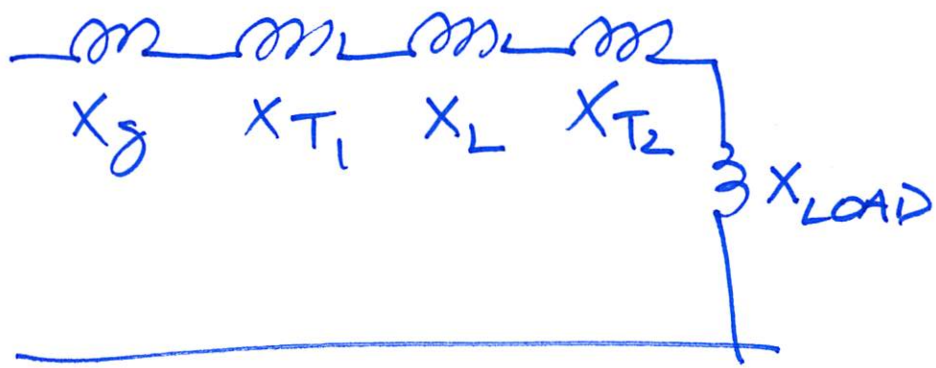
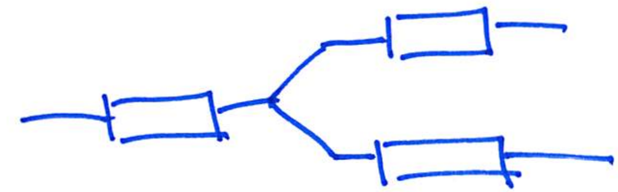
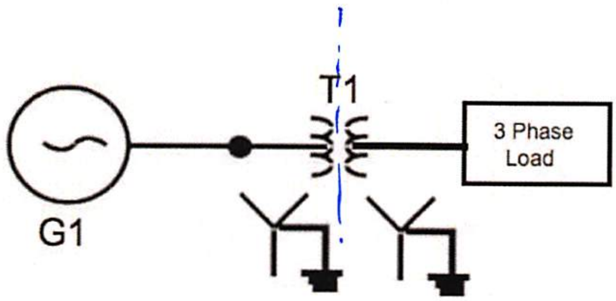


X_g
X_{T_1}
X_L
X_{T_2}
X_{LOAD}



1. The simple three phase system shown below is comprised of the following elements:

Generator : Xg = 15% on a 15kV, 120MVA base
Transformer : Three phase bank Xt = 10 % on its ratings of 13.2kV/132kV, 100MVA
Load: Three 200Ω resistors connected in wye



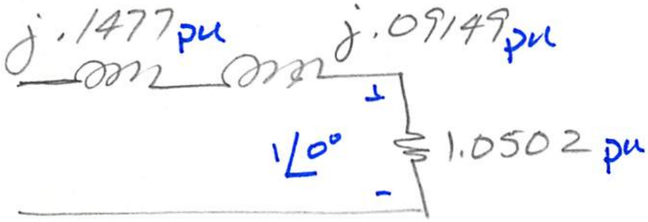
A. Draw the p.u. reactance diagram, indicating all element values. Choose base values of 138kV and 100MVA at the load.

$$V_{BASE\,gen} = 138kV \times \frac{13.2kV}{132kV} = 13.8kV$$

$$X_t = .1\left(\frac{132}{138}\right)^2 \times \frac{100}{100} = .09149$$

$$Z_g = .15\left(\frac{15kV}{13.8kV}\right)^2 \times \frac{100MVA}{120MVA} = .1477\ pu$$

$$R_{LOAD} = \frac{200}{\frac{(138kV)^2}{100MVA}} = 1.0502$$



B. If the voltage at the load is 138kV /0°, determine the magnitude of the generator current in actual amps.

$$V_{L\,pu} = \frac{138kV\angle 0°}{138kV} = 1\angle 0°$$

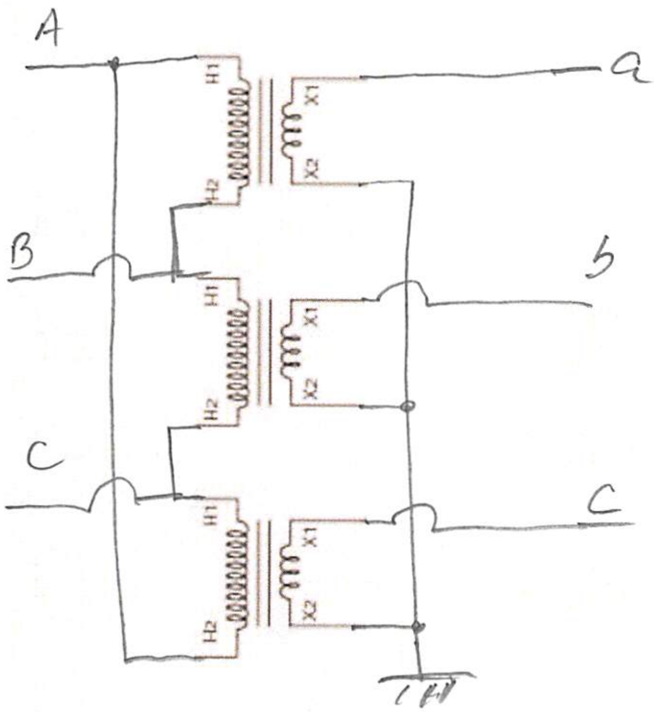
$$I_L = \frac{1\angle 0°}{1.0502} = .9522\angle 0°\ pu$$

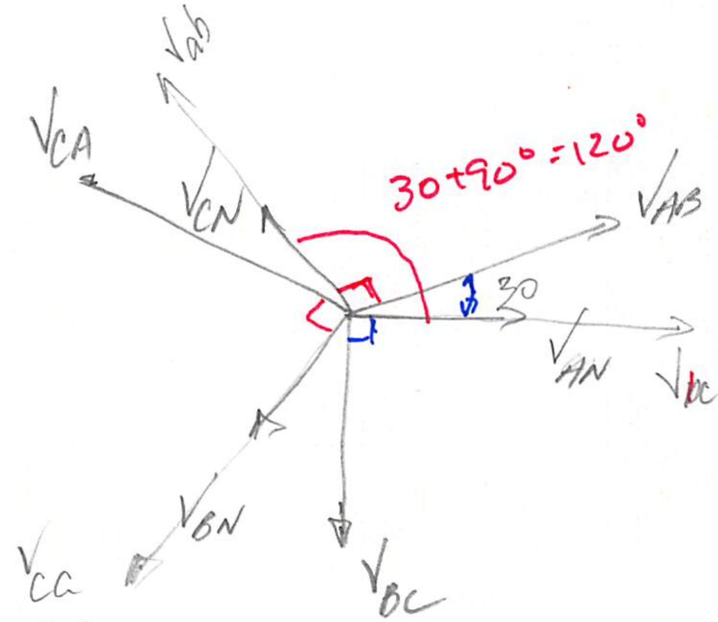
$$I_{BASE\,gen} = \frac{100MVA}{\sqrt{3}\ 13.8kV} = 4183.6976\ A$$

$$|S_{T3\phi}| = \sqrt{3}\,|V_{LL}|\,|I_L|$$

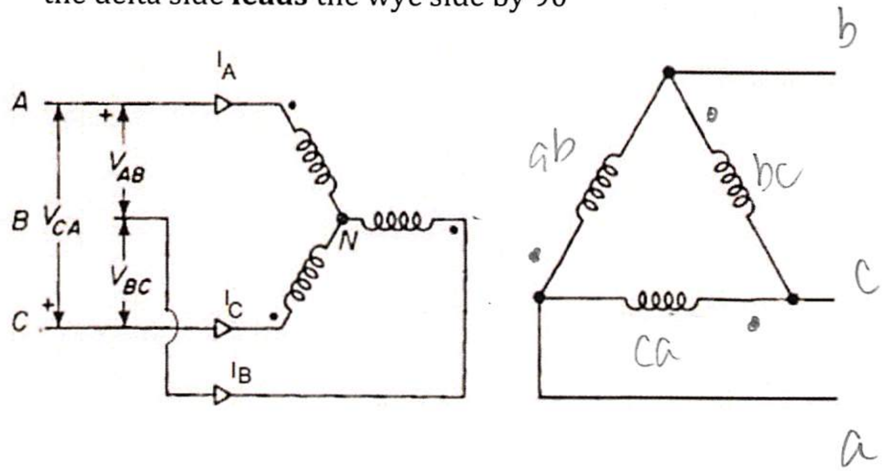
$$I_{gen\,ACT} = 3983.72\ A$$

2a. "Engineer" a three-phase transformer configuration utilizing the three single phase transformers shown below. Draw in the connections such that the high voltage side is connected in delta and low voltage side is in grounded wye. Also show the connections to the outside world i.e., lines "A", "B", "C", "a", "b" and "c"





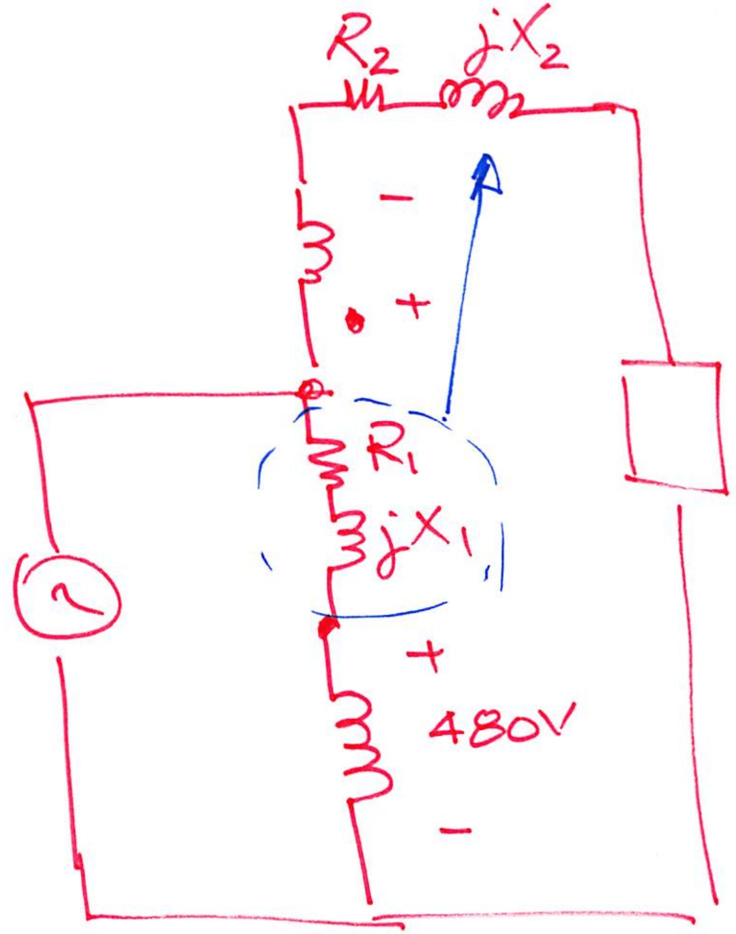
2b. Add the polarity markings and identify the lines such that:
the delta side **leads** the wye side by 90°



2c. The North American standard for wye/delta or delta/wye, three phase transformer connections states :

The line-to line voltages on the high side must ___lead___ the line-to-line low side voltages
(lead or lag)

by ___30°___ degrees.



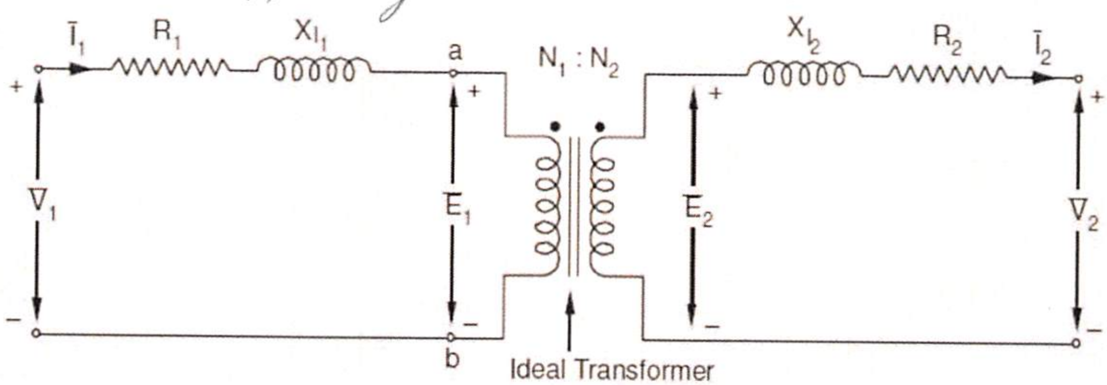
$$\underbrace{\frac{R_1}{a^2} + R_2}_{R_{eq_2}} \qquad \underbrace{j\frac{X_1}{a^2} + jX_2}_{jX_{eq_2}}$$

a = 4

3. A 480V/120V, 60Hz, 6kVA single phase transformer has the following winding parameters:

$Z_{480V\ winding} = 0.6 + j0.8\Omega$ $\qquad$ $Z_{120V\ winding} = 0.0375 + j0.050\Omega$

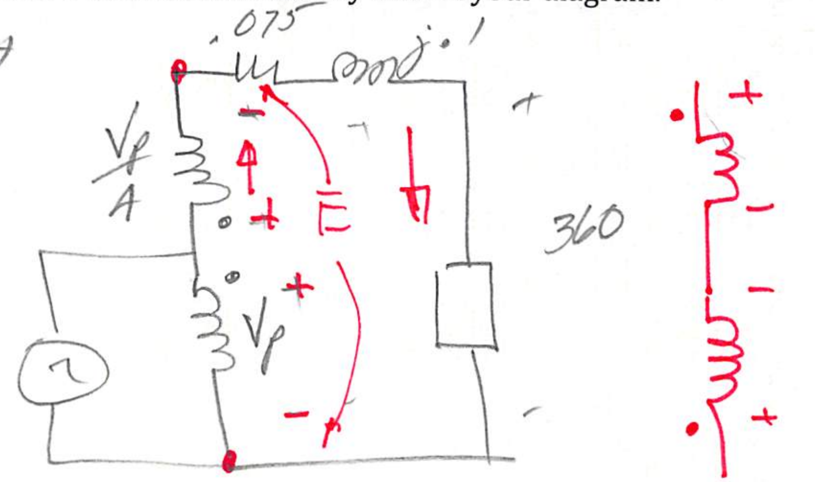
$.0375 + j.05$



The transformer is to be connected as a 480V/360V autotransformer.

Draw the transformer connection required to implement the A-T configuration. Be sure to use polarity markings in your diagram. Also include a source and arbitrary load in your diagram.

$I_{RATED\,LV} = \dfrac{6000VA}{120V} = 50A$



Determine the rated kVA for the autotransformer.

$360V \times 50A = S_{A-T} = 18kV$

Given the autotransformer is driving a 15kVA load at a power factor of 0.8 lagging and the voltage at the load is 360$\underline{/0°}$ V, determine the source voltage/

$$\bar{I}_L = \left(\frac{15{,}000\angle \cos .8}{360\angle 0}\right)^* = 41.667\angle -36.87A$$

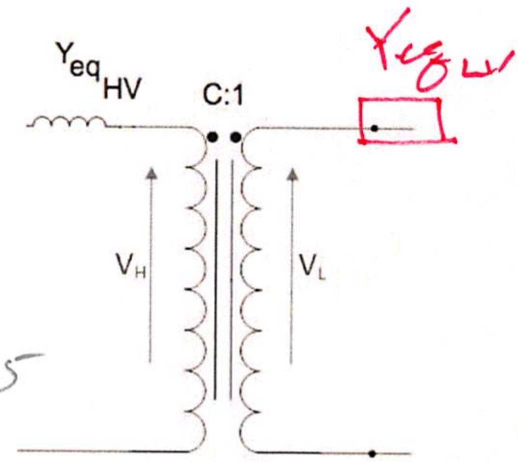
$$E = 360\angle 0° + 41.667\angle -36.87\,(.075 + j.1) = 365.003\angle .23°$$

$$E = V_p - \frac{V_p}{4} = (1 - .25)V_p \qquad V_p = 486.67\angle .23°$$

4. A three-phase transformer has nameplate data: 125kV/15.625kV, 60Hz, 100MVA, X_T =6.4%

The transformer is put into service in a system with a base power of 100MVA, Vbase(HV) = 100kV and Vbase (LV) = 10kV

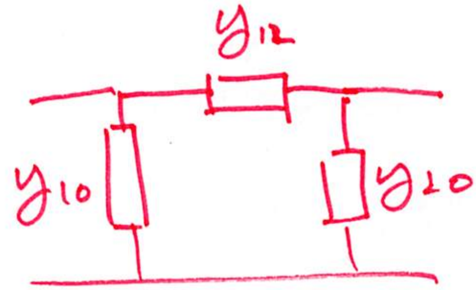
Determine the values for Yeq_{HV} and C that could be used to model to model the transformer when placed in this "off nominal turns" application. [You **do not** have to determine the pi equivalent parameters y_{10}, y_{12} and y_{20}]



$$Z_{pu_{HV}} = .064\left(\frac{125}{100}\right)^2 \times \frac{100}{100} = .1$$

$$Z_{pu_{LV}} = .064\left(\frac{15.625}{10}\right)^2 \times \frac{100}{100} = .15625$$

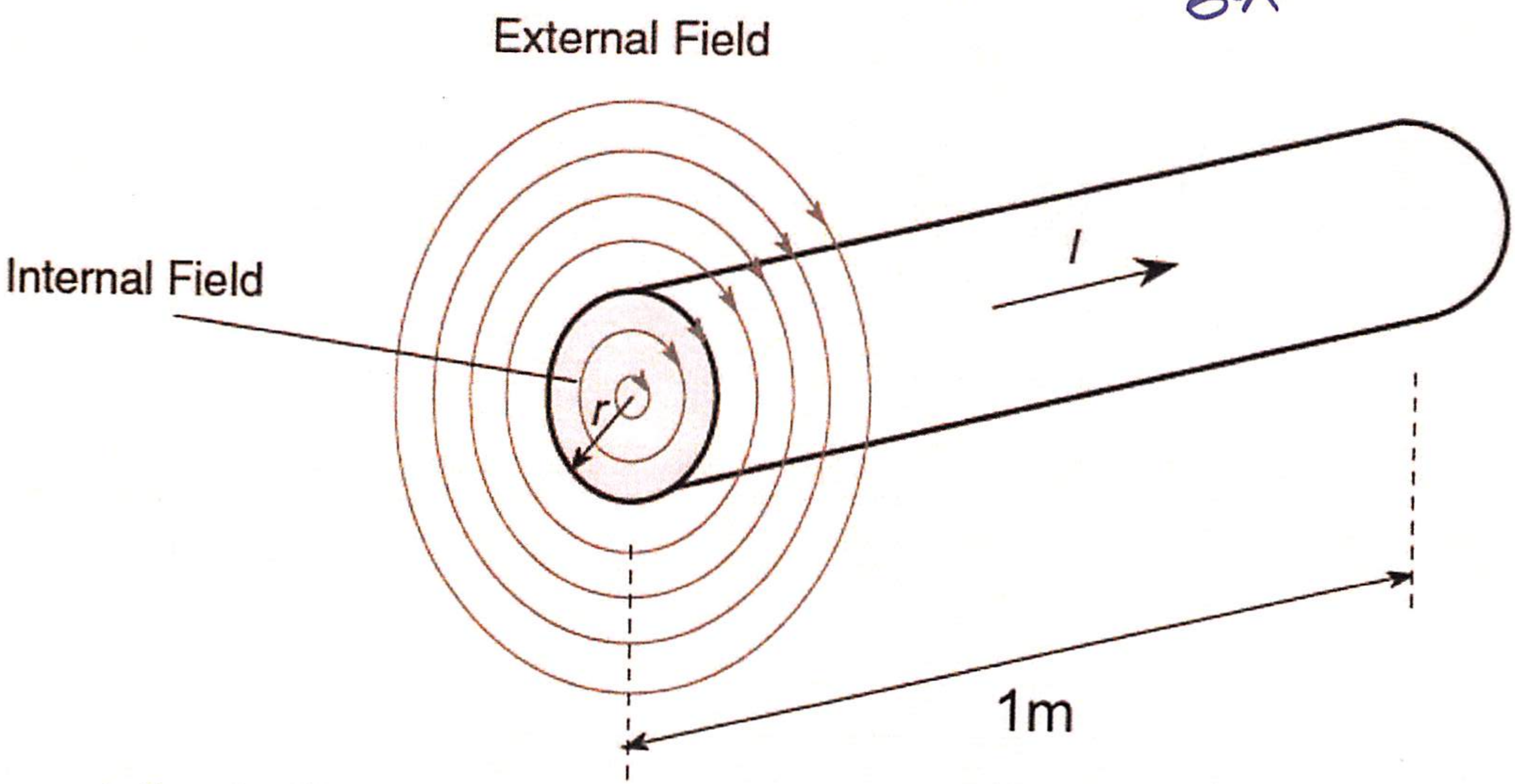
$$.1 = C^2 \times .15625 \longrightarrow C = .8$$



Yeq_{HV} = $-j10$ pu

C = 0.8

$$R = \frac{\rho l}{A} = \frac{l}{\sigma A} \qquad \mathcal{R} = \frac{l}{\mu A}$$



$\mu = \mu_0 \mu_R$

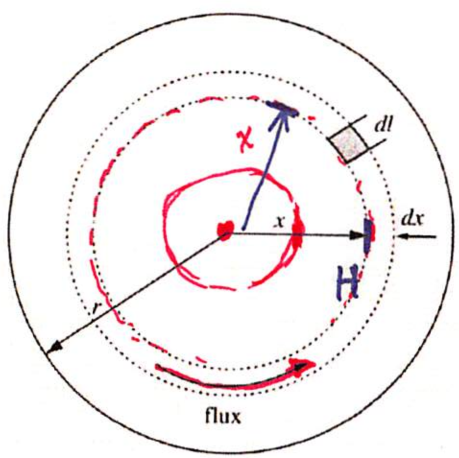
if conductor is Al or Cu $\mu_R \approx 1$

Inductance and Inductive Reactance

The series inductance of a transmission line consists of two components: internal and external inductances, which arise from the magnetic flux inside and outside the conductor respectively. The inductance of a transmission line is defined as the number of flux linkages [Wb-turns] produced per ampere of current flowing through the line:

$$L = \frac{\lambda}{I}$$

Internal Inductance:



Consider a cylindrical conductor of radius r carrying a current of I amperes (left). At a distance x from the center of this conductor, the magnetic field intensity H_x can be found from Ampere's law:

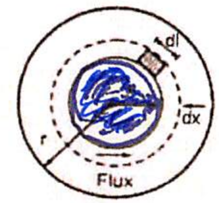
$$\oint H_x \cdot dl = I_{x\ (enclosed)}$$

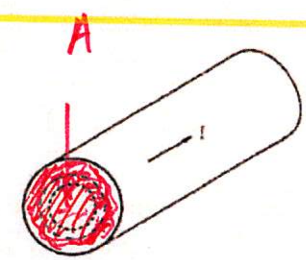
$H_x\ \ell_{\text{at distance } x}$

$2\pi x$

Ampere's Law Continued

$$\oint H_x \cdot dl = I_{x\ (enclosed)}$$





H_x is the magnetic field intensity at each point along a closed path, dl is a unit vector along that path and I_x is the net current enclosed in the path. For a homogeneous material and a circular path of radius x, the magnitude of H_x is constant, and dl is always parallel to H_x.

$$2\pi x H_x = I_x \quad or \quad H_x = \frac{I_{\cancel{x}}}{2\pi \cancel{x}} \frac{x^2}{r^2}$$

Now (ignoring skin effect) assuming that the current is distributed uniformly in the conductor:

$$I_x = \frac{\cancel{\pi} x^2}{\cancel{\pi} r^2} I$$

TOTAL CURRENT

Making the magnetic field intensity inside the conductor at radius x:

flux density
$B = \mu H$

$$H_x = \frac{I_x}{2\pi x} = \frac{xI}{2\pi r^2}$$

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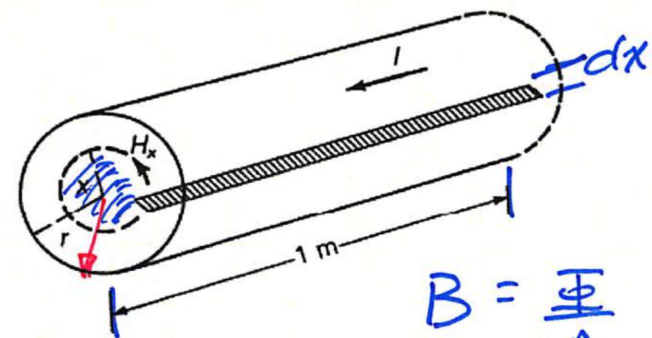
Inductance and Inductive Reactance

The flux density at a distance x from the center of the conductor is:

$$B_x = \mu H_x = \frac{\mu x I}{2\pi r^2} \; [Tesla]$$



$B = \frac{\Phi}{A}$

If a differential magnetic flux is contained in a unit length circular tube of thickness dx, at a distance x from the center of the conductor

$B\,dA = d\phi$

$$\underline{d\phi} = \frac{\mu x I}{2\pi r^2} dx \cdot 1 \; [Wb/m]$$

The flux linkages per meter of length due to flux in the tube is the product of the differential flux and the fraction of current linked:

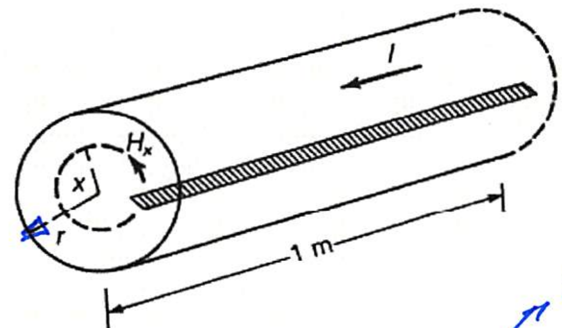
$$d\lambda = \frac{\pi x^2}{\pi r^2} d\phi = \frac{\mu x^3 I}{2\pi r^4} dx \; [Wb-turns/m]$$

Inductance and Inductive Reactance

$$\frac{x^4}{4}\Big|_0^r \qquad \frac{r^4}{4} - \frac{0}{4}$$

The total internal flux linkages per meter can be found by integration:

$$\lambda_{\text{int}} = \int_0^r d\lambda_x = \frac{\mu I}{2\pi \cancel{r^4}} \int_0^r x^3 dx = \frac{\mu I}{8\pi}$$



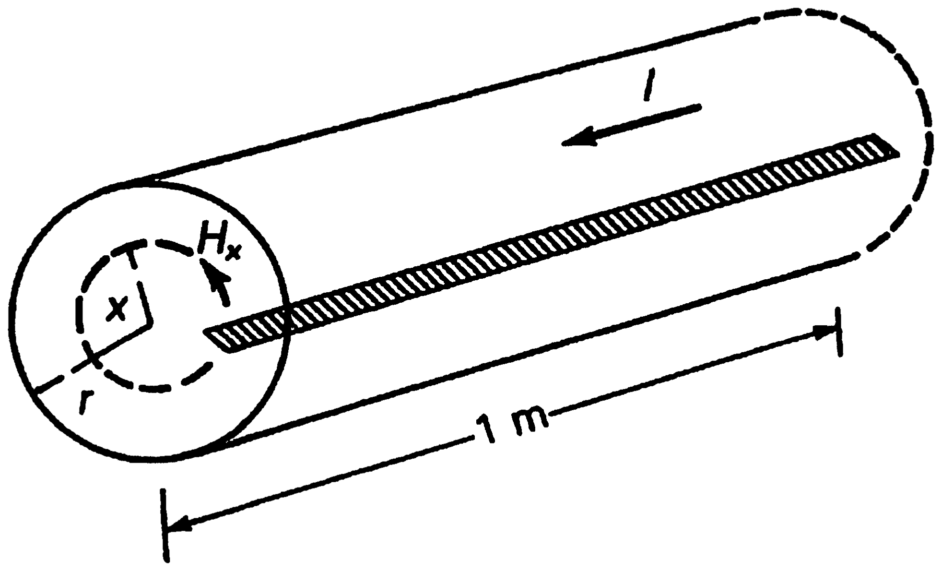
By definition, the internal inductance per meter is:

$$L_{\text{int}} = \frac{\lambda_{\text{int}}}{I} = \frac{\mu}{8\pi} [H/m]$$

$\mu = \mu_0 \mu_R$ 1.0

If the relative permeability of the conductor is 1 (i.e., non-ferromagnetic materials, such as copper and aluminum), the inductance per meter reduces to

$$L_{\text{int}} = \frac{\lambda_{\text{int}}}{I} = \frac{\mu_0}{8\pi} = \frac{4\pi \cdot 10^{-7}}{8\pi} = \underline{0.5 \cdot 10^{-7} [H/m]}$$



$$\oint H_x \cdot dl = I_{x\ (enclosed)}$$

$$2\pi x H_x = I_x \quad or \quad H_x = \frac{I_x}{2\pi x}$$

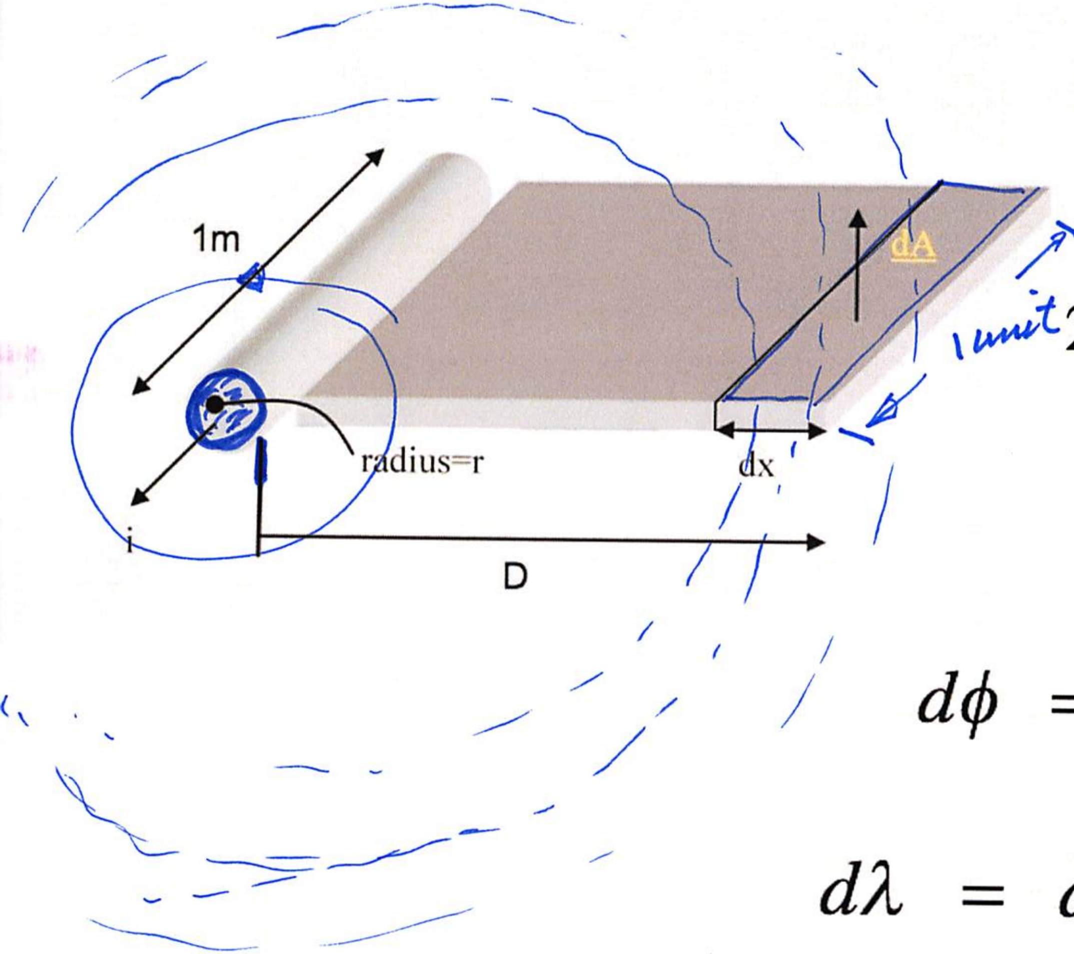
$$B = \mu H \quad \Phi = B \cdot A$$

$$d\phi = B \cdot dA = \frac{\mu x I}{2\pi r^2} dx \cdot 1 \ [Wb/m]$$

$$d\lambda = \frac{\pi x^2}{\pi r^2} d\phi = \frac{\mu x^3 I}{2\pi r^4} dx \ [Wb-turns/m]$$

$$\lambda_{\text{int}} = \int_0^r d\lambda_x = \frac{\mu I}{2\pi r^4} \int_0^r x^3 dx = \frac{\mu I}{8\pi}$$

$$L_{\text{int}} = \frac{\lambda_{\text{int}}}{I} = \frac{\mu}{8\pi} [H/m]$$



$$\oint H_x \cdot dl = I_{x\ (enclosed)}$$

$$2\pi x H_x = I \quad or \quad H_x = \frac{I}{2\pi x}$$

$$B = \mu H \quad \Phi = B \cdot A$$

$$d\phi = B \cdot dA = \frac{\mu I}{2\pi x} dx \cdot 1 \ [Wb/m]$$

$$d\lambda = d\phi = \frac{\mu I}{2\pi x} dx \ [Wb-turns/m]$$

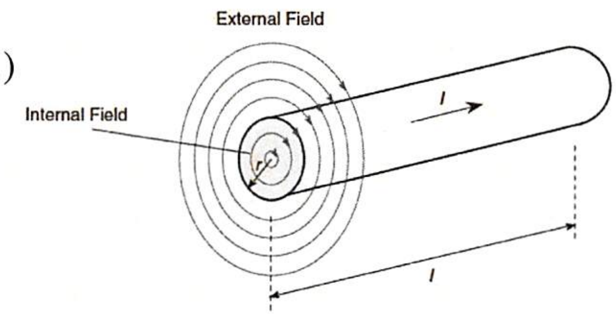
$$\ln x \Big|_r^D = \ln D - \ln r$$

$$\lambda ext = \int_r^D d\lambda = \int_r^D \frac{\mu I}{2\pi x} dx = \frac{\mu I}{2\pi} \int_r^D \frac{1}{x} dx = \frac{\mu I}{2\pi} \ln\left(\frac{D}{r}\right)$$

$$L_{ext} = \frac{\lambda_{ext}}{I} = \frac{\mu}{2\pi} \ln\left(\frac{D}{r}\right) [H/m]$$

Total Flux Linkage and Total Inductance

$$\lambda_{total(per\ \ meter)} = \lambda_{total(\text{external})} + \lambda_{total(\text{internal})}$$



$$\lambda_{total(per\ \ meter)} = \frac{\mu_0}{2\pi} I \left(\ln \frac{D}{r} \right) + \frac{\mu_0}{8\pi} I$$

$$L(per\ \ meter) = \frac{\lambda_{total(per\ \ meter)}}{I} = \frac{\mu_0}{2\pi} \left(\ln \frac{D}{r} + \frac{1}{4} \right) \rightarrow \ln\left(e^{1/4}\right)$$

$$L(per\ \ meter) = \frac{4\pi \cdot 10^{-7}}{2\pi} \left(\ln \frac{D}{r} + \ln\left(e^{\frac{1}{4}} \right) \right) = 2 \cdot 10^{-7} \ln\left(\frac{D \cdot e^{\frac{1}{4}}}{r} \right) = 2 \cdot 10^{-7} \ln\left(\frac{D}{r \cdot e^{\frac{-1}{4}}} \right) = 2 \cdot 10^{-7} \ln\left(\frac{D}{r'} \right)$$

r’ = 0.7788r

Two important results to note:

r' effective radius

The inductance is proportional to the natural logarithm of the ratio of distances
The result is problematic as the inductance per meter goes to infinity as D → ∞

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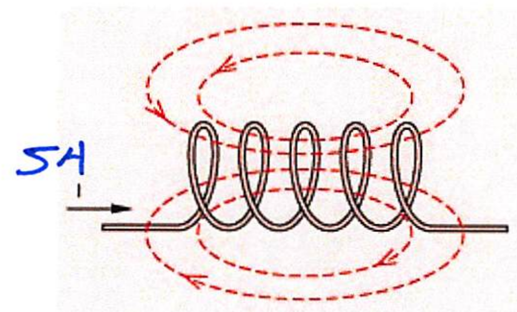
Flux Linkage

In an ideal inductor the flux generated by one of its turns would encircle all the other other turns.

Real coils come close to ideal behavior when the cross sectional dimensions of the windings are small compared with its diameter, or if the coil is wound on a high relative permeability core.

In longer air-core coils the situation is likely to be nearer to that shown in the figure (below).

Here we see that the flux density decreases towards the ends of the coil as some flux takes a 'short cut' bypassing the outer turns.



Given the current into the coil is 5 amps and eack line of flux represents 8mWb:

The three center turns link all four lines of flux = 4 x 8 = 32mWb
The two outer turns link only two lines of flux = 2 x 8 =16 mWb

$\lambda = 3 \times 32 + 2 \times 16 = 128$mWb –t $\qquad L = \lambda/I = 128/5 = 25.6$mH

Law of Logarithms

Throughout the investigation to follow, many expressions can be simplified utilizing a few simple identities that you already know:

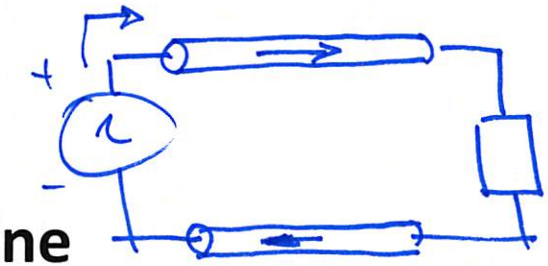
$$\ln(a \cdot b) = \ln(a) + \ln(b)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

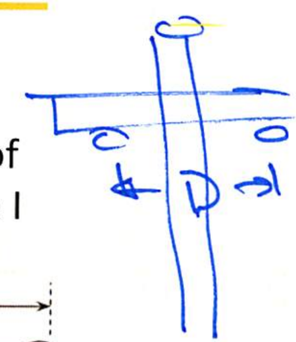
$$\ln(a^b) = b \cdot \ln(a)$$

$$\ln(e^a) = a \ln(e) = a \cdot 1 = a$$

$$\ln(1) = 0$$

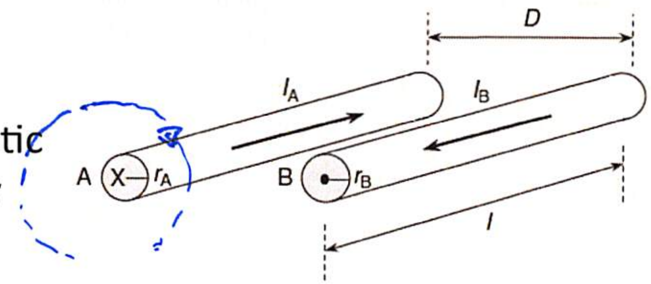


Inductance of a Single Phase Line



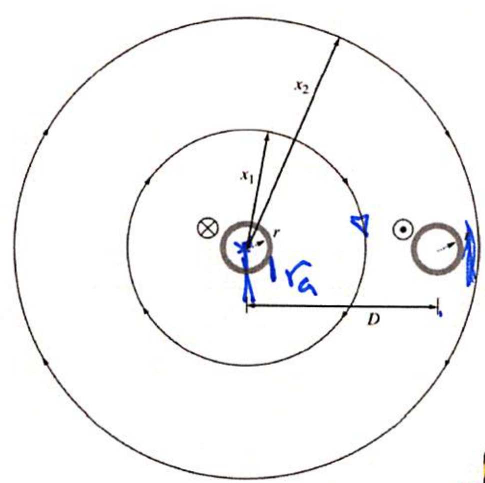
We next determine the series inductance of a single-phase line consisting of two conductors of radii r seperated by a distance D. The conductor "A" carries a current of magnitude I flowing into the page, and conductor "B" carries a current of magnitude I flowing out of the page.

Considering two circular integration paths; we find that the line integral along x_1 produces a net magnetic intensity since a non-zero net current is enclosed by x_1.



$$\oint H_x \cdot dl \;=\; I_{x\ (enclosed)}$$

However the path of radius x_2 encloses both conductors and the currents are equal and opposite, the net current enclosed is 0 and, therefore, there can be **NO** contributions to the total inductance from the magnetic fields at distances greater than D!



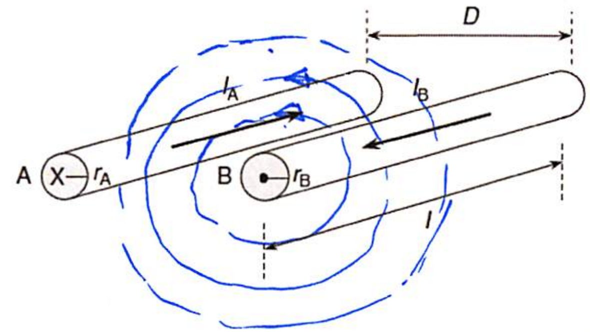
Inductance of a Single Phase Line

The total inductance of **a single wire** per unit length in a single phase transmission line is a sum of the internal inductance and the external inductance between the conductor surface (r) and the separation distance (D):

$$L_A = L_{\text{int}} + L_{ext} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln\frac{D}{r} \quad [H/m]$$

substituting $\mu_0 = 4\pi\cdot10^{-7}$

$$L = 2\cdot10^{-7}\left[\frac{1}{4} + \ln\frac{D}{r}\right] \quad [H/m]$$



re-writing the ¼ factor in a more convenient form yields:

$$L = 2\cdot10^{-7}\left[\ln(e^{\frac{1}{4}}) + \ln\frac{D}{r}\right] \quad [H/m]$$

$\ln a + \ln b = \ln ab$

Source: http://www.accessscience.com