4 Lectures Left $\frac{4}{28} = \frac{2}{14} = \frac{1}{7} = 14.3\%$

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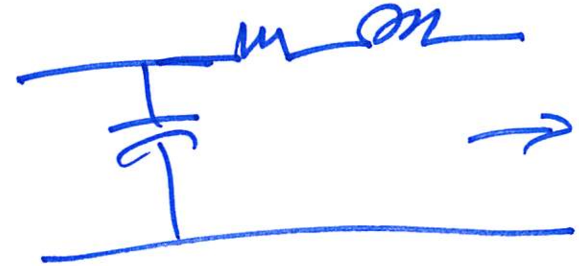
S →  → S_{LOAD}

Exam 3 Thursday @ 9:35

Due Monday @ 12:00 (Noon)

Box 49 7th Floor for on Campus Students

Chapt 5 → 3 Lectures

$\mu_0 = 4\pi \times 10^{-7}\ H/m$

$L = \frac{\lambda}{I}$

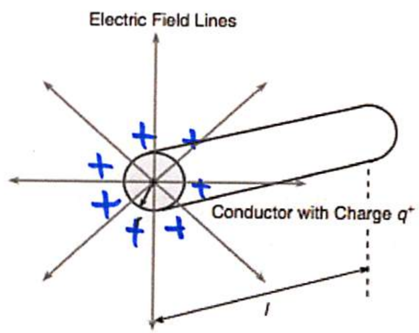
Transmission Line Parameters - Part 2

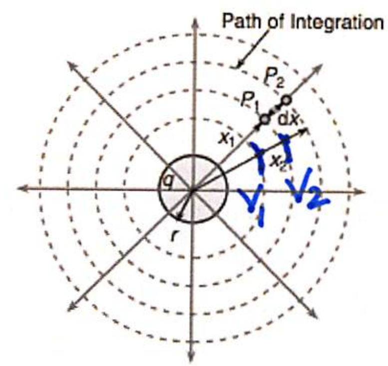
Capacitance and Capacitive Reactance

Voltage on a solid, single conductor

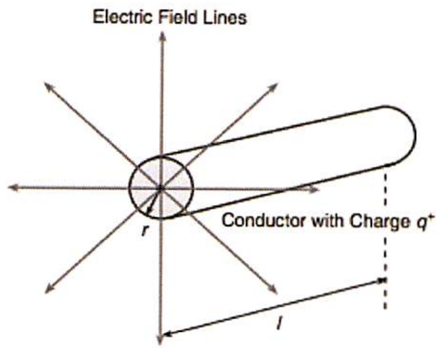
Consider a long, solid, cylindrical conductor of radius "r" immersed in a media with permittivity ε_0. (Where $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m) A charge of +q coulombs per meter exits and is uniformly distributed on the surface of the conductor. The conductor is a perfect conductor with resistivity assumed to be zero, so there is no **internal** electric field due to the charge on the conductor.

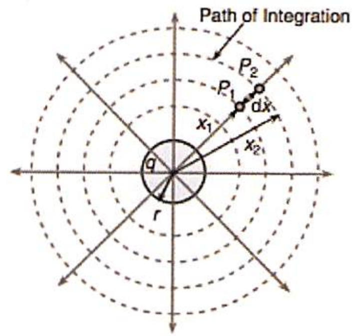
$q = CV$



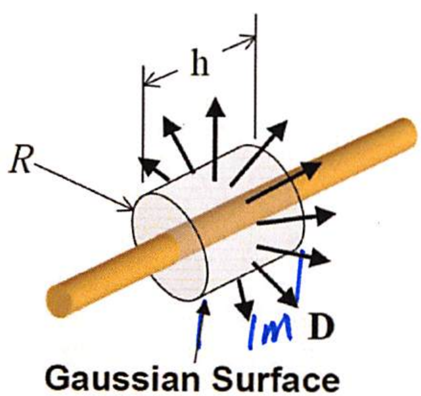


Capacitance





An electric field will be produced radial to the conductor due to the charge on the conductor, with equipotential surfaces concentric to the surface.



$$q_e = \int_A D \cdot da$$

According to Gauss's Law, the total electric flux leaving a closed surface is equal to the total charge inside the volume enclosed by the surface.

$B = \mu_0 H$

Capacitance

$$D_p = \frac{q}{A} = \frac{q}{2\pi x \cdot 1} (\mathrm{C/m^2})$$ The electric flux density D at point p = Charge/Area

field

$$D_p = \varepsilon \cdot E_p$$ The electric flux density = permittivity x electric flux intensity

$$E_p = \frac{q}{2\pi\varepsilon_0 x} \ (\mathrm{V/m})$$

The potential difference between any two points (P1 and P2) located outside the conductor surface at distance x_1 and x_2 from the center of the conductor respectively can be determined by integrating the electric field intensity from x_1 to x_2

$$V_{1-2} = \int_{X1}^{X2} \frac{q}{2\pi\varepsilon_0 x} \, dx = \frac{q}{2\pi\varepsilon_0} \ln\left(\frac{X_2}{X_1}\right) \ (\mathrm{V})$$

$\ln x \Big|_{x1}^{x2}$ $\ln x_2 - \ln x_1$

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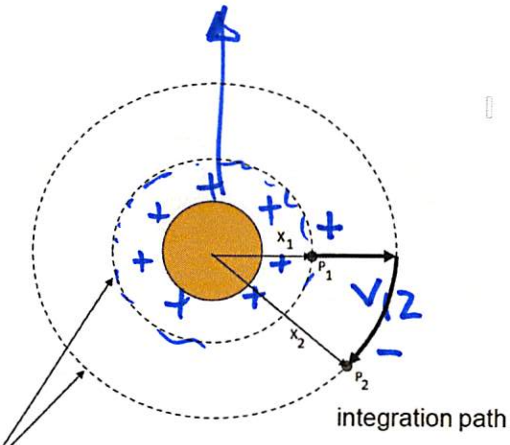
Capacitance

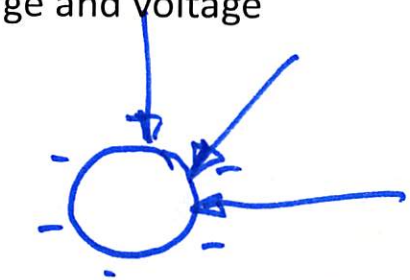
$$V_{1-2} = \frac{q}{2\pi\varepsilon_0} \ln\left(\frac{X_2}{X_1}\right) \quad \text{(V)}$$



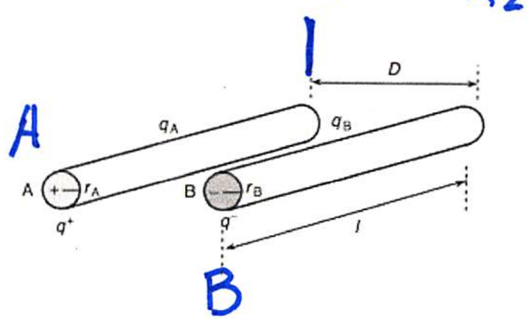
but capacitance is the proportionality constant relating charge and voltage

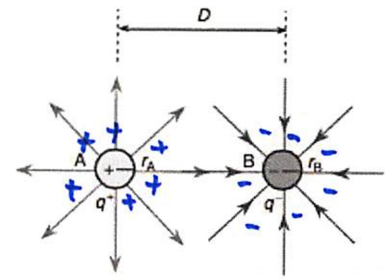
$$q = C \cdot V$$

$$C_{1-2} = \frac{q}{V_{1-2}} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{X_2}{X_1}\right)} \quad \text{(F/m)}$$



Capacitance of a Two Wire Single Phase Line





The voltage arising due to a charge on a single conductor was given by:

$$V_{1-2} = \frac{q}{2\pi\varepsilon_0} \ln\left(\frac{X_2}{X_1}\right) \quad (\mathrm{V})$$

Using the principle of superposition -

D_{AA} D_{BB}

r_A r_B

$$V_{A-B} = \frac{q}{2\pi\varepsilon_0} \ln\left(\frac{D_{AB}}{D_{AA}}\right) + \frac{-q}{2\pi\varepsilon_0} \ln\left(\frac{D_{BB}}{D_{BA}}\right) \quad (\mathrm{V})$$

$$V_{A-B} = \frac{q}{2\pi\varepsilon_0} \left(\ln\left(\frac{D}{r_A}\right) \mp \ln\left(\frac{r_B}{D}\right)^{-1}\right) = \frac{q}{2\pi\varepsilon_0}\left(\ln \frac{D^2}{r_A r_B}\right) (\mathrm{V}) = \frac{q}{\pi\varepsilon_0}\left(\ln \frac{D}{\sqrt{r_A r_B}}\right)$$

$\ln \frac{D}{r_B}$

$(\sqrt{r_A r_B})^2$

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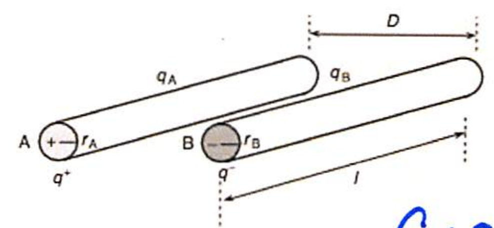
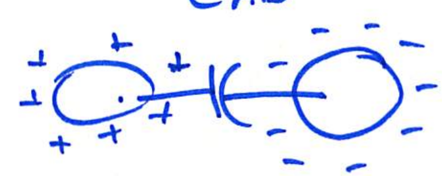
Capacitance of a Two Wire Single Phase Line

$$C_{AB} = \frac{q}{V_{A-B}}$$

$$C_{AB} = \frac{2\pi\varepsilon_0}{\ln \frac{D^2}{r_A r_B}} \quad \text{(F/m)}$$





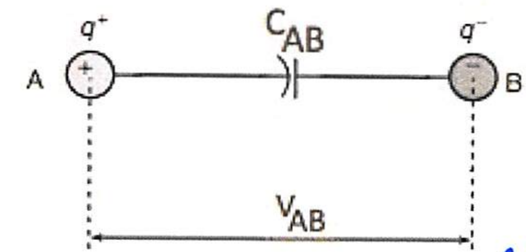
if $r_A = r_B = r$

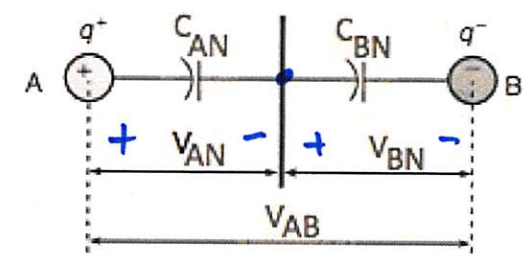
$$C_{AB} = \frac{2\pi\varepsilon_0}{\ln \frac{D^2}{r^2}} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D}{r}\right)^2} = \frac{2\pi\varepsilon_0}{2\ln\left(\frac{D}{r}\right)} = \frac{\pi\varepsilon_0}{\ln\left(\frac{D}{r}\right)} \quad \text{(F/m)}$$

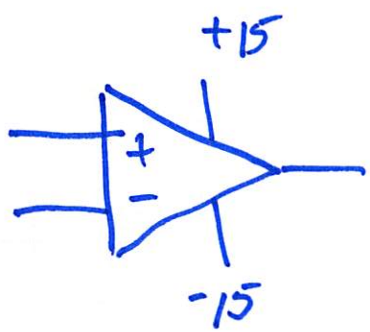
Capacitance to Neutral

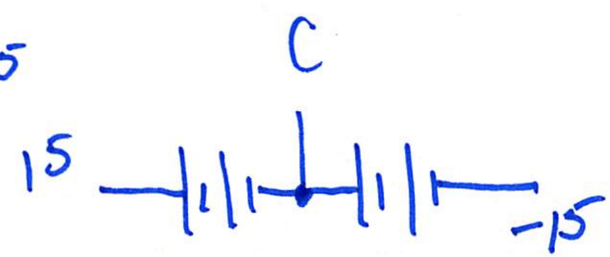
The potential difference to neutral is half the difference between the two conductors.

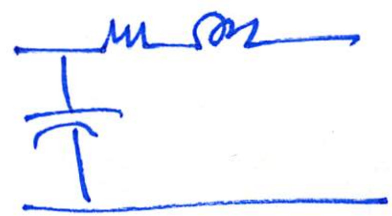
$$C_{AN} = C_{BN} = \frac{q}{\frac{(V_{AB})}{2}} = \frac{2\pi\varepsilon_0}{\ln\frac{D}{r}} \quad (F/m)$$











Capacitance and Inductance Comparison

Previously we derived the inductance for a two wire single phase line (both conductors having radius r)

$$L = 2 \cdot 10^{-7} \ln\left(\frac{D}{r'}\right) \quad H/m \qquad \frac{\mu_0}{2\pi} \qquad r' = r e^{-1/4}$$

The capacitance to neutral is given by:

$$C_{AN} = \frac{2\pi\varepsilon_0}{\ln \dfrac{D}{r}} \quad \text{F/m}$$

Note: the inductance is calculated using an "effective radius"
the capacitance is calculated using the **actual** radius

Capacitance to Neutral

Having determined the capacitance to neutral, we can now calculate the capacitive reactance

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi f \left(\frac{2\pi\varepsilon_0}{\ln \frac{D}{r}} \right)} = \frac{2.861 \cdot 10^9}{f} \left(\ln \frac{D}{r} \right) \ \Omega m$$

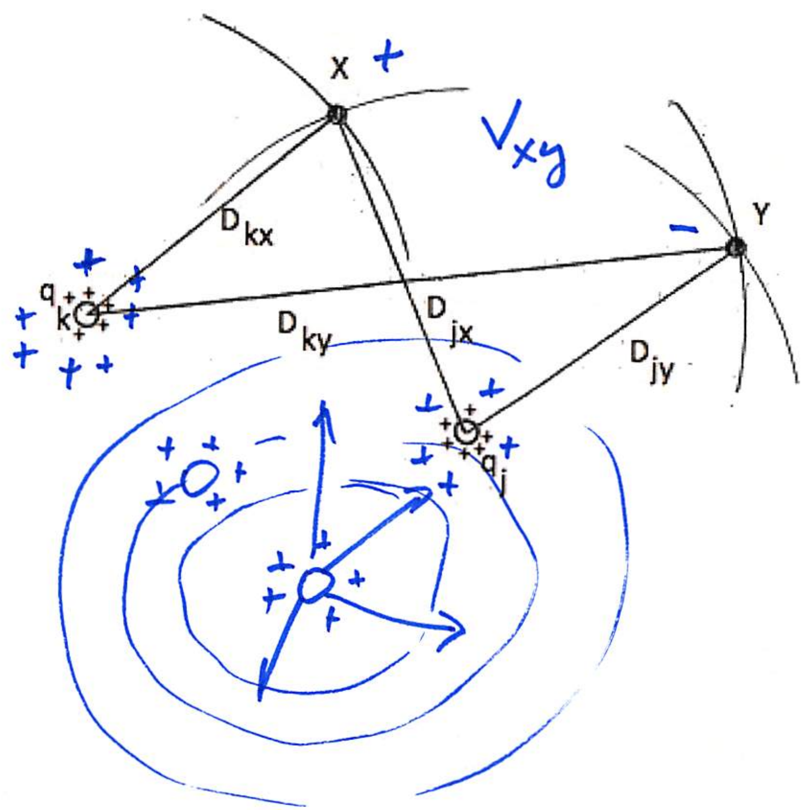
What happens to capacitive reactance as the line gets longer? (Hint: you need to divide by distance.)

$= 2\pi f \cdot L$

Contrast this to R and X_L

$$R = \frac{\rho \cdot l}{A} \qquad X_L = 4\pi \cdot 10^{-7} \ f \ \ln\left(\frac{D}{r'}\right) \ \Omega / m$$

Potential Difference Multiple Charges

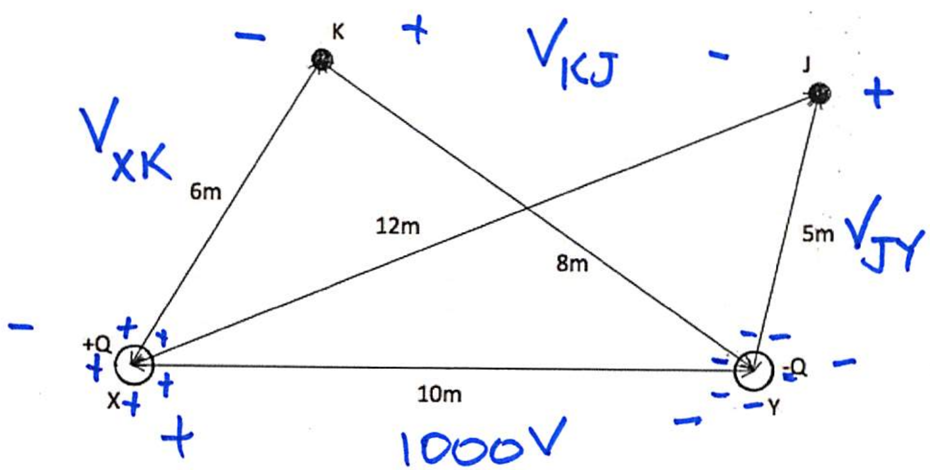


$$V_{XY} = \frac{q_k}{2\pi\varepsilon_0}\ln\left(\frac{D_{kY}}{D_{kX}}\right) \quad due \ \ to \ \ q_k$$

$$V_{XY} = \frac{q_j}{2\pi\varepsilon_0}\ln\left(\frac{D_{jY}}{D_{jX}}\right) \quad due \ \ to \ \ q_j$$

$$V_{XY} = \frac{1}{2\pi\varepsilon_0}\sum_{m=1}^{M} q_m \ln\frac{D_{mY}}{D_{mX}}$$

Potential Difference



Given $Q_x = Q = -Q_y$ & $r_x = r_y = 0.01$m
$V_{xy} = 1000$V

Determine: V_{xk}, V_{kj} and V_{jy}

$$V_{XY} = \frac{1}{2\pi\varepsilon_0}\sum_{m=1}^{M} q_m \ln\frac{D_{mY}}{D_{mX}}$$

$$V_{XY} = \frac{Q_x}{2\pi\varepsilon_0}\ln\left(\frac{D_{XY}}{D_{XX}}\right) + \frac{-Q_Y}{2\pi\varepsilon_0}\ln\left(\frac{D_{YY}}{D_{YX}}\right) = \frac{Q}{2\pi\varepsilon_0}\left[\ln\left(\frac{D_{XY}}{r_x}\right) \cancel{-} + \left(\ln\frac{r_y}{D_{YX}}\right)^{-1}\right] = \frac{Q}{2\pi\varepsilon_0}\ln\left(\frac{D_{XY}^2}{r_X r_Y}\right)$$

$$V_{XY} = \frac{Q}{2\pi\varepsilon_0}\ln\left(\frac{D_{XY}^2}{r_X r_Y}\right) \qquad \frac{Q}{2\pi\varepsilon_0} = \frac{1000}{\ln\frac{10^2}{.01^2}} = \underline{72.382}$$

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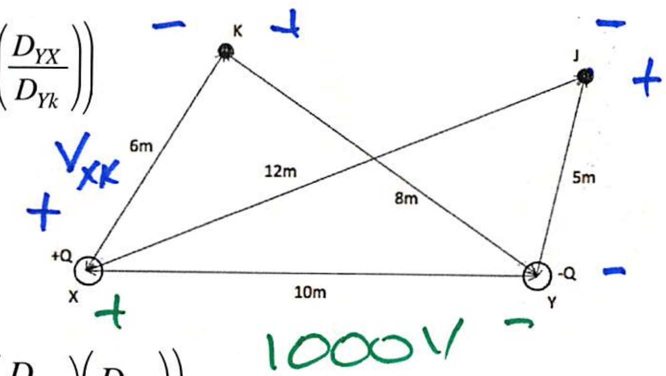
Potential Difference

$$V_{Xk} = \frac{1}{2\pi\varepsilon_0}\sum_{m=1}^{M} q_m \ln\frac{D_{mk}}{D_{mX}} = \frac{Q}{2\pi\varepsilon_0}\left(\ln\frac{D_{Xk}}{D_{XX}} - \ln\frac{D_{Yk}}{D_{YX}}\right) = \frac{Q}{2\pi\varepsilon_0}\ln\left(\left(\frac{D_{Xk}}{D_{XX}}\right)\left(\frac{D_{YX}}{D_{Yk}}\right)\right)$$



$$V_{Xk} = 72.382\ln\left(\left(\frac{6}{.01}\right)\left(\frac{10}{8}\right)\right) = 479.177 \text{ V}$$

$$V_{kj} = \frac{1}{2\pi\varepsilon_0}\sum_{m=1}^{M} q_m \ln\frac{D_{mj}}{D_{mk}} = \frac{Q}{2\pi\varepsilon_0}\left(\ln\frac{D_{Xj}}{D_{Xk}} - \ln\frac{D_{Yj}}{D_{Yk}}\right) = \frac{Q}{2\pi\varepsilon_0}\ln\left(\left(\frac{D_{Xj}}{D_{Xk}}\right)\left(\frac{D_{Yk}}{D_{Yj}}\right)\right)$$

$$V_{kj} = 72.382\ln\left(\left(\frac{12}{6}\right)\left(\frac{8}{5}\right)\right) = 84.192 \text{ V}$$

$$V_{jY} = \frac{1}{2\pi\varepsilon_0}\sum_{m=1}^{M} q_m \ln\frac{D_{mY}}{D_{mj}} = \frac{Q}{2\pi\varepsilon_0}\left(\ln\frac{D_{XY}}{D_{Xj}} - \ln\frac{D_{YY}}{D_{Yj}}\right) = \frac{Q}{2\pi\varepsilon_0}\ln\left(\left(\frac{D_{XY}}{D_{Xj}}\right)\left(\frac{D_{Yj}}{D_{YY}}\right)\right)$$

$$V_{jY} = 72.382\ln\left(\left(\frac{10}{12}\right)\left(\frac{5}{.01}\right)\right) = 436.631 \text{ V}$$

1000

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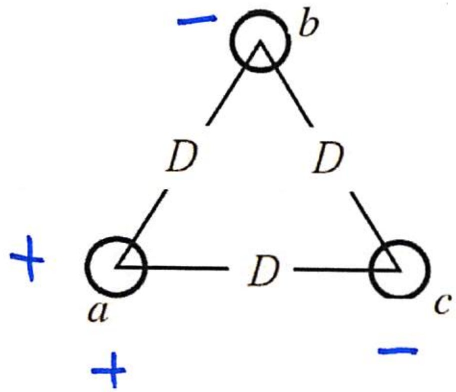
Capacitance of a Three Phase Line

Consider a balanced, abc positive phase sequence three phase line where $q_A + q_B + q_C = 0$

The space betwee phase conductors is given as D_{AB} , D_{BC} and D_{CA} = D
The conductor radii are given as r_A, r_B, r_C = r where the radii are small compared to D



$$V_{AB} = \frac{q_A}{2\pi\varepsilon_0}\ln\left(\frac{D_{AB}}{D_{AA}}\right) + \frac{q_B}{2\pi\varepsilon_0}\ln\frac{D_{BB}}{D_{BA}} + \frac{q_C}{2\pi\varepsilon_0}\ln\frac{D_{CB}}{D_{CA}}$$

$$V_{AC} = \frac{q_A}{2\pi\varepsilon_0}\ln\left(\frac{D_{AC}}{D_{AA}}\right) + \frac{q_B}{2\pi\varepsilon_0}\ln\frac{D_{BC}}{D_{BA}} + \frac{q_C}{2\pi\varepsilon_0}\ln\frac{D_{CC}}{D_{CA}}$$

$$V_{AB} = \frac{q_A}{2\pi\varepsilon_0}\ln\left(\frac{D}{r}\right) + \frac{q_B}{2\pi\varepsilon_0}\ln\frac{r}{D} + \frac{q_C}{2\pi\varepsilon_0}\ln\frac{D}{D} \to 0$$

$$V_{AC} = \frac{q_A}{2\pi\varepsilon_0}\ln\left(\frac{D}{r}\right) + \frac{q_B}{2\pi\varepsilon_0}\ln\frac{D}{D} + \frac{q_C}{2\pi\varepsilon_0}\ln\frac{r}{D}$$

0

$\frac{q}{t} = i$

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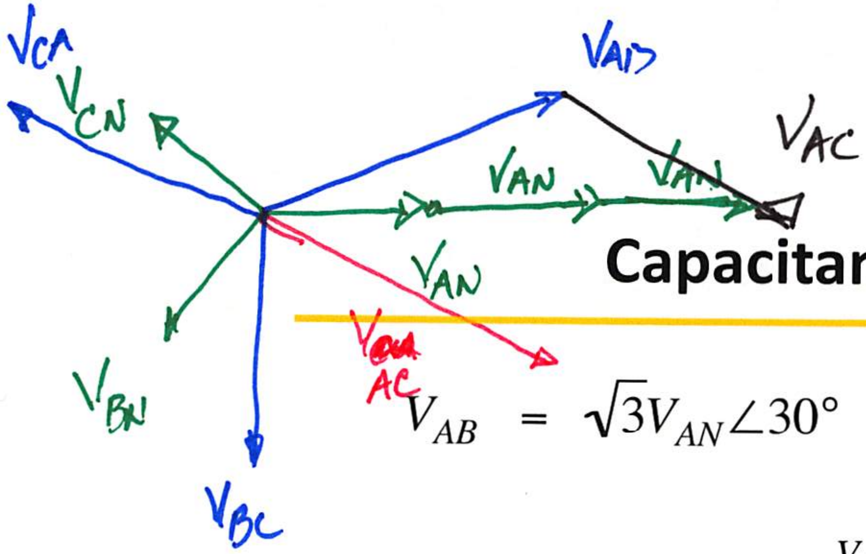


Capacitance of a Three Phase Line

$$V_{AB} = \sqrt{3}V_{AN}\angle 30^\circ \qquad V_{AC} = -V_{CA} = \sqrt{3}V_{AN}\angle -30^\circ$$

$$V_{AN} = \frac{V_{AB} + V_{AC}}{3}$$

$$V_{AB} = \frac{q_A}{2\pi\varepsilon_0}\ln\left(\frac{D}{r}\right) + \frac{q_B}{2\pi\varepsilon_0}\ln\frac{r}{D} + \cancel{\frac{q_C}{2\pi\varepsilon_0}\ln\frac{D}{D}} \to 0 \qquad V_{AC} = \frac{q_A}{2\pi\varepsilon_0}\ln\left(\frac{D}{r}\right) + \cancel{\frac{q_B}{2\pi\varepsilon_0}\ln\frac{D}{D}} \to 0 + \frac{q_C}{2\pi\varepsilon_0}\ln\frac{r}{D}$$

$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{6\pi\varepsilon_0}\left(q_A\ln\frac{D}{r} + \underline{q_B\ln\frac{r}{D}} + q_A\ln\frac{D}{r} + \underline{q_C\ln\frac{r}{D}}\right)$$

$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{6\pi\varepsilon_0}\left(2q_A\ln\frac{D}{r} + (\overbrace{q_B + q_C}^{-q_a})\ln\left(\frac{r}{D}\right)^{-1}\right)$$

$$q_a + q_b + q_c = 0 + q_a \ln\frac{D}{r}$$

Capacitance of a Three Phase Line

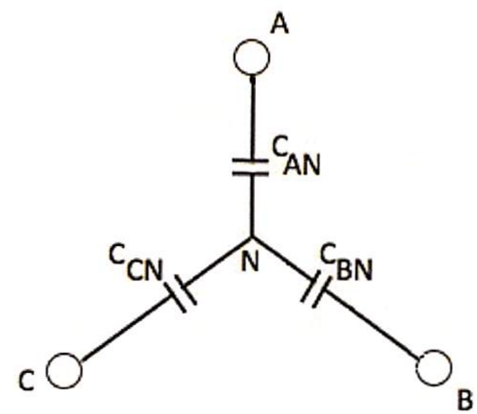
$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{6\pi\varepsilon_0}\left(2q_A \ln\frac{D}{r} + (q_B + q_C)\ln\frac{r}{D}\right)$$

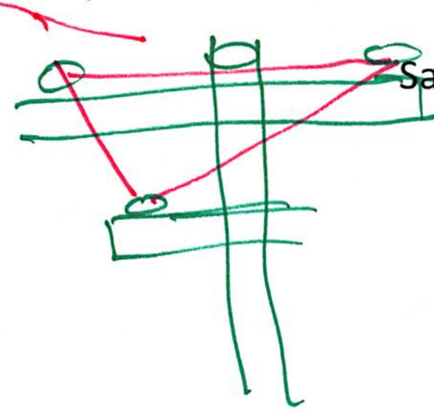
$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{6\pi\varepsilon_0}\left(2q_A \ln\frac{D}{r} + (-q_A)\ln\frac{r}{D}\right)$$

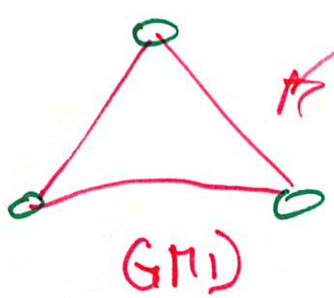
$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{2\cancel{6}\pi\varepsilon_0}\left(\cancel{3}q_A \ln\frac{D}{r}\right)$$

$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{2\pi\varepsilon_0}\left(q_A \ln\frac{D}{r}\right)$$

$$C_{AN} = \frac{q_A}{V_{AN}} = \frac{2\pi\varepsilon_0}{\ln\frac{D}{r}} \quad \text{F/m}$$

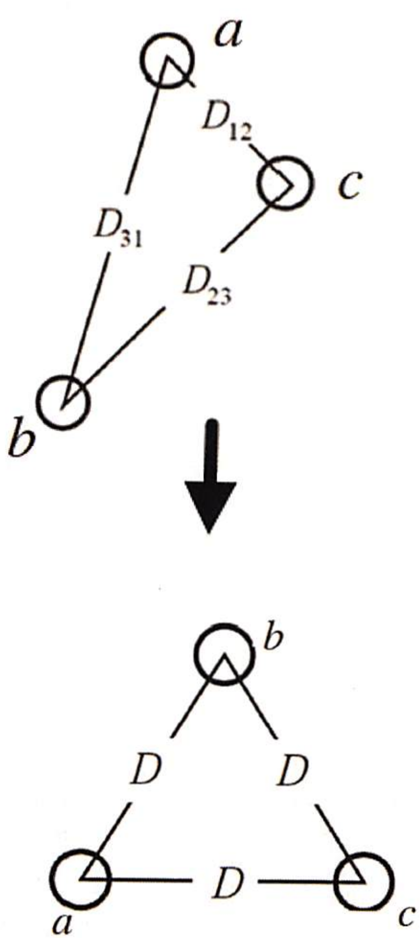






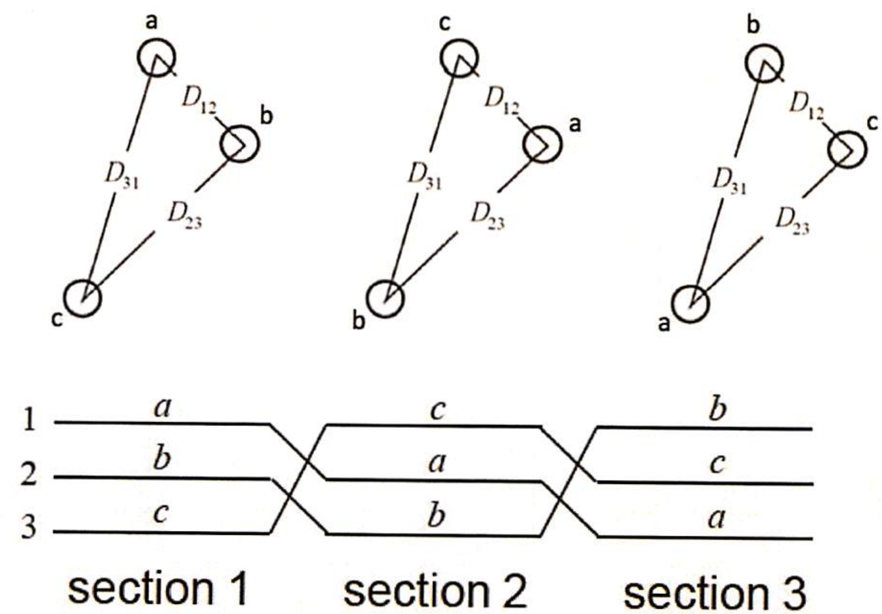
Same as the single phase result!!
Outstanding!!!

Capacitance Asymmetrical Spacing



Use the same approach we used for inductance

Force the asymmetry into a symmetric system by utilizing transposition.



$$\bar{V}_{AB} = \frac{V_{AB_1} + V_{AB_2} + V_{AB_3}}{3}$$

Capacitance

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{12}}{r}\right) + q_b \ln\left(\frac{r}{D_{12}}\right) + q_c \ln\left(\frac{D_{23}}{D_{31}}\right)\right) \text{ in section 1}$$

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{23}}{r}\right) + q_b \ln\left(\frac{r}{D_{23}}\right) + q_c \ln\left(\frac{D_{31}}{D_{12}}\right)\right) \text{ in section 2}$$

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{31}}{r}\right) + q_b \ln\left(\frac{r}{D_{31}}\right) + q_c \ln\left(\frac{D_{12}}{D_{23}}\right)\right) \text{ in section 3}$$

$$\overline{V_{ab}} = \frac{1}{6\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{12}D_{23}D_{31}}{r^3}\right) + q_b \ln\left(\frac{r^3}{D_{12}D_{23}D_{31}}\right) + \cancelto{0}{q_c \ln\left(\frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}\right)}\right)$$

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Capacitance

$$V_{ab} = \frac{1}{6\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{12}D_{23}D_{31}}{r^3}\right) + q_b \ln\left(\frac{r^3}{D_{12}D_{23}D_{31}}\right) + q_c \ln\left(\frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}\right)\right)$$

$$V_{ab} = \frac{1}{6\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{12}D_{23}D_{31}}{r^3}\right)^{1/3} + q_b \ln\left(\frac{r^3}{D_{12}D_{23}D_{31}}\right)\right)$$

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{eq}}{r}\right) + q_b \ln\left(\frac{r}{D_{eq}}\right)\right) \quad where \;\; D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$

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Capacitance

Similarly:

$$V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{eq}}{r}\right) + q_c \ln\left(\frac{r}{D_{eq}}\right)\right) \quad where \quad D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$

$$3V_{an} = V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(2q_a \ln\left(\frac{D_{eq}}{r}\right) + q_b \ln\left(\frac{r}{D_{eq}}\right) \quad q_c \ln\left(\frac{r}{D_{eq}}\right)\right)$$

$$3V_{an} = V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(2q_a \ln\left(\frac{D_{eq}}{r}\right) - q_a \ln\left(\frac{r}{D_{eq}}\right)\right)$$

$$\underbrace{(q_b+q_c)}_{+q_a}\ln\left(\frac{r}{D_{eq}}\right)^{-1}$$

$$3V_{an} = V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(2q_a \ln\left(\frac{D_{eq}}{r}\right) + q_a \ln\left(\frac{D_{eq}}{r}\right)\right)$$

$$\cancel{3}V_{an} = \frac{\cancel{3}}{2\pi\varepsilon_0} q_a \ln\left(\frac{D_{eq}}{r}\right)$$

$$C_{an} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D_{eq}}{r}\right)}$$

GMD

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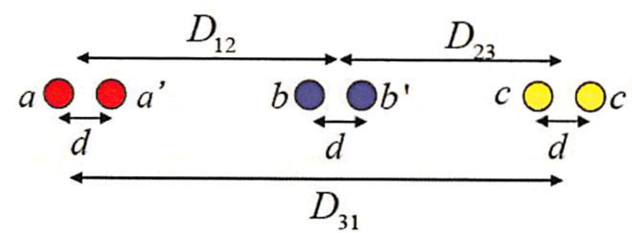
Bundled Solid Conductors Asymmetric Spacing

Assume D_{12}, D_{23} and D_{31} are much greater than d



Qa, Qa' + Qb, Qb' + Qc, Qc' = 0

Charge is equally divided and charge neutrality is preserved.

All conducrtors have radius r and the lines are fully transposed.

GMD

b

$$V_{AN_{trnasposed}} = \frac{V_{AB_{transposed}} + V_{AC\,transposed}}{3}$$

a

c

$$V_{AN_{transposed}} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\left(\frac{D_{eq}}{D_{sc}}\right)\right)$$

$D_{aa} \cdot D_{aa'} \cdot D_{a'a'} \, D_{a'a}$

$$C_{AN} = \frac{q_a}{V_{AN_{transposed}}} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D_{eq}}{D_{sc}}\right)} \qquad D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}} \quad and \quad D_{sc} = \sqrt[4]{r \cdot d \cdot r \cdot d}$$

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$D_{SL} = R_{b}' = GMR$

Quick Reminder

When we calculated the inductance for a fully transposed line we found:

$$L_a = \frac{\lambda_a}{I_a} = 2\cdot10^{-7}\left(\ln\frac{D_{eq}}{D_{sL}}\right) \quad D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}} \quad D_{sL} = \sqrt[4]{r'\cdot d\cdot r'\cdot d}$$

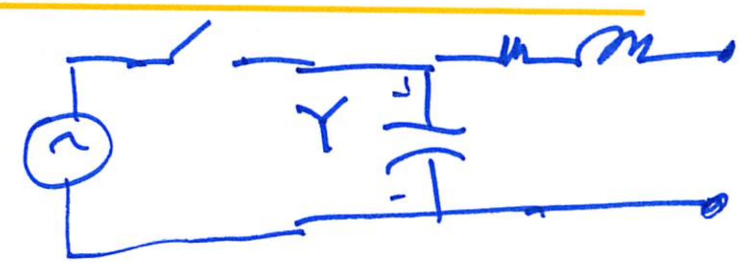
When calculating the capacitance for a fully transposed line:

$$C_{AN} = \frac{q_a}{V_{AN_{transposed}}} = \frac{2\pi\varepsilon_0}{\ln\left(\dfrac{D_{eq}}{D_{sC}}\right)} \quad D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}} \; and \; D_{sC} = \sqrt[4]{r\cdot d\cdot r\cdot d}$$

D_{eq} – the same!!!
The geometric radius of the bundle different!!!
Inductance uses r’ the **effective radius** of the conductor
Capacitrance uses r the **actual radius** of the conductor

Capacitance and Capacitive Reactance

$$C_{AN} = \frac{q_a}{V_{AN_{transposed}}} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D_{eq}}{D_{sc}}\right)}$$



$$X_C = \frac{1}{2\pi f C} = \frac{\ln\left(\frac{D_{eq}}{D_{sc}}\right)}{4\pi^2 f\varepsilon_0}\ \Omega\cdot m = 1.778\cdot 10^6 \frac{1}{f}\ln\left(\frac{D_{eq}}{D_{sc}}\right)\ \Omega\cdot mile$$

For a completely transposed line connected to a balanced positive sequence set of voltages, a "charging current" will result:

$$I_{chrg} = Y\,V_{an}$$

$$I_{chrg} = j2\pi f C_{an}\cdot l\cdot V_{an}$$

The reactive power associated with the charging current (per phase):
$Q_{C1\phi} = Y\,Van^2 = \omega\, C_{AN} V_{LN}^2$

The total reactive is three times the power per phase:
$Q_{C3\phi} = 3Y\,Van^2 = 3\omega\, C_{AN} V_{LN}^2 = \omega\, C_{AN} V_{LL}^2$