INTEGRATION

BY PARTS

(student version)

Question 1

Carry out the following integrations:

1. $\displaystyle\int x\mathrm{e}^{2x}\,dx$

2. $\displaystyle\int 3x\cos 2x\,dx$

3. $\displaystyle\int x\sin 4x\,dx$

4. $\displaystyle\int -2x\sin 5x\,dx$

5. $\displaystyle\int (1-2x)\mathrm{e}^{-x}\,dx$

6. $\displaystyle\int x^2\,\mathrm{e}^{-3x}\,dx$

7. $\displaystyle\int 16x^3\ln x\,dx$

8. $\displaystyle\int \ln x\,dx$

9. $\displaystyle\int x\cos\left(\tfrac{1}{2}x\right)\,dx$

10. $\displaystyle\int (3x-1)\sin(3x-1)\,dx$

Question 2

Carry out the following integrations:

1. $\displaystyle\int 6x\,\mathrm{e}^{3x}\ dx$

2. $\displaystyle\int 12x\cos 3x\ dx$

3. $\displaystyle\int x\sin 6x\ dx$

4. $\displaystyle\int -x\sin 2x\ dx$

5. $\displaystyle\int (2-x)\,\mathrm{e}^{-3x}\ dx$

6. $\displaystyle\int x^2\,\mathrm{e}^{4x}\ dx$

7. $\displaystyle\int x^2\,\mathrm{e}^{-\frac{1}{2}x}\ dx$

8. $\displaystyle\int 25x^4\ln x\ dx$

9. $\displaystyle\int 24x\cos\left(\tfrac{2}{3}x\right)\ dx$

10. $\displaystyle\int x^2\sin(1-x)\ dx$

Question 3

Carry out the following integrations:

1. $\displaystyle\int \tfrac{1}{2}x\,e^{4x}\ dx$

2. $\displaystyle\int 5x\sin 4x\ dx$

3. $\displaystyle\int (2x+1)\cos 2x\ dx$

4. $\displaystyle\int -3x\cos 4x\ dx$

5. $\displaystyle\int x^2\,e^{-2x}\ dx$

6. $\displaystyle\int x^2\sin 5x\ dx$

7. $\displaystyle\int x^2\cos\tfrac{1}{3}x\ dx$

8. $\displaystyle\int \tfrac{1}{2}x^3\ln x\ dx$

9. $\displaystyle\int x\ln 3x\ dx$

10. $\displaystyle\int \frac{\ln x}{x^3}\ dx$

Question 4

Carry out the following integrations:

1. $\displaystyle\int x\,\mathrm{e}^{5x}\;dx$

2. $\displaystyle\int 2x\cos 3x\;dx$

3. $\displaystyle\int x\sin 3x\;dx$

4. $\displaystyle\int x\sin 4x\;dx$

5. $\displaystyle\int 2\ln x\;dx$

6. $\displaystyle\int x^2\ln x\;dx$

7. $\displaystyle\int x\sin\left(\tfrac{1}{2}x\right)dx$

8. $\displaystyle\int x\sin\left(2x-1\right)dx$

9. $\displaystyle\int \frac{\ln x}{x^2}\;dx$

10. $\displaystyle\int x\sec^2 x\;dx$

Question 5

Carry out the following integrations:

1. $\displaystyle\int x^2 \sin x \; dx$

2. $\displaystyle\int x^3 \ln x \; dx$

3. $\displaystyle\int \sin x \ln(\sec x) \; dx$

4. $\displaystyle\int x \cos 5x \; dx$

5. $\displaystyle\int x^2 \sin 3x \; dx$

6. $\displaystyle\int 4x \, e^{-\frac{2}{3}x} \; dx$

7. $\displaystyle\int x^2 \cos\left(\frac{1}{3}x\right) dx$

8. $\displaystyle\int 2x^2 \sec^2 x \tan x \; dx$

9. $\displaystyle\int x^2 \, e^{\frac{1}{2}x} \; dx$

10. $\displaystyle\int x \sec x \tan x \; dx$

Question 6

Carry out the following integrations:

1. $\displaystyle\int x^2\, e^{-\frac{1}{4}x}\, dx$

2. $\displaystyle\int x^2\, e^{-x}\, dx$

3. $\displaystyle\int e^x \cos x\, dx$

4. $\displaystyle\int (\ln x)^2\, dx$

5. $\displaystyle\int e^x \sin x\, dx$

6. $\displaystyle\int (x^3 + 5x^2 - 2)e^{2x}\, dx$

7. $\displaystyle\int x\cos^2 x\, dx$

8. $\displaystyle\int x\ln 2x^3\, dx$

Question 7

Carry out the following integrations, to the answer given:

1. $\displaystyle\int_0^{\ln 2} x\mathrm{e}^{2x}\ dx = \ln 4 - \frac{3}{4}$

2. $\displaystyle\int_0^{\frac{\pi}{3}} 6x\sin 3x\ dx = \frac{2\pi}{3}$

3. $\displaystyle\int_0^{\frac{\pi}{2}} x^2\cos x\ dx = \frac{1}{4}\left(\pi^2 - 8\right)$

4. $\displaystyle\int_1^{\mathrm{e}} x\ln x\ dx = \frac{1}{4}\left(\mathrm{e}^2 + 1\right)$

5. $\displaystyle\int_0^1 4x\mathrm{e}^{3x}\ dx = \frac{4}{9}\left(2\mathrm{e}^3 + 1\right)$

6. $\displaystyle\int_0^{\frac{\pi}{4}} x\sin 4x\ dx = \frac{\pi}{16}$

7. $\displaystyle\int_1^2 x^3\ln x\ dx = 4\ln 2 - \frac{15}{16}$

8. $\displaystyle\int_0^1 x\mathrm{e}^{-2x}\ dx = \frac{1}{4}\left(1 - 3\mathrm{e}^{-2}\right)$

9. $\displaystyle\int_0^{\frac{\pi}{4}} 12x\cos 2x\ dx = \frac{3}{2}\left(\pi - 2\right)$

10. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 4x\sin 2x\ dx = \pi - 1$

Question 8

Carry out the following integrations, to the answer given:

1. $\displaystyle\int_0^{\frac{\pi}{3}} x\sin 3x \ dx = \frac{\pi}{9}$

2. $\displaystyle\int_0^{\frac{\pi}{4}} 2x\cos 4x \ dx = -\frac{1}{4}$

3. $\displaystyle\int_0^{\ln 2} 4x\,\mathrm{e}^{-x} \ dx = 2 - \ln 4$

4. $\displaystyle\int_1^{\mathrm{e}} \ln x \ dx = 1$

5. $\displaystyle\int_0^{\frac{\pi}{2}} x\sin 2x \ dx = \frac{\pi}{4}$

6. $\displaystyle\int_0^{\ln 4} x\,\mathrm{e}^{\frac{1}{2}x} \ dx = 8\ln 2 - 4$

7. $\displaystyle\int_0^{\pi} x\cos\left(\frac{1}{4}x\right) \ dx = 2\sqrt{2}\,(\pi+4) - 16$

8. $\displaystyle\int_0^1 (2x+1)\mathrm{e}^{2x} \ dx = \mathrm{e}^2$

9. $\displaystyle\int_{\frac{1}{\mathrm{e}}}^1 x\ln x \ dx = \frac{1}{4}\left(\frac{3}{\mathrm{e}^2}-1\right)$

10. $\displaystyle\int_{-1}^0 3\ln(2x+3) \ dx = \frac{3}{2}(\ln 27 - 2)$ **REQUIRES ADDITIONAL TECHNIQUES**

Question 9

Carry out the following integrations, to the answer given:

1. $\displaystyle\int_0^{\frac{\pi}{4}} x\sec^2 x \ dx = \frac{1}{4}(\pi - \ln 4)$

2. $\displaystyle\int_1^2 \frac{\ln x}{x} \ dx = \frac{1}{2}(\ln 2)^2$

3. $\displaystyle\int_0^{\frac{\pi}{2}} x\sin^2 x \ dx = \frac{1}{16}(\pi^2 + 4)$