PSHTRG - Editorial

10

PROBLEM LINK:

Div1, Div2
Practice

5

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DIFFICULTY:

Medium

PREREQUISITES:

Segment tree

PROBLEM:

You are given an array with N elements. You need to support two types of operations on this array. First one is point update. For the second operation, you will be given query range l, r and you need to find the maximum possible perimeter of a triangle with non-zero area whose sides are A_x, A_y, A_z, where $l \leq x < y < z \leq r$. If no triangle can be formed, you just need to output 0.

QUICK EXPLANATION:

Build a segment tree where each node stores largest K elements from the interval. Value of K for given constraint will be around 50. Update function on the tree can now be supported in $O(KlogN)$.

Claim: For a given range, triangle with maximum perimeter can be formed using largest K elements from that interval. Answer will be 0, only for the case, when there will be less than K elements in the interval and no triangle formation is possible.

You can query for largest K elements in $O(KlogN)$. Finding the triangle with maximum perimeter afterward is relatively easy and procedure for the same can be found in detailed explanation.

EXPLANATION:

Let's try to solve a simpler version first.

Solve a simple version first:
In this version, you are given an array of N integers and you need to find the maximum possible perimeter of a triangle.
To solve this problem, first, we sort our array in non-ascending order. Next, we iterate over each $A[i]$ (where $1 \leq i < N-1$) and fix the longest side, $c = A[i]$. To satisfy the condition $a + b > c$ and maximize $a + b + c$, we must choose $b = A[i+1]$ and $a = A[i+2]$. $a + b + c$ will be the largest perimeter for minimal i satisfying the condition $a + b > c$. The answer will be 0 if no such i exist.

Claim: If $N > K$, answer always exist and can be found using largest K elements, i.e. $i \leq K$. Here $K = O(log_\phi(\sqrt{5}.MAX))$, where $\phi = \frac{1+\sqrt{5}}{2}$ and MAX is the maximum value which can be present in the array.
Proof: Lets say you keep adding elements in an array in increasing order such that no triangle formation is possible at any instant. Say there are r elements currently in the array, when you add $(r+1)^{th}$ element, it must not satisy triangle inequality with any elements already present in the array. Hence we can write,

$$A[r+1] \geq A[r] + A[r-1]$$

If we keep on adding elements in this manner, we will end up on fibonacci sequence. Fibonacci numbers grows very fast and soon they will exceed the maximum value that can be present in the array. If we do not exceed the maximum value that can be present in the array, triangle can definitely be formed using K elements. You can compute the exact value of K, but we will just use approximation here for our purpose:

$$fib(K) > MAX$$
$$\Rightarrow \frac{\phi^K}{\sqrt{5}} > MAX \text{ (Using approximation)}$$
$$\Rightarrow K > log_\phi(\sqrt{5}.MAX)$$

Conclusion: For our original purpose, we will only need largest K elements from the query range. We need a Data Structure that can support point update and can report largest K elements from any query range in logarithmic time complexity or similar. For current constraints $K = 50$.

Using segment tree for our purpose

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We can perform our required operation with the help of segment tree. In each node of segment tree, you need to store largest K elements from that interval. If size of interval is less than K then store all elements from that interval.

Merge function for two nodes can be implemented in a manner similar to merge sort algorithm. Merging of two nodes can be done in $O(K)$. For each update/query, you will need to change/visit atmax $logN$ nodes. Hence the time complexity for each update/query will be $O(KlogN)$. For more implementation details, you can have a look at attached solutions.

Time Complexity:
$O(Q.K.logN)$

AUTHOR'S AND TESTER'S SOLUTIONS
Setter's solution
Tester's solution

mergesort medium pshtrg march18 segment-tree editorial triangle

This question is marked "community wiki".

edited 13 Mar, 15:04
admin ♦♦
[19.0k]•348•495•531

asked 05 Mar, 18:21
7★ adkroxx
[306]•7•18
accept rate: 7%

4 You can also simply use Max Segment Tree: Query for the largest element, remove it (by setting the value to -infinity), and repeat it K times. Afterwards you just restore the values. This is also $O(K \log n)$ per query.

4★ gdisastery1 (12 Mar, 15:54)

3 very nice editorial @adkroxx

4★ droy0528 (12 Mar, 18:07)

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12 Wouldn't a much simpler solution be to just make a segtree which returns the index of the max element in a range? After extracting the max index, we can save it and then point update it to 0 and then do the max query again to get the second largest. This is $O(KlogN)$ query and $O(logN)$ update but is much simpler.

link | award points

answered 12 Mar, 18:07
2★ ista2000 ♦
[2.3k]•4•19
accept rate: 20%

1 Wow such a different idea . Never thought of this.

6★ sdssudhu (12 Mar, 20:18)

4 I knew about this solution too. But for me something else was simpler, hence I decided to describe that solution. But anyway good job. :)

7★ adkroxx (12 Mar, 23:26)

1 Ah, yes the intended solution uses something different too. Its a cross between a segtree and a merge sort tree. Thanks for describing the intended one. :D

2★ ista2000 ♦ (14 Mar, 22:30)

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1 I have used square root decomposition for the problem. https://www.codechef.com/viewsolution/17760397

link | award points

answered 12 Mar, 18:59
5★ nilmadhab1
[11]•2
accept rate: 0%

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1 I am not getting how this O(Q*N) submission of mine is getting 100 points. My approach : First I created a vector of pair in which i stored every element's value and index. Then I sorted this vector. For first query I used an algorithm similar to one used in insertion sort. And for query 2 I just traversed the array from backward and checked the conditions for an element, whether its index is between l and r .

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edited 13 Mar, 11:31

answered 12 Mar, 23:06
5★ saurabh18213
[11]•2
accept rate: 0%

The only reason I can think of -
Weak test cases (lengths of required(valid) triangle were very close to r) i.e. **number of queries giving required triangle in less number of iterations >> number of queries giving required triangle in more number of iterations**

4★ shawnbam_96 (14 Mar, 05:05)

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0 Can anybody give a few testcases for my solutions (with complexity $O(QN \log N)$) this or this which gives WA ?

link | award points

answered 12 Mar, 16:17
3★ kayak
[2]•1
accept rate: 0%

BTW, how long does it takes for the rating to change ?

3★ kayak (12 Mar, 16:29)

@saurabh18213 I don't see why not using long may cause trouble because since signed int allows values upto $2^{31} > 3 * 10^9 \geq \text{perimetre}$.

3★ kayak (12 Mar, 16:40)

0

we are not able to access solutions.

link | award points

answered **12 Mar, 16:19**
2★ hideintree
[1]●1
accept rate: 0%

0

@kayak use long long for calculating answer as the answer can be the sum of three maximum integers, which will result in overflow.

link | award points

answered **12 Mar, 16:28**
5★ saurabh18213
[11]●2
accept rate: 0%

0

Having a priority queue at each node of the segment tree would be easier I guess, my solution . Also $K = 45$ worked fine.

link | award points

answered **12 Mar, 16:46**

5★ bhishma
[296]●7
accept rate: 12%

0

Solutions are not visible :(

link | award points

answered **12 Mar, 17:15**
3★ b123eginner
[11]●1
accept rate: 0%

0

@kayak 2^31 is less than 3*10^9.

link | award points

answered **12 Mar, 19:08**
5★ praveenkumar12
[170]●7
accept rate: 0%

0

Can I get a link to nice tutorial on Segment tree ? Btw I passes this question using sqrt decomposition. https://www.codechef.com/viewsolution/17743850 With little optimisation.

link | award points

answered **12 Mar, 19:25**
4★ aryanc403
[977]●2●11
accept rate: 15%

1 http://codeforces.com/blog/entry/15890

4★ kailashnath199 (12 Mar, 21:40)

https://www.codechef.com/viewsolution/17834749 @kailashnath199 Thank you Man. Now, i am able to do this question using segment tree. Thank You.

4★ aryanc403 (14 Mar, 22:35)

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