

ECE 311 Lab 3

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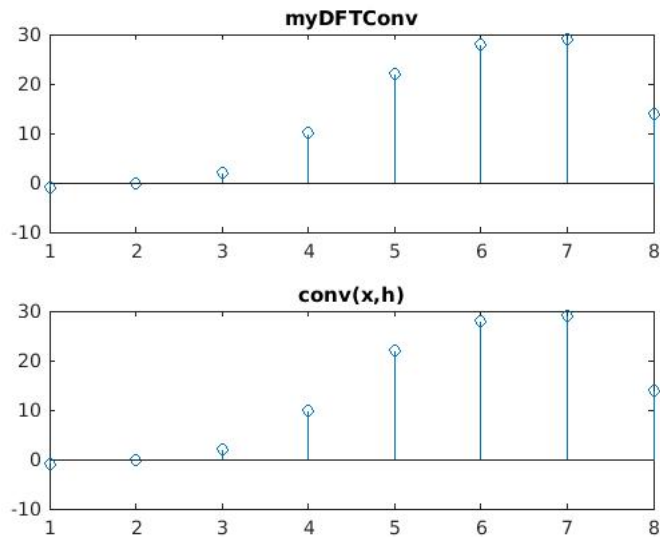
Report Item 1

```

1 function [ y ] = myDFTConv( x, h )
3 %transfer over to frequency domain
  l = size(x,2) + size(h,2) - 1;
5 x_w = fft(x,l);
  h_w = fft(h,l);
7
9 %element multiplication to get y_w
  y_w = x_w.*h_w;
11 %return to discrete domain
  y = ifft(y_w);
13
15 %plot my result
  figure;
  subplot(211);
  stem(y);
  title('myDFTConv');
19
21 %plot key
  subplot(212);
  stem(conv(x,h));
  title('conv(x,h)');
23
25 end

```

myDFTConv.m



Because of the use of fft and ifft, the time complexity is also $O(N \log(N))$.

Report Item 2

```

1 function [ y_n ] = sys1( a,N )
3 %get delta function
  x_n = zeros(1,N);
5 x_n(1) = 1;

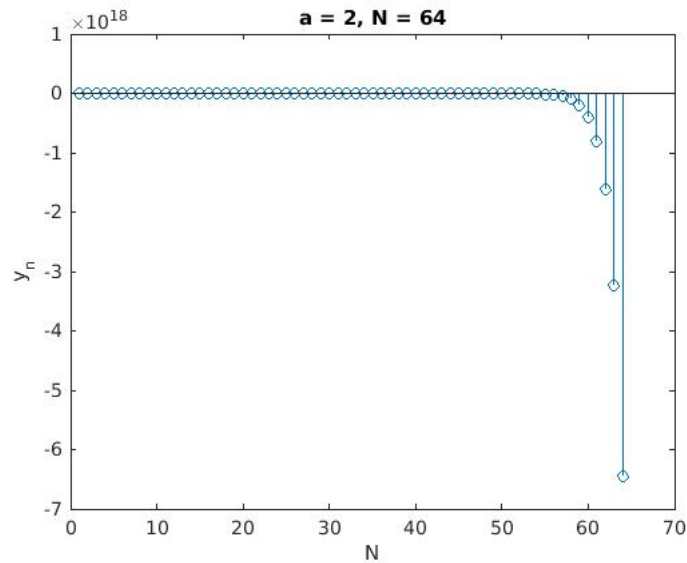
7 %get shifted delta function
  x_n_1 = zeros(1,N);
9 x_n_1(2) = 1;

11 y_n = zeros(1,N+1);

13 for i = 1:N,
    y_n(i+1) = a.*y_n(i) + 0.3.*x_n(i) -2.*x_n_1(i);
15 end
  y_n = y_n(2:length(y_n)); % remove first element to get to N
17 figure;
  stem(y_n);
19 title('a = 2, N = 64');
  ylabel('y_n');
21 xlabel('N');
  end

```

sys1.m



Notice that this function is in the order of a negative exponential, therefore, it is not stable $\lim_{N \rightarrow \infty} y_n = -\infty$. Because the system is LSI, we can also say that the sum of $|h[n]|$ over all integers N does not converge to a finite sum. The system is causal, however, by the finite difference equation, output only depends on past or current inputs.

Report Item 3

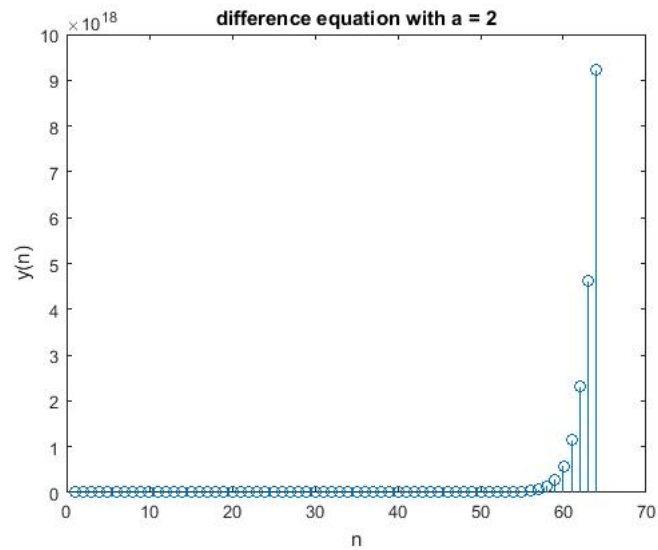
```

1 function [ y_n ] = sys2( x_n, a )
N = 64; % length always 64
3 y_n = zeros(1,N+1);

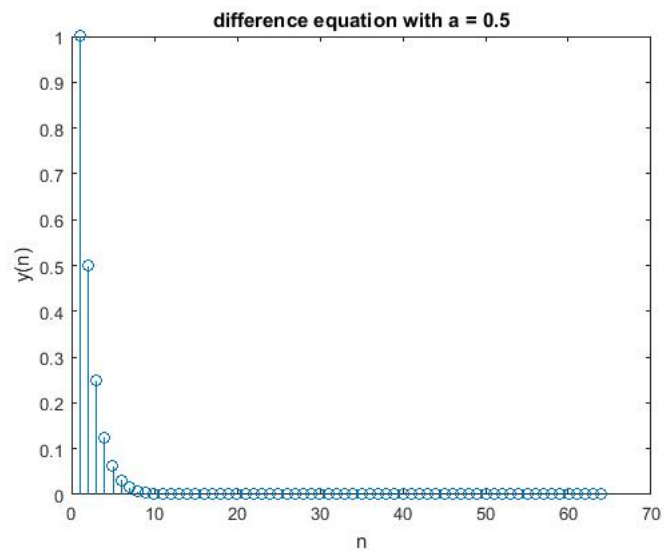
5 for i = 1:64,
    y_n(i+1) = a*y_n(i) + x_n(i).^2;
7 end
    y_n = y_n(2:length(y_n)); % remove first element to shorten to
    64
9
    figure;
11 stem(y_n);
    title('difference equation with a = 2');
13 xlabel('n');
    ylabel('y(n)');
15 end

```

sys2.m

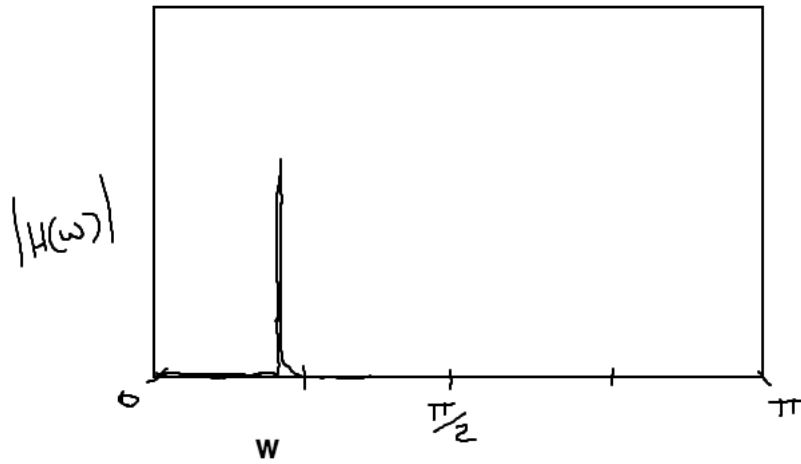


With $a = 2$ and $x(n) = (1, 0, 0, \dots)$, or the delta function, we can see that the equation is not linear because of $x(n)^2$. The system is causal because of the fact that the finite difference equation output only depends on past or current inputs. Because the output graph is in the form of an exponential, the equation is unstable.



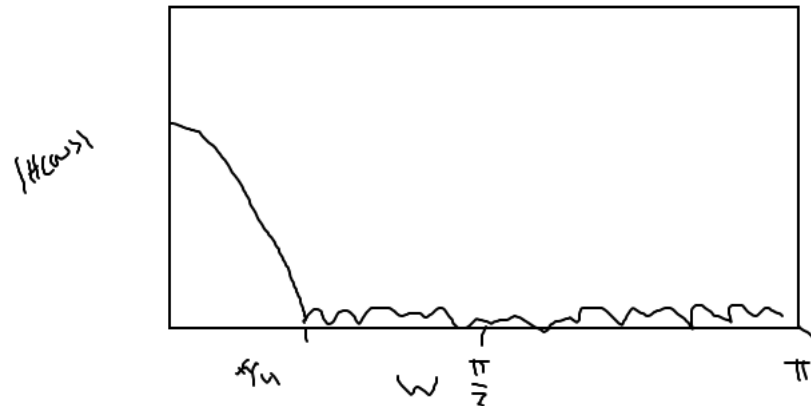
With $a = 0.5$, the equation is not linear, causal, and is stable because $\lim_{N \rightarrow \infty} y_n = 0$.

Because the equation is not linear, the system is not LSI so you cannot use convolution to find the output to either system.



Report Item 4

We can see on the interval from $\omega = 0$ to π , that there is only one point where the magnitude is not zero. Therefore, there is a spike at that point.



We can see on the interval from $\omega = 0$ to π that there is a minimum at $\omega = 0$ and a maximum at $\omega = \pi/4$ so the graph is decreasing until that point.

Report Item 5

```

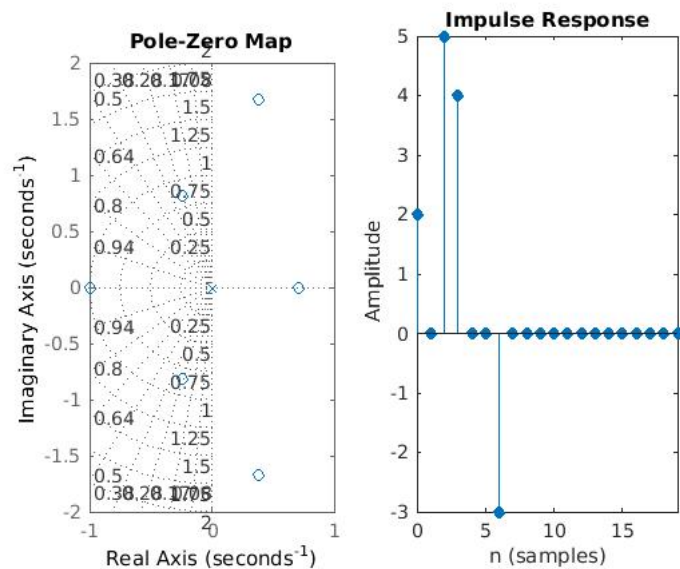
function [ ] = pzplot_impz( a,b )
2 S = tf(b,a);
  N = 20;

4
6 figure;
  subplot(121);

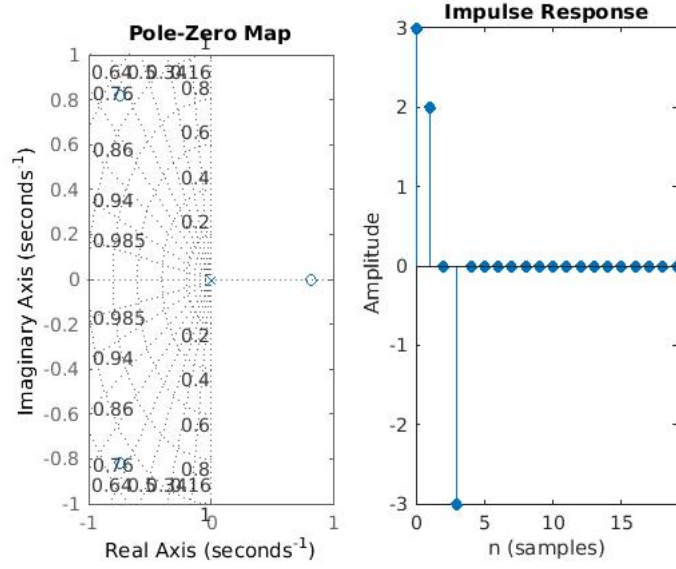
8 pzplot(S);
  grid on;
10 subplot(122);
  impz(b,a,N);
12
end

```

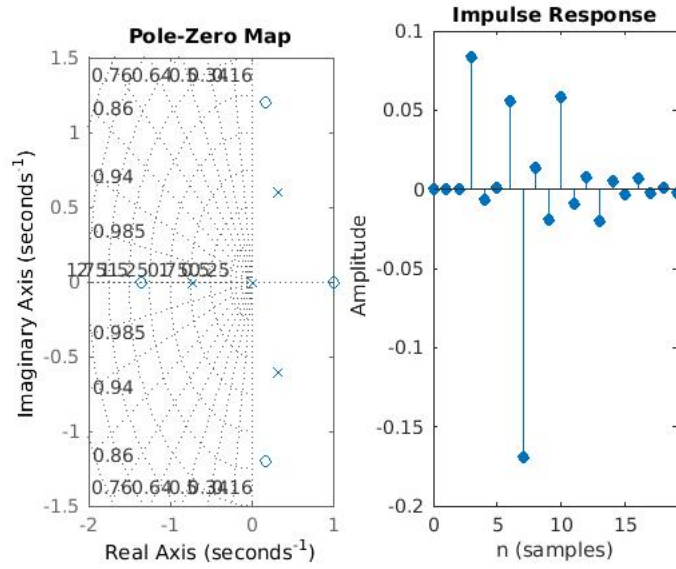
pzplot_impz.m



With $a = [1, 0, 0, 0, 0, 0, 0]$ and $b = [2, 0, 5, 4, 0, 0, -3]$ There are no poles on or outside the unit circle so the system is stable.



With $a = [1, 0, 0, 0]$ and $b = [3, 2, 0, -3]$, there are no poles on or outside the unit circle. Therefore, the system is stable.



With $a = [12, 1, 0, 4, 0, 0, 0, 0]$ and $b = [0, 0, 0, 1, 0, 0, 1, -2]$, there are no poles outside or on the unit circle. Therefore, the system is stable.

Report Item 6

$$\begin{aligned}
 H(z) &= \frac{z}{(z+e^{-\frac{i8\pi}{10}})(z+e^{\frac{i8\pi}{10}})} \\
 &= \frac{z}{z^2+ze^{-\frac{i8\pi}{10}}+ze^{\frac{i8\pi}{10}}+1} \\
 &= \frac{z}{z^2+2z\cos(4\pi/5)+1} \\
 &= \frac{z^{-1}}{1+z^{-1}2\cos(4\pi/5)+z^{-2}} \\
 a &= [1, 2\cos(4\pi/5), 1] \quad b = [0, 1, 0] \quad N = 35
 \end{aligned}$$

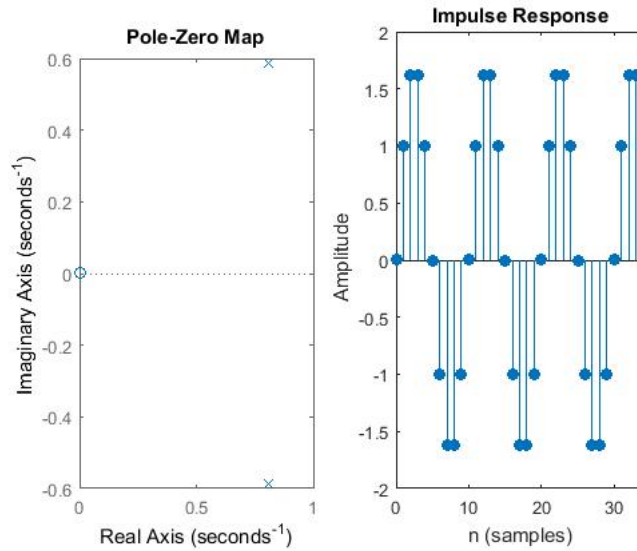
```

function [ ] = pzplot_impz2( a,b )
2 S = tf(b,a);
  N = 35;
4
  figure;
6 subplot(121);

8 pzplot(S);
  grid on;
10 subplot(122);
  impz(b,a,N);
12
end

```

pzplot_impz2.m



The system is not BIBO stable, we have two poles on the unit circle. The system is stable when $x[n]$ does not have poles that overlap with our poles from $h[n]$. The system is unstable when $|z| = -e^{\pm \frac{i8\pi}{10}}$ or whenever $x[n]$ has poles that overlap with the poles from $h[n]$.