Forums

Forums Algorithm Discussions Skill-building and Educational Discussion for Algorithm

World Finals 08 Warmup - GCD Summation

World Finals 08 Warmup - GCD Summation | Feedback: (+0/-1) | [+] [-] | Reply Sat, Mar 15, 2008 at 5:58 PM EDT

Khaled_912
389 posts

So what's the idea behind evaluating the GCD summation formula quickly for problem H and problem J?

```
int G(int N)
{
        int ret=0;
        for(i=1;i<N;i++)
        for(j=i+1;j<=N;j++)
        {
                ret+=gcd(i,j);
        }
        return ret;
}
```

The best I can do is calculating G(N) from G(N - 1) in O(N), which is obvious to deduce !!

Is there some weird formula or a method like the Totient function that help in calculating this formula?

Google made me find this. Not sure if it may help, but I don't even understand the formula!

Any hints plz?

Re: World Finals 08 Warmup - GCD Summation (response to post by **Khaled_912**) | Feedback: (+0/-0) | [+] [-] | Reply — Sat, Mar 15, 2008 at 6:00 PM EDT

jaher
339 posts

Well I used Euler's formula and also I found the divisors of a number by means of prime factorization and backtracking.

Re: World Finals 08 Warmup - GCD Summation (response to post by **jaher**) | Feedback: (+0/-0) | [+] [-] | Reply — Sat, Mar 15, 2008 at 6:16 PM EDT

Khaled_912
389 posts

WOW... Thanks for the very fast reply !!!
Euler's formula? You mean this? How can that help?

Re: World Finals 08 Warmup - GCD Summation (response to post by **Khaled_912**) | Feedback: (+0/-0) | [+] [-] | Reply — Sat, Mar 15, 2008 at 6:21 PM EDT

jaher
339 posts

I meant this one: http://en.wikipedia.org/wiki/Euler%27s_totient_function :)

Re: World Finals 08 Warmup - GCD Summation (response to post by **jaher**) | Feedback: (+0/-0) | [+] [-] | Reply — Sat, Mar 15, 2008 at 6:30 PM EDT

But the Totient function calculates the number of relatively prime numbers to N, how will I know the value of the rest of the summation?

Khaled_912
389 posts

Re: World Finals 08 Warmup - GCD Summation (response to post by jaher) | Feedback: (+0/-0) | [+] [-] | Reply — Sat, Mar 15, 2008 at 6:33 PM EDT

vlad89
67 posts

No, you do not need to do any prime factorization nor backtracking at all. First you have to calculate Euler's totient function. You can do it in O(n * log n) with algorithm similar to Eratosthenes sieve. The second part of algorithm is for every number P from 1 to N with divisors d1,d2,...,dn calculate sum phi[P/di] * di (excluding di = 1). This part is also uses technique of Eratosthenes sieve (you fix divisor di in outer cycle, and in inner you iterate through all possible P/di).

Re: World Finals 08 Warmup - GCD Summation (response to post by Khaled_912) | Feedback: (+0/-0) | [+] [-] | Reply — Sat, Mar 15, 2008 at 6:35 PM EDT

jaher
339 posts

sum{i=1..N, sum{d|i, phi(i/d) * d }}
Where d|i means d divides i.

Re: World Finals 08 Warmup - GCD Summation (response to post by jaher) | Feedback: (+0/-0) | [+] [-] | Reply — Sat, Mar 15, 2008 at 6:39 PM EDT

Khaled_912
389 posts

what will phi(i/d)*d exactly calculate with respect to i??

Re: World Finals 08 Warmup - GCD Summation (response to post by Khaled_912) | Feedback: (+0/-0) | [+] [-] | Reply — Sat, Mar 15, 2008 at 6:44 PM EDT

jaher
339 posts

phi(i/d) calculates the amount of numbers j with respect to a fixed i whose gcd(i,j)=d.

Re: World Finals 08 Warmup - GCD Summation (response to post by Khaled_912) | Feedback: (+0/-0) | [+] [-] | Reply — Sat, Mar 15, 2008 at 6:48 PM EDT

wongiseng
369 posts

You could have a look at this for a story behind that.

Re: World Finals 08 Warmup - GCD Summation (response to post by jaher) | Feedback: (+1/-0) | [+] [-] | Reply — Sat, Mar 15, 2008 at 6:49 PM EDT

Khaled_912
389 posts

That's one important peice of info that I didn't now...
Thanks for all those "almost chatting" responses :)

Re: World Finals 08 Warmup - GCD Summation (response to post by **wongiseng**) | Sat, Mar 15, 2008 at 10:01 PM EDT
Feedback: (+0/-0) | [+] [-] | Reply

broken_arrow
357 posts

The problems were written without reading this paper. So I thought the idea is not old :|.

Re: World Finals 08 Warmup - GCD Summation (response to post by **broken_arrow**) | Sun, Mar 16, 2008 at 12:31 AM EDT
Feedback: (+0/-0) | [+] [-] | Reply

sohelH
617 posts

Our code for H and J were identical.
We preprocessed 'euler phi' and for every divisor of X, res[X] += divisor*phi[X/divisor].

I think two different types of solutions were expected for these two problems. For H, number of test cases were very high so the requirement was to print the output of each test case in O(1) time.
But for J, number of test cases were only 100. Was a different solution expected for J that doesn't involve any preprocessing?

Re: World Finals 08 Warmup - GCD Summation (response to post by sohelH) | Feedback: (+0/-0) | [+] [-] | Reply Sun, Mar 16, 2008 at 2:34 AM EDT

broken_arrow
357 posts

Yes! we expected two different solutions. But I guess we were dumb founded :(. And that may have caused a downgrade to the contest.

Re: World Finals 08 Warmup - GCD Summation (response to post by **broken_arrow**) | Sun, Mar 16, 2008 at 3:13 AM EDT
Feedback: (+1/-7) | [+] [-] | Reply

Khaled_912
389 posts

Thanks, **wongiseng**, the paper was really helpful.
The problem really has a very nice idea and I enjoyed thinking of it and I was excited to know the solution. It would have been a great problem if googling "GCD-sum function" wouldn't return this paper as the first result :(
I'm sure the problem setters will not let us down and that we'll have a great warmup 2 and 3 and even a more great world finals problem set ;)

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