Problem 4: Working backwards: From the pole-zero plot… back to the ODE !

Pertinent readings for Problem #4:

Nise - Control systems engineering (6^{th} ed)

(Located under /Resources + textbooks section on Blackboard)

Ch. 4: Pages 163 - 169 (Intro to pole-zero plots for Laplace domain anaysis)

<u>Your tasks for this section:</u> *(Turn in any paper-written work (blue-highlighted parts)*

Part A: Going backwards from $H(s)$ to the original ODE

Suppose your boss gave you access to the pole and zeros of $H(s)$ for a system that a competing company is also building:

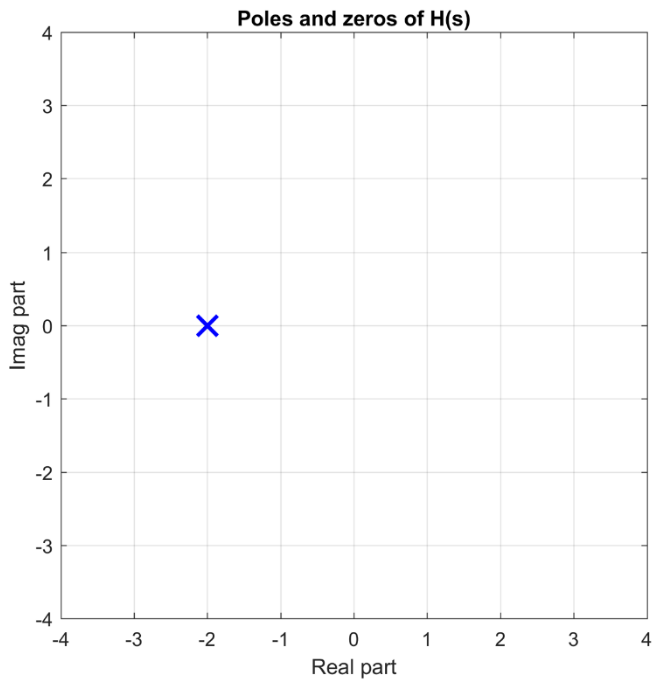

Figure 1: For this $H(s)$, we only have one pole at $p_1 = -2$ (and it does not contain zeros)

From class, you know if we only have 1 pole for our transfer function $H(s)$, we can translate the diagram in Figure 1 to this equation:

$$H(s) = \frac{K}{s - p_1} \quad , \quad where \quad \begin{cases} Zeros: & None \,!! \\ Poles: & p_1 = -2 \\ Also: & K = some\ constant \end{cases}$$

1) Working backwards, re-engineer your competitor's design by finding the equivalent differential equation for this $H(s)$.

2) Now, consider our $H(s)$ again. Using the inverse Laplace transform, find the impulse response $y(t) = h(t)$ and make a crude sketch of what $h(t)$ should look like.

Part B:	Complex conjugate poles… with 1 zero !

Suppose your boss gave you a pole-zero diagram of $H(s)$ from another system :

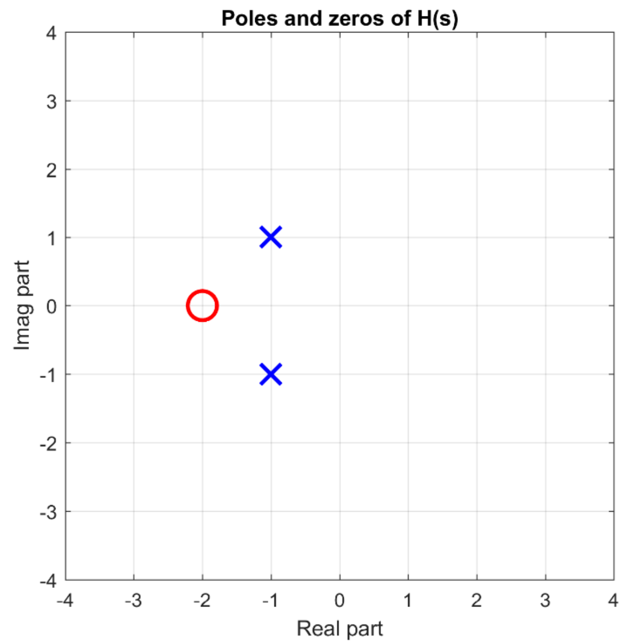

Figure 2: For this $H(s)$, we have one zero and a pair of complex conjugate poles.

From class, you know if we only have 1 pole for our transfer function $H(s)$, we can translate the diagram in Figure 1 to this equation:

$$H(s) = \frac{s - q_1}{(s-p_1)(s-p_2)} \quad , \quad where \begin{cases} Zeros: & q_1 = -2 \\ Poles: & p_{1,2} = -1 \pm j \\ \textit{Assume there are no other terms in the numerator} \end{cases}$$

1) Working backwards, find the equivalent differential equation for this $H(s)$.

2) Now, consider our $H(s)$ again. Using the inverse Laplace transform, find the impulse response $y(t) = h(t)$ and make a crude sketch of what $h(t)$ should look like. Your answer for $h(t)$ must be written in a compact, θ —phase-shifted exponential cosine format, where:

$$h(t) = 2\,|K|\,e^{\alpha t}\cos(\omega t + \theta) \quad , \quad where \quad K = |K|\,e^{j\theta} \quad \begin{pmatrix} \textit{Complex number in} \\ \textit{Euler form} \end{pmatrix}$$