RIYA Program 2024

PROBLEM FORMULATION REPORT

Dynamics of the QZS isolator with Coned Disks

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OVERVIEW

- **Vibration Isolation** - Necessary for many engineering scenarios [1]

- **Quasi-Zero Static (QZS) isolators** - Isolate **low frequency** vibration without sacrificing **load bearing** capability [2]

- **QZS Isolator Design Strategies -**

    1. Combining a **positive** and **negative** stiffness elements [2]

    2. Compact isolators using **coned disk** springs, which have **dynamic stiffness** and **nonlinear force-deflection** regimes [2-3]

- **Series Spring Stack (Figure 1) -** Used to achieve the desired stroke displacement, since a single spring has insufficient stroke

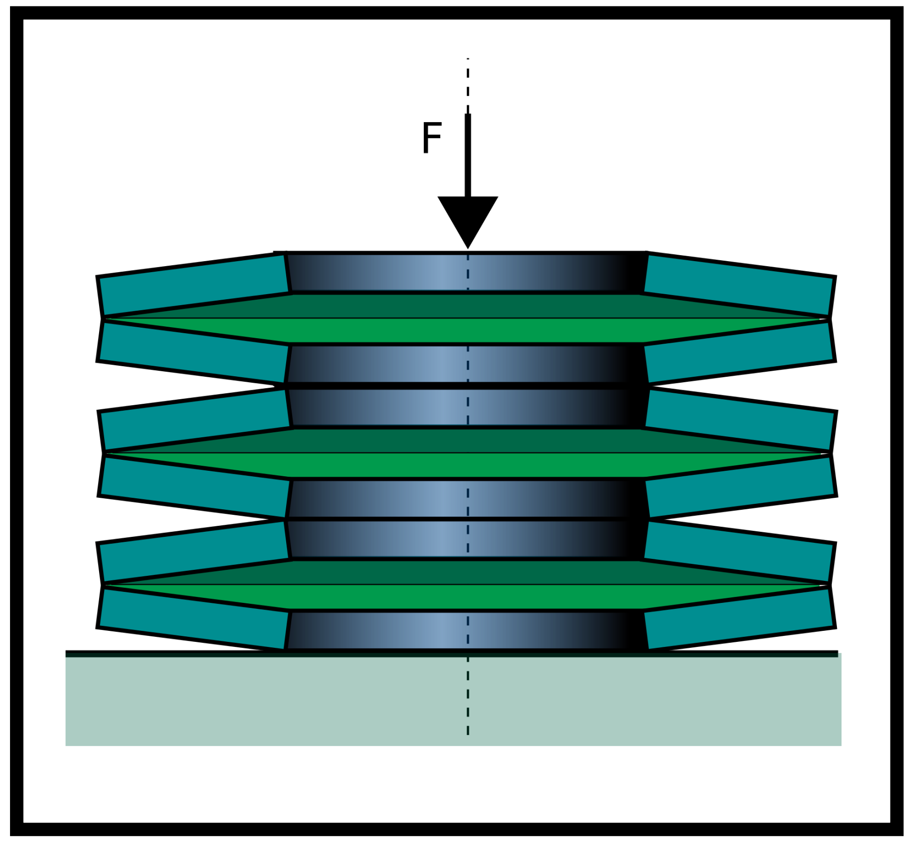


Figure 1:
Schematic of a disk spring stack[4]

Figure 2:
Physical prototype of a disk spring stack[2]

PROPERTIES OF CONED-DISK SPRINGS

- **Analytical expression** [3] for Force $P(\delta)$ vs deflection δ (in the absence of edge friction)

  $$P(\delta) = \frac{E\delta\pi}{a^2}\left(\frac{\alpha}{\alpha-1}\right)^2\left[(h-\delta)\left(h-\frac{\delta}{2}\right)M+N\right]$$

  where

  $$\alpha = \frac{a}{b},\ M = \left(\frac{\alpha+1}{\alpha-1}-\frac{2}{\ln(\alpha)}\right)\tau,\ N = \frac{\tau^3}{6}\ln(\alpha)$$

  E is the Young's modulus

- For $h/\tau \approx 1.41$, the spring stiffness in the vicinity of $\delta = h$ is close to zero, highlighting the **QZS nature**

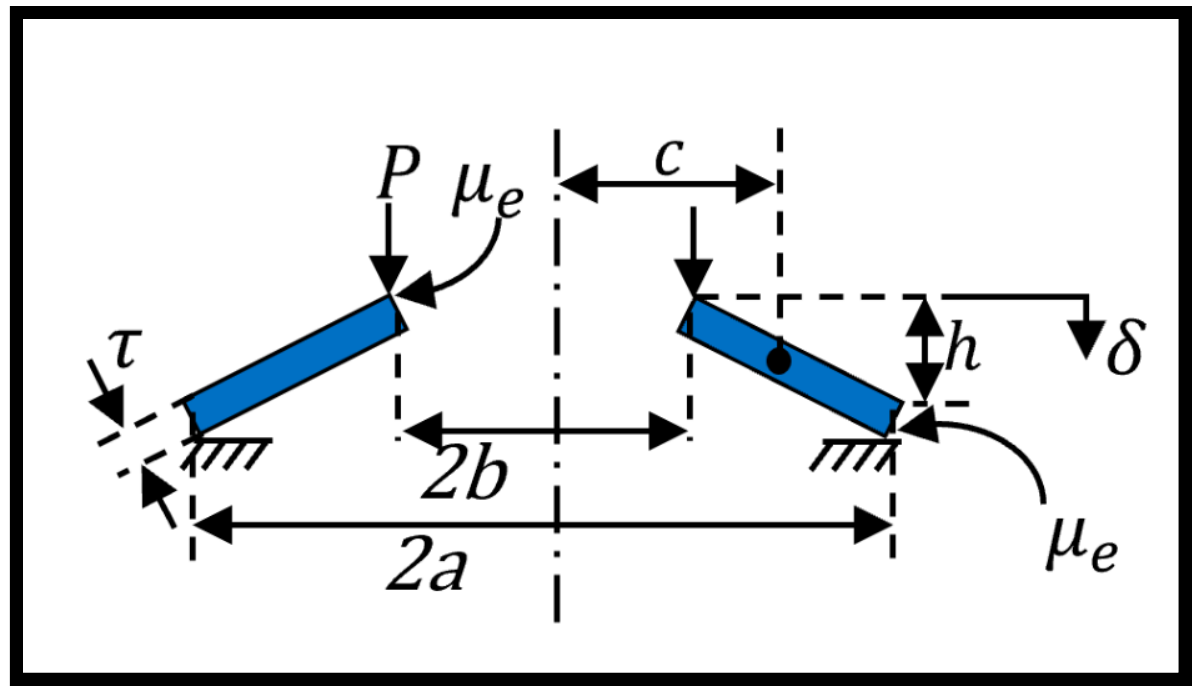


Figure 3:
Cross-section of a coned-disk spring[3]

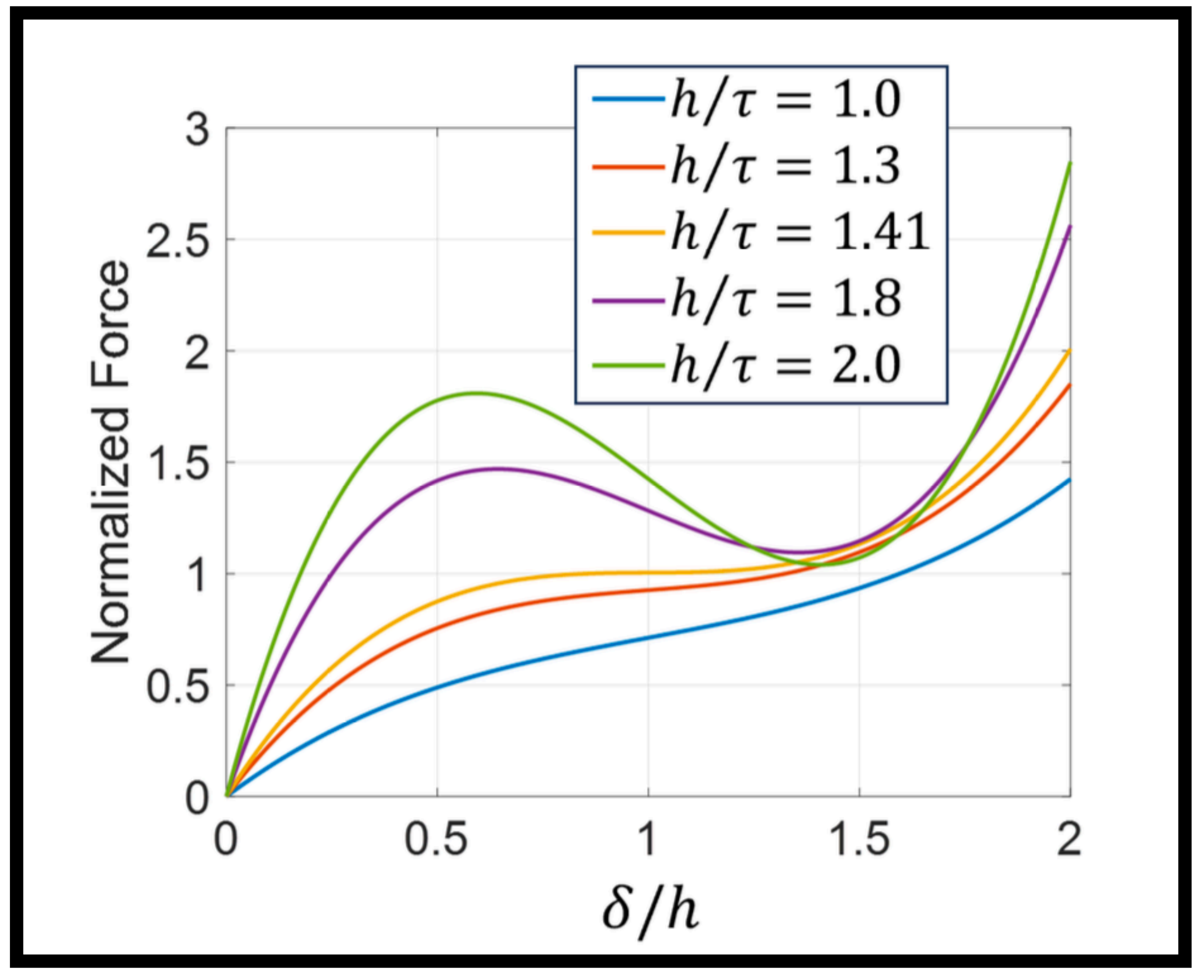
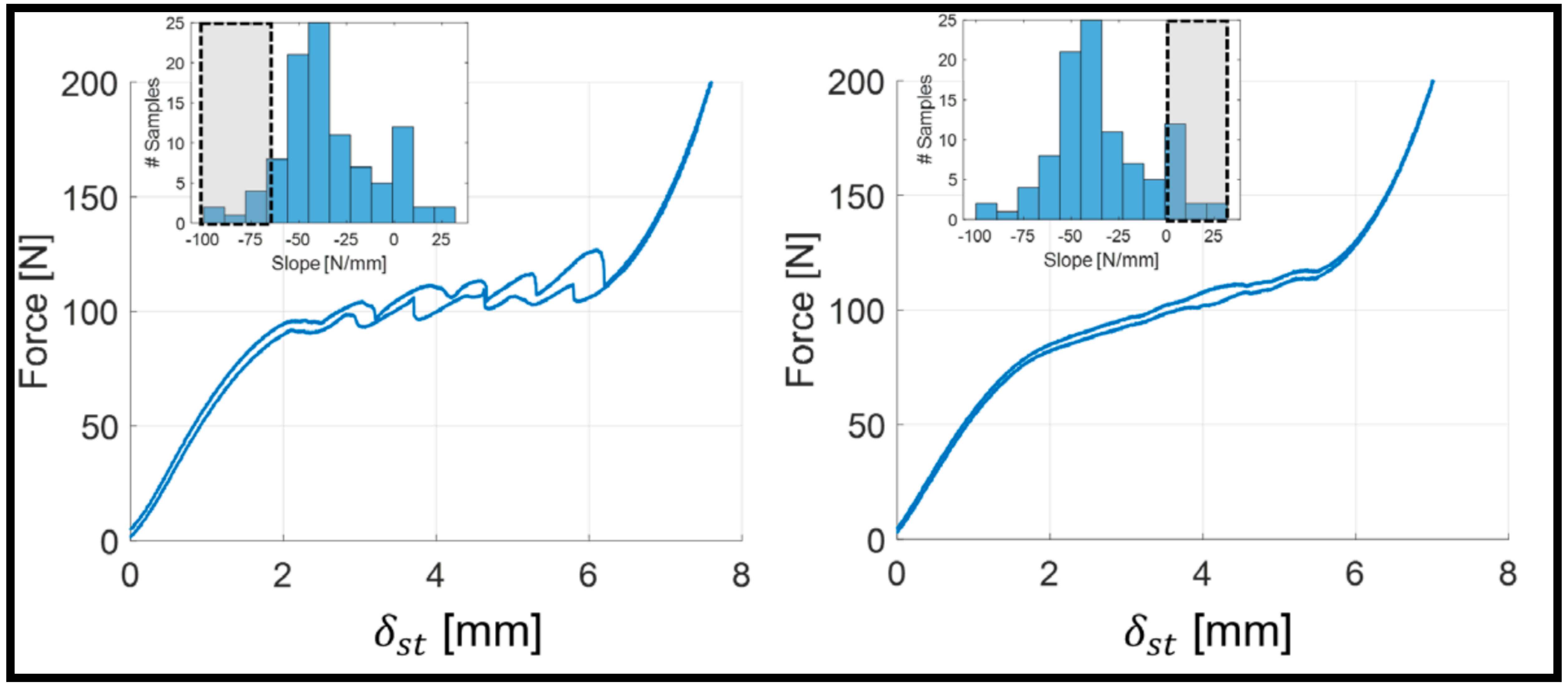


Figure 4:
Force-deflection curves for coned-disk springs[3]

EFFECT OF DISK-TO-DISK VARIATIONS

- **Major Design assumption -** Identical springs in a stack, but hard to achieve in practice due to manufacturing constraints [3].

- **Static load-deflection** behavior of spring stack is **sensitive** to variation in h/τ ratio of different springs, especially when $h/\tau > 1.414$ [3] (**snap-through behavior** due to negative-stiffness regimes)



(a) (b)

Figure 7:
Force-deflection curves for 6-spring stacks

Case (a) - Springs with Negative minimum stiffness
Case (b) - Springs with Positive minimum stiffness

Snap-through behavior and **directional dependence** are clearly visible in Case (a)

ANALYSIS OF A 2-SPRING STACK

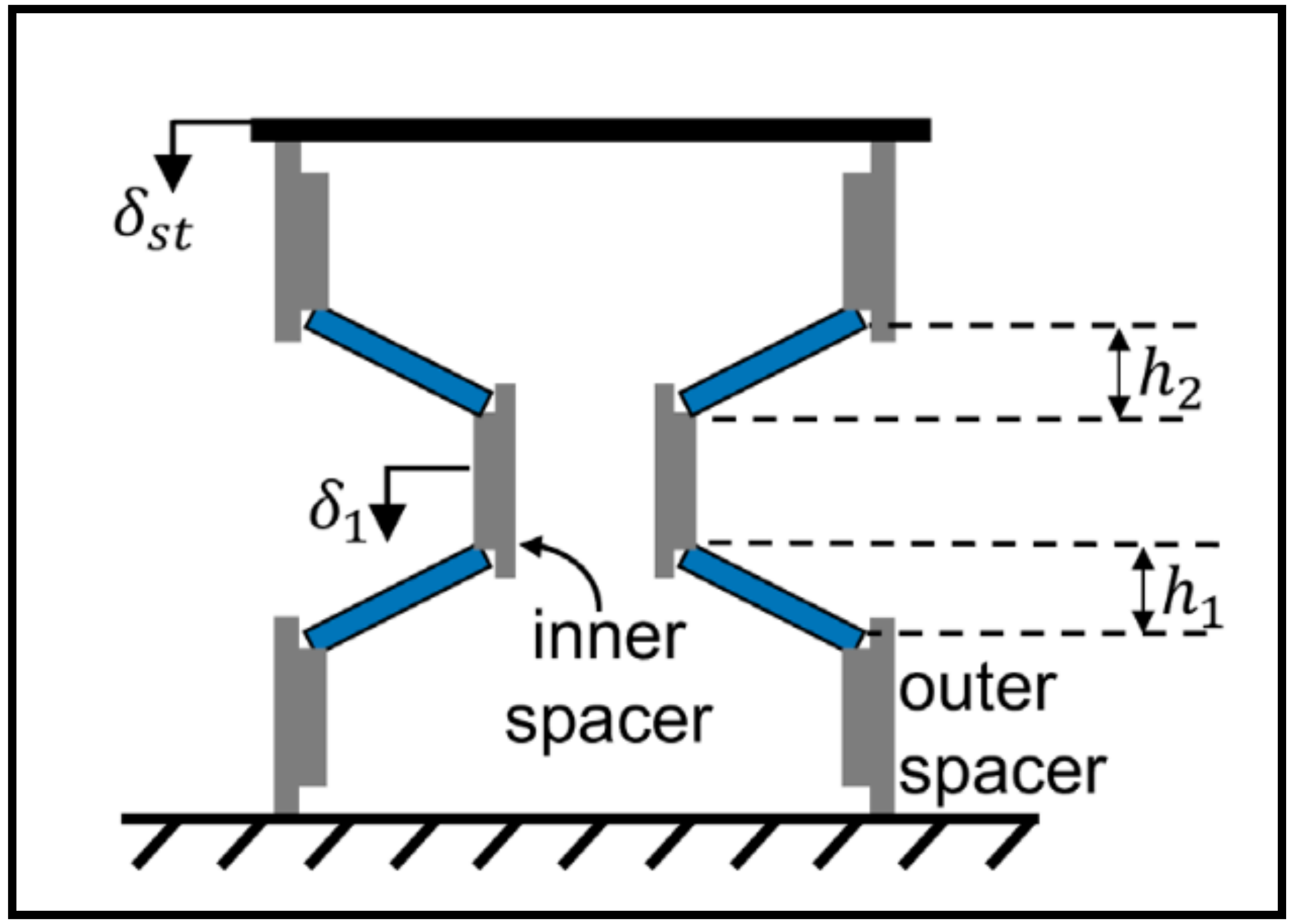


Figure 6:
Static model of 2-spring stack[3]

The height/thickness ratios of the top and bottom springs are h_1/τ and h_2/τ respectively, with force deflection behaviors given by $P_1(\delta)$ and $P_2(\delta)$ respectively.

- δ_{st} is the input displacement and δ_1 is the displacement response coordinate.

- For a given value of δ_{st}, the value of δ_1 can be determined using the **equality of forces** in both springs [3].

  $$P_1(\delta_{st} - \delta_1) = P_2(\delta_1)$$

  The solution(s) of the above non-linear equation yield the value(s) of δ_1.

- For a 2-spring stack, there can be **1 or 3** equilibrium-points.

- The **stability** of each equilibrium point can be determined using the sign of second-derivative of the potential energy.

TYPICAL STABLE AND UNSTABLE SOLUTIONS

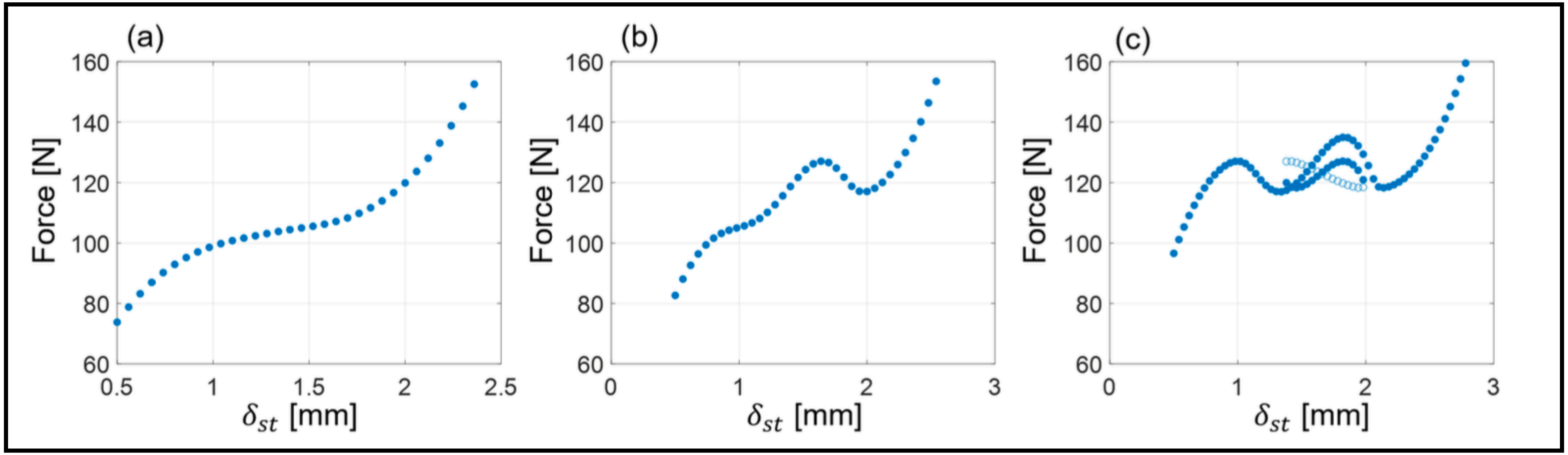


Figure 7:
Analytical force-deflection curves of different 2-spring stacks

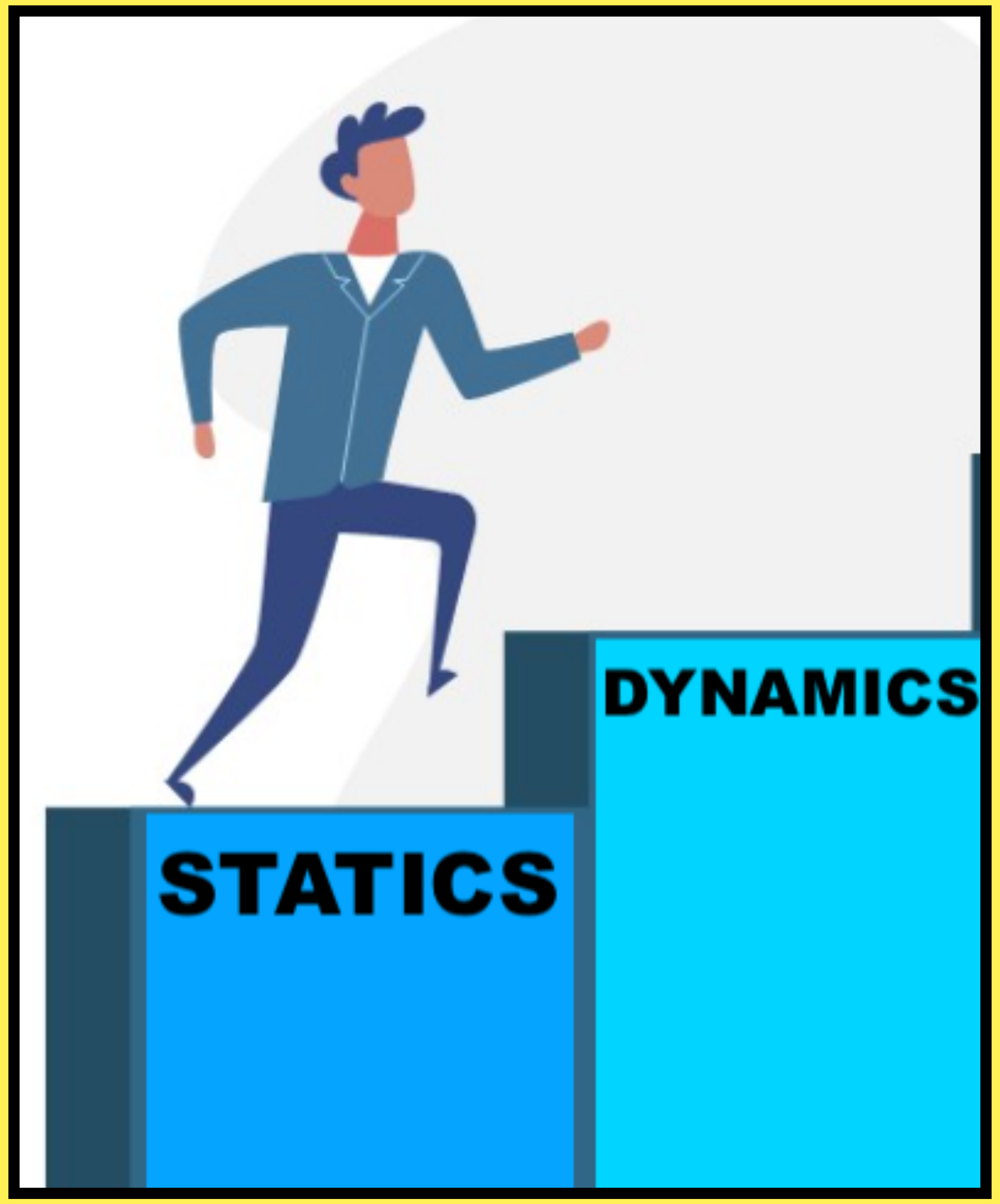
Case (a) $[h_1/\tau, h_2/\tau] = [1.32,1.36]$ - Only stable equilibria
Case (b) $[h_1/\tau, h_2/\tau] = [1.36,1.58]$ - Only stable equilibria
Case (c) $[h_1/\tau, h_2/\tau] = [1.58,1.64]$ - Both stable and unstable equilibria

Stable and **Unstable** equilibrium points are marked with **filled** and **unfilled** circles respectively.

THE NEXT STEP …

- **Goal -** To go beyond static analysis and venture into the **dynamics of the system**!

OBJECTIVES



MODELING & SIMULATION

EXPERIMENTAL VALIDATION

IN-DEPTH INVESTIGATION

PROBLEM FORMULATION

- **Objective 1-**
  A) Model the non-linear dynamics of a spring stack-mass-damper system with base-excitation
  B) Numerically simulate the model to obtain displacement, velocity, and acceleration profiles of the system for time-domain analysis, as well as Force-displacement profiles for physical-domain analysis

- **Objective 2 -**
  A) Validate simulation results for different base excitation amplitudes/frequencies through experiments
  B) Examine how simulation and experimental results can be brought closer through additional model elements
  C) Understand the key parameters that help in effective vibration isolation using a coned-disk spring stack

- **Objective 3 -**
  A) Investigate the effect of damping, hysteresis and other non-linearities on the dynamics
  B) Analyse the non-linear system in frequency domain for sub-harmonics and super-harmonics
  C) Compare the response of the 2 spring-stack with that of a single spring and the linearized system
  D) Compare the response of a 2 spring-stack consisting of identical springs vs non-identical springs

OBJECTIVE 1A - MODELING

System Sketch

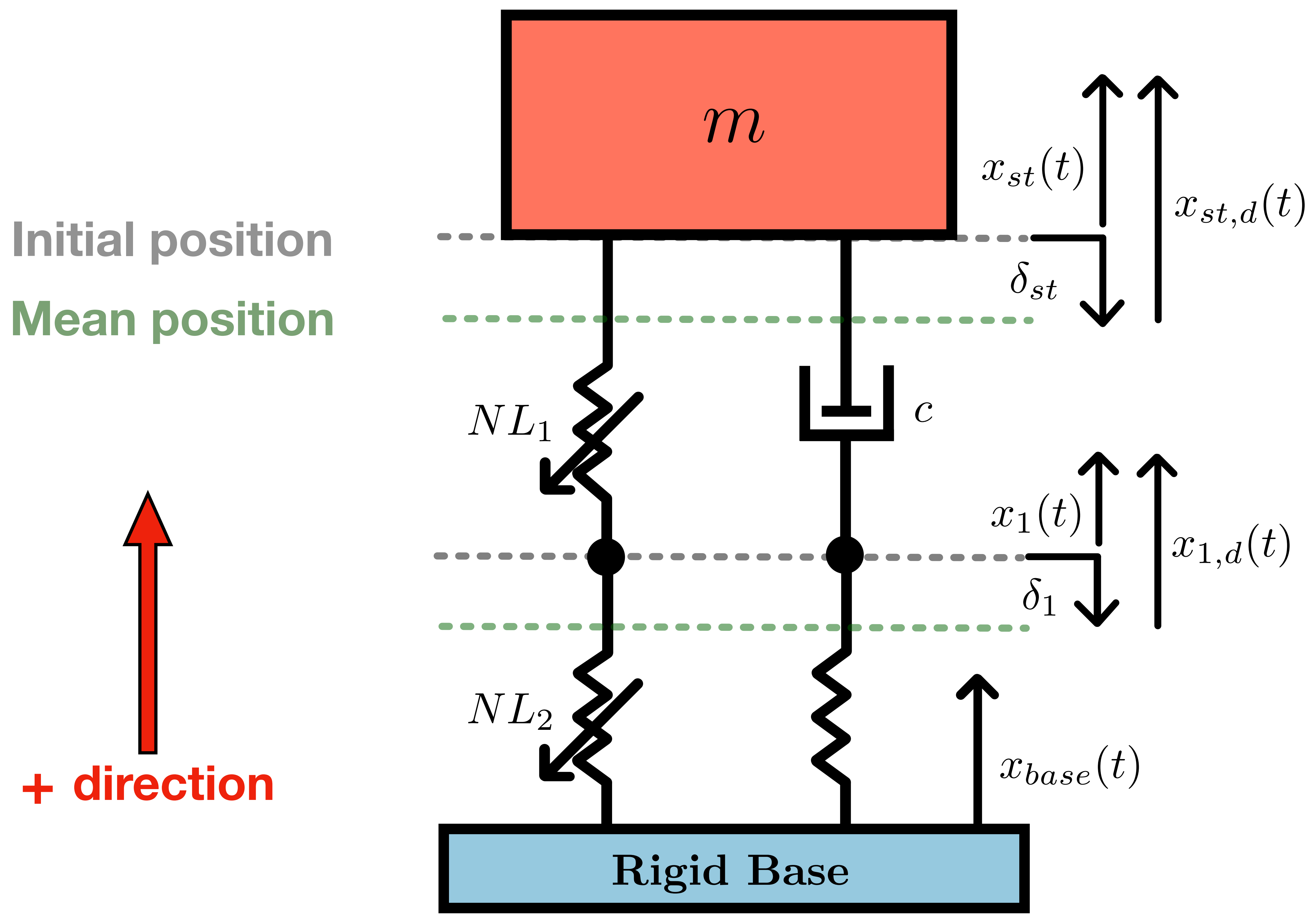


Figure 8 -
Sketch of a **spring stack-mass-damper** system

In **Figure 8**,
m - Mass of the suspended block
c - Damping coefficient of the Damper
NL_1 - Top conical disk spring with ratio h_1/τ
NL_2 - Bottom conical disk spring with ratio h_2/τ

For the mass m,
δ_{st} is the mean displacement in **static equilibrium**
$x_{st,d}(t)$ is the **dynamic displacement**
$x_{st}(t) = \delta_{st} + x_{st,d}(t)$ is the **total displacement**

For the bottom of NL_1,
δ_1 is the mean displacement in **static equilibrium**
$x_{1,d}(t)$ is the **dynamic displacement**
$x_1(t) = \delta_1 + x_{1,d}(t)$ is the **total displacement**

OBJECTIVE 1A - EQUATIONS OF MOTION

In the absence of damping

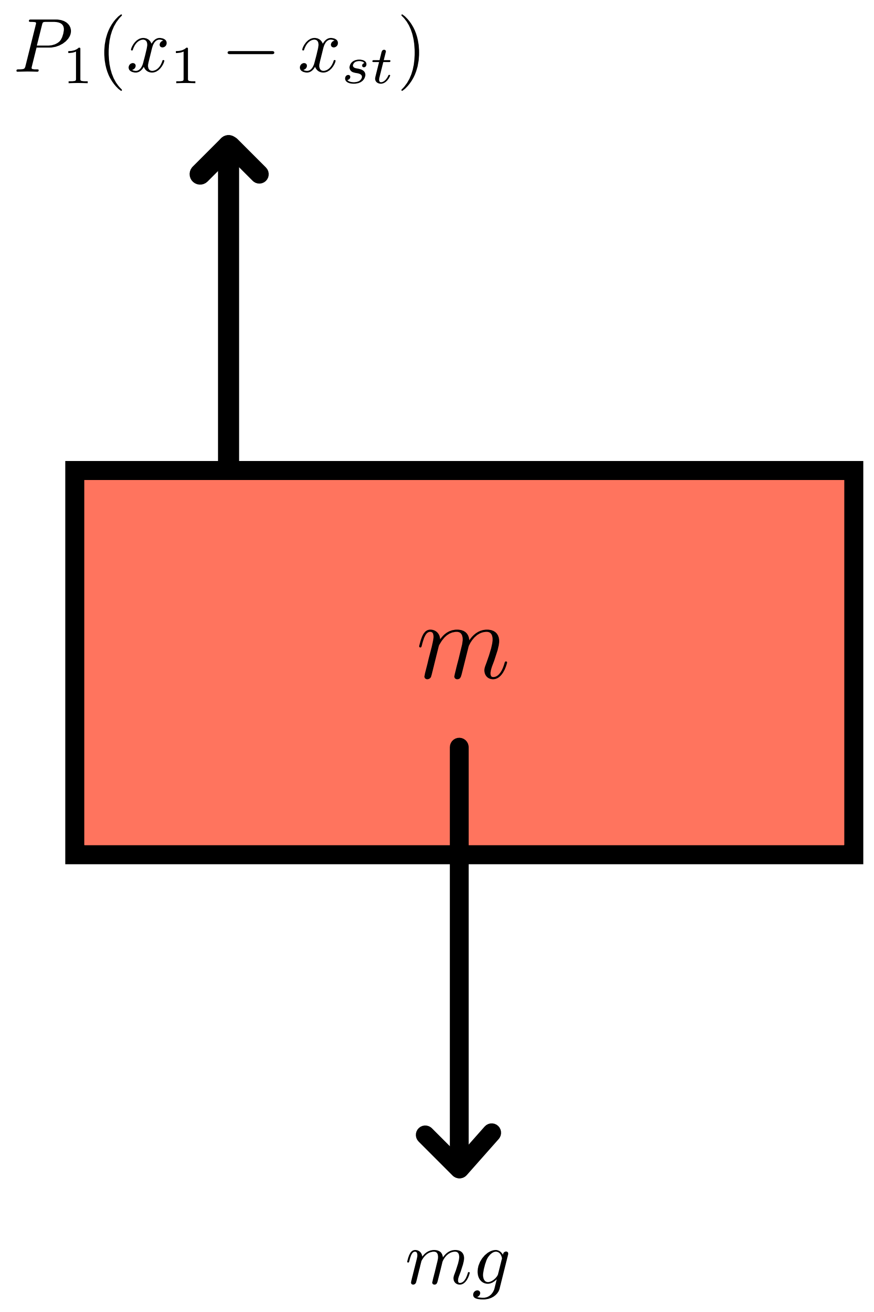


Figure 9 -
FBD of mass m

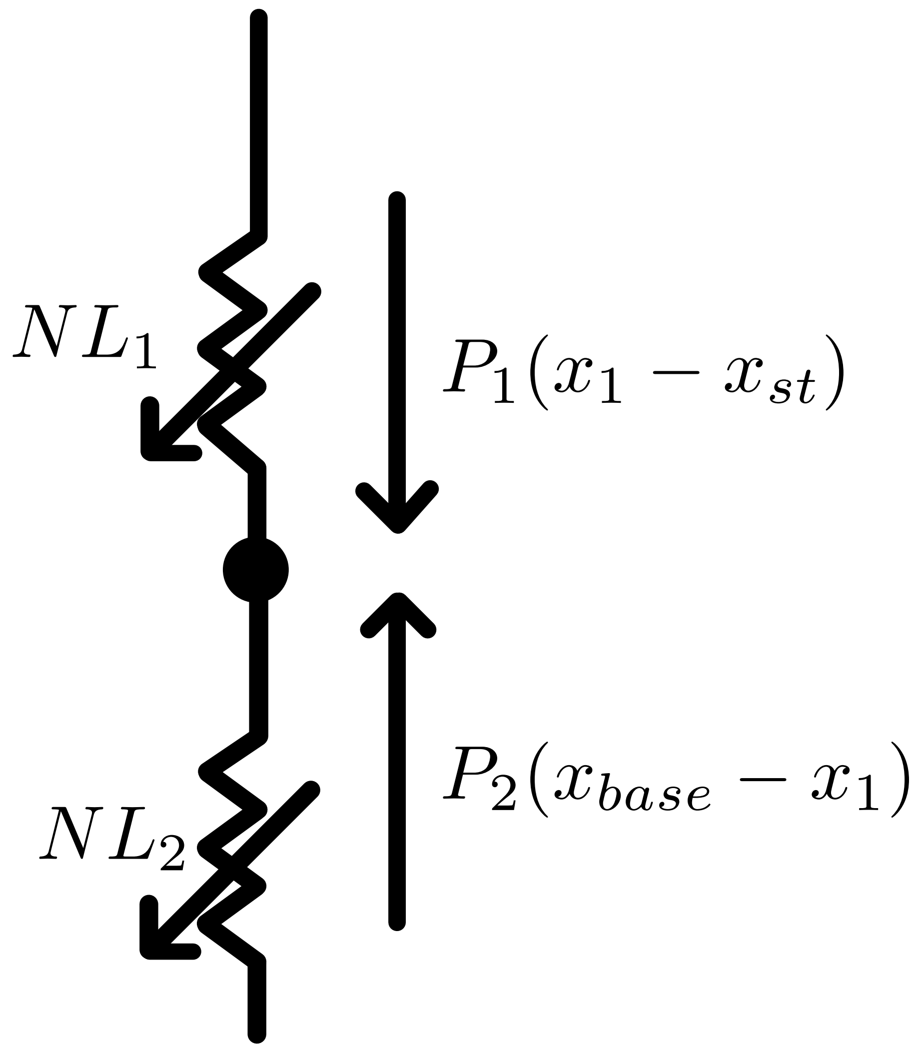


Figure 10 -
FBD of springs at the connection

From **Figure 9**,

$$m\ddot{x}_{st} = -mg + P_1(x_1 - x_{st})$$

From **Figure 10**,

$$P_1(x_1 - x_{st}) = P_2(x_{base} - x_1)$$

The state vector $x(t)$ is defined as

$$x(t) = \begin{bmatrix} x_{st}(t) \\ \dot{x}_{st}(t) \end{bmatrix}$$

OBJECTIVE 1B - SIMULATION PIPELINE

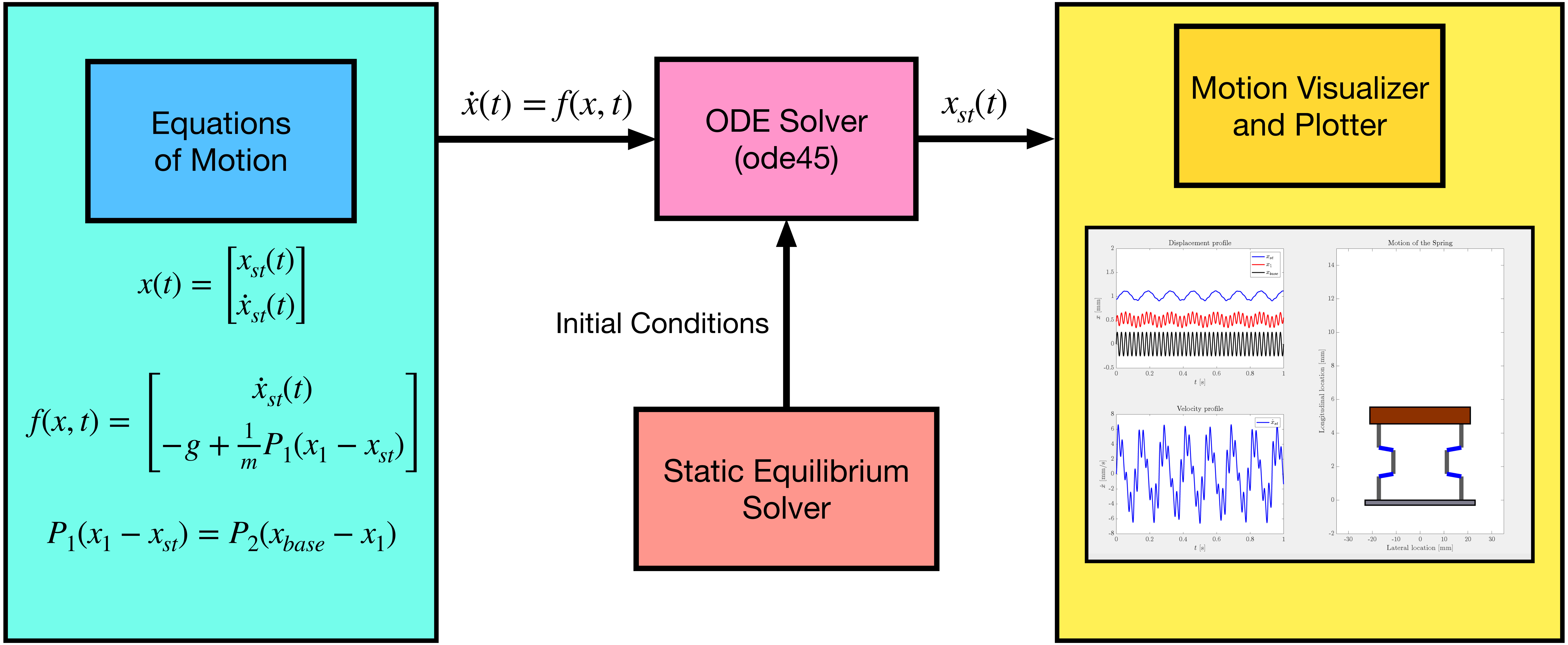


Figure 11 - Simulation pipeline in MATLAB

OBJECTIVE 1B - PRELIMINARY RESULTS

Identical Springs

Case 1: $h_1/\tau = 1.41$, $h_2/\tau = 1.41$, $c = 0$, $f = 40$ Hz, $x_{base}(t) = 0.25\sin(2\pi f t)$ (in mm)

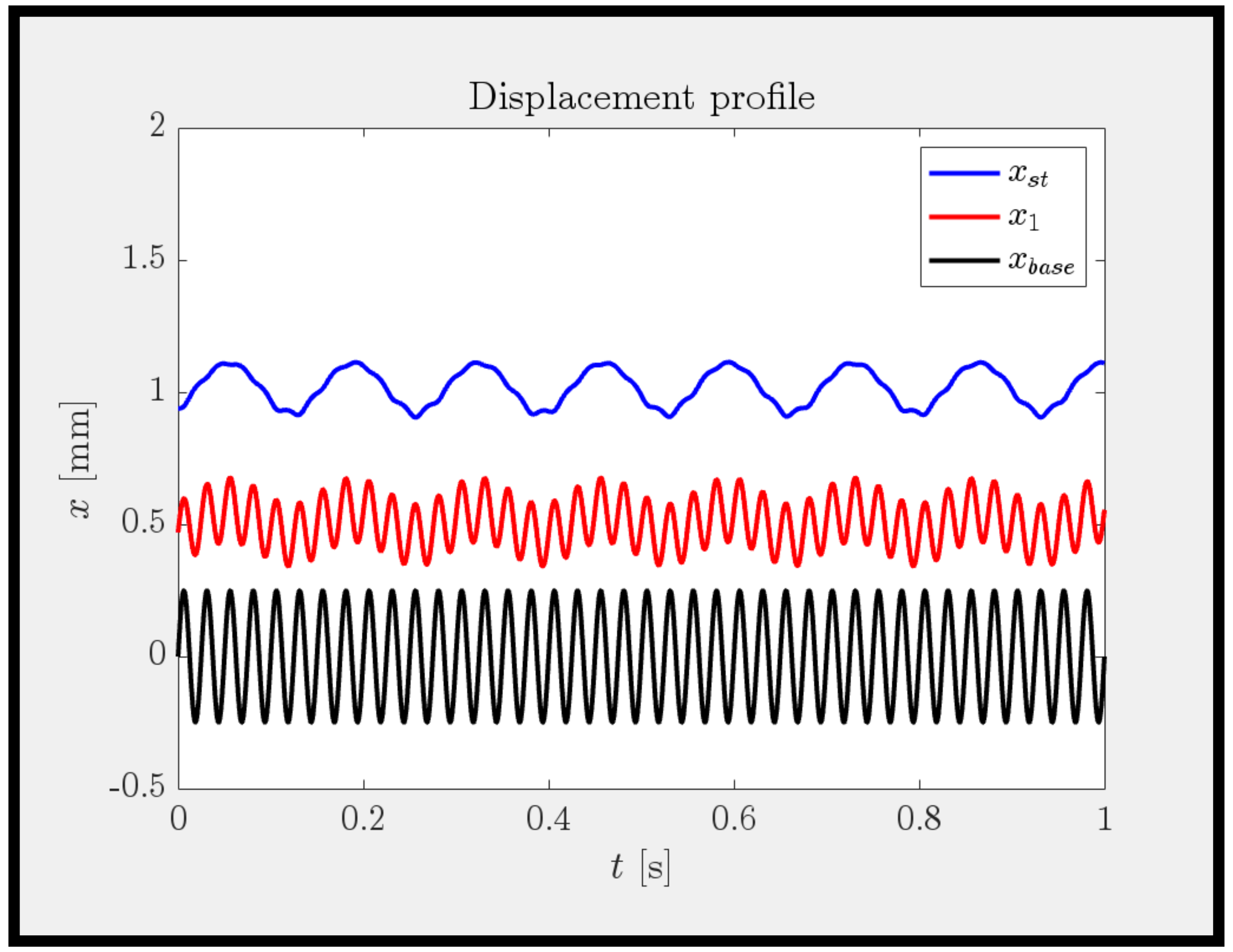


Figure 12 - Displacement profile in Case 1

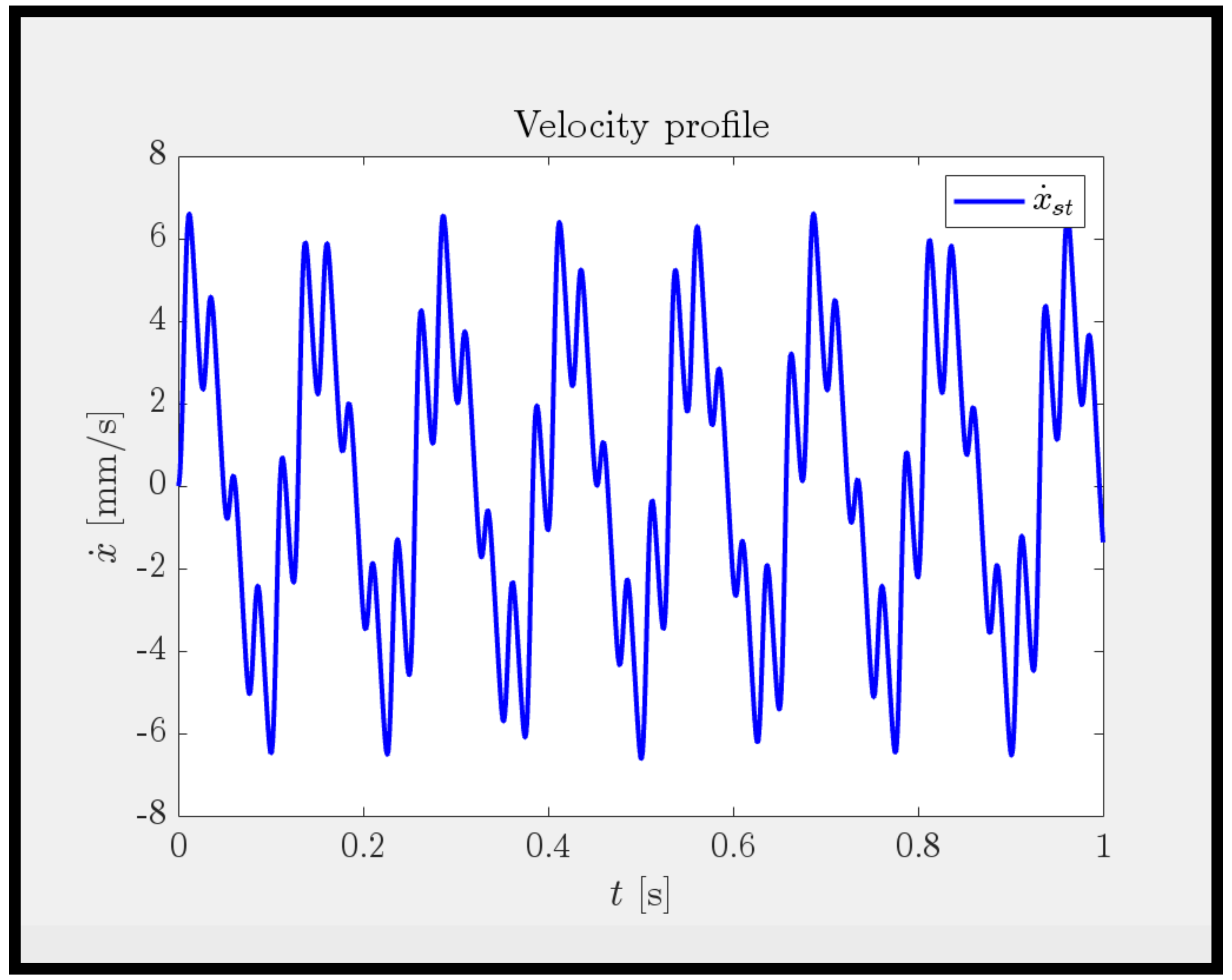


Figure 13 - Velocity profile in Case 1

OBSERVATIONS

- At 40 Hz base excitation, approximately 5 cycles of input lead to 1 cycle of output displacement
- The output amplitude is reduced (~0.21 mm peak-to-peak) compared to the input amplitude (0.5 mm peak-to-peak)
- The peak magnitude of velocity is approximately 6.6 mm/s

OBJECTIVE 1B - PRELIMINARY RESULTS

Non-Identical Springs

Case 2: $h_1/\tau = 1.32$, $h_2/\tau = 1.36$, $c = 0$, $f = 40$ Hz, $x_{base}(t) = 0.25\sin(2\pi f t)$ (in mm)

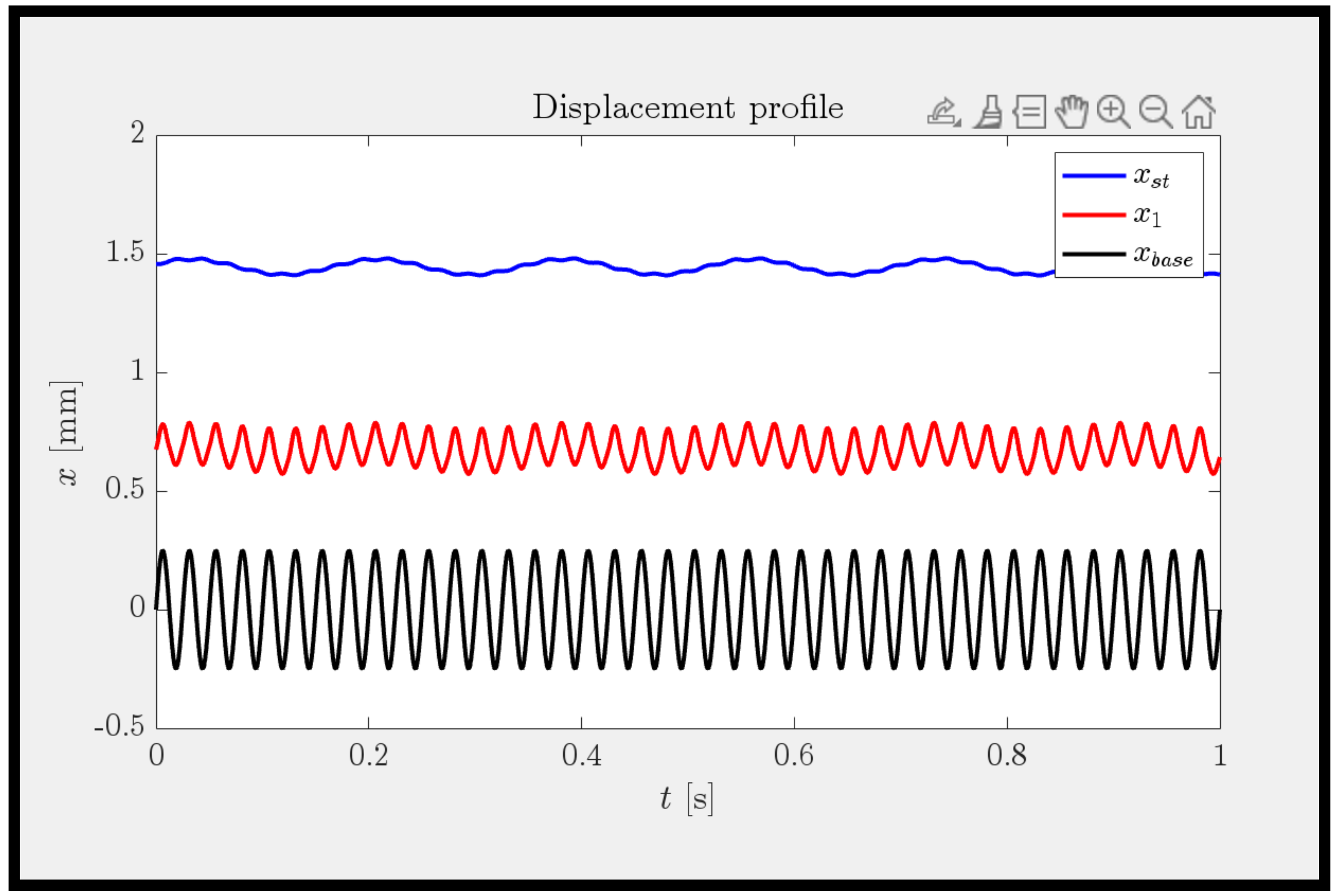


Figure 14 - Displacement profile in Case 2

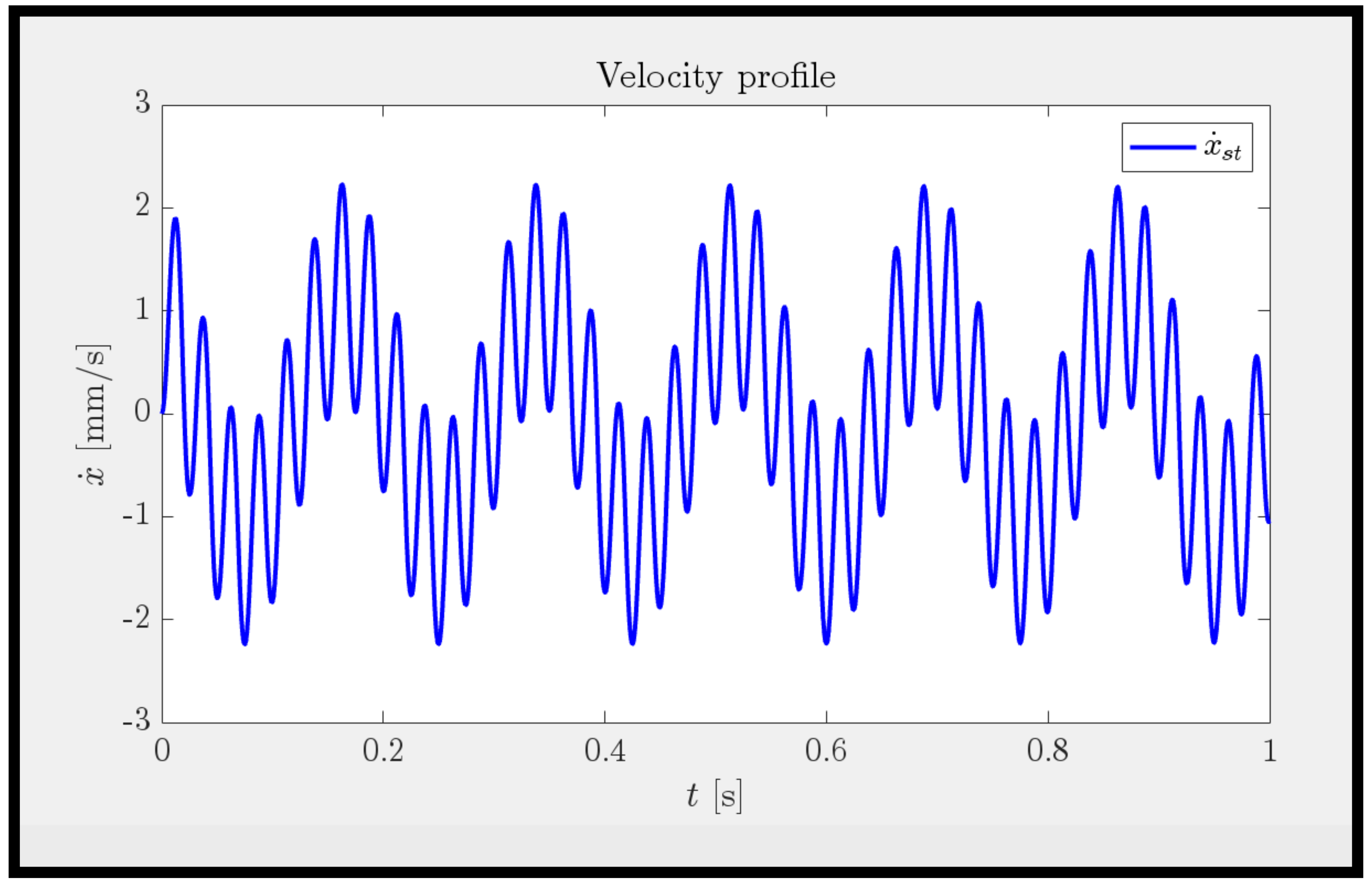


Figure 15 - Velocity profile in Case 2

OBSERVATIONS

- At 40 Hz base excitation, approximately 5 cycles of input lead to 1 cycle of output displacement
- The output amplitude is reduced (~ 0.07 mm) compared to the input amplitude (0.5 mm peak-to-peak)
- The peak magnitude of velocity is approximately 2.24 mm/s

EXPECTED CONCLUSIONS

1. When the **base excitation amplitude is very small** (<< 1 mm), the output response is expected to be very **similar to that of a linear spring** with the same local stiffness. Consequently, the frequency spectrum is expected to have only **1 peak** with no sub/super harmonics

2. When the **base excitation amplitude** is on the order of **0.1 mm** and the frequency of excitation is **less** (with respect to the natural or resonance frequency), the output displacement would have amplitudes comparable to the base excitation. The frequency spectrum is expected to have multiple peaks corresponding to sub/super harmonics

3. When the **base excitation amplitude** is on the order of **0.1 mm** and the frequency of excitation is **high** (with respect to the natural or resonance frequency), the output displacement would have amplitudes less than the base excitation. The frequency spectrum is expected to have multiple peaks corresponding to sub/super harmonics

APPENDIX A - REFERENCES

Journal / Conference Papers

1. Liu, C., Zhang, W., Yu, K., Liu, T., & Zheng, Y. (2024). Quasi-zero-stiffness vibration isolation: Designs, improvements and applications. In *Engineering Structures*, *301*, 117282.

2. Gilmore, P., & Gandhi, U. (2021, August). Development of disc spring stack containment methods for vibration isolation. In *INTER-NOISE and NOISE-CON Congress and Conference Proceedings* (Vol. 263, No. 2, pp. 4871-4879). Institute of Noise Control Engineering.

3. Gilmore, P., Gandhi, U., & Singh, R. Effect of Disk-to-Disk Variations on the Nonlinear Static Characteristics and Stability Regimes of Coned Disk Spring Stacks: Experimental and Computational Studies of Quasi-Zero-Stiffness Isolators. *Available at SSRN 4799335*.

4. "Belleville washer". In *Wikipedia.com.* URL: https://en.wikipedia.org/wiki/Belleville_washer

Software

1. Gilmore, P. (2024), MATLAB Code (stack_equations_solver_mm_2springs.m). *Personal Communication*

2. Singh, R. (2024), MATLAB Code (basicFFT.m). *Personal Communication*

APPENDIX B - LEARNING OUTCOMES

- Understanding the **principles** behind QZS Isolators, Coned-Disk Springs and Stack Design aspects
- **Static analysis** of a Coned-Disk Spring Stack to study the Force-deflection behavior
- Effect of **Disk-to-Disk variations** on the quasi-static force-deflection in terms of factors like **stability**
- **Dynamic analysis** of a 2-Spring stack using **Newton's Laws of Motion**
- **Simulating** a Non-Linear Dynamical system in MATLAB using numerical methods like **ode45**
- Enhanced **literature review skills** such as the ability to extract relevant information from papers
- Improved **PPT making and demonstration skills** through weekly progress presentations
- Boosted my ability to comprehend other's MATLAB code through **systematic annotation**

APPENDIX C - PROBLEMS ENCOUNTERED

- **Numerical errors and instability** in the **virtual mass implementation** of the dynamics, where a second block with mass $m \approx 10^{-6}$ kg was inserted between the two springs.

- Problems in finding solutions in cases where there are **multiple displacement solutions**, which typically occur when one spring in the stack has $h/\tau > \sqrt{2}$.

- Problems in frequency domain analysis such as **spectral-leakage** and **identifying harmonics**, especially in cases where the base excitation amplitude is on the order of 0.5 mm