Final Project Computational Physics

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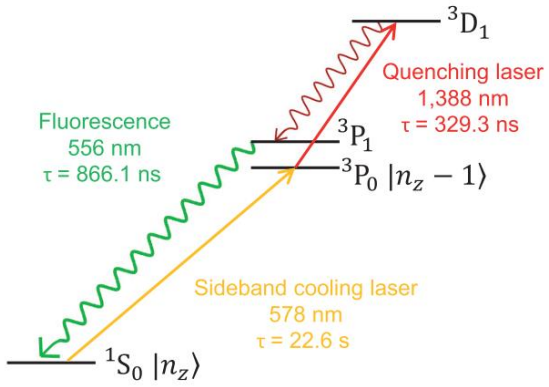
My final project simulates a type of Sisyphus cooling for the 171Yb atom. The level structure is as follows:



We can use the narrow line sideband cooling laser, or clock laser, to excite atoms to the 3P0 state from the 1S0 (ground) state.

This transition is very long lived so I approximate that spontaneous decay never happens.

Normally the 1388nm quenching laser is applied on resonance to excite atoms to the 3D1 state, where then can then spontaneously decay 1st to 3P1, then back to 1S0. However, if we apply the 1388nm laser off resonance it will cause an AC stark shift of the 3P0 and 3D1 levels by mixing them.

Since we use the F=1/2 state for both 3P0 and 3D1 we can write the Hamiltonian for both mF states for both 3P0 and 3D1 in the electric dipole approximation for the 1388nm coupling:

$$\begin{pmatrix} \hbar\,\omega_c & 0 & -\frac{e^{i\,t\,(\delta_s+\omega_s)}\,\hbar\,c_\pi\,\Omega_s}{2\sqrt{3}} & \frac{e^{i\,t\,(\delta_s+\omega_s)}\,\hbar\,c_m\,\Omega_s}{\sqrt{6}} \\ 0 & \hbar\,\omega_c & -\frac{e^{i\,t\,(\delta_s+\omega_s)}\,\hbar\,c_p\,\Omega_s}{\sqrt{6}} & \frac{e^{i\,t\,(\delta_s+\omega_s)}\,\hbar\,c_\pi\,\Omega_s}{2\sqrt{3}} \\ -\frac{e^{-i\,t\,(\delta_s+\omega_s)}\,\hbar\,c_\pi\,\Omega_s}{2\sqrt{3}} & -\frac{e^{-i\,t\,(\delta_s+\omega_s)}\,\hbar\,c_p\,\Omega_s}{\sqrt{6}} & \hbar\,(\omega_c+\omega_s) & 0 \\ \frac{e^{-i\,t\,(\delta_s+\omega_s)}\,\hbar\,c_m\,\Omega_s}{\sqrt{6}} & \frac{e^{-i\,t\,(\delta_s+\omega_s)}\,\hbar\,c_\pi\,\Omega_s}{2\sqrt{3}} & 0 & \hbar\,(\omega_c+\omega_s) \end{pmatrix}$$

This is done in the attached mathmatica notebook. The s subscript stands for Sisyphus, and refers to the 1388nm laser, the c subscript stands for clock and refers to the 578nm laser, w is an optical frequency, Ω is a Rabi frequency, δ is a detuning from resonance, and c represents the fraction of the 1388nm laser light with the polarization specified by the subscript (π, m= left hand circularly polarized, += right hand circularly polarized). This is too cumbersome to use, so we apply a unitary transformation

$$H' = UHU^{+} - U\frac{dU^{+}}{dT}$$

with U given by

$$\begin{pmatrix} e^{i\,t\,(-\delta_S+\omega_C)} & 0 & 0 & 0 \\ 0 & e^{i\,t\,(-\delta_S+\omega_C)} & 0 & 0 \\ 0 & 0 & e^{i\,t\,(\omega_C+\omega_S)} & 0 \\ 0 & 0 & 0 & e^{i\,t\,(\omega_C+\omega_S)} \end{pmatrix}$$

Mathmatica will apply this transformation to get the final light atom interaction Hamiltonian:

$$\begin{pmatrix} \hbar\,\delta_S & 0 & -\frac{\hbar\,c_\pi\,\Omega_S}{2\sqrt{3}} & \frac{\hbar\,c_m\,\Omega_S}{\sqrt{6}} \\ 0 & \hbar\,\delta_S & -\frac{\hbar\,c_p\,\Omega_S}{\sqrt{6}} & \frac{\hbar\,c_\pi\,\Omega_S}{2\sqrt{3}} \\ -\frac{\hbar\,c_\pi\,\Omega_S}{2\sqrt{3}} & -\frac{\hbar\,c_p\,\Omega_S}{\sqrt{6}} & 0 & 0 \\ \frac{\hbar\,c_m\,\Omega_S}{\sqrt{6}} & \frac{\hbar\,c_\pi\,\Omega_S}{2\sqrt{3}} & 0 & 0 \end{pmatrix}$$

which has no optical frequency time dependence and is thus tractable.

The goal is to use the AC stark shifts specified by this Hamiltonian to create a Sisyphus lattice, a lattice shaped potential on the 3P0 level. We can use the narrow line clock laser to selectively pump atoms into the bottom of the potential, and, if we are clever enough, engineer the scattering rate from the 1388nm laser to be highest at the top of the potential. This means as atoms get pumped (and spontaneously decay) around the 1S0, 3P0, 3D1, 3P1, 1S0 loop they will always go up hill while in the 3P0 state. This will cause them to cool in a process called Sisyphus cooling. However, there is some recoil heating associated with absorbing the 1388nm photon, and the recoils from the 3D1 to 3P1 and 3P1 to 1S0 spontaneous decays. The point where these recoil heating mechanisms balance the cooling from climbing the Sisyphus lattice is the equilibrium temperature and is essentially what this simulation desires as an ouput.

The simulation operates as follows:

1. Treat the positions of the atoms classically in the X and Y plane (the direction we apply Sisyphus cooling) and ignore motion in the Z plane. Our atoms are trapped in an optical lattice trap with a far off resonant 759nm laser. In the Z direction the atoms are in the Lamb-Dicke regime meaning they are recoil free. So we can assume that there will be no heating in the Z direction, since all our heating is from recoil heating. In addition the Z direction is in the resolved sideband regime, so a classical simulation has no hope of accurately capturing dynamics. The X and Y dimensions are not in the resolved sideband regime, if we approximate the trapping potential as harmonic the levels are spaced by ~4nK, which is low compare to the experimentally measured final temperature after Sisyphus cooling of ~800nK, meaning a classical simulation has hope. So we do a classical symplectic MC for newtons laws on X and Y positions. Symplectic because as the atoms oscillate back and forth in the harmonic trap (for X and Y) with trapping frequency 50Hz we don’t want the energy to grow making it seem like we heated the atoms.

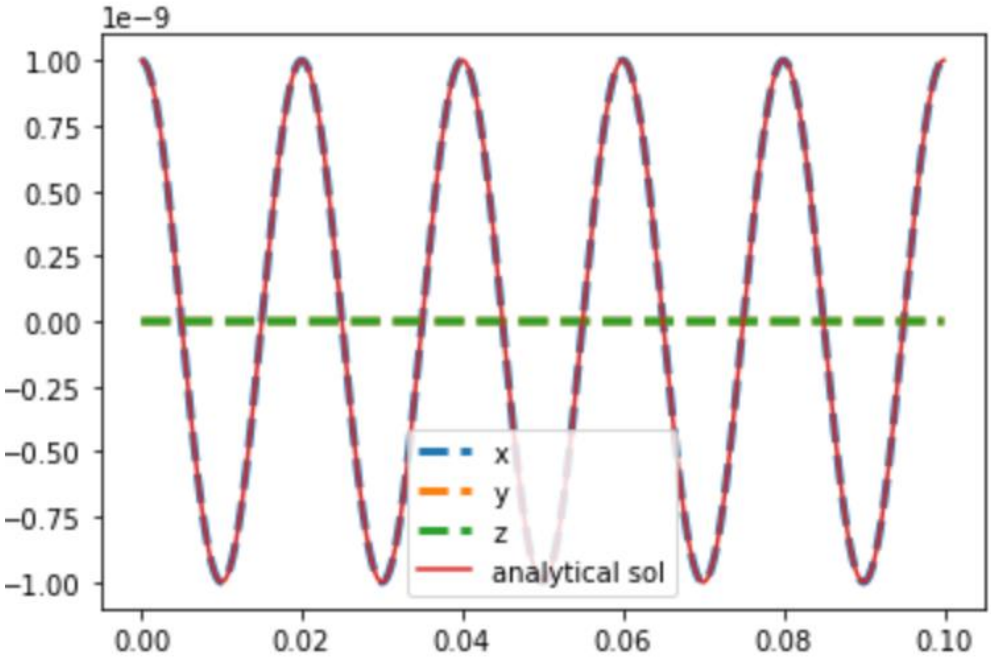
2. Come up with the dressed state energies as a function of the 1388nm detuning and intensity (in the code I use rabi frequency). I diagonalize the Hamiltonian in mathmatica (mostly for convince with symbolic math) and use the eigenvalues as the dressed state (SC stark shifted energies). This is the get_dressed_state_ernergy subroutine.
3. Use the dressed state energies to see how large the Sisyphus potential on the 3P0 state is. This is important in its own right.
4. The scattering rate from the 1388nm laser is given by 1/decay_time * the projection onto the 3D1 states. The eigenvectors (calculated in python this time) give us the projection.
5. Treat the internal state of the atom classically (this sounds like a terrible idea, but I've seen a few papers that do this for qualitatively correct results). And shift the atom between internal electronic states based on the rabi frequencies and spontaneous decay times.

Currently this is where I had to stop due to lack of time. Everything described thus far has been tested (results shown below), except for the lin perp lin configuration and step 5.

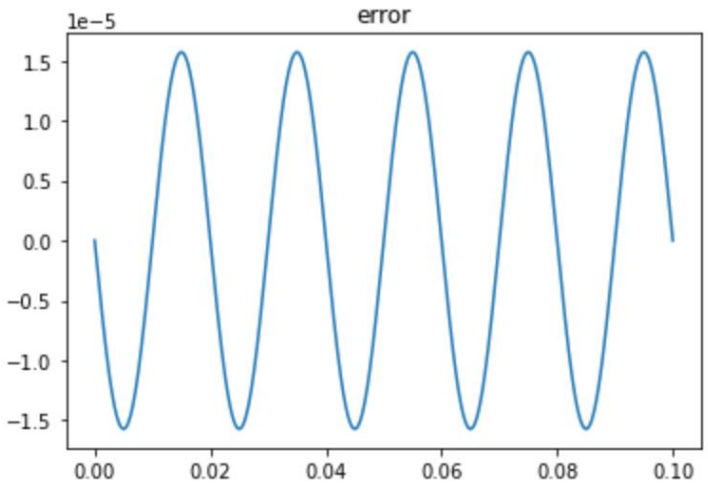
Testing Results:

I test the symplectic integrator of newtons laws and compare to the analytical solution for a harmonic oscillator (since our trapping potential is approximated to be a classical harmonic oscillator).

This test shows the atoms oscillating after being placed at 1nm from the trap center in the x direction:

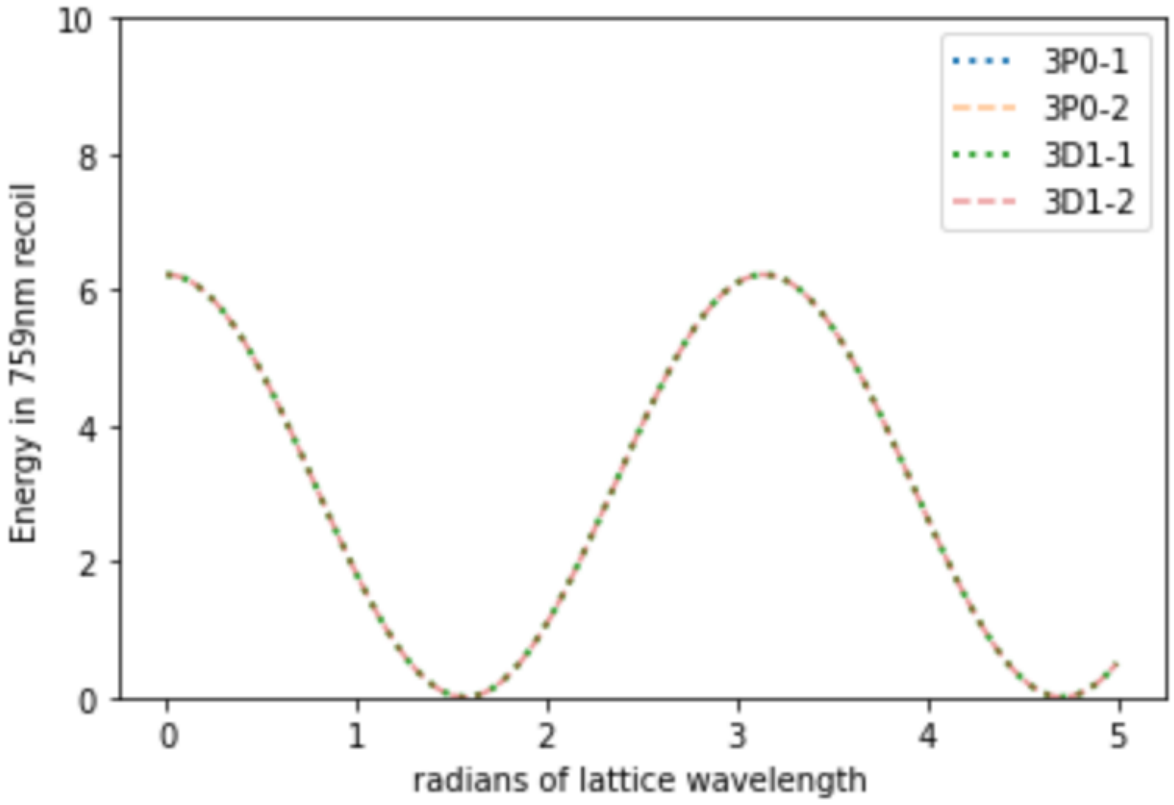


The trap frequency is 50Hz (x axis in seconds), and as we can see the integrator matches the analytical solution. Furthermore the relative error is plotted to show this is a true symplecic integrator:



After a few other similar tests of the integrator I move on to testing the dressed state energies.

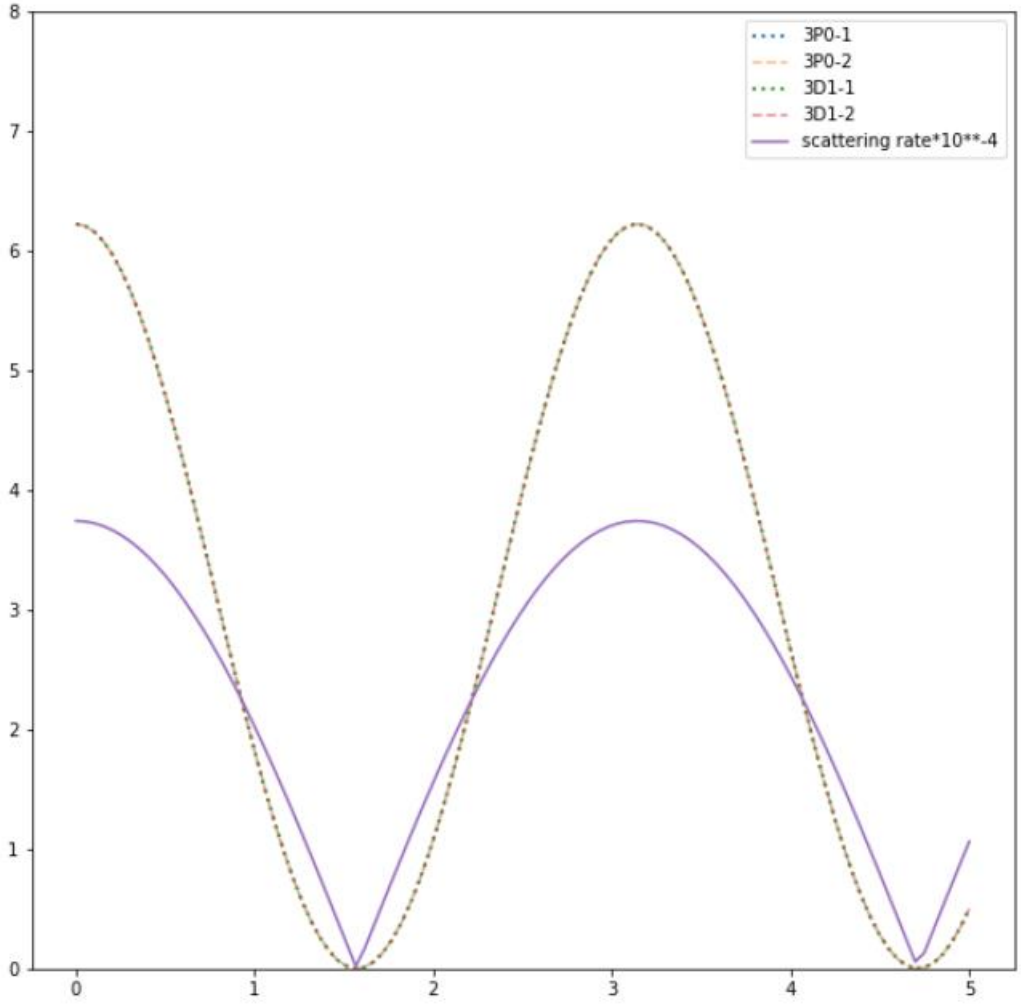
Applying lin parallel lin 1388nm light (both pi polarized) will create an intensity lattice who's pi polarization varies like cos(k*x). Python has a problem dealing with the small values created by nm scale lattices, so I just used cos(x) and will scale the values later. In this case we should see the two 3P0 mF states be degenerate and vary approximately sinusoidally.



We can see that the lattice behaves as it should, it goes higher than 0 (un-AC stark shifted energy) for blue detunings, varies sinusoidally, and is roughly the correct height (we don't have a great measurement of this from the lab, but as a rough estimate 10Er is decent).

Other tests show red detuning makes the lattice lower than 0, as it should, and large detunings make the stark shift almost negligible, as they should because the 1388nm laser basically induces no coupling between states in that case.

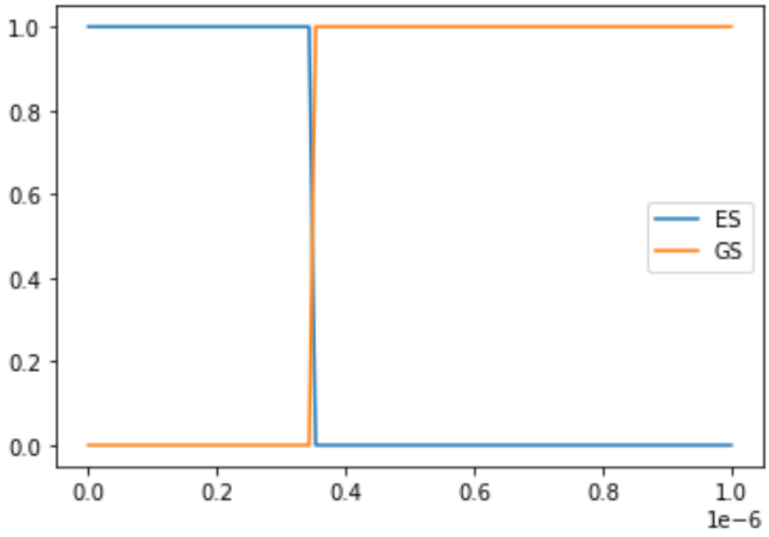
I test the scattering rate, for the blue detuned lattice case it should be highest at maximum AC stark shift and 0 with no AC stark shift, since both are reliant between mixing of the states.



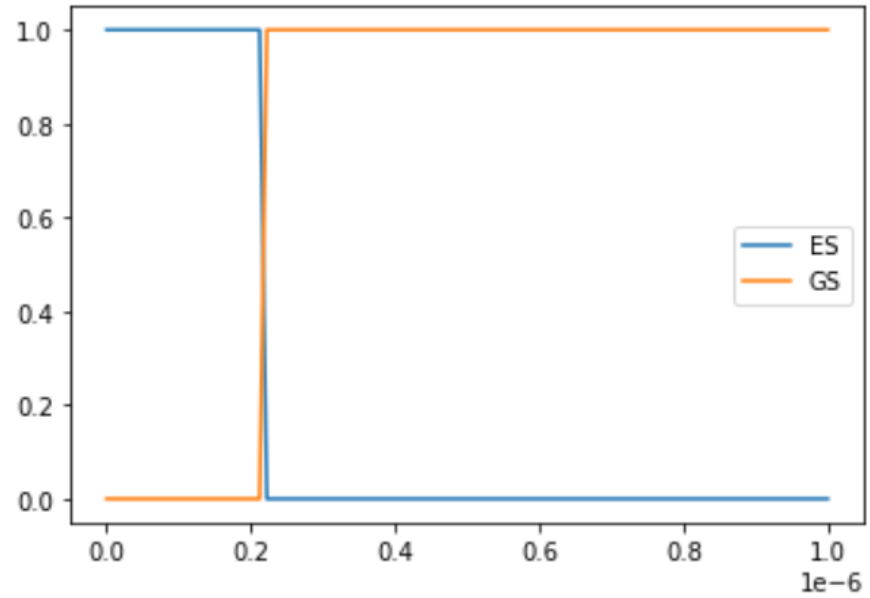
I've scaled the y axis units to make the scattering rate fit in the plot. We can see the scattering rate behaves correctly. This is what makes the cooling possible, we preferentially pump out at the top of the hill.

The last test I was able to successfully complete was beginning to test the Monte-Carlo part of this simulation.

I put the atom in the excited state and ran the monte carlo with no lasers. It should spontaneously decay to the ground state and stay there. Lifetime is about 300ns.



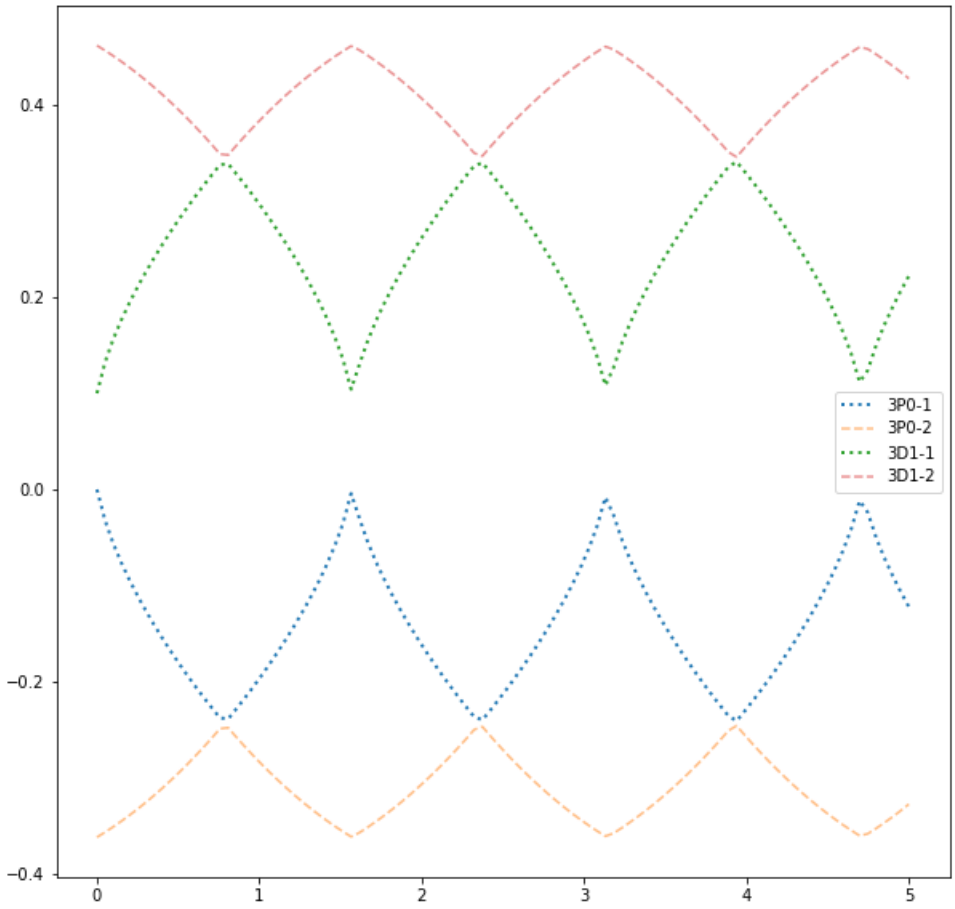
That is exactly what happens. I can run the same test again and see that the random nature is indeed working, because it won't decay at exactly the same time.



Failed tests:

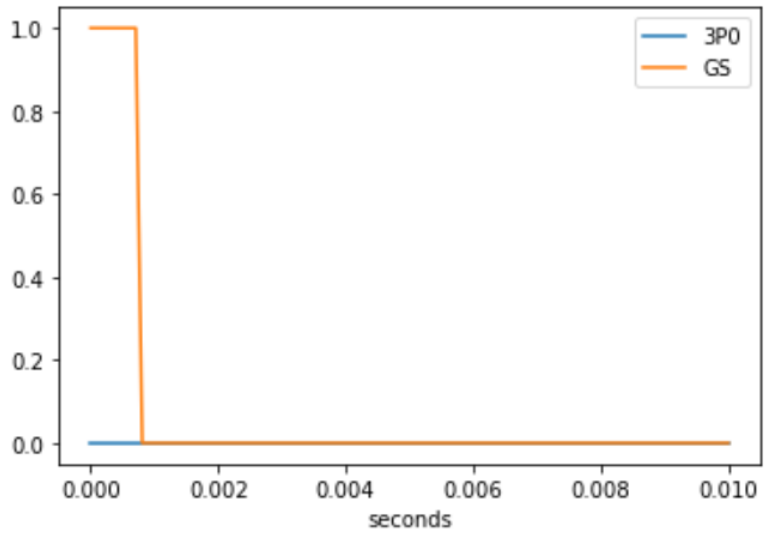
I'd love to fix these, but I just ran out of time.

1. For lin perp lin 1388nm light, both pi polarized we will see the Cp go as sin(x) and Cm go as cos(x). In this case we expect the mF=+- levels to have opposite stark shifts (ie. mF=+1/2 will be maximally stark shifted as mF=-1/2 is minimally stark shifted).
   My simulation shows this to be the case for some random number I plug in for the 1388nm rabi frequency and detuning as we can see in the April 30th commit in GitHub:



   Yet for the actualy measured rabi frequency and detuning we just get constant AC stark shifts. Perhaps this is merely a result of the physics, but since in the actual experiment we get cooling for this case there must be a spatially varying AC stark shift. I strongly suspect this is a result of one of the numbers in this part of the code being way off by many orders of magnitude, but as of now I am unable to track this down.

2. The rabi flopping test of the Monte-Carlo. We should see the atom go back and forth between the ground state and 3P0 at about the clock laser rabi frequency, but instead I see



So there must be something going wrong with transferring the atom between the ground and the 3P0 state.

Further things to do:

I understand this simulation is very incomplete, given its goal was to estimate a final temperature and it can't even evolve the internal state correctly now, which will certainly result in a lower grade, but I still have many plans to finish the simulation.

1. Fix broken tests outlined above.
2. Add in more tests for the update internal state. I will need to test the scattering rate from 1388nm laser works correctly and transfers atoms between 3P0 and 3D1. I will also need to make sure that all these things play nicely together, and that I could see an atom rabi flop from 1S0 to 3P0, scatter 1388nm to go to 3D1 and finally spontaneously decay back to 1S0 correctly.
3. Actually perform a simplified Monte Carlo. Create 1 atom at some random position in the lattice, with a random initial velocity given by the boltzman distribution for the starting temperature of our atoms (~4uK). Start it in the ground state. Evolve the internal state. Any time we enter into 3P0 record the AC stark shifted energy of the level. Once we leave it record that energy too, and subtract the energies to get the amount of kinetic energy the atom has lost. Subtract the appropriate velocity in the direction of the 1388nm lattice (given by the polarization). Any time we scatter a 1388nm photon add a recoil in a random direction. Likewise for a spontaneous decay. Record the final velocity of the atom once it has roughly stabilized.
4. Do this with many atoms. Fit the distribution of final velocities to the Boltzman distribution to extract the final temperature. Ideally this matches reasonably well with the experimentally measured 800nK.

There are other ways I'd eventually like to make this fancier. Add in 2 direction of 1388nm laser. Use the Monte-Carlo Wavefunction Method to evolve the internal state in a more quantum manner than the simplified classical thing I do here.

Results I'd like to eventually get from the simulation:
As it stands knowing the Sisyphus lattice on 3P0 is ~6Er deep is valuable information for our experiment. In the future I'd like to see that we observe similar final temperatures for red and blue detuned 1388nm beams (detuning the clock laser appropriately to hit the bottom of the Sisyphus lattice), and for lin perp lin and lin parallel lin 1388nm light configurations. This is what is observed in the lab and is a somewhat perplexing result that would be nice to recreate in simulation.
Finally I'd like to play with all of the parameters to see if there exists some better thing than what we currently do in the lab.
As a super reach goal this same general idea has been proposed for anti-hydrogen, so perhaps I could do this simulation for the anti-hydrogen atom and convince CERN that it is worth implementing over the current UV doppler cooling that takes about 5 hours.

Again sorry that I couldn't complete a reasonable amount of the monte carlo. I really thought I could at least get the velocity of one particle over time until it hit some equilibrium. I understand that the code as it is isn't great for my grade, but I guess that's the way life is sometimes. Thanks for a fun class!