Caso práctico

a) $J = \int \left[(1+t^2) \times (t) + y(t) + (x(t))^2 + (y'(t))^3 \right] dt$

Para x

$\dfrac{\partial F}{\partial x} = 1+t^2$ $\qquad$ $\dfrac{\partial F}{\partial x'} = 2x' \to \dfrac{d}{dt}\left(\dfrac{\partial F}{\partial x'}\right) = 2x''$

$\dfrac{\partial F}{\partial x} - \dfrac{d}{dt}\left(\dfrac{\partial F}{\partial x'}\right) = 1+t^2 - 2x'' = 0 \to x'' = \dfrac{t^2+1}{2}$

Para y

$\dfrac{\partial F}{\partial y} = 1$ $\qquad$ $\dfrac{\partial F}{\partial y'} = 3(y')^2 = \dfrac{d}{dt}\left(\dfrac{\partial F}{\partial y'}\right) = 6y'y''$

$\dfrac{\partial F}{\partial y} - \dfrac{d}{dt}\left(\dfrac{\partial F}{\partial y'}\right) = 1 - 6y'y'' = 0 \to 6y'y'' = 1$

Resolvemos las EDO:

Para x:

$x' = \dfrac{t^3}{6} + \dfrac{t}{2} + c_1 \to \boxed{x = \dfrac{t^4}{24} + \dfrac{t^2}{4} + c_1 t + c_2}$

Para y:

$6y'y'' = 1 \to$ sustituimos $\quad y' = u(y) \quad y'' = uu'$

$6u^2 u' = 1$

$6 u^2 \dfrac{du}{dy} = 1 \to 6u^2 du = dy$

$\int u^2 du = \int \tfrac{1}{6} dy$

$u = \dfrac{\sqrt[3]{y+c_1}}{\sqrt[3]{2}} \leftarrow \dfrac{u^3}{3} = \dfrac{y}{6} + c_1 \xrightarrow[\text{otra vez}]{\text{Sustituimos}} y' = \dfrac{\sqrt[3]{y+c_1}}{\sqrt[3]{2}}$

$\sqrt[3]{2}\, y' = \sqrt[3]{y+c_1}$

$\sqrt[3]{2}\, \dfrac{dy}{dx} = \sqrt[3]{y+c_1} \longrightarrow \dfrac{dy}{\sqrt[3]{y+c_1}} = \dfrac{dx}{\sqrt[3]{2}}$

$\int \dfrac{dy}{\sqrt[3]{y+c_1}} = \int \dfrac{dx}{\sqrt[3]{2}} \longrightarrow \dfrac{3\sqrt[3]{(y+c_1)^2}}{2} = \dfrac{x}{\sqrt[3]{2}} + c_2$

$(y+c_1)^2 = \left(\dfrac{\sqrt[3]{y} x}{3} + c_2\right)^3 \xrightarrow[\text{la } y]{\text{aislamos}} \boxed{y = 2\left(\dfrac{t}{3} + 2c_1\right)^{3/2} + c_2}$

b) $y(0,5) = 0,5$ $\qquad$ $x(0,5) = 0,5$

$\quad$ $y(1) = 1$ $\qquad\qquad$ $x(1) = 1$

Sistema para hallar las ctes de x:

$x(0,5) = \dfrac{0,5^4}{24} + \dfrac{0,5^2}{4} + c_1 \cdot 0,5 + c_2 = 0,5$ $\Big\{$ $c_1 = 0,516875$

$x(1) = \dfrac{1^4}{24} + \dfrac{1^2}{4} + c_1 \cdot 1 + c_2 = 1$ $\qquad$ $c_2 = 0,161458$

Sistema para hallar las ctes de y:

$y(0,5) = 2\left(\dfrac{0,5}{3} + 2c_1\right)^{3/2} + c_2 = 0,5$ $\Big\{$ $c_1 = 0,375289$

$y(1) = 2\left(\dfrac{1}{3} + 2c_1\right)^{3/2} + c_2 = 1$ $\qquad$ $c_2 = -1,256938$