PROBLEM LINK : **http://www.spoj.com/problems/TRENDGCD/**

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DIFFICULTY : Medium

PREREQUISITES : Mobius Inversion , Fast summation.

PROBLEM :

Given A and B you need to calculate S(A,B).

$$S(A, B) = \sum_{a=1}^{A} \sum_{b=1}^{B} a*b* f(GCD(a,b))$$

Here, f(n)=n, if n is square free otherwise 0. Also f(1)=1

QUICK EXPLANATION :

Eliminate the gcd part by writing it in the form of its sum function using mobius inversion ! Rearrange to get the computable expression.

EXPLANATION :

Assume throughout that A<=B.

Mobius Inversion Theorem :

If g and f are arithmetic functions satisfying

$$f(n) = \sum_{d|n} g(d) \ \ for\ every\ integer\ n >= 1$$

Then

$$g(n) = \sum_{d|n} \mu(d) f(n/d)$$

Here μ is mobius function.

So using this above expression reduces to

$$\sum_{a=1}^{A}\sum_{b=1}^{B} a * b * \sum_{d|n} g(d)$$

$$= \sum_{i=1}^{|(A/d)|} id \sum_{j=1}^{|(B/d|} jd \sum_{d|GCD(a,b)} g(d)$$

$$= \sum_{d=1}^{A} d^2 g(d) \sum_{i=1}^{|A/d|} i \sum_{j=1}^{|B/d|} j$$

$$= \sum_{d=1}^{A} d^2 g(d)|A/d|(|A/d|+1)|B/d|(|B/d|+1)/4$$

Now problem reduces to finding g(d) function
Using inversion formula

$$g(n) = \sum_{d|n} \mu(d) f(n/d) = n \sum_{d|n} (\mu(d)/d)\mu^2(n/d) \qquad \{\mu^2(k) = 1 \, if\ k\ is\ square-free, otherwise\ 0\}$$

If n is divisible by p^3 for some p,then g(n)=0.
So n can take form of $p_1{}^2 p_2{}^2 p\,{}^2{}_3 \ .....p_s{}^2 q_1 q_2 ......q_t$ for g(n) to be non-zero.
Let k=$p_1{}^2 p_2{}^2 p\,{}^2{}_3 \ .....p_s{}^2$ and m=$q_1 q_2 ......q_t$

So $g(n) = n * \prod_{p|k}(-1/p) \prod_{p|m}(1-1/p)$

Substituting value of n

$$g(n) = \prod_{p|k}(-p) \prod_{p|m}(p-1)$$

Now Main task left is to find g(n).
We can precompute g for each n<=1000000 by calculating a prime factor of p using segmented sieve.If p divides n then $g(n) = g(n/p) * (-p)$ else if p^2 divides n then $g(n) = g(n/p^{2)}(p-1)$ else g(n)=0.

To compute $\sum_{d=1}^{A} d^2 g(d)|A/d|(|A/d|+1)|B/d|(|B/d|+1)/4$ as |A/d| can take at

$2\sqrt{A}+1$ values and |B/d| can take at most $2\sqrt{B}+1$ values, so we can precompute $\sum_{d=1}^{k} d^2 g(d)$ for all k<=1000000 and can group terms which can take same values for |A/d| and |B/d|.

Take all answers mod 10^9+7.

COMPLEXITY : O(N + T * N$^{1/2}$)