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Development of disc spring stacks for vibration isolation

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ABSTRACT

Cone disc springs exhibit quasi-zero stiffness behavior that is useful in isolating objects from low frequency vibrations. However, the stroke of a single disc spring is too low for most applications, and springs are stacked to increase the displacement. A method to contain the isolator stack then becomes critical for practical uses. Many challenges in developing these containment methods have been identified and can be collectively described as how to appropriately contain the stack without affecting isolation performance. In this work, three designs are considered: a retaining ring design, tube and shaft design, and zero poisson ratio sleeve design. Disc spring stacks with containment method are built, and load-deflection curves are measured and compared with standalone stacks. Under quasi-static compression testing, each containment method has minimal effect on the standalone stack load-deflection curve. However, significant differences in isolation performance are observed in vibration testing and found to depend on characteristics such as lateral stability, lateral strength, and degrees of freedom. Lastly, advantages, disadvantages, and appropriate applications for each containment method are summarized. The conclusions of this work are that containment method is an important variable in the application of disc spring isolators and robust, versatile containment designs have been demonstrated.

1. INTRODUCTION

Quasi-zero stiffness (QZS) is a well-known concept to isolate objects from low frequency vibrations without sacrificing load bearing capability. The isolator has high stiffness at low deflection for load bearing capability and zero stiffness at higher deflection to achieve a low natural frequency. This characteristic is sometimes referred to as high-static-low-dynamic-stiffness (HSLDS) [1].

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Many QZS structures have been proposed, and many of these are mechanical linkages composed of different springs, bi-stable elements, cams, and rollers in order to create the desired nonlinear stiffness in a particular direction [2–6]. Pneumatic and magnetic components have also be used to improve upon the limitations of purely mechanical devices [7–9].

The way in which many of these mechanisms achieve a QZS effect is by combining a positive stiffness element with a negative stiffness element . Then with proper design, the net stiffness becomes zero at a certain deflection. A vertical spring combined with two lateral springs is the most common implementation of this strategy [10, 11]. The two lateral springs provide negative stiffness in the vertical direction over a small displacement range as they rotate through the horizontal position.

Many other negative stiffness elements exist. One example is cone disc springs, which have been combined with positive vertical stiffness elements to make QZS isolators [12–15]. These are commercially available springs with a truncated cone shape and are typically loaded by applying a uniform compressive force to the inner and outer edges as shown in (Figure 1a). Cone discs exhibit a negative stiffness region when the height/thickness ratio is greater than $\sqrt{2}$. On the other hand, when the ratio is equal to $\sqrt{2}$, the force-deflection curve becomes flat resulting in zero stiffness. As a result, QZS isolators can be made solely from disc springs without the need for separate positive and negative stiffness elements. This allows cone disc QZS isolators to be significatly more simple and compact than the existing QZS structures. However, very little work has been done on this particular implementation. This goals of this paper are therefore to (1) demonstrate the advantage of disc spring QZS isolators in terms of compactness, and (2) develop effective approaches to assemble and contain the isolator stacks for practical applications.

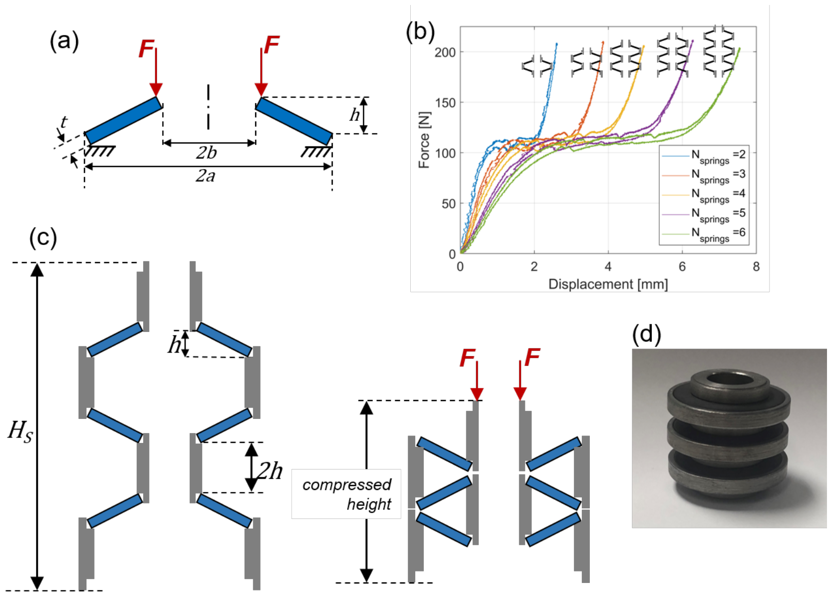


Figure 1: Geometry and loading method of a cone disc spring (a), load-defelection behavior of disc spring stacks for different number of springs (b), geometry and compression of disc spring stack with spacers (c), and physical prototype of disc spring stack (d).

2. DISC SPRING STACK STRAIN

As mentioned, the cone discs should have height/thickness ratio of $\sqrt{2}$ to achieve the quasi-zero stiffness effect, and then the thickness, inner diameter, and outer diameter can be easily designed to target any QZS force level based on existing analytical models [16]. Next, assuming the equilibrium point of the mass-isolator system is in the center of the QZS region, the required length of the

QZS region can be approximately set by the maximum expected vibration amplitude. The vibration amplitude may be larger than the length of the QZS region of a single disc spring. Multiple springs can be stacked in series to increase the length of the QZS region as shown in Figure 1b. This particular plot shows load-deflection curves for disc springs with the geometry given in Table 1, which is used throughout this paper. The springs are made from steel and and have a QZS force level of approximatley 110 N as shown.

Table 1: Geometry of disc spring used throughout this paper defined according to Figure 1a.

Parameter	**Value [mm]**
h	0.5
t	0.5
a	17.3
b	11.1

The springs are stacked in series using inner and outer steel spacers as illustrated in Figure 1c and d [17]. Typical disc spring stacks for preload applications do not have such spacers, and the springs cannot be compressed past the flat position [18, 19]. Spacers allow the full displacement of each disc spring to be exploited as shown in Figure 1c, extending the length of the QZS region. This assembly of springs and spacers will be subsequently referred to as the standard stack.

In order to estimate the compactness of disc spring QZS isolators, we perform strain calculations of disc spring stacks and compare it to typical strain values for QZS isolators based on a linear vertical spring plus negative stiffness disc spring. The full displacement of a single disc spring is two times its height, h, regardless of the other dimensions. Therefore, a minimum spacer height of $2h$ is required to prevent interference of adjacent springs during stack compression (see Figure 1c and d). The total displacement of the stack is given by:

$$d_S = 2hN_S \tag{1}$$

where N_S is the number of springs in the stack. Assuming each spacer has the minimum height, the total height of the stack, H_s, is given by Equation 2. It should be noted that the height H in this equation is the overall height of each disc spring, and is different from the cone height defined in Figure 1. H can be calculated from h according to Equation 3 assuming small angle approximation and height/thickness ratio is 1.41. The first term on the right hand side of Equation 2 is the total height of the springs, and the second term is the total height of the spacers.

$$H_S = HN_s + 2h(N_s + 1) \tag{2}$$

$$H = h + \frac{h}{1.41} \tag{3}$$

Now, using Eqns.1 and Eqns.2, the overall stack strain can be calculated by Equation 4. It is independent of the spring dimensions, and therefore can be plotted for an arbitrary spring stack in Figure 2. It is observed that the strain asymptotically approaches 0.54, or $2\sqrt(2)/(3\sqrt(2)+1)$. The effect of the two end spacers is to lower the strain when N_S is small but becomes less significant as N_S increases. Based on estimates from the previous literature, the QZS isolators made from a linear vertical spring plus negative stiffness cone disc have strains in the range of 10-20% [12]. Alternatively, we see that QZS isolators made from disc spring stacks can achieve strains of 35% for a single disc spring and > 50% strain for stacks with at least 7 springs. This is an important advantage that helps to minimize the overall size of the isolator.

$$\epsilon_S = \frac{d_S}{H_S} \tag{4}$$

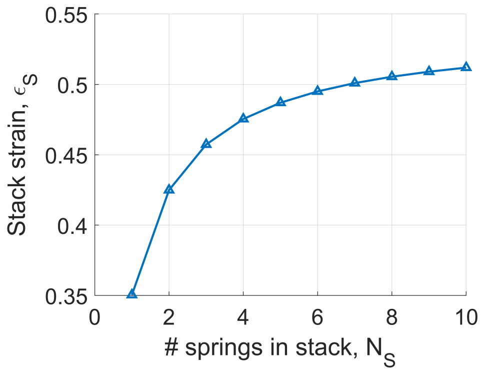


Figure 2: Maximum strain for a stack of disc springs and spacers. Spacers have the minimum height of $2h$

Lastly, our calculation of strain has included both the top and bottom spacers that each have thickness $2h$. In reality, these spacers only need thickness h because they are adjacent to only one disc spring. Therefore, there is the possibility to further increase the strain, especially at low N_s.

3. DEVELOPMENT OF STACK CONTAINMENT METHODS

3.1. Engineering requirements for stack containment

The springs and spacers as described in the previous section are simply stacked on top of each other and are not fixed together in any other way. Therefore a robust containment method that integrates these parts into a single component is essential for practical applications. The containment method should have minimal effect on the performance of the QZS isolator. In particular, the important engineering requirements are:

1. **Does not substantially reduce the overall stack strain:** The containment method should not affect the overall strain because compactness is one of the key advantages of disc spring QZS stacks
2. **Has minimal effect on the standard stack load-deflection curve:** The containment method may introduce some additional vertical stiffness into the system, and this stiffness should not significantly alter the overall QZS behavior.
3. **Has sufficient lateral strength:** The uncontained stack has poor resistance to lateral loads or mixed loading conditions. One of the primary functions of the containment method is to provde strength in directions other than vertical.
4. **Simple and quick assembly** This requirement is important not only for research purposes to allow quick testing of different configurations and parameters, but also from a production and cost standpoint
5. **Tolerant to misalignment** In many applications, the load may not be perfectly centered with this central axis of the stack or may not be purely translational. Under these types of loading, the containment method should not hinder the function of the stack, which naturally has some ability to compress asymmetrically.

3.2. Overview of Stack Containment Methods

Here, we introduce three stack containment methods and analyze them with respect to the five engineering requirements. The methods are introduced in Figure 3 a-c, and are called "tube and shaft design", "retaining spacer design", and "zero possion ratio (ZPR) sleeve design" respectively. The idea for the tube and shaft design is to insert the standard stack into a tube as shown in Figure 3a. A fixed shaft in the center of the tube acts as a linear guide for the piston which is connected to the load and compresses the stack. Horizontal screws at the top of the tube act as a stopper for the piston in a scenario where the assmebly may be subject to a tensile load. These screws also act to apply a very small preload (1-2 N) to the stack such that the stack will always be in compression. While the outer spacers are centered by the tube, the small preload reduces the possibility for the inner spacers or springs to slip out of concentric alignment.

The retaining spacer design in Figure 3b consists of augmented inner and outer spacers that can hold adjacent disc springs together. Standard inner and outer spacers are modified with a lip on the top and bottom to contain the disc springs as shown. In order to insert the springs, the retaining spacers must contain a small gap. Outer and inner retaining spacers can then be either expanded to compressed respectively to enable insertion of the springs. This concept was inspired by snap rings which are installed by a similar technique.

The third method is called the zero poisson ratio (ZPR) sleeve and was inspired from work on negative stiffness honeycombs which have a curved beam as the unit cell [20]. We recognized that this geometry may be useful as a sleeve for the isolator stack because it has both zero poisson ratio and zero stiffness region, making it consistent with disk spring characteristics. We converted the rectangular shape of the honeycomb to a circular one by giving the unit cell a small curvature out of plane and having an integer number of unit cells around the circumference (Figure 3c). The standard stack is inserted into the sleeve, and then the assembly can be compressed as as a whole.

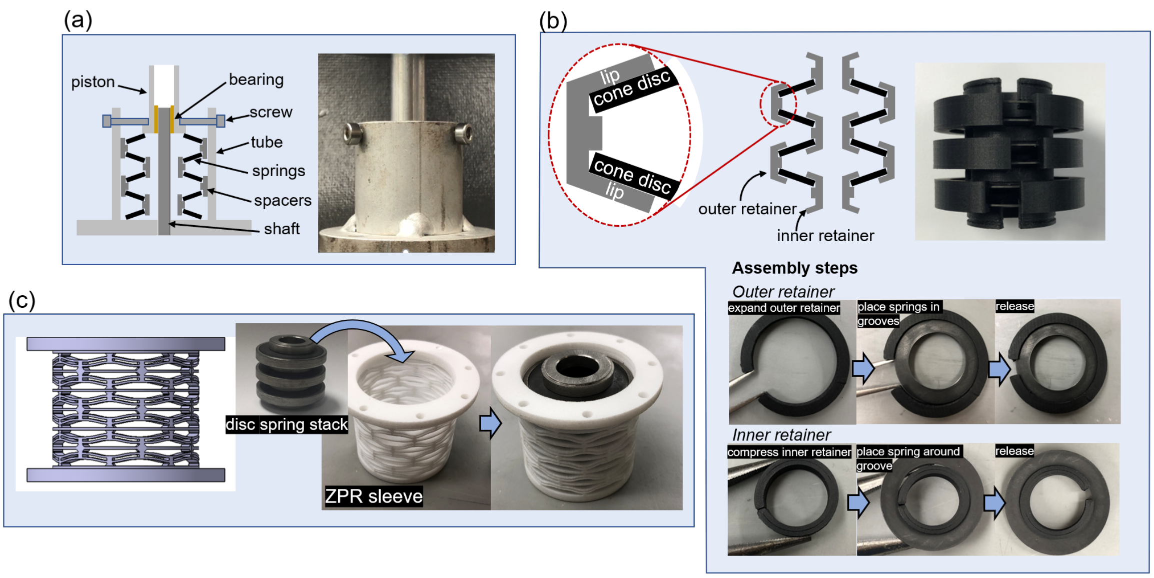


Figure 3: Overview of three different stack containment methods: tube and shaft design (a), retaining spacer design (b), and zero poisson ratio sleeve design (c).

3.3. Analysis of Stack Containment Methods

Effect on stack strain

Method 1 (tube and shaft assembly) results in a 30-50% reduction in the maximum possible stack strain in Figure 2 for a given number of springs. The overall height of springs and spacers remains the same, but the piston adds substantially to the overall assembly height. It must extend beyond the top of the tube by at least the stack displacement so that the object to be isolated does not interfere with the tube during compression. For example, the strain for a stack of six springs is reduced by 32% from 0.49 to 0.33 due to the height of the piston. Other aspects of the design such as the screws and base may also add to the overall height.

The strain of the retaining spacer method is also reduced from that of the standalone stack. The minimum spacer height must be increased to account for the thickness of the lip on the outer and inner retainers. Lip thickness will depend on the retainer material and manufacturing method. For example, a realistic lip thickness for the 3d-printed nylon retaining rings in Figure3b is 0.5 mm. The disc spring height is 0.7mm, and therefore the minimum spacer height increases from 1.4 to 2.4 mm. Strain for a stack of six springs is reduced by 28%, from 0.49 to 0.35.

The ZPR sleeve does not require components to extend above the stack or increases in spacer height. However, the curved beam geometry has its own inherent strain limits based on the buckling-type deformation. Maximum deformation of the unit cell is twice the curved beam height. A similar strain analysis as in Section 2 can be performed for the ZPR sleeve except without the assumption that the height/thickness ratio of the unit cell is $\sqrt{2}$. If the unit cell features a double beam as in Figure 3c, we have found that strains of 0.30-0.35 can be realistically achieved without sacrificing the other requirements. However, if the unit cell has a single beam, strains of .45-0.50 are feasible. Since the purpose of the double beam is to induce a certain buckling mode to increase the force [20], it is not necessary in this application. Consequently, the ZPR sleeve, which is mechanically in parallel to the spring stack, will cause only a small decrease of the maximum possible strain of 54%. In summary, the tube and shaft method and retaining spacer method reduce the overall stack strain by significant percentages while the ZPR sleeve can have negligible effect through careful design and selection of number of springs.

Effect on load-deflection curve

Load-deflection curves of the disc spring stack were measured with and without the containment methods, and the results are plotted in Figure 4. Each stack contains 6 disc springs. The tube and shaft concept has minimal effect on the load-deflection curve, maintaing the same QZS force and only slightly increasing the hysteresis. The retaining spacer concept also does not significantly impact the load-deflection behavior. The springs are observed to have a more significant snap-through behavior which may be due to the gap in the retaining rings disrupting the uniform load distribution. The ZPR sleeve increases the overall QZS force from 110 N to 150 N due to the stiffness of the sleeve, which is in parallel with the stack. However, the QZS effect is maintained because the curved beams in the ZPR sleeve also exhibit nonlinear stiffness, creating good synergy with the QZS disc springs. In summary, all the containment methods create minimal effect on the QZS beavhior in controlled quasi-static testing.

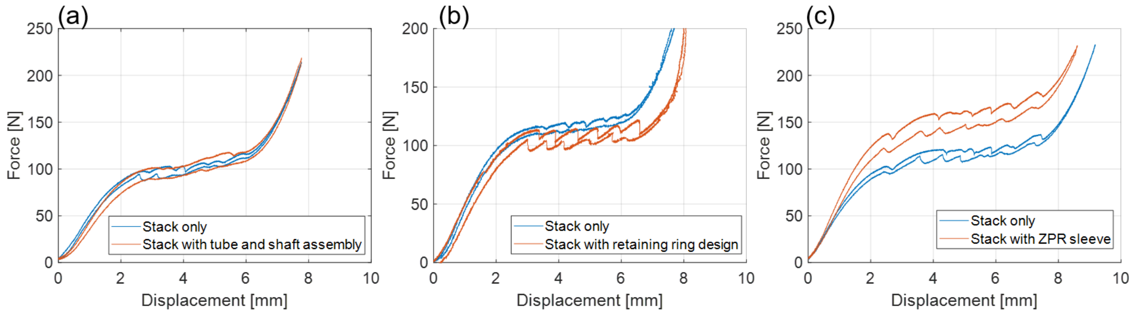
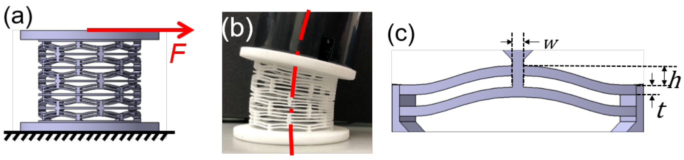


Figure 4: Load-deflection curves of standard stack compared to stack with containment method. Tube and shaft containment method (a), retaining spacer containment method (b), and ZPR sleeve containment method (c).

Lateral Strength and Misalignment The requirements for lateral strength and tolerance to misalignment are considered together because they typically have a dependent relationship. Lateral strength was evaluated using a simple load test shown in Figure 5a. Ability to perform well under misalignment is evaluated by vibration testing, but the results are only qualitatively reported here. The lateral strength of the tube and shaft assembly is the highest because lateral force is directly transmitted to the rigid center shaft and linear bearing. However, this rigid 1 DOF structure also has a downfall in that the isolator tends to bind if the base motion has any rotational or non-vertical translational component or the isolated object has multiple DOF. Small misalignment between the isolator axis and the object center of gravity can also lead to a binding problem. Alternatively, the retaining spacer method has the capability to move in more degrees of freedom within a certain range, but has poor lateral stability because the disc springs easily snap out of the retaining spacer grooves upon application of mixed loading conditions. Retaining spacer stiffness can be increased to compensate, but at the cost of increasing difficulty of assembly.

The ZPR sleeve is the most interesting with regard to meeting both requirements simultaneously. It can rotate as demonstrated in Figure 5b to accomodate some misalignment or small rotational input, but is also a single continuous part. The challenge was to maximize the lateral strength while keeping the vertical stiffness low to minimize the effect on the overall isolator load-deflection curve. ZPR sleeves were 3D printed by multi-jet fusion from Nylon (E = 1.9 GPa). Figure 5c shows the geometric parameters of the unit cell that were tuned to achieve this goal. In general, the stiffness is reduced by decreasing the thickness, decreasing the height, and increasing the number of unit cells around the circumference. In iteration 1 (Figure 5d), the stiffness was too high (50% disc spring stack stiffness), but thickness of only 0.5 mm caused poor durability. Thickness was increased in iteration 2 to improve durability, and the height was reduced in an effort to lower the stiffness. Stiffness was reduced more substantially in iteration 3 by decreasing number of units around circumference from 8 to 7 and further reducing height. Lateral strength of iteration 3 was very low, and the center support width was increased in iteration 4 to compensate. Finally, iteration 4 has low stiffness and high lateral force tolerance of 122 N. These results are summarized in Figure 5e.



(d)

Iteration #	thickness, t [mm]	height, h [mm]	center support width, w [mm]	unit cells around circumference
1	0.55	1.47	0.75	8
2	0.65	1.25	.75	8
3	0.65	1.0	1.0	7
4	0.65	1.0	1.6	7

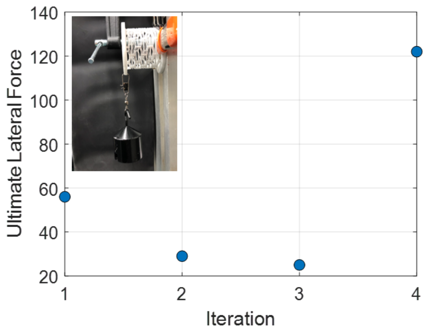


Figure 5: Lateral strength test of ZPR sleeve (a), demonstration of rotational capability of ZPR sleeve (b), geometric parameters of ZPR unit cell tuned to increase lateral strength (c), summary of ZPR geometry design iterations to increase lateral strength (d), and failure load of ZPR sleeve as a function of design iterations (e).

Assembly

Assembly of the tube and shaft method and ZPR sleeve method are straighforward and involve simply inserting the standard stack into the appropriate tube or sleeve. Assembly of the retaining spacer concept is more involved because each springs must be individually placed as shown in Figure 1b. As the stack is built up, it also becomes more difficult to access and compress the inner retainers for spring placement. If the retainers are made from metal, repeated assembly and disassembly of the stack leads to fatigue of the retainers. Lastly, tight tolerances are needed for the discs springs to fit correclty while also allowing the outer and inner retainers to sufficiently expand and compress for assembly. These qualitative observations lead to the conclusion that the retaining ring design is the most challenging from an assembly and manufacturing perspective.

4. CONTAINMENT METHOD SELECTION

Selection of an appropriate containment method will depend on the application and the relative importance of the various requirements. If the base vibration and mass are expected to move in 1DOF translation along the isolator axis and space is not a constraint, then the tube and shaft method is a suitable option. A more likely scenario is that the vibration input of the base has multiple components of acceleration and that the object is supported by multiple isolators. For example, the tube and shaft assembly may not be appropriate for an automotive seat which has a complex vibration response. Overall, the ZPR sleeve assembly is the most versatile option for many applications by being the most compact, maintaining the isolator compressibility under some misalignment and multi DOF motion, and having good lateral strength. There is also opportunity for further improvement of this concept including maximizing the lateral strength through CAE and exploring other unit cells besides the curved beam.

5. CONCLUSIONS

Quasi-zero stiffness isolator made from a stack of disc springs and spacers is proposed and is shown to have good QZS behavior while being very compact. Three containment methods for the stack are introduced and analyzed in terms of five engineering requirements. Each of the methods has minimal effect on the quasi-zero stiffness effect based on quasi-static testing, but only the ZPR sleeve maintains the high strain of the stack. Furthermore, the ZPR sleeve lateral load capacity is increased to 110 N, and the isolator performs well under different types of base motion and with multi-DOF objects. It is anticipated that these containment methods will be an important aspect of disc spring QZS isolator development for practical applications.

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