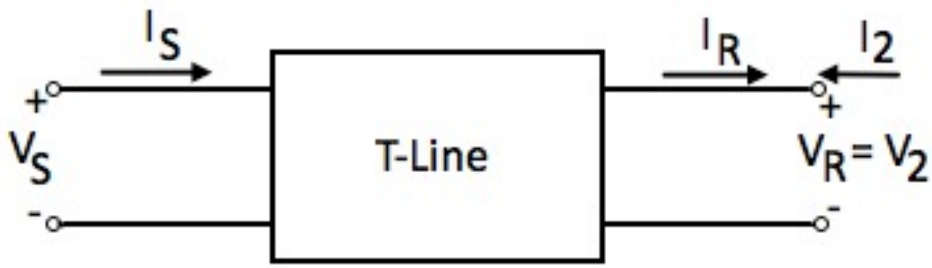
Two Ports



$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Admittance parameters

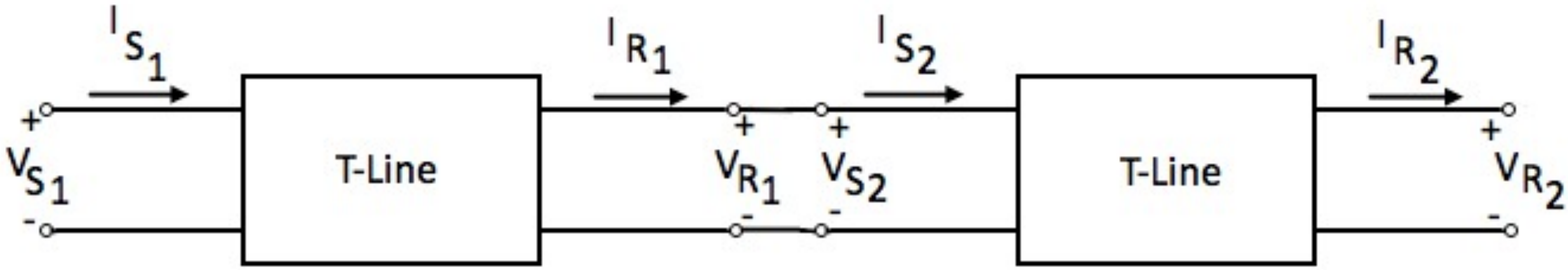
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Z - parameters

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

h - parameters

Two Ports

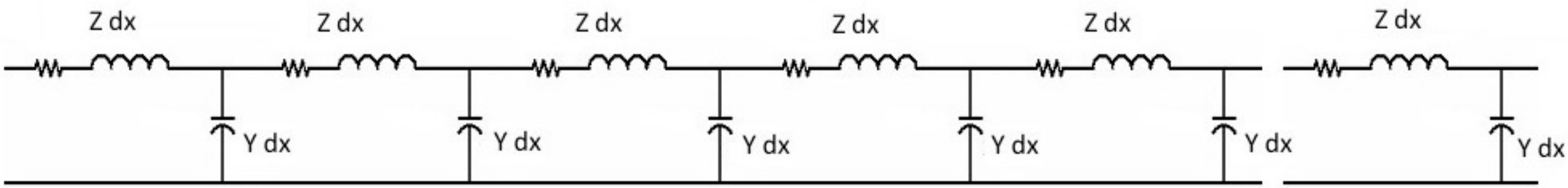


$$\begin{bmatrix} V_{S_1} \\ I_{S_1} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_{R_1} \\ I_{R_1} \end{bmatrix} \qquad \begin{bmatrix} V_{S_2} \\ I_{S_2} \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{R_2} \\ I_{R_2} \end{bmatrix}$$

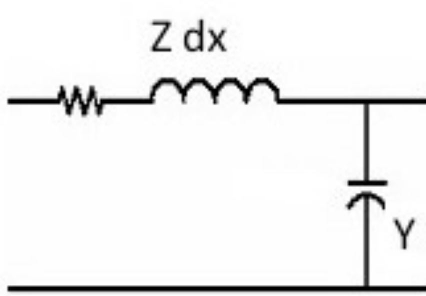
$$but \quad \begin{bmatrix} V_{R_1} \\ I_{R_1} \end{bmatrix} = \begin{bmatrix} V_{S_2} \\ I_{S_2} \end{bmatrix}$$

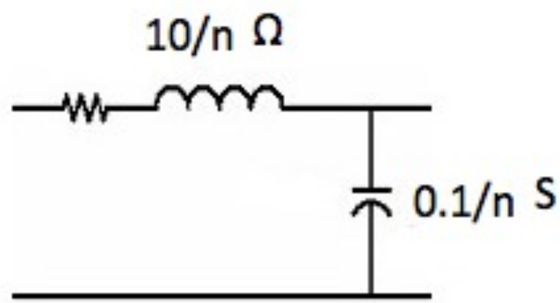
$$\begin{bmatrix} V_{S_1} \\ I_{S_1} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{R_2} \\ I_{R_2} \end{bmatrix}$$

The Distributed Network

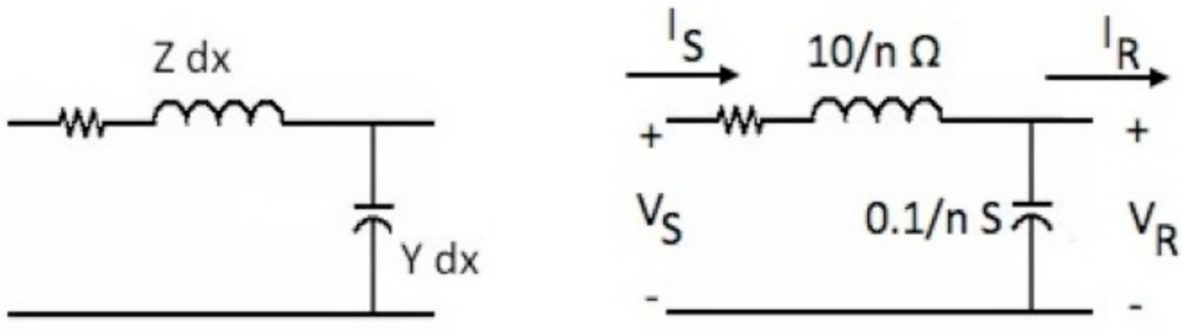


Let - $Z_{line\ (total)} = 10\Omega$ and $Y_{line\ (total)} = .1S$
Begin by dividing the line into n equal segments
The admittance and impedance per increment will be 1/n th of the total





The Distributed Network



$V_s = (10/n)\ I_s + V_R$

$I_s = I_R + V_R\ (0.1/n)$

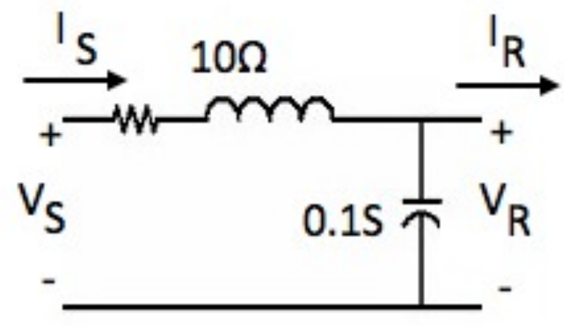
$V_s = (10/n)\ (I_R + V_R\ (0.1/n)) + V_R = (1 + 1/n^2)V_R + (10/n)I_R$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \left(1 + \dfrac{1}{n^2}\right) & \dfrac{10}{n} \\ \dfrac{0.1}{n} & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Distributed Network

For n = 1

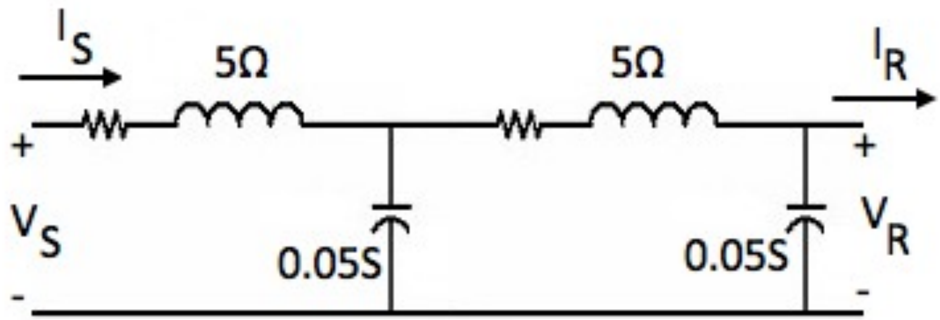
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \dfrac{1}{n^2}\right) & \dfrac{10}{n} \\ \dfrac{0.1}{n} & 1 \end{bmatrix} = \begin{bmatrix} 2.0 & 10.0 \\ 0.1 & 1 \end{bmatrix}$$



$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Distributed network

For n = 2

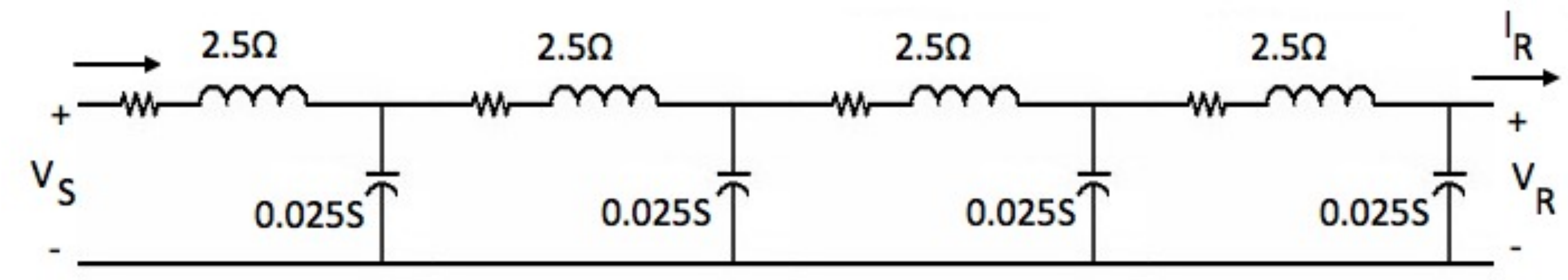


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{1}{n^2}\right) & \frac{10}{n} \\ \frac{0.1}{n} & 1 \end{bmatrix} = \begin{bmatrix} 1.25 & 5.0 \\ 0.05 & 1 \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1.25 & 5.0 \\ 0.05 & 1 \end{bmatrix} \begin{bmatrix} 1.25 & 5.0 \\ 0.05 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1.8125 & 11.2500 \\ 0.1125 & 1.2500 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Distributed Network

For n = 4



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \dfrac{1}{n^2}\right) & \dfrac{10}{n} \\ \dfrac{0.1}{n} & 1 \end{bmatrix} = \begin{bmatrix} 1.0625 & 2.5 \\ 0.025 & 1 \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1.0625 & 2.5 \\ 0.025 & 1 \end{bmatrix} \begin{bmatrix} 1.0625 & 2.5 \\ 0.025 & 1 \end{bmatrix} \begin{bmatrix} 1.0625 & 2.5 \\ 0.025 & 1 \end{bmatrix} \begin{bmatrix} 1.0625 & 2.5 \\ 0.025 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1.6853 & 11.6217 \\ 0.1162 & 1.3948 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Distributed Network

For n = 8

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \dfrac{1}{n^2}\right) & \dfrac{10}{n} \\ \dfrac{0.1}{n} & 1 \end{bmatrix} = \begin{bmatrix} 1.0156 & 1.25 \\ 0.0125 & 1 \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1.0156 & 1.25 \\ 0.0125 & 1 \end{bmatrix}^8 \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1.6156 & 11.7191 \\ 0.1172 & 1.4691 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

For n = 16

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \dfrac{1}{n^2}\right) & \dfrac{10}{n} \\ \dfrac{0.1}{n} & 1 \end{bmatrix} = \begin{bmatrix} 1.0039 & 0.625 \\ 0.00625 & 1 \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1.0039 & 0.625 \\ 0.00625 & 1 \end{bmatrix}^{16} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1.5796 & 11.7438 \\ 0.1174 & 1.5062 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Distributed Network

For n = infinity

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1.5431 & 11.7520 \\ 0.1175 & 1.5431 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

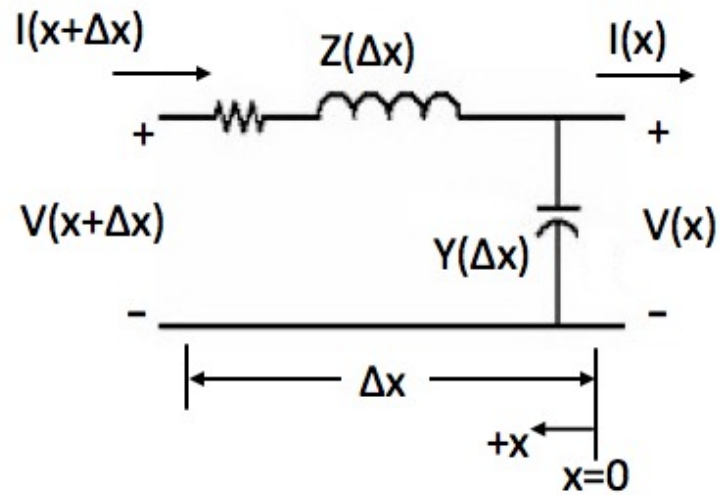
How do we know that???????

Defining the Differential Eq.

Consider a differential length of line:

$$V_{(X+\Delta X)} = V_{(X)} + Z_{\Delta X} \cdot I_{(X+\Delta X)}$$

$$I_{(X+\Delta X)} = I_X + Y_{\Delta X} \cdot V_X$$



$$\frac{V_{(X+\Delta X)} - V_{(X)}}{\Delta X} = Z_{\Delta X} \cdot I_{(X+\Delta X)}$$

$$\lim_{\Delta X \to 0} \frac{dV}{dX} = Z \cdot I_X$$

$$\frac{I_{(X+\Delta X)} - I_X}{\Delta X} = Y_{\Delta X} \cdot V_X$$

$$\lim_{\Delta X \to 0} \frac{dI}{dX} = Y \cdot V_X$$

Solving the Differential Eq.

$$\frac{dV}{dX} = Z_X \cdot I_X \qquad \frac{dI}{dX} = Y \cdot V_X$$

$$s \cdot V_{(S)} - V(0^+) = Z \cdot I_{(S)}$$

$$s \cdot I_{(S)} - I(0^+) = Y \cdot V_{(S)}$$

$$s \cdot V_{(S)} - Z \cdot I_{(S)} = V(0^+)$$

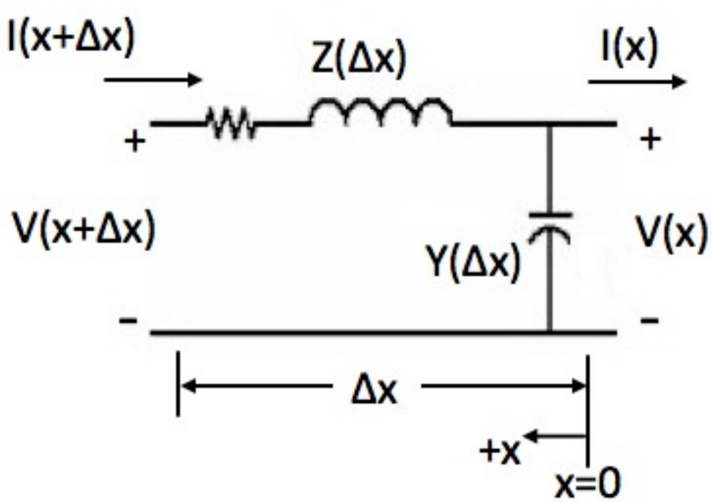
$$s \cdot I_{(S)} - Y \cdot V_{(S)} = I(0^+)$$

$$\begin{bmatrix} s & -Z \\ -Y & s \end{bmatrix} \begin{bmatrix} V_{(S)} \\ I_{(S)} \end{bmatrix} = \begin{bmatrix} V(0^+) \\ I(0^+) \end{bmatrix}$$

Solving the Diff Eq.

$$\begin{bmatrix} s & -Z \\ -Y & s \end{bmatrix}^{-1} \begin{bmatrix} s & -Z \\ -Y & s \end{bmatrix} \begin{bmatrix} V_{(S)} \\ I_{(S)} \end{bmatrix} = \begin{bmatrix} s & -Z \\ -Y & s \end{bmatrix}^{-1} \begin{bmatrix} V(0^+) \\ I(0^+) \end{bmatrix}$$

$$\begin{bmatrix} V_{(S)} \\ I_{(S)} \end{bmatrix} = \left(\frac{1}{s^2 - YZ} \right) \begin{bmatrix} s & Z \\ Y & s \end{bmatrix} \begin{bmatrix} V(0^+) \\ I(0^+) \end{bmatrix}$$



but $V(0^+) = V_R$
and $I(0^+) = I_R$

$$\begin{bmatrix} V_{(S)} \\ I_{(S)} \end{bmatrix} = \left(\frac{1}{s^2 - YZ} \right) \begin{bmatrix} s & Z \\ Y & s \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Solving the Diff Eq

Expanding $V_{(S)}$

$$V_{(S)} = \frac{sV_R + ZI_R}{s^2 - YZ} \quad but \quad YZ = \left(\sqrt{YZ}\right)^2 \quad and \quad s^2 - \left(\sqrt{YZ}\right)^2 = \left(s-\sqrt{YZ}\right)\left(s+\sqrt{YZ}\right)$$

$$V_{(S)} = \frac{sV_R + ZI_R}{\left(s-\sqrt{YZ}\right)\left(s+\sqrt{YZ}\right)} = \frac{A}{\left(s-\sqrt{YZ}\right)} + \frac{B}{\left(s+\sqrt{YZ}\right)}$$

$$sV_R + ZI_R = A\left(s+\sqrt{YZ}\right) + B\left(s-\sqrt{YZ}\right) = s(A+B) + (A-B)\sqrt{YZ}$$

$$V_R = A+B \qquad and \quad \frac{ZI_R}{\sqrt{YZ}} = A-B \quad or \quad \sqrt{\frac{Z}{Y}}I_R = A-B$$

$$V_R + \sqrt{\frac{Z}{Y}}I_R = 2A \qquad V_R - \sqrt{\frac{Z}{Y}}I_R = 2B$$

Solving the Diff Eq

$$V_R + \sqrt{\frac{Z}{Y}} I_R = 2A \qquad V_R - \sqrt{\frac{Z}{Y}} I_R = 2B$$

define characteristic impedance $Z_C = \sqrt{\frac{Z}{Y}}$ and propagation constant $\gamma = \sqrt{YZ} = \alpha + j\beta$

$$A = \frac{V_R}{2} + \frac{Z_C I_R}{2} \qquad B = \frac{V_R}{2} - \frac{Z_C I_R}{2} \qquad V_{(S)} = \frac{A}{\left(s - \sqrt{YZ}\right)} + \frac{B}{\left(s + \sqrt{YZ}\right)}$$

$$V_{(s)} = \frac{\frac{V_R}{2} + \frac{Z_C I_R}{2}}{s - \gamma} + \frac{\frac{V_R}{2} - \frac{Z_C I_R}{2}}{s + \gamma}$$

$$LaPlace^{-1}\left[\frac{K}{s + a}\right] = Ke^{-ax}$$

$$V_{(x)} = \left[\frac{V_R}{2} + \frac{Z_C I_R}{2}\right] \cdot e^{+\gamma x} + \left[\frac{V_R}{2} - \frac{Z_C I_R}{2}\right] \cdot e^{-\gamma x}$$

Solving the Diff Eq

$$V_{(x)} = \left[\frac{V_R}{2} + \frac{Z_C I_R}{2}\right]\cdot e^{+\gamma x} + \left[\frac{V_R}{2} - \frac{Z_C I_R}{2}\right]\cdot e^{-\gamma x}$$

when x $= l$ we are at the sending end of the line

$$V_s = \left[\frac{V_R}{2} + \frac{Z_C I_R}{2}\right]\cdot e^{+\gamma l} + \left[\frac{V_R}{2} - \frac{Z_C I_R}{2}\right]\cdot e^{-\gamma l}$$

$$V_s = \left[\frac{e^{+\gamma l} + e^{-\gamma l}}{2}\right] V_R + \left[\frac{e^{+\gamma l} - e^{-\gamma l}}{2}\right] Z_C I_R$$

Repeating the process and solving for I_S gives :

$$I_s = \left[\frac{e^{+\gamma l} - e^{-\gamma l}}{2}\right] \frac{V_R}{Z_C} + \left[\frac{e^{+\gamma l} + e^{-\gamma l}}{2}\right] I_R$$

Solving the Diff Eq

$$V_s = \left[\frac{e^{+\gamma l} + e^{-\gamma l}}{2}\right] V_R + \left[\frac{e^{+\gamma l} - e^{-\gamma l}}{2}\right] Z_C I_R$$

$$I_s = \left[\frac{e^{+\gamma l} - e^{-\gamma l}}{2}\right] \frac{V_R}{Z_C} + \left[\frac{e^{+\gamma l} + e^{-\gamma l}}{2}\right] I_R$$

This looks somewhat famaliar? Eulers Identity??

$$\cos(x) = \frac{e^{+jx} + e^{-jx}}{2} \qquad \sin(x) = \frac{e^{+jx} - e^{-jx}}{2j}$$

Not quite!

Solving the Diff Eq

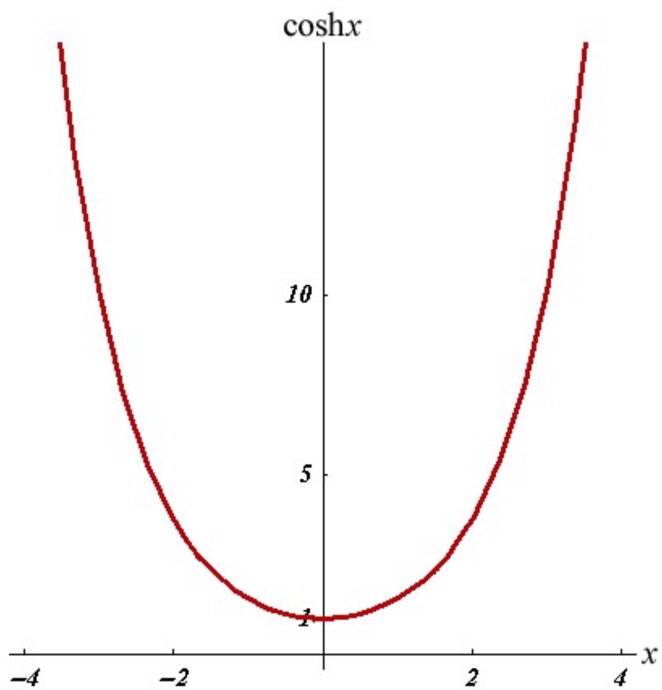
$$\left[\frac{e^{+\gamma l} + e^{-\gamma l}}{2}\right] = \cosh(\gamma l) \qquad \left[\frac{e^{+\gamma l} - e^{-\gamma l}}{2}\right] = \sinh(\gamma l)$$

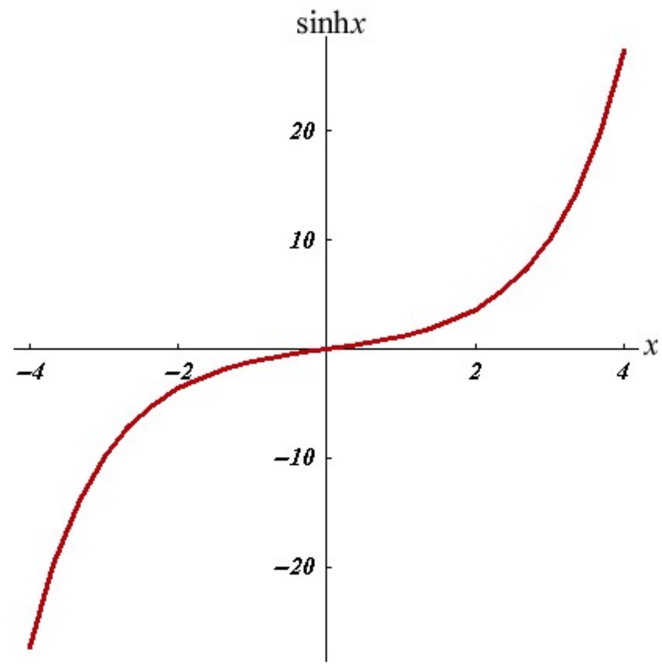
$$V_S = \cosh(\gamma l) \cdot V_R + Z_C \sinh(\gamma l) \cdot I_R$$

$$I_S = \frac{1}{Z_C}\sinh(\gamma l) \cdot V_R + \cosh(\gamma l) \cdot I_R$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

The Hyperbolic Functions





$\cosh(x) = (e^x+e^{-x})/2$ $\qquad$ $\sinh(x) = (e^x-e^{-x})/2$

$\frac{d}{dx}\cosh(x) = \sinh(x)$ $\qquad$ $\frac{d}{dx}\sinh(x) = \cosh(x)$ $\qquad$ and $\cosh^2(x) - \sinh^2(x) = 1$

Example

A transmission line is 100 football fields long:

The series impedance is 0.1Ω per football field (ohms per unit)

The shunt admittance is .001S per football field (ohms^{-1} per unit)

$$Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{0.1\Omega / \mathit{football\ \ field}}{0.001S / \mathit{football\ \ field}}} = 10\Omega$$

$$\gamma = \sqrt{ZY} = \sqrt{0.1 \cdot 0.001} = 0.01 / \mathit{football\ \ field}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

$$A = D = \cosh((0.01)(100)) = 1.543081$$

$$B = 10\sinh((0.01)(100)) = 11.752 \ \ \Omega$$

$$C = \frac{1}{10}\sinh((0.01)(100)) = .11752 \ \ S$$

Michigan Tech

Warning – Warnung - 警告 – Varoitus - Onyo

You will need to be able to calculate sinh(x), cosh(x) and tanh(x) as well as $\sinh^{-1}(x)$, $\cosh^{-1}(x)$ and $\tanh^{-1}(x)$

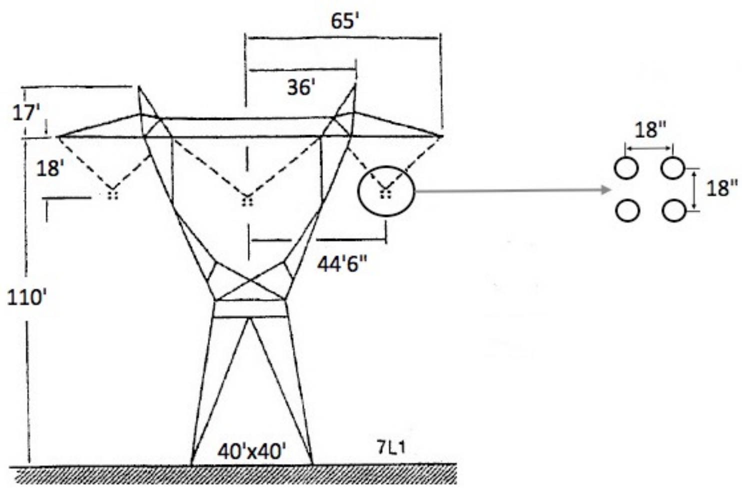
Where x is a complex number !!!!

$$\sinh(0.9\angle 80^\circ) = 0.7905\angle 82.79^\circ$$

$$\tanh(0.5\angle 75^\circ) = 0.5387\angle 72.35^\circ$$

$$\cosh^{-1}(0.8\angle 80^\circ) = 1.632\angle 63.56^\circ$$

Example



A 765 kV transmissionn line is 400km long. The “flat” configuration features 4 Canary conductors spaced at the corners of a square 18” per side. The bundles spacing is 44’6”.

Per Table A4:

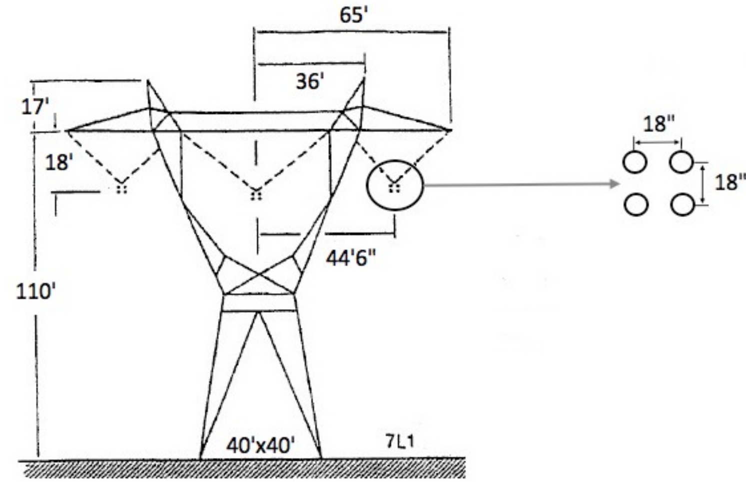
Outside Diameter = 1.162”

GMR = 0.0391’

$R_{50C\ 60Hz}$ = 0.1185 ohms/mile

Example

Per Table A4:
Outside Diameter = 1.162”
GMR = 0.0391’
R50C 60Hz = 0.1185 ohms/mile



$$D_{eq} = \sqrt[3]{44.5 \cdot 44.5 \cdot 89} = 56.066 \ \ feet$$

$$D_{SL} = \sqrt[4]{1.5 \cdot 1.5 \cdot 1.5\sqrt{2} \cdot 0.0391} = 0.65727 \ \ feet$$

$$D_{SC} = \sqrt[4]{1.5 \cdot 1.5 \cdot 1.5\sqrt{2} \cdot 1.162/24} = 0.69334 \ \ feet$$

$$R = 0.1184/4 \ \ \Omega/mile \cdot 1 mile/1.609 km = 0.0184 \ \ \Omega/km$$

$$X_L = j(2\pi \cdot 60) \cdot 2 \cdot 10^{-7} \ln\frac{56.066}{0.65727} = 3.35236 \cdot 10^{-4} \ \ \Omega/m \cdot 1000 m/km = 0.335236 \ \ \Omega/km$$

$$Y = \frac{j(2\pi \cdot 60) \cdot (2\pi) 8.854 \cdot 10^{-12}}{\ln\frac{56.066}{0.69339}} = 4.77432 \cdot 10^{-9} \ \ \Omega/m \cdot 1000 m/km = 4.77432 \cdot 10^{-6} \angle 90° \ \ \Omega/km$$

$$Z = 0.018395 + j0.335236 \ \ \Omega/km = 0.33574 \angle 86.859°$$

Example

$$\gamma = \sqrt{ZY} = \sqrt{(0.33574\angle 86.856°)(4.77441\cdot 10^{-6}\angle 90°)} = 0.0012661\angle 88.429°/km$$

$$\gamma l = 0.50643\angle 88.4289°$$

$$Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{(0.33574\angle 86.856°)}{(4.77441\cdot 10^{-6}\angle 90°)}} = 265.183\angle -1.571°$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8747\angle 0.4409° & 128.6374\angle 86.996° \\ 1.8293\cdot 10^{-3}\angle 90.137° & 0.8747\angle 0.4409° \end{bmatrix}$$

Example

Given we're delivering 1000MVA at rated voltage, pf = 0.8 lagging

Determine the sending end voltage and current.

$$V_{R\ (per\ \ phase)} = \frac{765000}{\sqrt{3}} = 441673\angle 0°$$

$$I_R = \left(\frac{1000 \cdot 10^6 \angle 36.87°}{\sqrt{3}\ \ 765000\angle 0°}\right)^* = 754.7\angle -36.87°$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 0.8747\angle 0.4409° & 128.6374\angle 86.996° \\ 1.8293 \cdot 10^{-3}\angle 90.137° & 0.8747\angle 0.4409° \end{bmatrix} \cdot \begin{bmatrix} 441673\angle 0° \\ 754.7\angle -36.87° \end{bmatrix} = \begin{bmatrix} 455202\angle 9.8° \\ 673.105\angle 38.17° \end{bmatrix}$$

$$S_{in} = 3 \cdot (455202\angle 9.8°)(673.105\angle -38.17°) = 919.196\angle -28.37°\ \ MVA$$

$$S_{in} = 808.798MW\ \ - j\ \ 436.768MVAR$$

Example

$$S_{out} = 1000\angle 36.87 \;\; MVA = 800MW + j600MVAR$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{800}{808.798} \cdot 100\% = 98.91\%$$

$|V_S| = 455202$ $\quad |V_R| = 441763$ $\quad V_S$ is about 3% larger than V_R

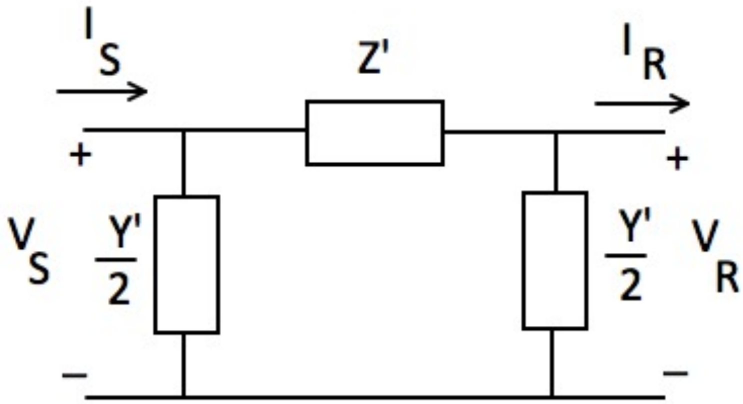
$$however$$

$$V_S = AV_R + BI_R \;\; under \;\; no-load \;\; conditions \;\; I_R = 0$$

$$|V_{R \;\; no-load}| = \left|\frac{V_S}{A}\right| = \left|\frac{455202}{0.8747}\right| = 520409.28$$

$$V_{REG} = \frac{V_{NL} - V_{FL}}{V_{FL}} \cdot 100\% = \frac{520409.28 - 441763}{441763} \cdot 100\% = 17.8\%$$

Lumped Parameter Network



$$V_S = V_R + \left[V_R \left(\frac{Y'}{2} \right) + I_R \right] Z'$$

$$V_S = \left(1 + \frac{Z'Y'}{2} \right) V_R + Z' I_R$$

$$V_S = AV_R + BI_R$$

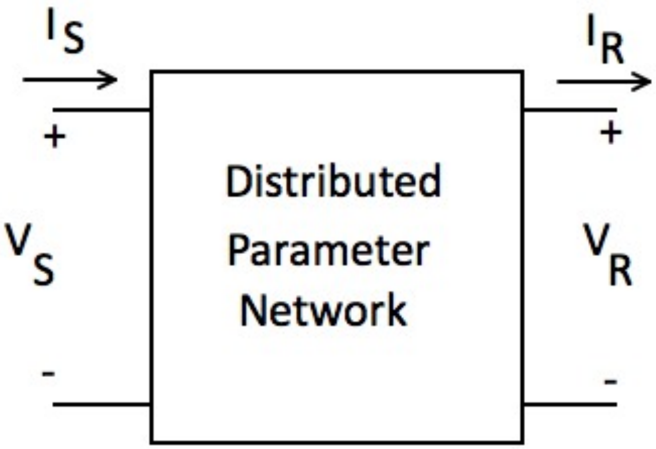
$$I_S = V_S \left(\frac{Y'}{2} \right) + V_R \left(\frac{Y'}{2} \right) + I_R$$

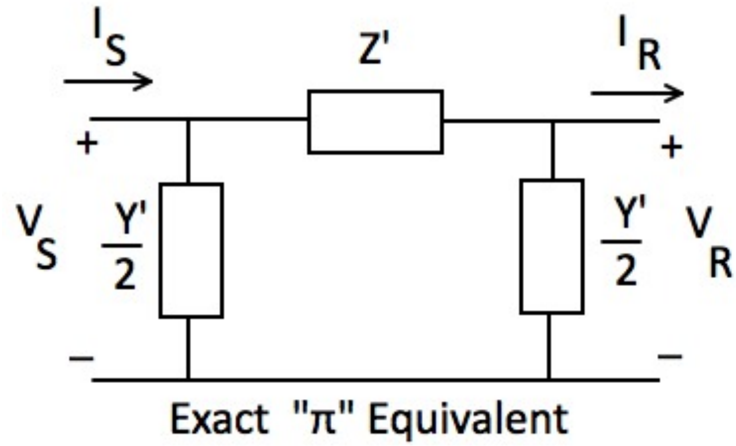
$$I_S = \left(\frac{Y'}{2} \right) \left(1 + \frac{Z'Y'}{2} \right) V_R + \left(\frac{Y'}{2} \right) Z' I_R + V_R \left(\frac{Y'}{2} \right) + I_R$$

$$I_S = \left(Y' + \frac{Z'Y'^2}{4} \right) V_R + \left(1 + \frac{Z'Y'}{2} \right) I_R$$

$$I_S = CV_R + DI_R$$

Lumped Parameter Network





$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{Z'Y'}{2}\right) & Z' \\ \left(Y' + \frac{Z'Y'^2}{4}\right) & \left(1 + \frac{Z'Y'}{2}\right) \end{bmatrix}$$

Lumped Parameter Network

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

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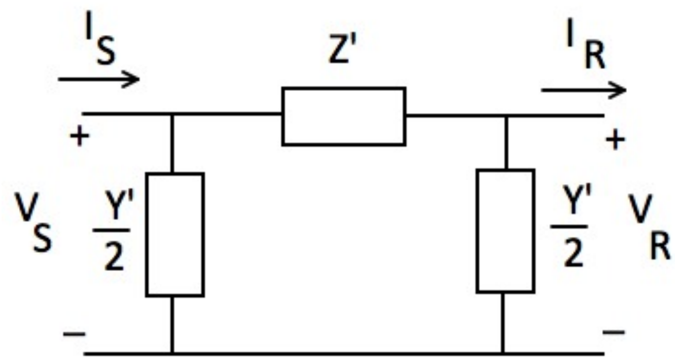
$$Z' = Z_C \cdot \sinh(\gamma l)$$

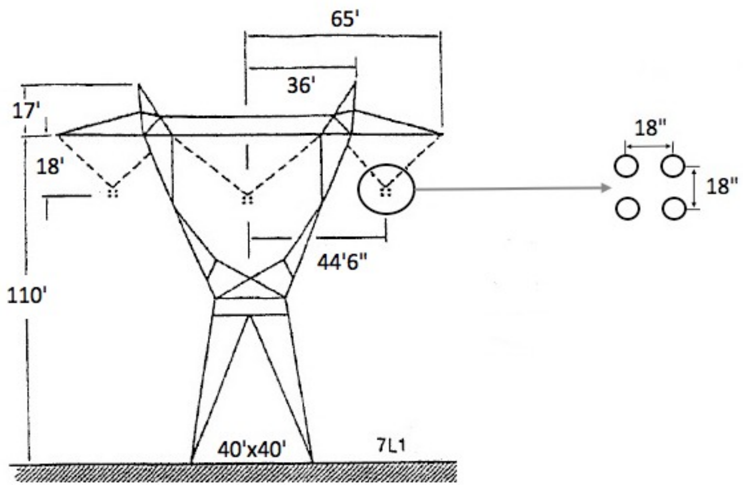
$$\left(1 + \frac{Z' Y'}{2}\right) = \cosh(\gamma l) \qquad \frac{Z' Y'}{2} = \cosh(\gamma l) - 1$$

$$\frac{Y'}{2} = \frac{\cosh(\gamma l) - 1}{Z_C \cdot \sinh(\gamma l)} = \frac{1}{Z_C}\tanh\left(\frac{\gamma l}{2}\right)$$



Exact "π" Equivalent

Returning to our Example



Per Table A4:
Outside Diameter = 1.162"   GMR = 0.0391'
$R_{50C\ 60Hz}$ = 0.1185 ohms/mile

$$Y = 4.77432 \cdot 10^{-6} \angle 90^\circ \ \ \Omega / km$$

$$Z = 0.018412 + j0.335235 \ \ \Omega / km = 0.33574 \angle 86.859^\circ$$

$$\gamma = \sqrt{ZY} = \sqrt{\left(0.33574 \angle 86.859^\circ\right)\left(4.77432 \cdot 10^{-6} \angle 90^\circ\right)} = 0.0012661 \angle 88.430^\circ / km$$

$$\gamma l = 0.50643 \angle 88.430^\circ \qquad Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{\left(0.33574 \angle 86.859^\circ\right)}{\left(4.77432 \cdot 10^{-6} \angle 90^\circ\right)}} = 265.183 \angle -1.571^\circ$$

$$Z' = Z_C \sinh(\gamma l) = 265.183 \angle -1.571^\circ \sinh\left(0.50644 \angle 88.428^\circ\right) = 128.637 \angle 86.995^\circ$$

$$\frac{Y'}{2} = \frac{1}{Z_C} \tanh\left(\frac{\gamma l}{2}\right) = \frac{1}{265.183 \angle -1.571^\circ} \tanh\left(\frac{0.50644 \angle 88.428^\circ}{2}\right) = 9.758 \cdot 10^{-4} \angle 89.931^\circ$$

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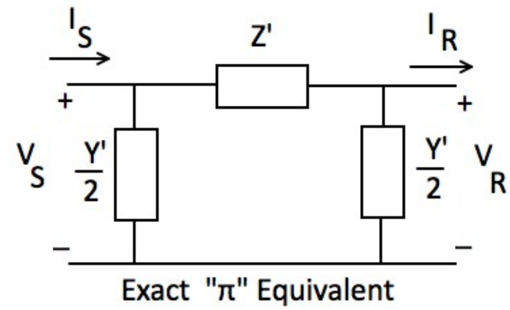
Lumped Parameter Solution

$$Z' = 128.6374\angle 86.995^\circ \ \ \Omega$$

$$\frac{Y'}{2} = 9.758\cdot 10^{-4}\angle 89.931^\circ \ \ S$$



$$V_{R\ (per\ \ phase)} = \frac{765000}{\sqrt{3}} = 441673\angle 0^\circ$$

$$I_R = \left(\frac{1000\cdot 10^6\angle 36.87^\circ}{\sqrt{3}\ \ 765000\angle 0^\circ}\right)^* = 754.71\angle -36.87^\circ A$$

$$I_{\frac{Y'}{2}(R)} = 441673\angle 0^\circ\cdot 9.758\cdot 10^{-4}\angle 89.931^\circ \ \ S = 430.985\angle 89.931^\circ A$$

$$I_{Z'} = I_R + I_{\frac{Y'}{2}(R)} = 754.7\angle -36.87^\circ + 430.985\angle 89.931^\circ = 604.673\angle -2.070 A$$

$$V_{Z'} = I_{Z'}\cdot Z' = 604.673\angle -2.070\cdot 128.6374\angle 86.995^\circ \ \ \Omega = 77783.52\angle 84.925^\circ V$$

$$V_S = V_{Z'} + V_R = 77783.52\angle 84.925^\circ + 441673\angle 0^\circ = 455195.38\angle 9.80^\circ V$$

$$I_S = V_S\cdot\frac{Y'}{2} + I_{Z'} = 455195.38\angle 9.80^\circ\cdot 9.758\cdot 10^{-4}\angle 89.931^\circ + 604.673\angle -2.070$$

$$= 673.102\angle 38.167^\circ A$$

The Nominal Pi Equivalent Circuit

$$Z' = Z_C \sinh(\gamma l) = \sqrt{\frac{Z}{Y}} \sinh\left(\sqrt{ZY} \cdot l\right)$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ .....$$

$$Z' = \sqrt{\frac{Z}{Y}} \left[\left(\sqrt{ZY} \cdot l\right) + \frac{\left(\sqrt{ZY} \cdot l\right)^3}{3!} + \frac{\left(\sqrt{ZY} \cdot l\right)^5}{5!} + \ ..... \right]$$

$$Z' \approx \sqrt{\frac{Z}{Y}} \left(\sqrt{ZY} \cdot l\right) \approx Zl$$

$$Z_{\text{nominal } \pi} = Zl$$

The Nominal Pi Equivalent Circuit

$$\frac{Y'}{2} = \frac{\cosh(\gamma l) - 1}{Z_C \sinh(\gamma l)}$$

$$\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \;.....$$

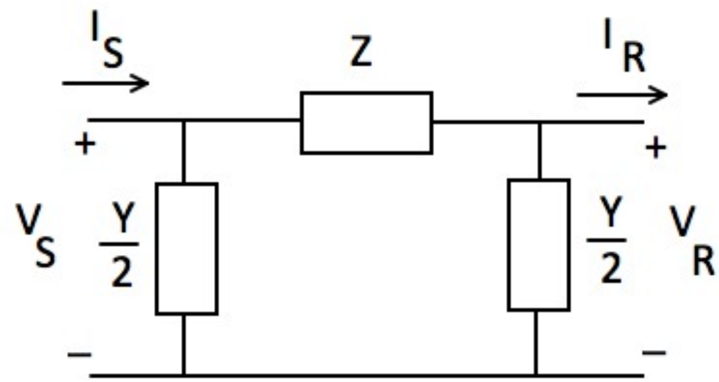
$$\frac{Y'}{2} = \frac{\left[1 + \frac{\left(\sqrt{ZY}\cdot l\right)^2}{2!} + \frac{\left(\sqrt{ZY}\cdot l\right)^4}{4!} + \;.....\right] - 1}{\sqrt{\frac{Z}{Y}}\left[\left(\sqrt{ZY}\cdot l\right) + \frac{\left(\sqrt{ZY}\cdot l\right)^3}{3!} + \frac{\left(\sqrt{ZY}\cdot l\right)^5}{5!} + \;.....\right]}$$

$$\frac{Y'}{2} \approx \frac{\frac{\left(\sqrt{ZY}\cdot l\right)^2}{2}}{\sqrt{\frac{Z}{Y}}\left(\sqrt{ZY}\cdot l\right)} \approx \frac{\sqrt{ZY}\cdot l}{2\cdot\sqrt{\frac{Z}{Y}}} \approx \frac{Y\cdot l}{2} = \frac{Y}{2}$$

$$\frac{Y}{2}_{\text{nominal }\pi} = \frac{Y\cdot l}{2}$$

The Nominal Pi EquivalentCircuit

$$Z_{\text{nominal } \pi} = Zl \qquad \frac{Y}{2}_{\text{nominal } \pi} = \frac{Y \cdot l}{2}$$



"Nominal π" Equivalent

$$Zl = 0.33574\angle 86.995° \cdot 400 = 134.296\angle 86.995° = Z_{\text{nominal } \pi}$$

$$\frac{Y \cdot l}{2} = \frac{4.77432 \cdot 10^{-6}\angle 90° \cdot 400}{2} = 9.5486 \cdot 10^{-4}\angle 90° = \frac{Y}{2}_{\text{nominal } \pi}$$

Recall the exact parameters:

$$Z' = Z_C \sinh(\gamma l) = 265.183\angle -1.571° \sinh(0.50644\angle 88.428°) = 128.637\angle 86.995°$$

$$\frac{Y'}{2} = \frac{1}{Z_C}\tanh\left(\frac{\gamma l}{2}\right) = \frac{1}{265.183\angle -1.571°}\tanh\left(\frac{0.50644\angle 88.428°}{2}\right) = 9.758 \cdot 10^{-4}\angle 89.931°$$

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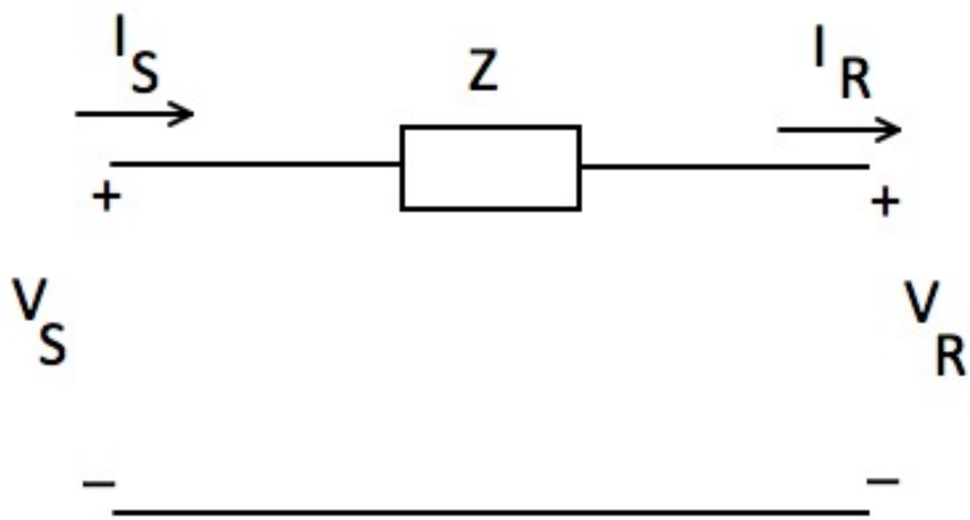
Short Line Approximation

Since the shunt admittance is normally quite small, as the line length gets small
$Yl/2$ becomes negligible and $Y/2 \Rightarrow 0$. If the admittance approaches zero, the impedance approaches infinity. The short line equivalent circuit is shown below.



Short Line Equivalent

Three Lumped Parameter Line Models

Exact π

$$Z' = Z_C \sinh(\gamma l)$$

$$\frac{Y'}{2} = \frac{1}{Z_C}\tanh\left(\frac{\gamma l}{2}\right)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

Nominal π

$$Z = Zl$$

$$\frac{Y}{2} = \frac{Yl}{2}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\begin{bmatrix} \left(1 + \frac{ZY}{2}\right) & Z \\ \left(Y + \frac{ZY^2}{4}\right) & \left(1 + \frac{ZY}{2}\right) \end{bmatrix}$$

Short Line

$$Z = Zl$$

$$\frac{Y}{2} = 0$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

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How do I know what to use??

Line Length	Appropriate Model
80km or less	Short line
80km – 250km	Nominal π
greater than 250 km	Exact π

Table 5.1 pg 258

Or – whatever I tell you to use !!!

Comparison Calculations

400 km long line

	Exact	Nominal	Short
A=D	$0.8747\angle.4407$	$0.8720\angle.4615$	1.0000
Z' (Z)=B	$128.6\angle 87.00$	$134.3\angle 86.86$	$134.3\angle 86.86$
C	$1.829\times10^{-3}\angle 90.14$	$1.665\times10^{-3}\angle 90.46$	0.0000
Y'/2 (Y/2)	$9.757\times10^{-4}\angle 89.93$	$9.548\times10^{-4}\angle 90.00$	0.0000
V_S	455.2 kV $\angle 9.80$	457.5 kV $\angle 10.2$	512.8 kV $\angle 8.7$
I_S	$673.0\angle 38.16$	$627.0\angle 33.36$	$754.7\angle -36.87$

100 km long line

	Exact	Nominal	Short
A=D	$0.9920\angle.0253$	$0.9920\angle.0254$	1.0000
Z' (Z)=B	$33.49\angle 86.87$	$33.58\angle 86.86$	$33.58\angle 86.86$
C	$4.761\text{x}10\text{-}4\angle 90.01$	$4.736\times10^{-4}\angle 90.03$	0.0000
Y'/2 (Y/2)	$2.390\text{x}10\text{-}4\angle 90.00$	$2.387\times10^{-4}\angle 90.00$	0.0000
V_S	454.8 kV $\angle 2.464$	454.9 kV $\angle 2.470$	458.4 kV $\angle 2.427$
I_S	$644.9\angle -21.72$	$645.2\angle -21.81$	$754.7\angle -36.87$

		25 km long line	
	Exact	Nominal	Short
A=D	$0.9995\angle.00157$	$1.0000\angle.0034$	1.0000
Z' (Z)=B	$8.392\angle 86.86$	$8.394\angle 86.86$	$8.394\angle 86.86$
C	$1.193\text{x}10\text{-}4\angle 90.00$	$1.194\times10^{-4}\angle 90.00$	0.0000
Y'/2 (Y/2)	$5.968\text{x}10\text{-}5\angle 90.00$	$5.968\times10^{-5}\angle 90.00$	0.0000
V_S	445.6 kV $\angle.625$	445.7 kV $\angle.627$	445.8 kV $\angle.624$
I_S	$723.9\angle -33.5$	$724.3\angle -33.5$	$754.7\angle -36.87$

765 kV line:

$z = .33575\angle 86.86\ \Omega/\text{km}$

$y = 4.7740\times10^{-6}\angle 90.00$ S/km

$Z_C = 265.2\angle -1.572\ \Omega$

$\gamma = 1.266\times10^{-3}\angle 88.43$ /km

$V_R = 441{,}673$ V/phase

$I_R = 754.7\angle -36.87$ A

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Lossless Lines

$$Z = 0.18396 + j0.33524 \quad \Omega / km$$

$$Y = j4.77432 \cdot 10^{-6} \quad S / km$$

$$if \quad we \quad neglect \quad R$$

$$Z = j0.33524 \quad \Omega / km$$

$$Z_C = \sqrt{\frac{Z}{Y}} = 264.986\Omega \quad \text{"Surge Impedance" of a lossless line (purely real)}$$

$$\gamma = \sqrt{ZY} = j0.0012649 \ / km = \alpha + j\beta \qquad (\alpha = 0 \text{ purely imaginary})$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \cosh(j\beta) = \frac{e^{j\beta} + e^{-j\beta}}{2} = \cos(\beta)$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \sinh(j\beta) = \frac{e^{j\beta} - e^{-j\beta}}{2} = j\sin(\beta)$$

Lossless Lines

$$\begin{bmatrix} A_{(x)} & B_{(x)} \\ C_{(x)} & D_{(x)} \end{bmatrix} = \begin{bmatrix} \cos(0.0012649 \cdot x) & j264.986 \sin(0.0012649 \cdot x) \\ j\dfrac{1}{264.986} \sin(0.0012649 \cdot x) & \cos(0.0012649 \cdot x) \end{bmatrix}$$

$$but \quad 1 \quad radian \quad = \quad 57.296°$$

$$\begin{bmatrix} A_{(x)} & B_{(x)} \\ C_{(x)} & D_{(x)} \end{bmatrix} = \begin{bmatrix} \cos(0.072473 \cdot x°) & j264.986 \sin(0.072473 \cdot x°) \\ j\dfrac{1}{264.986} \sin(0.072473 \cdot x°) & \cos(0.072473 \cdot x°) \end{bmatrix}$$

$$converting \quad to \quad p.u. \quad V_{base} \quad = \quad 765kV \quad and \quad S_{base} \quad = \quad 1000MVA$$

$$Z_{base} \quad = \quad \frac{\left(765 \cdot 10^3\right)^2}{1000 \cdot 10^6} \quad = \quad 585.225$$

$$\begin{bmatrix} A_{(x)} & B_{(x)} \\ C_{(x)} & D_{(x)} \end{bmatrix} = \begin{bmatrix} \cos(0.072473 \cdot x°) & j0.45279 \sin(0.072473 \cdot x°) \\ j2.20851 \ \sin(0.072473 \cdot x°) & \cos(0.072473 \cdot x°) \end{bmatrix}$$

Lossless Line

$$V_{(x)} = A_{(x)} \cdot V_R + B_{(x)} \cdot I_R$$

$$V_{(x)} = \cos\left(0.072473 \cdot x°\right) \cdot V_R + j264.986 \sin\left(0.072473 \cdot x°\right) \cdot I_R$$

if we terminate the line with a surge impedance load

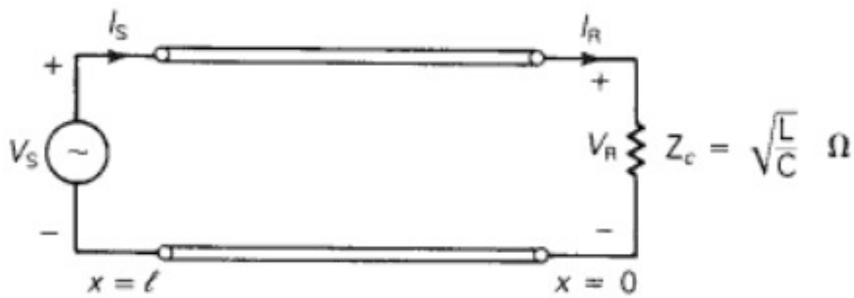
$$\mathrm{I_R} = \frac{V_R}{Z_C}$$

$$V_{(x)} = \cos\left(0.072473 \cdot x°\right) \cdot V_R + j \sin\left(0.072473 \cdot x°\right) \cdot V_R$$

$$V_{(x)} = \left[\cos\left(0.072473 \cdot x°\right) + j \sin\left(0.072473 \cdot x°\right)\right] \cdot V_R$$

$$V_{(x)} = \left(1\angle 0.072473 \cdot x°\right) \cdot V_R$$

Lossless Line



$$V_{(x)} = \cos(0.072473 \cdot x°) \cdot V_R + j\ Z_C \sin(0.072473 \cdot x°) \cdot I_R$$

$$I_{(x)} = j \frac{1}{Z_C} \sin(0.072473 \cdot x°) \cdot V_R + \cos(0.072473 \cdot x°) \cdot I_R$$

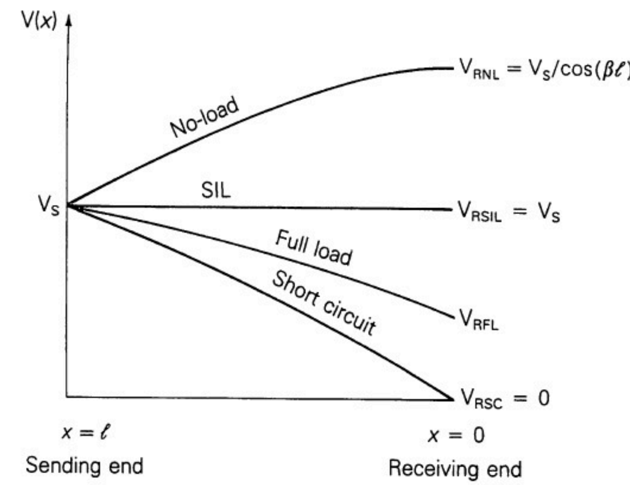
under surge impedance loading, $I_R = \dfrac{V_R}{Z_C}$

$$V_{(x)} = \cos(0.072473 \cdot x°) \cdot V_R + j\ \sin(0.072473 \cdot x°) \cdot V_R$$

$$V_{(x)} = (1\angle 0.072473 \cdot x°) V_R$$

$$I_{(x)} = j \frac{1}{Z_C} \sin(0.072473 \cdot x°) \cdot V_R + \cos(0.072473 \cdot x°) \cdot \frac{V_R}{Z_C}$$

$$I_{(x)} = (1\angle 0.072473 \cdot x°) V_R$$



$$S_{(x)} = V_{(x)} \cdot (I_{(x)})^* = \frac{|V_R|^2}{Z_C}$$

The real power flow is constant along the length of the line.
The reactive power flow is zero.

Wavelength

$$V_{(x)} = \left(1\angle 0.072473 \cdot x^\circ\right) \cdot V_R$$

$$V_{(x)} = \left(1\angle 0.072473 \cdot (400)^\circ\right) \cdot V_R$$

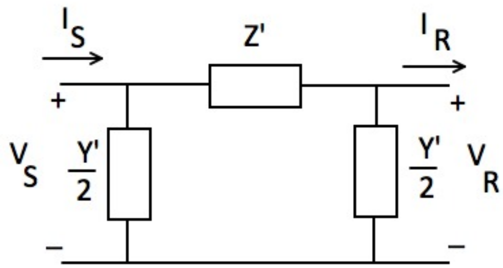
$$V_{(x)} = \left(1\angle 28.99^\circ\right) \cdot V_R$$

Wavelength is defined as the distance required to change the phase of the voltage or current by 360°.

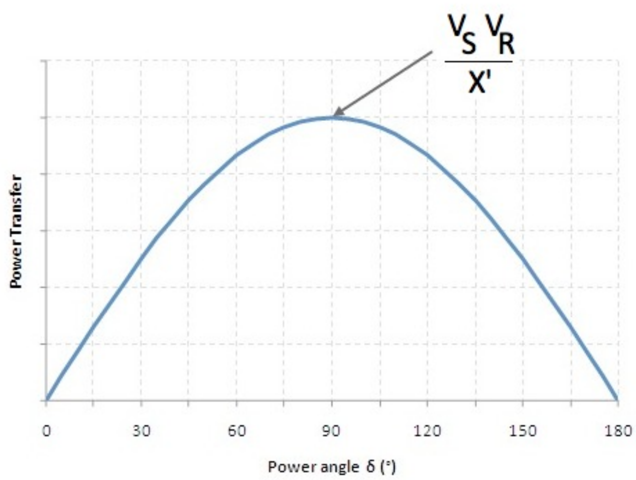
$$0.072473 \cdot \lambda^\circ = 360^\circ \qquad \lambda = 4{,}968 km = 3088 \ miles$$

Typical transmission lines are only a small fraction of a wavelength.

Steady State Stability Limit



Exact "π" Equivalent



$$I_R = \frac{V_S - V_R}{Z'} - \frac{Y'}{2} \cdot V_R$$

$$S_R = V_R \cdot I_R^{\,*} = V_R \cdot \left(\frac{V_S - V_R}{Z'} - \frac{Y'}{2} \cdot V_R \right)^*$$

$$V_R = V_R \angle 0^\circ \quad V_S = V_S \angle \delta^\circ \quad Z' = jX' \;\; and \;\; Y' = j\omega C' l$$

$$S_R = V_R \left(\frac{V_S \angle \delta - V_R}{jX'} \right)^* + \frac{j\omega C' l}{2} V_R^{\,2}$$

$$S_R = V_R \left(\frac{V_S \angle -\delta - V_R}{-jX'} \right) + \frac{j\omega C' l}{2} V_R^{\,2}$$

$$S_R = \frac{V_R V_S \angle 90 - \delta}{X'} - j \frac{V_R^{\,2}}{X'} + \frac{j\omega C' l}{2} V_R^{\,2}$$

$$S_R = P_R + jQ_R$$

$$P_R = \frac{|V_R| \cdot |V_S|}{|X'|} \cos(90 - \delta^\circ) = \frac{|V_R| \cdot |V_S|}{|X'|} \sin \delta$$

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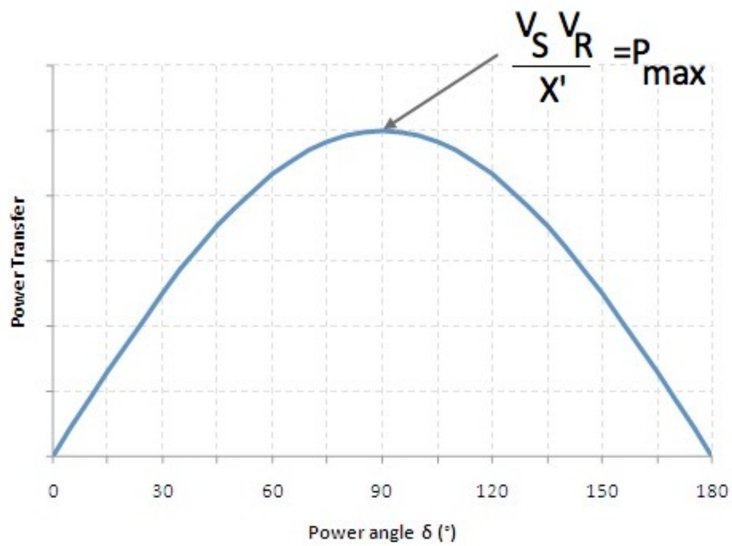
Steady State Stability Limit

$$P_R = \frac{|V_R| \cdot |V_S|}{|X'|} \sin\delta$$



When δ = 90°, sinδ = 1.0 and $P_R = P_{R\,MAX}$

$P_{R\,MAX}$ = the "theoretical steady state stability limit" of a lossless line.

If one were to try and exceed this limit the generators at the sending end of the line would lose synchronization with the machines on the receiving end of the line and that would be really really bad!

Exceeding The Stability Limit

Exceeding the Stability Limit

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Exceeding the Stability Limit

Steady State Stability Limit

We could do the same analysis using the ABCD parameters:

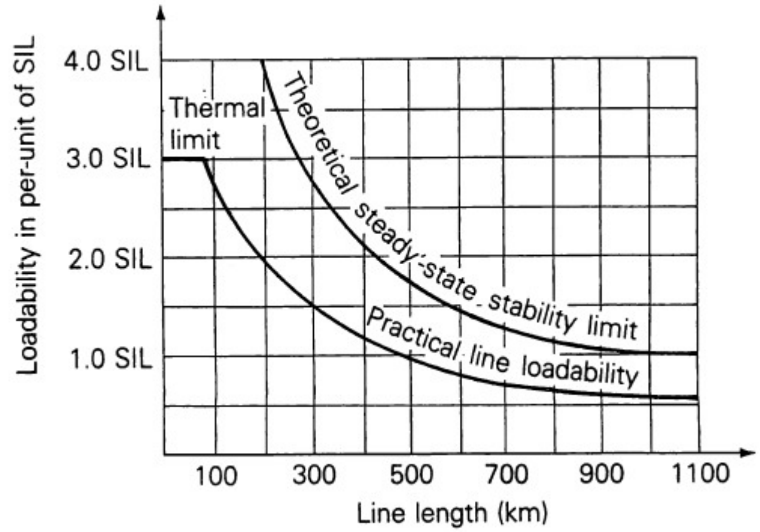
$$V_S = AV_R + BI_R \qquad I_R = \frac{V_S - AV_R}{B}$$

$$S_R = V_R \cdot (I_R)^*$$

$$P_R = \frac{|V_R| \cdot |V_S|}{B} \sin\delta = \frac{|V_R| \cdot |V_S|}{Z_C \sin\beta l} \sin\delta$$

$$P_R = \frac{|V_R|}{V_{Rated}} \cdot \frac{|V_S|}{V_{Rated}} \cdot \frac{(V_{Rated})^2}{Z_C} \cdot \frac{\sin\delta}{\sin\beta l}$$

$$P_R = V_{R(pu)} \cdot V_{S(pu)} \cdot SIL \cdot \frac{\sin\delta}{\sin\beta l}$$



The result shows the receiving end power is proportional to the square of the voltage i.e., a doubling of voltage produces a fourfold increase in maximum power.

The result also shows the receiving end power is inversely proportional to line length.

Line Loadability

Voltage kV	Zc(range) ohms	Zc(used) ohms	S.I.L (MW)	400 km line		200 km line	
				P(max) (MW)	P(limit) (MW)	P(max) (MW)	P(limit) (MW)
230	365-395	380	139	289	166	559	321
345	280-366	320	372	772	443	1496	858
500	233-294	260	961	1995	1144	3864	2216
765	254-266	260	2,250	4671	2679	9047	5190

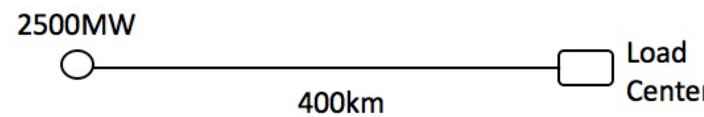
5,000 km wavelength used in calculations

$$P_{R\ MAX} = \frac{V_R V_S}{Z_C \sin \beta l} = \frac{V_R V_S}{Z_C \sin \frac{2\pi l}{\lambda}}$$

$$P_{R\ MAX} = \frac{V_R V_S}{Z_C \sin\left(\frac{360°}{5000km}\right)400km}$$

$$P_{R\ MAX} = \frac{\left(230 \cdot 10^3\right)^2}{380(0.4818)} = 289MW$$

$$P_{R\ LIMIT} = P_{R\ MAX} \sin 35° = 166MW$$



Volts	Lines Required
230kV	16 (15.06)
345kV	6 (5.64)
500kV	3 (2.18)
765kV	1

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Receiving End Power

$$V_S = AV_R + BI_R \qquad I_R = \frac{V_S - AV_R}{B}$$

$$I_S = CV_R + DI_R$$

$$S_R = V_R \cdot (I_R)^*$$

$$V_R = |V_R|\angle 0° \qquad V_S = |V_S|\angle \delta \qquad B = |B|\angle \beta \qquad A = |A|\angle \alpha$$

$$S_R = \frac{|V_R|\cdot|V_S|}{|B|}\angle \beta - \delta - \frac{|V_R|^2|A|}{|B|}\angle \beta - \alpha$$

$$S_R = \frac{|V_R|\cdot|V_S|}{|B|}\angle \beta - \delta + \frac{|V_R|^2|A|}{|B|}\angle \beta - \alpha - 180$$

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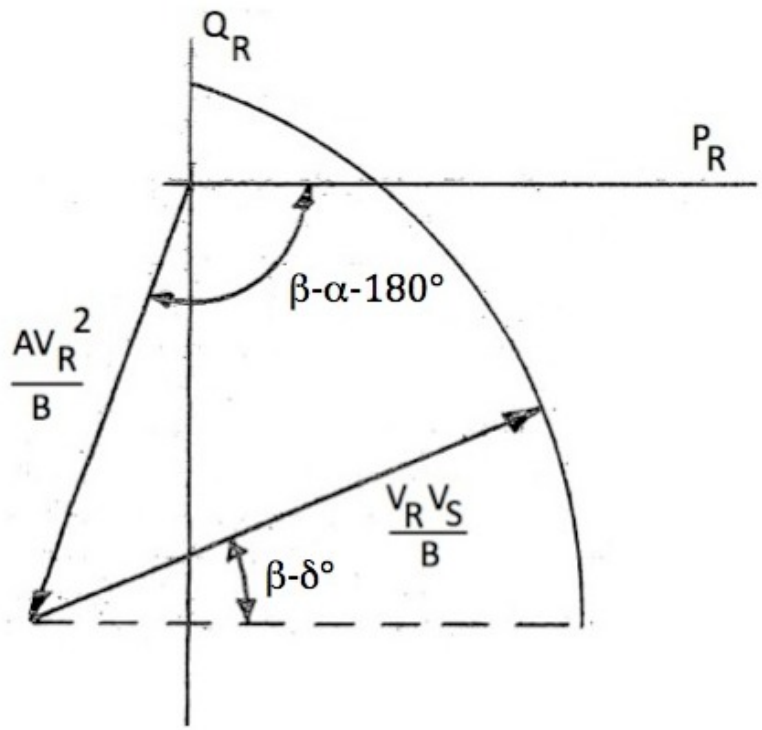
Receiving End Circle Diagram

$$S_R = \frac{|V_R| \cdot |V_S|}{|B|} \angle \beta - \delta + \frac{|V_R|^2 |A|}{|B|} \angle \beta - \alpha - 180$$

$$S_R = P_R + jQ_R$$

$$S_R = \underbrace{\frac{|V_R|^2 |A|}{|B|} \angle \beta - \alpha - 180}_{\text{fixed}} + \underbrace{\frac{|V_R| \cdot |V_S|}{|B|} \angle \beta - \delta}_{\text{varies with } \delta}$$



Example

400km 765kV line B = 128.6∠86.99° A = 0.8747∠0.4412°

Sbase = 1000MVA Vbase = 765kV Zbase = 585.2Ω

$B_{p.u.}$ = 0.219745∠86.99° $A_{p.u.}$ = 0.8747∠0.4412°

The magnitude of $V_R = V_S = 1.0_{p.u.}$

$$S_R = \frac{|V_R|^2|A|}{|B|}\angle\beta-\alpha-180 + \frac{|V_R|\cdot|V_S|}{|B|}\angle\beta-\delta$$

$$S_R = \frac{1^2\cdot 0.8747}{0.219745}\angle 86.99-0.4412-180 + \frac{1\cdot 1}{0.219745}\angle 86.99-\delta$$

$$S_R = 3.980531\angle-93.46 + 4.550739\angle 86.99-\delta$$

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Receiving End Circle Diagram

$$S_R = 3.980531\angle -93.46 + 4.550739\angle 86.99 - \delta$$

$$P_R = 3.98\cos(-93.46) + 4.55\cos(86.99 - \delta)$$

$$P_R = -0.24 + 4.55\cos(86.99 - \delta)$$

$$Q_R = 3.98\sin(-93.46) + 4.55\sin(86.99 - \delta)$$

$$Q_R = -3.97 + 4.55\sin(86.99 - \delta)$$

When will $P_R = P_{R\ MAX}$???

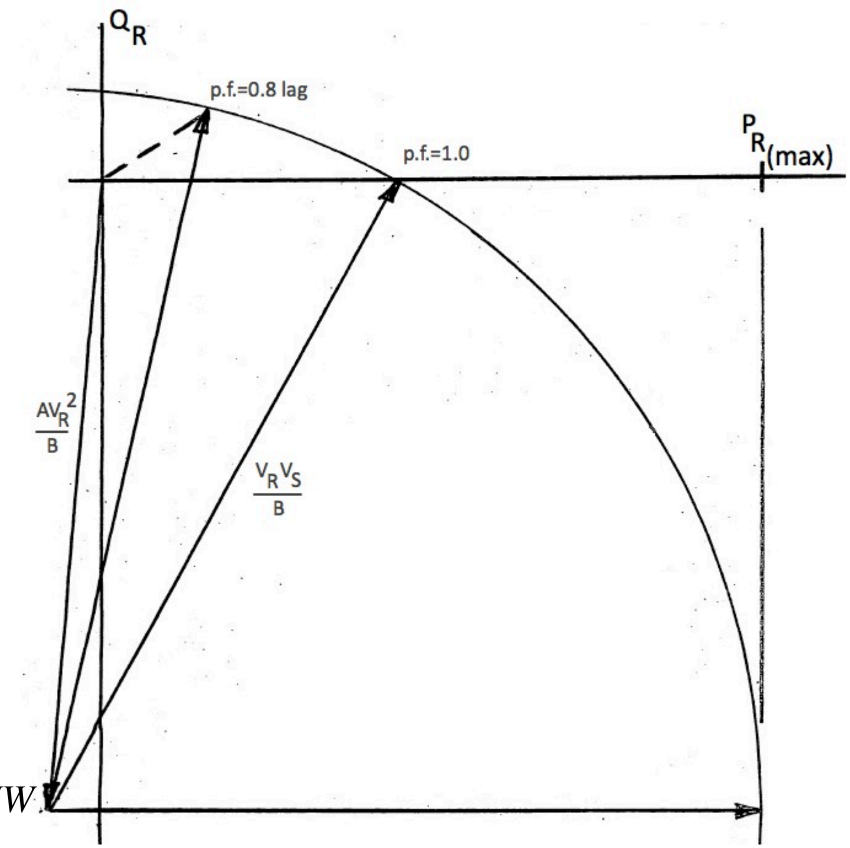
δ = 86.99 and cos(0°) = 1

$$P_{R\ MAX} = -0.24 + 4.55\cos(0°) = 4.31\,p.u. = 4310.5MW$$

$$Q_R = -3.97 + 4.55\sin(0°) = -3.97\,p.u. = 3972.6MVAR$$

$$p.f. = P/S = 0.736 \quad leading$$

Recall from the lossless calculations: $P_{R\ MAX} = 4558MW$

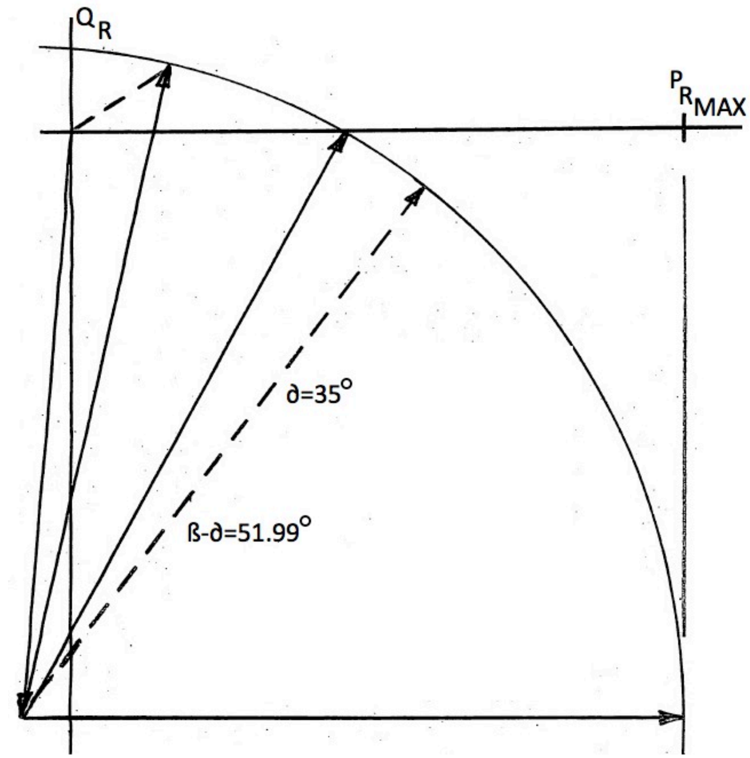


Receiving End Circle Diagram

$P_R = P_{R\,MAX}$ when $\delta = 86.99°$ and $\cos(0°) = 1$

This is too close to the limit of $\delta = 90°$

What is the receiving end power when $\delta = 35°$?



$$S_R = 3.980531\angle -93.46 + 4.550739\angle 86.99 - \delta$$

$$P_R = 3.98\cos(-93.46) + 4.55\cos(86.99 - 35)$$

$$P_R = -0.24 + 2.80 = 2.562\,p.u. = 2562MW$$

$$P_{R\ lossless} = 2598MW$$

$$Q_R = 3.98\sin(-93.46) + 4.55\sin(86.99 - 35)$$

$$Q_R = -3.97 + 3.58 = -0.3877\,p.u. = 387.7MVAR$$

Receiving End Circle Diagram

How much power can we deliver at unity power factor?

When p.f. = 1.0 Q_R = 0VARs

$$S_R = 3.980531\angle -93.46 + 4.550739\angle 86.99-\delta$$

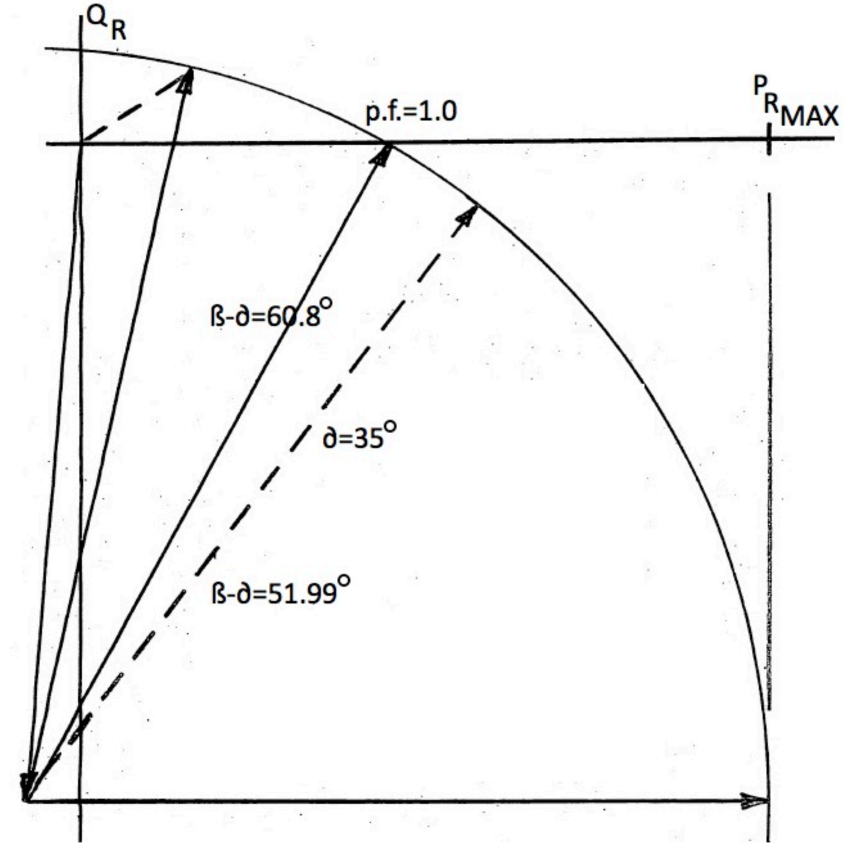
$$Q_R = 3.98\sin(-93.46) + 4.55\sin(86.99-\delta) = 0$$

$$(86.99-\delta) = \sin^{-1}\left(\frac{-3.98\sin(-93.46)}{4.55}\right) = 60.8^\circ$$

$$\delta = 26.2^\circ$$

$$P_R = 3.98\cos(-93.46) + 4.55\cos(86.99-26.2)$$

$$P_R = -0.24 + 2.22 = 1.983 p.u. = 1983 MW$$



Receiving End Circle Diagram

What is the receiving end power at p.f. = 0.8 lagging?

$$p.f. = \frac{P}{S} \quad S = \frac{P}{p.f.} = \frac{P}{.8}$$

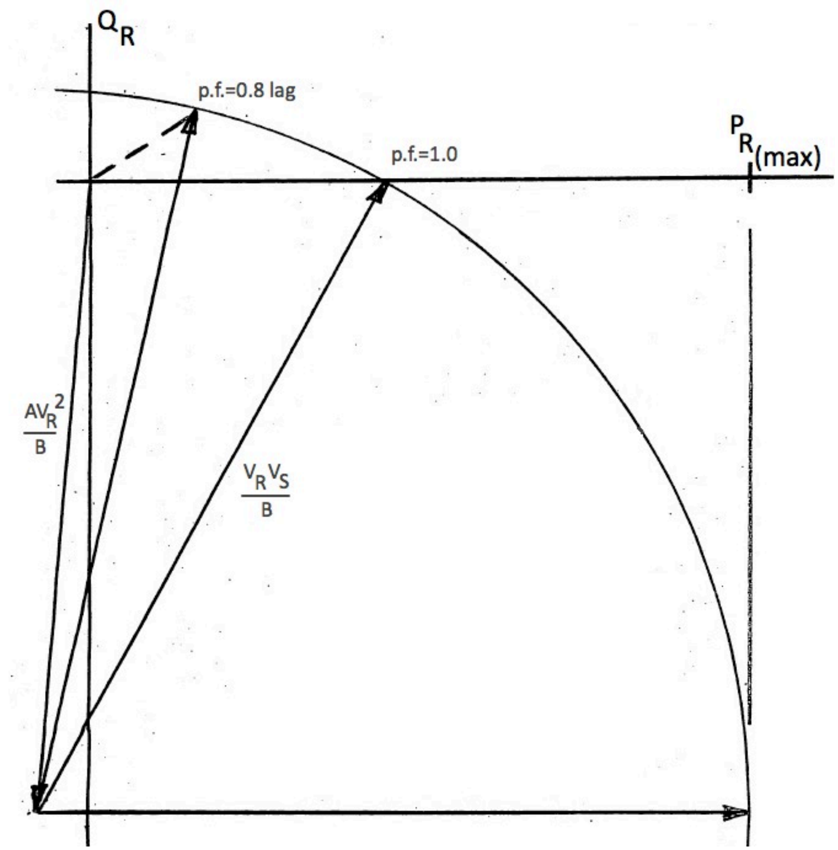
$$Q = \sqrt{S^2 - P^2} = \sqrt{\frac{P^2}{.64} - P^2} = 0.75P$$

$$Q_R = 0.75P_R = 3.98\sin(-93.46) + 4.55\sin(86.99 - \delta)$$

$$P_R = 3.98\cos(-93.46) + 4.55\cos(86.99 - \delta)$$

$$cSolve(0.75X = 3.98\sin(-93.46) + 4.55\sin(86.99 - Y)\,and$$
$$X = 3.98\cos(-93.46) + 4.55\cos(86.99 - Y),\{X, Y\})$$

$x = -4.77 \quad y = -4768.1 \quad or \quad x = -4.77 \quad y = -88.06 \quad or$
$x = -4.77 \quad y = 271.94 \quad or \quad x = -4.77 \quad y = 5671.94 \quad or$
$x = 0.652 \quad y = -2871.7 \quad or \quad x = 0.652 \quad y = 8.297 \quad or$
$x = 0.652 \quad y = 368.30$



$$P_R = 0.652\,p.u. = 652MW$$

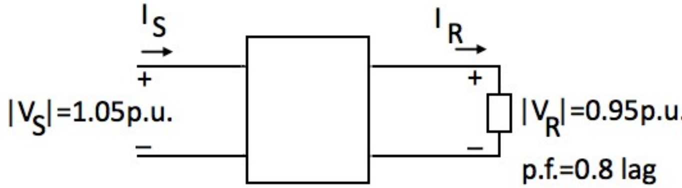
$$Q_R = 0.75P_R = 0.489\,p.u. = 489VAR$$

$$Q_R = 3.98\sin(-93.46) + 4.55\sin(86.99 - 8.297) = 0.489 \quad check!$$

Receiving End Power - ABCD Parameters

	Exact	Per Unit
A = D	$0.8747\angle 0.4412°$	$0.8747\angle 0.4412°$
Z' = B	$128.6\angle 86.99°$	$0.219745\angle 86.99°$
C	$1.829\text{x}10^{-3}\angle 90.14°$	$1.0704\angle 90.14°$
Y'/2	$9.757\text{x}10^{-4}\angle 89.93°$	$0.571004\angle 89.93°$
Vbase = 765kV	Sbase = 1000MVA	Zbase = 585.225
	Length = 400km	

765 kV line:
$z = .33575\angle 86.86\ \Omega/\text{km}$
$y = 4.7740\text{x}10^{-6}\angle 90.00\ \text{S/km}$
$Z_C = 265.2\angle -1.572\ \Omega$
$\gamma = 1.266\text{x}10^{-3}\angle 88.43\ /\text{km}$



$$S_R = \frac{|V_R|^2 |A|}{|B|}\angle \beta - \alpha - 180 + \frac{|V_R|\cdot|V_S|}{|B|}\angle \beta - \delta$$

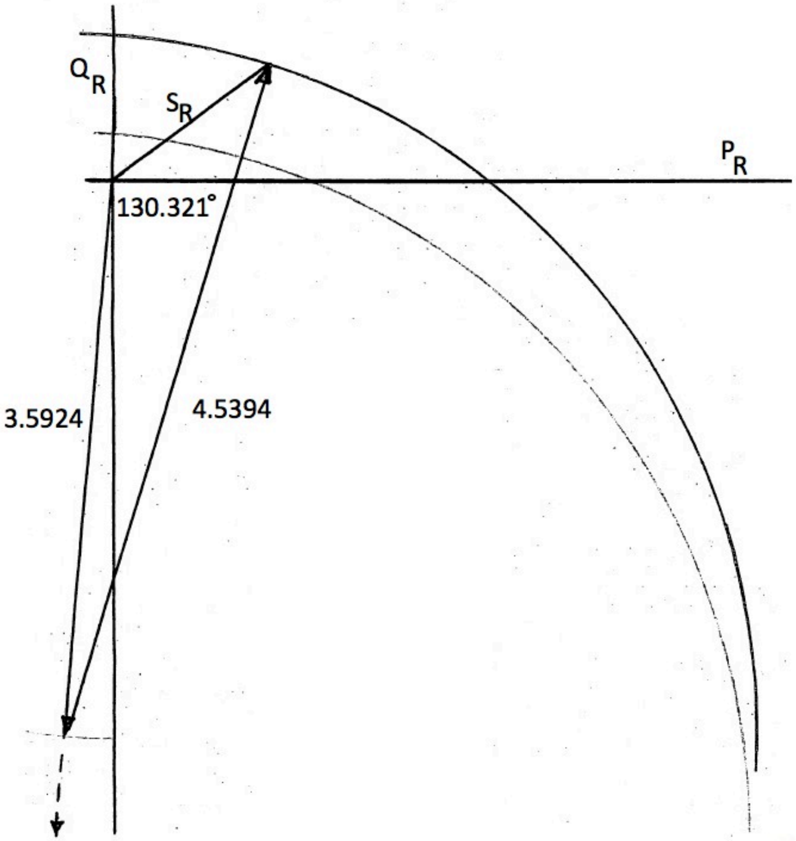
$$S_R = \frac{0.95^2\cdot 0.8747}{0.219745}\angle 86.99 - 0.4412 - 180 + \frac{0.95\cdot 1.05}{0.219745}\angle 86.99 - \delta$$

$$S_R = 3.59242\angle -93.451 + 4.53935\angle 86.99 - \delta$$

$$V_S\angle\delta = 0.8747\angle 0.4412°\cdot 0.95\angle 0° + 0.219745\angle 86.99°\cdot I_R\angle -36.87°$$

$$V_S\angle\delta = 0.830965\angle 0.4412° + 0.219745|I_R|\angle 50.12°$$

Receiving End Circle Diagram



$$S_R = 3.59242\angle -93.451 + 4.53935\angle 86.99 - \delta$$

Applying the Law of Cosines

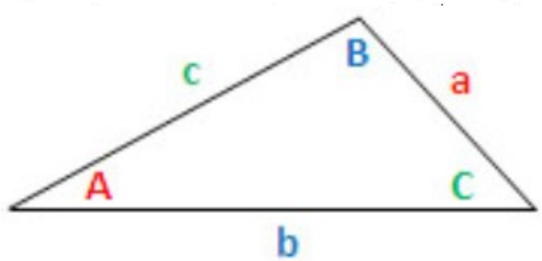
$$4.53935^2 = 3.59242^2 + |S_R|^2 - 2\cdot |S_R| \cdot 3.59242\cos(130.312)$$

Using cSolve

$$x = -5.9458 \quad or \quad x = 1.2953$$

$$P_R = .8\cdot S_R = 1.0362\,p.u. = 1036.2MW$$

$$I_R = \left(\frac{S_R}{V_R}\right)^* = \left(\frac{1.2953\angle 36.87°}{0.95\angle 0°}\right)^* = 1.3635\angle -36.87°$$



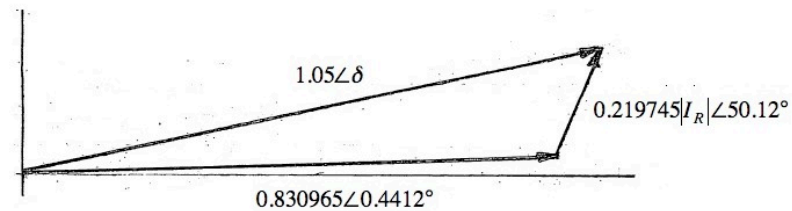
$$a^2 = b^2 + c^2 - 2bc\cos A$$

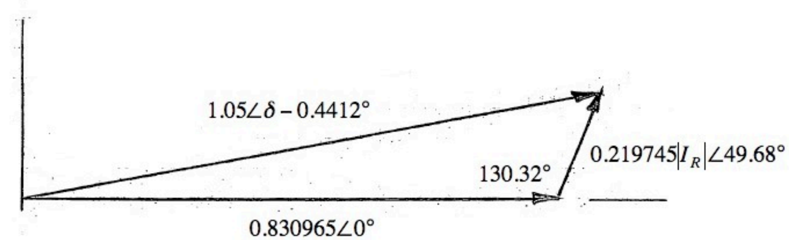
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Receiving End Power - ABCD Parameters

$$V_S\angle\delta \;=\; 0.8747\angle 0.4412° \cdot 0.95\angle 0° \;+\; 0.219745\angle 86.99° \cdot I_R\angle -36.87°$$





$$V_S\angle\delta \;=\; 0.830965\angle 0.4412° \;+\; 0.219745|I_R|\angle 50.12°$$

$$1.05\angle\delta \;=\; 0.830965\angle 0.4412° \;+\; 0.219745|I_R|\angle 50.12°$$

$$1.05\angle\delta - 0.4412° \;=\; 0.830965\angle 0.4412° - 0.4412° \;+\; 0.219745|I_R|\angle 50.12° - 0.4412°$$

$$1.05\angle\delta - 0.4412° \;=\; 0.830965\angle 0° \;+\; 0.219745|I_R|\angle 49.68°$$

$$1.05^2 \;=\; 0.830965^2 + \left(0.219745|I_R|\right)^2 - \; 2\cdot 0.219745|I_R|\cdot 0.830965\cos(130.32)$$

$$x \;=\; -6.2572 \quad or \quad x \;=\; 1.3636$$

$$S_R \;=\; V_R \cdot I_R^{\;*} \;=\; 0.95\angle 0° \cdot 1.3636\angle 36.87°^{*} \;=\; 1.29542\angle 36.87°$$

$$P_R \;=\; p.f.\,|S_R| \;=\; .8\cdot 1.29542\cdot \;=\; 1.0363 p.u. \;=\; 1036.3 MW$$

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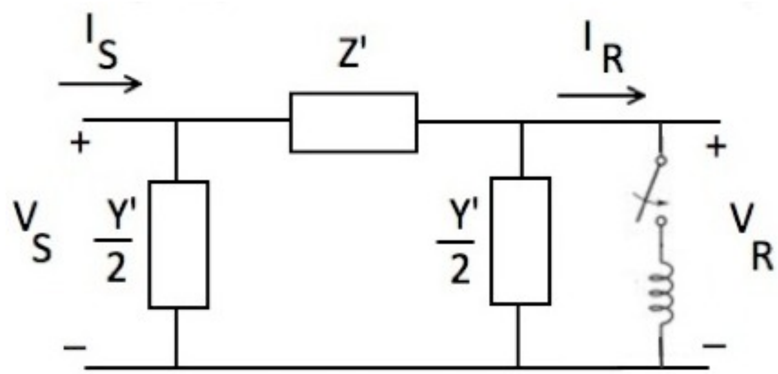
Voltage Regulation

$$V_S = AV_R + BI_R$$

$$but \;\; at \;\; no-load \;\; I_R = 0A$$

$$V_S = AV_R \qquad |V_R| = \frac{|V_S|}{|A|}$$

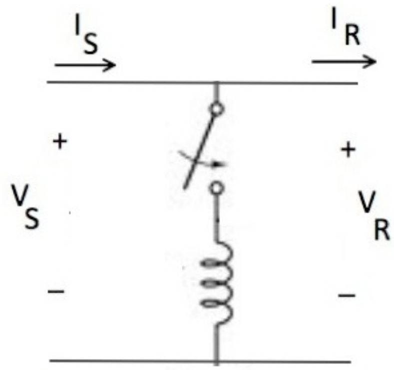


$$for \;\; our \;\; 765kV, \; 400km \;\; line \;\; |A| = .8747$$

$$V_R = \frac{1.05}{0.8747} = 1.20\,p.u. \quad 1.2 \cdot 765kV = 918kV$$

$$V_{REG} = \frac{\frac{1.05}{0.8747} - 1.0}{1.0} \cdot 100\% = 20.04\%$$

The Inductor as a Two Port



$$V_S = V_R \qquad I_S = \frac{V_R}{jX_L} + I_R$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{jX_L} & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \frac{1}{jX_L} & 1 \end{bmatrix}$$

$$A = A_1 \cdot 1 + B_1 \cdot \frac{1}{jX_L} = 0.8747\angle 0.4412^\circ + 0.219745\angle 86.99 \cdot \frac{1}{X_L}\angle -90^\circ$$

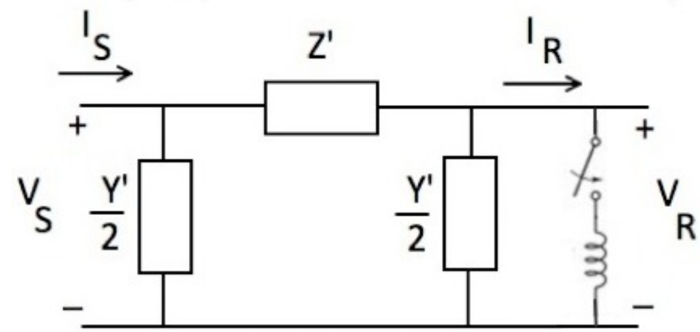
$$A = 0.8747\angle 0.4412 + \frac{0.219745}{X_L}\angle -3.01$$

$$|1.0| = 0.8747\angle 0.4412 + \frac{0.219745}{X_L}\angle -3.01 \qquad X_L = 1.751 p.u.$$

$$|V_R|_{no \quad load} = \frac{|V_S|}{|A|} = \frac{1.05}{1} = 1.05$$

Shunt Compensation

We may also determine X_L through power considerations:



$$if \quad X_L \quad is \quad chosen \quad properly \ |A| = 1.0 \Rightarrow V_{R(no \quad load)} = V_S$$

$$S_R = \frac{|V_R|^2 |A|}{|B|} \angle \beta - \alpha - 180 + \frac{|V_R| \cdot |V_S|}{|B|} \angle \beta - \delta$$

$$S_R = \frac{1.05^2 \cdot 0.8747}{0.219745} \angle 86.99 - 0.4412 - 180 + \frac{1.05 \cdot 1.05}{0.219745} \angle 86.99 - \delta$$

$$S_R = 4.385526 \angle -93.45° + 5.017179 \angle 86.99 - \delta°$$

$$P_R = 0$$

$$0 = 4.385526 \cos \angle -93.45° + 5.017179 \cos \angle 86.99 - \delta°$$

$$\angle 86.99 - \delta° = \frac{4.385526 \cos \angle -93.45°}{5.017179} = 86.9827$$

$$Q_R = 4.385526 \sin \angle -93.45° + 5.017179 \sin \angle 86.9827 = 0.62952$$

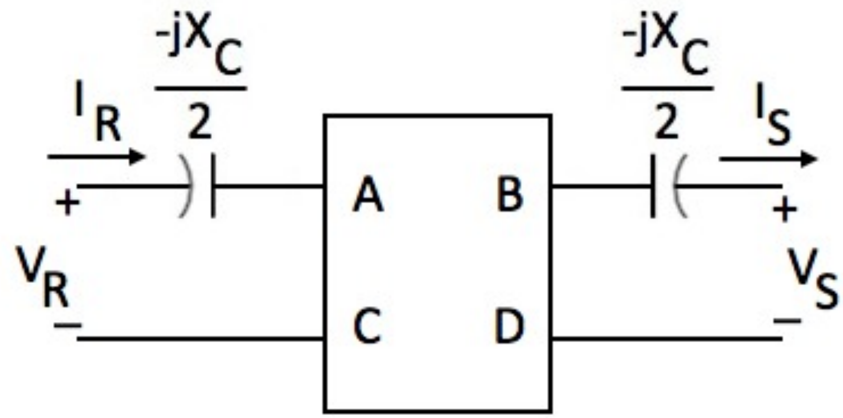
$$Q_R = \frac{V_R{}^2}{X_L} \Rightarrow X_L = \frac{1.05^2}{0.62952} = 1.75097 \, p.u.$$

Shunt Compensation

500kV 40MVAR Shunt Reactor Bank

Series Compensation

Series capacitors can be employed on long transmission lines to increase loadability. They work by reducing the total series reactance of the line. The compensation is expresses in the percent of the inductive reactance that is cancelled by the capacitors. Typically half the compensation is placed on each end of the line.



T-Line

For out 765kV, 400km line

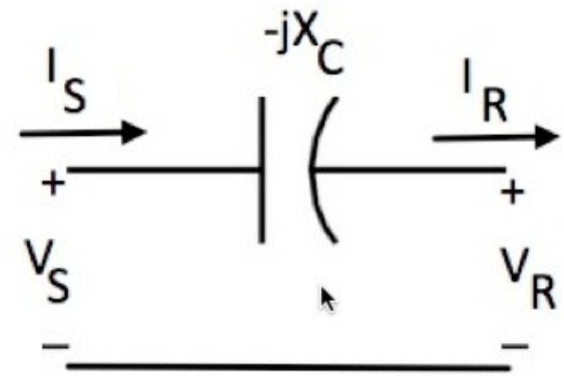
$Z' = 0.011539 + j0.219422\Omega$

$X_C = 30\%(0.219422\Omega) = 0.066\Omega$

$X_C/2 = 0.033\Omega$

The Capacitor as a Two-Port

$$V_S = V_R + (-jX_C)I_R \qquad I_S = I_R$$



$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & -jX_C \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & -jX_C \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} 1 & -jX_C \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & -j0.033 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.8747\angle 0.4412^\circ & 0.219745\angle 86.99^\circ \\ 1.0704\angle 90.14^\circ & 0.8747\angle 0.4412^\circ \end{bmatrix} \begin{bmatrix} 1 & -j0.033 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{compensated} = \begin{bmatrix} 0.9100\angle 0.4295^\circ & 0.1610\angle 85.7385^\circ \\ 1.0704\angle 90.14^\circ & 0.9100\angle 0.4295^\circ \end{bmatrix}$$

Series Compensation

$$1.05\angle\delta = 0.9100\angle 0.4295° \cdot 0.95\angle 0° + 0.1610\angle 85.7385° \cdot I_R\angle -36.87°$$

$$1.05\angle\delta = 0.8654\angle 0.4295° + 0.1610 \cdot |I_R|\angle 48.8586°$$

$$1.05\angle\delta - 0.4295° = 0.8654\angle 0° + 0.1610 \cdot |I_R|\angle 48.439°$$

$$1.05^2 = 0.8654^2 + \left(0.1610 \cdot |I_R|\right)^2 - 2 \cdot 0.1610 \cdot |I_R| \cdot 0.8654 \cos(131.561°)$$

$$x = -8.6999 \quad or \quad x = 1.56793$$

$$S_R = V_R \cdot I_R^{\ *} = \left(0.95\angle 0° \cdot 1.56793\angle -36.87°\right)^* = 1.48953\angle 36.87°$$

$$P_R = p.f.\,|S_R| = .8 \cdot 1.48953 \cdot = 1.1916\,p.u. = 1191.6MW$$

Recall the uncompensated results :

$$S_R = V_R \cdot I_R^{\ *} = 0.95\angle 0° \cdot 1.3636\angle 36.87°^* = 1.29542\angle 36.87°$$

$$P_R = p.f.\,|S_R| = .8 \cdot 1.29542 \cdot = 1.0363\,p.u. = 1036.3MW$$

Series Compensation - R.E. Circle Diagram

$$S_R = \frac{|V_R|^2 |A|}{|B|}\angle\beta-\alpha-180 + \frac{|V_R|\cdot|V_S|}{|B|}\angle\beta-\delta$$

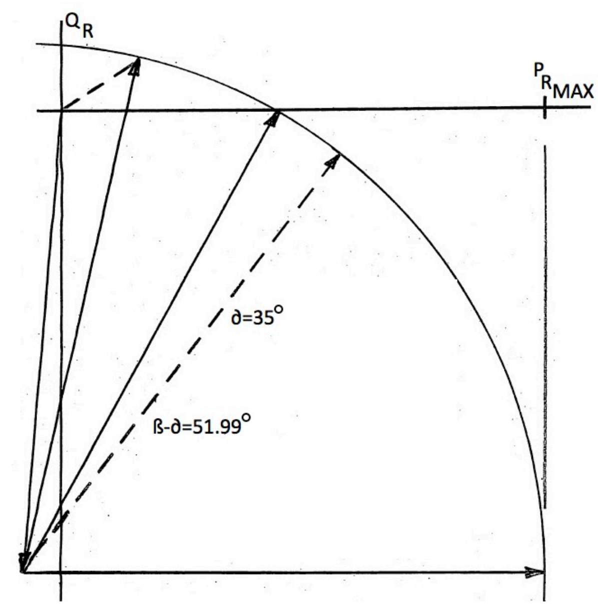
$$S_R = \frac{0.95^2\cdot 0.9100}{0.1610}\angle 85.7385-0.4295-180 + \frac{0.95\cdot 1.05}{0.1610}\angle 85.7385-\delta$$

$$S_R = 5.1011\angle -94.691 + 6.195652\angle 85.7385-\delta$$

$$6.195652^2 = 5.1011^2 + |S_R|^2 - 2\cdot|S_R|\cdot 5.1011\cos(131.564)$$

$$x = -8.2648 \quad or \quad x = 1.4961$$

$$P_R = 0.8\cdot S_R = 1.1969\,p.u. = 1196.9MW$$



Trust No One – Test Everything!

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Two test were performed on a 300km transmission line

Open Ckt Test (I_R=0) $Z_{OC} = V_S/I_S = 991.202\underline{/-89.9722}$
Short Ckt Test (V_R=0) $Z_{SC} = V_S/I_S = 160.751\underline{/+89.4422}$

$$(A \cdot D) - (B \cdot C) = 1 \quad and \quad A = D$$

$$A^2 = (B \cdot C) + 1$$

$$Z_{OC} = \frac{A}{C} = 991.202\angle -89.9722° \quad or \quad C = \frac{A}{991.202\angle -89.9722°}$$

$$Z_{SC} = \frac{B}{D} = \frac{B}{A} = 160.751\angle 89.4422° \quad or \quad B = A \cdot 160.751\angle 89.4422°$$

$$A^2 = \frac{160.751\angle 89.4422°}{991.202\angle -89.9722°} \cdot A^2 + 1 = 0.162178\angle 179.4144 \cdot A^2 + 1$$

$$A = 0.927609\angle 0.04086°$$

$$B = 0.927609\angle 0.04086° \cdot 160.751\angle 89.4422° = 149.1141\angle 89.4831°$$

$$C = \frac{0.927609\angle 0.04086°}{991.202\angle -89.9722°} = 9.3584 \cdot 10^{-4}\angle 90.0131°$$

Now Find Z (per km) and Y (per km)

$$A = 0.927609\angle 0.04086^\circ$$

$$B = 149.1141\angle 89.4831^\circ$$

$$C = 9.3584\cdot 10^{-4}\angle 90.0131^\circ$$

$$but \quad A = \cosh\gamma l \quad \Rightarrow \quad \gamma l = \cosh^{-1}\left(0.927609\angle 0.04086^\circ\right) = 0.38284\angle 89.7350$$

$$\gamma = \frac{\cosh^{-1}\left(0.927609\angle 0.04086^\circ\right)}{300} = 1.2761\cdot 10^{-3}\angle 89.7350^\circ = \sqrt{Z\cdot Y}$$

$$B = Z_C\cdot\sinh\left(\gamma l\right)$$

$$Z_C = \frac{149.1141\angle 89.4831^\circ}{\sinh\left(0.38284\angle 89.7350\right)} = 399.17386\angle -0.26497 = \sqrt{\frac{Z}{Y}}$$

$$Z = Z_C\cdot\gamma = 1.2761\cdot 10^{-3}\angle 89.7350^\circ \cdot 399.17386\angle -0.26497 = 0.5094\angle 89.47^\circ \;\; \Omega / km$$

$$Y = \frac{\gamma}{Z_C} = \frac{1.2761\cdot 10^{-3}\angle 89.7350}{399.17386\angle -0.26497} = 3.1969\cdot 10^{-6}\angle 90^\circ \;\; S / km$$