Name ____________________

E-mail ____________________

EE 4221

Hour Exam 3

December 7, 2017

Directions:

1. DO NOT START until told to do so.

2. There are 4 problems in this examination. All problems are equal valued.

3. The correct answer is a necessary but not sufficient condition to receive full credit for a problem. You MUST show you work! Disorderly or illegible work cannot and thus will not be graded.

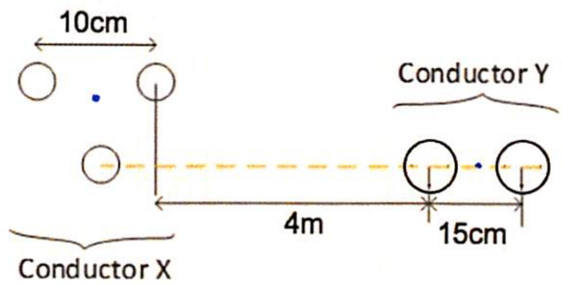
4. You are allowed use of any reference materials of your choice during the exam. You may not however classify your fellow classmate(s) as "reference materials" The exam is meant to be an individual effort!

5. Please return the exam by noon on Monday, December 11th, 2017. On-campus students submit to box 49, 7th floor EERC. Off-campus students submit as an e-mail attachment.

1. I plan to use a 7 strand all copper conductor (cable) for a low-loss, sub-transmission application.
Each strand in the cable is 3.0mm in diameter.
The spiral construction adds 2.25% to the actual length of each strand.

Determine the resistance for 1km of cable at 20°C and 50°C given:
The resistivity of copper is 1.72μΩ cm at 20°C and the temperature constant for copper is 234.5°C.

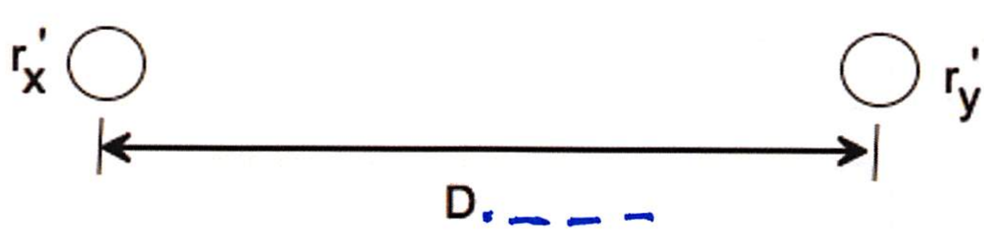
2. The conductor configuration of a bundled, single phase transmission line is shown below.



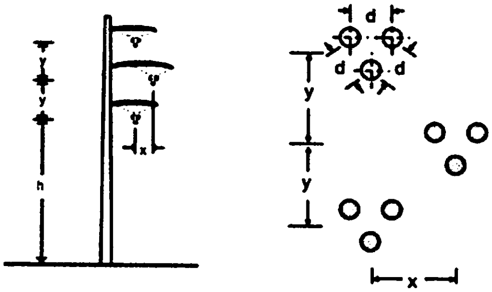
The "X" line features three solid conductors at the corners of an equilateral triangle with 10 cm spacing. Each X conductor has a diameter of 2.0 cm.

The "Y" line is comprised of two solid conductors in a horizontal configuration. The conductors are separated by 15 cm and each has a diameter of 4.0cm.

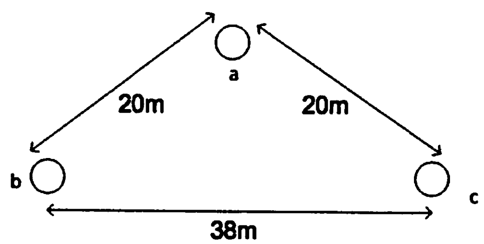
Assuming we wish to calculate the loop inductance of this single phase line, determine:
a) the effective radius of a single equivalent X conductor
b) the effective radius of a single equivalent Y conductor
c) the equivalent spacing between the single equivalent X and Y conductors.



3. The fully-transposed, 60Hz, 500kV, transmission line shown below features 3 "Cardinal" conductors on corners of an equilateral triangle with 35 centimeter spacing (d), between conductors. The bundles are arranged vertically as shown below with x = 3.0m, y = 6.5m and h = 20m. Determine the per phase, 60Hz, 50°C, series impedance (R + jX) for the line in ohms per mile. Use ACSR wire table provided after question #4.



4. Determine the per phase capacitance to neutral (in F/mile) and, shunt admittance (in S/mile) for the fully transposed, 3 phase, 60Hz, 220kV line shown (below). Each phase is comprised of a single "Flicker" conductor. You may neglect the effect of the earth.



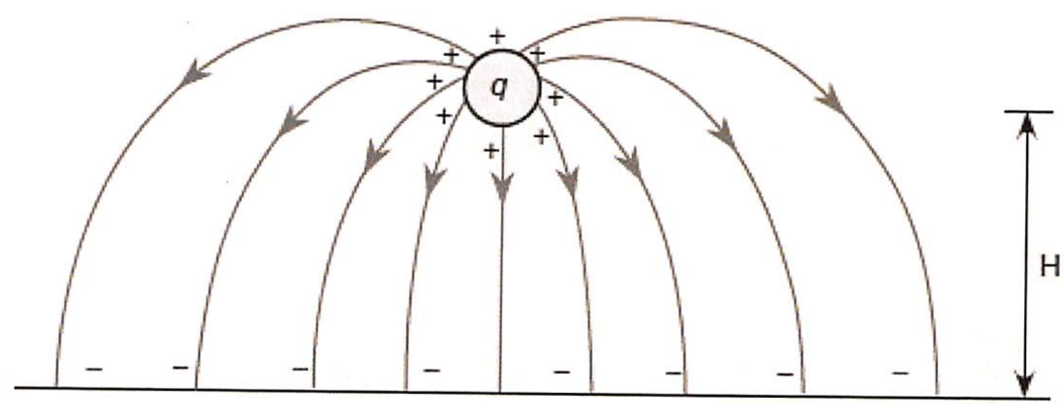
4b. Given the line is 100 miles long, determine the charging current at the nominal line-to-line system voltage of 220kV.

Code word	Aluminum	Stranding	Layers	Outside	Resistance	R - AC 60 Hz	R - Ac, 60 Hz	GMR	Reactance per conductor 1-ft spacing, 60 Hz		
	Area	Al/St	of	diameter	DC, 20°C,	20°C,	50°C,	feet	Inductive Xa	Capacitive Xa'	
	CMIL		Al	inches	Ω/1,000ft	Ω/mile	Ω/mile		Ω/mile	MΩ-mile	
Waxwing	266,800	18/1	2	0.609	0.0646	0.3488	0.3831	0.0198	0.476	0.109	Waxwing
Partridge	266,800	26/7	2	0.642	0.064	0.3452	0.3792	0.0217	0.465	0.1074	Partridge
Ostrich	300,000	26/7	2	0.68	0.0569	0.307	0.3372	0.0229	0.458	0.1057	Ostrich
Merlin	336,400	18/1	2	0.684	0.0512	0.2767	0.3037	0.0222	0.462	0.1055	Merlin
Linnet	336,400	26/7	2	0.721	0.0507	0.2737	0.3006	0.0243	0.451	0.104	Linnet
Oriole	336,400	30/7	2	0.741	0.0504	0.2719	0.2987	0.0255	0.445	0.1032	Oriole
Chickadee	397,500	18/1	2	0.743	0.0433	0.2342	0.2572	0.0241	0.452	0.1031	Chickadee
Ibis	397,500	26/7	2	0.783	0.043	0.2323	0.2551	0.0264	0.441	0.1015	Ibis
Pelican	477,000	18/1	2	0.814	0.0361	0.1957	0.2148	0.0264	0.441	0.1004	Pelican
Flicker	477,000	24/7	2	0.846	0.0359	0.1943	0.2134	0.0284	0.432	0.0992	Flicker
Hawk	477,000	26/7	2	0.858	0.0357	0.1931	0.212	0.0289	0.43	0.0988	Hawk
Hen	477,000	30/7	2	0.883	0.0355	0.1919	0.2107	0.0304	0.424	0.098	Hen
Osprey	556,500	18/1	2	0.879	0.0309	0.1679	0.1843	0.0284	0.432	0.0981	Osprey
Parakeet	556,500	24/7	2	0.914	0.0308	0.1669	0.1832	0.0306	0.423	0.0969	Parakeet
Dove	556,500	26/7	2	0.927	0.0307	0.1663	0.1826	0.0314	0.42	0.0965	Dove
Rook	636,000	24/7	2	0.977	0.0269	0.1461	0.1603	0.0327	0.415	0.095	Rook
Grosbeak	636,000	26/7	2	0.99	0.0268	0.1454	0.1596	0.0335	0.412	0.0946	Grosbeak
Drake	795,000	26/7	2	1.108	0.0215	0.1172	0.1284	0.0373	0.399	0.0912	Drake
Tern	795,000	45/7	3	1.063	0.0217	0.1188	0.1302	0.0352	0.406	0.0925	Tern
Rail	954,000	45/7	3	1.165	0.0181	0.0997	0.1092	0.0386	0.395	0.0897	Rail
Cardinal	954,000	54/7	3	1.196	0.018	0.0988	0.1082	0.0402	0.39	0.08	Cardinal
Ortolan	1,033,500	45/7	3	1.213	0.0167	0.0924	0.1011	0.0402	0.39	0.0885	Ortolan
Bluejay	1,113,000	45/7	3	1.259	0.0155	0.0861	0.0941	0.0415	0.386	0.0874	Bluejay
Finch	1,113,000	54/19	3	1.293	0.0155	0.0856	0.0937	0.0436	0.38	0.0866	Finch
Bittern	1,272,000	45/7	3	1.345	0.0136	0.0762	0.0832	0.0444	0.378	0.0855	Bittern
Pheasant	1,272,000	54/19	3	1.382	0.0135	0.0751	0.0821	0.0466	0.372	0.0847	Pheasant
Bobolink	1,431,000	45/7	3	1.427	0.0121	0.0684	0.0746	0.047	0.371	0.0837	Bobolink
Plover	1,431,000	54/19	3	1.465	0.012	0.0673	0.0735	0.0494	0.365	0.0829	Plover
Lapwing	1,590,000	45/7	3	1.502	0.0109	0.0623	0.0678	0.0498	0.364	0.0822	Lapwing
Falcon	1,590,000	54/19	3	1.545	0.0108	0.0612	0.0667	0.0523	0.358	0.0814	Falcon
Bluebird	2,156,000	84/19	4	1.762	0.008	0.0476	0.0515	0.0586	0.344	0.0776	Bluebird

Effect of the Earth

A single overhead conductor with a return path through the earth, seperated a distance H from the earth's surface would have a charge equal in magnitude but opposite in sign as the charge on the conductor.

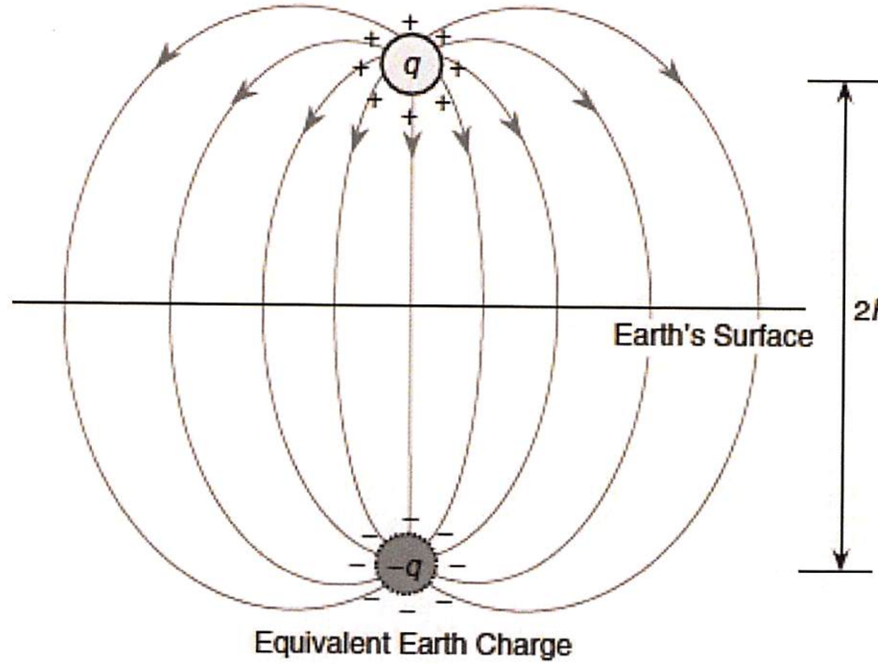
If the earth is assumed to be a conductive plane of infinite length, the electric field lines would go from the conductor to the earth, striking perpendicular to the earth as shown below.



Earth Surface

Effect of the Earth

Someone very smart determined a similar electric field distribution would be established if the negative charge on the surface of the earth was replaced by an "image conductor" with the same radius as the overhead conductor fixed a distance H directly below the overhead conductor.



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Effect of the Earth

The same principle can be applied to calculate the capacitance per phase of a three phase system.

Consider three identical conductors in the symmetric equilateral arramgement shown below along with their image parameter counterparts.

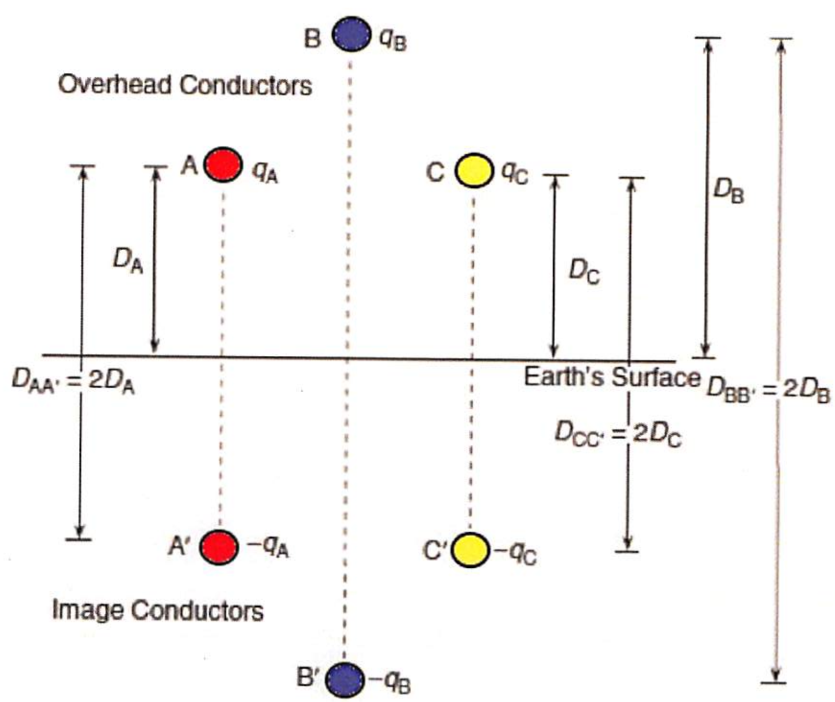


Image Parameters

$$V_{AB} = \frac{1}{2\pi\varepsilon_0}\begin{bmatrix} q_A \ln\frac{D_{AB}}{r_A} + q_B \ln\frac{r_A}{D_{AB}} + q_C \ln\frac{D_{BC}}{D_{AC}} \\ -q_A \ln\frac{D_{AB'}}{D_{AA'}} - q_B \ln\frac{D_{BB'}}{D_{AB'}} - q_C \ln\frac{D_{BC'}}{D_{AC'}} \end{bmatrix}$$

Recall $D_{AB} = D_{BC} = D_{CA} = D$ and $r_a = r_B = r_C = r$

$$V_{AB} = \frac{1}{2\pi\varepsilon_0}\left[q_A\left(\ln\frac{D}{r} - \ln\frac{D_{AB'}}{D_{AA'}}\right) + q_B\left(\ln\frac{r}{D} - \ln\frac{D_{BB'}}{D_{AB'}}\right) + q_C\left(\ln\frac{D}{D} - \ln\frac{D_{BC'}}{D_{AC'}}\right)\right] \quad (V)$$

$$V_{AB} = \frac{1}{2\pi\varepsilon_0}\left[q_A\left(\ln\frac{D}{r} - \ln\frac{D_{AB'}}{D_{AA'}}\right) + q_B\left(\ln\frac{r}{D} - \ln\frac{D_{BB'}}{D_{AB'}}\right) - q_C\left(\ln\frac{D_{BC'}}{D_{AC'}}\right)\right] \quad (V)$$

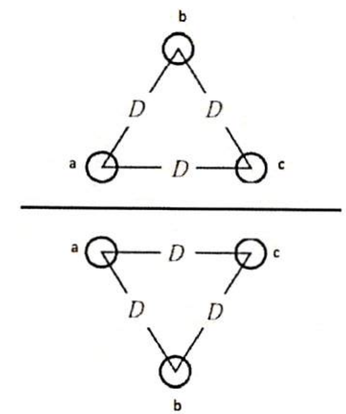
A similar expression can be derived for V_{AC}

$$V_{AC} = \frac{1}{2\pi\varepsilon_0}\left[q_A\left(\ln\frac{D}{r} - \ln\frac{D_{CA'}}{D_{AA'}}\right) + q_B\left(\ln\frac{D}{D} - \ln\frac{D_{CB'}}{D_{AB'}}\right) + q_C\left(\ln\frac{r}{D} - \ln\frac{D_{CC'}}{D_{AC'}}\right)\right] \quad (V)$$

$$V_{AC} = \frac{1}{2\pi\varepsilon_0}\left[q_A\left(\ln\frac{D}{r} - \ln\frac{D_{CA'}}{D_{AA'}}\right) - q_B\left(\ln\frac{D_{CB'}}{D_{AB'}}\right) + q_C\left(\ln\frac{r}{D} - \ln\frac{D_{CC'}}{D_{AC'}}\right)\right] \quad (V)$$

Image Parameters

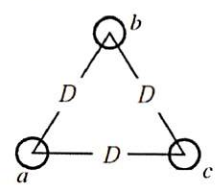
$$V_{AN} = \frac{V_{AB} + V_{AC}}{3} = \frac{1}{2\pi\varepsilon_0} q_A \left(\ln\left[\frac{D}{r}\right] - \ln\left[\frac{\sqrt[3]{D_{AB'}D_{BC'}D_{CA'}}}{\sqrt[3]{D_{AA'}D_{BB'}D_{CC'}}}\right] \right) \quad (V)$$



$$C_{AN} = \frac{q_A}{V_{AN}} = \frac{2\pi\varepsilon_0}{\left(\ln\left[\frac{D}{r}\right] - \ln\left[\frac{\sqrt[3]{D_{AB'}D_{BC'}D_{CA'}}}{\sqrt[3]{D_{AA'}D_{BB'}D_{CC'}}}\right] \right)} \quad (F/m)$$

recall that the result for the capacitance of the equally spaced conductors:

$$C_{AN} = \frac{q_A}{V_{AN}} = \frac{2\pi\varepsilon_0}{\ln\frac{D}{r}}$$



the results are similar with the image parameter denominator being smaller by the factor

$$\ln\left[\frac{\sqrt[3]{D_{AB'}D_{BC'}D_{CA'}}}{\sqrt[3]{D_{AA'}D_{BB'}D_{CC'}}}\right]$$

Capacitance increases but (for real lines) not by much!

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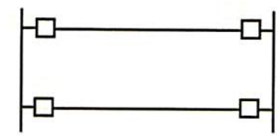
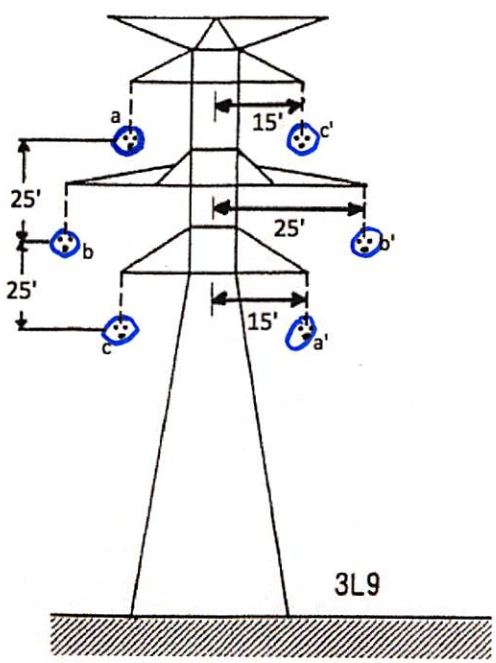
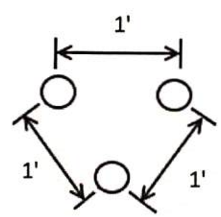
Double Circuit Lines

Right of way is expensive and they're not making any new land, so we have to utilize the space we have to the best of our ability. Frequently we carry two transmission lines on a single tower. These are referred to as double circuit lines.

We will only consider the case where the two lines operate in parallel and at the same voltage and frequency.







Each phase consists of three "Eagle" conductors on the cornes of an equilateral triangle, with 1 foot spacing.

From the ACSR table A4:
GMR = 0.0328'
Diameter = 0.953"

c_{eq}

a_{eq} GMD b_{eq}

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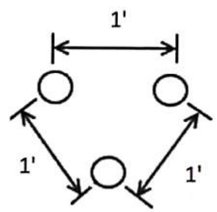
Double Circuit Lines



From the ACSR table A4:
GMR = 0.0328'
Diameter = 0.953"

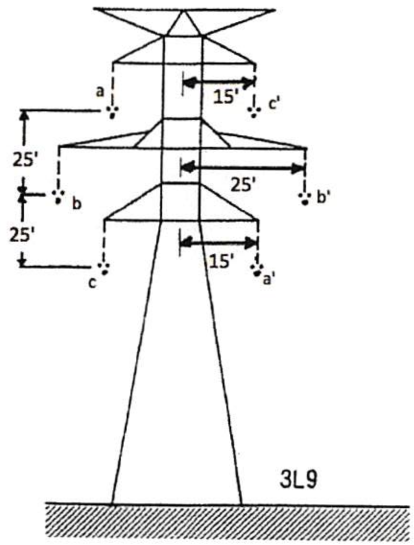
Step 1 : Reduce the bundles down to a single conductor.

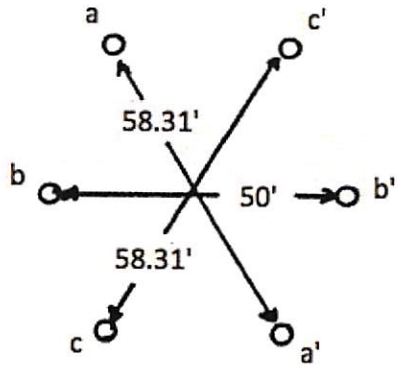
$$D_{SL} = R_B = r'_{eq} = \sqrt[9]{(0.0328)(1)(1)\cdot(0.0328)(1)(1)\cdot(0.0328)(1)(1)} = 0.3201'$$

for DSC we'll need the actual radius (in feet) r = 0.953"/(2 x 12) = 0.0397
(Note that 0.3412 x 0.7788 ≠ 0.3201 as the conductor is stranded NOT solid)

$$D_{SC} = \sqrt[9]{(0.0397)(1)(1)\cdot(0.0397)(1)(1)\cdot(0.0397)(1)(1)} = 0.3412'$$

Double Circuit Lines





To determine the **inductance** we'll need the average **effective** radius of the two conductor bundle.

$$D_{SL(a)} = \sqrt[4]{D_{aa} \cdot D_{aa'} \cdot D_{a'a'} \cdot D_{a'a}} = \sqrt[4]{0.3201 \cdot 58.31 \cdot 0.3201 \cdot 58.31} = 4.320'$$

$$D_{SL(b)} = \sqrt[4]{D_{bb} \cdot D_{bb'} \cdot D_{b'b'} \cdot D_{b'b}} = \sqrt[4]{0.3201 \cdot 50 \cdot 0.3201 \cdot 50} = 4.001'$$

$$D_{SL(c)} = \sqrt[4]{D_{cc} \cdot D_{cc'} \cdot D_{c'c'} \cdot D_{c'c}} = \sqrt[4]{0.3201 \cdot 58.31 \cdot 0.3201 \cdot 58.31} = 4.320'$$

$$D_{SL} = \sqrt[3]{4.320 \cdot 4.001 \cdot 4.320} = 4.211'$$

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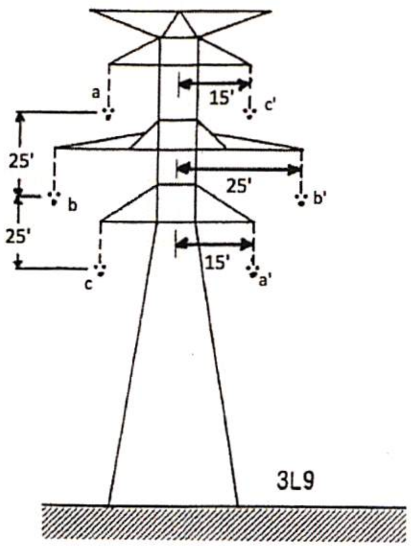
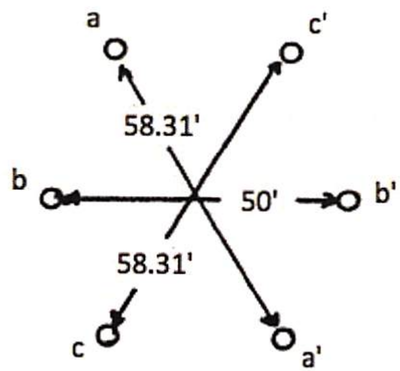
Double Circuit Lines





To determine the **capacitance** we'll need the average radius of the two conductor bundle.

$$D_{SC(a)} = \sqrt[4]{D_{aa} \cdot D_{aa'} \cdot D_{a'a'} \cdot D_{a'a}} = \sqrt[4]{0.3412 \cdot 58.31 \cdot 0.3412 \cdot 58.31} = 4.4604'$$

$$D_{SC(b)} = \sqrt[4]{D_{bb} \cdot D_{bb'} \cdot D_{b'b'} \cdot D_{b'b}} = \sqrt[4]{0.3412 \cdot 50 \cdot 0.3412 \cdot 50} = 4.1304'$$

$$D_{SC(c)} = \sqrt[4]{D_{cc} \cdot D_{cc'} \cdot D_{c'c'} \cdot D_{c'c}} = \sqrt[4]{0.3412 \cdot 58.31 \cdot 0.3412 \cdot 58.31} = 4.4604'$$

$$D_{SC} = \sqrt[3]{4.4604 \cdot 4.1304 \cdot 4.4604} = 4.348'$$

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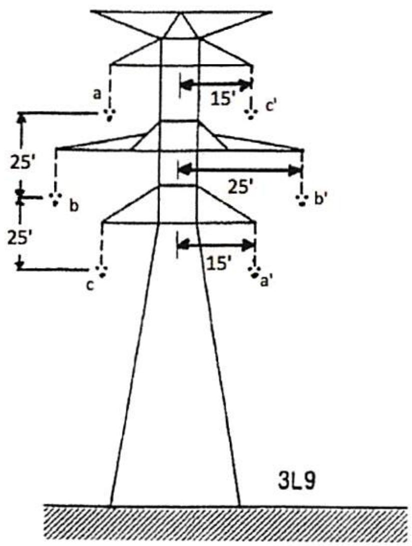
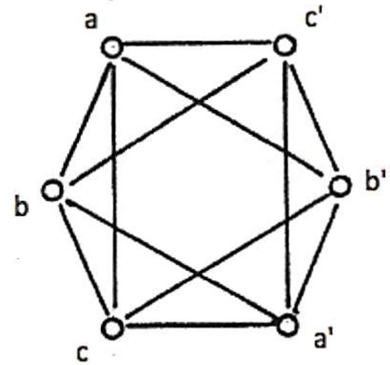
Double Circuit Line





Lastly, we'll need D_{eq}

$$D_{eq} = GMD = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$$

Dab	
a-b	26.93'
a-b'	47.17'
a'-b	47.17'
a'-b'	26.93'
Dbc	
b-c	26.93'
b-c'	47.17'
b'-c	47.17'
b'-c'	26.93'
Dca	
c-a	50'
c-a'	30'
c'-a	30'
c'-a'	50'

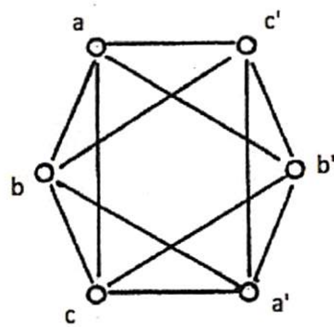
$$D_{ab} = \sqrt[4]{26.93 \cdot 47.17 \cdot 47.17 \cdot 26.93} = 35.641'$$

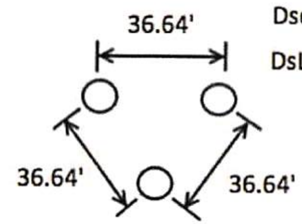
$$D_{bc} = \sqrt[4]{26.93 \cdot 47.17 \cdot 47.17 \cdot 26.93} = 35.641'$$

$$D_{ca} = \sqrt[4]{50 \cdot 30 \cdot 50 \cdot 30} = 35.641' = 38.730'$$

$$D_{eq} = \sqrt[3]{35.641 \cdot 35.641 \cdot 38.730} = 36.642'$$

Double Circuit Line





Dsc = 4.348' = r

DsL = 4.211 = r'

$$X_L = \left(2\cdot 10^{-7}\right)(2\pi 60)(1609m/mile)\ln\frac{D_{eq}}{D_{SL}} = \left(2\cdot 10^{-7}\right)(2\pi 60)(1609m/mile)\ln\frac{36.64}{4.211} = 0.2625\Omega/mile$$

$$Y = j\omega C = \frac{j(2\pi\cdot 60)\left(2\pi\cdot 8.854\cdot 10^{-12}\right)(1609m/mile)}{\ln\frac{D_{eq}}{D_{SC}}} = \frac{33.745\cdot 10^{-6}}{\ln\frac{36.64}{4.348}} = 15.832\cdot 10^{-6}S/mile$$

Double Circuit Line

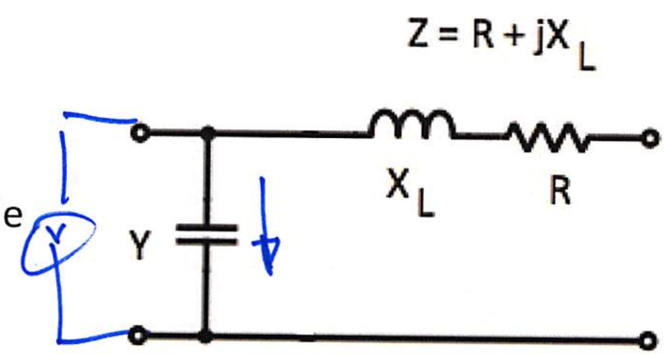
We’re almost done but we can’t forget about resistance.

From Table A4 Eagle conductors have $R_{(60Hz)} = 0.1859\ \Omega$/mile.

But we have three conductors per phase and two parallelled phases

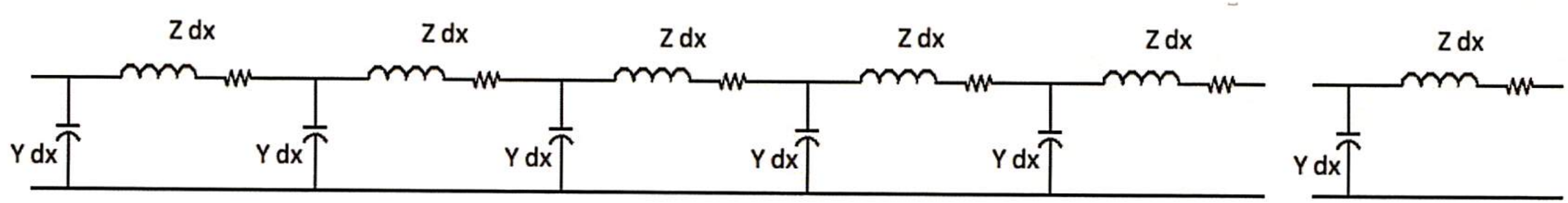
R(per phase) = 0.1859/6 = 0.03098 Ω/mile

$Z = 0.03098 + j0.2625\ \Omega$/mile

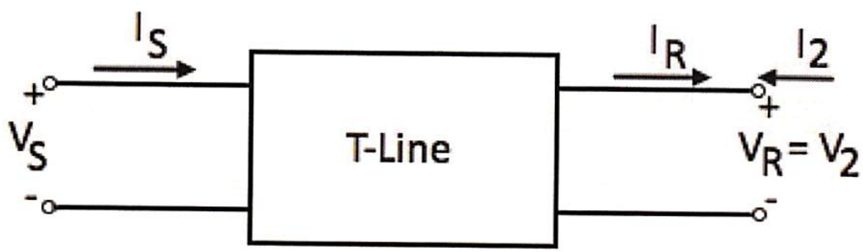


$Y = j15.83\text{x}10^{-6}$ S/mile

How Long is the Line?



Two Ports



$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Admittance parameters

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Z - parameters

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

h - parameters

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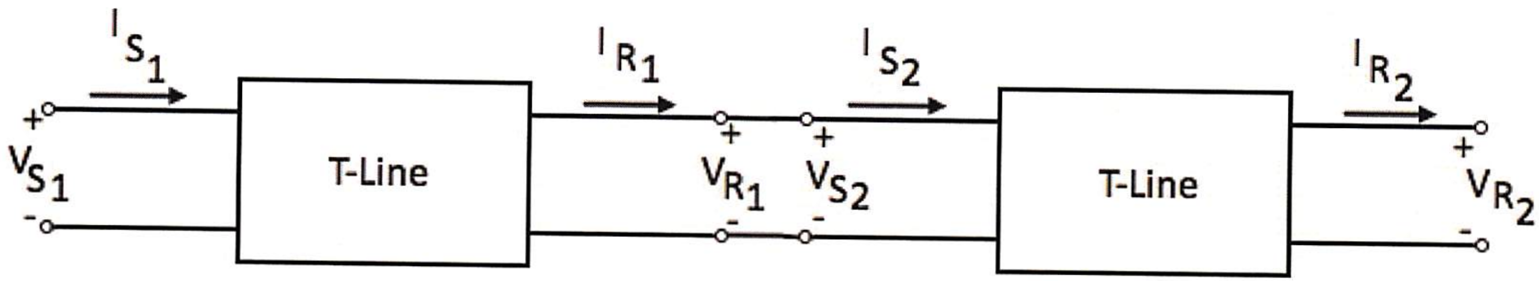
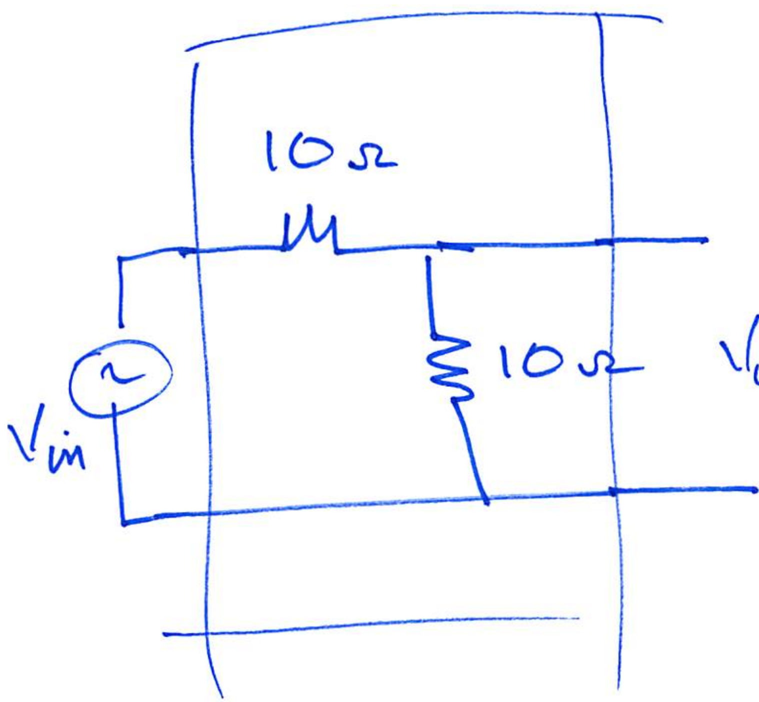
Two Ports



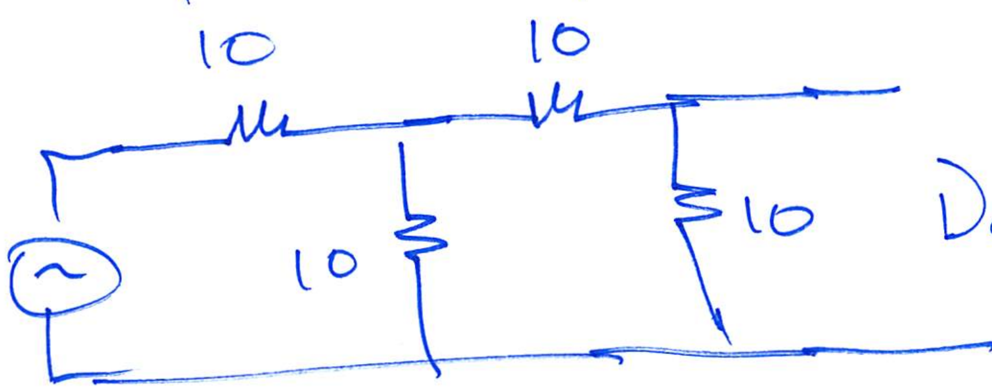
$$\begin{bmatrix} V_{S_1} \\ I_{S_1} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_{R_1} \\ I_{R_1} \end{bmatrix} \qquad \begin{bmatrix} V_{S_2} \\ I_{S_2} \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{R_2} \\ I_{R_2} \end{bmatrix}$$

$$but \begin{bmatrix} V_{R_1} \\ I_{R_1} \end{bmatrix} = \begin{bmatrix} V_{S_2} \\ I_{S_2} \end{bmatrix}$$

$$\begin{bmatrix} V_{S_1} \\ I_{S_1} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_{R_2} \\ I_{R_2} \end{bmatrix}$$

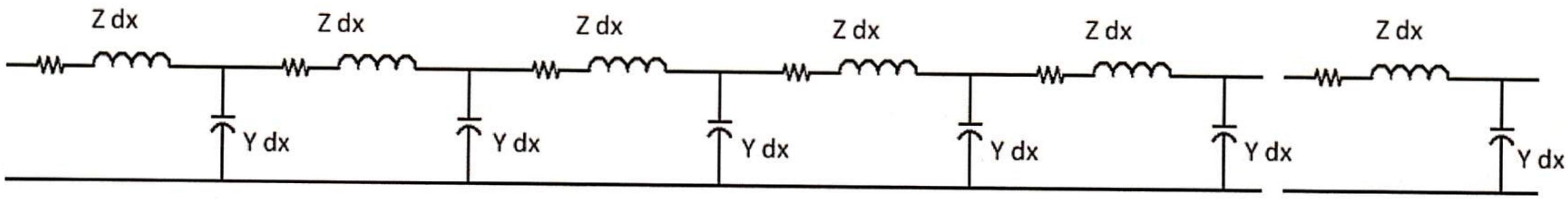


$V_{out} = \frac{V_{in}}{2}$

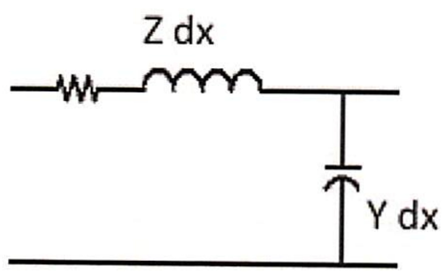


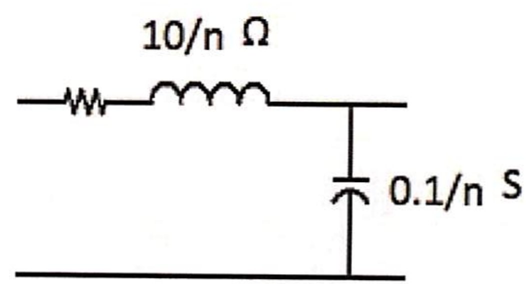
Does $V_o = \frac{V_{in}}{4}$ No!

The Distributed Network



Let - $Z_{line\ (total)} = 10\Omega$ and $Y_{line\ (total)} = .1S$
Begin by dividing the line into n equal segments
The admittance and impedance per increment will be 1/n th of the total





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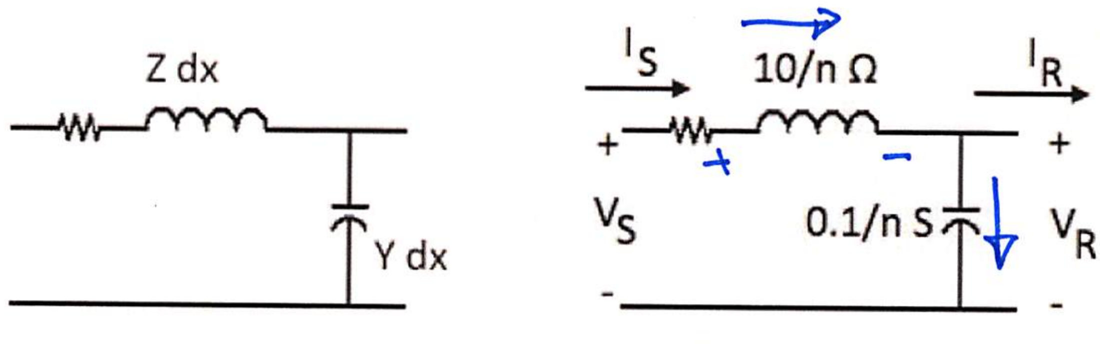
The Distributed Network



$V_R \cdot \frac{1}{n}$

$V_s = (10/n)\ I_s + V_R$

$I_s = I_R + V_R\ (0.1/n)$

D C A B

$V_s = (10/n)\ (I_R + V_R\ (0.1/n)) + V_R = (1 + 1/n^2)V_R + (10/n)I_R$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{1}{n^2}\right) & \frac{10}{n} \\ \frac{0.1}{n} & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

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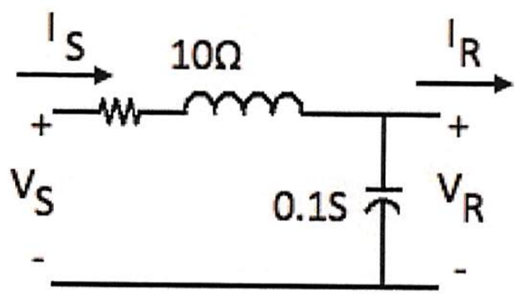
Distributed Network

For n = 1

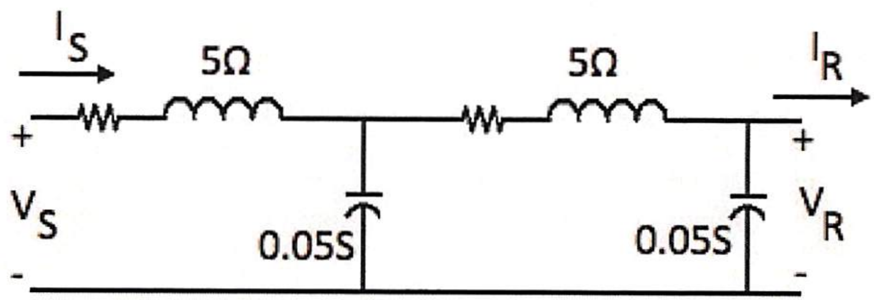
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{1}{n^2}\right) & \frac{10}{n} \\ \frac{0.1}{n} & 1 \end{bmatrix} = \begin{bmatrix} 2.0 & 10.0 \\ 0.1 & 1 \end{bmatrix}$$



$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Distributed network

For n = 2

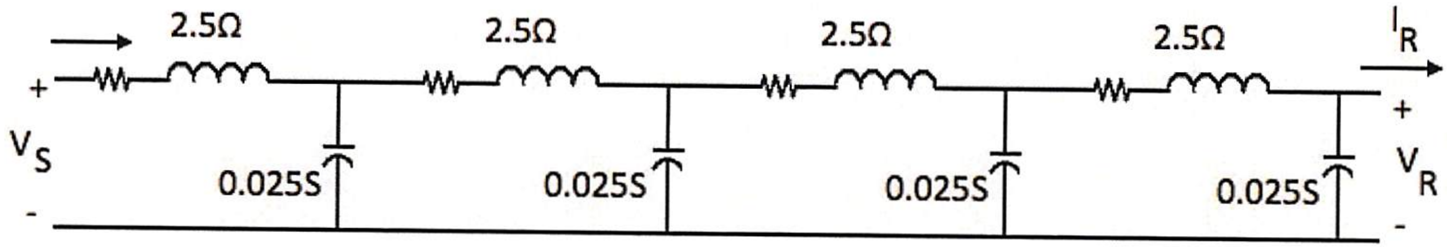


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \dfrac{1}{n^2}\right) & \dfrac{10}{n} \\ \dfrac{0.1}{n} & 1 \end{bmatrix} = \begin{bmatrix} 1.25 & 5.0 \\ 0.05 & 1 \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1.25 & 5.0 \\ 0.05 & 1 \end{bmatrix} \begin{bmatrix} 1.25 & 5.0 \\ 0.05 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1.8125 & 11.2500 \\ 0.1125 & 1.2500 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Distributed Network

For n = 4



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{1}{n^2}\right) & \frac{10}{n} \\ \frac{0.1}{n} & 1 \end{bmatrix} = \begin{bmatrix} 1.0625 & 2.5 \\ 0.025 & 1 \end{bmatrix}^{4}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1.0625 & 2.5 \\ 0.025 & 1 \end{bmatrix} \begin{bmatrix} 1.0625 & 2.5 \\ 0.025 & 1 \end{bmatrix} \begin{bmatrix} 1.0625 & 2.5 \\ 0.025 & 1 \end{bmatrix} \begin{bmatrix} 1.0625 & 2.5 \\ 0.025 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1.6853 & 11.6217 \\ 0.1162 & 1.3948 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Distributed Network

For n = 8

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{1}{n^2}\right) & \frac{10}{n} \\ \frac{0.1}{n} & 1 \end{bmatrix} = \begin{bmatrix} 1.0156 & 1.25 \\ 0.0125 & 1 \end{bmatrix}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1.0156 & 1.25 \\ 0.0125 & 1 \end{bmatrix}^8 \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1.6156 & 11.7191 \\ 0.1172 & 1.4691 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

For n = 16

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{1}{n^2}\right) & \frac{10}{n} \\ \frac{0.1}{n} & 1 \end{bmatrix} = \begin{bmatrix} 1.0039 & 0.625 \\ 0.00625 & 1 \end{bmatrix}$$

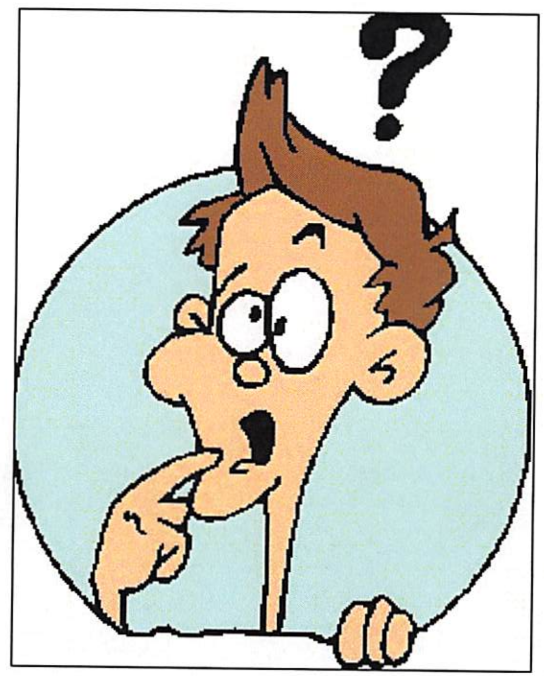
$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1.0039 & 0.625 \\ 0.00625 & 1 \end{bmatrix}^{16} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1.5796 & 11.7438 \\ 0.1174 & 1.5062 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Distributed Network

For n = infinity

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1.5431 & 11.7520 \\ 0.1175 & 1.5431 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

How do we know that???????



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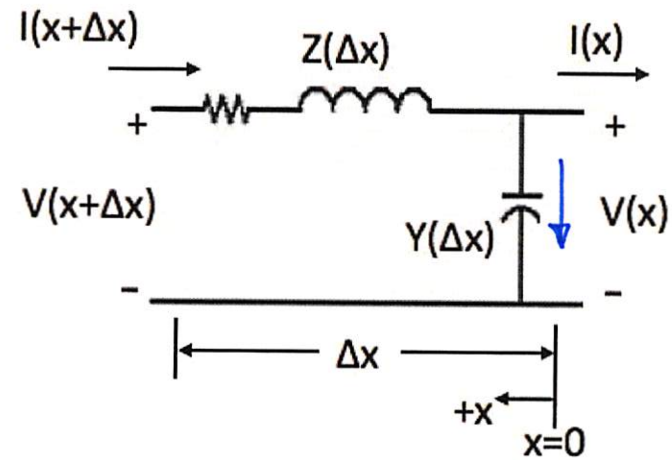
Defining the Differential Eq.

Consider a differential length of line:

$$V_{(X+\Delta X)} = V_{(X)} + Z_{\Delta X} \cdot I_{(X+\Delta X)}$$
$$I_{(X+\Delta X)} = I_X + Y_{\Delta X} \cdot V_X$$



$$\frac{V_{(X+\Delta X)} - V_{(X)}}{\Delta X} = Z_{\Delta X} \cdot I_{(X+\Delta X)}$$
$$\lim_{\Delta X \to 0} \frac{dV}{dX} = Z \cdot I_X$$

$$\frac{I_{(X+\Delta X)} - I_X}{\Delta X} = Y_{\Delta X} \cdot V_X$$
$$\lim_{\Delta X \to 0} \frac{dI}{dX} = Y \cdot V_X$$

Solving the Differential Eq.

$$\frac{dV}{dX} = Z_X \cdot I_X \qquad \frac{dI}{dX} = Y \cdot V_X$$

$$s \cdot V_{(S)} - V(0^+) = Z \cdot I_{(S)}$$

$$s \cdot I_{(S)} - I(0^+) = Y \cdot V_{(S)}$$

$$s \cdot V_{(S)} - Z \cdot I_{(S)} = V(0^+)$$

$$s \cdot I_{(S)} - Y \cdot V_{(S)} = I(0^+)$$

$$\begin{bmatrix} s & -Z \\ -Y & s \end{bmatrix} \begin{bmatrix} V_{(S)} \\ I_{(S)} \end{bmatrix} = \begin{bmatrix} V(0^+) \\ I(0^+) \end{bmatrix}$$

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Inverse of a 2 x 2 matrix

- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

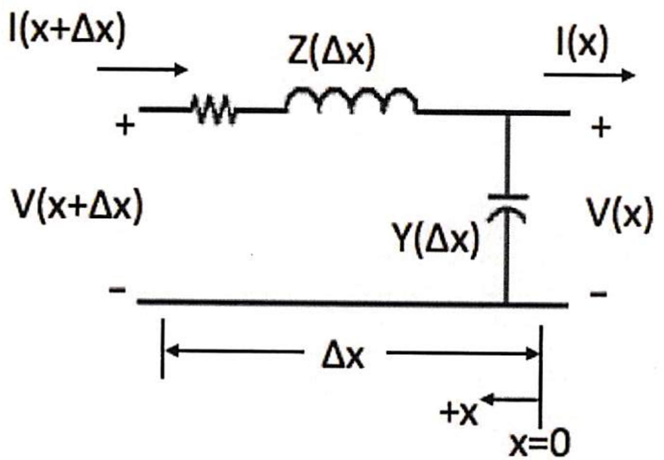
  then $A^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

- The value of $ad - bc$ is known as the determinant of matrix A and can be written as det A or $|A|$.

Solving the Diff Eq.

$$\begin{bmatrix} s & -Z \\ -Y & s \end{bmatrix}^{-1} \begin{bmatrix} s & -Z \\ -Y & s \end{bmatrix} \begin{bmatrix} V_{(S)} \\ I_{(S)} \end{bmatrix} = \begin{bmatrix} s & -Z \\ -Y & s \end{bmatrix}^{-1} \begin{bmatrix} V(0^+) \\ I(0^+) \end{bmatrix}$$

$$\begin{bmatrix} V_{(S)} \\ I_{(S)} \end{bmatrix} = \left(\frac{1}{s^2 - YZ}\right) \begin{bmatrix} s & Z \\ Y & s \end{bmatrix} \begin{bmatrix} V(0^+) \\ I(0^+) \end{bmatrix}$$



but $V(0^+) = V_R$
and $I(0^+) = I_R$

$$\begin{bmatrix} V_{(S)} \\ I_{(S)} \end{bmatrix} = \left(\frac{1}{s^2 - YZ}\right) \begin{bmatrix} s & Z \\ Y & s \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Solving the Diff Eq

Expanding $V_{(S)}$

$$V_{(S)} = \frac{sV_R + ZI_R}{s^2 - YZ} \quad but \quad YZ = \left(\sqrt{YZ}\right)^2 \quad and \quad s^2 - \left(\sqrt{YZ}\right)^2 = \left(s - \sqrt{YZ}\right)\left(s + \sqrt{YZ}\right)$$

$$V_{(S)} = \frac{sV_R + ZI_R}{\left(s - \sqrt{YZ}\right)\left(s + \sqrt{YZ}\right)} = \frac{A}{\left(s - \sqrt{YZ}\right)} + \frac{B}{\left(s + \sqrt{YZ}\right)}$$

$$\frac{K}{s+a} \xrightarrow{\mathcal{L}^{-1}} Ke^{-at}$$

$$Ke^{-ax}$$

$$sV_R + ZI_R = A\left(s + \sqrt{YZ}\right) + B\left(s - \sqrt{YZ}\right) = s(A+B) + (A-B)\sqrt{YZ}$$

$$V_R = A + B \qquad and \quad \frac{ZI_R}{\sqrt{YZ}} = A - B \quad or \quad \sqrt{\frac{Z}{Y}}I_R = A - B$$

$$V_R + \sqrt{\frac{Z}{Y}}I_R = 2A \qquad V_R - \sqrt{\frac{Z}{Y}}I_R = 2B$$

Solving the Diff Eq

$$V_R + \sqrt{\frac{Z}{Y}} I_R = 2A \qquad V_R - \sqrt{\frac{Z}{Y}} I_R = 2B$$

define characteristic impedance $Z_C = \sqrt{\frac{Z}{Y}}$ and propagation constant $\gamma = \sqrt{YZ} = \alpha + j\beta$

$$A = \frac{V_R}{2} + \frac{Z_C I_R}{2} \qquad B = \frac{V_R}{2} - \frac{Z_C I_R}{2} \qquad V_{(S)} = \frac{A}{\left(s - \sqrt{YZ}\right)} + \frac{B}{\left(s + \sqrt{YZ}\right)}$$

$$V_{(s)} = \frac{\frac{V_R}{2} + \frac{Z_C I_R}{2}}{s - \gamma} + \frac{\frac{V_R}{2} - \frac{Z_C I_R}{2}}{s + \gamma}$$

$$LaPlace^{-1}\left[\frac{K}{s + a}\right] = Ke^{-ax}$$

$$V_{(x)} = \left[\frac{V_R}{2} + \frac{Z_C I_R}{2}\right] \cdot e^{+\gamma x} + \left[\frac{V_R}{2} - \frac{Z_C I_R}{2}\right] \cdot e^{-\gamma x}$$

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Solving the Diff Eq

$$V_{(x)} = \left[\frac{V_R}{2} + \frac{Z_C I_R}{2}\right] \cdot e^{+\gamma x} + \left[\frac{V_R}{2} - \frac{Z_C I_R}{2}\right] \cdot e^{-\gamma x}$$

when x $= l$ we are at the sending end of the line

$$V_s = \left[\frac{V_R}{2} + \frac{Z_C I_R}{2}\right] \cdot e^{+\gamma l} + \left[\frac{V_R}{2} - \frac{Z_C I_R}{2}\right] \cdot e^{-\gamma l}$$

$$V_s = \left[\frac{e^{+\gamma l} + e^{-\gamma l}}{2}\right] V_R + \left[\frac{e^{+\gamma l} - e^{-\gamma l}}{2}\right] Z_C I_R$$

Repeating the process and solving for I_S gives :

$$I_s = \left[\frac{e^{+\gamma l} - e^{-\gamma l}}{2}\right] \frac{V_R}{Z_C} + \left[\frac{e^{+\gamma l} + e^{-\gamma l}}{2}\right] I_R$$

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Solving the Diff Eq

$$V_s = \left[\frac{e^{+\gamma l} + e^{-\gamma l}}{2}\right] V_R + \left[\frac{e^{+\gamma l} - e^{-\gamma l}}{2}\right] Z_C I_R$$

$$I_s = \left[\frac{e^{+\gamma l} - e^{-\gamma l}}{2}\right] \frac{V_R}{Z_C} + \left[\frac{e^{+\gamma l} + e^{-\gamma l}}{2}\right] I_R$$

This looks somewhat famaliar? Eulers Identity??

$$\cos(x) = \frac{e^{+jx} + e^{-jx}}{2} \qquad \sin(x) = \frac{e^{+jx} - e^{-jx}}{2j}$$

Not quite!

Solving the Diff Eq

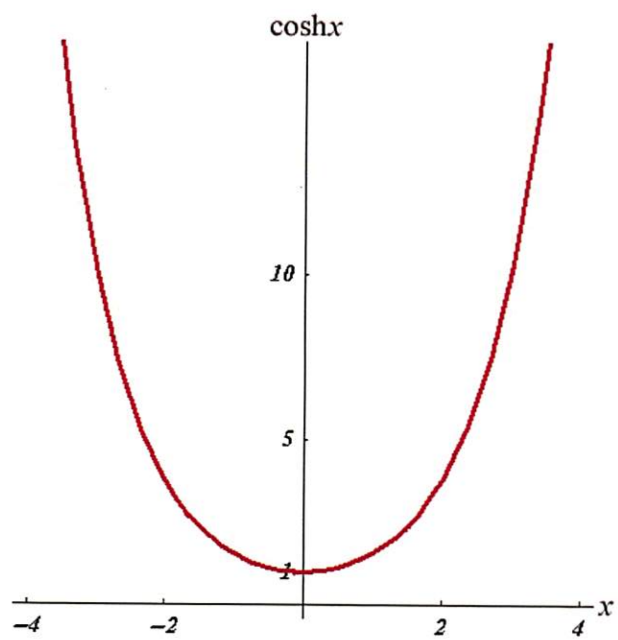
$$\left[\frac{e^{+\gamma l} + e^{-\gamma l}}{2}\right] = \cosh(\gamma l) \qquad \left[\frac{e^{+\gamma l} - e^{-\gamma l}}{2}\right] = \sinh(\gamma l)$$

$$V_S = \cosh(\gamma l)\cdot V_R + Z_C \sinh(\gamma l)\cdot I_R$$

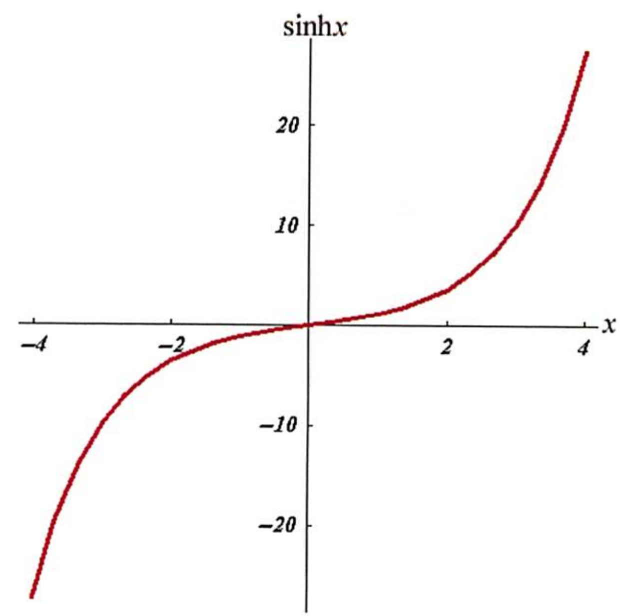
$$I_S = \frac{1}{Z_C}\sinh(\gamma l)\cdot V_R + \cosh(\gamma l)\cdot I_R$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

The Hyperbolic Functions



$\cosh(x) = (e^x+e^{-x})/2$



$\sinh(x) = (e^x-e^{-x})/2$

$d/dx \cosh(x) = \sinh(x)$ $d/dx \sinh(x) = \cosh(x)$ and $\cosh^2(x) - \sinh^2(x) = 1$

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Example

A transmission line is 100 football fields long:

The series impedance is 0.1Ω per football field (ohms per unit)

The shunt admittance is .001S per football field (ohms^{-1} per unit)

$$Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{0.1\Omega / football \quad field}{0.001S / football \quad field}} = 10\Omega$$

$$\gamma = \sqrt{ZY} = \sqrt{0.1 \cdot 0.001} = 0.01 / football \quad field$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$A = D = \cosh\left((0.01)(100)\right) = 1.543081$$

$$B = 10\sinh\left((0.01)(100)\right) = 11.752 \ \Omega$$

$$C = \frac{1}{10}\sinh\left((0.01)(100)\right) = .11752 \ S$$

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Warning – Warnung - 警告 – Varoitus - Onyo

You will need to be able to calculate sinh(x), cosh(x) and tanh(x) as well as $\sinh^{-1}(x)$, $\cosh^{-1}(x)$ and $\tanh^{-1}(x)$

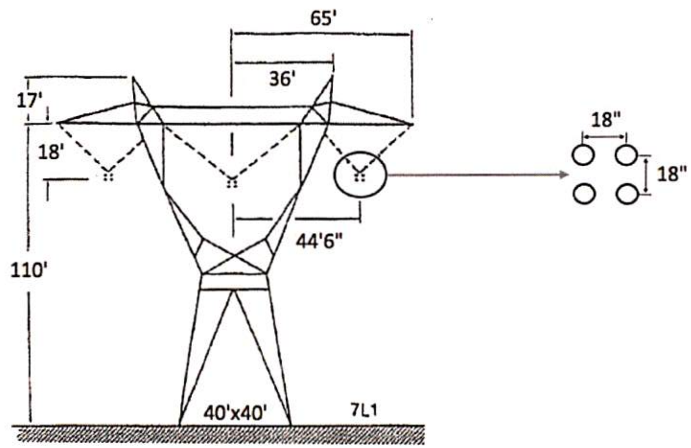
Where x is a complex number !!!!

$$\sinh(0.9\angle 80°) \;=\; 0.7905\angle 82.79°$$

$$\tanh(0.5\angle 75°) \;=\; 0.5387\angle 72.35°$$

$$\cosh^{-1}(0.8\angle 80°) \;=\; 1.632\angle 63.56°$$

Example



A 765 kV transmissionn line is 400km long. The "flat" configuration features 4 Canary conductors spaced at the corners of a square 18" per side. The bundles spacing is 44'6".

Per Table A4:

Outside Diameter = 1.162"

GMR = 0.0391'

$R_{50C\ 60Hz}$ = 0.1185 ohms/mile

Z

Y

R L

C

D_{eq} D_{eq} D_{eq}

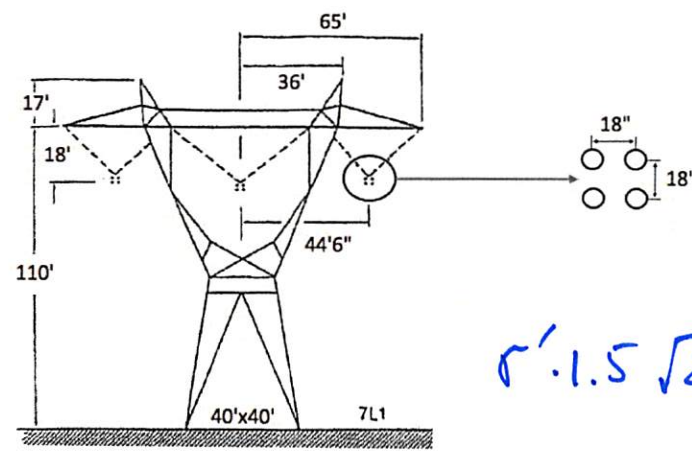
$$\sqrt[3]{D_{AB} \cdot D_{BC} \cdot D_{CA}}$$

$$44.5 \cdot 44.5 \cdot 89$$

56.066 ft

Example

Per Table A4:
Outside Diameter = 1.162”
GMR = 0.0391’
R50C 60Hz = 0.1185 ohms/mile



$r' \cdot 1.5 \sqrt{2}\ 1.5\ 1.5$

$$D_{eq} = \sqrt[3]{44.5 \cdot 44.5 \cdot 89} = 56.066 \quad feet$$

$$D_{SL} = \sqrt[4]{1.5 \cdot 1.5 \cdot 1.5\sqrt{2} \cdot 0.0391} = 0.65727 \quad feet$$

$$D_{SC} = \sqrt[4]{1.5 \cdot 1.5 \cdot 1.5\sqrt{2} \cdot 1.162/24} = 0.69334 \quad feet$$

$$R = 0.1184/4 \quad \Omega/mile \cdot 1mile/1.609km = 0.0184 \quad \Omega/km$$

$$X_L = j(2\pi \cdot 60) \cdot 2 \cdot 10^{-7} \ln \frac{56.066}{0.65727} = 3.35236 \cdot 10^{-4} \quad \Omega/m \cdot 1000m/km = 0.335236 \quad \Omega/km$$

$$Y = \frac{j(2\pi \cdot 60) \cdot (2\pi) 8.854 \cdot 10^{-12}}{\ln \frac{56.066}{0.69339}} = 4.77432 \cdot 10^{-9} \quad \Omega/m \cdot 1000m/km = 4.77432 \cdot 10^{-6} \angle 90° \quad \Omega/km$$

$$Z = 0.018395 + j0.335236 \quad \Omega/km = 0.33574 \angle 86.859°$$

Example

$$\gamma = \sqrt{ZY} = \sqrt{(0.33574\angle 86.856°)(4.77441 \cdot 10^{-6}\angle 90°)} = 0.0012661\angle 88.429° / km$$

$$\gamma l = 0.50643\angle 88.4289°$$

$$Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{(0.33574\angle 86.856°)}{(4.77441 \cdot 10^{-6}\angle 90°)}} = 265.183\angle -1.571°$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C \cdot \sinh(\gamma l) \\ \frac{1}{Z_C}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8747\angle 0.4409° & 128.6374\angle 86.996° \\ 1.8293 \cdot 10^{-3}\angle 90.137° & 0.8747\angle 0.4409° \end{bmatrix}$$

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