shubhamgoyal__'s blog

Problem on Möbius function

By **shubhamgoyal__**, history, 21 month(s) ago,

> Can somebody please explain how to use the Möbius function to solve this problem.
> https://www.hackerrank.com/contests/w3/challenges/gcd-product

+21

shubhamgoyal__ 21 month(s) ago 3

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jtnydv25

21 month(s) ago, # | ← Rev. 4 +23

Let f(n,m) denote the number of pairs (x, y), such that $x \le n$, $y \le m$, and gcd(x,y) = 1. WLOG, assume $n \ge m$

Using inclusion exclusion,

$$f(n,m) = \sum_{i=1}^{m} \mu(i) \left\lfloor \frac{n}{i} \right\rfloor \left\lfloor \frac{m}{i} \right\rfloor$$

If we store an array of prefix sums of mobius function, then $f(n, m)$ can be calculated in $O(\sqrt{n})$.

Now let $G(g, n, m)$ be the number of pairs (x, y), such that $x \le n$, $y \le m$, and $gcd(x, y) = g$.

Clearly, $G(g,n,m) = F\left(\left\lfloor \frac{n}{g} \right\rfloor, \left\lfloor \frac{m}{g} \right\rfloor\right)$. There are clearly $O(\sqrt{n})$ different values of this for all the g's

Our required answer is $\prod_{g=1}^{min(N,M)} g^{G(g,N,M)}$. Considering the different ranges in which $G(g, n, m)$ is same, this can be calculated in $O(n)$.

→ Reply


shubhamgoyal__

21 month(s) ago, # ^ | +3

Thanks for the nice explanation.

→ Reply


shas20

8 months ago, # | ← Rev. 2 +20

I solved it little differently. After the step

$$ans = \Pi_{g=1} pow(g, |\{(p, q): gcd(p, q) = g\}|)$$

Now I define a function f, such that for any prime p, $f(p^a) = p$ else $f(n) = 1$. It must be noted that

$$g = \Pi_{d|g} f(d)$$

, Now If I substitute this expression in above expression and rearrange the multiplication we get

$$ans = \Pi \ldots pow(f(x), |\{(p, q): gcd(p, q)\%x = 0\}|)$$

→ Pay attention

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06:38:11

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$$ans = \Pi_{x=1} pow(f(x), |\{(p,q) : gcd(p,q) \% x = 0\}|)$$

which reduces to

$$ans = \Pi_{x=1} pow(f(x), (n / i)(m / i))$$

. Now this is very simple to evaluate.

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