# EXPONENTIALS & LOGARITHMS practice

## **Question 1**

Solve each of the following equations.

- **a**)  $e^{x+1} = 17$
- **b**)  $e^{4-3y} = 20$
- c)  $5e^{2z} + 3 = 38$
- **d**)  $2e^{1-w} + 7 = 23$
- $e) \quad 7 e^{2t+3} 2 = 47$

$$\boxed{x = -1 + \ln 17 \approx 1.83}, \quad \boxed{y = \frac{1}{3} (4 - \ln 20) \approx 0.335}, \quad \boxed{z = \frac{1}{2} \ln 7 \approx 0.973},$$

$$\boxed{w = 1 - \ln 8 \approx -1.08}, \quad \boxed{t = \frac{1}{2} (-3 + \ln 7) \approx -0.527}$$

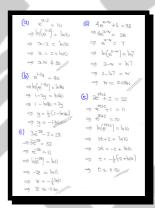


## **Question 2**

Solve each of the following equations.

- **a**)  $e^{x-2} = 10$
- **b**)  $e^{1-2y} = 40$
- $\mathbf{c)} \quad 3e^{-2z} 5 = 28$
- **d**)  $4e^{2-w} + 5 = 33$
- **e**)  $2e^{2t+2} + 2 = 22$

$$x = 2 + \ln 10 \approx 4.30$$
,  $y = \frac{1}{2}(1 - \ln 40) \approx -1.34$ ,  $z = -\frac{1}{2}\ln 11 \approx -1.20$ ,  $w = 2 - \ln 7 \approx 0.0541$ ,  $t = \frac{1}{2}(-2 + \ln 10) \approx 0.151$ 



## **Question 3**

Solve each of the following equations.

- **a**)  $e^{2x} = 9$
- **b**)  $5e^{1-y} = 30$
- **c**)  $4-3e^{2z}=3$
- **d**)  $13 e^{4-w} = 10$
- **e**)  $2e^{3t+2} + 2 = 20$

$$x = \ln 3 \approx 1.10$$
,  $y = 1 - \ln 6 \approx -0.792$ ,  $z = -\frac{1}{2} \ln 3 \approx -0.549$ ,  $w = 4 - \ln 3 \approx 2.90$ ,  $t = \frac{1}{3} (-2 + \ln 9) \approx 0.0657$ 

```
(a) e = 9
\Rightarrow h(e^2) = h(9)
\Rightarrow 2 = h(9)
\Rightarrow 2 = h(1)^2
\Rightarrow 2 = h(1)^3
\Rightarrow 1 =
```

## **Question 4**

Solve each of the following equations, leaving your final answers as expressions involving natural logarithms in their simplest form.

- **a**)  $e^x = 4$
- **b**)  $e^{2y} = 9$
- c)  $2e^{-z}+1=9$
- **d**)  $4e^{2w} 7 = 57$
- **e**)  $2e^{-3t} 7 = 243$

 $x = 2 \ln 2$ ,  $y = \ln 3$ ,  $z = -2 \ln 2$ ,  $w = 2 \ln 2$ ,  $t = -\ln 5$ 



## **Question 5**

Solve each of the following equations, leaving your final answers as expressions involving natural logarithms in their simplest form.

- **a**)  $e^{2x} = 16$
- **b**)  $e^{-2y} 1 = 8$
- c)  $3e^{2z} 20 = 88$
- **d**)  $e^{-3w} + 5 = 32$
- **e)**  $2e^{2t-2} + 2 = 10$

$$x = 2 \ln 2$$
,  $y = -\ln 3$ ,  $z = \ln 6$ ,  $w = -\ln 3$ ,  $t = 1 + \ln 2 = \ln (2e)$ 

```
(a) e^{2x} = 16 (b) e^{2x} = 18 (c) 3e^{2x} = 20 = 88

\Rightarrow 2x = 16 16 \Rightarrow e^{2x} = 9 \Rightarrow 3e^{2x} = 160

\Rightarrow 3e = 16 12^{4} \Rightarrow -2e = 163 \Rightarrow 2x = 1636

\Rightarrow 2x = 4162 \Rightarrow -3e = 1636

\Rightarrow 2x = 2162 \Rightarrow 2e = 1636

\Rightarrow 2e = 1636
```

## **Question 6**

Solve each of the following equations, leaving your final answers as expressions involving natural logarithms in their simplest form.

**a**) 
$$e^{4x} = 16$$

**b)** 
$$2e^{3y}-1=127$$

c) 
$$3e^{\frac{z}{2}} + 5 = 14$$

**d**) 
$$1-25e^{-4w} = \frac{24}{25}$$

$$e) \quad \frac{7 + 16807 \,\mathrm{e}^{-2t}}{35} = 10$$

$$x = \ln 2$$
,  $y = 2 \ln 2$ ,  $z = 2 \ln 3$ ,  $w = \ln 5$ ,  $t = \ln 7$ 

```
(a) e^{4k} = 16 (b) 3e^{4k} + 5 = 14 (c) 7 + 168 \times 16^{2k} = 10 (c) 3e^{4k} + 5 = 14 (d) 3e^{4k} + 5 = 16 (e) 3e^{4k} + 5 = 16 (f) 3e^{4k} + 6 = 1
```

## **Question 7**

Simplify each of the following expressions, giving the answer to the required form.

a) 
$$2 \ln 9 - \ln 6 - 4 \ln \sqrt{3} + \ln 2 \equiv a \ln 3$$

- **b)**  $2 \ln 54 \ln 12 \equiv b \ln 3$
- c)  $\frac{7}{4} \ln 16 \frac{2}{3} \ln 8 \equiv c \ln 2$
- **d**)  $2 \ln 56 \left[ \ln 168 \ln \left( \frac{3}{7} \right) \right] \equiv k \ln 2$
- e)  $2\ln 108 3\ln 48 \equiv p\ln 3 + q\ln 2$

$$a=1$$
,  $b=5$ ,  $c=5$ ,  $k=3$ ,  $p=3, q=-8$ 

```
(a) 2 \ln n - \ln G - 4 \ln 3 + \ln 2 = \ln 61 - \ln G - \ln 3 + \ln 2 = \ln 61 - \ln G - \ln 3 + \ln 2 = \ln 61 - \ln G - \ln 3 + \ln 2 = \ln 61 - \ln 6 - \ln 3 + \ln 2 = \ln 61 - \ln
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## **Question 8**

- $\mathbf{a)} \quad \ln\left(x+1\right) = 2$
- **b**)  $\ln(4-y)=2$
- c)  $\ln(3z-1)+6=7$
- **d**)  $2\ln(1-2w)+2=6$
- e)  $\ln(3-2t)+4=-2$

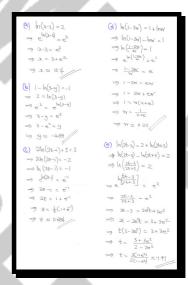
$$x = e^2 - 1 \approx 6.39$$
,  $y = 4 - e^2 \approx -3.39$ ,  $z = \frac{1}{3}(e+1) \approx 1.24$ ,  $w = \frac{1}{2}(1 - e^2) \approx -3.19$ ,  $t = \frac{1}{2}(3 - e^{-6}) \approx 1.50$ 

	_	
$\Rightarrow x+1 = e_x$ $\Rightarrow e_{\mu(x+1)} = e_x$ $\Rightarrow  \mu(x+1)  = 5$	(C) \( \langle (32-1) + C=7 \\ \( \Rightarrow \langle (32	> W= {(1-€)
=> 2 = 6 <sup>2</sup> -1 => 2 ≈ 6.39	} ⇒ e <sup>M(34-1)</sup> = e <sup>1</sup> ( ⇒ 32-1 = e	@ h(3-2+)+4=-2
(b) ln(4-y)= 2	32 = e+1 ⇒ 2= flex1)=12t/	h(3-2t) = -6 $e^{h(3-2t)} = -6$
$\Rightarrow e^{h(k-k)} = e^{2}$ $\Rightarrow 4-y = e^{2}$	$\begin{cases} (d) & 2\ln(1-2n)+2=6 \\ \Rightarrow & 2\ln(1-2n)=4 \end{cases}$	3-2t = e <sup>4</sup> 3 ~ e <sup>4</sup> = 2t
⇒ 4-e²=y		t= \frac{1}{2}(3-\tilde{e}^6)
$\Rightarrow y = 4 - e^2$ $\Rightarrow y = 4 - e^2$	$\Rightarrow e_{\mu(1-3\nu)} = e_5$	£# 1.20
~ /	⇒  -e=2w	

## **Question 9**

- a)  $\ln(x-3) = 2$
- **b**)  $1 \ln(3 y) = -1$
- c)  $2\ln(2z-1)+5=3$
- **d**)  $\ln(1-2w) = 1 + \ln w$
- e)  $\ln(2t-3) = 2 + \ln(2t+3)$

$$x = e^2 + 3 \approx 10.4$$
,  $y = 3 - e^2 \approx -4.39$ ,  $z = \frac{1}{2} (e^{-1} + 1) \approx 0.684$ ,  $w = \frac{1}{e+2} \approx 0.212$ ,  $t = \frac{3(1+e^2)}{2(1-e^2)} \approx -1.97$ 



#### **Question 10**

- a)  $\ln(x+3) = 2$
- **b)**  $2\ln(1-y)+1=3$
- c)  $3-2\ln(2z-1)=5$
- **d**)  $\ln(2w+1) = 1 + \ln(w-1)$
- e)  $\ln(t+1) = 2 + \ln(3t)$

$$x = e^2 - 3 \approx 4.39$$
,  $y = 1 - e \approx -1.72$ ,  $z = \frac{1}{2}(e^{-1} + 1) \approx 0.684$ ,  $w = \frac{e+1}{e-2} \approx 5.18$ ,  $t = \frac{1}{3e^2 - 1} \approx 0.0472$ 

```
(9) |N(3M+1) = 1 + |N(M-1)|
     [h(x+3) = 2
                                  ⇒ ln(2N+1)-ln(N-1)=1
   ⇒ 2+3 = e²
                                   \Rightarrow \ln \left( \frac{2W+1}{W-1} \right) = 1
                                   = e (N-1) = e1
   ⇒ 2 ≈ 439/
                                  \Rightarrow \frac{2N+1}{N-1} = e
                                   ⇒ 2W+1= eW-e
  \Rightarrow 2\ln(1-y) = 2
\Rightarrow \ln(1-y) = 1
\Rightarrow e^{\ln(1-y)} = e^{1}
                                   => I+e = ew-2w
                                   ⇒ 1+e = w(e-2)
                                   ⇒ W = e+1
                                   → N≈ 5.18/
                                (e) (n(t+1) = 2 + (n/3t)
       9 ≈-1.72/
                                => ln(t+1) - ln(3t) = 2
(c) 3-2ln(22-1)=5
                                 \Rightarrow \left| h\left(\frac{+1}{3t}\right) = 2.
 → - 2 = 214(22-1)
                                => e ( ( +4) = e2

⇒ -1 = In (22-1)
                                = ++1 = e2
                                ⇒ t+1 = 3te²
 =) 1+e<sup>-1</sup> = 22
                                 => 1=3te2-t
 = = 1 (1+e1)
                                  = 1 = +(3e-1)
```

#### **Question 11**

a) 
$$\ln(3x) - \ln(x-1) = 1$$

**b)** 
$$2 \ln y = \ln(2y + 8)$$

c) 
$$2 \ln z = \ln (4z + 12)$$

**d**) 
$$\ln(2w) = 2 + \ln(w+3)$$

e) 
$$\ln(t+7) = 1 + \ln(t-2)$$

$$x = \frac{e}{e - 3} \approx -9.65$$
,  $y = 4$ ,  $y \neq -2$ ,  $z = 6$ ,  $z \neq -2$ ,  $w = \frac{3e^2}{2 - e^2} \approx -4.11$ ,  $t = \frac{2e + 7}{e - 1} \approx 7.24$ 

```
(a) \ln(3x) - \ln(2x-1) = 1
\Rightarrow \ln(\frac{3x}{3x-1}) = 1
\Rightarrow \ln(\frac{3x}{3x-1}) = e^{1}
\Rightarrow \frac{3x}{3x-1} = e
\Rightarrow \frac{3x}{
```

## **Question 12**

a) 
$$\ln(2x) - \ln(x+2) = 1$$

**b)** 
$$2 \ln y = \ln (y + 20)$$

$$c) \quad 2\ln z = \ln(5z - 6)$$

**d**) 
$$\ln(2w-1) = 2 + \ln(w-2)$$

e) 
$$\ln(4t+1) = 1 + \ln(3t-2)$$

$$x = \frac{2e}{2 - e} \approx -7.57$$
,  $y = 5$ ,  $y \neq -4$ ,  $z = 2$ ,  $z = 3$ ,  $w = \frac{2e^2 - 1}{e^2 - 2} \approx 2.56$ ,  $t = \frac{2e + 1}{3e - 4} \approx 1.55$ 

## **Question 13**

Solve each of the following exponential equations.

**a**) 
$$e^x + 8e^{-x} = 6$$

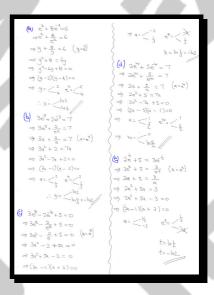
**b**) 
$$3e^y + 2e^{-y} = 7$$

c) 
$$3e^z - 2e^{-z} + 5 = 0$$

**d**) 
$$2e^{w} + 5e^{-w} = 7$$

**e**) 
$$2e^t + 5 = 3e^{-t}$$

$$x = \ln 2, 2 \ln 2$$
,  $y = \ln 2, -\ln 3$ ,  $z = -\ln 3$ ,  $w = 0, \ln \left(\frac{5}{2}\right)$ ,  $t = -\ln 2$ 



## **Question 14**

Solve each of the following exponential equations.

**a**) 
$$e^x + 3e^{-x} = 4$$

**b**) 
$$3e^y + 20e^{-y} = 19$$

$$c) 3e^z + 5e^{-z} = 16$$

**d**) 
$$e^w + e^{-w} = \frac{10}{3}$$

**e**) 
$$e^{2t} + 15 = 8e^t$$

$$x = 0, \ln 3$$
,  $y = \ln 5, \ln \left(\frac{4}{3}\right)$ ,  $z = \ln 5, -\ln 3$ ,  $w = \pm \ln 3$ ,  $t = \ln 3, \ln 5$ 

```
(a) \frac{e^{x}}{e^{x}} + 3e^{x} = \frac{1}{4}

\Rightarrow e^{x} + \frac{3}{4}e^{x} = \frac{1}{4}

\Rightarrow e^{x} + \frac{3}{4}e^{x} = \frac{1}{4}

\Rightarrow a + \frac{1}{4}e^{x} = 4

\Rightarrow a - \frac{1}{4}e^{x} = 4

\Rightarrow a -
```

## **Question 15**

Solve each of the following exponential equations.

**a**) 
$$e^x - 8e^{-x} = 2$$

**b**) 
$$2(e^y + e^{-y}) = 5$$

$$\mathbf{c}) \quad \frac{8e^z}{e^{2z}-1} = 3$$

**d**) 
$$e^w - 3 = \frac{8}{e^w - 1}$$

**e**) 
$$e^{2t} + e^{-2t} = 4$$

$$\boxed{x=2\ln 2}$$
,  $\boxed{y=\pm \ln 2}$ ,  $\boxed{z=\ln 3}$ ,  $\boxed{w=\ln 5}$ ,  $\boxed{t=\frac{1}{2}\ln\left(2\pm\sqrt{3}\right)=\pm\frac{1}{2}\ln\left(2+\sqrt{3}\right)}$