> This problem set has 16 questions, for a total of 110 points. Answer the questions
> below and mark your answers in the spaces provided. If the question asks for
> showing your work, you must provide details on how your answer was calculated.

Your Name: _____

1. [5 points] Which of the following descriptions best describes what **mystery** does?

```
int mystery(int *arr, int n) {
    if(n == 1) return arr[0];
    int val = mystery(arr + 1, n - 1)
    return (arr[0] < val) ? arr[0] : val;
}
```

A. find the minimum element of arr   B. find the maximum element of arr   C. find the the sum of all elements of arr   D. sort all elements of arr

1. _____

2. [5 points] Which of the following descriptions best describes what **mystery** does?

```
bool mystery(int n, int i) {
    if (n <= 2)
        return (n == 2) ? true : false;
    if (n % i == 0)
        return false;
    if (i * i > n)
```

```
        return true;

    return mystery(n, i + 1);
}
```
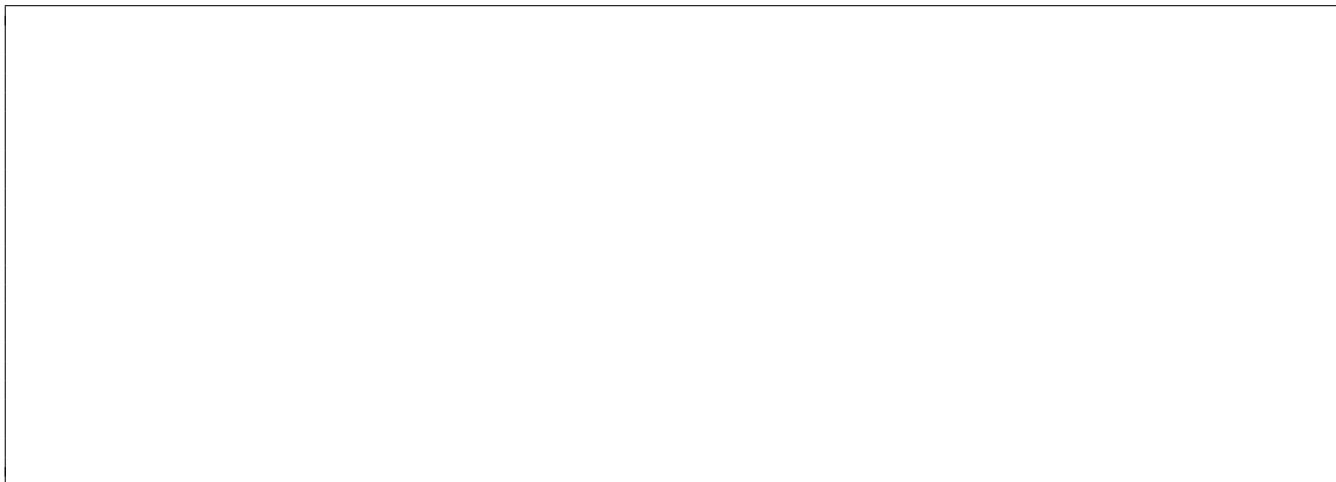

A. determine if n is an even number   B. determine if n is a prime number   C. determine if i evenly divides n   D. determine if n is an odd number

2. ─────────────

3. [5 points] Given the following sorting algorithm, determine if it is **stable**, **in-place**, **both**, or **neither**.
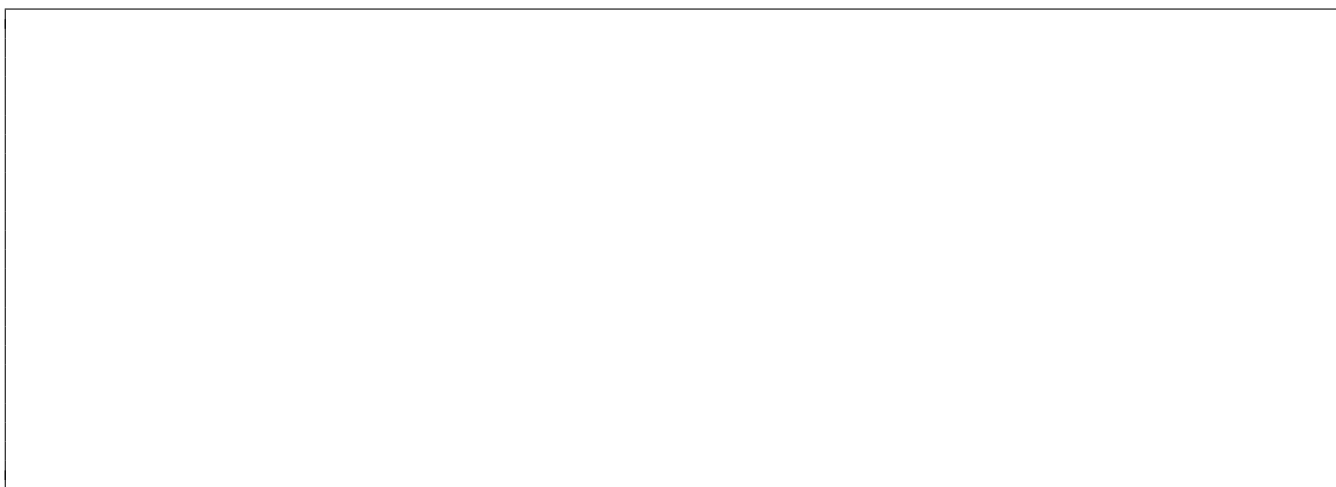
```
int sort(int *arr, int n) {
    if (n <= 1) return;
    sort(arr, n-1);
    int tmp = arr[n-1];
    int j = n-2;
    while (j >= 0 && arr[j] > tmp) {
        arr[j+1] = arr[j];
        j--;
    }
    arr[j+1] = tmp;
}
```

A. stable   B. in-place   C. both   D. neither

3. _____

4. [10 points] Solve the following recurrence relation: $T(0) = 1; T(n) = T(n-1) + 3$

A. $3n + 1$ B. $3n - 1$ C. $3n$

4. _____

5. [10 points] Solve the following recurrence relation: $T(1) = 1; T(n) = 2T(n/2) + n$

A. $n + logn$ B. $nlogn$ C. $n + nlogn$ D. $n^2 + nlogn$

5. _____

6. [5 points] Is a linked list the best underlying structure to implement a queue with? Justify your answer.

A. Yes   B. No

6. _____

7. Would a stack or queue be more efficient for the following:

   (a) [3 points] An undo button in a text editor

   (a) _____

   (b) [3 points] A web server

   (b) _____

(c) [3 points] A breadth-first search

(c) ―――――――

(d) [3 points] A depth-first search

(d) ―――――――

8. [5 points] Given the following function **mystery**, determine its output assuming **stack** has had the following elements inserted in order: 3, 10, 100, 5, 8

```cpp
int mystery(std::stack<int> stack) {
    int result = 0;
    int loop = stack.size();
    for(int i = 0 ; i < loop; i++) {
        if(!(i % 2)) {
            result += stack.top();
        }
        else {
            result *= stack.top();
        }
        stack.pop();
    }
    return result;
}
```

A. 1403   B. 658   C. 1530   D. 8040

8. ―――――――

9. [7 points] If a Binary Tree is complete, does that necessarily mean it is also full? Justify your answer with drawings of trees.

A. Yes   B. No

9. ⎯⎯⎯⎯⎯⎯

10. [8 points] If a Binary Tree is full, does that necessarily mean it is also complete? Justify your answer with drawings of trees.

A. Yes   B. No

10. ⎯⎯⎯⎯⎯⎯

11. [5 points] Assume a binary search tree has undergone the following insertions in order: 10, 7, 15, 12, 13, 4, 8, 1, 19, 20, 5. Which of the following represents the output of a post-order traversal on the resulting tree?

A. 10, 7, 15, 4, 8, 12, 19, 1, 5, 13, 20   B. 1, 4, 5, 7, 8, 10, 12, 13, 15, 19, 20   C. 10, 7, 4, 1, 5, 8, 15, 12, 13, 19, 20   D. 1, 5, 4, 8, 7, 13, 12, 20, 19, 15, 10

11. _____

12. [5 points] Assume a binary search tree has undergone the following insertions in order: 20, 10, 30, 5, 4, 1, 6, 24, 52, 28, 13. Which of the following represents the output of a pre-order traversal on the resulting tree?

A. 1, 4, 5, 6, 10, 13, 20, 24, 28, 30, 52   B. 20, 10, 5, 4, 1, 6, 13, 30, 24, 28, 52   C. 1, 4, 6, 5, 13, 10, 28, 24, 52, 30, 20   D. 10, 7, 4, 1, 5, 8, 15, 12, 13, 19, 20

12. _____

For questions 13 - 16, let T be a full k-ary tree, where $k = 2$ (a.k.a. *binary tree*), with n nodes. Let h denote the height of T.

.5.5.5

13. [7.5 points] What is the minimum number of leaves for T of height h? Justify your answer.

Example when $h = 0 : T$, being a *full tree* can have a minimum of 1 leaf.

A. 2^h B. $2h$ C. 2^{h-1} D. $2^h - 1$ E. $h + 1$

13. _____

.5.5.5

14. [7.5 points] What is the maximum number of leaves for T? Justify your answer.

A. 2^h B. $2h$ C. 2^{h-1} D. $2^h - 1$ E. $h + 1$

14. _____

.5.5.5

15. [7.5 points] What is the minimum number of internal nodes for T? Justify your answer.

A. 2^h B. $2h$ C. 2^{h-1} D. h E. $h+1$

15. _____

.5.5.5

16. [7.5 points] What is the maximum number of internal nodes for T? Justify your answer.

A. 2^h B. $2h$ C. 2^{h-1} D. $2^h - 1$ E. $h+1$

16. _____