



RIYA Week 5 Presentation

**A study of the non-linear dynamics of a 2-spring
stack under different situations**

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Tasks Accomplished

- Performed a **linearized system analysis** about an operating point to get a crude estimate about natural frequency, etc
- Obtained **non-linear time-domain** and **frequency domain** results for different **initial displacement conditions** in the **absence of base excitation**
- Obtained **non-linear time-domain** and **frequency domain results** for different **base excitation scenarios** for **equilibrium initial conditions**

Linearized System Analysis

Case :

$$h_1/\tau = h_2/\tau = 1.41,$$

$x_{base}(t) = 0$ (No base excitation)

Initial conditions -

$$x_{st}(0) = \delta_{st} + 0.001 \text{ mm}$$

$$\dot{x}_{st}(0) = 0$$

δ_{st} is the value of x_{st} in static equilibrium

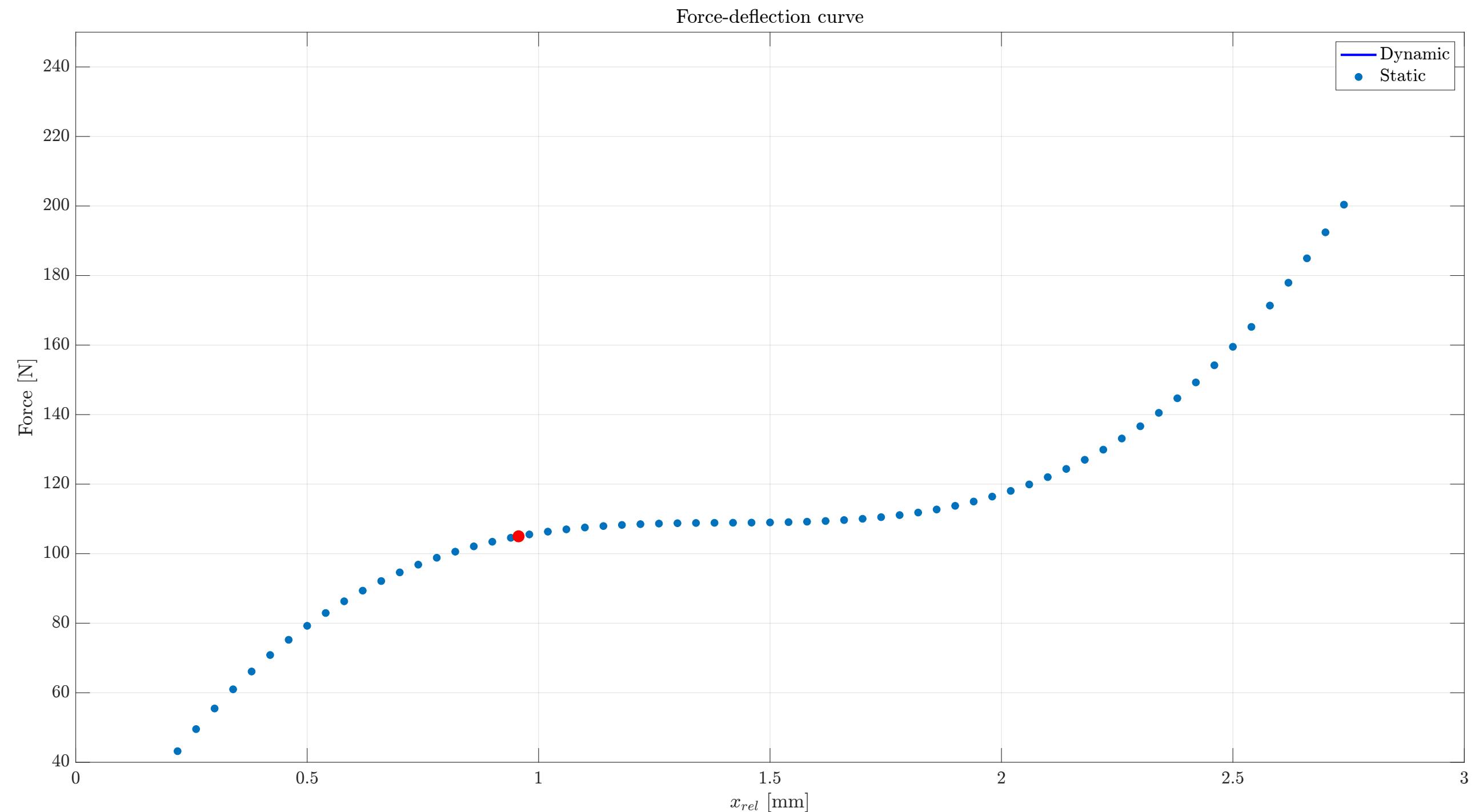


Figure 1 -
Linearized system behavior on Force-deflection curve
for very small initial deflection

Linearized System Analysis - Results

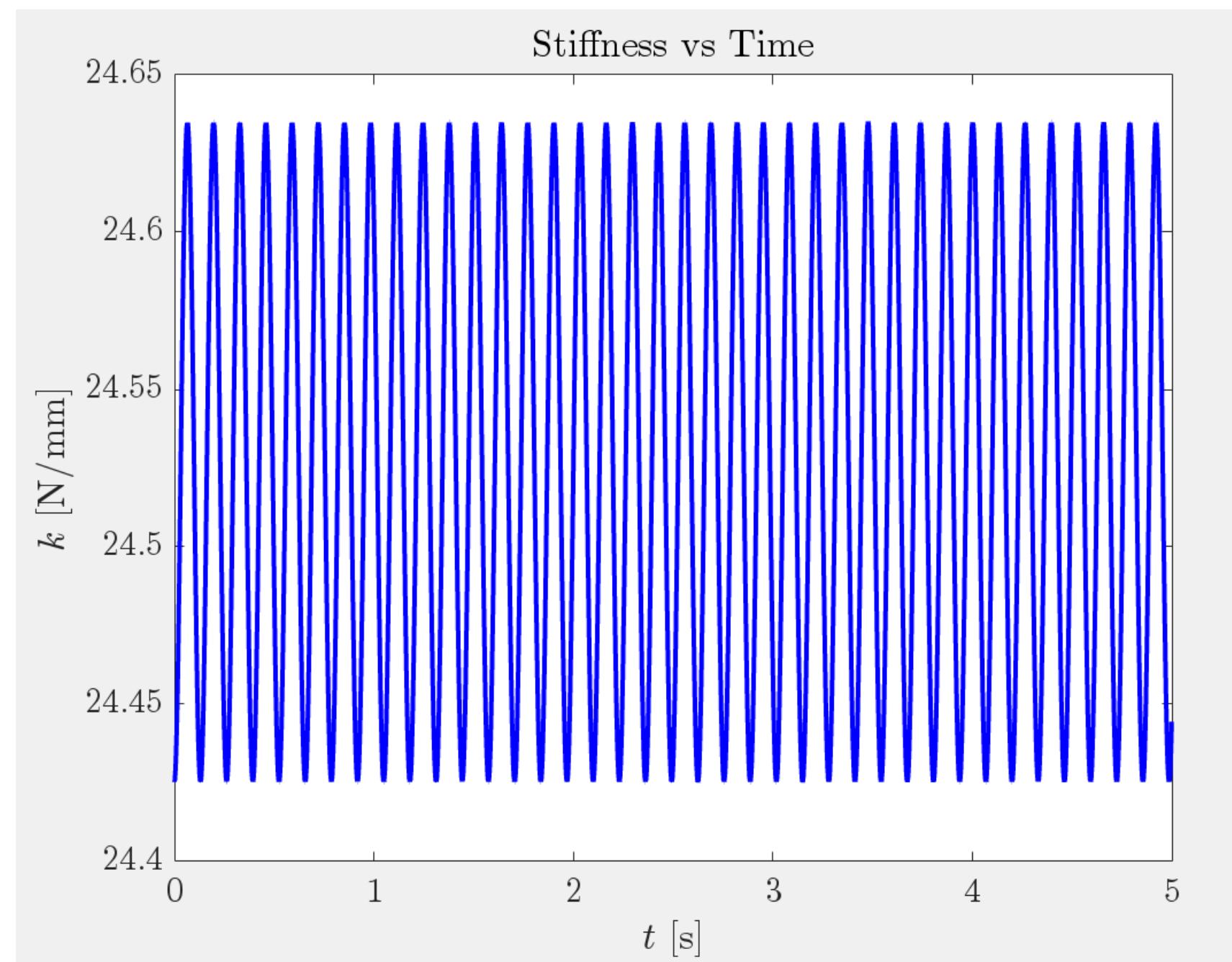


Figure 2 - Stiffness profile for the linear approximation

Mean stiffness $\bar{k} \approx 24.5$ N/mm

Stiffness at a given point of time is calculated
numerically (source of error !)

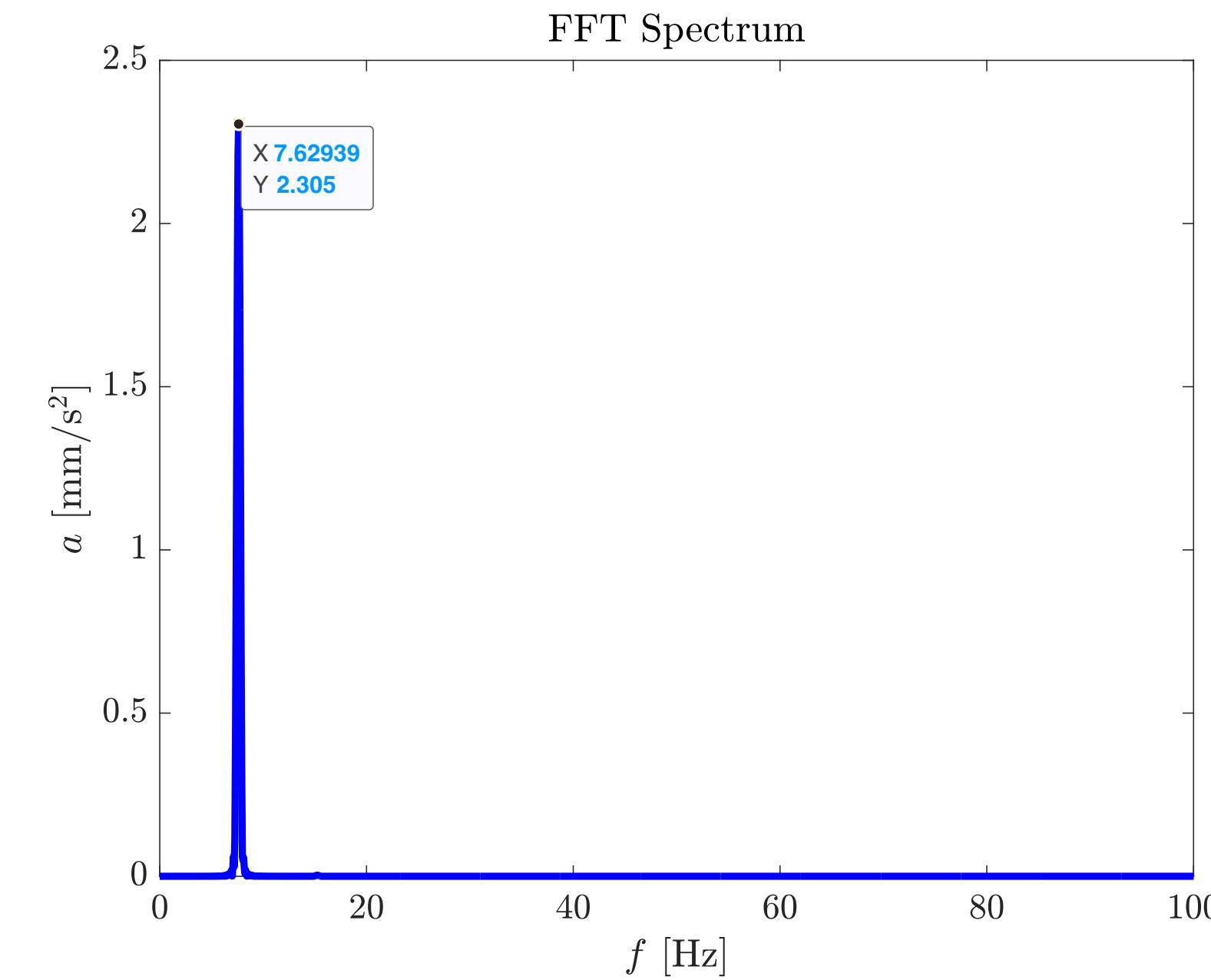


Figure 3 - FFT Spectrum of the output response

$$f_{natural} \approx \frac{1}{2\pi} \sqrt{\frac{\bar{k}}{m}} \approx \frac{1}{2\pi} \sqrt{\frac{24.5(10^3)}{10.7}} \approx 7.615 \text{ Hz}$$

Linearized System Analysis - Results

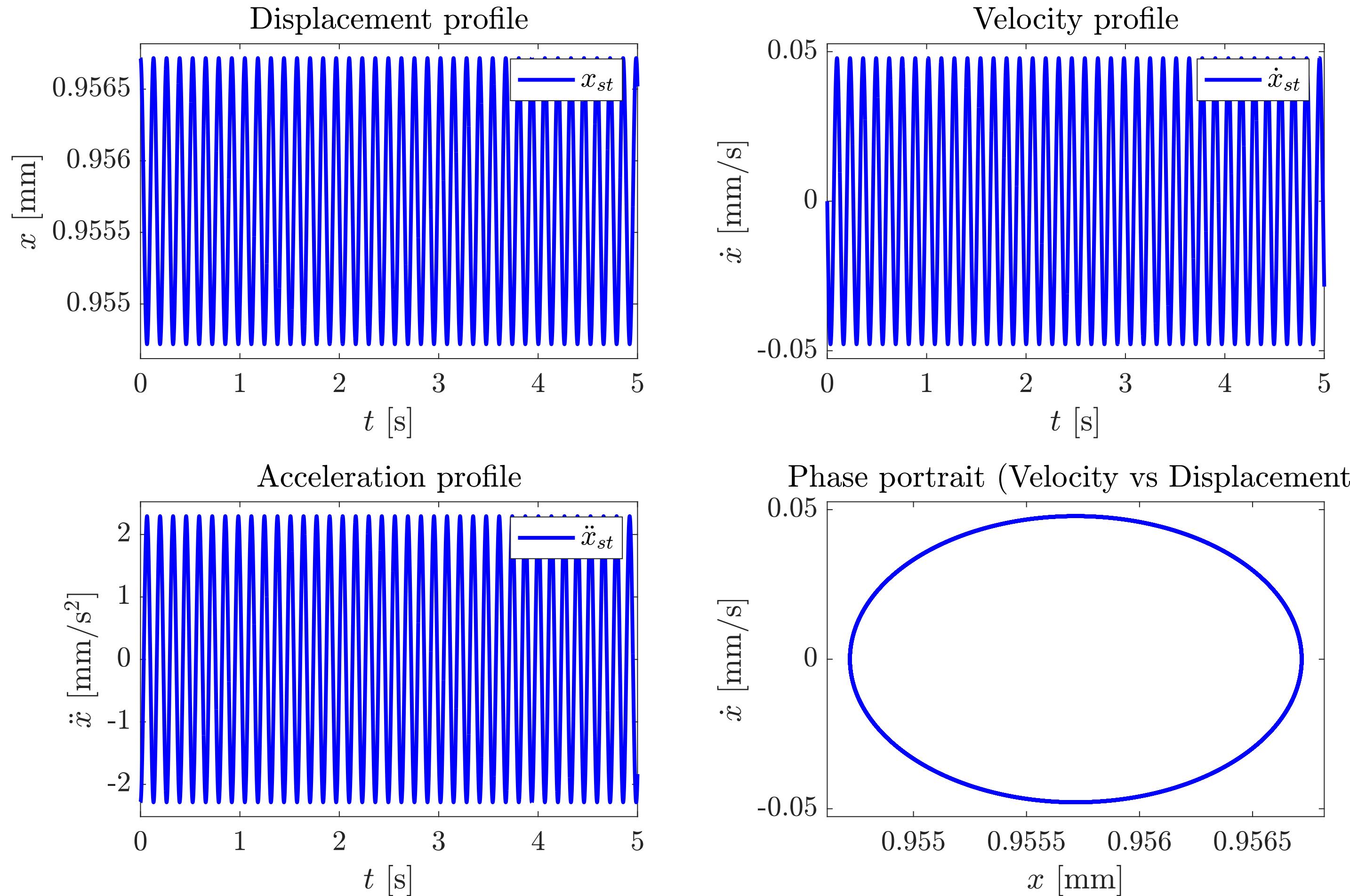


Figure 4 - Displacement, Velocity and Acceleration profiles

- Amplitude $A \approx 0.001 \text{ mm}$
- Amplitude of velocity = $\omega A \approx 0.047 \text{ mm/s}$
- Amplitude of acceleration = $\omega^2 A \approx 2.29 \text{ mm/s}^2$

OBSERVATIONS

- Linearized System results are promising and closely match the standard analytical results for a linear spring
- Single peak in the FFT spectrum corresponding to the natural frequency associated with the local stiffness
- The **peak associated with approximately 7-8 Hz** last time, probably wasn't a subharmonic !

Non-Linear System Analysis - Part 1 (NLA1)

Parameters : $h_1/\tau = h_2/\tau = 1.41$, $x_{base}(t) = 0$ (No base excitation)

Case No.	Initial displacement from equilibrium point
1	0.1 mm
2	0.5 mm
3	1 mm

NLA1 Case 1 - Time Domain

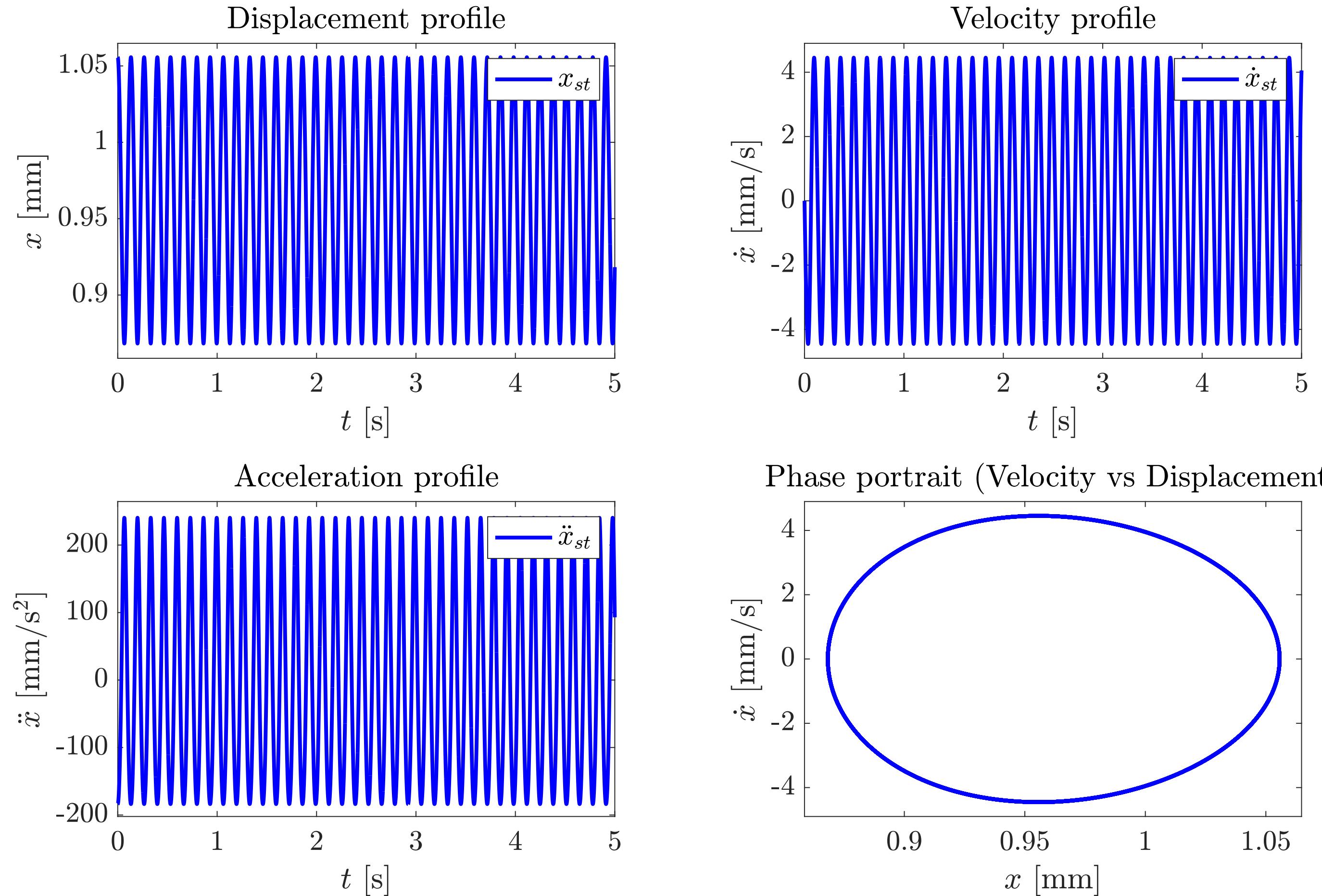


Figure 5 -
Time domain results for Case 1

For a **linearized system**

- Peak to Peak Amplitude ≈ 0.2 mm
- Amplitude of velocity ≈ 4.78 mm/s
- Amplitude of acceleration ≈ 229.22 mm/s²
-

In the **non-linear system**

- Peak to Peak Amplitude ≈ 0.1875 mm
- Amplitude of velocity ≈ 4.45 mm/s
- Amplitude of acceleration ≈ 240.47 mm/s²
-

Deviations from ideal linear behavior observed !

NLA1 Case 1 - Physical and Frequency Domain

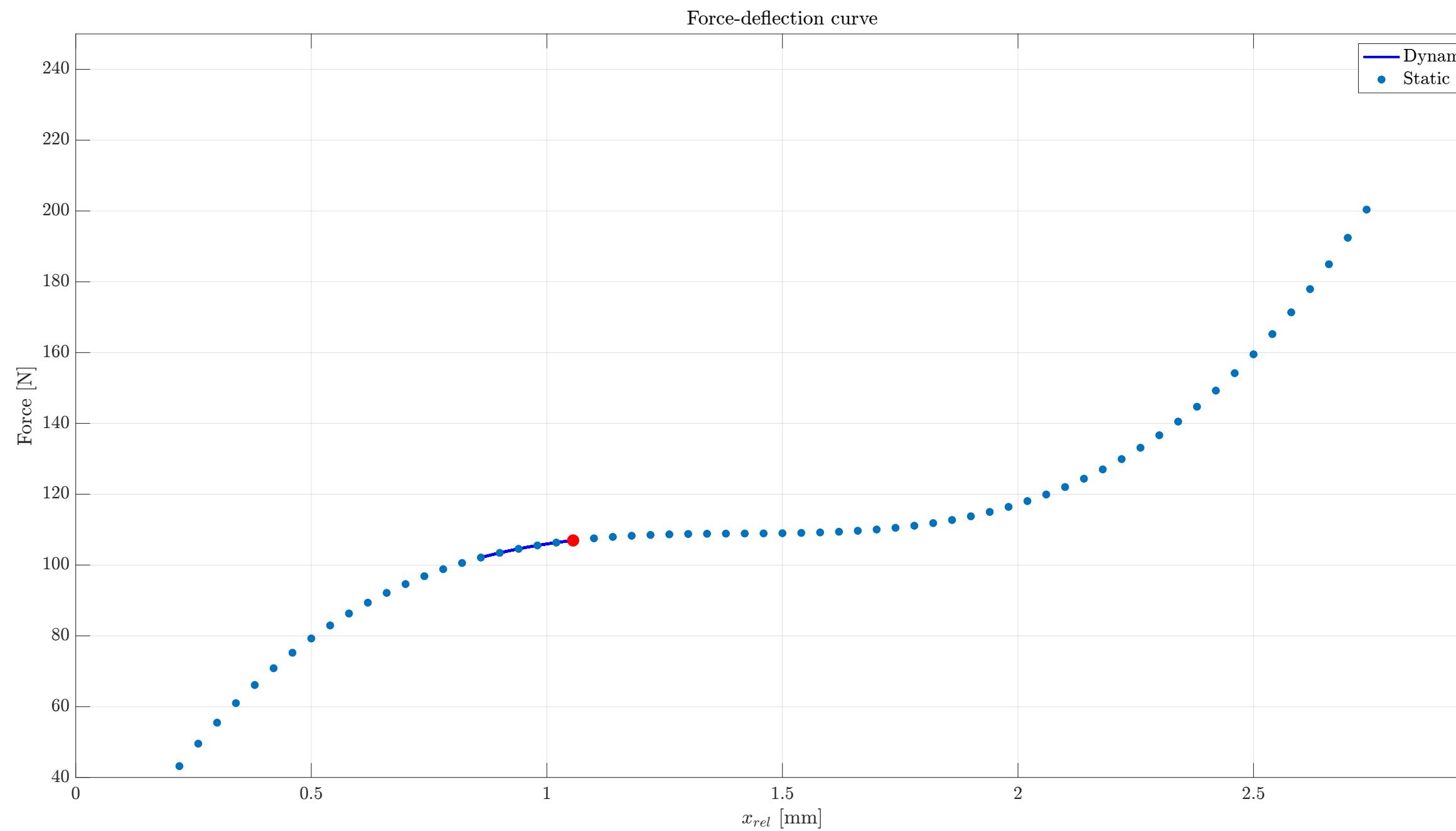


Figure 6 - Physical domain behavior for Case 1

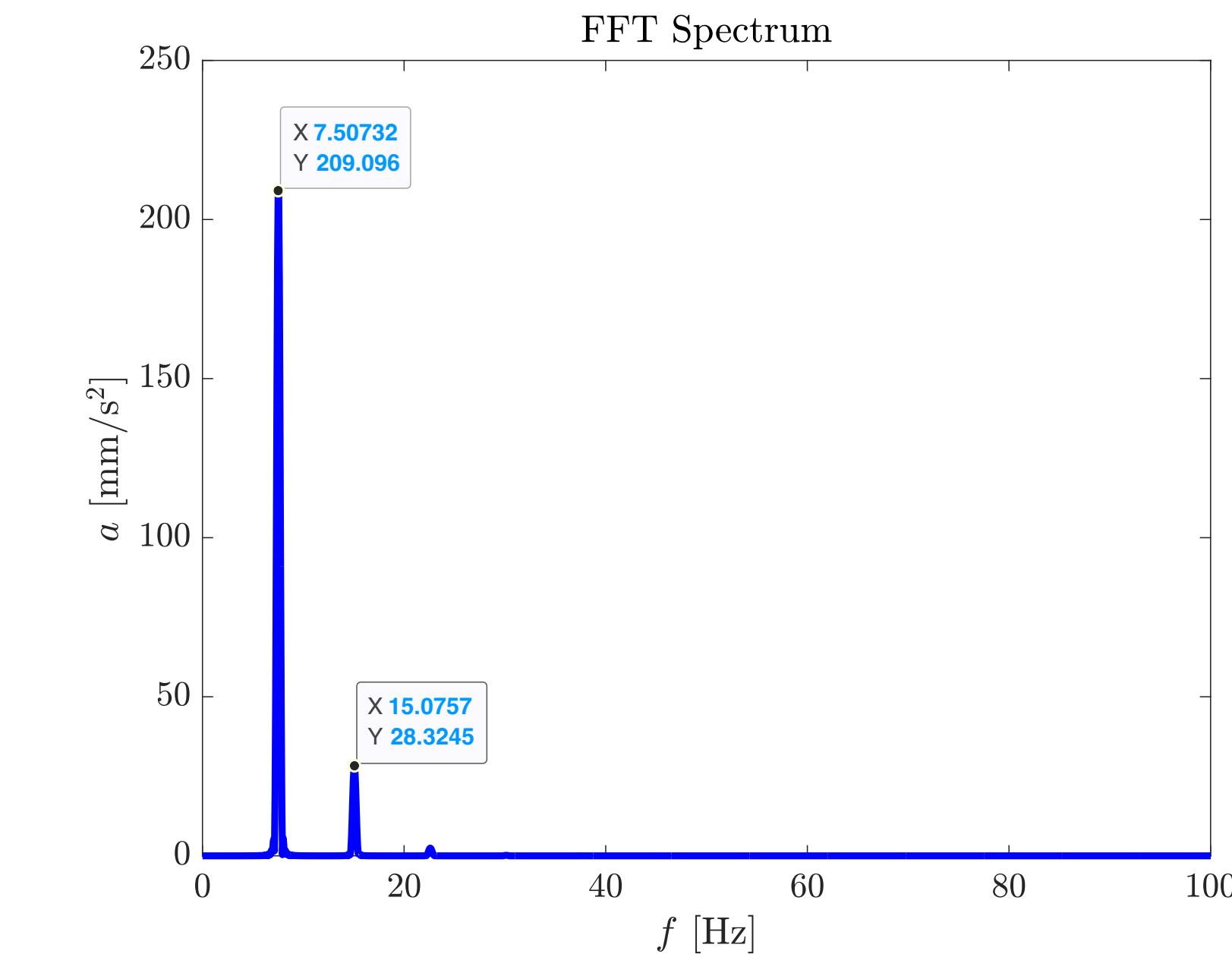


Figure 7 - FFT Spectrum for Case 1

Super harmonics emerge visibly!

First peak shifted from ~ 7.62 Hz to 7.5 Hz

NLA1 Case 2 - Time Domain

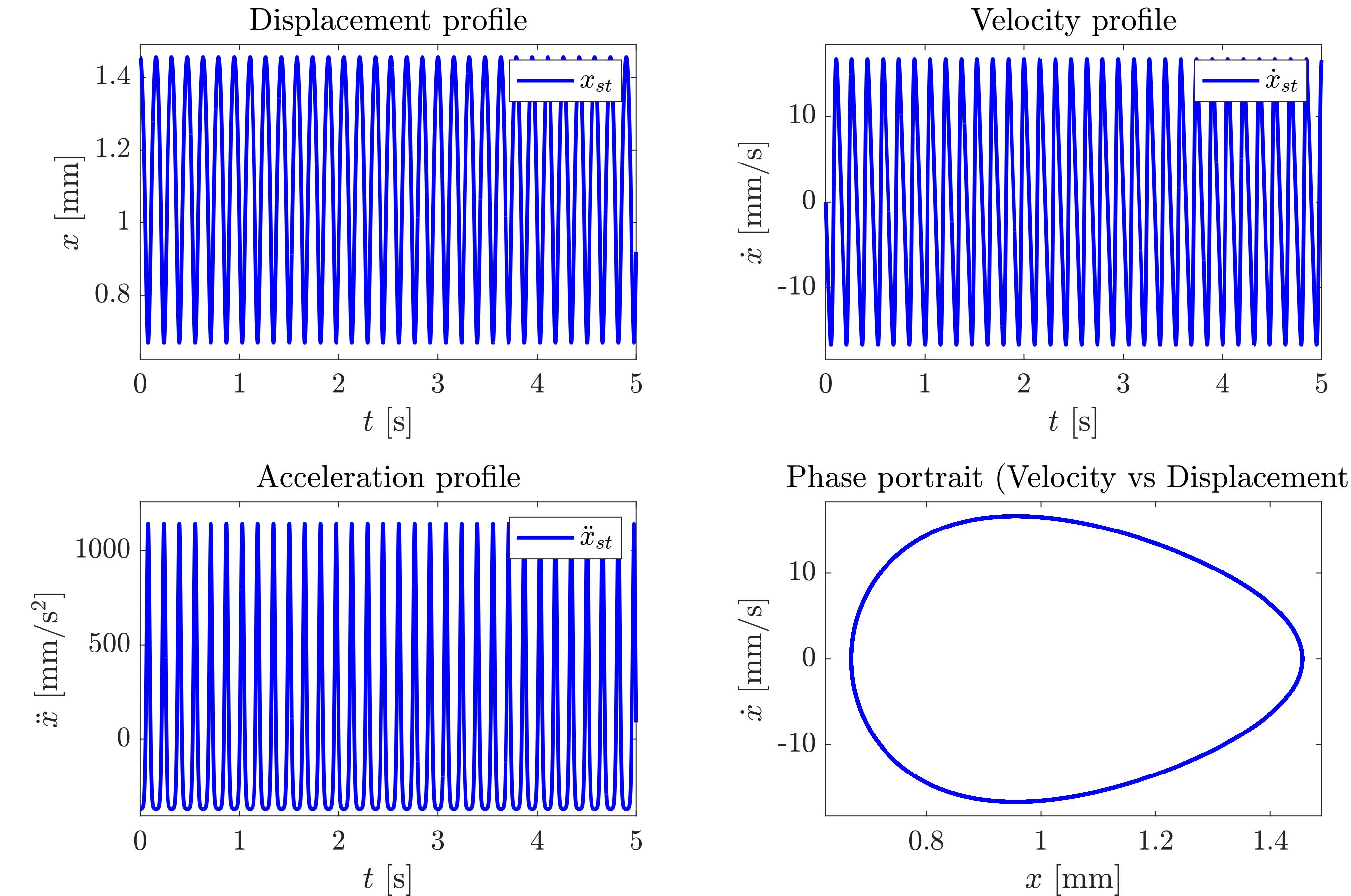


Figure 8 - Time domain results for Case 2

NLA1 Case 2 - Physical and Frequency Domain

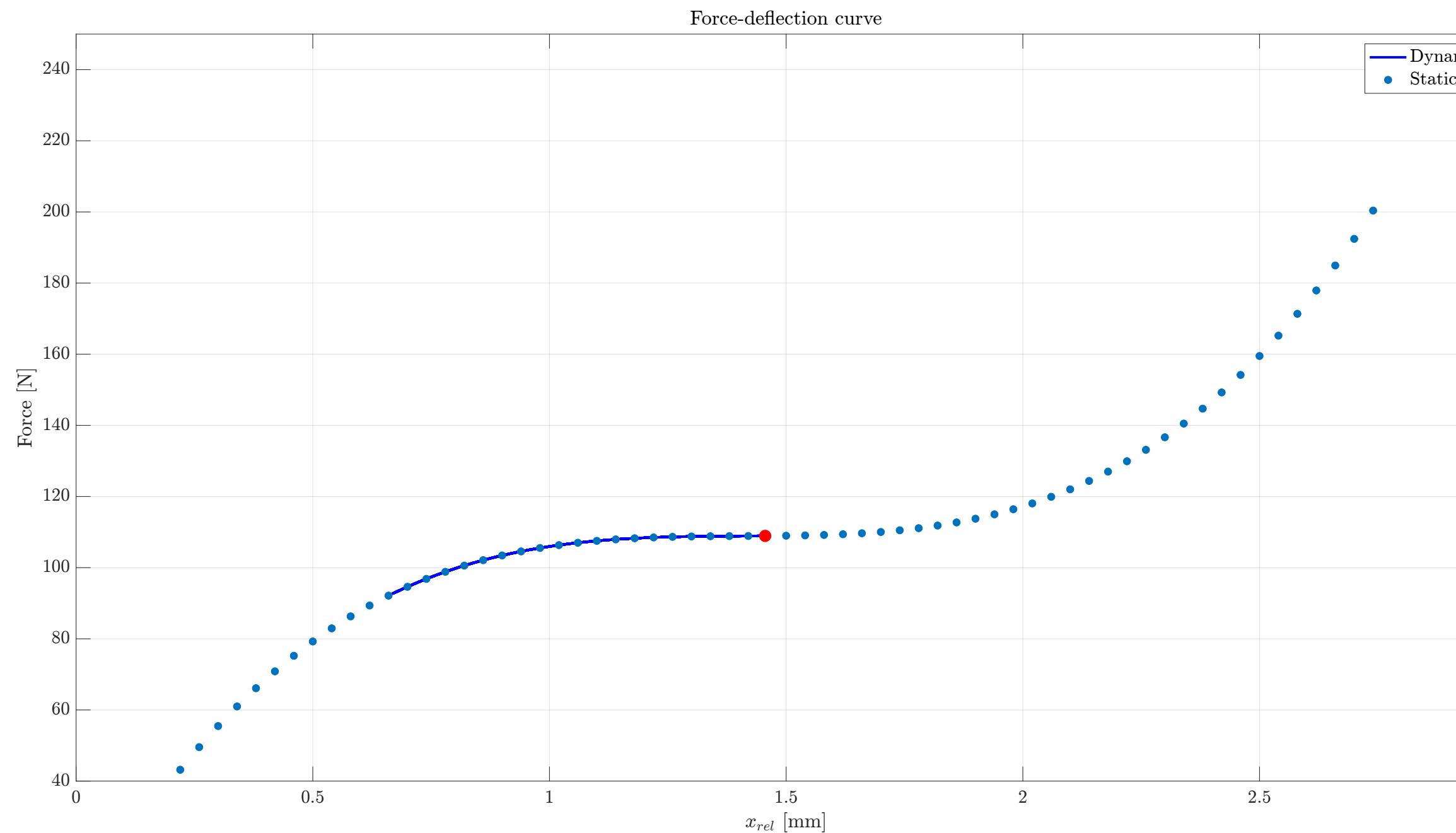


Figure 9 - Physical domain behavior for Case 2

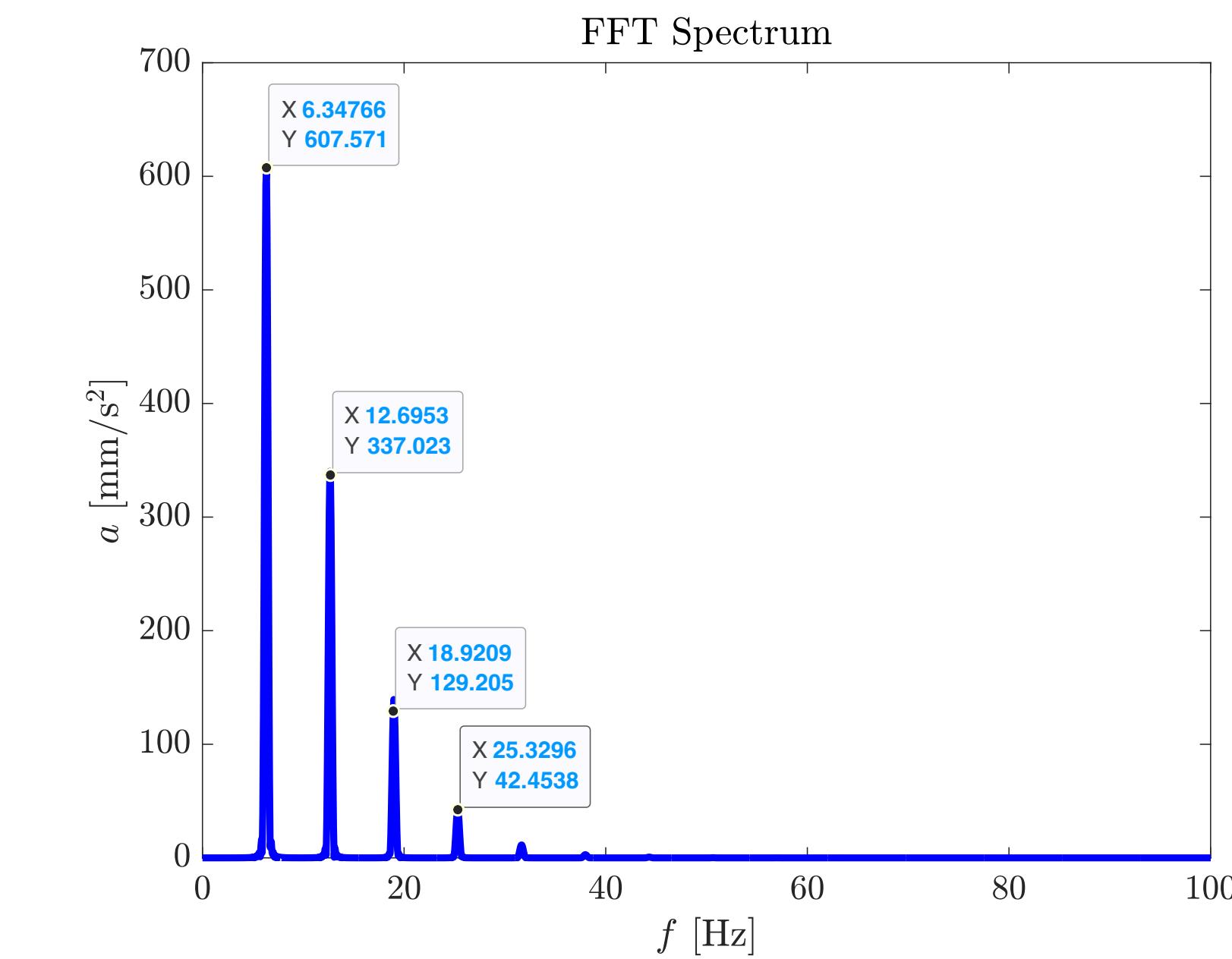


Figure 10 - FFT Spectrum for Case 2

More super harmonics !

First peak at **6.34 Hz**

NLA1 Case 3 - Time Domain

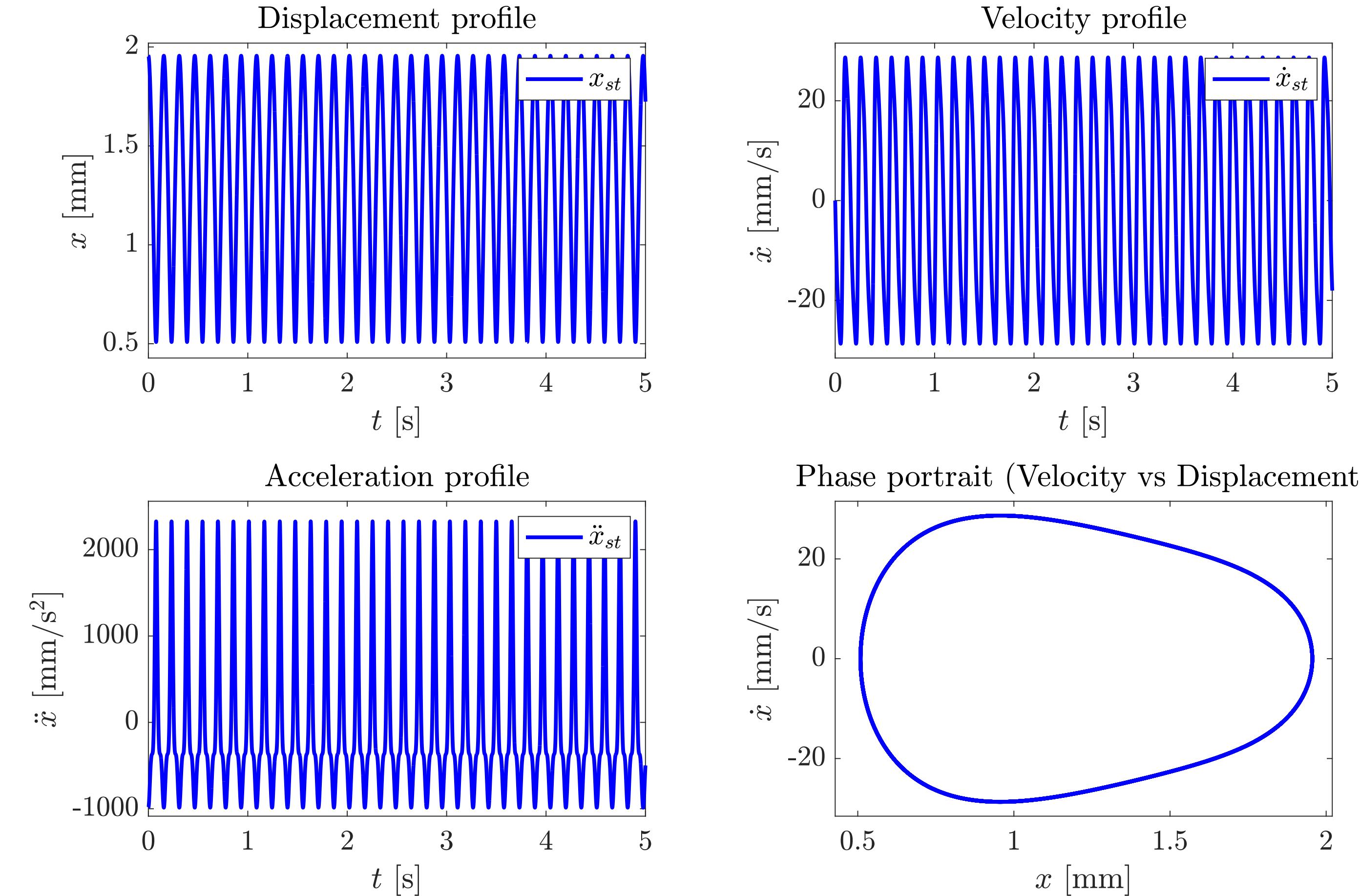


Figure 11 - Time domain results for Case 2

NLA1 Case 3 - Physical and Frequency Domain

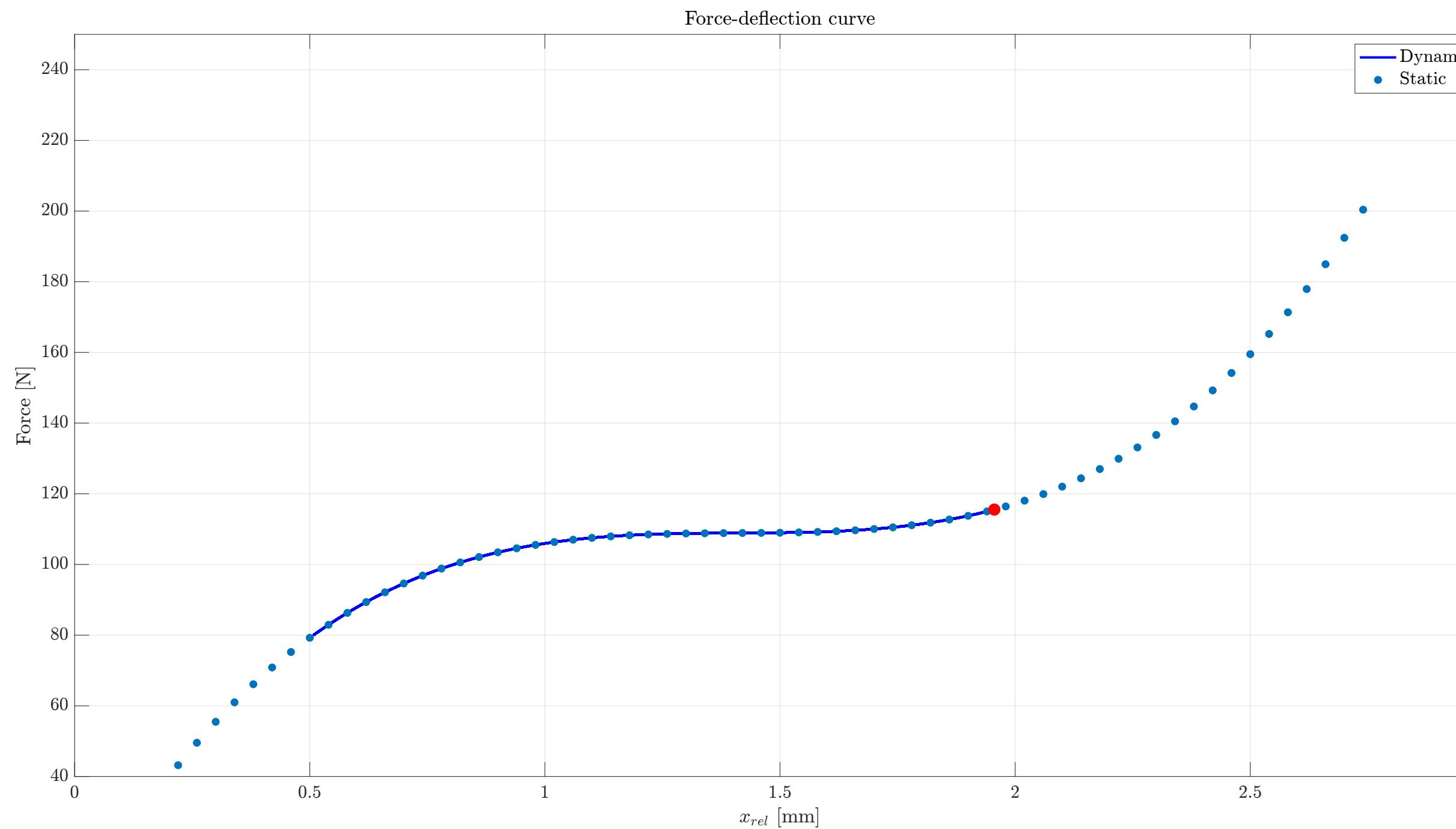


Figure 12 - Physical domain behavior for Case 1

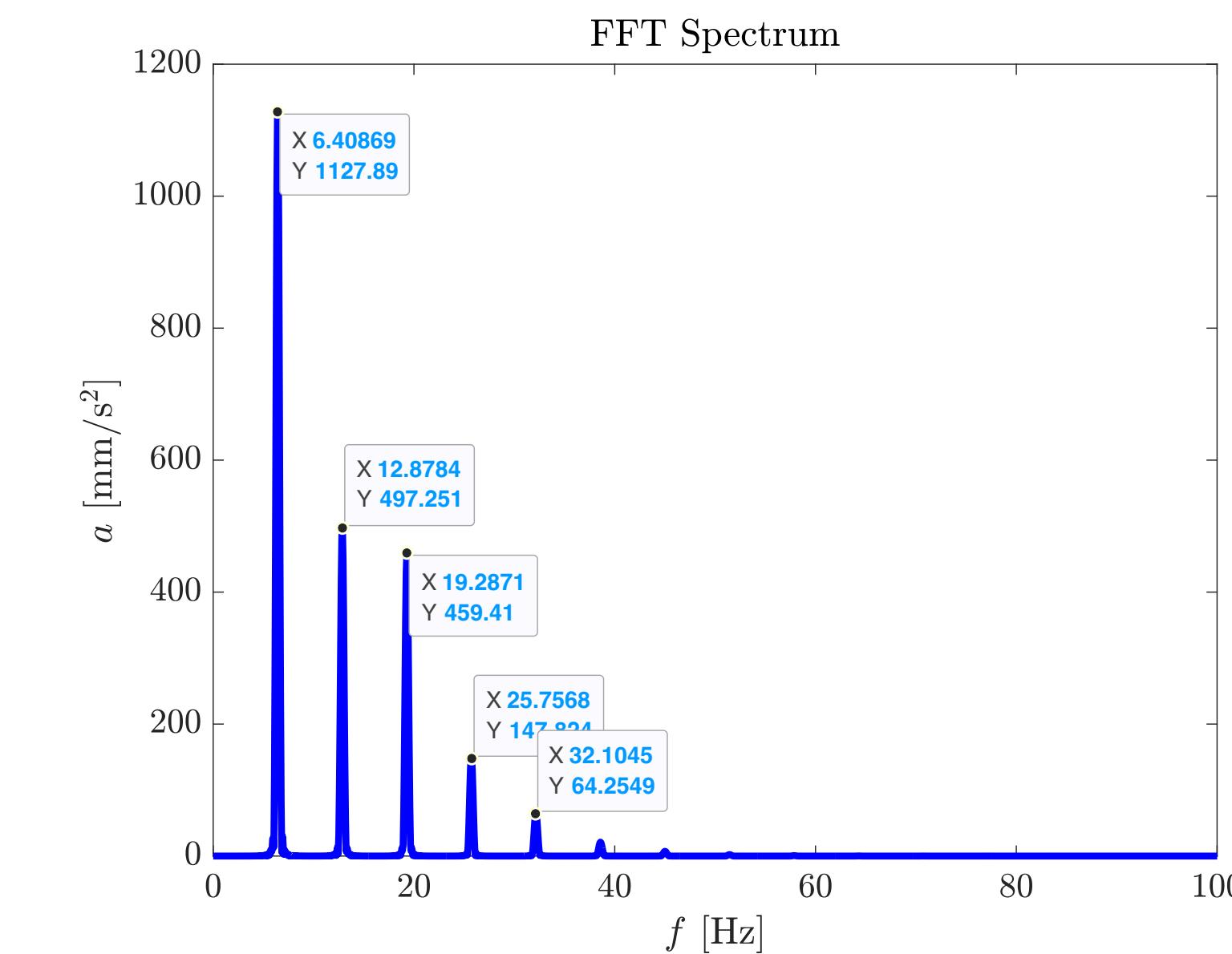


Figure 13 - FFT Spectrum for Case 1

More super harmonics !

First peak at **6.4 Hz**

Non-Linear System Analysis - Part 2 (NLA2)

Parameters : $h_1/\tau = h_2/\tau = 1.41$, $x_{base}(t) = A \sin(2\pi ft)$ (in mm)

A is the amplitude (mean to peak) and f is the frequency of the base excitation

Amplitude (peak to peak)			
	0.001 mm	0.1 mm	0.5 mm
5 Hz	Case 1	Case 5	Case 9
7 Hz	Case 2	Case 6	Case 10
8 Hz	Case 3	Case 7	Case 11
10 Hz	Case 4	Case 8	Case 12

NLA2 Case 1 ($A = 0.001, f = 5$) - Time Domain

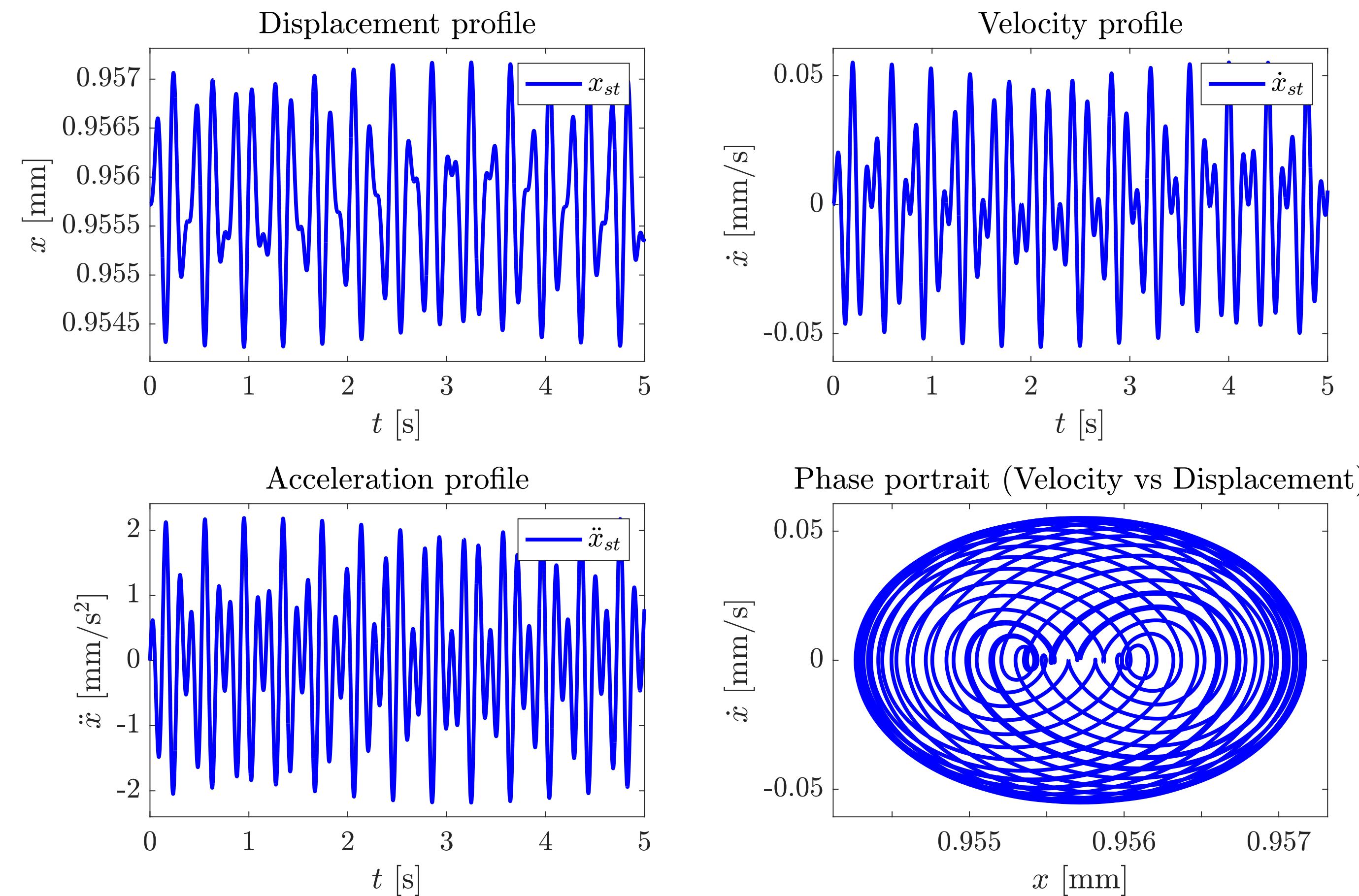


Figure 14 - Time domain results for Case 1

NLA2 Case 1 - Physical and Frequency Domain

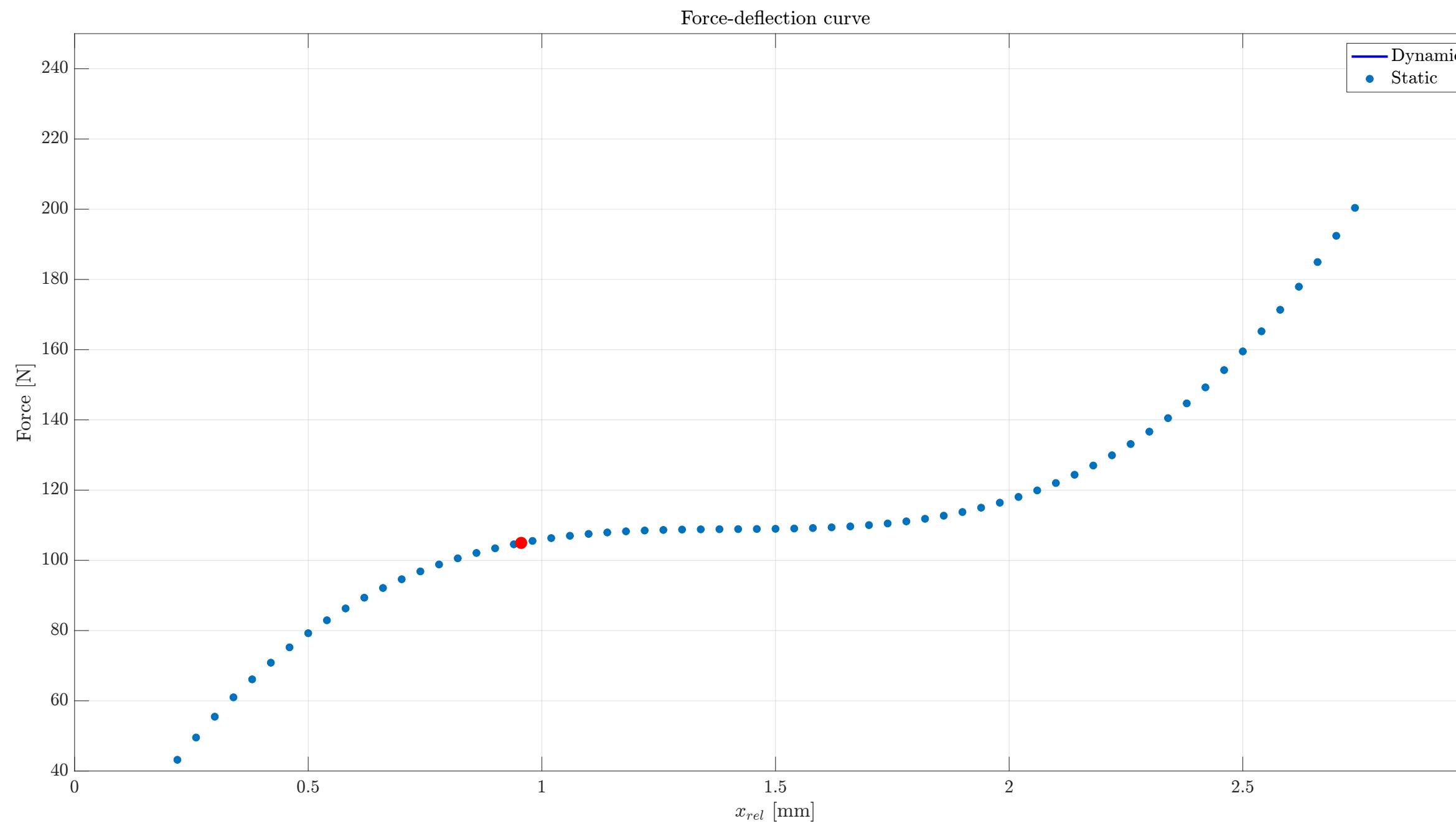


Figure 15 - Physical domain behavior for Case 1

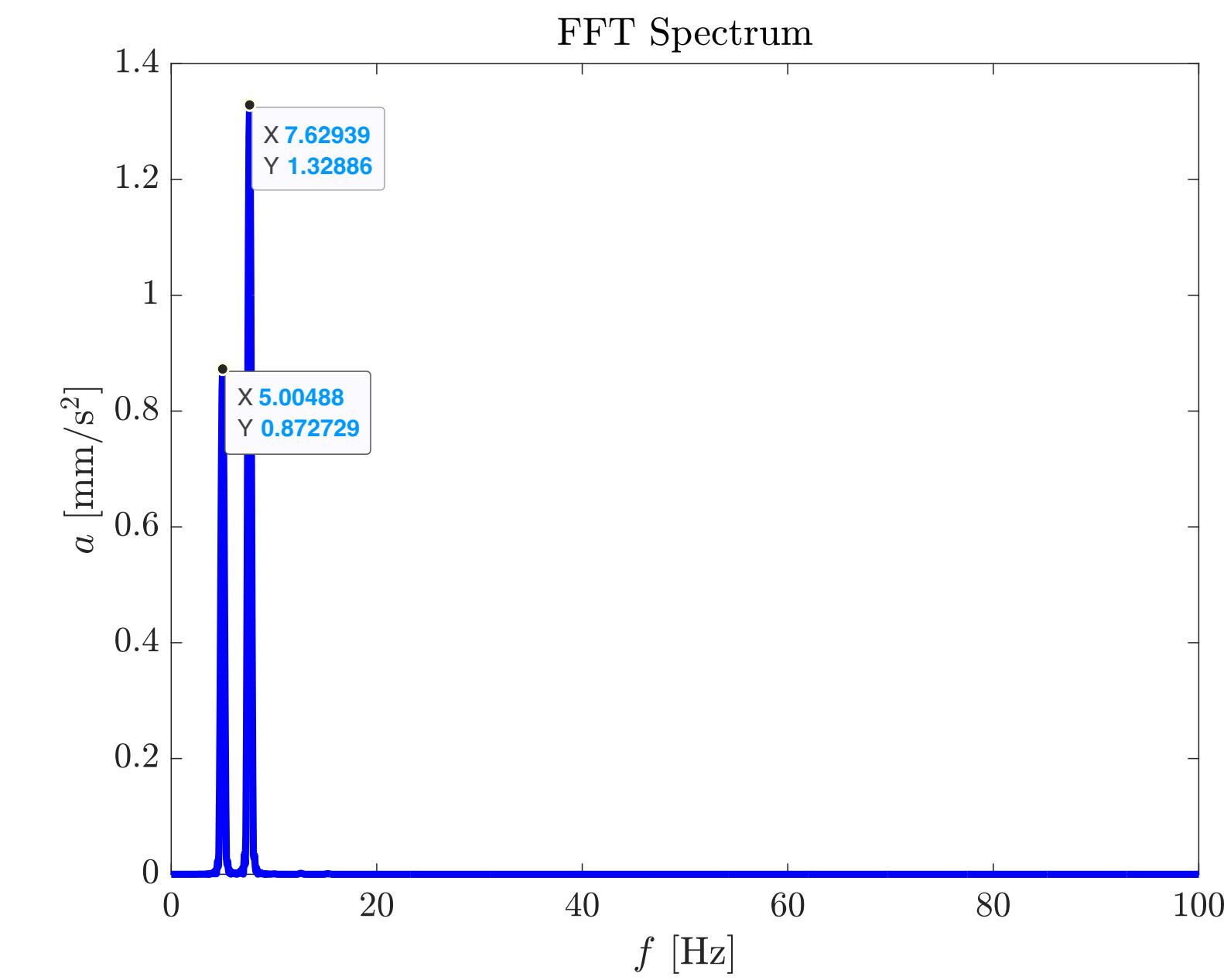
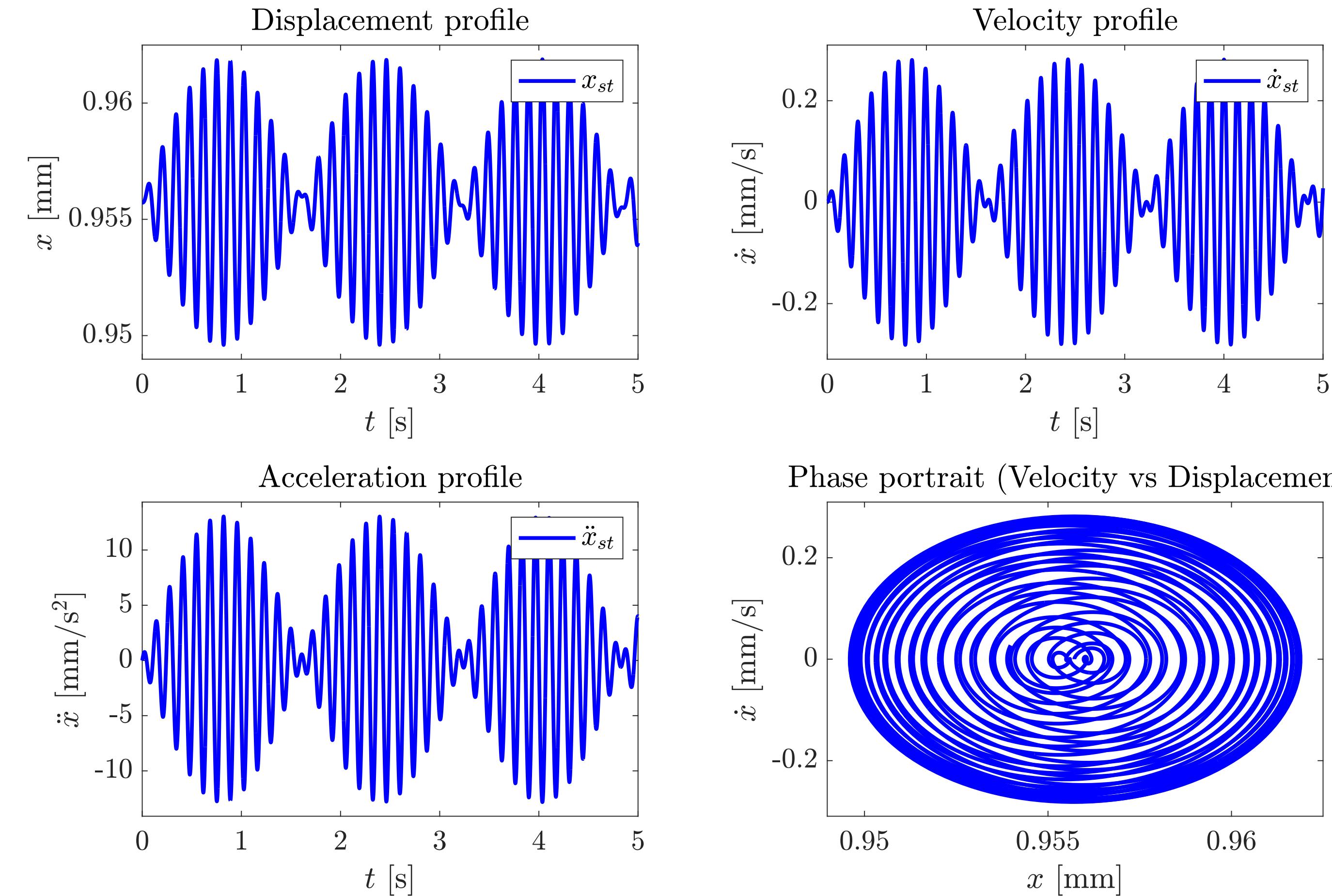


Figure 16 - FFT Spectrum for Case 1

Peaks corresponding to the “natural frequency” and the driving frequency

Can be difficult to find the period of the signal as a result

NLA2 Case 2 ($A = 0.001, f = 7$) - Time Domain



**Beats phenomenon
visible quite clearly !**

Figure 17 - Time domain results for Case 2

NLA2 Case 2 - Physical and Frequency Domain

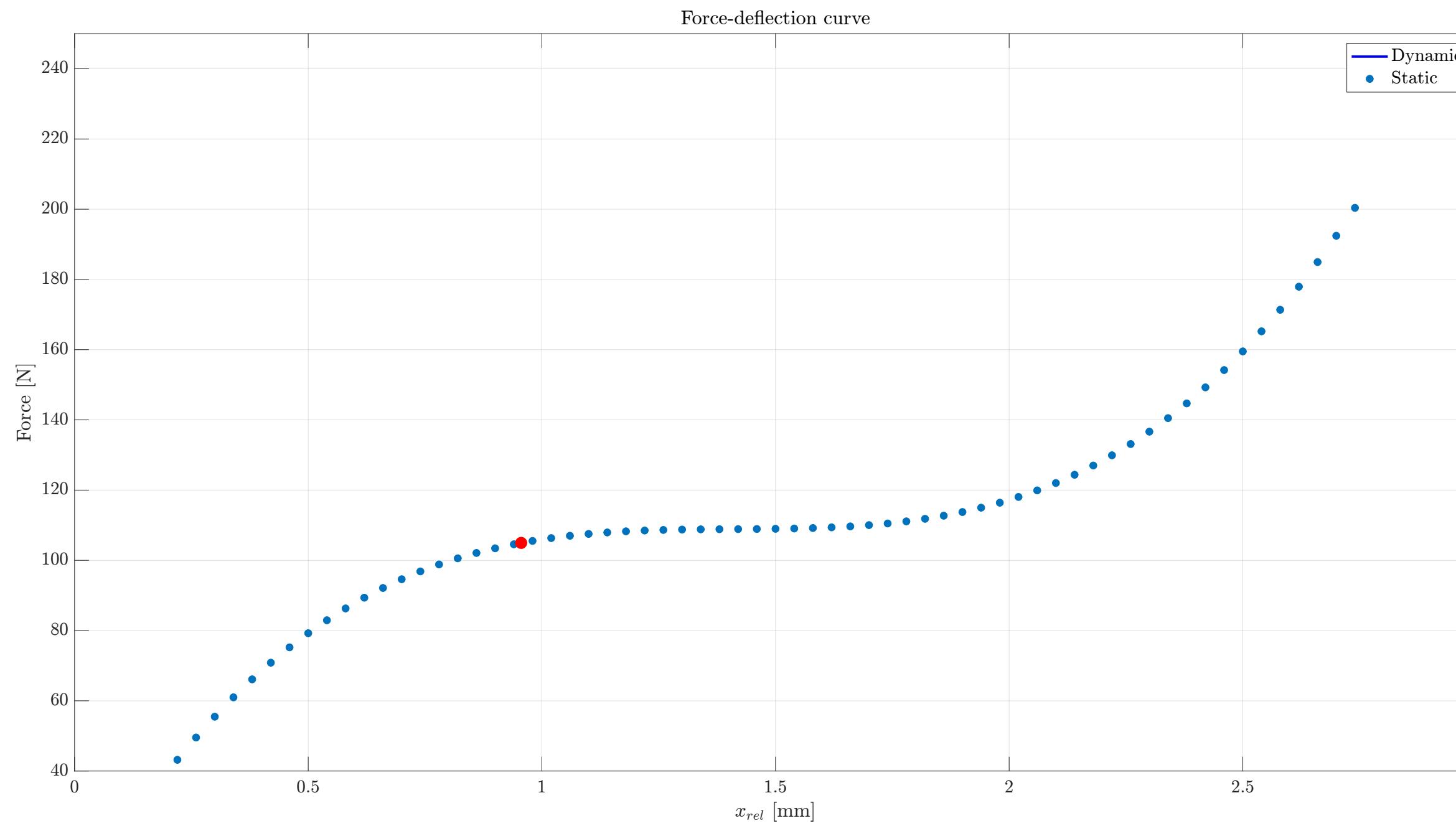


Figure 18 - Physical domain behavior for Case 2

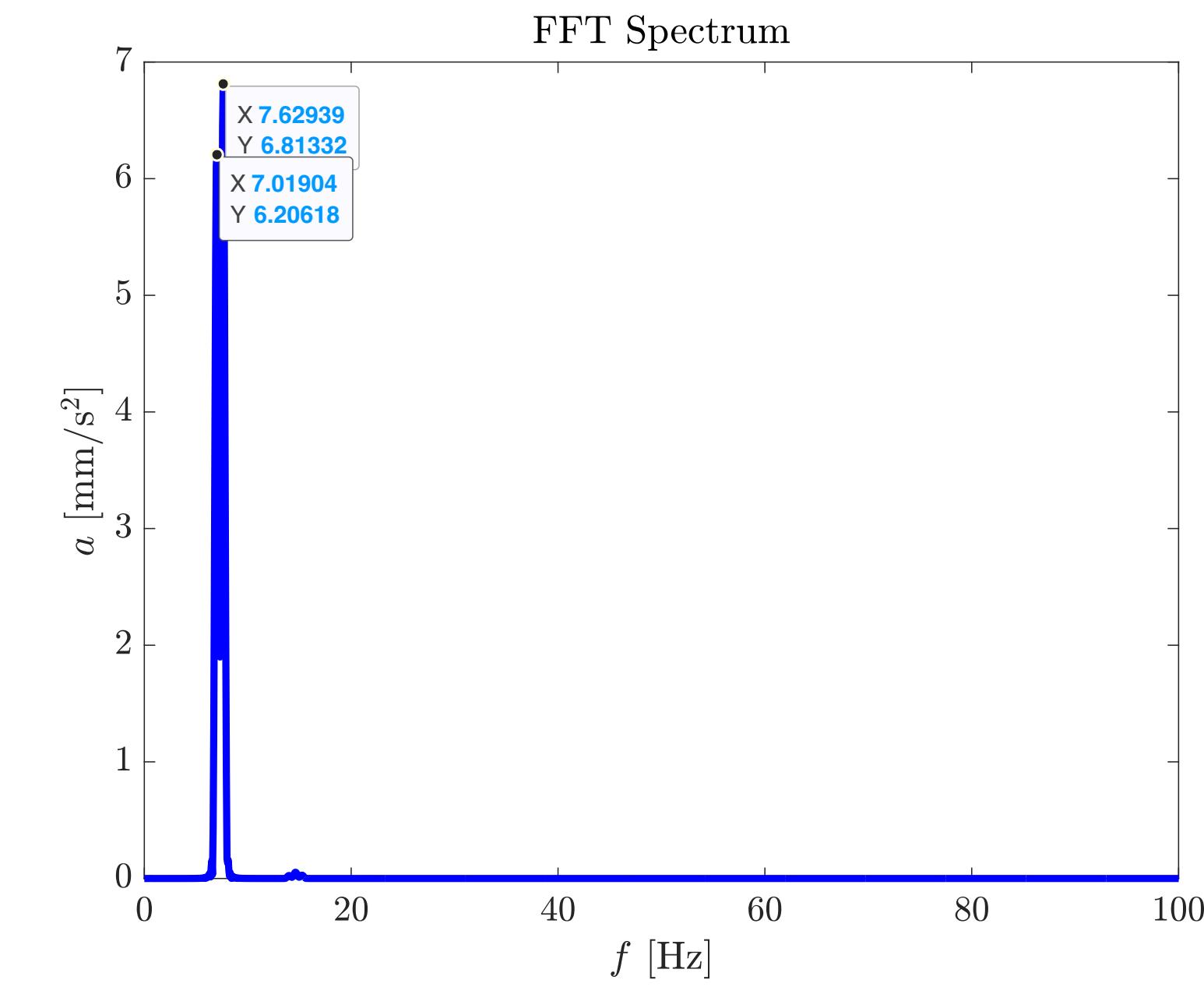


Figure 19 - FFT Spectrum for Case 2

NLA2 Case 3 ($A = 0.001, f = 8$) - Time Domain

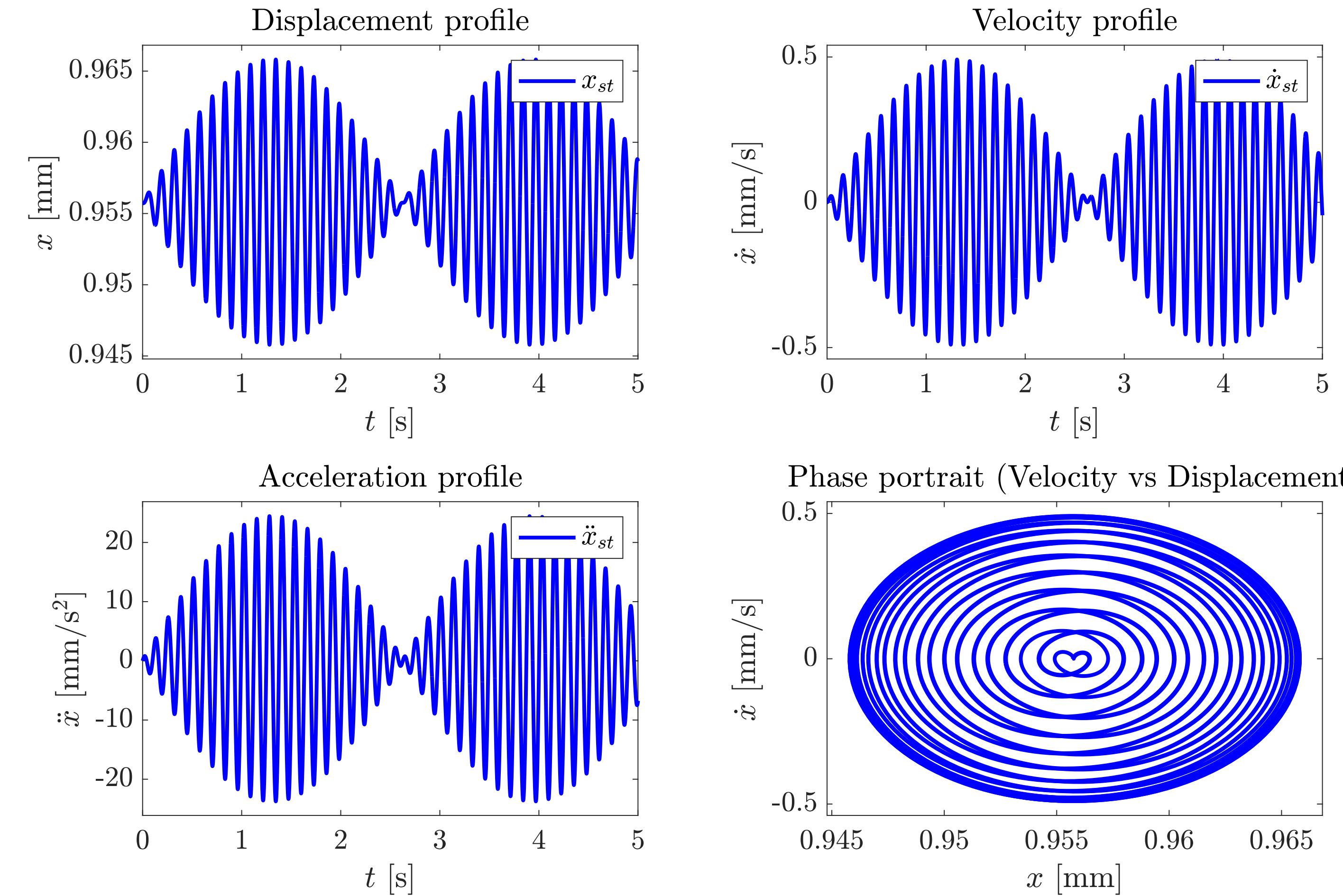


Figure 20 - Time domain results for Case 3

NLA2 Case 3 - Physical and Frequency Domain

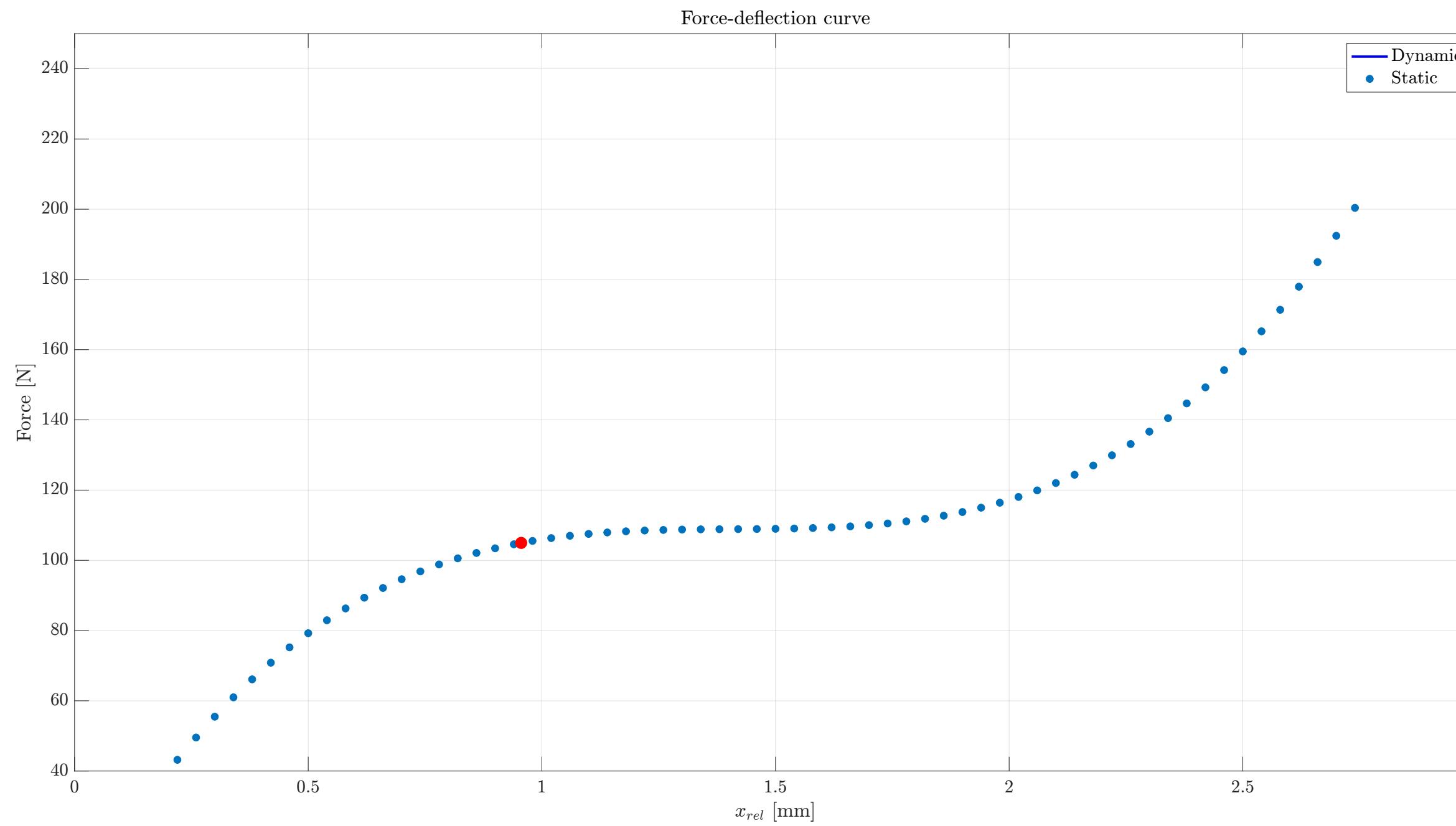


Figure 21 - Physical domain behavior for Case 3

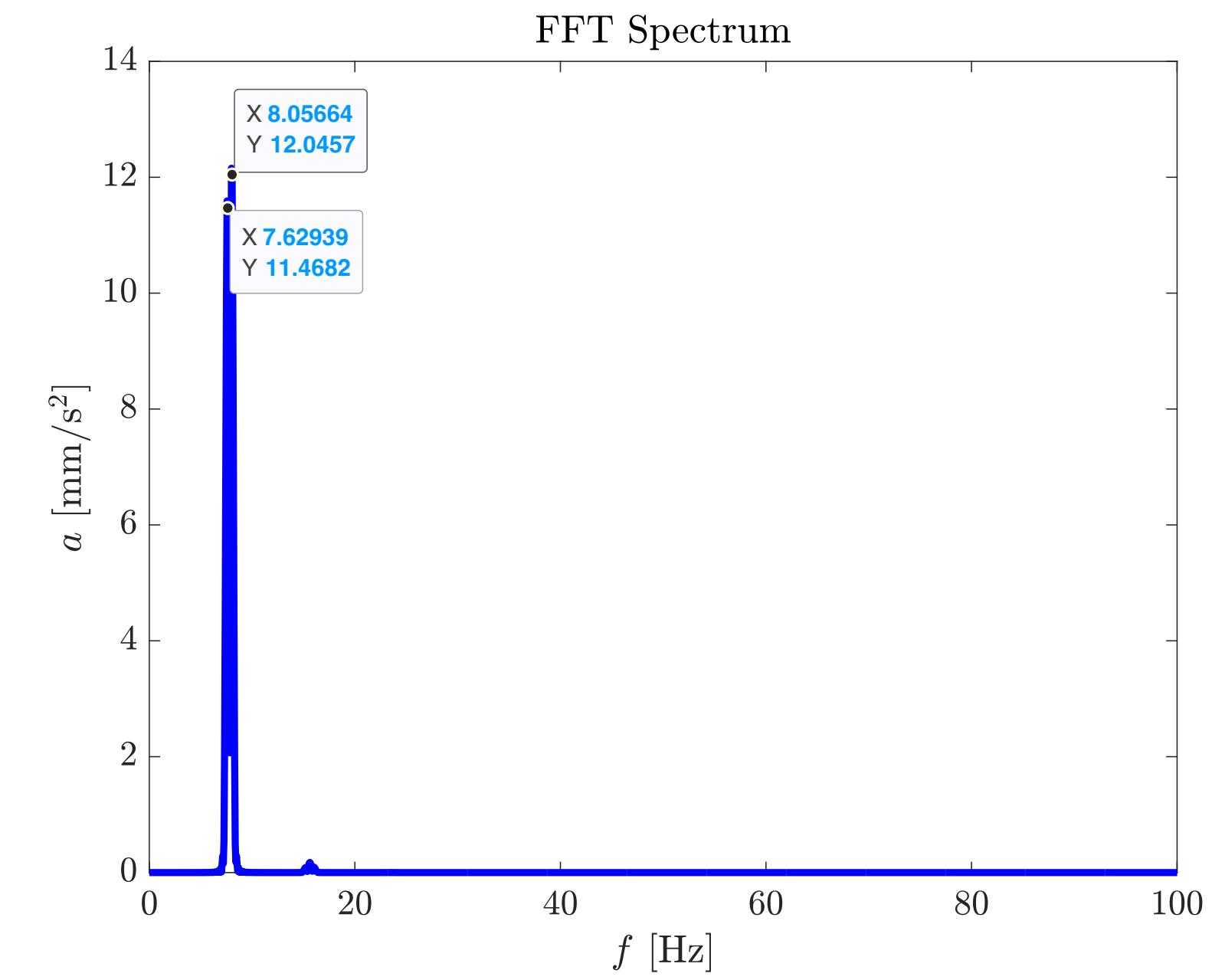


Figure 22 - FFT Spectrum for Case 3

NLA2 Case 4 ($A = 0.001, f = 10$) - Time Domain

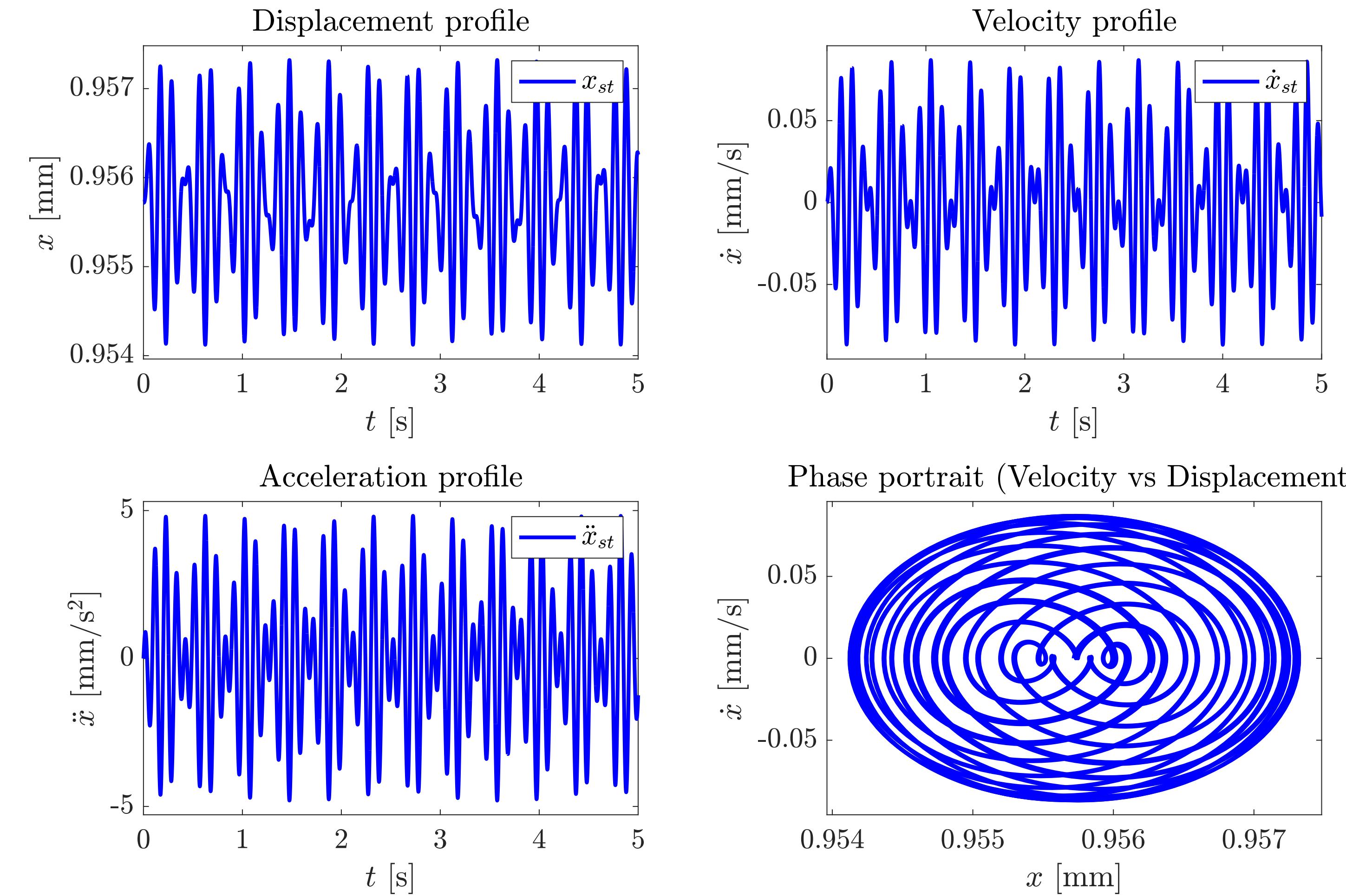


Figure 23 - Time domain results for Case 4

NLA2 Case 4 - Physical and Frequency Domain

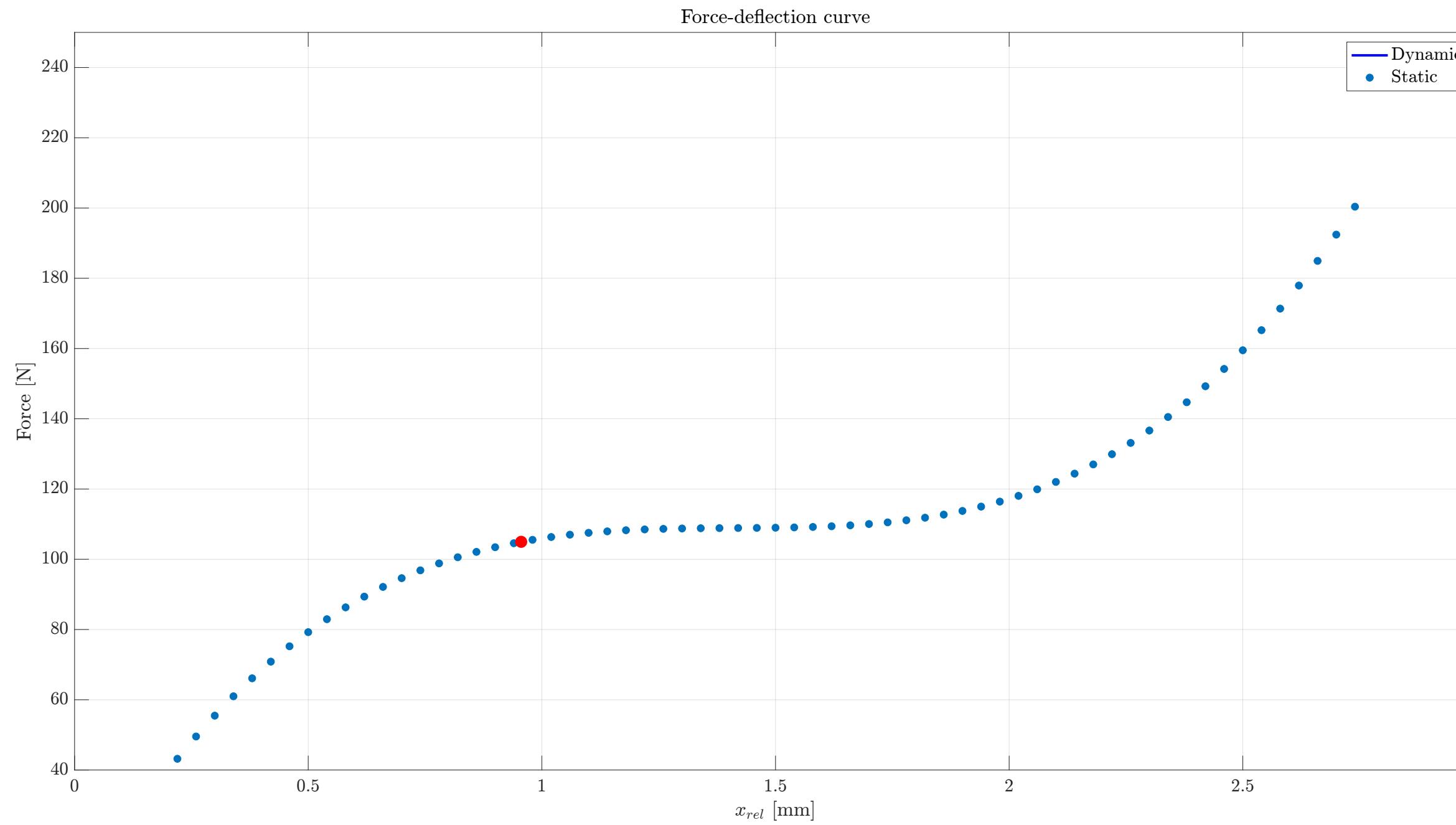


Figure 24 - Physical domain behavior for Case 4

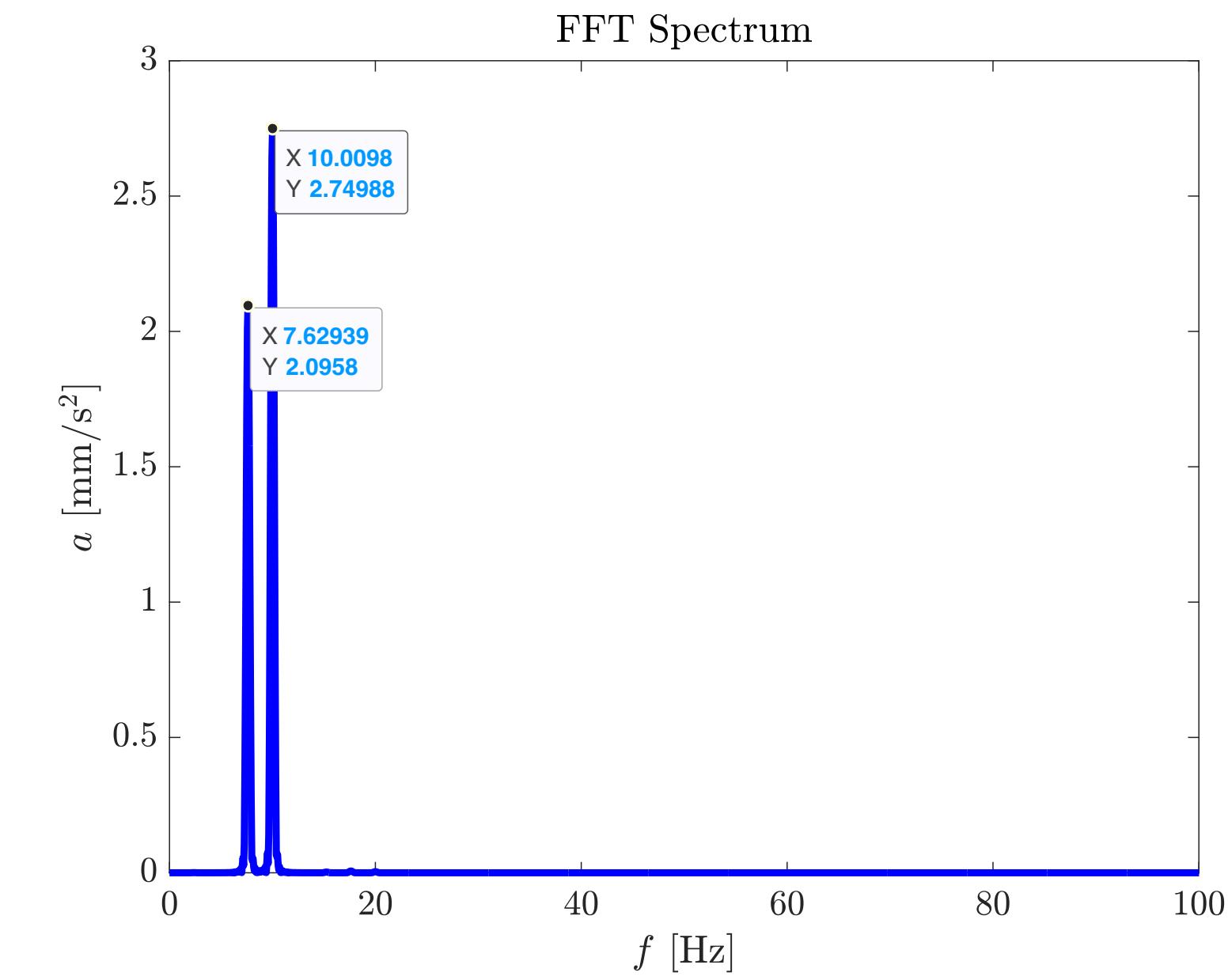


Figure 25 - FFT Spectrum for Case 4

NLA2 Case 5 ($A = 0.1, f = 5$) - Time Domain

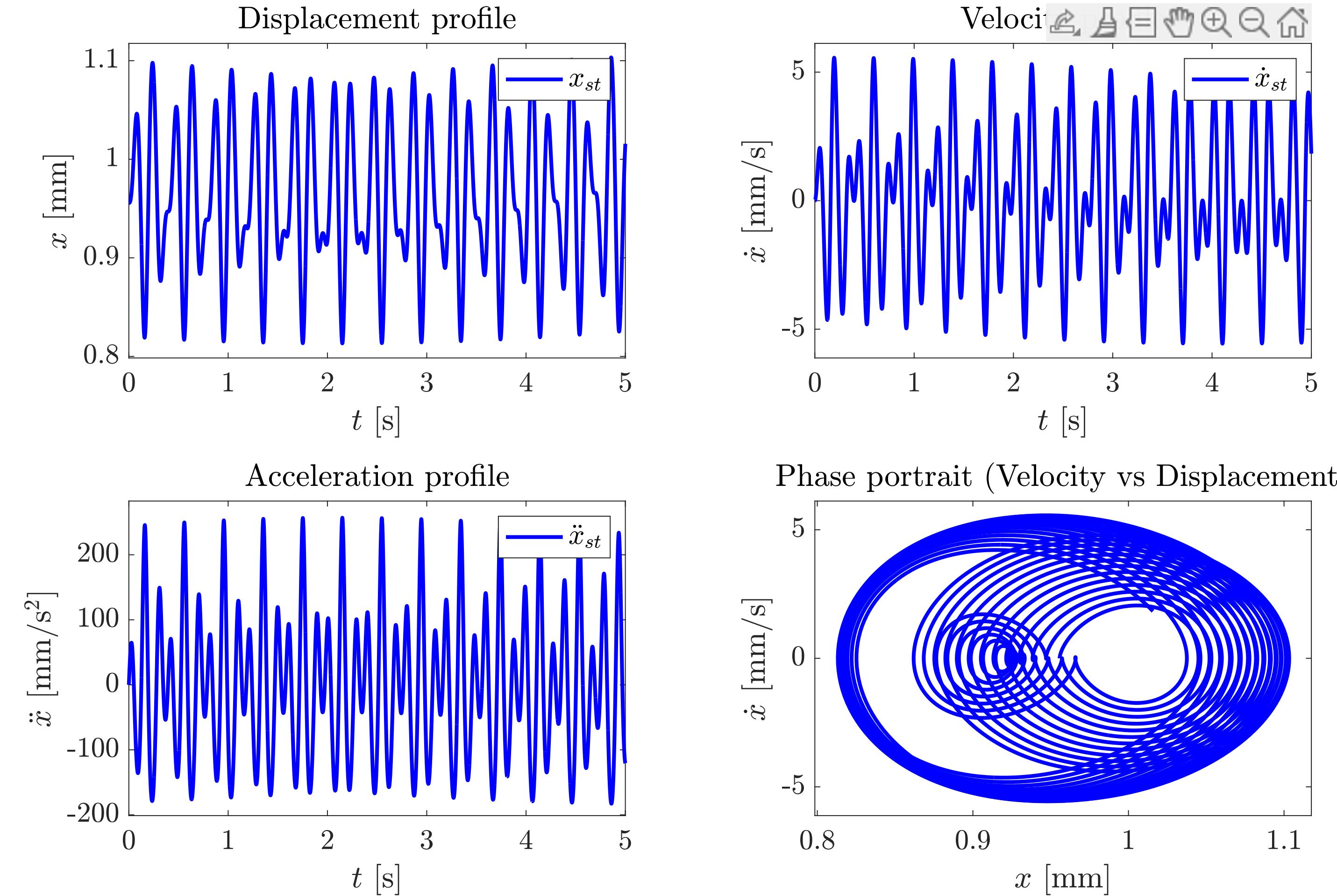


Figure 26 - Time domain results for Case 5

NLA2 Case 5 - Physical and Frequency Domain

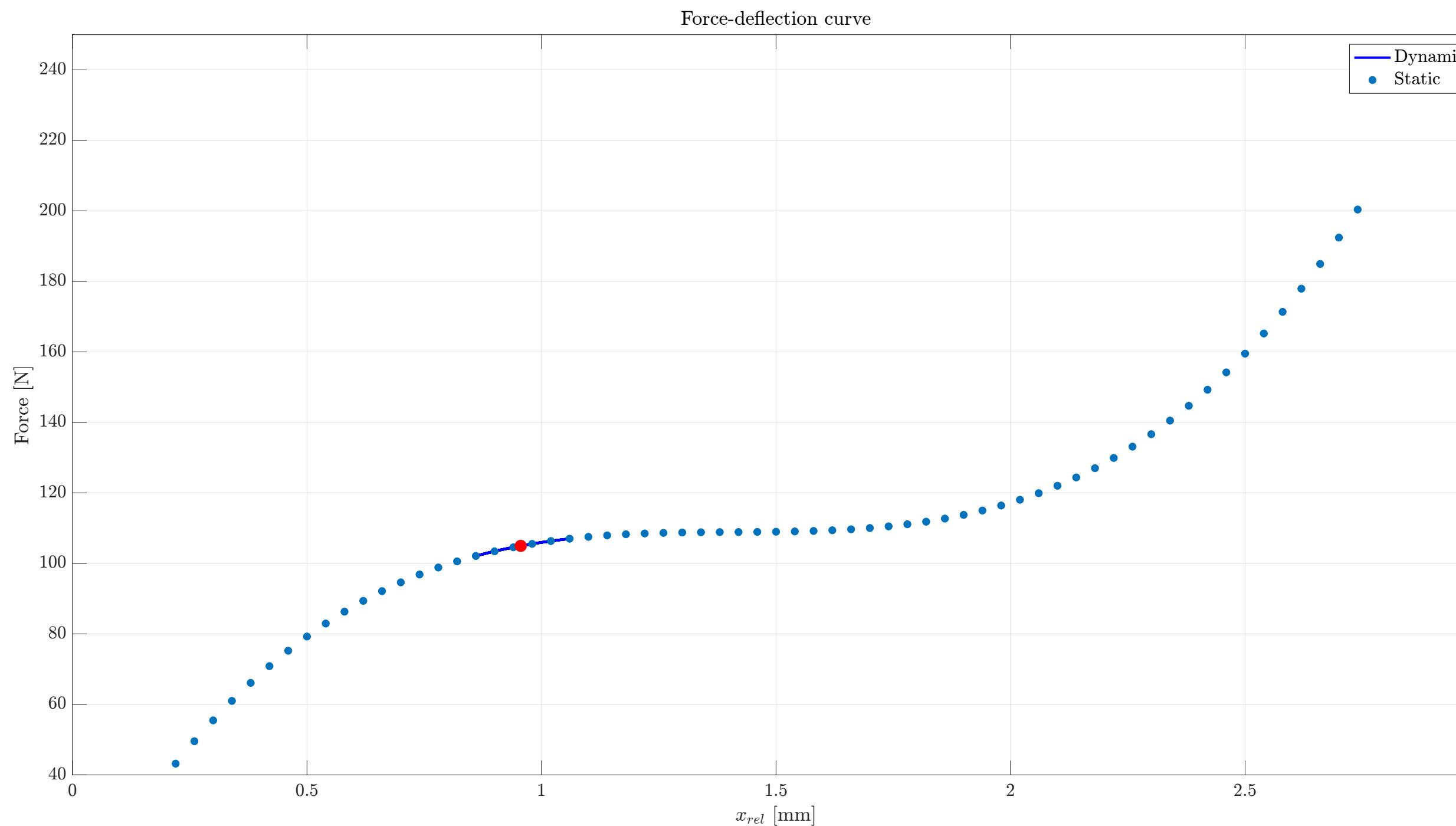


Figure 27 - Physical domain behavior for Case 5

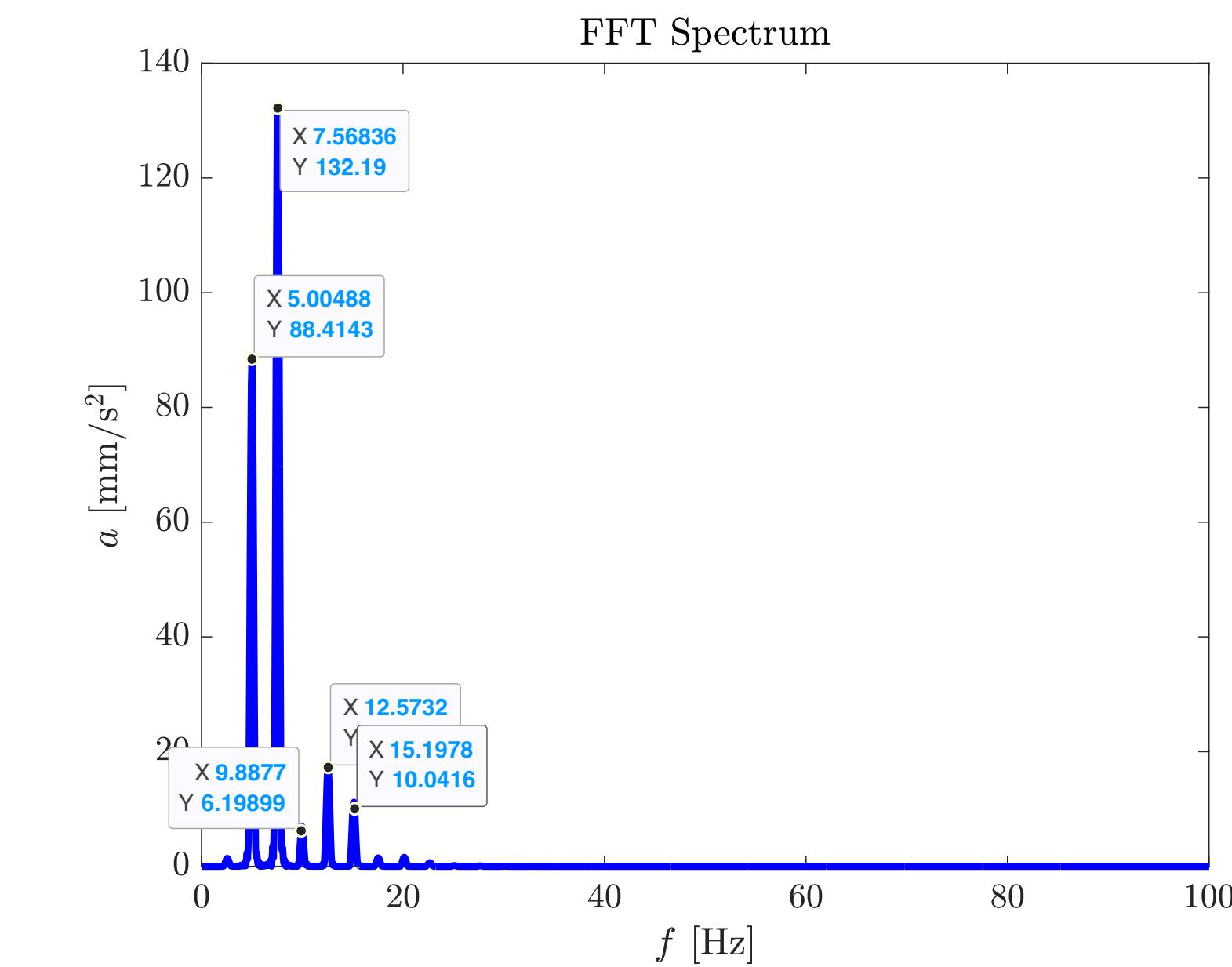


Figure 28 - FFT Spectrum for Case 5

Multiple frequency peaks !

NLA2 Case 6 ($A = 0.1, f = 7$) - Time Domain

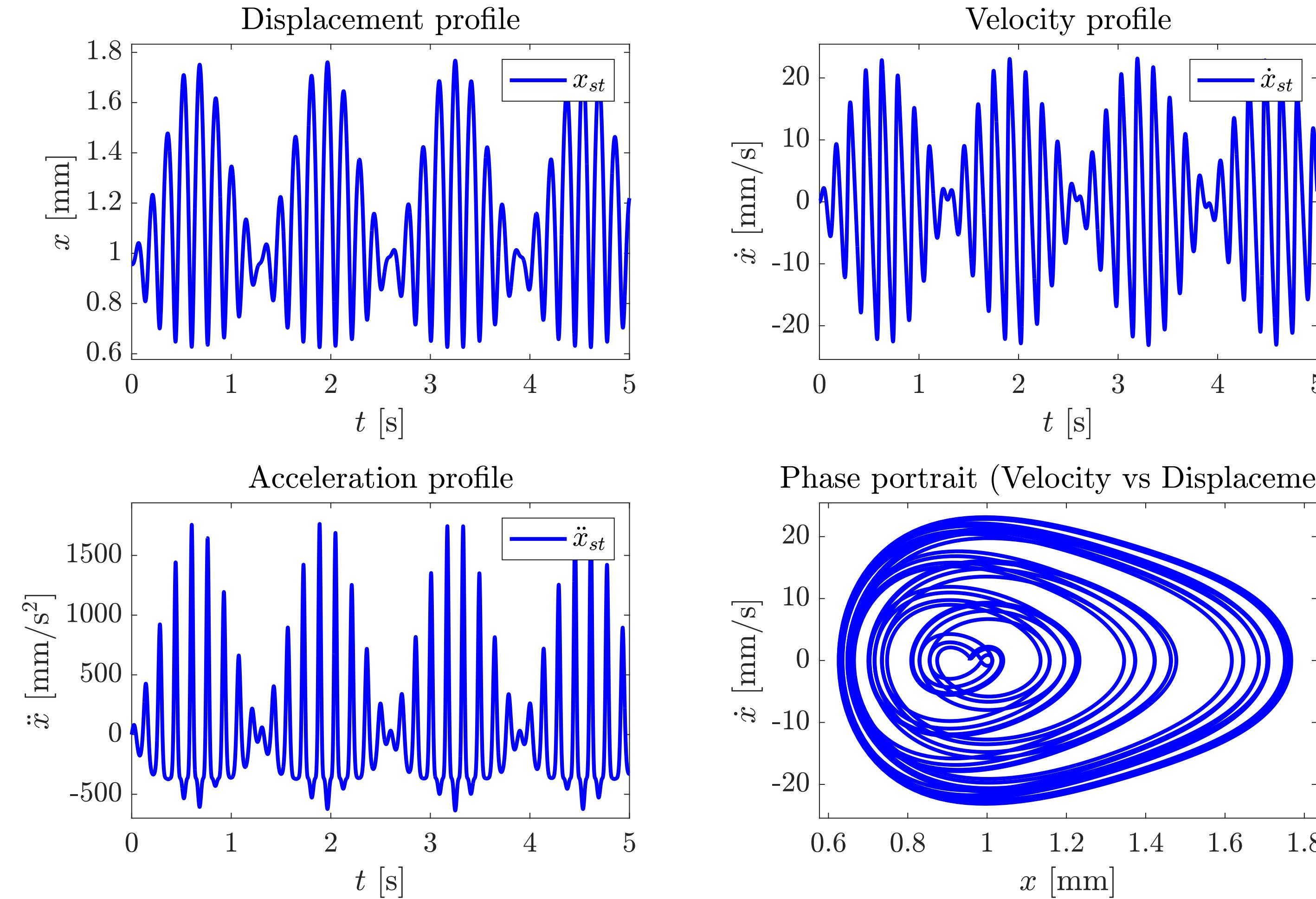


Figure 29 - Time domain results for Case 6

NLA2 Case 6 - Physical and Frequency Domain

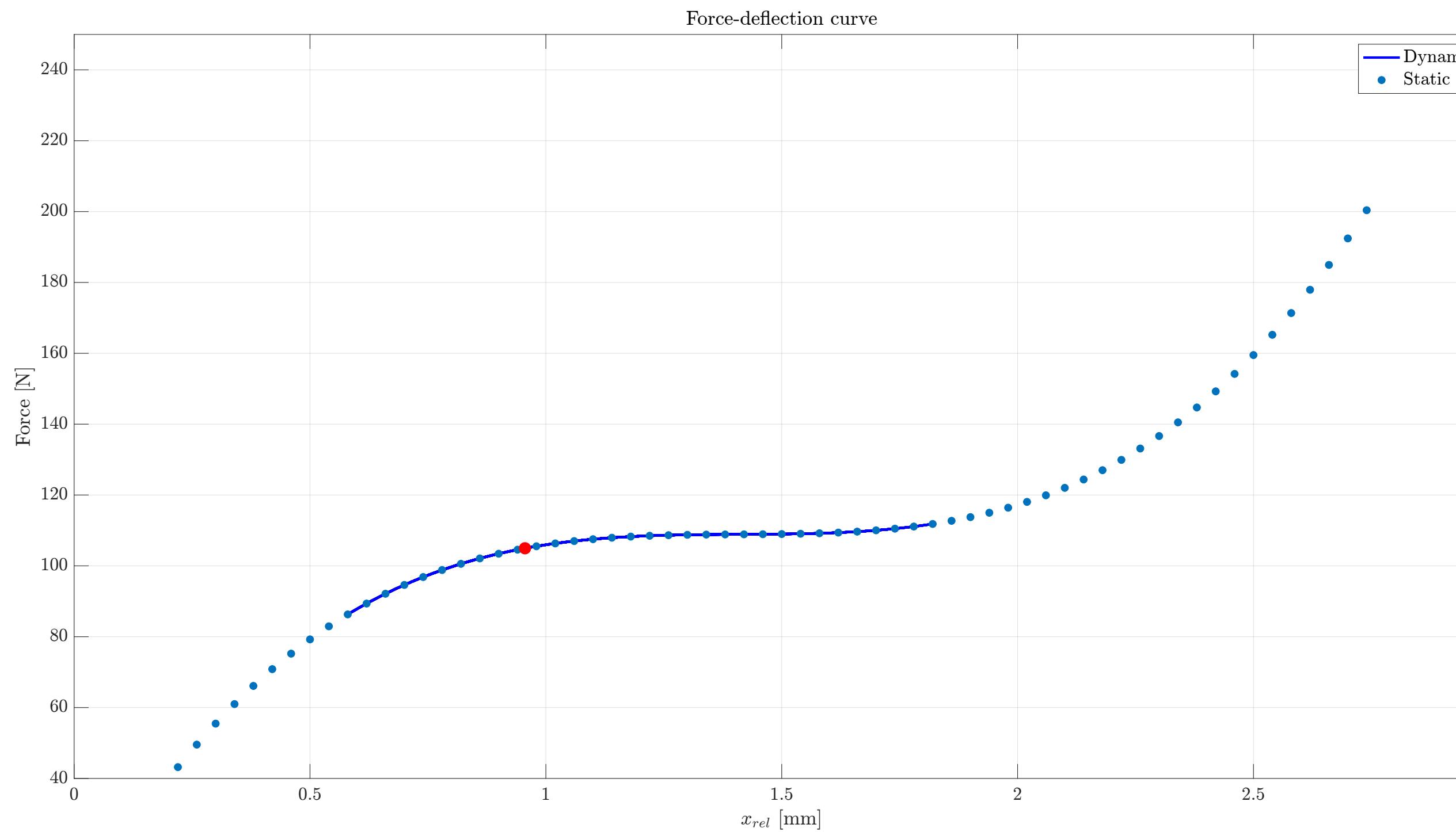


Figure 30 - Physical domain behavior for Case 6

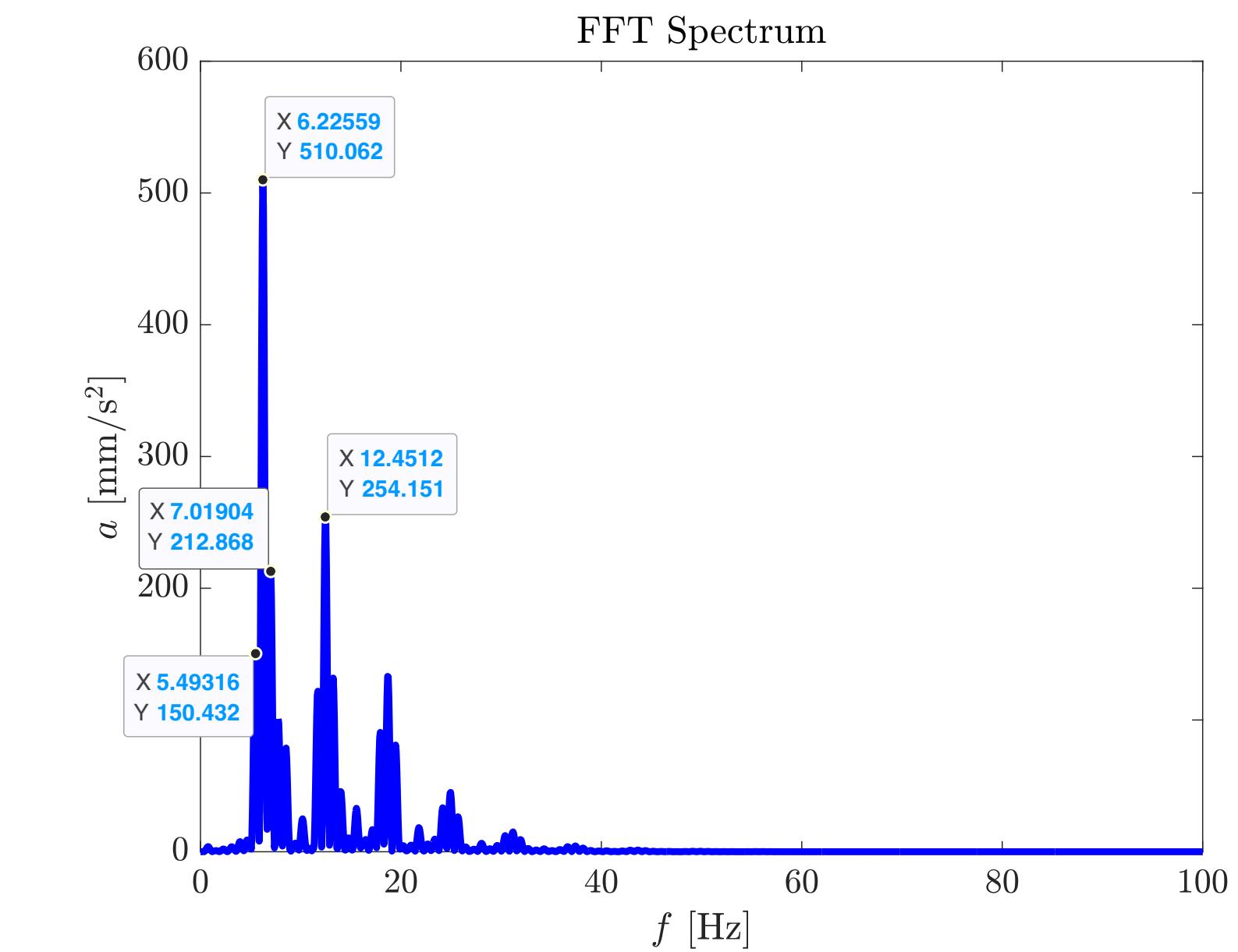


Figure 31 - FFT Spectrum for Case 6

Even more frequency peaks !

NLA2 Case 7 ($A = 0.1, f = 8$) - Time Domain

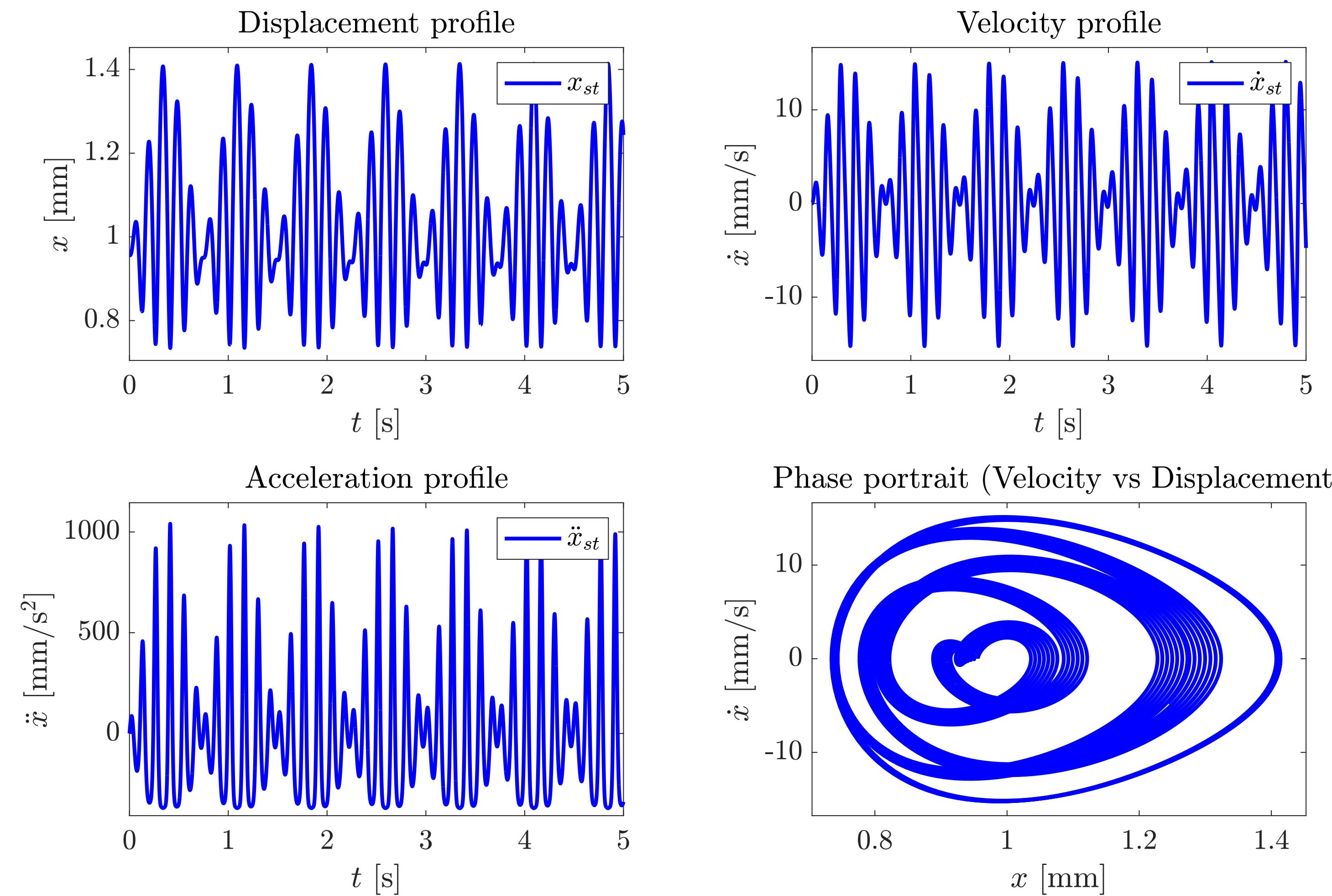


Figure 32 - Time domain results for Case 7

NLA2 Case 7 - Physical and Frequency Domain

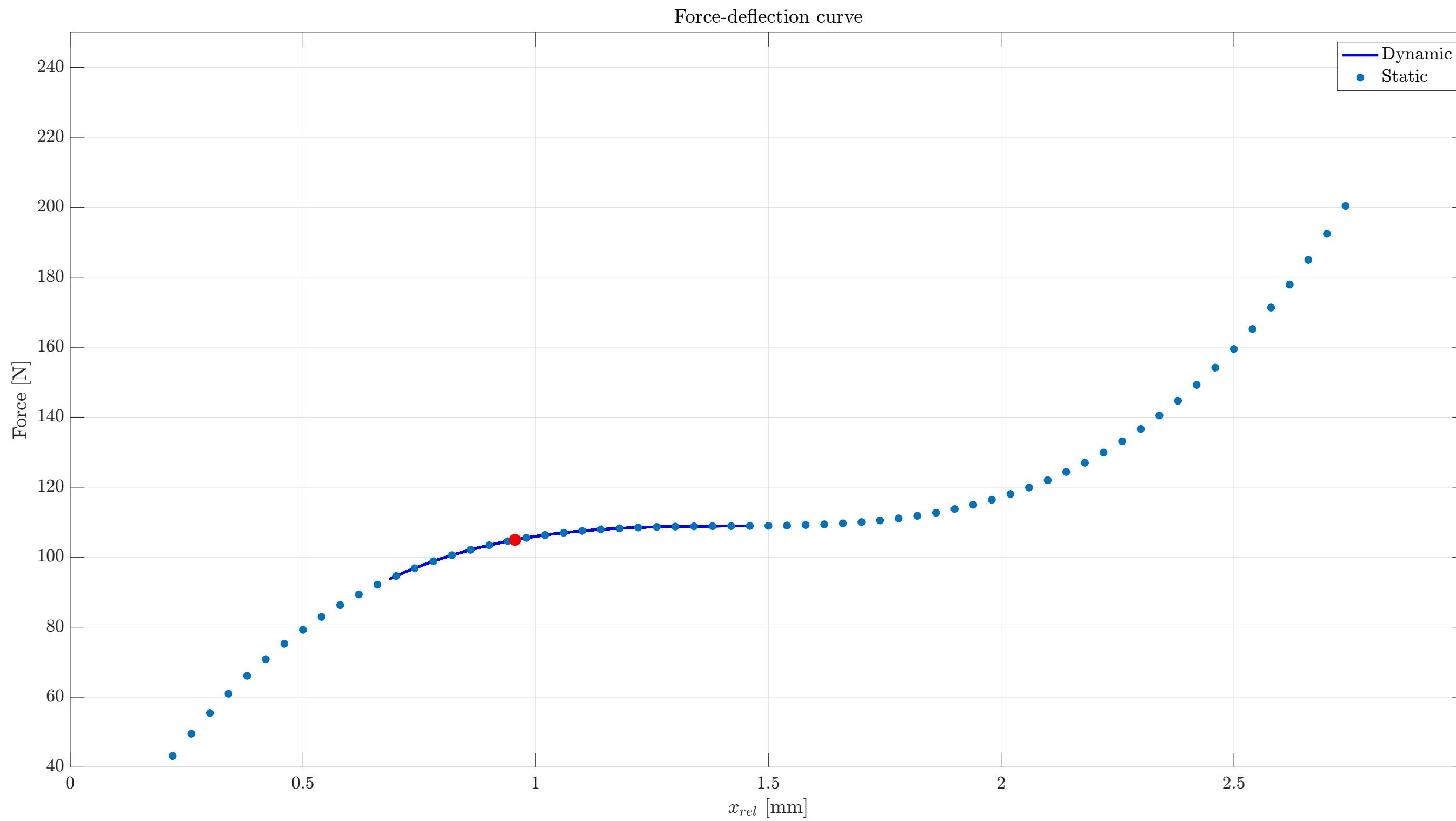


Figure 33 - Physical domain behavior for Case 7

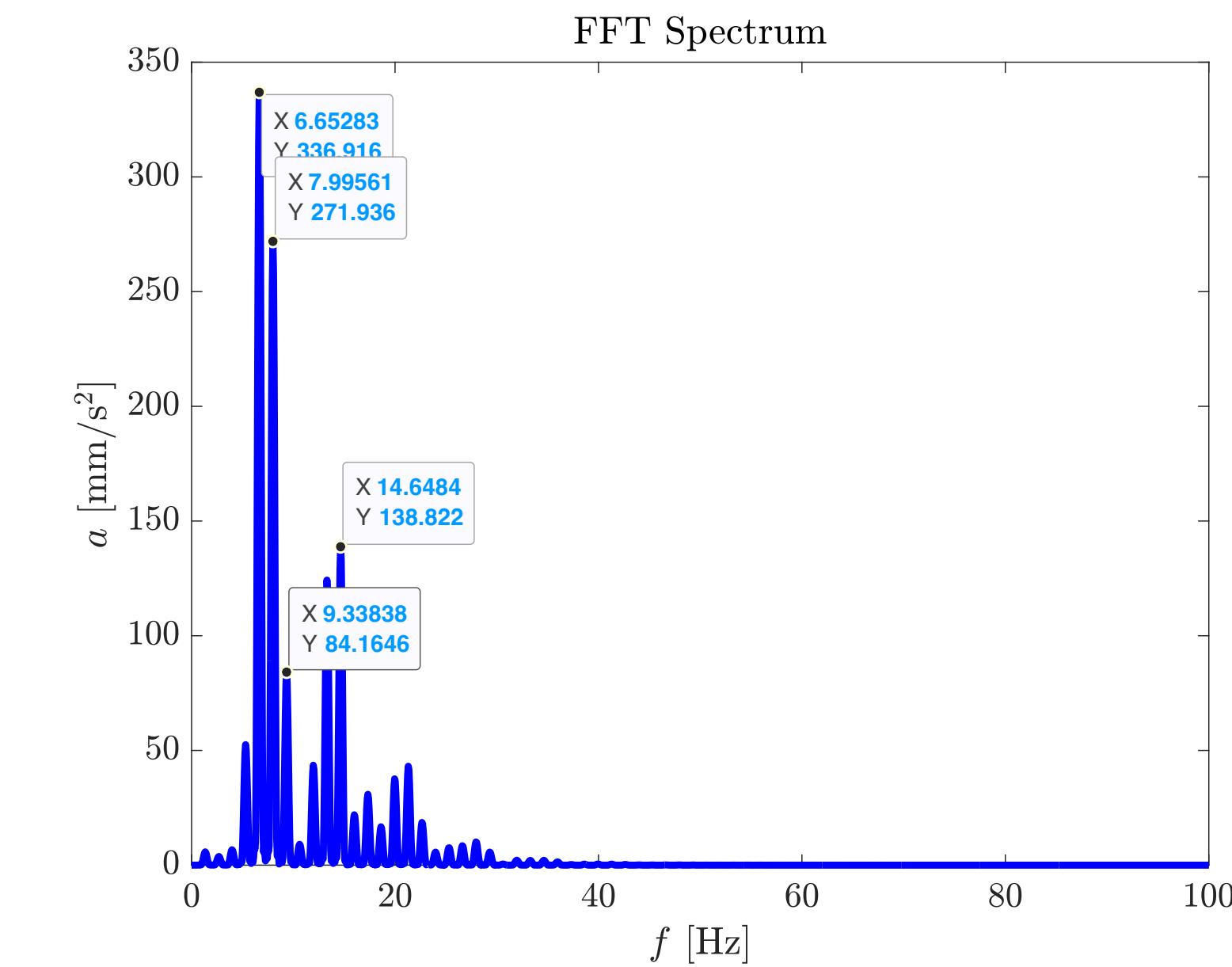


Figure 34 - FFT Spectrum for Case 7

NLA2 Case 8 ($A = 0.1, f = 10$) - Time Domain

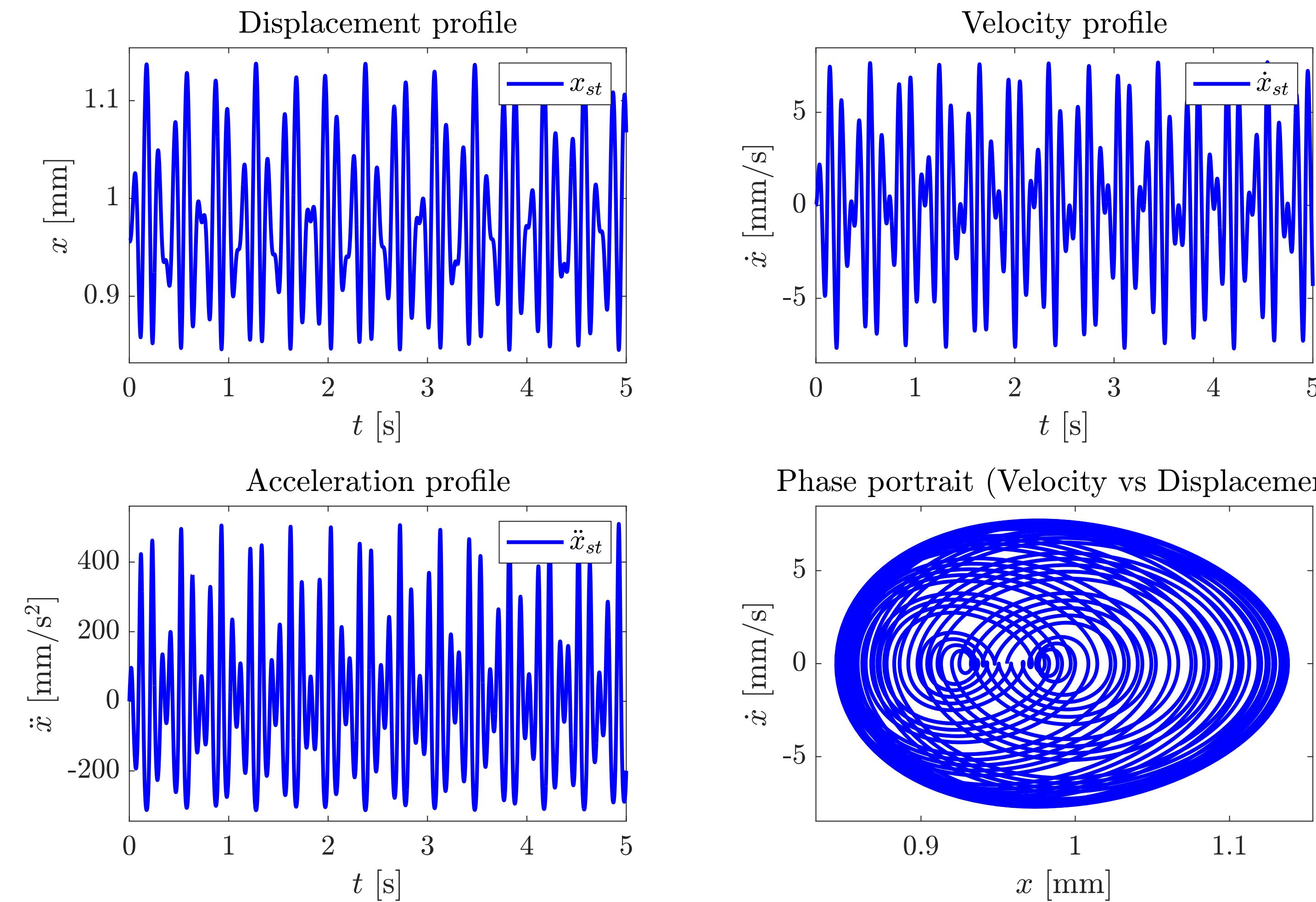


Figure 35 - Time domain results for Case 8

NLA2 Case 8 - Physical and Frequency Domain

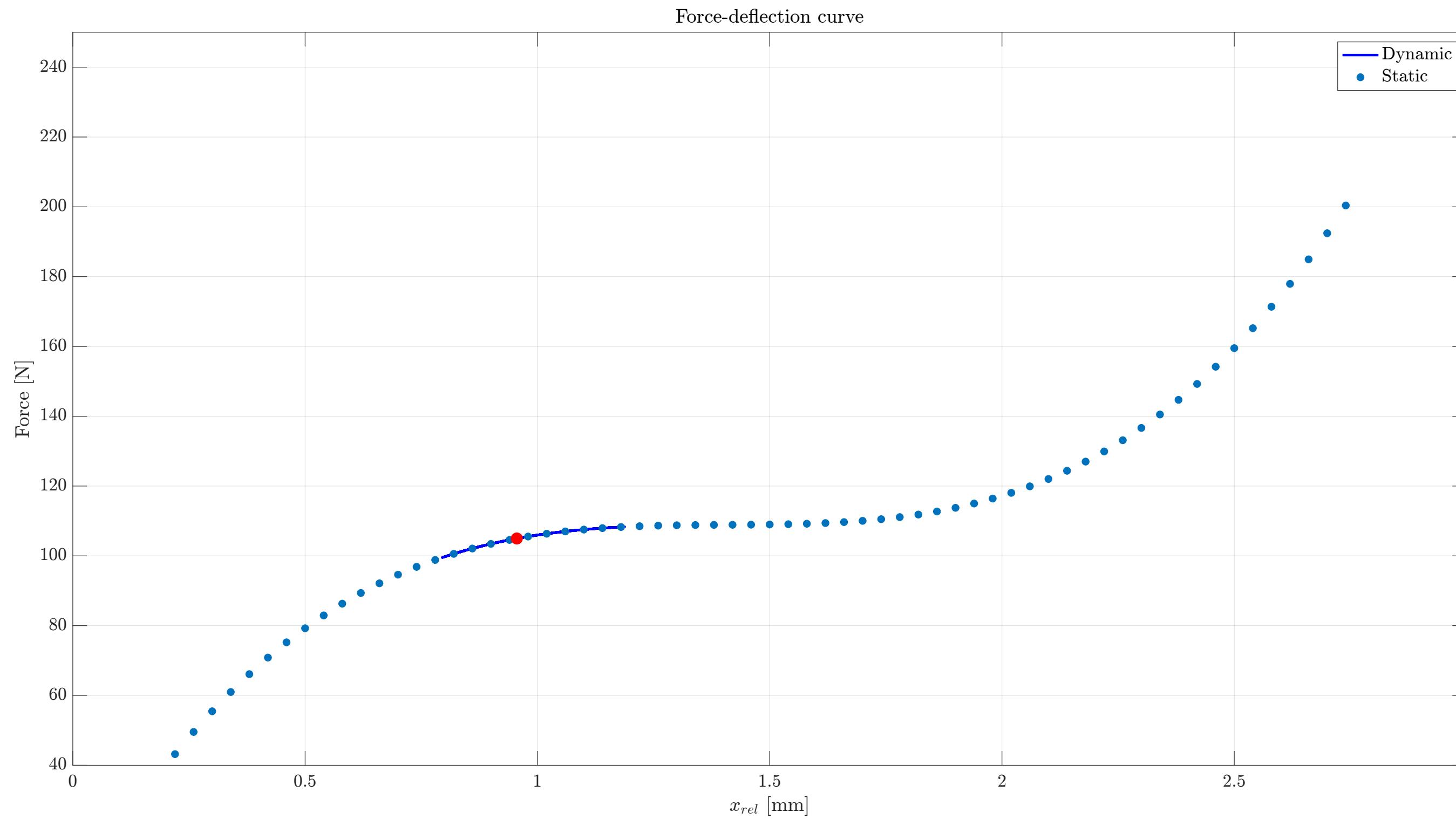


Figure 36 - Physical domain behavior for Case 8

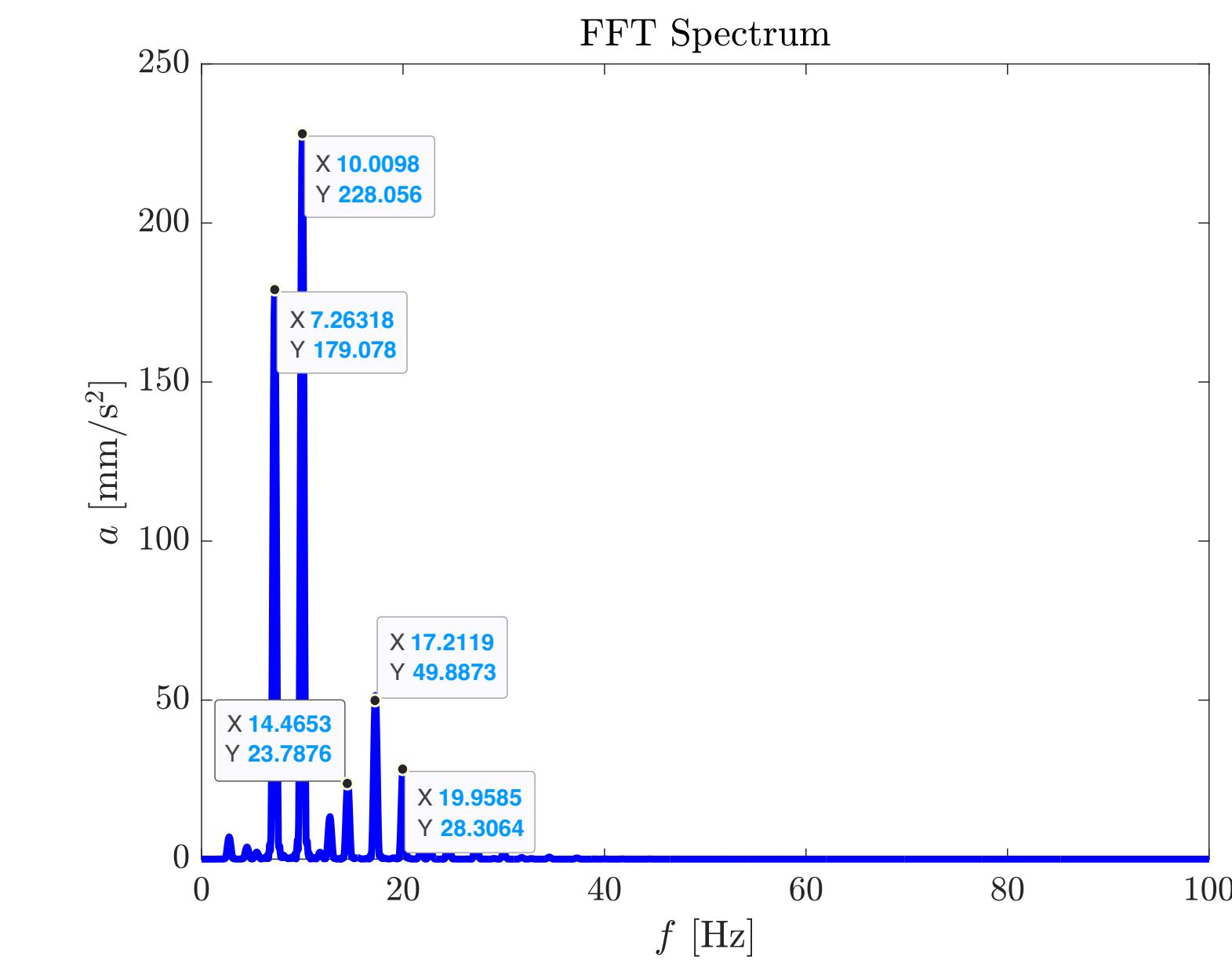


Figure 37 - FFT Spectrum for Case 8

NLA2 Case 9 ($A = 0.5, f = 5$) - Time Domain

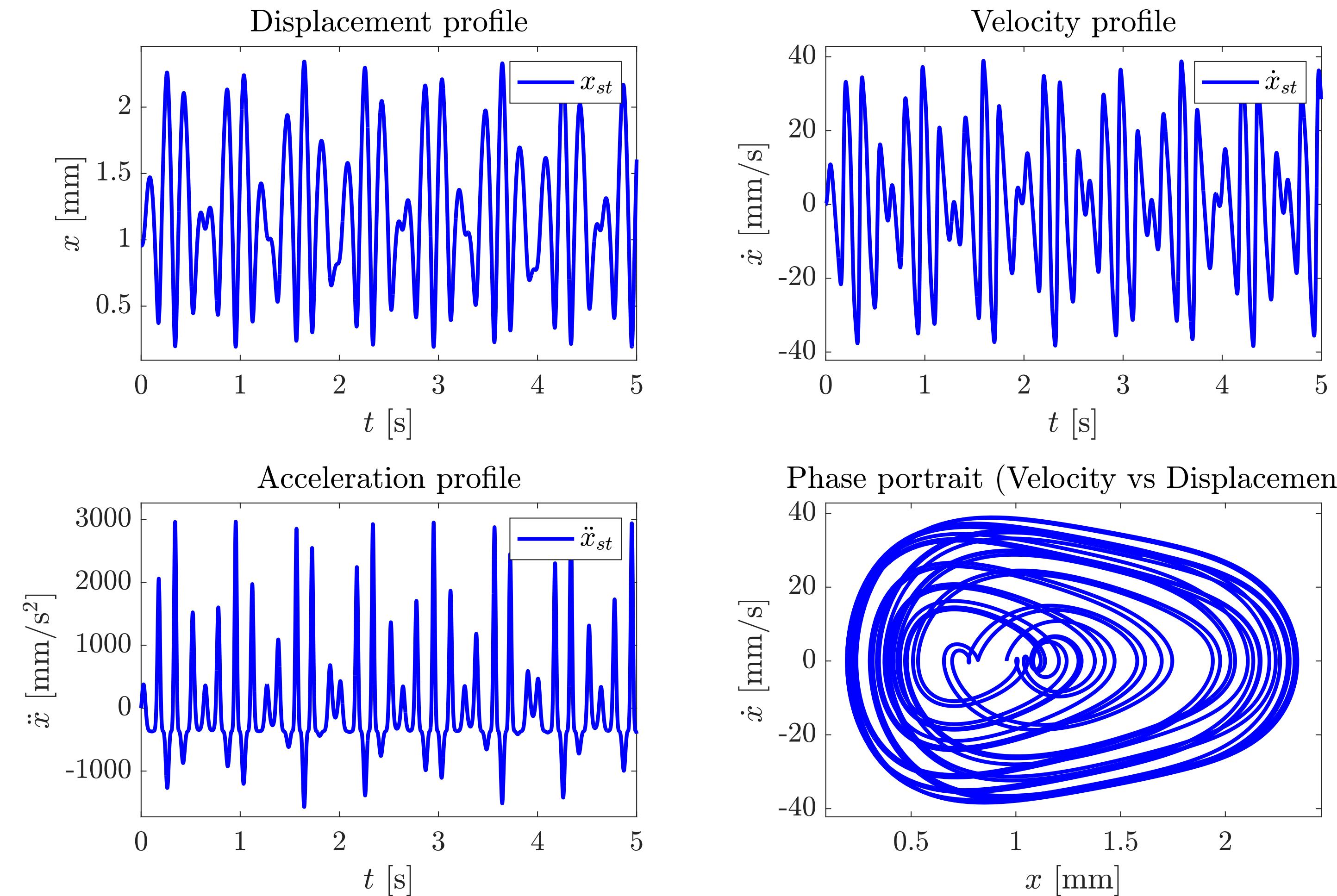


Figure 38 - Time domain results for Case 9

NLA2 Case 9 - Physical and Frequency Domain

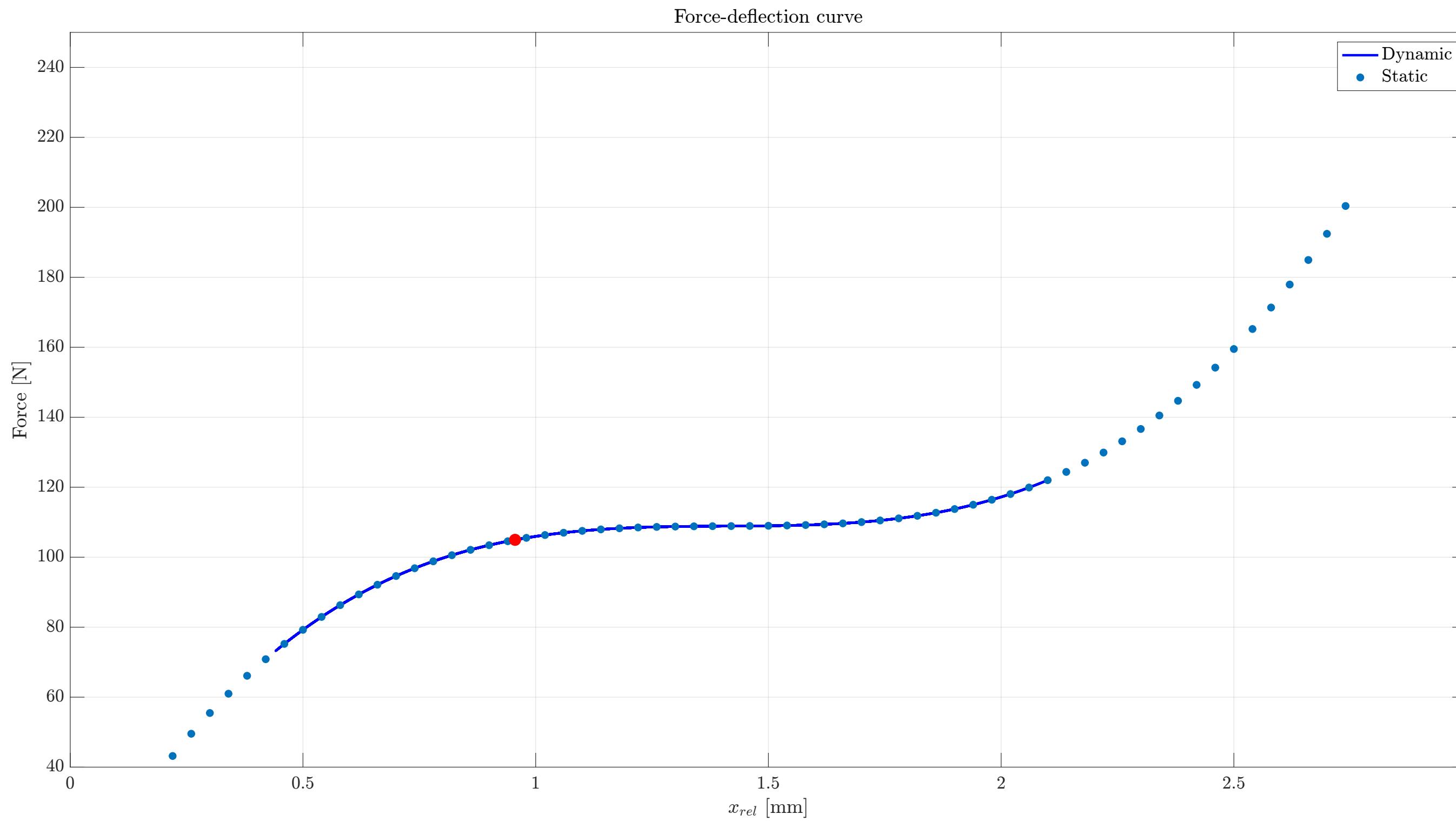


Figure 39 - Physical domain behavior for Case 9

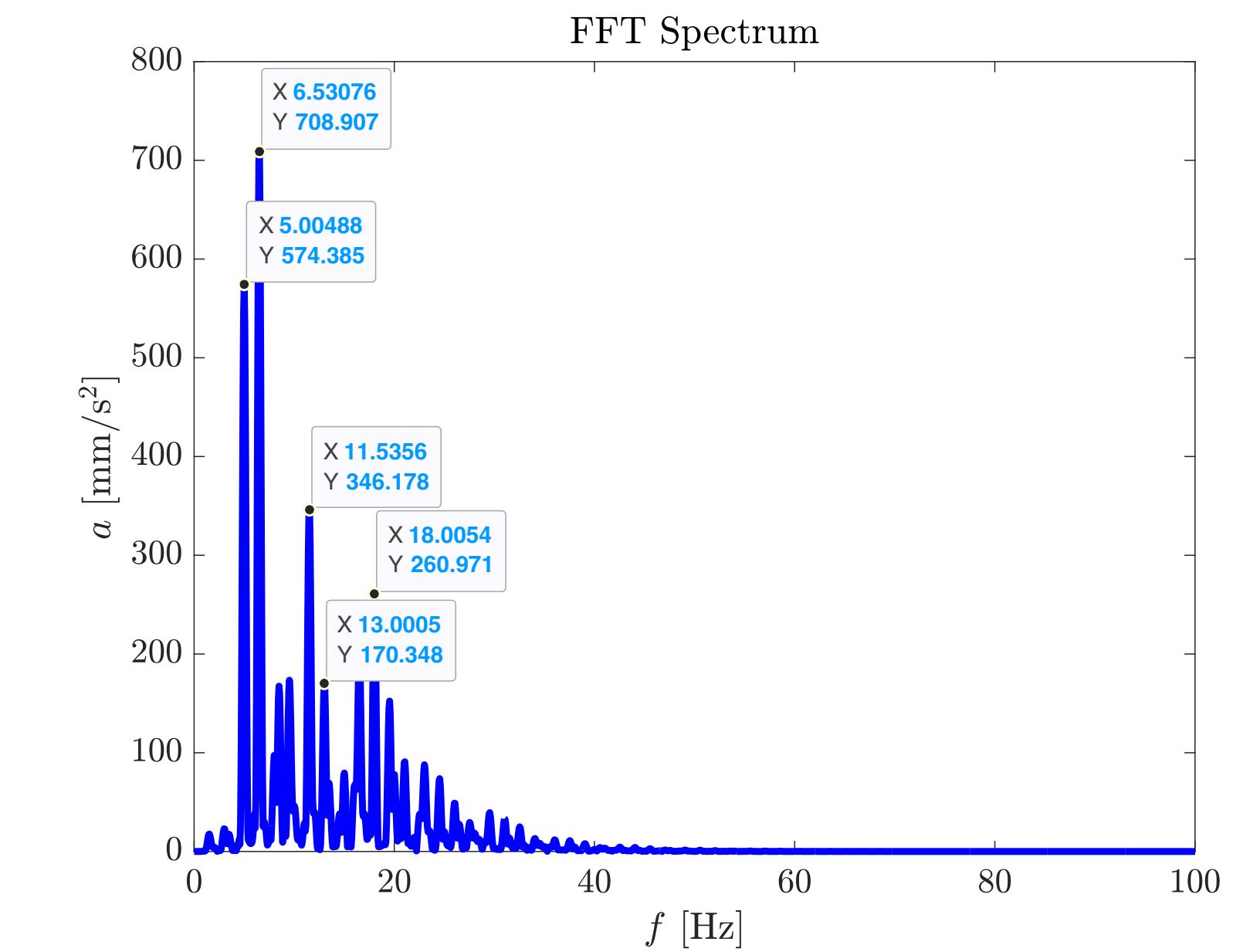


Figure 40 - FFT Spectrum for Case 9

NLA2 Case 10 ($A = 0.5, f = 7$) - Time Domain

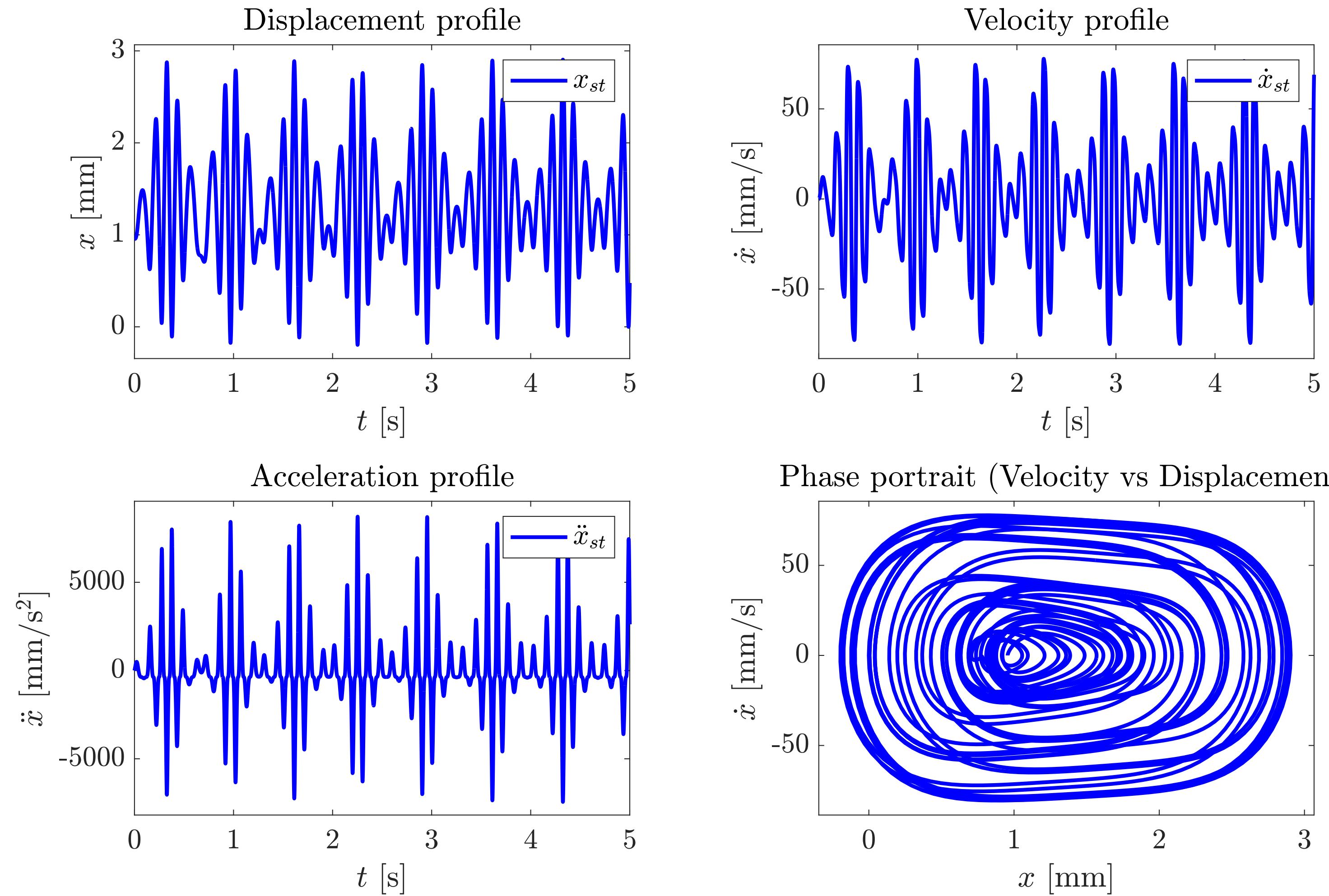


Figure 41 - Time domain results for Case 10

NLA2 Case 10 - Physical and Frequency Domain

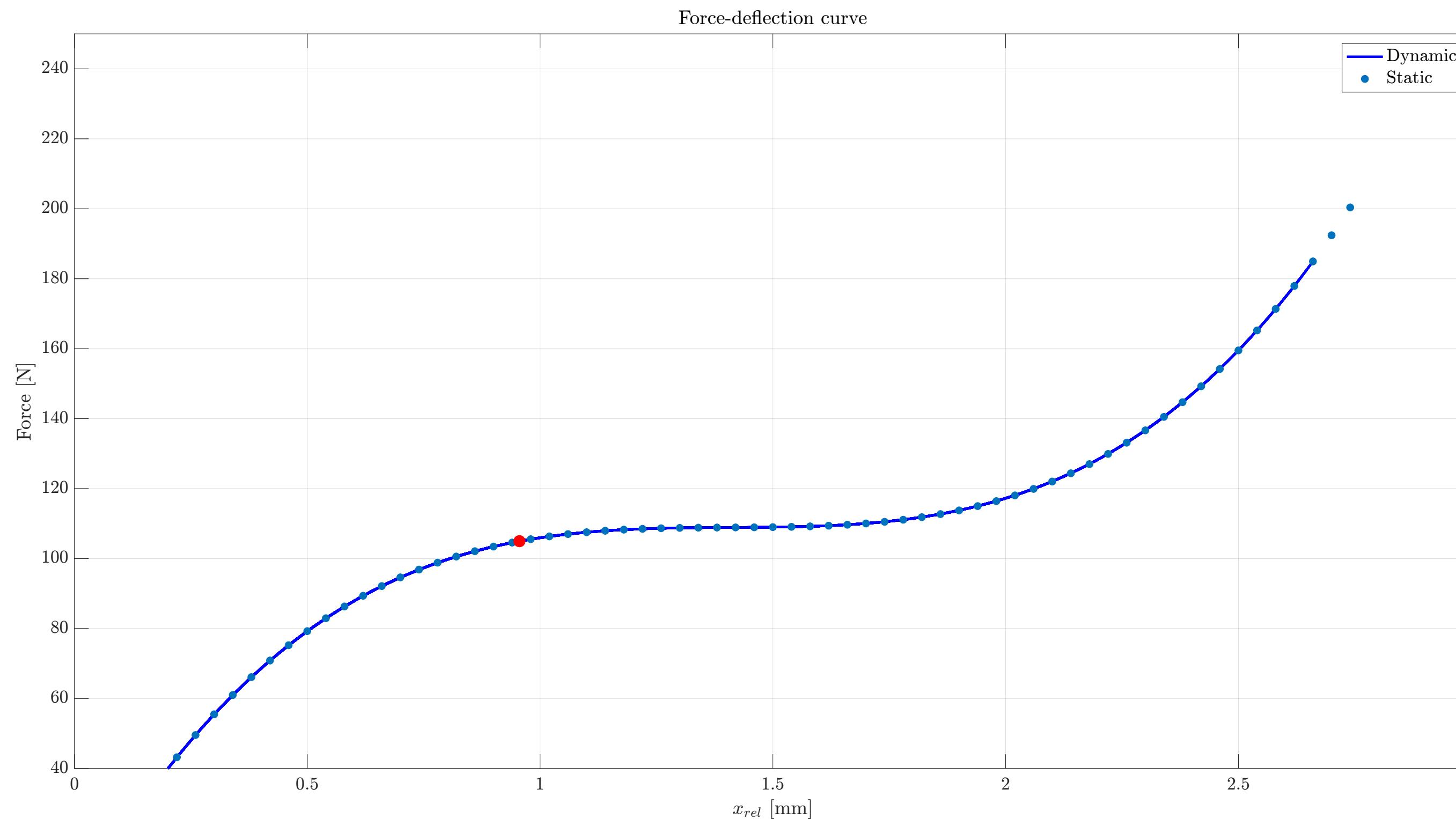


Figure 42 - Physical domain behavior for Case 10

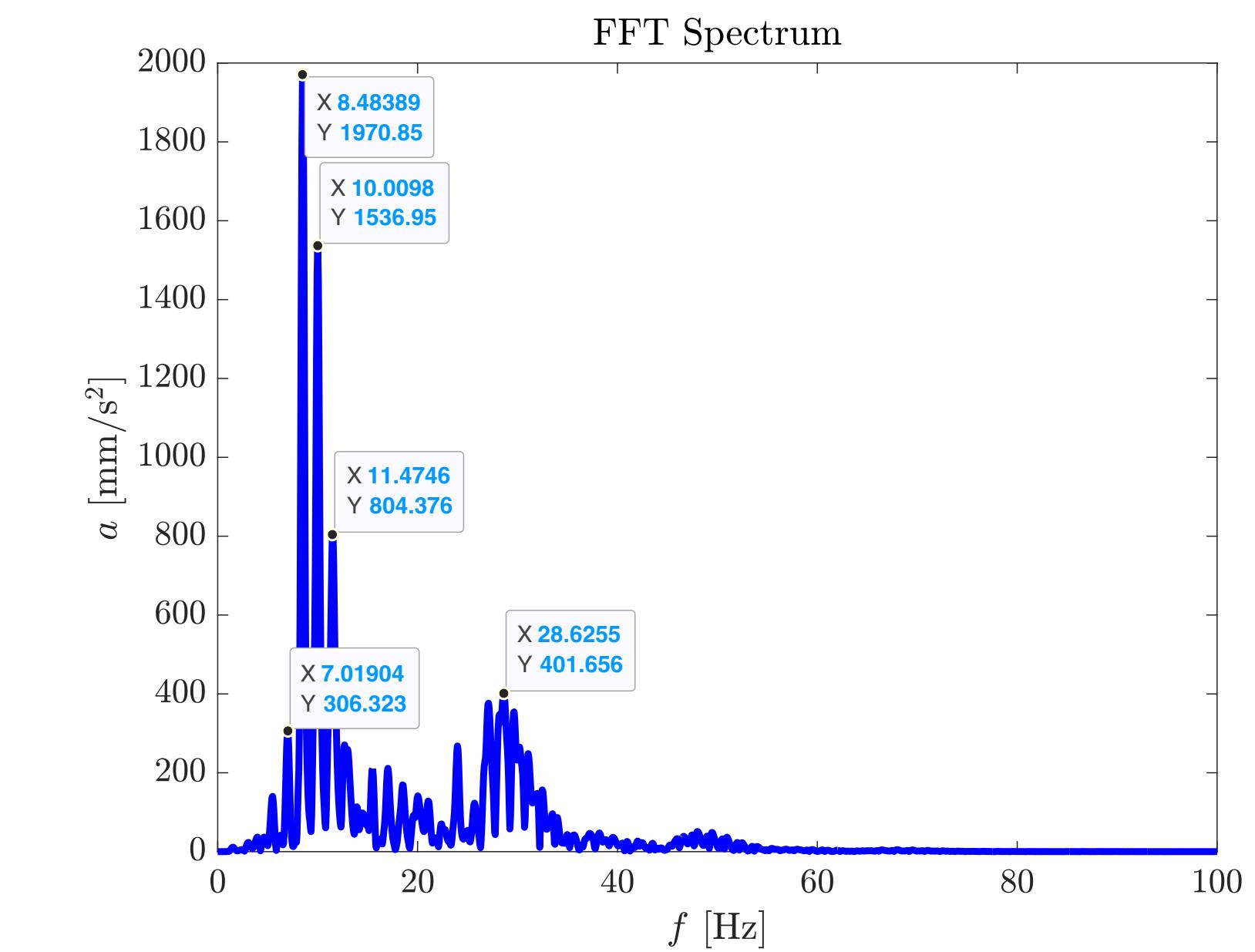


Figure 43 - FFT Spectrum for Case 10

NLA2 Case 11 ($A = 0.5, f = 8$) - Time Domain

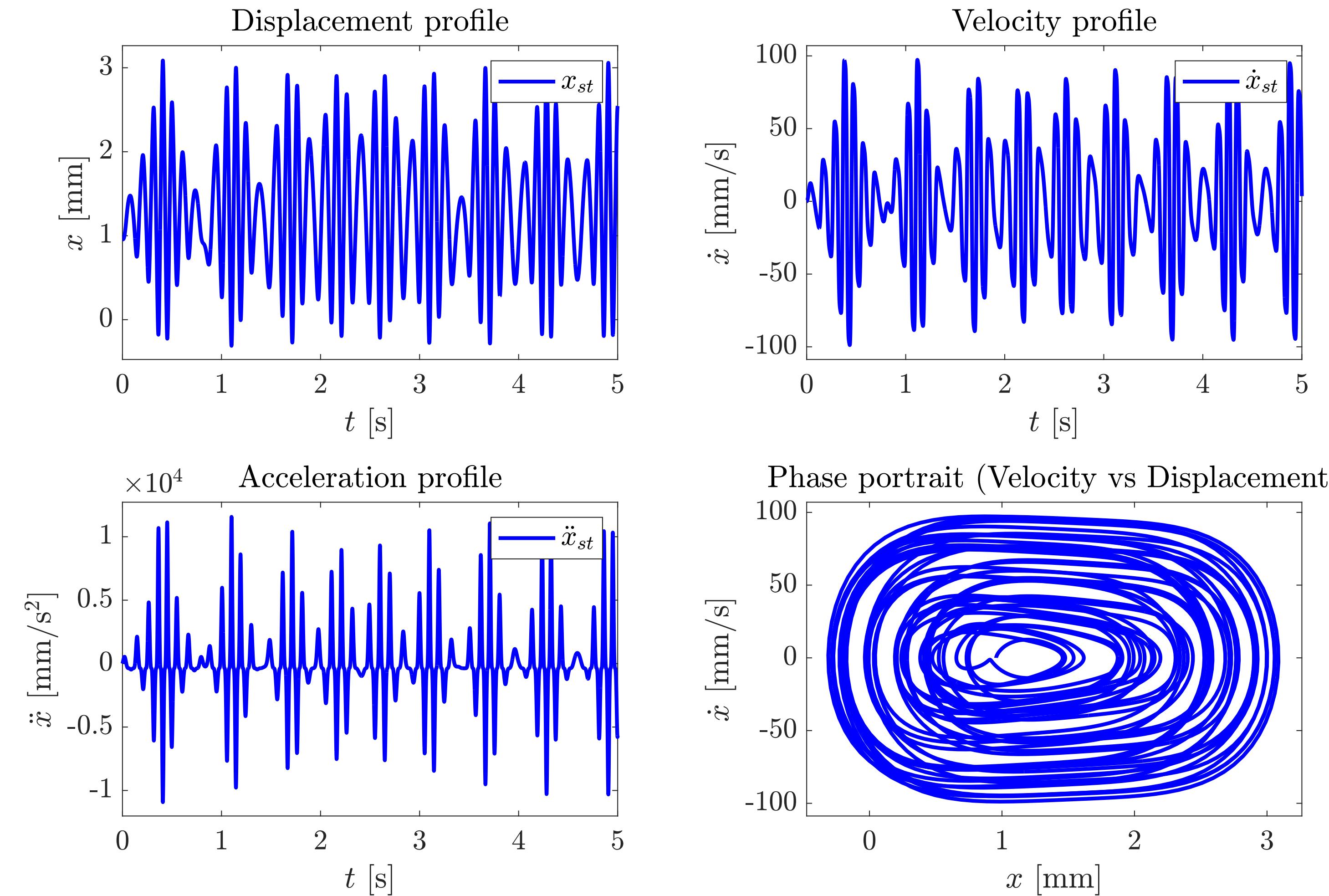


Figure 44 - Time domain results for Case 11

NLA2 Case 11 - Physical and Frequency Domain

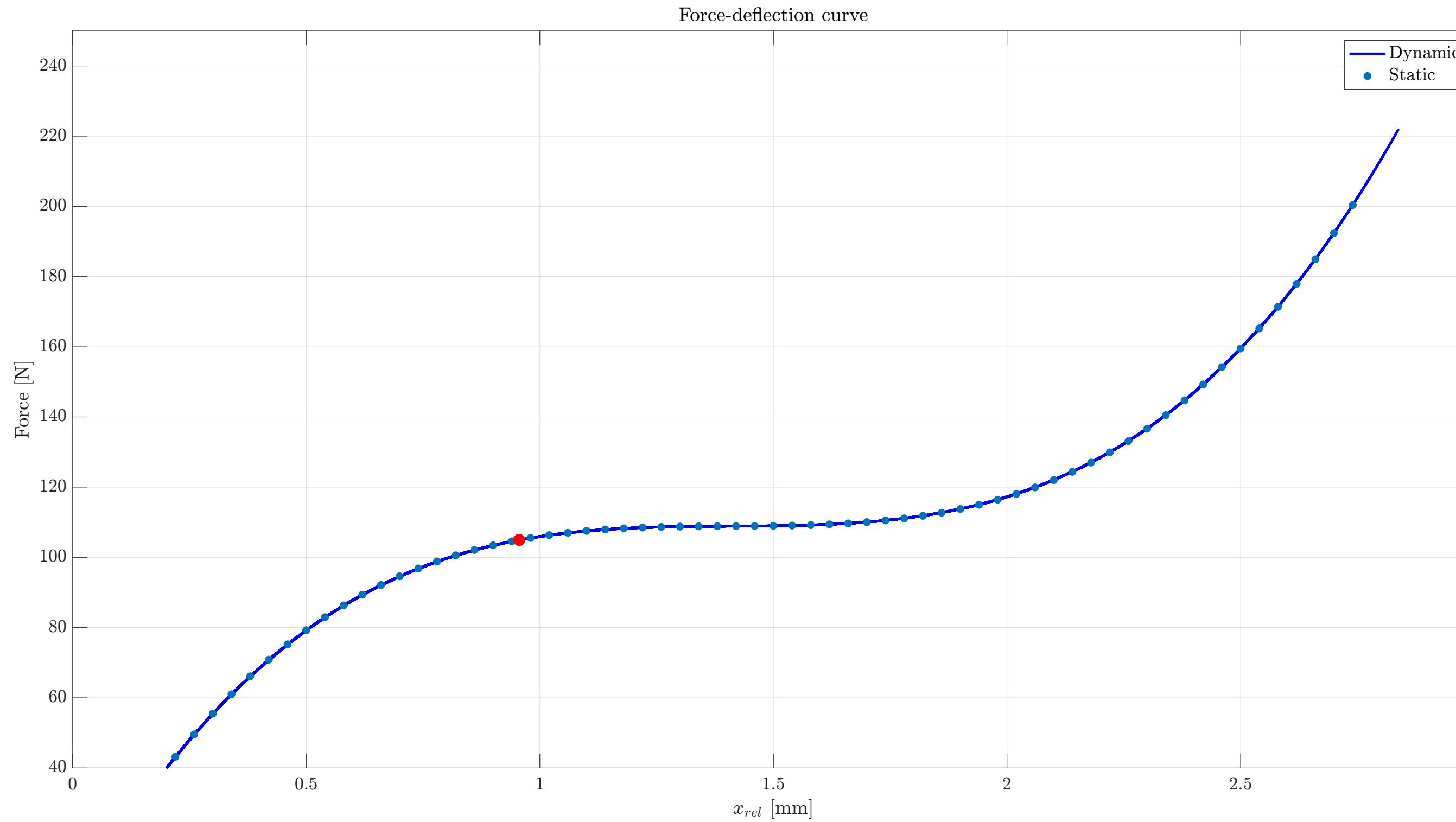


Figure 45 - Physical domain behavior for Case 11

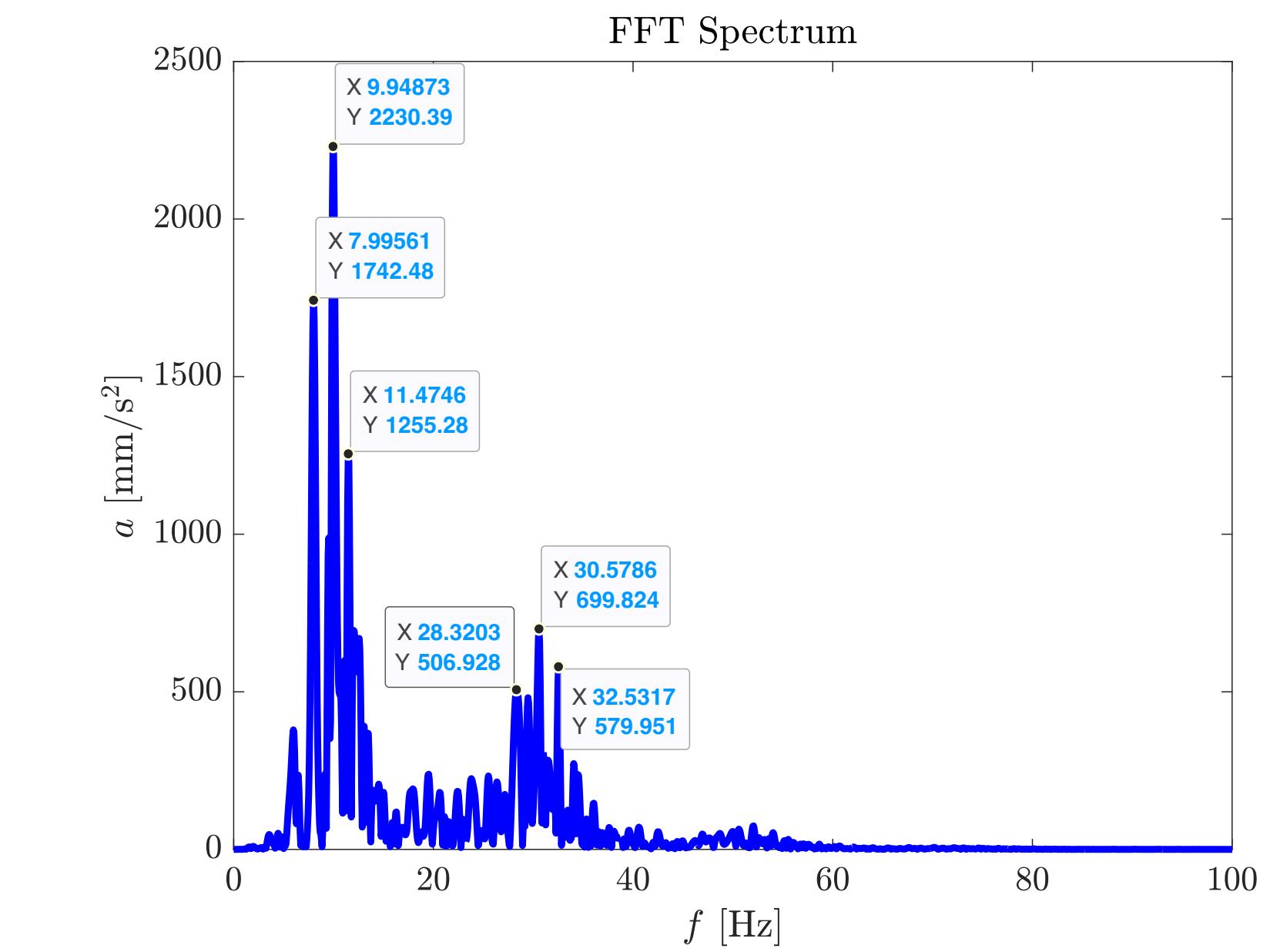


Figure 46 - FFT Spectrum for Case 11

NLA2 Case 12 ($A = 0.5, f = 10$) - Time Domain

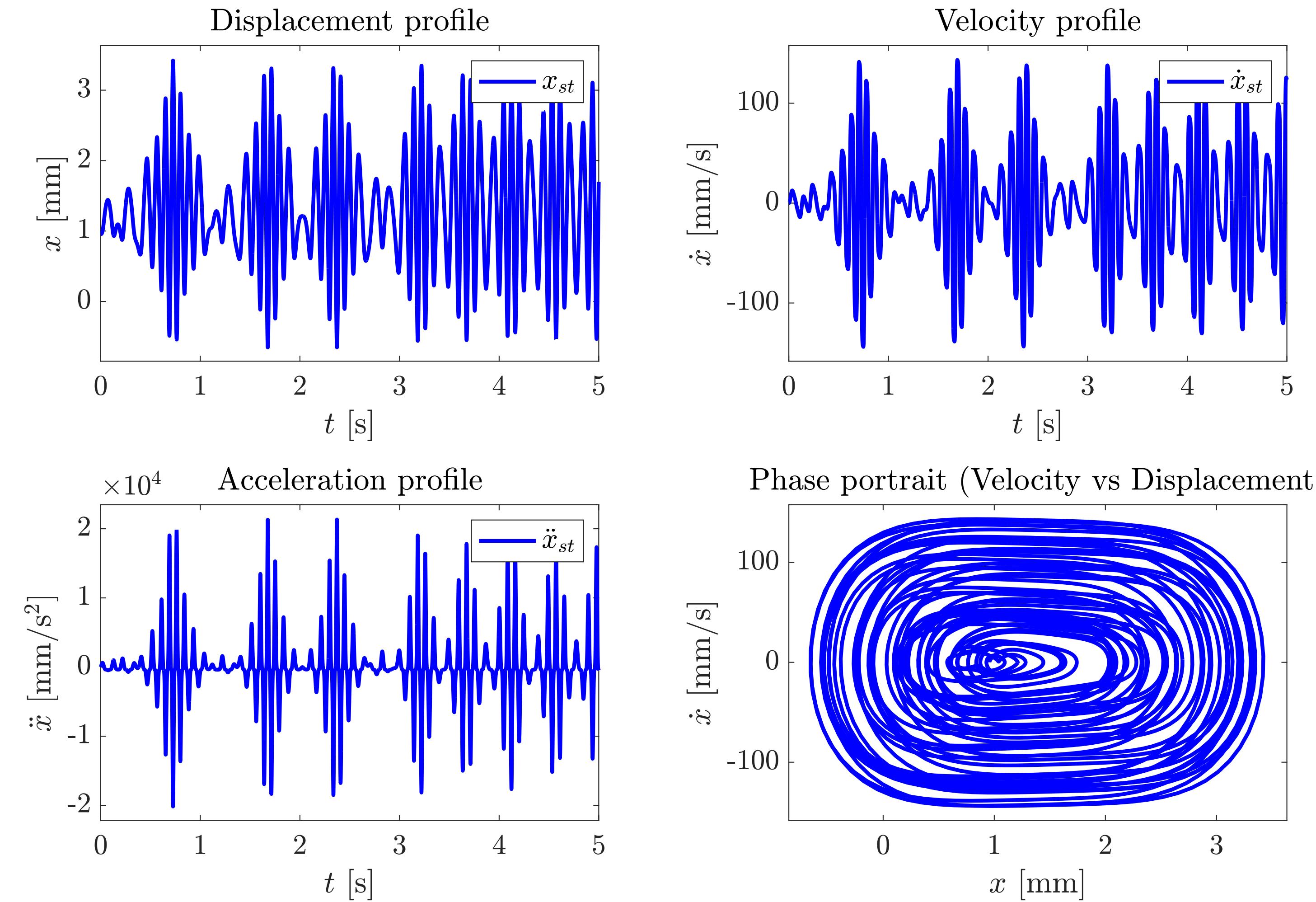


Figure 47 - Time domain results for Case 12

NLA2 Case 12 - Physical and Frequency Domain

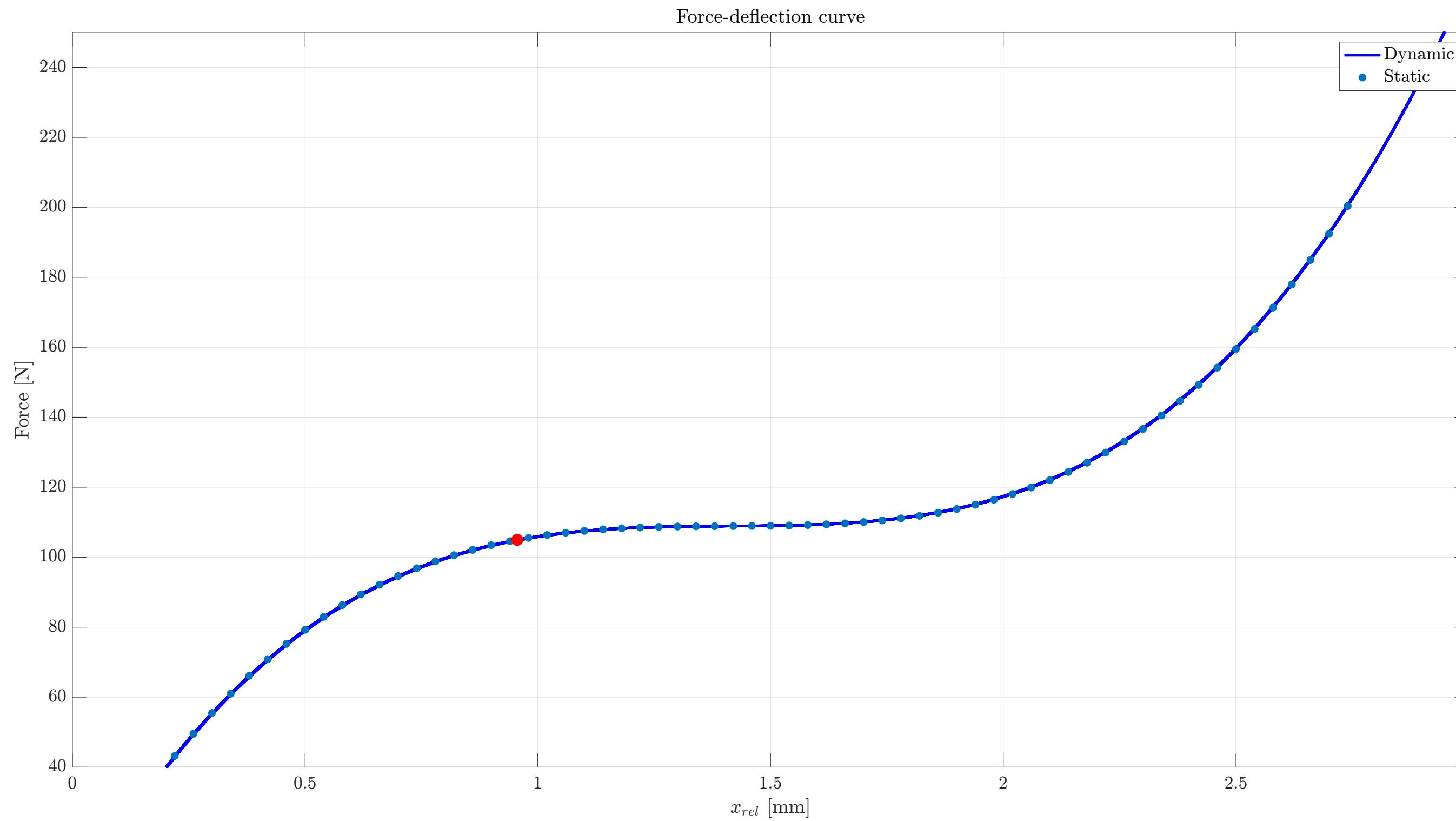


Figure 48 - Physical domain behavior for Case 12

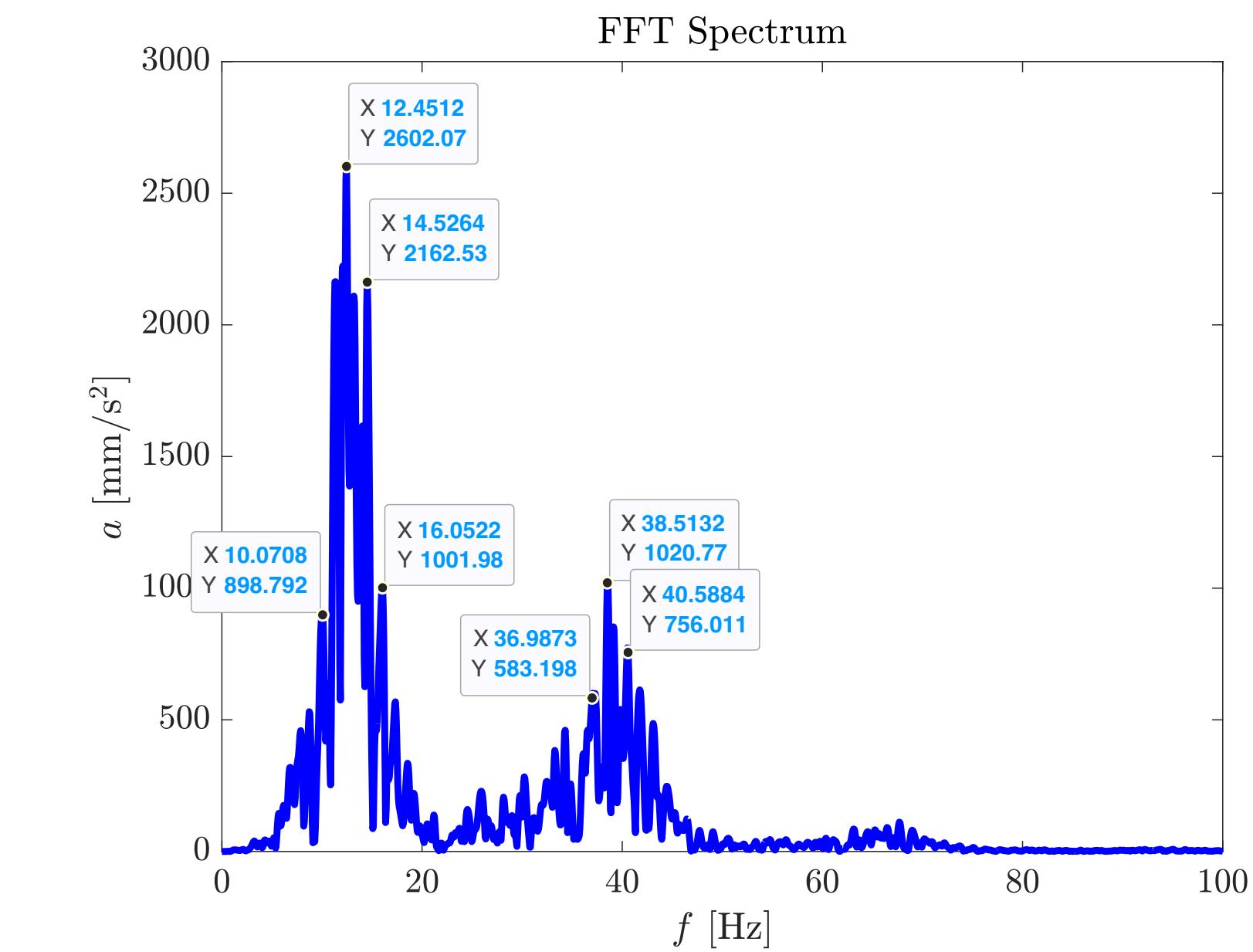


Figure 49 - FFT Spectrum for Case 12

Future Work

- Perform similar analysis for other spring combinations
- Continue working on in-depth investigation of dynamics