GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 6271 Summer 2018
Problem Set #1

Assigned: 15-May-18
Due Date: 22-May-18

Please turn in HW at the beginning of class on Tuesday.

PROBLEM 1.1:
Suppose you given a time series, $u(n) = \alpha \exp(\jmath\omega n) + \mathbf{v}(n)$, for $n = 0, 1, ..., N-1$. Assume that $\mathbf{v}(n)$ is a zero-mean noise signal with

$$E[\mathbf{v}(n)\mathbf{v}^*(n-k)] = \begin{cases} \sigma_v^2, & k = 0 \\ 0, & k \neq 0. \end{cases}$$

Find the following

(a) The mean ($E[u(n)]$) of $u(n)$,

(b) The autocorrelation function ($r(k) = E[u(n)u^*(n-k)]$) of $u(n)$, and

(c) The correlation matrix ($\mathbf{R}$) for $N = 3$. The correlation matrix is defined as $R_{ij} = r(i-j)$

PROBLEM 1.2:
An "unknown" system has a system function $y(n) = w_0\mathbf{x}(n) + w_1\mathbf{x}(n-1)$ where $w_0 = 1$ and $w_1 = 2$. $\mathbf{x}(n)$ is a zero-mean *real* random variable with

$$E[\mathbf{x}(n)\mathbf{x}(n-k)] = \begin{cases} \sigma^2, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

Suppose we model this "unknown" system using a filter with a different set of weights,

$$\hat{y}(n) = \hat{w}_0\mathbf{x}(n) + \hat{w}_1\mathbf{x}(n-1).$$

Since this system is "unknown" we can assume that our $\hat{w}_i$'s are not necessarily correct.

(a) Find an expression for the expected squared error, $E[|y(n) - \hat{y}(n)|^2]$ as a function of $\hat{w}_0$ and $\hat{w}_1$.

(b) Let $\sigma^2 = 1$, use MATLAB to plot the expected squared error as a function of $\hat{w}_0$ and $\hat{w}_1$ in the range $[-5, 5]$. Hint: this will be a 3-D plot similar to those shown in class.

PROBLEM 1.3:

Suppose $\mathbf{x}$ is a uniform random variable on the interval [-1,1] and $\mathbf{y}$ is a random variable that obeys the relationship $\mathbf{y} = \mathbf{x}^2$. Compute the following:

(a) $\mu_x = E[\mathbf{x}]$ (mean)

(b) $\mu_y = E[\mathbf{y}]$ (mean)

(c) $\sigma_x^2 = var[x] = E\left[|\mathbf{x} - \mu_x|^2\right]$ (variance)

(d) σ_y^2 (variance)

(e) $\phi_{xy} = E[\mathbf{xy}]$ (correlation)

(f) $\gamma_{xy} = E[(\mathbf{x} - \mu_x)(\mathbf{y}] - \mu_y)]$ (covariance)

(g) Are the two random variables *independent*?

(h) Are the two random variables *uncorrelated*?