

DIFFERENTIATION PRACTICE

THE CHAIN RULE WITH ALGEBRAIC FUNCTIONS

Question 1

1. $y = (2x+1)^4$

2. $y = (3x-2)^6$

3. $y = (8x-1)^{-1}$

4. $y = (6x+1)^{\frac{1}{2}}$

5. $y = (4x+3)^{\frac{3}{2}}$

6. $y = (1-x)^4$

7. $y = 2(5x-2)^3$

8. $y = 2(3x+1)^{\frac{1}{2}}$

9. $y = 4(1-5x)^{-\frac{1}{2}}$

10. $y = 6(1-2x)^{\frac{1}{3}}$

11. $y = (3x^2+1)^5$

12. $y = (2x^2-3x)^4$

13. $y = (5-2x+x^2)^{-2}$

14. $y = \frac{4}{(5x+9)^3}$

15. $y = \frac{3}{\sqrt{3-2x}}$

16. $y = \frac{1}{4x+1}$

17. $y = \frac{2}{3(2x+7)^2}$

18. $y = \frac{3}{x^2+1}$

19. $y = \frac{4}{\sqrt{4-3x^2}}$

20. $y = \frac{1}{1-x^3}$

Question 2

1. $y = (4x + 1)^3$

2. $y = (2x - 1)^7$

3. $y = (6x - 5)^{-1}$

4. $y = (4x + 1)^{\frac{1}{2}}$

5. $y = (6x - 3)^{\frac{5}{2}}$

6. $y = (1 - 2x)^8$

7. $y = 4(2x - 3)^5$

8. $y = 3(6x - 1)^{\frac{1}{2}}$

9. $y = 6(1 - 3x)^{-\frac{1}{2}}$

10. $y = 9(1 - 5x)^{\frac{1}{3}}$

11. $y = (2x^2 - 1)^5$

12. $y = (3x^2 - 4x)^3$

13. $y = (1 - 4x + x^2)^{-2}$

14. $y = \frac{2}{(4x - 1)^3}$

15. $y = \frac{2}{\sqrt{5 - 2x}}$

16. $y = \frac{3}{2x+1}$

17. $y = \frac{3}{2(4x+1)^2}$

18. $y = \frac{4}{x^2-3}$

19. $y = \frac{5}{6\sqrt{1-3x^2}}$

20. $y = \frac{4}{4-x^3}$

Question 3

1. $y = (6x - 5)^3$

2. $y = (1 - 3x)^6$

3. $y = (2x - 5)^{-2}$

4. $y = (8x + 3)^{\frac{1}{2}}$

5. $y = 3(4x - 1)^{\frac{5}{2}}$

6. $y = 4\left(1 - \frac{1}{2}x\right)^6$

7. $y = 8\left(\frac{15}{16}x - 3\right)^{1.2}$

8. $y = 5\left(\frac{3}{5}x - 1\right)^{\frac{1}{3}}$

9. $y = 2\left(1 - \frac{5}{2}x\right)^{1.4}$

10. $y = \frac{2}{4x - 1}$

11. $y = (4x^2 - 3)^3$

12. $y = (4x^2 + 2x)^5$

13. $y = (5 - 4x - 2x^2)^{-3}$

14. $y = \frac{5}{(4x^2 - 3)^2}$

15. $y = \frac{2}{x^4 + 1}$

16. $y = \frac{5}{2\sqrt{2x+1}}$

17. $y = \frac{5}{6\sqrt{1-x^3}}$

18. $y = \frac{4}{1+2\sqrt{x}}$

19. $y = \sqrt{2+\sqrt{x}}$

20. $y = \sqrt{3x+(2x+1)^4}$

THE CHAIN RULE WITH EXPONENTIALS & LOGS

Question 4

1. $y = e^{2x}$

2. $y = \ln 2x$

3. $y = e^{4x-1}$

4. $y = \ln(3x-4)$

5. $y = e^{-3x}$

6. $y = \ln(x^2 - 4)$

7. $y = 3e^{x^2}$

8. $y = (\ln x)^4$

9. $y = (1 + e^x)^4$

10. $y = \ln x^4$

11. $y = 3(1 + e^{2x})^3$

12. $y = (2x + \ln x)^4$

13. $y = (4 + e^{-5x})^4$

14. $y = (x^3 - 3\ln x)^5$

15. $y = (x^2 + e^{2x})^4$

16. $y = \sqrt{5x - 3 \ln x}$

17. $y = \sqrt{1 + e^{2x}}$

18. $y = \ln(e^{2x} + 3)$

19. $y = e^{(2x+1)^5}$

20. $y = \ln(e^4 + 1)$

Question 5

1. $y = e^{4x}$

2. $y = \ln 4x$

3. $y = e^{2x-5}$

4. $y = \ln(5x-4)$

5. $y = 3e^{-2x}$

6. $y = 3\ln(x^2+1)$

7. $y = 2e^{-x^2}$

8. $y = 2(\ln x)^3$

9. $y = 2(1+e^x)^3$

10. $y = 2\ln x^{\frac{1}{2}}$

11. $y = 4\sqrt{2+e^{-2x}}$

12. $y = \frac{3}{2x - \ln x}$

13. $y = 10\left(x^2 + e^{-\frac{1}{2}x}\right)^3$

14. $y = \frac{1}{2}\left(e^{3x} - 3\ln x\right)^{\frac{4}{3}}$

15. $y = \left(e^{2x} + e^{-2x}\right)^4$

16. $y = \frac{2}{\sqrt{4x - \ln x}}$

17. $y = \sqrt{1 + 2e^{2x^2}}$

18. $y = \ln(\ln x)$

19. $y = e^{e^x}$

20. $y = \ln(e^{4+\ln^2} + 1)$

Question 6

1. $y = e^{3x}$

2. $y = \ln 5x$

3. $y = e^{3x+2}$

4. $y = \ln(2x+7)$

5. $y = 4e^{-5x}$

6. $y = 2\ln(4x^2 - 3)$

7. $y = e^{3x} - 3e^{-x^2}$

8. $y = \ln x^5 + (\ln x)^5$

9. $y = 3(2 - e^x)^4$

10. $y = \ln \sqrt{x} + \sqrt{\ln x}$

11. $y = 3\sqrt{1 + e^{-3x}}$

12. $y = \frac{1}{x^3 + \ln x}$

13. $y = 10\left(x^4 - 2e^{-\frac{1}{2}x}\right)^4$

14. $y = \frac{1}{3}\left(e^{6x} + 3\ln x\right)^{\frac{1}{3}}$

15. $y = \left(2e^{3x} - 3e^{-2x}\right)^3$

16. $y = \frac{4}{\sqrt{2x - 3\ln x}}$

17. $y = \sqrt{1 - 2e^{-3x^2}}$

18. $y = \ln(\ln(\ln x))$

19. $y = 2e^{2e^{2x}}$

20. $y = 3\ln(e^{\sqrt{5+\ln 2}} - \ln 3)$

THE CHAIN RULE WITH SINES & COSINES

Question 7

1. $y = \sin 4x$

2. $y = 2\sin 3x$

3. $y = \cos 3x$

4. $y = 6\cos\left(\frac{2}{3}x\right)$

5. $y = 4\sin\left(\frac{x}{2}\right)$

6. $y = 3\sin(5x-1)$

7. $y = 3\cos\left(2x - \frac{\pi}{3}\right)$

8. $y = 2\cos\left(\frac{\pi}{4} - 2x\right)$

9. $y = 6\cos\left(1 - \frac{3x}{2}\right)$

10. $y = 2\sin x^4$

11. $y = 2\sin^4 x$

12. $y = 4\cos x^3$

13. $y = 4\cos^3 x$

14. $y = 3\sin^5 x$

15. $y = 2\cos\sqrt{x}$

16. $y = 2\sin^3 2x$

17. $y = 2(3\cos 2x + 1)^4$

18. $y = \sqrt{1 - 2\cos x}$

19. $y = (2\sin 3x - 3\cos 2x)^3$

20. $y = \sin^3\left(\frac{\pi}{2}\right)$

Question 8

1. $y = \sin 5x$

2. $y = 4 \sin 2x$

3. $y = \cos 4x$

4. $y = 8 \cos\left(\frac{1}{2}x\right)$

5. $y = 4 \sin\left(\frac{x}{4}\right)$

6. $y = 2 \sin(3x - 2)$

7. $y = 5 \cos\left(3x - \frac{\pi}{4}\right)$

8. $y = 4 \cos\left(\frac{\pi}{2} - 3x\right)$

9. $y = 8 \cos\left(3 - \frac{5x}{2}\right)$

10. $y = \frac{1}{2} \sin x^6$

11. $y = \frac{1}{2} \sin^6 x$

12. $y = 10 \cos x^5$

13. $y = 10 \cos^5 x$

14. $y = 3 \sin^7 x$

15. $y = 2 \cos\left(x^{\frac{3}{2}}\right)$

16. $y = \frac{1}{6} \sin^4 3x$

17. $y = \frac{1}{3} (2 \sin 3x + 3)^5$

18. $y = \sqrt{1 - \cos 6x}$

19. $y = (4 \sin 3x - 3 \cos 4x)^3$

20. $y = \sin\left(\frac{\pi}{4}\right)$

THE CHAIN RULE WITH TAN, COT, SEC, COSEC

Question 9

1. $y = 4 \tan 3x$

2. $y = 2 \tan \left(2x + \frac{\pi}{4} \right)$

3. $y = 3 \tan^4 x$

4. $y = 3 \tan 2x$

5. $y = 12 \tan \left(\frac{\pi x}{4} \right)$

6. $y = \cot 2x$

7. $y = 3 \tan 2x - \cot 3x$

8. $y = 4 \sec 2x$

9. $y = 2 \operatorname{cosec} 3x$

10. $y = 4 \sec \frac{x}{2} - 6 \operatorname{cosec} \frac{2x}{3}$

11. $y = 2 \cot 4x - 2 \sec 3x$

12. $y = 3 \tan 5x - 6 \operatorname{cosec} 2x$

13. $y = \tan^6 x$

14. $y = 3 \cot^4 x$

15. $y = 3 \sec^2 x$

16. $y = 4 \operatorname{cosec}^4 x$

17. $y = 2 \tan^4 3x$

18. $y = 2 \cot^2 4x$

19. $y = 4 \sec^4 2x$

20. $y = 6 \operatorname{cosec}^3 \left(\frac{x}{2} \right)$

Question 10

1. $y = 3 \tan 5x$

2. $y = 4 \tan \left(3x + \frac{\pi}{3} \right)$

3. $y = \frac{1}{2} \tan^6 x$

4. $y = 10 \tan \frac{1}{2} x$

5. $y = 12 \tan \left(\frac{5\pi x}{6} \right)$

6. $y = \cot 7x$

7. $y = 4 \tan 3x - 2 \cot 2x$

8. $y = 3 \sec 4x$

9. $y = 6 \operatorname{cosec} 2x$

10. $y = 6 \sec \frac{x}{3} - 4 \operatorname{cosec} \frac{3x}{4}$

11. $y = 7 \cot 2x - 3 \sec 3x$

12. $y = 2 \tan 7x - 7 \operatorname{cosec} 2x$

13. $y = \tan^3 x$

14. $y = 8 \cot^5 x$

15. $y = \frac{1}{2} \sec^4 x$

16. $y = \frac{3}{4} \operatorname{cosec}^6 x$

17. $y = 2 \tan^6 2x$

18. $y = 2 \cot^3 3x$

19. $y = 3\sec^3 3x$

20. $y = 12\operatorname{cosec}^3\left(\frac{x}{4}\right)$

THE CHAIN RULE WITH TRIGONOMETRIC FUNCTIONS

Question 11

1. $y = \sin 2x$

2. $y = 3 \cos 2x$

3. $y = 4 \tan 3x$

4. $y = 6 \sin\left(\frac{1}{2}x\right)$

5. $y = 3 \cos\left(\frac{x}{3}\right)$

6. $y = 2 \sin(3x-1)$

7. $y = 2 \cos\left(4x - \frac{\pi}{3}\right)$

8. $y = 2 \tan\left(2x + \frac{\pi}{4}\right)$

9. $y = 9 \cos\left(\frac{\pi}{6} - 3x\right)$

10. $y = 2 \sin x^3$

11. $y = 2 \sin^3 x$

12. $y = 4 \cos x^2$

13. $y = 5 \cos^2 x$

14. $y = 3 \tan^4 x$

15. $y = 4 \sin \sqrt{x}$

16. $y = \sin^5 2x$

17. $y = (3 \sin x + 2)^4$

18. $y = \sqrt{1 + 4 \sin x}$

19. $y = (\sin x - \cos x)^3$

20. $y = \sin^3 \left(\frac{\pi}{6} \right)$

Question 12

1. $y = 3 \sin 3x$

2. $y = 2 \cos 4x$

3. $y = 3 \tan 2x$

4. $y = 4 \sin\left(\frac{3}{2}x\right)$

5. $y = 2 \cos\left(\frac{x}{4}\right)$

6. $y = \frac{2}{3} \sin(6x - 5)$

7. $y = 3 \cos\left(\frac{\pi}{4} - 4x\right)$

8. $y = 12 \tan\left(\frac{\pi x}{4}\right)$

9. $y = \frac{\sqrt{3}}{3} \cos\left(\frac{\pi}{6} - \sqrt{3}x\right)$

10. $y = \sin x^4$

11. $y = \sin^4 x$

12. $y = \cos^3 x$

13. $y = 2 \sin^5 x$

14. $y = \tan^6 x$

15. $y = 4 \cos^2 x$

16. $y = 3 \cot^4 x$

17. $y = 3\sec^2 x$

18. $y = 4\operatorname{cosec}^4 x$

19. $y = 2\sin^5 2x$

20. $y = 4\cos^3 2x$

21. $y = 2\tan^4 3x$

22. $y = 2\cot^2 4x$

23. $y = 4\sec^4 2x$

24. $y = 6\operatorname{cosec}^3\left(\frac{x}{2}\right)$

**THE CHAIN RULE WITH TRIGONOMETRIC FUNCTIONS,
EXPONENTIALS AND LOGARITHMS**

Question 13

1. $y = 4e^{\sin x}$

2. $y = \sin(e^{2x})$

3. $y = \ln(\sin x)$

4. $y = \sin(\ln x)$

5. $y = e^{2 \tan x}$

6. $y = \tan(e^{-x})$

7. $y = \cos(3 \ln x)$

8. $y = 2e^{\cos 2x}$

9. $y = \sin^4(e^x)$

10. $y = e^{\sin^2 x}$

MIXED CHAIN RULE

Question 14

1. $y = (3x+1)^8$

2. $y = \ln 2x$

3. $y = 3 \sin 2x + 2 \cos 3x$

4. $y = e^{3-2x}$

5. $y = \ln(x^2 + 1)$

6. $y = \frac{4}{(2x-1)^2}$

7. $y = 4 \cot 3x$

8. $y = \ln(4 - x^3)$

9. $y = \tan(2x^2 + 3)$

10. $y = 2 \sin 4x - 3 \cos 2x$

11. $y = 2e^{3x}$

12. $y = \ln(\sin x)$

13. $y = \cos(\ln x)$

14. $y = e^{\sin x}$

15. $y = 4 \cos 3x - 2 \sin 4x$

16. $y = \frac{3}{(4x-2)^3}$

17. $y = (3-6x)^{\frac{5}{2}}$

18. $y = \ln(2x^2 + 3x - 1)$

19. $y = e^{x^2}$

20. $y = \sin(x^2)$

21. $y = \sin^3 x$

22. $y = \cos(x^2 - 1)$

23. $y = 3 \cot 4x$

24. $y = e^{\tan x}$

25. $y = (e^{2x} + 2)^3$

26. $y = 3(2x+1)^6$

27. $y = 3 \ln 4x$

28. $y = 4 \sin 3x - 3 \cos 2x$

29. $y = 4e^{1-4x}$

30. $y = 3 \ln(2x^2 + 1)$

31. $y = \frac{4}{\sqrt{2x-1}}$

32. $y = 4 \tan 2x$

33. $y = \ln(\sin 2x)$

34. $y = \tan^4 x$

35. $y = 2 \sin\left(\frac{1}{2}x\right) - 3 \cos\left(\frac{2}{3}x\right)$

36. $y = 2e^{x^2}$

37. $y = \ln(3 \sin 2x)$

38. $y = 2 \cos(2 \ln x)$

39. $y = 2e^{\sin 3x}$

40. $y = 2 \sin^4 3x$

41. $y = \frac{2}{(2x-1)^4}$

42. $y = (3 - 6e^x)^{\frac{3}{2}}$

43. $y = \ln(\sec x + \tan x)$

44. $y = 2e^{-x^4}$

45. $y = 4 \sin(\sqrt{x})$

46. $y = 4 \sin^3 2x$

47. $y = \cos(e^{2x} - 1)$

48. $y = 6 \tan 2x - 2 \cot 3x$

49. $y = e^{4 \tan^2 x}$

50. $y = \ln(\cos 3x)$

THE PRODUCT RULE

Question 15

1. $y = x \sin x$

2. $y = 4x^2 \cos x$

3. $y = x^4 e^x$

4. $y = x^3 e^{-2x}$

5. $y = x^2 (2x-1)^5$

6. $y = 2e^{3x} (3x-1)^4$

7. $y = 3e^{-4x} \sin 2x$

8. $y = \cos 2x \tan 2x$

9. $y = (2x-1)^4 \sqrt{x}$

10. $y = (2x+1)^{\frac{3}{2}} (6x-1)^{\frac{1}{2}}$

Question 16

1. $y = x \cos x$

2. $y = 2x^4 \sin x$

3. $y = x^3 e^{-x}$

4. $y = 4x e^{2x}$

5. $y = x^2 (3x-1)^4$

6. $y = 3e^{-2x} (2x-1)^3$

7. $y = e^{-2x} \tan 2x$

8. $y = \sin 2x \cot 2x$

9. $y = 2(1-4x)^3 \sqrt{x}$

10. $y = (6x+1)^{\frac{1}{2}} (2x-1)^{-\frac{1}{2}}$

MIXED PRODUCT RULE

Question 17

1. $y = x^4 (4x - 1)^3$

2. $y = 2x^3 (2x + 3)^5$

3. $y = 6x^{\frac{1}{2}} (2x - 1)^4$

4. $y = e^{3x} \cos x$

5. $y = x^2 e^{4x}$

6. $y = (4x + 1)e^{2x}$

7. $y = x^2 \tan x$

8. $y = 3x^2 \sin 2x$

9. $y = x^3 \tan 2x$

10. $y = x^4 \ln x$

11. $y = e^{4x} \cos x$

12. $y = 4x^{\frac{1}{2}} \ln x$

13. $y = 4e^{-x} \tan 2x$

14. $y = e^{2x} (4 \sin 2x + 3 \cos 2x)$

15. $y = (2x + 1) \cot 4x$

16. $y = (3x^2 - 4x) \tan 2x$

17. $y = x^4 \sin^2 x$

18. $y = 3x^2 \sec 2x$

19. $y = (4x+5)^{\frac{3}{2}} e^{-2x}$

20. $y = x^3 (\sin 2x - 3 \cos 2x)$

21. $y = e^{6x} \cos^3 x$

22. $y = \sin x \tan^2 x$

23. $y = 4x^3 \operatorname{cosec} 3x$

24. $y = 3e^{-4x} \cot 6x$

25. $y = 4x^{\frac{5}{2}} \sin^5 x$

26. $y = 5x \ln(x^2 - 2)$

27. $y = \sin x \tan x$

28. $y = x^2 \sin 4x$

29. $y = e^{2x} \cos 3x$

30. $y = (4x-1)e^{-x}$

31. $y = x^3 \tan 2x$

32. $y = x^4 (4x-1)^3$

33. $y = (3-2x^2) \cos 2x$

34. $y = (3x-1)^{\frac{1}{2}} e^x$

35. $y = 2x^4 \ln x$

36. $y = e^x (\sin x - \cos x)$

37. $y = e^{-2x} (4x - 1)^3$

38. $y = (x^2 - 2x + 1)e^{2x}$

39. $y = (4x + 1)^3 (1 - 3x)^2$

40. $y = x^4 \sqrt{4x - 1}$

41. $y = x \sin^2 x$

42. $y = x^{-2} \ln x$

43. $y = \operatorname{cosec} x \cot x$

44. $y = \sqrt{x} \sin 2x$

45. $y = (4x + 3) \tan 2x$

46. $y = (3x - 1)^5 (2x + 1)^{\frac{3}{2}}$

47. $y = x^2 \sin^4 x$

48. $y = 6e^{2x} \cos^3 x$

49. $y = x^2 (e^x + e^{-x})$

50. $y = (1 - 2x)^{\frac{3}{2}} (3x + 1)^{-\frac{1}{2}}$

51. $y = x^5 \sqrt{x^2 - 1}$

THE QUOTIENT RULE

Question 18

1. $y = \frac{2x-5}{3x-1}$

2. $y = \frac{4x-1}{1-5x}$

3. $y = \frac{2x^2+1}{3x^2-1}$

4. $y = \frac{\ln x}{x}$

5. $y = \frac{\sin x}{x}$

6. $y = \frac{e^{2x}}{x}$

7. $y = \frac{x^2}{(3x-1)^2}$

8. $y = \frac{\sin x}{\cos x}$

9. $y = \frac{x^2-1}{2x+3}$

10. $y = \frac{8x^2+8x+3}{(2x+1)^2}$

Question 19

1. $y = \frac{4x-3}{2x-5}$

2. $y = \frac{3x-1}{1-2x}$

3. $y = \frac{5x^2+1}{2x^2-3}$

4. $y = \frac{\ln x}{x^3}$

5. $y = \frac{\sin 2x}{x^2}$

6. $y = \frac{e^{3x}}{2x}$

7. $y = \frac{4x^2}{(2x+3)^2}$

8. $y = \frac{\cos x}{\sin x}$

9. $y = \frac{3x^2+2}{x-1}$

10. $y = \frac{3x^2-6x+4}{(x-1)^2}$

MIXED QUOTIENT RULE

Question 20

1. $y = \frac{4x+3}{2x-3}$

2. $y = \frac{3-4x}{2x+1}$

3. $y = \frac{2x^2+1}{3x^2+1}$

4. $y = \frac{1+\cos x}{1+\sin x}$

5. $y = \frac{\ln x}{x^2}$

6. $y = \frac{x^2+1}{x^2-2}$

7. $y = \frac{3x^2+2}{x^2+5}$

8. $y = \frac{1-\cos x}{1+\cos x}$

9. $y = \frac{\sec x}{\tan x}$

10. $y = \frac{e^x+2}{e^x-2}$

11. $y = \frac{2x-1}{\sqrt{x+1}}$

12. $y = \frac{\sin^2 x}{\tan x}$

13. $y = \frac{\ln x}{x^4}$

14. $y = \frac{\sin 2x}{x}$

15. $y = \frac{x^2 - 1}{\sqrt{x+1}}$

16. $y = \frac{3x}{(4x-2)^3}$

17. $y = \frac{3e^x}{2e^x - 1}$

18. $y = \frac{4x-1}{2x+1}$

19. $y = \frac{1-2x}{3x+2}$

20. $y = \frac{4x^3 + 1}{2x^3 + 1}$

21. $y = \frac{1 - \sin x}{1 + \sin x}$

22. $y = \frac{\ln 2x}{x^3}$

23. $y = \frac{2x^2 + 3}{x^2 - 1}$

24. $y = \frac{2x^2 + 3}{x+1}$

25. $y = \frac{1 + \cos x}{1 - \cos x}$

26. $y = \frac{\sec x}{\sin x}$

27. $y = \frac{e^{2x} + 2}{e^{2x} - 1}$

28. $y = \frac{2x - 5}{\sqrt{4x + 1}}$

29. $y = \frac{\cos^2 x}{\tan x}$

30. $y = \frac{\ln x}{\sqrt{x}}$

31. $y = \frac{\sin 2x}{x^2}$

32. $y = \frac{4x}{\sqrt{2x + 1}}$

33. $y = \frac{x^3}{(2x - 1)^4}$

34. $y = \frac{4e^x}{e^x + 2}$

Question 21

Prove that:

$$1. \quad \frac{d}{dx} \left(e^{2x} (4 \sin 2x + 3 \cos 2x) \right) = 2e^{2x} (\sin 2x + 7 \cos 2x) \quad (**)$$

$$2. \quad \frac{d}{dx} \left(e^{2x} (x^2 - 4x - 2) \right) = 2e^{2x} (x^2 - 3x - 4) \quad (**)$$

$$3. \quad \frac{d}{dx} \left(\frac{4x}{4x-3} \right) = -\frac{12}{(4x-3)^2} \quad (**)$$

$$4. \quad \frac{d}{dx} \left(\frac{4x+3}{2x-3} \right) = -\frac{18}{(2x-3)^2} \quad (**)$$

$$5. \quad \frac{d}{dx} \left(\frac{3-4x}{2x+1} \right) = -\frac{10}{(2x+1)^2} \quad (**)$$

$$6. \quad \frac{d}{dx} \left(\frac{2x^2+1}{3x^2+1} \right) = -\frac{2x}{(3x^2+1)^2} \quad (**)$$

$$7. \quad \frac{d}{dx} (\ln(\sec x + \tan x)) = \sec x \quad (***)$$

$$8. \quad \frac{d}{dx} (x^4 (4x-1)^3) = 4x^3 (7x-1) (4x-1)^2 \quad (***)$$

$$9. \quad \frac{d}{dx} (2x^3 (2x+3)^5) = 2x^2 (16x+9) (2x+3)^4 \quad (***)$$

$$10. \quad \frac{d}{dx} (x^4 (4x-1)^3) = 4x^3 (7x-1) (4x-1)^2 \quad (***)$$

$$11. \quad \frac{d}{dx} \left(2e^{-3x} (2x+1)^{\frac{3}{2}} \right) = -12xe^{-3x} (2x+1)^{\frac{1}{2}} \quad (****)$$

$$12. \quad \frac{d}{dx} \left(\frac{x-4}{\sqrt{x}+2} \right) = \frac{1}{2\sqrt{x}} \quad (****+)$$

$$13. \frac{d}{dx} \left(e^{-2x} (4x-1)^3 \right) = 2(7-4x)(4x-1)^2 e^{-2x} \quad (***)$$

$$14. \frac{d}{dx} \left((x^2 - 2x + 1) e^{2x} \right) = 2x(x-1) e^{2x} \quad (***)$$

$$15. \frac{d}{dx} \left(x^4 \sqrt{4x-1} \right) = \frac{2x^3(9x-2)}{\sqrt{4x-1}} \quad (****)$$

$$16. \frac{d}{dx} \left(4\sqrt{x} \ln x \right) = \frac{2(2+\ln x)}{\sqrt{x}} \quad (****)$$

$$17. \frac{d}{dx} \left((4x+5)^{\frac{3}{2}} e^{-2x} \right) = -4(2x+1)\sqrt{4x+5} e^{-2x} \quad (****)$$

$$18. \frac{d}{dx} \left(6\sqrt{x} (2x-1)^4 \right) = \frac{3(18x-1)(2x-1)^3}{\sqrt{x}} \quad (****)$$

$$19. \frac{d}{dx} \left(\frac{1-\cos x}{1+\cos x} \right) = \frac{2\sin x}{(1+\cos x)^2} \quad (***)$$

$$20. \frac{d}{dx} \left(\frac{3+\sin 2x}{2+\cos 2x} \right) = \frac{6\sin 2x + 4\cos 2x + 2}{(2+\cos 2x)^2} \quad (***)$$

$$21. \frac{d}{dx} \left(\frac{5x^2 - 10x + 8}{(x-1)^2} \right) = -\frac{6}{(x-1)^3} \quad (***)$$

$$22. \frac{d}{dx} \left(\frac{\sec x}{\tan x} \right) = -\operatorname{cosec} x \cot x \quad (****)$$

$$23. \frac{d}{dx} \left(\frac{e^x + 2}{e^x - 2} \right) = -\frac{4}{(e^x - 2)^2} \quad (***)$$

$$24. \frac{d}{dx} \left(\frac{2x+1}{\sqrt{x+1}} \right) = \frac{2x+3}{2(x+1)^{\frac{3}{2}}} \quad (****)$$

$$25. \frac{d}{dx} \left(\frac{x-1}{\sqrt{x+1}} \right) = \frac{1}{2\sqrt{x}} \quad (****)$$

$$26. \frac{d}{dx} \left(\frac{\sin^2 x}{\tan x} \right) = \cos 2x \quad (****)$$

$$27. \frac{d}{dx} \left(\frac{3e^x}{2e^x - 1} \right) = -\frac{3e^x}{(2e^x - 1)^2} \quad (***)$$

$$28. \frac{d}{dx} \left(\frac{4x-1}{2x+1} \right) = \frac{6}{(2x+1)^2} \quad (**)$$

$$29. \frac{d}{dx} \left(\frac{1-2x}{3x+2} \right) = -\frac{7}{(3x+2)^2} \quad (**)$$

$$30. \frac{d}{dx} \left(\frac{4x^3+1}{2x^3+1} \right) = \frac{6x^2}{(2x^3+1)^2} \quad (**)$$

$$31. \frac{d}{dx} \left(\frac{1-\sin x}{1+\sin x} \right) = -\frac{2\cos x}{(1+\sin x)^2} \quad (**)$$

$$32. \frac{d}{dx} \left(\ln \left(\frac{1-\sin x}{1+\sin x} \right) \right) = -\sec x \quad (****)$$

$$33. \frac{d}{dx} \left(\ln \left(\frac{x+1}{x-1} \right) \right) = -\frac{2}{x^2-1} \quad (***)$$

$$34. \frac{d}{dx} \left(\frac{2\sin 3x}{1+\cos 3x} \right) = \frac{6}{1+\cos 3x} \quad (***)$$

$$35. \frac{d}{dx} \left(\frac{2x^2+3}{x^2-1} \right) = -\frac{10x}{(x^2-1)^2} \quad (**)$$

$$36. \frac{d}{dx} \left(\frac{2x^2+3}{x+1} \right) = \frac{2x^2+4x-3}{(x+1)^2} \quad (***)$$

$$37. \frac{d}{dx} \left(\frac{3x^2+6x-5}{(x+1)^2} \right) = \frac{16}{(x+1)^3} \quad (***)$$

$$38. \frac{d}{dx} \left(\frac{1 + \cos x}{1 - \cos x} \right) = -\cot \frac{x}{2} \operatorname{cosec}^2 \frac{x}{2} \quad (*****)$$

$$39. \frac{d}{dx} \left(\frac{\sec x}{\sin x} \right) = \sec^2 x - \operatorname{cosec}^2 x \quad (****)$$

$$40. \frac{d}{dx} \left(\ln \left(\frac{1}{1 - \sin x} \right) \right) = \frac{1 + \sin x}{\cos x} \quad (****)$$

$$41. \frac{d}{dx} \left(\frac{e^{2x} + 2}{e^{2x} - 1} \right) = -\frac{6e^{2x}}{(e^{2x} - 1)^2} \quad (****)$$

$$42. \frac{d}{dx} \left(\frac{2x - 5}{\sqrt{4x + 1}} \right) = \frac{4(x + 3)}{(4x + 1)^{\frac{3}{2}}} \quad (****)$$

$$43. \frac{d}{dx} \left(\frac{\ln x}{\sqrt{x}} \right) = \frac{2 - \ln x}{2x^{\frac{3}{2}}} \quad (****)$$

$$44. \frac{d}{dx} \left(\frac{4x}{\sqrt{2x + 1}} \right) = \frac{4(x + 1)}{(2x + 1)^{\frac{3}{2}}} \quad (****)$$

$$45. \frac{d}{dx} \left(\frac{x^3}{(2x - 1)^4} \right) = -\frac{x^2(2x + 3)}{(2x - 1)^5} \quad (****)$$

$$46. \frac{d}{dx} (\ln \sqrt{4x + 1}) = \frac{2}{4x + 1} \quad (**)$$

$$47. \frac{d}{dx} \left(\frac{4e^x}{e^x + 2} \right) = \frac{8e^x}{(e^x + 2)^2} \quad (****)$$

$$48. \frac{d}{dx} \left(\frac{\sec x}{\sin x} \right) = -4 \operatorname{cosec} 2x \cot 2x \quad (****)$$

$$49. \frac{d}{dx} (\ln(\tan x)) = 2 \operatorname{cosec} 2x \quad (****)$$

$$50. \frac{d}{dx} \left(\frac{2x(x^2 + 6x + 12)}{(x+2)^3} \right) = \frac{48}{(x+2)^4} \quad (****)$$

$$51. \frac{d}{dx} \left(\sqrt{\frac{x+1}{x-1}} \right) = -\frac{1}{(x-1)\sqrt{x^2-1}} \quad (****)$$

$$52. \frac{d}{dx} \left(\ln \left(\frac{1+\cos x}{1-\cos x} \right) \right) = -2 \operatorname{cosec} x \quad (****)$$

$$53. \frac{d}{dx} (\cos 2x + \tan x \sin 2x) = 0 \quad (*****)$$

$$54. \frac{d}{dx} \left(\ln \left(\frac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1} \right) \right) = \frac{1}{\sqrt{e^x+1}} \quad (*****)$$

$$55. \frac{d}{dx} \left(\ln \left(\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1} \right) \right) = \frac{2}{x\sqrt{1-x^2}} \quad (*****)$$

$$56. \frac{d}{dx} \left(\frac{\cos 2x}{\sqrt{1+\sin 2x}} \right) = -\sin x - \cos x \quad (*****)$$

$$57. \frac{d}{dx} \left(\ln \left(x + \sqrt{x^2+8} \right) - \frac{x}{x^2+8} \right) = \frac{x^2}{(x^2+8)^{\frac{3}{2}}} \quad (*****)$$

$$58. \frac{d}{dx} \left(\frac{\sqrt{e^{2x}-9}}{e^x} \right) = \frac{9}{e^x \sqrt{e^{2x}-9}} \quad (****)$$

$$59. \frac{d}{dx} \left(\ln \left(e^x + \sqrt{e^{2x}-9} \right) \right) = \frac{e^x}{\sqrt{e^{2x}-9}} \quad (*****)$$

$$60. \frac{d}{dx} \left(\frac{\sqrt{e^{2x}+4}}{e^x} \right) = -\frac{4}{e^x \sqrt{e^{2x}+4}} \quad (****)$$

$$61. \frac{d}{dx} \left(e^x \sqrt{e^{2x}-1} - \ln \left(e^x + \sqrt{e^{2x}-1} \right) \right) = 2e^x \sqrt{e^{2x}-1} \quad (*****)$$

THE CHAIN RULE WITH ALGEBRAIC FUNCTIONS

Question 1

1. $\frac{d}{dx}((2x+1)^4) = 8(2x+1)^3$

2. $\frac{d}{dx}((3x-2)^6) = 18(3x-2)^5$

3. $\frac{d}{dx}((8x-1)^{-1}) = -8(8x-1)^{-2}$

4. $\frac{d}{dx}((6x+1)^{\frac{1}{2}}) = 3(6x+1)^{-\frac{1}{2}}$

5. $\frac{d}{dx}((4x+3)^{\frac{3}{2}}) = 6(4x+3)^{\frac{1}{2}}$

6. $\frac{d}{dx}((1-x)^4) = -4(1-x)^3$

7. $\frac{d}{dx}(2(5x-2)^3) = 30(5x-2)^2$

8. $\frac{d}{dx}(2(3x+1)^{\frac{1}{2}}) = 3(3x+1)^{-\frac{1}{2}}$

9. $\frac{d}{dx}(4(1-5x)^{-\frac{1}{2}}) = 10(1-5x)^{-\frac{3}{2}}$

10. $\frac{d}{dx}(6(1-2x)^{\frac{1}{3}}) = -4(1-2x)^{-\frac{2}{3}}$

11. $\frac{d}{dx}((3x^2+1)^5) = 30x(3x^2+1)^4$

12. $\frac{d}{dx}((2x^2-3x)^4) = 4(4x-3)(2x^2-3x)^3$

13. $y = (5 - 2x + x^2)^{-2} = 4(1 - x)(5 - 2x + x^2)^{-3}$

14. $\frac{d}{dx} \left(\frac{4}{(5x+9)^3} \right) = -\frac{60}{(5x+9)^4}$

15. $\frac{d}{dx} \left(\frac{3}{\sqrt{3-2x}} \right) = \frac{3}{(3-2x)^{\frac{3}{2}}}$

16. $\frac{d}{dx} \left(\frac{1}{4x+1} \right) = -\frac{4}{(4x+1)^2}$

17. $\frac{d}{dx} \left(\frac{2}{3(2x+7)^2} \right) = -\frac{8}{3(2x+7)^3}$

18. $\frac{d}{dx} \left(\frac{3}{x^2+1} \right) = -\frac{6x}{(x^2+1)^2}$

19. $\frac{d}{dx} \left(\frac{4}{\sqrt{4-3x^2}} \right) = \frac{12x}{(4-3x^2)^{\frac{3}{2}}}$

20. $\frac{d}{dx} \left(\frac{1}{1-x^3} \right) = \frac{3x^2}{(1-x^3)^2}$

Question 2

1. $\frac{d}{dx}((4x+1)^3) = 12(4x+1)^2$

2. $\frac{d}{dx}((2x-1)^7) = 14(2x-1)^6$

3. $\frac{d}{dx}((6x-5)^{-1}) = -6(6x-5)^{-2}$

4. $\frac{d}{dx}((4x+1)^{\frac{1}{2}}) = 2(4x+1)^{-\frac{1}{2}}$

5. $\frac{d}{dx}((6x-3)^{\frac{5}{2}}) = 15(6x-3)^{\frac{3}{2}}$

6. $\frac{d}{dx}((1-2x)^8) = -16(1-2x)^7$

7. $\frac{d}{dx}(4(2x-3)^5) = 40(2x-3)^4$

8. $\frac{d}{dx}(3(6x-1)^{\frac{1}{2}}) = 9(6x-1)^{-\frac{1}{2}}$

9. $\frac{d}{dx}(6(1-3x)^{-\frac{1}{2}}) = 9(1-3x)^{-\frac{3}{2}}$

10. $\frac{d}{dx}(9(1-5x)^{\frac{1}{3}}) = -15(1-5x)^{-\frac{2}{3}}$

11. $\frac{d}{dx}((2x^2-1)^5) = 20x(2x^2-1)^4$

12. $\frac{d}{dx}((3x^2-4x)^3) = 6(3x-2)(3x^2-4x)^2$

13. $\frac{d}{dx}((1-4x+x^2)^{-2}) = -4(x-2)(1-4x+x^2)^{-3}$

14. $\frac{d}{dx} \left(\frac{2}{(4x-1)^3} \right) = \frac{-24}{(4x-1)^4}$

15. $\frac{d}{dx} \left(\frac{2}{\sqrt{5-2x}} \right) = \frac{2}{(5-2x)^{\frac{3}{2}}}$

16. $\frac{d}{dx} \left(\frac{3}{2x+1} \right) = -\frac{6}{(2x+1)^2}$

17. $\frac{d}{dx} \left(\frac{3}{2(4x+1)^2} \right) = -\frac{12}{(4x+1)^3}$

18. $\frac{d}{dx} \left(\frac{4}{x^2-3} \right) = -\frac{8x}{(x^2-3)^2}$

19. $\frac{d}{dx} \left(\frac{5}{6\sqrt{1-3x^2}} \right) = -\frac{5x}{2(1-3x^2)^{\frac{3}{2}}}$

20. $\frac{d}{dx} \left(\frac{4}{4-x^3} \right) = \frac{12x^2}{(4-x^3)^2}$

Question 3

1. $\frac{d}{dx}((6x-5)^3) = 18(6x-5)^2$

2. $\frac{d}{dx}((1-3x)^6) = -18(1-3x)^5$

3. $\frac{d}{dx}((2x-5)^{-2}) = -4(2x-5)^{-3}$

4. $\frac{d}{dx}((8x+3)^{\frac{1}{2}}) = 4(8x+3)^{-\frac{1}{2}}$

5. $\frac{d}{dx}(3(4x-1)^{\frac{5}{2}}) = 30(4x-1)^{\frac{3}{2}}$

6. $\frac{d}{dx}(4(1-\frac{1}{2}x)^6) = -12(1-\frac{1}{2}x)^5$

7. $\frac{d}{dx}(8(\frac{15}{16}x-3)^{1.2}) = 9(\frac{15}{16}x-3)^{0.2}$

8. $\frac{d}{dx}(5(\frac{3}{5}x-1)^{\frac{1}{3}}) = (\frac{3}{5}x-1)^{-\frac{2}{3}}$

9. $\frac{d}{dx}(2(1-\frac{5}{2}x)^{1.4}) = -7(1-\frac{5}{2}x)^{0.4}$

10. $\frac{d}{dx}(\frac{2}{4x-1}) = -\frac{8}{(4x-1)^2}$

11. $\frac{d}{dx}((4x^2-3)^3) = 24x(4x^2-3)^2$

12. $\frac{d}{dx}((4x^2+2x)^5) = 10(4x+1)(4x^2+2x)^4$

13. $\frac{d}{dx}((5-4x-2x^2)^{-3}) = 12(x+1)(5-4x-2x^2)^{-4}$

$$14. \frac{d}{dx} \left(\frac{5}{(4x^2 - 3)^2} \right) = -\frac{80x}{(4x^2 - 3)^3}$$

$$15. \frac{d}{dx} \left(\frac{2}{x^4 + 1} \right) = -\frac{8x^3}{(x^4 + 1)^2}$$

$$16. \frac{d}{dx} \left(\frac{5}{2\sqrt{2x+1}} \right) = -\frac{5}{2(2x+1)^{\frac{3}{2}}}$$

$$17. \frac{d}{dx} \left(\frac{5}{6\sqrt{1-x^3}} \right) = \frac{5x^2}{4(1-x^3)^{\frac{3}{2}}}$$

$$18. \frac{d}{dx} \left(\frac{4}{1+2\sqrt{x}} \right) = -\frac{4}{\sqrt{x}(1+2\sqrt{x})^2}$$

$$19. \frac{d}{dx} \left(\sqrt{2+\sqrt{x}} \right) = \frac{1}{4\sqrt{x}\sqrt{2+\sqrt{x}}} = \frac{1}{4\sqrt{2x+x\sqrt{x}}}$$

$$20. \frac{d}{dx} \left(\sqrt{3x+(2x+1)^4} \right) = \frac{3+8(2x+1)^3}{2\sqrt{3x+(2x+1)^4}}$$

THE CHAIN RULE WITH EXPONENTIALS & LOGS

Question 4

1. $\frac{d}{dx}(e^{2x}) = 2e^{2x}$

2. $\frac{d}{dx}(\ln 2x) = \frac{1}{x}$

3. $\frac{d}{dx}(e^{4x-1}) = 4e^{4x-1}$

4. $\frac{d}{dx}(\ln(3x-4)) = \frac{3}{3x-4}$

5. $\frac{d}{dx}(e^{-3x}) = -3e^{-3x}$

6. $\frac{d}{dx}(\ln(x^2-4)) = \frac{2x}{x^2-4}$

7. $\frac{d}{dx}(3e^{x^2}) = 6xe^{x^2}$

8. $\frac{d}{dx}((\ln x)^4) = \frac{4(\ln x)^3}{x}$

9. $\frac{d}{dx}((1+e^x)^4) = 4e^x(1+e^x)^3$

10. $\frac{d}{dx}(\ln x^4) = \frac{4}{x}$

11. $\frac{d}{dx}(3(1+e^{2x})^3) = 18e^{2x}(1+e^{2x})^2$

12. $\frac{d}{dx}((2x+\ln x)^4) = 4(2x+\ln x)^3\left(2+\frac{1}{x}\right)$

$$13. \frac{d}{dx} \left((4 + e^{-5x})^4 \right) = -20e^{-5x} (4 + e^{-5x})^3$$

$$14. \frac{d}{dx} \left((x^3 - 3 \ln x)^5 \right) = 15(x^3 - 3 \ln x)^4 \left(x^2 - \frac{1}{x} \right)$$

$$15. \frac{d}{dx} \left((x^2 + e^{2x})^4 \right) = 8(x + e^{2x})(x^2 + e^{2x})^3$$

$$16. \frac{d}{dx} (\sqrt{5x - 3 \ln x}) = \frac{1}{2\sqrt{5x - 3 \ln x}} \left(5 - \frac{3}{x} \right) = \frac{5x - 3}{2x\sqrt{5x - 3 \ln x}}$$

$$17. \frac{d}{dx} (\sqrt{1 + e^{2x}}) = \frac{e^{2x}}{\sqrt{1 + e^{2x}}}$$

$$18. \frac{d}{dx} (\ln(e^{2x} + 3)) = \frac{2e^{2x}}{e^{2x} + 3}$$

$$19. \frac{d}{dx} (e^{(2x+1)^5}) = 10(2x+1)^4 e^{(2x+1)^5}$$

$$20. \frac{d}{dx} (\ln(e^4 + 1)) = 0$$

Question 5

1. $\frac{d}{dx}(e^{4x}) = 4e^{4x}$

2. $\frac{d}{dx}(\ln 4x) = \frac{1}{x}$

3. $\frac{d}{dx}(e^{2x-5}) = 2e^{2x-5}$

4. $\frac{d}{dx}(\ln(5x-4)) = \frac{5}{5x-4}$

5. $\frac{d}{dx}(3e^{-2x}) = -6e^{-2x}$

6. $\frac{d}{dx}(3\ln(x^2+1)) = \frac{6x}{x^2+1}$

7. $\frac{d}{dx}(2e^{-x^2}) = -4xe^{-x^2}$

8. $\frac{d}{dx}(2(\ln x)^3) = \frac{6}{x}(\ln x)^2$

9. $\frac{d}{dx}(2(1+e^x)^3) = 6e^x(1+e^x)^2$

10. $\frac{d}{dx}(2\ln x^{\frac{1}{2}}) = \frac{1}{x}$

11. $\frac{d}{dx}(4\sqrt{2+e^{-2x}}) = -\frac{4e^{-2x}}{\sqrt{2+e^{-2x}}}$

12. $\frac{d}{dx}\left(\frac{3}{2x-\ln x}\right) = \frac{3\left(\frac{1}{x}-2\right)}{(2x-\ln x)^2} = \frac{3(1-2x)}{x(2x-\ln x)^2}$

13. $\frac{d}{dx}\left(10\left(x^2+e^{-\frac{1}{2}x}\right)^3\right) = 15\left(4x-e^{-\frac{1}{2}x}\right)\left(x^2+e^{-\frac{1}{2}x}\right)^2$

$$14. \frac{d}{dx} \left(\frac{1}{2} (e^{3x} - 3 \ln x)^{\frac{4}{3}} \right) = 2 \left(e^{3x} - \frac{1}{x} \right) (e^{3x} - 3 \ln x)^{\frac{1}{3}}$$

$$15. \frac{d}{dx} \left((e^{2x} + e^{-2x})^4 \right) = 8(e^{2x} - e^{-2x})(e^{2x} + e^{-2x})^3$$

$$16. \frac{d}{dx} \left(\frac{2}{\sqrt{4x - \ln x}} \right) = \left(\frac{1}{x} - 4 \right) (4x - \ln x)^{-\frac{3}{2}} = \frac{1 - 4x}{x(4x - \ln x)^{\frac{3}{2}}}$$

$$17. \frac{d}{dx} \left(\sqrt{1 + 2e^{2x^2}} \right) = \frac{4xe^{2x^2}}{\sqrt{1 + 2e^{2x^2}}}$$

$$18. \frac{d}{dx} (\ln(\ln x)) = \frac{1}{x \ln x}$$

$$19. \frac{d}{dx} (e^{e^x}) = e^x e^{e^x}$$

$$20. \frac{d}{dx} (\ln(e^{4+\ln^2} + 1)) = 0$$

Question 6

1. $\frac{d}{dx}(e^{3x}) = 3e^{3x}$

2. $\frac{d}{dx}(\ln 5x) = \frac{1}{x}$

3. $\frac{d}{dx}(e^{3x+2}) = 3e^{3x+2}$

4. $\frac{d}{dx}(\ln(2x+7)) = \frac{2}{2x+7}$

5. $\frac{d}{dx}(4e^{-5x}) = -20e^{-5x}$

6. $\frac{d}{dx}(2\ln(4x^2-3)) = \frac{16x}{4x^2-3}$

7. $\frac{d}{dx}(e^{3x} - 3e^{-x^2}) = 3e^{3x} + 6xe^{-x^2}$

8. $\frac{d}{dx}(\ln x^5 + (\ln x)^5) = \frac{5}{x} + \frac{5}{x}(\ln x)^4 = \frac{5}{x}[1 + (\ln x)^4]$

9. $\frac{d}{dx}(3(2-e^x)^4) = -12e^x(2-e^x)^4$

10. $\frac{d}{dx}(\ln \sqrt{x} + \sqrt{\ln x}) = \frac{1}{2x} + \frac{1}{2x\sqrt{\ln x}} = \frac{1}{2x}\left(1 + \frac{1}{\sqrt{\ln x}}\right)$

11. $\frac{d}{dx}(3\sqrt{1+e^{-3x}}) = -\frac{9e^{-3x}}{2\sqrt{1+e^{-3x}}}$

12. $\frac{d}{dx}\left(\frac{1}{x^3 + \ln x}\right) = -(x^3 + \ln x)^{-2} \times \left(3x^2 + \frac{1}{x}\right) = -\frac{3x^3 + 1}{x(x^3 + \ln x)^2}$

13. $\frac{d}{dx}\left(10\left(x^4 - 2e^{-\frac{1}{2}x}\right)^4\right) = 40\left(4x^3 + e^{-\frac{1}{2}x}\right)\left(x^4 - 2e^{-\frac{1}{2}x}\right)^3$

$$14. \frac{d}{dx} \left(\frac{1}{3} (e^{6x} + 3 \ln x)^{\frac{1}{3}} \right) = \frac{1}{9} \left(6e^{6x} + \frac{3}{x} \right) (e^{6x} + 3 \ln x)^{-\frac{2}{3}} = \frac{1}{3} \left(2e^{6x} + \frac{1}{x} \right) (e^{6x} + 3 \ln x)^{-\frac{2}{3}}$$

$$15. \frac{d}{dx} \left((2e^{3x} - 3e^{-2x})^3 \right) = 3(2e^{3x} - 3e^{-2x})^2 (6e^{3x} + 6e^{-2x}) = 18(2e^{3x} - 3e^{-2x})^2 (e^{3x} + e^{-2x})$$

$$16. \frac{d}{dx} \left(\frac{4}{\sqrt{2x - 3 \ln x}} \right) = -4 \left(2 - \frac{3}{x} \right) (2x - 3 \ln x)^{-\frac{3}{2}} = \frac{4(3 - 2x)}{x(2x - 3 \ln x)^{\frac{3}{2}}}$$

$$17. \frac{d}{dx} \left(\sqrt{1 - 2e^{-3x^2}} \right) = 6xe^{-3x^2} (1 - 2e^{-3x^2})^{-\frac{1}{2}} = \frac{6xe^{-3x^2}}{\sqrt{1 - 2e^{-3x^2}}}$$

$$18. \frac{d}{dx} (\ln(\ln(\ln x))) = \frac{1}{x(\ln x)[\ln(\ln x)]}$$

$$19. \frac{d}{dx} (2e^{2e^{2x}}) = 8e^{2x} e^{2e^{2x}}$$

$$20. \frac{d}{dx} (3 \ln(e^{\sqrt{5 + \ln 2}} - \ln 3)) = 0$$

THE CHAIN RULE WITH SINES & COSINES

Question 7

1. $\frac{d}{dx}(\sin 4x) = 4 \cos 4x$

2. $\frac{d}{dx}(2 \sin 3x) = 6 \cos 3x$

3. $\frac{d}{dx}(\cos 3x) = -3 \sin 3x$

4. $\frac{d}{dx}\left(6 \cos\left(\frac{2}{3}x\right)\right) = -4 \sin\left(\frac{2}{3}x\right)$

5. $\frac{d}{dx}\left(4 \sin\left(\frac{x}{2}\right)\right) = 2 \cos\left(\frac{x}{2}\right)$

6. $\frac{d}{dx}(3 \sin(5x-1)) = 15 \cos(5x-1)$

7. $\frac{d}{dx}\left(3 \cos\left(2x - \frac{\pi}{3}\right)\right) = -6 \sin\left(2x - \frac{\pi}{3}\right)$

8. $\frac{d}{dx}\left(2 \cos\left(\frac{\pi}{4} - 2x\right)\right) = 4 \sin\left(\frac{\pi}{4} - 2x\right)$

9. $\frac{d}{dx}\left(6 \cos\left(1 - \frac{3x}{2}\right)\right) = 9 \sin\left(1 - \frac{3x}{2}\right)$

10. $\frac{d}{dx}(2 \sin x^4) = 8x^3 \cos x^4$

11. $\frac{d}{dx}(2 \sin^4 x) = 8 \sin^3 x \cos x$

12. $\frac{d}{dx}(4 \cos x^3) = -12x^2 \sin x^3$

13. $\frac{d}{dx}(4\cos^3 x) = -12\cos^2 x \sin x$

14. $\frac{d}{dx}(3\sin^5 x) = 15\sin^4 x \cos x$

15. $\frac{d}{dx}(2\cos\sqrt{x}) = -\frac{\sin\sqrt{x}}{\sqrt{x}}$

16. $\frac{d}{dx}(2\sin^3 2x) = 12\sin^2 2x \cos 2x$

17. $\frac{d}{dx}(2(3\cos 2x + 1)^4) = -48\sin 2x(3\cos 2x + 1)^3$

18. $\frac{d}{dx}(\sqrt{1-2\cos x}) = \frac{\sin x}{\sqrt{1-2\cos x}}$

19. $\frac{d}{dx}((2\sin 3x - 3\cos 2x)^3) = 18(\cos 3x + \sin 2x)(2\sin 3x - 3\cos 2x)^2$

20. $\frac{d}{dx}(\sin^3(\frac{\pi}{2})) = 0$

Question 8

1. $\frac{d}{dx}(\sin 5x) = 5 \cos 5x$

2. $\frac{d}{dx}(4 \sin 2x) = 8 \cos 2x$

3. $\frac{d}{dx}(\cos 4x) = -4 \sin 4x$

4. $\frac{d}{dx}\left(8 \cos\left(\frac{1}{2}x\right)\right) = -4 \sin\left(\frac{1}{2}x\right)$

5. $\frac{d}{dx}\left(4 \sin\left(\frac{x}{4}\right)\right) = \cos\left(\frac{x}{4}\right)$

6. $\frac{d}{dx}(2 \sin(3x - 2)) = 6 \cos(3x - 2)$

7. $\frac{d}{dx}\left(5 \cos\left(3x - \frac{\pi}{4}\right)\right) = -15 \sin\left(3x - \frac{\pi}{4}\right)$

8. $\frac{d}{dx}\left(4 \cos\left(\frac{\pi}{2} - 3x\right)\right) = 12 \sin\left(\frac{\pi}{2} - 3x\right)$

9. $\frac{d}{dx}\left(8 \cos\left(3 - \frac{5x}{2}\right)\right) = 20 \sin\left(3 - \frac{5x}{2}\right)$

10. $\frac{d}{dx}\left(\frac{1}{2} \sin x^6\right) = 3x^5 \cos x^6$

11. $\frac{d}{dx}\left(\frac{1}{2} \sin^6 x\right) = 3 \sin^5 x \cos x$

12. $\frac{d}{dx}(10 \cos x^5) = -50x^4 \sin x^5$

13. $\frac{d}{dx}(10 \cos^5 x) = -50 \cos^4 x \sin x$

14. $\frac{d}{dx}(3\sin^7 x) = 21\sin^6 x \cos x$

15. $\frac{d}{dx}\left(2\cos\left(x^{\frac{3}{2}}\right)\right) = -3x^{\frac{1}{2}}\sin\left(x^{\frac{3}{2}}\right)$

16. $\frac{d}{dx}\left(\frac{1}{6}\sin^4 3x\right) = 2\cos 3x \sin^3 3x$

17. $\frac{d}{dx}\left(\frac{1}{3}(2\sin 3x + 3)^5\right) = 10\cos 3x(2\sin 3x + 3)^4$

18. $\frac{d}{dx}(\sqrt{1 - \cos 6x}) = \frac{3\sin 6x}{\sqrt{1 - \cos 6x}}$

19. $\frac{d}{dx}\left((4\sin 3x - 3\cos 4x)^3\right) = 36(\cos 3x + \sin 4x)(4\sin 3x - 3\cos 4x)^2$

20. $\frac{d}{dx}\left(\sin\left(\frac{\pi}{4}\right)\right) = 0$

THE CHAIN RULE WITH TAN, COT, SEC, COSEC

Question 9

1. $\frac{d}{dx}(4 \tan 3x) = 12 \sec^2 3x$

2. $\frac{d}{dx}\left(2 \tan\left(2x + \frac{\pi}{4}\right)\right) = 4 \sec^2\left(2x + \frac{\pi}{4}\right)$

3. $\frac{d}{dx}(3 \tan^4 x) = 12 \tan^3 x \sec^2 x$

4. $\frac{d}{dx}(3 \tan 2x) = 6 \sec^2 2x$

5. $\frac{d}{dx}\left(12 \tan\left(\frac{\pi x}{4}\right)\right) = 3\pi \sec^2\left(\frac{\pi x}{4}\right)$

6. $\frac{d}{dx}(\cot 2x) = -2 \operatorname{cosec}^2 2x$

7. $\frac{d}{dx}(3 \tan 2x - \cot 3x) = 6 \sec^2 2x + 3 \operatorname{cosec}^2 3x$

8. $\frac{d}{dx}(4 \sec 2x) = 8 \sec 2x \tan 2x$

9. $\frac{d}{dx}(2 \operatorname{cosec} 3x) = -6 \operatorname{cosec} 3x \cot 3x$

10. $\frac{d}{dx}\left(4 \sec \frac{x}{2} - 6 \operatorname{cosec} \frac{2x}{3}\right) = 2 \sec \frac{x}{2} \tan \frac{x}{2} + 4 \operatorname{cosec} \frac{2x}{3} \cot \frac{2x}{3}$

11. $\frac{d}{dx}(2 \cot 4x - 2 \sec 3x) = -8 \operatorname{cosec}^2 4x - 6 \sec 3x \tan 3x$

12. $\frac{d}{dx}(3 \tan 5x - 6 \operatorname{cosec} 2x) = 15 \sec^2 5x + 12 \operatorname{cosec} 2x \cot 2x$

13. $\frac{d}{dx}(\tan^6 x) = 6 \tan^5 x \sec^2 x$

14. $\frac{d}{dx}(3 \cot^4 x) = -12 \cot^3 x \operatorname{cosec}^2 x$

15. $\frac{d}{dx}(3 \sec^2 x) = 6 \sec^2 x \tan x$

16. $\frac{d}{dx}(4 \operatorname{cosec}^4 x) = -16 \operatorname{cosec}^4 x \cot x$

17. $\frac{d}{dx}(2 \tan^4 3x) = 24 \tan^3 3x \sec^2 3x$

18. $\frac{d}{dx}(2 \cot^2 4x) = -16 \cot 4x \operatorname{cosec}^2 4x$

19. $\frac{d}{dx}(4 \sec^4 2x) = 32 \sec^4 2x \tan 2x$

20. $\frac{d}{dx}\left(6 \operatorname{cosec}^3\left(\frac{x}{2}\right)\right) = -9 \operatorname{cosec}^3\left(\frac{x}{2}\right) \cot\left(\frac{x}{2}\right)$

Question 10

1. $\frac{d}{dx}(3 \tan 5x) = 15 \sec^2 5x$

2. $\frac{d}{dx}\left(4 \tan\left(3x + \frac{\pi}{3}\right)\right) = 12 \sec^2\left(3x + \frac{\pi}{3}\right)$

3. $\frac{d}{dx}\left(\frac{1}{2} \tan^6 x\right) = 3 \tan^5 x \sec^2 x$

4. $\frac{d}{dx}\left(10 \tan \frac{1}{2} x\right) = 5 \sec^2 \frac{1}{2} x$

5. $\frac{d}{dx}\left(12 \tan\left(\frac{5\pi x}{6}\right)\right) = 10\pi \sec^2\left(\frac{5\pi x}{6}\right)$

6. $\frac{d}{dx}(\cot 7x) = -7 \operatorname{cosec}^2 7x$

7. $\frac{d}{dx}(4 \tan 3x - 2 \cot 2x) = 12 \sec^2 3x + 4 \operatorname{cosec}^2 2x$

8. $\frac{d}{dx}(3 \sec 4x) = 12 \sec 4x \tan 4x$

9. $\frac{d}{dx}(6 \operatorname{cosec} 2x) = -12 \operatorname{cosec} 2x \cot 2x$

10. $\frac{d}{dx}\left(6 \sec \frac{x}{3} - 4 \operatorname{cosec} \frac{3x}{4}\right) = 2 \sec \frac{x}{3} \tan \frac{x}{3} + 3 \operatorname{cosec} \frac{3x}{4} \cot \frac{3x}{4}$

11. $\frac{d}{dx}(7 \cot 2x - 3 \sec 3x) = -14 \operatorname{cosec}^2 2x - 9 \sec 3x \tan 3x$

12. $\frac{d}{dx}(2 \tan 7x - 7 \operatorname{cosec} 2x) = 14 \sec^2 7x + 14 \operatorname{cosec} 2x \cot 2x$

13. $\frac{d}{dx}(\tan^3 x) = 3 \tan^2 x \sec^2 x$

14. $\frac{d}{dx}(8\cot^5 x) = -40\cot^4 x \operatorname{cosec}^2 x$

15. $\frac{d}{dx}\left(\frac{1}{2}\sec^4 x\right) = 2\sec^4 x \tan x$

16. $\frac{d}{dx}\left(\frac{3}{4}\operatorname{cosec}^6 x\right) = -\frac{9}{2}\operatorname{cosec}^6 x \cot x$

17. $\frac{d}{dx}(2\tan^6 2x) = 24\tan^5 2x \sec^2 2x$

18. $\frac{d}{dx}(2\cot^3 3x) = -18\cot^2 3x \operatorname{cosec}^2 3x$

19. $\frac{d}{dx}(3\sec^3 3x) = 27\sec^3 3x \tan 3x$

20. $\frac{d}{dx}\left(12\operatorname{cosec}^3\left(\frac{x}{4}\right)\right) = -9\operatorname{cosec}^3\left(\frac{x}{4}\right)\cot\left(\frac{x}{4}\right)$

THE CHAIN RULE WITH TRIGONOMETRIC FUNCTIONS

Question 11

1. $\frac{d}{dx}(\sin 2x) = 2 \cos 2x$

2. $\frac{d}{dx}(3 \cos 2x) = -6 \sin 2x$

3. $\frac{d}{dx}(4 \tan 3x) = 12 \sec^2 3x$

4. $\frac{d}{dx}\left(6 \sin\left(\frac{1}{2}x\right)\right) = 3 \cos\left(\frac{1}{2}x\right)$

5. $\frac{d}{dx}\left(3 \cos\left(\frac{x}{3}\right)\right) = -\sin\left(\frac{x}{3}\right)$

6. $\frac{d}{dx}(2 \sin(3x-1)) = 6 \cos(3x-1)$

7. $\frac{d}{dx}\left(2 \cos\left(4x - \frac{\pi}{3}\right)\right) = -8 \sin\left(4x - \frac{\pi}{3}\right)$

8. $\frac{d}{dx}\left(2 \tan\left(2x + \frac{\pi}{4}\right)\right) = 4 \sec^2\left(2x + \frac{\pi}{4}\right)$

9. $\frac{d}{dx}\left(9 \cos\left(\frac{\pi}{6} - 3x\right)\right) = 27 \sin\left(\frac{\pi}{6} - 3x\right)$

10. $\frac{d}{dx}(2 \sin x^3) = 6x^2 \cos x^3$

11. $\frac{d}{dx}(2 \sin^3 x) = 6 \sin^2 x \cos x$

12. $\frac{d}{dx}(4 \cos x^2) = -8x \sin x^2$

13. $\frac{d}{dx}(5\cos^2 x) = -10\cos x \sin x$

14. $\frac{d}{dx}(3\tan^4 x) = 12\tan^3 x \sec^2 x$

15. $\frac{d}{dx}(4\sin\sqrt{x}) = \frac{2}{\sqrt{x}}\cos\sqrt{x}$

16. $\frac{d}{dx}(\sin^5 2x) = 10\sin^4 2x \cos 2x$

17. $\frac{d}{dx}((3\sin x + 2)^4) = 12\cos x(3\sin x + 2)^3$

18. $\frac{d}{dx}(\sqrt{1+4\sin x}) = \frac{2\cos x}{\sqrt{1+4\sin x}}$

19. $\frac{d}{dx}((\sin x - \cos x)^3) = 3(\cos x + \sin x)(\sin x - \cos x)^2$

20. $\frac{d}{dx}\left(\sin^3\left(\frac{\pi}{6}\right)\right) = 0$

Question 12

1. $\frac{d}{dx}(3\sin 3x) = 9\cos 3x$

2. $\frac{d}{dx}(2\cos 4x) = -8\sin 4x$

3. $\frac{d}{dx}(3\tan 2x) = 6\sec^2 2x$

4. $\frac{d}{dx}\left(4\sin\left(\frac{3}{2}x\right)\right) = 6\cos\left(\frac{3}{2}x\right)$

5. $\frac{d}{dx}\left(2\cos\left(\frac{x}{4}\right)\right) = -\frac{1}{2}\sin\left(\frac{x}{4}\right)$

6. $\frac{d}{dx}\left(\frac{2}{3}\sin(6x-5)\right) = 4\cos(6x-5)$

7. $\frac{d}{dx}\left(3\cos\left(\frac{\pi}{4}-4x\right)\right) = 12\sin\left(\frac{\pi}{4}-4x\right)$

8. $\frac{d}{dx}\left(12\tan\left(\frac{\pi x}{4}\right)\right) = 3\pi\sec^2\left(\frac{\pi x}{4}\right)$

9. $\frac{d}{dx}\left(\frac{\sqrt{3}}{3}\cos\left(\frac{\pi}{6}-\sqrt{3}x\right)\right) = \sin\left(\frac{\pi}{6}-\sqrt{3}x\right)$

10. $\frac{d}{dx}(\sin x^4) = 4x^3 \cos x^4$

11. $\frac{d}{dx}(\sin^4 x) = 4\sin^3 x \cos x$

12. $\frac{d}{dx}(\cos^3 x) = -3\cos^2 x \sin x$

13. $\frac{d}{dx}(2\sin^5 x) = 10\sin^4 x \cos x$

14. $\frac{d}{dx}(\tan^6 x) = 6 \tan^5 x \sec^2 x$

15. $\frac{d}{dx}(4 \cos^2 x) = -8 \cos x \sin x$

16. $\frac{d}{dx}(3 \cot^4 x) = -12 \cot^3 x \operatorname{cosec}^2 x$

17. $\frac{d}{dx}(3 \sec^2 x) = 6 \sec^2 x \tan x$

18. $\frac{d}{dx}(4 \operatorname{cosec}^4 x) = -16 \cot x \operatorname{cosec}^4 x$

19. $\frac{d}{dx}(2 \sin^5 2x) = 20 \sin^4 2x \cos 2x$

20. $\frac{d}{dx}(4 \cos^3 2x) = -24 \cos^2 2x \sin 2x$

21. $\frac{d}{dx}(2 \tan^4 3x) = 24 \tan^3 3x \sec^2 3x$

22. $\frac{d}{dx}(2 \cot^2 4x) = -16 \cot 4x \operatorname{cosec}^2 4x$

23. $\frac{d}{dx}(4 \sec^4 2x) = 32 \sec^4 2x \tan 2x$

24. $\frac{d}{dx}\left(6 \operatorname{cosec}^3\left(\frac{x}{2}\right)\right) = -9 \operatorname{cosec}^3\left(\frac{x}{2}\right) \cot\left(\frac{x}{2}\right)$

**THE CHAIN RULE WITH TRIGONOMETRIC FUNCTIONS,
EXPONENTIALS AND LOGARITHMS**

Question 13

1. $\frac{d}{dx}(4e^{\sin x}) = 4e^{\sin x} \cos x$

2. $\frac{d}{dx}(\sin(e^{2x})) = 2e^{2x} \cos(e^{2x})$

3. $\frac{d}{dx}(\ln(\sin x)) = \cot x$

4. $\frac{d}{dx}(\sin(\ln x)) = \frac{\cos(\ln x)}{x}$

5. $\frac{d}{dx}(e^{2 \tan x}) = 2e^{2 \tan x} \sec^2 x$

6. $\frac{d}{dx}(\tan(e^{-x})) = -e^{-x} \sec^2(e^{-x})$

7. $\frac{d}{dx}(\cos(3 \ln x)) = -\frac{3 \sin(3 \ln x)}{x}$

8. $\frac{d}{dx}(2e^{\cos 2x}) = -4e^{\cos 2x} \sin 2x$

9. $\frac{d}{dx}(\sin^4(e^x)) = 4e^x \sin^3(e^x) \cos(e^x)$

10. $\frac{d}{dx}(e^{\sin^2 x}) = 2e^{\sin^2 x} \sin x \cos x$

MIXED CHAIN RULE

Question 14

1. $\frac{d}{dx} \left((3x+1)^8 \right) = 24(3x+1)^7$

2. $\frac{d}{dx} (\ln 2x) = \frac{1}{x}$

3. $\frac{d}{dx} (3 \sin 2x + 2 \cos 3x) = 6 \cos 2x - 6 \sin 3x$

4. $\frac{d}{dx} (e^{3-2x}) = -2e^{3-2x}$

5. $\frac{d}{dx} \left(\ln(x^2 + 1) \right) = \frac{2x}{x^2 + 1}$

6. $\frac{d}{dx} \left(\frac{4}{(2x-1)^2} \right) = -\frac{16}{(2x-1)^3}$

7. $\frac{d}{dx} (4 \cot 3x) = -12 \operatorname{cosec}^2 3x$

8. $\frac{d}{dx} \left(\ln(4 - x^3) \right) = -\frac{3x^2}{4 - x^3}$

9. $\frac{d}{dx} \left(\tan(2x^2 + 3) \right) = 4x \sec^2(2x^2 + 3)$

10. $\frac{d}{dx} (2 \sin 4x - 3 \cos 2x) = 8 \cos 4x + 6 \sin 2x$

11. $\frac{d}{dx} (2e^{3x}) = 6e^{3x}$

12. $\frac{d}{dx} (\ln(\sin x)) = \cot x$

$$13. \frac{d}{dx}(\cos(\ln x)) = -\frac{1}{x} \sin(\ln x)$$

$$14. \frac{d}{dx}(e^{\sin x}) = e^{\sin x} \cos x$$

$$15. \frac{d}{dx}(4 \cos 3x - 2 \sin 4x) = -12 \sin 3x - 8 \cos 4x$$

$$16. \frac{d}{dx}\left(\frac{3}{(4x-2)^3}\right) = -\frac{36}{(4x-2)^4}$$

$$17. \frac{d}{dx}\left((3-6x)^{\frac{5}{2}}\right) = -15(3-6x)^{\frac{3}{2}}$$

$$18. \frac{d}{dx}(\ln(2x^2 + 3x - 1)) = \frac{4x+3}{2x^2 + 3x - 1}$$

$$19. \frac{d}{dx}(e^{x^2}) = 2x e^{x^2}$$

$$20. \frac{d}{dx}(\sin(x^2)) = 2x \cos(x^2)$$

$$21. \frac{d}{dx}(\sin^3 x) = 3 \sin^2 x \cos x$$

$$22. \frac{d}{dx}(\cos(x^2 - 1)) = -2x \sin(x^2 - 1)$$

$$23. \frac{d}{dx}(3 \cot 4x) = -12 \operatorname{cosec}^2 4x$$

$$24. \frac{d}{dx}(e^{\tan x}) = e^{\tan x} \sec^2 x$$

$$25. \frac{d}{dx}((e^{2x} + 2)^3) = 6e^{2x}(e^{2x} + 2)^2$$

$$26. \frac{d}{dx}(3(2x+1)^6) = 36(2x+1)^5$$

27. $\frac{d}{dx}(3 \ln 4x) = \frac{3}{x}$

28. $\frac{d}{dx}(4 \sin 3x - 3 \cos 2x) = 12 \cos 3x + 6 \sin 2x$

29. $\frac{d}{dx}(4e^{1-4x}) = -16e^{1-4x}$

30. $\frac{d}{dx}(3 \ln(2x^2 + 1)) = \frac{12x}{2x^2 + 1}$

31. $\frac{d}{dx}\left(\frac{4}{\sqrt{2x-1}}\right) = -4(2x-1)^{-\frac{3}{2}}$

32. $\frac{d}{dx}(4 \tan 2x) = 8 \sec^2 2x$

33. $\frac{d}{dx}(\ln(\sin 2x)) = 2 \cot 2x$

34. $\frac{d}{dx}(\tan^4 x) = 4 \tan^3 x \sec^2 x$

35. $\frac{d}{dx}\left(2 \sin\left(\frac{1}{2}x\right) - 3 \cos\left(\frac{2}{3}x\right)\right) = \cos\left(\frac{1}{2}x\right) + 2 \sin\left(\frac{2}{3}x\right)$

36. $\frac{d}{dx}(2e^{x^2}) = 4xe^{x^2}$

37. $\frac{d}{dx}(\ln(3 \sin 2x)) = 2 \cot 2x$

38. $\frac{d}{dx}(2 \cos(2 \ln x)) = -\frac{4}{x} \sin(2 \ln x)$

39. $\frac{d}{dx}(2e^{\sin 3x}) = 6e^{\sin 3x} \cos 3x$

40. $\frac{d}{dx}(2 \sin^4 3x) = 24 \sin^3 3x \cos 3x$

$$41. \frac{d}{dx} \left(\frac{2}{(2x-1)^4} \right) = -\frac{16}{(2x-1)^5}$$

$$42. \frac{d}{dx} \left((3-6e^x)^{\frac{3}{2}} \right) = -9e^x (3-6e^x)^{\frac{1}{2}}$$

$$43. \frac{d}{dx} (\ln(\sec x + \tan x)) = \sec x$$

$$44. \frac{d}{dx} (2e^{-x^4}) = -8x^3 e^{-x^4}$$

$$45. \frac{d}{dx} (4 \sin(\sqrt{x})) = \frac{2 \cos \sqrt{x}}{\sqrt{x}}$$

$$46. \frac{d}{dx} (4 \sin^3 2x) = 24 \sin^2 2x \cos 2x$$

$$47. \frac{d}{dx} (\cos(e^{2x}-1)) = -2e^{2x} \sin(e^{2x}-1)$$

$$48. \frac{d}{dx} (6 \tan 2x - 2 \cot 3x) = 12 \sec^2 2x + 6 \operatorname{cosec}^2 3x$$

$$49. \frac{d}{dx} (e^{4 \tan^2 x}) = 8e^{4 \tan^2 x} \tan x \sec^2 x$$

$$50. \frac{d}{dx} (\ln(\cos 3x)) = -3 \tan 3x$$

THE PRODUCT RULE**Question 15**

$$1. \quad \frac{d}{dx}(x \sin x) = \sin x + x \cos x$$

$$2. \quad \frac{d}{dx}(4x^2 \cos x) = 8x \cos x - 4x^2 \sin x$$

$$3. \quad \frac{d}{dx}(x^4 e^x) = 4x^3 e^x + x^4 e^x = x^3(x+4)e^x$$

$$4. \quad \frac{d}{dx}(x^3 e^{-2x}) = 3x^2 e^{-2x} - 2x^3 e^{-2x} = x^2(3-2x)e^{-2x}$$

$$5. \quad \frac{d}{dx}(x^2(2x-1)^5) = 2x(2x-1)^5 + 10x^2(2x-1)^4 = 2x(2x-1)^4(7x-1)$$

$$6. \quad \frac{d}{dx}(2e^{3x}(3x-1)^4) = 6e^{3x}(3x-1)^4 + 24e^{3x}(3x-1)^3 = 18e^{3x}(3x-1)^3(x+1)$$

$$7. \quad \frac{d}{dx}(3e^{-4x} \sin 2x) = -12e^{-4x} \sin 2x + 6e^{-4x} \cos 2x = 6e^{-4x}(\cos 2x - 2 \sin 2x)$$

$$8. \quad \frac{d}{dx}(\cos 2x \tan 2x) = -2 \sin 2x \tan 2x + 2 \cos 2x \sec^2 2x = 2 \cos 2x$$

$$9. \quad \frac{d}{dx}((2x-1)^4 \sqrt{x}) = 8(2x-1)^3 x^{\frac{1}{2}} + \frac{1}{2}(2x-1)^4 x^{-\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}}(2x-1)^3(18x-1)$$

$$10. \quad \frac{d}{dx}((2x+1)^{\frac{3}{2}}(6x-1)^{\frac{1}{2}}) = 3(2x+1)^{\frac{1}{2}}(6x-1)^{\frac{1}{2}} + 3(2x+1)^{\frac{3}{2}}(6x-1)^{-\frac{1}{2}} = 24x(2x+1)^{\frac{1}{2}}(6x-1)^{-\frac{1}{2}}$$

Question 16

$$1. \quad \frac{d}{dx}(x \cos x) = \cos x - x \sin x$$

$$2. \quad \frac{d}{dx}(2x^4 \sin x) = 8x^3 \sin x + 2x^4 \cos x = 2x^3(4 \sin x + x \cos x)$$

$$3. \quad \frac{d}{dx}(x^3 e^{-x}) = 3x^2 e^{-x} - x^3 e^{-x} = x^2(3 - x)e^{-x}$$

$$4. \quad \frac{d}{dx}(4x e^{2x}) = 4e^{2x} + 8x e^{2x} = 4(2x + 1)e^{2x}$$

$$5. \quad \frac{d}{dx}(x^2(3x - 1)^4) = 2x(3x - 1)^4 + 12x^2(3x - 1)^3 = 2x(9x - 1)(3x - 1)^3$$

$$6. \quad \frac{d}{dx}(3e^{-2x}(2x - 1)^3) = -6e^{-2x}(2x - 1)^3 + 18e^{-2x}(2x - 1)^2 = 12e^{-2x}(2 - x)(2x - 1)^2$$

$$7. \quad \frac{d}{dx}(e^{-2x} \tan 2x) = -2e^{-2x} \tan 2x + 2e^{-2x} \sec^2 2x = 2e^{-2x}(\sec^2 2x - \tan 2x)$$

$$8. \quad \frac{d}{dx}(\sin 2x \cot 2x) = 2 \cos 2x \cot 2x - 2 \sin 2x \operatorname{cosec}^2 2x = -2 \sin 2x$$

$$9. \quad \frac{d}{dx}(2(1 - 4x)^3 \sqrt{x}) = -24x^{\frac{1}{2}}(1 - 4x)^2 + x^{-\frac{1}{2}}(1 - 4x)^3 = x^{-\frac{1}{2}}(1 - 28x)(1 - 4x)^2$$

$$10. \quad \frac{d}{dx}\left((6x + 1)^{\frac{1}{2}}(2x - 1)^{-\frac{1}{2}}\right) = 3(6x + 1)^{-\frac{1}{2}}(2x - 1)^{-\frac{1}{2}} - (6x + 1)^{\frac{1}{2}}(2x - 1)^{-\frac{3}{2}} = -4(6x + 1)^{-\frac{1}{2}}(2x - 1)^{-\frac{3}{2}}$$

MIXED PRODUCT RULE**Question 17**

$$1. \frac{d}{dx} \left(x^4 (4x-1)^3 \right) = 4x^3 (4x-1)^3 + 12x^4 (4x-1)^2 = 4x^3 (7x-1)(4x-1)^2$$

$$2. \frac{d}{dx} \left(2x^3 (2x+3)^5 \right) = 6x^2 (2x+3)^5 + 20x^3 (2x+3)^4 = 2x^2 (16x+9)(2x+3)^4$$

$$3. \frac{d}{dx} \left(6x^{\frac{1}{2}} (2x-1)^4 \right) = 3x^{-\frac{1}{2}} (2x-1)^4 + 48x^{\frac{1}{2}} (2x-1)^3 = 3x^{-\frac{1}{2}} (2x-1)^3 (18x-1)$$

$$4. \frac{d}{dx} \left(e^{3x} \cos x \right) = 3e^{3x} \cos x - e^{3x} \sin x = e^{3x} (3 \cos x - \sin x)$$

$$5. \frac{d}{dx} \left(x^2 e^{4x} \right) = 2x e^{4x} + 4x^2 e^{4x} = 2x(2x+1)e^{4x}$$

$$6. \frac{d}{dx} \left((4x+1)e^{2x} \right) = 4e^{2x} + 2(4x+1)e^{2x} = 2(4x+3)e^{2x}$$

$$7. \frac{d}{dx} \left(x^2 \tan x \right) = 2x \tan x + x^2 \sec^2 x = x(2 \tan x + x \sec^2 x)$$

$$8. \frac{d}{dx} \left(3x^2 \sin 2x \right) = 6x \sin 2x + 6x^2 \cos 2x = 6x(\sin 2x + x \cos 2x)$$

$$9. \frac{d}{dx} \left(x^3 \tan 2x \right) = 3x^2 \tan 2x + 2x^3 \sec^2 2x = x^2(3 \tan 2x + 2x \sec^2 2x)$$

$$10. \frac{d}{dx} \left(x^4 \ln x \right) = 4x^3 \ln x + x^3 = x^3(4 \ln x + 1)$$

$$11. \frac{d}{dx} \left(e^{4x} \cos x \right) = 4e^{4x} \cos x - e^{4x} \sin x = e^{4x}(4 \cos x - \sin x)$$

$$12. \frac{d}{dx} \left(4x^{\frac{1}{2}} \ln x \right) = 2x^{-\frac{1}{2}} \ln x + 4x^{-\frac{1}{2}} = 2x^{-\frac{1}{2}}(\ln x + 2)$$

$$13. \frac{d}{dx} (4e^{-x} \tan 2x) = -4e^{-x} \tan 2x + 8e^{-x} \sec^2 2x = 4e^{-x} (2 \sec^2 2x - \tan 2x)$$

$$14. \frac{d}{dx} (e^{2x} (4 \sin 2x + 3 \cos 2x)) = 2e^{2x} (4 \sin 2x + 3 \cos 2x) + e^{2x} (8 \cos 2x - 6 \sin 2x) \\ = 2e^{2x} (\sin 2x + 7 \cos 2x)$$

$$15. \frac{d}{dx} ((2x+1) \cot 4x) = 2 \cot 4x - 4(2x+1) \operatorname{cosec}^2 4x$$

$$16. \frac{d}{dx} ((3x^2 - 4x) \tan 2x) = (6x - 4) \tan 2x + 2(3x^2 - 4x) \sec^2 2x$$

$$17. \frac{d}{dx} (x^4 \sin^2 x) = 2x^3 \sin^2 x + 2x^4 \sin x \cos x = 2x^3 \sin x (2 \sin x + x \cos x)$$

$$18. \frac{d}{dx} (3x^2 \sec 2x) = 6x \sec 2x + 6x^2 \sec 2x \tan 2x = 6x \sec 2x (1 + x \tan 2x)$$

$$19. \frac{d}{dx} ((4x+5)^{\frac{3}{2}} e^{-2x}) = 6(4x+5)^{\frac{1}{2}} e^{-2x} - 2(4x+5)^{\frac{3}{2}} e^{-2x} = -4(2x+1)(4x+5)^{\frac{1}{2}} e^{-2x}$$

$$20. \frac{d}{dx} (x^3 (\sin 2x - 3 \cos 2x)) = 3x^2 (\sin 2x - 3 \cos 2x) + 2x^3 (\cos 2x + 3 \sin 2x)$$

$$21. \frac{d}{dx} (e^{6x} \cos^3 x) = 6e^{6x} \cos^3 x - 3e^{6x} \cos^2 x \sin x = 3e^{6x} \cos^2 x (2 \cos x - \sin x)$$

$$22. \frac{d}{dx} (\sin x \tan^2 x) = \cos x \tan^2 x + 2 \sin x \tan x \sec^2 x$$

$$23. \frac{d}{dx} (4x^3 \operatorname{cosec} 3x) = 12x^2 \operatorname{cosec} 3x - 12x^3 \operatorname{cosec} 3x \cot 3x = 12x^2 \operatorname{cosec} 3x (1 - x \cot 3x)$$

$$24. \frac{d}{dx} (3e^{-4x} \cot 6x) = -12e^{-4x} \cot 6x - 18e^{-4x} \operatorname{cosec}^2 6x = -6e^{-4x} (2 \cot 6x + 3 \operatorname{cosec}^2 6x)$$

$$25. \frac{d}{dx} (4x^{\frac{5}{2}} \sin^5 x) = 10x^{\frac{3}{2}} \sin^5 x + 20x^{\frac{5}{2}} \sin^4 x \cos x = 10x^{\frac{3}{2}} \sin^4 x (\sin x + 2x \cos x)$$

$$26. \frac{d}{dx} \left(5x \ln(x^2 - 2) \right) = 5 \ln(x^2 - 2) + \frac{10x^2}{x^2 - 2}$$

$$27. \frac{d}{dx} (\sin x \tan x) = \sin x \sec^2 x + \cos x \tan x$$

$$28. \frac{d}{dx} (x^2 \sin 4x) = 2x(\sin 4x + 2x \cos 4x)$$

$$29. \frac{d}{dx} (e^{2x} \cos 3x) = 2e^{2x} \cos 3x - 3e^{2x} \sin 3x = e^{2x} (2 \cos 3x - 3 \sin 3x)$$

$$30. \frac{d}{dx} ((4x-1)e^{-x}) = 4e^{-x} - (4x-1)e^{-x} = e^{-x} (5-4x)$$

$$31. \frac{d}{dx} (x^3 \tan 2x) = 3x^2 \tan 2x + 2x^3 \sec^2 2x = x^2 (3 \tan 2x + 2x \sec^2 2x)$$

$$32. \frac{d}{dx} (x^4 (4x-1)^3) = 4x^3 (4x-1)^3 + 12x^4 (4x-1)^2 = 4x^3 (7x-1)(4x-1)^2$$

$$33. \frac{d}{dx} ((3-2x^2) \cos 2x) = -4x \cos 2x - 2(3-2x^2) \sin 2x$$

$$34. \frac{d}{dx} ((3x-1)^{\frac{1}{2}} e^x) = \frac{3}{2} (3x-1)^{-\frac{1}{2}} e^x + (3x-1)^{\frac{1}{2}} e^x = \frac{(6x+1)e^x}{2\sqrt{3x-1}}$$

$$35. \frac{d}{dx} (2x^4 \ln x) = 2x^3 (1 + 4 \ln x)$$

$$36. \frac{d}{dx} (e^x (\sin x - \cos x)) = e^x (\sin x - \cos x) + e^x (\cos x + \sin x) = 2e^x \sin x$$

$$37. \frac{d}{dx} (e^{-2x} (4x-1)^3) = -2e^{-2x} (4x-1)^3 + 12e^{-2x} (4x-1)^2 = 2(7-4x)(4x-1)^2 e^{-2x}$$

$$38. \frac{d}{dx} ((x^2 - 2x + 1)e^{2x}) = 2(x^2 - 2x + 1)e^{2x} + (2x - 2)e^{2x} = 2x(x-1)e^{2x}$$

$$39. \frac{d}{dx} ((4x+1)^3 (1-3x)^2) = 12(4x+1)^2 (1-3x)^2 - 6(4x+1)^3 (1-3x) = 6(1-3x)(1-10x)(4x+1)^2$$

$$40. \frac{d}{dx} \left(x^4 \sqrt{4x-1} \right) = 4x^3 (4x-1)^{\frac{1}{2}} + 2x^4 (4x-1)^{-\frac{1}{2}} = \frac{2x^3 (9x-2)}{\sqrt{4x-1}}$$

$$41. \frac{d}{dx} (x \sin^2 x) = \sin x (\sin x + 2x \cos x)$$

$$42. \frac{d}{dx} (x^{-2} \ln x) = -2x^{-3} + x^{-3} = \frac{1-2 \ln x}{x^3}$$

$$43. \frac{d}{dx} (\operatorname{cosec} x \cot x) = -\operatorname{cosec} x (\cot^2 x + \operatorname{cosec}^2 x)$$

$$44. \frac{d}{dx} (\sqrt{x} \sin 2x) = \frac{1}{2} x^{-\frac{1}{2}} \sin 2x + 2x^{\frac{1}{2}} \cos 2x = \frac{\sin 2x + 4x \cos 2x}{2\sqrt{x}}$$

$$45. \frac{d}{dx} ((4x+3) \tan 2x) = 4 \tan 2x + 2(4x+3) \sec^2 2x$$

$$46. \frac{d}{dx} \left((3x-1)^5 (2x+1)^{\frac{3}{2}} \right) = 15(3x-1)^4 (2x+1)^{\frac{3}{2}} + 3(3x-1)^5 (2x+1)^{\frac{1}{2}} \\ = 3(13x+4)(3x-1)^4 (2x+1)^{\frac{1}{2}}$$

$$47. \frac{d}{dx} (x^2 \sin^4 x) = 2x \sin^4 x + 4x^2 \sin^3 x \cos x = 2x \sin^3 x (\sin x + 2x \cos x)$$

$$48. \frac{d}{dx} (6e^{2x} \cos^3 x) = 12e^{2x} \cos^3 x - 18e^{2x} \cos^2 x \sin x = 6e^{2x} \cos^2 x (2 \cos x - 3 \sin x)$$

$$49. \frac{d}{dx} (x^2 (e^x + e^{-x})) = 2x(e^x + e^{-x}) + x^2(e^x - e^{-x}) = x(x+2)e^x + 2x(1-x)e^{-x}$$

$$50. \frac{d}{dx} \left((1-2x)^{\frac{3}{2}} (3x+1)^{-\frac{1}{2}} \right) = -3(1-2x)^{\frac{1}{2}} (3x+1)^{-\frac{1}{2}} - \frac{3}{2}(1-2x)^{\frac{3}{2}} (3x+1)^{-\frac{3}{2}} \\ = -\frac{3}{2}(4x+3)(1-2x)^{\frac{1}{2}} (3x+1)^{-\frac{3}{2}}$$

$$51. \frac{d}{dx} (x^5 \sqrt{x^2-1}) = 5x^4 (x^2-1)^{\frac{1}{2}} + x^6 (x^2-1)^{-\frac{1}{2}} = \frac{x^4 (6x^2-5)}{\sqrt{x^2-1}}$$

THE QUOTIENT RULE**Question 18**

$$1. \frac{d}{dx} \left(\frac{2x-5}{3x-1} \right) = \frac{13}{(3x-1)^2}$$

$$2. \frac{d}{dx} \left(\frac{4x-1}{1-5x} \right) = -\frac{1}{(1-5x)^2}$$

$$3. \frac{d}{dx} \left(\frac{2x^2+1}{3x^2-1} \right) = -\frac{10x}{(3x^2-1)^2}$$

$$4. \frac{d}{dx} \left(\frac{\ln x}{x} \right) = \frac{1-\ln x}{x^2}$$

$$5. \frac{d}{dx} \left(\frac{\sin x}{x} \right) = \frac{x \cos x - \sin x}{x^2}$$

$$6. \frac{d}{dx} \left(\frac{e^{2x}}{x} \right) = \frac{2xe^{2x} - e^{2x}}{x^2} = \frac{e^{2x}(2x-1)}{x^2}$$

$$7. \frac{d}{dx} \left(\frac{x^2}{(3x-1)^2} \right) = \frac{2x(3x-1)^2 - 6x^2(3x-1)}{(3x-1)^4} = -\frac{2x}{(3x-1)^3}$$

$$8. \frac{d}{dx} \left(\frac{\sin x}{\cos x} \right) = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$9. \frac{d}{dx} \left(\frac{x^2-1}{2x+3} \right) = \frac{2x(2x+3) - 2(x^2-1)}{(2x+3)^2} = \frac{2x^2+6x+2}{(2x+3)^2}$$

$$10. \frac{d}{dx} \left(\frac{8x^2+8x+3}{(2x+1)^2} \right) = \frac{(16x+8)(2x+1)^2 - 4(8x^2+8x+3)(2x+1)}{(2x+1)^4} = -\frac{4}{(2x+1)^3}$$

Question 19

$$1. \frac{d}{dx} \left(\frac{4x-3}{2x-5} \right) = -\frac{14}{(2x-5)^2}$$

$$2. \frac{d}{dx} \left(\frac{3x-1}{1-2x} \right) = \frac{1}{(1-2x)^2}$$

$$3. \frac{d}{dx} \left(\frac{5x^2+1}{2x^2-3} \right) = -\frac{34x}{(2x^2-3)^2}$$

$$4. \frac{d}{dx} \left(\frac{\ln x}{x^3} \right) = \frac{1-3\ln x}{x^4}$$

$$5. \frac{d}{dx} \left(\frac{\sin 2x}{x^2} \right) = \frac{2x^2 \cos 2x - 2x \sin 2x}{x^4} = \frac{2(x \cos 2x - \sin 2x)}{x^3}$$

$$6. \frac{d}{dx} \left(\frac{e^{3x}}{2x} \right) = \frac{6xe^{3x} - 2e^{3x}}{4x^2} = \frac{e^{3x}(3x-1)}{2x^2}$$

$$7. \frac{d}{dx} \left(\frac{4x^2}{(2x+3)^2} \right) = \frac{8x(2x+3)^2 - 16x^2(2x+3)}{(2x+3)^4} = \frac{24x}{(2x+3)^3}$$

$$8. \frac{d}{dx} \left(\frac{\cos x}{\sin x} \right) = \frac{\sin x(-\sin x) - \cos x \cos x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$$

$$9. \frac{d}{dx} \left(\frac{3x^2+2}{x-1} \right) = \frac{6x(x-1) - (3x^2+2)}{(x-1)^2} = \frac{3x^2-6x-2}{(x-1)^2}$$

$$10. \frac{d}{dx} \left(\frac{3x^2-6x+4}{(x-1)^2} \right) = \frac{(6x-6)(x-1)^2 - 2(3x^2-6x+4)(x-1)}{(x-1)^4} = -\frac{2}{(x-1)^3}$$

MIXED QUOTIENT RULE**Question 20**

$$1. \frac{d}{dx} \left(\frac{4x+3}{2x-3} \right) = \frac{4(2x-3) - 2(4x+3)}{(2x-3)^2} = -\frac{18}{(2x-3)^2}$$

$$2. \frac{d}{dx} \left(\frac{3-4x}{2x+1} \right) = \frac{-4(2x+1) - 2(3-4x)}{(2x+1)^2} = -\frac{10}{(2x+1)^2}$$

$$3. \frac{d}{dx} \left(\frac{2x^2+1}{3x^2+1} \right) = \frac{4x(3x^2+1) - 6x(2x^2+1)}{(3x^2+1)^2} = -\frac{2x}{(3x^2+1)^2}$$

$$4. \frac{d}{dx} \left(\frac{1+\cos x}{1+\sin x} \right) = \frac{-\sin x(1+\sin x) - (1+\cos x)\cos x}{(1+\sin x)^2} = -\frac{1+\sin x+\cos x}{(1+\sin x)^2}$$

$$5. \frac{d}{dx} \left(\frac{\ln x}{x^2} \right) = \frac{1-2\ln x}{x^3}$$

$$6. \frac{d}{dx} \left(\frac{x^2+1}{x^2-2} \right) = \frac{2x(x^2-2) - 2x(x^2+1)}{(x^2-2)^2} = -\frac{6x}{(x^2-2)^2}$$

$$7. \frac{d}{dx} \left(\frac{3x^2+2}{x^2+5} \right) = \frac{6x(x^2+5) - 2x(3x^2+2)}{(x^2+5)^2} = \frac{26x}{(x^2+5)^2}$$

$$8. \frac{d}{dx} \left(\frac{1-\cos x}{1+\cos x} \right) = \frac{(1+\cos x)\sin x - (-\sin x)(1-\cos x)}{(1+\cos x)^2} = \frac{2\sin x}{(1+\cos x)^2}$$

$$9. \frac{d}{dx} \left(\frac{\sec x}{\tan x} \right) = \frac{\tan x(\sec x \tan x) - \sec x(\sec^2 x)}{\tan^2 x} = -\operatorname{cosec} x \cot x$$

$$10. \frac{d}{dx} \left(\frac{e^x+2}{e^x-2} \right) = \frac{e^x(e^x-2) - e^x(e^x+2)}{(e^x-2)^2} = -\frac{4e^x}{(e^x-2)^2}$$

$$11. \frac{d}{dx} \left(\frac{2x+1}{\sqrt{x+1}} \right) = \frac{2(x+1)^{\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}(2x+1)}{x+1} = \frac{2x+3}{2(x+1)^{\frac{3}{2}}}$$

$$12. \frac{d}{dx} \left(\frac{\sin^2 x}{\tan x} \right) = \frac{\tan x (2 \sin x \cos x) - \sin^2 x \sec^2 x}{\tan^2 x} = \cos^2 x - \sin^2 x = \cos 2x$$

$$13. \frac{d}{dx} \left(\frac{\ln x}{x^4} \right) = \frac{x^4 \times \frac{1}{x} - 4x^3 \ln x}{x^8} = \frac{x^3(1-4\ln x)}{x^8} = \frac{1-4\ln x}{x^5}$$

$$14. \frac{d}{dx} \left(\frac{\sin 2x}{x} \right) = \frac{2x \cos 2x - \sin 2x}{x^2}$$

$$15. \frac{d}{dx} \left(\frac{x^2-1}{\sqrt{x+1}} \right) = \frac{2(x+1)^{\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}(x^2-1)}{x+1} = \frac{5+4x-x^2}{2(x+1)^{\frac{3}{2}}}$$

$$16. \frac{d}{dx} \left(\frac{3x}{(4x-2)^3} \right) = \frac{3(4x-2)^3 - 36x(4x-2)^2}{(4x-2)^6} = -\frac{24x+6}{(4x-2)^4} = -\frac{3(4x+1)}{8(2x-1)^4}$$

$$17. \frac{d}{dx} \left(\frac{3e^x}{2e^x-1} \right) = \frac{3(2e^x-1)e^x - 2e^x(3e^x)}{(2e^x-1)^2} = -\frac{3e^x}{(2e^x-1)^2}$$

$$18. \frac{d}{dx} \left(\frac{4x-1}{2x+1} \right) = \frac{6}{(2x+1)^2}$$

$$19. \frac{d}{dx} \left(\frac{1-2x}{3x+2} \right) = -\frac{7}{(3x+2)^2}$$

$$20. \frac{d}{dx} \left(\frac{4x^3+1}{2x^3+1} \right) = \frac{6x^2}{(2x^3+1)^2}$$

$$21. \frac{d}{dx} \left(\frac{1-\sin x}{1+\sin x} \right) = \frac{-\cos x(1+\sin x) - \cos x(1-\sin x)}{(1+\sin x)^2} = -\frac{2\cos x}{(1+\sin x)^2}$$

$$22. \frac{d}{dx} \left(\frac{\ln 2x}{x^3} \right) = \frac{x^2 - 3x^2 \ln 2x}{x^6} = \frac{1 - 3 \ln 2x}{x^4}$$

$$23. \frac{d}{dx} \left(\frac{2x^2 + 3}{x^2 - 1} \right) = \frac{4x(x^2 - 1) - 2x(2x^2 + 3)}{(x^2 - 1)^2} = -\frac{10x}{(x^2 - 1)^2}$$

$$24. \frac{d}{dx} \left(\frac{2x^2 + 3}{x + 1} \right) = \frac{4x(x + 1) - (2x^2 + 3)}{(x + 1)^2} = \frac{2x^2 + 4x - 3}{(x + 1)^2}$$

$$25. \frac{d}{dx} \left(\frac{1 + \cos x}{1 - \cos x} \right) = \frac{-\sin x(1 - \cos x) - \sin x(1 + \cos x)}{(1 - \cos x)^2} = \frac{-2 \sin x}{(1 - \cos x)^2} = -\cot \frac{x}{2} \operatorname{cosec}^2 \frac{x}{2}$$

$$26. \frac{d}{dx} \left(\frac{\sec x}{\sin x} \right) = \frac{\sin x \sec x \tan x - \sec x \cos x}{\sin^2 x} = \sec^2 x - \operatorname{cosec}^2 x = -4 \operatorname{cosec} 2x \cot 2x$$

$$27. \frac{d}{dx} \left(\frac{e^{2x} + 2}{e^{2x} - 1} \right) = \frac{2e^{2x}(e^{2x} - 1) - 2e^{2x}(e^{2x} + 2)}{(e^{2x} - 1)^2} = -\frac{6e^{2x}}{(e^{2x} - 1)^2}$$

$$28. \frac{d}{dx} \left(\frac{2x - 5}{\sqrt{4x + 1}} \right) = \frac{2(4x + 1)^{\frac{1}{2}} - 2(4x + 1)^{-\frac{1}{2}}(2x - 5)}{4x + 1} = \frac{4(x + 3)}{(4x + 1)^{\frac{3}{2}}}$$

$$29. \frac{d}{dx} \left(\frac{\cos^2 x}{\tan x} \right) = \frac{-2 \sin x \cos x \tan x - \cos^2 x \sec^2 x}{\tan^2 x} = -\frac{2 \sin^2 x + 1}{\tan^2 x}$$

$$30. \frac{d}{dx} \left(\frac{\ln x}{\sqrt{x}} \right) = \frac{x^{\frac{1}{2}} \times \frac{1}{x} - \frac{1}{2} x^{-\frac{1}{2}} \ln x}{x} = \frac{2 - \ln x}{2x^{\frac{3}{2}}}$$

$$31. \frac{d}{dx} \left(\frac{\sin 2x}{x^2} \right) = \frac{2x^2 \cos 2x - 2x \sin 2x}{x^4} = \frac{2x \cos 2x - 2 \sin 2x}{x^3}$$

$$32. \frac{d}{dx} \left(\frac{4x}{\sqrt{2x + 1}} \right) = \frac{4(2x + 1)^{\frac{1}{2}} - 4x(2x + 1)^{-\frac{1}{2}}}{2x + 1} = \frac{4(x + 1)}{(2x + 1)^{\frac{3}{2}}}$$

$$33. \frac{d}{dx} \left(\frac{x^3}{(2x - 1)^4} \right) = \frac{3x^2(2x - 1)^4 - 8x^3(2x - 1)^3}{(2x - 1)^8} = -\frac{x^2(2x + 3)}{(2x - 1)^5}$$

34. $\frac{d}{dx}\left(\frac{4e^x}{e^x+2}\right) = \frac{8e^x}{(e^x+2)^2}$