RIYA Program 2024

PROBLEM FORMULATION REPORT

Dynamics of the QZS isolator with Coned Disks

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INTRODUCTION

Overview

- **Vibration Isolation** - Necessary for many engineering scenarios [1]
- **Quasi-Zero Static (QZS) isolators** - Isolate **low frequency** vibration without sacrificing **load bearing** capability [2]
- **QZS Isolator Design Strategies -**
    1. Combining a **positive** and **negative** stiffness elements [2]
    2. Compact isolators using **coned disk** springs, which have **dynamic stiffness** and **nonlinear force-deflection** regimes [2-3]
- **Series Spring Stack (Figure 1) -** Used to achieve the desired stroke displacement, since a single spring has insufficient stroke

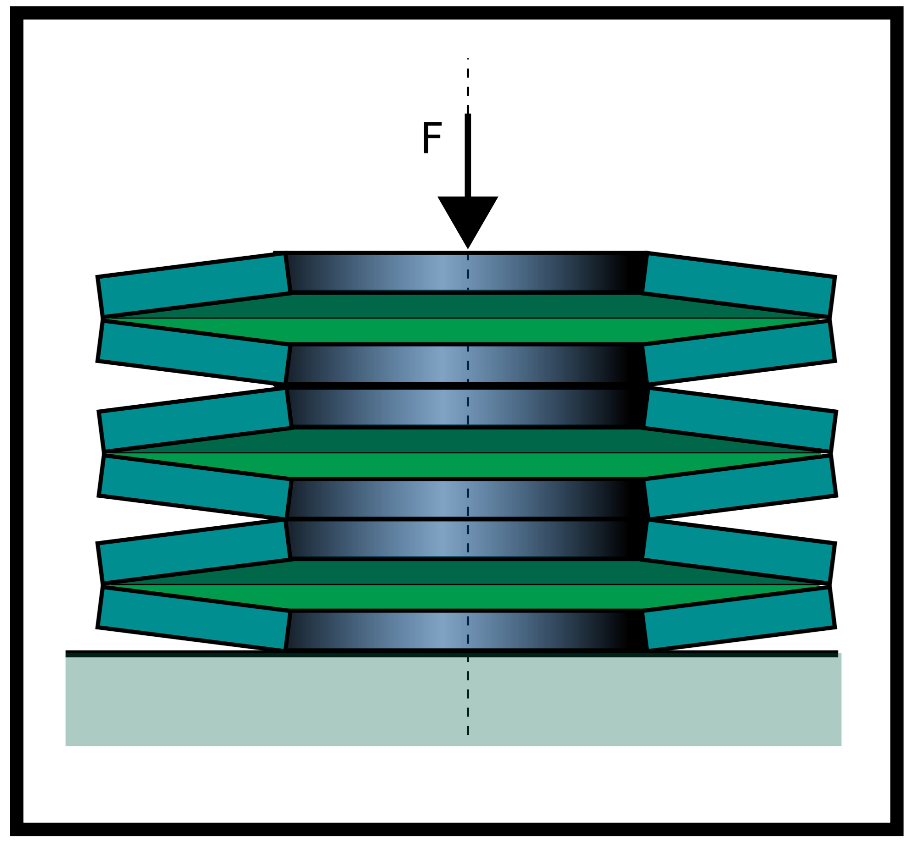


Figure 1:
Schematic of a disk spring stack[4]

Figure 2:
Physical prototype of a disk spring stack[2]

INTRODUCTION

Properties of Coned-Disk Springs

- **Analytical expression** [3] for Force $P(\delta)$ vs deflection δ (in the absence of edge friction)

$$P(\delta) = \frac{E\delta\pi}{a^2}\left(\frac{\alpha}{\alpha-1}\right)^2\left[(h-\delta)\left(h-\frac{\delta}{2}\right)M+N\right]$$

where

$$\alpha = \frac{a}{b},\ M = \left(\frac{\alpha+1}{\alpha-1}-\frac{2}{\ln(\alpha)}\right)\tau,\ \ N = \frac{\tau^3}{6}\ln(\alpha)$$

E is the Young's modulus

- For $h/\tau \approx 1.41$, the spring stiffness in the vicinity of $\delta = h$ is close to zero, highlighting the **QZS nature**

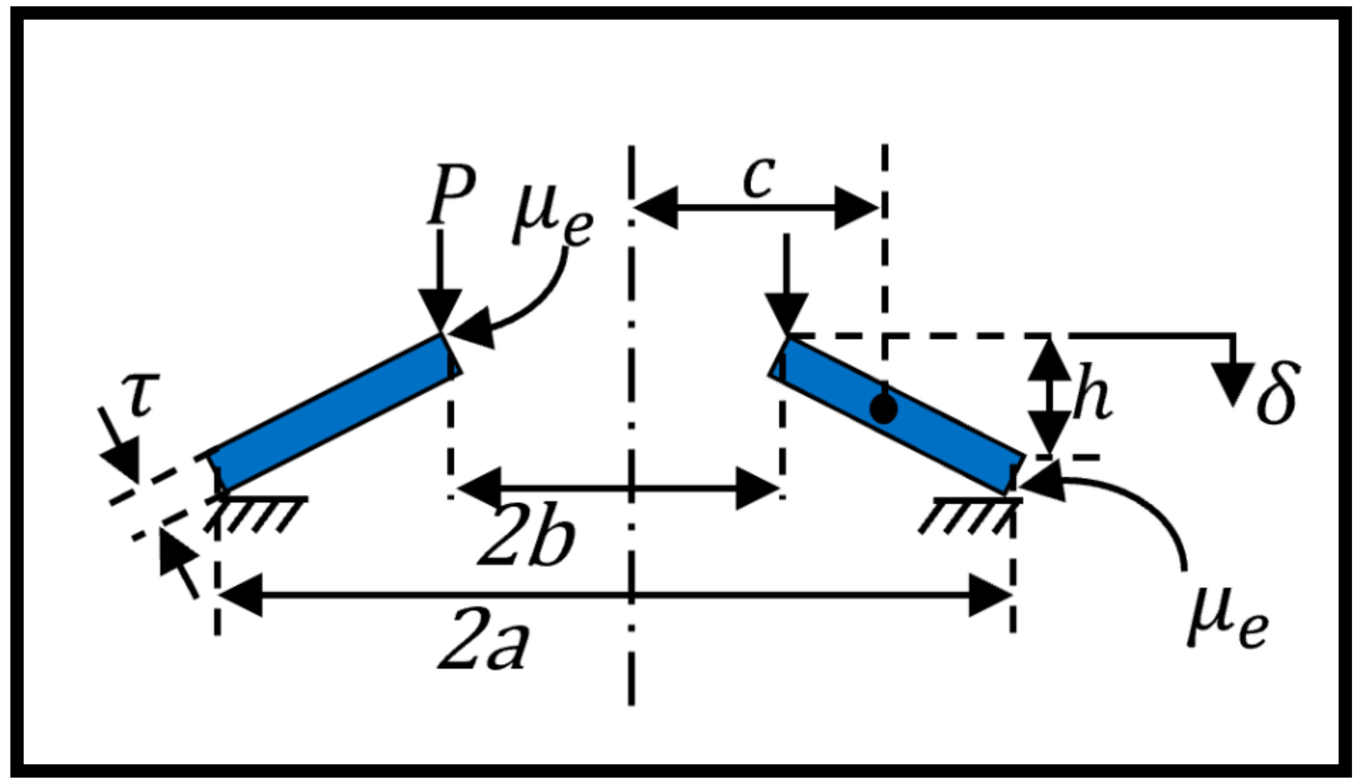


Figure 3:
Cross-section of a coned-disk spring[3]

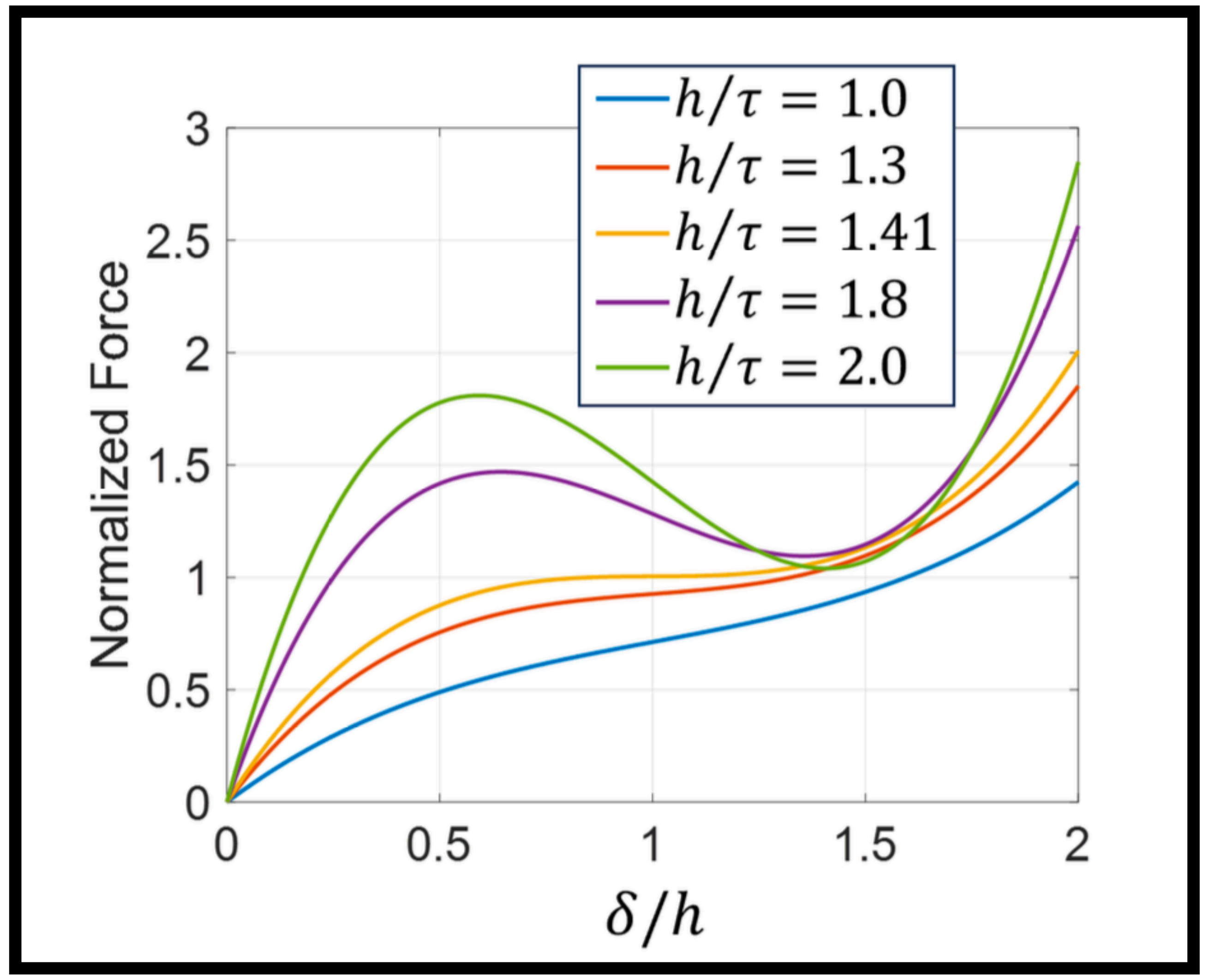


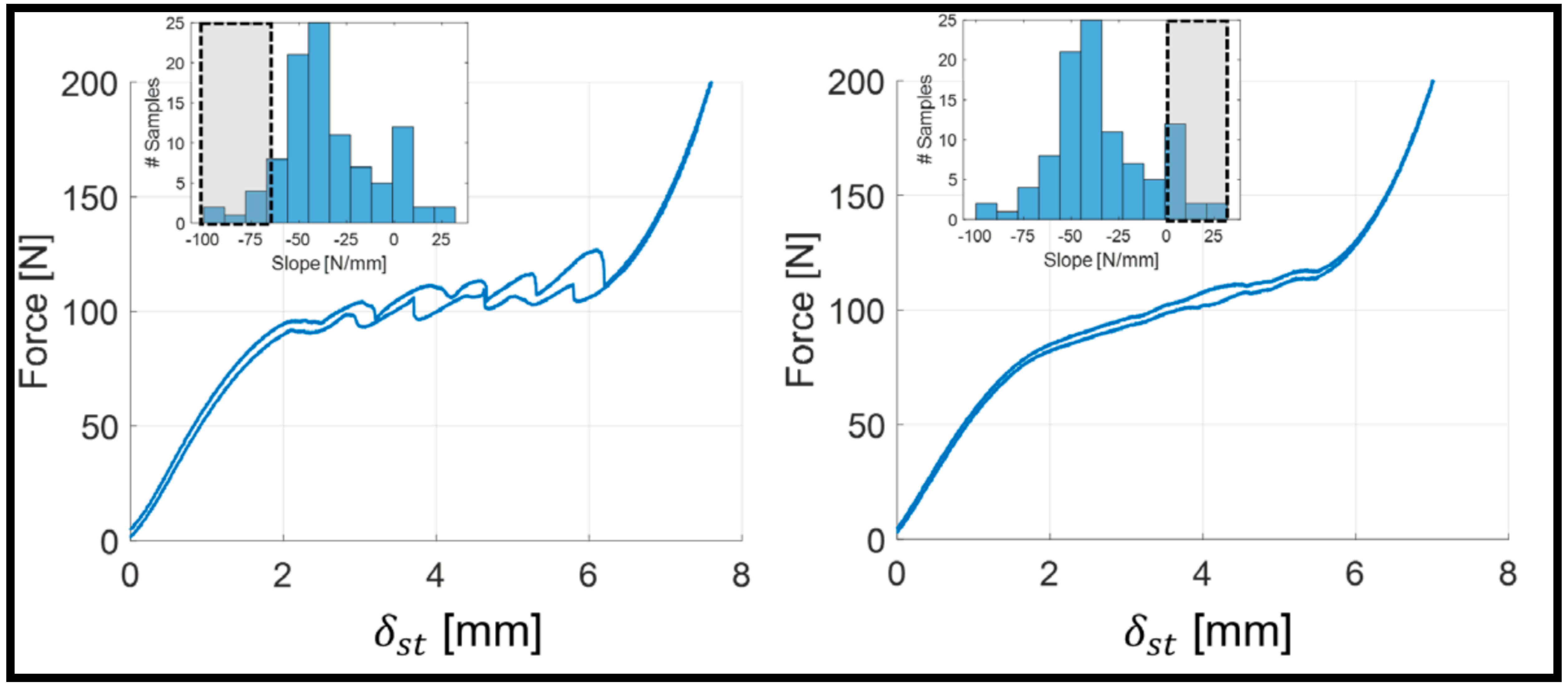
Figure 4:
Force-deflection curves for coned-disk springs[3]

INTRODUCTION

Effect of Disk-to-Disk variations

- **Major Design assumption -** Identical springs in a stack, but hard to achieve in practice due to manufacturing constraints [3].

- **Static load-deflection** behavior of spring stack is **sensitive** to variation in h/τ ratio of different springs, especially when $h/\tau > 1.414$ [3] (**snap-through behavior** due to negative-stiffness regimes)



(a) (b)

Figure 7:
Force-deflection curves for 6-spring stacks

Case (a) - Springs with Negative minimum stiffness
Case (b) - Springs with Positive minimum stiffness

Snap-through behavior and **directional dependence** are clearly visible in Case (a)

INTRODUCTION

Analysis of a 2-Spring Stack

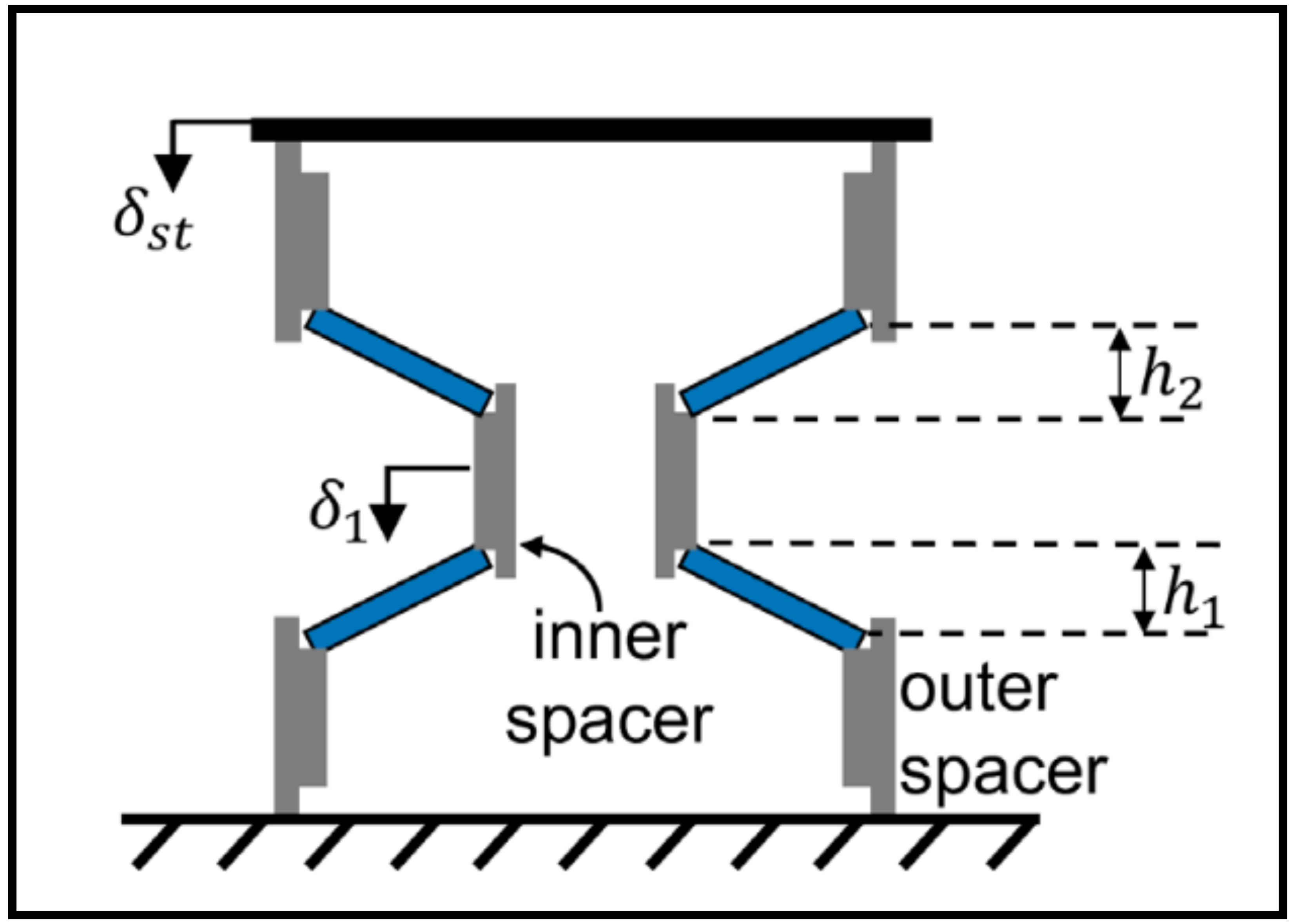


Figure 6:
Static model of 2-spring stack[3]

The height/thickness ratios of the top and bottom springs are h_1/τ and h_2/τ respectively, with force deflection behaviors given by $P_1(\delta)$ and $P_2(\delta)$ respectively.

- δ_{st} is the input displacement and δ_1 is the displacement response coordinate.

- For a given value of δ_{st}, the value of δ_1 can be determined using the **equality of forces** in both springs [3].

$$P_1(\delta_{st} - \delta_1) = P_2(\delta_1)$$

  The solution(s) of the above non-linear equation yield the value(s) of δ_1.

- For a 2-spring stack, there can be **1 or 3** equilibrium-points.

- The **stability** of each equilibrium point can be determined using the sign of second-derivative of the potential energy.

INTRODUCTION

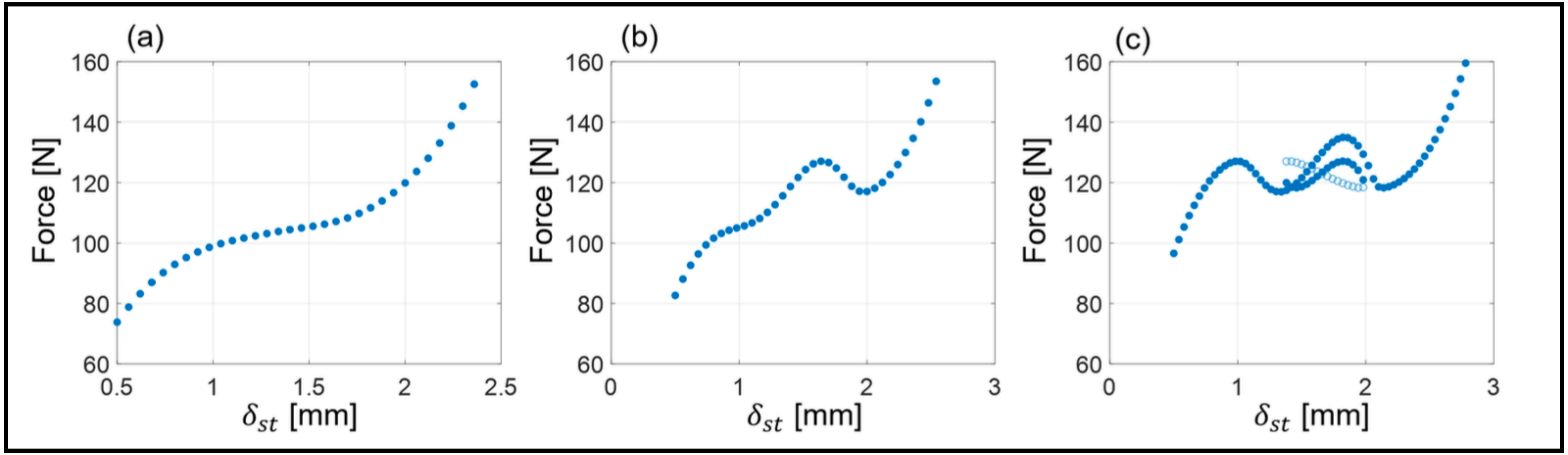


Figure 7:
Analytical force-deflection curves of different 2-spring stacks

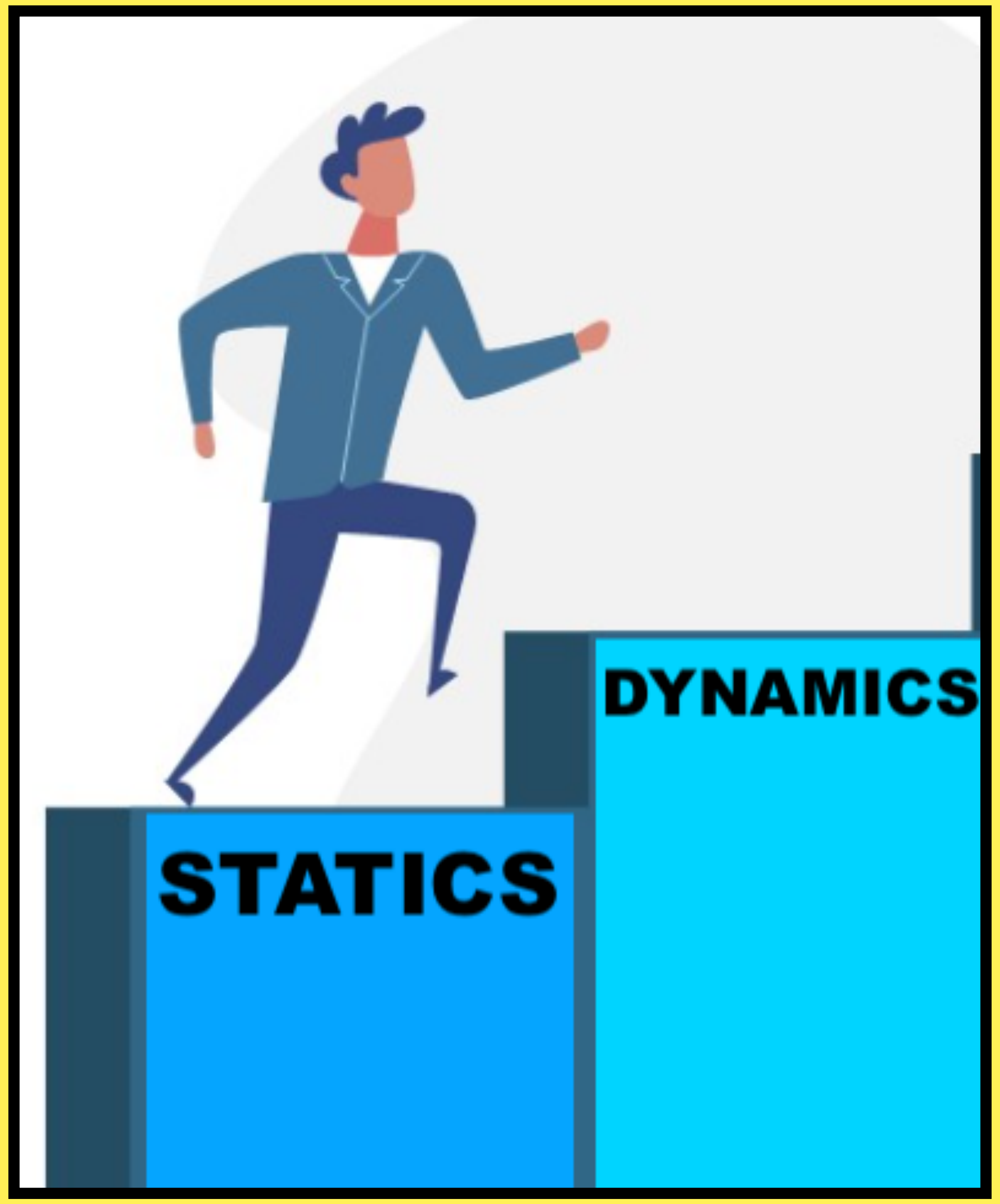
Case (a) $[h_1/\tau, h_2/\tau] = [1.32, 1.36]$ - Only stable equilibria
Case (b) $[h_1/\tau, h_2/\tau] = [1.36, 1.58]$ - Only stable equilibria
Case (c) $[h_1/\tau, h_2/\tau] = [1.58, 1.64]$ - Both stable and unstable equilibria

Stable and **Unstable** equilibrium points are marked with **filled** and **unfilled** circles respectively.

PROBLEM FORMULATION

- **Goal -** To go beyond static analysis and venture into the **dynamics of the system**!

OBJECTIVES



MODELING & SIMULATION

EXPERIMENTAL VALIDATION

IN-DEPTH INVESTIGATION

PROBLEM FORMULATION

- **Objective 1-**
  A. Modeling the non-linear dynamics of a spring stack-mass-damper system with base-excitation
  B. Numerical Simulation of the model to obtain displacement, velocity, and acceleration profiles of the system for time-domain analysis, as well as Force-displacement profiles for physical-domain analysis

- **Objective 2 -**
  A. Validation of results obtained in simulation through experiments

- **Objective 3 -**
  A. Investigate the effect of damping, hysteresis and other non-linearities on the dynamics
  B. Frequency domain analysis of the non-linear system
  C. Compare the response of the 2 spring-stack with that of a single spring and the linearized system
  D. Compare the response of a 2 spring-stack consisting of identical springs with that of a spring-stack consisting of non-identical springs.

OBJECTIVE 1A - MODELING

System Sketch

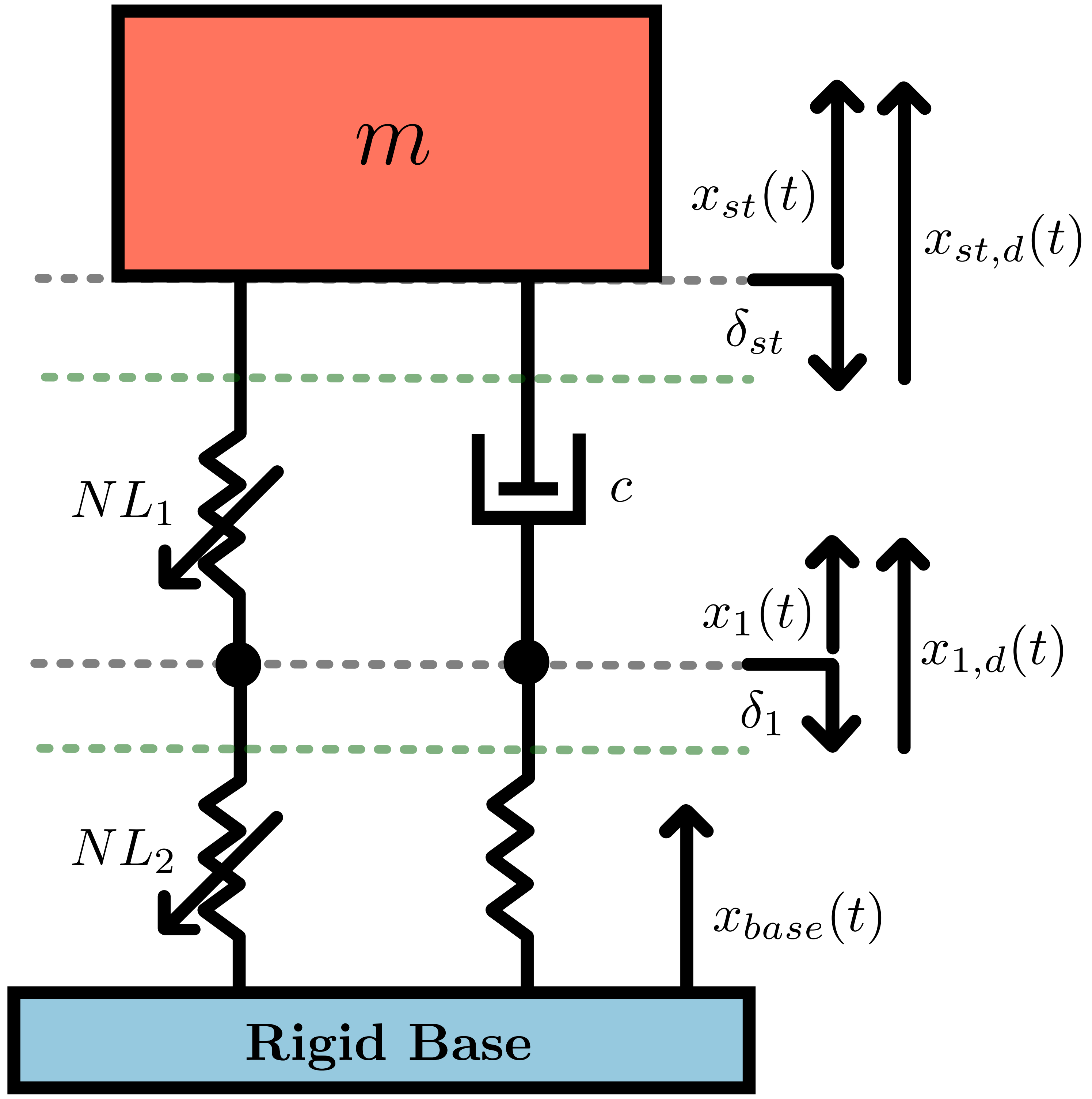


Figure 8 -
Sketch of a **spring stack-mass-damper** system

In **Figure 8**,
m - Mass of the suspended block
c - Damping coefficient of the Damper
NL_1 - Top conical disk spring with ratio h_1/τ
NL_2 - Bottom conical disk spring with ratio h_2/τ

For the mass m,
δ_{st} is the mean displacement in **static equilibrium**
$x_{st,d}(t)$ is the **dynamic displacement**
$x_{st}(t) = \delta_{st} + x_{st,d}(t)$ is the **total displacement**

For the bottom of NL_1,
δ_1 is the mean displacement in **static equilibrium**
$x_{1,d}(t)$ is the **dynamic displacement**
$x_1(t) = \delta_1 + x_{1,d}(t)$ is the **total displacement**

OBJECTIVE 1A - MODELING

Equations of Motion in the absence of damping

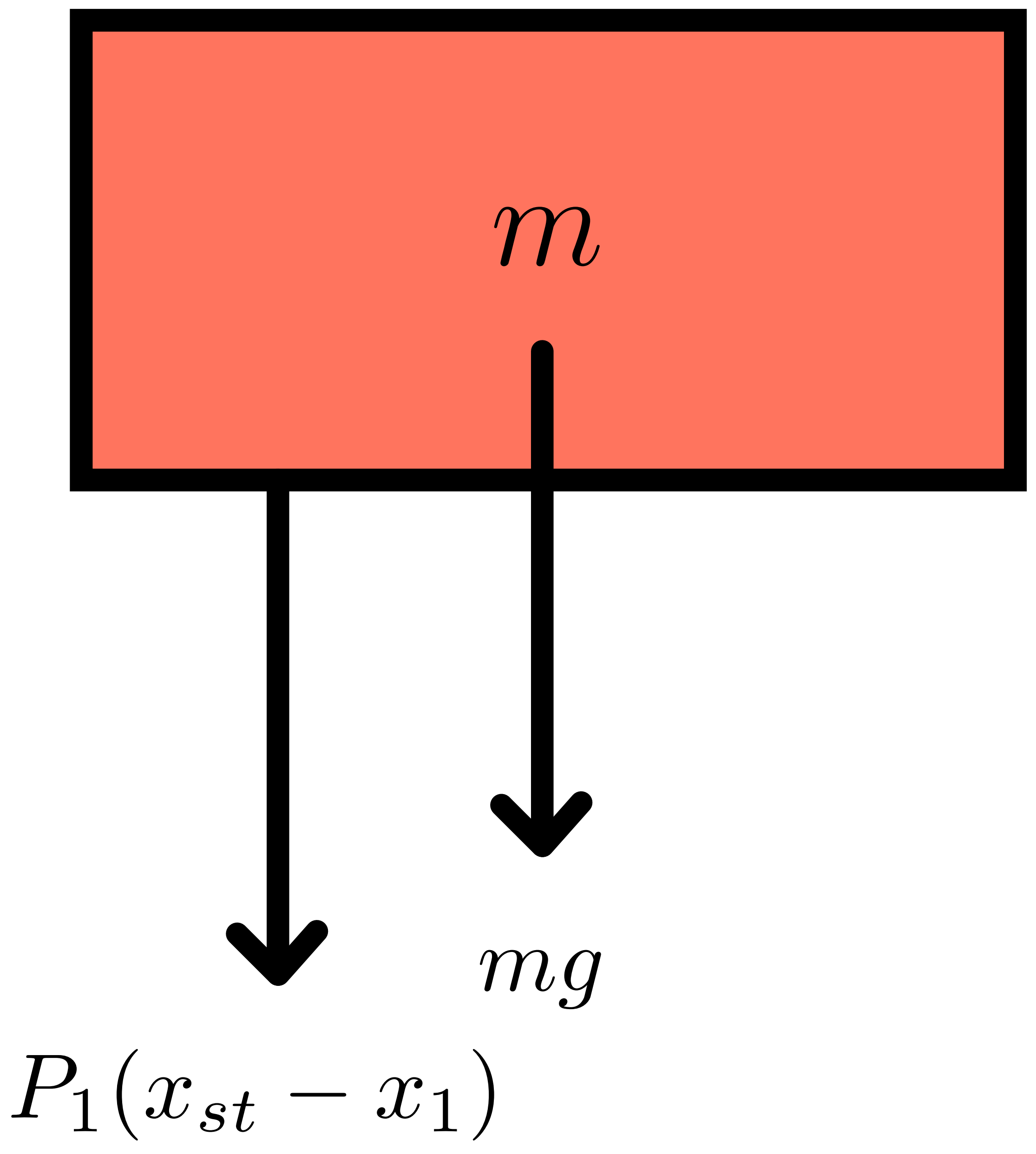


Figure 9 -
FBD of mass m

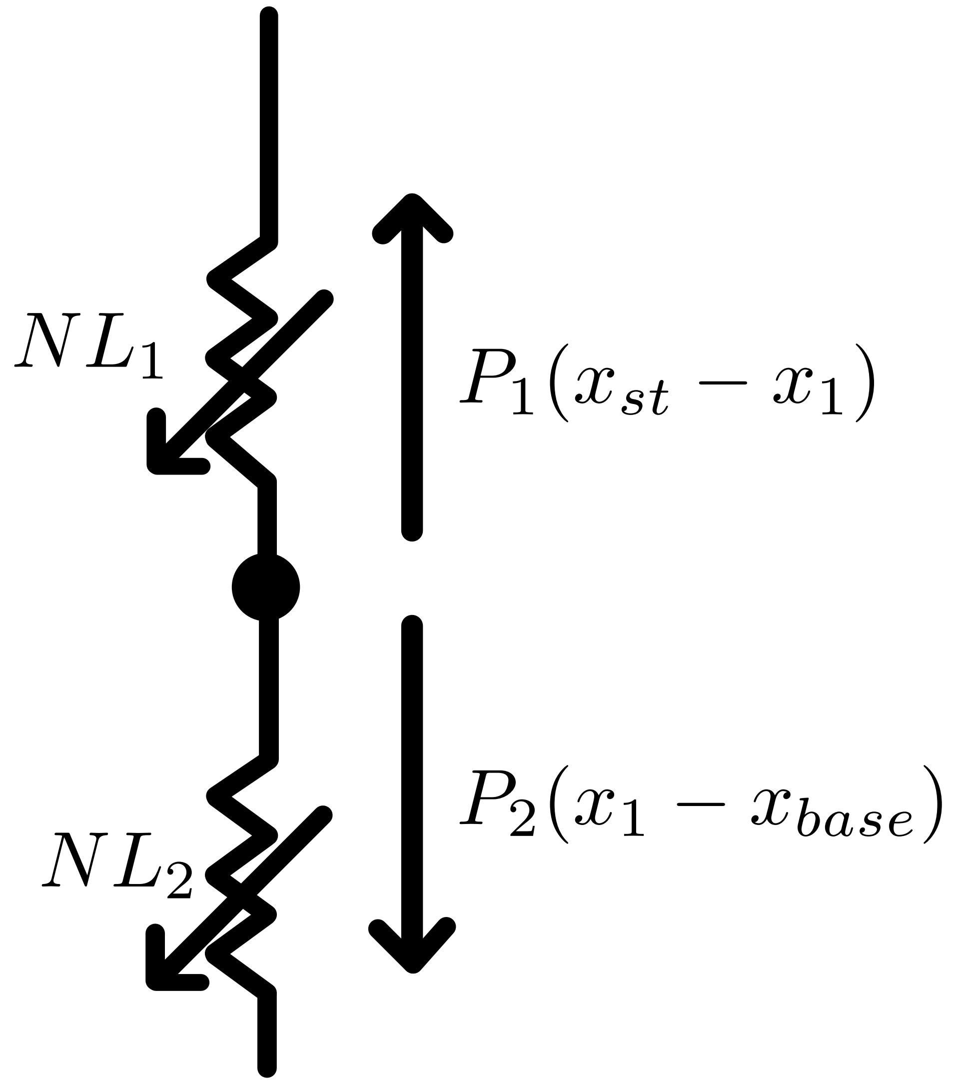


Figure 10 -
FBD of springs

From **Figure 9**,

$$m\ddot{x}_{st} = -mg - P_1(x_{st} - x_1)$$

From **Figure 10**,

$$P_1(x_{st} - x_1) = P_2(x_1 - x_{base})$$

The state vector $x(t)$ is defined as

$$x(t) = \begin{bmatrix} x_{st}(t) \\ \dot{x}_{st}(t) \end{bmatrix}$$

OBJECTIVE 1B - SIMULATION

MATLAB Simulator Pipeline

Equations of Motion

$$x(t) = \begin{bmatrix} x_{st}(t) \\ \dot{x}_{st}(t) \end{bmatrix}$$

$$f(x,t) = \begin{bmatrix} \dot{x}_{st}(t) \\ -g - \frac{1}{m}P_1(x_{st} - x_1) \end{bmatrix}$$

$$P_1(x_{st} - x_1) = P_2(x_1 - x_{base})$$

$\dot{x}(t) = f(x,t)$

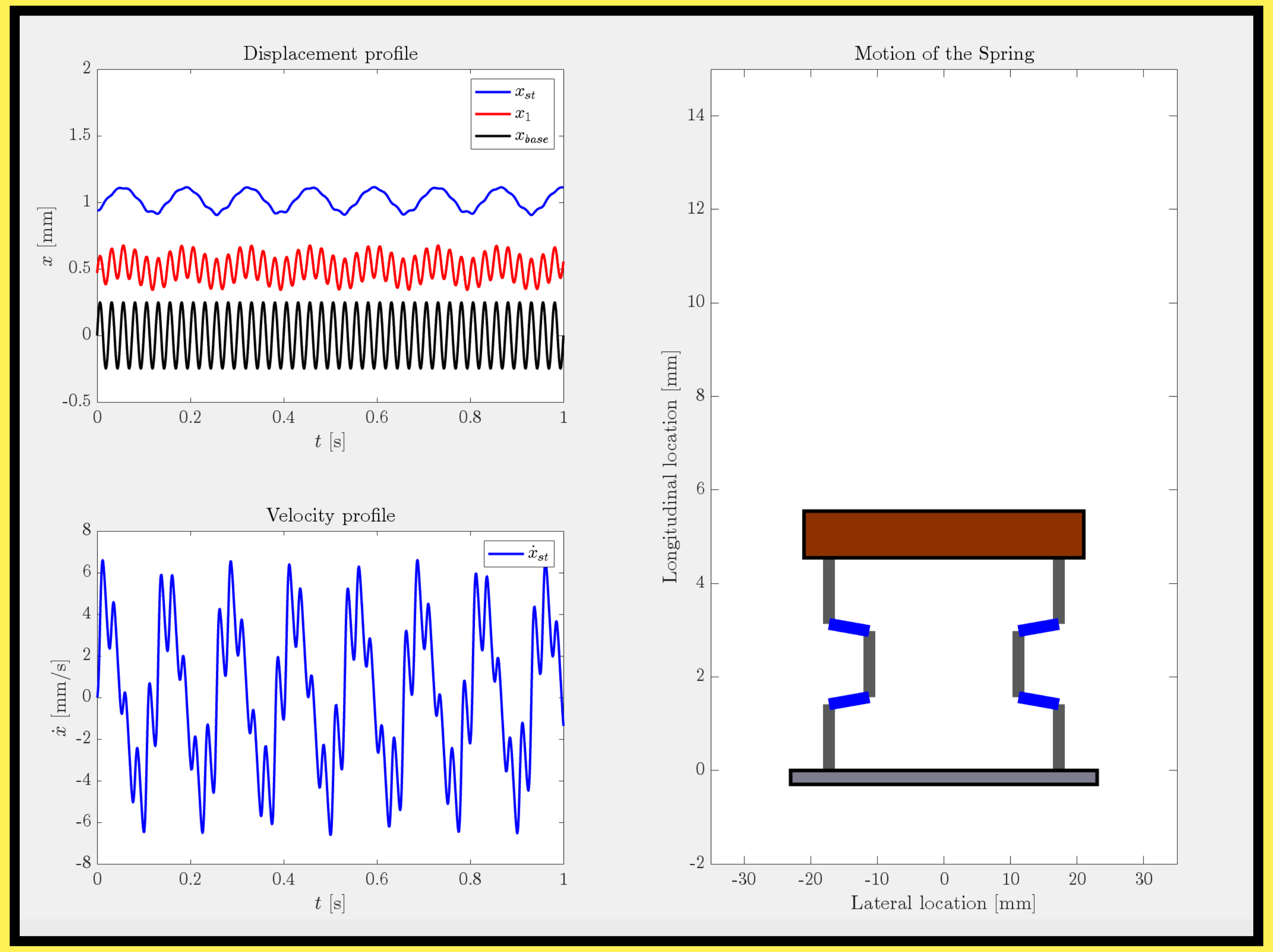
ODE Solver (ode45)

Initial Conditions

Static Equilibrium Solver

$x_{st}(t)$

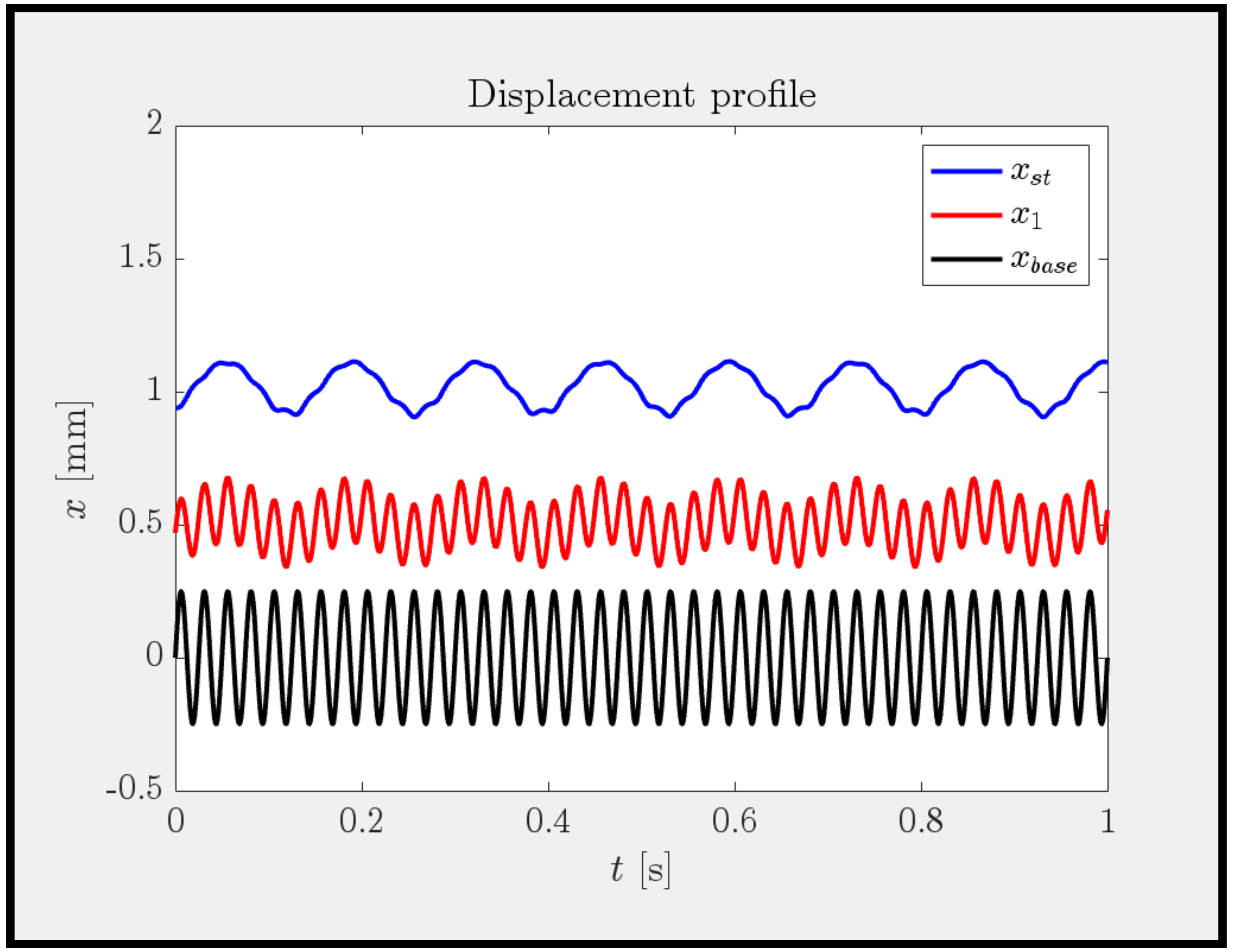
Motion Visualizer and Plotter

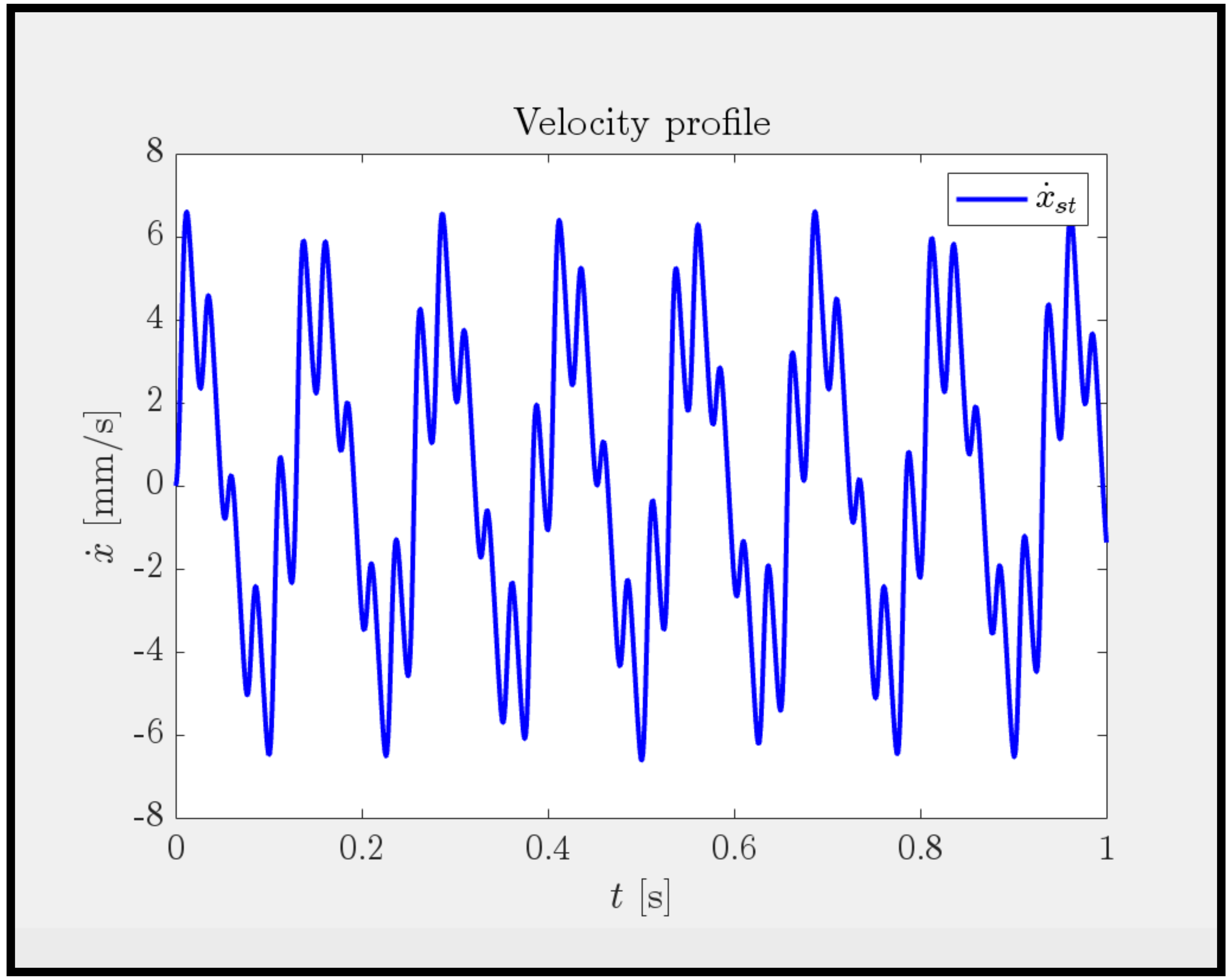


OBJECTIVE 1B - SIMULATION

Preliminary Results

Case 1: $h_1/\tau = 1.41$, $h_2/\tau = 1.41$, $c = 0$, $x_{base}(t) = 0.25\sin(80\pi t)$ (in mm) **(Identical Springs)**

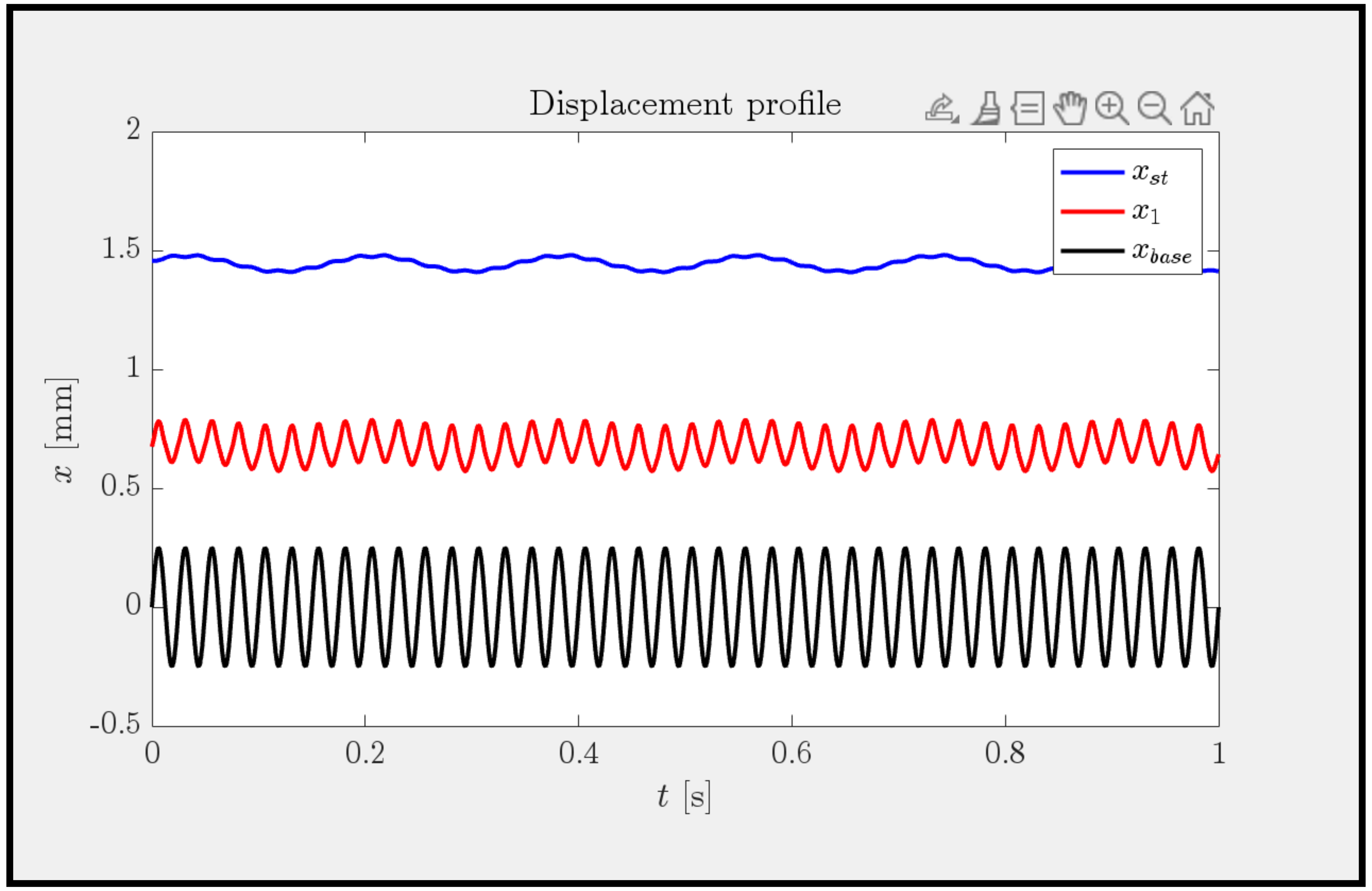


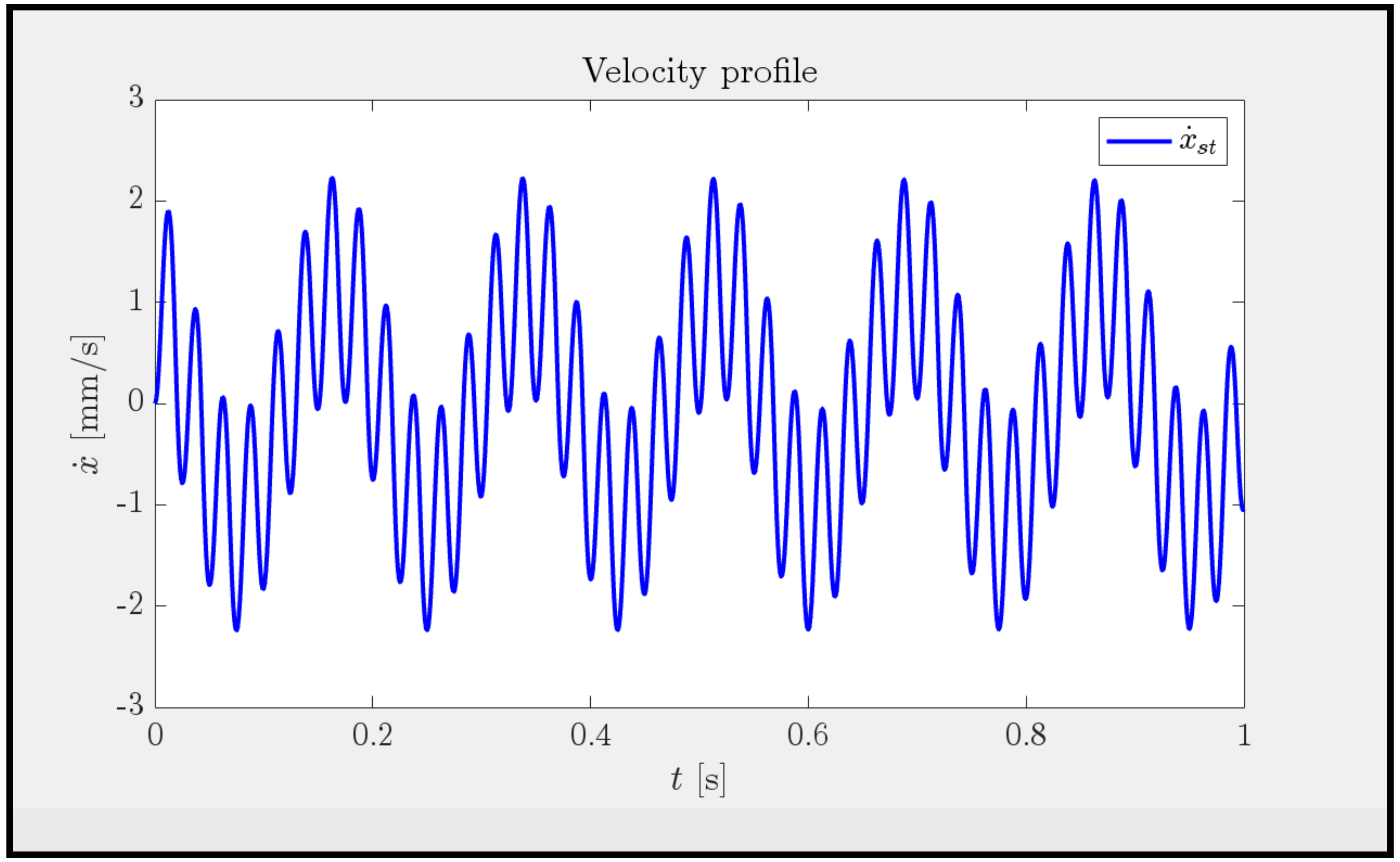


OBJECTIVE 1B - SIMULATION

Preliminary Results

Case 2: $h_1/\tau = 1.32$, $h_2/\tau = 1.36$, $c = 0$, $x_{base}(t) = 0.25\sin(80\pi t)$ (in mm) **(Non-Identical Springs)**





APPENDIX A - REFERENCES

Journal / Conference Papers

1. Liu, C., Zhang, W., Yu, K., Liu, T., & Zheng, Y. (2024). Quasi-zero-stiffness vibration isolation: Designs, improvements and applications. In *Engineering Structures*, *301*, 117282.

2. Gilmore, P., & Gandhi, U. (2021, August). Development of disc spring stack containment methods for vibration isolation. In *INTER-NOISE and NOISE-CON Congress and Conference Proceedings* (Vol. 263, No. 2, pp. 4871-4879). Institute of Noise Control Engineering.

3. Gilmore, P., Gandhi, U., & Singh, R. Effect of Disk-to-Disk Variations on the Nonlinear Static Characteristics and Stability Regimes of Coned Disk Spring Stacks: Experimental and Computational Studies of Quasi-Zero-Stiffness Isolators. *Available at SSRN 4799335*.

4. "Belleville washer". In *Wikipedia.com.* URL: https://en.wikipedia.org/wiki/Belleville_washer

Software

1. Gilmore, P. (2024), MATLAB Code (stack_equations_solver_mm_2springs.m). *Personal Communication*

2. Singh, R. (2024), MATLAB Code (basicFFT.m). *Personal Communication*

APPENDIX B - LEARNING OUTCOMES

- Understanding the **fundamental ideas** behind vibration isolation, QZS Isolators and Coned-Disk Springs
- **Design aspects** of a Coned-Disk Spring Stack such as Containment methods
- **Static analysis** of a Coned-Disk Spring Stack to study the Force-deflection behavior
- Effect of **Disk-to-Disk variations** on the quasi-static force-deflection in terms of factors like **stability**
- **Dynamic analysis** of a 2-Spring stack using **Newton's Laws of Motion**
- **Simulating** a Non-Linear Dynamical system in MATLAB using numerical methods like **ode45** with appropriate tolerances.