



RIYA Week 6 Presentation

**Study of the Non-Linear Dynamics of a 2-spring
stack in the presence of Damping**

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Tasks Accomplished

- Obtained **non-linear time-domain** and **frequency domain** results for the two-spring stack system **with damping** under different **base excitation conditions** for **equilibrium ICs** in the **QZS** and **Non-QZS** regimes.

Calculation of Damping Coefficient

Assumed damping ratio, $\zeta = 0.05$

The approximate value of damping coefficient is given by $c = 2\zeta\sqrt{mk}$

In the **Non-QZS regime** ($m = 10.7$ kg, $k = 24.5$ N/mm)

$$c_{NQZS} \approx 2(0.05)\sqrt{(10.7)(24500)} \approx 51.2 \text{ Ns/m}$$

In the **QZS regime** ($m = 11.1$ kg, $k = 0.7$ N/mm)

$$c_{QZS} \approx 2(0.05)\sqrt{(11.1)(700)} \approx 8.81 \text{ Ns/m}$$

The **natural frequencies** of the linearized system about the operating point in the **Non-QZS** and **QZS regimes** are **7.62 Hz** and **1.26 Hz** respectively

Non-Linear Analysis in the Non-QZS Regime

Parameters : $h_1/\tau = h_2/\tau = 1.41$, $x_{base}(t) = A \sin(2\pi ft)$ (in mm)

A is the amplitude (mean to peak) and f is the frequency of the base excitation

Amplitude (peak to peak)	
	0.1 mm
5 Hz	Case 1
7 Hz	Case 2
8 Hz	Case 3
9 Hz	Case 4
11 Hz	Case 5
13 Hz	Case 6

Non QZS Case 1 ($f = 5$) - Time Domain

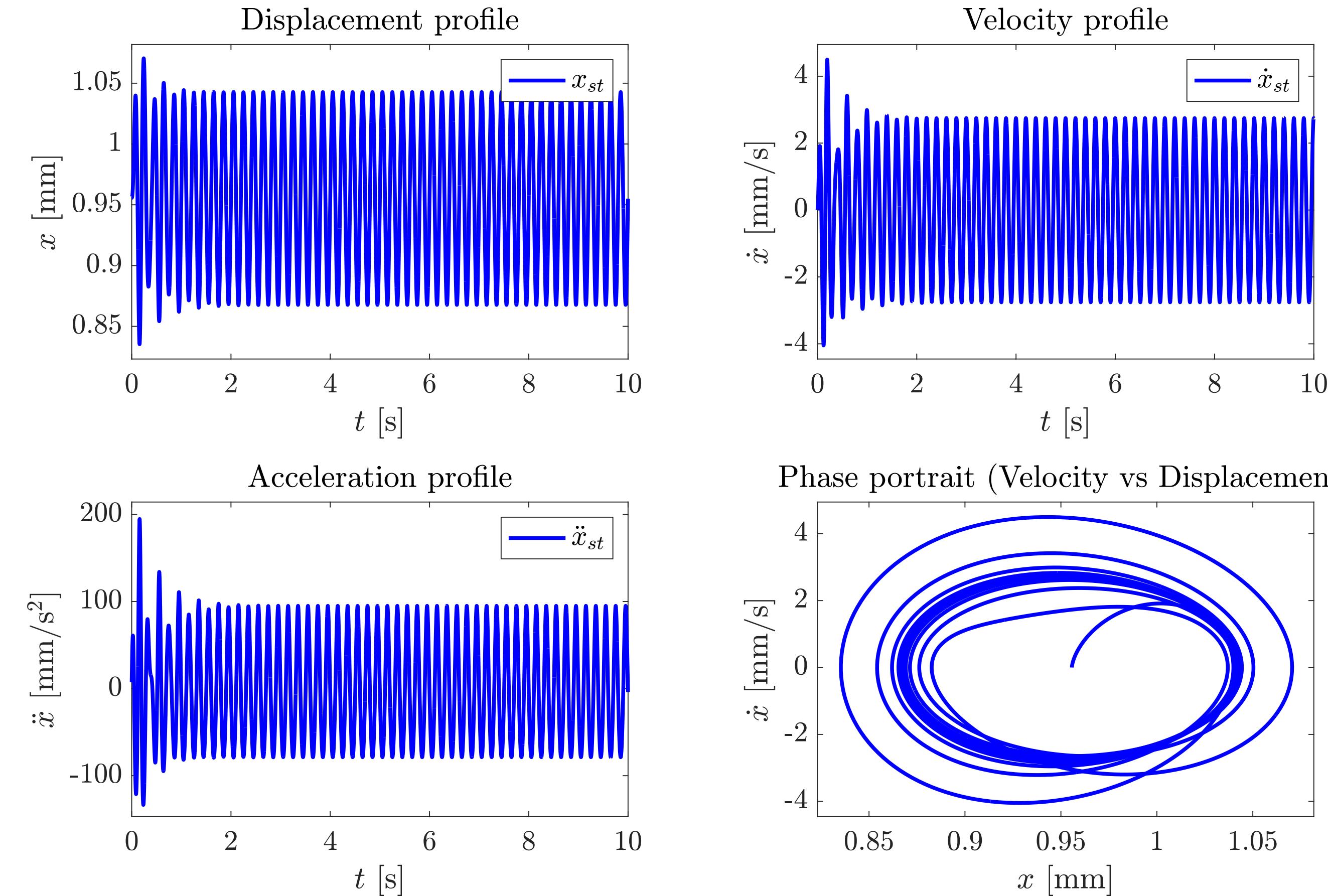


Figure 1 - Time domain results for Case 1

Non QZS Case 1 - Physical and Frequency Domain

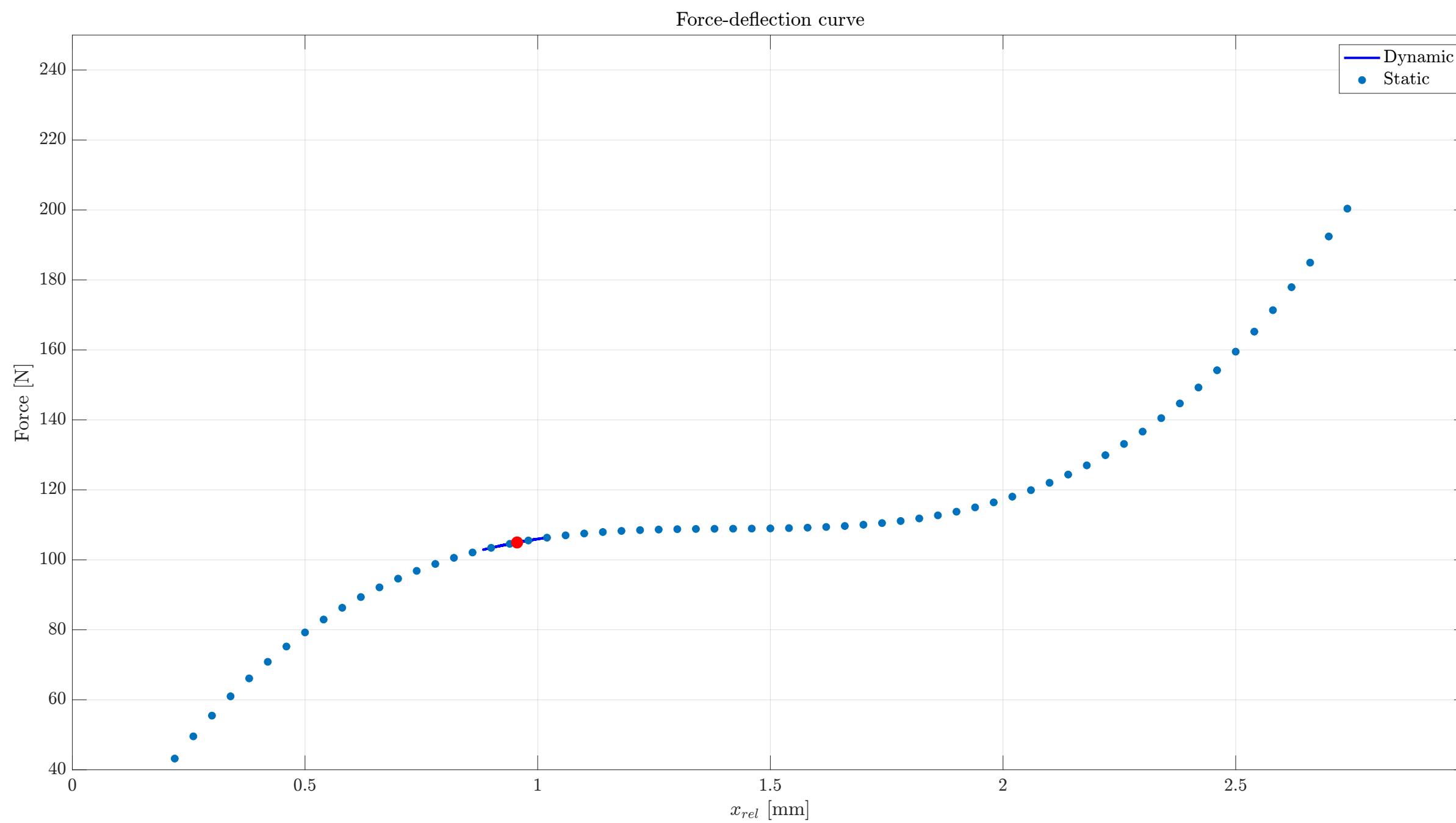


Figure 2 - Physical domain behavior for Case 1

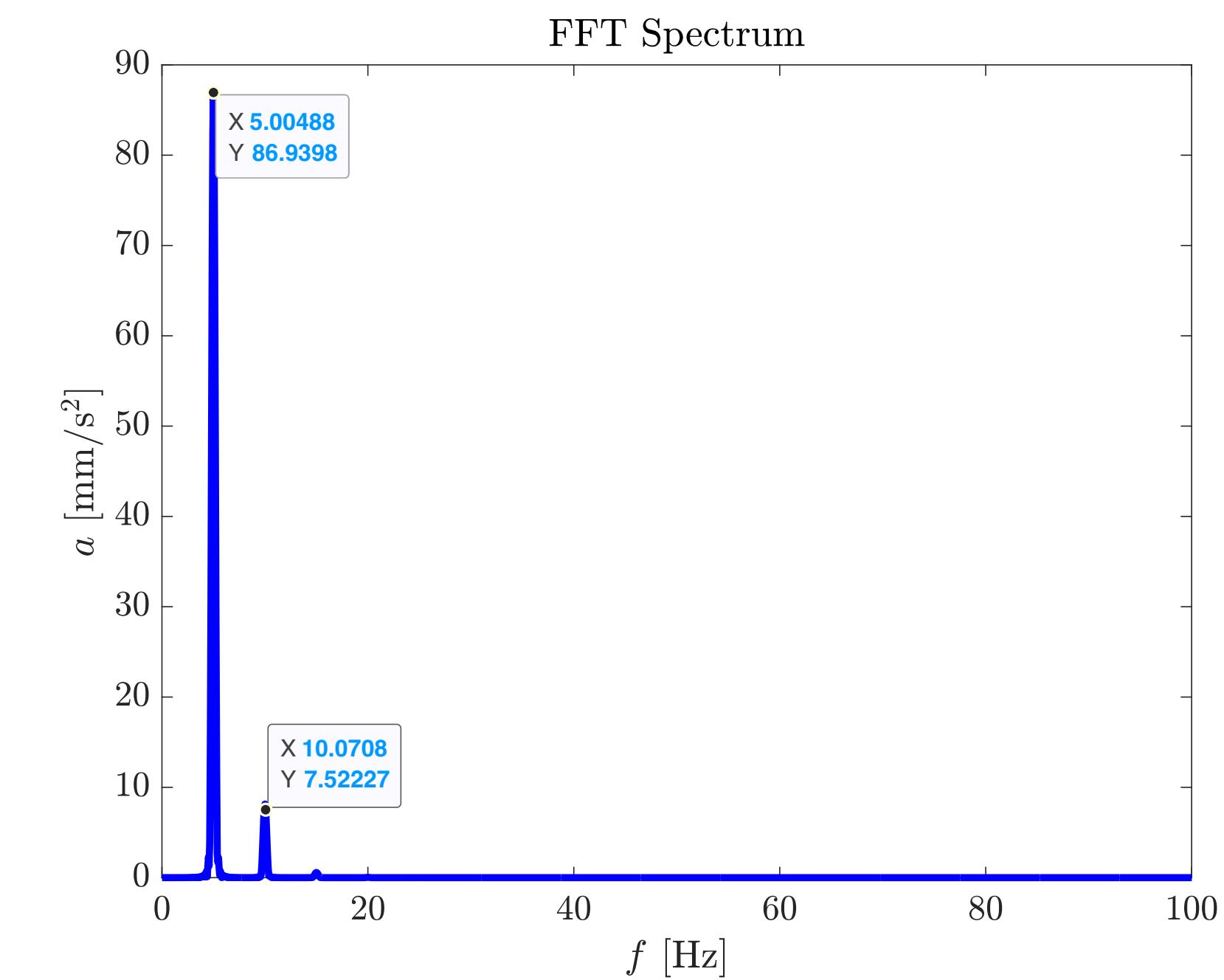


Figure 3 - FFT Spectrum for Case 1

Super harmonics corresponding to **driving frequency** present

Non QZS Case 2 ($f = 7$) - Time Domain

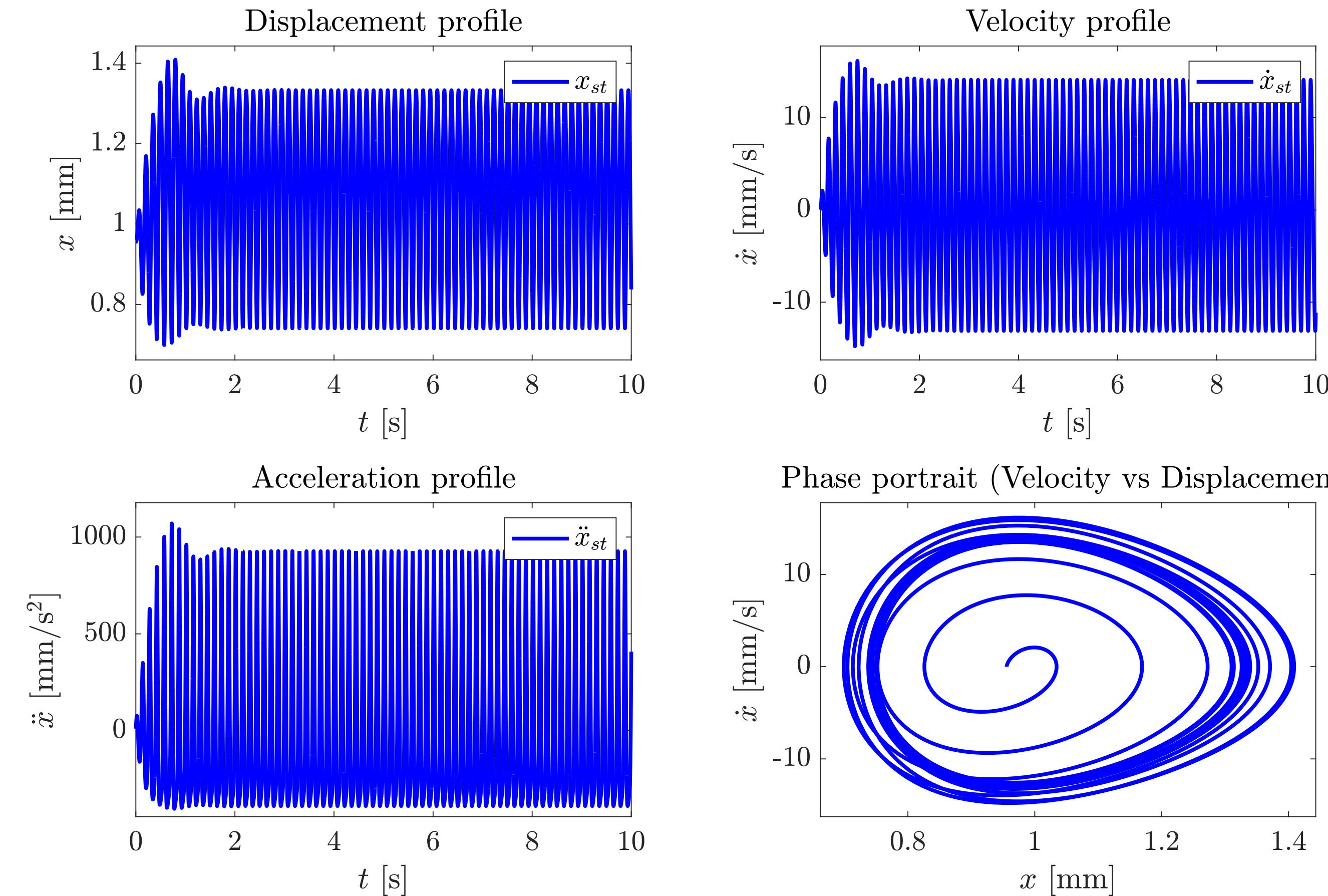


Figure 4 - Time domain results for Case 2

Non QZS Case 2 - Physical and Frequency Domain

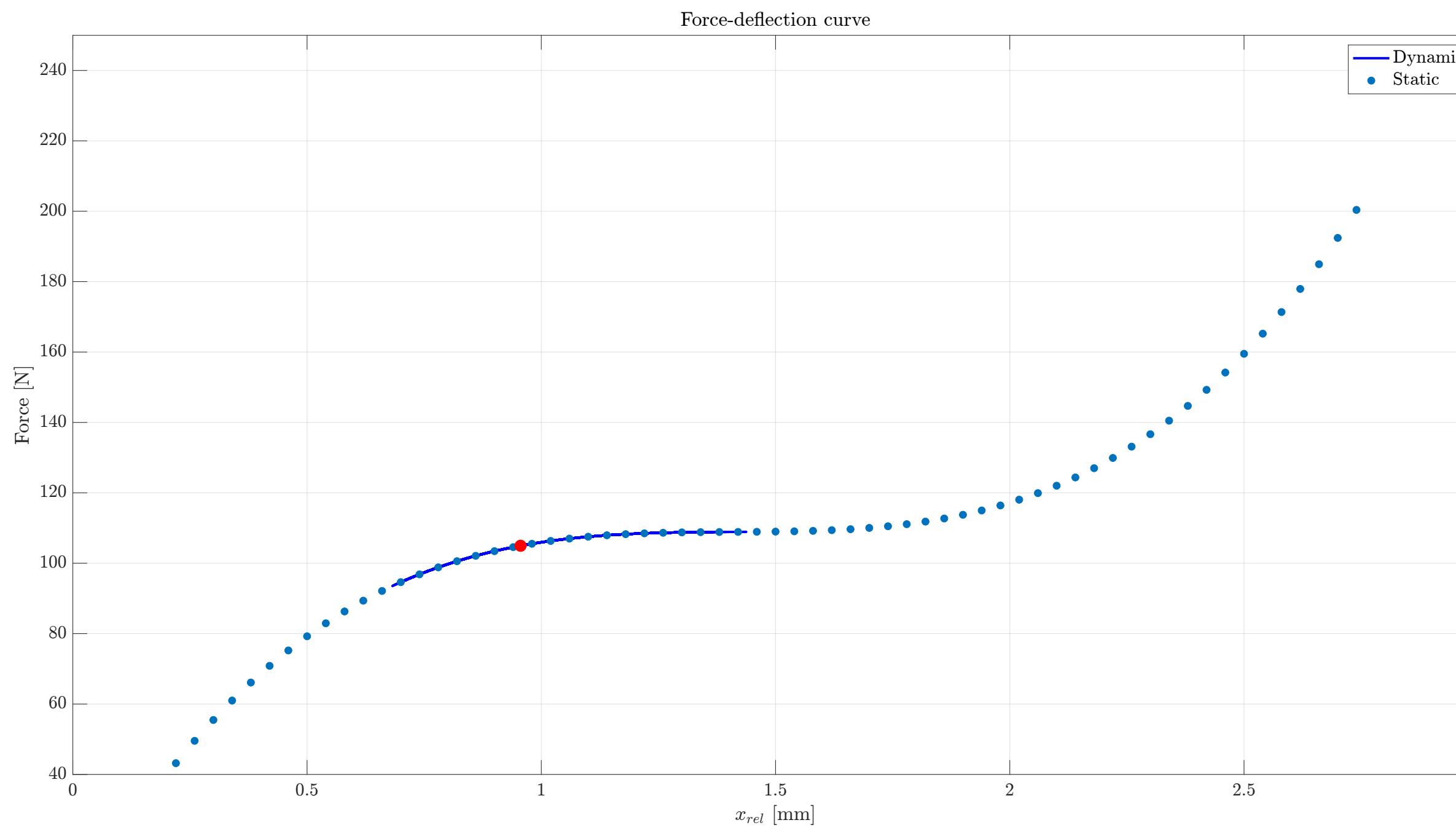


Figure 5 - Physical domain behavior for Case 2

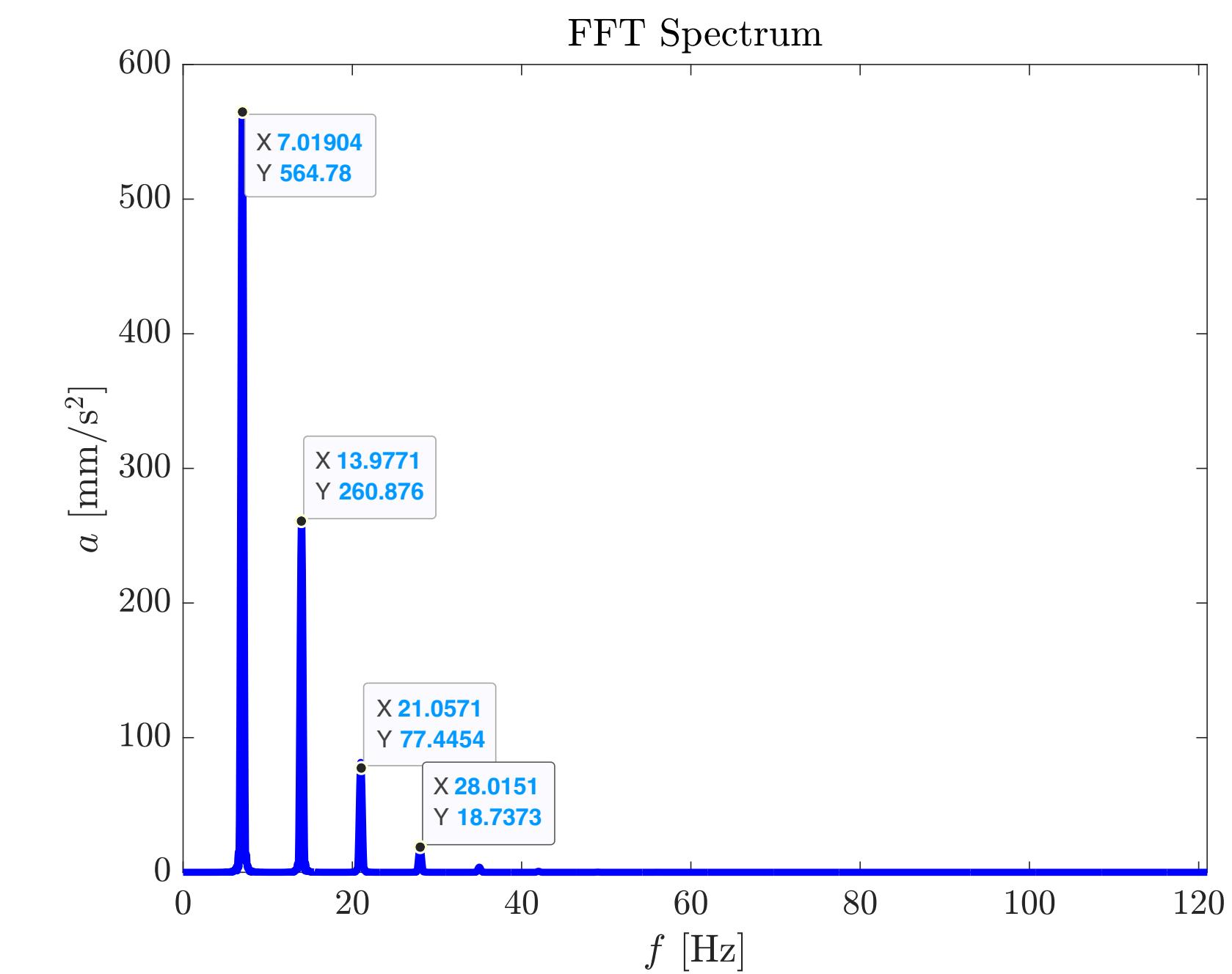


Figure 6 - FFT Spectrum for Case 2

Many peaks corresponding to super harmonics !

Non QZS Case 3 ($f = 8$) - Time Domain

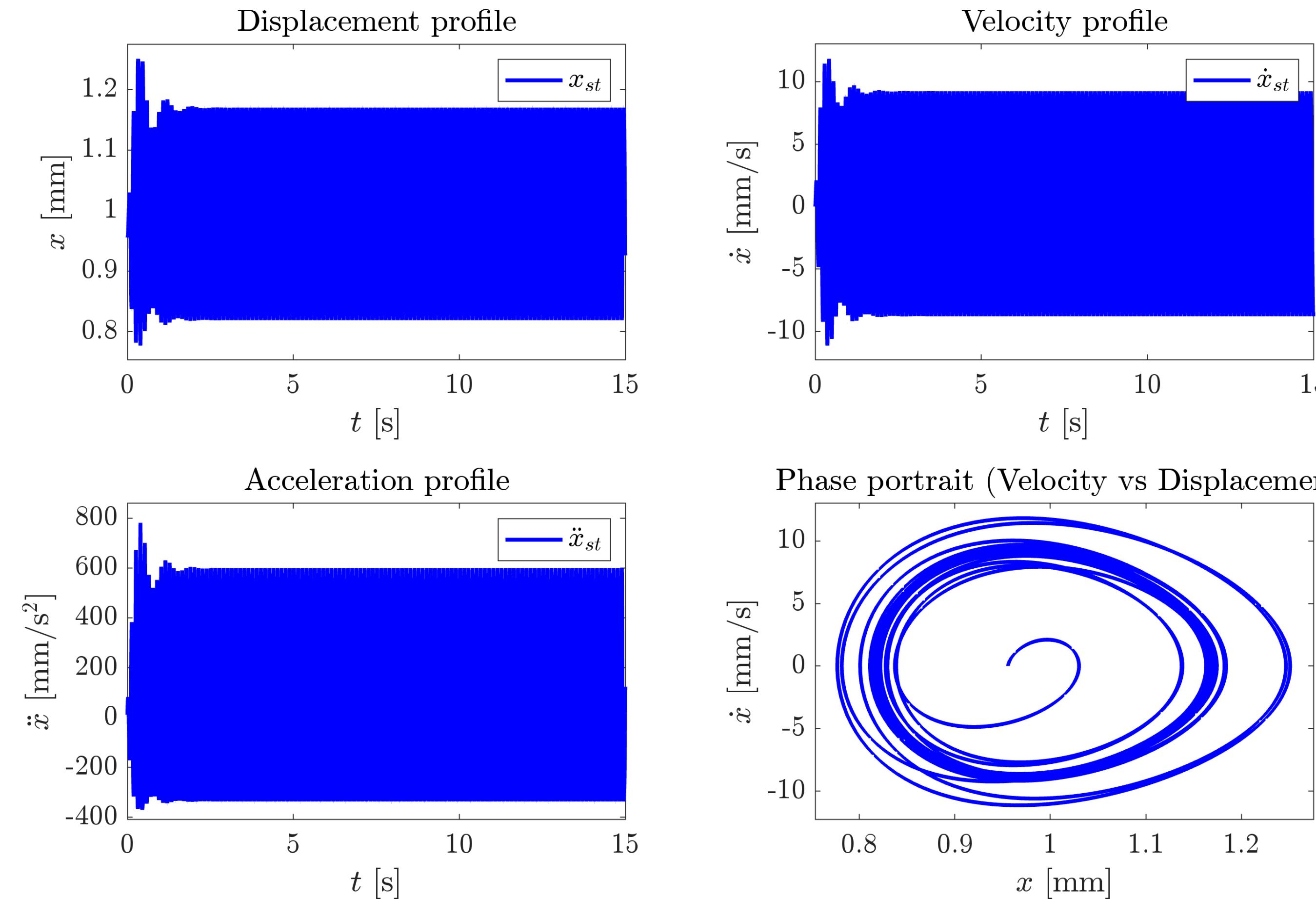


Figure 7 - Time domain results for Case 3

Non QZS Case 3 - Physical and Frequency Domain

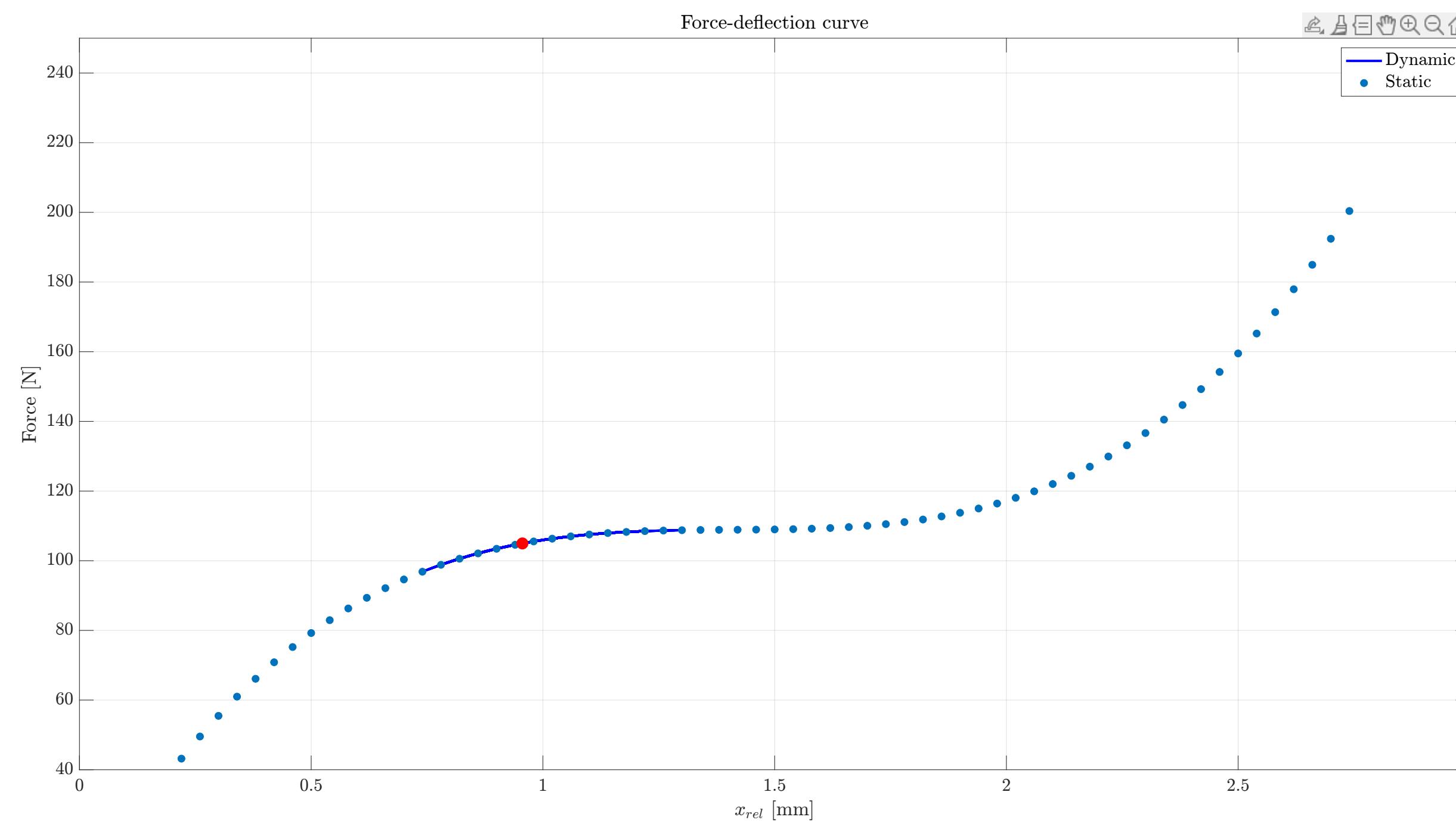


Figure 8 - Physical domain behavior for Case 3

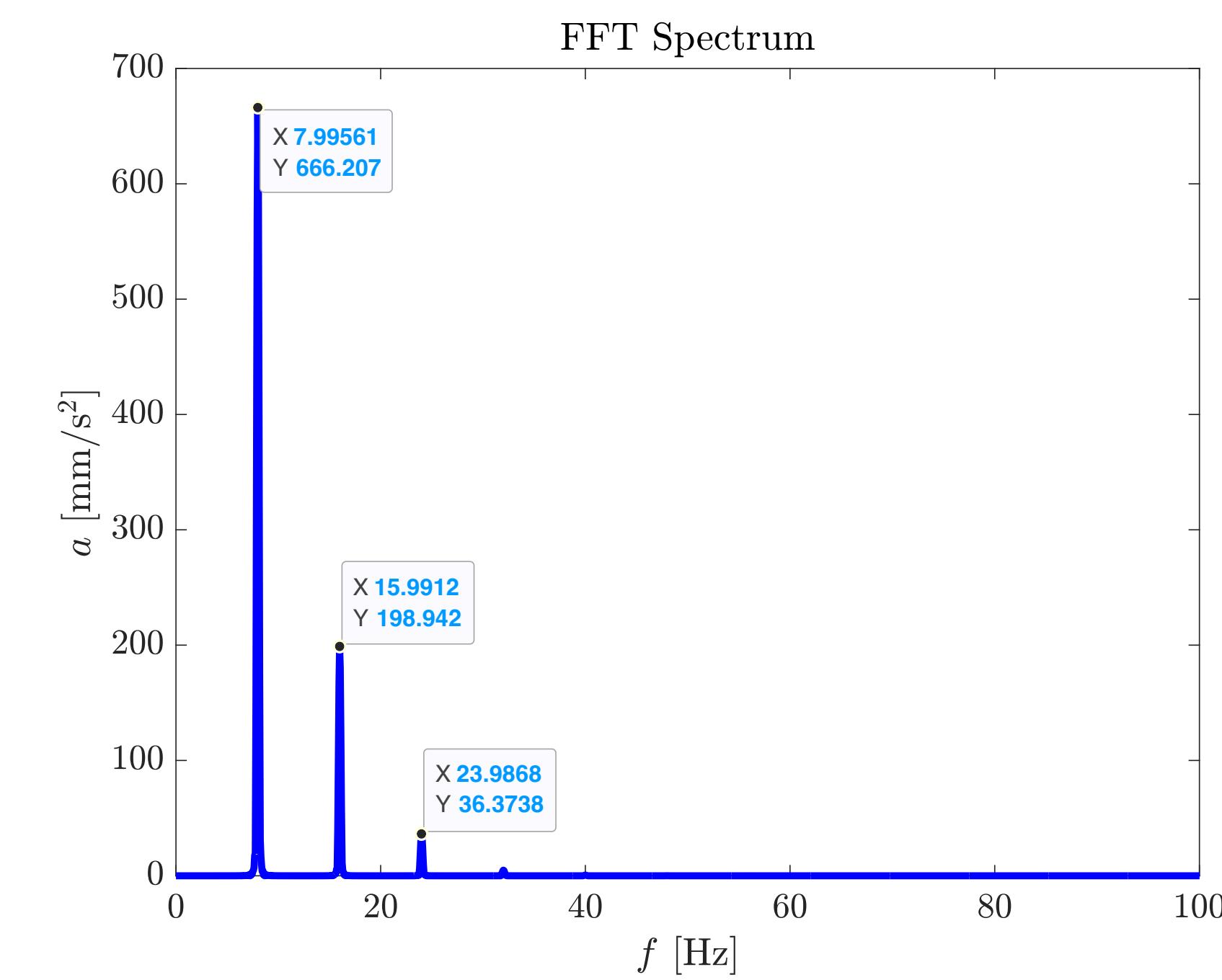


Figure 9 - FFT Spectrum for Case 3

Non QZS Case 6 ($f = 13$) - Time Domain

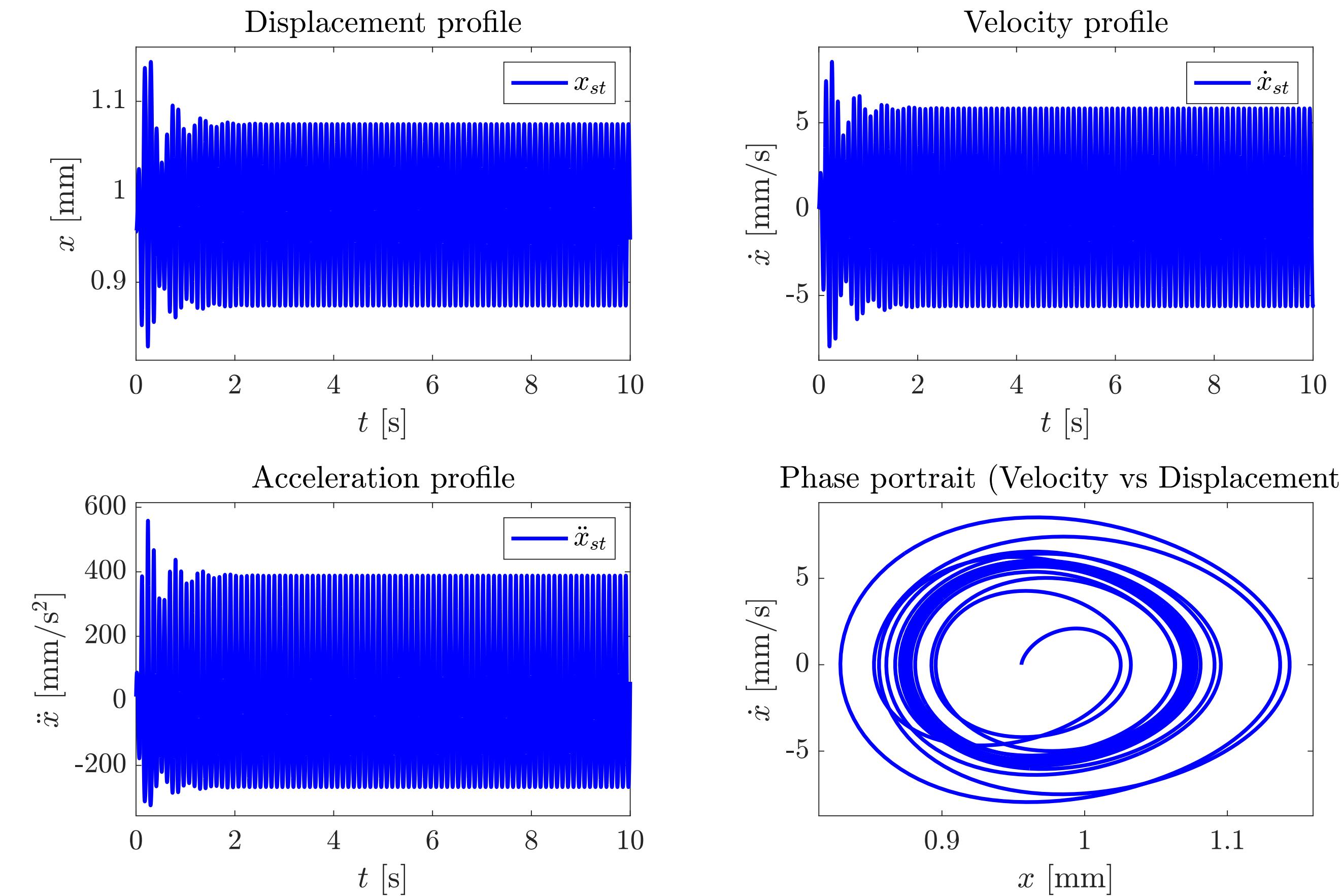


Figure 10 - Time domain results for Case 6

Non QZS Case 6 - Physical and Frequency Domain

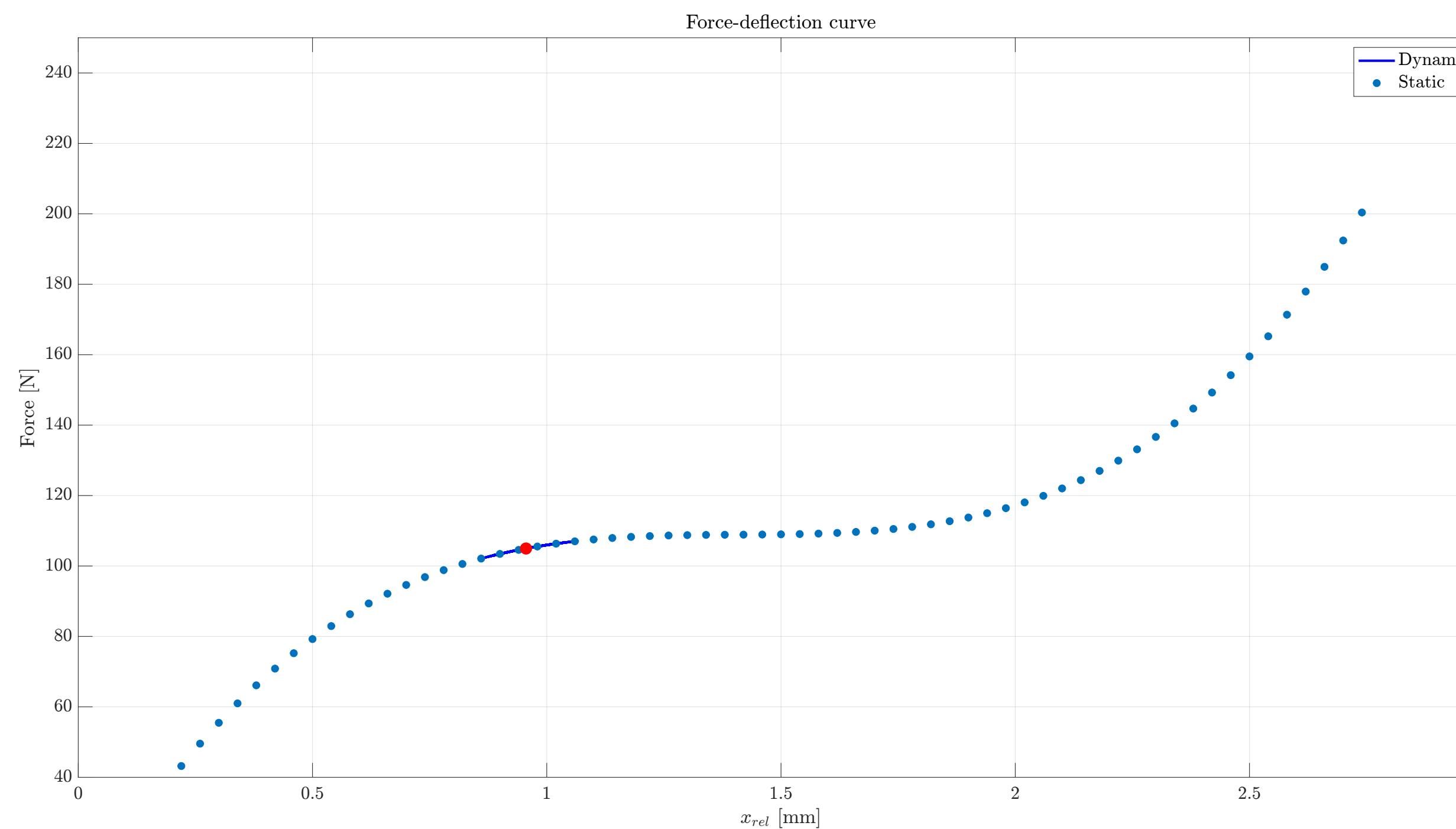


Figure 11 - Physical domain behavior for Case 6

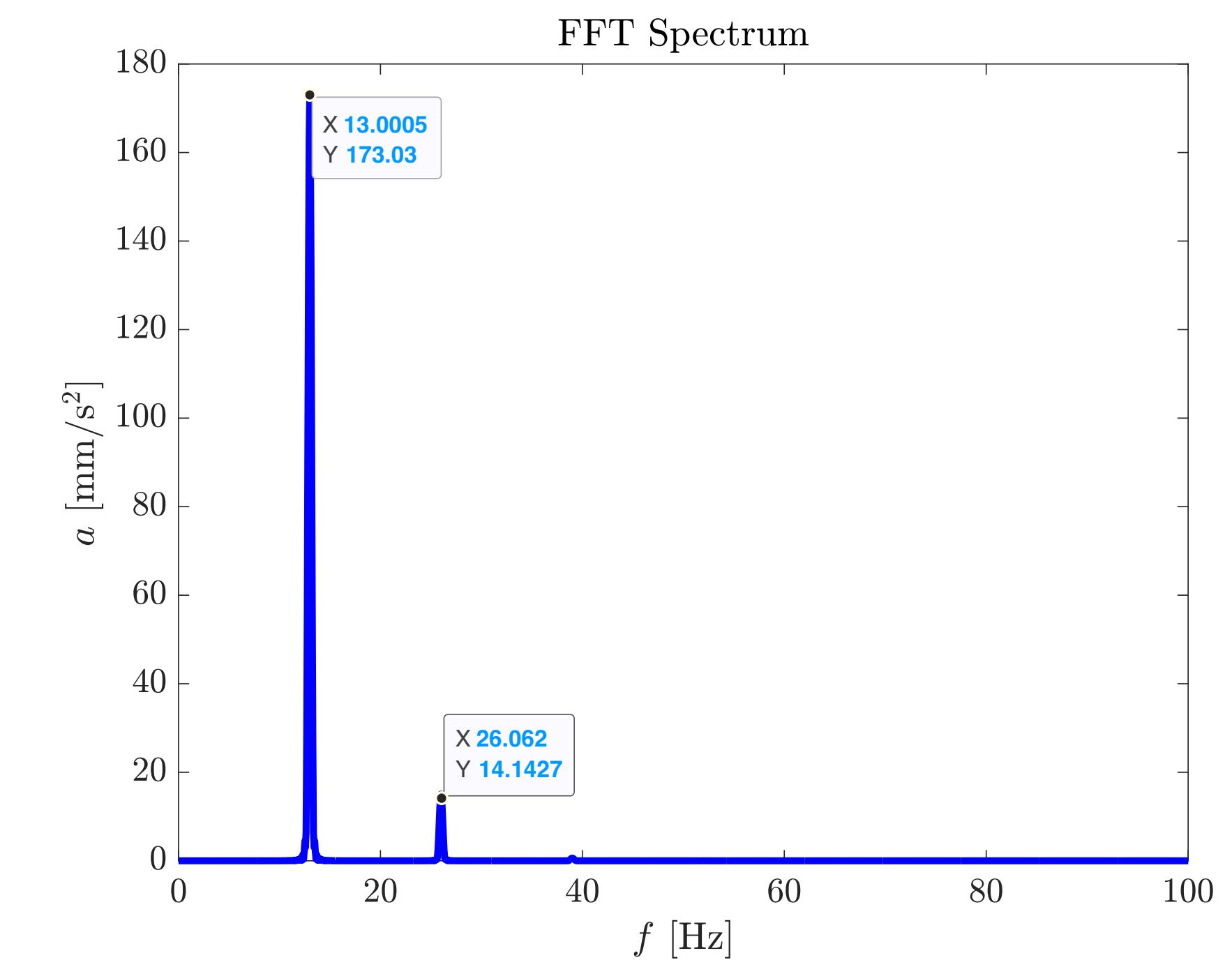


Figure 12 - FFT Spectrum for Case 6

Number of “dominant” peaks reduced ?

Results - Motion Transmissibility (NQZS)

Amplitude (peak to peak)

Frequency	Case Number	Motion Transmissibility (dB)
5 Hz	Case 1	4.91
7 Hz	Case 2	16.65
8 Hz	Case 3	11.37
9 Hz	Case 4	6.23
11 Hz	Case 5	-1.05
13 Hz	Case 6	-5.72

Non-Linear Analysis in the QZS Regime

Parameters : $h_1/\tau = h_2/\tau = 1.41$, $x_{base}(t) = A \sin(2\pi ft)$ (in mm)

A is the amplitude (mean to peak) and f is the frequency of the base excitation

Amplitude (peak to peak)	
Frequency	0.1 mm
1 Hz	Case 1
3 Hz	Case 2
5 Hz	Case 3
7 Hz	Case 4
9 Hz	Case 5

QZS Case 1 ($f = 1$) - Time Domain

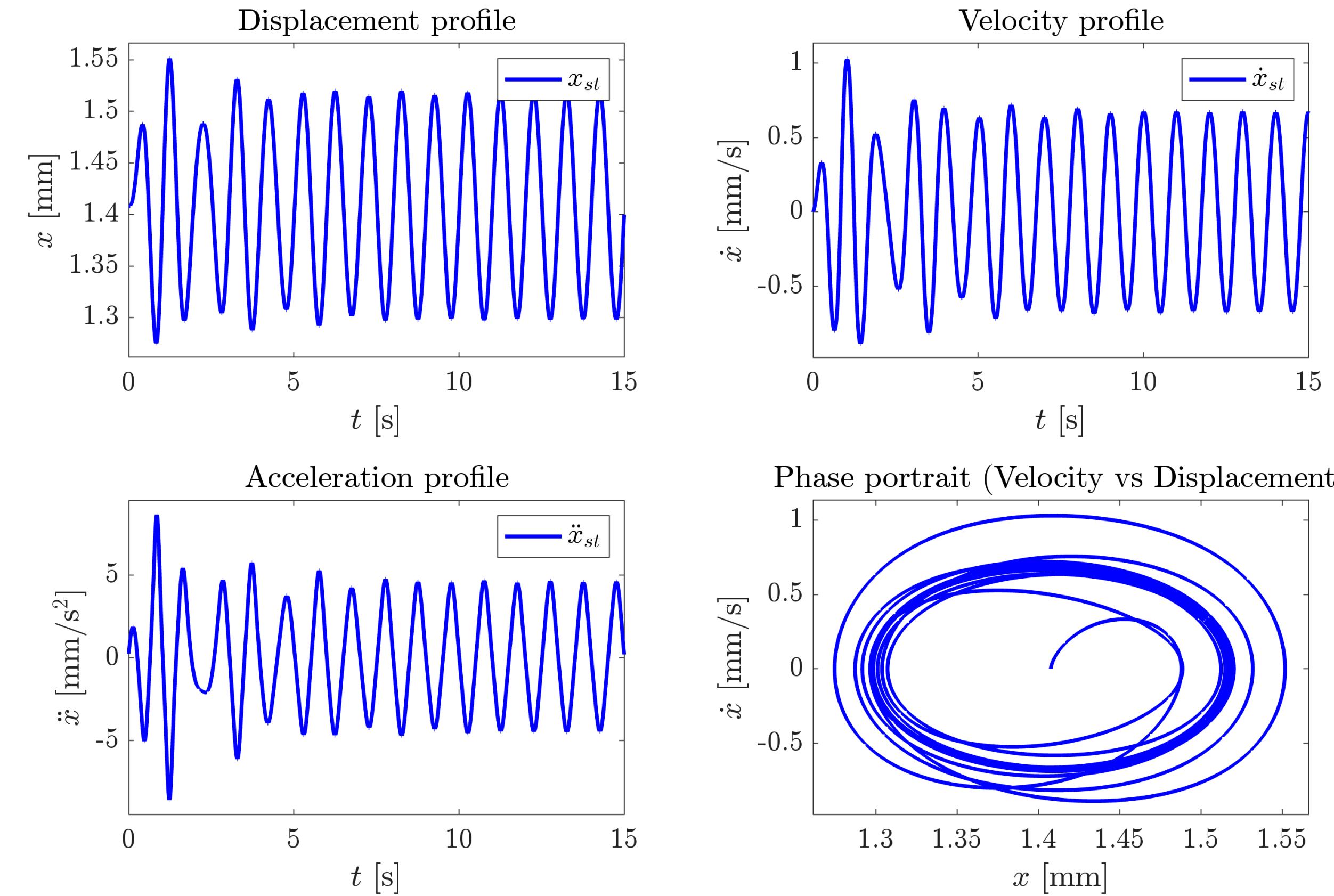


Figure 13 - Time domain results for Case 1

QZS Case 1 - Physical and Frequency Domain

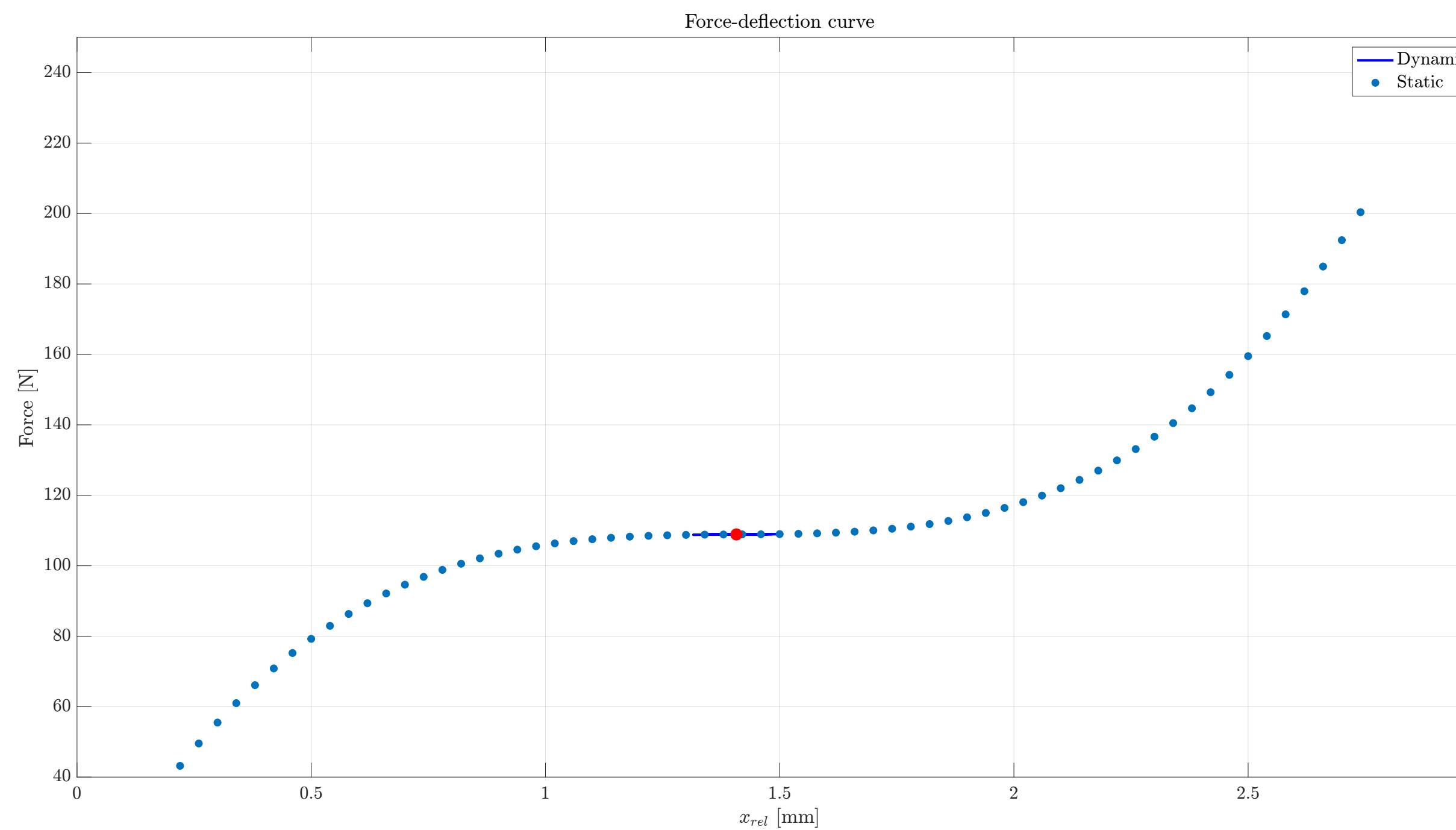


Figure 14 - Physical domain behavior for Case 1

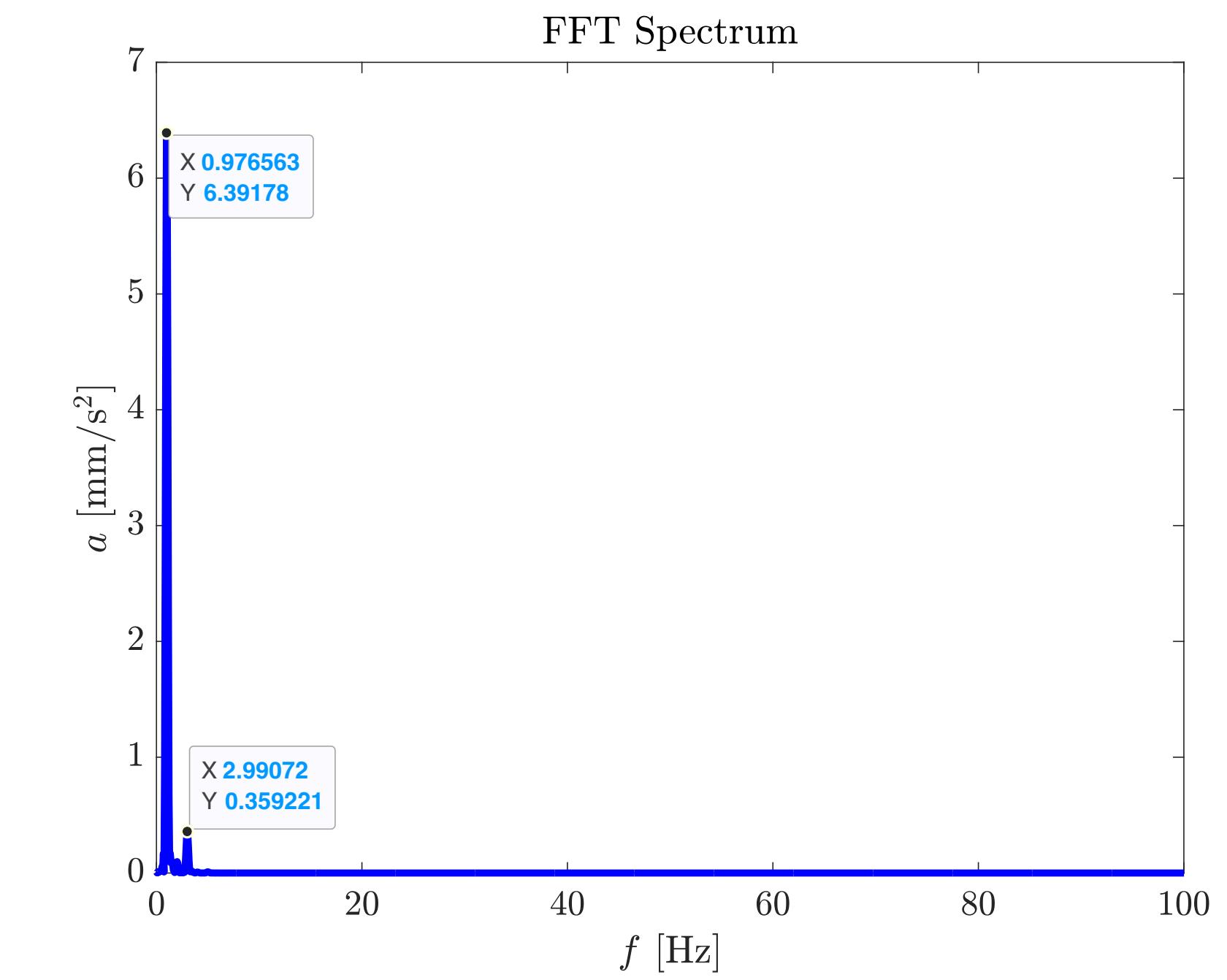


Figure 15 - FFT Spectrum for Case 1

QZS Case 2 ($f = 2$) - Time Domain

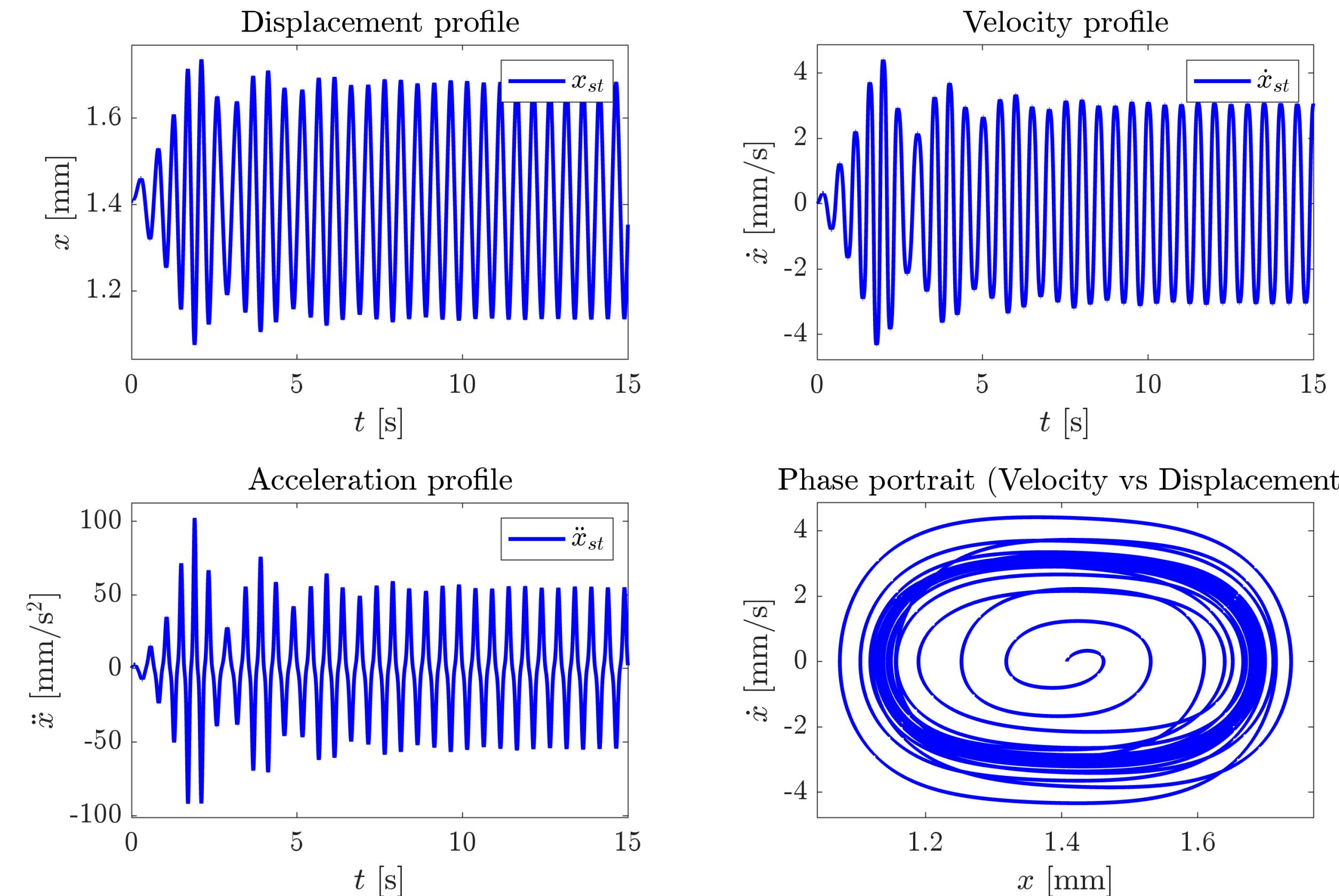


Figure 16 - Time domain results for Case 2

QZS Case 2 - Physical and Frequency Domain

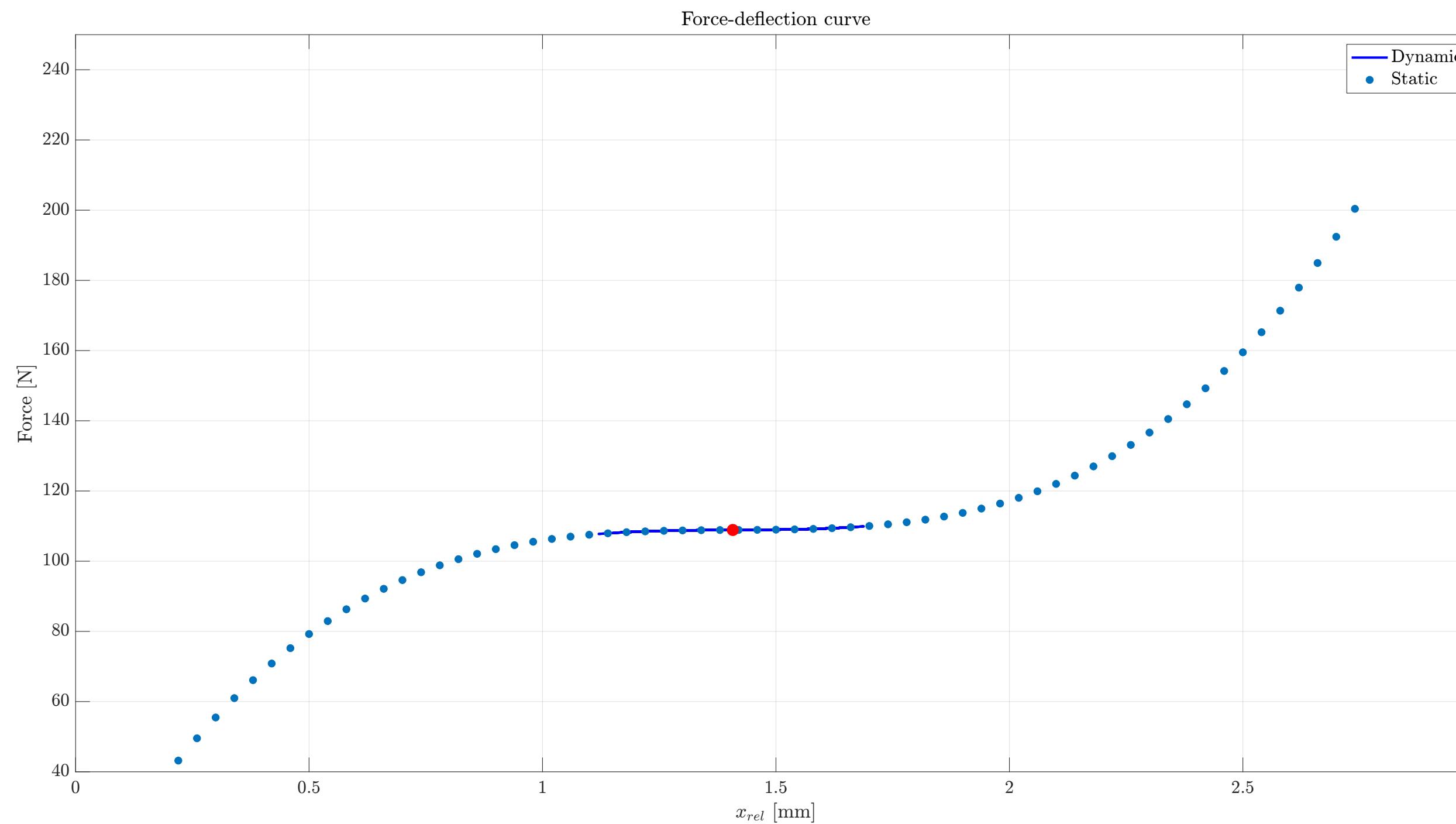


Figure 17 - Physical domain behavior for Case 2

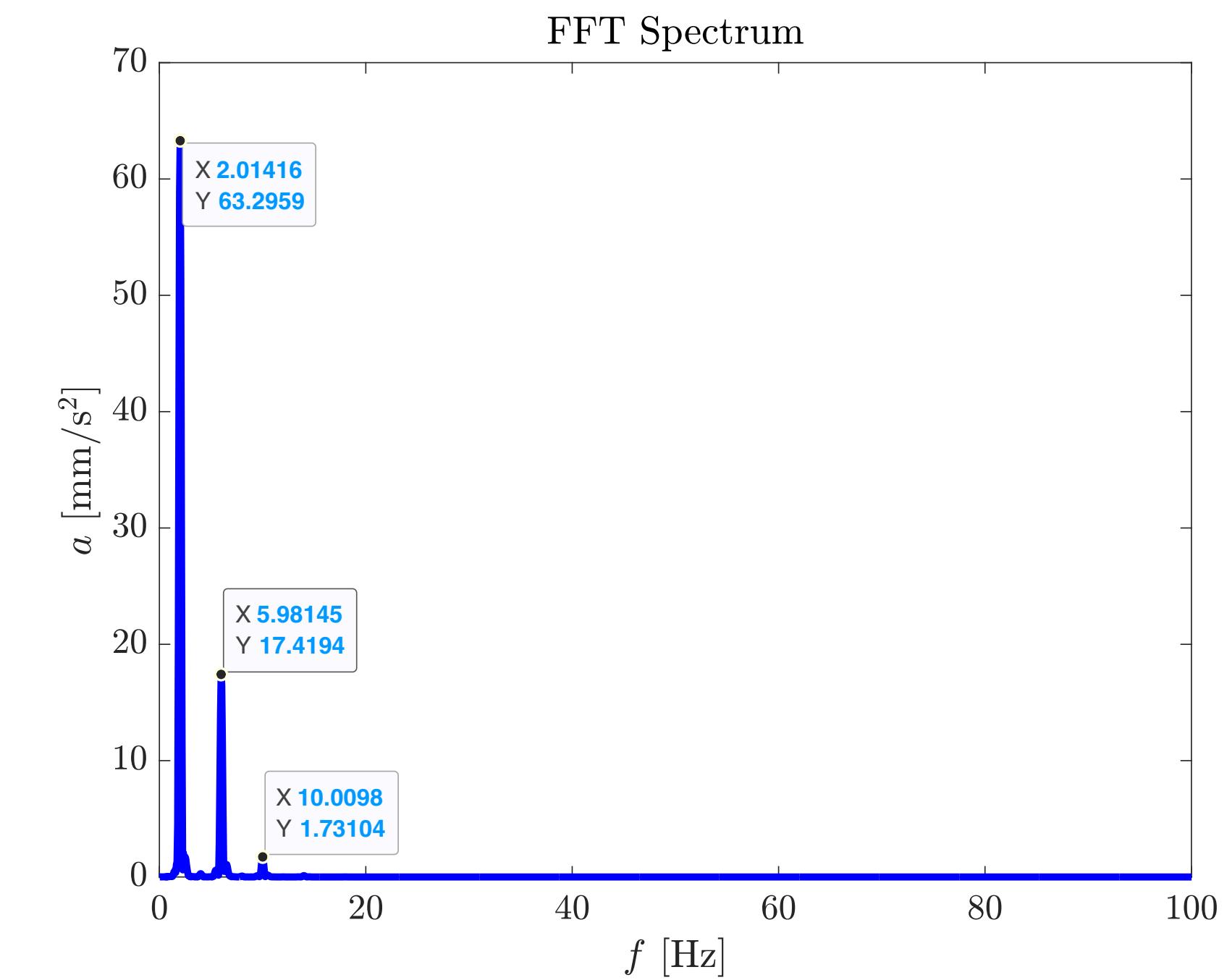


Figure 18 - FFT Spectrum for Case 2

QZS Case 3 ($f = 3$) - Time Domain

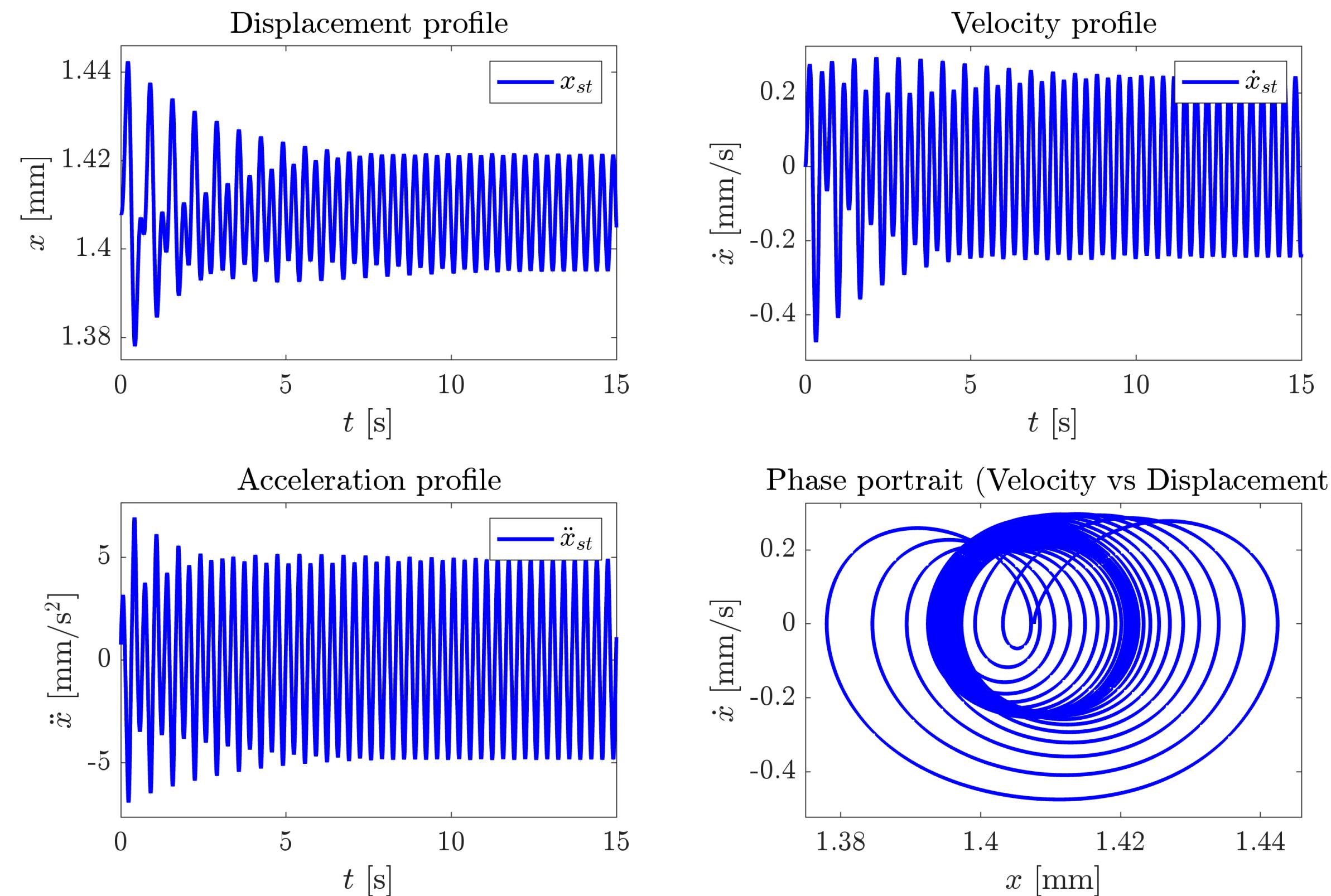


Figure 19 - Time domain results for Case 3

QZS Case 3 - Physical and Frequency Domain

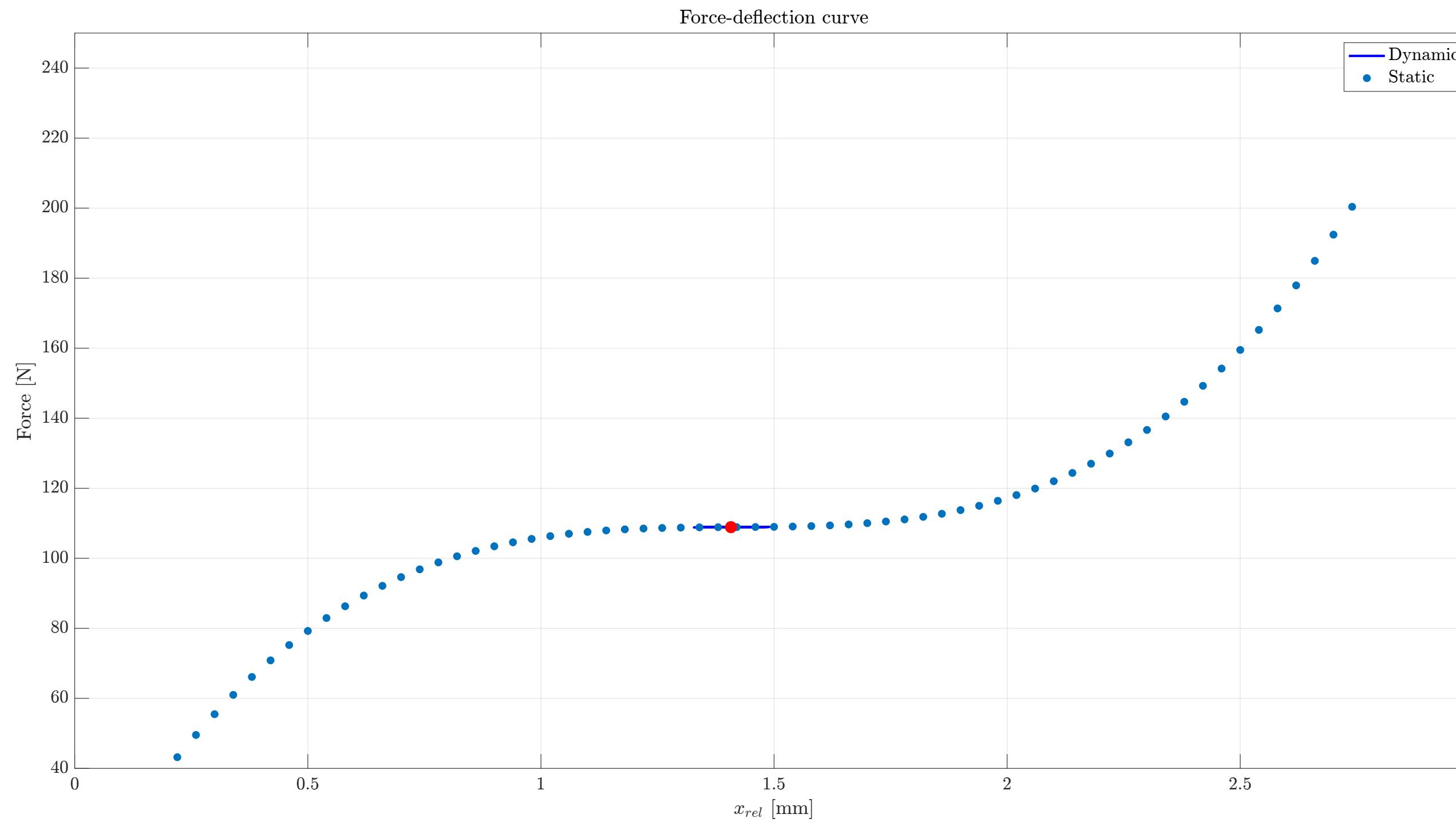


Figure 20 - Physical domain behavior for Case 3

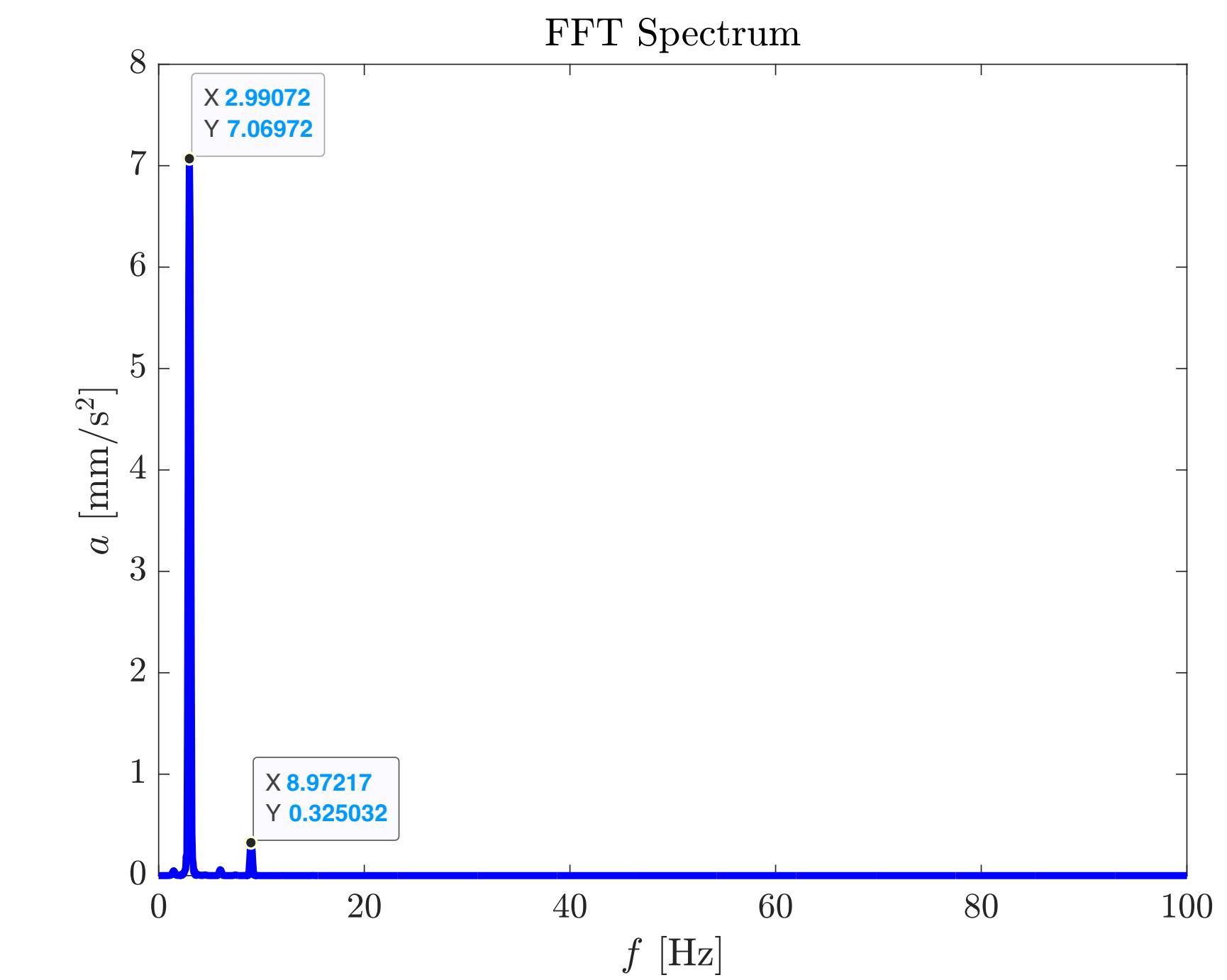


Figure 21 - FFT Spectrum for Case 3

Results - Motion Transmissibility (QZS)

Amplitude (peak to peak)		
	Case Number	Motion Transmissibility (dB)
1 Hz	Case 1	5.42
2 Hz	Case 2	17.48
3 Hz	Case 3	-11.06
5 Hz	Case 4	-21.49
7 Hz	Case 5	-27.41

Results - Motion Transmissibility comparison

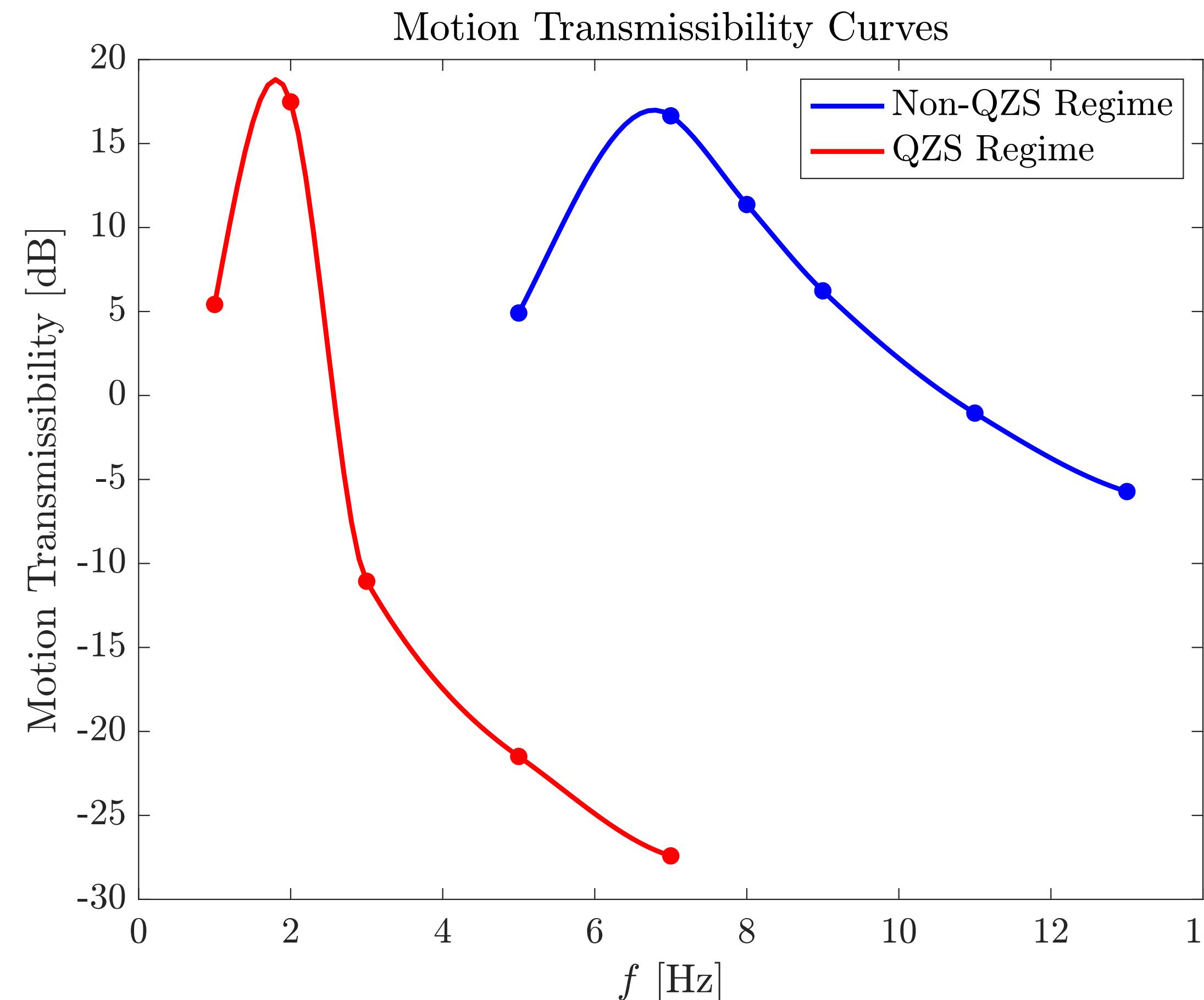


Figure - Motion Transmissibility Curves in Non-QZS (Blue) and QZS (Red) regimes

In the **Non-QZS Regime**,
Peak between **7-8 Hz** approximately

In the **QZS Regime**,
Peak between **1-2 Hz** approximately

Lower Motion Transmissibility values
for $f > 5$ Hz in the QZS regime
⇒ Effective vibration isolation

Future Work

- Continue working on in-depth investigation of dynamics
- Validation of Simulation results with Experimental results (Objective 2)