ECE220
Honors Lab Section

Lab 7: Dynamic Programming

Problem

- Given some array A[0… n-1], what is the sum of the largest subsequence of size k in the array. Note that a subsequence does not have to be contiguous!
- For example, suppose that A is:

Index	0	1	2	3
Value	1	6	2	5

- What is the largest subsequence of size 2?
- A[1] + A[3] = 6 + 5 = 11

- How can I go about solving this problem?

Algorithm

- How can I formulate this problem recursively?

Index	0	1	2	3
Value	1	6	2	5

- Consider what should happen at each index

$$sub(i,m) \begin{cases} 0 \; if \; i \geq n \vee m \geq k \\ \max \begin{cases} sub(i+1,m), \\ A[i] + sub(i+1,m+1) \end{cases} \end{cases}$$

Proof

- Let's show that the recursive algorithm actually works

Index	0	1	2	3
Value	1	6	2	5

sub(0, 0) = max[sub(1, 0), 1 + sub(1, 1)] = max[11, 6] = 11

sub(1, 0) = max[sub(2, 0), 6 + sub(2, 1)] = max[7, 6 + 5] = 11

sub(1, 1) = max[sub(2, 1), 2 + sub(2, 2)] = max[sub(2, 1), 2 + 0] = max[5, 2] = 5

sub(2, 0) = max[sub(3, 0), 2 + sub(3, 1)] = max[5, 2 + 5] = 7

sub(2, 1) = max[sub(3, 1), 2 + sub(3, 2)] = max[sub(3, 1), 2 + 0] = max[5, 2] = 5

sub(3, 0) = max[sub(4, 0), 5 + sub(4, 1)] = max[0, 5] = 5

sub(3, 1) = max[sub(4, 1), 5 + sub(4, 2)] = max[0, 5] = 5

- 11 is the answer we were looking for!

Memoization

- How can we eliminate recursive calls?
- Store values into an array instead. Essentially replace '()' with '[]' in the recursive definition.

$$sub(i,m)\begin{cases} 0\ if\ i \geq n \vee m \geq k \\ \max\begin{cases} sub(i+1,m), \\ A[i] + sub(i+1,m+1) \end{cases} \end{cases}$$

Index	0	1	2	3
Value	1	6	2	5

	n				
k	11	11	7	5	0
	6	6	5	5	0
	0	0	0	0	0

Iterative solution

	n				
k	11	11	7	5	0
	6	6	5	5	0
	0	0	0	0	0

```
sub(A, n,k)
    int arr_sub[k+1][n+1] = {0};

    for (int i = 0; i < k; i++)
        for (int j = 0; i < n; j++)
            arr_sub = max(arr_sub[i][j+ 1], A[j] + arr_sub[i+ 1][j + 1])

    return arr_sub[0][0];
```

Other, Lab 7, and Lab Help

- Fibonacci
  - Use an array to store the previous values
- My recent CS 374 homework (if you guys want to see)
- Lab 7 demo!
  - Conway's Game of Life
- Otherwise, there won't be any office hours this week so if you have any questions regarding the labs, ask now!