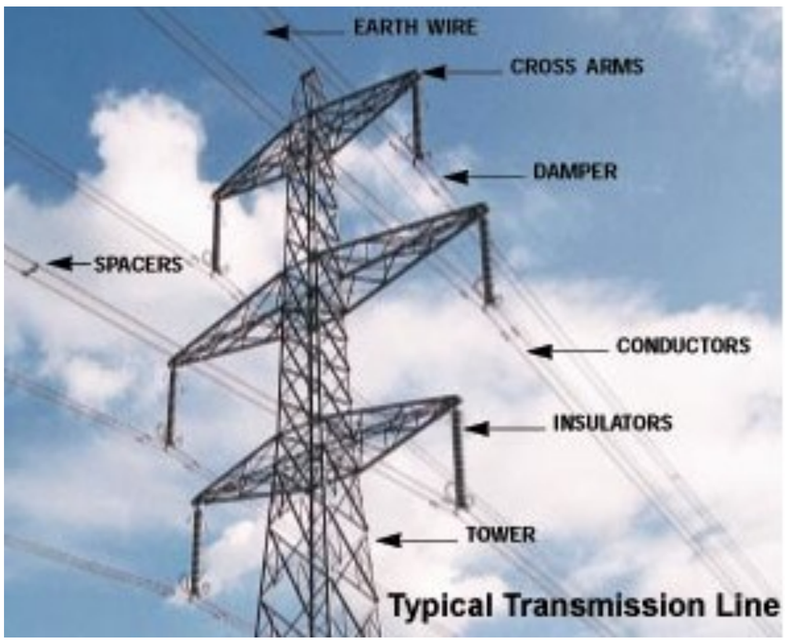
Transmission Line Parameters

The task: Provide a highly reliable, low loss, low impedance path from the generators that are producing electricity to the loads that are consuming electricity.



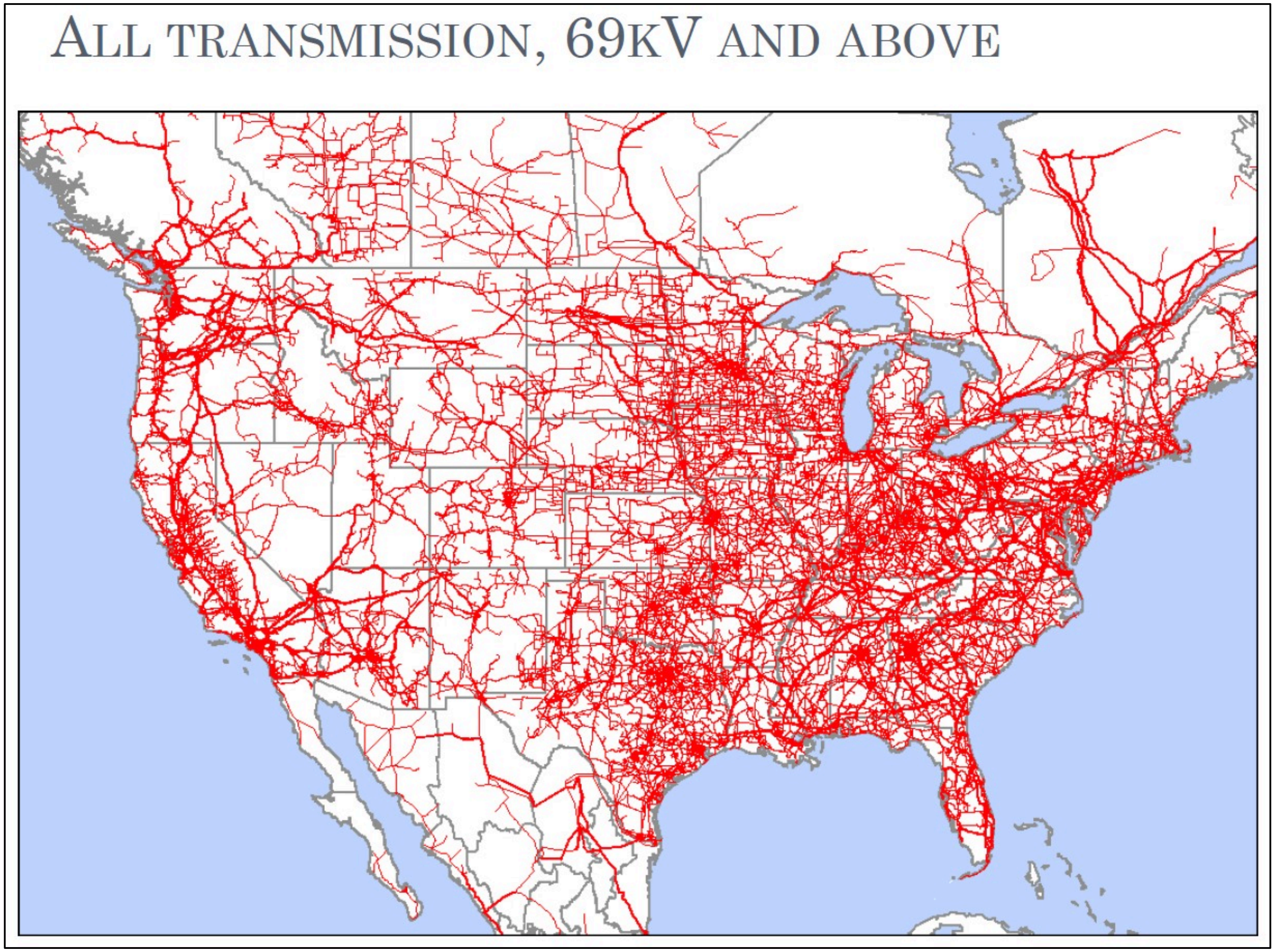
nationalgrid.com

Georgia Transmission

high-voltage power lines are rarely installed underground: prohibitive construction costs, shorter life expectancy, and slower repair time during outages.

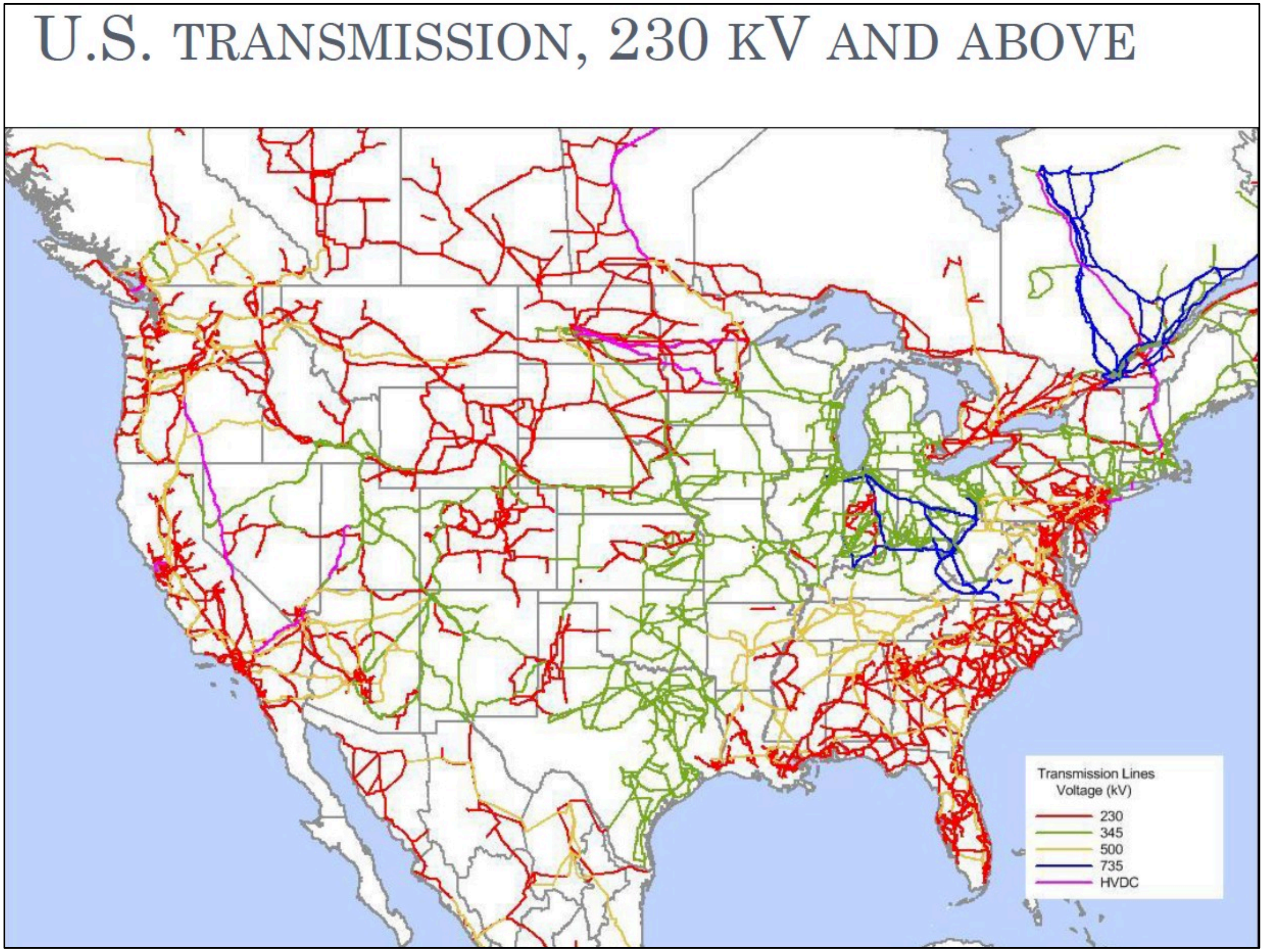
Michigan Tech

Transmission Lines

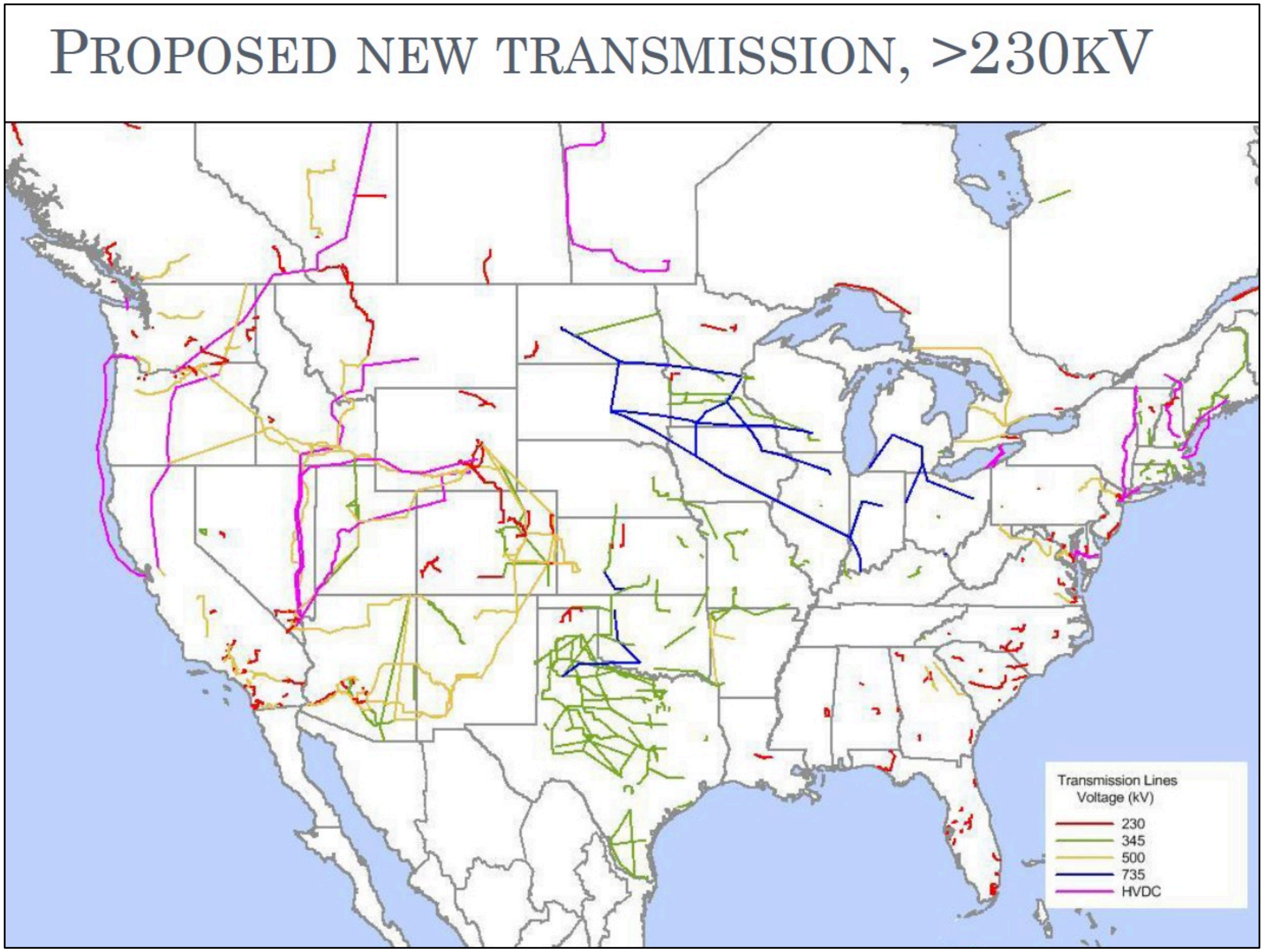


naruc.org

Transmission Lines

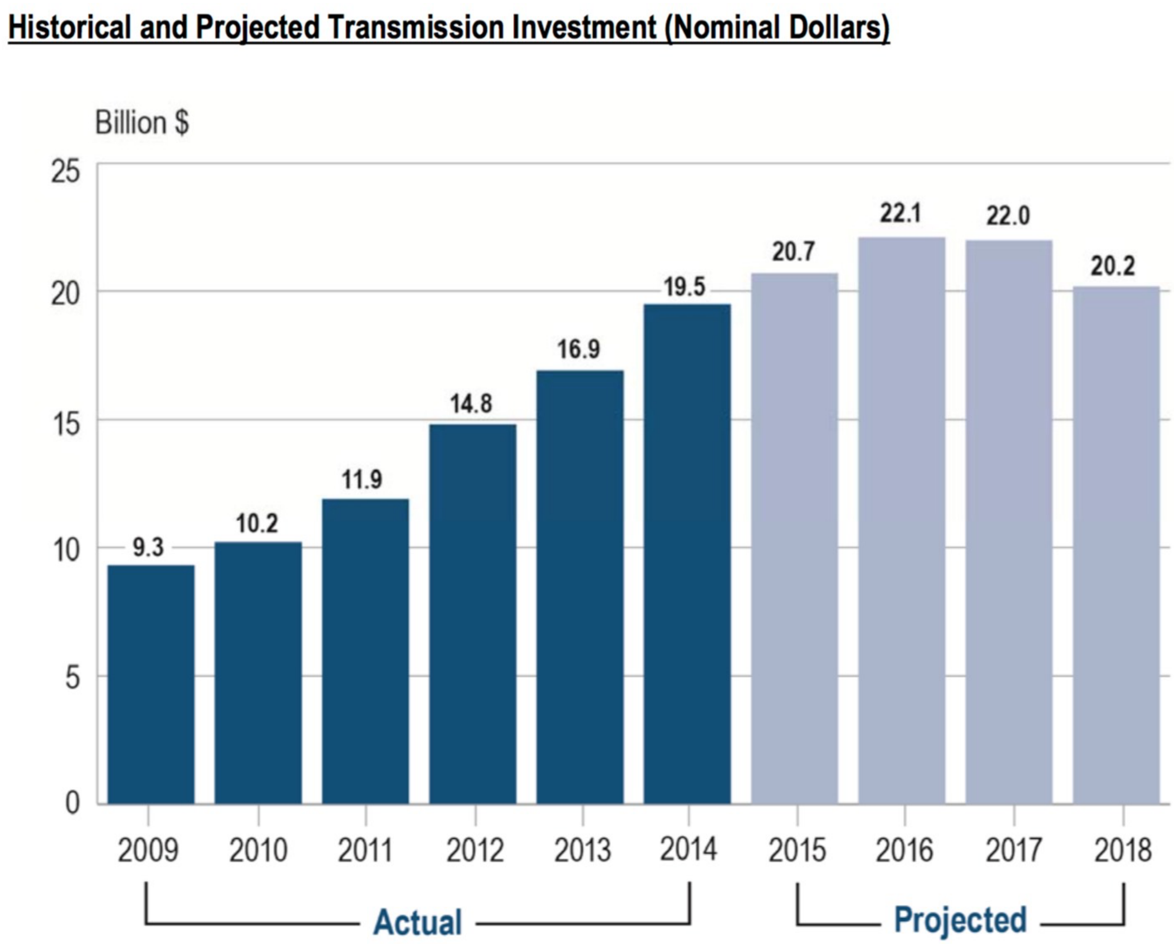


Transmission Lines



Transmission Lines

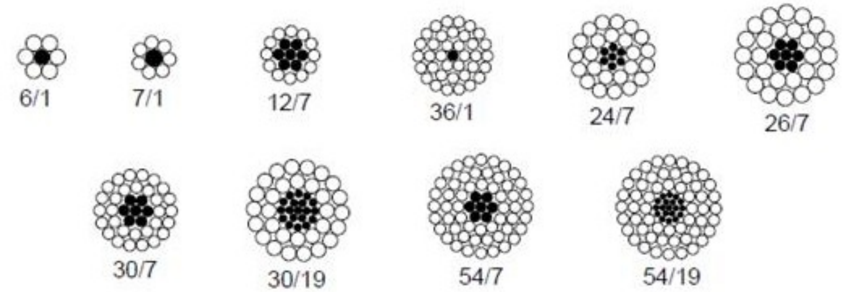
Actual and Planned Transmission Investment by Shareholder Owned Utilities



Edison Electric Institute

Transmission Line Conductors

AAC – All Aluminum Conductor



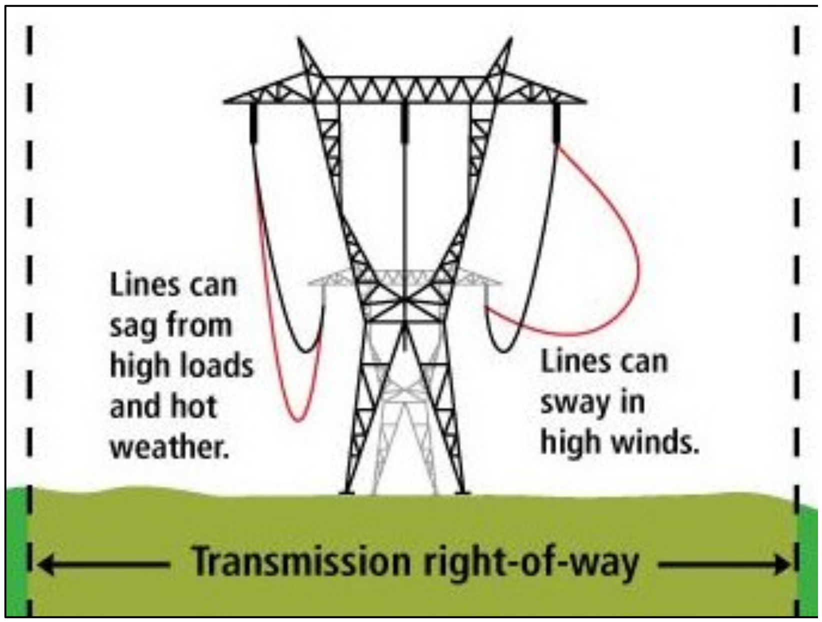
ACSR

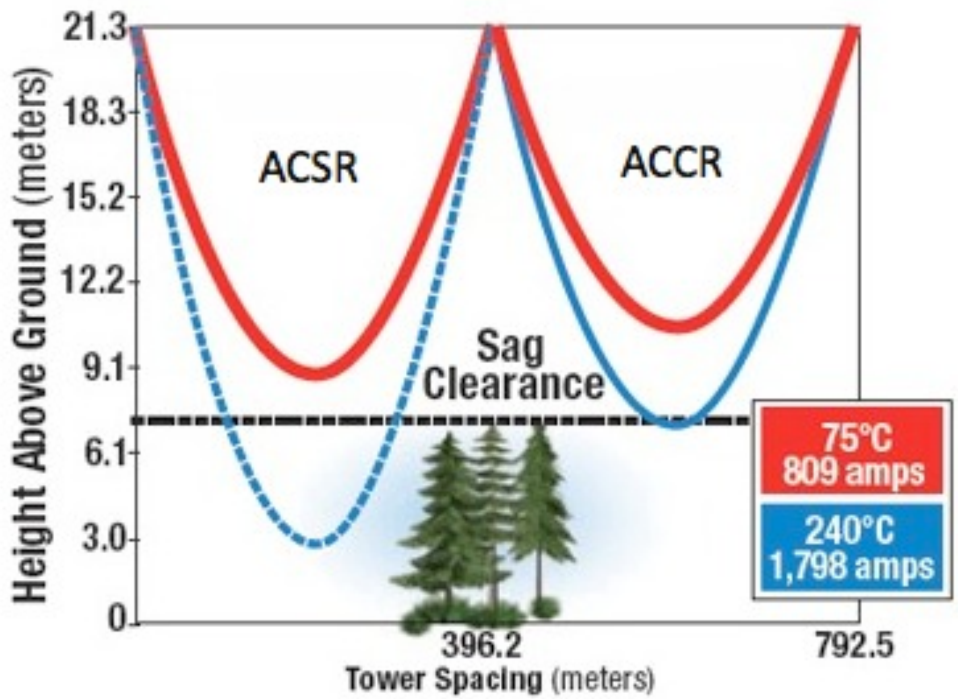
Aluminum Conductor
Steel Reinforced

ACCR

Aluminum Conductor Composite Reinforced

Transmission Line Sag/Sway





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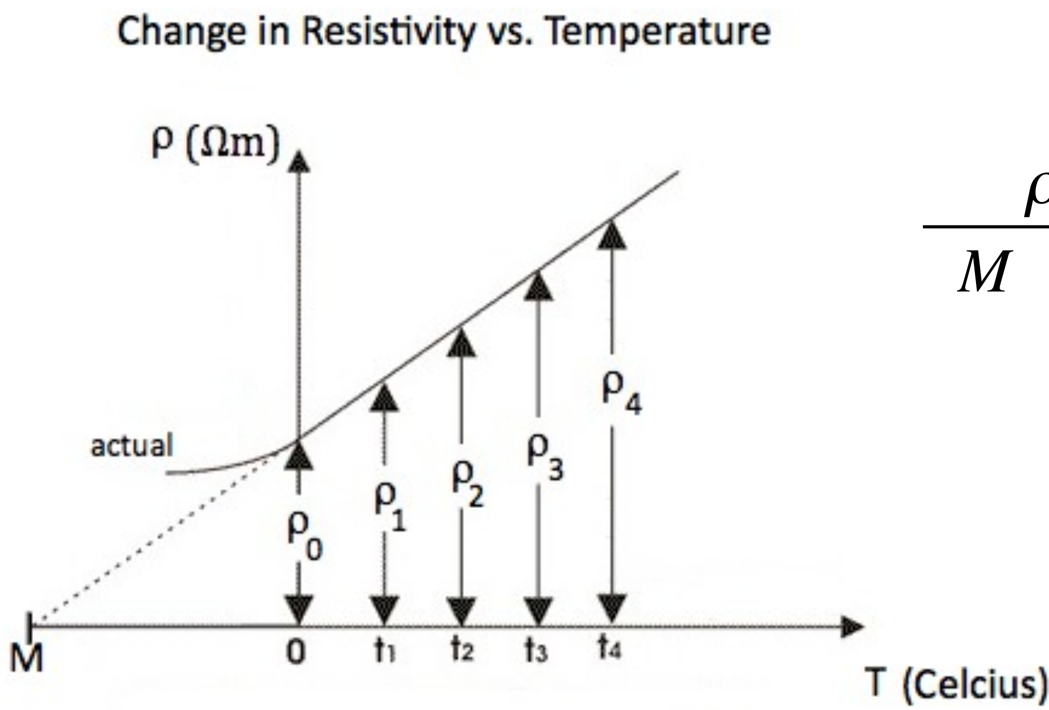
Resistivity vs. Temperature

Resistivity increases linearly with temperature over the normal range of system temperatures. If the resistivity is known at one temperature, the resistivity can be calculated at another temperature from the ratio of "similar triangles."



$$\frac{\rho_1}{M + T_1} = \frac{\rho_2}{M + T_2}$$

Resistivity vs. Temperature

Rearranging:

$$\rho_{T_2} = \rho_{T_1}\left(\frac{M + T_2}{M + T_1}\right)$$

where ρ_{T2} and ρ_{T1} are resistivities at temperatures T_2 and T_1 respectively and M is an extrapolated temperature constant that depends on the conductor material.

Material	Resistivity at 20°C [Ω·m]	Temperature constant [°C]
Annealed copper	$1.72 \cdot 10^{-8}$	234.5
Hard-drawn copper	$1.77 \cdot 10^{-8}$	241.5
Aluminum	$2.83 \cdot 10^{-8}$	228.1
Iron	$10.00 \cdot 10^{-8}$	180.0
Silver	$1.59 \cdot 10^{-8}$	243.0

Resistance

$$R = \frac{\rho_T \; l}{A}$$

ρ_T = conductor resistivity at temperature T	(Ωm)	(Ω-cmil/ft)
l = conductor length	(m)	(ft)
A = conductor cross-sectional area	(m^2)	cmil

Area (cmils) = D^2 where d is diameter in mils (1 mil = 0.001 in.)

Michigan Tech

Resistivity

Material	ρ (Ω•m) at 20 °C Resistivity
Silver	1.59×10^{-8}
Copper	1.68×10^{-8}
Annealed copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.82×10^{-8}
Calcium	3.36×10^{-8}
Tungsten	5.60×10^{-8}
Zinc	5.90×10^{-8}
Nickel	6.99×10^{-8}
Lithium	9.28×10^{-8}
Iron	1.0×10^{-7}
Platinum	1.06×10^{-7}
Tin	1.09×10^{-7}
Carbon steel	(10^{10})
Lead	2.2×10^{-7}
Titanium	4.20×10^{-7}
Grain oriented electrical steel	4.60×10^{-7}
Manganin	4.82×10^{-7}
Constantan	4.9×10^{-7}
Stainless steel	6.9×10^{-7}
Mercury	9.8×10^{-7}
Nichrome	1.10×10^{-6}
GaAs	5×10^{-7} to 10×10^{-3}

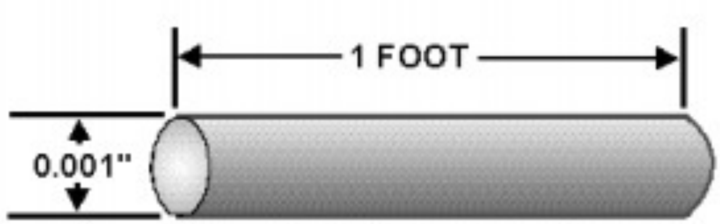
The data from the table (left) shows silver and copper to be among the best conductors. However aluminum, being much cheaper and lighter, is most often used for transmission line conductors.

Of course, for the same rated current, conductors made out of aluminum will have bigger diameter than copper conductors to offset the higher resistivity of the material.

Specific Resistance

Resistance presented in ohms per unit volume

Substance	Specific resistance at 20ºC.	
	Centimeter cube (microhoms)	Circular-mil-foot (ohms)
Silver	1.629	9.8
Copper (drawn)	1.724	10.37
Gold	2.44	14.7
Aluminum	2.828	17.02
Carbon (amorphous)	3.8 to 4.1	....
Tungsten	5.51	33.2
Brass	7.0	42.1
Steel (soft)	15.9	95.8
Nichrome	109.0	660.0



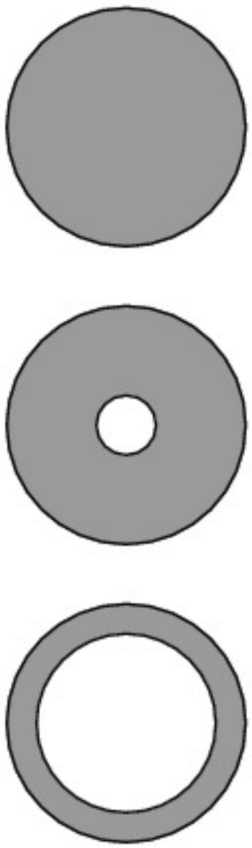
A circular-mil-foot is a unit of volume. A "unit conductor" 1 foot in length and has a cross-sectional area of 1 circular mil is depicted to the left. Because it is a unit conductor, the circular-mil-foot is useful in making comparisons between wires consisting of different metals.

Spiraling

To provide the mechanical force to hold a conductor together alternate layers of wire strands are spiraled in opposite directions.

The effect of spiraling is to make the "resistive length" of a conductor 1-2% longer than the measured physical length.

Skin Effect



Cross-sectional area of a round conductor available for conducting DC current

"DC resistance"

Cross-sectional area of the same conductor available for conducting low-frequency AC

"AC resistance"

Cross-sectional area of the same conductor available for conducting high-frequency AC

"AC resistance"

AC resistance of a conductor is always higher than its DC resistance due to the skin effect forcing more current flow near the outer surface of the conductor.

The higher the frequency of current, the more noticeable skin effect.

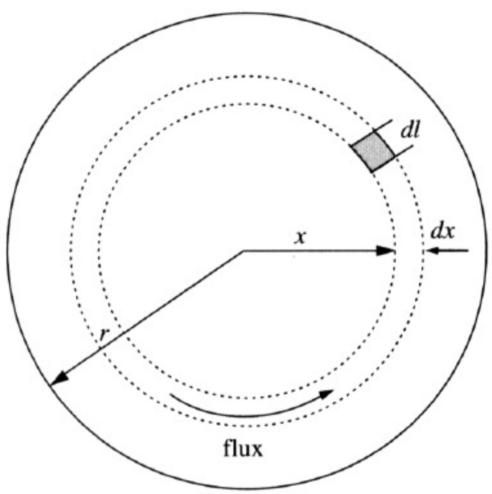
At frequencies of interest to power engineers (50-60 Hz), skin effect is not very strong.

Inductance and Inductive Reactance

The series inductance of a transmission line consists of two components: internal and external inductances, which arise from the magnetic flux inside and outside the conductor respectively. The inductance of a transmission line is defined as the number of flux linkages [Wb-turns] produced per ampere of current flowing through the line:

$$L = \frac{\lambda}{I}$$

Internal Inductance:

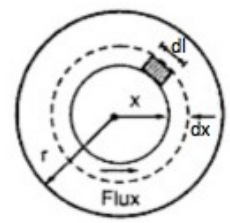


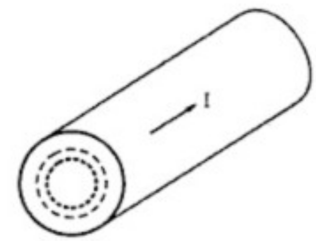
Consider a cylindrical conductor of radius r carrying a current of I amperes (left). At a distance x from the center of this conductor, the magnetic field intensity H_x can be found from Ampere's law:

$$\oint H_x \cdot dl = I_{x\ (enclosed)}$$

Ampere's Law Continued

$$\oint H_x \cdot dl = I_{x\,(enclosed)}$$





H_x is the magnetic field intensity at each point along a closed path, dl is a unit vector along that path and I_x is the net current enclosed in the path. For a homogeneous material and a circular path of radius x, the magnitude of H_x is constant, and dl is always parallel to H_x.

$$2\pi x H_x = I_x \quad or \quad H_x = \frac{I_x}{2\pi x}$$

Now (ignoring skin effect) assuming that the current is distributed uniformly in the conductor:

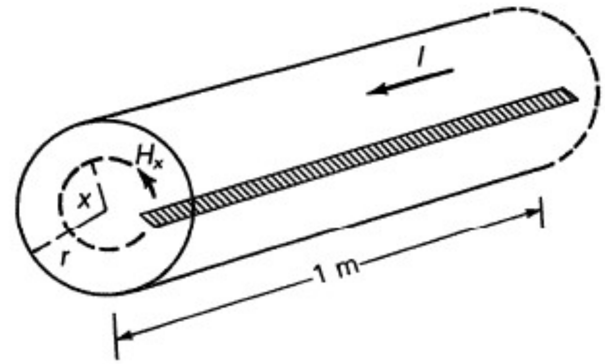
$$I_x = \frac{\pi x^2}{\pi r^2} I$$

Making the magnetic field intensity inside the conductor at radius x:

$$H_x = \frac{I_x}{2\pi x} = \frac{xI}{2\pi r^2}$$

Inductance and Inductive Reactance

The flux density at a distance x from the center of the conductor is:



$$B_x = \mu H_x = \frac{\mu x I}{2\pi r^2} \; [Tesla]$$

If a differential magnetic flux is contained in a unit length circular tube of thickness dx, at a distance x from the center of the conductor

$$d\phi = \frac{\mu x I}{2\pi r^2} dx \cdot 1 \; [Wb/m]$$

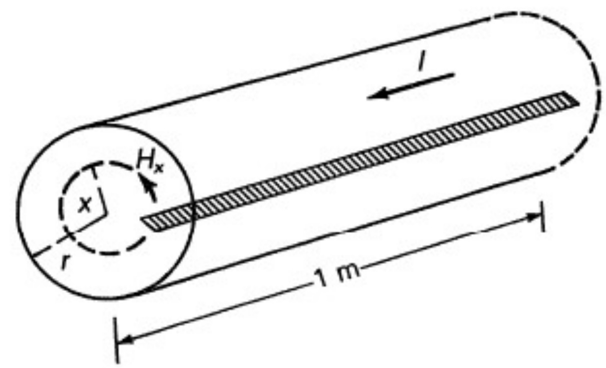
The flux linkages per meter of length due to flux in the tube is the product of the differential flux and the fraction of current linked:

$$d\lambda = \frac{\pi x^2}{\pi r^2} d\phi = \frac{\mu x^3 I}{2\pi r^4} dx \; [Wb - turns/m]$$

Inductance and Inductive Reactance

The total internal flux linkages per meter can be found by integration:

$$\lambda_{\text{int}} = \int_0^r d\lambda_x = \frac{\mu I}{2\pi r^4}\int_0^r x^3 dx = \frac{\mu I}{8\pi}$$



By definition, the internal inductance per meter is:

$$L_{\text{int}} = \frac{\lambda_{\text{int}}}{I} = \frac{\mu}{8\pi}[H/m]$$

If the relative permeability of the conductor is 1 (i.e., non-ferromagnetic materials, such as copper and aluminum), the inductance per meter reduces to

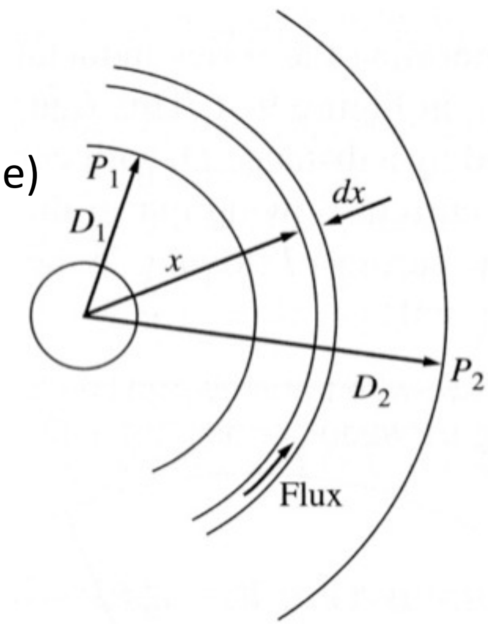
$$L_{\text{int}} = \frac{\lambda_{\text{int}}}{I} = \frac{\mu_0}{8\pi} = \frac{4\pi\cdot 10^{-7}}{8\pi} = 0.5\cdot 10^{-7}[H/m]$$

External Inductance

To find the inductance **external** to a conductor, we need to calculate the flux linkages of the conductor due only the portion of flux between two points P1 and P2 that lie at distances D1 and D2 from the center of the conductor.

In the region external to the conductor, the magnetic intensity at a distance x from the center of conductor is:

$$H_x = \frac{I_x}{2\pi x} = \frac{I}{2\pi x}$$ (since all the current is internal to the tube)



The flux density at a distance x from the center of conductor is:

$$B_x = \mu H_x = \frac{\mu I}{2\pi x} \ [Tesla]$$

The differential magnetic flux contained in a circular tube of thickness dx and at a distance x from the center of the conductor is:

$$d\phi = \frac{\mu I}{2\pi x} dx \ [Wb/m]$$

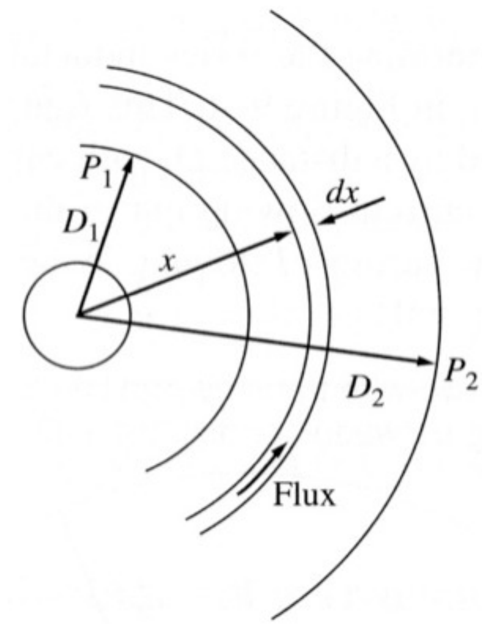
External Inductance

Outside the conductor surface, the flux links the full current carried by the conductor:

$$d\lambda \;=\; d\phi \;=\; \frac{\mu I}{2\pi x}\,dx \;\; [Wb - turns/m]$$

The total external flux linkages per meter can be found by integration:

$$\lambda_{ext} \;=\; \int d\lambda \;=\; \int_{D_1}^{D_2} \frac{\mu I}{2\pi x}dx \;=\; \frac{\mu I}{2\pi}\ln\frac{D_2}{D_1} \;\; [Wb - turns/m]$$

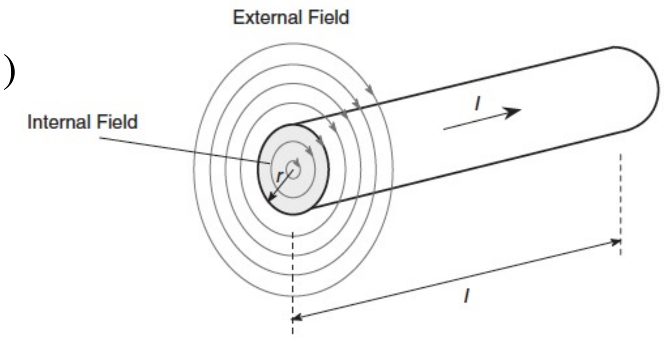


The external inductance per meter is:

$$L_{ext} \;=\; \frac{\lambda_{ext}}{I} = \frac{\mu}{2\pi}\ln\frac{D_2}{D_1} \;\; [H/m]$$

Total Flux Linkage and Total Inductance

$$\lambda_{total(per\ \ meter)} = \lambda_{total(\text{external})} + \lambda_{total(\text{internal})}$$



$$\lambda_{total(per\ \ meter)} = \frac{\mu_0}{2\pi} I \left(\ln \frac{D}{r} \right) + \frac{\mu_0}{8\pi} I$$

$$L(per\ \ meter) = \frac{\lambda_{total(per\ \ meter)}}{I} = \frac{\mu_0}{2\pi} \left(\ln \frac{D}{r} + \frac{1}{4} \right)$$

$$L(per\ \ meter) = \frac{4\pi \cdot 10^{-7}}{2\pi} \left(\ln \frac{D}{r} + \ln \left(e^{\frac{1}{4}} \right) \right) = 2 \cdot 10^{-7} \ln \left(\frac{D \cdot e^{\frac{1}{4}}}{r} \right) = 2 \cdot 10^{-7} \ln \left(\frac{D}{r \cdot e^{\frac{-1}{4}}} \right) = 2 \cdot 10^{-7} \ln \left(\frac{D}{r'} \right)$$

r’ = 0.7788r

Two important results to note:

The inductance is proportional to the natural logarithm of the ratio of distances
The result is problematic as the inductance per meter goes to infinity as D → ∞

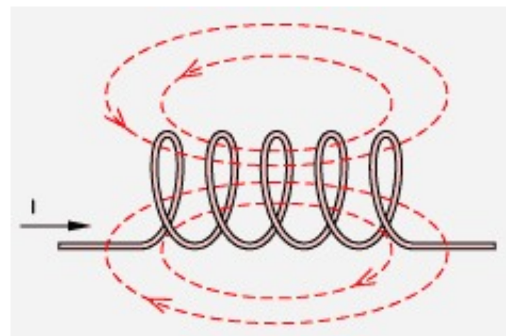
Flux Linkage

In an ideal inductor the flux generated by one of its turns would encircle all the other other turns.

Real coils come close to ideal behavior when the cross sectional dimensions of the windings are small compared with its diameter, or if the coil is wound on a high relative permeability core.

In longer air-core coils the situation is likely to be nearer to that shown in the figure (below).

Here we see that the flux density decreases towards the ends of the coil as some flux takes a 'short cut' bypassing the outer turns.



Given the current into the coil is 5 amps and eack line of flux represents 8mWb:

The three center turns link all four lines of flux = 4 x 8 = 32mWb
The two outer turns link only two lines of flux = 2 x 8 =16 mWb

λ = 3 x 32 + 2 x 16 = 128mWb –t L = λ/I =128/5 = 25.6mH

Law of Logarithms

Throughout the investigation to follow, many expressions can be simplified utilizing a few simple identities that you already know:

$$\ln(a \cdot b) = \ln(a) + \ln(b)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln(a^b) = b \cdot \ln(a)$$

$$\ln(e^a) = a \ln(e) = a \cdot 1 = a$$

$$\ln(1) = 0$$

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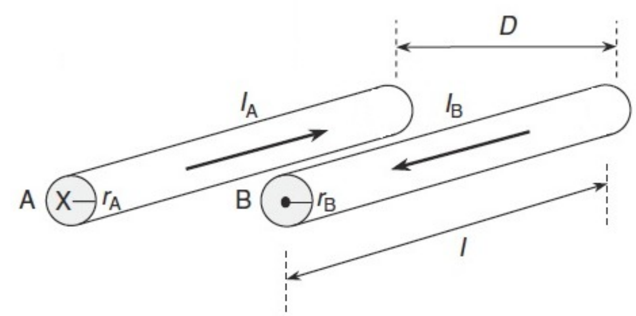
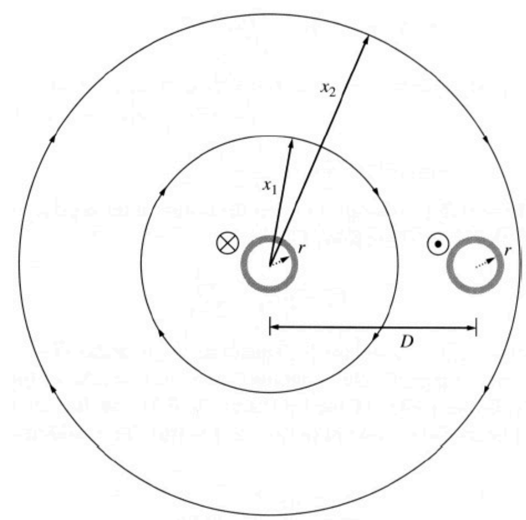
Inductance of a Single Phase Line

We next determine the series inductance of a single-phase line consisting of two conductors of radii r seperated by a distance D. The conductor "A" carries a current of magnitude I flowing into the page, and conductor "B" carries a current of magnitude I flowing out of the page.



Considering two circular integration paths; we find that the line integral along x_1 produces a net magnetic intensity since a non-zero net current is enclosed by x_1.

$$\oint H_x \cdot dl = I_{x\ (enclosed)}$$



However the path of radius x_2 encloses both conductors and the currents are equal and opposite, the net current enclosed is 0 and, therefore, there can be **NO** contributions to the total inductance from the magnetic fields at distances greater than D!

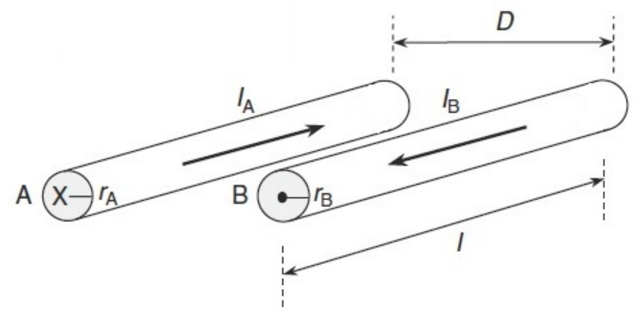
Inductance of a Single Phase Line

The total inductance of **a single wire** per unit length in a single phase transmission line is a sum of the internal inductance and the external inductance between the conductor surface (r) and the separation distance (D):

$$L = L_{\text{int}} + L_{ext} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln\frac{D}{r} \; [H/m]$$

substituting $\mu_0 = 4\pi \cdot 10^{-7}$

$$L = 2 \cdot 10^{-7}\left[\frac{1}{4} + \ln\frac{D}{r}\right] [H/m]$$



re-writing the ¼ factor in a more convenient form yields:

$$L = 2 \cdot 10^{-7}\left[\ln(e^{\frac{1}{4}}) + \ln\frac{D}{r}\right] [H/m]$$

Source: http://www.accessscience.com

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Inductance of a Single Phase Line

$$L = 2 \cdot 10^{-7} \left[\ln \frac{D}{re^{\frac{-1}{4}}} \right] [H/m]$$

$$L = 2 \cdot 10^{-7} \left[\ln \frac{D}{r'} \right] \; [H/m] \; \; where \; \; r' = re^{\frac{-1}{4}} or \; \; r' = \; 0.7788r$$

By symmetry, the inductance of the return wire is the same, therefore, the total inductance (sometimes called the loop inductance) of a two-wire transmission line is

$$L_{total} = 2L = 4 \cdot 10^{-7} \left[\ln \frac{D}{r'} \right] [H/m] \; \; where \; \; r' = re^{\frac{-1}{4}} or \; \; r' = \; 0.7788r$$

Where r’ is the “effective radius” of the conductors and D is the distance between conductors.

Inductance of a single phase line

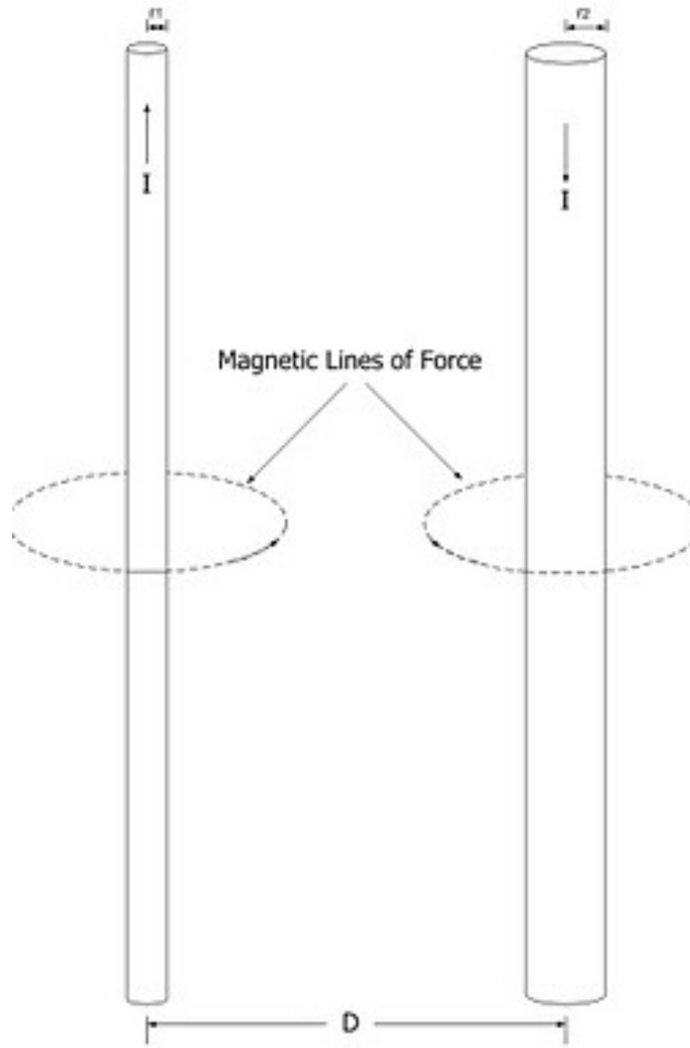
If one conductor had radius r_x and the other r_y

$$L_{total} = L_x + L_y = 2 \cdot 10^{-7}\left[\ln\frac{D}{r_x'}\right] + 2 \cdot 10^{-7}\left[\ln\frac{D}{r_y'}\right] \; [H/m]$$

$$L_{total} = L_x + L_y = 2 \cdot 10^{-7}\left[\ln\frac{D^2}{r_x' r_y'}\right] \; [H/m]$$

$$L_{total} = L_x + L_y = 2 \cdot 10^{-7}\left[\ln\frac{D^2}{\left(\sqrt{r_x' r_y'}\right)^2}\right] \; [H/m]$$

$$L_{total} = L_x + L_y = 4 \cdot 10^{-7}\left[\ln\frac{D}{\sqrt{r_x' r_y'}}\right] \; [H/m]$$



Multiple Current Carrying Conductors

Consider a point P external to the conductors,

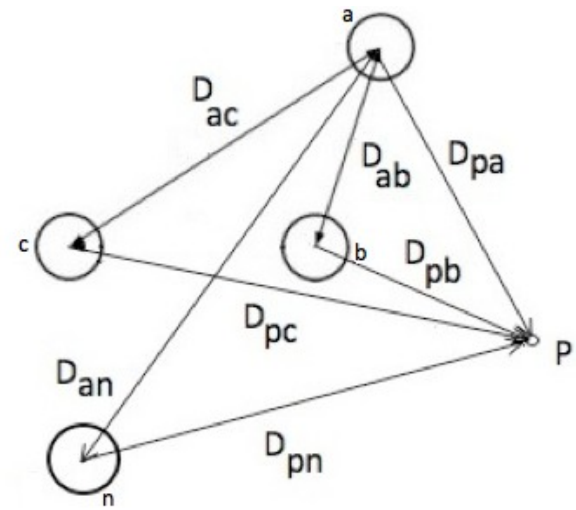
The flux linked by conductor **a** due to the current Ia includes all internal flux linkages

$$\lambda_{apa} = 2\cdot 10^{-7} I_a \ln \frac{D_{pa}}{r'}$$

The flux linked by conductor **a** due to the current $\mathbf{I_b}$ in conductor b

$$\lambda_{apb} = 2\cdot 10^{-7} I_b \ln \frac{D_{pb}}{D_{ab}}$$

The flux linked by conductor **a** due to the current $\mathbf{I_c}$



$$\lambda_{apc} = 2\cdot 10^{-7} I_c \ln \frac{D_{pc}}{D_{ac}}$$

$$\lambda_a = \lambda_{apa} + \lambda_{apb} + \lambda_{apc} + \ldots + \lambda_n$$

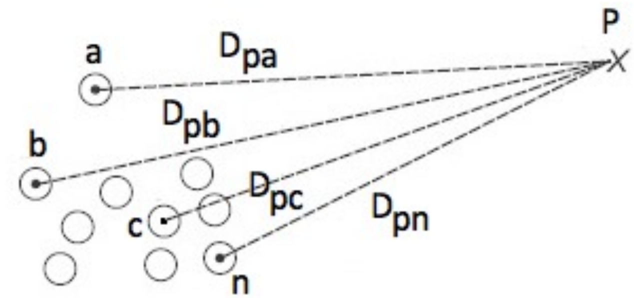
$$\lambda_a = 2\cdot 10^{-7}\left(I_a \ln \frac{D_{pa}}{r'} + I_b \ln \frac{D_{pb}}{D_{ab}} + I_c \ln \frac{D_{pc}}{D_{ac}} + \ldots + I_n \ln \frac{D_{pn}}{D_{an}} \right)$$

Multiple Carrying Conductors

$$\lambda_a = 2\cdot 10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{ab}} + I_c \ln\frac{1}{D_{ac}} + \ldots + I_n \ln\frac{1}{D_{an}}\right) +$$

$$2\cdot 10^{-7}\left(I_a \ln D_{pa} + I_b \ln D_{pb} + I_c \ln D_{pc} + \ldots + I_n \ln D_{pn}\right)$$

However as P → ∞ $\quad D_{pa} = D_{pb} = D_{pc} = D_{pn} = D$



$$\lambda_a = 2\cdot 10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{ab}} + I_c \ln\frac{1}{D_{ac}} + \ldots + I_n \ln\frac{1}{D_{an}}\right) +$$

$$2\cdot 10^{-7}\left(I_a + I_b + I_c + \ldots + I_n\right)\ln D$$

But the currents must sum to zero to satisfy KCL, therefore:

$$\lambda_a = 2\cdot 10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{ab}} + I_c \ln\frac{1}{D_{ac}} + \ldots + I_n \ln\frac{1}{D_{an}}\right)$$

Inductance of a symmetrically spaced 3 Phase Line

For a system with multiple conductors carrying currents that do not violate Kirchoff's Current Law, the flux linkages on the "kth" conductor can be calculated as:

$$\lambda_k = 2\cdot 10^{-7}\sum_{m=1}^{M} I_m \ln\frac{1}{D_{km}} \; [Wb-t/m]$$

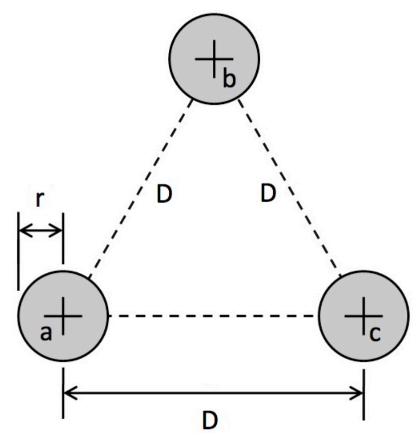
See pages 181-182 of you text

Applying this general result to a three phase three wire line seperated by equal spacing D

$$\lambda_a = 2\cdot 10^{-7}\left[I_a \ln\frac{1}{D_{aa}} + I_b \ln\frac{1}{D_{ab}} + I_c \ln\frac{1}{D_{ac}}\right] [Wb-t/m]$$

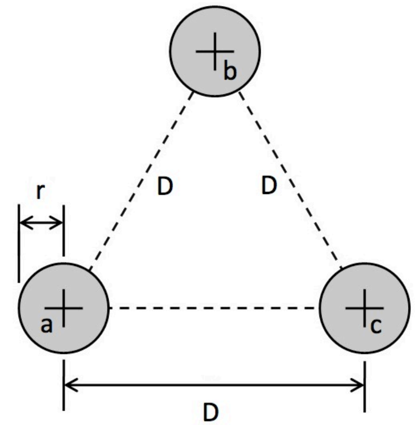
$$\lambda_a = 2\cdot 10^{-7}\left[I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D} + I_c \ln\frac{1}{D}\right] [Wb-t/m]$$

$$\lambda_a = 2\cdot 10^{-7}\left[I_a \ln\frac{1}{r'} + (I_b + I_c) \ln\frac{1}{D}\right] [Wb-t/m]$$



Inductance of a symmetrically spaced 3 Phase Line

but $I_a + I_b + I_c = 0$ therefore $I_b + I_c = -I_a$



$$\lambda_a = 2 \cdot 10^{-7} \left[I_a \ln \frac{1}{r'} + (-I_a) \ln \frac{1}{D} \right] [Wb - t/m]$$

$$\lambda_a = 2 \cdot 10^{-7} \left[I_a \ln \frac{D}{r'} \right] [Wb - t/m]$$

$$L_a = \frac{\lambda_a}{I_a} = 2 \cdot 10^{-7} \left[\ln \frac{D}{r'} \right] [H/m] \quad per \quad phase$$

Inductane is not a friend of the power engineer.
To reduce its effect we could:

a) reduce the distance between conductors
b) increase the diameter of the conductors

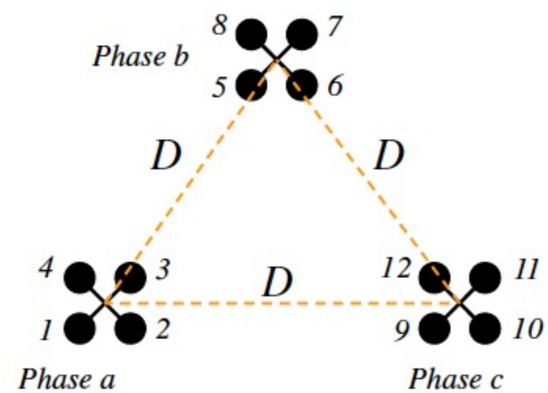
Bundled Conductors

Instead of using a single conductor per phase lets use “n” closely spaced conductors.
(Closely can be interperted as the distance between conductors is small compared to the distance between phases.)

Consider the a,b and c phase “conductors” are comprised of 4 conductors all of radius r

The flux linkages in conductor 1 of the **a phase** bundle can be calculated by:

$$\lambda_1 = 2\cdot 10^{-7}\left[\begin{array}{l} \frac{I_a}{4}(\ln\frac{1}{r'} + \ln\frac{1}{D_{12}} + \ln\frac{1}{D_{13}} + \ln\frac{1}{D_{14}}) + \\ \frac{I_b}{4}(\ln\frac{1}{D_{15}} + \ln\frac{1}{D_{16}} + \ln\frac{1}{D_{17}} + \ln\frac{1}{D_{18}}) + \\ \frac{I_c}{4}(\ln\frac{1}{D_{19}} + \ln\frac{1}{D_{110}} + \ln\frac{1}{D_{111}} + \ln\frac{1}{D_{112}}) \end{array}\right]$$



Bundled Conductors

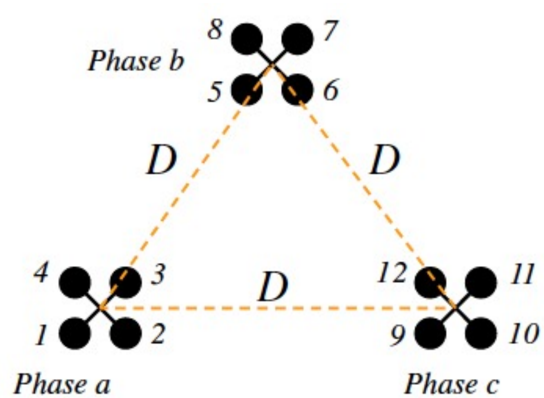
$$\lambda_1 = 2\cdot 10^{-7}\left[\begin{array}{l} I_a\left(\ln \dfrac{1}{\left(r' D_{12}D_{13}D_{14}\right)^{\frac{1}{4}}}\right) + \\ I_b\left(\ln \dfrac{1}{(D_{15}D_{16}D_{17}D_{18})^{\frac{1}{4}}}\right) + \\ I_c\left(\ln \dfrac{1}{(D_{19}D_{110}D_{111}D_{112})^{\frac{1}{4}}}\right) \end{array}\right]$$

The denominators by definition:

$GMR = R_b = \sqrt[4]{\left(r' D_{12}D_{13}D_{14}\right)}$ = geomatic mean radius of the square bundle

$D_{1b} = \sqrt[4]{(D_{15}D_{16}D_{17}D_{18})}$ = geometric mean distance from conductor 1 to phase b

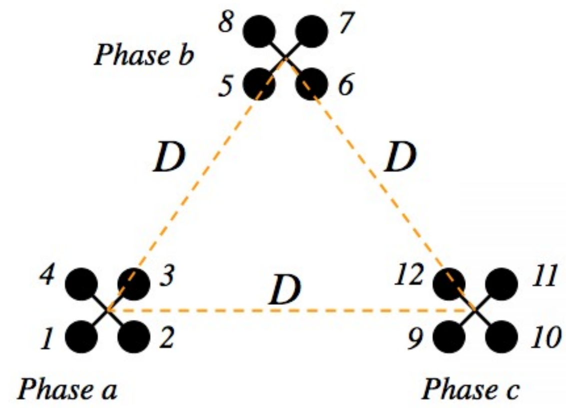
$D_{1c} = \sqrt[4]{(D_{19}D_{110}D_{111}D_{112})}$ = geometric mean distance from conductor 1 to phase c



Bundled Conductors

From the schematic

$$D_{1b} \approx D_{2b} \approx D_{3b} \approx D_{4b} \approx D_{ab}$$
$$D_{1c} \approx D_{2c} \approx D_{3c} \approx D_{4c} \approx D_{ac}$$
$$D_{ab} = D_{ac} = D_{bc} = D$$



$$\lambda_1 = 2 \cdot 10^{-7} \left[I_a \left(\ln \frac{1}{R_b} \right) + (I_b + I_c) \left(\ln \frac{1}{D} \right) \right]$$

but $(I_b + I_c) = -I_a$

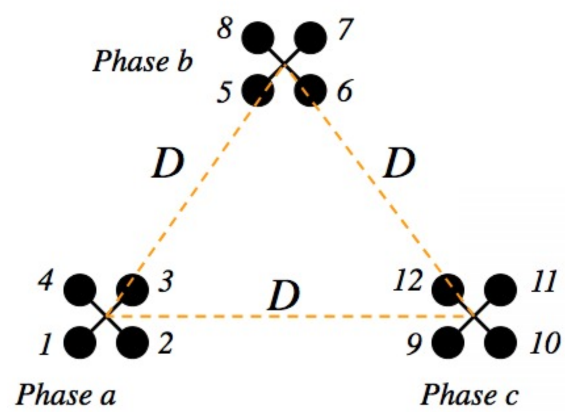
$$\lambda_1 = 2 \cdot 10^{-7} \left[I_a \left(\ln \frac{1}{R_b} \right) - I_a \left(\ln \frac{1}{D} \right) \right]$$

$$\lambda_1 = 2 \cdot 10^{-7} \left[I_a \left(\ln \frac{D}{R_b} \right) \right]$$

Bundled Conductors

$$\lambda_1 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{D}{R_b}\right)\right]$$

If the current divides evenly in the bundled conductors $I_a = 4I_1$



$$\lambda_1 = 2\cdot 10^{-7}\left[4I_1\left(\ln\frac{D}{R_b}\right)\right]$$

$$L_1 = \frac{\lambda_1}{I_1} = 2\cdot 10^{-7}\left[4\left(\ln\frac{D}{R_b}\right)\right]$$

This is the self inductance of wire 1

There are four conductors per bundle in our example and by symmetry $L_1 = L_2 = L_3 = L_4$

The bundled conductors are in parallel, therefore the inductance of the **a phase** $L_a = L_1/4$

$$L_a = 2\cdot 10^{-7}\left(\ln\frac{D}{R_b}\right) \quad or \quad L_a = 2\cdot 10^{-7}\left(\ln\frac{GMD}{GMR}\right) \quad H/m$$

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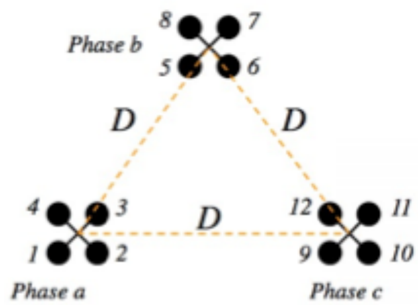
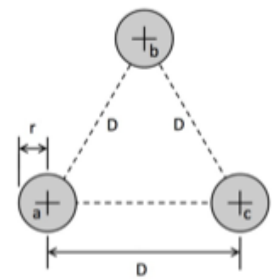
Bundled Conductors





GMR - Geometric Mean Radius $= R_b$ = the effective radius of the bundle

$$\text{GMR} = R_b = \sqrt[16]{(D_{11}D_{12}D_{13}D_{14})(D_{22}D_{21}D_{23}D_{24})(D_{33}D_{31}D_{32}D_{34})(D_{44}D_{41}D_{42}D_{43})}$$

$$\text{GMR} = R_b = \sqrt[16]{(r'_1\,D_{12}D_{13}D_{14})(r_2'\,D_{21}D_{23}D_{24})(r_3'\,D_{31}D_{32}D_{34})(r_4'1D_{41}D_{42}D_{43})}$$

$$\text{GMR} = R_b = \sqrt[16]{(r'_1\,D_{12}D_{13}D_{14})^4}$$

$$\text{GMR} = R_b = \sqrt[4]{(r'_1\,D_{12}D_{13}D_{14})}$$

GMD - Geometric Mean Distance the mean distance between bundles

$$\text{GMD} = D = \sqrt[3]{(D_{ab}D_{ac}D_{bc})}$$

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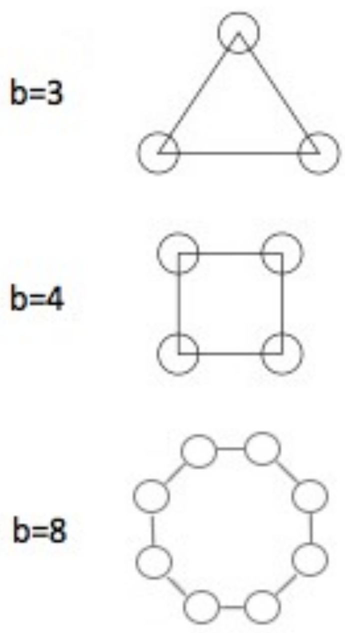
Bundled Conductors



As b (the number of conductors in the bundle) gets large, the bundle begins to take on the configuration of a circle, with all current flowing at the circumference.

The bundle resembles a hollow conductor!

So the effective bundle radius R_b increases from r' (for the case of b=1) to the radius of the circle comprised by the bundle as b gets large.

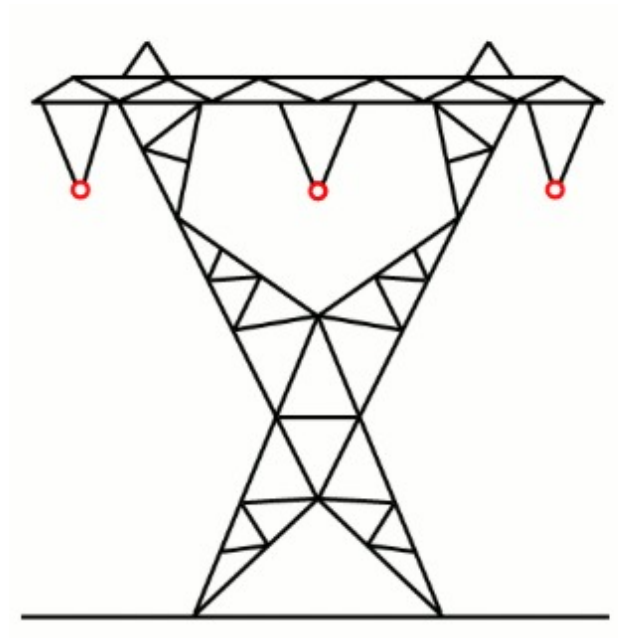
As R_b increases the inductance decreases.

This is the effect we desire!

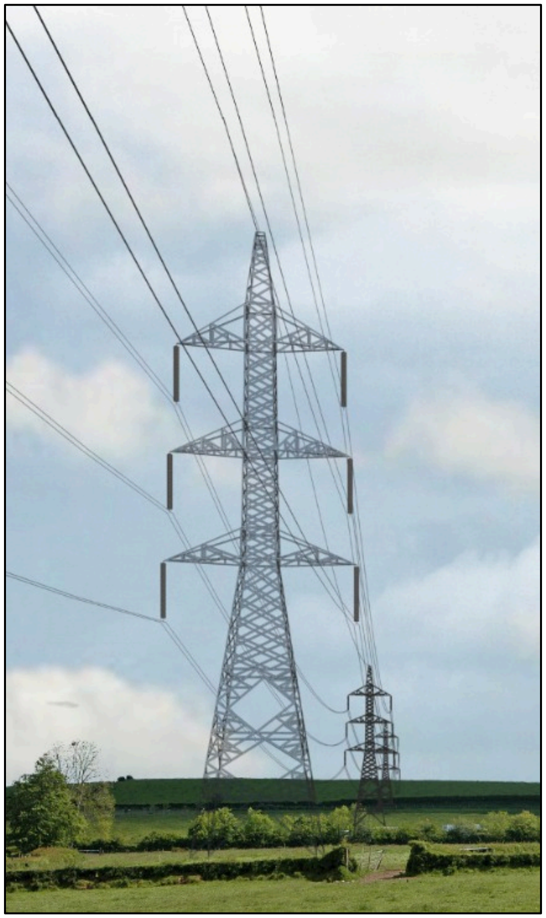
Practical Tower Configurations

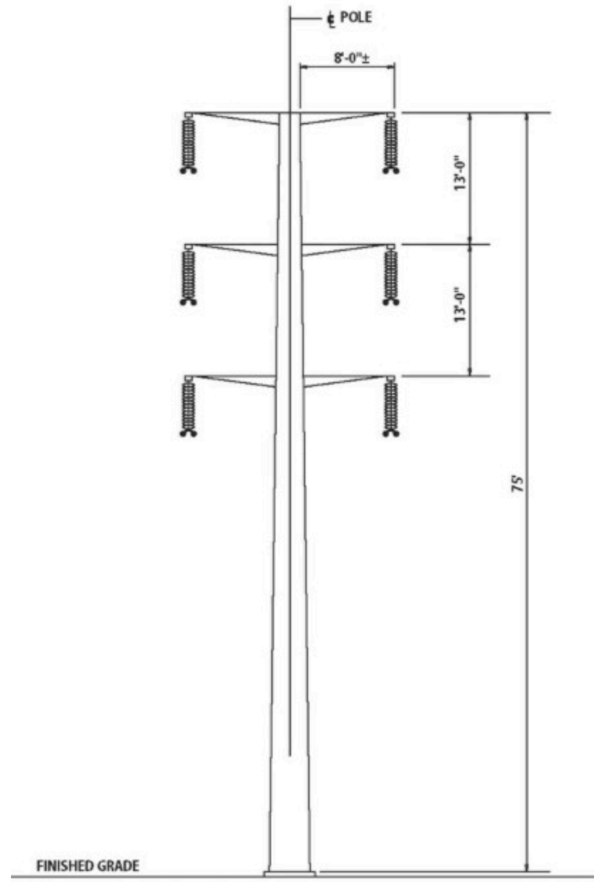
I fooled you (once again) into believing transmission lines will all have symmetrically spaced conductors.

In fact this is seldom the case!



Practical Tower Configurations





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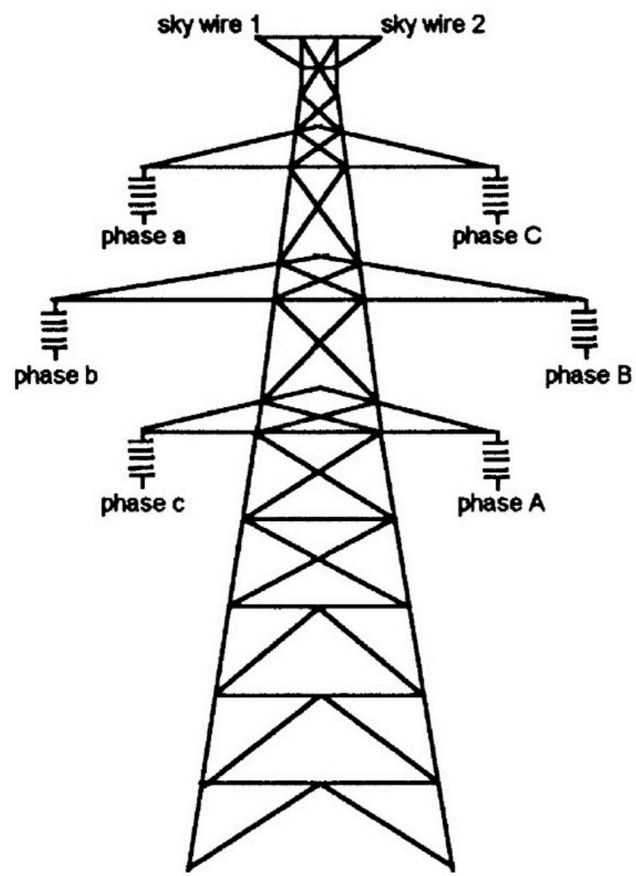
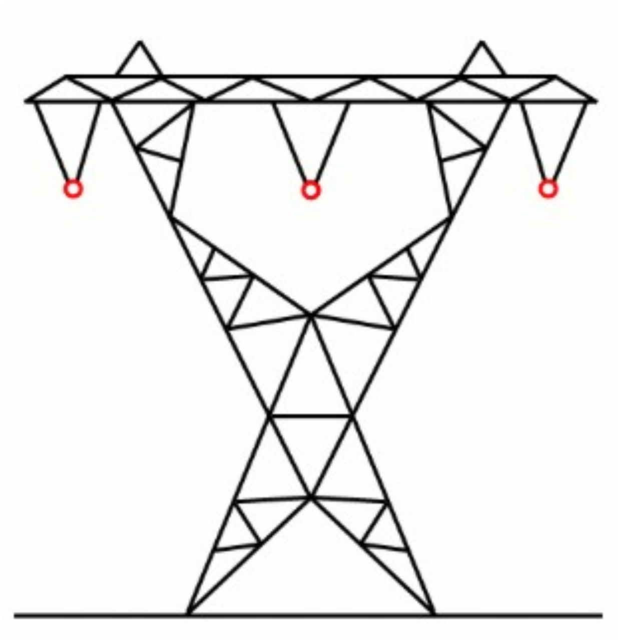
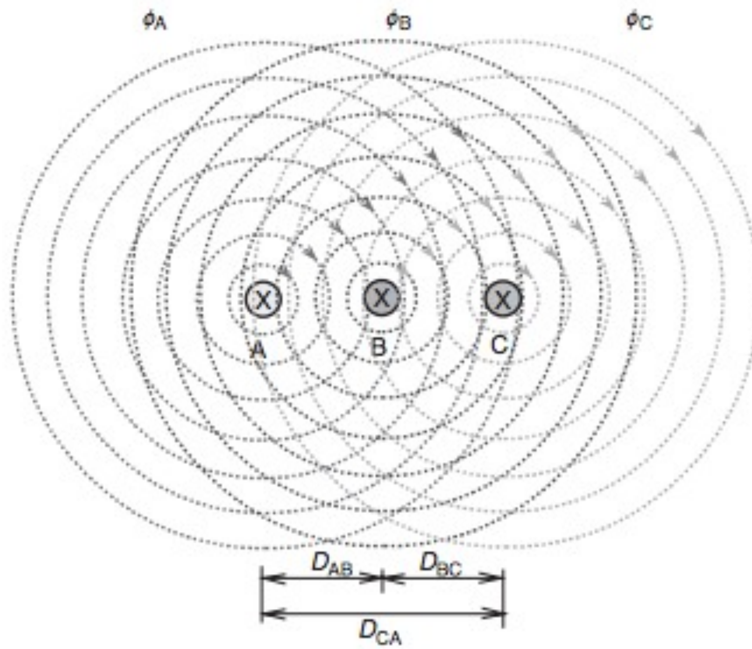
Practical Tower Configurations



Unbalance in the system





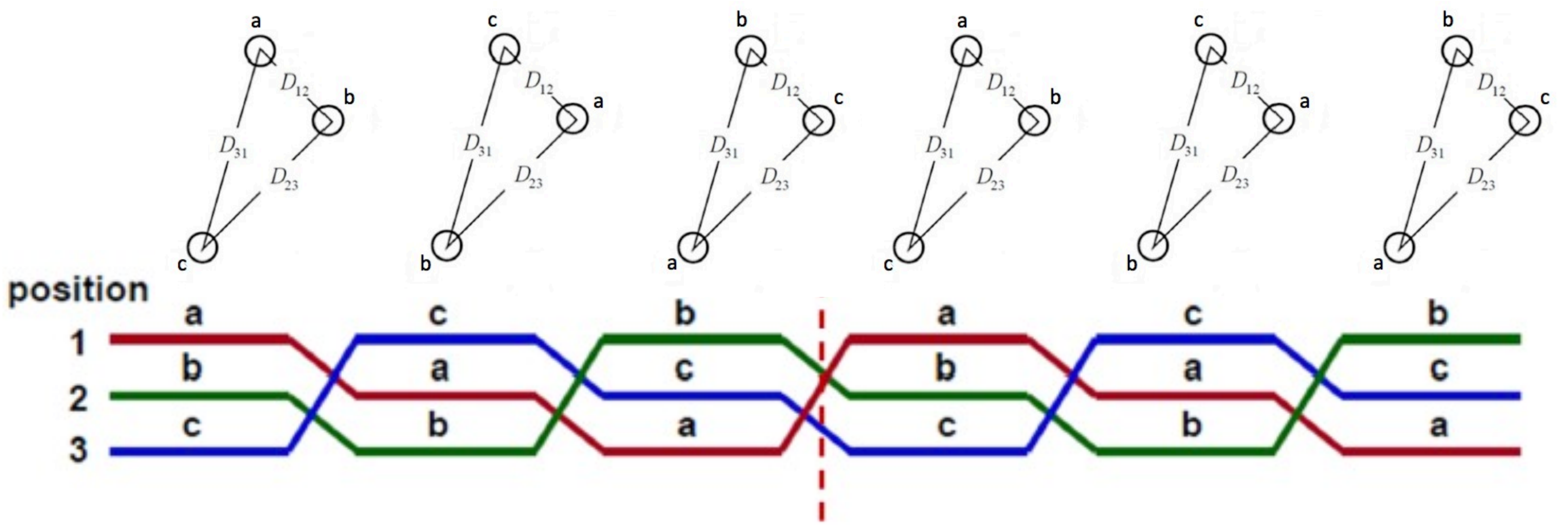
Transposition

Arranging the phase conductors in an equilateral triangle configuration is vot very practical from a construction ease, maintenance or cost consideration.

It is easily seen in the vertical or horizontal configurations that all symmetry is lost.

$$D_{ab} \neq D_{bc} \neq D_{ca}$$

Symmetry is regained by employing **TRANSPOSITION**



Think of this as looking at a flat T-line configuration from the top or a vertical T-line configuration from the side.

Transposition

To keep the system balanced, the conductors are "rotated" over the length of a transmission line so each phase occupies each position on the tower for an equal distance.

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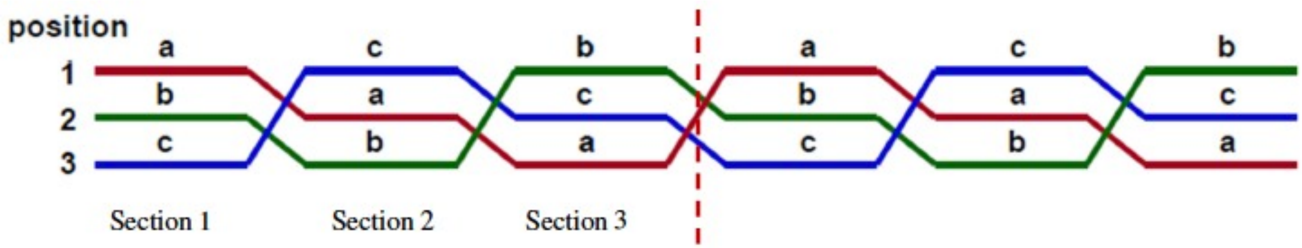
Transposition

Transposition

In a completely transposed each conductor spends 1/3 of the time in each of the three positions.

For a single conductor of radiur r in each of the phases:



$$\lambda a_1 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{1}{r'}\right) + I_b\left(\ln\frac{1}{D_{12}}\right) + I_c\left(\ln\frac{1}{D_{13}}\right)\right]$$ "a" phase in position 1

$$\lambda a_2 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{1}{r'}\right) + I_b\left(\ln\frac{1}{D_{23}}\right) + I_c\left(\ln\frac{1}{D_{21}}\right)\right]$$ "a" phase in position 2

$$\lambda a_3 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{1}{r'}\right) + I_b\left(\ln\frac{1}{D_{31}}\right) + I_c\left(\ln\frac{1}{D_{32}}\right)\right]$$ "a" phase in position 3

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Transposition

$$\overline{\lambda_a} = \frac{\lambda a_1 + \lambda a_2 + \lambda a_3}{3}$$

$$\lambda_a = \frac{2 \cdot 10^{-7}}{3}\left[3I_a\left(\ln\frac{1}{r'}\right) + I_b\left(\ln\frac{1}{D_{12}D_{23}D_{31}}\right) + I_c\left(\ln\frac{1}{D_{21}D_{32}D_{13}}\right)\right]$$

but Ib + Ic = -Ia

$$\lambda_a = \frac{2 \cdot 10^{-7}}{3}\left[3I_a\left(\ln\frac{1}{r'}\right) - I_a\left(\ln\frac{1}{D_{12}D_{23}D_{31}}\right)\right] = \frac{2 \cdot 10^{-7}}{3}\left[3I_a\left(\ln\frac{1}{r'}\right) + 3I_a\left(\ln\sqrt[3]{D_{12}D_{23}D_{31}}\right)\right]$$

$$\lambda_a = 2 \cdot 10^{-7}\left[I_a\left(\ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'}\right)\right] \qquad L_a = \frac{\lambda_a}{I_a} = 2 \cdot 10^{-7}\left(\ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'}\right)$$

$$Deq = \sqrt[3]{D_{12}D_{23}D_{31}} \ \text{(the GMD between the phases)}$$

For solid conductors use r’, for stranded conductors or bundles use the **GMR** of the phase

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