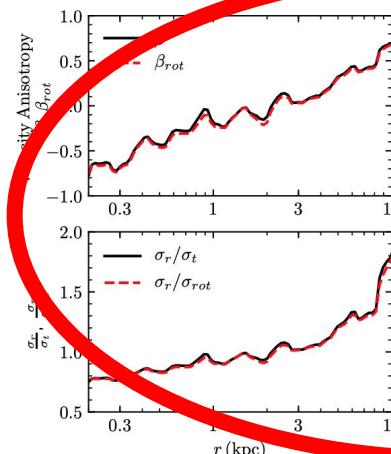
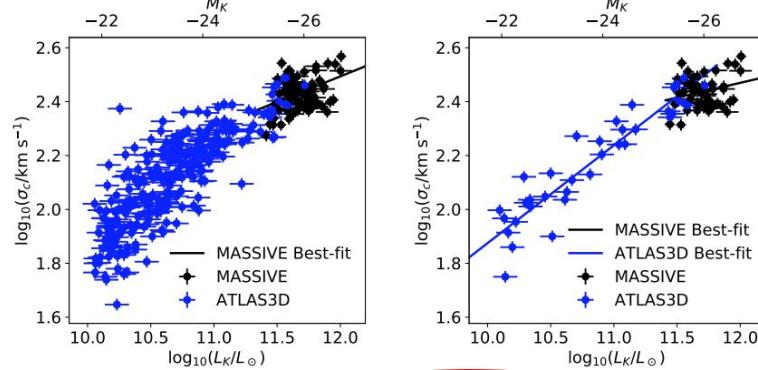
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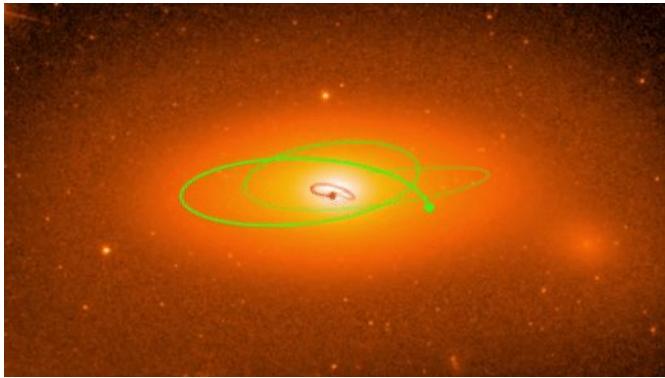


dynamical modeling to constrain SMBH masses, shapes (Liepold+2020)

stellar dynamics pt. 2

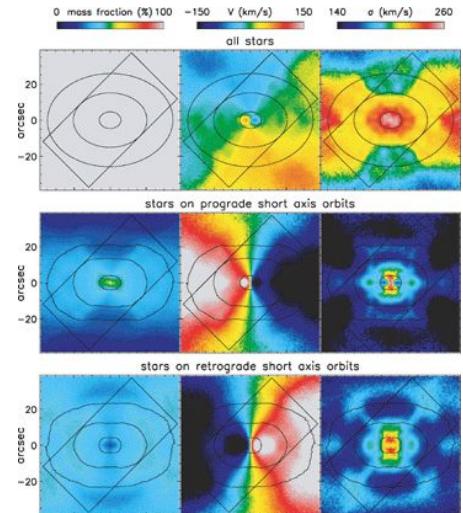
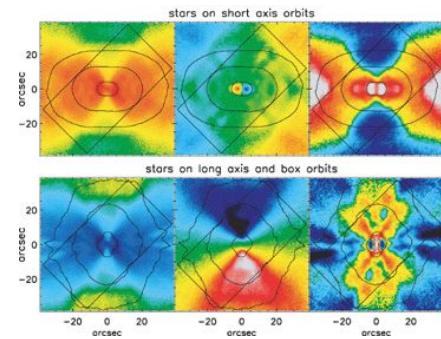
1. Choose a trial potential:

$$\text{Galaxy} = (\text{DM}) + (\text{STARS}) + (\text{BH}) + \dots$$



2. Integrate orbits in a given potential
– store “observations” (i.e., positions,
velocities of tracers)

3. Assign weights to orbits →
Reproduce kinematics

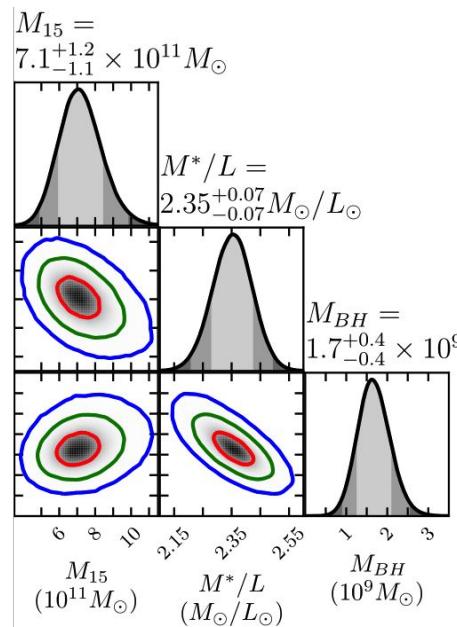


4. Repeat for many potentials
(BH, M/L, DM halo, galaxy
shape, etc...) and find best fit
to data.

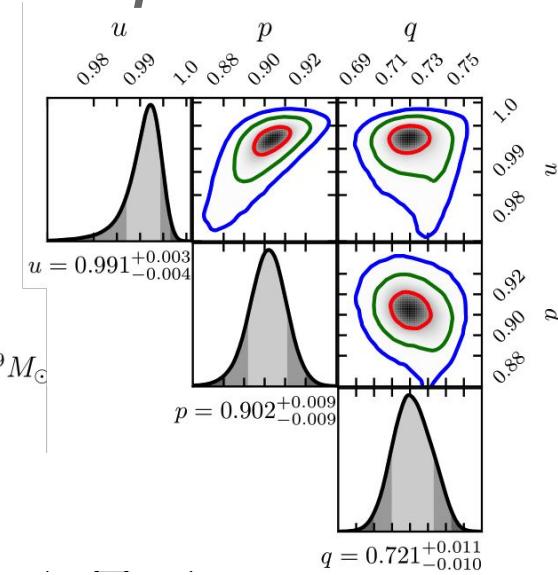
stellar dynamical modeling: applications to NGC 2693

- first simultaneous measurements of DM mass, BH mass, **and intrinsic shapes**
- **NGC 2693 has a moderately triaxial intrinsic shape,**
 - Intermediate-to-major axis ratio: $p = b/a \sim 0.9$
 - Minor-to-major axis ratio: $q = c/a \sim 0.7$
- **NGC 2693 $M_{BH} = (1.7 \pm 0.4) \times 10^9 M_{\odot}$**

*mass
parameters*



*shape
parameters*

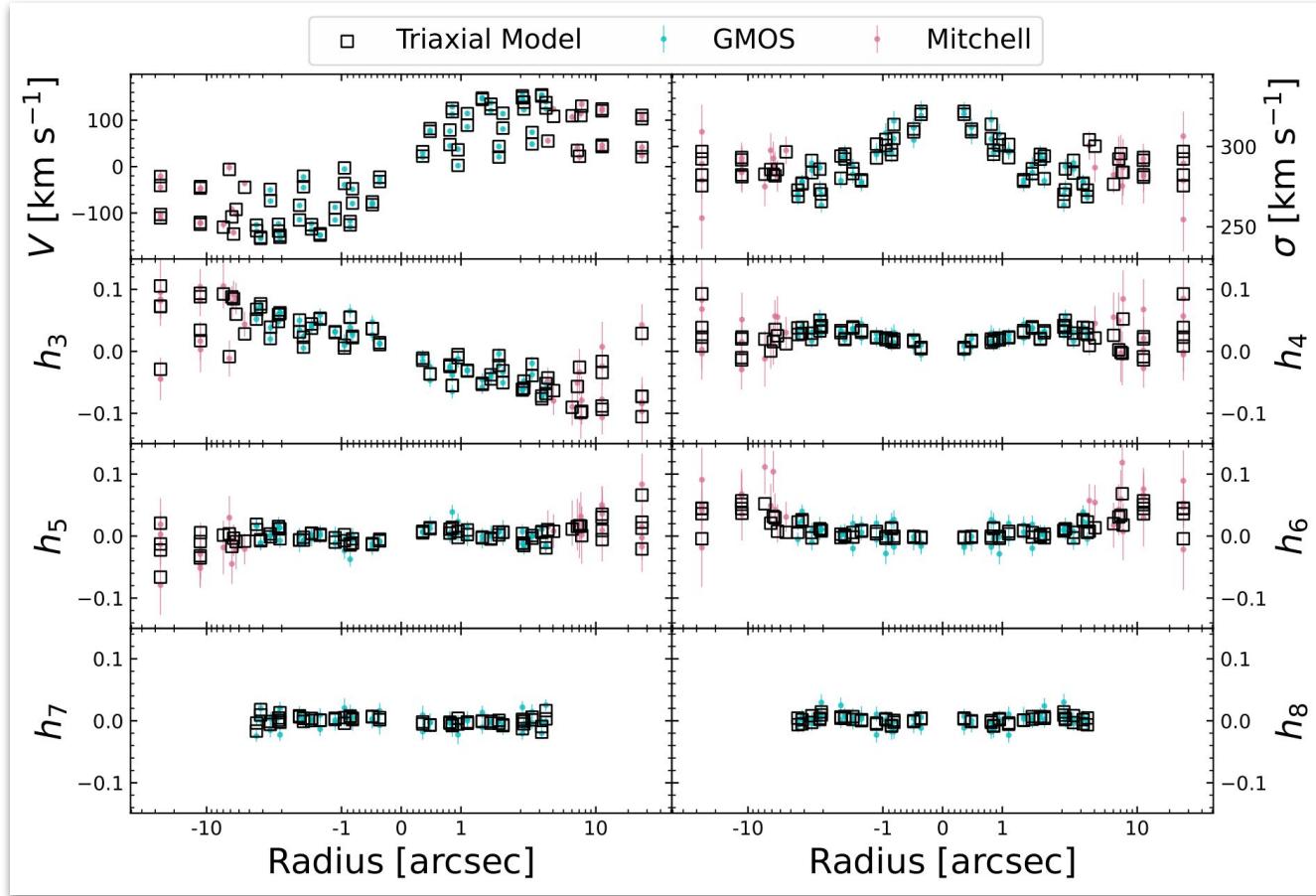


TRIAXIAL

AXISYMMETRIC

NGC 2693 BH: $(1.7 \pm 0.4) \times 10^9 M_{\odot} \rightarrow (2.4 \pm 0.6) \times 10^9 M_{\odot}$

modeling results: applications to NGC 2693



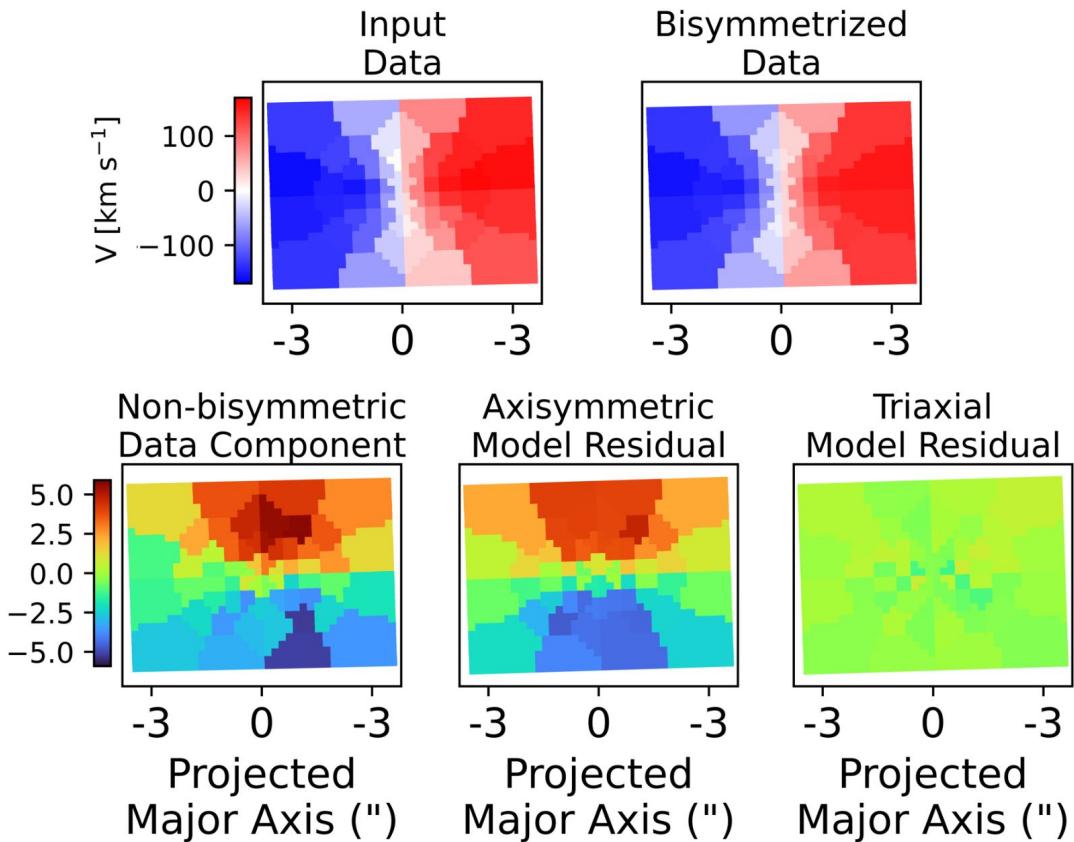
1''~350pc

axisymmetric vs. triaxial models

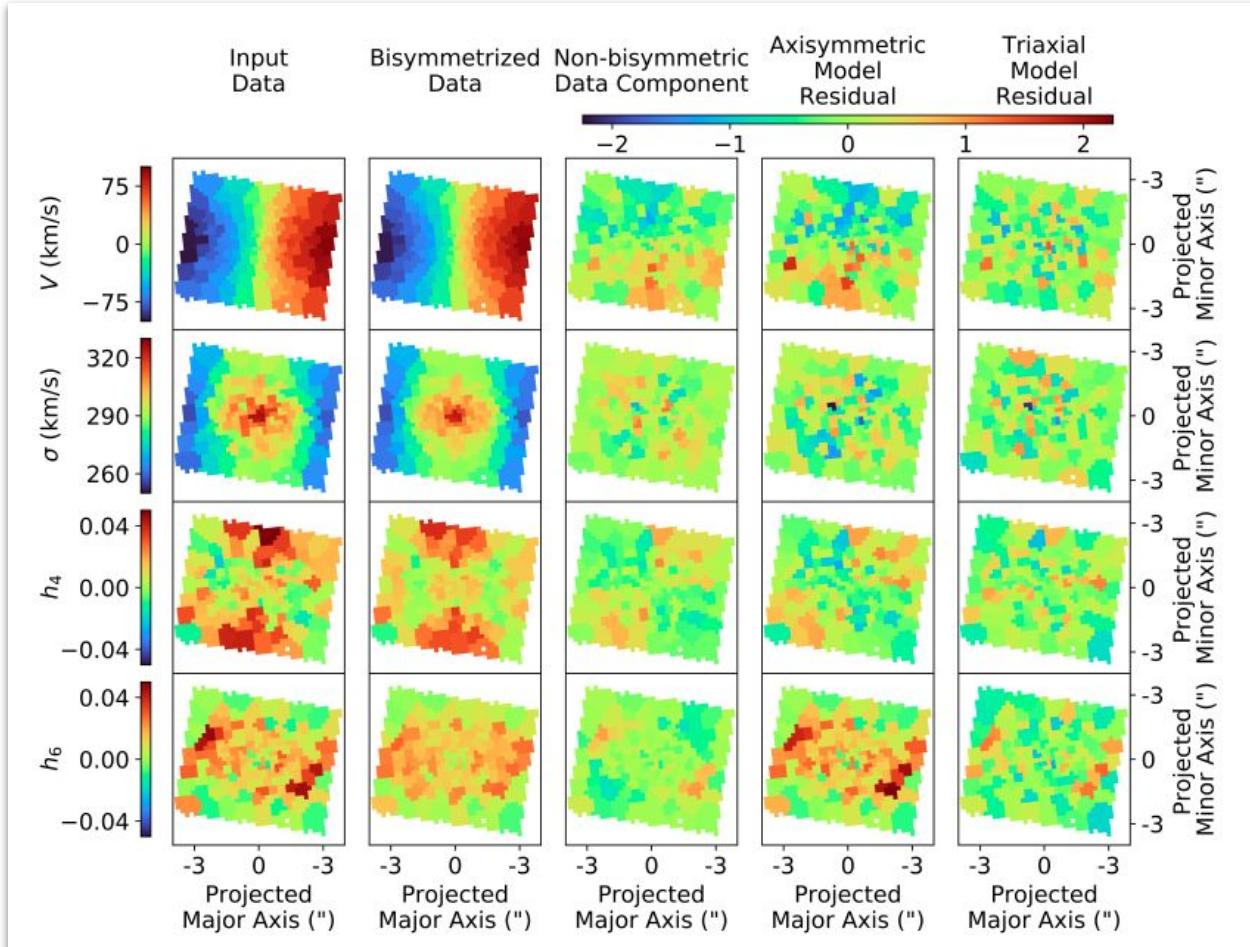
- triaxial model fits data better
- triaxial model also reproduces more complicated velocity structure



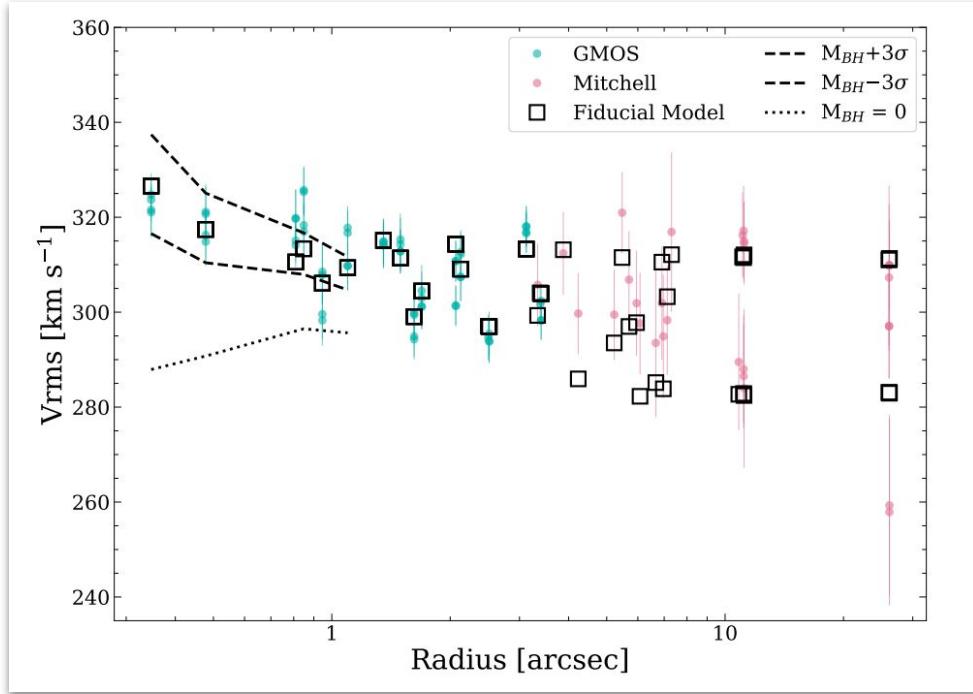
axisymmetric models
cannot reproduce
non-bisymmetric
velocity components



axisymmetric vs. triaxial models



Jeans Modeling



Galaxy Parameter	Triaxial Orbit Model	Axisymmetric Orbit Model	JAM Model
$M_{\text{BH}} [10^9 M_{\odot}]$	1.7 ± 0.4	2.4 ± 0.6	2.9 ± 0.3
$M^*/L [M_{\odot}/L_{\odot}]$	2.35 ± 0.07	2.27 ± 0.1	2.17 ± 0.03
$M_{15} [10^{11} M_{\odot}]$	$7.1^{+1.2}_{-1.1}$	7.9 ± 1.3	4.7 ± 0.2
β_z	See caption. [†]	See caption. [†]	0.07 ± 0.01
T	0.39 ± 0.04		
T_{maj}	$0.09^{+0.04}_{-0.03}$		
T_{min}	$0.17^{+0.04}_{-0.05}$		
u	$0.991^{+0.003}_{-0.004}$		
p	0.902 ± 0.009		
q	$0.721^{+0.011}_{-0.010}$		
$\theta (\text{°})$	66^{+4}_{-3}		
$\phi (\text{°})$	72 ± 3		
$\psi (\text{°})$	$93.0^{+0.7}_{-0.6}$		

Table 2. Summary of best-fit galaxy models for NGC 2693. For each parameter, we marginalize over the other dimensions and report the 1σ uncertainties. The axisymmetric orbit models and JAM models have fixed inclination of 70° . In orbit models, θ is the inclination angle in the oblate axisymmetric limit ($\psi = 90^\circ$, or equivalently $p = 1$), with $\theta = 90^\circ$ being edge-on and $\theta = 0^\circ$ being face-on. [†]We measure β_z in the orbit model as a function of radius, shown in the bottom panel of Figure 6. The best-fit JAM value of $\beta_z = 0.07 \pm 0.01$ is consistent with the range of β_z values measured from this best-fit model, with values ranging from $\beta_z = -0.27$ at small radii to $\beta_z = 0.28$ at large radii in both the triaxial and axisymmetric Schwarzschild models.

Schwarzschild Modeling Power

our typical scenario:

- (~a few hundred kinematic a)
* (~8 Gaussian moments per aperture)

~ 10^3 kinematic constraints

(a) how well can we recover a series of known inputs from our **triaxial** models?

(b) are there analogous biases in our triaxial modeling scheme?

typical Schwarzschild model:

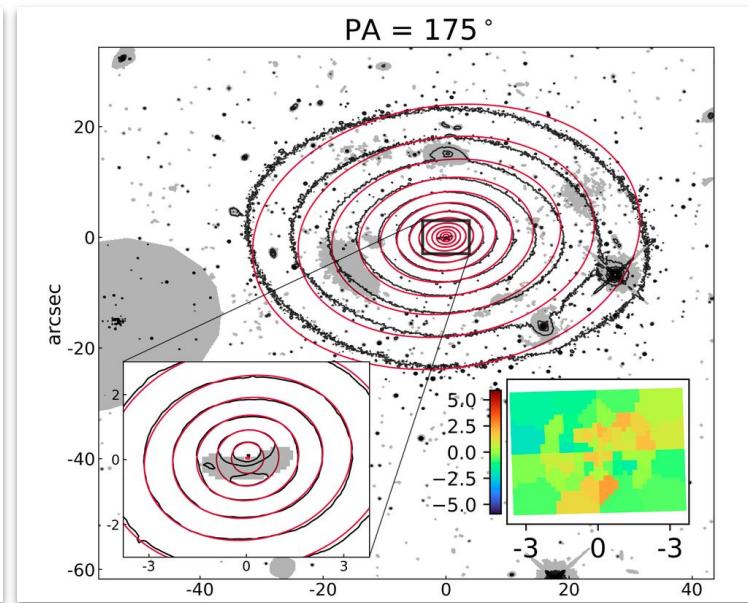
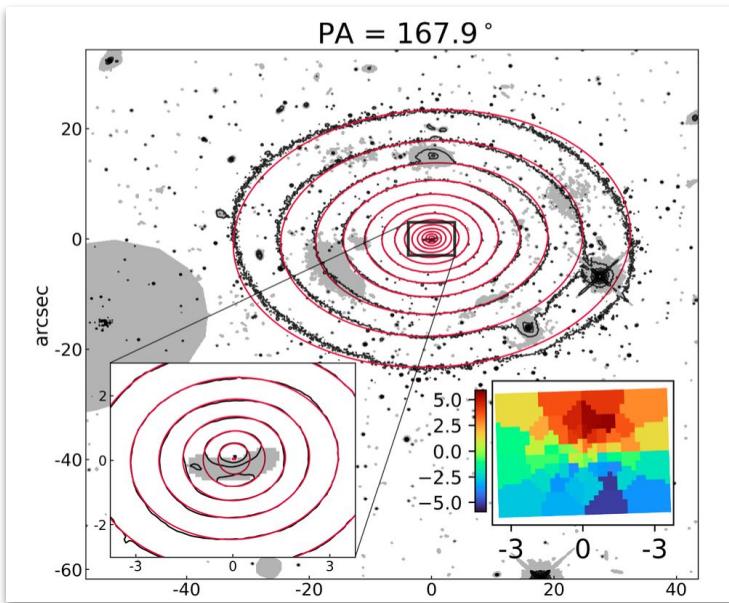
- $\geq 10^4$ orbits with a free weight parameter

If this were a simple linear model

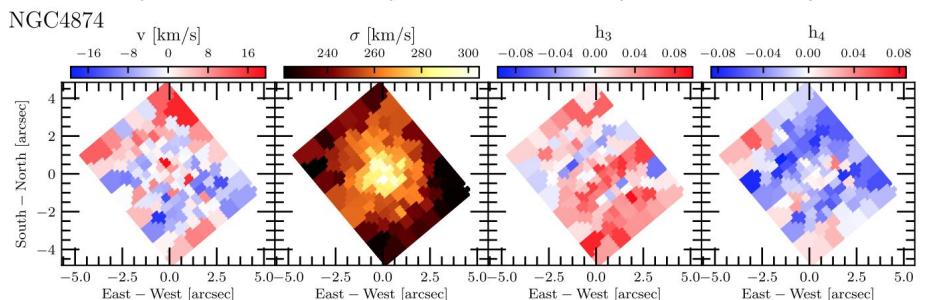
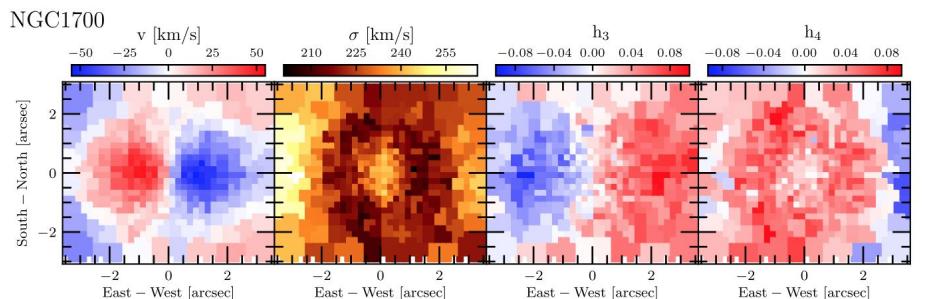
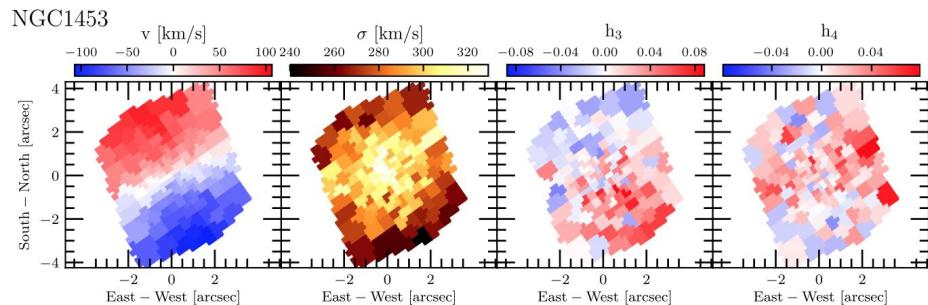
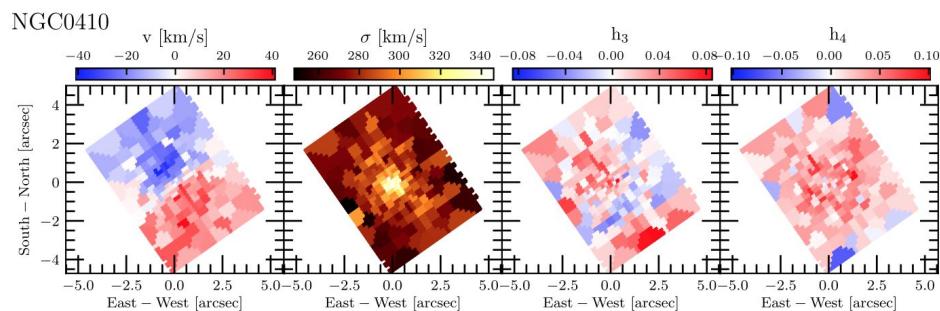
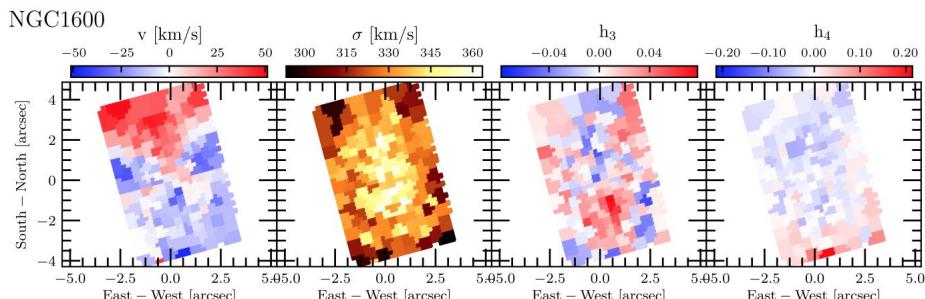
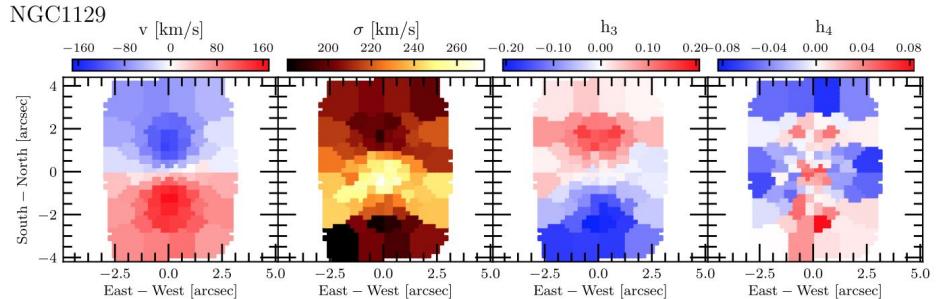
axisymmetric vs. triaxial models



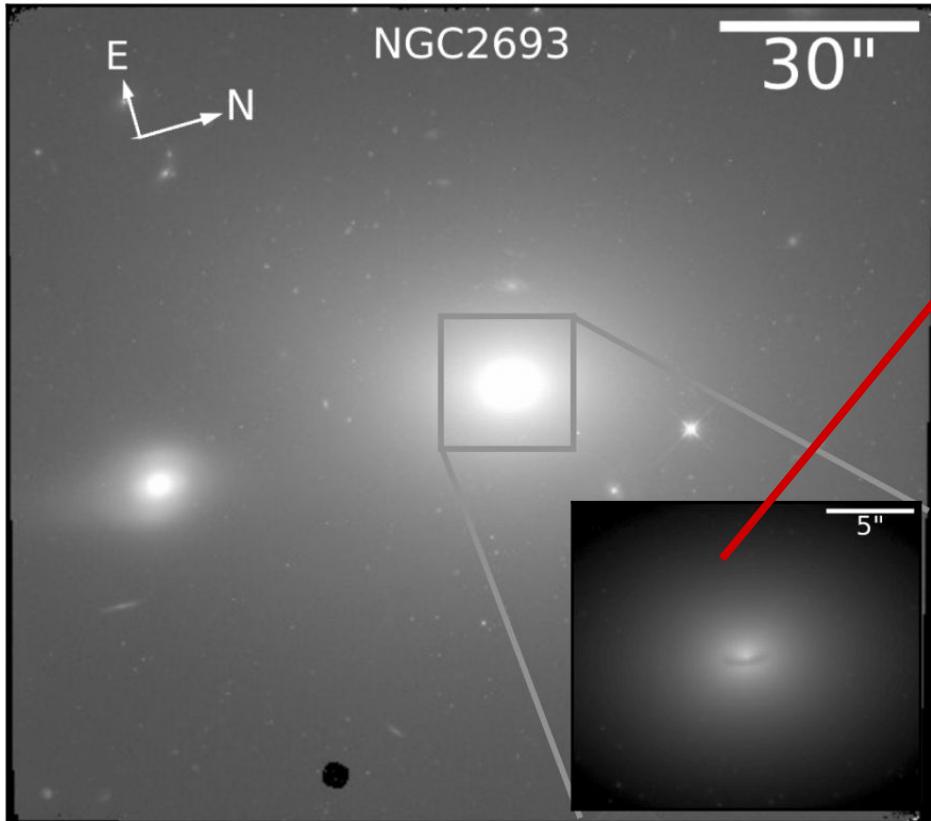
“rotating away” the non-bisymmetric component gives an inconsistent surface brightness profile



The MASSIVE Survey



a nice surprise:



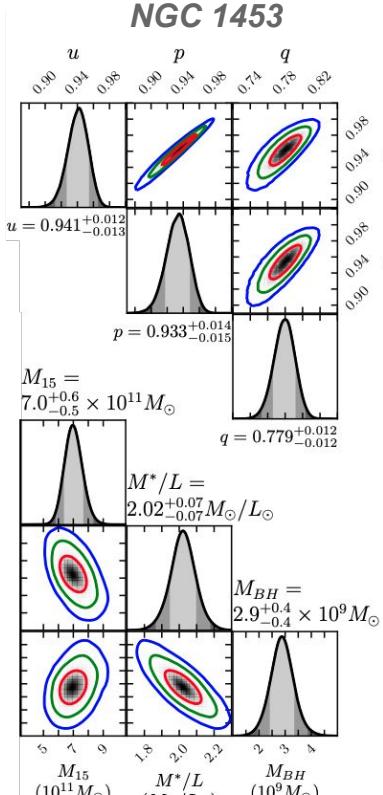
dust disk
inclination:

$\theta \sim 70$ degrees

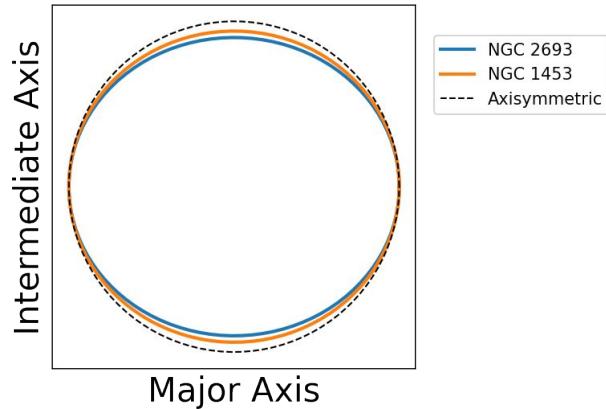
“inclination”
from the triaxial
model:

$$\theta = 66^{+4}_{-3} \text{ degrees}$$

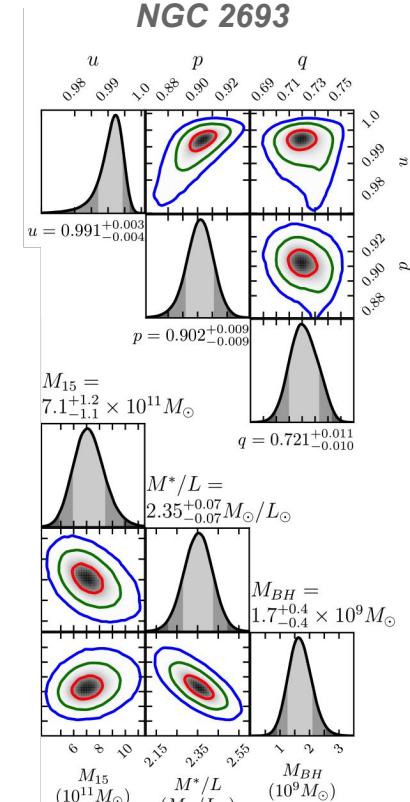
stellar dynamical modeling: applications to NGC 1453 and 2693



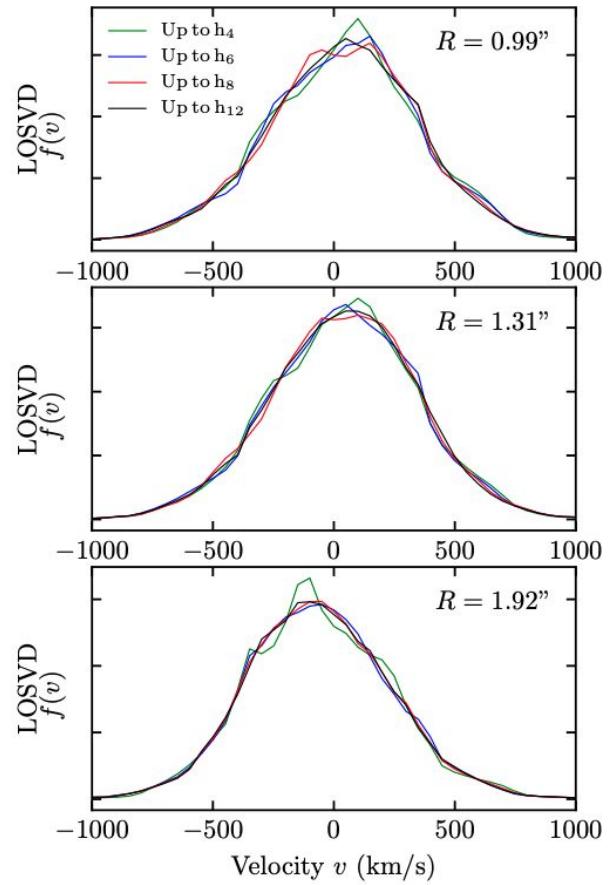
- first simultaneous measurements of DM halo, BH mass, and intrinsic shapes
- neither galaxy is axisymmetric, despite properties suggesting so



- NGC 1453 BH: $2.9 \times 10^9 M_\odot$
- NGC 2693 BH: $1.7 \times 10^9 M_\odot$



LOSVDs



why else are SMBHs *important*?

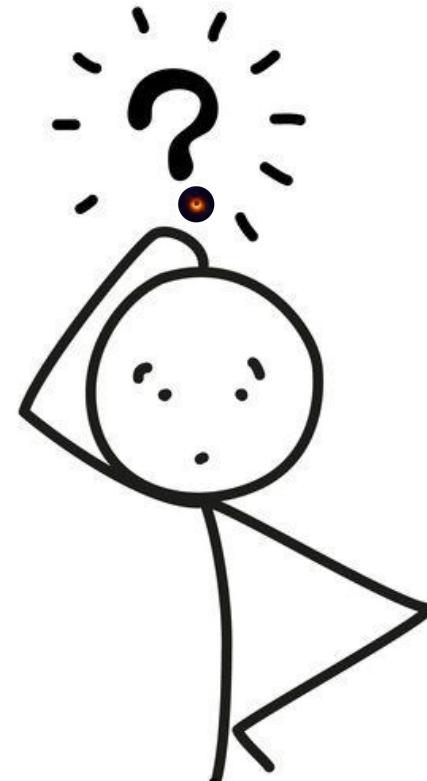
1. Estimate BH mass from galaxy properties
where we can't resolve SOI

2. Cross-checking of gas dynamics, mega-maser disks, and reverberation mapping BH masses
[i.e., Yu+2019 (reverberation mapping), Thater+21 (gas dynamics)]

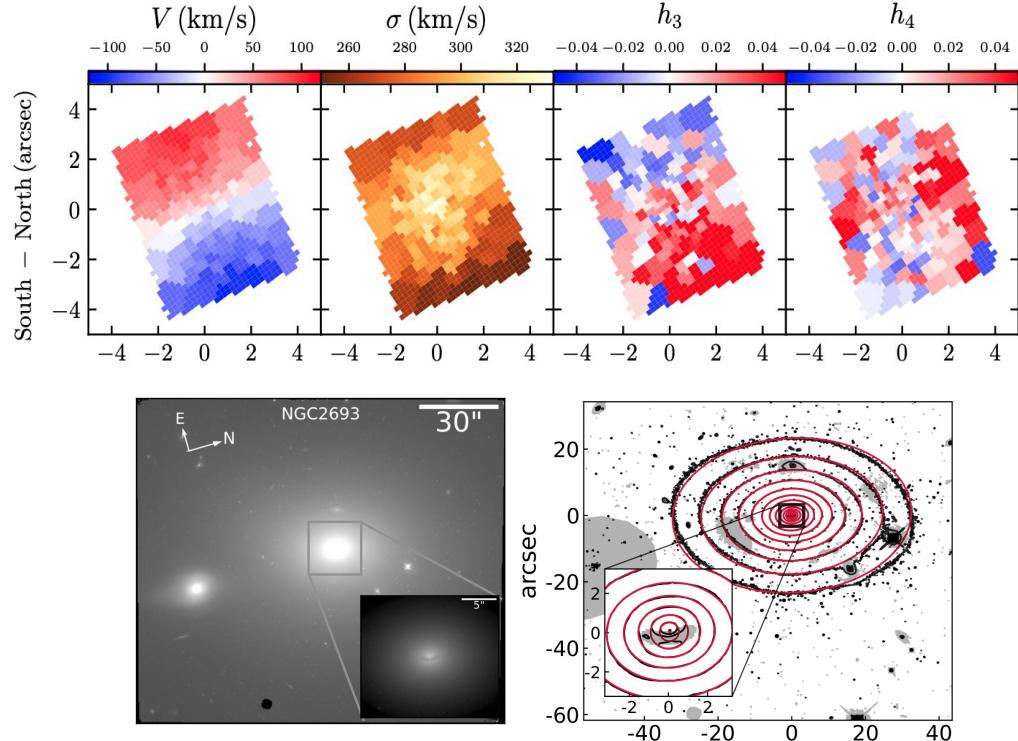
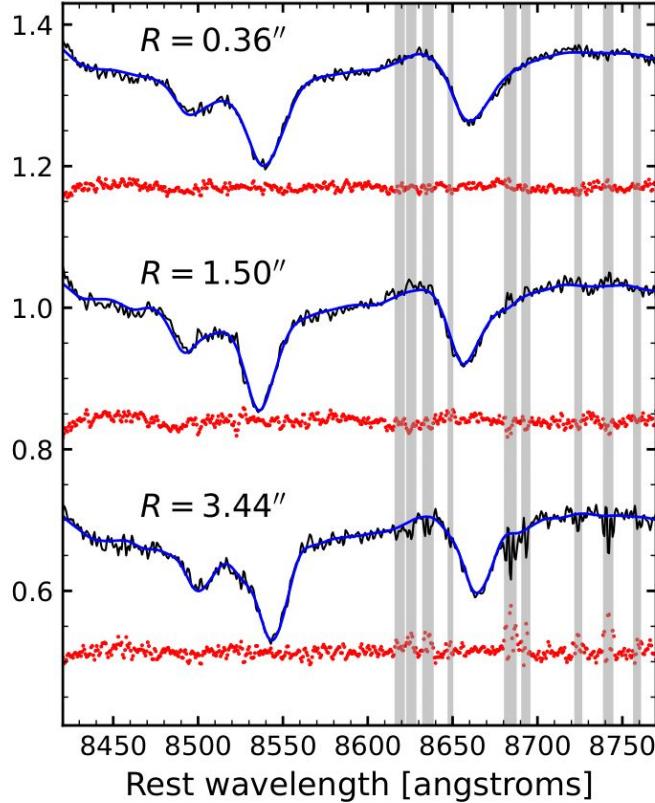
3. $z \sim 0$ mass function/number density → predictions
for long- λ gravitational wave signal from Pulsar
Timing Arrays/LISA
[i.e., Shannon+2015, Arzoumanian+2019]

4. Comparison to simulations of AGN feedback
modes and mechanisms
[i.e., Li+2019, Habouzit+2020]

...



stellar dynamical modeling: applications to NGC 1453 and 2693



+ ~10,000 individual
galaxy models

Nested Sampling in A Single Slide

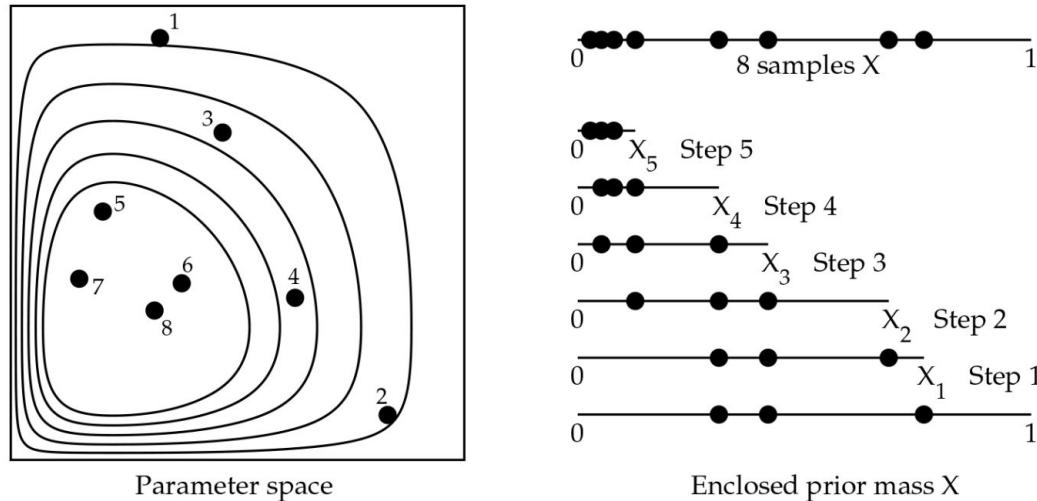


Figure 4: Nested sampling for five steps with a collection of three points. Likelihood contours shrink by factors $\exp(-1/3)$ in area and are roughly followed by successive sample points.

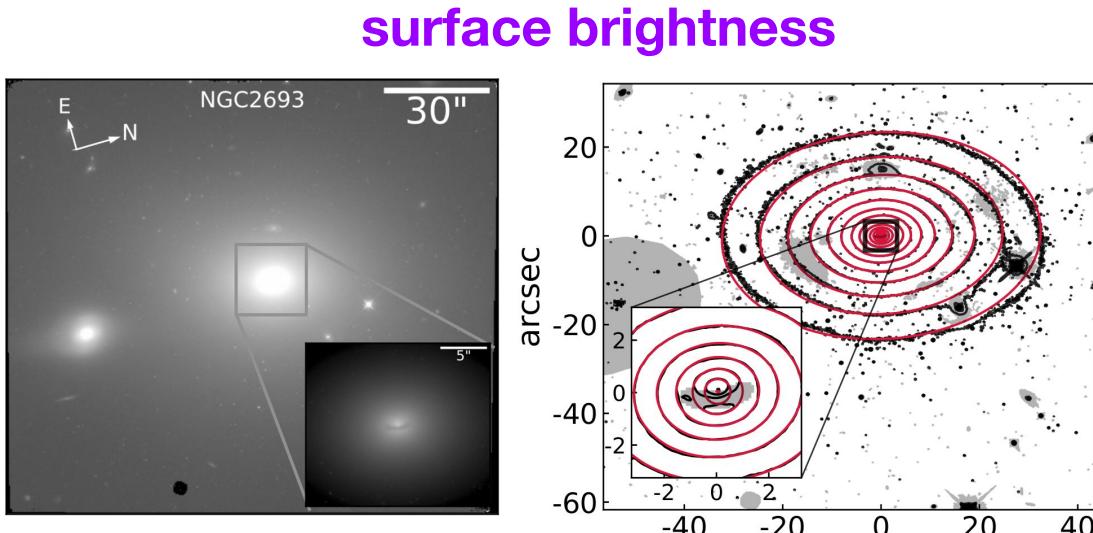
*J. Skilling 2006

idea : iteratively sample points, getting rid of lowest likelihood at each step → volume shrinks to maxima of distributions

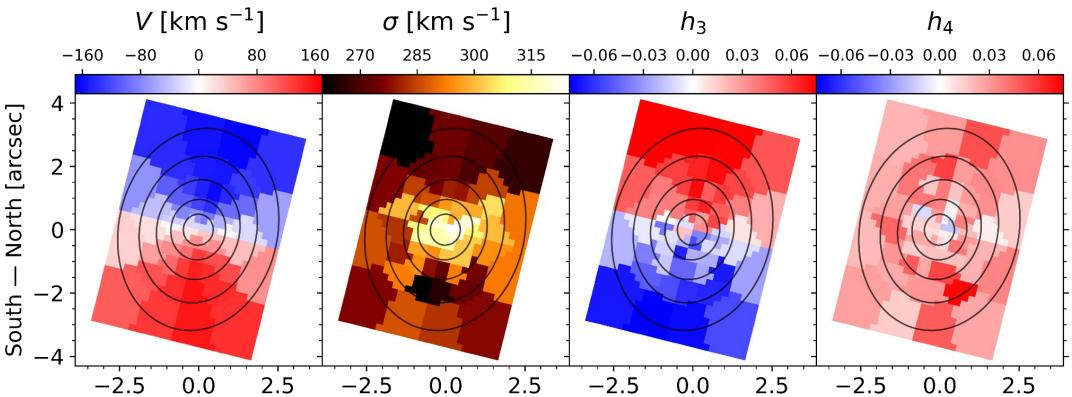
stellar dynamical models

CENTRAL IDEA:

given kinematics + surface brightness of NGC 2693,
can we determine the galaxy's mass components
by integrating the orbits of stars in a gravitational potential?



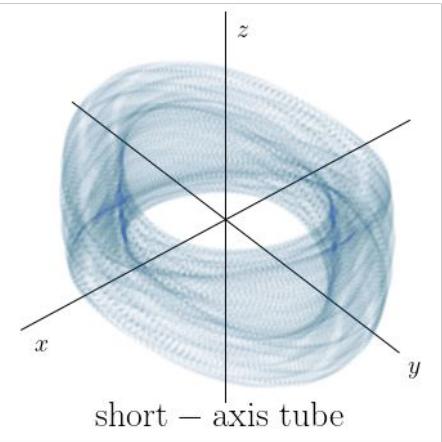
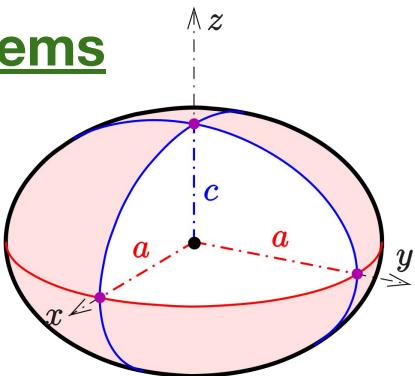
line of sight velocity distribution



triaxial dynamical modeling

axisymmetric systems

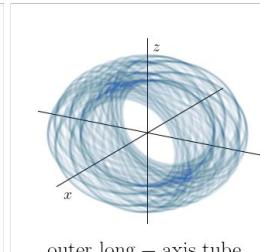
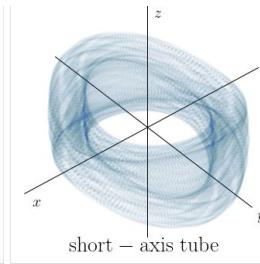
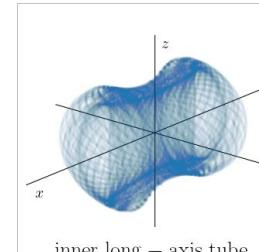
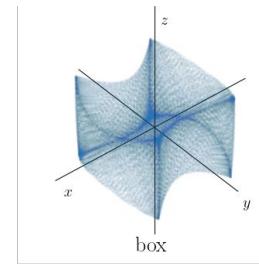
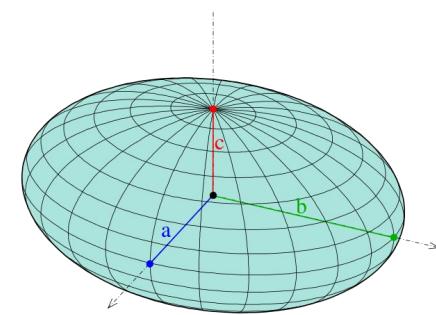
- 4 parameters:
 - BH
 - M/L
 - DM Halo
 - Inclination



important:
axisymmetric
systems are, by
construction,
bisymmetric

triaxial systems

- 6 parameters:
 - BH
 - M/L
 - DM Halo
 - 3 Shape Parameters

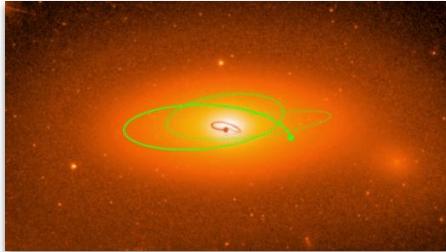


triaxial dynamical modeling

1. Choose a trial potential:

$$\text{Galaxy} = (\text{DM}) + (\text{STARS}) + (\text{BH}) + \dots$$

2. Generate stellar orbits in trial potential

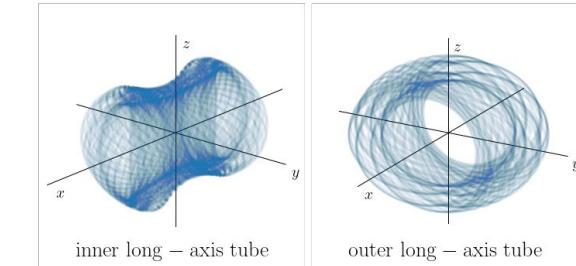
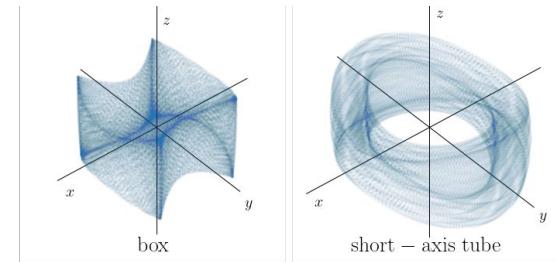
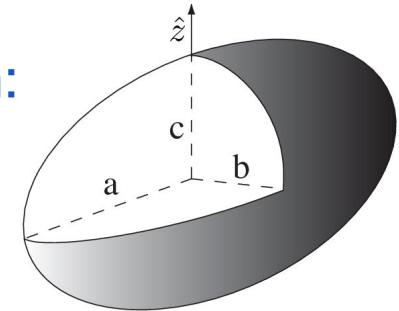


3. Determine which orbits most accurately reproduce **kinematics** + **photometry** for a *single* trial potential

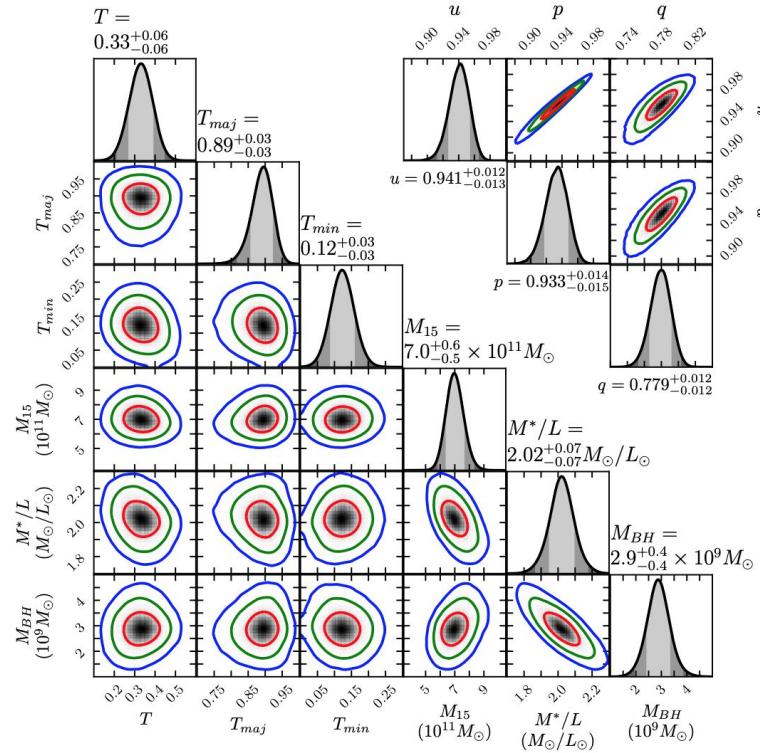
4. Find which assumed potential fits **kinematics** + **photometry** best **across trial potentials (BH, ML, Shapes)**

triaxial systems

- 6 knobs to turn:
 - BH
 - M/L
 - DM Halo
 - 3 Shape Parameters

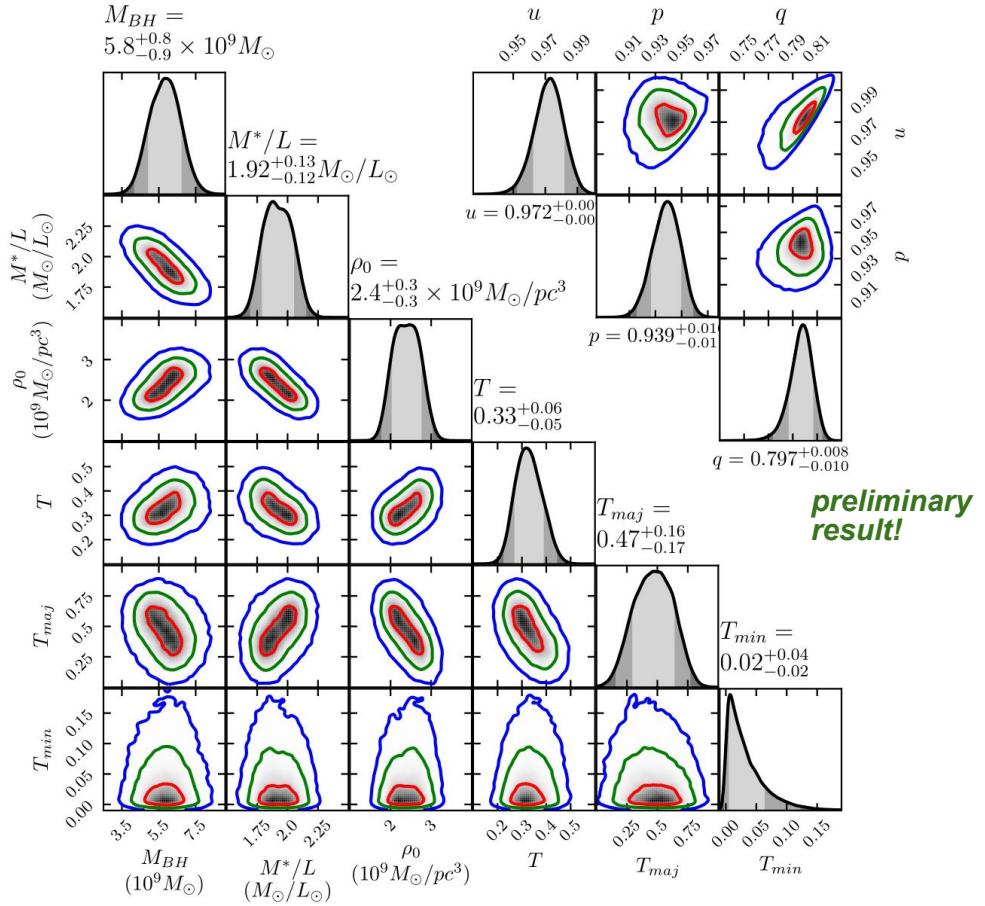
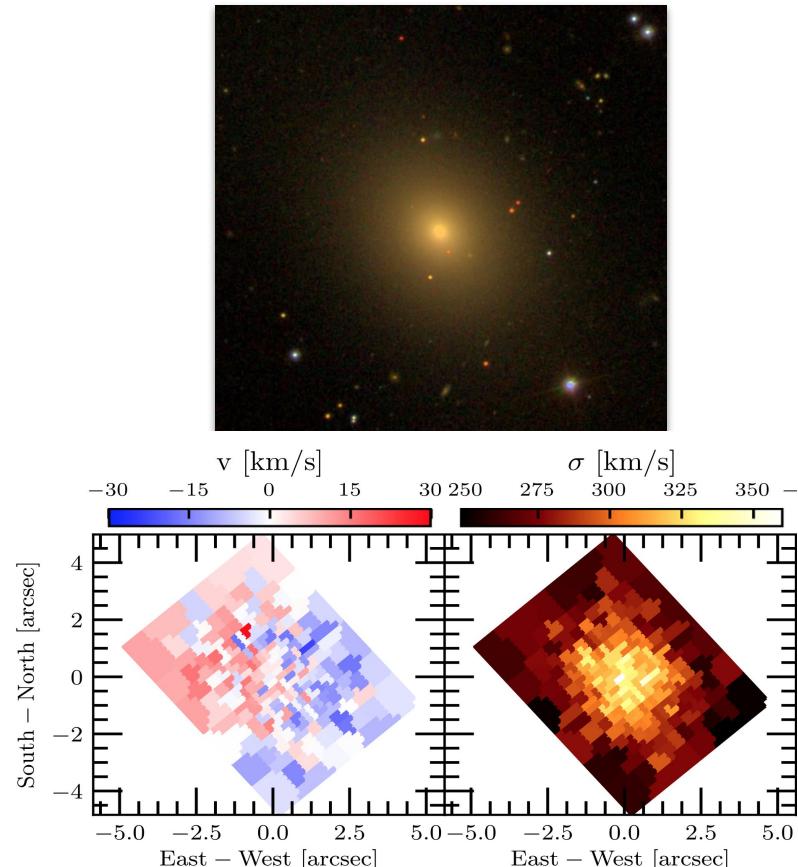


stellar dynamical modeling: other applied cases



stellar dynamical modeling: ongoing work

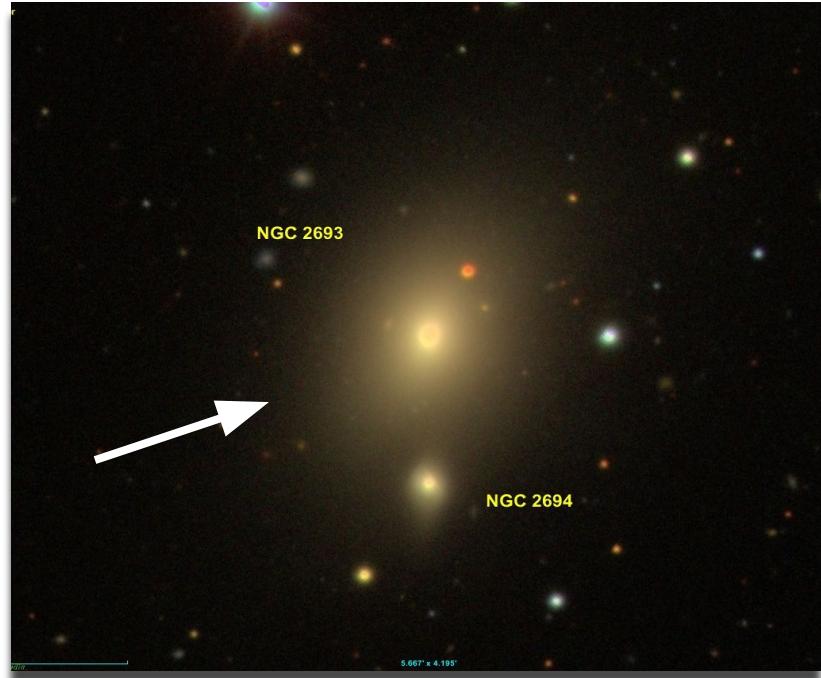
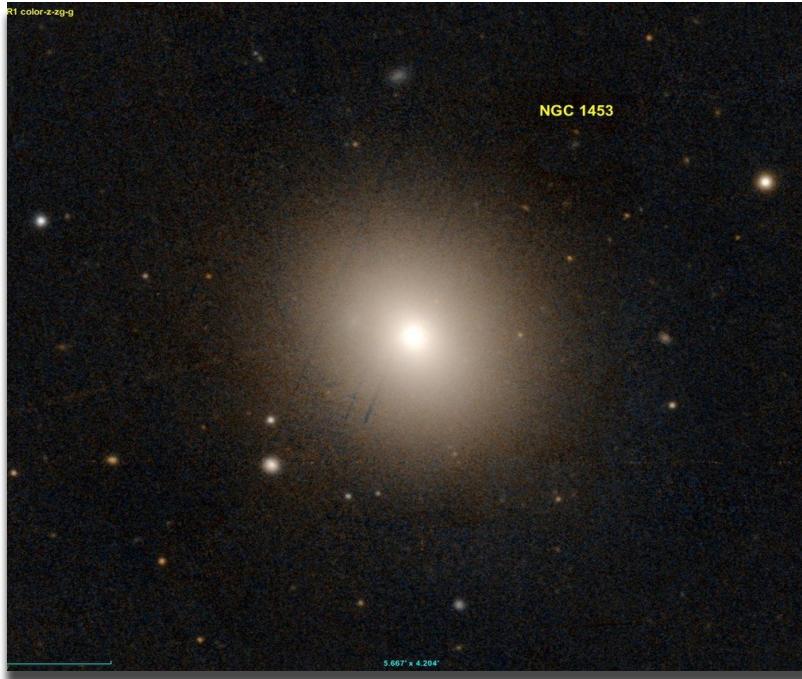
NGC 57



Triaxial Result: $M_{BH} \sim 6 \times 10^9 M_{\text{sun}}$,

- Another simultaneous measurement of mass components + intrinsic shape of a massive elliptical

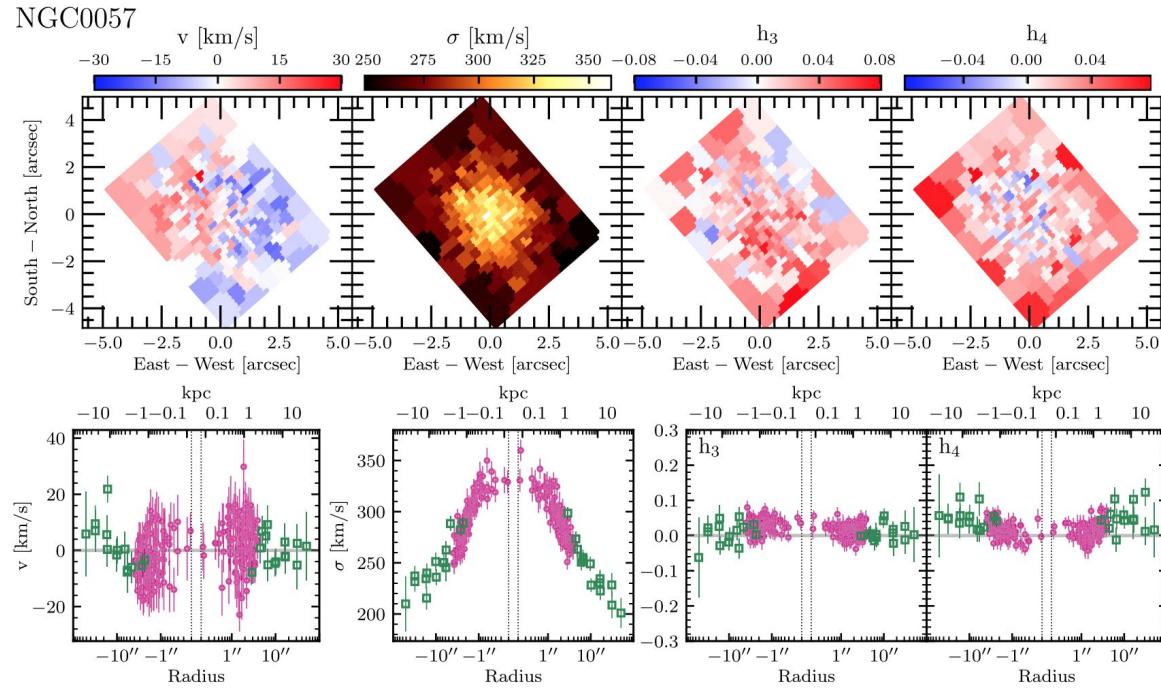
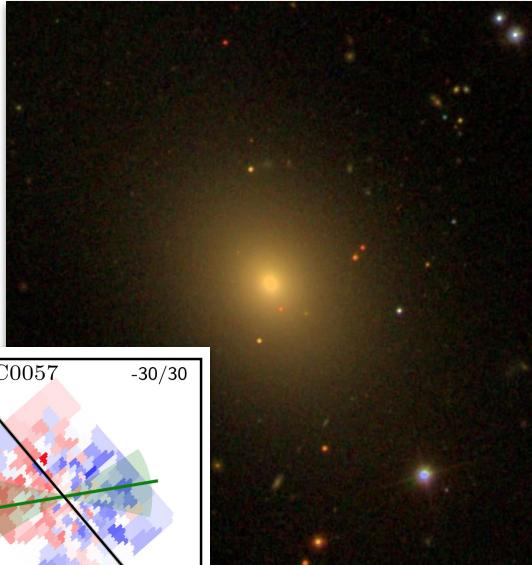
stellar kinematics of NGC 1453 and 2693



SDSS DR9 Images

stellar dynamical modeling: ongoing work

NGC 57

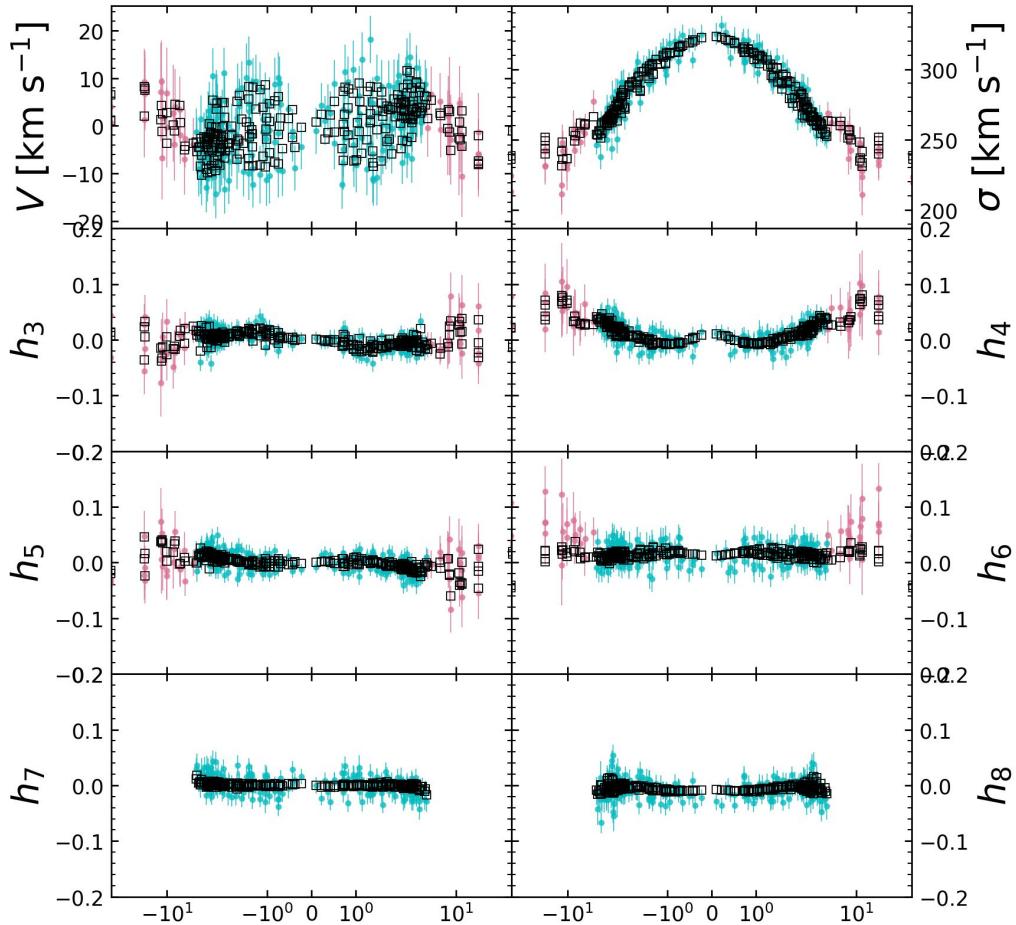
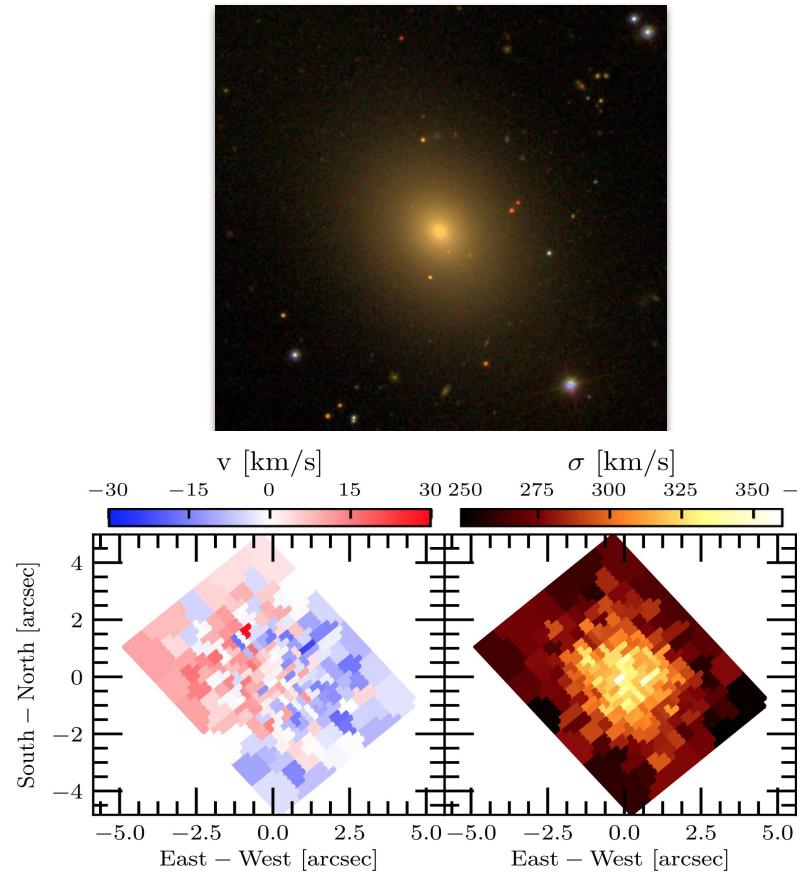


- Slow rotation (~ 25 km/s),
kinematically and
photometrically aligned

stellar dynamical modeling: ongoing work

□ Triaxial Model ▪ Mitchell ▲ GMOS

NGC 57

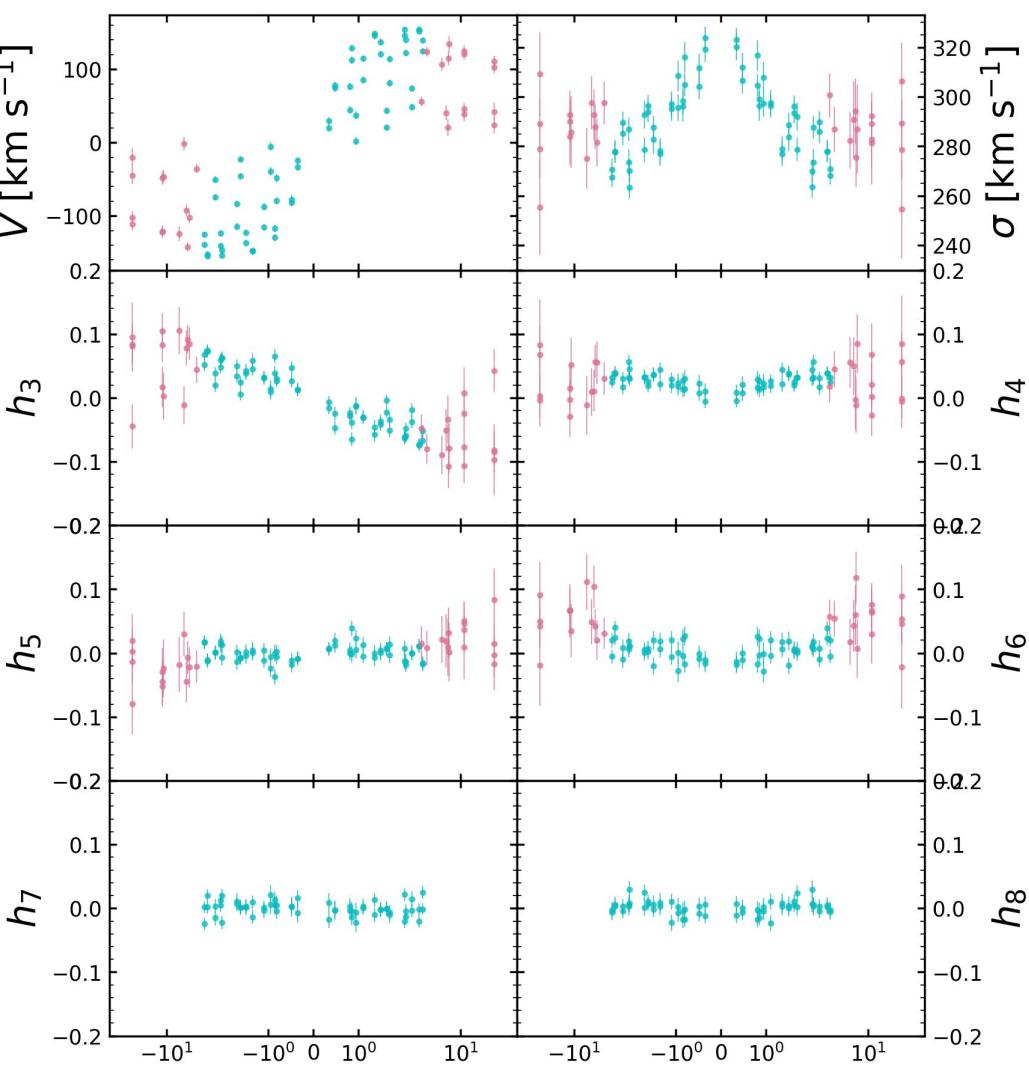


Transition to Mock Results

- Mention contrasting axisymmetric Mbh/difference with triaxial might be because of modeling differences, with inclination as the example case of parameter bias – could this be the underlying difference between recovered mass parameters?
 - Need to be a bit more elaborate on axisymmetric modeling if this is the case. Currently only describe triaxial modeling
- Motivate this with an example of fits with different numbers of d.o.f to show the difference (something simple), and a primer on m_{eff}
- Our machinery seems to reliably recover the inputs, and that our parameter recovery are not strongly dependent on the degrees of freedom in our models
- Additionally have started to investigate more interpretable measurements of model flexibility for schwarzchild models,

Schwarzschild Model Flexibility: Axisymmetric Models

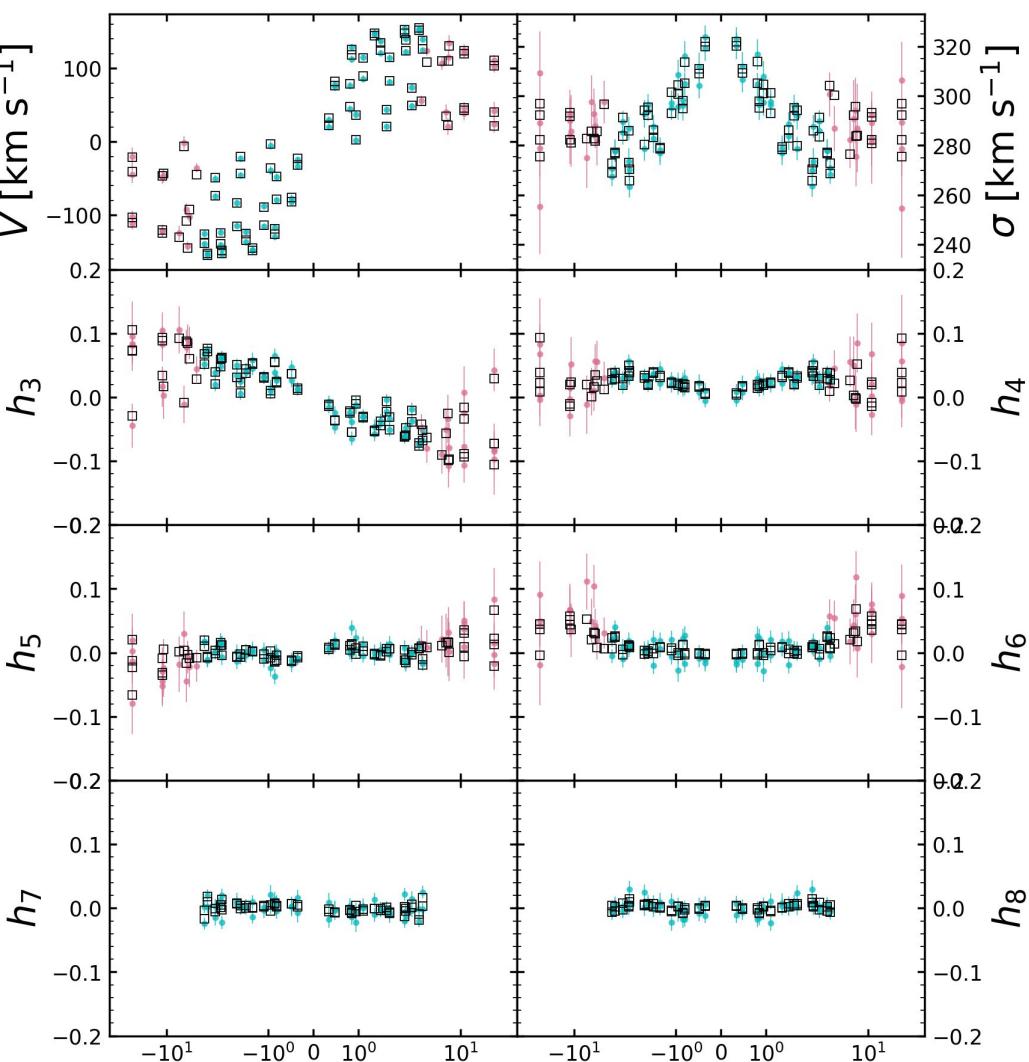
consider the following scenario:



Schwarzschild Model Flexibility: Axisymmetric Models

consider the following scenario:

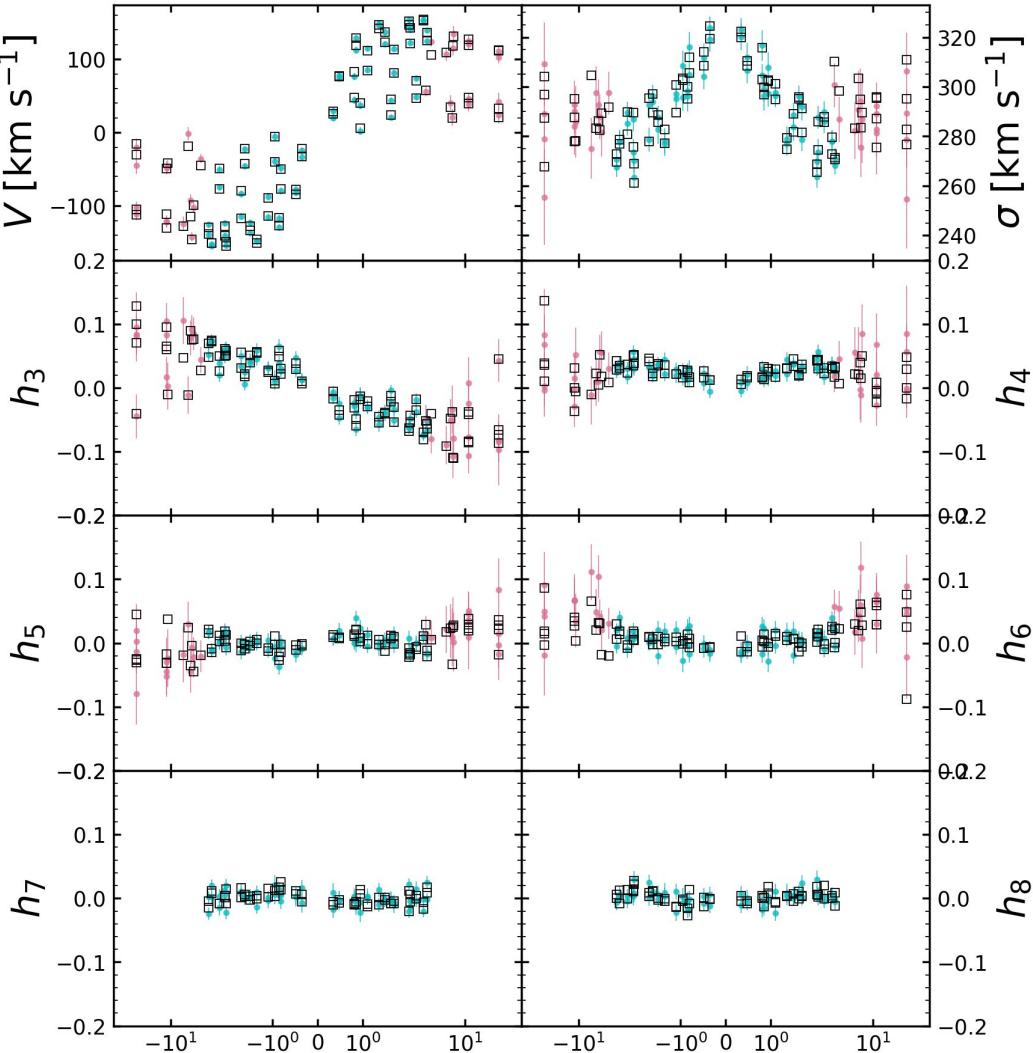
- We fit a Schwarzschild model to NGC 2693's data with the measured M_{BH} , M/L, DM halo, and $i = 70^\circ$



Schwarzschild Model Flexibility: Axisymmetric Models

consider the following scenario:

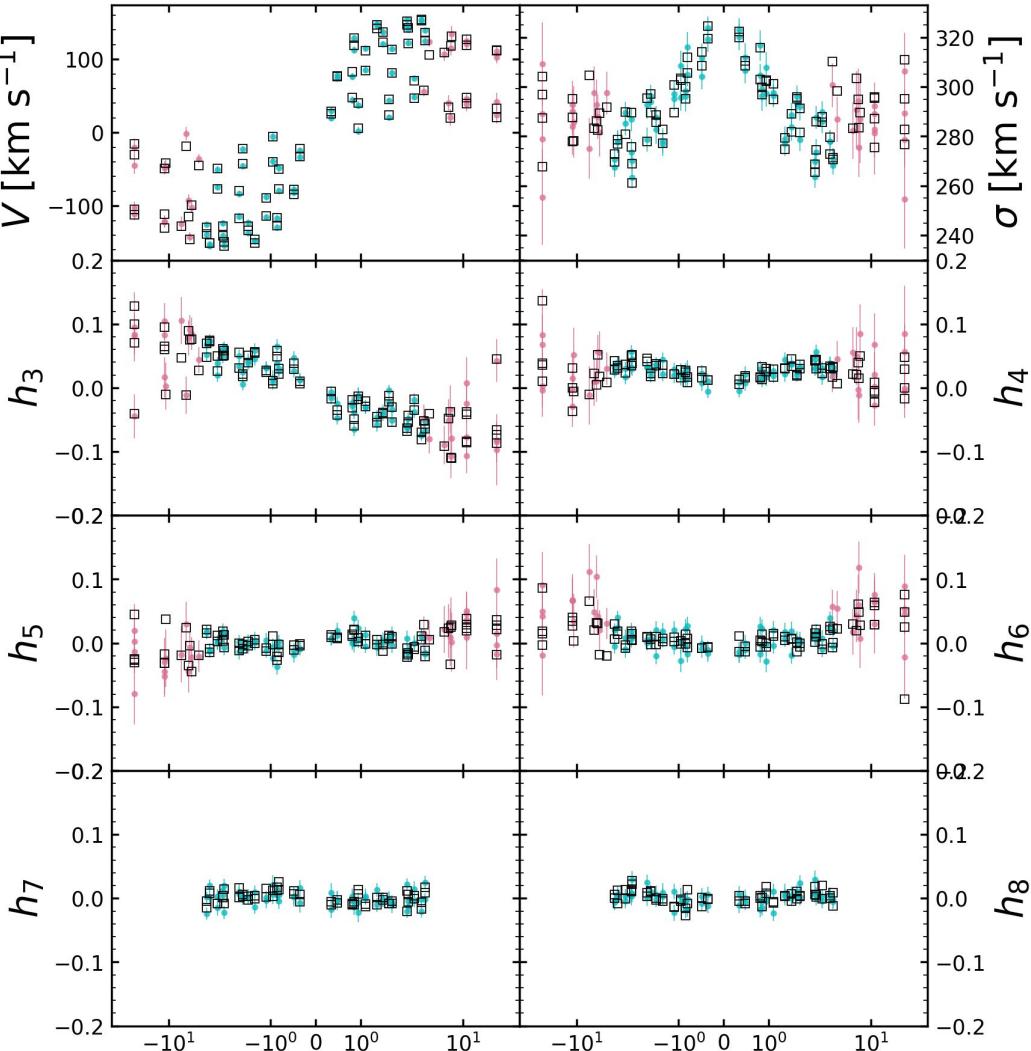
- We fit a Schwarzschild model to NGC 2693's data with the measured M_{BH} , M/L, DM halo, and $i = 70^\circ$
- Perturb the model fit by the measurement uncertainty



Schwarzschild Model Flexibility: Axisymmetric Models

consider the following scenario:

- We fit a Schwarzschild model to NGC 2693's data with the measured M_{BH} , M/L, DM halo, and $i = 70^\circ$
- Perturb the model fit by the measurement uncertainty
- Feed this as “input” to our code architecture and try to recover the correct inputs



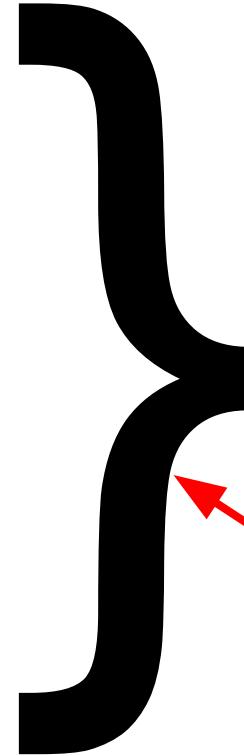
Schwarzschild Modeling Power

our typical scenario:

- (~a few hundred kinematic apertures)
* (~8 Gauss-Hermite moments per aperture)
- ~ 10^3 kinematic constraints

typical schwarzschild model:

- $\geq 10^4$ orbits with a free weight parameter



$$\chi_k^2 \rightarrow 0$$

k (degrees of freedom)
= (# data points) - (# params)
= negative

if this were a simple linear model

Schwarzschild Model Flexibility (pt 2)

motivating why we might expect this in axisymmetric models:

- both prograde and retrograde orbits are included in our orbits libraries, which have opposite signs and contribute maximally to the LOSVD in edge-on orientations; this in turn gives a higher effective degrees of freedom in these high i models
- In other words, they're more **flexible** as you move toward $i \rightarrow 90^\circ$
- Solution: apply a penalty which quantifies the flexibility of these models (have a slide motivating a penalty like $m_{\text{eff}}/\text{Norb}$, show results of penalizing the models as such)

Schwarzschild Model Flexibility (pt 3)

Triaxial models and current work:

- Currently evaluating if there is a similar bias in triaxial models which don't have the same symmetries in the orbits, plus have different families of orbits entirely
- Initial results seems to suggest that we don't need a penalty term for our triaxial models, and that we can accurately recover the parameters associated with a set of input kinematics