C3, IYGB, PAPER F

1. a) $y = (1-x^2)^6$

$$\frac{dy}{dx} = 6(1-x^2)^5 \times (-2x)$$

$$\frac{dy}{dx} = -12x(1-x^2)^5$$

b) $y = x^3 \sin 3x$

$$\frac{dy}{dx} = 3x^2 \sin 3x + x^3(3\cos 3x)$$

$$\frac{dy}{dx} = 3x^2(\sin 3x + x\cos 3x)$$

c) $y = \frac{5x}{x^3+2}$

$$\frac{dy}{dx} = \frac{(x^3+2)\times 5 - 5x(3x^2)}{(x^3+2)^2}$$

$$\frac{dy}{dx} = \frac{5x^3+10-15x^3}{(x^3+2)^2} = \frac{10-x^3}{(x^3+2)^2}$$

$$= \frac{10(1-x^3)}{(x^3+2)^2}$$

2.

$$\sec 2x = \frac{1}{\cos 2x} = \frac{1}{1-2\sin^2 2x} = \frac{1}{1-2\left(\frac{3}{5}\right)^2} = \frac{1}{1-2\times\frac{9}{25}}$$

$$= \frac{1}{1-\frac{18}{25}} = \frac{1}{\frac{7}{25}} = \frac{25}{7}$$

3.

$$|x^2-2x-4| = 4$$

$x^2-2x-4 = 4$

$x^2-2x-8 = 0$

$(x-4)(x+2) = 0$

$x = -2, 4$

OR

$x^2-2x-4 = -4$

$x^2-2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$\therefore x = -2, 0, 2, 4$

C3, IYGB, PAPER F

4. a) $f(x) = 4x - 3\sin x - 1$

$f(0.7) = -0.13265\ldots$
$f(0.8) = 0.04793\ldots$

As $f(x)$ is continuous and changes sign between 0.7 & 0.8, there must be a solution between 0.7 & 0.8

b) $4x - 3\sin x - 1 = 0$

$4x = 1 + 3\sin x$

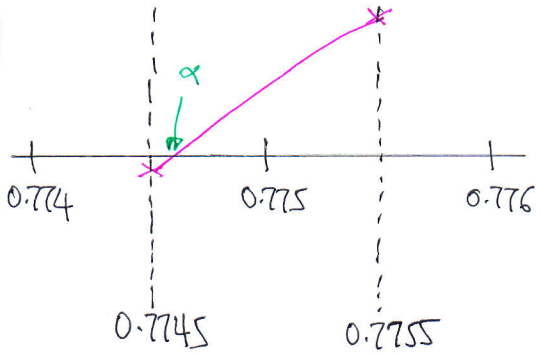
$x = \frac{1}{4} + \frac{3}{4}\sin x$

Thus

$x_{n+1} = \frac{1}{4} + \frac{3}{4}\sin x_n$

$x_1 = 0.75$
$x_2 = 0.76123$
$x_3 = 0.76739$
$x_4 = 0.77068$
$x_5 = 0.77247$

c)



$f(0.7745) = 0.0000076$
$f(0.7755) = 0.0017804$

As $f(x)$ changes sign between 0.7745 & 0.7755,

$0.7745 < \alpha < 0.7755$

$\alpha \approx 0.775$

5. a) $P = 400\,e^{\frac{1}{12}(t-8)}$

When $t=0$

$P = 400\,e^{\frac{1}{12}(0-8)}$

$P = 205$

b) When $t=8$

$P = 400\,e^{\frac{1}{12}(8-8)}$

$P = 400$

c) $1000 = 400e^{\frac{1}{12}(t-8)}$

$\Rightarrow \frac{5}{2} = e^{\frac{1}{12}(t-8)}$

$\Rightarrow \ln\frac{5}{2} = \frac{1}{12}(t-8)$

$\Rightarrow 12\ln\frac{5}{2} = t-8$

$\Rightarrow t = 8 + 12\ln\frac{5}{2}$

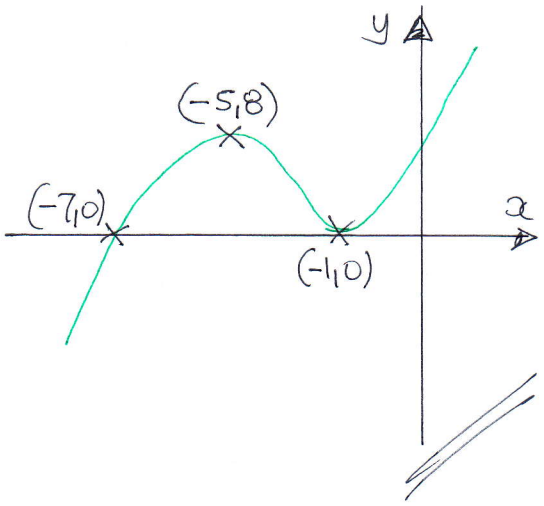
$\Rightarrow t \approx 18.945\ldots$

$(t \approx 19)$

C3, IYGB, PAPER F

6. $y = 4f(x+1)$

- REFLECTION IN THE y AXIS
- VERTICAL STRETCH BY S.F. 4
- TRANSLATION 1 UNIT TO THE LEFT



7.

$$2\cos 2\theta = 4\cos\theta - 3$$
$$\Rightarrow 2(2\cos^2\theta - 1) = 4\cos\theta - 3$$
$$\Rightarrow 4\cos^2\theta - 2 = 4\cos\theta - 3$$
$$\Rightarrow 4\cos^2\theta - 4\cos\theta + 1 = 0$$
$$\Rightarrow (2\cos\theta - 1)^2 = 0$$
$$\Rightarrow \cos\theta = \frac{1}{2}$$

$\arccos\left(\frac{1}{2}\right) = 60°$

$\theta = 60 \pm 360n$
$\theta = 300 \pm 360n$ $\quad n = 0, 1, 2, 3, \ldots$

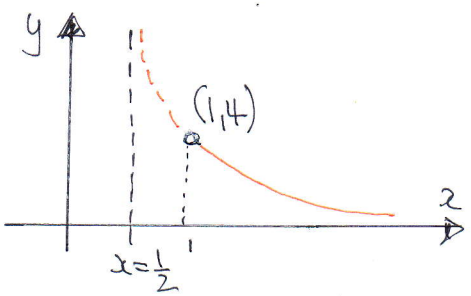
$\theta_1 = 60$
$\theta_2 = 300$

8. a) $y = \frac{2}{x-2} - \frac{6}{(x-2)(2x-1)} = \frac{2(2x-1)-6}{(x-2)(2x-1)} = \frac{4x-2-6}{(x-2)(2x-1)}$

$= \frac{4x-8}{(x-2)(2x-1)} = \frac{4(x-2)}{(x-2)(2x-1)} = \frac{4}{2x-1}$ AS REQUIRED

b) VERTICAL ASYMPTOTE AT $x = \frac{1}{2}$ (DIVISION BY ZERO)

c)



RANGE $0 < f(x) < 4$

d) Let $y = \dfrac{4}{2x-1}$

$2xy - y = 4$

$2xy = y + 4$

$x = \dfrac{y+4}{2y}$

$\therefore f^{-1}(x) = \dfrac{x+4}{2x}$ //

e)

	f	f^{-1}
D	$x>1$	$0<x<4$
R	$0<f(x)<4$	$f^{-1}(x)>1$

$\therefore$ DOMAIN $0<x<4$

RANGE $f^{-1}(x)>1$ //

9. a) $y = \dfrac{4x+k}{4x-k}$

$\dfrac{dy}{dx} = \dfrac{(4x-k)\times 4 - (4x+k)\times 4}{(4x-k)^2} = \dfrac{16x - 4k - 16x - 4k}{(4x-k)^2}$

$\therefore \dfrac{dy}{dx} = -\dfrac{8k}{(4x-k)^2}$ //

b) $\left.\dfrac{dy}{dx}\right|_{x=3} = \dfrac{8}{27}$

$\Rightarrow \dfrac{-8k}{(12-k)^2} = \dfrac{8}{27}$

$\Rightarrow -8(12-k)^2 = -216k$

$\Rightarrow -(12-k)^2 = -27k$

$\Rightarrow [144 - 24k + k^2] = 27k$

$\Rightarrow k^2 - 51k + 144 = 0$

$\Rightarrow (k-48)(k-3) = 0$

$\therefore k = 3, 48$ //

10. a) LHS $= \sin 3x$

$= \sin(2x+x)$

$= \sin 2x\cos x + \cos 2x \sin x$

$= (2\sin x\cos x)\cos x + (1-2\sin^2 x)\sin x$

$= 2\sin x\cos^2 x + \sin x - 2\sin^3 x$

$= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$

$=$ R+S

b) $\sin 3x = 3\sin x - 4\sin^3 x$

Differentiate w.r.t x

$\frac{d}{dx}(\sin 3x) = \frac{d}{dx}[3\sin x - 4\sin^3 x]$

$3\cos 3x = 3\cos x - 12\sin^2 x\cos x$

$3\cos 3x = 3\cos x - 12\cos x(1-\cos^2 x)$

$3\cos 3x = 3\cos x - 12\cos x + 12\cos^3 x$

$3\cos 3x = 12\cos^3 x - 9\cos x$

$\cos 3x = 4\cos^3 x - 3\cos x$