DarthPrince's blog

Algorithm Gym :: Data structures

By DarthPrince, 3 years ago,

Today I want to introduce you some very very useful data structures.

In this lecture, we are trying to improve your data structures skills, stay with us and click on **read more**. Important data structures :

Trees

Trees are one of the most useful data structures.A tree is a connected-acyclic graph.There are too many types of trees, like : rooted trees, weighted trees, directed trees, tries, etc.

Partial sum

There are two types of problems solvable by partial sum.

1.Problems which you are asked to answer some queries about the sum of a part of elements (without modify queries).

Solution of all of this problems are the same. You just need to know how to solve one of them.

Example : You are asked some queries on an array $a_1, a_2, ...a, _n$. Each query give you numbers l and r and you should print $a_l + a_{l+1} + ... + a_r$.

Solution : You need to build another array $s_1, s_2, ..., s_n$ which $s_i = a_1 + a_2 + ... + a_i$ and answer is $s_r - s_{l-1}$.

2.Problems which you are asked to perform some queries asking you to modify a part of elements (without printing queries.)

Solution of all of this problems are the same. You just need to know how to solve one of them.

Example : You need to perform some queries on an array $a_1, a_2, ...a, _n$. Each query give you numbers l, r and v and for each i such that $l \le i \le r$ you should increase a_i by v, and then after performing all queries, you should print the whole array.

Solution : You should have another array $p_1, p_2, ..., p_n$ which, all of its members are initially 0, for each query, you should increase p_l by v and decrease p_{r+1} by v .

An then, for each i, starting from 1 you should increase p_i by p_{i-1}. So, final array would be $a_1 + p_1, a_2 + p_2, ..., a_n + p_n$.

Hard problem of partial sum : Troynacci Query

Disjoint sets

Disjoint sets are also useful data structures. Using them is fast and easy. We use theme in many algorithms, like Kruskal's and Prim's.

Disjoint sets, or DSU (Disjoint Sets Union) as their name, are sum sets. Imagine we have some boxes and some tools and initially each tool is in one box. Mostly, we are given some queries and ask to merge two boxes or print the members of a box or find which box is some tool in.

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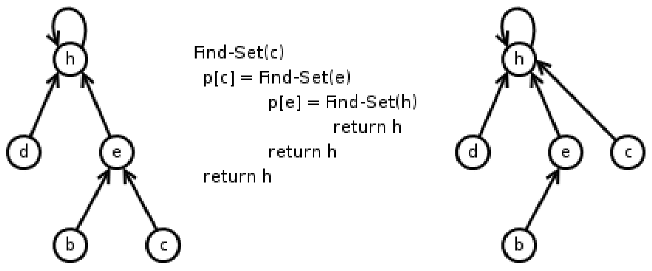
For rest of these, let's consider that initially there is exactly one tool in a box.That is, we have n tools and n boxes and initially, tool number i is in box number i.

For this propose, we can use so many containers.Like :

Trees

Trees are the most useful containers for DSU. For each vertex, we keep it's parent (and parrent of the root is -1). So, initially are parents are set to -1, and we have queries to find the root of each box(having the root, we can easily find the box's index) and queries for merging two trees. For better time complexity, every time we want to find the root of each vertex, we set it's parent to the root for the next queries.And while merging, we always want to minimize the height of the tree, so when we want to merge the boxes, it's like we put all the tools of the box with fewer tools in the other box.



The best way I've seen to code this kind of DSU, is style of **bmerry** : (C++)

```
int root(int v){return par[v] < 0 ? v : (par[v] = root(par[v]));}
void merge(int x,int y){        //      x and y are some tools
(vertices)
        if((x = root(x)) == (y = root(y))     return ;
        if(par[y] < par[x])      // balancing the height of the tree
                swap(x, y);
        par[x] += par[y];
        par[y] = x;
}
```

In the code above, for each root v, $par[v]$ equals the negative of number of tools in that box.

Arrays, vectors

We keep tools in a vector (or an array) and when we have a query to merge two boxes, we put all the tools of the box with fewer tools in the other box.

The time complexity is good because for each tool, we take and put it in an other box at most $log(n)$ times (each time the size of the vector will be at least doubled).

So time complexity would be $O(n.log(n))$.

Sets (red-black trees)

Other way is to keep them in a red-black tree (in C++ it's `set`). We do exactly like vectors, so time complexity would be $O(n.log^2(n))$. (One log is for inserting).

Problems : Hamro and tools, TROY Query (Join the group ACM-OI first)

Tries

Tries are some kind of rooted trees in which each edge has a character on it. Actually, trie is some kind of DFA (Determining Finite Automata). For a bunch of strings, their trie is the smallest rooted tree with a character on each edge and each of these strings can be build by writing down the characters in the path from the root to some node.

It's advantage is, LCP (Longest Common Prefix) of two of these strings is the LCA (Lowest Common Ancestor) of their nodes in the trie(a node that we can build the string by writing

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Detailed →

down the characters in the path from the root to that node).

Generating the trie :

Root is vertex number 0 (C++)

```
int x[MAX_NUMBER_OF_NODES][MAX_ASCII_CODE], next = 1; //initially all
numbers in x are -1
void build(string s){
	int i = 0, v = 0;
	while(i < s.size()){
		if(x[v][s[i]] == -1)
			v = x[v][s[i++]] = next ++;
		else
			v = x[v][s[i++]];
	}
}
```

Problem : A lot of games

Suffix array

Suffix array is a data structure that helps you sort all the suffixes in lexicography order.

This array consists of integers, the beginning of suffixes.

There are two ways to achieve this goal :

One) Non-deterministic algorithm : Use Robin-Carp and for check if a suffix is lexicographically less than another one, find their LCP using binary search + hash and then check the next character after their LCP.

Code :

```
namespace HashSuffixArray
{
	const int
		MAXN = 1 << 21;

	typedef unsigned long long hash;

	const hash BASE = 137;

	int N;
	char * S;
	int sa[MAXN];
	hash h[MAXN], hPow[MAXN];

	#define getHash(lo, size) (h[lo] - h[(lo) + (size)] *
hPow[size])

	inline bool sufCmp(int i, int j)
	{
		int lo = 1, hi = min(N - i, N - j);
		while (lo <= hi)
		{
			int mid = (lo + hi) >> 1;
			if (getHash(i, mid) == getHash(j, mid))
				lo = mid + 1;
			else
				hi = mid - 1;
		}
		return S[i + hi] < S[j + hi];
	}
	void buildSA()
	{
		N = strlen(S);
		hPow[0] = 1;
		for (int i = 1; i <= N; ++i)
			hPow[i] = hPow[i - 1] * BASE;
		h[N] = 0;
		for (int i = N - 1; i >= 0; --i)
			h[i] = h[i + 1] * BASE + S[i], sa[i] = i;

		stable_sort(sa, sa + N, sufCmp);
	}

} // end namespace HashSuffixArray
```

Two) Deterministic algorithm : We sort them $log(MaxLength)$ steps, in the i - th step (counting from 0), we sort them according to their first 2^i characters and put the suffixes whit the same prefix with 2^i characters in the same buckets.

Code :

```
/*
Suffix array O(n lg^2 n)
LCP table O(n)
*/
#include <cstdio>
#include <algorithm>
#include <cstring>

using namespace std;

#define REP(i, n) for (int i = 0; i < (int)(n); ++i)

namespace SuffixArray
{
	const int MAXN = 1 << 21;
	char * S;
	int N, gap;
	int sa[MAXN], pos[MAXN], tmp[MAXN], lcp[MAXN];

	bool sufCmp(int i, int j)
	{
		if (pos[i] != pos[j])
			return pos[i] < pos[j];
		i += gap;
		j += gap;
		return (i < N && j < N) ? pos[i] < pos[j] : i > j;
	}

	void buildSA()
	{
		N = strlen(S);
		REP(i, N) sa[i] = i, pos[i] = S[i];
		for (gap = 1;; gap *= 2)
		{
			sort(sa, sa + N, sufCmp);
			REP(i, N - 1) tmp[i + 1] = tmp[i] +
sufCmp(sa[i], sa[i + 1]);
			REP(i, N) pos[sa[i]] = tmp[i];
			if (tmp[N - 1] == N - 1) break;
		}
	}

	void buildLCP()
```

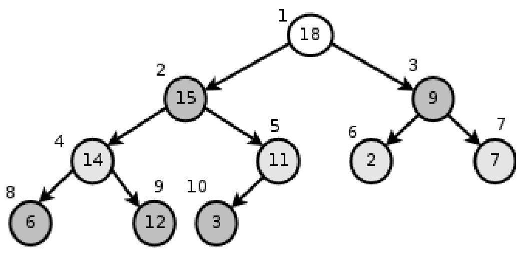
```
        {
                for (int i = 0, k = 0; i < N; ++i) if (pos[i] != N - 1)
                {
                        for (int j = sa[pos[i] + 1]; S[i + k] == S[j +
k];)
                        ++k;
                        lcp[pos[i]] = k;
                        if (k)--k;
                }
        }
} // end namespace SuffixArray
```

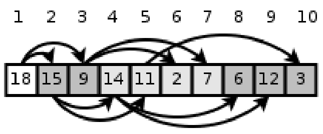
(Codes by **mukel**)

Heaps

A heap is a binary rooted tree (a rooted tree that each node has at most 2 children) and each vertex has a value.

Heap property : Heap usually has a property, like the value of each vertex is equal to or greater than the value of its child(ren) (we call this a max heap). We can use heaps in heap sort.



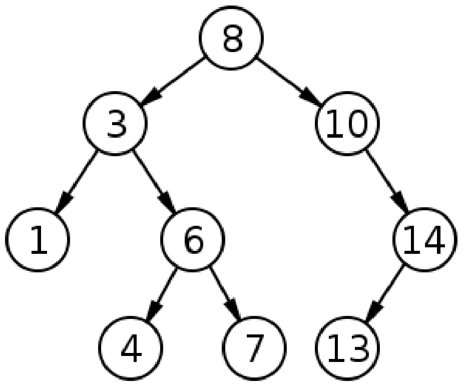


Fibonacci heaps

A fibonacci heap is a kind of heap with better complexities. We don't need to know what a fibonacci heap is.C++ already has one, `priority_queue` .

Binary Search Tree (BST)

A binary search tree (BST) is a binary rooted tree that every node has a value, and for each node, the value of every node in its left child's subtree is less than its value and the value of every node in its right child's subtree is greater than that. Usually we perform some queries on BSTs, like inserting, deleting, asking and ... .



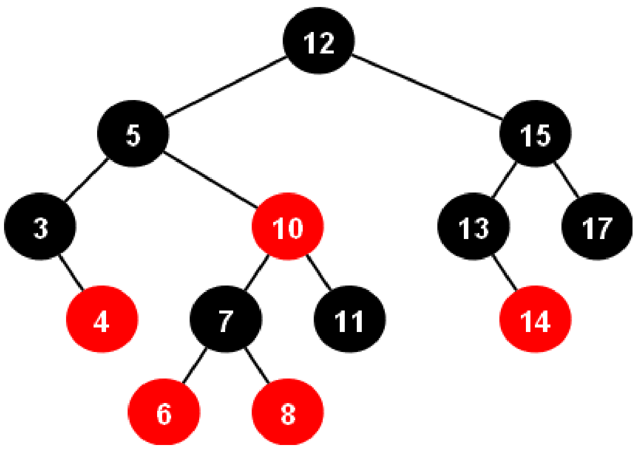
Binary search trees are too useful.

Red-black trees

A red-black tree is a kind of BST that after each query, BST will be balanced in such a way that it's height remains $O(log(n))$.

C++ already has a red-black tree inside, `set` .

You can read about them in C++ references.



Unfortunately, `set` has not any function to find the $k-th$ smallest minimum or find the index of an element, bust there is a data structure in C++ with does it in $O(log(n))$(also contains all `set` functions), `tree` :

```
#include<bits/stdc++.h>
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
using namespace std;
template <typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
tree_order_statistics_node_update>;

int main(){
        ordered_set<int>  s;
        s.insert(1);
        s.insert(3);
        cout << s.order_of_key(2) << endl; // the number of elements in
the s less than 2
        cout << *s.find_by_order(0) << endl; // print the 0-th smallest
number in s(0-based)
}
```

(Thanks to **Swift** for syntax `using` !)

This works even in C++ 98 !

You can read more about it, just google `sgi STL` .

SQRT Decomposition

Suppose we have an array $a_1, a_2, ..., a_n$ and $k = \sqrt{n}$. We partition this array into k pieces each containing k elements of a.

Doing this, we can do a lot of things in $O(\sqrt{n})$. Usually we use them in the problems with modify and ask queries.

Problems : Holes, DZY Loves Colors, RMQ (range minimum query) problem

Sparse Table

The main problem that we can solve is RMQ problem, we have an array $a_1, a_2, ..., a_n$ and some queries. Each query gives you numbers l and r ($l \le r$) and you should print the value of $min(a_l, a_{l+1}, ..., a_r)$.

Solving using Sparse Table : For each i that $1 \le i \le n$ and for each j that $0 \le j$ and $i + 2^j - 1 \le n$, we keep the value of $min(a_i, a_{i+1}, ..., a_{i+2^j-1})$ in $st[i][j]$ (preprocess) : (code is 0-based)

```
for(int j = 0;j < MAX_LOG;j++)
        for(int i = 0; i < n;i ++)if(i + (1 << j) - 1 < n)
                st[i][j] = (j ? min(st[i][j-1], st[i + (1 << (j-1)) - 1]
[j-1]): a[i]);
```

And then for each query, first of all, find the maximum x such that $2^x \le r - l + 1$ and answer is $min(st[l][x], st[r - 2^x + 1][x])$.

So, the main idea of Sparse Table, is to keep the value for each interval of length 2^k (for each k).

You can use the same idea for LCA problem and so many other problems.

So preprocess will be in $O(n.log(n))$ and query will be in $O(1)$

Problems : Strip, GCDSSQ, LCM Query .

Heavy light decomposition

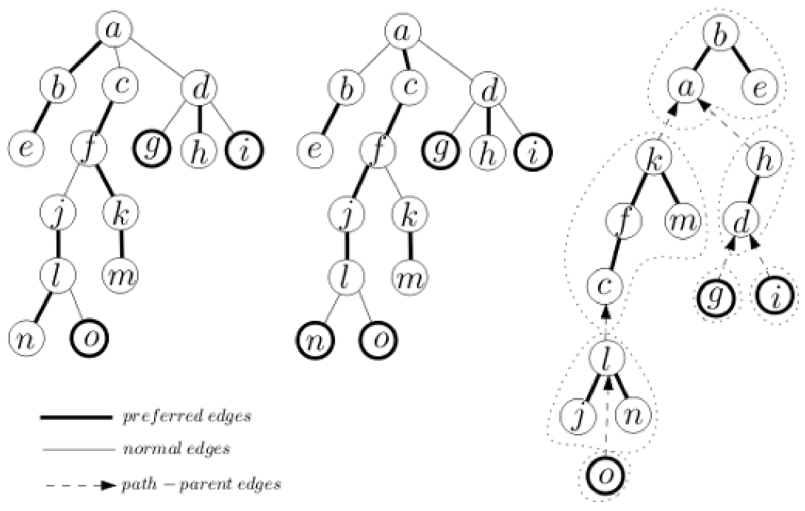
Heavy light decomposition is a way to partition a tree's vertices (or edges) in a good way.

In this kind of decomposition, we have some chains, and each vertex belongs to only one chain.

If vertex v is the parent of u size_of_subtree_of(v)/2 < size_of_subtree_of(u), u and v are in a chain and we call the edge uv, heavy, otherwise light.

There is at most one such child for each vertex v. If we consider the path from any vertex v to the root, there will be at most $log(n)$ light edges there (go from v to the root, every time we see a light edge, size of subtree will be at least doubled). So, the number of chains on the way = $O(log(n))$.

In each of these chains, we can contain a container or another data structure like segment tree or etc.



Problem : GRASS PLANTING

Fenwick

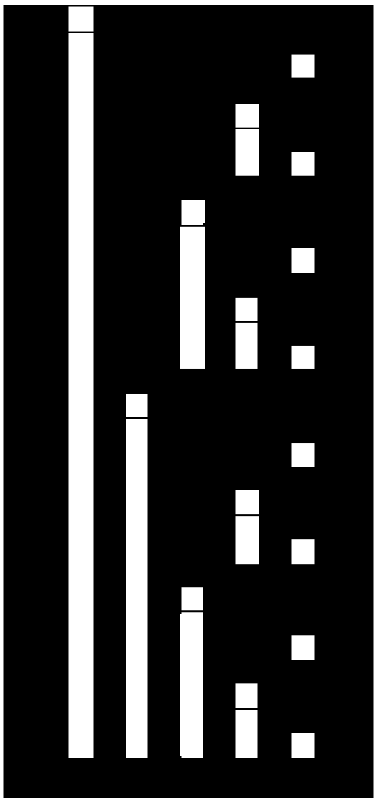
Suppose that we have n elements numbered from 1 to n.

Fenwick or BIT(Binary Indexed Tree) is a data structure with n nodes that node number i has some information about elements in the interval $(i - i\& - i, i]$.

Actually, you don't need to know what each node contains. The only thing you should know, it this (then you can change and convert it) :

We have an array $a_1, a_2, ..., a_n$ and all of them are initially 0. We are gives some queries, 1.increase a_p by val and print $a_1 + a_2 + ... + a_p$.

Only thing you should know is that how to solve this problem using Fenwick (and then you can change it and solve so many problems).



We perform each query in $O(log(n))$. Code : (1-based)

```
int fen[MAX_N];
void update(int p,int val){
	for(int i = p;i <= n;i += i & -i)
		fen[i] += val;
}
int sum(int p){
	int ans = 0;
	for(int i = p;i;i -= i & -i)
		ans += fen[i];
	return ans;
}
```

Please note that it should be **1-based**. It can't be done 0-based.

Problems : Inversions, Pashmak and Parmida's problem, BST .

Segment tree

We have an array of elements and some queries on intervals. So, we will be glad if we can split this interval to $O(log(n))$ intervals that we have actually some information about them.

Segment tree does that for you. Segment tree is a tree that each of it's nodes belongs to an interval.

Root of the tree belongs to the interval $[0, n)$ (0-based).

Each node, has 0 or two children. Left and right. If a node's interval is $[l, r)$ and $l+1 \neq r$, the interval of its children will be $[l, mid)$ and $[mid, r)$ in order where $mid = \frac{l+r}{2}$, so the height of this tree will be $O(log(n))$.

Each node has an index, we consider that root has index 1 and the children of a vertex x will have indices $2x$ and $2x+1$ in order.

Segment tree is the most useful data structure and every problem solvable by Fenwick is also solvable by Segment tree.

If the size of the root's interval is n, segment tree could have up to $4n$ nodes.

To split an interval to some nodes of this tree, we will act like this :

Suppose that S is the set of nodes which their union is $[x, y)$ and no two different nodes in S have nonempty intersection.

A node i with interval $[l, r)$ is in S if and only if $x \leq l \leq r \leq y$ and if it has a parent with interval $[b, e)$, $x > l$ or $r > y$.

C++ code :

```
vector<int> s;
void split(int x,int y, int id = 1,int l = 0, int r = n){//     id is
the index of the node
        if(x >= r or l >= y)    return ;        // in this case,
intersect of [l,r) and [x,y) is empty
        if(x <= l && r <= y){
                s.push_back(id);
                return ;
        }
        int mid = (l+r)/2;
        split(x,y,id * 2,l,mid);
        split(x,y,id * 2 + 1,mid,r);
}
```

Example :

We have an array $a_1, a_2, ..., a_n$ and q queries. There are 2 types of queries.

1. $S\ l\ r$, Print $a_l, a_{l+1}, ..., a_r$
2. $M\ p\ x$, Modify a_p to x, it means $a_p = x$.

First of all we need to build the segment tree, for each node we keep the sum of its interval, for node i we call it $s[i]$, so we should build the initial segment tree.

```
void build(int id = 1,int l = 0,int r = n){
        if(r - l < 2){  //      l + 1 == r
                s[id] = a[l];
                return ;
        }
        int mid = (l+r)/2;
        build(id * 2, l, mid);
        build(id * 2 + 1, mid, r);
        s[id] = s[id * 2] + s[id * 2 + 1];
}
```

So, before reading the queries, we should call $build()$.

Modify function :

```
void modify(int p,int x,int id = 1,int l = 0,int r = n){
        s[id] += x - a[p];
        if(r - l < 2){  //      l = r - 1 = p
                a[p] = x;
                return ;
        }
        int mid = (l + r)/2;
        if(p < mid)
                modify(p, x, id * 2, l, mid);
        else
                modify(p, x, id * 2 + 1, mid, r);
}
```

(We should call $modify(p,\ x)$)

Ask for sum function :

```
int sum(int x,int y,int id = 1,int l = 0,int r = n){
        if(x >= r or l >= y)    return 0;
        if(x <= l && r <= y)    return s[id];
        int mid = (l+r)/2;
        return sum(x, y, id * 2, l, mid) +
               sum(x, y, id * 2 + 1, mid, r);
}
```

(We should call $sum(l,\ r)$)

Lazy propagation

Imagine we have updates on intervals, what should we do ?

Example :

We have an array $a_1, a_2, ..., a_n$ and q queries. There are 2 types of queries.

1. $S\ l\ r$, Print $a_l, a_{l+1}, ..., a_r$
2. $I\ l\ r\ x$, for each i such that $l \le i < r$, increase a_i by x.

We shouldn't update all the nodes in this interval, just the maximal ones, then pass it to children when we need. This trick is called **Lazy Propagation**, so we should have another array $lazy$ (for nodes) which are initially 0 and every time we want to perform increase query, increase $lazy[id]$ with x.

As above, we also should have an array s for nodes.

So, $build$ function will be same as above. But we need some more functions :

A function to update a node :

```
void upd(int id,int l,int r,int x){//   increase all members in this
interval by x
        lazy[id] += x;
        s[id] += (r - l) * x;
}
```

A function to pass the update information to its children :

```
void shift(int id,int l,int r){//pass update information to the children
        int mid = (l+r)/2;
        upd(id * 2, l, mid, lazy[id]);
        upd(id * 2 + 1, mid, r, lazy[id]);
        lazy[id] = 0;// passing is done
}
```

A function to perform increase queries :

```
void increase(int x,int y,int v,int id = 1,int l = 0,int r = n){
        if(x >= r or l >= y)    return ;
        if(x <= l && r <= y){
                upd(id, l, r, v);
                return ;
        }
        shift(id, l, r);
        int mid = (l+r)/2;
        increase(x, y, v, id * 2, l, mid);
        increase(x, y, v, id*2+1, mid, r);
        s[id] = s[id * 2] + s[id * 2 + 1];
}
```

(We should call $increase(l\ r\ x)$)

A function to answer to queries asking about the sum :

```
int sum(int x,int y,int id = 1,int l = 0,int r = n){
        if(x >= r or l >= y)    return 0;
        if(x <= l && r <= y)    return s[id];
        shift(id, l, r);
        int mid = (l+r)/2;
        return sum(x, y, id * 2, l, mid) +
               sum(x, y, id * 2 + 1, mid, r);
}
```

(We should call $sum(l,\ r)$)

Problems : GSS1, GSS3, MULTQ3, DQUERY, KQUERY, POSTERS, PATULJCI, New Year Domino, Copying Data, DZY Loves Fibonacci Numbers, FRBSUM

Persistent data structures

Consider we have some elements, you perform some updates on it and then,and after performing all of them, you want to have the information about the elements, after each update.

For this propose, you got a data structure and somehow, you save the version of that data structure.

The most useful data structure for this propose is segment tree, I will explain persistent segment tree and all other data structures (like Fenwick) are like that.

Persistent segment tree

Example problem :

We have an array $a_1, a_2, ..., a_n$ and at first q update queries and then u ask queries which you have to answer online.

Each update query gives you numbers p and v and asks you to increase a_p by v .

Each ask query, gives you three numbers i and x and y and asks you to print the value of $a_x + a_{x+1} + ... + a_y$ after performing i - th query.

Each update query, changes the value of $O(log(n))$ nodes in the segment tree, so you should keep rest of nodes (not containing p) and create $log(n)$ new nodes. Totally, you need to have $q.log(n)$ nodes. So, you can not use normal segment's indexing, you should keep the index of children in the arrays L and R.

If you update a node, you should assign a new index to its interval (for i - th query).

You should keep an array $root[q]$ which gives you the index of the interval of the root ($[0, n)$) after performing each query and a number $ir = 0$ which is its index in the initial segment tree (ans of course, an array $s[MAX_{NODES}]$ which is the sum of elements in that node). Also you should have a *NEXT_FREE_INDEX = 1* which is always the next free index for a node.

First of all, you need to build the initial segment tree :

(In these codes, all arrays and queries are **0-based**)

```
void build(int id = ir,int l = 0,int r = n){
        if(r - l < 2){
                s[id] = a[l];
                return ;
        }
        int mid = (l+r)/2;
        L[id] = NEXT_FREE_INDEX ++;
        R[id] = NEXT_FREE_INDEX ++;
        build(L[id], l, mid);
        build(R[id], mid, r);
        s[id] = s[L[id]] + s[R[id]];
}
```

(So, we should call $build()$)

Update function : (its return value, is the index of the interval in the new version of segment tree and id is the index of old one)

```
int upd(int p, int v,int id,int l = 0,int r = n){
        int ID =  NEXT_FREE_INDEX ++; // index of the node in new
version of segment tree
        if(r - l < 2){
                s[ID] = (a[p] += v);
                return ID;
        }
        int mid = (l+r)/2;
        L[ID] = L[id], R[ID] = R[id]; // in case of not updating the
interval of left child or right child
        if(p < mid)
                L[ID] = upd(p, v, L[ID], l, mid);
        else
                R[ID] = upd(p, v, R[ID], mid, r);
        return ID;
}
```

(For the first query (with index 0) we should run $root[0] = upd(p,\ v,\ ir)$ and for the rest of them, for j - th query se should run $root[j] = upd(p,\ v,\ root[j-1])$)

Function for ask queries :

```
int sum(int x,int y,int id,int l = 0,int r = n){
        if(x >= r or l >= y)    return 0;
        if(x <= l && r <= y)    return s[id];
        int mid = (l+r)/2;
        return sum(x, y, L[id], l, mid) +
               sum(x, y, R[id], mid, r);
}
```

(So, we should print the value of $sum(x, y, root[i])$)

Problems : Sign on Fence, MKTHNUM, COT, The Classic Problem

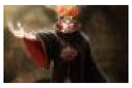
Statements of KBO preparation kamp day2 (string)

algorithms, **data structures**, **tutorial**

+608 DarthPrince 3 years ago 116

Comments (116)

Write comment?

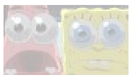


Kei.One

3 years ago, # | **+3**

Thank you for this Useful toturial

→ Reply



Majid

3 years ago, # | -11

thanks :)

→ Reply

nic11

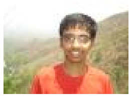
3 years ago, # | ← Rev. 3 **+27**

Thanks for the article, especially for problems!

I didn't know that priority_queue is a Fibonacci heap. BTW, are you sure? cplusplus.com and cppreference say that push() works in logarithmic time.

P.S.1 This link (Hamro and tools problem in DSU section) gives access error: link
P.S.2 Isn't "sparse" a correct spelling?

→ Reply



nitishch

2 years ago, # ^ | **+3**

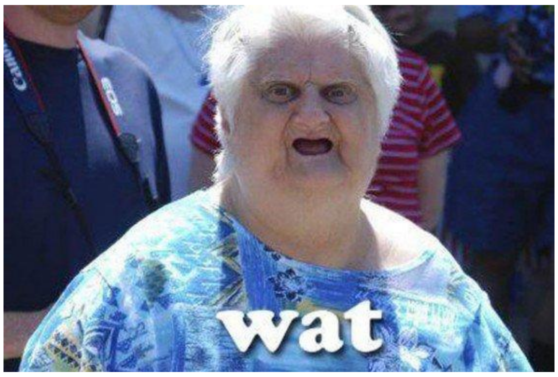
As he said, you have to register youself in the ACM-OI group.

→ Reply

nic11

2 years ago, # ^ | **0**

.



→ Reply

atique

new, 2 months ago, # ^ | **0**

@PrinceOfPersia, thanks for the great article. Do you have any other article on Fibonacci Heap, do you mind extending on the topic a bit with details such as inner working and implement of fibonacci heap?

→ Reply

edogrigqv2

3 years ago, # | **0**

Great intro. Still have 1 question. Does order_set works only in g++? How it is going to work in Visual C++ ?

→ Reply

adamant

3 years ago, # ^ | ☆ 0

Does order_set works only in g++

Yes. It's part of SGI STL extensions of G++.

→ Reply

adamant

3 years ago, # | ☆ +16

Btw, `tree` isn't part of C++ standard, it is extension of GNU C++. You can read more about this data structures here and here :)

Also about persistent data structures. One can be interested in this: #TphcLk (k-th order statistics in the array segment using persistent bit trie. $O(n \log C)$. Fully online, works with negative numbers also!)

Also you can use this structure to answer all queries from this problem.

→ Reply

speedy03

3 years ago, # ^ | ☆ 0

How to apply your code to the XOR query please?

→ Reply

adamant

3 years ago, # ^ | ☆ +1

My full solution of Xor Queries: #QI8sxL

Xor query can be done greedy — we iterate through implicit bit trie of $l..r$ subsegment and each time we try to take k-th bit in answer which is not equal to k-th bit in query.

→ Reply

speedy03

3 years ago, # ^ | ☆ 0

Got it, thanks! The official solution is also based on trie. Any specific advantages over segment tree?

→ Reply

adamant

3 years ago, # ^ | ☆ 0

Actually segment tree is a kind of trie... Trie just a bit more generic

→ Reply

edogrigqv2

3 years ago, # | ☆ 0

The is probably mistake in DSU implementation. par[x] += par[y] ??? What is this ?

→ Reply

DarthPrince

3 years ago, # ^ | ☆ +1

Read it !

In the code above, for each root v, par[v] equals the negative of number of tools in that box.

So, par[x] = -sizeofbox(x), par[y] = -sizeofbox(y). so, par[x] + par[y] = -sizeofbox(x unuion y).

→ Reply

edogrigqv2

3 years ago, # ^ | ☆ 0

Have I understood it correct now ? par[v] shows the parent of v, if v is not the root, otherwise it shows negative number of nodes in group. 2 in 1 array!!!

→ Reply

DarthPrince

3 years ago, # ^ | ☆ 0

Yep !

→ Reply

natsukagami

3 years ago, # ^ | ☆ 0

Actually there's an official name for the technique. It's called `union by rank`. More info can be found here.

→ Reply

sheri.mori

3 years ago, # ^ | ☆ 0

Why are some comments in blue rectangles in codeforces (like the comment above) ?

→ Reply

VKundas

3 years ago, # ^ | ☆ 0

It is new comments that you hadn't read before. If you update webpage it'll be like any other comment

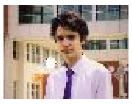
→ Reply

sheri.mori

3 years ago, # ^ | ☆ 0

OK tnx :-)

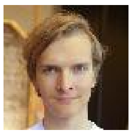
→ Reply


ikbal

3 years ago, # | ☆ +25

I thought c++ priority queues are binary heap. As far as I remember Fibonacci heaps has huge constant factor. Could you share source where you get this information?

→ Reply


yeputons

3 years ago, # ^ | ☆ +24

cplusplus.com thinks that `priority_queue` is just a wrapper around push_heap/ `pop_heap` / `make_heap`, which works with standard binary heap in a random access container and have logarithmic complexity.

→ Reply

edogrigqv2

3 years ago, # ^ | ☆ +21

In my Visual C++ 2013 header of I have found make_heap and push_heap that use the same vector.

```
        void push(const value_type& _Val)
                {       // insert value in priority
order
                c.push_back(_Val);
                push_heap(c.begin(), c.end(), comp);
                }
```

Maybe g++ uses Fibonacci heap. But it is no more part of STL. cplusplus.com usually hints if in different compilers something maybe different. Here it only states that priority_queue uses make_heap, push_heap, and pop_heap.

I also have found this.

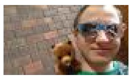
→ Reply

Bugman

3 years ago, # | ☆ 0

Fenwick can be 0-based! Change `i+=i&-i` to `i|=i+1` and change `i-=i&-i` to `i=(i&(i+1))-1`

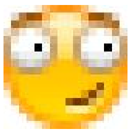
→ Reply


Swistakk

3 years ago, # ^ | ☆ ← Rev. 2 +7

Why would I want to shoot in my foot :P?

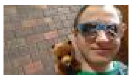
→ Reply


adamant

3 years ago, # ^ | ☆ 0

Because e-maxx says so :)

→ Reply


Swistakk

3 years ago, # ^ | ☆ +40

Fenwick can be 12-based. `i+=i&-i` -> `i += ((i-11)&-(i-11))` :P

→ Reply

Bugman

3 years ago, # ^ | ☆ +5

:)

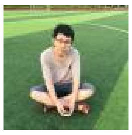
→ Reply

OmaeWaMouShenDeiru

3 years ago, # | ☆ 0

just in time :D thanx

→ Reply


meijun

3 years ago, # | ☆ ← Rev. 4 +3

At "Arrays, vectors", you said, "So time complexity would be O(n.log(n)) ."

However, I think the time complexity is O(n). The count of copied item is (1 + 2 + 4 + ... + N) where N < n and N = 2^k k is non-negative integer. So the count of copied item is (N * 2 — 1), if original array-modify involved, the count is (N * 2 — 1 + n). Time complexity is O(n).

→ Reply

tom

3 years ago, # ^ | ☆ 0

Yes, and we can simply do $vector.reserve(N)$ and problem disappears at all.

→ Reply

DarthPrince

3 years ago, # ^ | ☆ +9

$n = O(n.log(n))$, I didn't say $\Theta(n.log(n))$

→ Reply

remidinishanth

new, 6 weeks ago, # ^ | ☆ 0

And I think, the complexity will be O(nlogn) in cases where the merge operation is done like merge sort? And if we consider the merging two sets of size s1 and s2 takes O(s1+s2), is this the case?

→ Reply

remidinishanth

new, 6 weeks ago, # ^ | ☆ 0

In arrays, vectors what exactly we are doing can you please elaborate? Are we trying to implement Union-find DS using arrays or vectors? Are we using a vector at each index? So that at max each vertex has to placed O(logn) times? Thank you.

→ Reply

3 years ago, # | ☆ 0

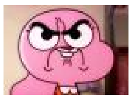
In case 2 of partial sum, can you apply your formula to binary index tree if sum speedy03

In case 2 of partial sum, can you apply your formula to binary index tree if sum within the range [l,r] is queried?
→ Reply

DarthPrince

3 years ago, # ^ | ☆ ← Rev. 3 0

Yes. Just increase the value of a_l by v and increase the value of a_{r+1} by $-v$ (as I said in Fenwick).
→ Reply


joaquingc123

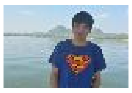
3 years ago, # | ☆ -9

Thank you >:)
→ Reply

erikgrabljevec5

3 years ago, # | ☆ -9

Amazing! Thank you so much.
→ Reply


AtomRush

3 years ago, # | ☆ 0

How to solve inversion count using BIT
→ Reply

tom

3 years ago, # ^ | ☆ 0

For every number check how many numbers are before it? To do this, add numbers to BIT sorted from the largest to smallest.
→ Reply

Shayan.To

3 years ago, # ^ | ☆ 0

Can you explain more?
→ Reply

surjection

3 years ago, # ^ | ☆ -8

This blog post explains how to use a BIT for counting inversions.
→ Reply

cristi.dospra

3 years ago, # | ☆ -8

Thank you very much for the great tutorial! :D
→ Reply

LittleDreamer

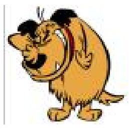
3 years ago, # | ☆ -13

good read. thank you!
→ Reply

amarveer

3 years ago, # | ☆ -8

Thank you
→ Reply


jairsaidds

3 years ago, # | ☆ 0

Dou you have any example in code of TRIES?

Thanx.
→ Reply

3 years ago, # ^ | ☆ 0

Refer this and this.
→ Reply

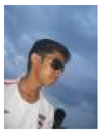
xpertcoder → Reply

Sonechko

3 years ago, # | ☆ ← Rev. 2 0

/

→ Reply


sachithg

3 years ago, # | ☆ 0

Nice blog/tutorial.. it would be useful for beginners as they will get an idea of what all to study.... Would have been nice if something like this was there when I started...

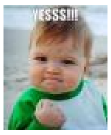
→ Reply

vaishious

3 years ago, # | ☆ -8

Good stuff. Cheers

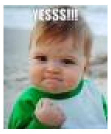
→ Reply


buGMaster

3 years ago, # | ☆ ← Rev. 2 0

Hi!
I don't get how suffix array (deterministic version) works.
Could you please give me some more clear description about it?
What does "tmp" store? It seems it contains something like [0,1,...,N-1], doesn't it?!
What about "pos"?
What's the initialization of "tmp"?
Thanks...

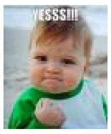
→ Reply


buGMaster

3 years ago, # ^ | ☆ ← Rev. 2 0

?!

→ Reply


buGMaster

3 years ago, # | ☆ 0

http://codeforces.com/blog/entry/16541

→ Reply

sazzad8867

3 years ago, # | ☆ ← Rev. 3 0

Another problem of partial sum http://codeforces.com/problemset/problem/433/B

→ Reply

CrazzyBeer

3 years ago, # | ☆ ← Rev. 11 0

Speaking of DSU, here's the same function, but in a more understandable way.

```
int root(int v){
  while (par[v]!=-1) {
     par[v]=par[par[v]];
     v=par[v];
  }
  return v;
}
```

UPD: This is a way to compress paths, but not as efficient as the presented algorithm.

→ Reply

Koderok

3 years ago, # ^ | ☆ 0

If you observe carefully, the code actually does update all the nodes in the path which is what you want to do.

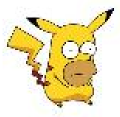
→ Reply

Reyna

3 years ago, # ^ | 0

the actual code compresses better All of the nodes parent will be the root but in crazzybeer's code it doesnt work also there is a closed bracket which leads to infinite loop

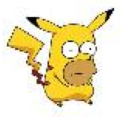
→ Reply


CrazzyBeer

3 years ago, # ^ | 0

Mistake fixed. Now it should work. (Actually, this is the algorithm from Coursera so it works pretty well)

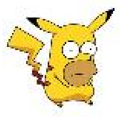
→ Reply


CrazzyBeer

3 years ago, # ^ | 0

It works, but, I guess, it does a bit less work than the presented algorithm.

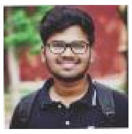
→ Reply


CrazzyBeer

3 years ago, # ^ | 0

I see now. Thanks.

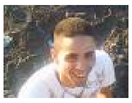
→ Reply


adnanSharif

3 years ago, # | 0

Bookmarked! Thanks for this tutorial! :)

→ Reply


ypizarroza

3 years ago, # | 0

Thank you for this Useful toturial.

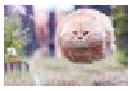
→ Reply

suraj021

3 years ago, # | 0

What does the modify(p, x) function do in the Segment tree section ?

→ Reply


kaifa

3 years ago, # ^ | 0

"Modify a_p to x, it means $a_p = x$."

→ Reply

suraj021

3 years ago, # ^ | 0

Thanks

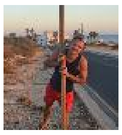
→ Reply

sheri.mori

3 years ago, # | 0

"The only thing you should know, **it** this (then you can change and convert it) :" I think "it" has to be "is",right?

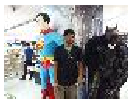
→ Reply


dev_il

3 years ago, # | 0

Can please someone explain Partial sum part 2? What is the problem it solve? We need to find sum of elements of array a with indexes [l, r) ?

→ Reply


tirupati

3 years ago, # ^ | ← Rev. 2 0

Partial Sum part 2 solves the problem where you are given an array and all you have to do is to print the array after performing q queries where each query asks you to change all the elements between [l,r]. Say you have an array of size of the order 10^5 and you have 10^5

queries. Now Naive method is to iterate over all the r-l+1 elements for

queries. Now Naive method is to iterate over all the r-l+1 elements for each query hence the solution would be O(n^2). So according to part 2 of the solution all you do is to update two elements for each query and O(n) in the end. So the solution would be O(n).
→ Reply

HitmanBBa
3 years ago, # | ☆ 0

Thanks PrinceOfPersia for this very very useful article. :)

Note: please edit `line 1` after R/B Tree image `bust` --> `but` . :)
→ Reply

manuag
3 years ago, # | ☆ 0

Can someone please elaborate the method used for building trie? I tried to dry run it on some test cases but could not make out how the resulting array is a trie. Thanks.
→ Reply

manuag
3 years ago, # ^ | ☆ 0

Got it!
→ Reply

Bazinga112
3 years ago, # | ☆ 0

Very informative. Thanks a lot.
→ Reply

Inactivated
3 years ago, # | ☆ 0

how can multiple assignment modifications be done this way?please suggest an idea ,source code bingo !!
→ Reply

singh_iitian
3 years ago, # | ☆ ← Rev. 2 0

Is there a way to solve a problem which is same as partial sum instead of adding value, we need to change values in the interval [l,r] to v. Is there any good solution for this.
→ Reply

radoslav11
3 years ago, # ^ | ☆ 0

Segment Tree + Lazy Propagation
→ Reply

slycelote
3 years ago, # ^ | ☆ +13

Astrologers proclaimed the Week of CodeChef Challenge. Amount of similar questions doubled.
→ Reply

Adkay
3 years ago, # | ☆ 0

In The persistent data structure function we should have a

s[id] = s[L[id]] + s[R[id]]; before the return statement
→ Reply

eshaankuls25
2 years ago, # | ☆ 0

If the size of the root's interval is n, segment tree could have up to 4n nodes.How can we justify it? When I constructed an example, I got 2n-1 nodes.In what cases it will have 4n nodes.
→ Reply

alllleksssa

2 years ago, # ^ | ← Rev. 4 +3

Sorry, I didn't explain well in the previous post.

First, interval tree is balanced binary tree. So first level has 1 node, second 2, third 4... i-th level has 2^(i-1) nodes.

If your interval has lenght n, you must add some extra elements till n is not a power of two (condition for balanced tree). Now the last level has m nodes (m=2^k) and higer levels have 2^(k-1) nodes, 2^(k-2) nodes... Sum of all nodes is 2^(k+1)-1. That is equal with 2m -1 nodes. The worst case is when n=2^x+1 and in that case you should have about 4n nodes, in best case if n=2^x you will have 2n-1 nodes.

I hope that now everything is clear.

→ Reply

Arpa

2 years ago, # | 0

thank you for this useful algorithms.

is there any other posts like this?

→ Reply

Arpa

2 years ago, # | 0

in Lazy propagation you had a function shift and function update.

what if we compare them?

like this:

```
inline void add(int st,int en,int v,int l=0,int r=n,int node=1){
  s[node]+=v*(en-st);
  if(l+1==r)
    return;
  else if(st>=mid(l,r))
    add(st,en,v,mid(l,r),r,(node<<1)+1);
  else if(en<=mid(l,r))
    add(st,en,v,l,mid(l,r),node<<1);
  else
add(st,mid(l,r),v,l,mid(l,r),node<<1),add(mid(l,r),en,v,mid(l,r),r,
(node<<1)+1);
}
```

and this is sum function:

```
#define mid(x,y) (x+y)/2
inline int sum(int st,int en,int l=0,int r=n,int node=1){
  if(l==st && r==en)
    return s[node];
  if(st>=mid(l,r))
    return sum(st,en,mid(l,r),r,(node<<1)+1);
  if(en<=mid(l,r))
    return sum(st,en,l,mid(l,r),node<<1);
  return
sum(st,mid(l,r),l,mid(l,r),node<<1)+sum(mid(l,r),en,mid(l,r),r,
(node<<1)+1);
}
```

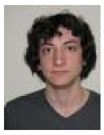
→ Reply

jaswanthi

2 years ago, # | ← Rev. 2 0

In the given Trie code (as posted by PrinceOfPersia) , It is only possible to search for prefix, How can we search if the word exist or not ?

```
public boolean search(String word) {
        int v = 0;
        for(int i = 0; i < word.length(); i++) {
            v  = x[v][word.charAt(i)];
            if(v == -1 )
                return false;
        }
        // THIS WON"T WORK
        return true;
    }
```
⟶ Reply


Diego1149

2 years ago, # ^ | ☆ 0

You can add a boolean array, call it something like ends and initialize it to false, for each word you insert you set ends to true only in the last character of the word, so in your search method change the "return true" line to "return ends[v]". If you insert the word partition, if you look for part, it will return false, instead of true (which is what your code is doing).
⟶ Reply

Dipanker

22 months ago, # | ☆ 0

In the upd() function in the persistent tree, S[ID] has not been updated after the recursive calls. Add s[ID] = S[L[ID]] + S[R[ID]]; before returning ID. Thank You for such a Useful tutorial !
⟶ Reply

aka.Sohieb

22 months ago, # | ☆ 0

You put this problem PATULJCI after explaining the segment tree with lazy propagation and before (Persistent).. but I can not solve it and my friend told me he solved it using (Persistent) segment tree!!!

could you please explain how to solve it without using Persistent ?!
⟶ Reply

DarthPrince

22 months ago, # ^ | ☆ 0

I have no idea how to solve it with persistent!!!
⟶ Reply

aka.Sohieb

22 months ago, # ^ | ☆ ← Rev. 2 +1

This code is my friend's solution (**OmarHashim**) using persistent,

but i wanna know how to solve it without using persistent, could you please explain or give a hint on how to solve it?!!
⟶ Reply

aka.Sohieb

22 months ago, # ^ | ☆ +1

UPD I got a O(N log N) solution with no segment tree at all, thanks :)
⟶ Reply

anh1l1ator

19 months ago, # ^ | ☆ 0

Hello, can you explain what are you doing , I have solved the problem using persistent segment tree(It was trivially a modification to MKTHNUM problem).
⟶ Reply

aka.Sohieb

19 months ago, # ^ | ☆ +3

`cntBit[i][j]` => means how many elements from 1 to i having 1 in the jth bit.

you can see it's easy to

you can see it's easy to construct that table in O(nlogn) time.

then when I get query [L,R] I iterate throw bits and see if there is more than (R-L+1)/2 elements having this bit 1, if so I set it, when I done I have a number every bit in it exists in more than siz/2 elements. so it is easy to notice that either this is the answer or there is no answer at all, to check if it is the answer or not, I count how many occurrences of this number in the interval [L,R] using binary search.

→ Reply

closedacc

21 month(s) ago, # | ☆ 0

why adding -1 to i + (1 << (j-1)) in the code of sparse table?

→ Reply

Kole

21 month(s) ago, # ^ | ☆ 0

I'll try to explain on small example.

Let's say that you have array of 10 elements, and you are on 3rd. If you want to increase 5 consecutive starting from position 3 you will have to increase elements on following positions: 3, 4, 5, 6, 7. You'd probably say 'ok, from 3 to 3+5=8', but that is not correct since 3 is considered to be one of these 5 consecutive elements.

Same is with sparse table. If you want to take 2^k consecutive elements starting from position *i*, last element is i+2^k-1 which is i+(1<<k)-1.

→ Reply

Thomas_94

new, 5 weeks ago, # ^ | ☆ 0

I believe it is an error. Let say i=4, j=1.

```
st[i][j] = min(st[i][j-1], st[i + (1 << (j-1)) - 1][j-1]);
st[4][1] = min(st[4][0], st[4][0]);
```

→ Reply

cipher0

18 months ago, # | ☆ +6

wonderful toturial:) can u please write one about DP?

→ Reply

Dpman

11 months ago, # ^ | ☆ 0

Sure :D

→ Reply

bhargav104

18 months ago, # | ☆ ← Rev. 2 0

Can somebody please explain how 444C - DZY Loves Colors is solved with sqrt decomposition? amd

→ Reply

rohit_0809

18 months ago, # | ☆ 0

Its extremely helpful.Thanks a lot!!!! :)

→ Reply

vatsalsharma376

18 months ago, # | ☆ 0

Well, Hamaro and Tool (UFDS) can be solved without Union Find but rather simple 2D array manipulation. :)

→ Reply

want_2b_expert

15 months ago, # | ☆ 0

amd Please clear me in Partial Sum Example-02,What will be the P array when r=n? Thanks advanced!!!.

→ Reply

Frez

14 months ago, # | ☆ ← Rev. 2 0

Hi! I have read that complexity of union find is `O(nlog*(n))` but you have written that it's complexity is `nlog(n)` .am i missing something?! (Can anybody help?!)

→ Reply

eightnoteight

14 months ago, # ^ | ☆ 0

it depends on the implementation, you will get `O(nlog*(n))` if you do path compression, but he just mentions height balancing so the complexity will be `O(nlog(n))`

→ Reply

elghareb

10 months ago, # | ☆ 0

Useful one

→ Reply

codePotato98

10 months ago, # | ☆ 0

This implementation of segment tree works with arrays only whose length is in form of (2^k) or it works for any n? Thanks in advance

→ Reply

vatsalsharma376

9 months ago, # | ☆ 0

I am not able to see the image under "Segment Tree" topic.It's just a plain black image. Anyone else experiencing the same?

→ Reply

mishi_gun

8 months ago, # | ☆ 0

Can any body tell me the condition when we can use lazy propagation on segment tree and when not ??

→ Reply

claudy

8 months ago, # | ☆ 0

In persistent segment tree update function, i think you forgot this line of code at the end :

s[ID] = s[L[ID]] + s[R[ID]];

→ Reply

nbz_11

7 months ago, # | ☆ 0

Why is it written as s[id] += (r — l) * x;

I am wondering why is the r-l factor there? Can someone help please

→ Reply

7 months ago, # ^ | ☆ 0

(r — l) is the length of segment [l , r) ! when we add x to each element in this segment the sum of this segment increase by (r — l) * x ! okay ?

nima10khodaveisi → Reply

nbz_11

7 months ago, # ^ | ☆ 0

Thanks , I actually got that after a while.

→ Reply

Informer

5 months ago, # | ☆ 0

Well it cover a lot of generally used Data structure, Thanks;

→ Reply

booleancode_01

4 months ago, # | ☆ 0

nice tutorial

→ Reply

hpkt.pkt

4 months ago, # | ☆ 0

Nice Tutorial overall !!! PrinceOfPersia

In sparse table I think : `st[i][j] = (j ? min(st[i][j-1], st[i + (1 << (j-1)) - 1][j-1]) : a[i]);` should be replaced by `st[i][j] = (j ? min(st[i][j-1], st[i + (1 << (j-1))][j-1]) : a[i]);`

The change is in the second component, reason is obvious, as per the representation written about rmq[i][j]

→ Reply

bharath

new, 7 weeks ago, # | ☆ 0

Really thank you so much !!

→ Reply

michelledilbert1

new, 5 weeks ago, # | ☆ 0

In DSU implementation using arrays/vectors why will the size of set double every time? Example: if I have an array a[] = {a,b,c,d,e,f} and I put queries like merge(a,b), merge(b,c), merge(c,d), merge(d,e) merge(e,f) clearly it takes n-1 steps and so the size of array will increase by 1 in every step.

Also we are adding more tools to less tools to keep the height of tree less, correct me if I am wrong...

→ Reply

michelledilbert1

new, 5 weeks ago, # ^ | ☆ 0

Got the answer to first question, the the author is taking about **each** tool.

→ Reply