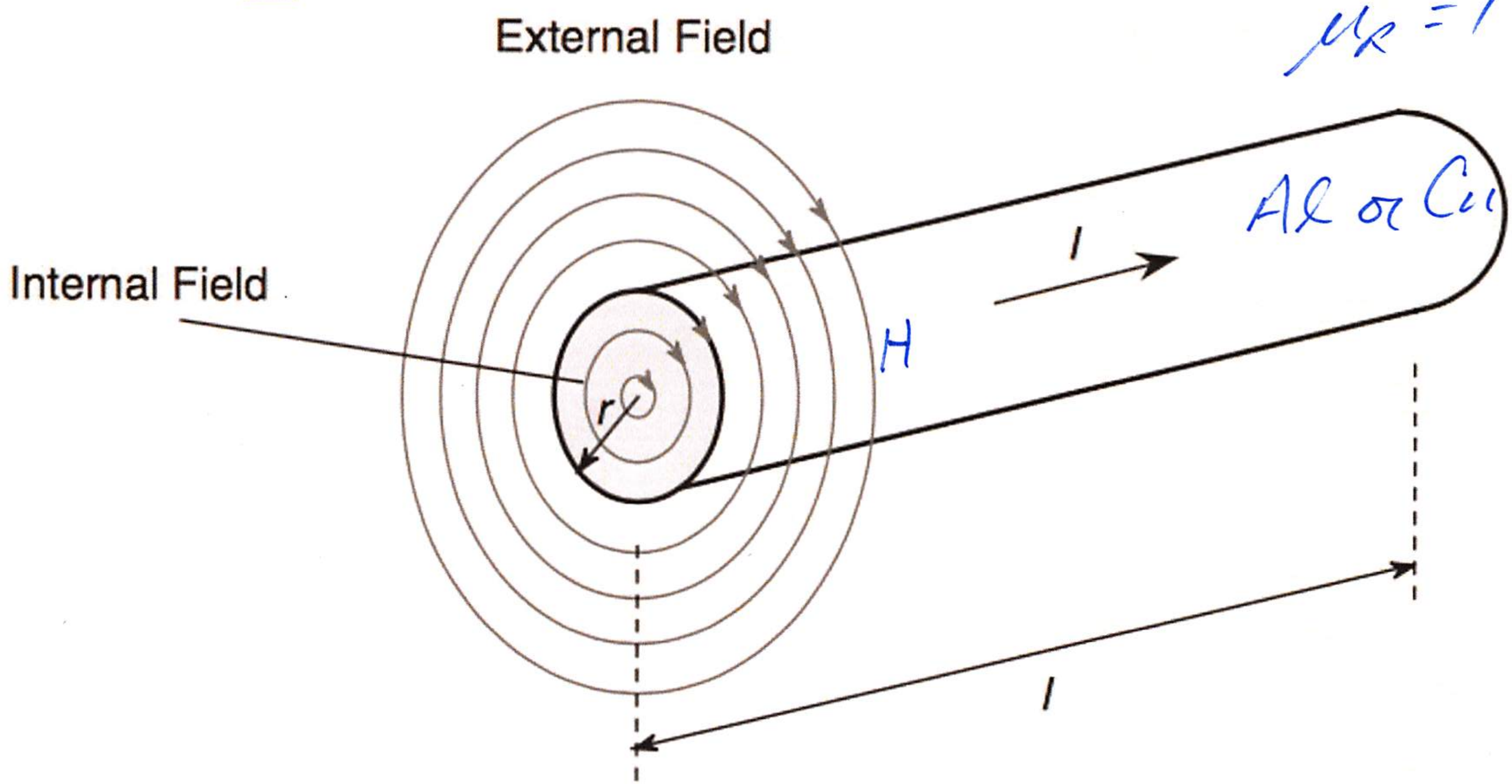
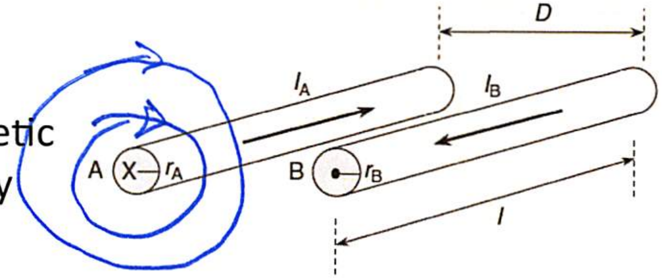
$$L = \frac{\lambda}{I}$$



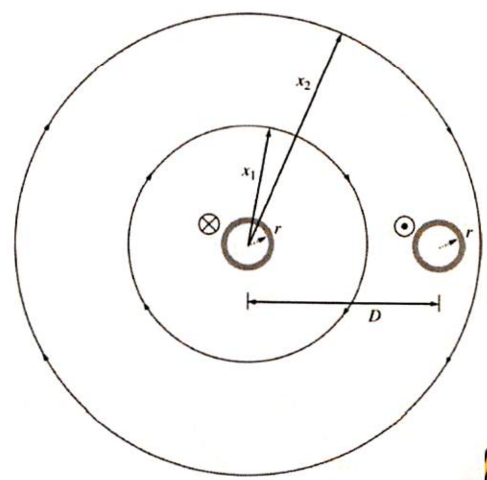
Inductance of a Single Phase Line

We next determine the series inductance of a single-phase line consisting of two conductors of radii r seperated by a distance D. The conductor "A" carries a current of magnitude I flowing into the page, and conductor "B" carries a current of magnitude I flowing out of the page.



Considering two circular integration paths; we find that the line integral along x_1 produces a net magnetic intensity since a non-zero net current is enclosed by x_1.

$$\oint H_x \cdot dl = I_{x\ (enclosed)}$$



However the path of radius x_2 encloses both conductors and the currents are equal and opposite, the net current enclosed is 0 and, therefore, there can be **NO** contributions to the total inductance from the magnetic fields at distances greater than D!

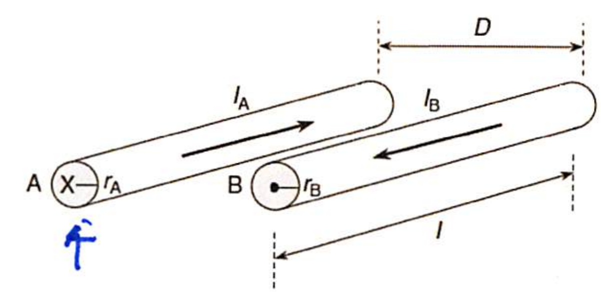
Inductance of a Single Phase Line

The total inductance of **a single wire** per unit length in a single phase transmission line is a sum of the internal inductance and the external inductance between the conductor surface (r) and the separation distance (D):

$$L_A = L_{\text{int}} + L_{ext} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln\frac{D}{r} \; [H/m]$$

substituting $\mu_0 = 4\pi \cdot 10^{-7}$ $\frac{\mu_0}{2\pi}$

$$L = 2\cdot 10^{-7}\left[\frac{1}{4} + \ln\frac{D}{r}\right] \; [H/m]$$

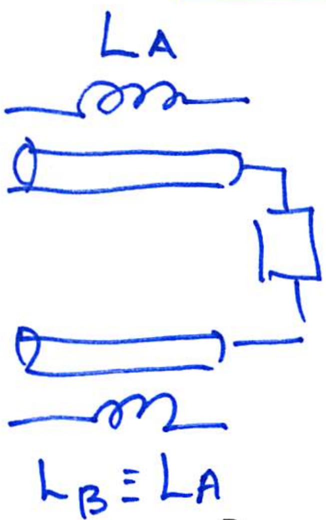


re-writing the ¼ factor in a more convenient form yields:

$$L = 2\cdot 10^{-7}\left[\ln(e^{\frac{1}{4}}) + \ln\frac{D}{r}\right] \; [H/m]$$

Source: http://www.accessscience.com MichiganTech

Inductance of a Single Phase Line



$$L_A = 2\cdot 10^{-7}\left[\ln \frac{D\ e^{\frac{1}{4}}}{r}\right] = 2\cdot 10^{-7}\left[\ln \frac{D}{re^{\frac{-1}{4}}}\right] [H/m]$$

r' effective radius

$$L_A = 2\cdot 10^{-7}\left[\ln \frac{D}{r'}\right]\ [H/m]\ \ where\ \ r' = re^{\frac{-1}{4}}\ or\ \ r' = 0.7788r$$

By symmetry, the inductance of the return wire is the same, therefore, the total inductance (sometimes called the loop inductance) of a two-wire transmission line is

$$L_{total} = 2L = 4\cdot 10^{-7}\left[\ln \frac{D}{r'}\right]\ [H/m]\ \ where\ \ r' = re^{\frac{-1}{4}}\ or\ \ r' = 0.7788r$$

Where r' is the "effective radius" of the conductors and D is the distance between conductors.

Inductance of a single phase line

If one conductor had radius r_x and the other r_y

$$L_{total} = L_x + L_y = 2\cdot 10^{-7}\left[\ln\frac{D}{r_x'}\right] + 2\cdot 10^{-7}\left[\ln\frac{D}{r_y'}\right] \ [H/m]$$

$\ln A + \ln B$

$\ln (A \cdot B)$

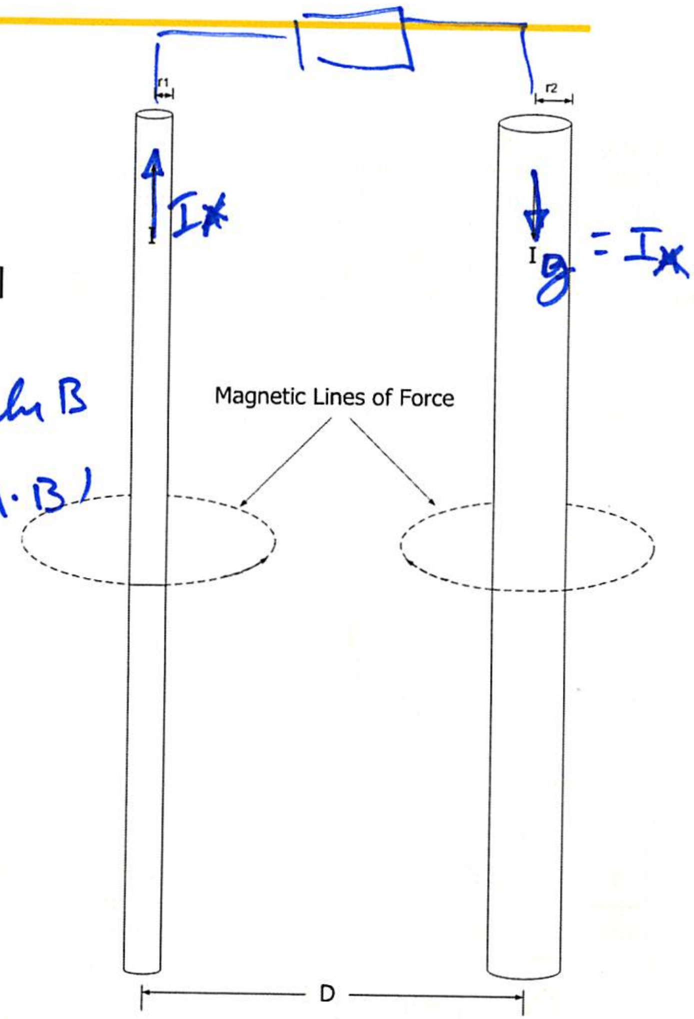
$$L_{total} = L_x + L_y = 2\cdot 10^{-7}\left[\ln\frac{D^2}{r_x' r_y'}\right] \ [H/m]$$

$$L_{total} = L_x + L_y = 2\cdot 10^{-7}\left[\ln\frac{D^2}{\left(\sqrt{r_x' r_y'}\right)^2}\right] \ [H/m]$$

$$L_{total} = L_x + L_y = 4\cdot 10^{-7}\left[\ln\frac{D}{\sqrt{r_x' r_y'}}\right] \ [H/m]$$

$\sqrt{r_x' r_x'} \therefore r_x'$

$4 \times 10^{-7} \ln \frac{D}{r_x'}$



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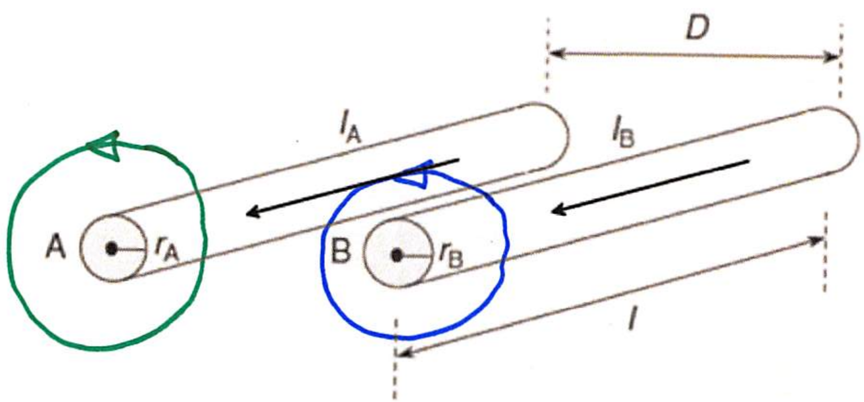
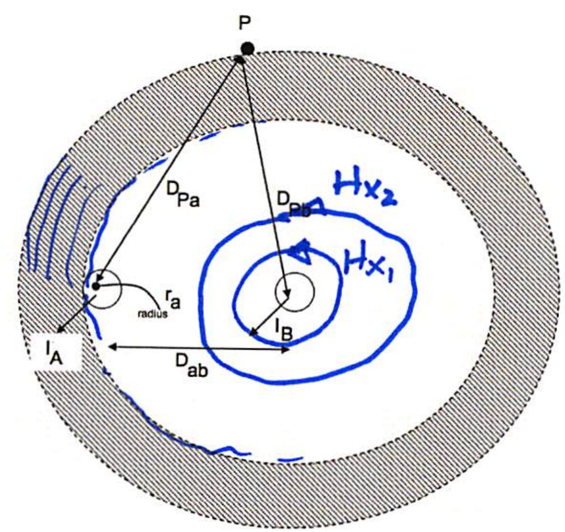
Two Parallel Conductors

In this example the currents in conductors A and B are in the same direction. We would like to compute the flux linkages from conductor B that link conductor A out to some arbitrary point P in space.

We note the flux from conductor B is radial about the conductor in the counter-clockwise direction as given by the right hand rule. Also, the flux from conductor B inside the white region (lower right diagram) does not link conductor A as it does not encircle the A conductor.





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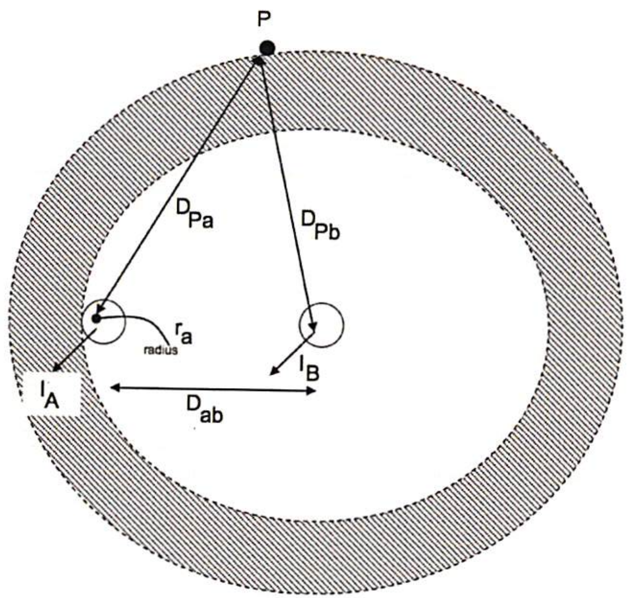
Two Parallel Conductors



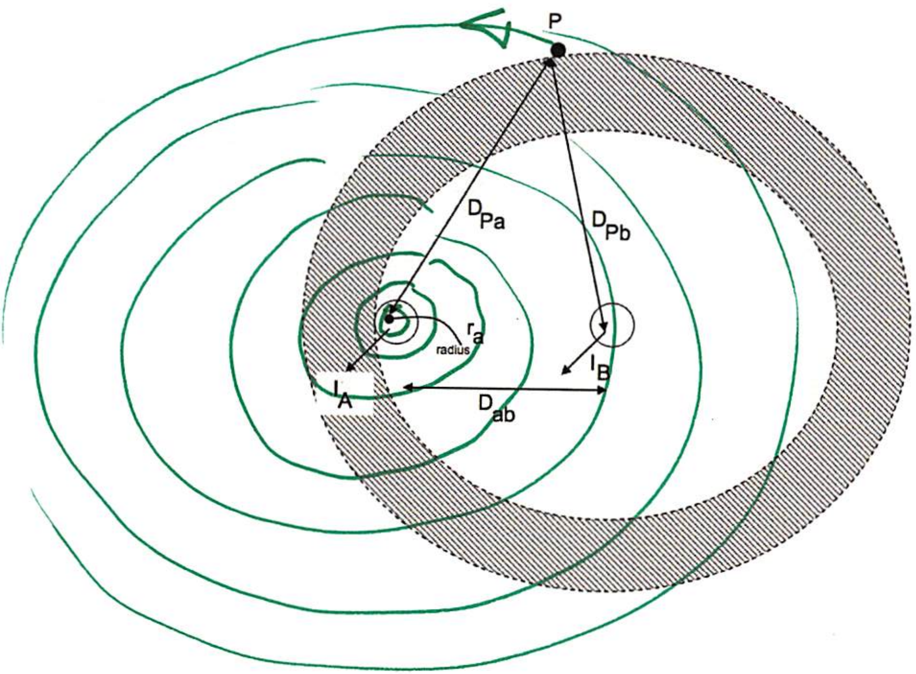
To calculate the flux linkages from conductor B that link conductor A out to some arbitrary point P in space, we integrate the differential flux linkages from the inner circle (where conductor A lies) to the outer circle (where point P is located)

$$\int \frac{1}{x}\,dx = \ln x \Big|_{D_{ab}}^{D_{Pb}}$$

$$\ln D_{Pb} - \ln D_{ab}$$

$$\lambda_{A,P,B} = \int d\lambda = \int_{D_{ab}}^{D_{Pb}} \frac{\mu I_B}{2\pi x}\,dx = \frac{\mu I_B}{2\pi} \ln \frac{D_{Pb}}{D_{ab}} \quad [Wb - turns/m]$$

Two Parallel Conductors



The **total** flux linking conductor A out to some point P will be that of the flux linkages from the current in conductor A along with the contribution from current in conductor B.

$$\lambda_{A(total)} = \lambda_{A,P,A} + \lambda_{A,P,B} = \frac{\mu I_A}{2\pi}\ln\frac{D_{Pa}}{r_a{}'} + \frac{\mu I_B}{2\pi}\ln\frac{D_{Pb}}{D_{ab}} \quad [Wb-turns/m]$$

Multiple Current Carrying Conductors

Consider a point P external to "n" conductors,

The flux linked by conductor **a** due to the current I_a includes all flux linkages internal to conductor a and all external flux linkages out to point **P**

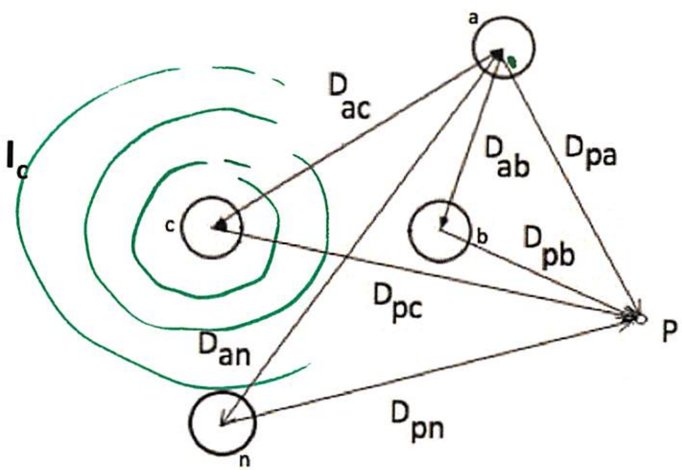
$$\lambda_{apa} = 2\cdot 10^{-7} I_a \ln\frac{D_{pa}}{D_{aa}} = 2\cdot 10^{-7} I_a \ln\frac{D_{pa}}{r'}$$

$D_{aa} = r'$

The flux linked by conductor **a** due to the current $\mathbf{I_b}$ in conductor b

$$\lambda_{apb} = 2\cdot 10^{-7} I_b \ln\frac{D_{pb}}{D_{ab}}$$

The flux linked by conductor **a** due to the current $\mathbf{I_c}$



$$\lambda_{apc} = 2\cdot 10^{-7} I_c \ln\frac{D_{pc}}{D_{ac}}$$

$$\lambda_a = \lambda_{apa} + \lambda_{apb} + \lambda_{apc} + \ldots + \lambda_n$$

$$\lambda_a = 2\cdot 10^{-7}\left(I_a \ln\frac{D_{pa}}{r'} + I_b \ln\frac{D_{pb}}{D_{ab}} + I_c \ln\frac{D_{pc}}{D_{ac}} + \ldots + I_n \ln\frac{D_{pn}}{D_{an}} \right)$$

ln

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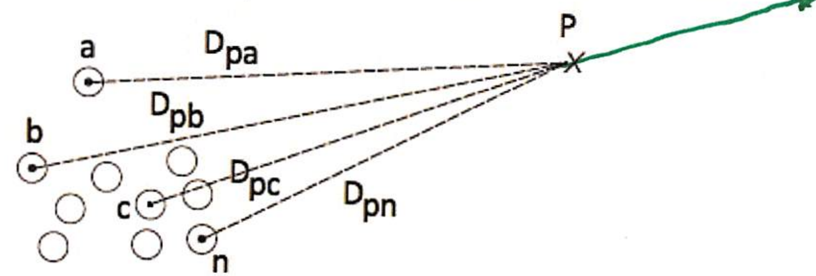
Multiple Carrying Conductors

$$\lambda_a = 2\cdot 10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{ab}} + I_c \ln\frac{1}{D_{ac}} + \dots + I_n \ln\frac{1}{D_{an}}\right) +$$
$$2\cdot 10^{-7}\left(I_a \ln D_{pa} + I_b \ln D_{pb} + I_c \ln D_{pc} + \dots + I_n \ln D_{pn}\right)$$

However as $P \to \infty$ $\quad D_{pa} = D_{pb} = D_{pc} = D_{pn} = D$



$$\lambda_a = 2\cdot 10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{ab}} + I_c \ln\frac{1}{D_{ac}} + \dots + I_n \ln\frac{1}{D_{an}}\right) +$$
$$2\cdot 10^{-7}\left(I_a + I_b + I_c + \dots + I_n\right)\ln D$$

0

But the currents must sum to zero to satisfy KCL, therefore:

$$\lambda_a = 2\cdot 10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{ab}} + I_c \ln\frac{1}{D_{ac}} + \dots + I_n \ln\frac{1}{D_{an}}\right)$$

Inductance of a symmetrically spaced 3 Phase Line

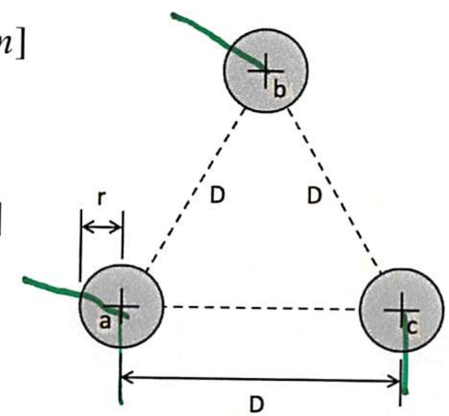
Formally stating what we have determined from the previous 2 examples:
For a system with multiple conductors carrying currents that do not violate Kirchoff's Current Law, the flux linkages on the "kth" conductor can be calculated as:

$$\lambda_k = 2\cdot 10^{-7}\sum_{m=a}^{\cancel{M}\,c} I_m \ln\frac{1}{D_{km}} \;[Wb-t/m]$$

See pages ~~181-182~~ 189 of you text

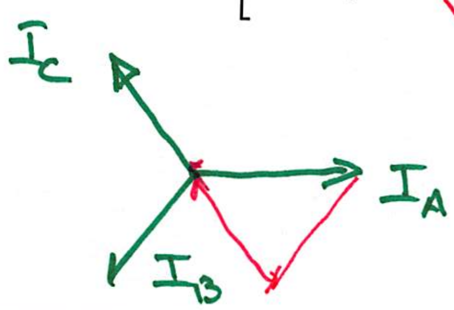
Applying this general result to a three phase three wire line seperated by equal spacing D *(let k =a and sum from m=a to m=c)*

$$\lambda_a = 2\cdot 10^{-7}\left[I_a \ln\frac{1}{D_{aa}} + I_b \ln\frac{1}{D_{ab}} + I_c \ln\frac{1}{D_{ac}}\right] [Wb-t/m]$$



$$\lambda_a = 2\cdot 10^{-7}\left[I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D} + I_c \ln\frac{1}{D}\right] [Wb-t/m]$$

$$\lambda_a = 2\cdot 10^{-7}\left[I_a \ln\frac{1}{r'} + \underbrace{(I_b + I_c)}_{-I_A} \ln\frac{1}{D}\right] [Wb-t/m]$$



$\bar{I}_A + \bar{I}_B + \bar{I}_C = 0 \quad \therefore\; \bar{I}_B + \bar{I}_C = -I_A$

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Inductance of a symmetrically spaced 3 Phase Line

but in a balanced 3 phase system $I_a + I_b + I_c = 0$ therefore $I_b + I_c = -I_a$

765kV

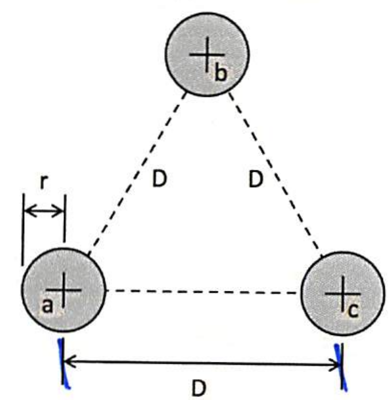
$$\lambda_a = 2\cdot 10^{-7}\left[I_a \ln\frac{1}{r'} + (-I_a)\ln\left(\frac{1}{D}\right)\right]\ [Wb-t/m]$$

-1 -1 → ln D

$$\lambda_a = 2\cdot 10^{-7}\left[I_a \ln\frac{D}{r'}\right]\ [Wb-t/m]$$

$$L_a = \frac{\lambda_a}{I_a} = 2\cdot 10^{-7}\left[\ln\frac{D}{r'}\right]\ [H/m]\ \ per\ \ phase$$

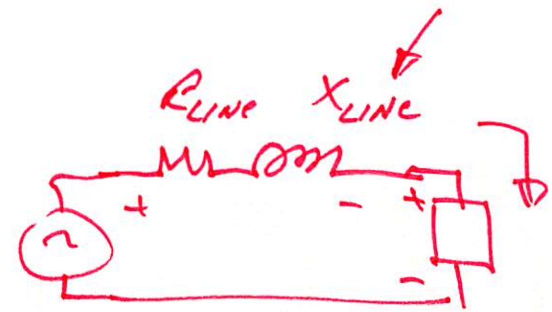
$\frac{\mu_0}{2\pi}$

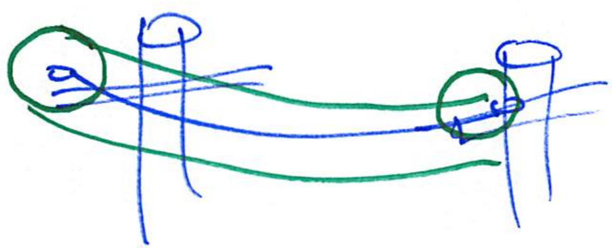


Inductane is not a friend of the power engineer.

To **reduce** its effect we could:

a) reduce the distance between conductors
b) increase the diameter of the conductors





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Bundled Conductors

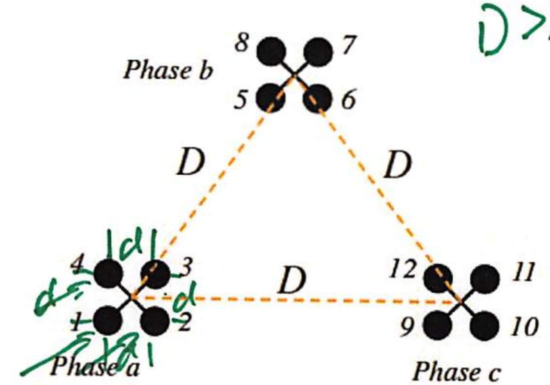
Instead of using a single conductor per phase lets use “n” closely spaced conductors.
(Closely can be interperted as the distance between conductors is small compared to the distance between phases.)

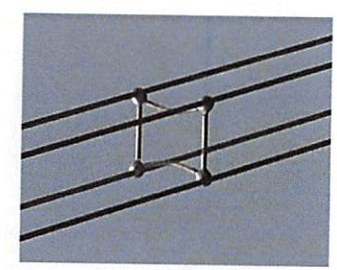
Consider the a,b and c phase “conductors” are comprised of 4 conductors all of radius r

$D >> d$

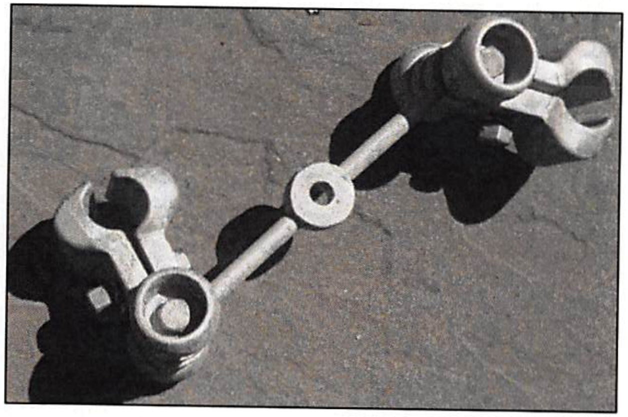
The flux linkages in conductor 1 of the a phase bundle can be calculated by:

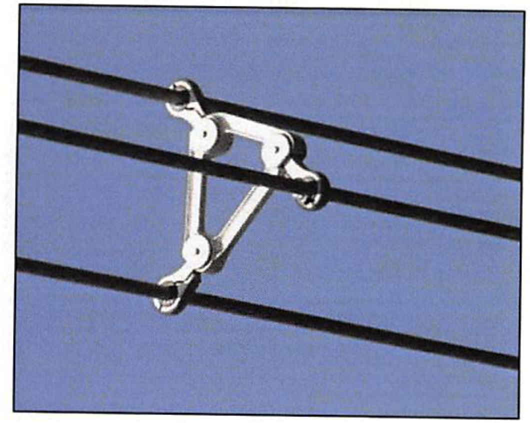
$$\lambda_1 = 2\cdot 10^{-7}\left[\begin{array}{l} \frac{I_a}{4}\left(\ln\frac{1}{r'} + \ln\frac{1}{D_{12}} + \ln\frac{1}{D_{13}} + \ln\frac{1}{D_{14}}\right) + \\ \frac{I_b}{4}\left(\ln\frac{1}{D_{15}} + \ln\frac{1}{D_{16}} + \ln\frac{1}{D_{17}} + \ln\frac{1}{D_{18}}\right) + \\ \frac{I_c}{4}\left(\ln\frac{1}{D_{19}} + \ln\frac{1}{D_{110}} + \ln\frac{1}{D_{111}} + \ln\frac{1}{D_{112}}\right) \end{array}\right]$$

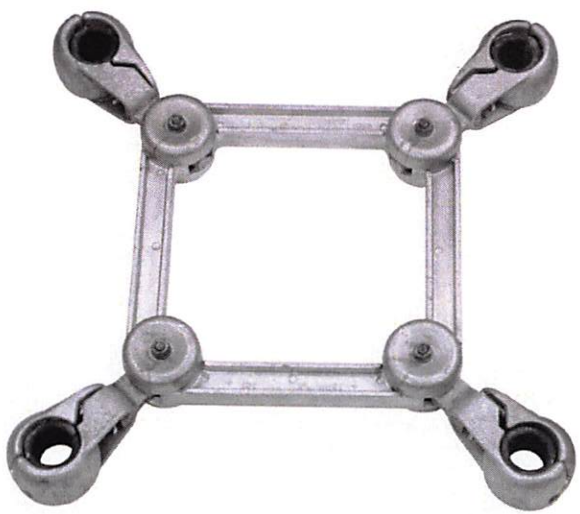


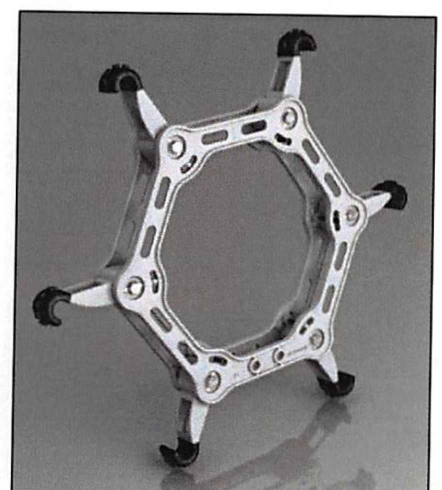


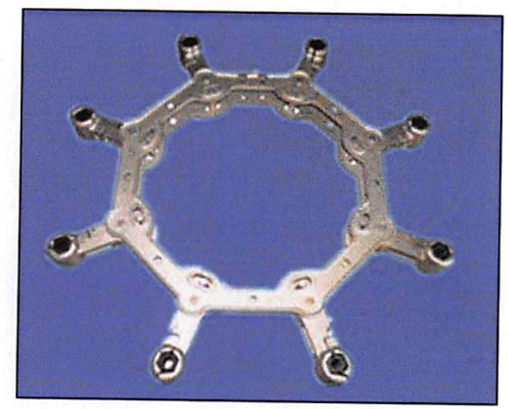
Bundle spacer/dampers











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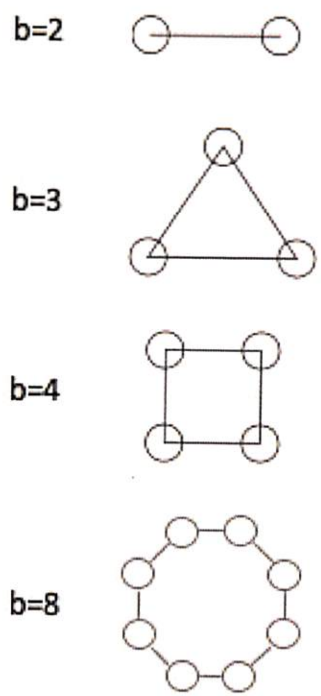
Bundled Conductors



As b (the number of conductors in the bundle) gets large, the bundle begins to take on the configuration of a circle, with all current flowing at the circumference.

The bundle resembles a hollow conductor!

So the effective bundle radius R_b increases from r' (for the case of b=1) to the radius of the circle comprised by the bundle as b gets large.

As R_b increases the inductance decreases.

This is the effect we desire!

$r_b' \uparrow$

so

$L_{per\ \phi} \downarrow$

phase

Bundled Conductors

Bringing the ¼ into the logarithm and using ln(a) + ln(b) = ln(a*b)

$$\lambda_1 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{1}{(r' D_{12}D_{13}D_{14})^{\frac{1}{4}}}\right) + I_b\left(\ln\frac{1}{(D_{15}D_{16}D_{17}D_{18})^{\frac{1}{4}}}\right) + I_c\left(\ln\frac{1}{(D_{19}D_{110}D_{111}D_{112})^{\frac{1}{4}}}\right)\right]$$

The denominators by definition:

$$GMR = R_b = \sqrt[4]{(r' D_{12}D_{13}D_{14})}$$ = geometric mean radius of the square bundle

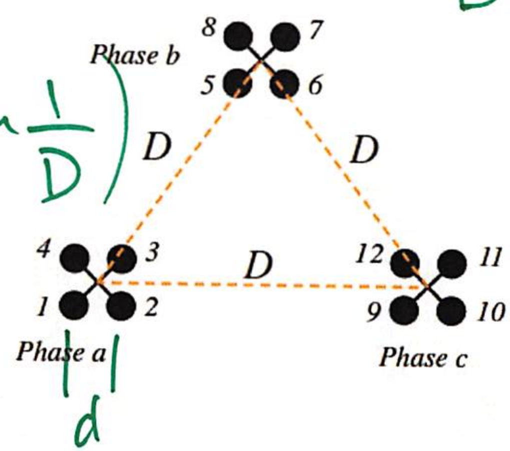
$$D_{1b} = \sqrt[4]{(D_{15}D_{16}D_{17}D_{18})}$$ = geometric mean distance from conductor 1 to phase b

$$D_{1c} = \sqrt[4]{(D_{19}D_{110}D_{111}D_{112})}$$ = geometric mean distance from conductor 1 to phase c

D

2×10^{-7}

$$\left(I_a \ln\frac{1}{R_b} + \underbrace{(I_b+I_c)}_{-I_a}\ln\frac{1}{D}\right)$$



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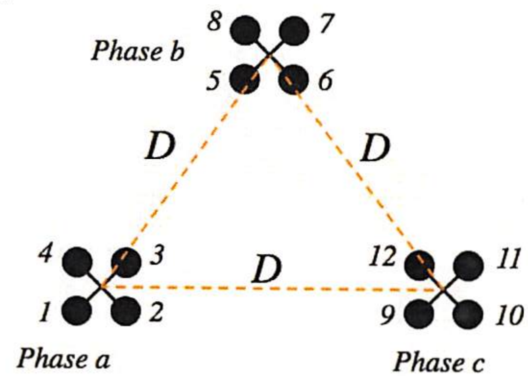
Bundled Conductors

In real systems, the conductor diameter is small compared to the distance between phases.

$$D_{1b} \approx D_{2b} \approx D_{3b} \approx D_{4b} \approx D_{ab}$$

$$D_{1c} \approx D_{2c} \approx D_{3c} \approx D_{4c} \approx D_{ac}$$

$$D_{ab} = D_{ac} = D_{bc} = D$$



$$\lambda_1 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{1}{R_b}\right) + (I_b + I_c)\left(\ln\frac{1}{D}\right)\right]$$

IA

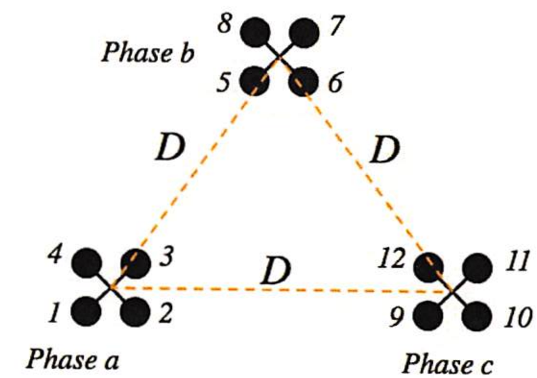
$I_1 = \frac{I_A}{4}$

but $(I_b + I_c) = -I_a$

$$\lambda_1 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{1}{R_b}\right) \overset{+}{-} I_a\left(\ln\frac{1}{D}\right)\right]$$

$$\lambda_1 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{D}{R_b}\right)\right]$$

Bundled Conductors



$$\lambda_1 = 2\cdot 10^{-7}\left[I_a\left(\ln\frac{D}{R_b}\right)\right]$$

If the current divides evenly in the bundled conductors $I_a = 4I_1$

$$\lambda_1 = 2\cdot 10^{-7}\left[4I_1\left(\ln\frac{D}{R_b}\right)\right]$$

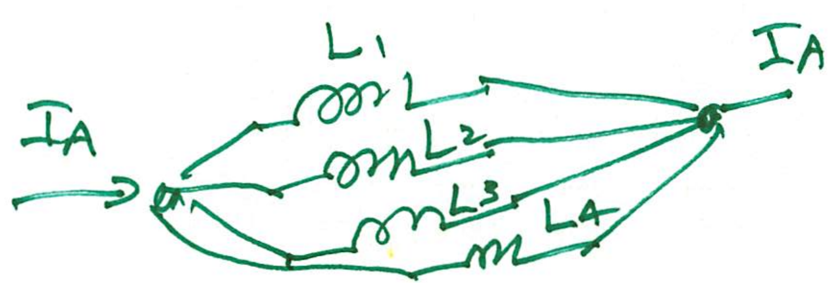
$$L_1 = \frac{\lambda_1}{I_1} = 2\cdot 10^{-7}\left[\cancel{4}\left(\ln\frac{D}{R_b}\right)\right]$$

This is the self inductance of wire 1

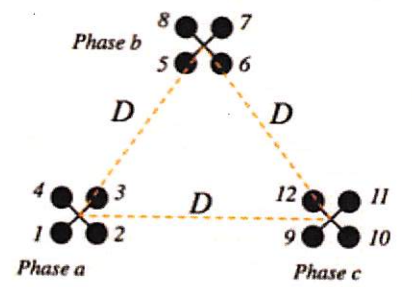
There are four conductors per bundle in our example and by symmetry $L_1 = L_2 = L_3 = L_4$

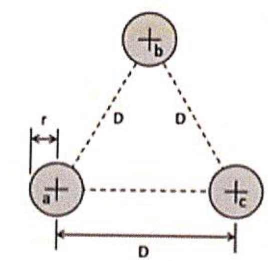
The bundled conductors are in parallel, therefore the inductance of the **a phase** $L_a = L_1/4$

$$L_a = 2\cdot 10^{-7}\left(\ln\frac{D}{R_b}\right) \quad or \quad L_a = 2\cdot 10^{-7}\left(\ln\frac{GMD}{GMR}\right) \quad H/m$$



Bundled Conductors





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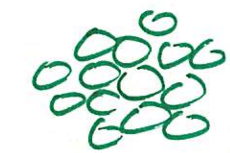

GMR - Geometric Mean Radius = R_b = the effective radius of the bundle

n^2

$$GMR = R_b = \sqrt[16]{(D_{11}D_{12}D_{13}D_{14})(D_{22}D_{21}D_{23}D_{24})(D_{33}D_{31}D_{32}D_{34})(D_{44}D_{41}D_{42}D_{43})}$$

$$GMR = R_b = \sqrt[16]{(r'_1 D_{12}D_{13}D_{14})(r_2' D_{21}D_{23}D_{24})(r_3' D_{31}D_{32}D_{34})(r_4' 1 D_{41}D_{42}D_{43})}$$

$$GMR = R_b = \sqrt[16]{(r'_1 D_{12}D_{13}D_{14})^4}$$

$$GMR = R_b = \sqrt[4]{(r'_1 D_{12}D_{13}D_{14})}$$

GMD - Geometric Mean Distance the mean distance between bundles

$$GMD = D = \sqrt[3]{(D_{ab}D_{ac}D_{bc})} \quad \sqrt[3]{D \cdot D \cdot D} = \sqrt[3]{D^3} = D$$

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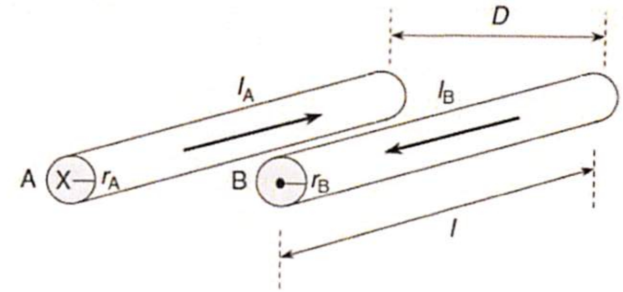
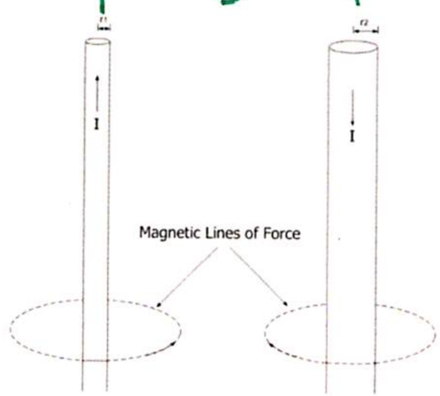
$$L = 2\cdot 10^{-7}\left[\ln\frac{D}{re^{\frac{-1}{4}}}\right] \qquad L = 2\cdot 10^{-7}\left[\ln\frac{D}{r'}\right]\ [H/m]$$



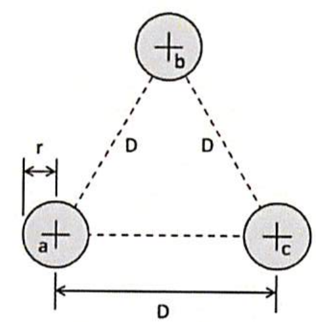
$$L_{total} = 2L = 4\cdot 10^{-7}\left[\ln\frac{D}{r'}\right]$$

D

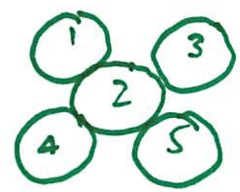


$$L_{total} = L_x + L_y = 4\cdot 10^{-7}\left[\ln\frac{D}{\sqrt{r_x' r_y'}}\right]\ [H/m]$$

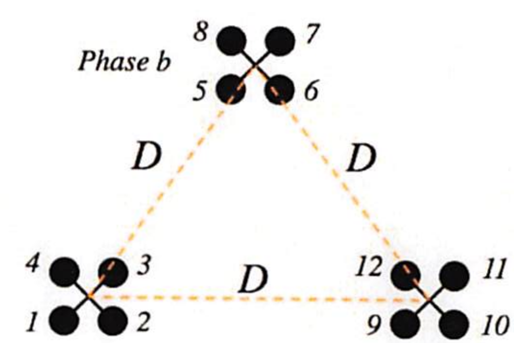
$$L_a = \frac{\lambda_a}{I_a} = 2\cdot 10^{-7}\left[\ln\frac{D}{r'}\right]\ [H/m]\ \ per\ \ phase$$

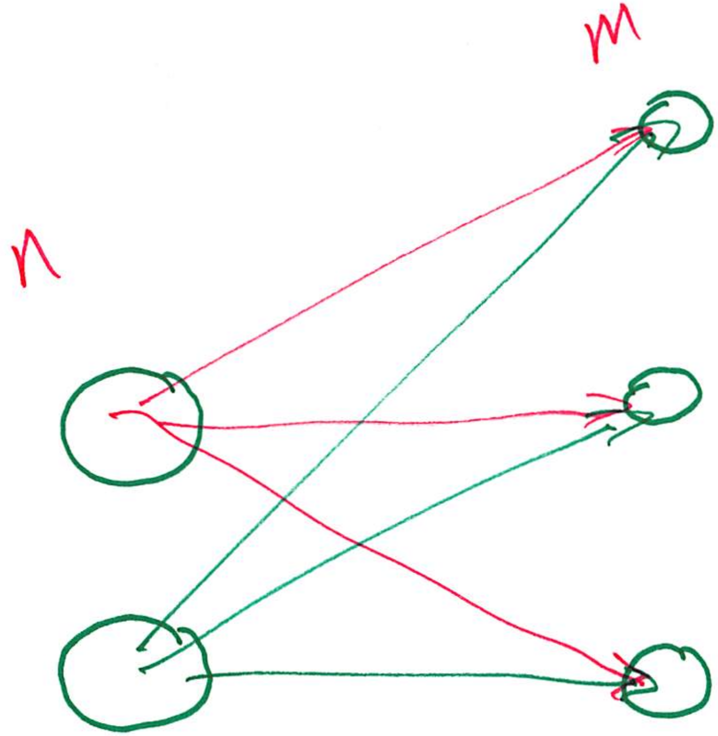


$$\sqrt[25]{D_{11}D_{12}D_{13}D_{14}D_{15}\cdot D_{22}D_{21}D_{23}D_{24}}$$



$$L_a = 2\cdot 10^{-7}\left(\ln\frac{D}{R_b}\right)\ or\ \ L_a = 2\cdot 10^{-7}\left(\ln\frac{GMD}{GMR}\right)\ \ H/m$$



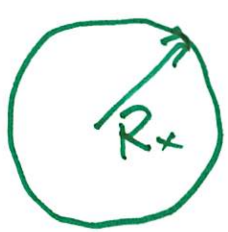


$n \times m$

$$GMD = \sqrt[6]{D_{x_1y_1} D_{x_1y_2} D_{x_1y_3} \; D_{x_2y_1} D_{x_2y_2} D_{x_2y_3}}$$

r_x

$$GMR_x = \sqrt[4]{}$$



r_y

$$GMR_y = \sqrt[9]{}$$