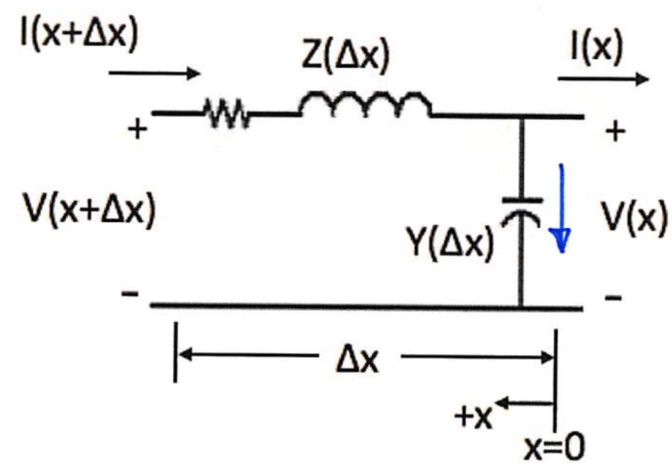
Defining the Differential Eq.

Consider a differential length of line:

$$V_{(X+\Delta X)} = V_{(X)} + Z_{\Delta X} \cdot I_{(X+\Delta X)}$$

$$I_{(X+\Delta X)} = I_X + Y_{\Delta X} \cdot V_X$$



$$\frac{V_{(X+\Delta X)} - V_{(X)}}{\Delta X} = Z_{\Delta X} \cdot I_{(X+\Delta X)}$$

$$\lim_{\Delta X \to 0} \frac{dV}{dX} = Z \cdot I_X$$

$$\frac{I_{(X+\Delta X)} - I_X}{\Delta X} = Y_{\Delta X} \cdot V_X$$

$$\lim_{\Delta X \to 0} \frac{dI}{dX} = Y \cdot V_X$$

Solving the Diff Eq

$$\left[\frac{e^{+\gamma l} + e^{-\gamma l}}{2}\right] = \cosh(\gamma l) \qquad \left[\frac{e^{+\gamma l} - e^{-\gamma l}}{2}\right] = \sinh(\gamma l)$$

$$V_S = \cosh(\gamma l)\cdot V_R + Z_C \sinh(\gamma l)\cdot I_R$$

$$I_S = \frac{1}{Z_C}\sinh(\gamma l)\cdot V_R + \cosh(\gamma l)\cdot I_R$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

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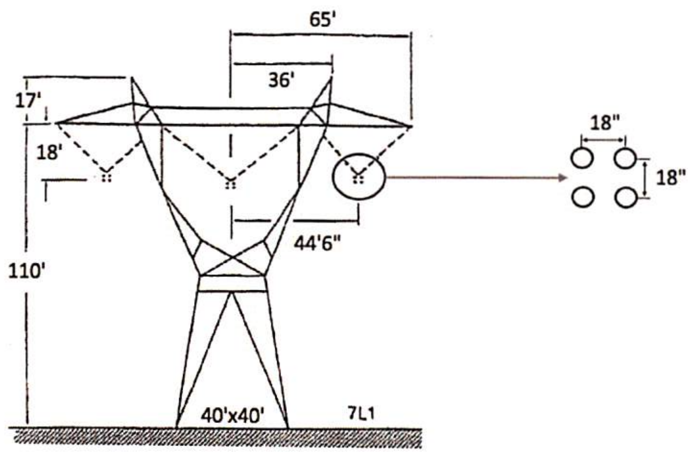
Example



A 765 kV transmissionn line is 400km long. The “flat” configuration features 4 Canary conductors spaced at the corners of a square 18” per side. The bundles spacing is 44’6”.

Per Table A4:

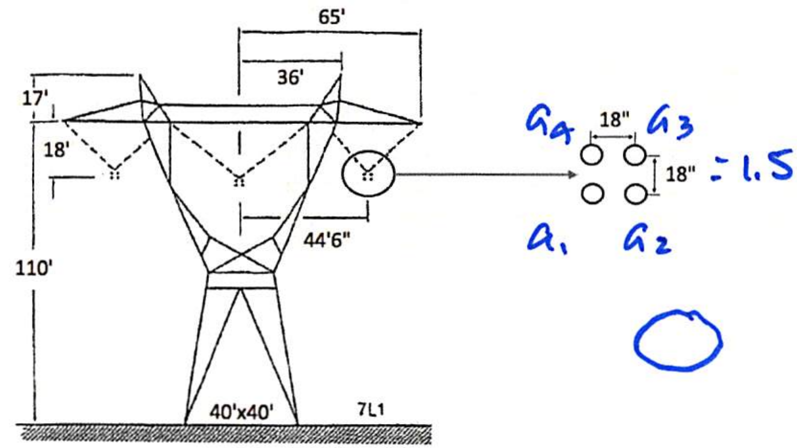
Outside Diameter = 1.162”

GMR = 0.0391’

$R_{50C\ 60Hz}$ = 0.1185 ohms/mile

Example

Per Table A4:
Outside Diameter = 1.162”
GMR = 0.0391’
R50C 60Hz = 0.1185 ohms/mile



$$D_{eq} = \sqrt[3]{44.5 \cdot 44.5 \cdot 89} = 56.066 \quad feet$$

R'_{bL} :

$$D_{SL} = \sqrt[4]{1.5 \cdot 1.5 \cdot 1.5\sqrt{2} \cdot 0.0391} = 0.65727 \quad feet$$

$$D_{SC} = \sqrt[4]{1.5 \cdot 1.5 \cdot 1.5\sqrt{2} \cdot 1.162/24} = 0.69334 \quad feet$$

$$R = 0.1184/4 \ \ \Omega/mile \cdot 1mile/1.609km = 0.0184 \ \ \Omega/km$$

$$X_L = j(2\pi \cdot 60) \cdot 2 \cdot 10^{-7} \ln\frac{56.066}{0.65727} = 3.35236 \cdot 10^{-4} \ \ \Omega/m \cdot 1000m/km = 0.335236 \ \ \Omega/km$$

$$Y = \frac{j(2\pi \cdot 60) \cdot (2\pi) 8.854 \cdot 10^{-12}}{\ln\frac{56.066}{0.69339}} = 4.77432 \cdot 10^{-9} \ \ \Omega/m \cdot 1000m/km = 4.77432 \cdot 10^{-6} \angle 90^\circ \ \ \Omega/km$$

$$Z = 0.018395 + j0.335236 \ \ \Omega/km = 0.33574 \angle 86.859^\circ$$

Example

$$\gamma = \sqrt{ZY} = \sqrt{(0.33574\angle 86.856^\circ)(4.77441\cdot 10^{-6}\angle 90^\circ)} = 0.0012661\angle 88.429^\circ\,/\,km$$

$$\gamma l = 0.50643\angle 88.4289^\circ$$

$$Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{(0.33574\angle 86.856^\circ)}{(4.77441\cdot 10^{-6}\angle 90^\circ)}} = 265.183\angle -1.571^\circ$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8747\angle 0.4409^\circ & 128.6374\angle 86.996^\circ \\ 1.8293\cdot 10^{-3}\angle 90.137^\circ & 0.8747\angle 0.4409^\circ \end{bmatrix}$$

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Example

Given we're delivering 1000MVA at rated voltage, pf = 0.8 lagging

Determine the sending end voltage and current.

$$V_{R\ (per\ \ phase)} = \frac{765000}{\sqrt{3}} = 441673\angle 0^\circ$$

$$I_R = \left(\frac{1000\cdot 10^6\angle 36.87^\circ}{\sqrt{3}\ \ 765000\angle 0^\circ}\right)^* = 754.7\angle -36.87^\circ$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 0.8747\angle 0.4409^\circ & 128.6374\angle 86.996^\circ \\ 1.8293\cdot 10^{-3}\angle 90.137^\circ & 0.8747\angle 0.4409^\circ \end{bmatrix} \cdot \begin{bmatrix} 441673\angle 0^\circ \\ 754.7\angle -36.87^\circ \end{bmatrix} = \begin{bmatrix} 455202\angle 9.8^\circ \\ 673.105\angle 38.17^\circ \end{bmatrix}$$

$$S_{in} = 3\cdot(455202\angle 9.8^\circ)(673.105\angle -38.17^\circ) = 919.196\angle -28.37^\circ\ \ MVA$$

$$S_{in} = 808.798MW\ - j\ \ 436.768MVAR$$

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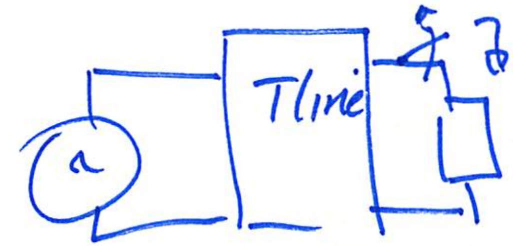
Example

$$S_{out} = 1000\angle 36.87 \ MVA = 800MW + j600MVAR$$



$$\eta = \frac{P_{out}}{P_{in}} = \frac{800}{808.798} \cdot 100\% = 98.91\%$$

$|V_S| = 455202 \qquad |V_R| = 441763 \qquad V_S$ is about 3% larger than V_R

however

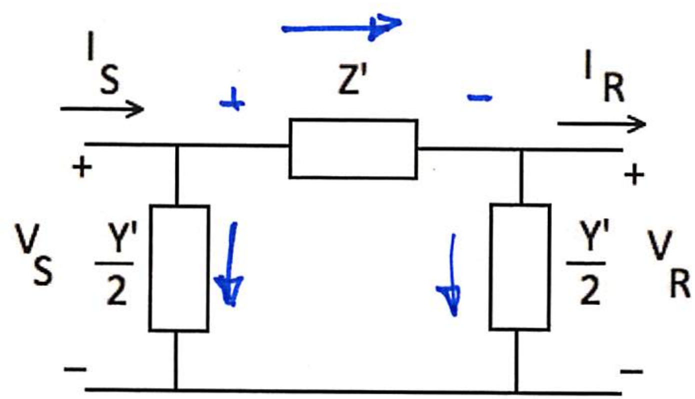
$$V_S = AV_R + BI_R \quad under \ \ no-load \ \ conditions \ \ I_R = 0$$

$$|V_{R \ no-load}| = \left|\frac{V_S}{A}\right| = \left|\frac{455202}{0.8747}\right| = 520409.28$$

$A = \cosh \gamma l$

$$V_{REG} = \frac{V_{NL} - V_{FL}}{V_{FL}} \cdot 100\% = \frac{520409.28 - 441763}{441763} \cdot 100\% = \underline{17.8\%}$$

Lumped Parameter Network



$$V_S = V_R + \left[V_R\left(\frac{Y'}{2}\right) + I_R\right]Z'$$

$$V_S = \left(1 + \frac{Z'Y'}{2}\right)V_R + Z'I_R$$

$$V_S = AV_R + BI_R$$

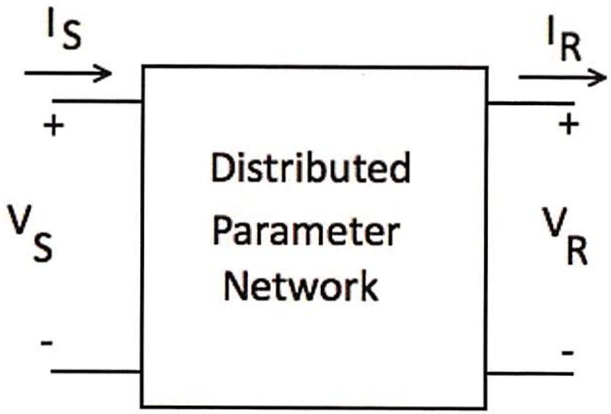
$$I_S = V_S\left(\frac{Y'}{2}\right) + V_R\left(\frac{Y'}{2}\right) + I_R$$

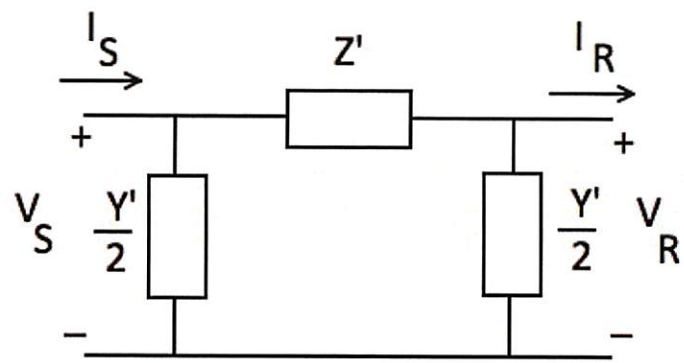
$$I_S = \left(\frac{Y'}{2}\right)\left(1 + \frac{Z'Y'}{2}\right)V_R + \left(\frac{Y'}{2}\right)Z'I_R + V_R\left(\frac{Y'}{2}\right) + I_R$$

$$I_S = \left(Y' + \frac{Z'Y'^2}{4}\right)V_R + \left(1 + \frac{Z'Y'}{2}\right)I_R$$

$$I_S = CV_R + DI_R$$

Lumped Parameter Network





Exact "π" Equivalent

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{Z'Y'}{2}\right) & Z' \\ \left(Y' + \frac{Z'Y'^2}{4}\right) & \left(1 + \frac{Z'Y'}{2}\right) \end{bmatrix}$$

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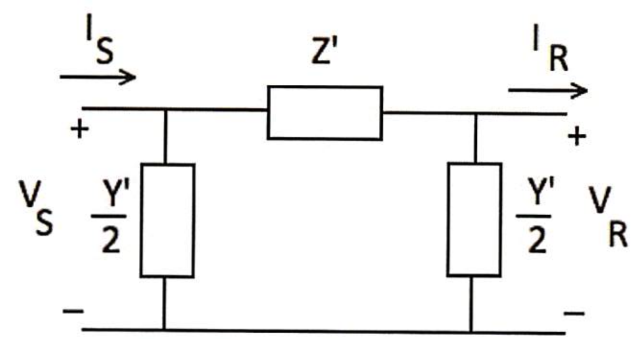
Lumped Parameter Network

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix} \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{Z'Y'}{2}\right) & Z' \\ \left(Y' + \frac{Z'Y'^2}{4}\right) & \left(1 + \frac{Z'Y'}{2}\right) \end{bmatrix}$$

$$Z' = Z_C \cdot \sinh(\gamma l)$$

$$\left(1 + \frac{Z'Y'}{2}\right) = \cosh(\gamma l) \qquad \frac{Z'Y'}{2} = \cosh(\gamma l) - 1$$

$$\frac{Y'}{2} = \frac{\cosh(\gamma l) - 1}{Z_C \cdot \sinh(\gamma l)} = \frac{1}{Z_C}\tanh\left(\frac{\gamma l}{2}\right)$$



Exact "π" Equivalent

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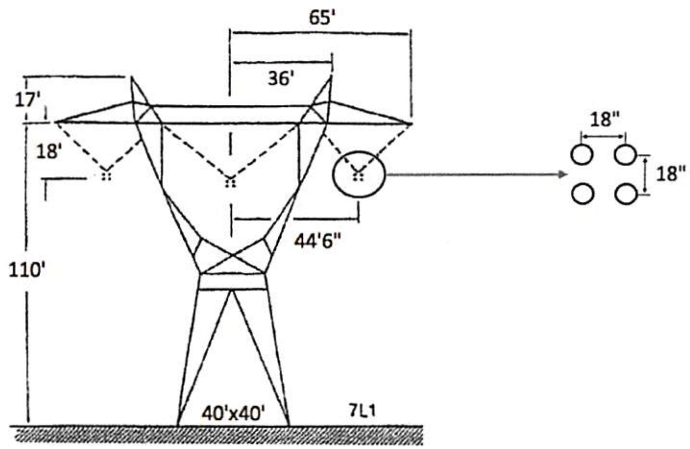
Returning to our Example



Per Table A4:
Outside Diameter = 1.162" GMR = 0.0391'
$R_{50C\ 60Hz}$ = 0.1185 ohms/mile

$$Y = 4.77432 \cdot 10^{-6} \angle 90° \quad \Omega / km$$

$$Z = 0.018412 + j0.335235 \ \Omega / km = 0.33574 \angle 86.859°$$

$$\gamma = \sqrt{ZY} = \sqrt{(0.33574\angle 86.859°)(4.77432 \cdot 10^{-6}\angle 90°)} = 0.0012661\angle 88.430° / km$$

$$\gamma l = 0.50643\angle 88.430° \qquad Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{(0.33574\angle 86.859°)}{(4.77432\cdot 10^{-6}\angle 90°)}} = 265.183\angle -1.571°$$

$$Z' = Z_C \sinh(\gamma l) = 265.183\angle -1.571° \sinh(0.50644\angle 88.428°) = 128.637\angle 86.995° \ \Omega$$

$$\frac{Y'}{2} = \frac{1}{Z_C}\tanh\left(\frac{\gamma l}{2}\right) = \frac{1}{265.183\angle -1.571°}\tanh\left(\frac{0.50644\angle 88.428°}{2}\right) = 9.758 \cdot 10^{-4}\angle 89.931° \ S$$

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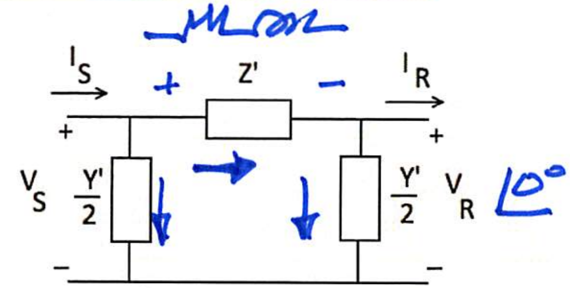
Lumped Parameter Solution

$$Z' = 128.6374\angle 86.995°\ \Omega$$

$$\frac{Y'}{2} = 9.758 \cdot 10^{-4}\angle 89.931°\ S$$



Exact "π" Equivalent

$$V_{R\ (per\ \ phase)} = \frac{765000}{\sqrt{3}} = 441673\angle 0°$$

$$I_R = \left(\frac{1000 \cdot 10^6\angle 36.87°}{\sqrt{3}\ \ 765000\angle 0°}\right)^* = 754.71\angle -36.87°\,A$$

$$I_{\frac{Y'}{2}(R)} = 441673\angle 0° \cdot 9.758 \cdot 10^{-4}\angle 89.931°\ \ S = 430.985\angle 89.931°\,A$$

$$I_{Z'} = I_R + I_{\frac{Y'}{2}(R)} = 754.7\angle -36.87° + 430.985\angle 89.931° = 604.673\angle -2.070\,A$$

$$V_{Z'} = I_{Z'} \cdot Z' = 604.673\angle -2.070 \cdot 128.6374\angle 86.995°\ \ \Omega = 77783.52\angle 84.925° V$$

$$V_S = V_{Z'} + V_R = 77783.52\angle 84.925° + 441673\angle 0° = 455195.38\angle 9.80° V$$

$$I_S = V_S \cdot \frac{Y'}{2} + I_{Z'} = 455195.38\angle 9.80° \cdot 9.758 \cdot 10^{-4}\angle 89.931° + 604.673\angle -2.070$$

$$= 673.102\angle 38.167°\,A$$

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The Nominal Pi Equivalent Circuit

$$Z' = Z_C \sinh(\gamma l) = \sqrt{\frac{Z}{Y}} \sinh\left(\sqrt{ZY} \cdot l\right)$$

$$\sinh(x) = x + \cancel{\frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!}} + \ldots\ldots$$

$$Z' = \sqrt{\frac{Z}{Y}} \left[\left(\sqrt{ZY} \cdot l\right) + \frac{\left(\sqrt{ZY} \cdot l\right)^3}{3!} + \frac{\left(\sqrt{ZY} \cdot l\right)^5}{5!} + \ldots\ldots \right]$$

$$Z' \approx \sqrt{\frac{Z}{\cancel{Y}}} \left(\sqrt{Z\cancel{Y}} \cdot l\right) \approx \underline{Zl}$$

$$Z_{\text{nominal } \pi} = \underline{Zl}$$

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The Nominal Pi Equivalent Circuit

$$\frac{Y'}{2} = \frac{\cosh(\gamma l) - 1}{Z_C \sinh(\gamma l)}$$

$$\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \; .....$$

$$\frac{Y'}{2} = \frac{\left[1 + \frac{\left(\sqrt{ZY}\cdot l\right)^2}{2!} + \cancel{\frac{\left(\sqrt{ZY}\cdot l\right)^4}{4!}} + \; .....\right] - 1}{\sqrt{\frac{Z}{Y}}\left[\left(\sqrt{ZY}\cdot l\right) + \cancel{\frac{\left(\sqrt{ZY}\cdot l\right)^3}{3!} + \frac{\left(\sqrt{ZY}\cdot l\right)^5}{5!}} + \; .....\right]}$$

$$\frac{Y'}{2} \approx \frac{\frac{\left(\sqrt{ZY}\cdot l\right)^2}{2}}{\sqrt{\frac{Z}{Y}}\left(\sqrt{ZY}\cdot l\right)} \approx \frac{\sqrt{ZY}\cdot l}{2\cdot\sqrt{\frac{Z}{Y}}} \approx \frac{Y\cdot l}{2} = \frac{Y}{2}$$

S $\quad \frac{Y}{2}_{\text{nominal } \pi} = \frac{Y \cdot l}{2}$

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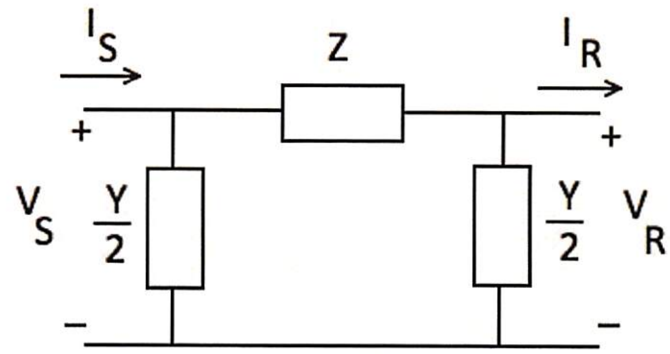
The Nominal Pi EquivalentCircuit

$$Z_{\text{nominal } \pi} = Zl \qquad \frac{Y}{2}_{\text{nominal } \pi} = \frac{Y \cdot l}{2}$$



"Nominal π" Equivalent

$$Zl = 0.33574\angle 86.995° \cdot 400 = 134.296\angle 86.995° = Z_{\text{nominal } \pi}$$ Ω

$$\frac{Y \cdot l}{2} = \frac{4.77432 \cdot 10^{-6}\angle 90° \cdot 400}{2} = 9.5486 \cdot 10^{-4}\angle 90° = \frac{Y}{2}_{\text{nominal } \pi}$$ S

Recall the exact parameters:

$$Z' = Z_C \sinh(\gamma l) = 265.183\angle -1.571° \sinh(0.50644\angle 88.428°) = 128.637\angle 86.995°$$ Ω

$$\frac{Y'}{2} = \frac{1}{Z_C}\tanh\left(\frac{\gamma l}{2}\right) = \frac{1}{265.183\angle -1.571°}\tanh\left(\frac{0.50644\angle 88.428°}{2}\right) = 9.758 \cdot 10^{-4}\angle 89.931°$$

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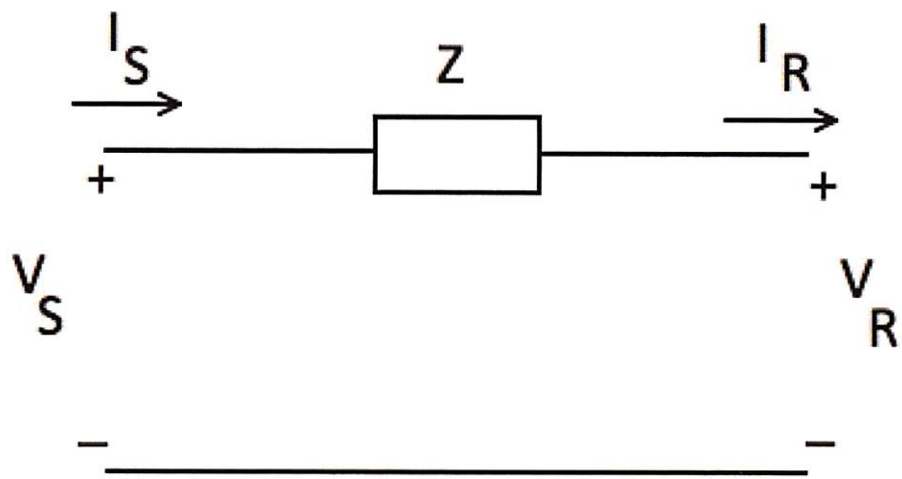
Short Line Approximation

Since the shunt admittance is normally quite small, as the line length gets small
Yl/2 becomes negligible and $Y/2 \Rightarrow 0$. If the admittance approaches zero, the impedance approaches infinity.
The short line equivalent circuit is shown below.



Short Line Equivalent

Three Lumped Parameter Line Models

Exact π

$$Z' = Z_C \sinh(\gamma l)$$

$$\frac{Y'}{2} = \frac{1}{Z_C}\tanh\left(\frac{\gamma l}{2}\right)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

Nominal π

$$Z = Zl$$

$$\frac{Y}{2} = \frac{Yl}{2}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\begin{bmatrix} \left(1 + \frac{ZY}{2}\right) & Z \\ \left(Y + \frac{ZY^2}{4}\right) & \left(1 + \frac{ZY}{2}\right) \end{bmatrix}$$

Short Line

$$Z = Zl$$

$$\frac{Y}{2} = 0$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

How do I know what to use??

Line Length	Appropriate Model
80km or less	Short line
80km – 250km	Nominal π
greater than 250 km	Exact π

Or – whatever I tell you to use !!!

Table 5.1 pg ~~258~~ 269

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Comparison Calculations

400 km long line

	Exact	Nominal	Short
A=D	$0.8747\angle.4407$	$0.8720\angle.4615$	1.0000
Z' (Z)=B	$128.6\angle 87.00$	$134.3\angle 86.86$	$134.3\angle 86.86$
C	$1.829\text{x}10^{-3}\angle 90.14$	$1.665\text{x}10^{-3}\angle 90.46$	0.0000
Y'/2 (Y/2)	$9.757\text{x}10^{-4}\angle 89.93$	$9.548\text{x}10^{-4}\angle 90.00$	0.0000
V_S	455.2 kV $\angle 9.80$	457.5 kV $\angle 10.2$	512.8 kV $\angle 8.7$
I_S	$673.0\angle 38.16$	$627.0\angle 33.36$	$754.7\angle -36.87$

100 km long line

	Exact	Nominal	Short
A=D	$0.9920\angle.0253$	$0.9920\angle.0254$	1.0000
Z' (Z)=B	$33.49\angle 86.87$	$33.58\angle 86.86$	$33.58\angle 86.86$
C	$4.761\text{x}10\text{-}4\angle 90.01$	$4.736\text{x}10^{-4}\angle 90.03$	0.0000
Y'/2 (Y/2)	$2.390\text{x}10\text{-}4\angle 90.00$	$2.387\text{x}10^{-4}\angle 90.00$	0.0000
V_S	454.8 kV $\angle 2.464$	454.9 kV $\angle 2.470$	458.4 kV $\angle 2.427$
I_S	$644.9\angle -21.72$	$645.2\angle -21.81$	$754.7\angle -36.87$

		25 km long line	
	Exact	Nominal	Short
A=D	$0.9995\angle.00157$	$1.0000\angle.0034$	1.0000
Z' (Z)=B	$8.392\angle 86.86$	$8.394\angle 86.86$	$8.394\angle 86.86$
C	$1.193\text{x}10\text{-}4\angle 90.00$	$1.194\text{x}10^{-4}\angle 90.00$	0.0000
Y'/2 (Y/2)	$5.968\text{x}10\text{-}5\angle 90.00$	$5.968\text{x}10^{-5}\angle 90.00$	0.0000
V_S	445.6 kV $\angle.625$	445.7 kV $\angle.627$	445.8 kV $\angle.624$
I_S	$723.9\angle -33.5$	$724.3\angle -33.5$	$754.7\angle -36.87$

765 kV line:

$z = .33575\angle 86.86\ \Omega/\text{km}$

$y = 4.7740\text{x}10^{-6}\angle 90.00$ S/km

$Z_C = 265.2\angle -1.572\ \Omega$

$\gamma = 1.266\text{x}10^{-3}\angle 88.43$ /km

V_R = 441,673 V/phase

$I_R = 754.7\angle -36.87$ A

Lossless Lines

$$Z = 0.\cancel{1839}6 + j0.33524 \quad \Omega / km$$

$$Y = j4.77432 \cdot 10^{-6} \quad S / km$$

if we neglect R

$$Z = j0.33524 \quad \Omega / km$$

$$Z_C = \sqrt{\frac{Z}{Y}} = 264.986\Omega$$ "Surge Impedance" of a lossless line (purely real)

$$\gamma = \sqrt{ZY} = j0.0012649 \ / km = \alpha + j\beta$$ ($\alpha = 0$ purely imaginary)

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2} \qquad \cosh(j\beta) = \frac{e^{j\beta} + e^{-j\beta}}{2} = \cos(\beta)$$

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2} \qquad \sinh(j\beta) = \frac{e^{j\beta} - e^{-j\beta}}{2} = j\sin(\beta)$$

Lossless Lines

$$\begin{bmatrix} A_{(x)} & B_{(x)} \\ C_{(x)} & D_{(x)} \end{bmatrix} = \begin{bmatrix} \cos(0.0012649 \cdot x) & j264.986 \sin(0.0012649 \cdot x) \\ j\frac{1}{264.986} \sin(0.0012649 \cdot x) & \cos(0.0012649 \cdot x) \end{bmatrix}$$

$$but \;\; 1 \;\; radian \;\; = \;\; 57.296^\circ$$

$$\begin{bmatrix} A_{(x)} & B_{(x)} \\ C_{(x)} & D_{(x)} \end{bmatrix} = \begin{bmatrix} \cos(0.072473 \cdot x^\circ) & j264.986 \sin(0.072473 \cdot x^\circ) \\ j\frac{1}{264.986} \sin(0.072473 \cdot x^\circ) & \cos(0.072473 \cdot x^\circ) \end{bmatrix}$$

$$converting \;\; to \;\; p.u. \;\; V_{base} = 765kV \;\; and \;\; S_{base} = 1000MVA$$

$$Z_{base} = \frac{\left(765 \cdot 10^3\right)^2}{1000 \cdot 10^6} = 585.225$$

$$\begin{bmatrix} A_{(x)} & B_{(x)} \\ C_{(x)} & D_{(x)} \end{bmatrix} = \begin{bmatrix} \cos(0.072473 \cdot x^\circ) & j0.45279 \sin(0.072473 \cdot x^\circ) \\ j2.20851 \;\sin(0.072473 \cdot x^\circ) & \cos(0.072473 \cdot x^\circ) \end{bmatrix}$$

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Lossless Line

$$V_{(x)} = A_{(x)} \cdot V_R + B_{(x)} \cdot I_R$$

$$V_{(x)} = \cos(0.072473 \cdot x°) \cdot V_R + j264.986 \sin(0.072473 \cdot x°) \cdot I_R$$

if we terminate the line with a surge impedance load

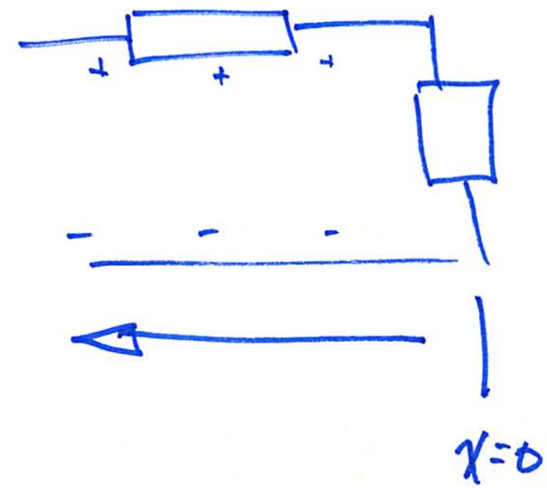
$$\mathrm{I_R} = \frac{V_R}{Z_C}$$

$$V_{(x)} = \cos(0.072473 \cdot x°) \cdot V_R + j \sin(0.072473 \cdot x°) \cdot V_R$$

$$V_{(x)} = \left[\cos(0.072473 \cdot x°) + j \sin(0.072473 \cdot x°)\right] \cdot V_R$$

$$V_{(x)} = (1\angle 0.072473 \cdot x°) \cdot V_R$$

$\frac{765{,}000}{\sqrt{3}} \angle 0°$



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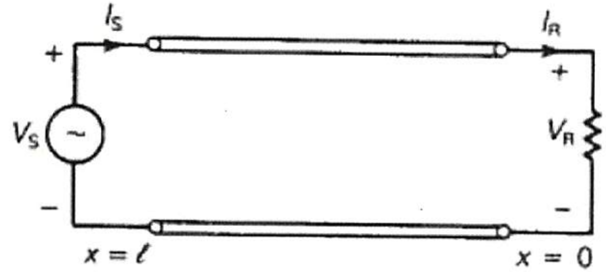
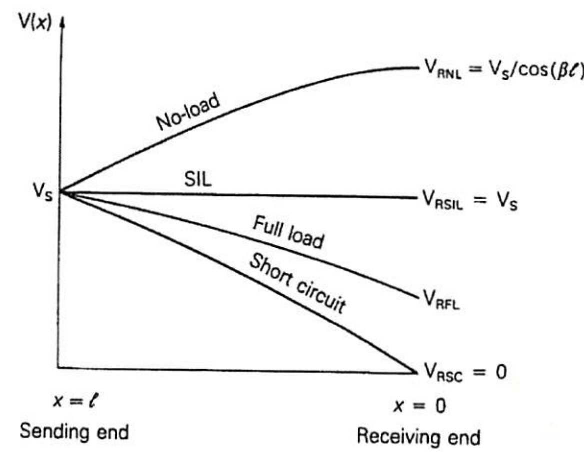
Lossless Line



$Z_c = \sqrt{\frac{L}{C}} \ \Omega$



$$V_{(x)} = \cos(0.072473 \cdot x^\circ) \cdot V_R + j \ Z_C \sin(0.072473 \cdot x^\circ) \cdot I_R$$

$$I_{(x)} = j\frac{1}{Z_C}\sin(0.072473 \cdot x^\circ) \cdot V_R + \cos(0.072473 \cdot x^\circ) \cdot I_R$$

under surge impedance loading, $I_R = \dfrac{V_R}{Z_C}$

$$V_{(x)} = \cos(0.072473 \cdot x^\circ) \cdot V_R + j \ \sin(0.072473 \cdot x^\circ) \cdot V_R$$

$$V_{(x)} = (1\angle 0.072473 \cdot x^\circ) V_R$$

$$I_{(x)} = j\frac{1}{Z_C}\sin(0.072473 \cdot x^\circ) \cdot V_R + \cos(0.072473 \cdot x^\circ) \cdot \frac{V_R}{Z_C}$$

$$I_{(x)} = (1\angle 0.072473 \cdot x^\circ) V_R$$

$$S_{(x)} = V_{(x)} \cdot \left(I_{(x)}\right)^* = \frac{|V_R|^2}{Z_C}$$

The real power flow is constant along the length of the line.
The reactive power flow is zero.

Wavelength

$$V_{(x)} = \left(1\angle 0.072473 \cdot x^\circ\right) \cdot V_R$$

$$V_{(x)} = \left(1\angle 0.072473 \cdot (400)^\circ\right) \cdot V_R$$

$$V_{(x)} = \left(1\angle 28.99^\circ\right) \cdot V_R$$

Wavelength is defined as the distance required to change the phase of the voltage or current by 360°.

$$0.072473 \cdot \lambda^\circ = 360^\circ \qquad \lambda = 4{,}968 km = 3088 \;\; miles$$

Typical transmission lines are only a small fraction of a wavelength.

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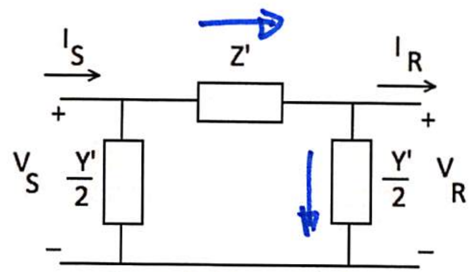
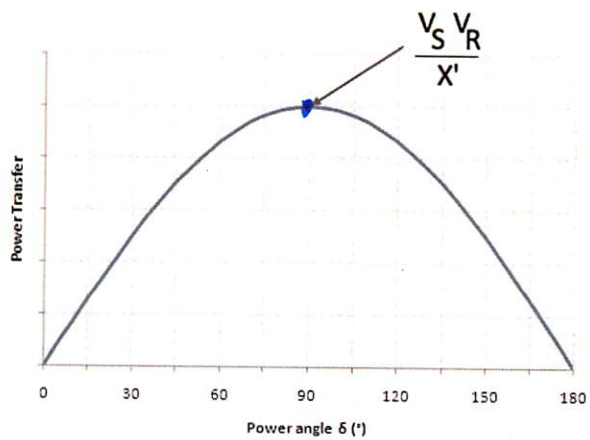
Steady State Stability Limit



Exact "π" Equivalent



$$I_R = \frac{V_S - V_R}{Z'} - \frac{Y'}{2}\cdot V_R$$

$$S_R = V_R \cdot I_R^{\,*} = V_R \cdot \left(\frac{V_S - V_R}{Z'} - \frac{Y'}{2}\cdot V_R\right)^*$$

$$V_R = V_R\angle 0^\circ \quad V_S = V_S\angle \delta^\circ \quad Z' = jX' \ \ and \ \ Y' = j\omega C' l$$

$$S_R = V_R\left(\frac{V_S\angle\delta - V_R}{jX'}\right)^* + \frac{j\omega C' l}{2}V_R^{\,2}$$

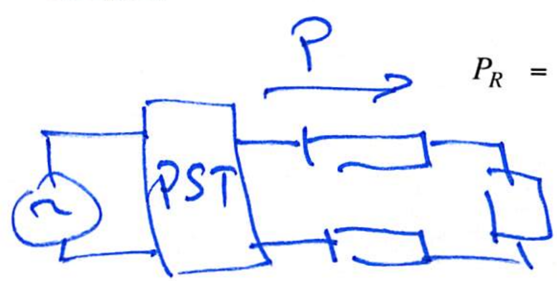
$$S_R = V_R\left(\frac{V_S\angle -\delta - V_R}{-jX'}\right) + \frac{j\omega C' l}{2}V_R^{\,2}$$

$S = P + jQ$

$$S_R = \frac{V_R V_S\angle 90-\delta}{X'} - j\frac{V_R^{\,2}}{X'} + \frac{j\omega C' l}{2}V_R^{\,2}$$

$$S_R = P_R + jQ_R$$

$$P_R = \frac{|V_R|\cdot|V_S|}{|X'|}\cos(90-\delta^\circ) = \frac{|V_R|\cdot|V_S|}{|X'|}\sin\delta$$



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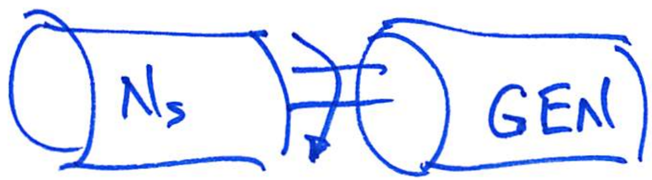
$\uparrow\downarrow\uparrow$

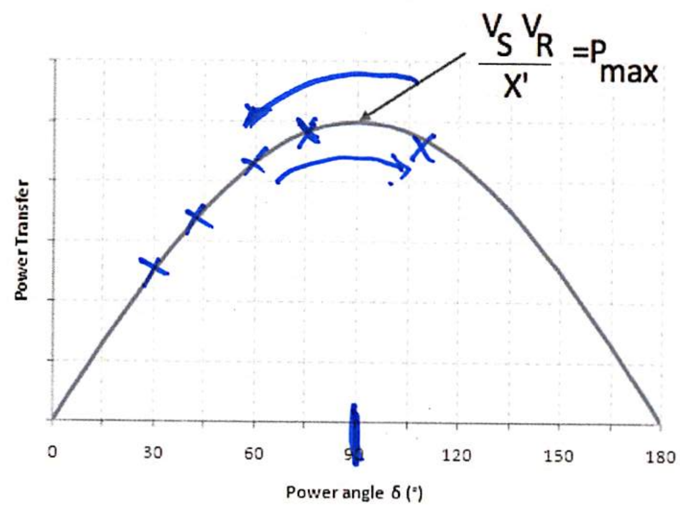
$P_{mech} = \tau \cdot N_s$

Steady State Stability Limit



$f_0 = 60Hz$

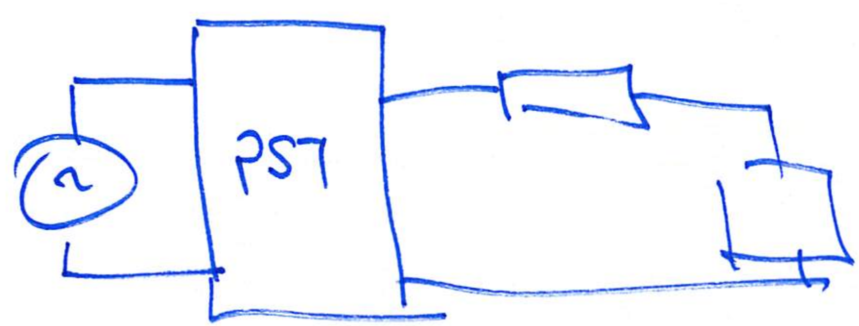
$$P_R = \frac{|V_R| \cdot |V_S|}{|X'|} \sin\delta$$



When $\delta = 90°$, $\sin\delta = 1.0$ and $P_R = P_{R\,MAX}$

$P_{R\,MAX}$ = the "theoretical steady state stability limit" of a lossless line.

If one were to try and exceed this limit the generators at the sending end of the line would lose synchronization with the machines on the receiving end of the line and that would be really really bad!



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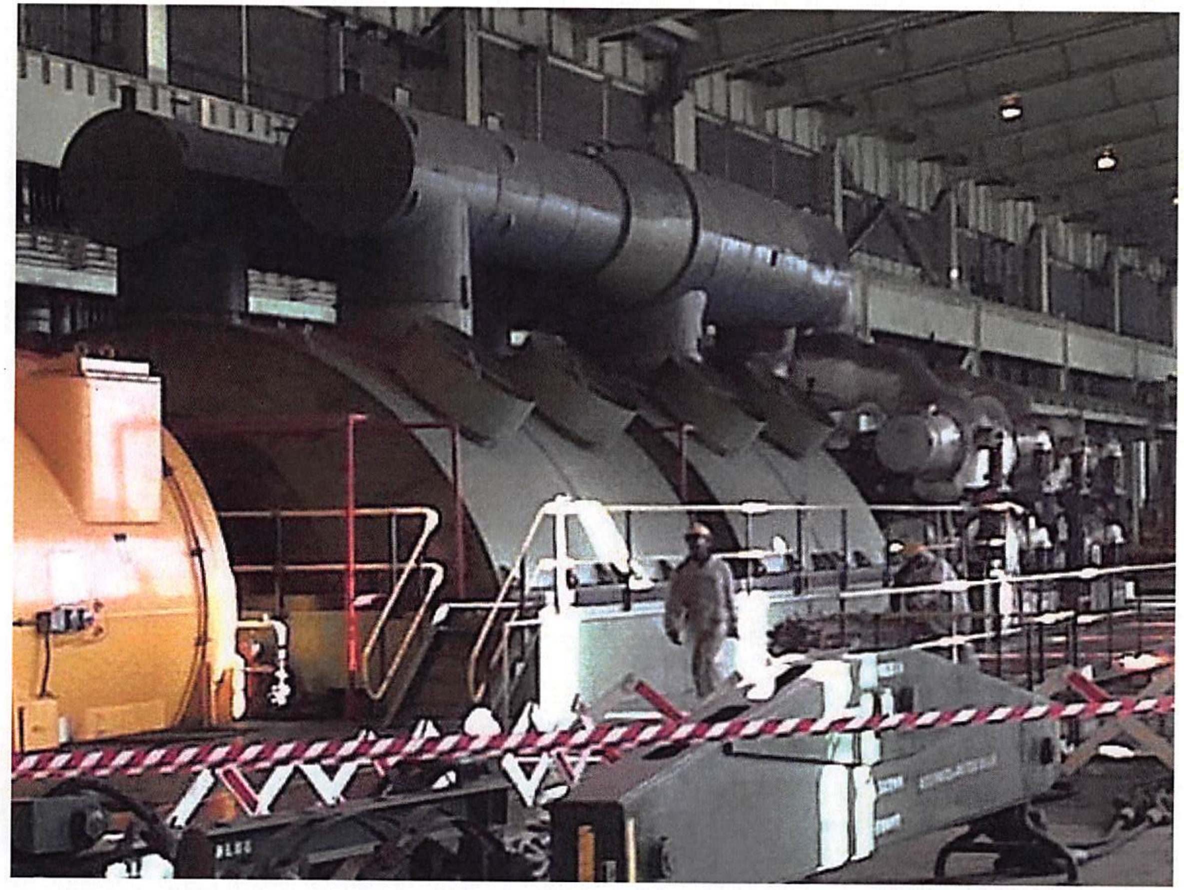
Exceeding The Stability Limit



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Exceeding the Stability Limit

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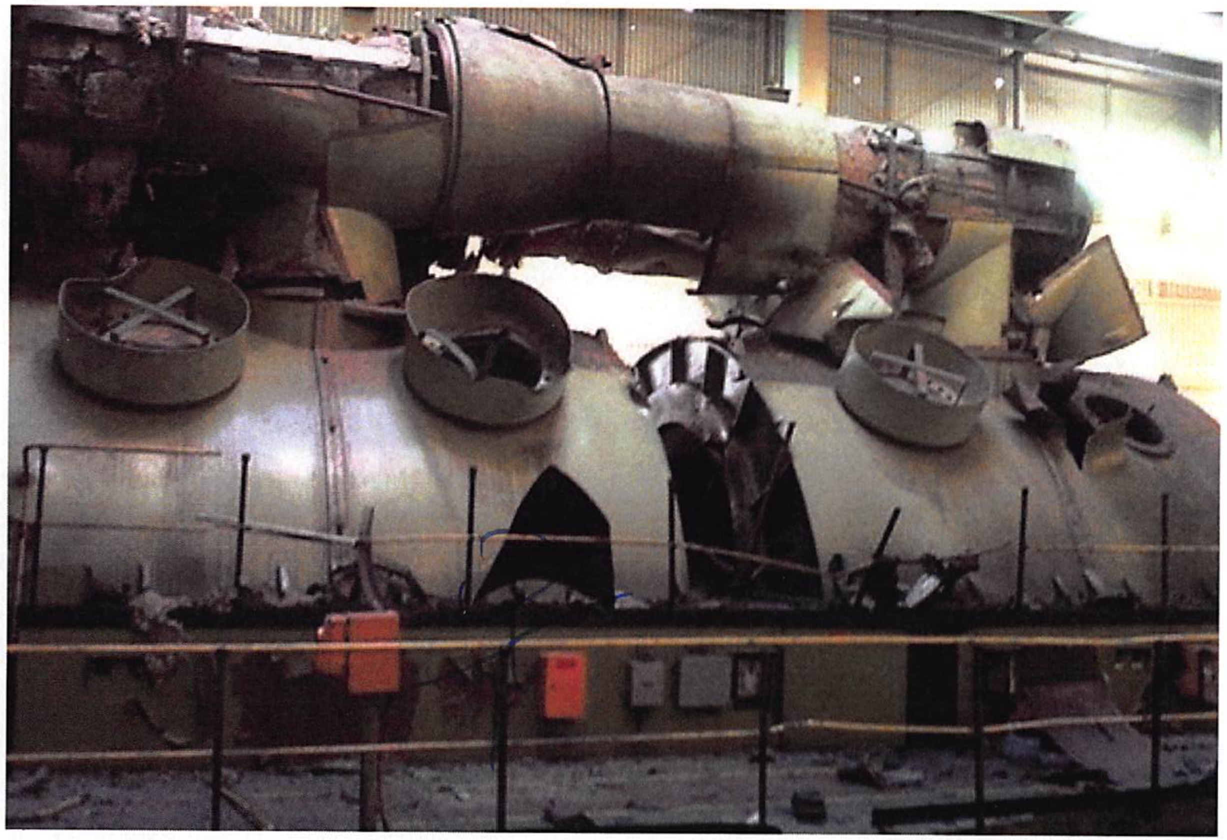
Exceeding the Stability Limit



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