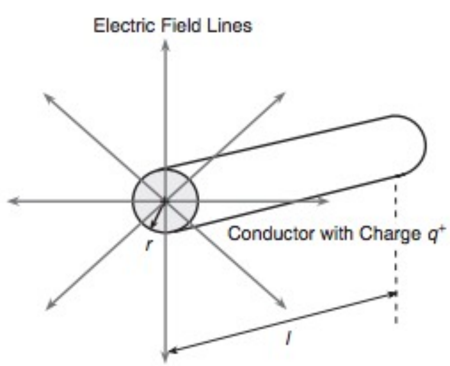
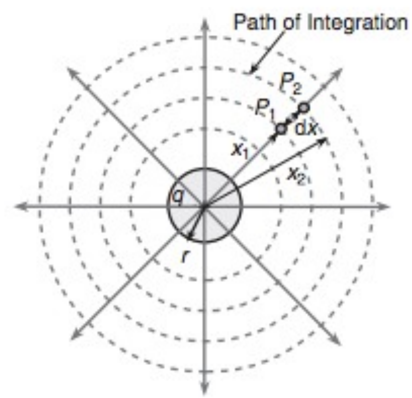
Transmission Line Parameters - Part 2

Capacitance and Capacitive Reactance

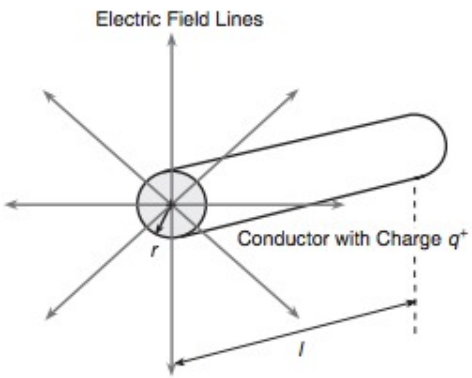
Voltage on a solid, single conductor

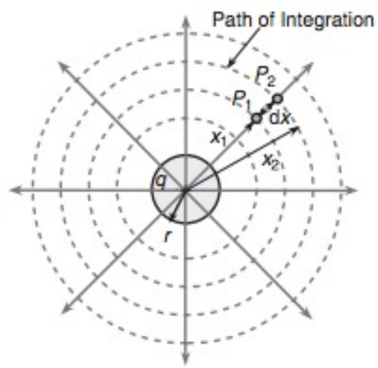
Consider a long, solid, cylindrical conductor of radius "r" immersed in a media with permittivity ε_0. (Where $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m) A charge of +q coulombs per meter exits and is uniformly distributed on the surface of the conductor. The conductor is a perfect conductor with resistivity assumed to be zero, so there is no **internal** electric field due to the charge on the conductor.



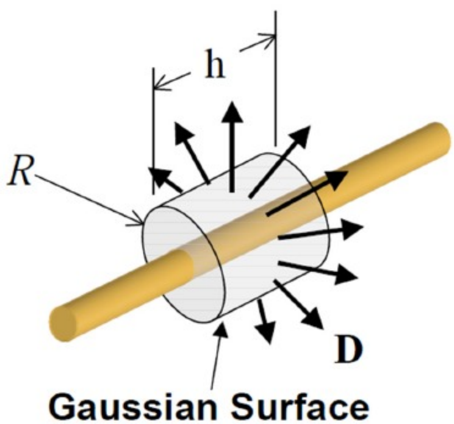


Capacitance





An electric field will be produced radial to the conductor due to the charge on the conductor, with equi-potential surfaces concentric to the surface.



$$q_e = \int_A D \cdot da$$

According to Gauss's Law, the total electric flux leaving a closed surface is equal to the total charge inside the volume enclosed by the surface.

Capacitance

$$D_p = \frac{q}{A} = \frac{q}{2\pi x} \ (\mathrm{C/m^2})$$ The electric flux density D at point p = Charge/Area

$$D_p = \varepsilon \cdot E_p$$ The electric flux density = permittivity x electric flux intensity

$$E_p = \frac{q}{2\pi\varepsilon_0 x} \ (\mathrm{V/m})$$

The potential difference between any two points (P1 and P2) located outside the conductor surface at distance x_1 and x_2 from the center of the conductor respectively can be determined by integrating the electric field intensity from x_1 to x_2

$$V_{1-2} = \int_{X1}^{X2} \frac{q}{2\pi\varepsilon_0 x} \, dx = \frac{q}{2\pi\varepsilon_0} \ln\left(\frac{X_2}{X_1}\right) \ (\mathrm{V})$$

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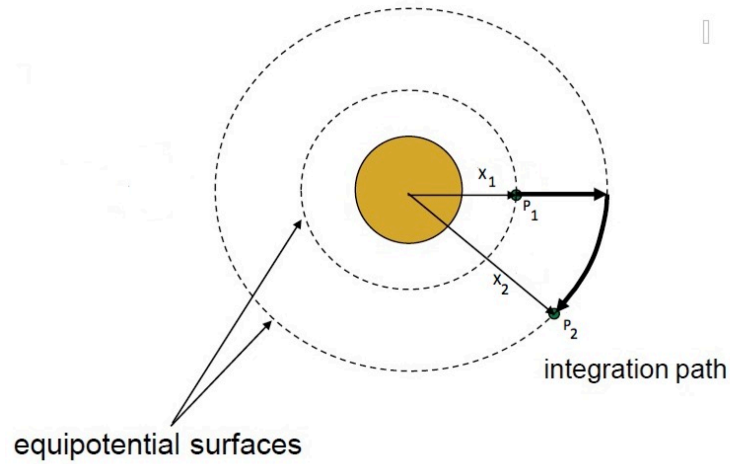
Capacitance

$$V_{1-2} = \frac{q}{2\pi\varepsilon_0} \ln\left(\frac{X_2}{X_1}\right) \quad \text{(V)}$$

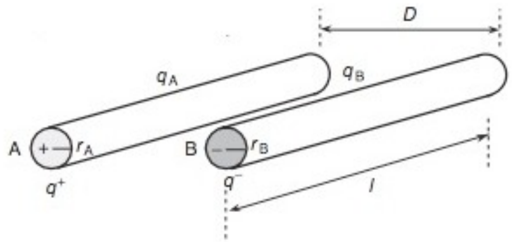


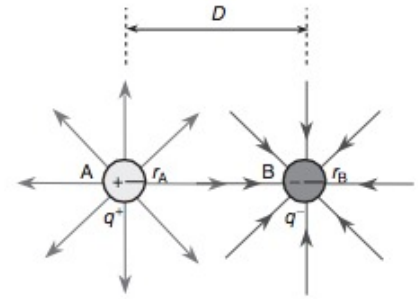
but capacitance is the proportionality constant relating charge and voltage

q = C · V

$$C_{1-2} = \frac{q}{V_{1-2}} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{X_2}{X_1}\right)} \quad \text{(F/m)}$$

Capacitance of a Two Wire Single Phase Line





The voltage arising due to a charge on a single conductor was given by:

$$V_{1-2} = \frac{q}{2\pi\varepsilon_0} \ln\left(\frac{X_2}{X_1}\right) \quad \text{(V)}$$

Using the principle of superposition -

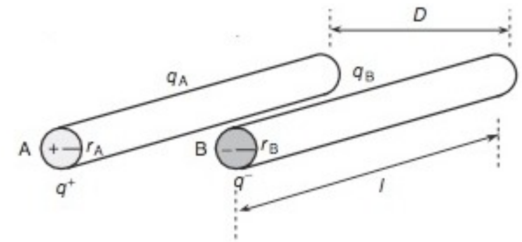
$$V_{A-B} = \frac{q}{2\pi\varepsilon_0} \ln\left(\frac{D_{AB}}{D_{AA}}\right) + \frac{-q}{2\pi\varepsilon_0} \ln\left(\frac{D_{BB}}{D_{BA}}\right) \quad \text{(V)}$$

$$V_{A-B} = \frac{q}{2\pi\varepsilon_0} \left(\ln\left(\frac{D}{r_A}\right) - \ln\left(\frac{r_B}{D}\right)\right) = \frac{q}{2\pi\varepsilon_0} \left(\ln \frac{\mathrm{D}^2}{r_\mathrm{A} r_B}\right) \text{(V)} = \frac{q}{\pi\varepsilon_0} \left(\ln \frac{\mathrm{D}}{\sqrt{r_\mathrm{A} r_B}}\right)$$

Capacitance of a Two Wire Single Phase Line

$$C_{AB} = \frac{q}{V_{A-B}}$$

$$C_{AB} = \frac{2\pi\varepsilon_0}{\ln \frac{D^2}{r_A r_B}} \text{ (F/m)}$$



if $r_A = r_B = r$

$$C_{AB} = \frac{2\pi\varepsilon_0}{\ln \frac{D^2}{r^2}} = \frac{2\pi\varepsilon_0}{\ln \left(\frac{D}{r}\right)^2} = \frac{2\pi\varepsilon_0}{2\ln \left(\frac{D}{r}\right)} = \frac{\pi\varepsilon_0}{\ln \left(\frac{D}{r}\right)} \text{ (F/m)}$$

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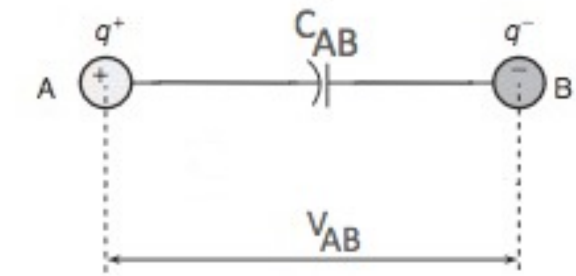
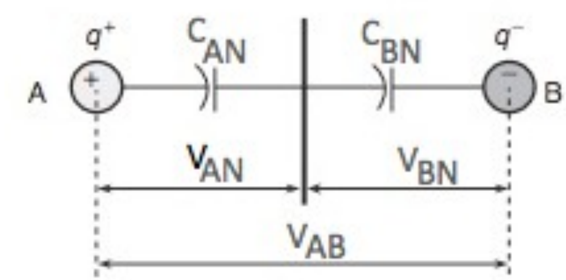
Capacitance to Neutral

The potential difference to neutral is half the difference between the two conductors.

$$C_{AN} = C_{BN} = \frac{q}{\frac{(V_{AB})}{2}} = \frac{2\pi\varepsilon_0}{\ln \frac{D}{r}} \quad \text{(F/m)}$$





Capacitance and Inductance Comparison

Previously we derived the inductance for a two wire single phase line (both conductors having radius r)

$$L = 2 \cdot 10^{-7} \ln\left(\frac{D}{r'}\right) \quad H/m$$

The capacitance to neutral is given by:

$$C_{AN} = \frac{2\pi\varepsilon_0}{\ln \frac{\mathrm{D}}{r}} \quad \mathrm{F/m}$$

Note: the inductance is calculated using an "effective radius"
the capacitance is calculated using the **actual** radius

Capacitance to Neutral

Having determined the capacitance to neutral,
we can now calculate the capacitive reactance

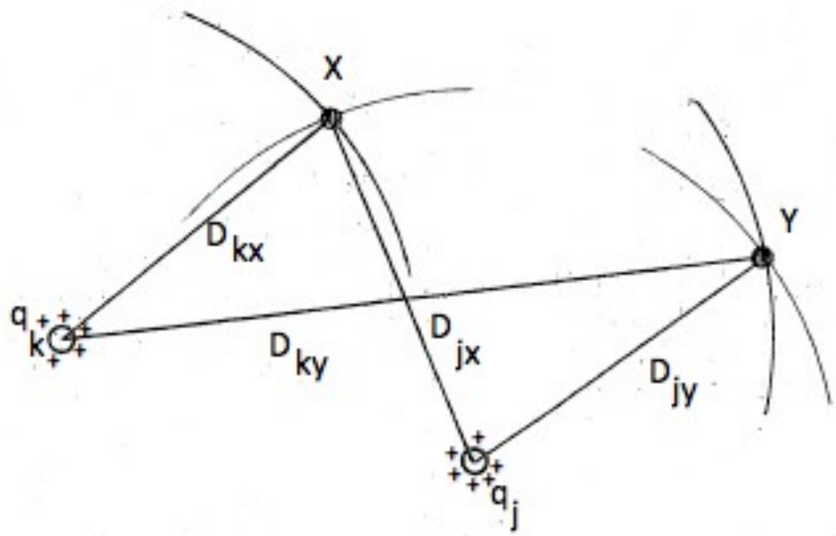
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi f\left(\dfrac{2\pi\varepsilon_0}{\ln \dfrac{\mathrm{D}}{r}}\right)} = \frac{2.861\cdot 10^9}{f}\left(\ln \frac{\mathrm{D}}{r}\right) \quad \Omega m$$

What happens to capacitive reactance as the line gets longer?
(Hint: you need to divide by distance.)

Contrast this to R and X_L

$$R = \frac{\rho \cdot l}{A} \qquad\qquad X_L = 4\pi \cdot 10^{-7} \; f \; \ln\left(\frac{D}{r'}\right) \quad \Omega / m$$

Potential Difference Multiple Charges

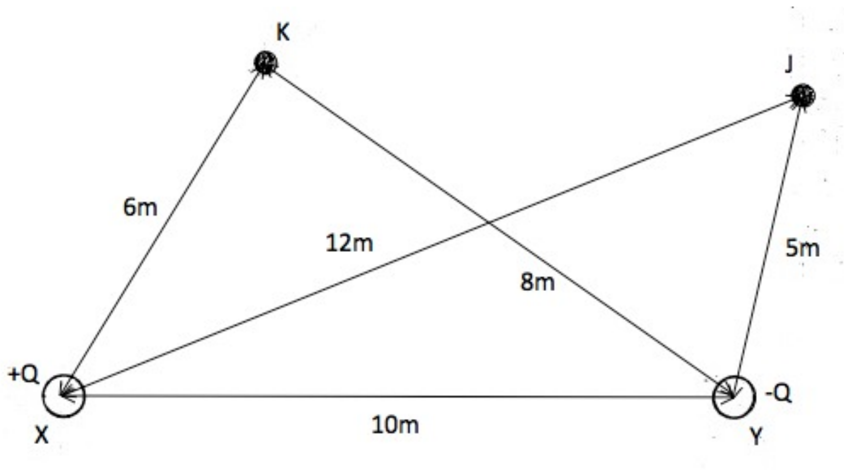


$$V_{XY} = \frac{q_k}{2\pi\varepsilon_0}\ln\left(\frac{D_{kY}}{D_{kX}}\right) \quad due \quad to \quad q_k$$

$$V_{XY} = \frac{q_j}{2\pi\varepsilon_0}\ln\left(\frac{D_{jY}}{D_{jX}}\right) \quad due \quad to \quad q_j$$

$$V_{XY} = \frac{1}{2\pi\varepsilon_0}\sum_{m=1}^{M} q_m \ln\frac{D_{mY}}{D_{mX}}$$

Potential Difference



Given $Q_x = Q = -Q_y$ & $r_x = r_y = 0.01m$
$V_{xy} = 1000V$

Determine: V_{xk}, V_{kj} and V_{jy}

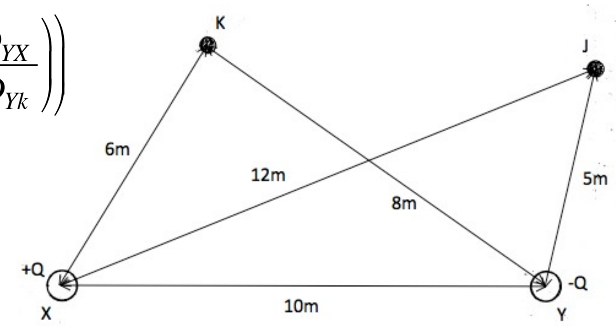
$$V_{XY} = \frac{1}{2\pi\varepsilon_0}\sum_{m=1}^{M} q_m \ln\frac{D_{mY}}{D_{mX}}$$

$$V_{XY} = \frac{Q_x}{2\pi\varepsilon_0}\ln\left(\frac{D_{XY}}{D_{XX}}\right) + \frac{-Q_Y}{2\pi\varepsilon_0}\ln\left(\frac{D_{YY}}{D_{YX}}\right) = \frac{Q}{2\pi\varepsilon_0}\left[\ln\left(\frac{D_{XY}}{r_x}\right) - \left(\ln\frac{r_y}{D_{YX}}\right)\right] = \frac{Q}{2\pi\varepsilon_0}\ln\left(\frac{D_{XY}{}^2}{r_X r_Y}\right)$$

$$V_{XY} = \frac{Q}{2\pi\varepsilon_0}\ln\left(\frac{D_{XY}{}^2}{r_X r_Y}\right) \qquad \frac{Q}{2\pi\varepsilon_0} = \frac{1000}{\ln\frac{10^2}{.01^2}} = 72.382$$

Potential Difference

$$V_{Xk} = \frac{1}{2\pi\varepsilon_0}\sum_{m=1}^{M} q_m \ln\frac{D_{mk}}{D_{mX}} = \frac{Q}{2\pi\varepsilon_0}\left(\ln\frac{D_{Xk}}{D_{XX}} - \ln\frac{D_{Yk}}{D_{YX}}\right) = \frac{Q}{2\pi\varepsilon_0}\ln\left(\left(\frac{D_{Xk}}{D_{XX}}\right)\left(\frac{D_{YX}}{D_{Yk}}\right)\right)$$



$$V_{Xk} = 72.382\ln\left(\left(\frac{6}{.01}\right)\left(\frac{10}{8}\right)\right) = 479.177$$

$$V_{kj} = \frac{1}{2\pi\varepsilon_0}\sum_{m=1}^{M} q_m \ln\frac{D_{mj}}{D_{mk}} = \frac{Q}{2\pi\varepsilon_0}\left(\ln\frac{D_{Xj}}{D_{Xk}} - \ln\frac{D_{Yj}}{D_{Yk}}\right) = \frac{Q}{2\pi\varepsilon_0}\ln\left(\left(\frac{D_{Xj}}{D_{Xk}}\right)\left(\frac{D_{Yk}}{D_{Yj}}\right)\right)$$

$$V_{kj} = 72.382\ln\left(\left(\frac{12}{6}\right)\left(\frac{8}{5}\right)\right) = 84.192$$

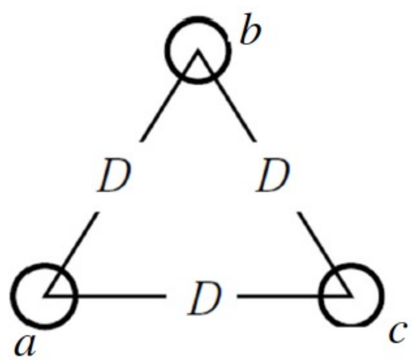
$$V_{jY} = \frac{1}{2\pi\varepsilon_0}\sum_{m=1}^{M} q_m \ln\frac{D_{mY}}{D_{mj}} = \frac{Q}{2\pi\varepsilon_0}\left(\ln\frac{D_{XY}}{D_{Xj}} - \ln\frac{D_{YY}}{D_{Yj}}\right) = \frac{Q}{2\pi\varepsilon_0}\ln\left(\left(\frac{D_{XY}}{D_{Xj}}\right)\left(\frac{D_{Yj}}{D_{YY}}\right)\right)$$

$$V_{jY} = 72.382\ln\left(\left(\frac{10}{12}\right)\left(\frac{5}{.01}\right)\right) = 436.631$$

Capacitance of a Three Phase Line

Consider a balanced, abc positive phase sequence three phase line where $q_A + q_B = q_C = 0$

The space betwee phase conductors is given as D_{AB} , D_{BC} and $D_{CA} = D$
The conductor radii are given as r_A, r_B, $r_C = r$ where the radii are small compared to D



$$V_{AB} = \frac{q_A}{2\pi\varepsilon_0}\ln\left(\frac{D_{AB}}{D_{AA}}\right) + \frac{q_B}{2\pi\varepsilon_0}\ln\frac{D_{BB}}{D_{BA}} + \frac{q_C}{2\pi\varepsilon_0}\ln\frac{D_{CB}}{D_{CA}}$$

$$V_{AC} = \frac{q_A}{2\pi\varepsilon_0}\ln\left(\frac{D_{AC}}{D_{AA}}\right) + \frac{q_B}{2\pi\varepsilon_0}\ln\frac{D_{BC}}{D_{BA}} + \frac{q_C}{2\pi\varepsilon_0}\ln\frac{D_{CC}}{D_{CA}}$$

$$V_{AB} = \frac{q_A}{2\pi\varepsilon_0}\ln\left(\frac{D}{r}\right) + \frac{q_B}{2\pi\varepsilon_0}\ln\frac{r}{D} + \frac{q_C}{2\pi\varepsilon_0}\ln\frac{D}{D}$$

$$V_{AC} = \frac{q_A}{2\pi\varepsilon_0}\ln\left(\frac{D}{r}\right) + \frac{q_B}{2\pi\varepsilon_0}\ln\frac{D}{D} + \frac{q_C}{2\pi\varepsilon_0}\ln\frac{r}{D}$$

Capacitance of a Three Phase Line

$$V_{AB} = \sqrt{3} V_{AN} \angle 30° \qquad V_{AC} = -V_{CA} = \sqrt{3} V_{AN} \angle -30°$$

$$V_{AN} = \frac{V_{AB} + V_{AC}}{3}$$

$$V_{AB} = \frac{q_A}{2\pi\varepsilon_0} \ln\left(\frac{D}{r}\right) + \frac{q_B}{2\pi\varepsilon_0} \ln\frac{r}{D} + \frac{q_C}{2\pi\varepsilon_0} \ln\frac{D}{D} \qquad V_{AC} = \frac{q_A}{2\pi\varepsilon_0} \ln\left(\frac{D}{r}\right) + \frac{q_B}{2\pi\varepsilon_0} \ln\frac{D}{D} + \frac{q_C}{2\pi\varepsilon_0} \ln\frac{r}{D}$$

$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{6\pi\varepsilon_0}\left(q_A \ln\frac{D}{r} + q_B \ln\frac{r}{D} + q_A \ln\frac{D}{r} + q_C \ln\frac{r}{D}\right)$$

$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{6\pi\varepsilon_0}\left(2q_A \ln\frac{D}{r} + \left(q_B + q_C\right) \ln\frac{r}{D}\right)$$

Capacitance of a Three Phase Line

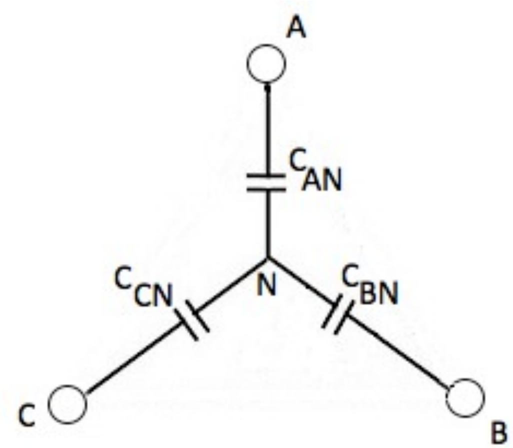
$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{6\pi\varepsilon_0}\left(2q_A \ln\frac{D}{r} + (q_B + q_C)\ln\frac{r}{D}\right)$$

$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{6\pi\varepsilon_0}\left(2q_A \ln\frac{D}{r} + (-q_A)\ln\frac{r}{D}\right)$$

$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{6\pi\varepsilon_0}\left(3q_A \ln\frac{D}{r}\right)$$

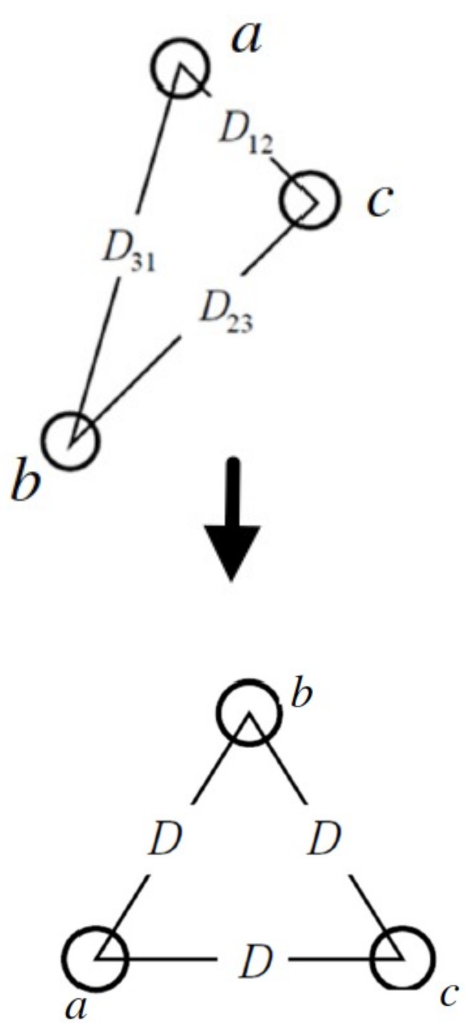
$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{2\pi\varepsilon_0}\left(q_A \ln\frac{D}{r}\right)$$

$$C_{AN} = \frac{q_A}{V_{AN}} = \frac{2\pi\varepsilon_0}{\ln\frac{D}{r}}$$ F/m



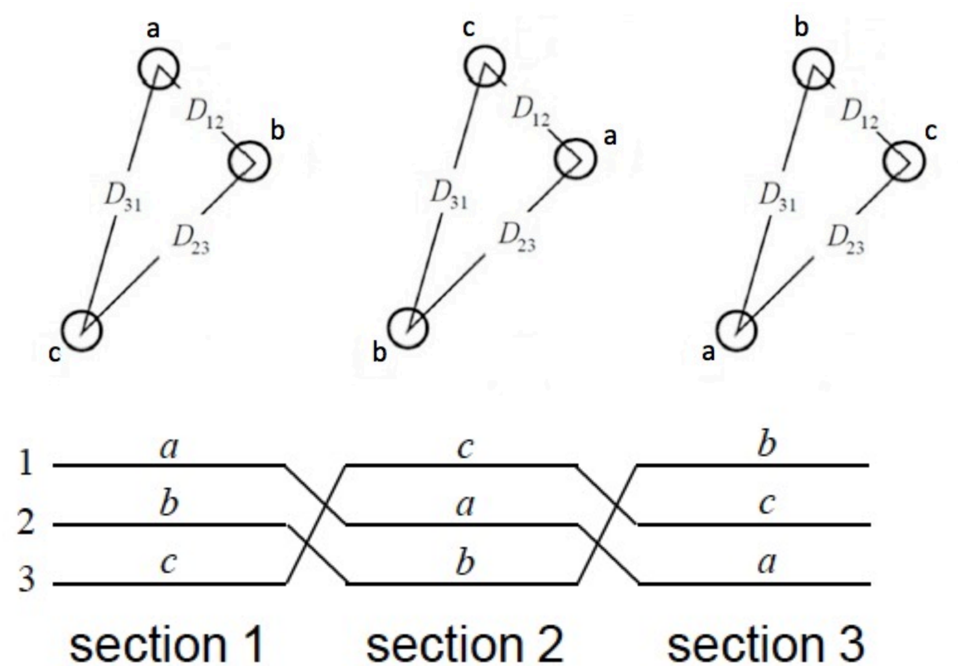
Same as the single phase result!!
Outstanding!!!

Capacitance Asymmetrical Spacing



Use the same approach we used for inductance

Force the asymmetry into a symmetric system by utilizing transposition.



Capacitance

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{12}}{r}\right) + q_b \ln\left(\frac{r}{D_{12}}\right) + q_c \ln\left(\frac{D_{23}}{D_{31}}\right)\right) \text{ in section 1}$$

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{23}}{r}\right) + q_b \ln\left(\frac{r}{D_{23}}\right) + q_c \ln\left(\frac{D_{31}}{D_{12}}\right)\right) \text{ in section 2}$$

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{31}}{r}\right) + q_b \ln\left(\frac{r}{D_{31}}\right) + q_c \ln\left(\frac{D_{12}}{D_{23}}\right)\right) \text{ in section 3}$$

$$\overline{V_{ab}} = \frac{1}{6\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{12}D_{23}D_{31}}{r^3}\right) + q_b \ln\left(\frac{r^3}{D_{12}D_{23}D_{31}}\right) + q_c \ln\left(\frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}\right)\right)$$

Capacitance

$$V_{ab} = \frac{1}{6\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{12}D_{23}D_{31}}{r^3}\right) + q_b \ln\left(\frac{r^3}{D_{12}D_{23}D_{31}}\right) + q_c \ln\left(\frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}\right)\right)$$

$$V_{ab} = \frac{1}{6\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{12}D_{23}D_{31}}{r^3}\right) + q_b \ln\left(\frac{r^3}{D_{12}D_{23}D_{31}}\right)\right)$$

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{eq}}{r}\right) + q_b \ln\left(\frac{r}{D_{eq}}\right)\right) \quad where \quad D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$

Capacitance

Similarly:

$$V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{eq}}{r}\right) + q_c \ln\left(\frac{r}{D_{eq}}\right)\right) \quad where \quad D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$

$$3V_{an} = V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(2q_a \ln\left(\frac{D_{eq}}{r}\right) + q_b \ln\left(\frac{r}{D_{eq}}\right) \quad q_c \ln\left(\frac{r}{D_{eq}}\right)\right)$$

$$3V_{an} = V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(2q_a \ln\left(\frac{D_{eq}}{r}\right) - q_a \ln\left(\frac{r}{D_{eq}}\right)\right)$$

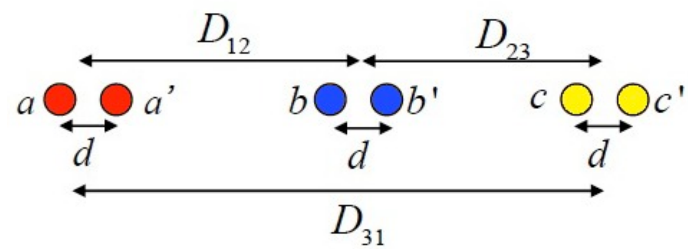
$$3V_{an} = V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(2q_a \ln\left(\frac{D_{eq}}{r}\right) + q_a \ln\left(\frac{D_{eq}}{r}\right)\right)$$

$$3V_{an} = \frac{3}{2\pi\varepsilon_0} q_a \ln\left(\frac{D_{eq}}{r}\right)$$

$$C_{an} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D_{eq}}{r}\right)}$$

Bundled Solid Conductors Asymmetric Spacing

Assume D_{12}, D_{23} and D_{31} are much greater than d



Qa, Qa' + Qb, Qb' + Qc, Qc' = 0

Charge is equally divided and charge neutrality is preserved.

All conducrtors have radius r and the lines are fully transposed.

$$V_{AN_{trnasposed}} = \frac{V_{AB_{transposed}} + V_{AC\,transposed}}{3}$$

$$V_{AN_{transposed}} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{eq}}{D_{sc}}\right)\right)$$

$$C_{AN} = \frac{q_a}{V_{AN_{transposed}}} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D_{eq}}{D_{sc}}\right)} \quad D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}} \quad and \quad D_{sc} = \sqrt[4]{r \cdot d \cdot r \cdot d}$$

Quick Reminder

When we calculated the inductance for a fully transposed line we found:

$$L_a = \frac{\lambda_a}{I_a} = 2\cdot 10^{-7}\left(\ln\frac{D_{eq}}{D_{sL}}\right) \quad D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}} \quad D_{sL} = \sqrt[4]{r'\cdot d\cdot r'\cdot d}$$

When calculating the capacitance for a fully transposed line:

$$C_{AN} = \frac{q_a}{V_{AN_{transposed}}} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D_{eq}}{D_{sC}}\right)} \quad D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}} \;\; and \;\; D_{sC} = \sqrt[4]{r\cdot d\cdot r\cdot d}$$

D_{eq} – the same!!!
The geometric radius of the bundle different!!!
Inductance uses r' the **effective radius** of the conductor
Capacitrance uses r the **actual radius** of the conductor

Capacitance and Capacitive Reactance

$$C_{AN} = \frac{q_a}{V_{AN_{transposed}}} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D_{eq}}{D_{sc}}\right)}$$

$$X_C = \frac{1}{2\pi fC} = \frac{\ln\left(\frac{D_{eq}}{D_{sc}}\right)}{4\pi^2 f\varepsilon_0}\ \Omega\cdot m = 1.778\cdot 10^6\frac{1}{f}\ln\left(\frac{D_{eq}}{D_{sc}}\right)\ \Omega\cdot mile$$

For a completely transposed line connected to a balanced positive sequence set of voltages, a "charging current" will result:

$I_{chrg} = Y\ V_{an}$

$$I_{chrg} = j2\pi fC_{an}\cdot l\cdot V_{an}$$

The reactive power associated with the charging current (per phase):
$Q_{C1\phi} = Y\ Van^2 = \omega\ C_{AN}V_{LN}^2$

The total reactive is three times the power per phase:
$Q_{C3\phi} = 3Y\ Van^2 = 3\omega\ C_{AN}V_{LN}^2 = \omega\ C_{AN}V_{LL}^2$

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Table Usage

$$X_C = 1.778 \cdot 10^6 \frac{1}{f} \ln\left(\frac{D_{eq}}{D_{sc}}\right) \;\; \Omega \cdot mile$$

$$X_C = 1.778 \cdot 10^6 \frac{1}{f} \ln\left(\frac{1}{D_{sc}}\right) + 1.778 \cdot 10^6 \frac{1}{f} \ln\left(D_{eq}\right) \;\; \Omega \cdot mile \;\; (line \;\; to \;\; nuetral)$$

$$X_C = \qquad X'_a \qquad + \qquad X_D$$

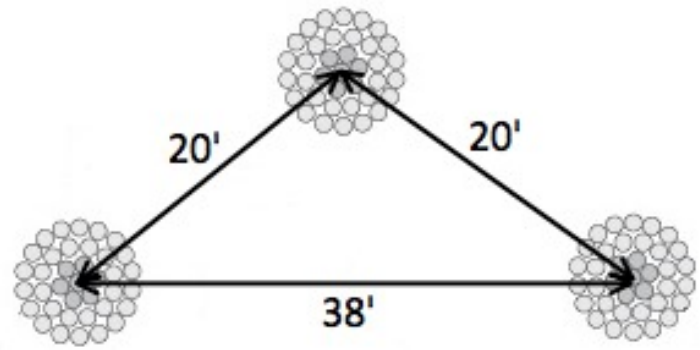
X'$_a$ = the shunt capacitive reactance per conductor per mile at 1 foot spacing

X'$_D$ = the capacitive reactance spacing factor

Example

A fully transposed 60Hz, three phase line has ACSR “Drake” conductors arranged as shown.

Drake
Area = 795000 cmil
Diameter = 1.108 inches
GMR = 0.0375 ft at 60Hz
X'_a = 0.0912 Megohms per conductor per mile



Note the picture is not to proportion but I wanted to emphasize the “Drakes” are stranded conductors

$$D_{eq} = \sqrt[3]{D_{AB} D_{BC} D_{CA}} = \sqrt[3]{20 \cdot 20 \cdot 38} = 24.771 \quad ft$$

Conductor radius = 1.108/2 = 0.554 inches = 0.04617 feet

$$X_C = 1.778 \cdot 10^6 \frac{1}{f} \ln\left(\frac{D_{eq}}{D_{sc}}\right) \ \Omega \cdot mile = 1.778 \cdot 10^6 \frac{1}{60} \ln\left(\frac{24.771}{0.04617}\right) = 1.86248 \cdot 10^5 \quad \Omega \cdot miles$$

or

$$X_C = 1.778 \cdot 10^6 \frac{1}{f} \ln\left(\frac{D_{eq}}{D_{sc}}\right) \ \Omega \cdot mile = X'_a + 1.778 \cdot 10^6 \frac{1}{f} \ln D_{eq}$$

$$= 0.0912 \cdot 10^6 + 1.778 \cdot 10^6 \frac{1}{60} \ln 24.771 = 1.863 \cdot 10^5 \quad \Omega \cdot miles$$

Example

If the "Drake conductor" line was 5 miles long, determine the charging current (per phase) and the total reactive power associated with the charging current.

$$X_C = 1.86248 \cdot 10^5 \ \ \Omega \cdot miles \cdot \frac{1}{5miles} = 37249.9 \ \ \Omega$$

$$Y = j\frac{1}{X_C} = 2.685 \cdot 10^{-5} \ \ S$$

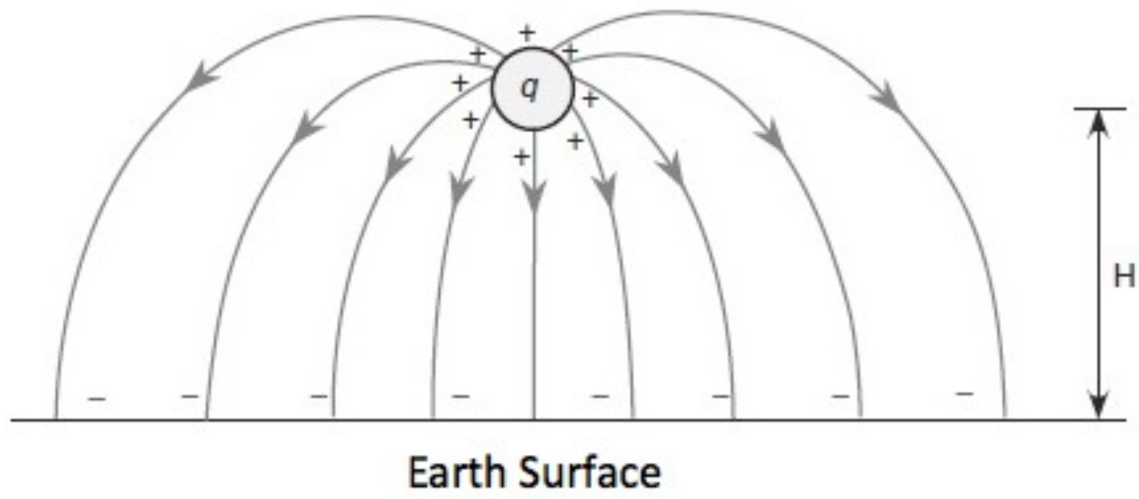
$$I_{chrg} = Y \cdot V_{LN} = 2.685 \cdot 10^{-5} \cdot \frac{345{,}000}{\sqrt{3}} = 5.347 \ \ A/phase$$

$$Q_{3\phi} = Y \cdot V_{LL}^{\ 2} = 2.685 \cdot 10^{-5} \cdot 345{,}000^2 = 3.164 MVARs$$

Effect of the Earth

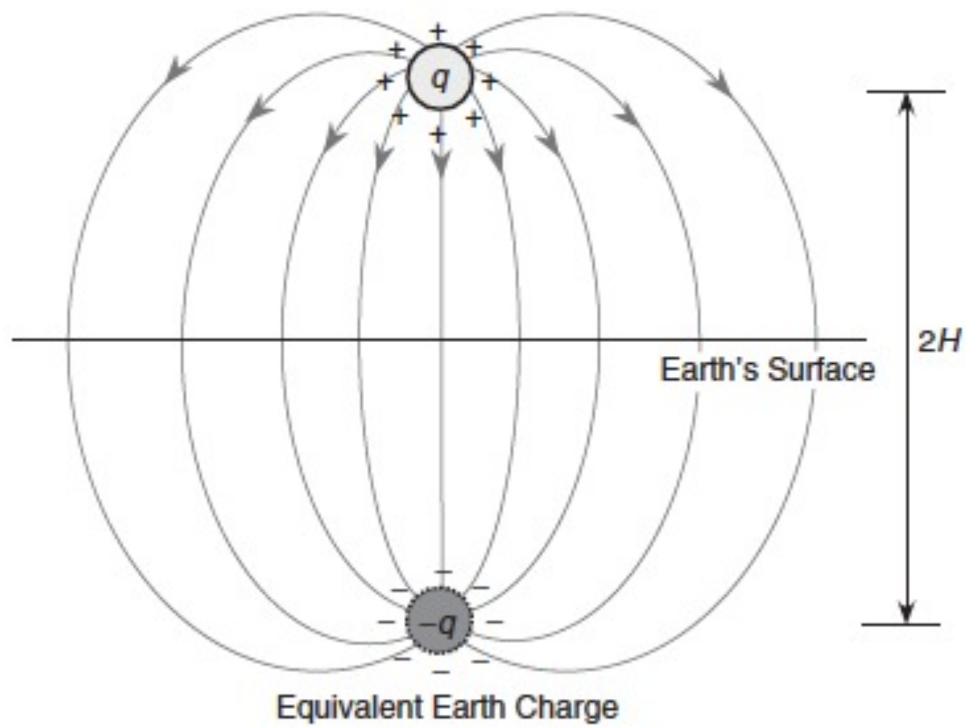
A single overhead conductor with a return path through the earth, seperated a distance H from the earth's surface would have a charge equal in magnitude but opposite in sign as the charge on the conductor.

If the earth is assumed to be a conductive plane of infinite length, the electric field lines would go from the conductor to the earth, striking perpendicular to the earth as shown below.



Effect of the Earth

Someone very smart determined a similar electric field distribution would be established if the negative charge on the surface of the earth was replaced by an "image conductor" with the same radius as the overhead conductor fixed a distance H directly below the overhead conductor.



Effect of the Earth

The same principle can be applied to calculate the capacitance per phase of a three phase system.
Consider three identical conductors in the symmetric equilateral arramgement shown below along with their image parameter counterparts.

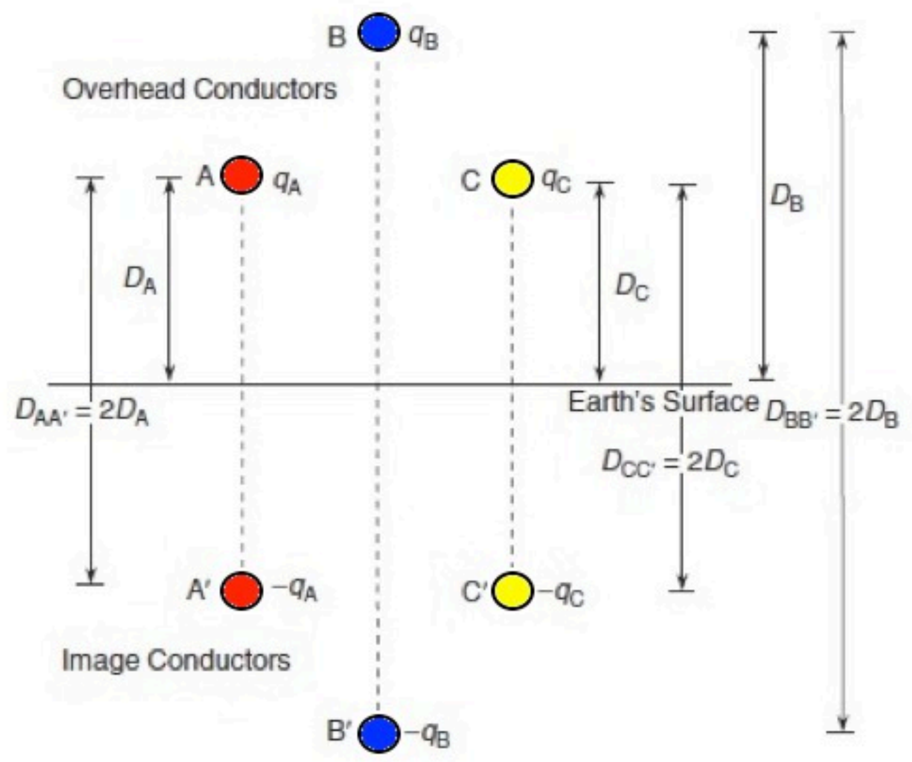


Image Parameters

$$V_{AB} = \frac{1}{2\pi\varepsilon_0}\begin{bmatrix} q_A \ln\dfrac{D_{AB}}{r_A} + q_B \ln\dfrac{r_A}{D_{AB}} + q_C \ln\dfrac{D_{BC}}{D_{AC}} \\ -q_A \ln\dfrac{D_{AB'}}{D_{AA'}} - q_B \ln\dfrac{D_{BB'}}{D_{AB'}} - q_C \ln\dfrac{D_{BC'}}{D_{AC'}} \end{bmatrix}$$

Recall $D_{AB} + D_{BC} = D_{CA} = D$ and $r_a = r_B = r_C = r$

$$V_{AB} = \frac{1}{2\pi\varepsilon_0}\left[q_A\left(\ln\frac{D}{r} - \ln\frac{D_{AB'}}{D_{AA'}}\right) + q_B\left(\ln\frac{r}{D} - \ln\frac{D_{BB'}}{D_{AB'}}\right) + q_C\left(\ln\frac{D}{D} - \ln\frac{D_{BC'}}{D_{AC'}}\right)\right] \quad (V)$$

$$V_{AB} = \frac{1}{2\pi\varepsilon_0}\left[q_A\left(\ln\frac{D}{r} - \ln\frac{D_{AB'}}{D_{AA'}}\right) + q_B\left(\ln\frac{r}{D} - \ln\frac{D_{BB'}}{D_{AB'}}\right) - q_C\left(\ln\frac{D_{BC'}}{D_{AC'}}\right)\right] \quad (V)$$

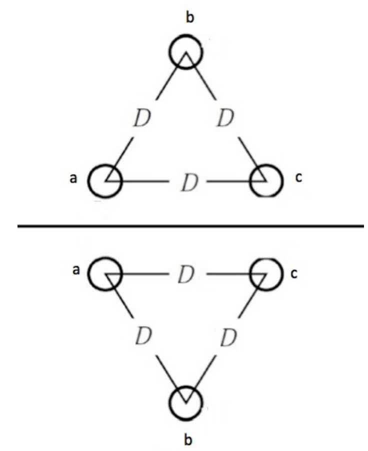
A similar expression can be derived for V_{AC}

$$V_{AC} = \frac{1}{2\pi\varepsilon_0}\left[q_A\left(\ln\frac{D}{r} - \ln\frac{D_{CA'}}{D_{AA'}}\right) + q_B\left(\ln\frac{D}{D} - \ln\frac{D_{CB'}}{D_{AB'}}\right) + q_C\left(\ln\frac{r}{D} - \ln\frac{D_{CC'}}{D_{AC'}}\right)\right] \quad (V)$$

$$V_{AC} = \frac{1}{2\pi\varepsilon_0}\left[q_A\left(\ln\frac{D}{r} - \ln\frac{D_{CA'}}{D_{AA'}}\right) - q_B\left(\ln\frac{D_{CB'}}{D_{AB'}}\right) + q_C\left(\ln\frac{r}{D} - \ln\frac{D_{CC'}}{D_{AC'}}\right)\right] \quad (V)$$

Image Parameters

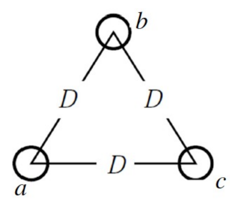
$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{2\pi\varepsilon_0} q_A \left(\ln\left[\frac{D}{r}\right] - \ln\left[\frac{\sqrt[3]{D_{AB'}D_{BC'}D_{CA'}}}{\sqrt[3]{D_{AA'}D_{BB'}D_{CC'}}}\right] \right) \quad (V)$$



$$C_{AN} = \frac{q_A}{V_{AN}} = \frac{2\pi\varepsilon_0}{\left(\ln\left[\frac{D}{r}\right] - \ln\left[\frac{\sqrt[3]{D_{AB'}D_{BC'}D_{CA'}}}{\sqrt[3]{D_{AA'}D_{BB'}D_{CC'}}}\right] \right)} \quad (F/m)$$

recall that the result for the capacitance of the equally spaced conductors:

$$C_{AN} = \frac{q_A}{V_{AN}} = \frac{2\pi\varepsilon_0}{\ln\frac{D}{r}}$$



the results are similar with the image parameter denominator being smaller by the factor

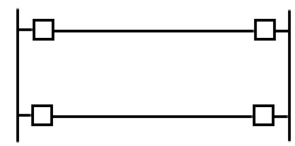
$$\ln\left[\frac{\sqrt[3]{D_{AB'}D_{BC'}D_{CA'}}}{\sqrt[3]{D_{AA'}D_{BB'}D_{CC'}}}\right]$$

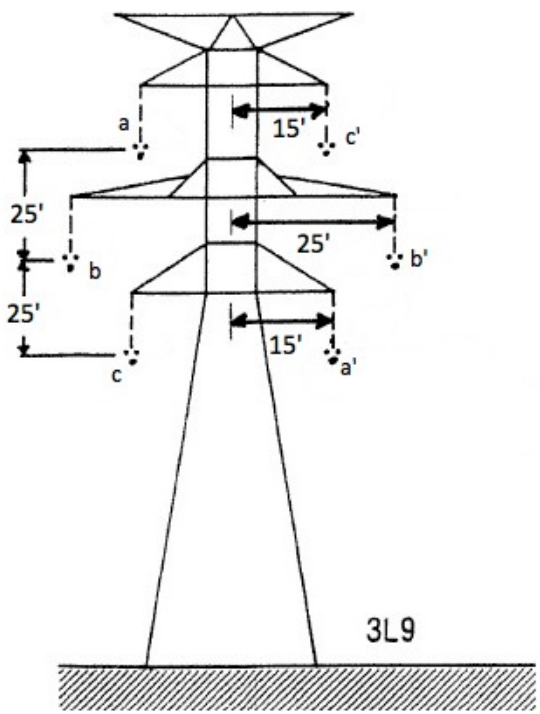
Capacitance increases but (for real lines) not by much!

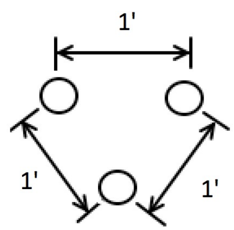
Double Circuit Lines

Right of way is expensive and they're not making any new land, so we have to utilize the space we have to the best of our ability. Frequently we carry two transmission lines on a single tower. These are referred to as double circuit lines.

We will only consider the case where the two lines operate in parallel and at the same voltage and frequency.



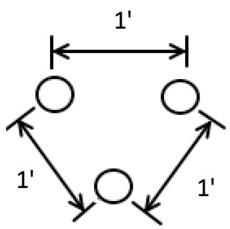




Each phase consists of three "Eagle" conductors on the cornes of an equilateral triangle, with 1 foot spacing.

From the ACSR table A4:
GMR = 0.0328'
Diameter = 0.953"

Double Circuit Lines



From the ACSR table A4:
GMR = 0.0328’
Diameter = 0.953”

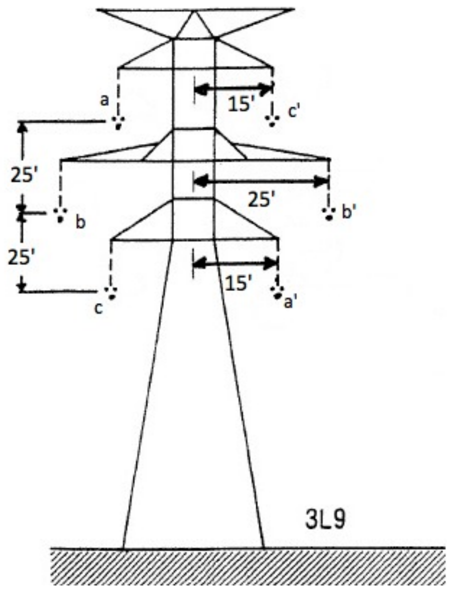
Step 1 : Reduce the bundles down to a single conductor.

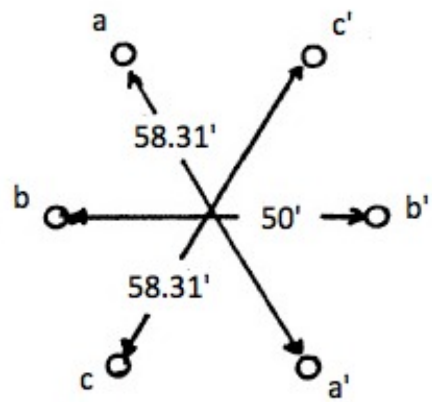
$$D_{SL} = R_B = r'_{eq} = \sqrt[9]{(0.0328)(1)(1)\cdot(0.0328)(1)(1)\cdot(0.0328)(1)(1)} = 0.3201'$$

for DSC we’ll need the actual radius (in feet) r = 0.953”/(2 x 12) = 0.0397
(Note that 0.3412 x 0.7788 ≠ 0.3201 as the conductor is stranded NOT solid)

$$D_{SC} = \sqrt[9]{(0.0397)(1)(1)\cdot(0.0397)(1)(1)\cdot(0.0397)(1)(1)} = 0.3412'$$

Double Circuit Lines





To determine the **inductance** we'll need the average **effective** radius of the two conductor bundle.

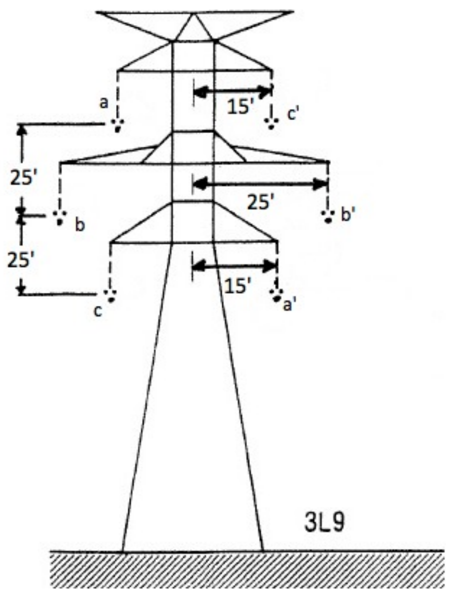
$$D_{SL(a)} = \sqrt[4]{D_{aa} \cdot D_{aa'} \cdot D_{a'a'} \cdot D_{a'a}} = \sqrt[4]{0.3201 \cdot 58.31 \cdot 0.3201 \cdot 58.31} = 4.320'$$

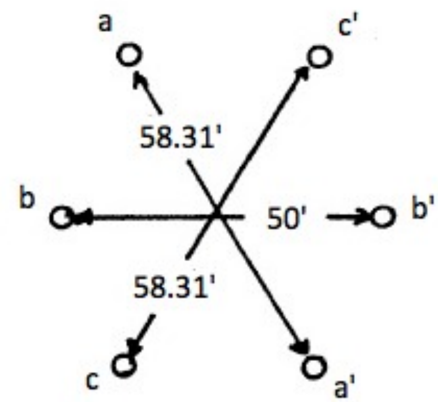
$$D_{SL(b)} = \sqrt[4]{D_{bb} \cdot D_{bb'} \cdot D_{b'b'} \cdot D_{b'b}} = \sqrt[4]{0.3201 \cdot 50 \cdot 0.3201 \cdot 50} = 4.001'$$

$$D_{SL(c)} = \sqrt[4]{D_{cc} \cdot D_{cc'} \cdot D_{c'c'} \cdot D_{c'c}} = \sqrt[4]{0.3201 \cdot 58.31 \cdot 0.3201 \cdot 58.31} = 4.320'$$

$$D_{SL} = \sqrt[3]{4.320 \cdot 4.001 \cdot 4.320} = 4.211'$$

Double Circuit Lines





To determine the **capacitance** we'll need the average radius of the two conductor bundle.

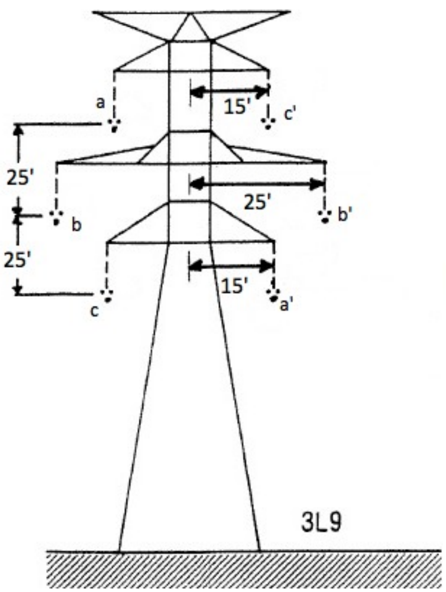
$$D_{SC(a)} = \sqrt[4]{D_{aa} \cdot D_{aa'} \cdot D_{a'a'} \cdot D_{a'a}} = \sqrt[4]{0.3412 \cdot 58.31 \cdot 0.3412 \cdot 58.31} = 4.4604'$$

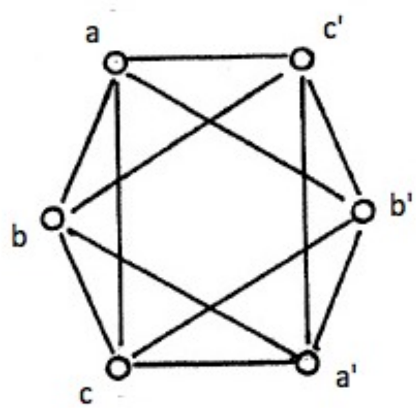
$$D_{SC(b)} = \sqrt[4]{D_{bb} \cdot D_{bb'} \cdot D_{b'b'} \cdot D_{b'b}} = \sqrt[4]{0.3412 \cdot 50 \cdot 0.3412 \cdot 50} = 4.1304'$$

$$D_{SC(c)} = \sqrt[4]{D_{cc} \cdot D_{cc'} \cdot D_{c'c'} \cdot D_{c'c}} = \sqrt[4]{0.3412 \cdot 58.31 \cdot 0.3412 \cdot 58.31} = 4.4604'$$

$$D_{SC} = \sqrt[3]{4.4604 \cdot 4.1304 \cdot 4.4604} = 4.348'$$

Double Circuit Line





Lastly, we'll need D_{eq}

$$D_{eq} = GMD = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$$

Dab	
a-b	26.93'
a-b'	47.17'
a'-b	47.17'
a'-b'	26.93'
Dbc	
b-c	26.93'
b-c'	47.17'
b'-c	47.17'
b'-c'	26.93'
Dca	
c-a	50'
c-a'	30'
c'-a	30'
c'-a'	50'

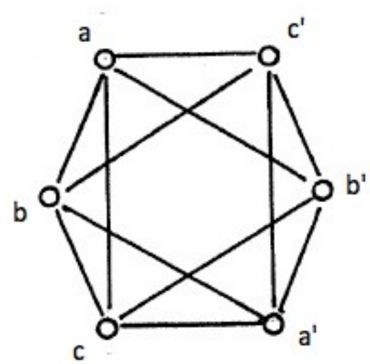
$$D_{ab} = \sqrt[4]{26.93 \cdot 47.17 \cdot 47.17 \cdot 26.93} = 35.641'$$

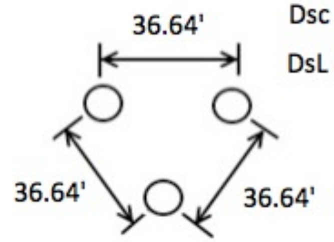
$$D_{bc} = \sqrt[4]{26.93 \cdot 47.17 \cdot 47.17 \cdot 26.93} = 35.641'$$

$$D_{ca} = \sqrt[4]{50 \cdot 30 \cdot 50 \cdot 30} = 35.641' = 38.730'$$

$$D_{eq} = \sqrt[3]{35.641 \cdot 35.641 \cdot 38.730} = 36.642'$$

Double Circuit Line





Dsc = 4.348' = r

DsL = 4.211 = r'

$$X_L = \left(2\cdot 10^{-7}\right)\left(2\pi 60\right)\left(1609m/mile\right)\ln\frac{D_{eq}}{D_{SL}} = \left(2\cdot 10^{-7}\right)\left(2\pi 60\right)\left(1609m/mile\right)\ln\frac{36.64}{4.211} = 0.2625\Omega/mile$$

$$Y = j\omega C = \frac{j\left(2\pi\cdot 60\right)\left(2\pi\cdot 8.854\cdot 10^{-12}\right)\left(1609m/mile\right)}{\ln\frac{D_{eq}}{D_{SC}}} = \frac{33.745\cdot 10^{-6}}{\ln\frac{36.64}{4.348}} = 15.832\cdot 10^{-6}\,S/mile$$

Double Circuit Line

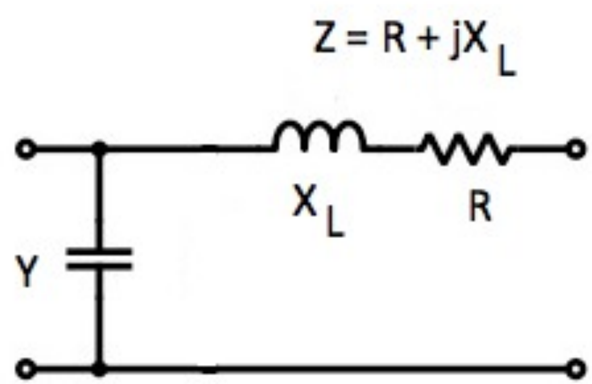
We're almost done but we can't forget about resistance.

From Table A4 Eagle conductors have $R_{(60Hz)}$ = 0.1859 Ω/mile.

But we have three conductors per phase and two parallelled phases

R(per phase) = 0.1859/6 = 0.03098 Ω/mile

Z = 0.03098 + j0.2625 Ω/mile



Y = $j15.83 \times 10^{-6}$ S/mile

How Long is the Line?