

Stellar Dynamics and Galaxies

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1 August 25, 2021: Introduction and Course Overview

The major themes and sections of the class are:

- General properties of the galaxy population
 - Classification
 - Observations
 - Physical properties
 - Correlations
 - Statistical properties.
- Key ingredients
 - Stellar populations
 - Star formation
 - Gas and dust (ISM/CGM/IGM)
 - Chemical evolution and baryon cycle
 - Dark matter halos
- Elliptical and Disk Galaxies
 - Structures
 - Dynamics
 - Scaling relations
 - Maybe dwarf galaxies
- Nuclear activity and black holes
 - Different AGN
 - Relations between BHs and host galaxies
 - Black hole accretion histories
- Galaxy interactions and environment
 - Galaxy Interactions
 - Clusters
 - Environment
 - Large Scale Structure
 - Galaxy clustering
- First galaxies and galaxy evolution
 - Observing over cosmic time
 - First galaxies
 - Cosmic star formation history
 - Assemblt history
 - Evolution models
 - Formation models and simulations

We now turn our attention to the **Milky Way**.

1.1 The Milky Way

- Disk
 - Components
 - * $M_\star \sim 6 \times 10^{10} M_\odot$
 - * Cold gas $m \sim 5 \times 10^9 M_\odot$
 - about 20% of this is H_2 . Cold and dense, associated with star formation.
 - other 80% is HI and often thought of as reservoirs for future star formation.
 - * Hot gas $\geq 10^6$ K.
 - Disk is kinematically cold ($\sim 10 \text{ km s}^{-1}$) and rotating
 - The youngest stars in the disk are about 10^6 years old
 - There's a radial metallicity gradient such that metals increase toward the center
 - MW has spiral arms!
 - The disk can be further divided into the thin and thick disk:
- Thin Disk
 - $M_\star \sim 5 \times 10^{10} M_\odot$
 - Radial scale height $h_r \sim 0.3 \text{ kpc}$, with a drop off in surface brightness
 - Vertical scale height $h_z \sim 0.3 \text{ kpc}$
 - Radius $r \sim 35 \text{ kpc}$
 - Mixed stellar ages
- Thick Disk
- Dark Matter Halo
 - $M_{DM} \sim 10^{12} M_\odot$, known within a factor 2 – 3 or so
 - Virial radius of roughly 150 kpc
- Rotation Curve
 - Sun is moving at roughly $V = 220 \text{ km s}^{-1}$. This speed is set by the enclosed mass which can be found by equating gravity to the Sun's centripetal acceleration:

$$\frac{mv^2(r)}{R} = \frac{GmM(r)}{R^2} \rightarrow \boxed{v_{circ}(r) = \sqrt{\frac{GM}{R}}}; \boxed{M_{enc} \propto r} \quad (1)$$

We care about classifying galaxies for a variety of reasons. There are a few classification schemes. Here's **Hubble's**:

There have been others developed since then, such as the deVaucouleur system or the T-type system.
Recently, we have started a **newer, fresher** classification based on rotation!
There are some difficulties in determining morphology, though:

- It depends on orientation.
- It depends on wavelength.
- Subjective.
- etc.

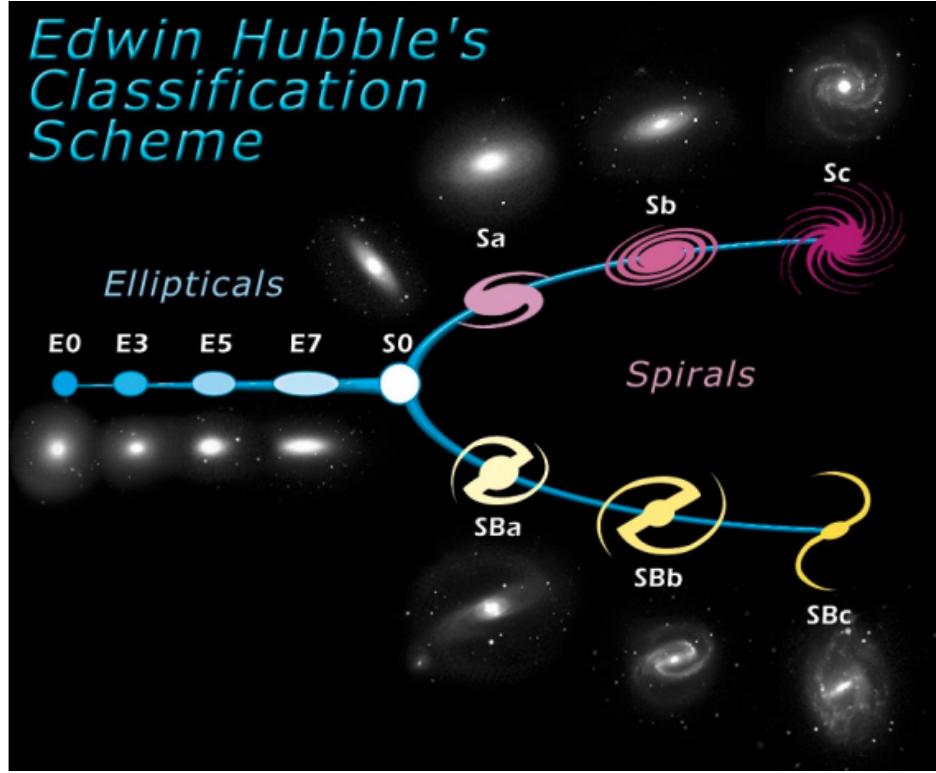


Figure 1: Hubble Classification Scheme

2 August 27, 2021: Global Properties and Galaxy Correlations

2.1 Morphology, Continued

Let's start by looking a little deeper at the morphological classifications from last time. First, let's look at spectra along the Hubble sequence, HI content, stellar mass, stellar surface density, oxygen abundance, and environment. The results are presented in Figures 4 through 7.

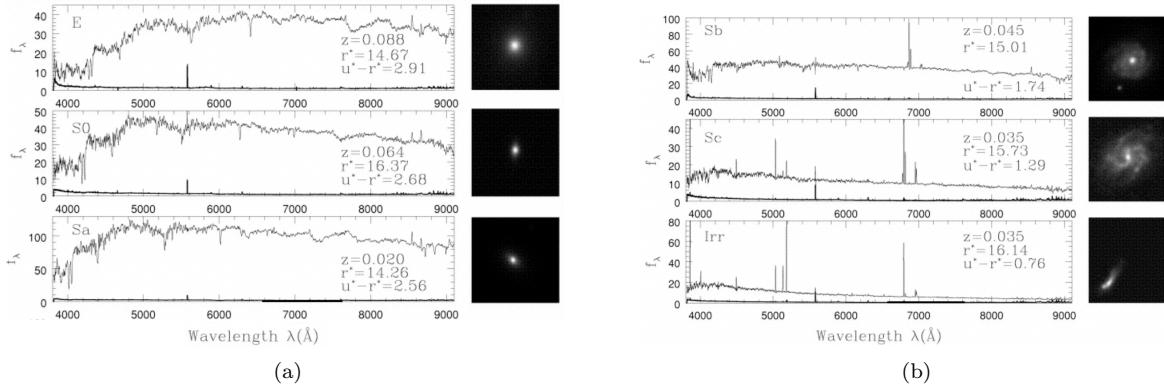


Figure 4: Spectra along the Hubble sequence. Early type galaxies are much redder and have deeper absorption features. Later-type galaxies have spectra that are dominated by emission from hot stars.

Table 2.5. Galaxy morphological types.

Hubble deV T	E	E-SO	SO	SO-Sa	Sa	Sa-b	Sb	Sb-c	Sc	Sc-Irr	Irr
	E	SO ⁻	SO ⁰	SO ⁺	Sa	Sab	Sb	Sbc	Scd	Sdm	Im
	-5	-3	-2	0	1	2	3	4	6	8	10

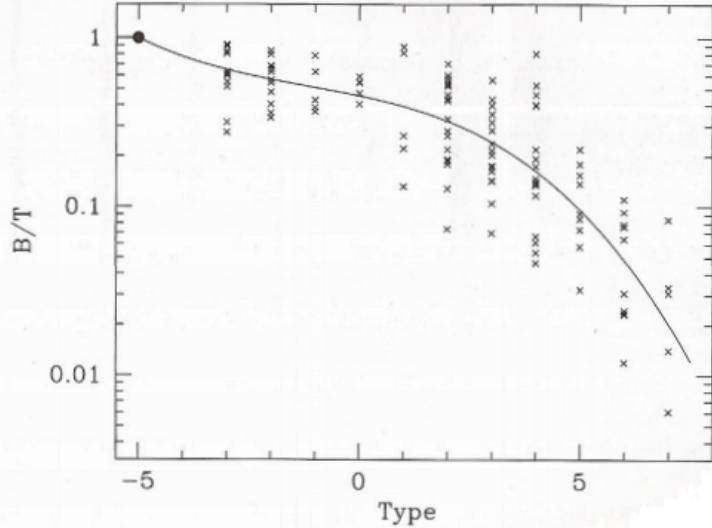


Figure 2: Other classification schemes.

Cappellari et al. (2011)

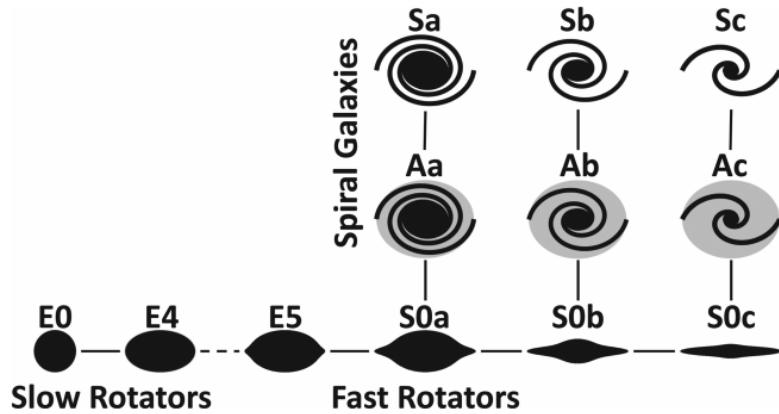


Figure 3: Fast and slow rotation scheme.

Here are the general trends. Star formation rate, gas fractions, and rotational support increase for later-type galaxies. Mass, metallicity, environment, and mass surface density increase for earlier-type galaxies.

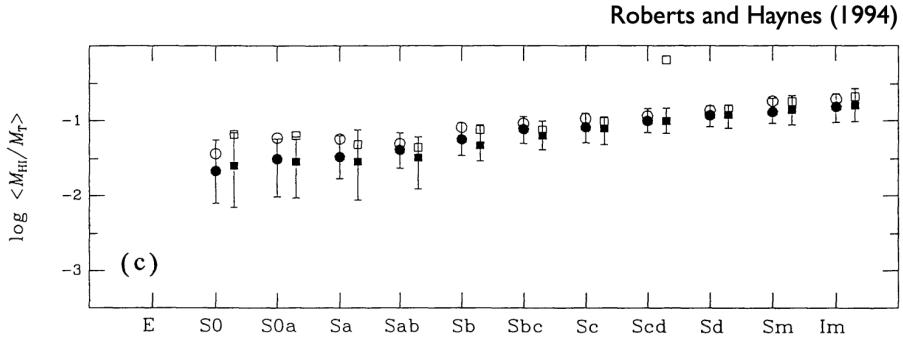


Figure 5: HI content as a function of morphology.

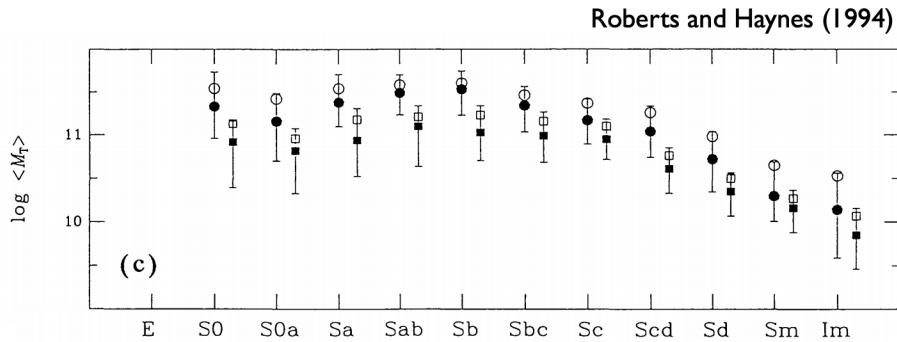


Figure 6: Total stellar mass as a function of morphology.

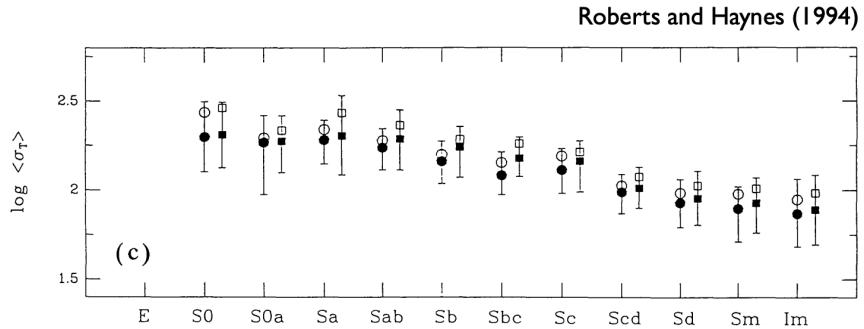


Figure 7: Stellar surface density as a function of morphology.

2.2 Units and Quantities

Let's do a quick review of units and other quantities we will use through the semester. Recall **specific intensity**:

$$[I_\nu] = \frac{\text{ergs}}{\text{s cm}^2 \text{sr Hz}} \quad (2)$$

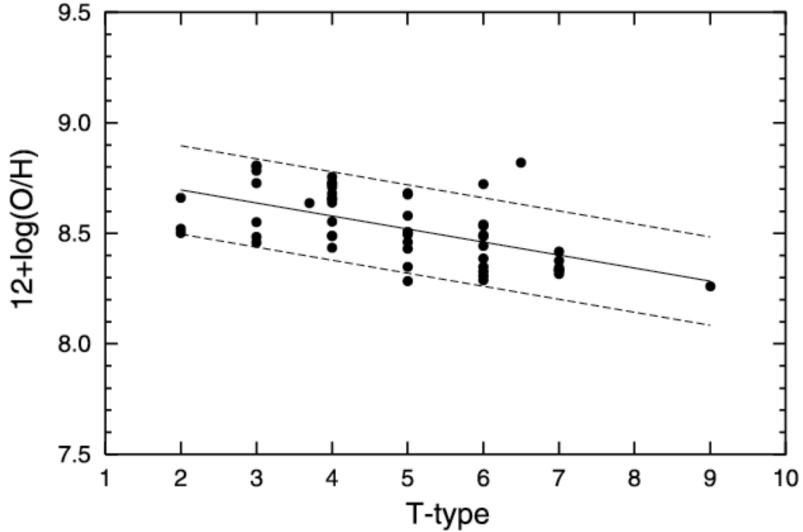


Figure 8: Oxygen abundance as a function of morphology. Note that the +12 is just to make things positive!

We also have the quantity of **flux density**:

$$[F_\lambda] = \frac{\text{ergs}}{\text{s cm}^2 \text{\AA}} \quad (3)$$

In general, remember that $F_\lambda \neq F_\nu$. Instead, $F_\lambda d\lambda = F_\nu d\nu$, and thus:

$$F_\nu = \frac{c}{\nu^2} F_\lambda \quad (4)$$

We also discussed magnitudes which are a terrible unit. But alas, we must use them. The apparent magnitude is defined as:

$$m_x = -2.5 \log_{10} \left(\frac{F_x}{F_{x,0}} \right) \quad (5)$$

where the subscript x represents a filter and $F_{x,0}$ is the flux of a reference in filter x . Historically, stellar astronomers have used Vega as the reference point. Galactic astronomers have used AB magnitudes (I have no idea what the reference is, here). In this **AB Magnitude system**, we have:

$$m_x = -2.5 \log_{10} (F_x) - 48.57 \quad (6)$$

Absolute magnitudes are defined as:

$$\underbrace{m_x - M_x}_{\text{distance modulus } \mu} = 5 \log_{10} \left(\frac{d}{d_0} \right), \text{ where } d_0 \equiv 10 \text{ pc} \quad (7)$$

We also need to be familiar with redshift, both due to expansion and relativistic velocities. We define the redshift z :

$$1 + z = \frac{\lambda_{obs}}{\lambda_{em}} \quad (8)$$

In the case of particles moving at relativistic speeds, we have:

$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}} \quad (9)$$

In the non-relativistic case, we have the **Hubble's Law**:

$$v \approx cz \quad (10)$$

Lastly, we defined the **luminosity distance** as:

$$d_L^2 \equiv \frac{L}{4\pi F} \quad (11)$$

2.3 Sloan Digital Sky Survey

The Sloan Digital Sky Survey (SDSS) revolutionized astronomy. Not only did it take spectra and photometry of millions of galaxies, but it also redefined the photometric system from the Johnson-Cousins system to the familiar *ugriz* system. First, let's discuss an important aspect of filters: **filter response functions**. This is basically like a sensitivity/efficiency, and these curves look like this:

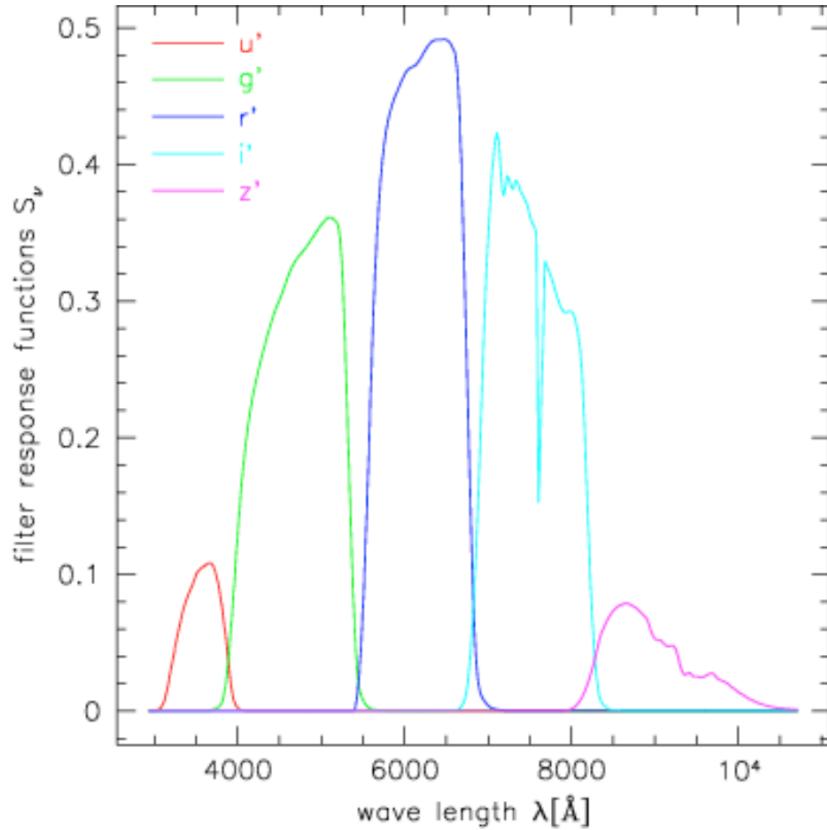


Figure 9: SDSS transmission curves.

Note that:

$$F_x = \frac{\int_0^\infty S_\nu F_\nu d\nu}{\int_0^\infty S_\nu d\nu} \quad (12)$$

SDSS had 5 band photometry for millions of objects, which means we need to look at spectral energy distributions (SEDs). These are basically low resolution spectra, but we can still learn a ton from these plots! We can match SEDs to synthetic galaxy spectra to measure photometric redshift or learn about broad properties of the galaxies.

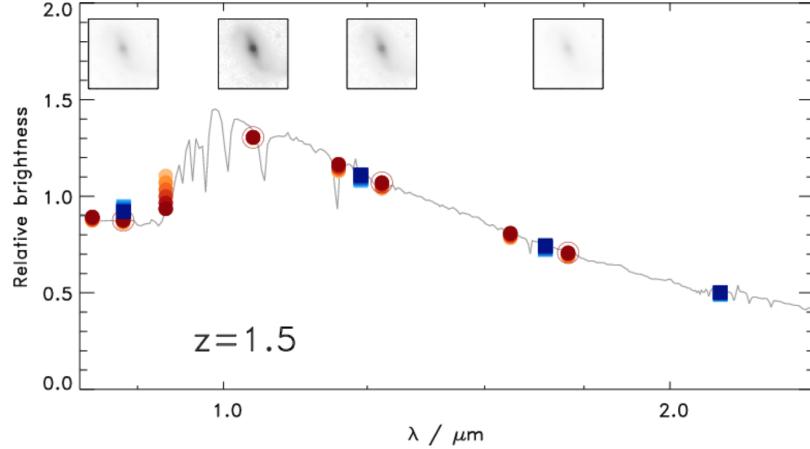


Figure 10: Example SED with fit for photometric redshift.

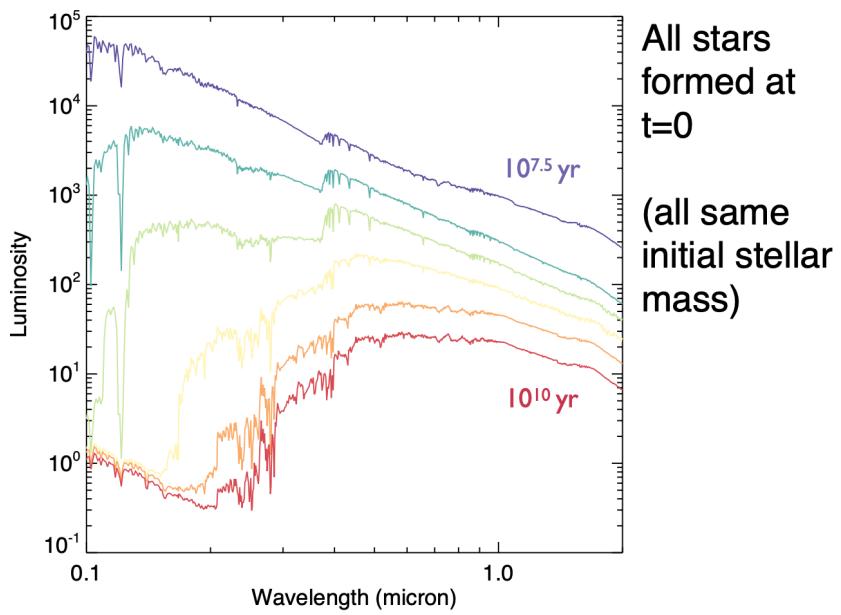


Figure 11: Synthetic galaxy spectra as a function of age.

One thing to note is that redder galaxies are less luminous for the same stellar mass, as we can see in the synthetic spectra. This result in redder galaxies having higher mass to light ratios!

2.4 K Corrections

Here's a figure to demonstrate K-corrections.

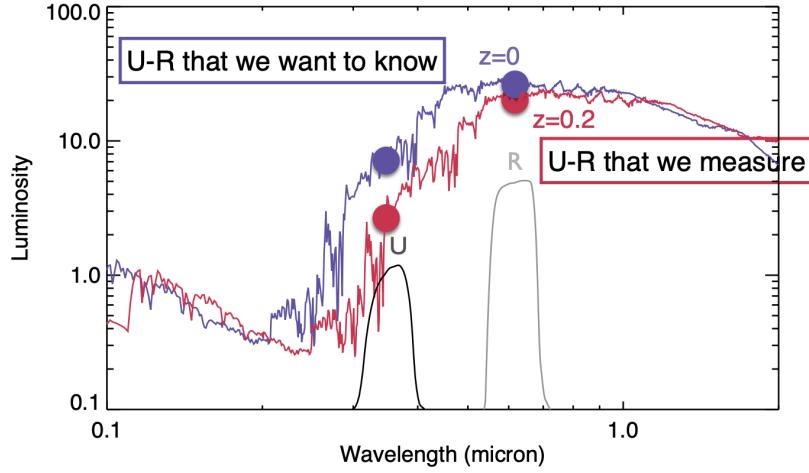


Figure 12: The SED changes as a function of redshift. We ultimately want the rest-frame $u - r$ value, but we need to correct the observed spectrum in some way since $u - r$ itself is a function of redshift. This is the concept of a K correction.

We model K corrections in this way:

$$m = M + \mu + K \quad (13)$$

$$\text{observed} = \text{intrinsic} + \text{distance} + \text{expansion} \quad (14)$$

2.5 Trends of Stellar Mass with Color

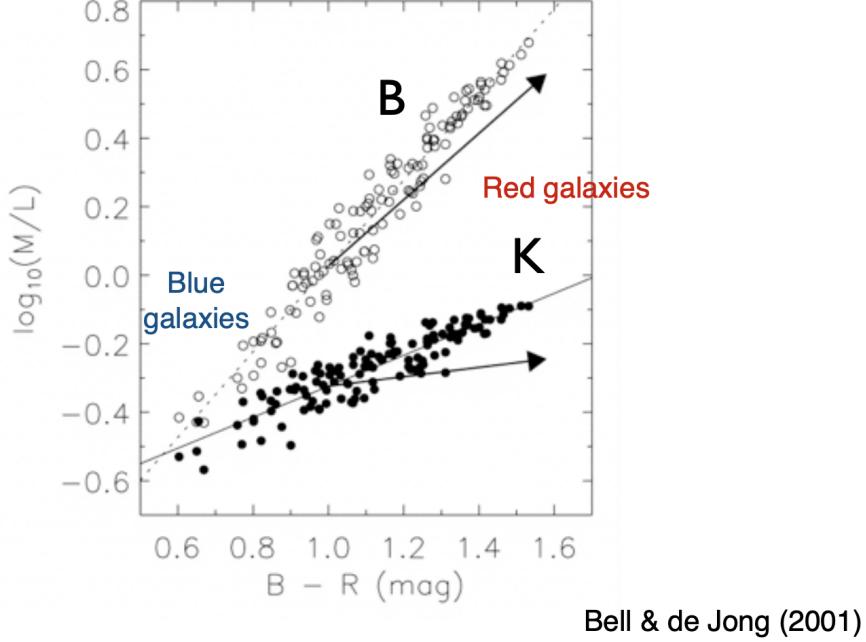


Figure 13: Trends of stellar mass to light ratio with color.

TABLE 1
STELLAR M/L RATIO AS A FUNCTION OF COLOR FOR THE FORMATION EPOCH MODEL WITH BURSTS, ADOPTING A SCALED SALPETER IMF

Color	a_B	b_B	a_V	b_V	a_R	b_R	a_I	b_I	a_J	b_J	a_H	b_H	a_K	b_K
$B-V$	-0.994	1.804	-0.734	1.404	-0.660	1.222	-0.627	1.075	-0.621	0.794	-0.663	0.704	-0.692	0.652
$B-R$	-1.224	1.251	-0.916	0.976	-0.820	0.851	-0.768	0.748	-0.724	0.552	-0.754	0.489	-0.776	0.452
$V-I$	-1.919	2.214	-1.476	1.747	-1.314	1.528	-1.204	1.347	-1.040	0.987	-1.030	0.870	-1.027	0.800
$V-J$	-1.903	1.138	-1.477	0.905	-1.319	0.794	-1.209	0.700	-1.029	0.505	-1.014	0.442	-1.005	0.402
$V-H$	-2.181	0.978	-1.700	0.779	-1.515	0.684	-1.383	0.603	-1.151	0.434	-1.120	0.379	-1.100	0.345
$V-K$	-2.156	0.895	-1.683	0.714	-1.501	0.627	-1.370	0.553	-1.139	0.396	-1.108	0.346	-1.087	0.314

NOTE— $\log_{10} (M/L) = a_\lambda + b_\lambda \text{Color}$. Note that the stellar M/L values can be estimated for any combination of the above colors by a simple linear combination of the above fits. Note also that if all (even very high surface brightness) disks are submaximal, the above zero points should be modified by subtracting a constant from the above relations.

$$\log_{10} (M/L) = a_\lambda + b_\lambda \text{Color}$$

Figure 14: Table which was a first attempt at quantifying this relationship. Much better ways of doing this now!

3 September 1, 2021: Global Properties and Galaxy Correlations

3.1 AB Magnitudes and Vega Magnitudes

The AB magnitude system is the **absolute** system.

$$m_{AB} = -2.5 \log_{10} \left(\frac{f_\nu}{3631 \text{ Jy}} \right) = -2.5 \log_{10} (f_\nu) - 48.57 \quad (15)$$

We have a differ net system called the **STmag**, which is similar but in wavelength; not in frequency. The conversion between the two is non-trivial. The corrections in the optical are small, but these increase at larger and smaller wavelengths.

3.2 Synthetic galaxy spectra as a function of age

This discussion primarily revolves around Figure 11, the figure of the synthetic spectra as a function of age. Make sure to be able to re-create this type of plot!

Imagine taking a stellar population synthesis model at the same time, with some mass. Young populations are brighter per unit mass, and they are bluer. Redder galaxies have higher mass to light ratios.

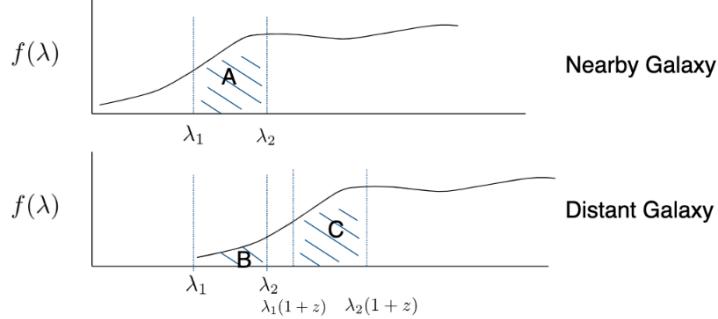
Let's now move onto K-corrections...

3.3 K-corrections

The idea: Normally we observe through filters, meaning over some specific $d\lambda$. For a typical galaxy spectra at $z = 0$:

$$L = \int_{\lambda_1}^{\lambda_2} f(\lambda) d\lambda \quad (16)$$

Due to redshift effects, the light emitted gets shifted by a factor of $(1+z)$. Looking at galaxies at higher z , we receive emission from shorter wavelengths at a slower rate. Knowing the spectra shape, we can correct for this via K-correction. The **assumption: we know something about the intrinsic spectrum**. Here's a figure explaining what we mean:



We measure B, but want to measure C to compare to A.

$$K(z) = \frac{\int_0^\infty T(\lambda/(1+z))f(\lambda/(1+z))d\lambda}{(1+z)\int_0^\infty T(\lambda)f(\lambda)d\lambda}$$

Made by Greg Taylor

Figure 15: K-corrections, by Greg Taylor.

Here's a derivation:

The flux we observe per unit wavelength, F_λ (the emitted luminosity is: $L_{\lambda/(1+z)}$):

$$F_\lambda = \frac{1}{1+z} \frac{L_{\lambda/(1+z)}}{4\pi D_L^2} = \frac{1}{1+z} \frac{L_{\lambda/(1+z)}}{L_\lambda} \frac{L_\lambda}{4\pi D_L^2} \quad (17)$$

Equivalently: $F_\nu \propto (1 + z)$. After some work, we can see that:

$$m = M + \mu + K, \text{ where } K \equiv -2.5 \log_{10} (F_\lambda) \quad (18)$$

Generally, K corrections are positive. There are unusual cases where the K correction is negative, but that's uncommon.

3.4 Stellar Masses (Bell & de Jong 2001)

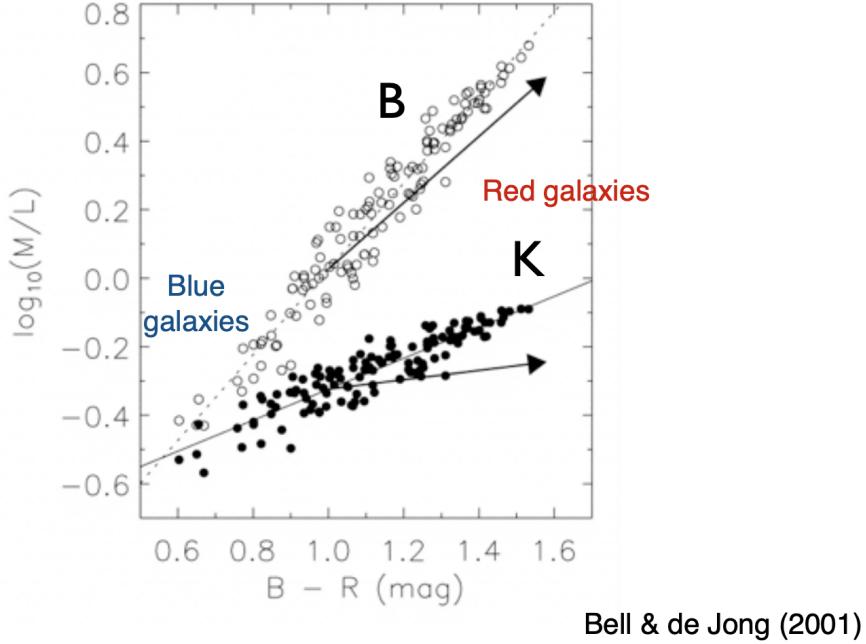


Figure 16: Re-visited on September 1!

We're discussing the figure here. The x-axis is the $B - R$ color, and the y-axis is the inferred mass-to-light ratio. Each dot is an individual galaxy. The two groups are B and K band mass to light ratios, with the key takeaway is that **there is no one-to-one mapping between mass and luminosity**. Instead, luminosity and color can give you a mass.

It's best to observe K band for measuring mass to light ratios so that the spectrum isn't dominated by **single O stars**. The ideal case is a filter for which the mass-to-light does not depend on the color. K is close, but it's not ideal!

However, bluer colors are better at disentangling the age of a stellar population.

3.5 SEDs for Different Metallicities

Here's an interesting figure:

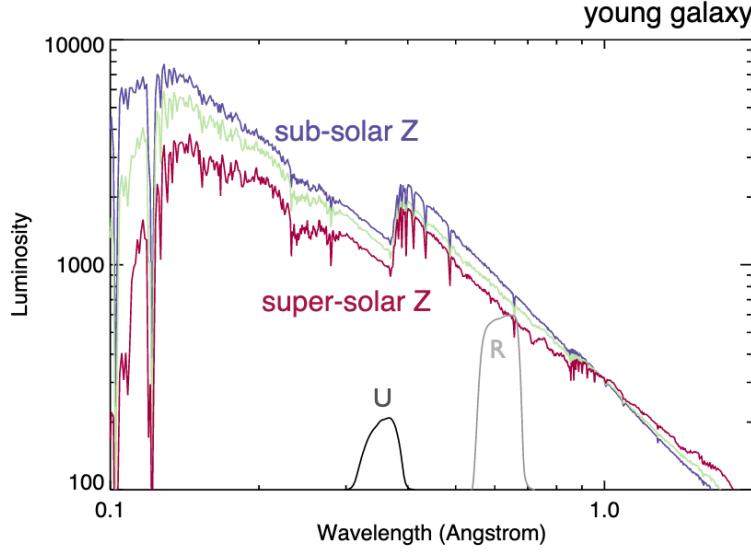


Figure 17: SEDs for different metallicities for a young galaxy. What are the takeaways?

For a fixed mass and galaxy type, lower-metallicities are brighter, particularly in the blue! As you go to the red end of the spectrum, the SEDs blend together better.

This is due to **opacity**. Fewer metallicites have a lower opacity, and so more light gets out. For higher metallicities, we have a higher opacity and thus less light gets out.

Also know that the age has a huge impact on the spectra, but metallicity has much less of an effect. This also means that metallicity is harder to measure than age. They're also **degenerate** with each other.

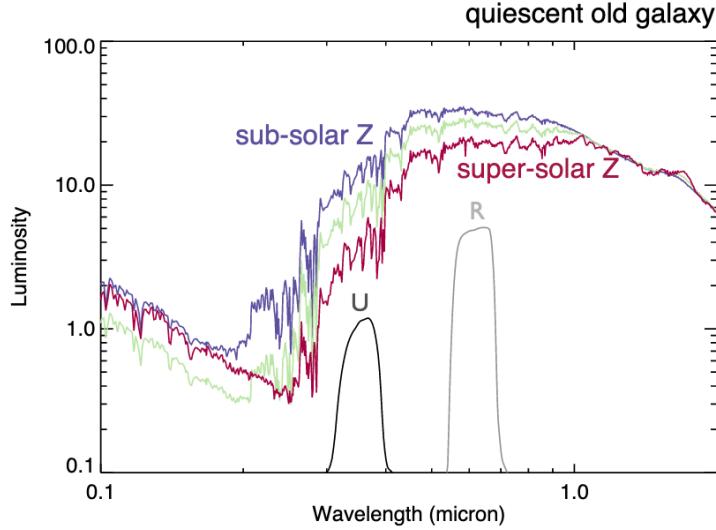


Figure 18: SEDs for different metallicities for an old galaxy. What are the takeaways?

What's different for the older, quiescent galaxy? At the blue end of the spectrum, solar metallicity populations are actually **weakest**! Detailed stellar physics of post-main sequence stars dictate this effect.

3.6 Color Sequences

Let's look at color vs. stellar mass of galaxies. The color of the galaxy $m_u - m_r$ can be thought of as the slope of the SED. Interpreted differently, this is:

$$u - r = -2.5 \log_{10}(F_u) + 2.5 \log_{10}(F_r) \rightarrow u - r = -2.5 \log_{10} \left(\frac{F_u}{F_R} \right) \quad (19)$$

Here's a cool plot:

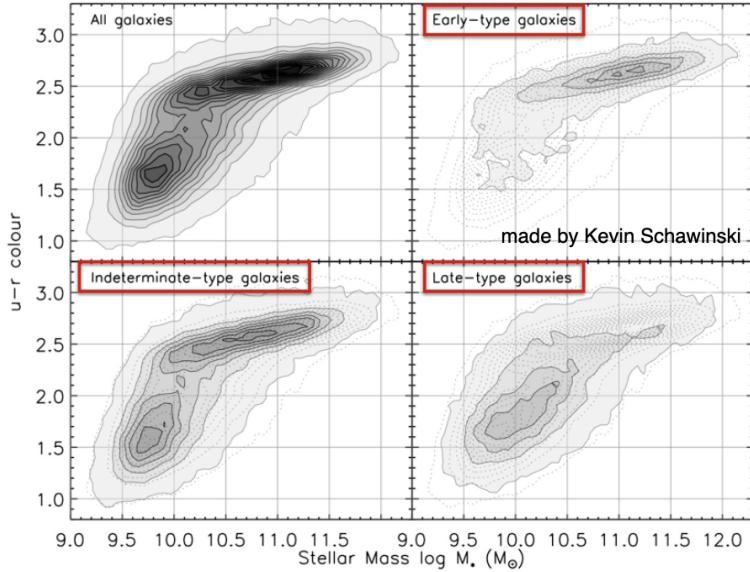


Figure 19: Color sequences. Note that there are two pileups: redder and more massive; blue and less massive.

If you measure a galaxy color, we can measure the mass to light ratio. **This is a key reason to measure color.** If we also measure the spectrum (or the SED with multi-band photometry), we can get the galaxy flux, redshift, and make a K-correction. Note we can also measure the mass-to-light (or mass) from the SED, too!

What are some other trends in this plot? The early-type galaxies hardly change in color but increase 2 dex in mass. This

3.7 Galaxy Correlations in the SDSS

Let's talk about the D-4000 decrement from Kauffmann et al., 2003:

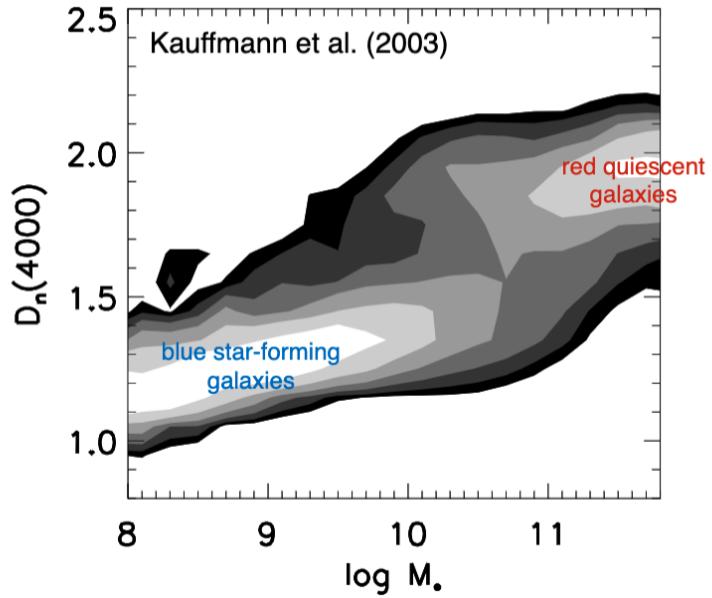


Figure 20: We can see that the D4000 feature very nicely separates the red, quiescent galaxies from the blue, star-forming galaxies.

So what is D4000? It's a flux ratio (color) between 4000Å and 4100Å as well as the flux between 3856Å to 3950Å. There are lots of age sensitive features here, which make it ideal. Note that these ranges are not exact.

$$D_n(4000\text{\AA}) = \frac{F(4000\text{\AA} - 4100\text{\AA})}{F(3856\text{\AA} - 3950\text{\AA})} \quad (20)$$

This looks like this:

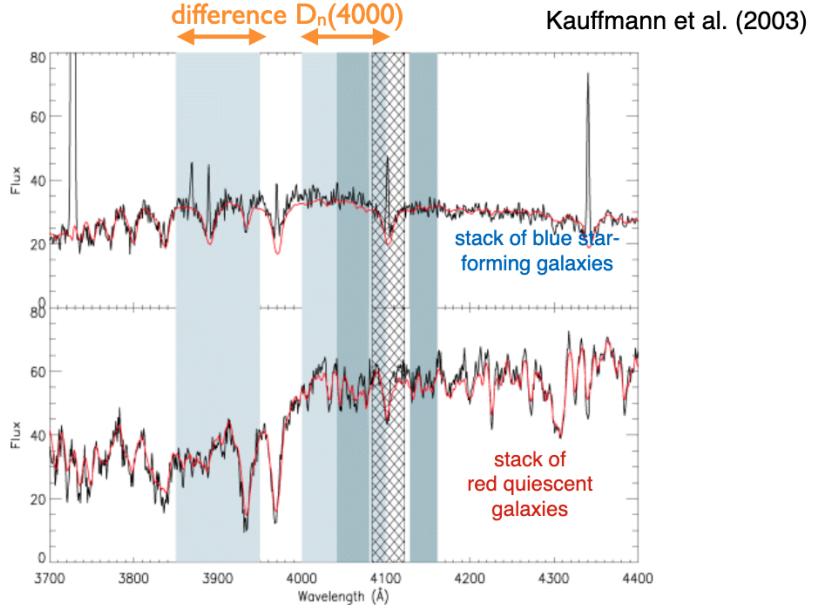


Figure 21: Star forming galaxies have a very small break, but the red galaxies have a very large break near 4000\AA .

Here is the 4000 Angstrom break as a function of both age and metallicity:

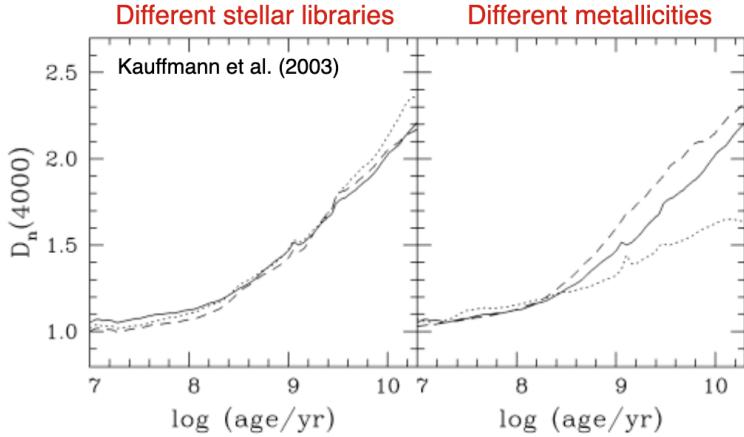


Figure 22: The 4000 Angstrom break.

So, the takeaway is that the 4000\AA break is a much better measurement of age compared to broadband photometry, but much more expensive. There's also a pretty strong signal to noise requirements needed for D_{4000} measurements.

D_{4000} measurements are tough with k-corrections in the mix as well since we need to know the intrinsic shape of the spectrum, which is much harder for high redshift galaxies.

3.8 Stellar metallicity - mass relation (Gallazzi et al., 2005)

At higher masses, stars are more metal-rich. As you go to lower masses, we have lower metallicity. Under the hood, star formation is generating metals. Massive galaxies have more star formation, and thus more metals, to zeroth order.

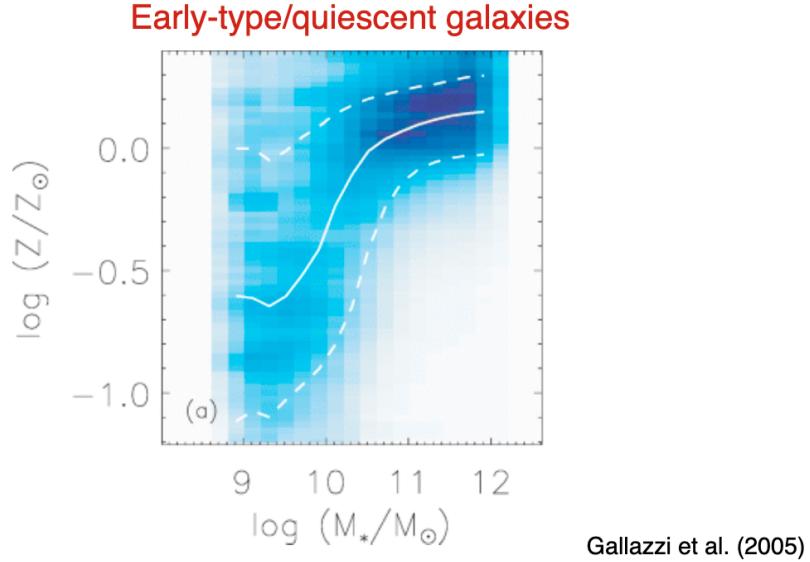


Figure 23: Stellar metallicity-mass relation for early-type galaxies.

For **late type galaxies**, this is more complicated but gives a similar trend.

Late-type / star-forming galaxies

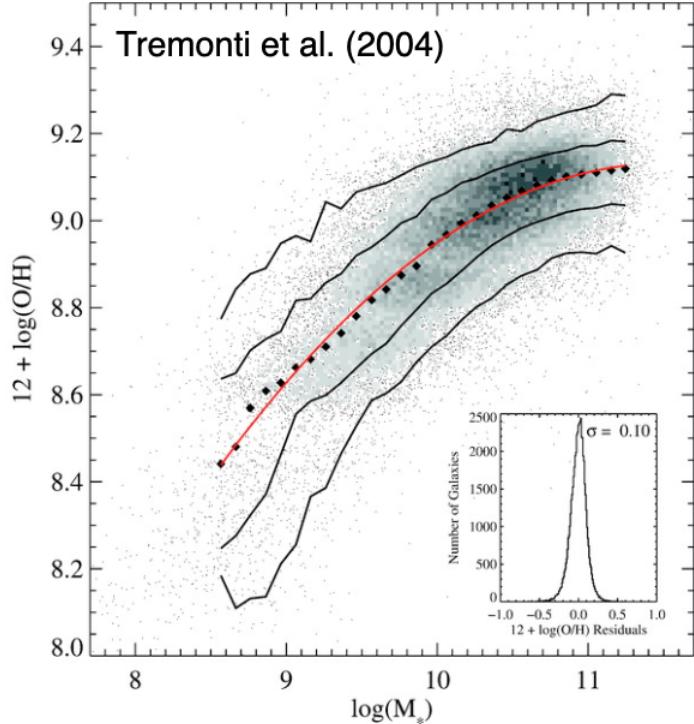


Figure 24: Stellar metallicity-mass relation for late-type galaxies.

How do you make a measurement like this? Ionized HII regions emit oxygen lines in late-type galaxies. This doesn't happen in early type galaxies!

We will continue next time with the late-type slope in the color-mass plot!

4 September 3, 2021: Global properties, galaxy correlations, and statistics

4.1 Measuring gas-phase metallicity

What is going on with the fiber versus total spectra in the SDSS data? SDSS is a fiber fed system, with each fiber being really tiny. As a result, only the central regions are getting spectra!

We are thus assuming that in the central region, oxygen emission comes only from star formation. It turns out that AGN also create emission line patterns which complicate things!

In our problem set, we are asked for gas-phase metallicity for our galaxies, so what do we do? For our homework, use total fluxes and not fiber fluxes!

4.2 Brief Review of Last Time: Color Sequences,

Remember the $u - r$ color slope is primarily driven by the metallicity (mass-metallicity relation) for early type galaxies.

Late-type galaxies are driven by star-formation properties. The way we can see this most clearly is the star formation rate-stellar mass relation (Samir et al, 2007):

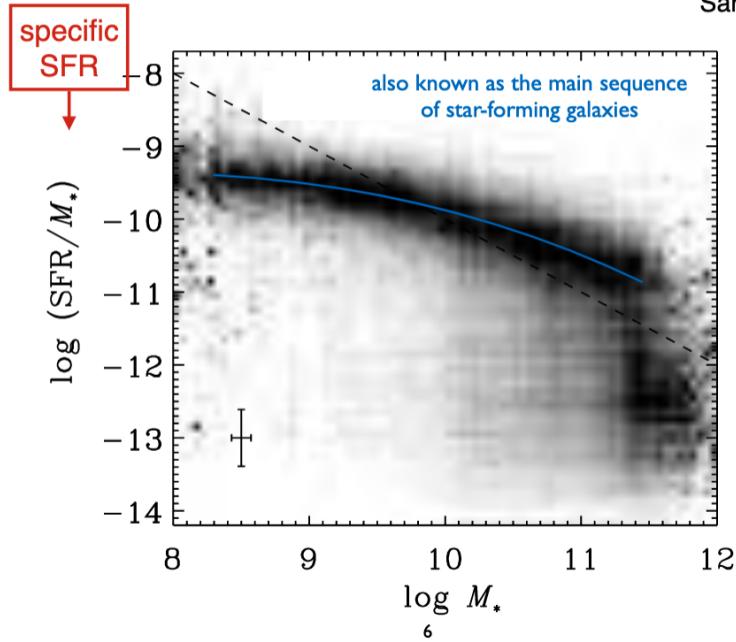


Figure 25: sSFR-stellar mass relation.

This isn't the most obvious thing to plot, however. It'd be more obvious to plot star formation rate versus the stellar mass (star-forming main sequence). The shape changes when we plot the specific star formation rate. So what does this tell us/what is the interpretation?

As we go to higher masses, the **star formation rate per unit mass is lower because it's less efficient**. One way to think about this (since the previous statement is pretty empty): low mass galaxies have younger populations which have formed more recently (and thus have no converted mass into stars yet because the potential is shallower). Another explanation is that larger galaxies have had time to spread mass out over the whole halo (where gas density is too low to form stars).

4.3 Stellar mass-halo mass relation

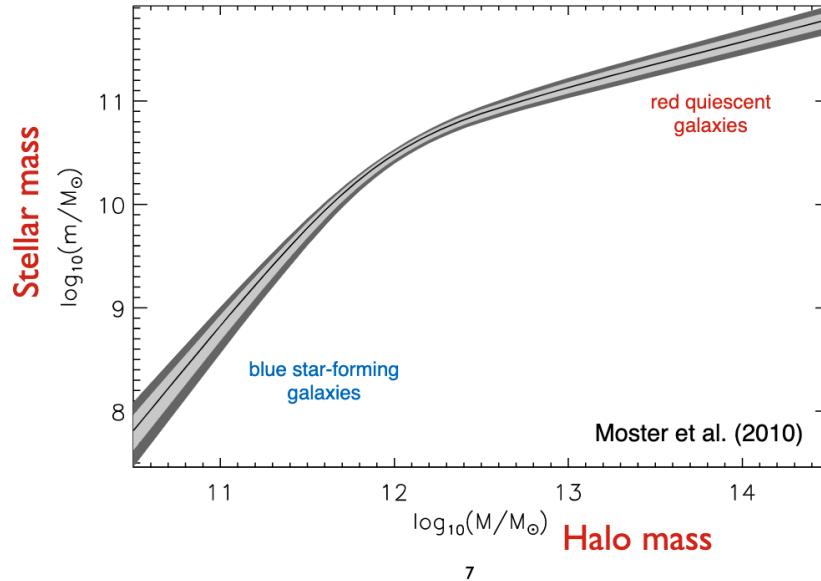
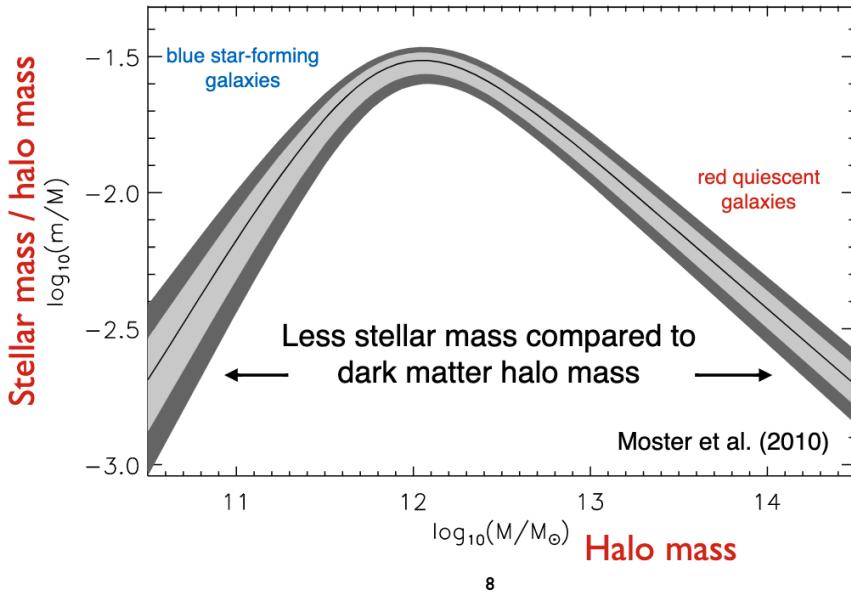


Figure 26: Stellar mass-halo mass relation (Moster 2010).

One of the ideas in galaxy formation is that the dark matter halo drives galaxy formation moreso than anything else. The halo mass should thus determine the evolution of the galaxy as a whole. It sets the potential, temperatures of gas (kinematic temperature), accretion rate (mass inflow rate), etc. **The problem is that measuring halo masses is really difficult!**

We can assign galaxies to halos with **abundance matching**, shown in Figure 26, giving rise to the stellar-mass halo-mass relation (SMHM, SMH relation). The most massive stellar and dark galaxies are red, quiescent galaxies. At the low end, there are blue star-forming galaxies.

This leads to one of the most important plots in the course:



8

Figure 27: Stellar mass-halo mass ratio vs. halo mass. This is saying that, as you go to really low or high mass haloes, there is less stellar mass! **Per unit halo mass, really massive and really low-mass galaxies are inefficient at star formation.** The peak is where star formation is the most efficient!

4.4 Galaxy Sizes

4.4.1 Sersic (1968) Profiles

The generalized version is:

$$I(R) = I_0 e^{-\beta_n (R/R_e)^{1/n}} = I_e e^{-\beta_n ((R/R_e)^{1/n} - 1)} \quad (21)$$

Here, I_0 is the central intensity, β_n is a scale factor, n is the Sersic index, $R_e = R_{1/2}$ is the effective radius or the half-light radius.

You often see this written in terms of surface brightness, which is just the magnitude form.

There are two specific cases of the Sersic profile. For early type galaxies, we have deVaucouleur's Law:

$$I(R) = I_e e^{-7.7[(R/R_e)^{1/4} - 1]} \quad (22)$$

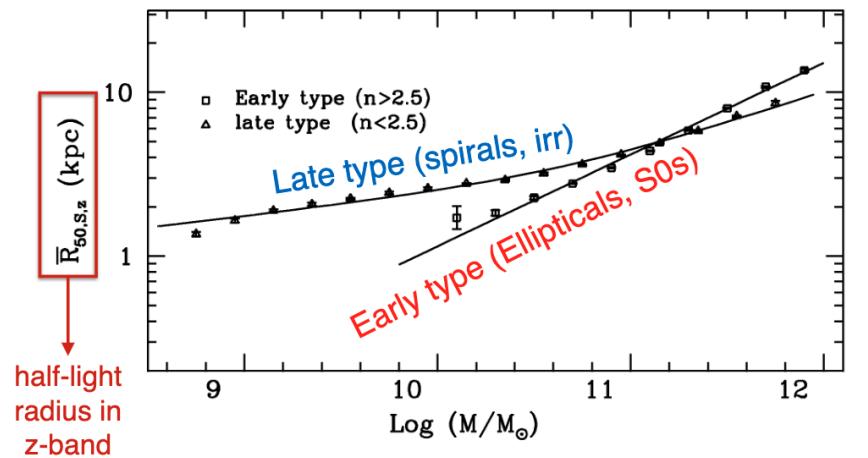
This is a Sersic with $n = 4$ and $\beta_n = -7.7$. If we plot μ versus $r^{1/4}$, this is a straight line:

Late-type galaxies are even more difficult to fit. For late-type galaxies, we have:

$$I(r) = I_0 e^{-r/h} \quad (23)$$

where I_0 is the central surface brightness, h is the scale length (1e folding of light). In this case, plotting μ versus r alone gives the straight line.

4.5 Stellar mass-size relation (Shen et al., 2003)



Shen et al. (2003)

Figure 28: Stellar mass-size relation.

4.6 Measuring galaxy sizes and Sersic indices

Large sersic indices are peaky toward the middle (ETGs), smaller values (late-type) are toward the outskirts.

4.7 Galaxy sizes over cosmic time

Good to know that sizes are not constant over time, and we will come back to this.

4.8 Relation with Environment

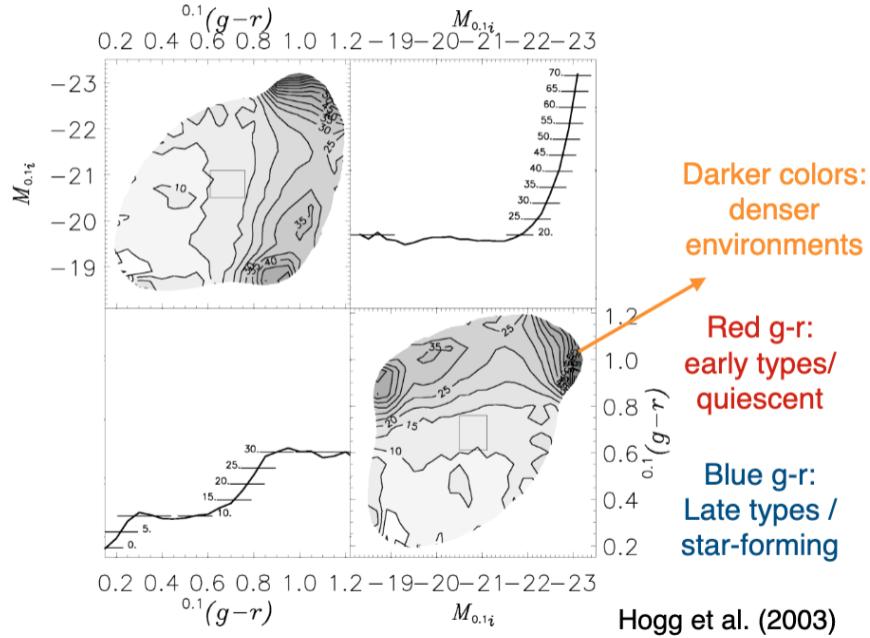


Figure 29: Relation with environment. Bottom right: Contours are density, with darker colors being denser environments (number per volume). In higher density environments, you find early type galaxies.

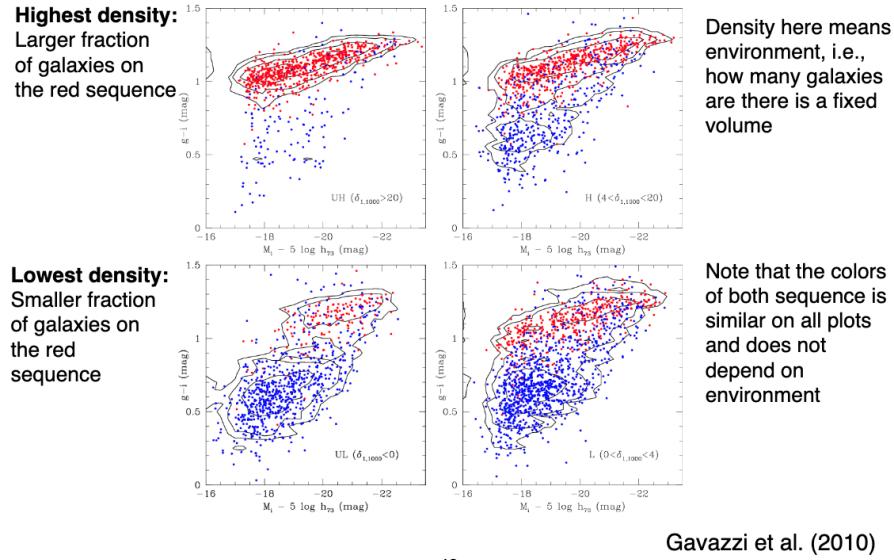


Figure 30: Color-magnitude diagrams in different environments (Gavazzi et al. (2010)).

4.9 Disentangling environment, mass, morphological type, and Sersic parameter (van der Wel 2008).

Left hand column x-axis is number per volume.

4.10 Staistical properties of the galaxy population

Look at the M87 field. We can start to plot the number of galaxies as a function of luminosity bin since they are all at the same (rough) distance. Our theories can make predictions on this!

4.11 The luminosity function of galaxies

First posited by Schechter (1975), we have the Schechter luminosity function. For better or worse, there are no physical bases for this even though it works really well!

$$\Phi(L) dL = \Phi_* \left(\frac{L}{L_*} \right)^\alpha e^{-L/L_*} d\left(\frac{L}{L_*} \right) \quad (24)$$

L_* is the inflection point in the luminosity function, α is the “faint-end slope”, Φ_* is a normalization. Note that the units of Φ are number/volume. Sometimes this is further divided into magnitude bins.

When $L \ll L_*$, we have power law behavior with α dominating. For $L \gg L_*$, we have exponential decline dominating. In the local universe, the Milky Way is approximately an L_* galaxy. In physical units, we have:

$$L_{\text{B-band}}^{\text{MW}} \sim 10^{10} L_\odot \quad (25)$$

4.12 Peculiar Motions

To get a luminosity, we first need a distance! Occasionally, you see a **huge** scatter in the relationship because of peculiar velocities of things like the Virgo cluster.

This introduces the concept of **Malmquist bias**. You can see brighter objects farther away:

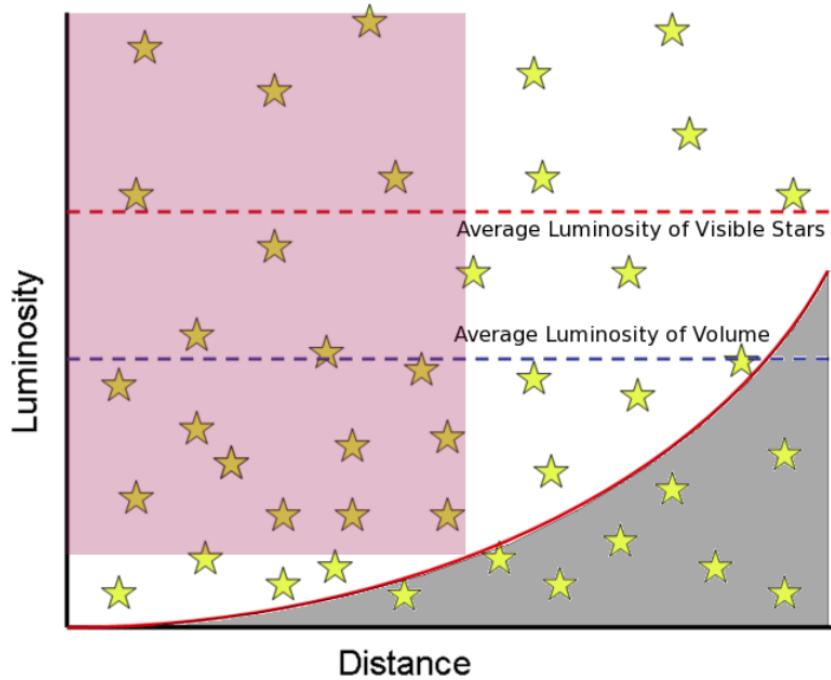


Figure 31: Malmquist bias.

Perhaps most obvious fix is a luminosity cutoff, but this has disadvantages in that you impose a distance cutoff and also you're throwing out data. The better solution is a $1/V_{max}$ correction:

$1/V_{max}$ corrections for Malmquist bias

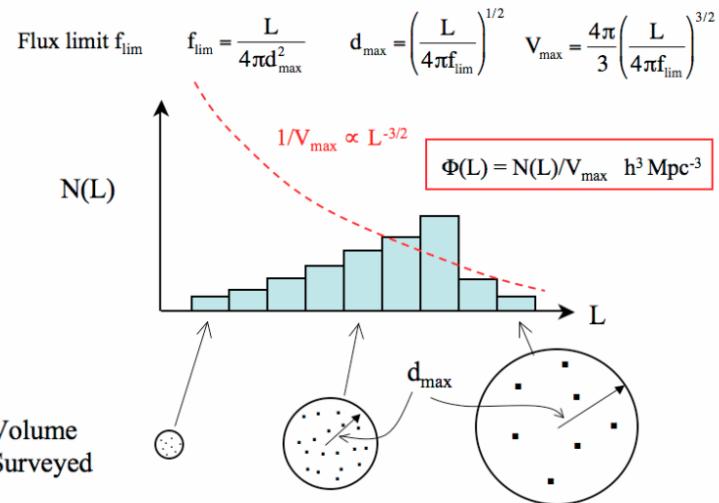


Figure 32: Malmquist bias solution as a $1/V_{max}$ correction.

5 September 8, 2021: Finish galaxy statistics; stellar populations; stellar evolution review

5.1 Luminosity Function for Different Hubble Types

Why does the Schechter function work? See Binggeli (1998). The composite of individual galaxy functions form a Schechter function, but each individual function does not! Is this accurate? Maybe not...are these complete at the faint end? To first order, yes. But nothing is first order.

5.1.1 Challenges

- Need to know distances; redshifts for low-z galaxies reflect distance and peculiar motion
- Low surface brightness galaxies are hard to detect.
- Malquist bias: brighter galaxies can be traced to larger distances than fainter galaxies. We can correct by only considering range of luminosity and distance; or you can apply volume corrections for each luminosity.

5.1.2 Vmax Correction, Revisited

5.2 Luminosity Functions as a function of color (Blanton 2001)

Red galaxies dominate the bright end of the luminosity function – the biggest are the brightest! The blue galaxies dominate the faint end. This is not totally a surprise! Remember the stellar mass versus color diagram!

5.3 Luminosity Function as a function of environment (Christlein 2000)

There are more galaxies in denser environments. Also, the faint end slope gets steeper in low density environments. There are many more low mass galaxies in dense environments.

5.4 Luminosity and Mass Functions (Yang et al. 2009)

Redder galaxies have higher mass-to-light ratios Υ , and thus the red function moves to the right compared to the blue function when going from luminosity to mass. Working in mass function space is **much better** than in luminosity space, but you need way more data to get masses.

5.5 Physical Origin of the Luminosity Function

Ratio of these two lines give us the plot from last time of the halo mass versus stellar mass per halo mass plot (with peak efficiency near the MW).

5.6 Evolution of the Mass Function (with redshift) [Muzzin et al 2013]

The normalization shifts, for sure, but you also start to see other patterns emerge. At high redshift ($z \sim 2$), the functions are more or less on top of each other. We can see changes in galaxy populations!

The mass function can be used to trace the build up of stellar mass in both star-forming and quiescent galaxies over cosmic time.

5.7 Evolution of the UV Luminosity Function (Bouwens et al 2000)

UV Luminosity functions (UVLF) can be used to trace the evolution of the SFR density over cosmic time. At high redshift, we have low number densities of galaxies because galaxies have not formed yet! We can also look at the confidence intervals for the Schechter fit parameters, and we see the evolution!

5.8 Star formation history of the Universe

Inferred by integrating the UV luminosity function. This is basically the star formation rate per unit volume as a function of redshift. Going back in time, cosmic SFR peaks around $z \sim 2 - 3$, and declines steeply to high redshift. The peak epoch was around 2 – 3 billion years after the Big Bang.

To make these measurements, we get a UVLF at a redshift, fit a Schechter function, integrate to some limit (giving total UV luminosity at that redshift), use a stellar population model which gives a number of stars and masses, and then get star formation rate.

The blue line above is the integrated UV luminosity. The red shows what happens when we dust correct. No dust was really present at a redshift of $z > 6$. Dustiest galaxies are near $z \sim 2 - 3$. Early galaxies were low metallicity and thus couldn't enrich with dust until SFR kicked in (with other chemical evolution).

5.9 Quiz!

- The specific star formation rate is slightly lower though for more massive star-forming galaxies on the star forming main sequence, and thus these galaxies are slightly more evolved and redder. Also think about the color-mass relation – smaller mass has lower $u - r$ and thus is bluer.

6 September 10, 2021: Stellar Evolution and Stellar Populations

6.1 Background

- Stellar spectra and spectral types
- Stellar evolution
- Stellar initial mass function
- Dust attenuation and emission
- Star formation histories
- Stellar population synthesis modeling

6.2 A quick thing about Problem 3e: Photon and Energy Counting Detectors

Here's the issue. We wrote down an equation:

$$\int_0^\infty S_\lambda F_\lambda d\lambda \quad (26)$$

This equation is ambiguous. It's not specific enough. This F_λ and S_λ have different units, actually. There are actually units attached to S_λ that are ignored, typically. The units don't make sense because S_λ is kind of like a "fraction of photons," and thus we have to convert to number of photons instead of the energy of photons.

Apparently, here is the solution – don't derive from first principles, but think dimensionally. Take F_λ and go from energy to number. We do this by:

$$F_\lambda \rightarrow \frac{F_\lambda}{E} = \frac{\lambda F_\lambda}{hc} \quad (27)$$

This is a **photon counting detector**, not an **energy counting detector**. In that case, it's more complicated, and we have $\lambda^2 F_\lambda S_\lambda d\lambda$.

6.3 Stellar Evolution

6.3.1 HR Diagrams: Main Sequence

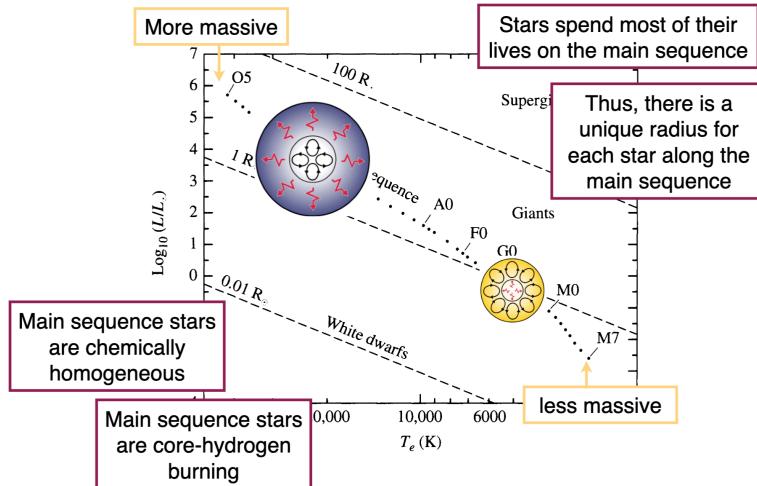


Figure 33: HR Diagram.

- Stars spend most of their life on the main sequence, defined as core hydrogen burning.
- Thus, there is a unique radius for each star along the main sequence. We infer a radius with the Stefan-Boltzmann law (derived from blackbodies, good for stars). We measure a luminosity $L = 4\pi Fd^2$ and temperature from spectra or colors:

$$L = 4\pi R^2 \sigma T^4 \quad (28)$$

- More massive stars in the top left; less massive in the bottom right.
- Main sequence stars are chemically homogeneous
 - Main sequence stars are in hydrostatic equilibrium.
- Low mass stars and high mass stars differ in energy transport. Low mass has convection exterior; high mass stars have convection in the core. Determined by adiabatic temperature gradients. If the temperature gradient is super adiabatic, you have convection.

6.3.2 Difference in Chemical Composition

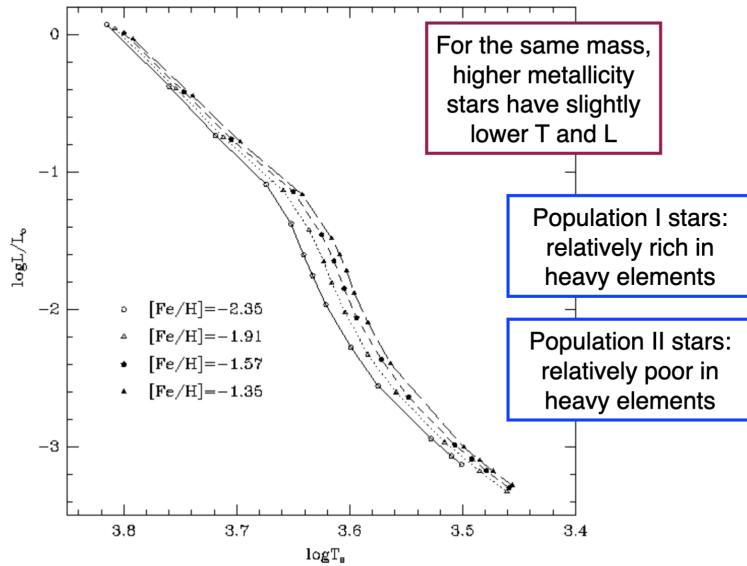


Figure 34: Differences in chemical composition.

Fe/H is the amount of iron-content of a star relative to the sun. The more negative, the more metal poor.

- For the same mass, high metallicity is cooler and less luminous.
 - Pop I: Relatively rich in heavy elements
 - Pop II: Relatively poor in heavy elements
 - Pop III: Metal free stars.

6.3.3 Relation between luminosity and mass

Over the entire main sequence, it's almost $L \sim M^{3.5}$.

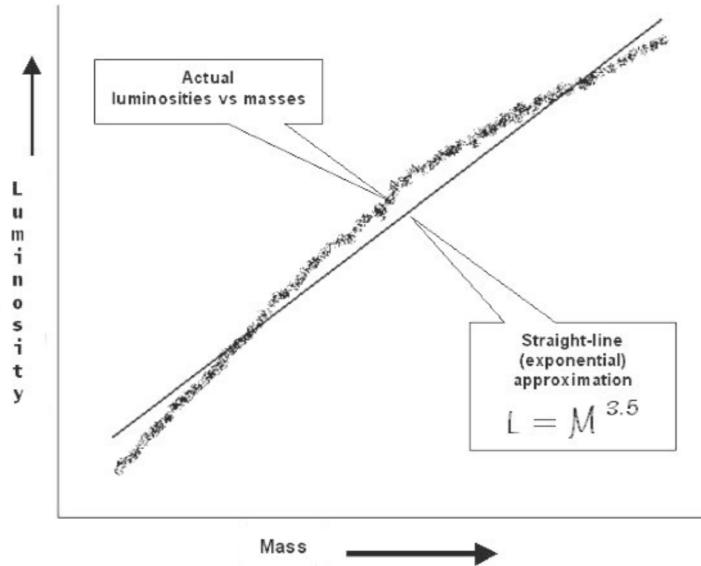


Figure 35: Proportionality.

6.3.4 Post MS Evolution

Stars spend about 90% of their time on the MS. After the core is exhausted of hydrogen, we enter the **sub-giant branch**. The core contracts and heats. Hydrogen burning shell gets formed around the Helium core. Energy release from H-envelope leads to expansion of intermediate layers, and thus a drop in temperature T .

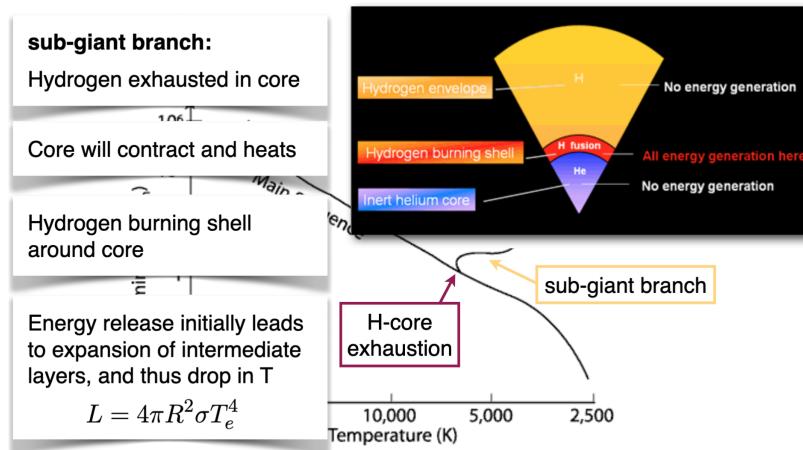


Figure 36: Sub-giant branch.

Then, we have vertical evolution on the HR diagram called the **red giant branch**. There is an ascent phase during which the core is contracting and hydrogen is burning in shells ferociously. The luminosity increases rapidly. You have mass loss because it's non-stable phase of evolution, shedding some of the outer envelopes (not much compared to the total mass of the star). The core continues to contract as material is dumped on the core.

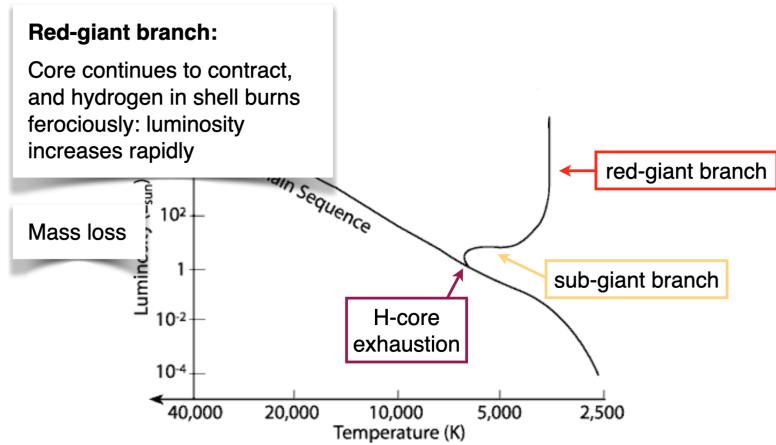


Figure 37: Red-giant branch.

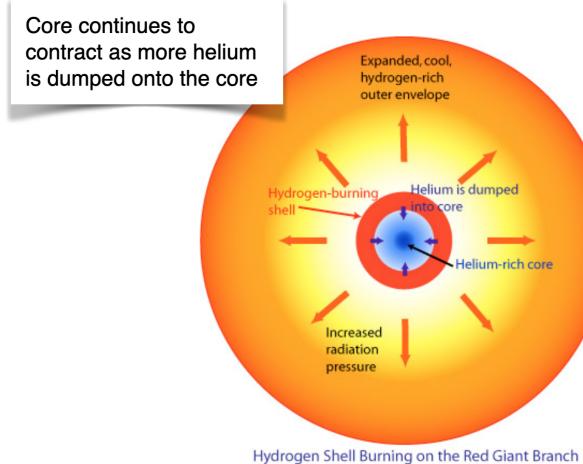


Figure 38: Red-giant branch 2.

We then hit the **helium flash**. This is when the core temperature gets to about $T \sim 10^8$ K. This is so high that helium starts to burn. The core is no longer in free-fall contraction. Boom, we have a burst of energy! The core expands, gravity weakens, and energy production rate goes down. The evolution is to higher temperature and lower luminosity. This is the **horizontal branch**.

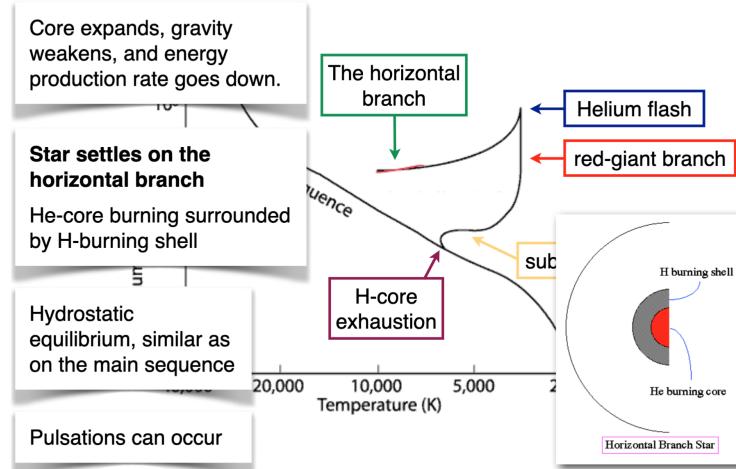
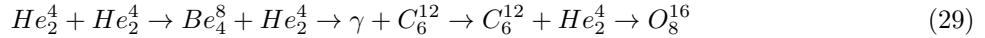


Figure 39: Horizontal branch.

The horizontal branch is characterized by core helium burning with a surrounding H-burning shell. Tons of details go here, and we don't really fully understand it. This is in hydrostatic equilibrium, however (getting us in the ballpark, sparing the details). This is when pulsations can occur! RR Lyrae are horizontal branch stars. Think of a pot of water with a top on and the pot-top is shaking, allowing for these pulsations.

6.3.5 Production of Carbon and Oxygen

We know how H turns into He. We fuse He with the triple alpha process (helium nucleus is an alpha particle). We basically have:



This is what is happening in horizontal branch stars! As you hit the end of the horizontal branch (i.e., not hot enough to fuse helium to carbon and oxygen). Carbon-oxygen core is not burning; the helium burning stars in a shell and the core mass increases. The star mass increases and we are now on the **asymptotic giant branch**. There is a lot of mass loss here. This is also the **double shell** burning phase, by the way.

6.3.6 Stellar Evolution Tracks

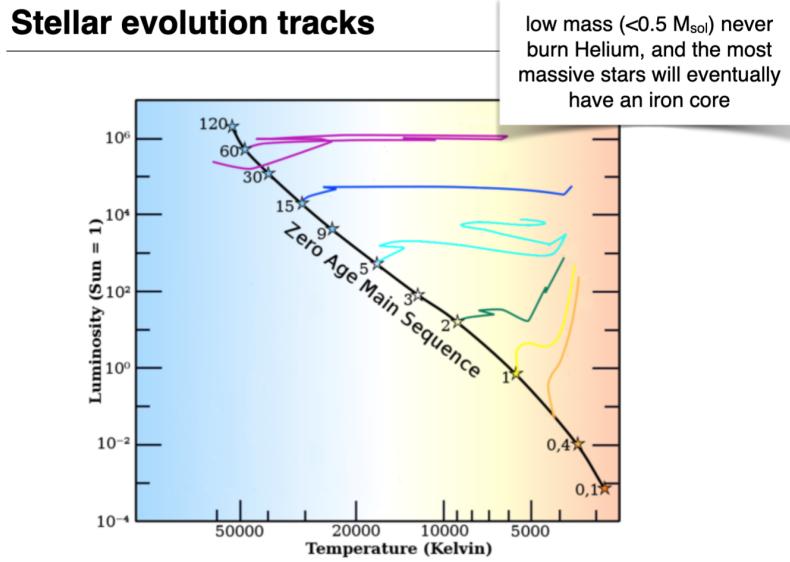


Figure 40: Stellar evolution tracks.

Evolution of a star at fixed initial mass is a “track.” The evolution of high mass stars is crazy! Low mass go mostly vertical, high mass stars bounce around horizontally.

6.3.7 Massive Stars

This is a crazy onion with tons of shells. They burn unbelievably fast, on the order of a few million years. The reactions are really complicated as well. The temperatures are ridiculous high, approaching $T > 10^9$ K in order to burn some of the elements.

6.3.8 Binding energy per nucleon

A is the number of nucleons. Less than Fe, energy is liberated via fusion. These reactions are exothermic and energy can be liberated with fusion. Elements heavier than Fe, energy is released with fission and these processes are exothermic.

Iron is most tightly bound and thus cannot fuse.

6.3.9 Evolution of a solar mass star

Eventually becomes a white dwarf which is supported by electron degeneracy pressure. White dwarfs are about the size of the Earth and about a solar mass.

6.3.10 Stellar remnants

Low-to-average mass ends in a white dwarf (up to 7 solar masses or so). Larger stars (between about 7 and 20 [roughly] solar masses) become neutron stars. They are a few solar masses and the size on Manhattan. They are supported by neutron degeneracy pressure. Lastly, the most massive stars become black holes.

7 September 15, 2021: Stellar Populations

7.1 Ingredients

- **Initial mass function:** How many stars of a certain mass M are born in a birth cloud $N(M)$? See below for the initial mass function discussion. This gives $N(m)$.
- **Isochrones:** Gives the luminosity L and temperature T of a star as a function of mass M , age t and metallicity Z of a star. This gives $L(M, t, Z)$ and $T(M, t, Z)$.
- **Spectral library:** Assigns a spectrum to each star using its luminosity L , temperature T , and metallicity Z . This gives $f_\nu(L, T, Z) = f_\nu(M, t, Z)$.

Simple stellar populations combine the IMF, isochrones, and stellar spectra to make $f_{\nu,ssp}(t, Z)$.

7.2 Initial Mass Function

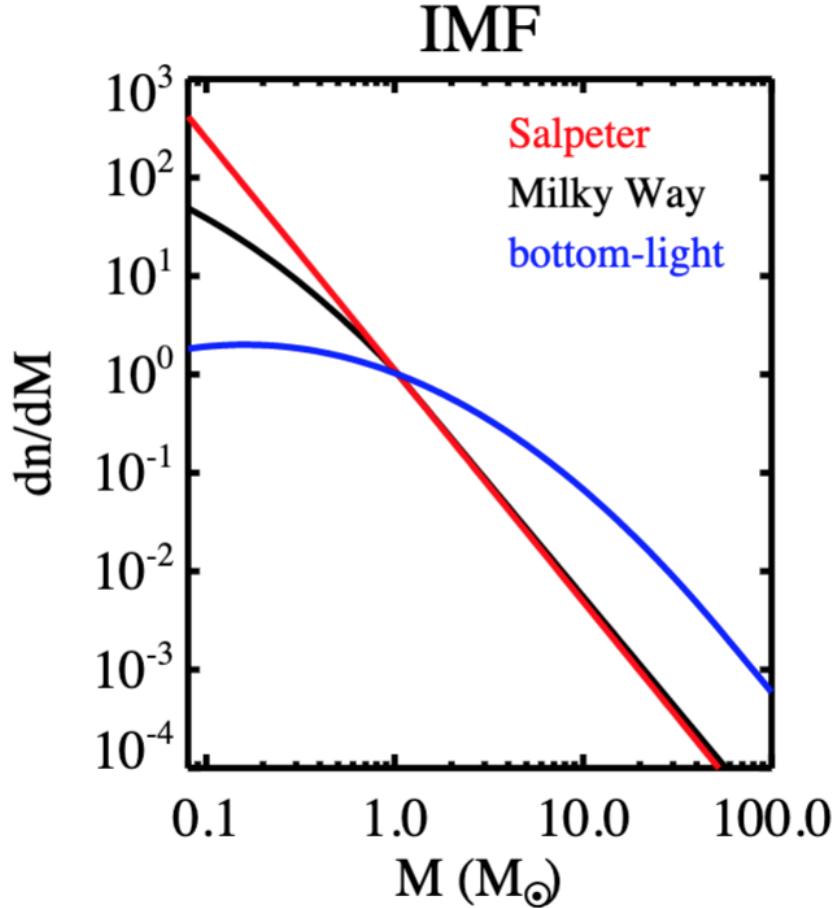


Figure 41: IMF

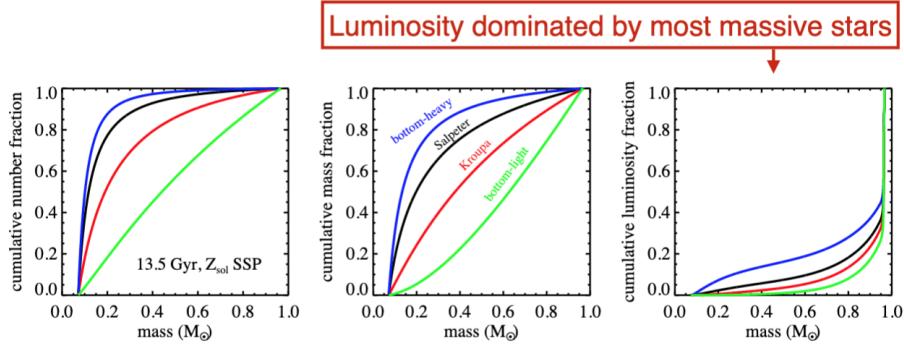


Figure 42: IMF cont.

This is the distribution of mass **at birth**, which makes it really difficult to measure. Qualitatively, the Salpeter IMF is a straight line in log-log space:

$$\xi(m) = \xi_0 M^{-\alpha}, \text{ where } \alpha = 2.35 \quad (30)$$

Sometimes we will see

$$\xi(m) = \xi_0 M^{-(1+\Gamma)}, \text{ where } \Gamma = 1.35 \quad (31)$$

The ξ_0 is a normalization. One question we might have: how many stars exist between two masses?

$$N = \int_{m_1}^{m_2} \xi(m) dm \quad (32)$$

For the Salpeter, we have:

$$N = \frac{\xi_0}{1.35} \left[\frac{1}{m_1^{1.35}} - \frac{1}{m_2^{1.35}} \right] \quad (33)$$

Sometimes you see:

$$\frac{dn}{dm} \propto m^{-2.35} \quad (34)$$

Note that we are assuming a universal IMF which we will take to be true. This is not necessarily the case.

7.3 Evolving a simple stellar population

A galaxy spectrum is dominated by the most massive stars still alive. The galaxy becomes redder and fainter over time, as massive stars die.

7.4 Evolution of Color and Mass-to-Light Ratio

Galaxies become redder in time, and the mass-to-light ratio increases in time.

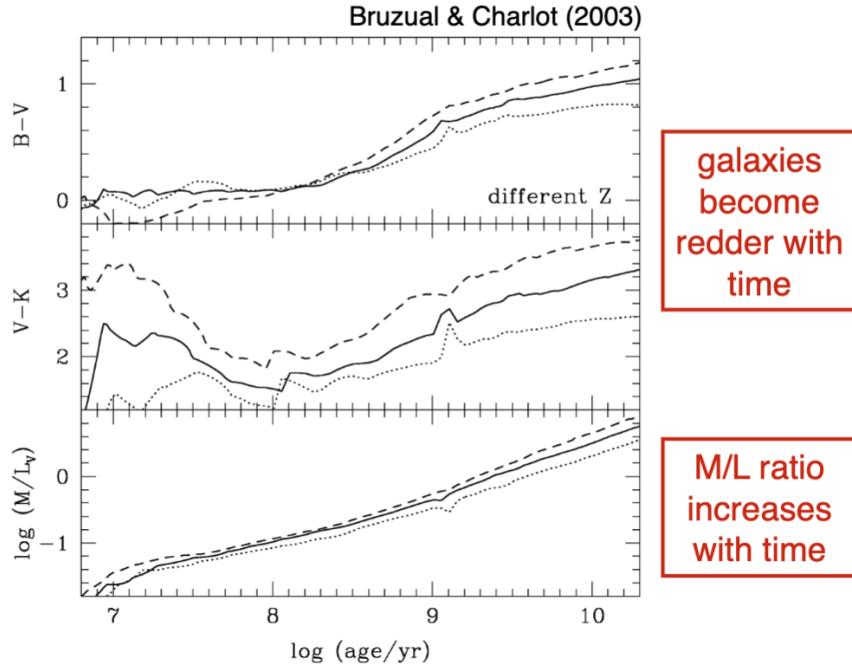


Figure 43: Evolution of color and M/L ratio.

7.5 Dust Attenuation Law

You typically see $A(\lambda)/A(V)$. This shows that the cross-section/optical depth at bluer wavelengths is larger than at redder wavelengths. In effect, you are losing blue light.

Attenuation is the net effect of the removal of light. There are many causes, however:

- Extinction (how much light is scattered out of the line of sight)
- Geometry (what's the geometry of a dust particle, what's the geometry of the galaxy relative to you)
- Radiative transfer

We typically describe attenuation with A_λ :

$$A_\lambda = \Delta m_\lambda = (m - m_0)_\lambda \quad (35)$$

This is really a magnitude difference (flux ratio) and can also be written as:

$$F_\lambda = F_{\lambda,0} 10^{-0.4A_\lambda} = F_{\lambda,0} e^{-\tau_\lambda} \rightarrow A_\lambda = 1.086 \tau_\lambda \quad (36)$$

It is also commonly to talk in terms of color excess $E(B - V)$:

$$E(B - V) = A_B - A_V \quad (37)$$

7.6 Dust Attenuation Curve

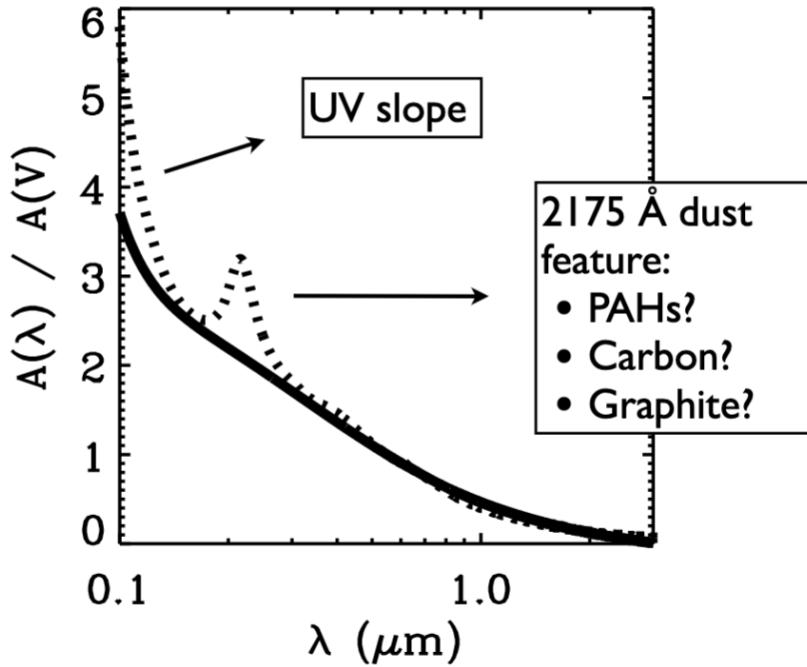


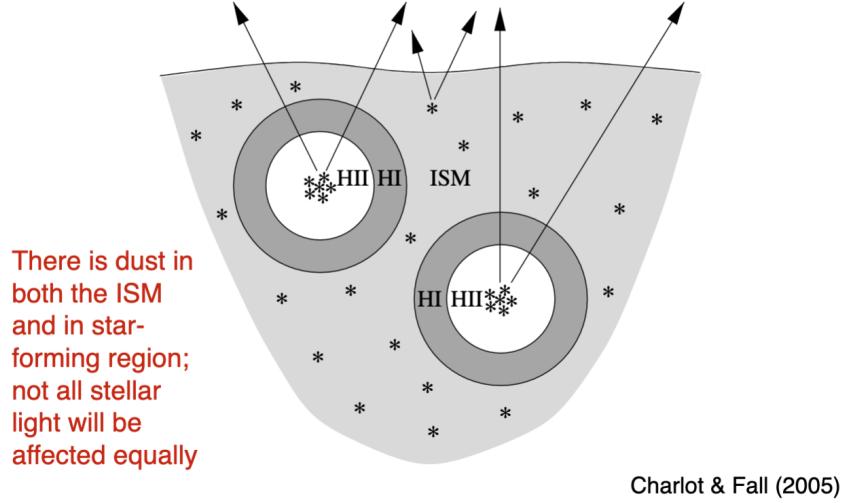
Figure 44: Dust attenuation law for starburst (solid) and MW.

You typically see $A(\lambda)/A(V)$ where the center of V is 5500\AA . We also have the “attenuation law” which is:

$$R_\lambda = \frac{A_\lambda}{E(B - \lambda)} \quad (38)$$

Here, R_λ is the slope of the attenuation curve. For the MW, this is $R_V = 3.1$. Note that often we assumed a grey atmosphere/slab of material.

Dust geometry



27

Figure 45: Dust geometry implications.

7.7 Calzetti Law

Not all attenuation laws are the same. The Calzetti law is popular for starburst galaxies, where the maximum of the attenuation is 3.5 as opposed to 6 for the MW. Notice, too, that the MW has the 2175\AA bump, which might be caused by:

- PAHs
- Graphite
- Carbon

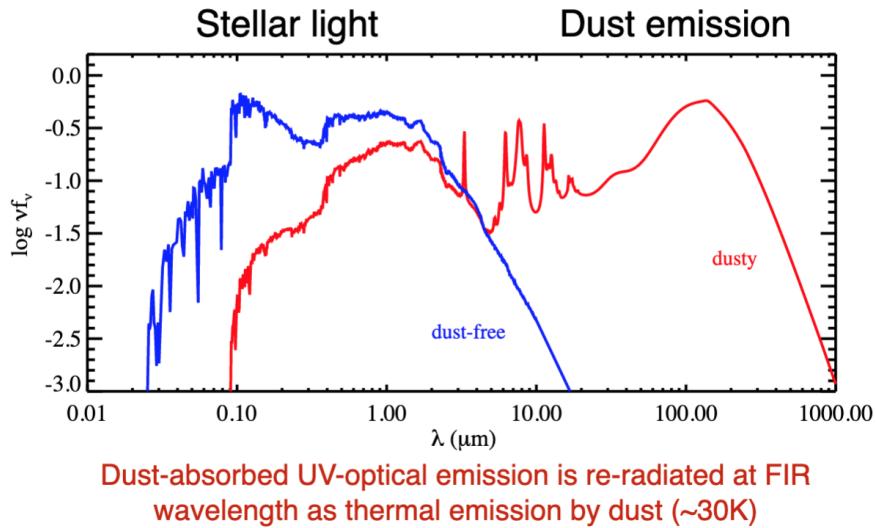


Figure 46: Sample Galaxy SED. Dust re-radiated at 30K which is warm dust.

8 September 17, 2021: Stellar Populations continued

8.1 An error from last time...

Last time, we had used a base e instead of a base 10 somewhere. Just correcting that.

$$A_\lambda = \Delta m_\lambda = (m - m_0)_\lambda = -2.5 \log_{10} \left(\frac{F_\lambda}{F_{\lambda,0}} \right) \quad (39)$$

8.2 Dust Emission Models (Draine and Li, 2007)

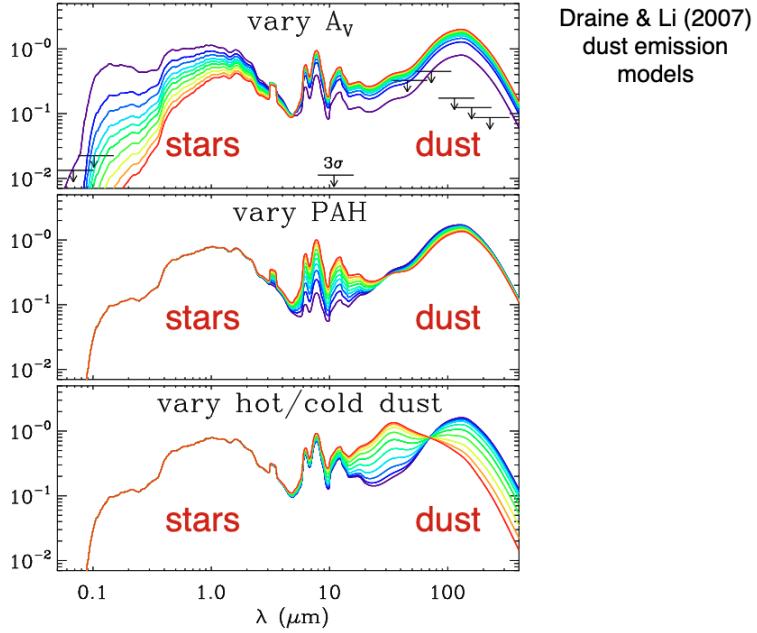


Figure 47: Varying A_V (increasing) dims starlight and increases dust light. Varying PAHs (increasing PAHs) peak near a few microns and cause many features there. Varying dust temperature (increasing temperature) peaks to smaller wavelengths. Note that varying PAH and temperature does not affect the stars that much.

8.3 Negative k-correction

For higher redshifts, the dust peak becomes fainter and shifts to longer wavelengths. However, the flux density at 1mm stays approximately constant and thus is independent of redshift. Because the spectrum won't fade/shift down, the k correction will be negative.

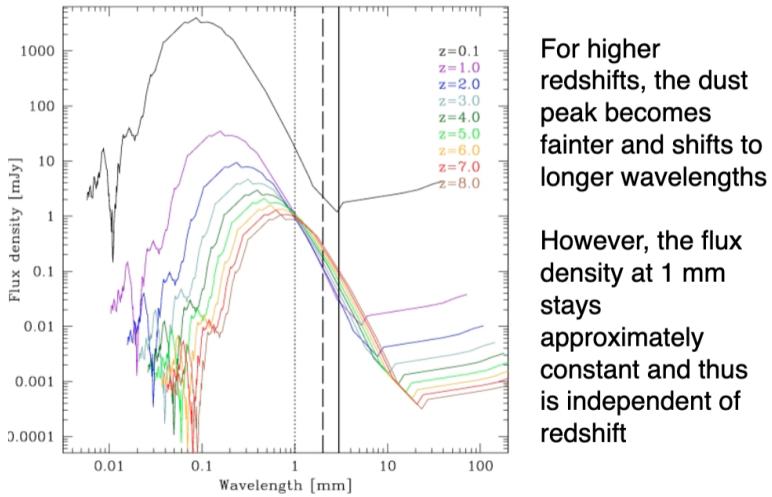


Figure 48: Negative K-correction

8.4 Simple Stellar Populations

For an SSP, we can write down the flux:

$$f_{SSP}(t, Z) = \underbrace{\int_{M=m_{lo}}^{M=m_{up}(t)} f_{\text{star}}[T_{eff}, \log g(M), t, Z] \Phi(M) dM}_{\text{integrate over masses}} \quad (40)$$

What are those integration limits? Well m_{lo} is the lowest mass star possible (about 0.08 solar masses). Higher mass stars die off and thus it is a function of time. Also notice that we are assuming the IMF is a function of mass alone. This is **controversial**.

8.5 Star formation history of a simulated galaxy

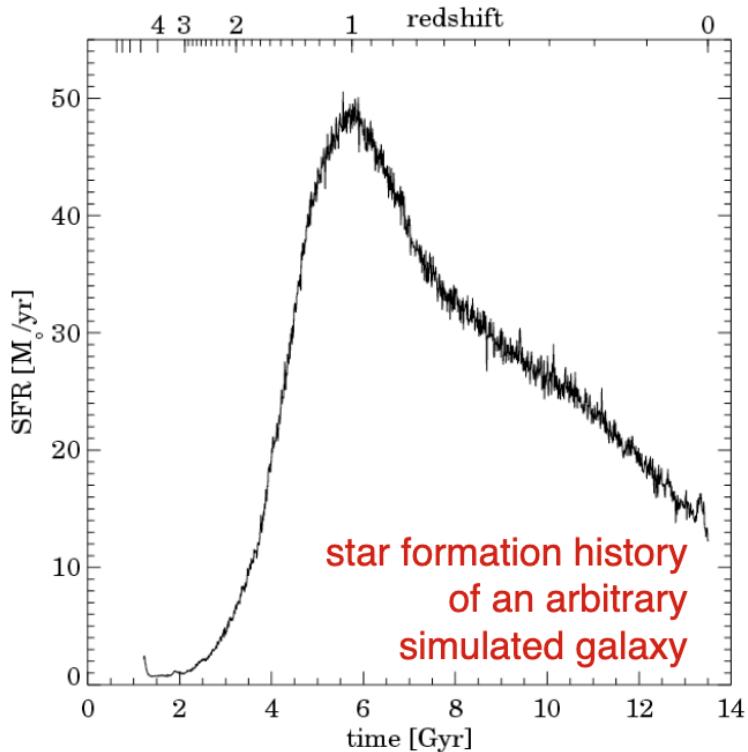


Figure 49: SFR of arbitrary galaxy.

Broadly, star formation rises rapidly, reaches a peak near $z \sim 1$, and then declines toward the present. There are wiggles over time, too, which might be bursty events.

Instead, we typically adopt a simplified “parametrized” star-formation rates (histories). It is usually parametrized by a smooth function. First, we have a **falling star formation history/exponential decay**:

$$SFR \propto e^{-t/0.1 \text{ Gyr}} \quad (41)$$

This is popular because it's a one-parameter model! This is also called a τ model.
Next, we have the **delayed exponential**:

$$SFR \propto te^{-t/\tau} \quad (42)$$

There is one other parametrization, called a **rising star formation history**:

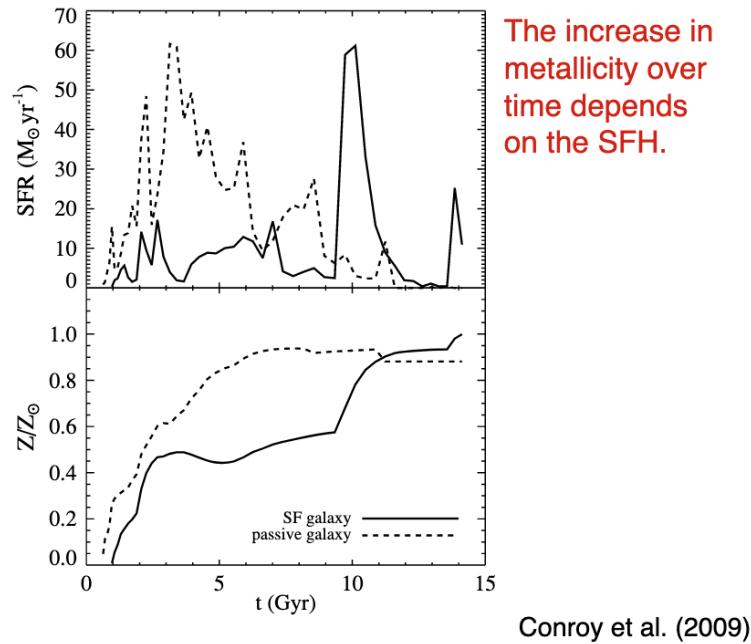
$$SFR \propto t \quad (43)$$

And a rising-tau model:

$$SFR \propto e^{t/\tau} \quad (44)$$

8.6 Star formation and enrichment history

The increase in metallicity over time depends on the star formation history.



Conroy et al. (2009)

Figure 50: Caption

8.7 Fractional contribution to the total flux

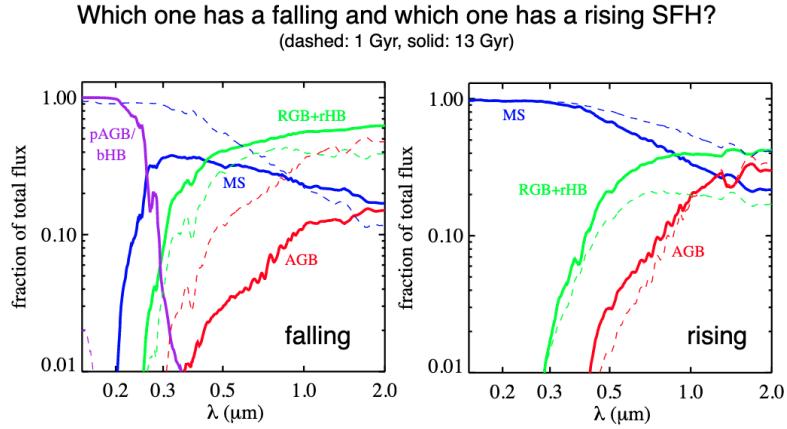


Figure 51: Fractional contribution to the total flux question.

Which one has a rising and a falling star formation history?

In a star forming galaxy, the main sequence always dominates. In a rising history, it's **always** dominated by young luminous stars. This makes the right side rising.

The left side shows that the other weird blue stars dominate instead of blue main sequence stars. This means that larger fractional contributions from post-MS evolution for a falling history.

This plot is important!

8.8 Fractional contribution to the total flux for a star forming galaxy

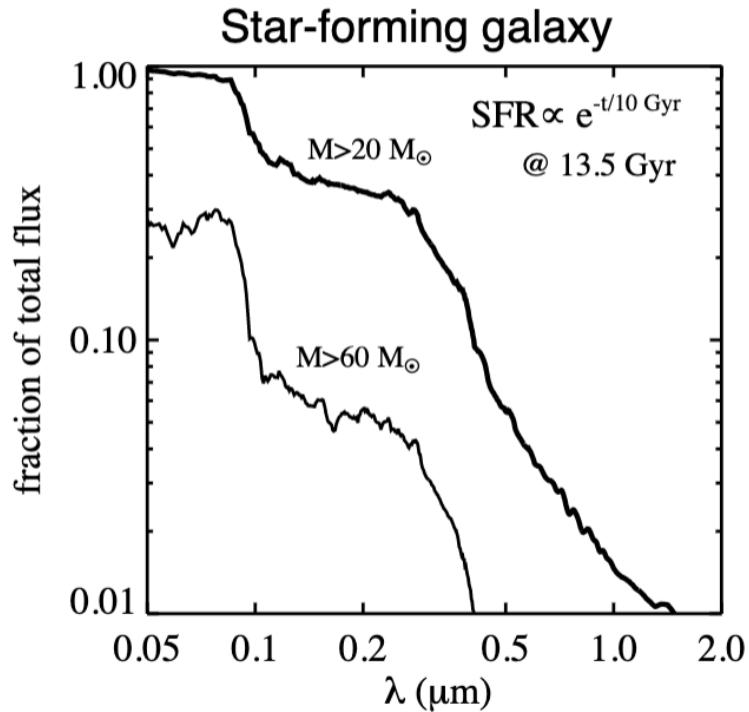


Figure 52: Fractional contribution 2.

Even though $60M_{\odot}$ stars are luminous, they are RARE. Thus, star forming galaxies are really characterized by $\sim 10 - 20M_{\odot}$ stars and not **super-massive** stars.

8.9 Light-weighted age as a function of wavelength

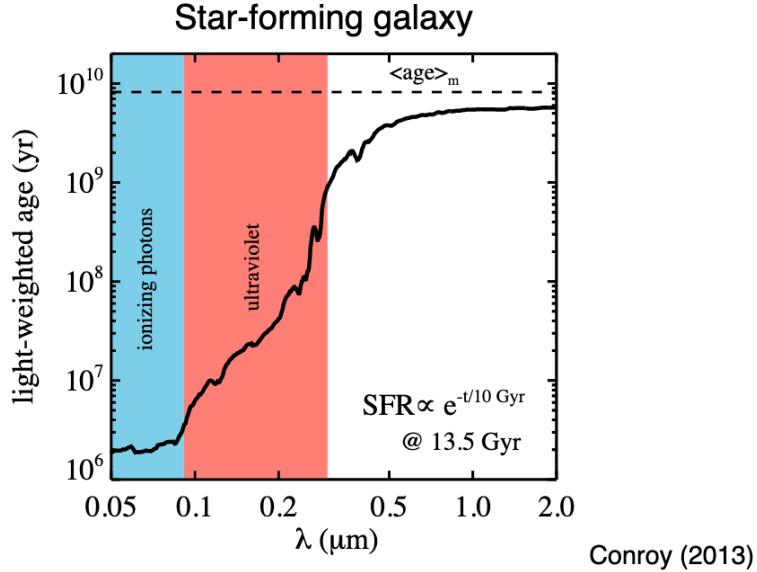


Figure 53: Light-weighted age as a function of wavelength.

The age that we measure is dependent on our wavelength, since everything is light-weighted. In the infrared, we are sensitive to older populations and thus get closer and closer to the real mass-weighted age. This **shows the importance of multi-wavelength observations**. It also shows how bad UV observations are at predicting galaxy ages. Also note that the light-weighted age only asymptotes to the mass-weighted age.

8.10 Nebular Emission

One thing we have totally neglected is the effect of nebular emission, which is one of the main ways that we see star-formation in young galaxies.

8.11 Complex Stellar Population

$$f_{csp}(t) = \int_{t'=0}^t \int_{Z=0}^{Z_{max}} \left[SFR(t-t') P(Z, t-t') f_{ssp}(t', Z) e^{-\tau(t')} + A f_{dust}(t', Z) \right] dt' dZ \quad (45)$$

where t' is the age of the population, P is the metallicity distribution function, f_{ssp} is the integral we had before, $e^{-\tau(t')}$ is an extinction term from dust, and the last term $A f_{dust}$ represents dust emission (with A being a normalization to ensure energy conservation).

Computationally, this is a very annoying calculation to do.

8.12 Fitting broadband photometry

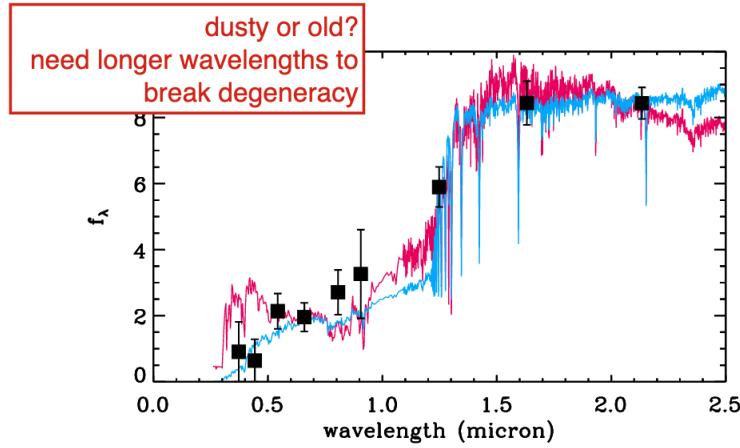


Figure 54: Need full SED to break degeneracies.

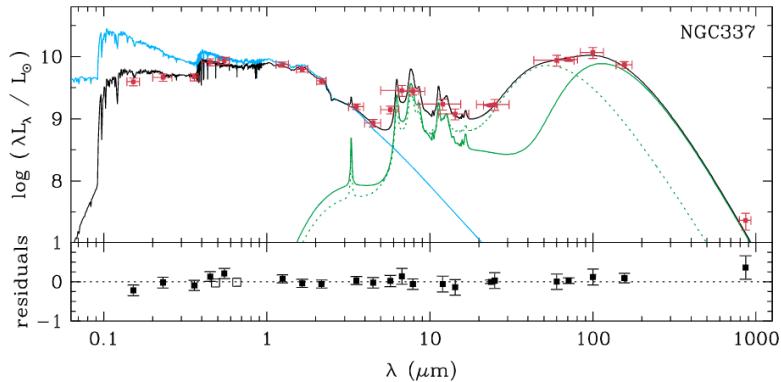


Figure 55: Full spectrum SED.

8.13 Fitting Spectra without parametrizing the SFH

8.14 Uncertainties in the stellar population modeling

8.15 Thermally pulsing AGB phase

These molecular features are imparted because of the AGB star. These are super bright and thus dominate the spectrum. The timescales over which they change their spectra can be short, too (days)!

8.16 Challenges of SPS Modeling

- Degeneracies between age and metallicity
- SPS models are not perfect; various stages of stellar evolution are poorly understood (AGB, HB, Wolf-Rayet stars, binary evolution, convection, rotation, etc)

- Dust law, initial mass function, and star formation history usually assumed
- Massive stars with low M/L will dominate the spectra
- Etc.

Challenges in stellar population synthesis (SPS) modeling

- Degeneracies between age and metallicity
- SPS models are not perfect, various stages of stellar evolution that are poorly understood:
 - (TP-)AGB, horizontal branch, blue stragglers, post-AGB, wolf-rayet stars, convection, rotation, binary evolution, etc.
- Dust law, initial mass function, and star formation history usually assumed
- Massive stars with low M/L will dominate the spectrum
- etc.

Figure 56: Challenges.

9 September 22, 2021: Gas and Star Formation

9.1 IMF Completion

We had forgotten a few things. The total mass from an IMF is given by:

$$M_* = \int_{m_1}^{m_2} m \xi(m) dm \quad (46)$$

An equivalent expression for luminosity:

$$L_* = \int_{m_1}^{m_2} L(m) \xi(m) dm \quad (47)$$

One other thing, related to the stars...what about the lifetime on the main sequence?

$$\tau \propto \frac{M}{L} \rightarrow \tau = 10^{10} \frac{M}{L} \quad (48)$$

Let's explicitly state something about star formation rates, too. Mathematically, a star formation rate is defined as:

$$SFR = \frac{dM}{dt} = \dot{M} \quad (49)$$

For example, a τ model is:

$$\frac{dM}{dt} = C e^{-t/\tau} \quad (50)$$

9.2 Star formation in the nearby Universe

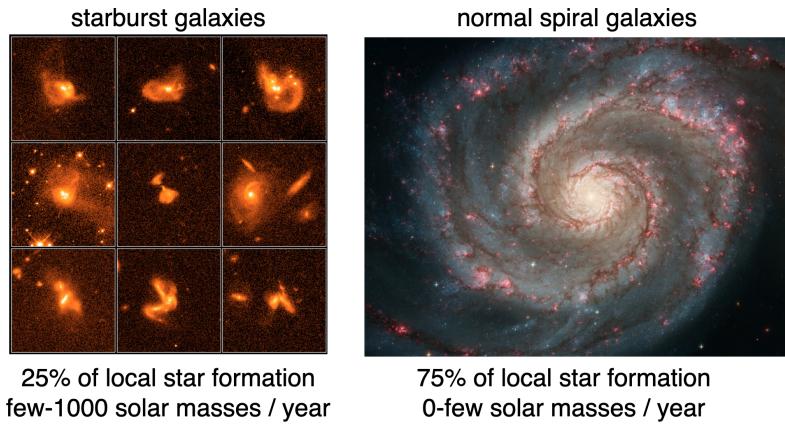


Figure 57: Starbursts vs. Normal Galaxy

At local z , roughly 3/4 of the star formation happen in normal spiral galaxies. The rates range from 0 to a few solar masses per year.

The other quarter of star formation is taking place in star burst galaxies. These have SFRs from a few to ~ 1000 solar masses per year.

Stars form in giant molecular clouds (GMCs). They are roughly 10^5 to 10^6 solar masses. They extend over a few tens of parsecs, and they have temperatures near 10 Kelvin.

We eventually get cold enough that molecules form. Thus the outsides of clouds are less dense than the insides. Outside of the cloud, hot stars pound the surface with radiation (lots of **self-shielding** interior). There's molecular hydrogen, too, which protects from dissociation of molecules. Eventually gravity takes over, and you form a star!

Giant Molecular Clouds (GMCs)



Figure 58: GMCs

9.3 Mass function of dense cores in GMCs (Alves+2007)

Star formation is very inefficient. For a couple of solar masses of stars, you need a TON of gas to make that happen.

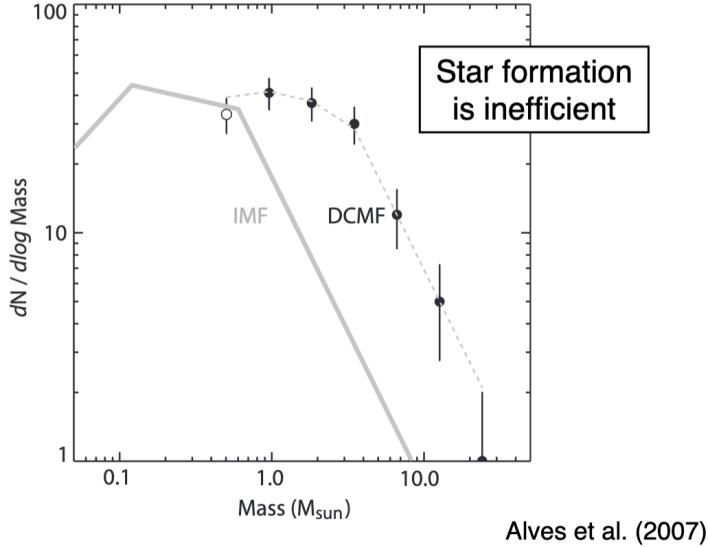


Figure 59: GMC mass function.

9.4 Jeans Mass and Cloud Fragmentation

If you balance gravity against the ideal gas law, you can get the Jeans mass!

The **Jeans mass**:

$$M_J \propto \frac{T^{3/2}}{\rho^{1/2}} \quad (51)$$

The cloud collapses if the mass of the cloud is greater than the Jeans mass:

$$M_{CLOUD} > M_J \rightarrow \text{COLLAPSE!} \quad (52)$$

In details:

$$\underbrace{\rho a}_{\text{accel.}} = \underbrace{-\frac{dP}{dr}}_{\text{pressure force per volume}} - \underbrace{\frac{GM_r}{r^2}\rho}_{\text{gravity per volume}} \quad (53)$$

There are two phases, roughly. This is summarized in the figure and lists. **In phase one...:**

- Photons can easily escape
- $T \sim \text{constant}$ (isothermal)
- Gravity force / volume $\propto \frac{M^2}{R^5}$
- Pressure force / volume $\propto \frac{MT}{R^4}$
- For constant T , gravity wins: keeps collapsing

In phase two...:

- Clouds become too dense for photons to escape
- Adiabatic: $T \propto \frac{M^{2/3}}{R^2}$
- Pressure force increases $\propto \frac{M^{5/3}}{R^6}$
- Hydrostatic equilibrium
- Further collapse using Kelvin-Helmholtz contraction

Note that this is all pre main sequence, so KH contraction really is powering at this point. Then, fusion takes over.

Another useful thing to keep in mind is the **freefall timescale** which is generally shorter by a few orders of magnitude than the KH timescale:

$$t_{ff} \propto \frac{1}{\sqrt{G\rho}} \quad (54)$$

Cloud collapse

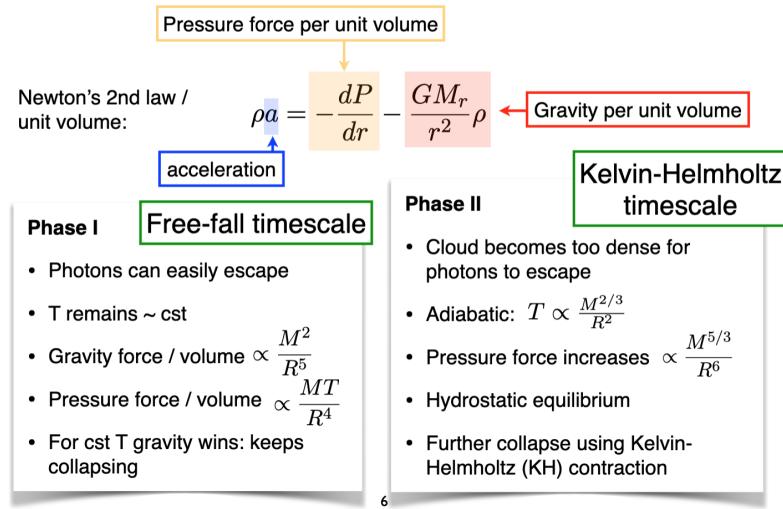
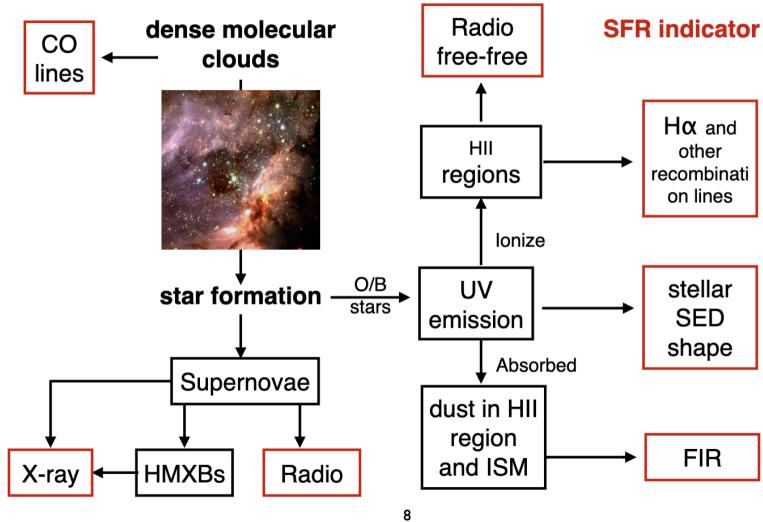


Figure 60: Cloud collapse.

9.5 Star formation rate indicators (recipes)

Star formation rate (SFR) indicators



8

Figure 61: SFR Indicators.

Fundamental question: My galaxy has massive stars that we cannot necessarily observe. Can we find another indicator that there is recent or ongoing star formation?

Note that none of this works if we don't have O and B type stars since they have UV emission.

Let's step through SFR indicators.

9.6 UV Emission

The UV is really great to see **massive stars greater than $3M_{\odot}$, or so** (main sequence stars with $T \gtrsim 10^4$ K with lifetimes of 10 – 200 million years. We can use this emission (corrected for dust with β slope) can be used to estimate the SFR of a galaxy.

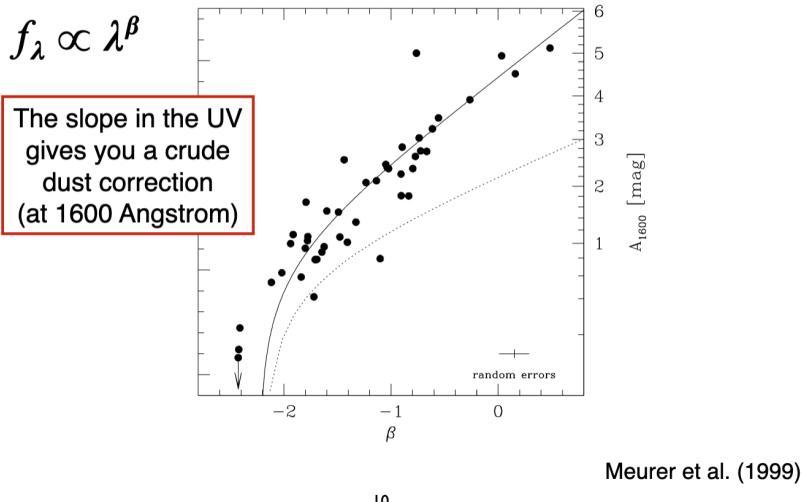


Figure 62: Correction with β slope.

Also note that $A_{UV} \sim 8E(B - V)$ so dust can be really damaging. In practice, we use a **beta index** which is the slope between a far-UV and near-UV channel. The slope is β . You can assume that $F_\lambda \propto \lambda^\beta$ to correct for the dust.

We are measuring the very high mass end of the IMF, and then extrapolating to lower and lower masses. This is part of the reason the IMF is so important.

Assumptions:

- Dust assumption for β correction.
- Need to assume an IMF.

If you take the SPS model, correct for dust, and assume an IMF, you get the **Kennicutt SFR (98)** relations:

$$SFR = 1.4 \times 10^{-28} L_\nu \frac{\text{ergs}}{\text{s} \cdot \text{Hz}} \quad (55)$$

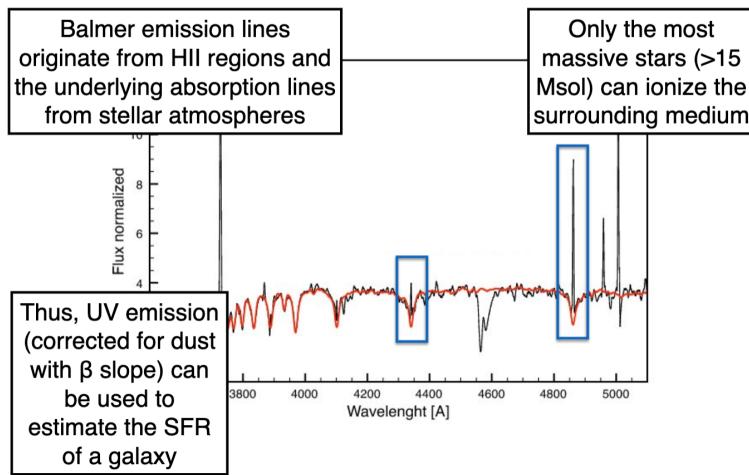
where L_ν is typically over $\lambda = [1500, 2800]\text{\AA}$.

9.6.1 Correction UV Emission using Beta Slope (Meurer+1999)

The slope in the UV gives you a crude dust correction (at 1600 Angstrom). This is all that is available at high z . Lower z has things like more SED points.

9.7 Recombination Lines

II. Recombination lines



II

Figure 63: Recombination lines as SFR indicator.

Balmer emission lines (lower state $n = 2$) originate from HII regions and the underlying absorption lines from stellar atmospheres. Only the most massive stars ($\gtrsim 15 M_{\odot}$) can ionize the surrounding medium. Thus, UV emission (corrected for dust with β slope) can be used to estimate the SFR of the galaxy.

The continuum is set by all the stars, and the emission lines are due to the most massive stars ionizing the environment. Let's first look at $H\alpha$.

Let's focus on stars with $M > 15 M_{\odot}$, $T \gtrsim 3 \times 10^4$ K, and thus only O-type and B0-type stars.

If you can measure both $H\alpha$ and $H\beta$, we know that $H\alpha/H\beta = 2.8$. If this is different, we need to correct for the dust changing this ratio.

There is an equivalent **Kennicutt Recipe (98)**:

$$\text{SFR} = \frac{L(H\alpha)}{1.26 \times 10^{41} \frac{\text{erg}}{\text{s}}} \quad (56)$$

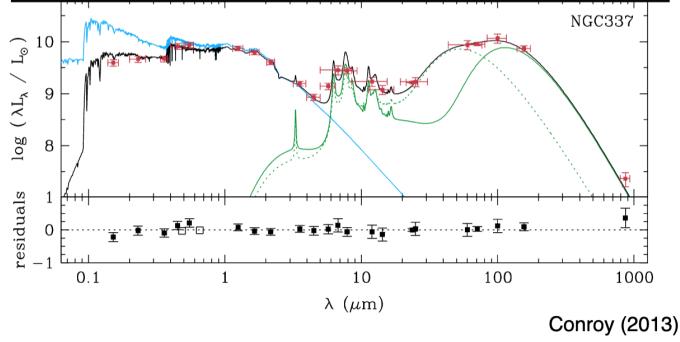
Note that this is only sensitive to the most massive stars and thus we are incredibly sensitive to the IMF. Here are some of the assumptions:

- Assume a metallicity.
- Dust correction.
- IMF is good since we are measuring only the most massive.
- Binary stars can effect this.
- Others.

9.8 FIR Emission

Full spectral energy distribution of a star-forming galaxy

FIR emission mostly originates from attenuated UV light from massive stars, i.e., it serves as a SFR indicator. In order to catch both the unobscured and obscured star formation activity, we need to combine the UV and FIR SFRs



13

Figure 64: FIR emission as SFR indicator.

FIR emission mostly originates from attenuated UV light from massive stars, i.e., it serves as an SFR indicator. In order to catch both the unobscured and obscured star formation acitivity, we need to combined UV and FIR SFRs.

There's a **Kennicutt relation**:

$$\text{SFR} = \frac{L_{FIR}}{2.2 \times 10^{43} \frac{\text{ergs}}{\text{s}}} \quad (57)$$

One strength of this is that we can probe regions of high extinction.

9.9 X-ray emission (Menou et al 2012)

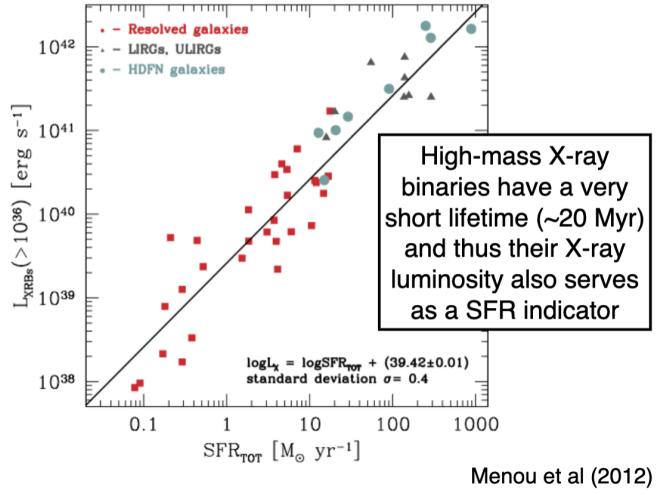


Figure 65: X-ray emission as SFR.

High-mass X-ray binaries have a very short lifetime (~ 20 Myr) and thus their X-ray luminosity also serves as a SFR indicator.

X-ray observations are hard to get, and thus this is not used that often.

9.10 Radio Emission

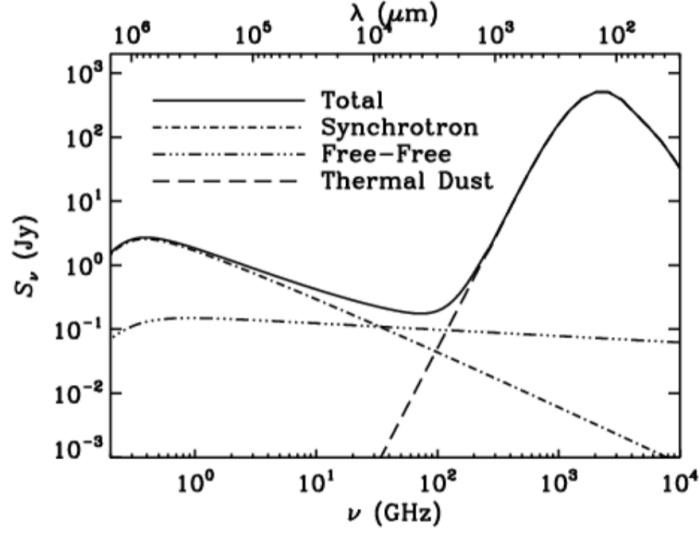


Figure 66: Free-free emission window where it is dominant.

Synchrotron, thermal free-free emission. There is a small, small window where free-free emission dominates. The wavelengths are really long and thus not affected by dust. There are very few radio telescopes measuring near 30 GHz, and thus this is hard.

If you could do it, this would be the gold standard since there's no dust and coming from HII regions.

9.11 Measuring gas surface densities

HI is diffuse cold case, CO traces stars forming.

We have two phases of gas – atomic hydrogen and molecular CO. The total gass surface density is thus $\Sigma_{gas} = \Sigma_{HI} + \sigma_{H_2}$. H_2 is really hard to observe, so we end up measuring:

$$\Sigma_{gas} = \Sigma_{HI} + \alpha_{CO} I_{CO} \quad (58)$$

where α_{CO} takes us from CO to H_2 .

The total gas content of $HI + CO$ turns out to be a good tracer of dust. Thus, α is calibrated locally and extrapolated outward.

This is **metallicity dependent** since clouds are less-shielded.

9.12 Kennicutt-Schmidt Law (1998)

There is a really tight correlation between gas surface density and star formation rate. Often written as the **KS Relation**:

$$\Sigma_{SFR} \propto \Sigma_{gas}^n, \text{ where } n = 1.4 \quad (59)$$

10 September 24, 2021: Gas and star formation, chemical evolution

10.1 Group Exercise

Challenges in stellar population synthesis (SPS) modeling

- Degeneracies between age and metallicity
- SPS models are not perfect, various stages of stellar evolution that are poorly understood:
 - (TP-)AGB, horizontal branch, blue stragglers, post-AGB, wolf-rayet stars, convection, rotation, binary evolution, etc.
- Dust law, initial mass function, and star formation history usually assumed
- Massive stars with low M/L will dominate the spectrum
- etc.

Figure 67: Group exercise.

Imagine we take two measurements in the UV at 1500\AA and 2500\AA (far and near UV). The SFR is given by:

$$\dot{m} = C \times L_{UV} \quad (60)$$

Well, we first need **distance** to go from magnitude to a flux. We also need an estimate of the **dust correction**. This gives us L_{UV} .

The constant comes from theory. How do we get that? We need to assume an **IMF**, first of all. We also need a **metallicity** and assumed **star formation history**. We need a mass-luminosity relation to get an observable, which we can get from isochrones (more generally, stellar evolution model). One last really subtle thing (not as important for the UV), but it's the **escape fraction** (fraction of photons coming from star).

This comes down to basically assuming a Salpeter IMF, one metallicity (solar), a constant star formation history, picking a stellar evolution model, and then an escape fraction of 1.

If we saw this edge on, we need to trust our dust correction EVEN MORE!

If you could pick one more wavelength to observe, what would you pick and why? It might be best to get FIR measurements so that we can improve our measurement of β by seeing how much light is obscured.

We might also instead measure high mass X-ray binaries. Thus we have an independent handle (aside from O and B stars). Thus we have two different measurements of \dot{M} .

Another common SFR tracer is $\text{H}\alpha$. If we wanted to measure the $\text{H}\alpha$ SFR of this galaxy, what do we do? We can go to a telescope and put a narrow band filter on our data and sum the entire image. The UV is produced from $M > 3M_\odot$, whereas $\text{H}\alpha$ is dominated by $M > 15M_\odot$ stars. **Do we expect that the SFRs agree between the X-ray and UV?** In an ideal world, yes they should be the same. However, if we plot the ratio SFR in $\text{H}\alpha$ to SFR in UV, we find that the data is a function of stellar mass at low mass galaxies. This tells us that some assumption is being violated, and this is an active part of research. **Discrepancy in $\text{H}\alpha$ and UV SFR tracers is huge!**

We can also take observations at the **free-free** part of the radio spectrum (where synchrotron is small and free-free dominates). This would be a dust-free measurement of the SFR.

10.2 Gas components of a galaxy

The easiest gas component of a galaxy to measure is HI from the 21 cm line. HI doesn't get dense enough on its own, so stars really form from H_2 (molecule hydrogen). This can get colder and thus denser (because pressure decreases due to temperature). H_2 only has a quadrupole moment because it's symmetric. This is **really weak**, and happens around $\lambda \sim 30\mu m$. Observing H_2 directly is really, really difficult. Instead, we observe CO, which is easily observed. We observe CO near $\lambda \sim 2.6$ mm. We want to know:

$$\Sigma_{gas} = \Sigma_{H_2} + \Sigma_{HI} \quad (61)$$

This is also written as:

$$\Sigma_{gas} = \alpha_{CO} I_{CO} + \Sigma_{HI} \quad (62)$$

Really, we can measure I_{CO} from the telescope and make assumptions on what α_{CO} is. Often, you do not really get a clean measurement.

Instead, you infer the gas content from the dust content. The dust traces the gas, so sometimes this is written as:

$$\Sigma_{gas} = \underbrace{\delta_{GDR}}_{\text{gas-to-dust ratio}} \quad \Sigma_{dust} = \alpha_{CO} I_{CO} + \Sigma_{HI} \quad (63)$$

We can infer δ_{GDR} to infer a value of $\alpha_{CO} I_{CO}$.

Challenges in stellar population synthesis (SPS) modeling

- Degeneracies between age and metallicity
- SPS models are not perfect, various stages of stellar evolution that are poorly understood:
 - (TP-)AGB, horizontal branch, blue stragglers, post-AGB, wolf-rayet stars, convection, rotation, binary evolution, etc.
- Dust law, initial mass function, and star formation history usually assumed
- Massive stars with low M/L will dominate the spectrum
- etc.

Figure 68: Looking at HI only, we don't see strong correlation. If we have CO only, the same is true. However, $HI + CO$ is very nicely correlated with gas surface density.

10.3 CO to H₂ Conversions

Challenges in stellar population synthesis (SPS) modeling

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- Dust law, initial mass function, and star formation history usually assumed
- Massive stars with low M/L will dominate the spectrum
- etc.

Figure 69: This has a metallicity dependence because at low metallicity, we expect less dust.

10.4 Kennicutt-Schmidt Law

$$\Sigma_{SFR} \propto (\Sigma_{gas})^n, \text{ where } n = 1.4 \quad (64)$$

Challenges in stellar population synthesis (SPS) modeling

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- SPS models are not perfect, various stages of stellar evolution that are poorly understood:
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- Dust law, initial mass function, and star formation history usually assumed
- Massive stars with low M/L will dominate the spectrum
- etc.

Figure 70: Kennicutt-Schmidt relation. Extremely famous. Relates star formation rate to gas density. What is going on in nature to tell us this correlation?

10.5 KS Law Explanations

One of the things we discussed was the gas consumption timescale (efficiency factor between gas conversion to stars). It turns out efficiency varies galaxy-to-galaxy. There are three theories.

- Efficiency should be constant (wrong).
- **Theory 1: Free Fall Collapse:** Star formation rate is proportional to gas density and free-fall time: $SFR \propto \frac{\rho}{t_{ff}} \propto \rho\sqrt{G\rho} \propto \rho^{3/2} \propto n^{1.5}$. This is close to Kennicutt-Schmidt! This was first posited in 92 by Larsen.
- **Theory 2: Orbital Argument:** Maybe the star formation rate scales with the dynamical (orbital time) of a galaxy. The result is $\Sigma_{SFR} = 0.017\Sigma_g\Omega_g$. Paper is 1997 from Elmegreen and Silk.
- **Theory 3: Molecular Gas Formation is Tough:** Maybe what varies is the ability to **make** gas. Maybe really $\Sigma_{SFR} \propto \Sigma_{H_2}$, and then we need to figure out how a galaxy can produce Σ_{H_2} and with what efficiency. Paper by Krumholz (with review in 2014). The corollary of this is that really low metallicity galaxies don't have CO, but they have stars! **At low metallicity, CO formation is not good and thus our story falls apart! Everyone has molecular hydrogen. Σ_{H_2} is probably just varying a lot!** Lots of theories for why.

Challenges in stellar population synthesis (SPS) modeling

- Degeneracies between age and metallicity
- SPS models are not perfect, various stages of stellar evolution that are poorly understood:
 - (TP-)AGB, horizontal branch, blue stragglers, post-AGB, wolf-rayet stars, convection, rotation, binary evolution, etc.
- Dust law, initial mass function, and star formation history usually assumed
- Massive stars with low M/L will dominate the spectrum
- etc.

Figure 71: Efficiency is more complicated. There is no one efficiency in galaxies.

10.6 The star formation law in atomic and molecular gas

THINGS team finds:

$$\Sigma_{SFR} = 10^{-2.1}\Sigma_{H_2}1.0 \quad (65)$$

Challenges in stellar population synthesis (SPS) modeling

- Degeneracies between age and metallicity
- SPS models are not perfect, various stages of stellar evolution that are poorly understood:
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- Dust law, initial mass function, and star formation history usually assumed
- Massive stars with low M/L will dominate the spectrum
- etc.

Figure 72: Data from THINGS team. We get the Kennicutt-Schmidt relation really well!

Challenges in stellar population synthesis (SPS) modeling

- Degeneracies between age and metallicity
- SPS models are not perfect, various stages of stellar evolution that are poorly understood:
 - (TP-)AGB, horizontal branch, blue stragglers, post-AGB, wolf-rayet stars, convection, rotation, binary evolution, etc.
- Dust law, initial mass function, and star formation history usually assumed
- Massive stars with low M/L will dominate the spectrum
- etc.

Figure 73: More dta

10.7 Molecular Fraction vs. Gas Surface Density

Really low surface densities have almost **NO H_2** . Less molecular gas is less star formation. Very tips of spiral arms is not as strong and thus less star formation. Same in dwarf galaxies (cannot pull gas in!).

Challenges in stellar population synthesis (SPS) modeling

- Degeneracies between age and metallicity
- SPS models are not perfect, various stages of stellar evolution that are poorly understood:
 - (TP-)AGB, horizontal branch, blue stragglers, post-AGB, wolf-rayet stars, convection, rotation, binary evolution, etc.
- Dust law, initial mass function, and star formation history usually assumed
- Massive stars with low M/L will dominate the spectrum
- etc.

Figure 74: Molecular gas fraction and surface density of gas.

10.8 Summary of Today

Challenges in stellar population synthesis (SPS) modeling

- Degeneracies between age and metallicity
- SPS models are not perfect, various stages of stellar evolution that are poorly understood:
 - (TP-)AGB, horizontal branch, blue stragglers, post-AGB, wolf-rayet stars, convection, rotation, binary evolution, etc.
- Dust law, initial mass function, and star formation history usually assumed
- Massive stars with low M/L will dominate the spectrum
- etc.

Figure 75: SUMMARY of today.