Name ____________________

E-mail ____________________

EE 4221

Hour Exam 3

December 7, 2017

Directions:

1. DO NOT START until told to do so.

2. There are 4 problems in this examination. All problems are equal valued.

3. The correct answer is a necessary but not sufficient condition to receive full credit for a problem. You MUST show you work! Disorderly or illegible work cannot and thus will not be graded.

4. You are allowed use of any reference materials of your choice during the exam. You may not however classify your fellow classmate(s) as "reference materials" The exam is meant to be an individual effort!

5. Please return the exam by noon on Monday, December 11th, 2017. On-campus students submit to box 49, 7th floor EERC. Off-campus students submit as an e-mail attachment.

1. I plan to use a 7 strand all copper conductor (cable) for a low-loss, sub-transmission application.
Each strand in the cable is 3.0mm in diameter.
The spiral construction adds 2.25% to the actual length of each strand.

M

Determine the resistance for 1km of cable at 20°C and 50°C given:
The resistivity of copper is 1.72μΩ cm at 20°C and the temperature constant for copper is 234.5°C.

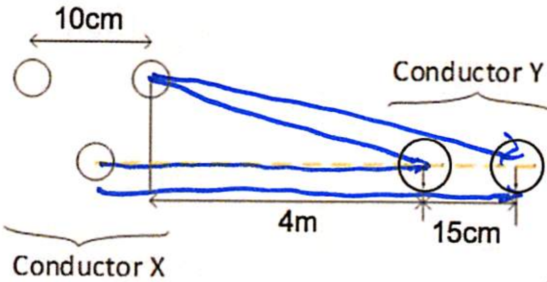
$$R_{\text{per strand } 20°C} = \frac{\rho_{20C} \cdot l_{+\text{SPIRAL}}}{A}$$

$$\frac{1.72 \times 10^{-6}\ \Omega \cdot cm \times \frac{1m}{100cm} \times 1022.5m}{\frac{\pi (.003)^2}{4}} = 2.48805 \frac{\Omega}{\text{conductor}}$$

$$R_{\text{CABLE } 20C} = R_{\text{per strand}} \div 7 = .35544 \frac{\Omega}{km} \text{ @ } 20°C$$

$$R_{\text{CABLE } 50C} = .35545 \left(\frac{234.5+50}{234.5+20}\right) = .39733 \frac{\Omega}{km} \text{ @ } 50°C$$

2. The conductor configuration of a bundled, single phase transmission line is shown below.



$r'_x = \sqrt[3]{1 \times .7788 \times 10 \times 10} = 4.2705 \text{ cm}$ $\qquad r'_y = \sqrt[2]{2 \times .7788 \times 15} = 4.8336 \text{ cm}$

The "X" line features three solid conductors at the corners of an equilateral triangle with 10 cm spacing. Each X conductor has a diameter of 2.0 cm.

The "Y" line is comprised of two solid conductors in a horizontal configuration. The conductors are separated by 15 cm and each has a diameter of 4.0cm.

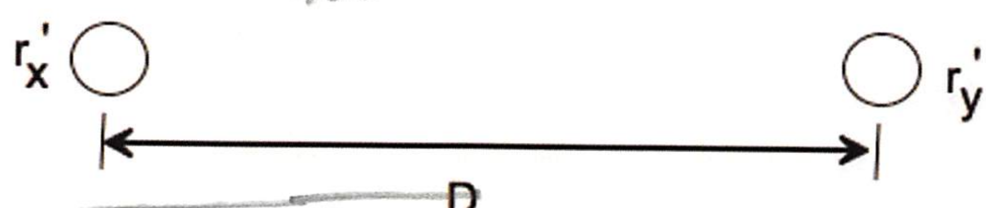
Assuming we wish to calculate the loop inductance of this single phase line, determine:
a) the effective radius of a single equivalent X conductor
b) the effective radius of a single equivalent Y conductor
c) the equivalent spacing between the single equivalent X and Y conductors.

$d_{x_1-y_1} = 4.05 \text{ m} \qquad d_{x_1-y_2} = 4.2 \text{ m}$

$d_{x_2-y_1} = \sqrt{4^2 + \left(\frac{5\sqrt{3}}{100}\right)^2} = 4.00094 \text{ m}$

$d_{x_2-y_2} = \sqrt{(4.15)^2 + \left(\frac{5\sqrt{3}}{100}\right)^2} = 4.15090 \text{ m}$

$d_{x_3-y_2} = \sqrt{(4.1)^2 + \left(\frac{5\sqrt{3}}{100}\right)^2} = 4.10091 \text{ m}$

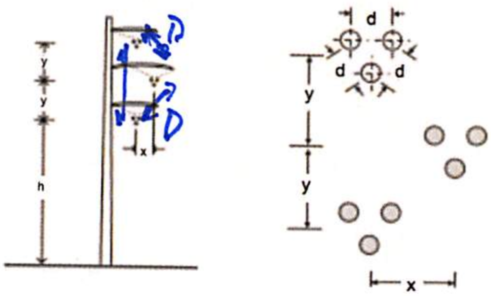


$d_{x_3-y_2} = \sqrt{(4.25)^2 + \left(\frac{5\sqrt{3}}{100}\right)^2} = 4.25088$

$D = \sqrt[6]{\text{product of distances}} = 4.12472 \text{ m}$

3. The fully-transposed, 60Hz, 500kV, transmission line shown below features 3 "Cardinal" conductors on corners of an equilateral triangle with 35 centimeter spacing (d), between conductors. The bundles are arranged vertically as shown below with x = 3.0m, y = 6.5m and h = 20m. Determine the per phase, 60Hz, 50°C, series impedance (R + jX) for the line in ohms per mile. Use ACSR wire table provided after question #4.

$R = .1082\,\Omega/\text{mile}$



$GMR = .0402\,ft \cdot \frac{12\,in}{1\,ft} \cdot \frac{2.54\,cm}{1\,in} = 1.2253\,cm$

$R_b' = \sqrt[3]{1.2253 \times 35 \times 35} = 11.44965\,cm$

$GMD = \sqrt[3]{13 \times 7.15891 \times 7.15891} = 8.733984\,m$

$L = 2\times10^{-7} \ln \frac{873.3984\,cm}{11.44965\,cm} = 8.66865\times10^{-7}\,\frac{H}{m} \times 1609\,\frac{m}{mile}$

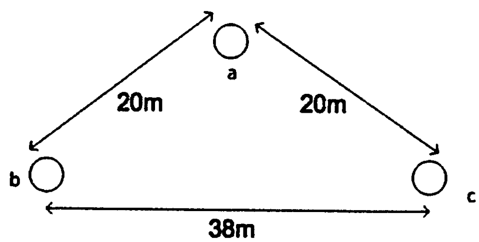
$= 1.39482\,mH/mile$

$X_L = j\,2\pi 60 L = j.5258\,\frac{\Omega}{mile}$

$R = \frac{.1082}{3} = .0361\,\frac{\Omega}{mile}$

$Z = (.0361 + j.5258)\,\frac{\Omega}{mile}$

4. Determine the per phase capacitance to neutral (in F/mile) and, shunt admittance (in S/mile) for the fully transposed, 3 phase, 60Hz, 220kV line shown (below). Each phase is comprised of a single "Flicker" conductor. You may neglect the effect of the earth.



$OD = .846\ in \quad r = \frac{OD}{2} = .423\ in \times \frac{2.54\ cm}{1\ in} \times \frac{1\ m}{100\ cm}$

$r = .0107442\ m$

$GMD = \sqrt[3]{20 \times 20 \times 38} = 24.7712\ m$

$$C_{AN} = \frac{2\pi\varepsilon_0}{\ln \frac{24.7712}{.0107442}} = 7.1846 \times 10^{-12} \frac{F}{m} \times \frac{1609\ m}{mile} = 1.156 \times 10^{-8} \frac{F}{mi}$$

$$Y = j\,2\pi 60\, C_{AN} = 4.3581 \times 10^{-6} \frac{S}{mi}$$

4b. Given the line is 100 miles long, determine the charging current at the nominal line-to-line system voltage of 220kV.

$$I_{CHARGE} = Y \cdot \ell \cdot V_{LN} = 4.3581 \times 10^{-4} \times \frac{220{,}000}{\sqrt{3}}$$

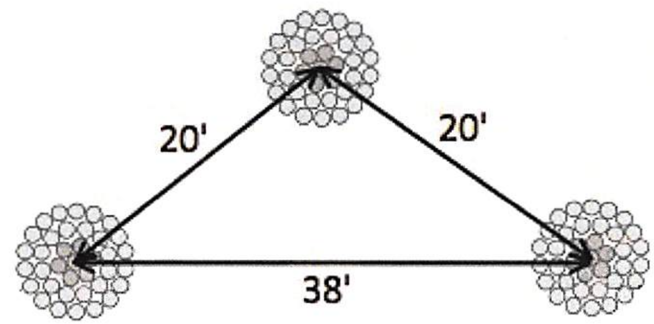
$55.355\ A/phase$

Code word	Aluminum	Stranding	Layers	Outside	Resistance	R - AC 60 Hz	R - Ac, 60 Hz	GMR	Reactance per conductor 1-ft spacing, 60 Hz		
	Area	Al/St	of	diameter	DC, 20°C,	20°C,	50°C,	feet	Inductive Xa	Capacitive Xa'	
	CMIL		Al	inches	Ω/1,000ft	Ω/mile	Ω/mile		Ω/mile	MΩ-mile	
Waxwing	266,800	18/1	2	0.609	0.0646	0.3488	0.3831	0.0198	0.476	0.109	Waxwing
Partridge	266,800	26/7	2	0.642	0.064	0.3452	0.3792	0.0217	0.465	0.1074	Partridge
Ostrich	300,000	26/7	2	0.68	0.0569	0.307	0.3372	0.0229	0.458	0.1057	Ostrich
Merlin	336,400	18/1	2	0.684	0.0512	0.2767	0.3037	0.0222	0.462	0.1055	Merlin
Linnet	336,400	26/7	2	0.721	0.0507	0.2737	0.3006	0.0243	0.451	0.104	Linnet
Oriole	336,400	30/7	2	0.741	0.0504	0.2719	0.2987	0.0255	0.445	0.1032	Oriole
Chickadee	397,500	18/1	2	0.743	0.0433	0.2342	0.2572	0.0241	0.452	0.1031	Chickadee
Ibis	397,500	26/7	2	0.783	0.043	0.2323	0.2551	0.0264	0.441	0.1015	Ibis
Pelican	477,000	18/1	2	0.814	0.0361	0.1957	0.2148	0.0264	0.441	0.1004	Pelican
Flicker	477,000	24/7	2	0.846	0.0359	0.1943	0.2134	0.0284	0.432	0.0992	Flicker
Hawk	477,000	26/7	2	0.858	0.0357	0.1931	0.212	0.0289	0.43	0.0988	Hawk
Hen	477,000	30/7	2	0.883	0.0355	0.1919	0.2107	0.0304	0.424	0.098	Hen
Osprey	556,500	18/1	2	0.879	0.0309	0.1679	0.1843	0.0284	0.432	0.0981	Osprey
Parakeet	556,500	24/7	2	0.914	0.0308	0.1669	0.1832	0.0306	0.423	0.0969	Parakeet
Dove	556,500	26/7	2	0.927	0.0307	0.1663	0.1826	0.0314	0.42	0.0965	Dove
Rook	636,000	24/7	2	0.977	0.0269	0.1461	0.1603	0.0327	0.415	0.095	Rook
Grosbeak	636,000	26/7	2	0.99	0.0268	0.1454	0.1596	0.0335	0.412	0.0946	Grosbeak
Drake	795,000	26/7	2	1.108	0.0215	0.1172	0.1284	0.0373	0.399	0.0912	Drake
Tern	795,000	45/7	3	1.063	0.0217	0.1188	0.1302	0.0352	0.406	0.0925	Tern
Rail	954,000	45/7	3	1.165	0.0181	0.0997	0.1092	0.0386	0.395	0.0897	Rail
Cardinal	954,000	54/7	3	1.196	0.018	0.0988	0.1082	0.0402	0.39	0.08	Cardinal
Ortolan	1,033,500	45/7	3	1.213	0.0167	0.0924	0.1011	0.0402	0.39	0.0885	Ortolan
Bluejay	1,113,000	45/7	3	1.259	0.0155	0.0861	0.0941	0.0415	0.386	0.0874	Bluejay
Finch	1,113,000	54/19	3	1.293	0.0155	0.0856	0.0937	0.0436	0.38	0.0866	Finch
Bittern	1,272,000	45/7	3	1.345	0.0136	0.0762	0.0832	0.0444	0.378	0.0855	Bittern
Pheasant	1,272,000	54/19	3	1.382	0.0135	0.0751	0.0821	0.0466	0.372	0.0847	Pheasant
Bobolink	1,431,000	45/7	3	1.427	0.0121	0.0684	0.0746	0.047	0.371	0.0837	Bobolink
Plover	1,431,000	54/19	3	1.465	0.012	0.0673	0.0735	0.0494	0.365	0.0829	Plover
Lapwing	1,590,000	45/7	3	1.502	0.0109	0.0623	0.0678	0.0498	0.364	0.0822	Lapwing
Falcon	1,590,000	54/19	3	1.545	0.0108	0.0612	0.0667	0.0523	0.358	0.0814	Falcon
Bluebird	2,156,000	84/19	4	1.762	0.008	0.0476	0.0515	0.0586	0.344	0.0776	Bluebird

Example

A fully transposed 60Hz, three phase line has ACSR "Drake" conductors arranged as shown.

Drake
Area = 795000 cmil
Diameter = 1.108 inches
GMR = 0.0375 ft at 60Hz
X'_a = 0.0912 Megohms per conductor per mile



Note the picture is not to proportion but I wanted to emphasize the "Drakes" are stranded conductors

$$D_{eq} = \sqrt[3]{D_{AB} D_{BC} D_{CA}} = \sqrt[3]{20 \cdot 20 \cdot 38} = 24.771 \;\; ft$$

Conductor radius = 1.108/2 = 0.554 inches = 0.04617 feet

$$X_C = 1.778 \cdot 10^6 \frac{1}{f} \ln\left(\frac{D_{eq}}{D_{sc}}\right) \; \Omega \cdot mile = 1.778 \cdot 10^6 \frac{1}{60} \ln\left(\frac{24.771}{0.04617}\right) = 1.86248 \cdot 10^5 \;\; \Omega \cdot miles$$

or

$$X_C = 1.778 \cdot 10^6 \frac{1}{f} \ln\left(\frac{D_{eq}}{D_{sc}}\right) \; \Omega \cdot mile = X'_a + 1.778 \cdot 10^6 \frac{1}{f} \ln D_{eq}$$

$$= 0.0912 \cdot 10^6 + 1.778 \cdot 10^6 \frac{1}{60} \ln 24.771 = 1.863 \cdot 10^5 \;\; \Omega \cdot miles$$

Steady State Stability Limit

We could do the same analysis using the ABCD parameters:

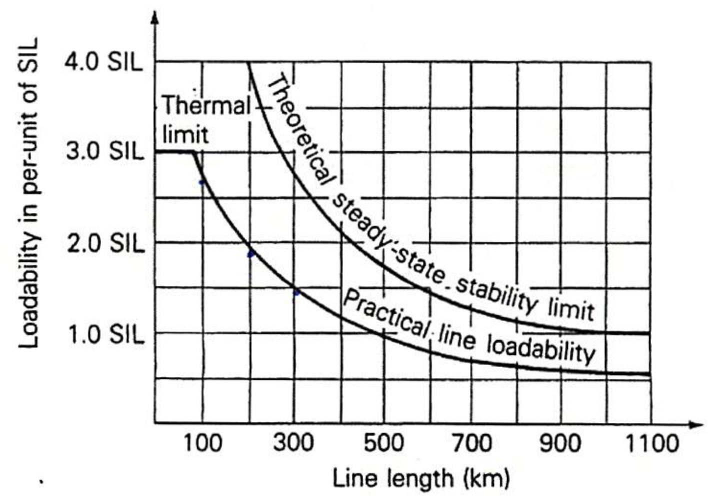
$$V_S = AV_R + BI_R \qquad I_R = \frac{V_S - AV_R}{B}$$

$$S_R = V_R \cdot (I_R)^*$$

$$P_R = \frac{|V_R| \cdot |V_S|}{B} \sin\delta = \frac{|V_R| \cdot |V_S|}{Z_C \sin\beta l} \sin\delta$$

$$P_R = \frac{|V_R|}{V_{Rated}} \cdot \frac{|V_S|}{V_{Rated}} \cdot \frac{(V_{Rated})^2}{Z_C} \cdot \frac{\sin\delta}{\sin\beta l}$$

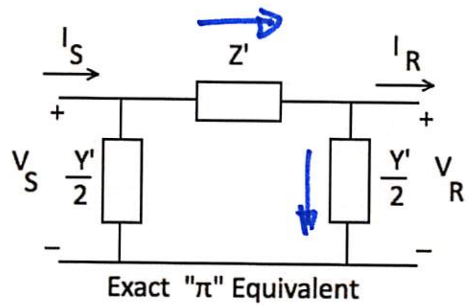
$$P_R = V_{R(pu)} \cdot V_{S(pu)} \cdot SIL \cdot \frac{\sin\delta}{\sin\beta l}$$



The result shows the receiving end power is proportional to the square of the voltage i.e., a doubling of voltage produces a fourfold increase in maximum power.

The result also shows the receiving end power is inversely proportional to line length.

Steady State Stability Limit

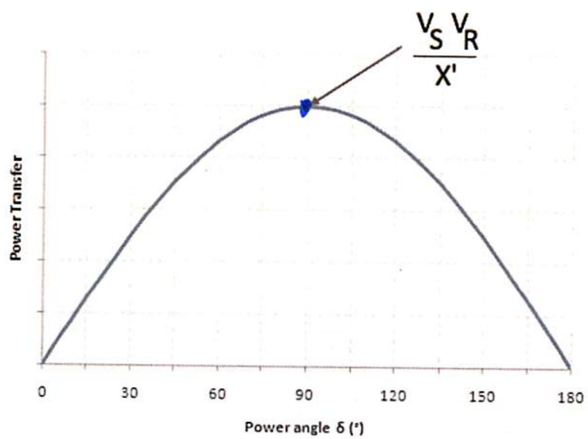


Exact "π" Equivalent

$$I_R = \frac{V_S - V_R}{Z'} - \frac{Y'}{2} \cdot V_R$$

$$S_R = V_R \cdot I_R^* = V_R \cdot \left(\frac{V_S - V_R}{Z'} - \frac{Y'}{2} \cdot V_R \right)^*$$

$$V_R = V_R \angle 0^\circ \quad V_S = V_S \angle \delta^\circ \quad Z' = jX' \ \ and \ \ Y' = j\omega C' l$$



$$S_R = V_R \left(\frac{V_S \angle \delta - V_R}{jX'} \right)^* + \frac{j\omega C' l}{2} V_R^2$$

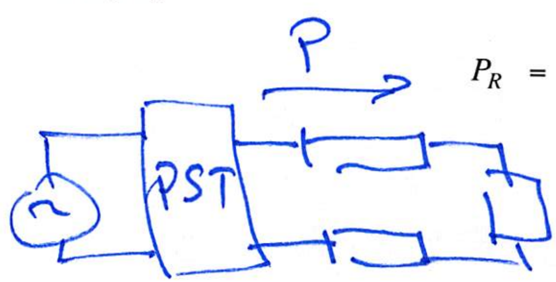
$$S_R = V_R \left(\frac{V_S \angle -\delta - V_R}{-jX'} \right) + \frac{j\omega C' l}{2} V_R^2$$

S = P + jQ

$$S_R = \frac{V_R V_S \angle 90 - \delta}{X'} \underbrace{- j\frac{V_R^2}{X'} + \frac{j\omega C' l}{2} V_R^2}$$

$$S_R = P_R + jQ_R$$

$$P_R = \frac{|V_R| \cdot |V_S|}{|X'|} \cos(90 - \delta^\circ) = \frac{|V_R| \cdot |V_S|}{|X'|} \sin \delta$$



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Line Loadability

Voltage kV	Zc(range) ohms	Zc(used) ohms	S.I.L (MW)	400 km line		200 km line	
				P(max) (MW)	P(limit) (MW)	P(max) (MW)	P(limit) (MW)
230	365-395	380	139	289	166	559	321
345	280-366	320	372	772	443	1496	858
500	233-294	260	961	1995	1144	3864	2216
765	254-266	260	2,250	4671	2679	9047	5190

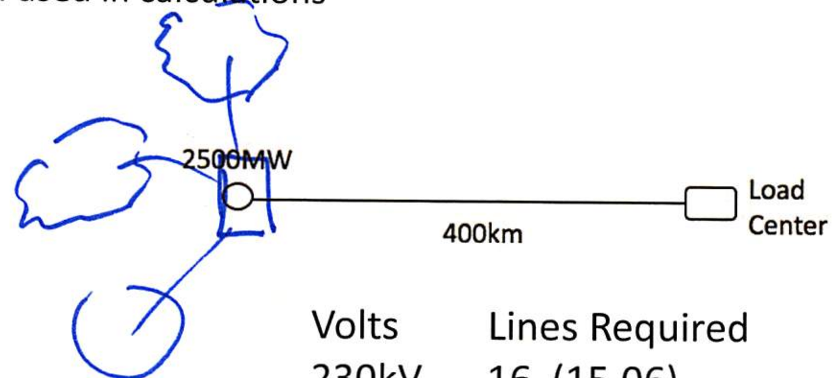
5,000 km wavelength used in calculations

$$P_{R\ MAX} = \frac{V_R V_S}{Z_C \sin \beta l} = \frac{V_R V_S}{Z_C \sin \frac{2\pi l}{\lambda}}$$

$$P_{R\ MAX} = \frac{V_R V_S}{Z_C \sin\left(\frac{360^\circ}{5000km}\right) 400km}$$

$$P_{R\ MAX} = \frac{\left(230 \cdot 10^3\right)^2}{380(0.4818)} = 289MW$$

$$P_{R\ LIMIT} = P_{R\ MAX} \sin 35^\circ = 166MW$$



Volts	Lines Required
230kV	16 (15.06)
345kV	6 (5.64)
500kV	3 (2.18)
765kV	1

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Receiving End Power

$$V_S = AV_R + BI_R \qquad I_R = \frac{V_S - AV_R}{B}$$

$$I_S = CV_R + DI_R$$

$$S_R = V_R \cdot (I_R)^*$$

$$V_R = |V_R|\angle 0^\circ \qquad V_S = |V_S|\angle\delta \qquad B = |B|\angle\beta \qquad A = |A|\angle\alpha$$

$$S_R = \frac{|V_R|\cdot|V_S|}{|B|}\angle\beta-\delta - \frac{|V_R|^2|A|}{|B|}\angle\beta-\alpha$$

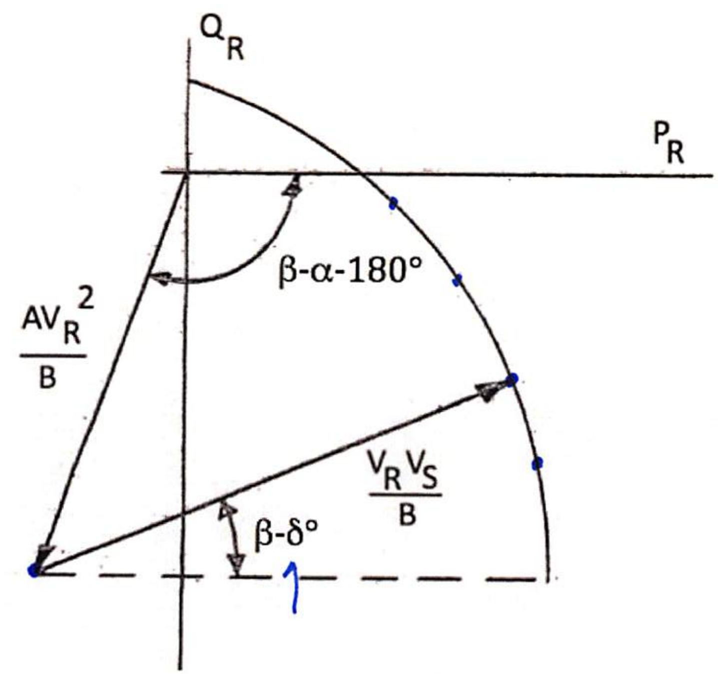
$$S_R = \frac{|V_R|\cdot|V_S|}{|B|}\angle\beta-\delta + \frac{|V_R|^2|A|}{|B|}\angle\beta-\alpha-180^\circ$$

Receiving End Circle Diagram

$$S_R = \frac{|V_R|\cdot|V_S|}{|B|}\angle\beta-\delta + \frac{|V_R|^2|A|}{|B|}\angle\beta-\alpha-180$$

$$S_R = P_R + jQ_R$$

$$S_R = \underbrace{\frac{|V_R|^2|A|}{|B|}\angle\beta-\alpha-180}_{\text{fixed}} + \underbrace{\frac{|V_R|\cdot|V_S|}{|B|}\angle\beta-\delta}_{\text{varies with }\delta}$$



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$B = Z_c \sinh \gamma \ell$

$Z_c = \sqrt{\frac{Z}{Y}}$ $\quad \gamma = \sqrt{ZY}$

Example

400km 765kV line B = 128.6∠86.99° A = 0.8747∠0.4412°

Sbase = 1000MVA Vbase = 765kV Zbase = 585.2Ω

$B_{p.u.} = 0.219745\angle 86.99°$ $A_{p.u.} = 0.8747\angle 0.4412°$

The magnitude of $V_R = V_S = 1.0_{p.u.}$

$$S_R = \frac{|V_R|^2|A|}{|B|}\angle\beta-\alpha-180 + \frac{|V_R|\cdot|V_S|}{|B|}\angle\beta-\delta$$

$$S_R = \frac{1^2\cdot 0.8747}{0.219745}\angle 86.99-0.4412-180 + \frac{1\cdot 1}{0.219745}\angle 86.99-\delta$$

$$S_R = \underline{3.980531\angle -93.46} + \underline{4.550739\angle 86.99-\delta}$$

Receiving End Circle Diagram

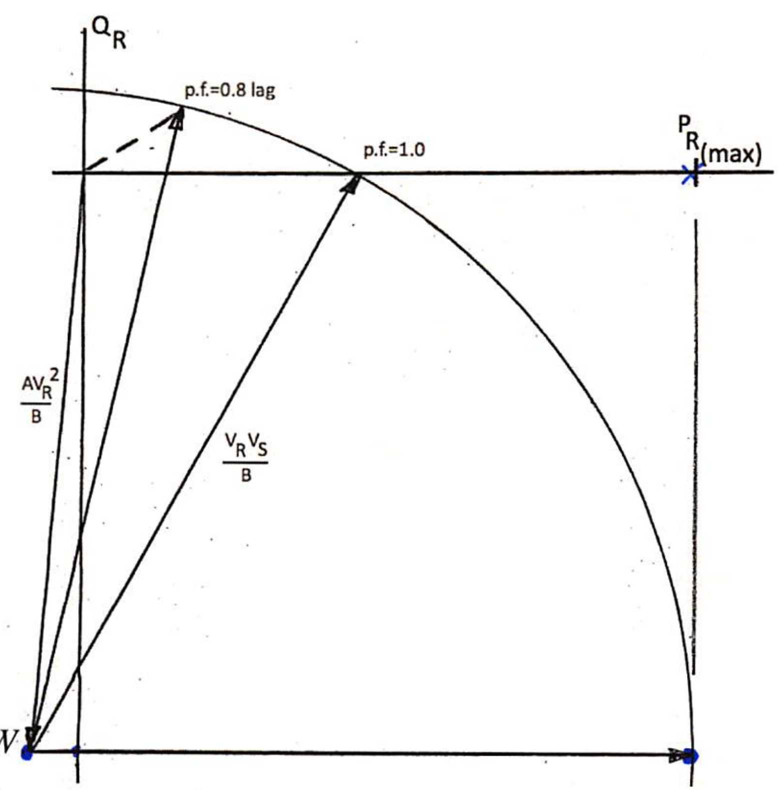
$$S_R = 3.980531\angle -93.46 + 4.550739\angle 86.99-\delta$$

$$P_R = 3.98\cos(-93.46) + 4.55\cos(86.99-\delta)$$

$$P_R = -0.24 + 4.55\cos(86.99-\delta)$$

$$Q_R = 3.98\sin(-93.46) + 4.55\sin(86.99-\delta)$$

$$Q_R = -3.97 + 4.55\sin(86.99-\delta)$$



When will $P_R = P_{R\ MAX}$???

$\delta = 86.99$ and $\cos(0°) = 1$

$$P_{R\ MAX} = -0.24 + 4.55\cos(0°) = 4.31 p.u. = 4310.5MW$$

$$Q_R = -3.97 + 4.55\sin(0°) = -3.97 p.u. = 3972.6MVAR$$

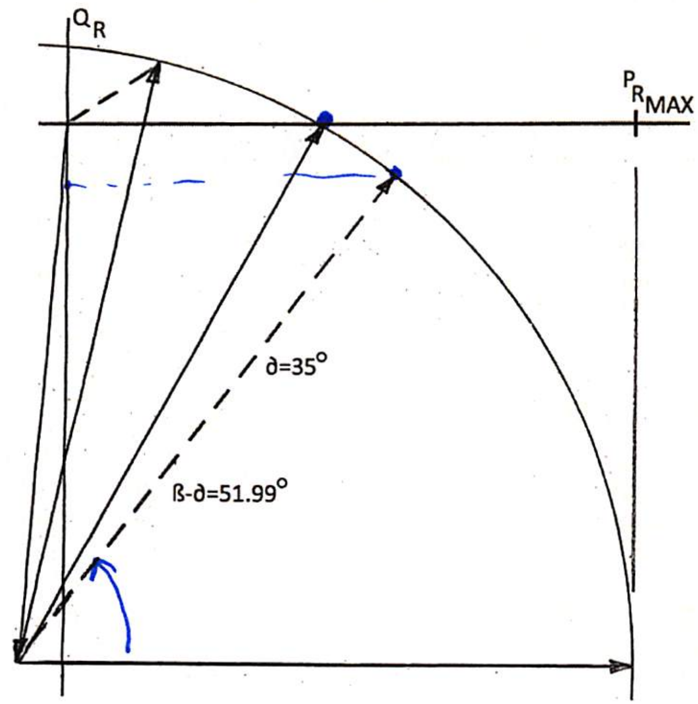
$$p.f. = P/S = 0.736 \ \ leading$$

Recall from the lossless calculations: $P_{R\ MAX} = 4558MW$

Receiving End Circle Diagram

$P_R = P_{R\,MAX}$ when $\delta =$ 86.99° and cos(0°) = 1
This is too close to the limit of $\delta =$ 90°

What is the receiving end power when $\delta =$ 35° ?



$$S_R = 3.980531\angle -93.46 + 4.550739\angle 86.99-\delta$$

$$P_R = 3.98\cos(-93.46) + 4.55\cos(86.99-35)$$

$$P_R = -0.24 + 2.80 = 2.562\,p.u. = 2562MW$$

$$P_{R\ lossless} = 2598MW$$

$$Q_R = 3.98\sin(-93.46) + 4.55\sin(86.99-35)$$

$$Q_R = -3.97 + 3.58 = -0.3877\,p.u. = 387.7MVAR$$

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Receiving End Circle Diagram

How much power can we deliver at unity power factor?

When p.f. = 1.0 Q_R = 0VARs

$$S_R = 3.980531\angle -93.46 + 4.550739\angle 86.99-\delta$$

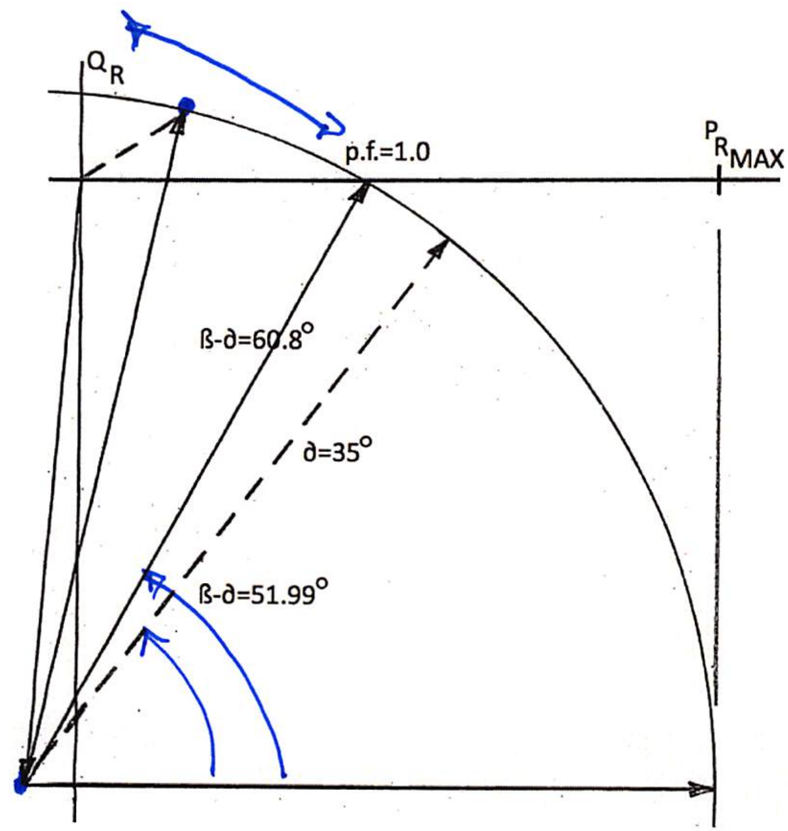
$$Q_R = 3.98\sin(-93.46) + 4.55\sin(86.99-\delta) = 0$$

$$(86.99-\delta) = \sin^{-1}\left(\frac{-3.98\sin(-93.46)}{4.55}\right) = 60.8°$$

$$\delta = 26.2°$$

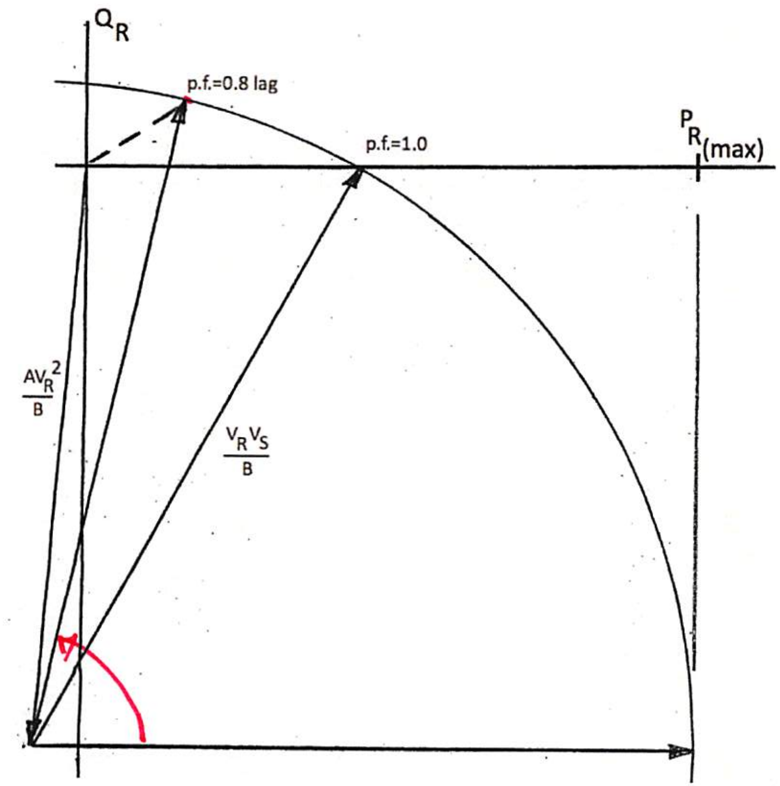
$$P_R = 3.98\cos(-93.46) + 4.55\cos(86.99-26.2)$$

$$P_R = -0.24 + 2.22 = 1.983\,p.u. = 1983MW$$



Receiving End Circle Diagram

What is the receiving end power at p.f. = 0.8 lagging?



$$p.f. = \frac{P}{S} \quad S = \frac{P}{p.f.} = \frac{P}{.8}$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{\frac{P^2}{.64} - P^2} = 0.75P$$

$$Q_R = 0.75P_R = 3.98\sin(-93.46) + 4.55\sin(86.99 - \delta) \checkmark$$

$$\underline{P_R} = 3.98\cos(-93.46) + 4.55\cos(86.99 - \delta) \checkmark$$

P

$$cSolve(0.75X = 3.98\sin(-93.46) + 4.55\sin(86.99 - Y)\,and$$
$$X = 3.98\cos(-93.46) + 4.55\cos(86.99 - Y),\{X,Y\})$$

$x = \cancel{-4.77 \quad y = -4768.1}$ *or* $x = \cancel{-4.77 \quad y = -88.06}$ *or*

$x = \cancel{-4.77 \quad y = 271.94}$ *or* $x = \cancel{-4.77 \quad y = 5671.94}$ *or*

$x = \cancel{0.652 \quad y = -2871.7}$ *or* $x = 0.652 \quad y = 8.297$ *or*

$x = 0.652 \quad y = 368.30$

$X = P$

$Y = \delta$

$$P_R = 0.652\,p.u. = 652MW$$

$$Q_R = 0.75P_R = 0.489\,p.u. = 489VAR$$

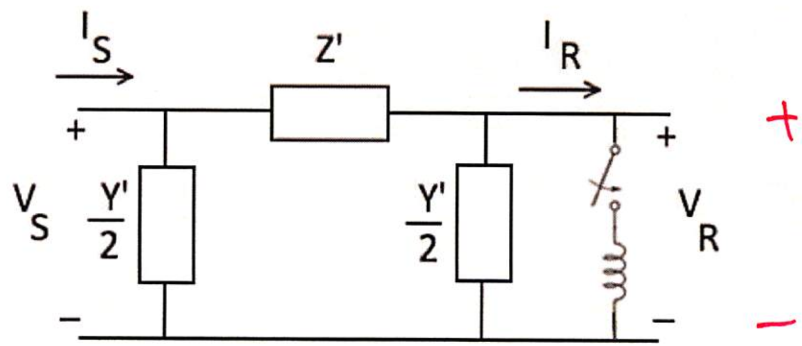
$$Q_R = 3.98\sin(-93.46) + 4.55\sin(86.99 - 8.297) = 0.489 \quad check!$$

Voltage Regulation

$$V_S = AV_R + BI_R$$

$$but \;\; at \;\; no-load \;\; I_R = 0A$$

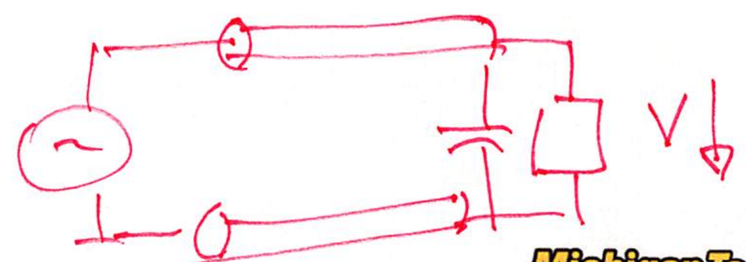
$$V_S = AV_R \qquad |V_R| = \frac{|V_S|}{|A|}$$



$$for \;\; our \;\; 765kV, \; 400km \;\; line \;\; |A| = .8747$$

$$V_R = \frac{1.05}{0.8747} = 1.20p.u. \quad 1.2 \cdot 765kV = 918kV$$

$$V_{REG} = \frac{\frac{1.05}{0.8747} - 1.0}{1.0} \cdot 100\% = 20.04\%$$



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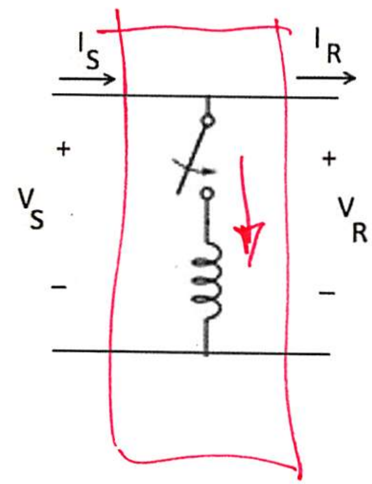
The Inductor as a Two Port



$$V_S = V_R \quad I_S = \frac{V_R}{jX_L} + I_R$$ ✓

$A_L \quad B_L$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{jX_L} & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$C_L \quad D_L$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \frac{1}{jX_L} & 1 \end{bmatrix}$$

$$A = A_1 \cdot 1 + B_1 \cdot \frac{1}{jX_L} = 0.8747\angle 0.4412^\circ + 0.219745\angle 86.99 \cdot \frac{1}{X_L}\angle -90^\circ$$

$$A = 0.8747\angle 0.4412 + \frac{0.219745}{X_L}\angle -3.01$$

$$|1.0| = 0.8747\angle 0.4412 + \frac{0.219745}{X_L}\angle -3.01 \qquad X_L = 1.751 p.u.$$

$$|V_R|_{no\ load} = \frac{|V_S|}{|A|} = \frac{1.05}{1} = 1.05$$

$$|V_R| = \frac{|V_S|}{A}$$

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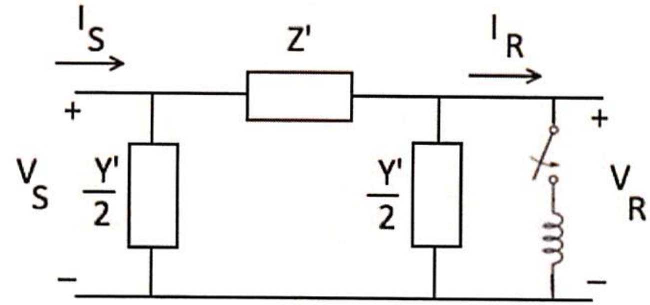
Shunt Compensation



We may also determine X_L through power considerations:

$$if \;\; X_L \;\; is \;\; chosen \;\; properly \; |A| = 1.0 \Rightarrow V_{R(no\;\;load)} = V_S$$

$$S_R = \frac{|V_R|^2 |A|}{|B|} \angle \beta - \alpha - 180 + \frac{|V_R| \cdot |V_S|}{|B|} \angle \beta - \delta$$

$$S_R = \frac{1.05^2 \cdot 0.8747}{0.219745} \angle 86.99 - 0.4412 - 180 + \frac{1.05 \cdot 1.05}{0.219745} \angle 86.99 - \delta$$

$$S_R = 4.385526 \angle -93.45^\circ + 5.017179 \angle 86.99 - \delta^\circ$$

$$P_R = 0$$

$$0 = 4.385526 \cos \angle -93.45^\circ + 5.017179 \cos \angle 86.99 - \delta^\circ$$

$$\angle 86.99 - \delta^\circ = \frac{4.385526 \cos \angle -93.45^\circ}{5.017179} = 86.9827$$

$$Q_R = 4.385526 \sin \angle -93.45^\circ + 5.017179 \sin \angle 86.9827 = 0.62952$$

$$Q_R = \frac{{V_R}^2}{X_L} \Rightarrow X_L = \frac{1.05^2}{0.62952} = 1.75097\, p.u.$$

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Shunt Compensation

500kV 40MVAR Shunt Reactor Bank