RIYA Program 2024

FINAL PRESENTATION

Dynamics of the QZS isolator with Coned Disks

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ACKNOWLEDGEMENT

I would like to wholeheartedly thank my mentors, **Prof. Rajendra Singh** (Ohio State University) along with **Dr. Paul Gilmore** (Toyota R&D) for the opportunity to pursue this project as well as for their valuable insights and guidance throughout the project. I would especially like to thank Dr. Gilmore and his staff for taking their valuable time to conduct the vibration experiments quickly and efficiently.

OVERVIEW

- **Vibration Isolation** - Necessary for many engineering scenarios [1]
- **Quasi-Zero Stiffness (QZS) Isolator** - **Low frequency** vibration isolation with **load-bearing capability** [2]
- **QZS Isolator Design Strategy -** Coned Disk Springs, which have dynamic stiffness and nonlinear force-deflection regimes (Figure 3) [2-3]
- **Series Spring Stack -** Used to achieve the desired stroke displacement

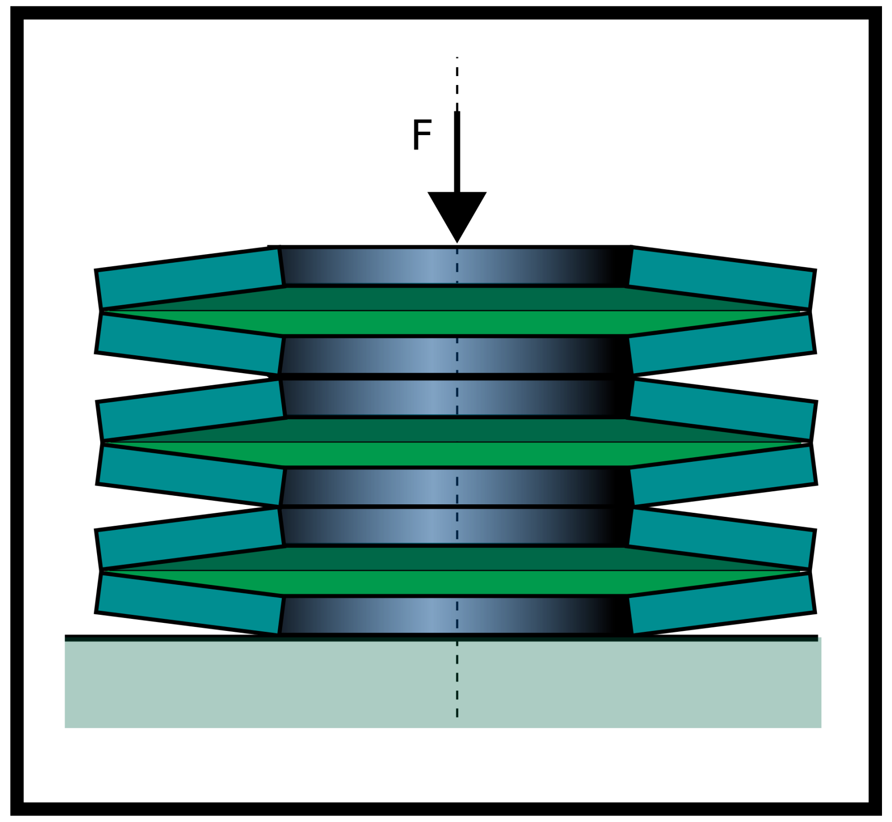


Figure 1:
Schematic of a disk spring stack[4]

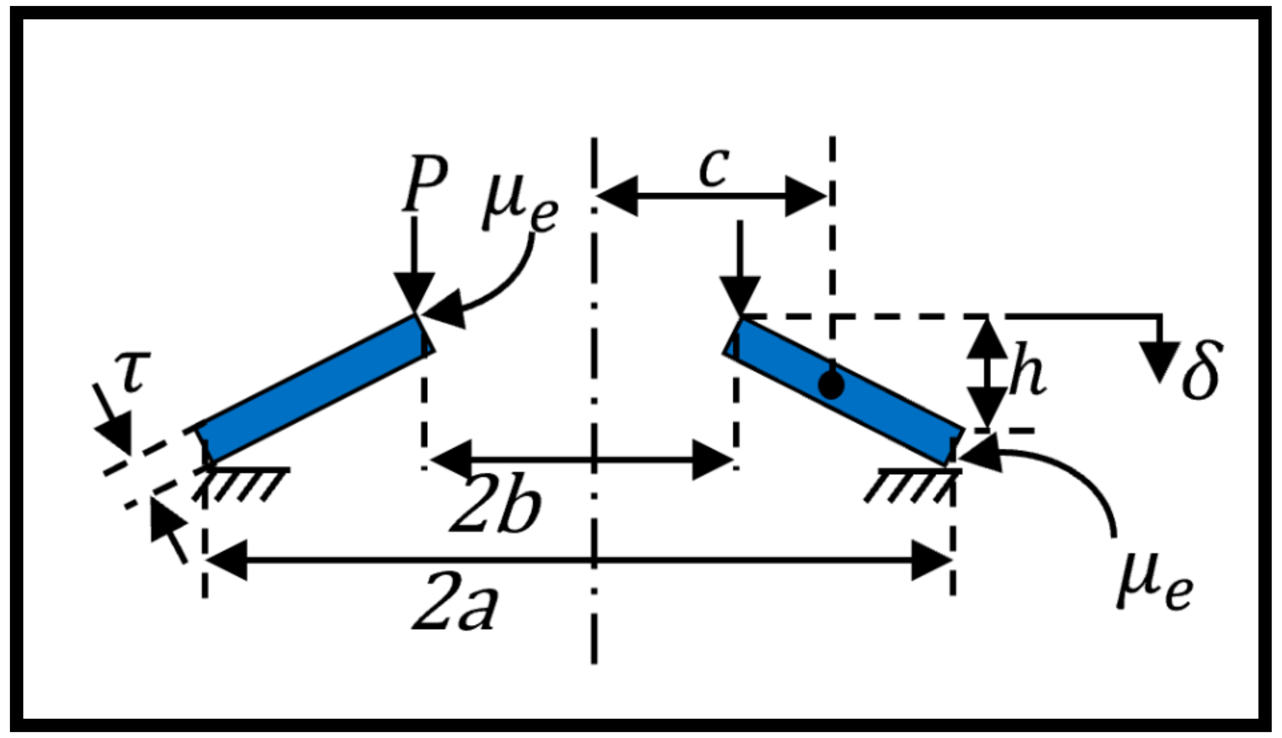


Figure 2:
Cross-section of a coned-disk spring[3]

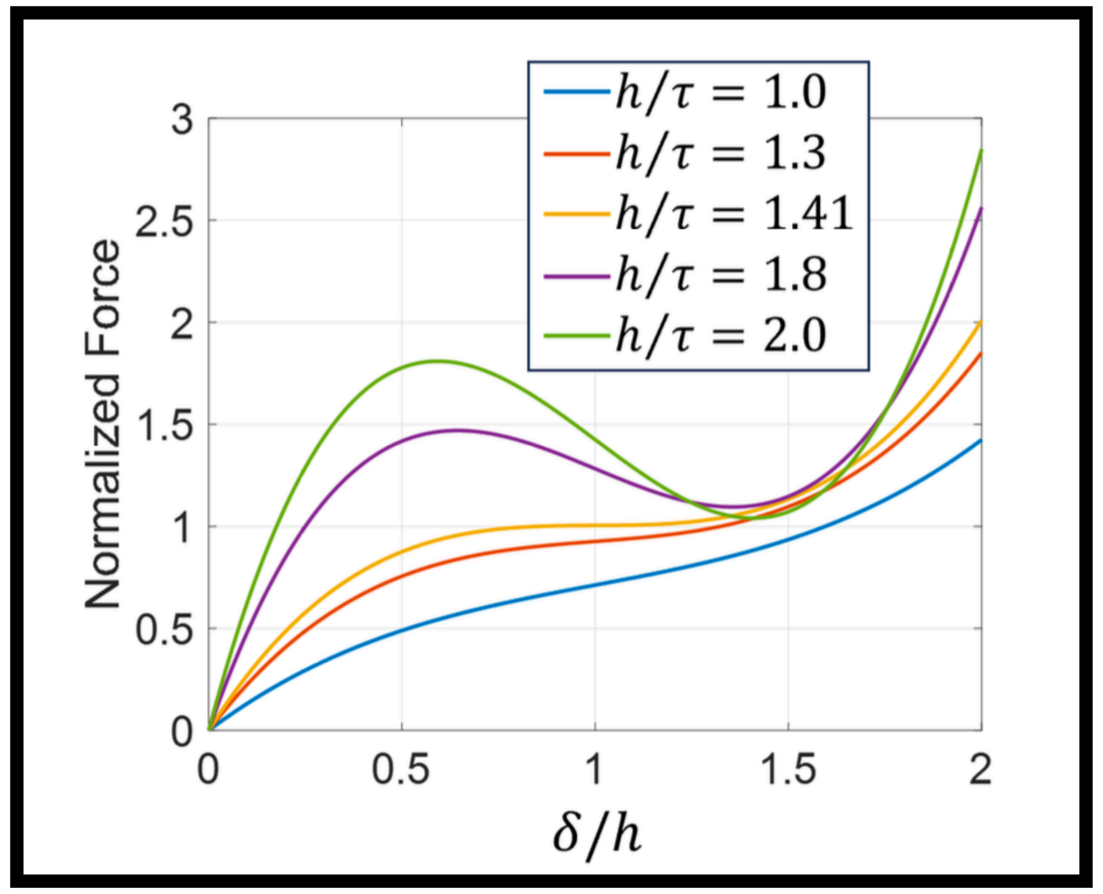


Figure 3:
Force-deflection curves for coned-disk springs[3]

PROBLEM FORMULATION

Goal - To go beyond **static analysis** and venture into the **dynamics of the system**

OBJECTIVE 1 - MODELING AND SIMULATION

A) Model the **non-linear dynamics** of a spring stack-mass-damper system with base-excitation
B) Simulate the model to obtain **time domain results** like displacement, velocity and acceleration profiles

OBJECTIVE 2 - INVESTIGATION AND VERIFICATION

A) Verify the dynamics of the system through **Linearized System Analysis**
B) Investigate the dynamics via **Physical Domain Analysis** and **Frequency Domain Analysis**

OBJECTIVE 3 - EXPERIMENTAL VALIDATION

A) Validate simulation results for different base excitation amplitudes/frequencies through **experiments**
B) Bring simulation and experimental results closer to each other through **parameter tuning**
C) Examine the role of **damping** and **operating region** on **Motion Transmissibility**

SYSTEM MODELING

System Sketch

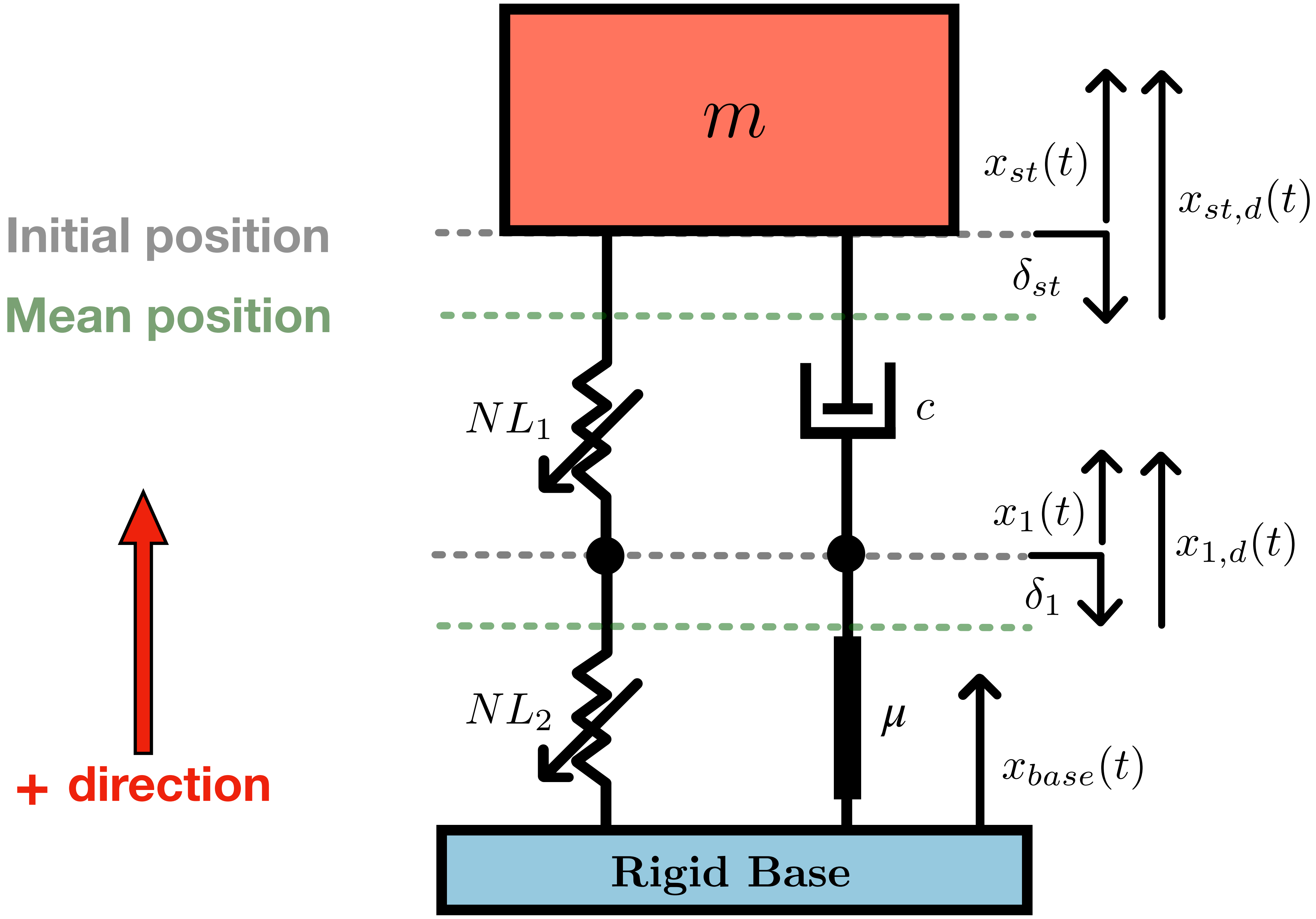


Figure 4 -
Sketch of a **spring stack-mass-damper** system

In **Figure 4**,
m - Mass of the suspended block
c - Damping coefficient of the Damper
μ - Coefficient of Coulomb friction
NL_1 - Top conical disk spring with ratio h_1/τ
NL_2 - Bottom conical disk spring with ratio h_2/τ
$x_{base}(t)$ is the **base excitation**

For the mass m,
δ_{st} is the mean displacement in **static equilibrium**
$x_{st,d}(t)$ is the **dynamic displacement**
$x_{st}(t) = \delta_{st} + x_{st,d}(t)$ is the **total displacement**

For the bottom of NL_1,
δ_1 is the mean displacement in **static equilibrium**
$x_{1,d}(t)$ is the **dynamic displacement**
$x_1(t) = \delta_1 + x_{1,d}(t)$ is the **total displacement**

EQUATIONS OF MOTION

In the presence of damping

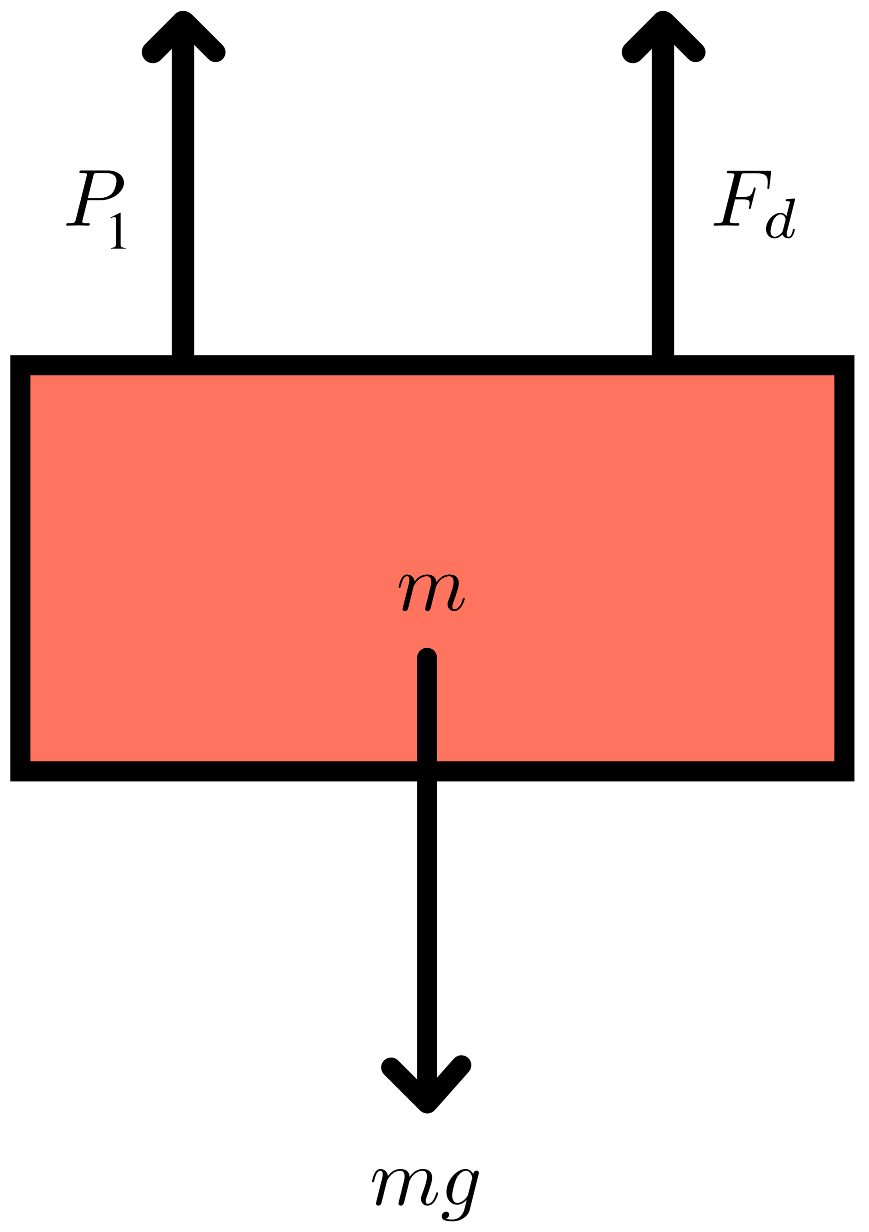


Figure 5 -
FBD of mass m

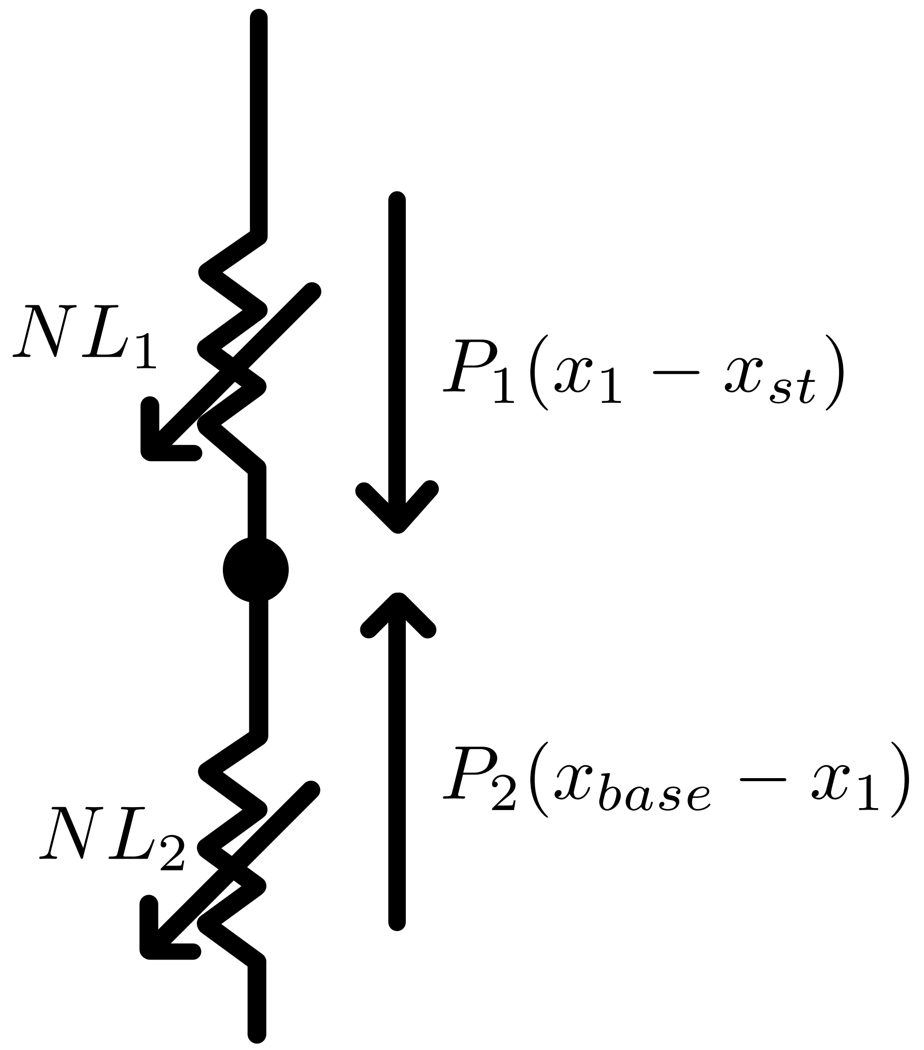


Figure 6 -
FBD of springs at the connection

From **Figure 5**,

$$m\ddot{x}_{st} = -mg + P_1(x_1 - x_{st}) + F_d$$

$$F_d = c(\dot{x}_{base} - \dot{x}_{st}) + \mu mg\, sgn(\dot{x}_{base} - \dot{x}_{st})$$

From **Figure 6**,

$$P_1(x_1 - x_{st}) = P_2(x_{base} - x_1)$$

The state vector $x(t)$ is defined as

$$x(t) = \begin{bmatrix} x_{st}(t) \\ \dot{x}_{st}(t) \end{bmatrix}$$

SIMULATION PIPELINE

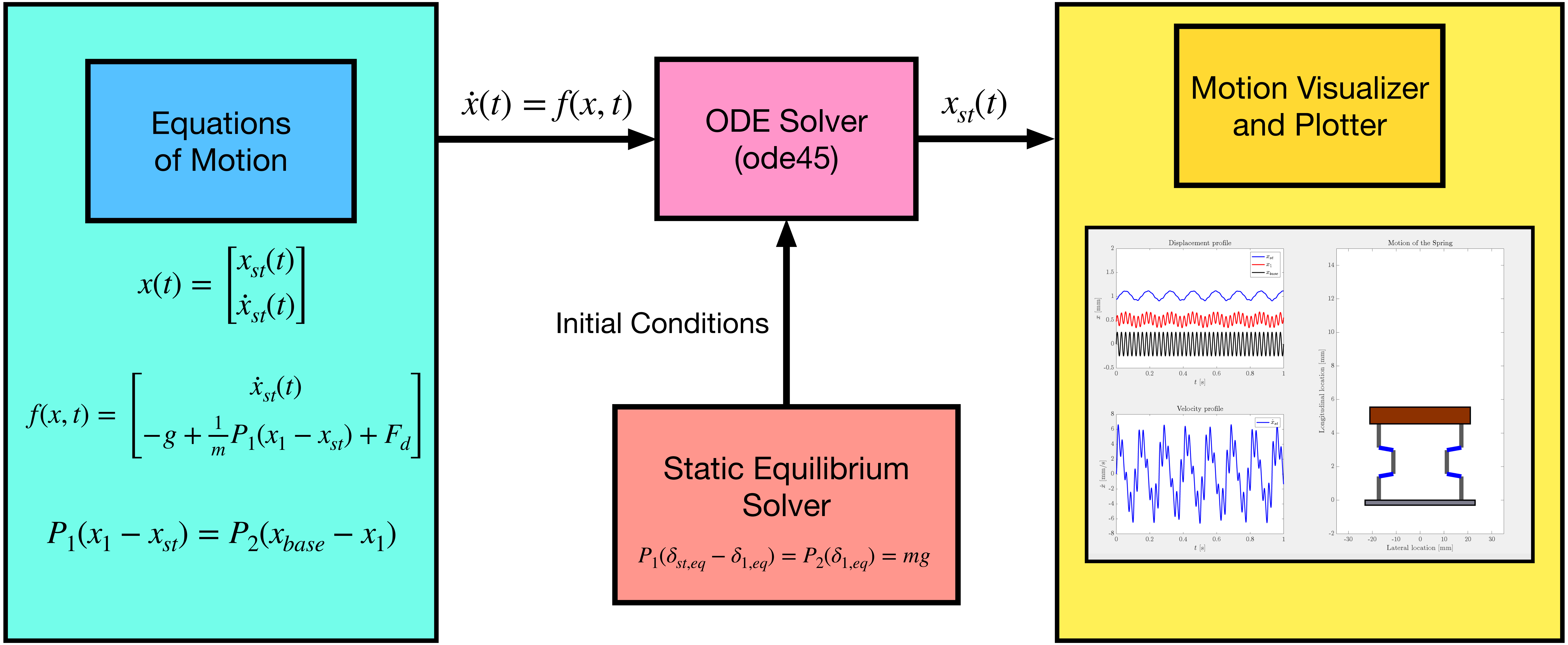


Figure 7 - Simulation pipeline in MATLAB

SIMULATION RESULTS

In the presence of damping

SIMULATION CASE :

$h_1/\tau = h_2/\tau = 1.41$

$E = 200$ GPa

$m = 10.7$ kg

$c = 0.05$ Ns/mm

$\mu = 0$

$f = 7$ Hz

$x_{base}(t) = 0.05\sin(2\pi f t)$ (in mm)

The results for other cases are present in the link referenced in **Appendix D**

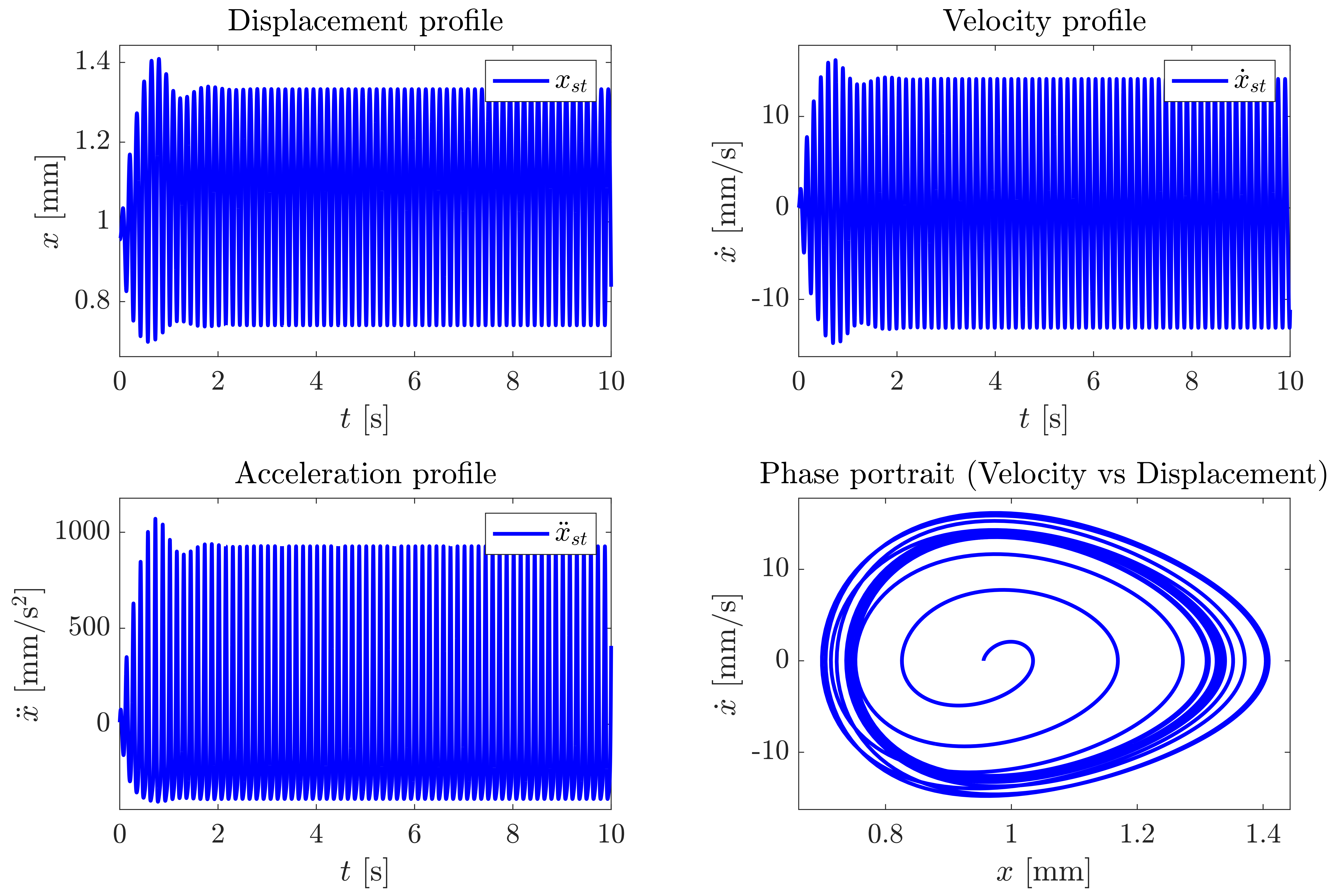


Figure 8 - Time domain results in the damped case

PHYSICAL DOMAIN ANALYSIS

SIMULATION CASE :

$h_1/\tau = h_2/\tau = 1.41$

$E = 200$ GPa

$m = 10.7$ kg

$c = 0.05$ Ns/mm

$\mu = 0$ N

$f = 7$ Hz

$x_{base}(t) = 0.05\sin(2\pi f t)$ (in mm)

The physical domain results for other cases are present in the link referenced in **Appendix D**

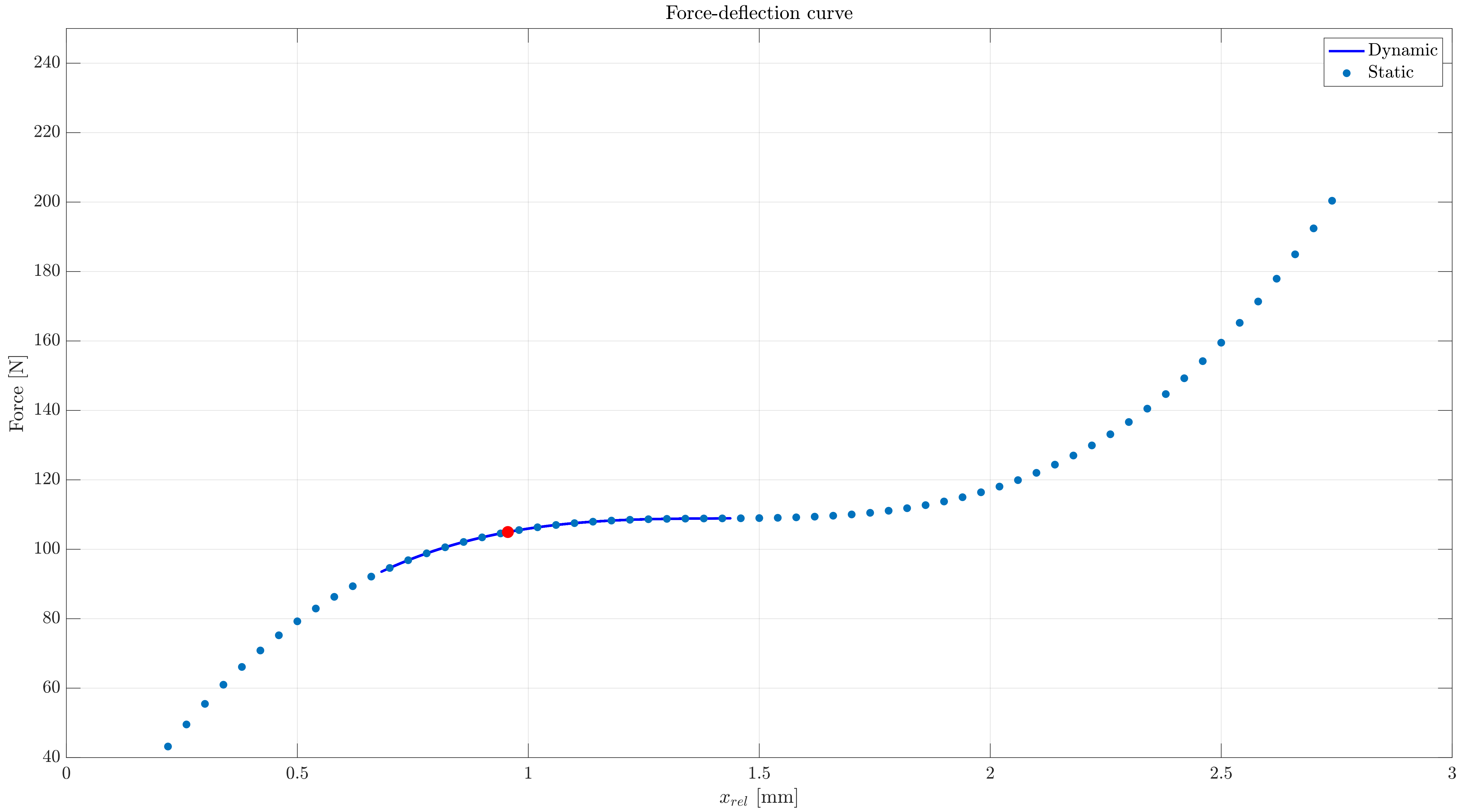


Figure 9 - Physical Domain behavior of the system

HIGHLY NONLINEAR BEHAVIOR SPANNING THE QZS AND NON-QZS REGIONS

EXPERIMENTAL PROCESS

- Base displacement excitation experiments performed to test the **simulator's reliability more rigorously**
- Spring stack with **relatively flat QZS region** desired $\Longrightarrow$ Experimentally determined and characterized
- **Acceleration** of the base and mass were acquired using **accelerometers** at a **2 kHz** sampling rate

<u>EXPERIMENT 1</u>

Mass $m = 11.2$ kg

Base Excitation Parameters
Peak-to-Peak Amplitude $A = 0.5$ mm pk-to-pk
Frequency range $f = [4 : 2 : 26]$ Hz

<u>EXPERIMENT 2</u>

Mass $m = 6.3$ kg

Base Excitation Parameters
Peak-to-Peak Amplitude $A = 0.2$ mm pk-to-pk
Frequency range $f = [4 : 2 : 20]$ Hz

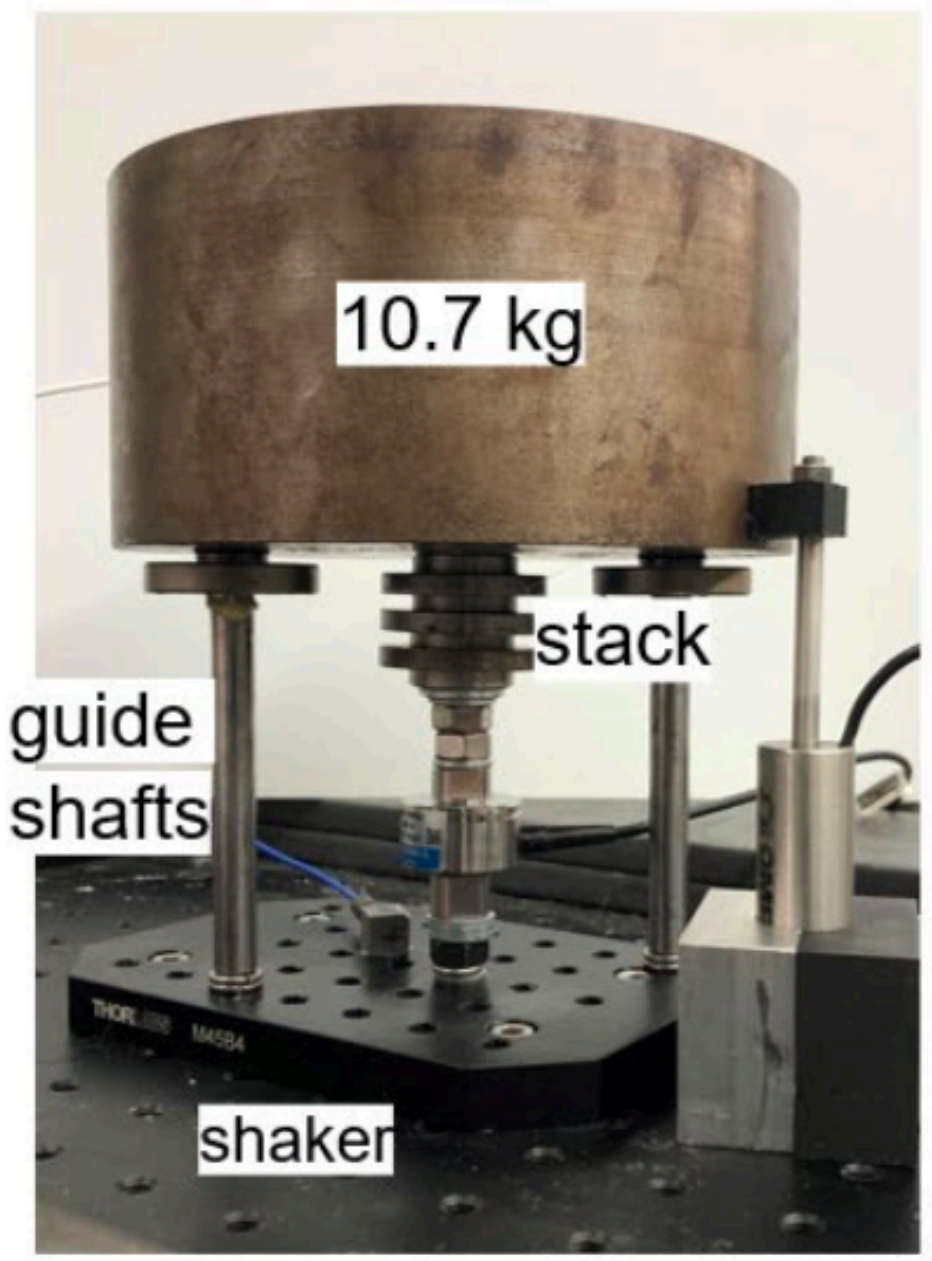


Figure 10 - Experimental setup[3]

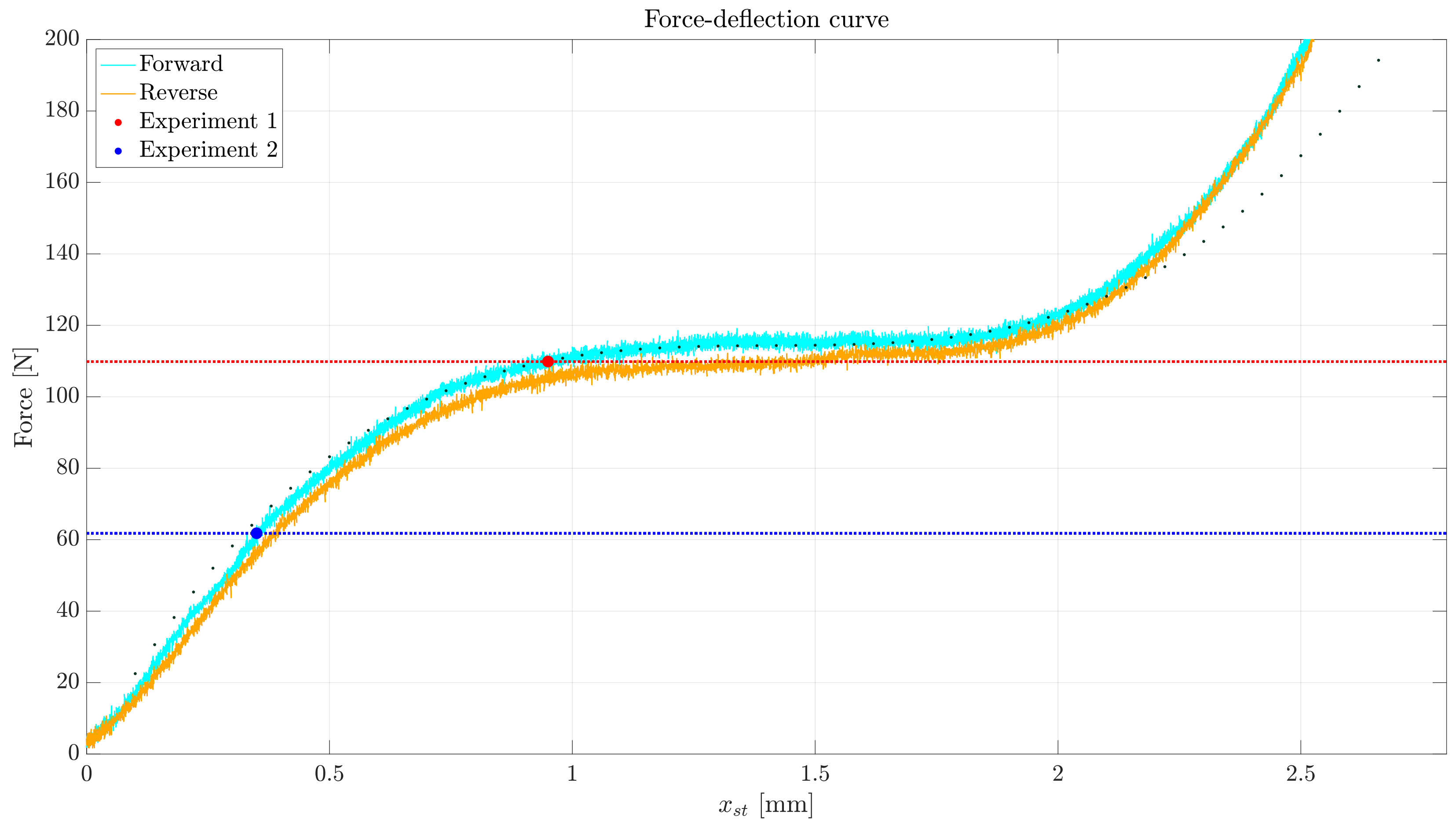


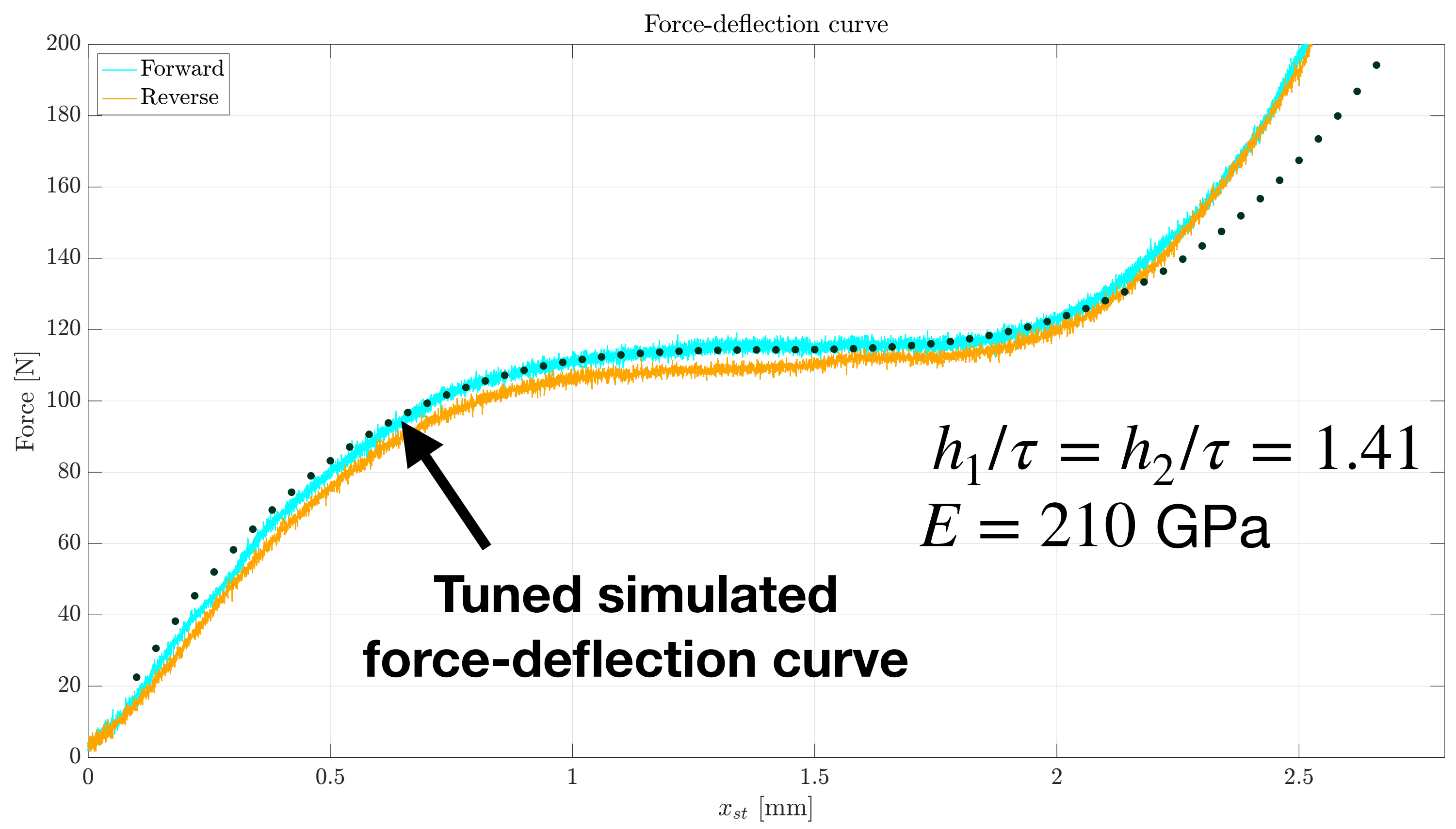
Figure 11 - Operating point of the system in Experiments 1 and 2

PARAMETER TUNING IN SIMULATION

MASS & SPRING PARAMETERS

The value of **mass** m was kept the same as that used in the experiment

h/τ and E are tuned so that simulated and experimental **Force-deflection curves** of the 2-spring stack largely match



DAMPING PARAMETERS

The **true nature** of damping is **unclear**

Linear Viscous Damping model was assumed initially and the value of c was chosen between **0.05-0.5 Ns/mm**

Coulomb friction was assumed thereafter (**smoothened** using $\tanh$ function) and μ was chosen between **0 - 0.1**

SIMULATION

Acceleration profiles of the mass were obtained for different **equilibrium ICs** and **base excitation frequencies (4 - 26 Hz at intervals of 2 Hz)**

The **base motion** is exactly **sinusoidal** in simulation but not in experiment

ITERATIVE VARIATION OF DAMPING PARAMETERS FOR BEST MATCH BETWEEN SIMULATION AND EXPERIMENT

SAMPLE RESULTS - EXPERIMENT 1

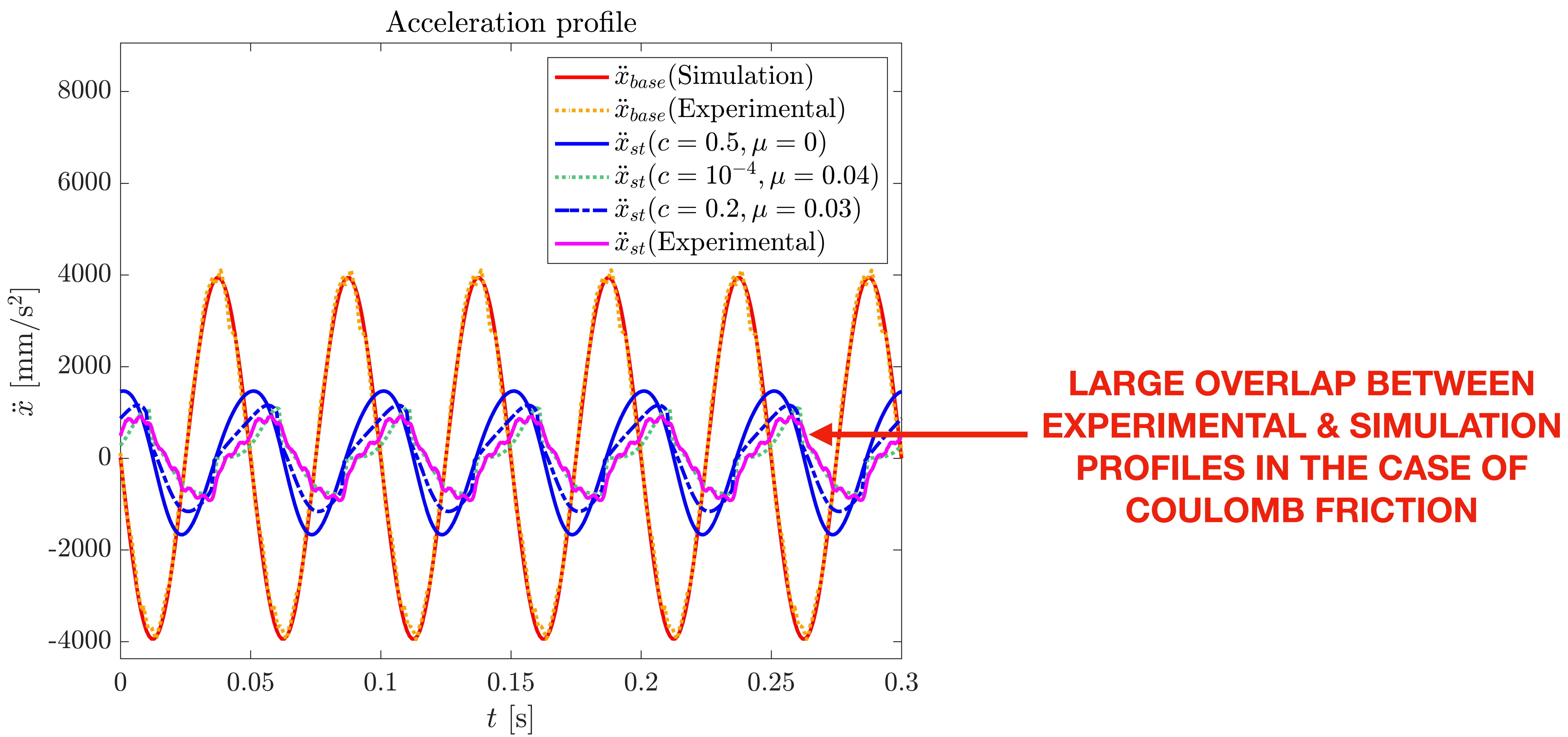


Figure 12 - Acceleration profiles in experiment and simulation for $f = 20$ Hz

The results for other cases are present in the link referenced in **Appendix D**

SAMPLE RESULTS - EXPERIMENT 2

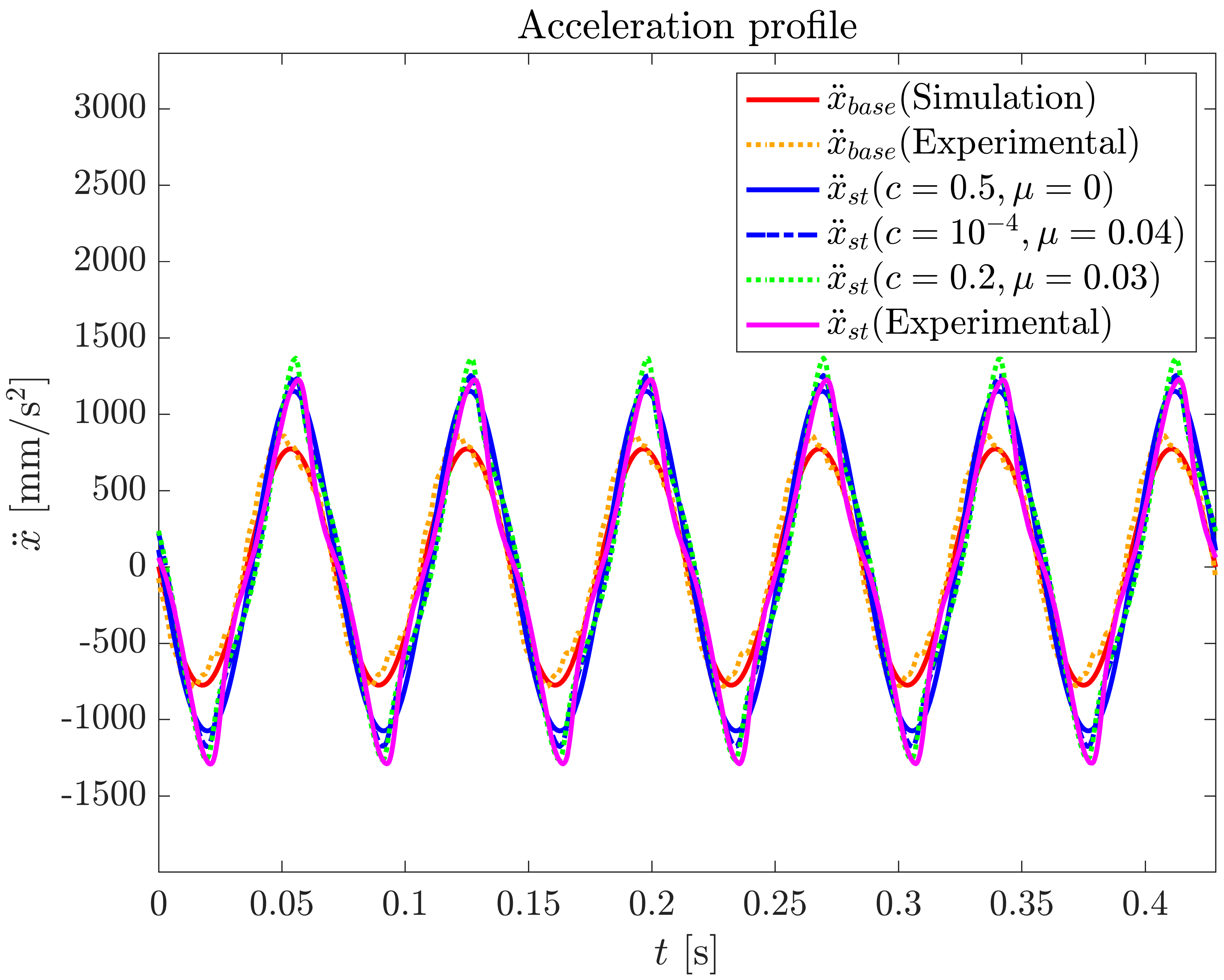
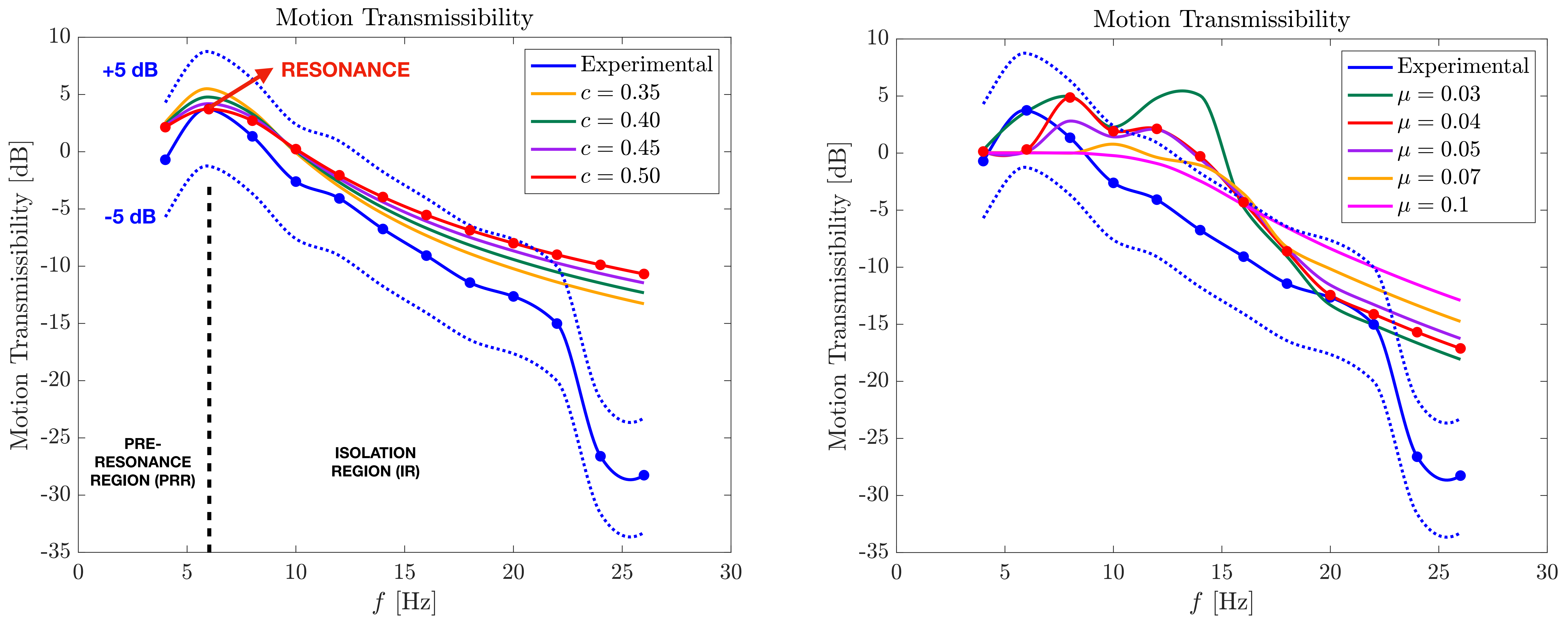


Figure 13 - Acceleration profiles in experiment and simulation for $f = 14$ Hz

The results for other cases are present in the link referenced in **Appendix D**

RESULTS - MOTION TRANSMISSIBILITY

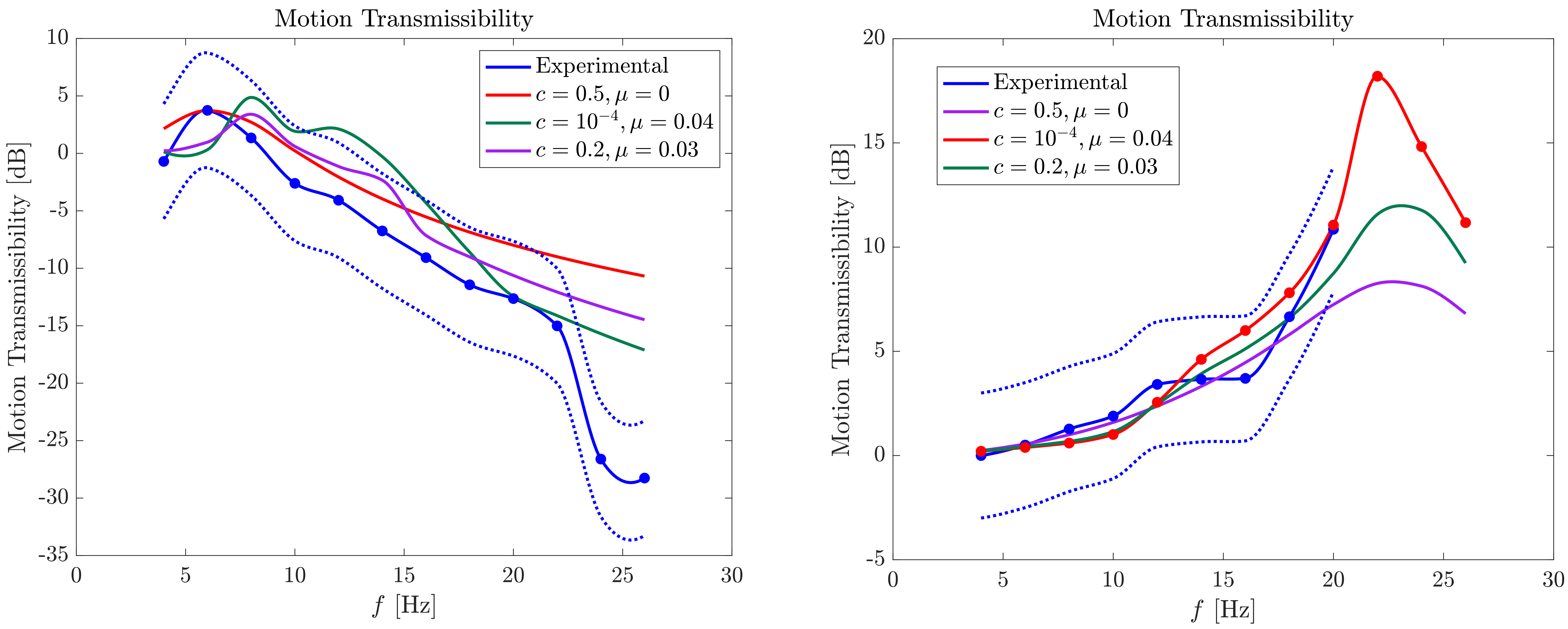


a) **b)**

Figure 14 - Motion Transmissibility Curves in Experiment 1 for different values of **a)** c and **b)** μ

AS DAMPING INCREASES, 'PEAK-TO-PEAK MOTION TRANSMISSIBILITY' DECREASES IN THE 'PRR' AND GENERALLY INCREASES IN THE 'IR'

EFFECT OF DAMPING



a) **b)**

Figure 15 - Motion Transmissibility Curves for different damping models in **a)** Experiment 1 and **b)** Experiment 2

VISCOUS DAMPING AND COULOMB FRICTION BEST CAPTURE THE MOTION TRANSMISSIBILITY BEHAVIOR IN ‘PRR’ AND ‘IR’ RESPECTIVELY

EFFECT OF OPERATING REGION

SIMULATION PARAMETERS :

$m_{NQZS} = 10.7$ kg (for Non-QZS)
$m_{QZS} = 11.2$ kg (for QZS)
$h_1/\tau = h_2/\tau = 1.41$
$E = 200$ GPa
$c = 0.05$ Ns/mm, $\mu = 0$
$x_{base}(t) = 0.05 \sin(2\pi f t)$ (in mm)

OBSERVATIONS :

- Lower Resonance Frequency in QZS region
- Lower Motion Transmissibility for $f > 5$ Hz in QZS $\Longrightarrow$ Effective Vibration Isolation

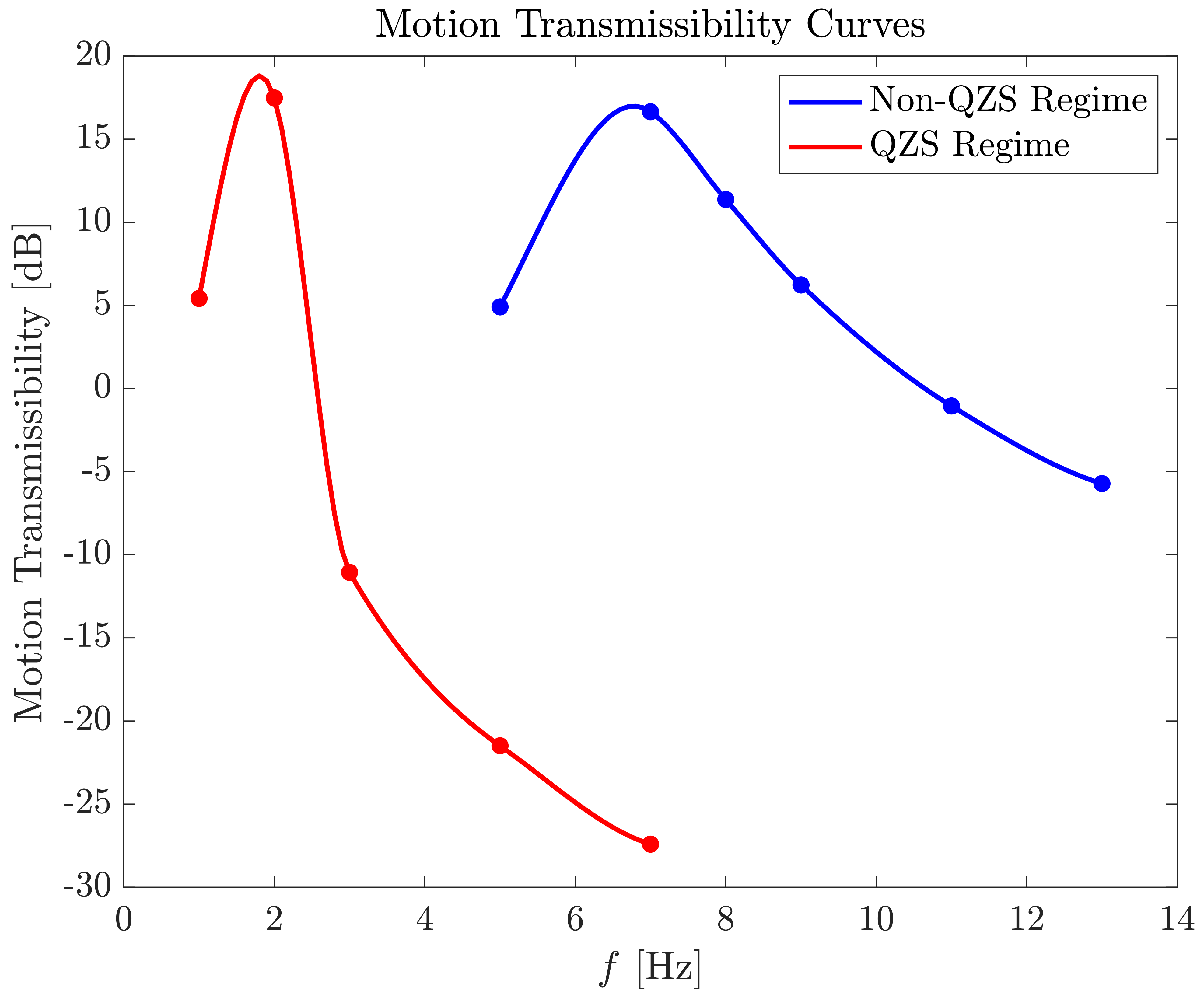


Figure 16 - Motion Transmissibility Curves in Non-QZS and QZS regimes

CONCLUSION

Summary

1. In this project, the **dynamics of a QZS isolator** stack consisting of **two coned-disk springs** in the presence of **base excitation** is modeled as a **single DoF** system and numerically simulated in **MATLAB**

2. A **linearized system analysis** was performed as a simple test. The close match between analytical and simulation results indicated the **reliability** of the simulator for small displacements

3. **Experimental studies** were conducted and simulations were performed after suitable **parameter tuning** in order to test the reliability of the simulator at different base excitation frequencies

Observations

1. The **motion transmissibility** results in simulation generally lie within **5 dB** of the experimental values, which is a good starting point. However, **no damping model** is able to capture all aspects of the **acceleration profiles** for all frequencies currently

2. Viscous damping and Coulomb friction best capture the motion transmissibility behavior in the low and high frequency regions, thus a **mixed non-linear damping** model has the potential to describe the transmissibility behavior better

3. The **lower motion transmissibility** at high frequencies in the **QZS region** compared to the Non-QZS region allows for **effective vibration isolation** as intended for the isolator stack

FUTURE WORK

- Perform simulation and experimental studies on stacks having **non-identical springs**, especially springs that have h/τ ratios $>$ 1.41. Similar studies need to be performed for stacks with **more than 2 springs**
- Investigate the effect of factors like **edge-friction coefficient, snap-through,** and **hysteresis** behavior
- Study the behavior of the system in scenarios where **multiple equilibrium states** exist for a spring-stack
- Improve the model by treating the spring stack and base together as a **2 DoF** system and performing simulations by using the **actual motion of the base** as the input after pre-processing
- Refine the **damping model** so that it accurately captures the damping behavior in the physical system
- Attempt to create a **systematic approach** for tuning the system parameters in an optimum manner, so that simulation and experimental results match more closely

APPENDIX A - REFERENCES

Journal / Conference Papers

1. Liu, C., Zhang, W., Yu, K., Liu, T., & Zheng, Y. (2024). Quasi-zero-stiffness vibration isolation: Designs, improvements and applications. In *Engineering Structures*, *301*, 117282.

2. Gilmore, P., & Gandhi, U. (2021, August). Development of disc spring stack containment methods for vibration isolation. In *INTER-NOISE and NOISE-CON Congress and Conference Proceedings* (Vol. 263, No. 2, pp. 4871-4879). Institute of Noise Control Engineering.

3. Gilmore, P., Gandhi, U., & Singh, R. Effect of Disk-to-Disk Variations on the Nonlinear Static Characteristics and Stability Regimes of Coned Disk Spring Stacks: Experimental and Computational Studies of Quasi-Zero-Stiffness Isolators. *Available at SSRN 4799335*.

4. "Belleville washer". In *Wikipedia.com.* URL: https://en.wikipedia.org/wiki/Belleville_washer

Software

1. Gilmore, P. (2024), MATLAB Code (stack_equations_solver_mm_2springs.m). *Personal Communication*

2. Singh, R. (2024), MATLAB Code (basicFFT.m). *Personal Communication*

APPENDIX B - LEARNING OUTCOMES

- Understanding the **principles** behind QZS Isolators, Coned-Disk Springs and Stack Design aspects
- **Static analysis** of a Coned-Disk Spring Stack to study the Force-deflection behavior
- Effect of **Disk-to-Disk variations** on the quasi-static force-deflection in terms of factors like **stability**
- **Dynamic analysis** of a 2-Spring stack using **Newton's Laws of Motion**
- **Simulating** a Non-Linear Dynamical system in MATLAB to obtain **time** and **frequency domain** results
- Greater understanding of how to match and compare **simulation and experimental** results
- Enhanced **literature review skills** such as the ability to extract relevant information from papers
- Improved **PPT making and demonstration skills** through weekly progress presentations
- Boosted my ability to comprehend other's MATLAB code through **systematic annotation**

APPENDIX C - PROBLEMS ENCOUNTERED

- **Numerical errors and instability** in the **virtual mass implementation** of the dynamics, where a second block with mass $m \approx 10^{-6}$ kg was inserted between the two springs.

- Problems in finding solutions in cases where there are **multiple displacement solutions**, which typically occur when one spring in the stack has $h/\tau > \sqrt{2}$.

- Problems in frequency domain analysis such as **spectral-leakage** and **identifying harmonics**, especially in cases where the base excitation amplitude is on the order of 0.5 mm

- Problems with **tuning** the **spring** and **damping parameters** initially in order to get results that matched with simulation reasonably

APPENDIX D - REPOSITORY

- Weekly Presentations
- Problem Formulation Report
- MATLAB Simulation Code
- Results
- Learning Resources

APPENDIX E - SPRING STACK STATIC ANALYSIS

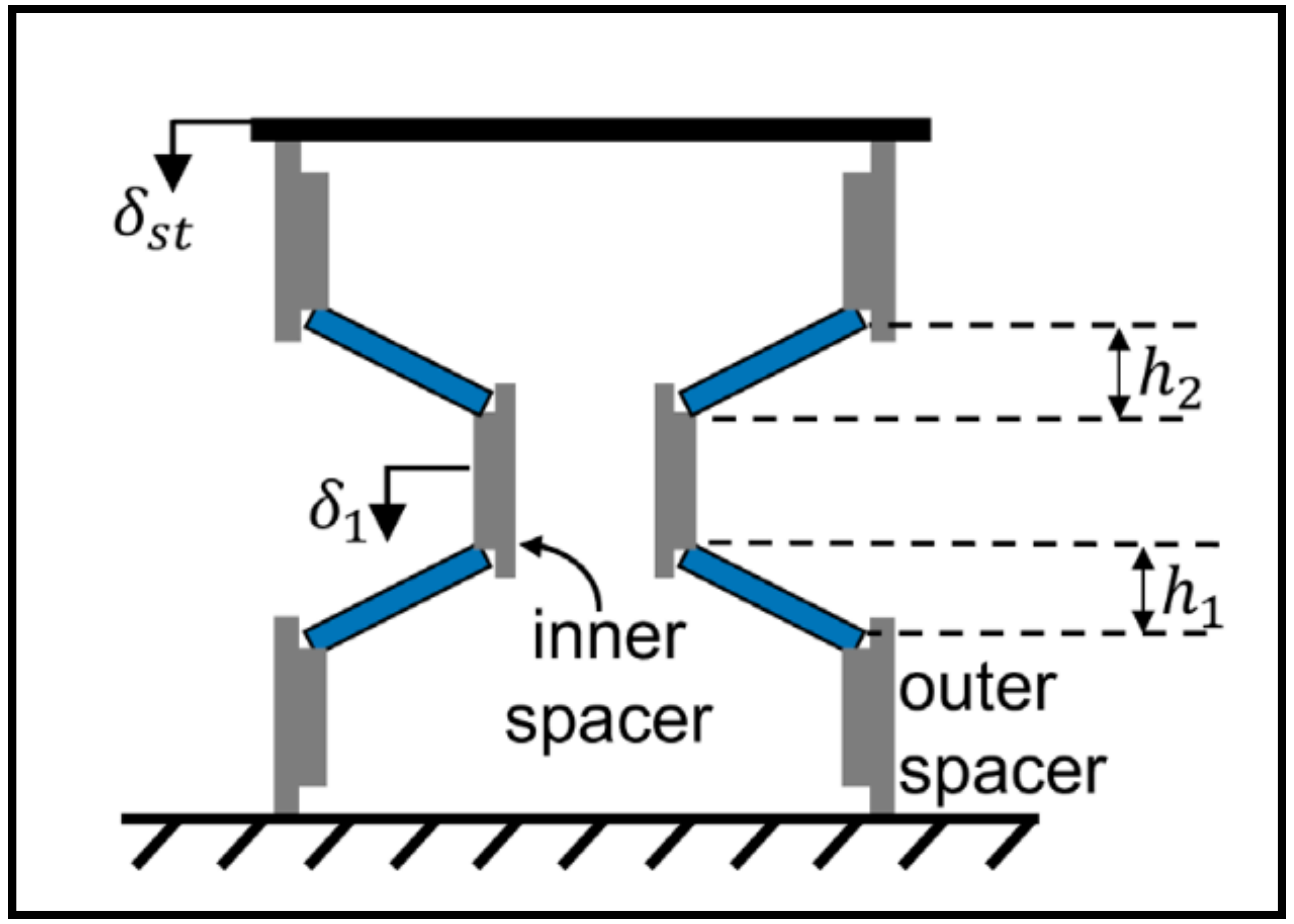


Figure 17:
Static model of 2-spring stack[3]

The height/thickness ratios of the top and bottom springs are h_1/τ and h_2/τ respectively, with force deflection behaviors given by $P_1(\delta)$ and $P_2(\delta)$ respectively

- **Analytical expression** [3] for Force $P(\delta)$ vs deflection δ of a spring (in the absence of edge friction) using the nomenclature defined in **Figure 2**

$$P(\delta) = \frac{E\delta\pi}{a^2}\left(\frac{\alpha}{\alpha-1}\right)^2\left[(h-\delta)\left(h-\frac{\delta}{2}\right)M+N\right] \text{ where}$$

$$\alpha = \frac{a}{b},\ M = \left(\frac{\alpha+1}{\alpha-1} - \frac{2}{\ln(\alpha)}\right)\tau,\ N = \frac{\tau^3}{6}\ln(\alpha)$$

E is the Young's modulus

- δ_{st} is the input displacement and δ_1 is the displacement response coordinate

- For a given value of δ_{st}, the value of δ_1 can be determined using the **equality of forces** in both springs [3]

$$P_1(\delta_{st} - \delta_1) = P_2(\delta_1)$$

The solution(s) of the above non-linear equation yield the value(s) of δ_1

APPENDIX F - FREQUENCY DOMAIN ANALYSIS I

SIMULATION CASE :

$h_1/\tau = h_2/\tau = 1.41$

$E = 200$ GPa

$m = 10.7$ kg

$c = 0.05$ Ns/mm

$\mu = 0$ N

$f = 7$ Hz

$x_{base}(t) = 0.05\sin(2\pi f t)$ (in mm)

Frequency domain analysis is performed on the **acceleration data** in **steady-state**

The frequency domain results for other cases are present in the link referenced in **Appendix D**

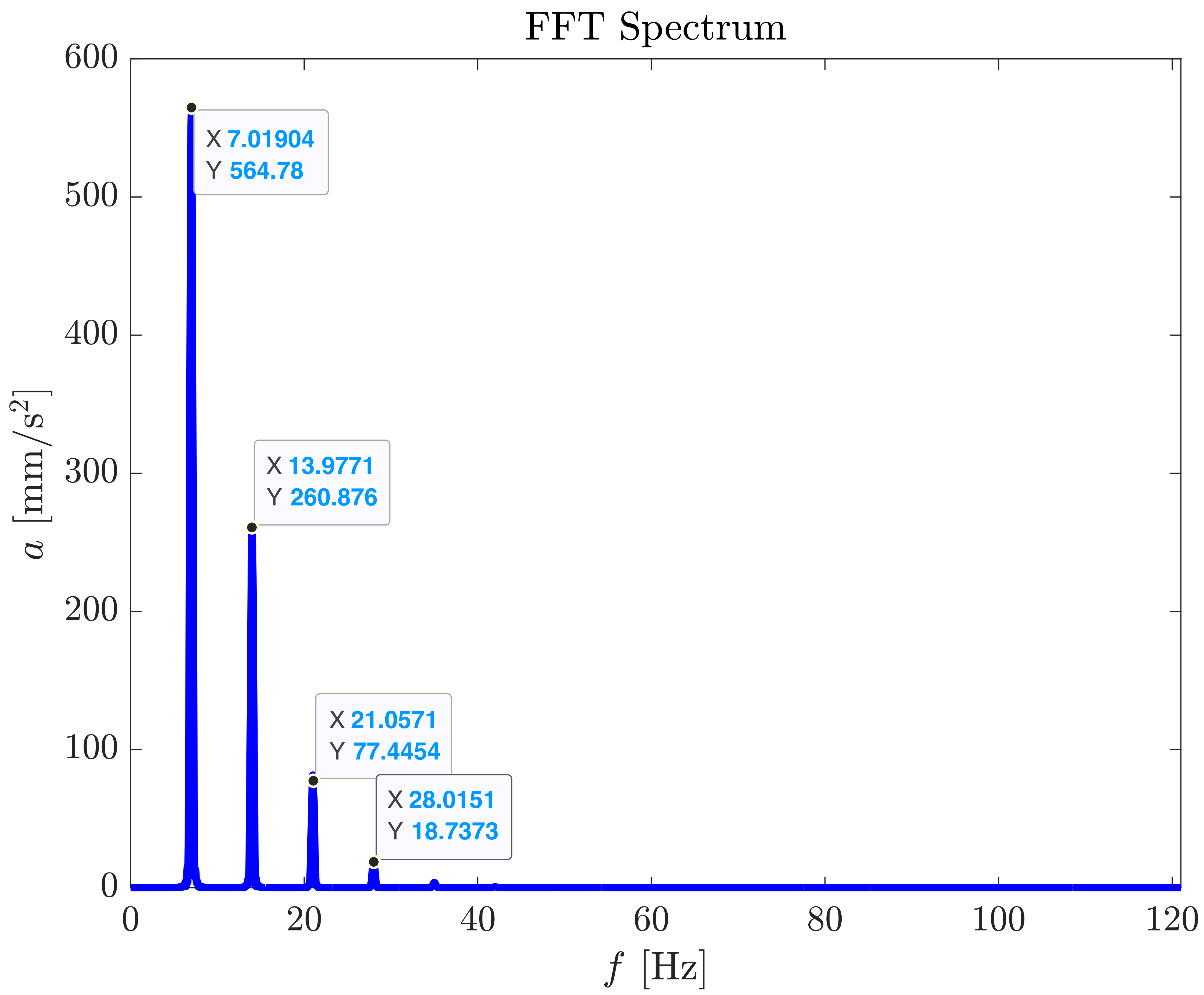


Figure 18 - FFT Spectrum of the output response in simulation

SUPER HARMONICS CORRESPONDING TO THE DRIVING FREQUENCY $\Longrightarrow$ HIGHLY NON-LINEAR

APPENDIX G - LINEARIZED SYSTEM ANALYSIS I

SIMULATION PARAMETERS :

$h_1/\tau = h_2/\tau = 1.41$,

$x_{base}(t) = 0$ (No base excitation)

Initial conditions -

$x_{st}(0) = \delta_{st} + 0.001$ mm

$\dot{x}_{st}(0) = 0$

δ_{st} is the value of x_{st} in static equilibrium

ANALYTICAL RESULTS:

- Amplitude $A \approx$ **0.001 mm**
- Velocity amplitude $= \omega A \approx$ **0.047 mm/s**
- Acceleration amplitude $= \omega^2 A \approx$ **2.29 mm/s²**

MATCH SIMULATION RESULTS CLOSELY

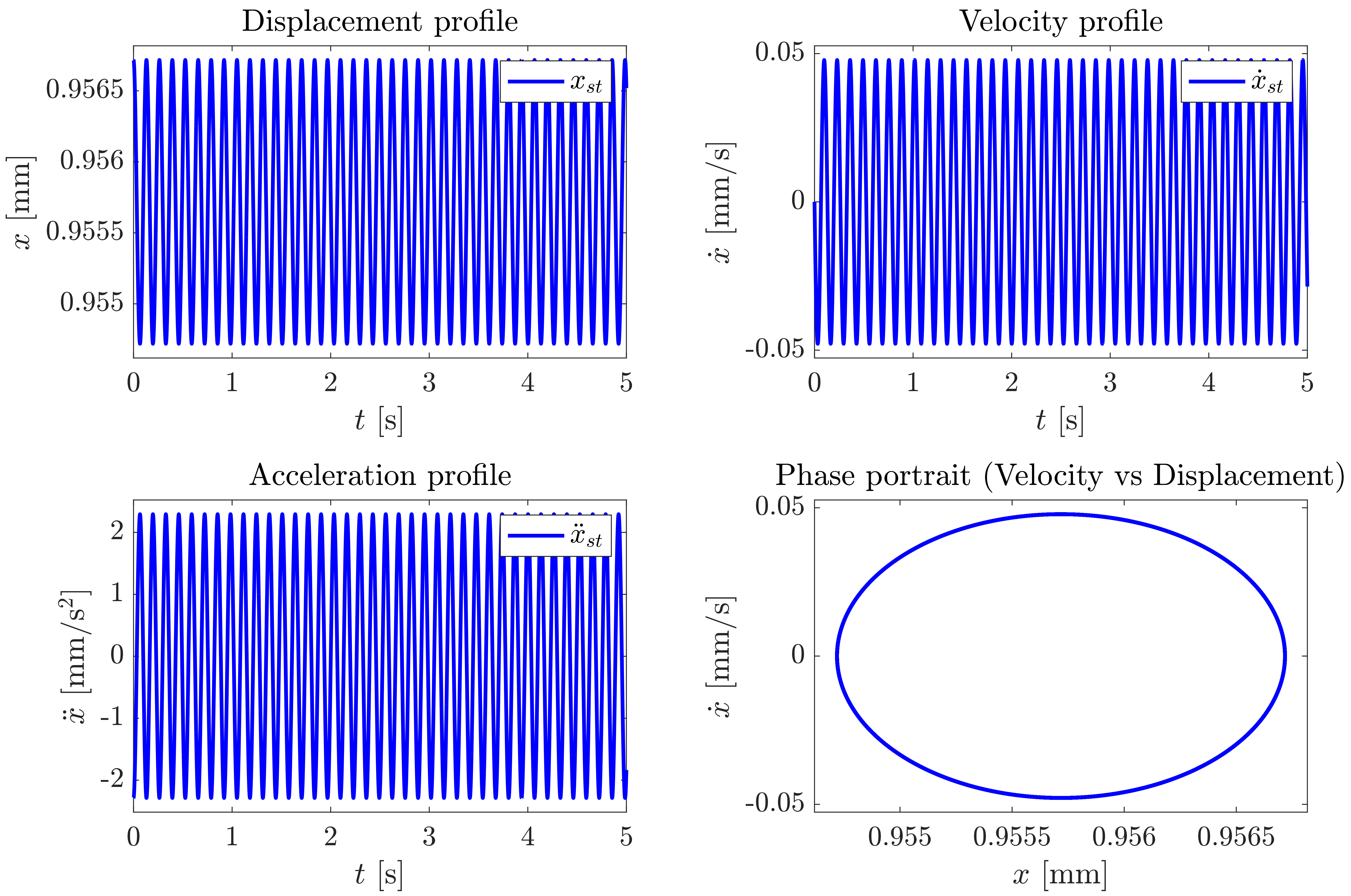


Figure 19 - Time domain results of Linearized system

AAPPENDIX G - LINEARIZED SYSTEM ANALYSIS II

Mean stiffness $\bar{k} \approx 24.5$ N/mm

ANALYTICAL NATURAL FREQUENCY

$$f_{natural} \approx \frac{1}{2\pi}\sqrt{\frac{\bar{k}}{m}} \approx \frac{1}{2\pi}\sqrt{\frac{24.5(10^3)}{10.7}} \approx \textbf{7.62 Hz}$$

OBSERVATIONS -

- **Single Peak** corresponding to the system natural frequency is present in the FFT spectrum, which closely matches **analytical natural frequency**
- Thus, the results of linearized system analysis in **both time and frequency domains** are promising

A SIMPLE VERIFICATION OF THE SIMULATOR'S RELIABILITY

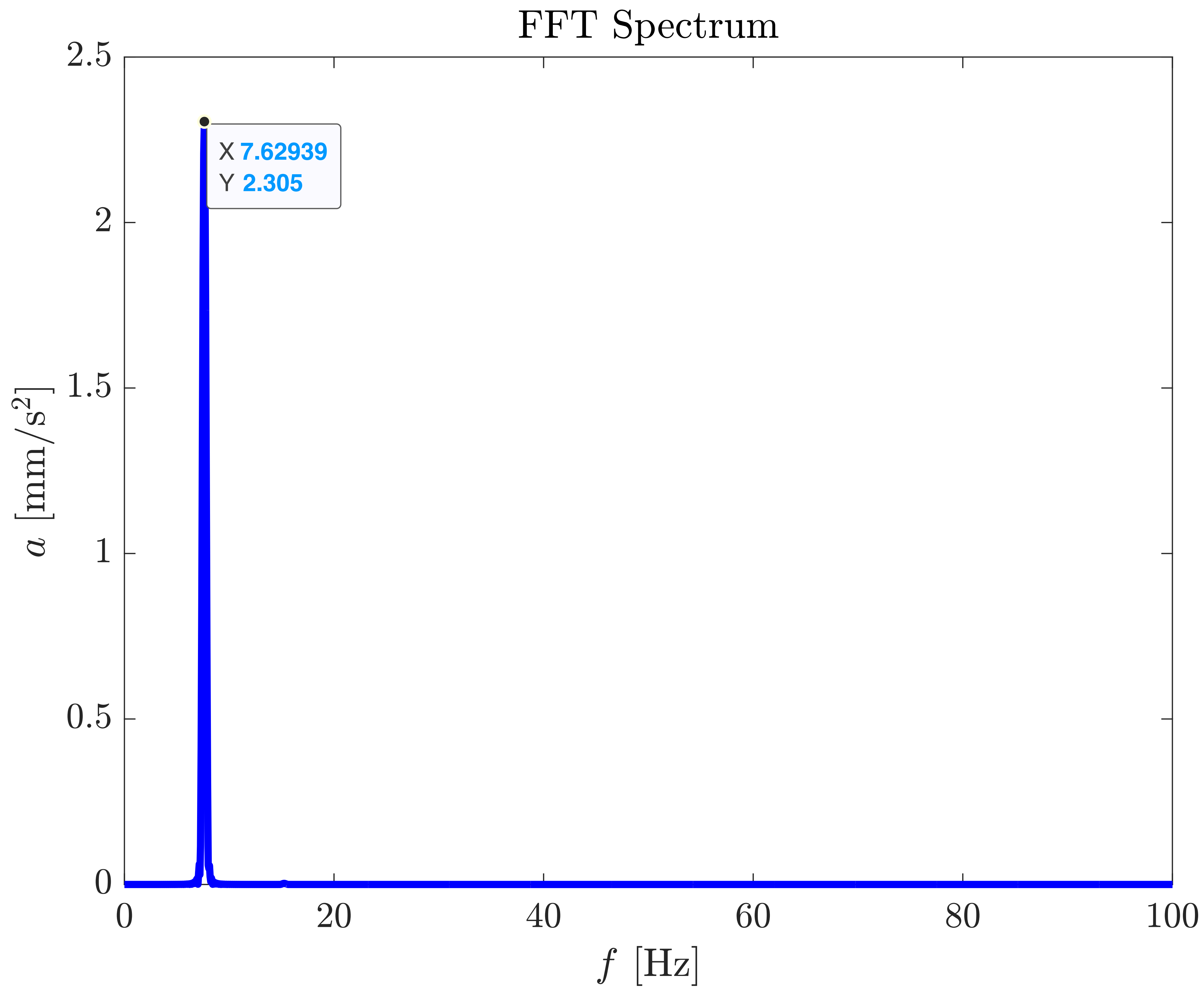


Figure 20 - FFT Spectrum of the output response in simulation