# TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

**PAPER 1** **D513/01**

**2022** **75 minutes**

Additional materials: Answer sheet

**INSTRUCTIONS TO CANDIDATES**

**Please read these instructions carefully, but do not open the question paper until you are told that you may do so.**

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

This paper is the first of two papers.

This paper contains 20 questions. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

You can use the question paper for rough working or notes, but **no extra paper** is allowed.

You **must** complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

**Please wait to be told you may begin before turning this page.**

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**1**    How many real solutions are there to the equation

$$2\cos^4\theta - 5\cos^2\theta + 3 = 0$$

in the interval $0 \le \theta \le 2\pi$?

**A**   1

**B**   2

**C**   3

**D**   4

**E**   5

**F**   6

**G**   7

**H**   8

**2**   Find the complete set of values of $p$ for which the equation

$$x^2 - 2px + y^2 - 6y - p^2 + 8p + 9 = 0$$

describes a circle in the $xy$-plane.

**A**   $p < -\dfrac{9}{4}$

**B**   $0 < p < 4$

**C**   $-1 < p < 9$

**D**   $p < 0$ or $p > 4$

**E**   $p < -1$ or $p > 9$

**F**   all real values of $p$

**3** Given the following statements about a function f

- $f''(x) = a$ for all $x$
- $f(0) = 1,\ f(1) = 2$
- $\int_0^1 f(x)\,dx = 1$

find the value of $a$.

A   $-6$

B   $-3$

C   $-2$

D   $2$

E   $3$

F   $6$

**4**
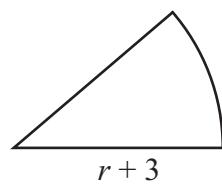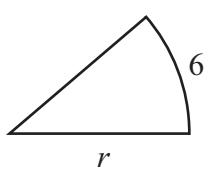


These sectors of circles are similar.

The arc length of the smaller sector is 6.

The difference between the areas of the sectors is 21.

Find the positive difference between the perimeters of the sectors.

**A**   4.5

**B**   7

**C**   8

**D**   9

**E**   10.5

**F**   14

**G**   15

**5**    The terms $x_n$ of a sequence follow the rule

$$x_{n+1} = \frac{x_n + p}{x_n + q}$$

where $p$ and $q$ are real numbers.

Given that $x_1 = 3$, $x_2 = 5$, and $x_3 = 7$, find the value of $x_4$

A    −5

B    5

C    $\dfrac{51}{7}$

D    $\dfrac{15}{2}$

E    $\dfrac{23}{3}$

F    9

G    11

H    13

$$x_{n+1} = 7$$

**6**   Given that

$$\int_{\log_2 5}^{\log_2 20} x \, \mathrm{d}x = \log_2 M$$

what is the value of $M$?

**A**  4

**B**  15

**C**  16

**D**  20

**E**  25

**F**  100

**G**  10 000

**7**   Find the finite area enclosed between the line $y = 0$ and the curve $y = x^2 - 4|x| - 12$

A   $\dfrac{128}{3}$

B   $\dfrac{176}{3}$

C   $\dfrac{256}{3}$

D   108

E   144

F   288

**8** A geometric sequence has first term $a$ and common ratio $r$, where $a$ and $r$ are positive integers and $r$ is greater than 1.

The sum of the first $n$ terms of this sequence is denoted by $S_n$

It is given that the terms of the sequence satisfy

$$S_{30} - S_{20} = k S_{10}$$

for some positive integer $k$.

What is the smallest possible value of $k$?

**A** $2^{10}$

**B** $2^{20}$

**C** $2^{30}$

**D** $\dfrac{2^{10}}{2^{10} - 1}$

**E** $2^{10}(2^{10} - 1)$

**9** This question is about pairs of functions f and g that satisfy

$$f(x) - g(x) = 2\sin x$$

$$f(x)g(x) = \cos^2 x$$

for all real numbers $x$.

Across all solutions for f($x$), what is the minimum value that f($x$) attains for any $x$?

A  $1 - \sqrt{2}$

B  $-1 - \sqrt{2}$

C  $0$

D  $-1$

E  $-2$

F  $-3$

G  $-4$

11

**10** A sequence of translations is applied to the graph of $y = x^3$

Which of the following graphs could be the result of this sequence of translations?

I   $y = x^3 - 3x^2 + 9x - 27$

II   $y = x^3 - 9x^2 + 27x - 3$

III   $y = 27x^3 - 9x^2 + x - 3$

**A**   none of them

**B**   I only

**C**   II only

**D**   III only

**E**   I and II only

**F**   I and III only

**G**   II and III only

**H**   I, II and III

11  Evaluate

$$\sum_{n=1}^{100} \log_{10}\left(3^{1-n}\right)$$

A   $-4950 \log_{10} 3$

B   $4950 \log_{10} 3$

C   $-5050 \log_{10} 3$

D   $5050 \log_{10} 3$

E   $1 - 4950 \log_{10} 3$

F   $1 + 4950 \log_{10} 3$

G   $1 - 5050 \log_{10} 3$

H   $1 + 5050 \log_{10} 3$

12  A family of quadratic curves is given by

$$y_k = 2\left(x - \frac{k}{2}\right)^2 + \frac{k^2}{2} + 4k + 3$$

where $k$ is any real number and $y_k$ is a function of $x$.

All these curves are sketched, and the point with the lowest $y$-coordinate among all the curves $y_k$ is $(a, b)$.

Find the value of  $a + b$

A   −1

B   −3

C   −5

D   −7

E   −9

$$y_k = 2\left(x - \frac{k}{2}\right)^2 \quad 14 \quad + \frac{k^2}{2} + 4k + 3$$

13    Given that

$$\left(a^3 + \frac{2}{b^3}\right)\left(\frac{2}{a^3} - b^3\right) = \sqrt{2}$$

where $a$ and $b$ are real numbers, what is the least value of $ab$?

A    $-\sqrt{2}$

B    $\sqrt{2}$

C    $-2\sqrt{2}$

D    $2\sqrt{2}$

E    $-\dfrac{\sqrt{2}}{2}$

F    $\dfrac{\sqrt{2}}{2}$

G    $-2^{\frac{1}{6}}$

H    $2^{\frac{1}{6}}$

**14**  A circle has centre $O$ and radius 6.

P, Q and R are points on the circumference with angle $POQ \geq \dfrac{\pi}{2}$

The area of the triangle $POQ$ is $9\sqrt{3}$

What is the greatest possible area of triangle $PRQ$?

A   $18 + 9\sqrt{3}$

B   $18\sqrt{3}$

C   $27 + 9\sqrt{3}$

D   $27\sqrt{3}$

E   $36 + 9\sqrt{3}$

F   $36\sqrt{3}$

**15** A rectangle is drawn in the region enclosed by the curves $p$ and $q$, where

$$p(x) = 8 - 2x^2$$

$$q(x) = x^2 - 2$$

such that the sides of the rectangle are parallel to the $x$- and $y$-axes.

What is the maximum possible area of the rectangle?

A $\dfrac{26}{9}$

B $\dfrac{52}{9}$

C $\dfrac{4\sqrt{6}}{3}$

D $\dfrac{8\sqrt{6}}{3}$

E $4\sqrt{2}$

F $8\sqrt{2}$

G $\dfrac{20\sqrt{10}}{9}$

H $\dfrac{40\sqrt{10}}{9}$

**16** The solutions to $7x^4 - 6x^2 + 1 = 0$ are $\pm\cos\theta$ and $\pm\cos\beta$.

Which one of the following equations has solutions $\pm\sin\theta$ and $\pm\sin\beta$?

A $\quad 7x^4 - 8x^2 - 5 = 0$

B $\quad 7x^4 - 8x^2 + 2 = 0$

C $\quad 7x^4 - 6x^2 - 2 = 0$

D $\quad 7x^4 - 6x^2 + 1 = 0$

E $\quad 7x^4 + 6x^2 - 1 = 0$

F $\quad 7x^4 + 6x^2 + 5 = 0$

**17**
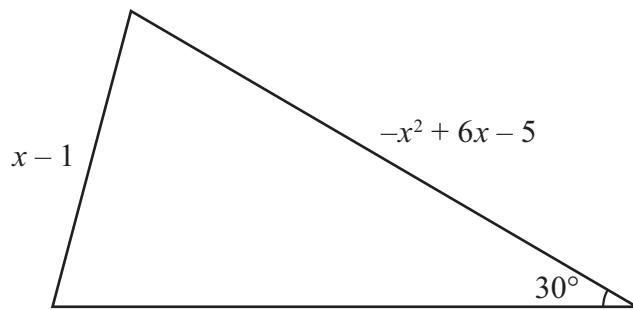


Find the complete set of values of $x$ for which there are two non-congruent triangles with the side lengths and angle as shown in the diagram.

A  $1 < x < 3$

B  $1 < x < 4$

C  $1 < x < 5$

D  $3 < x < 4$

E  $3 < x < 5$

F  $4 < x < 5$

**18** It is given that

$$f(x) = x^2(x-1)^2(x-2)$$

$$g(x) = -p(x-q)^2(x-r)^2$$

where $p$, $q$ and $r$ are positive and $q < r$

Find the set of values of $q$ and $r$ that guarantees the greatest number of distinct real solutions of the equation $f(x) = g(x)$ for all $p$.

**A**  $q < 1$ and $r < 1$

**B**  $q < 1$ and $1 < r < 2$

**C**  $q < 1$ and $r > 2$

**D**  $1 < q < 2$ and $1 < r < 2$

**E**  $1 < q < 2$ and $r > 2$

**F**  $q > 2$ and $r > 2$

**19** Circle $C_1$ is defined as $x^2 + y^2 = 25$

A second circle $C_2$ has radius 4 and centre $(a, b)$ where

$$-2 \leq a \leq 2 \quad \text{and} \quad -3 \leq b \leq 3$$

If the centre of $C_2$ is equally likely to be located anywhere within the given range, what is the probability that $C_2$ intersects $C_1$?

A  $\dfrac{1}{25}$

B  $\dfrac{9}{25}$

C  $\dfrac{16}{25}$

D  $\dfrac{6 - \pi}{6}$

E  $\dfrac{16 - \pi}{24}$

F  $\dfrac{24 - \pi}{24}$

**20**   $n$ is the number of points of intersection of the graphs

$$y = |x^2 - a^2| \text{ and } y = a^2|x - 1|$$

where $a$ is a real number.

What is the smallest value of $n$ that is **not** possible?

**A**   $n = 1$

**B**   $n = 2$

**C**   $n = 3$

**D**   $n = 4$

**E**   $n = 5$

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