Caso práctico

a) $J = \int \left[(1+t^2)\, x(t) + y(t) + (x'(t))^2 + (y'(t))^3 \right] dt$

Para x

$$\frac{\partial F}{\partial x} = 1+t^2 \qquad \frac{\partial F}{\partial x'} = 2x' \rightarrow \frac{d}{dt}\left(\frac{\partial F}{\partial x'}\right) = 2x''$$

$$\frac{\partial F}{\partial x} - \frac{d}{dt}\left(\frac{\partial F}{\partial x'}\right) = 1+t^2 - 2x'' = 0 \rightarrow x'' = \frac{t^2+1}{2}$$

Para y

$$\frac{\partial F}{\partial y} = 1 \qquad \frac{\partial F}{\partial y'} = 3(y')^2 \quad \frac{d}{dt}\left(\frac{\partial F}{\partial y'}\right) = 6y'y''$$

$$\frac{\partial F}{\partial y} - \frac{d}{dt}\left(\frac{\partial F}{\partial y'}\right) = 1 - 6y'y'' = 0 \rightarrow 6y'y'' = 1$$

Resolvemos las EDOs:

Para x:

$$x' = \frac{t^3}{6} + \frac{t}{2} + c_1 \longrightarrow \boxed{x = \frac{t^4}{24} + \frac{t^2}{4} + c_1 t + c_2}$$

Para y:

$6y'y'' = 1 \rightarrow$ sustituimos $y' = u(y)$ $\quad y'' = uu'$

$$6u^2u' = 1$$

$$\frac{6u^2\, du}{dy} = 1 \rightarrow 6u^2\, du = dy$$

$$\int u^2\, du = \int 1/6\, dy$$

$$u = \frac{\sqrt[3]{y+c_1}}{\sqrt[3]{2}} \leftarrow \frac{u^3}{3} = y/6 + c_1 \xrightarrow[\text{otra vez}]{\text{Sustituimos}} y' = \frac{\sqrt[3]{y+c_1}}{\sqrt[3]{2}}$$

$$\sqrt[3]{2}\, y' = \sqrt[3]{y+c_1}$$

$$\frac{\sqrt[3]{2}\, dy}{dx} = \sqrt[3]{y+c_1} \longrightarrow \frac{dy}{\sqrt[3]{y+c_1}} = \frac{dx}{\sqrt[3]{2}}$$

$$\int \frac{dy}{\sqrt[3]{y+c_1}} = \int \frac{dx}{\sqrt[3]{2}} \longrightarrow \frac{3\sqrt[3]{(y+c_1)^2}}{2} = \frac{x}{\sqrt[3]{2}} + c_2$$

$$(y+c_1)^2 = \left(\frac{\sqrt[3]{4}\,x}{3} + c_2\right)^3 \xrightarrow[\text{la } y]{\text{aislamos}} \boxed{y = 2\left(t/3 + 2c_1\right)^{3/2} + c_2}$$

b) $y(0,5) = 0,5 \qquad x(0,5) = 0,5$

$y(1) = 1 \qquad x(1) = 1$

Sistema para hallar las ctes de x:

$$\left.\begin{aligned} x(0,5) &= \frac{0,5^4}{24} + \frac{0,5^2}{4} + c_1 \cdot 0,5 + c_2 = 0,5 \\ x(1) &= \frac{1^4}{24} + \frac{1^2}{4} + c_1 \cdot 1 + c_2 = 1 \end{aligned}\right\} \begin{aligned} c_1 &= 0,546875 \\ c_2 &= 0,161458 \end{aligned}$$

Sistema para hallar las ctes de y:

$$\left.\begin{aligned} y(0,5) &= 2\left(\frac{0,5}{3} + 2c_1\right)^{3/2} + c_2 = 0,5 \\ y(1) &= 2\left(1/3 + 2c_1\right)^{3/2} + c_2 = 1 \end{aligned}\right\} \begin{aligned} c_1 &= 0,375289 \\ c_2 &= -1,256938 \end{aligned}$$

d) $D(t) = x(t) - y(t)$

$D'(t) = x'(t) - y'(t)$

$D''(t) = x''(t) - y''(t) \Rightarrow$ Se usa para hallar los ptos críticos

$$\cancel{D(t) = \frac{t^4}{24} + \frac{t^2}{4} + c_1 t + c_2 - 2\left(\frac{t}{3} + 2c_1\right.}$$

$$D(t) = \frac{t^4}{24} + \frac{t^2}{4} + 0,546875t + 0,161458 - 2\left(\frac{t}{3} + 2 \cdot 0,375289\right)^{3/2} - 1,256938$$

$$D'(t) = \frac{t^3}{6} + t/2 + 0,546875 - \frac{\sqrt[3]{t/3 + 0,375289}}{\sqrt[3]{2}}$$

$$t_{n+1} = t_n - \frac{\frac{t^4}{24} + \frac{t^2}{4} + 0,546875t + 0,161458 - 2\left(t/3 + 2 \cdot 0,375289\right)^{3/2} - 1,256938}{\frac{t^3}{6} + t/2 + 0,546875t - \frac{\sqrt[3]{2\left(t/3 + 2 \cdot 0,375289\right)^{3/2} - 1,256938 + 0,375289}}{\sqrt[3]{2}}}$$