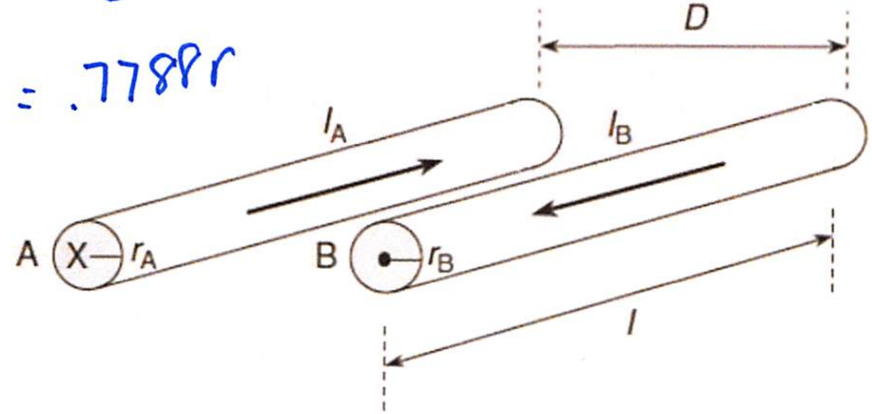
$r' = e^{-1/4} r$

$= .7788r$

$2\times10^{-7} = \frac{\mu_0}{2\pi} \quad \frac{H}{m}$



$R_A = \frac{\rho l}{A}$

$R_{T_2} = R_{T_1}$ ↓

$$\rho_{T_2} = \rho_{T_1}\left(\frac{M + T_2}{M + T_1}\right)$$

Find the (50C) resistance and inductive reactance of the single phase line comprised of two #6 AWG hard drawn copper wire:
Outside Diameter = 0.1620in
ρ = 10.66 ΩCM/ft @ 20C M= 241.5
The line length(l) = 4.0 miles line spacing(D) = 15feet

$A_{CM} = d^2_{MILS} = \quad .162in \frac{1 mil}{.001 in} = \quad 162 mils = d \quad d^2 = 26244\ CM$

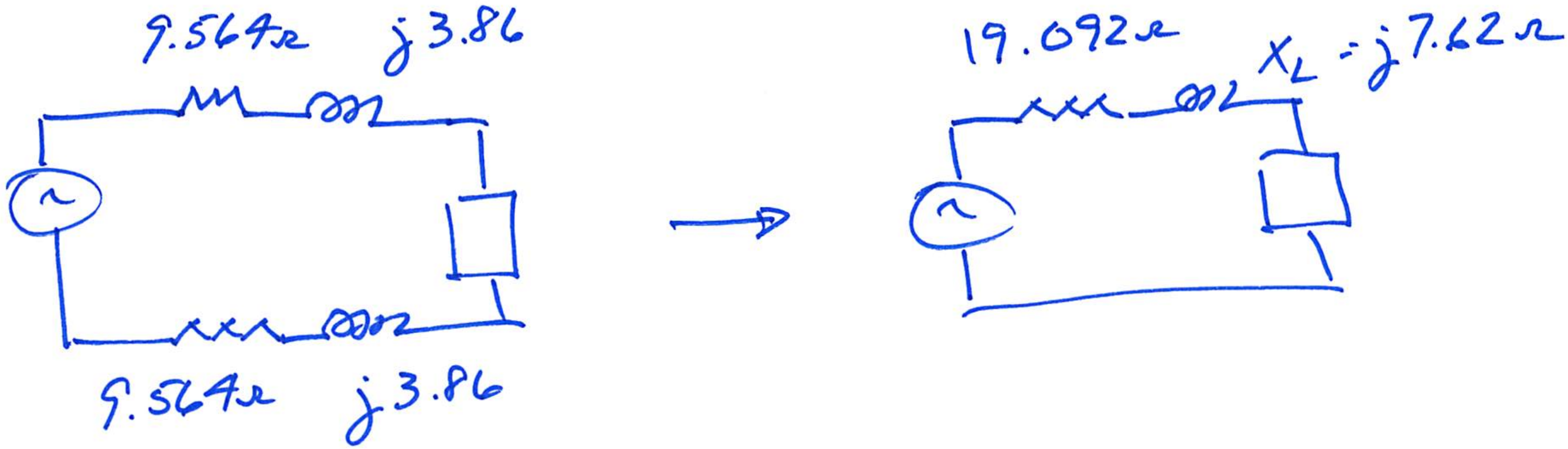
$10.66 \frac{\Omega\ CM}{ft} \times \frac{1}{26244\ CM} \times \frac{5280\ ft}{1\ mile} = 2.1447 \frac{\Omega}{mile}$ @ 20°C

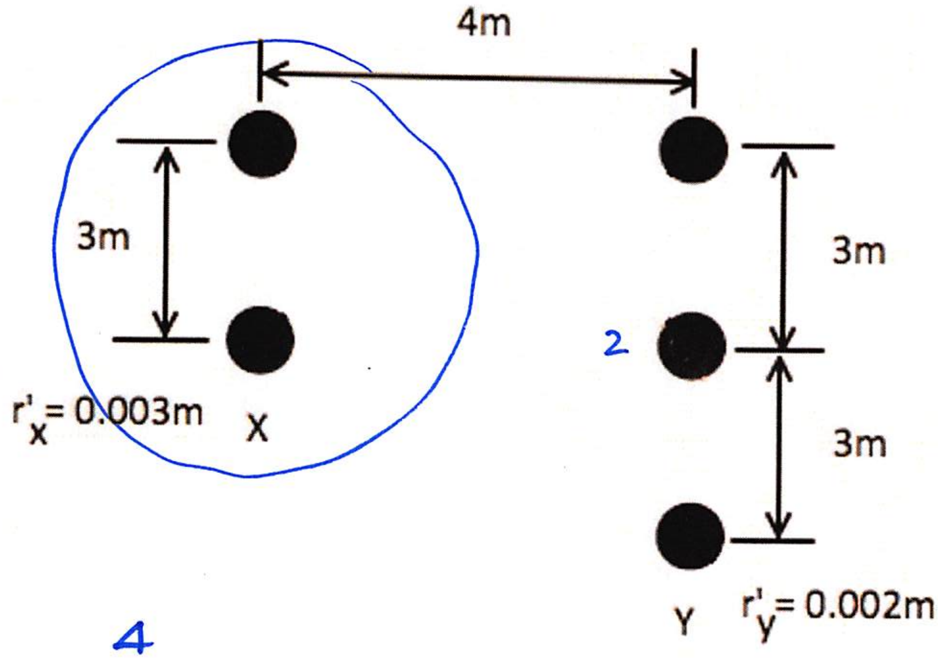
$R_{A_{50C}} = 2.1447\left(\frac{241.5+50}{241.5+20}\right) = 2.391 \frac{\Omega}{mi} \times 4mi = 9.564\ \Omega$/conductor

$L_A = 2\times10^{-7} \ln \frac{D}{r'} = 2\times10^{-7} \ln \frac{15 ft \times \left(\frac{12 in}{1 ft}\right)}{.7788 \times \frac{.162 in}{2}} = 1.591\times10^{-6}\ H/m$

$1.591\times10^{-6}\ H/m \times \frac{1609 m}{mi} = 2.56 \frac{mH}{mi} \times 4mi = 10.24\ mH$/conductor

$X_L = j\omega L = j2\pi f L = j2\pi\, 60 \times 10.24\times10^{-3} = j\,3.86\,\Omega$


9.564Ω
j3.86
9.564Ω
j3.86
19.092Ω
$X_L = j7.62\Omega$



$$L = 2\times10^{-7} \ln \frac{GMD}{GMR}$$

$$\ln \frac{GMD}{R_b'}$$

effective radius of the bundle

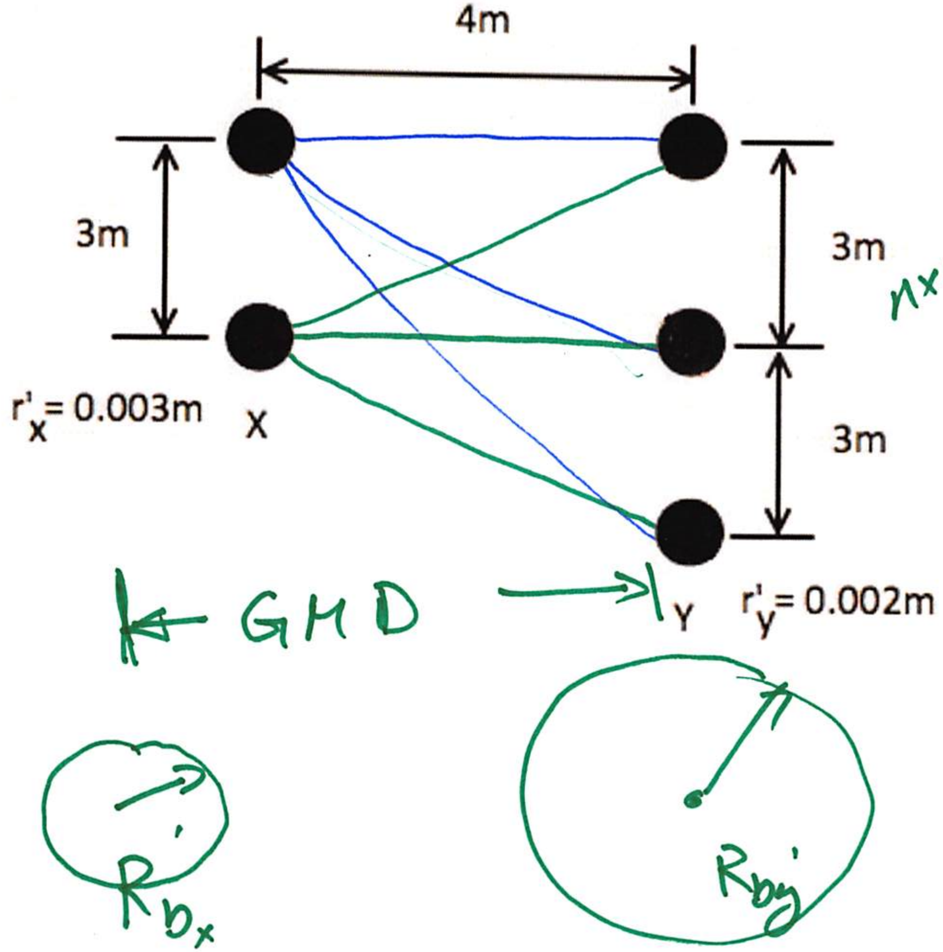
$$R_{b_x}' \quad \sqrt[2^2]{D_{x1-x1} D_{x1-x2} D_{x2-x2} D_{x2-x1}} = \sqrt[4]{(.003)(3)(.003)(3)}$$

$$= .09487\text{ m}$$

$$R_{b_y}' \quad \sqrt[9]{D_{y1-y1} D_{y1-y2} D_{y1-y3} D_{y2-y2} D_{y2-y1} D_{y2-y3} D_{y3-y3} D_{y3-y1} D_{y3-y2}}$$

$$\sqrt[9]{(.002)(3)(6)(.002)(3)(3)(.002)(6)(3)}$$

$$= .30571\text{ m}$$



MEAN DISTANCE BETWEEN ALL THE X CONDUCTORS TO ALL THE Y CONDUCTORS

$$\sqrt[6]{(4)(5)(\underbrace{\sqrt{4^2+6^2}}_{7.21})(5)(4)(5)} \quad n \times m$$

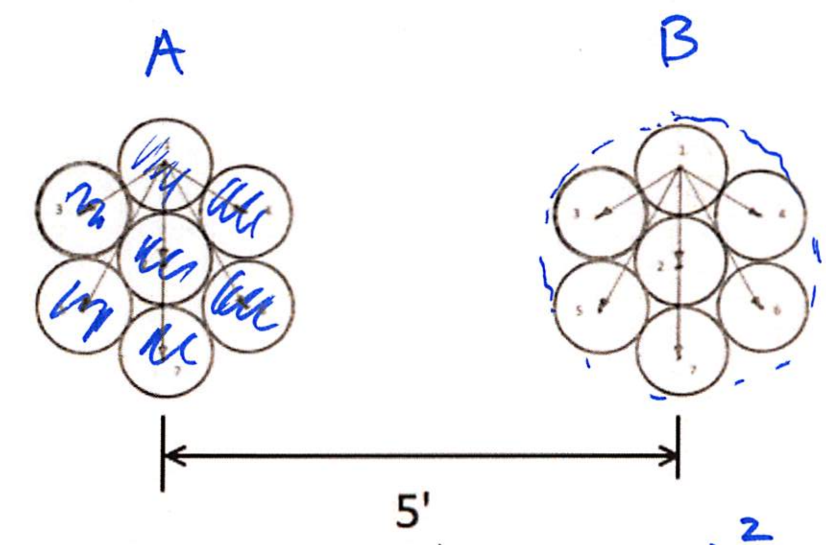
$$GMD = 4.9337\ m$$

$$L_x = 2\times10^{-7} \ln \frac{4.9337}{.0947}$$

$$= .7903\times10^{-6}\ H/m$$

$$L_y = 2\times10^{-7} \ln \frac{4.9337}{.3057}$$

$$= 1.3465\times10^{-6}\ H/m$$



$R = \frac{\rho l}{A}$

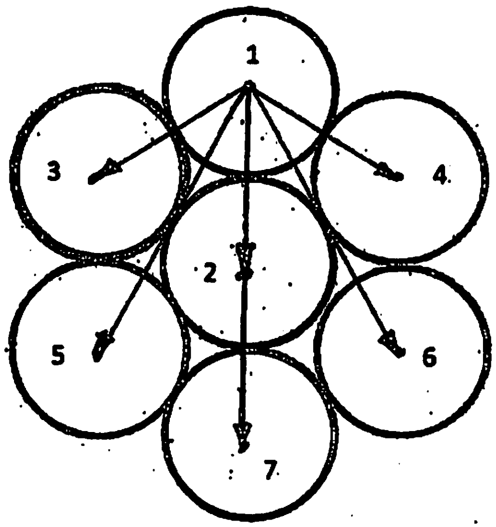
Table A3 pg 923
3/0 Aluminum conductor
7 strands of 0.1548" diameter conductors
Outside Diameter = 0.464"
ρ(20C) = 10.66 Ωcmil/ft   M = 241.5

$$A_{CM} = \left(\frac{.1548\text{in}}{} \Big| \frac{1\text{mil}}{.001\text{in}}\right)^2 \times 7 = A_{TOTAL_{CM}} = 167741 \text{ CM}$$

$$10.66 \frac{\Omega \text{CM}}{\text{ft}} \times \frac{1}{167741\text{CM}} \times \frac{5280\text{ft}}{1\text{mile}} = .3355 \frac{\Omega}{\text{mile}} \text{ @ } 20°C$$

$$L_A = L_B = 2\times10^{-7} \ln \frac{D_{AB}}{R_b'}$$

$$R_b' = \sqrt[49]{(D_{1-1} D_{1-2} D_{1-3} D_{1-4} D_{1-5} D_{1-6} D_{1-7})(D_{22} D_{21} D_{23} D_{24} D_{25} D_{26} D_{2-7})}$$



3/0 Copper conductor — 7 strands of 0.1548" diameter conductors
Outside Diameter = 0.464"
$\rho_{(20C)}$ = 10.66 Ωcmil/ft M = 241.5

Determine the resistance of the conductor per mile at 20°C and at 50°C.

Area (in cmil) = $7 \times D^2 = 7 \times 154.8^2 = 167{,}741$ cmils

(10.66 Ωcmil/ft / 167,741cmils) x 5280 ft/mile = 0.3355 Ω/mile

$$R(50°C) = 0.3355 \times \frac{241.5 + 50}{241.5 + 20} = 0.374\Omega/mi$$

Table A3 pg 816 reports 0.381Ω/mi 0.381/0.374 = 1.019
1.9% increase due to spiraling

GMR $[(D_{11}D_{12}D_{13}D_{14}D_{15}D_{16}D_{17})^6 \times (D_{22}\, D_{21}D_{23}D_{24}D_{25}D_{26}D_{27})]^{1/49}$

$$GMR = [(0.7788r \times 2r \times 2r \times 2r \times 2\sqrt{3}r \times 2\sqrt{3}r \times 4r)^6 \times (0.7788r \times 2r \times 2r \times 2r \times 2r \times 2r \times 2r)]^{1/49}$$

D = 0.1548" therefore r = 0.0774 GMR = 0.1685" = 0.01404' same as in the table

$L_x = 2 \times 10^{-7} \ln(D_{xy}/GMR) = 2 \times 10^{-7} \ln(5/\, 0.01404) = 1.17507\ \mu H/m$

$X_{Lx} = 2\pi 60 \times 1.17507\ \mu H/m \times 1609$ m/mile = 0.7128 Ω /mile

Or

$X_L = 2\pi 60 \times 2 \times 10\text{-}7 \times 1609$ m/mile $[\ln(Deq/\, Dsl)]$

$X_L = 0.121316 \ln(1/Dsl) + 0.121316 \ln Deq = 0.518 + 0.121316 \ln(5) = 0.713$ Ω/mile

X_a = Inductive reactance in ohms per conductor per mile at 1 ft spacing

SIZE AWG / kcmil	STRANDS No.	STRANDING CLASS	STRAND DIAMETER (mils)	CROSS SECTION (sq inches)	CONDUCTOR DIAMETER (inches)	TOTAL WEIGHT (lb/1000ft)	DC RESISTANCE AT 20°C [1] (ohm/ft)			NOMINAL TENSILE STRENGTH [2] (lb)		AMPACITY [3] (A)	GEOMETRIC MEAN RADIUS (inches)	INDUCTIVE REACTANCE [4] (ohm/mile)	CAPACITIVE REACTANCE [4] (Mohm-mile
							SOFT	MEDIUM HARD	HARD	MEDIUM HARD	HARD				
20	1	Sólido	31.97	0.000801	0.0320	3.09	10.2	10.5	10.6	N/A	N/A	15	0.0125	0.8337	0.1964
20	7	B	12.09	0.000801	0.0363	3.15	10.4	10.7	10.8	N/A	N/A	15	0.0132	0.8270	0.1927
18	1	Sólido	40.28	0.00127	0.0403	4.90	6.40	6.62	6.66	73	85	20	0.0157	0.8057	0.1895
18	7	B	15.24	0.00127	0.0457	5.00	6.53	6.76	6.79	N/A	N/A	21	0.0166	0.7989	0.1858
16	1	Sólido	50.83	0.00203	0.0508	7.81	4.02	4.16	4.18	117	135	27	0.0198	0.7775	0.1826
16	7	B	19.21	0.00203	0.0576	7.97	4.10	4.24	4.26	N/A	N/A	27	0.0209	0.7707	0.1789
14	1	Sólido	64.13	0.00323	0.0641	12.4	2.52	2.61	2.62	183	213	36	0.0250	0.7492	0.1757
14	7	B	24.25	0.00323	0.0728	12.7	2.57	2.66	2.68	N/A	N/A	37	0.0264	0.7425	0.1720
12	1	Sólido	80.83	0.00513	0.0808	19.8	1.59	1.64	1.65	291	339	48	0.0315	0.7212	0.1689
12	7	B	30.55	0.00513	0.0917	20.2	1.62	1.68	1.68	N/A	N/A	49	0.0333	0.7145	0.1652
10	1	Sólido	101.89	0.00815	0.102	31.4	1.00	1.03	1.04	463	526	64	0.0397	0.6931	0.1620
10	7	B	38.54	0.00815	0.116	32.0	1.02	1.05	1.06	N/A	N/A	65	0.0420	0.6863	0.1583
8	1	Sólido	128.50	0.0130	0.129	50.0	0.628	0.650	0.653	734	828	85	0.0501	0.6649	0.1551
8	7	B	48.58	0.0130	0.146	51.0	0.641	0.663	0.666	661	779	87	0.0529	0.6582	0.1514
6	1	Sólido	162.01	0.0206	0.162	79.4	0.395	0.409	0,411	1166	1286	113	0.0631	0.6368	0.1483
6	7	B	61.26	0.0206	0.184	81.0	0.403	0.417	0.419	1052	1225	116	0.0667	0.6300	0.1445
4	1	Sólido	204.33	0.0328	0.204	126	0.248	0.257	0.258	1856	1974	151	0.0796	0.6086	0.1414
4	7	B	77.24	0.0328	0.232	129	0.253	0.262	0.264	1671	1948	154	0.0841	0.6019	0.1376
2	7	B	97.40	0.0521	0.292	205	0.159	0.165	0.166	2657	3030	206	0.106	0.5738	0.1308
1	7	A	109.37	0.0657	0.328	258	0.126	0.131	0.131	3349	3820	238	0.119	0.5597	0.1273
1	19	B	66.38	0.0657	0.332	258	0.126	0.131	0.131	3349	3905	239	0.126	0.5531	0.1270
1/0	7	A	122.83	0.0829	0.369	326	0.100	0.104	0.104	4224	4764	276	0.134	0.5456	0.1239
1/0	19	B	74.57	0.0829	0.373	326	0.100	0.104	0.104	4224	4928	276	0.141	0.5390	0.1235
2/0	7	A	137.91	0.105	0.414	411	0.0795	0.0822	0.0827	5324	5938	318	0.150	0.5316	0.1204
2/0	19	B	83.70	0.105	0.419	411	0.0795	0.0822	0.0827	5324	6141	319	0.159	0.5249	0.1201
3/0	7	A	154.84	0.132	0.465	518	0.0630	0.0652	0.0656	6711	7399	368	0.169	0.5175	0.1170

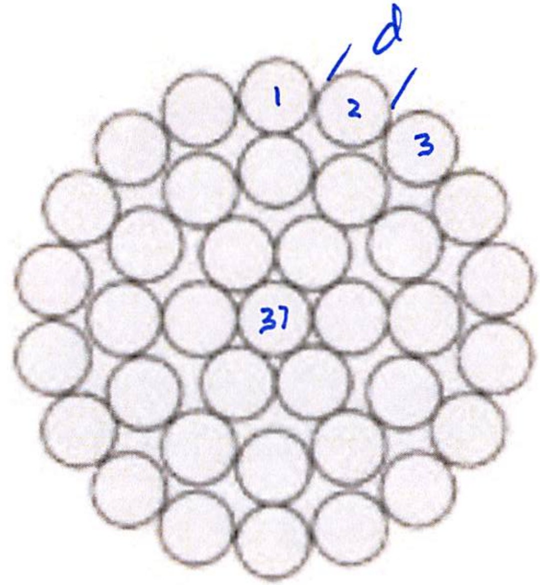
Characteristics of aluminum cable, steel, reinforced (Aluminum Company of America) ACSR

Code Word	Circular Mils Aluminum	Aluminum		Aluminum Strand Diameter (inches)	Steel	Steel Strand Diameter (inches)	Outside Diameter (inches)	Copper Equivalent* Circular Mils or A W G	Ultimate Strength (pounds)	Weight (pounds per mile)	Geometric Mean Radius at 60 Hz (feet)	Approx. Current Carrying Capacity† (amps)	r_a Resistance (Ohms per Conductor per Mile) 25°C (77°F) Small Currents dc	25 Hz	50 Hz	60 Hz	50°C (122°F) Current Approx 75% Capacity‡ dc	25 Hz	50 Hz	60 Hz	x_a Inductive Reactance (ohms per conductor per mile at 1 ft spacing all currents) 60 Hz	x'_a Shunt Capacitive Reactance (megohms per conductor per mile at 1 ft spacing) 60 Hz
Joree	2 515 000	76		0.1819	19	0.0849	1.880		61 700		0.0621									0.0450	0.337	0.0755
Thrasher	2 312 000	76		0.1744	19	0.0814	1.802		57 300		0.0595									0.0482	0.342	0.0767
Kiwi	2 167 000	72	4	0.1735	7	0.1157	1.735		49 800		0.0570									0.0511	0.348	0.0778
Bluebird	2 156 000	84	4	0.1602	19	0.0961	1.762		60 300		0.0588									0.0505	0.344	0.0774
Chukar	1 781 000	84	4	0.1456	19	0.0874	1.602		51 000		0.0534									0.0598	0.355	0.0802
Falcon	1 590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	56 000	10 777	0.0520	1 380	0.0587	0.0588	0.0590	0.0591	0.0646	0.0656	0.0675	0.0684	0.359	0.0814
Parrot	1 510 500	54	3	0.1673	19	0.1004	1.506	950 000	53 200	10 237	0.0507	1 340	0.0618	0.0619	0.0621	0.0622	0.0680	0.0690	0.0710	0.0720	0.362	0.0821
Plover	1 431 000	54	3	0.1628	19	0.0977	1.465	900 000	50 400	9 699	0.0493	1 300	0.0652	0.0653	0.0655	0.0656	0.0718	0.0729	0.0749	0.0760	0.365	0.0830
Martin	1 351 000	54	3	0.1582	19	0.0949	1.424	850 000	47 600	9 160	0.0479	1 250	0.0691	0.0692	0.0694	0.0695	0.0761	0.0771	0.0792	0.0803	0.369	0.0838
Pheasant	1 272 000	54	3	0.1535	19	0.0921	1.382	800 000	44 800	8 621	0.0465	1 700	0.0734	0.0735	0.0737	0.0738	0.0808	0.0819	0.0840	0.0851	0.372	0.0847
Grackle	1 192 500	54	3	0.1486	19	0.0892	1.338	750 000	43 100	8 082	0.0450	1 160	0.0783	0.0784	0.0786	0.0788	0.0862	0.0872	0.0894	0.0906	0.376	0.0857
Finch	1 113 000	54	3	0.1436	19	0.0862	1.293	700 000	40 700	7 544	0.0435	1 110	0.0839	0.0840	0.0842	0.0844	0.0924	0.0935	0.0957	0.0969	0.380	0.0867
Curlew	1 033 500	54	3	0.1384	7	0.1384	1.246	650 000	37 100	7 019	0.0420	1 060	0.0903	0.0905	0.0907	0.0909	0.0994	0.1005	0.1025	0.1035	0.385	0.0878
Cardinal	954 000	54	3	0.1329	7	0.1329	1.196	600 000	34 200	6 479	0.0403	1 010	0.0979	0.0980	0.0981	0.0982	0.1078	0.1088	0.1118	0.1128	0.390	0.0890
Canary	900 000	54	3	0.1291	7	0.1291	1.162	566 000	32 300	6 112	0.0391	970	0.104	0.104	0.104	0.104	0.1145	0.1155	0.1175	0.1185	0.393	0.0898
Crane	874 500	54	3	0.1273	7	0.1273	1.146	550 000	31 400	5 940	0.0386	950	0.107	0.107	0.107	0.108	0.1178	0.1188	0.1218	0.1228	0.395	0.0903
Condor	795 000	54	3	0.1214	7	0.1214	1.093	500 000	28 500	5 399	0.0368	900	0.117	0.118	0.118	0.119	0.1288	0.1308	0.1358	0.1378	0.401	0.0917
Drake	795 000	26	2	0.1749	7	0.1360	1.108	500 000	31 200	5 770	0.0375	900	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.399	0.0912
Mallard	795 000	30	2	0.1628	19	0.0977	1.140	500 000	38 400	6 517	0.0393	910	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.393	0.0904
Crow	715 500	54	3	0.1151	7	0.1151	1.036	450 000	26 300	4 859	0.0349	830	0.131	0.131	0.131	0.132	0.1442	0.1452	0.1472	0.1482	0.407	0.0932
Starling	715 500	26	2	0.1659	7	0.1290	1.051	450 000	28 100	5 193	0.0355	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.405	0.0928
Redwing	715 500	30	2	0.1544	19	0.0926	1.081	450 000	34 600	5 865	0.0372	840	0.131	0.131	0.131	0.131	0.1442	0.1447	0.1442	0.1447	0.399	0.0920
Flamingo	666 600	54	3	0.1111	7	0.1111	1.000	419 000	24 500	4 527	0.0337	800	0.140	0.140	0.141	0.141	0.1541	0.1571	0.1591	0.1601	0.412	0.0943
Rook	636 000	54	3	0.1085	7	0.1085	0.977	400 000	23 600	4 319	0.0329	770	0.147	0.147	0.148	0.148	0.1618	0.1638	0.1678	0.1688	0.414	0.0950
Grosbeak	636 000	26	2	0.1564	7	0.1216	0.990	400 000	25 000	4 616	0.0335	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.412	0.0946
Egret	636 000	30	2	0.1456	19	0.0874	1.019	400 000	31 500	5 213	0.0351	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.406	0.0937
Peacock	605 000	54	3	0.1059	7	0.1059	0.953	380 500	22 500	4 109	0.0321	750	0.154	0.155	0.155	0.155	0.1695	0.1715	0.1755	0.1775	0.417	0.0957
Squab	605 000	26	2	0.1525	7	0.1186	0.966	380 500	24 100	4 391	0.0327	760	0.154	0.154	0.154	0.154	0.1700	0.1720	0.1720	0.1720	0.415	0.0953
Dove	556 500	26	2	0.1463	7	0.1138	0.927	350 000	22 400	4 039	0.0313	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.420	0.0965
Eagle	556 500	30	2	0.1362	7	0.1362	0.953	350 000	27 200	4 588	0.0328	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.415	0.0957
Hawk	477 000	26	2	0.1355	7	0.1054	0.858	300 000	19 430	3 462	0.0290	670	0.196	0.196	0.196	0.196	0.216				0.430	0.0988
Hen	477 000	30	2	0.1261	7	0.1261	0.883	300 000	23 300	3 933	0.0304	670	0.196	0.196	0.196	0.196	0.216				0.424	0.0980
Ibis	397 500	26	2	0.1236	7	0.0961	0.783	250 000	16 190	2 885	0.0265	590	0.235				0.259				0.441	0.1015
Lark	397 500	30	2	0.1151	7	0.1151	0.806	750 000	19 980	3 277	0.0278	600	0.235		Same as dc		0.259		Same as dc		0.435	0.1006
Linnet	336 400	26	2	0.1138	7	0.0855	0.721	4/0	14 050	2 442	0.0244	530	0.278				0.306				0.451	0.1039
Oriole	336 400	30	2	0.1059	7	0.1059	0.741	4/0	17 040	2 774	0.0255	530	0.278				0.306				0.445	0.1032
Ostrich	300 000	26	2	0.1074	7	0.0835	0.680	188 700	12 650	2 178	0.0230	490	0.311				0.342				0.458	0.1057
Piper	300 000	30	2	0.1000	7	0.1000	0.700	188 700	15 430	2 473	0.0241	500	0.311				0.342				0.462	0.1049
Partridge	266 800	26	2	0.1013	7	0.0788	0.642	3/0	11 250	1 936	0.0217	460	0.350				0.385				0.465	0.1074

*Based on copper 97%, aluminum 61% conductivity

†For conductor at 75°C, air at 25°C, wind 1.4 miles per hour (2 ft/sec), frequency = 60 Hz

‡ Current Approx. 75% Capacity is 75% of the Approx. Current Carrying Capacity in Amps and is approximately the current which will produce 50°C conductor temp. (25°C rise) with 25°C air temp., wind 1.4 miles per hour



37 All Aluminium

GMR = ?

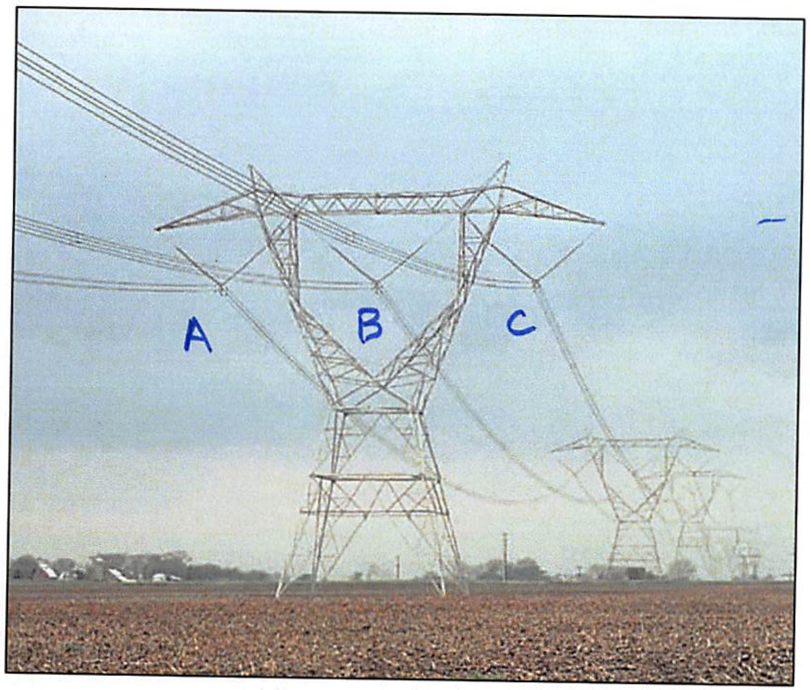
1369

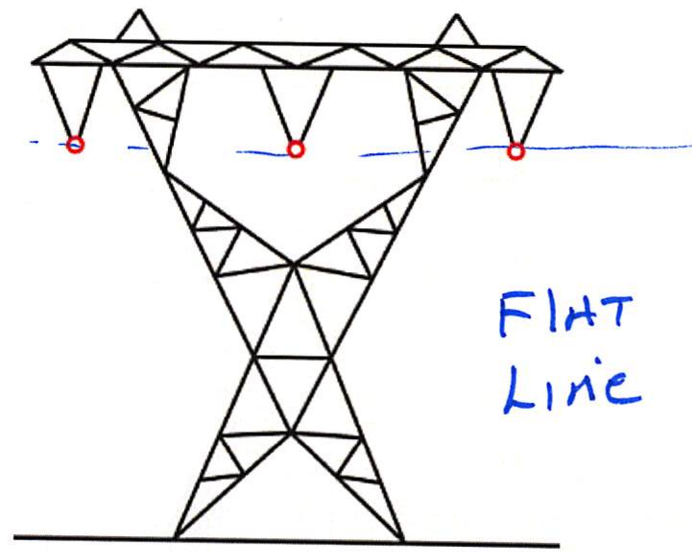
$$\sqrt[37^2]{D_{11} D_{12} \cdots D_{1-37}}$$

Practical Tower Configurations

I fooled you (once again) into believing transmission lines will all have symmetrically spaced conductors.

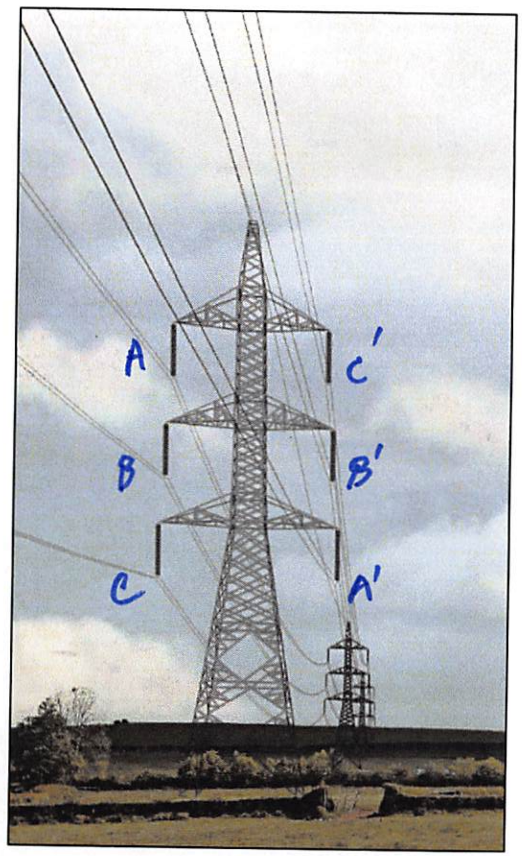
In fact this is seldom the case!

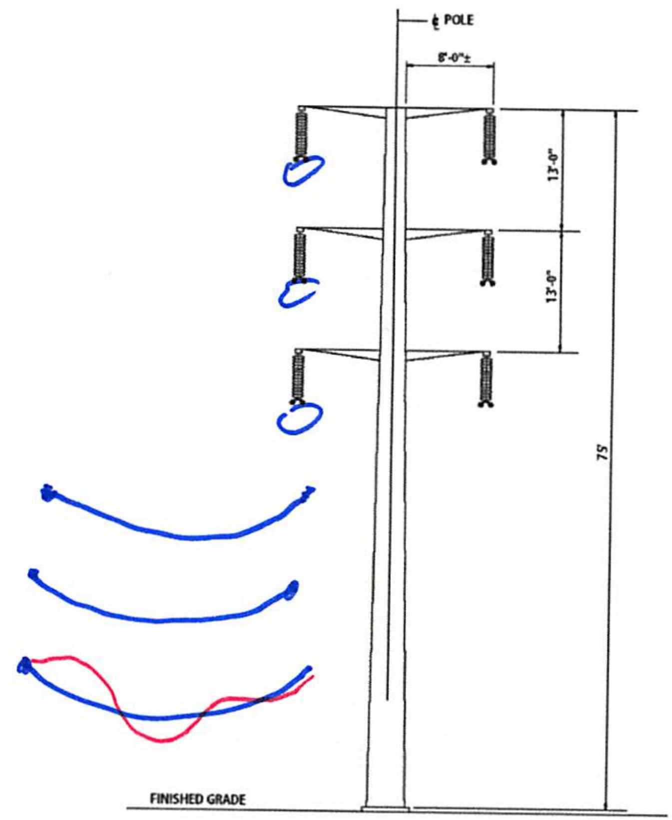




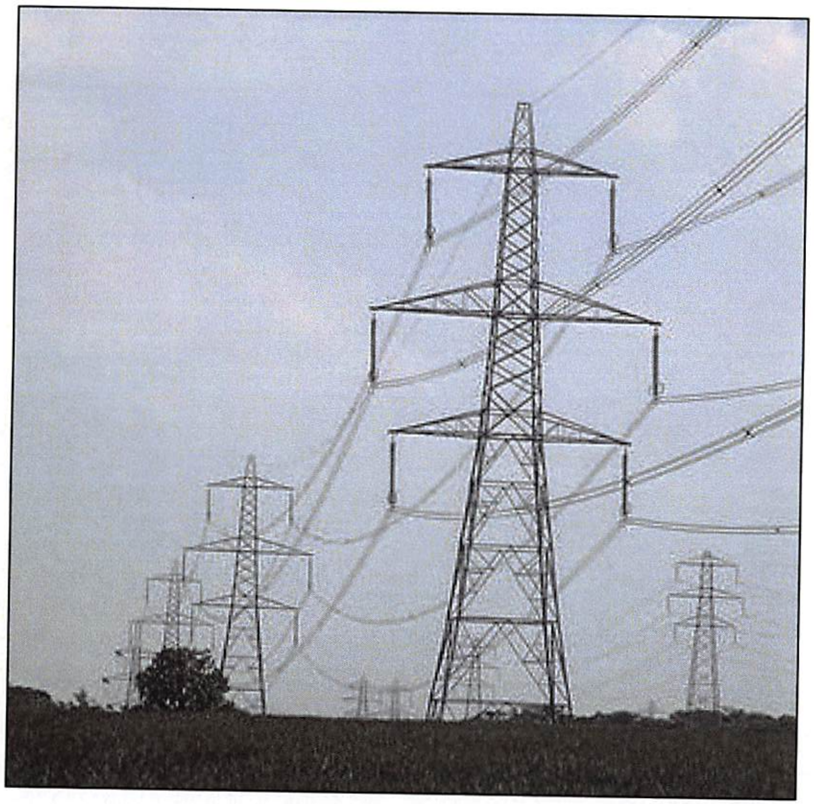
FlAT
Line

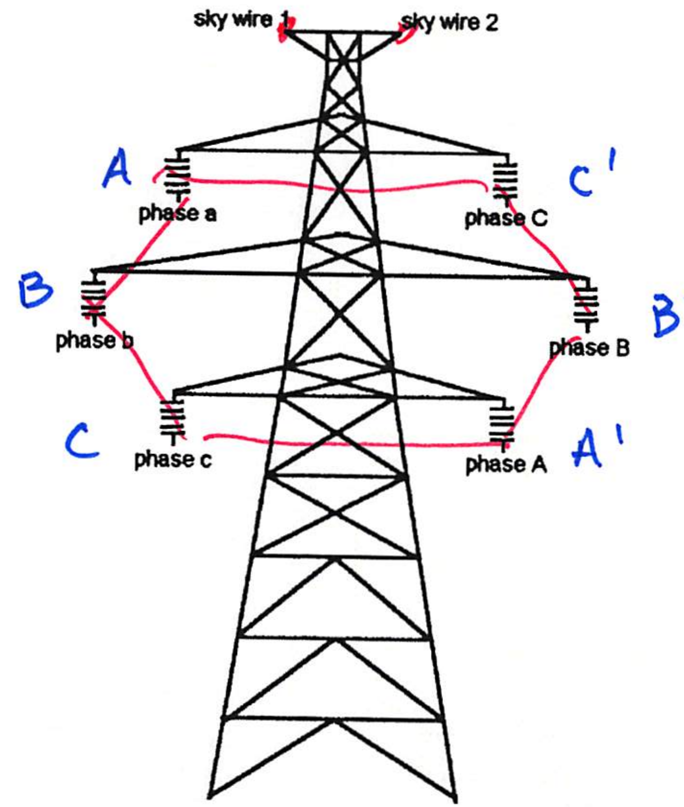
Practical Tower Configurations



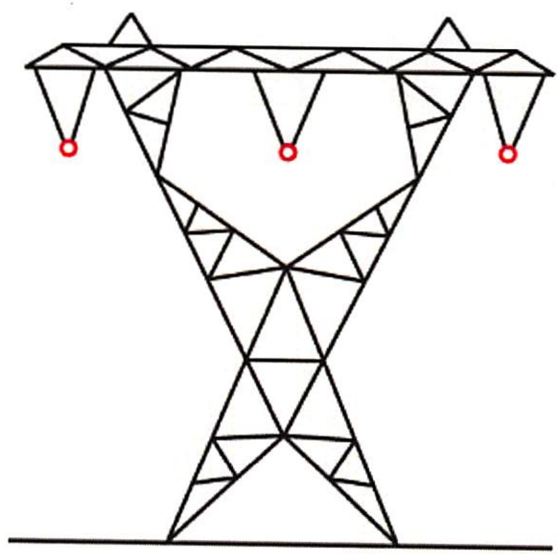


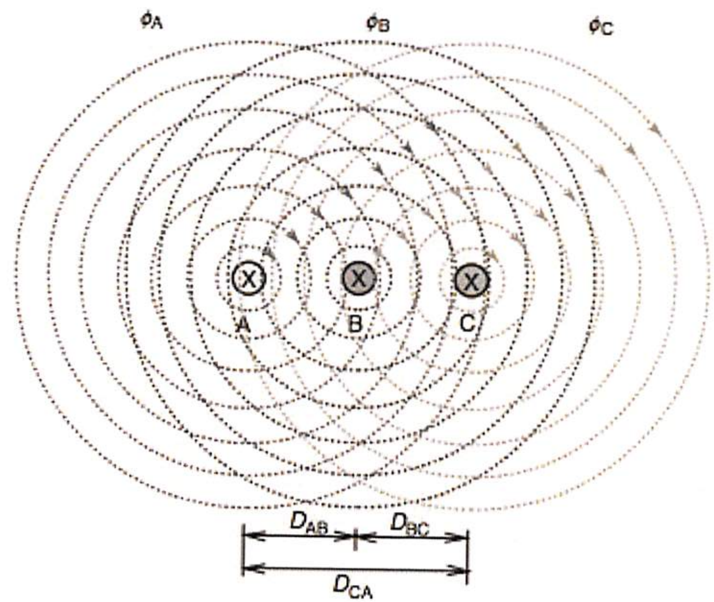
Practical Tower Configurations





Unbalance in the system





$L = \frac{\lambda}{I}$

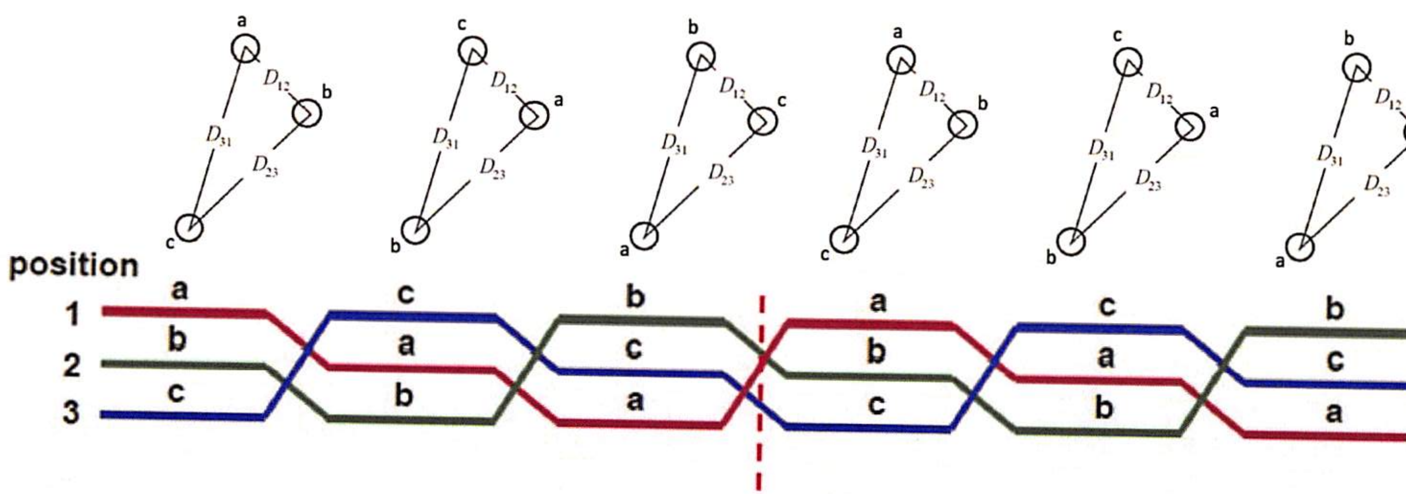
Transposition

Arranging the phase conductors in an equilateral triangle configuration is not very practical from a construction ease, maintenance or cost consideration.

It is easily seen in the vertical or horizontal configurations that all symmetry is lost.

$$D_{ab} \neq D_{bc} \neq D_{ca}$$

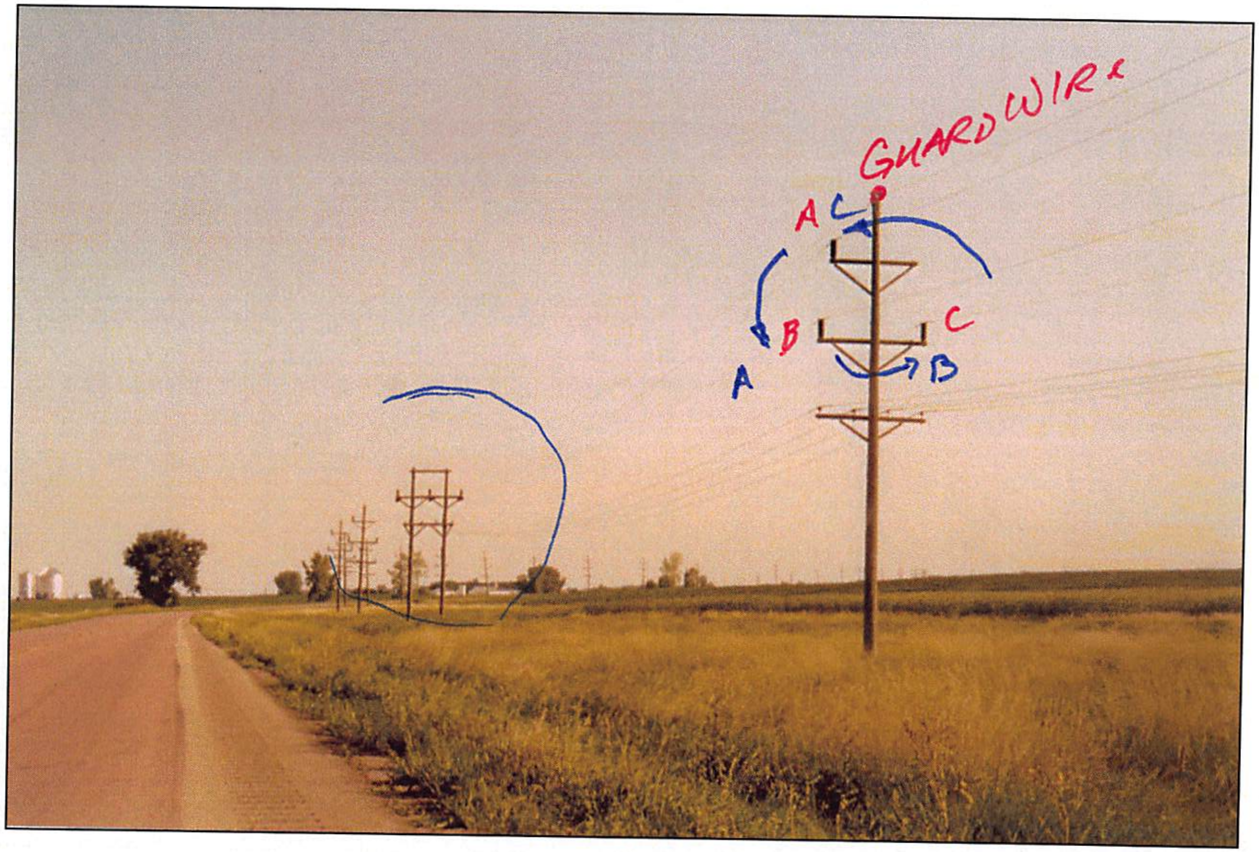
Symmetry is regained by employing **TRANSPOSITION**



Think of this as looking at a flat T-line configuration from the top or a vertical T-line configuration fromm the side.

Transposition

To keep the system balanced, the conductors are “rotated” over the length of a transmission line so each phase occupies each position on the tower for an equal distance.



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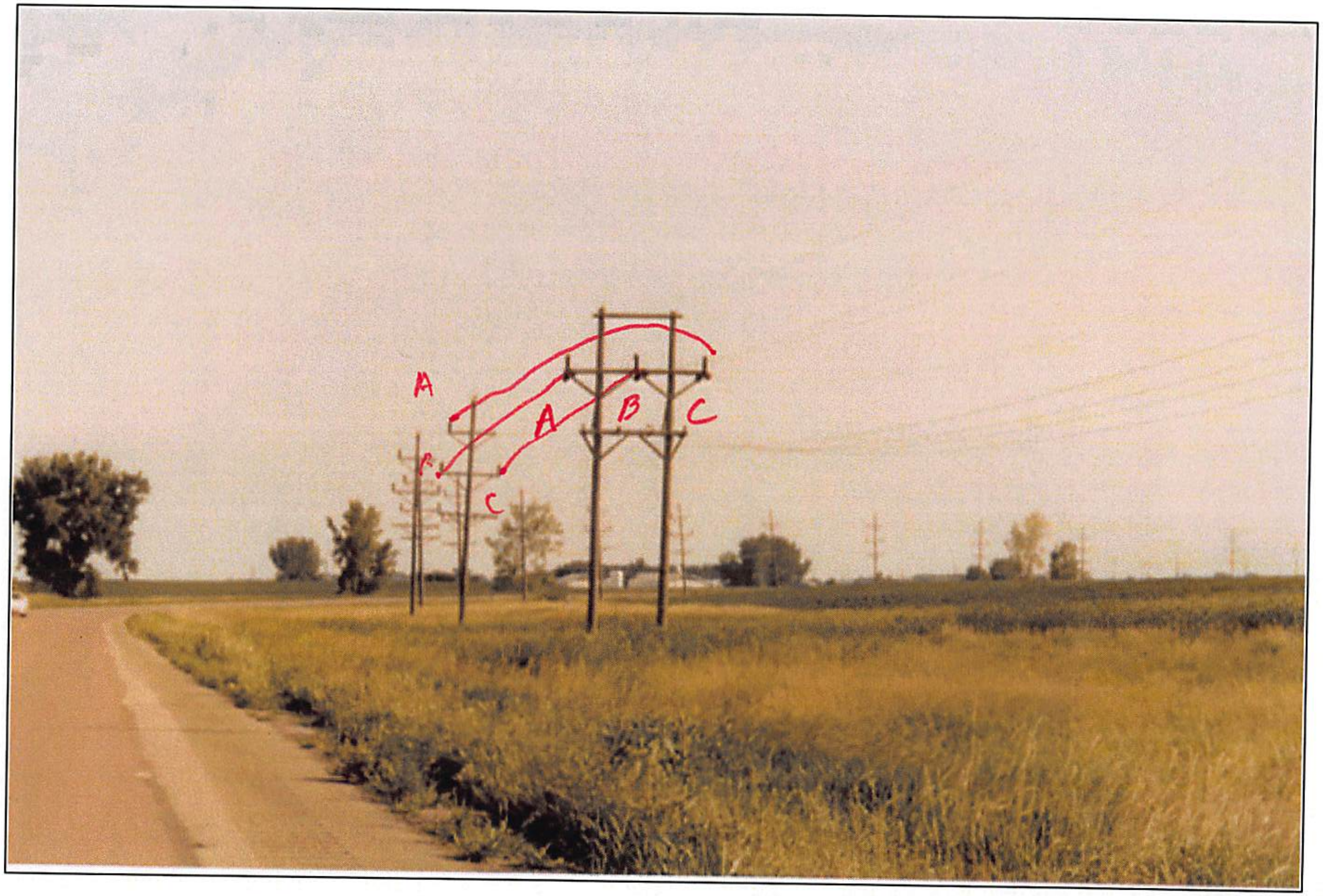
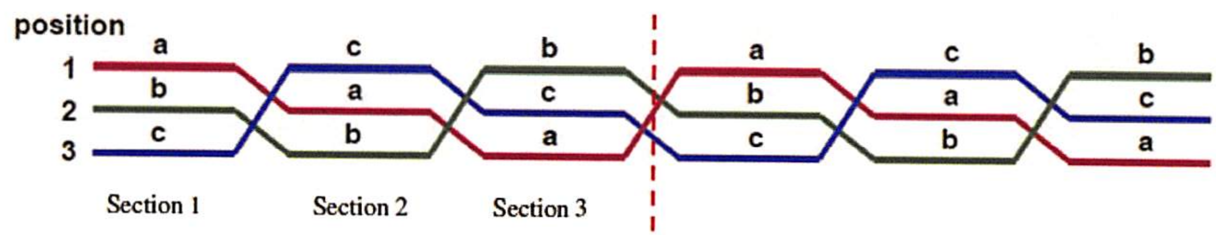
Transposition



Transposition

In a completely transposed each conductor spends 1/3 of the time in each of the three positions.

For a single conductor of radiur r in each of the phases:



$$\lambda a_1 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{1}{r'}\right) + I_b\left(\ln\frac{1}{D_{12}}\right) + I_c\left(\ln\frac{1}{D_{13}}\right)\right]$$ "a" phase in position 1

$$\lambda a_2 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{1}{r'}\right) + I_b\left(\ln\frac{1}{D_{23}}\right) + I_c\left(\ln\frac{1}{D_{21}}\right)\right]$$ "a" phase in position 2

$$\lambda a_3 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{1}{r'}\right) + I_b\left(\ln\frac{1}{D_{31}}\right) + I_c\left(\ln\frac{1}{D_{32}}\right)\right]$$ "a" phase in position 3

Transposition

$$\overline{\lambda_a} = \frac{\lambda a_1 + \lambda a_2 + \lambda a_3}{3}$$

$$\lambda_a = \frac{2 \cdot 10^{-7}}{3}\left[3I_a\left(\ln\frac{1}{r'}\right) + I_b\left(\ln\frac{1}{D_{12}D_{23}D_{31}}\right) + I_c\left(\ln\frac{1}{D_{21}D_{32}D_{13}}\right)\right]$$

but Ib + Ic = -Ia and $D_{12}=D_{21}$, $D_{23}=D_{32}$, $D_{31}=D_{13}$

$$\lambda_a = \frac{2 \cdot 10^{-7}}{3}\left[3I_a\left(\ln\frac{1}{r'}\right) - I_a\left(\ln\frac{1}{D_{12}D_{23}D_{31}}\right)\right] = \frac{2 \cdot 10^{-7}}{3}\left[3I_a\left(\ln\frac{1}{r'}\right) + 3I_a\left(\ln\sqrt[3]{D_{12}D_{23}D_{31}}\right)\right]$$

$$\lambda_a = 2 \cdot 10^{-7}\left[I_a\left(\ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'}\right)\right] \qquad L_a = \frac{\lambda_a}{I_a} = 2 \cdot 10^{-7}\left(\ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'}\right)$$

$$Deq = \sqrt[3]{D_{12}D_{23}D_{31}} \quad \text{(the GMD between the phases)}$$

For solid conductors use r’, for stranded conductors or bundles use the **GMR** of the phase

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