

CALCULUS KINEMATICS

CALCULUS KINEMATICS IN SCALAR FORM

Question 1 ()**

A particle P is moving on the x axis and its displacement from the origin, x m, t seconds after a given instant, is given by

$$x = \frac{1}{3}t(t^2 - 3t - 24), t \geq 0.$$

Determine the displacement of P when it is instantaneously at rest.

, $x = -26\frac{2}{3}$ m

Differentiating to obtain the velocity

$$\begin{aligned}x &= \frac{1}{3}t(t^2 - 3t - 24) \\&\Rightarrow x = \frac{1}{3}(t^3 - 3t^2 - 24t) \\V &= \frac{dx}{dt} = \frac{1}{3}(3t^2 - 6t - 24) \\&\Rightarrow V = t^2 - 2t - 8\end{aligned}$$

Instantaneously at rest $\Rightarrow V = 0$

$$\begin{aligned}\Rightarrow 0 &= t^2 - 2t - 8 \\&\Rightarrow (t+2)(t-4) = 0 \\&\Rightarrow t = -2 \quad \cancel{t=4}\end{aligned}$$

This displacement when $t=0$ can now be found

$$x(4) = \frac{1}{3} \times 4 \times (4^2 - 3 \times 4 - 24) = \frac{32}{3} \quad \cancel{(x=-26.7)}$$

Question 2 ()**

A particle P is moving on the x axis and its acceleration $a \text{ ms}^{-2}$, t seconds after a given instant, is given by

$$a = 6t - 18, t \geq 0.$$

The particle is initially at the origin O , moving with a speed of 15 ms^{-1} in the positive x direction.

- a) Determine the times when P is instantaneously at rest.

- b) Find the distance between the points, at which P is instantaneously at rest.

$$\boxed{\quad}, t=1, \boxed{t=5}, \boxed{d=32 \text{ m}}$$

a) FIND AN EXPRESSION FOR THE SPEED v

$$\begin{aligned} \Rightarrow a &= 6t - 18 \\ \Rightarrow v &= \int 6t - 18 \, dt \\ \Rightarrow v &= 3t^2 - 18t + C \end{aligned}$$

APPLY INITIAL CONDITIONS $t=0, v=15$

$$\begin{aligned} 15 &= 0 - 0 + C \\ C &= 15 \end{aligned}$$

USE THE DERIVATIVE EXPRESSION WITH $t=0$

$$\begin{aligned} \Rightarrow v &= 3t^2 - 18t + 15 \\ \Rightarrow 0 &= 3t^2 - 18t + 15 \\ \Rightarrow 0 &= t^2 - 6t + 5 \\ \Rightarrow 0 &= (t-1)(t-5) \\ \Rightarrow t &= 1 \quad \text{or} \quad t = 5 \end{aligned}$$

b) INTEGRATE THE SPEED EXPRESSION TO OBTAIN A DISTANCE

$$\begin{aligned} \Rightarrow v &= 3t^2 - 18t + 15 \\ \Rightarrow x &= \int 3t^2 - 18t + 15 \, dt \\ \Rightarrow x &= t^3 - 9t^2 + 15t + D \end{aligned}$$

WHEN $t=0, x=0$ (BECAUSE $\int D \, dt = 0$)

$$\Rightarrow x = t^3 - 9t^2 + 15t$$

ALTERNATIVE FOR PART (b) BY SPEED TIME GRAPH

SCRATCHING

$$\begin{aligned} v &= 3t^2 - 18t + 15 \\ v &= 3(t^2 - 6t + 5) \\ v &= 3(t-1)(t-5) \end{aligned}$$

\therefore DISTANCE = $\left| \int_0^5 3t^2 - 18t + 15 \, dt \right|$

$$\begin{aligned} &= \left| \left[t^3 - 9t^2 + 15t \right] \Big|_0^5 \right| \\ &= \left| (125 - 225 + 75) - (0 - 0 + 0) \right| \\ &= |-25 - 7| \\ &= |-32| \\ &= \boxed{32 \text{ m}} \end{aligned}$$

Question 3 ()**

A particle P is moving on the x axis and its velocity $v \text{ ms}^{-1}$, t seconds after a given instant, is given by

$$v = t^2 - 4t - 12, t \geq 0.$$

When $t = 0$, its displacement x from the origin O is 20 m.

- Find the acceleration of P when $t = 3$.
- Find the acceleration of P , when P is instantaneously at rest.
- Determine the distance of P from O , when P is instantaneously at rest.

, $a = 2 \text{ ms}^{-2}$, $a = 8 \text{ ms}^{-2}$, $d = 52 \text{ m}$

a) Differentiate w.r.t. t , to find an expression for the acceleration

$$v = t^2 - 4t - 12$$

$$a = \frac{dv}{dt} = 2t - 4$$

$$a|_{t=3} = 2(3) - 4$$

$$a = 2 \text{ ms}^{-2}$$

b) Since $v=0$

$$\Rightarrow t^2 - 4t - 12 = 0$$

$$\Rightarrow (t-6)(t+2) = 0$$

$$\Rightarrow t = -2 \cancel{\times} \quad \therefore a|_{t=6} = 2(6) - 4$$

$$= 8 \text{ ms}^{-2}$$

c) Integrate the velocity expression, to obtain a displacement expression

$$v = t^2 - 4t - 12$$

$$\Rightarrow x = \int v \, dt = \int t^2 - 4t - 12 \, dt$$

$$\Rightarrow x = \frac{1}{3}t^3 - 2t^2 - 12t + C$$

$$\begin{aligned} &\Rightarrow x = \frac{1}{3}t^3 - 2t^2 - 12t + 20 \\ &x|_0 = 20 \\ &x|_6 = \frac{1}{3}(6)^3 - 2(6)^2 - 12(6) + 20 = -52 \end{aligned}$$

$\therefore \text{Distance of } 52 \text{ m}$

c) Alternative by speed-time graph (for part c)

- Sketch the speed-time graph
- "Signed area" = $\int_0^6 (t^2 - 4t - 12) \, dt$
- $= \left[\frac{1}{3}t^3 - 2t^2 - 12t \right]_0^6$
- $= (72 - 72 - 72) = -72$
- Now the particle was $+20$ (displacement) when $t=0$
Since the displacement is $-72 + 20 = -52$
- This the distance is 52 m

Question 4 (*)**

A particle P is moving on the x axis and its velocity $v \text{ ms}^{-1}$, $t \text{ s}$ after a given instant, is given by

$$v = t^2(3-t), t \geq 0.$$

When $t = 2$, P is observed to be 4 m from the origin O , in the positive x direction.

- a) Find the acceleration of P when $t = 2$.

The particle is at instantaneous rest initially, and when $t = T$.

- b) Determine the distance of P from O when $t = T$.

, $a = 0$, $d = 6.75 \text{ m}$

a) DIFFERENTIATE VELOCITY TO OBTAIN ACCELERATION

$$v = 3t^2 - t^3$$

$$a = \frac{dv}{dt} = 6t - 3t^2$$

$$\left. a \right|_{t=2} = (6 \cdot 2) - (3 \cdot 2^2) = 12 - 12 = 0$$

// ZERO ACCELERATION

b) BY INSPECTION, AT REST WHEN $v=0$, NEEDS $t=0$ OR $t=3$

IF $v = t^2(3-t)$
 $0 = t^2(3-t)$
 $t = < 0$ $t = T = 3$

INTEGRATE TO OBTAIN DISPLACEMENT

$$v = 3t^2 - t^3$$

$$x = \int 3t^2 - t^3 \, dt$$

$$x = t^3 - \frac{1}{4}t^4 + C$$

APPLY CONDITION $t=2, x=4$

$$4 = 2^3 - \frac{1}{4} \cdot 2^4 + C$$

$$4 = 8 - 4 + C$$

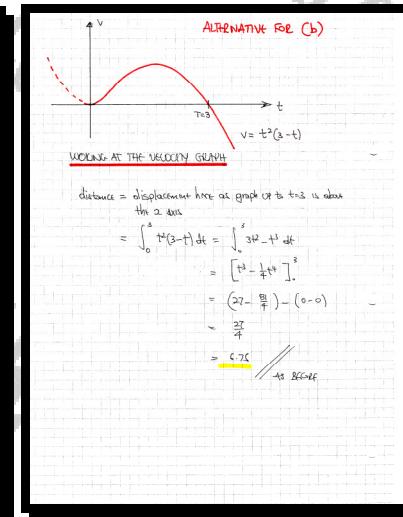
$$C = 0$$

$$\therefore x = t^3 - \frac{1}{4}t^4$$

FINALLY WHEN $t=T=3$

$$x = 3^3 - \frac{1}{4} \cdot 3^4 = 6.75$$

$\therefore 6.75 \text{ m}$



Question 5 (*)**

A particle P is moving on the x axis and its acceleration $a \text{ ms}^{-2}$, t seconds after a given instant, is given by

$$a = 8 - 2t, t \geq 0.$$

Initially, P is on the positive x axis 84 m away from the origin O , and is moving towards O with a speed of 7 ms^{-1} .

- a) Find an expression for the velocity of P .
- b) Calculate the maximum velocity of P .
- c) Determine the times when P is instantaneously at rest.
- d) Show that when $t = 12$, P is passing through O .

$$v = -t^2 + 8t - 7, v_{\max} = 9 \text{ ms}^{-1}, t = 1, 7$$

(a) $a = 8 - 2t, t \geq 0, x = 84, v = -7$

 $v = \int a \, dt$
 $v = \int 8 - 2t \, dt$
 $v = 8t - t^2 + C$

When $t = 0, v = -7$
 $-7 = 0 + C$
 $C = -7$

 $v = 8t - t^2 - 7$


(b) $v = f(t) \Rightarrow$

 $\Rightarrow \frac{dv}{dt} = 0$
 $\Rightarrow 8 - 2t = 0$
 $\Rightarrow t = 4$
 $\therefore v_{\max} = 8(4) - 4^2 - 7 = 9 \text{ ms}^{-1}$

(c) $v = 0$
 $\Rightarrow 8t - t^2 - 7 = 0$
 $\Rightarrow 0 = t^2 - 8t + 7$
 $\Rightarrow 0 = (t - 1)(t - 7)$
 $\Rightarrow t = 1, 7$

(d) $x = \int v \, dt$
 $\Rightarrow x = \int 8t - t^2 - 7 \, dt$
 $\Rightarrow x = 4t^2 - \frac{1}{3}t^3 - 7t + K$

When $t = 0, x = 84$
 $84 = 0 + K$
 $K = 84$

 $\Rightarrow x = 4t^2 - \frac{1}{3}t^3 - 7t + 84$

When $t = 12, x = ?$
 $x = 576 - 512 - 84 + 84$
 $x = 0$

Passed AF O

Question 6 (*)**

A particle is moving in a straight line.

At time t s, the particle has displacement x m from a fixed origin O and is moving with velocity v ms $^{-1}$.

When $t = 1$, $x = -5$ and $v = 1$.

The acceleration a of the particle is given by

$$a = (16 - 6t) \text{ ms}^{-2}, t \geq 0.$$

The particle passes through O with speed U when $t = T$, $T > 0$.

Find the possible values of U .

, $U = 8, 24$

SING INTEGRATION TO OBTAIN A VELOCITY EXPRESSION

$$a = \frac{dv}{dt} = 16 - 6t$$

$$v = \int 16 - 6t \, dt$$

$$v = 16t - 3t^2 + A$$

Using $t=1, v=1$

$$\Rightarrow 1 = 16 - 3 + A$$

$$\Rightarrow A = -12$$

$$\Rightarrow v = -3t^2 + 16t - 12$$

INTegrate again to get the displacement

$$x = \int -3t^2 + 16t - 12 \, dt$$

$$x = -t^3 + 16t^2 - 12t + B$$

Using $t=1, x=-5$

$$\Rightarrow -5 = -1 + 16 - 12 + B$$

$$\Rightarrow -5 = -5 + B$$

$$\Rightarrow B = 0$$

$$\Rightarrow x = -t^3 + 16t^2 - 12t$$

NOW SOLVING $x=0$ ("PASSED THROUGH THE ORIGIN")

$$\Rightarrow 0 = -t^3 + 16t^2 - 12t$$

$$\Rightarrow t^3 - 16t^2 + 12t = 0$$

$$\Rightarrow t(t^2 - 16t + 12) = 0$$

$$\Rightarrow t(t-2)(t-6) = 0$$

$t =$ 

From this we can find the velocity when $v = -3t^2 + 16t - 12$.

- $t=2$ • $t=6$
- $v_2 = 3(2)^2 + 16(2) - 12$ $v_6 = -3(6)^2 + 16(6) - 12$
- $v_2 = -12 + 32 - 12$ $v_6 = -108 + 96 - 12$
- $v_2 = 8$ $v_6 = -24$

\therefore THE REPORTED SPEEDS ARE 8 ms^{-1} & 24 ms^{-1}

Question 7 (*)+**

A particle P is moving on the x axis and its displacement from the origin, x m, t seconds after a given instant, is given by

$$x = 2t^3 - 3t^2 + At + B, \quad t \geq 0,$$

where A and B are constants.

- a) Find the value of t when the acceleration of P is zero.

When $t = 1.5$ s, P is passing through the origin O , and is moving in the negative x direction with speed 7.5 ms^{-1} .

- b) Determine the value of A and the value of B .
 c) Determine the time when P is instantaneously at rest.
 d) Calculate as an exact surd the value of t , when P is passing through O again.

$$\boxed{t = \frac{1}{2}}, \boxed{A = -12}, \boxed{B = 18}, \boxed{t = 2}, \boxed{t = \sqrt{6}}$$

The image shows handwritten working for Question 7, divided into four sections (a, b, c, d) enclosed in a black border.

- (a)** Shows the displacement equation $x = 2t^3 - 3t^2 + At + B$ and the derivative $V = \frac{dx}{dt} = 6t^2 - 6t + A$. It then solves for t when $V=0$, leading to $t = \frac{1}{2}$.
- (b)** Shows the velocity equation $V = 6t^2 - 6t + A$ and the condition $V=-7.5$ at $t=1.5$. Solving for A gives $A = -12$, and substituting back gives $B = 18$.
- (c)** Shows the displacement equation $x = 2t^3 - 3t^2 - 12t + 18$ and the condition $x=0$ at $t=1.5$. Solving for t gives $t = \sqrt{6}$.
- (d)** Shows the displacement equation $x = 2t^3 - 3t^2 - 12t + 18$ and the condition $x=0$. Solving for t gives $t = \sqrt{6}$, with a note "t=1.5 ← ALREADY KNOWN".

Question 8 (***)

A car is travelling on a straight horizontal road with constant velocity of 37.5 ms^{-1} .

The driver applies the brakes and the car decelerates at $(9.25 - t) \text{ ms}^{-2}$, where t s is the time since the instant when the brakes were first applied.

- a) Show that while the car is decelerating its velocity is given by

$$\frac{1}{4}(2t^2 - 37t + 150) \text{ ms}^{-1}.$$

- b) Hence find the time taken to bring the car to rest.
c) Determine the distance covered while the car was decelerating.

$$t = 6 \text{ s}, \quad d = 94.5 \text{ m}$$

C9 DECELERATION OF $(9.25 - t) \Rightarrow$ ACCELERATION OF $(t - 9.25)$

THUS $\frac{dv}{dt} = t - 9.25$

$$v = \int t - 9.25 \, dt$$

$$[v = \frac{1}{2}t^2 - 9.25t + C]$$

When $t=0, v=37.5 \Rightarrow C=37.5$

$$v = \frac{1}{2}t^2 - 9.25t + 37.5$$

$$v = \frac{1}{4}(2t^2 - 37t + 150)$$

As required

B) $v=0 \Rightarrow \frac{1}{2}t^2 - 37t + 150 = 0$
 $2t^2 - 74t + 300 = 0 \quad \text{BY QUADRATIC FORMULA OR FACTORIZATION}$
 $(t-6)(2t-50)=0$
 $t = <\cancel{6}> \quad <\cancel{25}> \quad \text{One has already stopped!!}$

C)

$v = \frac{1}{4}(2t^2 - 37t + 150)$

$$\begin{aligned} d &= \int_0^6 \frac{1}{4}(2t^2 - 37t + 150) \, dt = \frac{1}{4} \left[\frac{2}{3}t^3 - \frac{37}{2}t^2 + 150t \right]_0^6 \\ &= \frac{1}{4} \left[(48 - 666 + 900) - (0) \right] \\ &= 94.5 \text{ m} \end{aligned}$$

Question 9 (***)+

A particle P is moving on the x axis and its velocity v ms $^{-1}$ in the positive x direction, t seconds after a given instant, is given by

$$v = t^2 - 2t - 24, \quad t \geq 0.$$

When $t = 3$, P is observed passing through the origin.

- Find the acceleration of P when $t = 3$.
- Determine the distance of P from O when it is instantaneously at rest.
- Find the time at which P is passing through O again.

$$\boxed{}, \boxed{a = 4 \text{ ms}^{-2}}, \boxed{d = 36 \text{ m}}, \boxed{t = \sqrt{72} \approx 8.49}$$

<p>a) OBTAIN THE ACCELERATION BY DIFFERENTIATING THE VELOCITY</p> $a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 2t - 24)$ $a = 2t - 2$ $ _{t=3} \quad a = 4 \text{ ms}^{-2}$ <p>b) FIND THE TIME WHEN $V=0$</p> $\rightarrow 0 = t^2 - 2t - 24$ $\rightarrow 0 = (t+4)(t-6)$ $\rightarrow t = \cancel{-4}, \quad t = 6$ <p>USING INTEGRATION TO OBTAIN AN EXPRESSION FOR THE DISPLACEMENT x</p> $\rightarrow x = \int v \, dt = \int t^2 - 2t - 24 \, dt$ $\rightarrow x = \frac{1}{3}t^3 - t^2 - 24t + C$ <p>APPLY CONDITION $t=3, x=0$</p> $\rightarrow 0 = \frac{1}{3}(3)^3 - (3)^2 - 24(3) + C$ $\rightarrow 0 = 9 - 9 - 72 + C$ $\rightarrow C = 72$ $\rightarrow x = \frac{1}{3}t^3 - t^2 - 24t + 72$	<p>FINALLY WRITE $t=6$</p> $x(6) = \frac{1}{3}(6)^3 - 6^2 - 24(6) + 72$ $= 72 - 36 - 144 + 72$ $= -36$ <p>\therefore A DISTANCE OF 36m FROM O</p> <p>ALTERNATIVE FOR PART (b) USING VELOCITY TIME GRAPH</p> <p>DISPLACEMENT = $\int_{0}^{6} (t^2 - 2t - 24) \, dt$</p> $= \left[\frac{1}{3}t^3 - t^2 - 24t \right]_0^6$ $= (72 - 36 - 144) - (0 - 0)$ $= -36 \text{ m}$ <p>9) USING $x = \frac{1}{3}t^3 - t^2 - 24t + 72$ WITH $x=0$ & NOTING $t=3$ IS A DOUBLE SOLUTION (GRAPH AS CONDITN)</p> $\rightarrow \frac{1}{3}t^3 - t^2 - 24t + 72 = 0$ $\rightarrow t^2 - 2t^2 - 24t + 72 = 0$ $\rightarrow t(t-3)(t+8) = 0$ (OR USE ALGEBRAIC DIVISION) $\rightarrow t=3 \quad \text{OR} \quad t=-8$ $\therefore t = \sqrt{72} \approx 8.49 \text{ s}$
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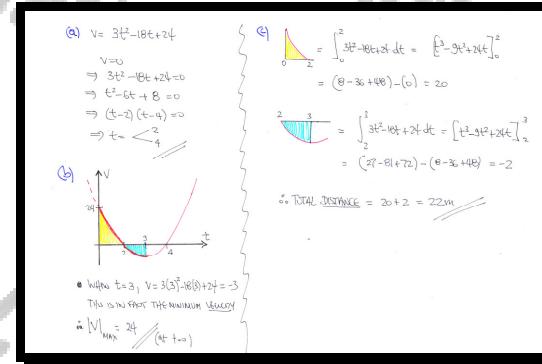
Question 10 (*)**

A particle P is moving on the x axis and its velocity v ms $^{-1}$ in the positive x direction, t seconds after a given instant, is given by

$$v = 3t^2 - 18t + 24, \quad t \geq 0.$$

- a) Find the times when P is instantaneously at rest.
- b) Determine the greatest speed of P in the interval $0 \leq t \leq 3$.
- c) Calculate the total distance covered by P in the interval $0 \leq t \leq 3$.

$$t = 2, 4, \quad |v|_{\max} = 24 \text{ ms}^{-1}, \quad d = 22 \text{ m}$$



Question 11 (*)**

A particle P is moving on a straight line.

At time t seconds, the distance of P from a fixed origin O is x metres and its acceleration is

$$(8 - 2t) \text{ ms}^{-1}$$

in the direction of x increasing.

It is further given that when $t = 0$, P was moving towards O with speed 7 ms^{-1} .

Determine the total distance covered by P in the first 7 seconds.

, $d = 39\frac{1}{3} \text{ m}$

• START BY OBTAINING AN EXPRESSION FOR THE VELOCITY.

$$\begin{aligned} a &= 8 - 2t \\ \Rightarrow \frac{dv}{dt} &= 8 - 2t \\ \Rightarrow 1 \, dv &= (8 - 2t) \, dt \\ \Rightarrow \int_7^v 1 \, dv &= \int_{t=0}^t 8 - 2t \, dt \\ \Rightarrow [v]_7^v &= [8t - t^2]_0^t \\ \Rightarrow v + 7 &= (8t - t^2) - 0 \\ \Rightarrow v &= -7 + 8t - t^2 \end{aligned}$$

• SECURE THE ORDER: $v = f(t)$

DISTANCE = $\int_0^7 -7 + 8t - t^2 \, dt + \int_7^{t=7} -7 + 8t - t^2 \, dt$

$$\begin{aligned} &= \int_0^7 -7 + 8t - t^2 \, dt + \int_7^{t=7} -7 + 8t - t^2 \, dt \\ &= \left[-7t + 4t^2 - \frac{t^3}{3} \right]_0^7 + \left[-7t + 4t^2 - \frac{t^3}{3} \right]_7^{t=7} \\ &= 0 - \left(7 + 4 - \frac{7^3}{3} \right) + \left(49 + 49 - \frac{49^3}{3} \right) - (7 + 4 - \frac{7^3}{3}) \\ &= \frac{118}{3} \\ &= 39\frac{1}{3} \text{ m} \end{aligned}$$

Question 12 (***)

A particle is moving in a straight line in an electromagnetic field.

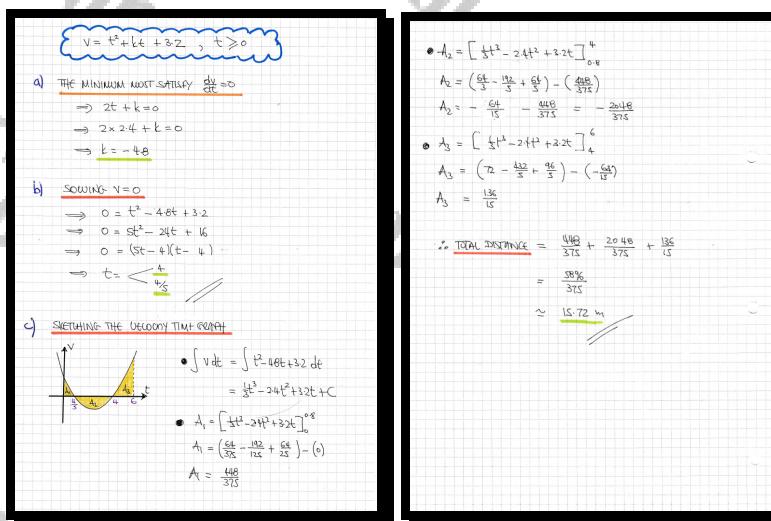
Its velocity, v ms $^{-1}$, at time t s, $t \geq 0$, is given by

$$v = t^2 + kt + 3.2,$$

where k is a non zero constant.

- a) Given that the particle achieves its minimum velocity when $t = 2.4$ s, show that $k = -4.8$.
- b) Determine the values of t when the particle is instantaneously at rest.
- c) Calculate the total distance covered by the particle for $0 \leq t \leq 6$.

	$t = 0.8, t = 4$	$d = \frac{5896}{375} \approx 15.72$ m
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Question 13 (***)**

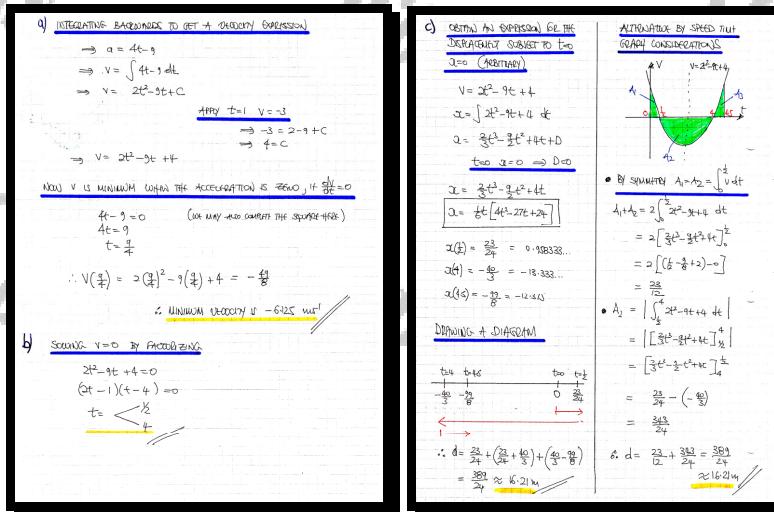
A particle P is moving on the x axis and its acceleration $a \text{ ms}^{-2}$, t seconds after a given instant, is given by

$$a = 4t - 9, t \geq 0.$$

When $t = 1$, P is moving with a velocity of -3 ms^{-1} .

- Find the minimum velocity of P .
- Determine the times when P is instantaneously at rest.
- Find the distance travelled by P in the first $4\frac{1}{2}$ seconds of its motion.

$v_{\min} = -6.125 \text{ ms}^{-1}$	$t = \frac{1}{2}, 4$	$d = \frac{389}{24} \approx 16.21 \text{ m}$	



Question 14 (***)**

A particle is moving in a straight line, so that its velocity, $v \text{ ms}^{-1}$, at time $t \text{ s}$ satisfies

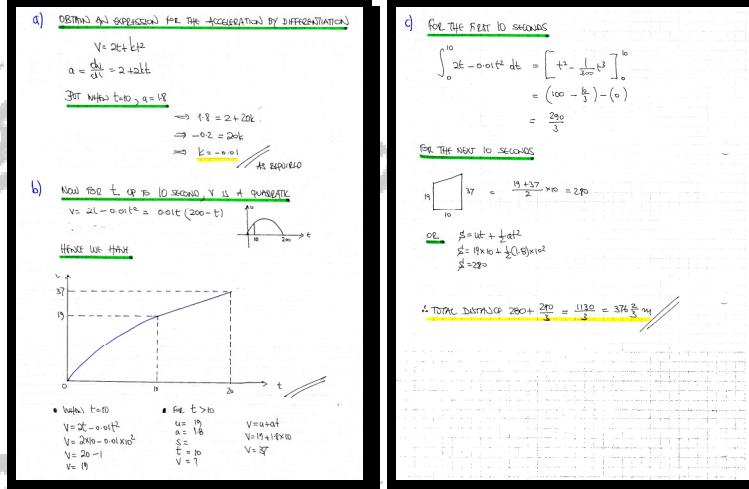
$$v = 2t + kt^2, \quad 0 \leq t \leq 10,$$

where k is a non zero constant.

When $t = 10$, the particle reaches an acceleration of 1.8 ms^{-2} , which it maintains for a further 10 s.

- Show that $k = -0.01$.
- Sketch a detailed velocity time graph, which describes the motion of this particle, for $0 \leq t \leq 20$.
- Find the distance travelled by the particle for $0 \leq t \leq 20$.

, $d = 376\frac{1}{3} \text{ m}$



Question 15 (****)

Russel is driving through the countryside, along a straight horizontal road at a constant speed of 22.5 ms^{-1} .

He sees a fallen tree blocking the road ahead, at a distance of 75 m ahead, so he immediately applies the brakes trying to stop his car before it hits the fallen tree.

The way he applies the brakes is such so that the **deceleration** of his car is given by $\left(3 + \frac{1}{4}t\right) \text{ ms}^{-2}$, where t is measured since the instant he first applied the brakes.

Russel's car stops $D \text{ m}$ **before** he hits the tree.

Determine the value of D .

$$\boxed{ } , D = 3$$

START BY EXPANDING EXPRESSIONS FOR VELOCITY & DISPLACEMENT

DECELERATION OF $3 + \frac{1}{4}t \Rightarrow a = -3 - \frac{1}{4}t$

$$v = -3t - \frac{1}{4}t^2 + 22.5$$

REARRANGE

$$\Rightarrow s = -\frac{1}{2}t^2 - \frac{1}{4}t^2 + 22.5t + C$$

DISPLACEMENT IS HORIZONTAL, SO THE INITIAL POSITION IS ZERO

Given to A starts initial $v=0$

$$\Rightarrow 0 = -3t - \frac{1}{4}t^2 + 22.5$$

$$\Rightarrow t^2 + 12t - 90 = 0$$

$$\Rightarrow (t+15)(t-6) = 0$$

$$\Rightarrow t = -15 \quad \cancel{\text{X}}$$

USING THE DISPLACEMENT EXPRESSION NOW WITH $t=6$

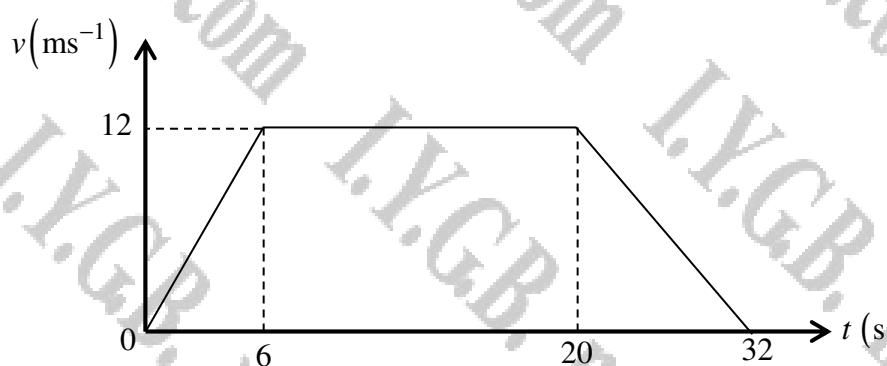
$$\Rightarrow s = -\frac{1}{2}t^2 - \frac{1}{4}t^2 + 22.5t$$

$$\Rightarrow s = -54 - 9 + 135$$

$$\Rightarrow s = 72$$

AS THE TREE WAS 75M AWAY

$D = 3$

Question 16 (***)

The figure above shows the speed time graph (t, v) of a car travelling along a straight horizontal road between two sets of traffic lights.

The car starts from rest at the first set of lights and accelerates uniformly for 6 s, reaching a speed of 12 ms^{-1} .

This speed is maintained for 14 s, before the car decelerates uniformly for 12 s, coming to rest as it reaches the second set of lights.

The distance of the car, $s(t)$, measured from the first set of traffic lights is given by

$$s(t) = \begin{cases} f_1(t) & 0 \leq t < 6 \\ f_2(t) & 6 \leq t < 20 \\ f_3(t) & 20 \leq t < 32 \end{cases}$$

where $f_1(t)$, $f_2(t)$ and $f_3(t)$ are functions of t .

Determine simplified expressions for $f_1(t)$, $f_2(t)$ and $f_3(t)$.

$$\boxed{\quad}, \boxed{f_1(t) = t^2}, \boxed{f_2(t) = 12t - 36}, \boxed{f_3(t) = \frac{1}{2}t^2 + 32t - 236}$$

LOOKING AT THE GRAPH OPPOSITE

- GRAD $v_1 = \frac{12-0}{6-0} = 2$
- $v_1(t) = 2t$
- GRAD $v_2 = 0$
- $v_2 = 12$
- GRAD $v_3 = -\frac{12}{32-20} = -1$
- $v_3 = -1(t-20)$
- $v_3 = 32-t$

NOW WE CAN TREAT THIS AS PARABOLAS

- $\int_0^6 2t \, dt = \left[t^2 \right]_0^6 = 6^2 - 0 = 36$
- $\int_6^{20} 12 \, dt = 12t \Big|_6^{20} = 12(20) - 12(6) = 36$
- $\int_{20}^{32} (32-t) \, dt = 36 + \left[12t \right]_{20}^{32} = 36 + (12(32) - 12(20)) = 126 - 36 = 90$
- $\int_0^{32} 2t \, dt = 12(32) - 36 = 204$

$$\begin{aligned} s_3(t) &= 204 + \int_{20}^t [32-t] \, dt = 204 + \left[32t - \frac{1}{2}t^2 \right]_{20}^t \\ &= 204 + \left[32t - \frac{1}{2}t^2 - (640 - 200) \right] \\ &= \underline{\underline{\frac{1}{2}t^2 + 32t - 236}} \end{aligned}$$

HENCE WE FINALLY OBTAIN

$$s(t) = \begin{cases} t^2 & 0 \leq t < 6 \\ 12t - 36 & 6 \leq t < 20 \\ \frac{1}{2}t^2 + 32t - 236 & 20 \leq t \leq 32 \end{cases}$$

Question 17 (***)+

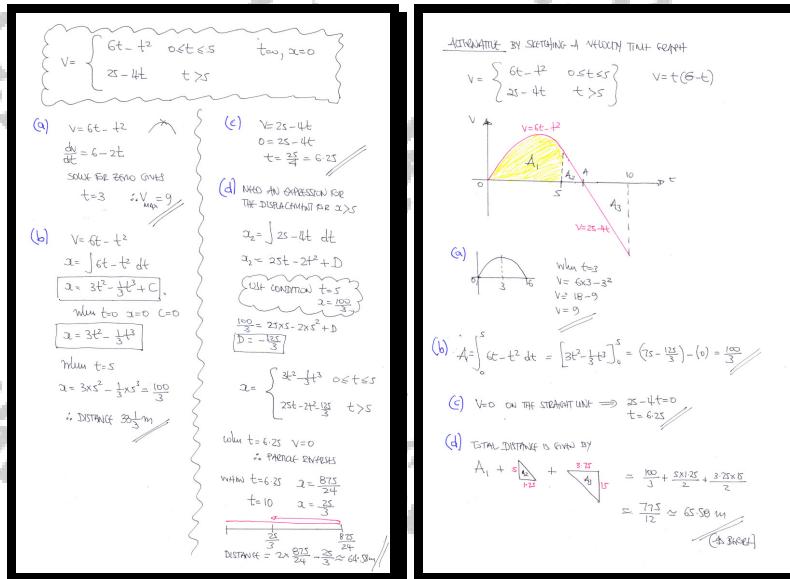
A particle P is moving on the x axis and its velocity $v \text{ ms}^{-1}$, t seconds after a given instant, is given by

$$v = \begin{cases} 6t - t^2 & 0 \leq t \leq 5 \\ 25 - 4t & t > 5 \end{cases}$$

The particle is initially at the origin O .

- Find the greatest speed of P for $0 \leq t \leq 5$.
- Show that the distance of P from O when $t = 5$ is $33\frac{1}{3} \text{ m}$.
- State the time at which P is instantaneously at rest for $t > 5$.
- Hence determine the **total distance** travelled by P during the first 10 seconds of its motion.

$$v_{\max} = 9 \text{ ms}^{-1}, \quad t = \frac{25}{4} = 6.25 \text{ s}, \quad d = \frac{775}{12} \approx 64.58 \text{ m}$$



Question 18 (*)+**

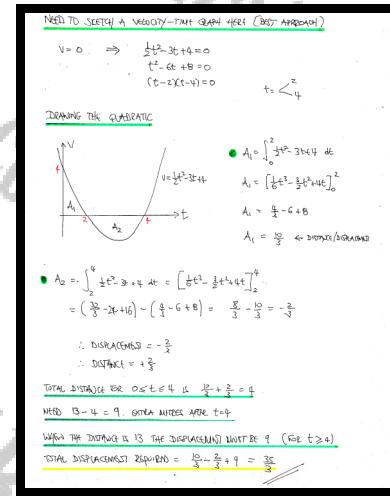
A particle P is moving on the x axis and its velocity v ms $^{-1}$ in the positive x direction, t seconds after a given instant, is given by

$$v = \frac{1}{2}t^2 - 3t + 4, \quad t \geq 0.$$

The particle is passing through the origin when $t = 0$

Determine the displacement of the particle from the origin, when it has covered a **total distance** of 13 m.

, $x = \frac{35}{3}$



Question 19 (***)+

A car moving on a straight road is modelled as a particle moving on the x axis, and its acceleration $a \text{ ms}^{-2}$, t seconds after a given instant, is given by

$$a = \begin{cases} 4 - \frac{1}{2}t & 0 \leq t \leq 8 \\ 0 & t > 8 \end{cases}$$

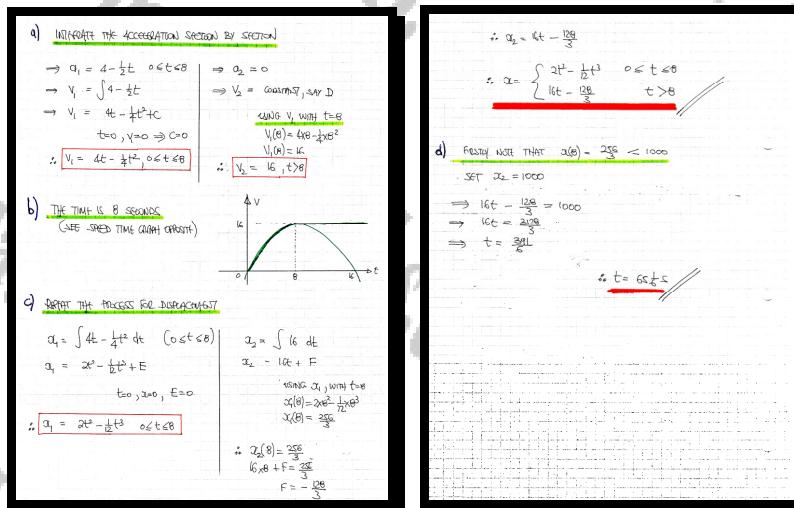
The car starts from rest at the origin O .

- Find a similar expression for the velocity of the car, as that of its acceleration.
- State the time it takes for the car to reach its maximum speed.
- Show that the displacement of P from O is given by

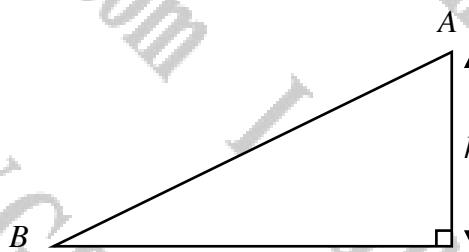
$$x = \begin{cases} 2t^2 - \frac{1}{12}t^3 & 0 \leq t \leq 8 \\ 16t - \frac{128}{3} & t > 8 \end{cases}$$

- Calculate the time it takes the car to cover the first 1000 m.

$$\boxed{\quad}, v = \begin{cases} 4t - \frac{1}{4}t^2 & 0 \leq t \leq 8 \\ 16 & t > 8 \end{cases}, \boxed{t = 8 \text{ s}}, \boxed{t = 65\frac{1}{6} \text{ s}}$$



Question 20 (***)+



A particle is sliding down the line of greatest slope of a **smooth** plane inclined at a fixed angle to the horizontal. The particle experiences no other resistances.

The particle is released from rest from a point *A* at the top of the plane and takes 12 seconds to slide down to a point *B* on the plane. Point *A* lies at a vertical distance of *h* above the level of *B*, as shown in the figure above.

The particle slides down by 1 cm during the first second of its motion, and in each subsequent second it slides down by an extra 3 cm than in the previous second.

Show that $h = 6\frac{3}{7}$, measured in millimetres.

[], [] proof

USING THE SUMMATION FORMULA FOR ARITHMETIC PROGRESSIONS WITH $a = 0.01$ AND $d = 0.03$ (in m)

$$S_t = \frac{1}{2} [2a + (n-1)d]$$

$$S_t = \frac{1}{2} [2(0.01) + (t-1) \times 0.03] \quad \leftarrow \text{DISPLACEMENT AT TIME } t \text{ SECONDS}$$

$$S_t = \frac{1}{2} [0.02 + 0.03t - 0.03]$$

$$S_t = \frac{1}{20} [0.03t - 0.01]$$

$$\frac{dS_t}{dt} = \frac{1}{20} (3t - 1)$$

$$\frac{ds}{dt} = \frac{1}{20} (3t - 1)$$

DIFFERENTIATE TO OBTAIN EXPRESSION FOR VELOCITY AND ACCELERATION

$$v_t = \frac{1}{20} (3t - 1)$$

$$a_t = \frac{3}{20} = 0.03$$

NOW LOOKING AT A DIAGRAM

$$mgs \sin \theta = ma$$

$$\sin \theta = \frac{h}{l}$$

$$\sin \theta = \frac{0.01}{\frac{l}{20}}$$

$$\sin \theta = \frac{20}{l}$$

NOW THE DISTANCE COACHED ON THE PLANE AFTER 12 SECONDS

$$S_{12} = \frac{1}{20} (3t^2 - t)$$

$$S_{12} = \frac{1}{20} (3(12)^2 - 12)$$

$$S_{12} = 2.1$$

FINALLY USE TRIG

$$\frac{h}{l} = \sin \theta$$

$$\frac{l}{S_{12}} = \frac{3}{20}$$

$$l = \frac{3}{20} \text{ (metres)}$$

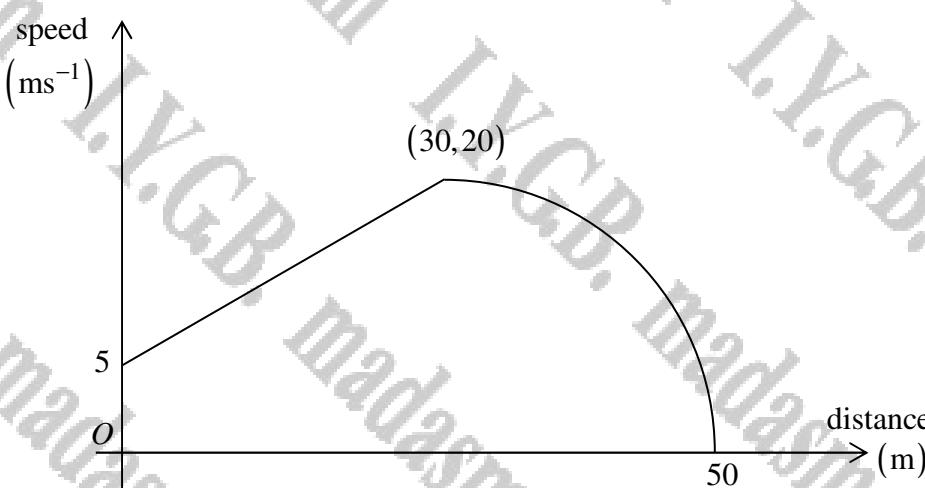
$$l = \frac{3}{14} \text{ (cm)}$$

$$l = \frac{45}{14} \text{ (cm)}$$

$$l = 6\frac{3}{7} \text{ mm}$$

Question 21

(*****)



The speed distance graph of the journey of a particle is shown above.

It consists of a straight line segment joining the point $(0,5)$ to $(30,20)$, joined to a quarter circle of radius 20 . The total distance covered by the particle is 50 m .

Determine in exact form the total journey time of the particle.

You may assume without proof that

$$\int \frac{1}{\sqrt{a^2 - (u-b)^2}} du = \arcsin\left(\frac{u-b}{a}\right) + \text{constant}$$

, $t = \left(\frac{1}{2}\pi + 4\ln 2\right)\text{ s}$

STARTING WITH THE DISTANCE-SPEED GRAPH

- GRADIENT OF LINE = $\frac{\Delta v}{\Delta s} = \frac{15}{30} = \frac{1}{2}$
- EQUATION OF LINE: $v = \frac{1}{2}s + 5$
- EQUATION OF THE CIRCLE IS: $GIVE BY$
 $(x-30)^2 + v^2 = 20^2$
 $v^2 = 400 - (x-30)^2$
 $v = \pm\sqrt{400 - (x-30)^2}$
- THE EQUATION OF THE SEMI-CIRCLE IS: $GIVEN BY$
 $v = \begin{cases} \frac{1}{2}s + 5 & 0 \leq s \leq 30 \\ \sqrt{400 - (x-30)^2} & 30 < s \leq 50 \end{cases}$

INTEGRATING THE FIRST SECTION

$$v = \frac{ds}{dt} = \frac{1}{2}s + 5$$

$$\int \frac{1}{\frac{1}{2}s+5} ds = \int dt$$

$$\int \frac{1}{2s+10} ds = \int dt$$

$$\int \frac{2}{2s+10} ds = \int dt$$

$$[2\ln|2s+10|]_0^{30} = [t]_0^t$$

$$2\ln|2(30+10)| - 2\ln|2(10)| = t - 0$$

$$2\ln|40| - 2\ln|20| = t$$

$$2\ln 2 = t$$

$$t = 2\ln 2$$

$$\underline{t = 4\ln 2}$$

MOVING INTO THE SECOND SECTION OF THE JOURNEY

$$v = \frac{ds}{dt} = \sqrt{400 - (s-30)^2}$$

$$\int \frac{1}{\sqrt{400 - (s-30)^2}} ds = \int dt$$

$$\int \frac{1}{\sqrt{20^2 - (s-30)^2}} ds = \int \frac{1}{40} dt$$

USING THE RESULT FROM

$$[\arcsin\left(\frac{s-30}{20}\right)]_0^{30} = [t]_{4\ln 2}^t$$

$$\arcsin(1) - \arcsin(0) = t - 4\ln 2$$

$$t = \frac{\pi}{2} + 4\ln 2$$

$$\underline{t = \frac{1}{2}(\pi + 8\ln 2)}$$

CALCULUS KINEMATICS IN VECTOR FORM

Question 1 ()**

The position vector, \mathbf{r} m, of a particle, t seconds after a given instant is given by

$$\mathbf{r} = (2t^2 - 1)\mathbf{i} + (6t - 5t^2)\mathbf{j}, t \geq 0,$$

where \mathbf{i} and \mathbf{j} are unit vectors pointing due east and due north, respectively.

Given that the mass of the particle is 0.5 kg, determine the magnitude of the resultant force acting on the particle.

$$F = \sqrt{29} \approx 5.39 \text{ N}$$

$$\begin{aligned} \mathbf{r} &= (2t^2 - 1)\mathbf{i} + (6t - 5t^2)\mathbf{j} \\ \mathbf{v} &= \frac{d\mathbf{r}}{dt} = (4t)\mathbf{i} + (6 - 10t)\mathbf{j} \\ \mathbf{a} &= \frac{d\mathbf{v}}{dt} = 4\mathbf{i} - 10\mathbf{j} \\ |\mathbf{a}| &= \sqrt{4^2 + (-10)^2} = \sqrt{116} \end{aligned} \quad \left. \begin{array}{l} F = ma \\ F = \frac{1}{2} \times \sqrt{116} \\ F = \frac{1}{2} \times 2\sqrt{29} \\ F = \sqrt{29} \\ \approx 5.39 \text{ N} \end{array} \right\}$$

Question 2 ()**

The position vector, \mathbf{r} m, of a particle P , t s after a given instant is given by

$$\mathbf{r} = (t^3 - 2t)\mathbf{i} + (4t^2 + t)\mathbf{j}, t \geq 0,$$

where \mathbf{i} and \mathbf{j} are unit vectors pointing due east and due north, respectively.

- Find the magnitude of the acceleration of the particle, when $t = 1$.
- Determine the value of t when P is moving parallel to the vector $\mathbf{i} + \mathbf{j}$.

$$a = 10 \text{ ms}^{-2}, [t = 3]$$

$$\begin{aligned} \text{(a)} \quad \mathbf{r} &= (t^3 - 2t)\mathbf{i} + (4t^2 + t)\mathbf{j} \\ \mathbf{v} &= \frac{d\mathbf{r}}{dt} = (3t^2 - 2)\mathbf{i} + (8t + 1)\mathbf{j} \\ \mathbf{a} &= \frac{d\mathbf{v}}{dt} = 6t\mathbf{i} + 8\mathbf{j} \\ \text{when } t &= 1 \\ \mathbf{a} &= 6\mathbf{i} + 8\mathbf{j} \\ |\mathbf{a}| &= \sqrt{6^2 + 8^2} = 10 \text{ ms}^{-2} \end{aligned} \quad \begin{aligned} \text{(b)} \quad \mathbf{v} &= (3t^2 - 2)\mathbf{i} + (8t + 1)\mathbf{j} \\ \text{IF MOVING IN THE DIRECTION} \\ \mathbf{i} + \mathbf{j} \text{ THEN} \\ \Rightarrow 3t^2 - 2 = 8t + 1 \\ \Rightarrow 3t^2 - 8t - 3 = 0 \\ \Rightarrow (3t + 1)(t - 3) = 0 \\ t = -\frac{1}{3} \quad \cancel{\text{or}} \quad t = 3 \end{aligned}$$

Question 3 (*)**

The velocity, \mathbf{v} ms $^{-1}$, of a particle P , t seconds after a given instant is given by

$$\mathbf{v} = (4t - 3)\mathbf{i} + (2t + 3)\mathbf{j}, t \geq 0,$$

where \mathbf{i} and \mathbf{j} are unit vectors pointing due east and due north, respectively.

- a) Find the magnitude of the acceleration of P .

When $t = 1$, the position vector of P is $8\mathbf{j}$ m.

- b) Determine the **initial distance** of P from the origin O .

$$a = \sqrt{20} \approx 4.47 \text{ ms}^{-2}, \quad d = \sqrt{17} \approx 4.12 \text{ m}$$

(a) $\mathbf{v} = (4t - 3)\mathbf{i} + (2t + 3)\mathbf{j}$ $t=1 \quad \mathbf{v}=8\mathbf{j}$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 4\mathbf{i} + 2\mathbf{j}$$

$$|\mathbf{a}| = \sqrt{4^2 + 2^2} = \sqrt{20} \approx 4.47 \text{ ms}^{-2}$$

(b) $\mathbf{v} = (4t - 3)\mathbf{i} + (2t + 3)\mathbf{j}$

$$\mathbf{s} = \int [(4t - 3)\mathbf{i} + (2t + 3)\mathbf{j}] dt$$

$$\mathbf{s} = (4t^2 - 3t + C)\mathbf{i} + (t^2 + 3t + D)\mathbf{j}$$

when $t=1 \quad \mathbf{s} = 8\mathbf{j}$

$$8\mathbf{j} = (-4 + C)\mathbf{i} + (4 + D)\mathbf{j} \quad \therefore C=4 \quad D=4$$

$$\mathbf{s} = (4t^2 - 3t + 4)\mathbf{i} + (t^2 + 3t + 4)\mathbf{j}$$

when $t=0, \quad \mathbf{s} = \mathbf{j} + 4\mathbf{j}$

DISTANCE FROM O IS $\sqrt{1^2 + 4^2} = \sqrt{17} \approx 4.12 \text{ m}$

Question 4 (*)**

The velocity, \mathbf{v} ms⁻¹, of a particle of mass 2 kg, t s after a given instant is given by

$$\mathbf{v} = 6t^2\mathbf{i} - 6t^{\frac{3}{2}}\mathbf{j}, \quad t \geq 0,$$

where \mathbf{i} and \mathbf{j} are unit vectors pointing due east and due north, respectively.

- a) Find the magnitude of the resultant force acting on the particle, when $t = 1$.

When $t = 0$, the particle is at the point A whose position vector is $(2\mathbf{i} + \mathbf{j})$ m and when $t = 1$ the particle is at the point B .

- b) Determine the distance AB .

$$F = 30 \text{ N}, \quad |AB| \approx 3.12 \text{ m}$$

<p>(a)</p> $\begin{cases} \mathbf{v} = 6t^2\mathbf{i} - 6t^{\frac{3}{2}}\mathbf{j} \\ \mathbf{a} = \frac{d\mathbf{v}}{dt} = 12t\mathbf{i} - 9t^{\frac{1}{2}}\mathbf{j} \end{cases}$ <ul style="list-style-type: none"> When $t=1$ $\mathbf{a} = 12\mathbf{i} - 9\mathbf{j}$ $a = \sqrt{12^2 + 9^2} = 15 \text{ m s}^{-2}$ $F = ma$ ($m=2$ kg) $F = 2 \times 15$ $F = 30 \text{ N}$ 	<p>(b)</p> $\begin{cases} \mathbf{v} = 6t^2\mathbf{i} - 6t^{\frac{3}{2}}\mathbf{j} \\ \mathbf{r} = \int \mathbf{v} dt = 6t^3\mathbf{i} - 6t^{\frac{5}{2}}\mathbf{j} \end{cases}$ <ul style="list-style-type: none"> $\mathbf{r} = (4t^3 + C)\mathbf{i} + (-\frac{2}{3}t^{\frac{5}{2}} + D)\mathbf{j}$ When $t=0$, $\mathbf{r}_A = 2\mathbf{i} + \mathbf{j}$ $2t^3 + C = 2\mathbf{i} + \mathbf{j}$ $C=2$ $D=1$ $\mathbf{r} = (2t^3 + 2)\mathbf{i} + (-\frac{2}{3}t^{\frac{5}{2}} + 1)\mathbf{j}$ When $t=1$ $\mathbf{r}_B = 4\mathbf{i} - \frac{2}{3}\mathbf{j} + 15 \left(4, \frac{2}{3}\right)$ $\mathbf{r}_B = 2\mathbf{i} + \frac{1}{3}\mathbf{j} + A(2, 1)$ $AB = \sqrt{(2-2)^2 + (\frac{1}{3}-1)^2} \approx 3.12$
--	---

Question 5 (***)

The velocity, \mathbf{v} ms $^{-1}$, of a particle of mass 5 kg, t s after a given instant is given by

$$\mathbf{v} = (12t^2 - 2)\mathbf{i} + (2t - 3t^2)\mathbf{j}, t \geq 0,$$

where \mathbf{i} and \mathbf{j} are unit vectors pointing due east and due north, respectively.

- Find the magnitude of the resultant force acting on the particle, when $t = 2$.
- Find the value of t when the particle's acceleration is parallel to the x axis.
- Determine the distance AB .

$$F \approx 245 \text{ N}, \quad t = \frac{1}{3}, \quad |AB| = 2 \text{ m}$$

(a) $\mathbf{v} = (12t^2 - 2)\mathbf{i} + (2t - 3t^2)\mathbf{j}$

 $\mathbf{a} = \frac{d\mathbf{v}}{dt} = 24t\mathbf{i} + (2 - 6t)\mathbf{j}$

- When $t = 2$,
- $\mathbf{a} = 48\mathbf{i} - 10\mathbf{j}$
- $|\mathbf{a}| = \sqrt{(48)^2 + (-10)^2} = \sqrt{2404}$
- $\mathbf{F} = m\mathbf{a}$
- $F = 5 \times \sqrt{2404}$
- $F \approx 245 \text{ N}$

(b) ACCELERATION PARALLEL TO x AXIS

$$\mathbf{a} = k\mathbf{i} + 0\mathbf{j}$$

$$\mathbf{a} = 24t\mathbf{i} + (2 - 6t)\mathbf{j}$$

$$2 - 6t = 0$$

$$t = \frac{1}{3}$$

(c) $\mathbf{r} = \int \mathbf{v} dt$

$$\mathbf{r} = \int ((12t^2 - 2)\mathbf{i} + (2t - 3t^2)\mathbf{j}) dt$$

$$\mathbf{r} = (4t^3 - 2t)\mathbf{i} + (t^2 - t^3)\mathbf{j}$$

when $t = 0$, $\mathbf{r}_0 = \mathbf{i} + \mathbf{e}_3$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{r}_1$$

$$\mathbf{r} = (4t^3 - 2t)\mathbf{i} + (t^2 - t^3)\mathbf{j}$$

when $t = 1$

$$\mathbf{r}_1 = 3\mathbf{i} + \mathbf{e}_3$$

$\therefore A(1, 6)$

$B(3, 6)$

$\therefore |AB| = 2$

Since they are at the same height?

Question 6 (*)**

The position vector, \mathbf{r} m, of a particle of mass 0.5 kg, t s after a given instant satisfies

$$\mathbf{r} = (3t^2 - 7t + 2)\mathbf{i} + (2t^2 - 5t + 2)\mathbf{j}, \quad t \geq 0,$$

where \mathbf{i} and \mathbf{j} are unit vectors pointing due east and due north, respectively.

- Find the value of t when the particle is at the origin.
- Determine the magnitude of the resultant force acting on the particle.
- Find the value of t when the particle is moving parallel to the vector $2\mathbf{i} + \mathbf{j}$.

$$t = 2, \quad F = \sqrt{13} \approx 3.61 \text{ N}, \quad t = 1.5$$

(a) $\Sigma = (3t^2 - 7t + 2)\mathbf{i} + (2t^2 - 5t + 2)\mathbf{j}$
 $\Sigma = (3t-1)(t-2)\mathbf{i} + (2t-1)(t-2)\mathbf{j}$
 BY INSPECTING MATHS $t=2$ BOTH
 COORDINATES ARE ZERO, IF PARTICLE IS
 AT THE ORIGIN
 $\therefore t=2$

(b) $\Sigma = \frac{d\Sigma}{dt} = (6t-7)\mathbf{i} + (4t-5)\mathbf{j}$
 $\Sigma = \frac{d\Sigma}{dt} = 6\mathbf{i} + 4\mathbf{j}$
 $|\Sigma| = \sqrt{6^2 + 4^2} = \sqrt{52}$
 $F = ma$
 $F = 0.5 \times \sqrt{52}$
 $F = \sqrt{13} \approx 3.61 \text{ N}$

(c) $\Sigma = (6t-7)\mathbf{i} + (4t-5)\mathbf{j}$
 "PARALLEL TO $2\mathbf{i} + \mathbf{j}$ "
 $\Rightarrow \frac{6t-7}{4t-5} = \frac{2}{1}$
 $\Rightarrow 6t-7 = 8t-10$
 $3 = 2t$
 $t = \frac{3}{2} = 1.5$

Question 7 (***)+

The acceleration \mathbf{a} ms^{-2} of a particle P of mass 0.2 kg, t s after a given instant is given by

$$\mathbf{a} = (2t-4)\mathbf{i} + 3\mathbf{j}, t \geq 0,$$

where \mathbf{i} and \mathbf{j} are unit vectors pointing along the positive x axis and along the positive y axis, respectively.

- a) Find the magnitude of the resultant force acting on P , when $t = 4$.

It is further given that when $t = 0$, P is at the point A with position vector $(-18\mathbf{i} - 24\mathbf{j})$ m and has velocity $(3\mathbf{i} - 9\mathbf{j}) \text{ ms}^{-1}$.

- b) Find the value of t when the particle is at rest.
 c) Show that when $t = 6$, P is on the y axis and state its distance from A .
 d) Determine the value of t when the particle is on the x axis.

[] , $F = 1\text{N}$, $t = 3$, 18 m , $t = 8$

a) $\mathbf{a} = (2t-4)\mathbf{i} + 3\mathbf{j}$

$$\mathbf{a}_4 = (2(4)-4)\mathbf{i} + 3\mathbf{j}$$

$$\mathbf{a}_4 = 4\mathbf{i} + 3\mathbf{j}$$

$$|\mathbf{a}_4| = \sqrt{4^2 + 3^2}$$

$$|\mathbf{a}_4| = 5 \text{ ms}^{-2}$$

using $F = ma$
 $F = 0.2 \times 5$
 $F = 1\text{N}$

b) INTEGRATE THE ACCELERATION VECTOR TO OBTAIN VELOCITY VECTOR

$$\Rightarrow \mathbf{v} = \int (2t-4)\mathbf{i} + 3\mathbf{j} dt$$

$$\Rightarrow \mathbf{v} = (t^2-4t+A)\mathbf{i} + (3t+B)\mathbf{j}$$

when $t=0$ $\mathbf{v} = 3\mathbf{i} - 9\mathbf{j}$

$$\therefore 3\mathbf{i} - 9\mathbf{j} = A\mathbf{i} + B\mathbf{j}$$

$$A=3$$

$$B=-9$$

$$\therefore \mathbf{v} = (t^2-4t+3)\mathbf{i} + (3t-9)\mathbf{j}$$

$$\mathbf{v} = (t-3)(t-1)\mathbf{i} + 3(t-3)\mathbf{j}$$

$$\therefore \text{BY INSPECTION } \mathbf{v}=0 \text{ WHEN } t=3$$

c) INTEGRATE AGAIN TO OBTAIN THE POSITION VECTOR

$$\mathbf{r} = \int (t^2-4t+3)\mathbf{i} + (3t-9)\mathbf{j} dt$$

$$\mathbf{r} = \left(\frac{1}{3}t^3-2t^2+3t+C\right)\mathbf{i} + \left(\frac{3}{2}t^2-9t+D\right)\mathbf{j}$$

when $t=0$ $\mathbf{r} = -18\mathbf{i} - 24\mathbf{j}$

$$\Rightarrow -18\mathbf{i} - 24\mathbf{j} = C\mathbf{i} + D\mathbf{j}$$

$$C = -18$$

$$D = -24$$

$$\therefore \mathbf{r} = \left(\frac{1}{3}t^3-2t^2+3t-18\right)\mathbf{i} + \left(\frac{3}{2}t^2-9t-24\right)\mathbf{j}$$

when $t=6$

$$\mathbf{r} = (72-72+18-18)\mathbf{i} + (54-54-24)\mathbf{j} = -24\mathbf{j}$$

METH ON THE Y AXIS

∴ DISTANCE FROM A IS 18 m

d) when $y=0$, $16 \perp$ COMPARE ZERO IN r

$$\Rightarrow \frac{3}{2}t^2-9t-16=0$$

$$\Rightarrow t^2-6t-16=0$$

$$\Rightarrow (t-8)(t+2)=0$$

$$\therefore t = 8$$

Question 8 (**)**

The position vector, velocity and acceleration of a particle P , t s after a given instant are denoted by \mathbf{r} m, \mathbf{v} ms $^{-1}$ and \mathbf{a} ms $^{-2}$.

When $t = 1$, $\mathbf{r} = 9\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = 13\mathbf{i} + \mathbf{j}$, where \mathbf{i} and \mathbf{j} are unit vectors pointing due east and due north, respectively.

It is further given that P has a constant acceleration of $6\mathbf{i}$ ms $^{-2}$.

- Determine the distance of P from the origin O , when $t = 3$.
- Show that P is moving on the curve with equation

$$x = 3y^2 + y - 5.$$

$$\boxed{\quad}, \approx 47.17 \text{ m}$$

a) INTEGRATE BACK FROM ACCELERATION TO VELOCITY

$$\begin{aligned}\rightarrow \mathbf{a} &= 6\mathbf{i} + \mathbf{c}\mathbf{j} \\ \Rightarrow \mathbf{v} &= \int (6\mathbf{i} + \mathbf{c}\mathbf{j}) \, dt \\ \Rightarrow \mathbf{v} &= (6t)\mathbf{i} + \mathbf{c}\mathbf{j}.\end{aligned}$$

With $t=1$ $\mathbf{v}=13\mathbf{i}+\mathbf{j}$

$$\begin{aligned}\Rightarrow 13\mathbf{i} + \mathbf{j} &= (6t)\mathbf{i} + \mathbf{c}\mathbf{j} \\ \Rightarrow 13 &= 6t \quad \mathbf{c} = \mathbf{j} \\ \Rightarrow \mathbf{v} &= (6t+1)\mathbf{i} + \mathbf{j}\end{aligned}$$

INTEGRATE AGAIN IN ORDER TO OBTAIN THE POSITION VECTOR

$$\begin{aligned}\Rightarrow \mathbf{s} &= \int ((6t+1)\mathbf{i} + \mathbf{j}) \, dt \\ \Rightarrow \mathbf{s} &= (3t^2+7t+C)\mathbf{i} + (t+D)\mathbf{j}.\end{aligned}$$

With $t=1$ $\mathbf{s}=9\mathbf{i}+2\mathbf{j}$

$$\begin{aligned}\Rightarrow 9\mathbf{i}+2\mathbf{j} &= (3t^2+7t+C)\mathbf{i} + (t+D)\mathbf{j} \\ \Rightarrow C &= -1 \quad D = 1 \\ \Rightarrow \mathbf{s} &= (3t^2+7t-1)\mathbf{i} + (t+1)\mathbf{j}.\end{aligned}$$

Now with $t=3$

$$\begin{aligned}\Rightarrow \mathbf{s} &= (27+21-1)\mathbf{i} + (3+1)\mathbf{j} = 49\mathbf{i} + 4\mathbf{j} \\ \Rightarrow |\mathbf{o}\mathbf{r}| &= \sqrt{49^2 + 4^2} = \sqrt{2225} = 47.17 \text{ m}\end{aligned}$$

b) $\mathbf{s} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3t^2+7t-1 \\ t+1 \end{pmatrix}$

$$\begin{aligned}t &= y-1 \\ \Rightarrow 2 &= 3(y-1)^2 + 7(y-1) - 1 \\ \Rightarrow 2 &= 3(y^2-2y+1) + 7y - 7 - 1 \\ \Rightarrow 2 &= 3y^2 + 4y + 3 + 7y - 8 \\ \Rightarrow 2 &= 3y^2 + 11y - 5\end{aligned}$$

ACTIVATION FOR PART (a) - REMODEL AS $t=1 \mapsto t=2$

$$\begin{aligned}\mathbf{a} &= 6\mathbf{i} \quad \text{constant} \\ t=1 \quad (\text{initial}) & \quad \mathbf{s} = \mathbf{s}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2 \\ \mathbf{v}_0 &= 13\mathbf{i} + \mathbf{j} \\ \mathbf{s}_0 &= 9\mathbf{i} + 2\mathbf{j}\end{aligned}$$

using time change
 $t \mapsto t-1$

With $t=3$, we use $t=2$

$$\begin{aligned}\mathbf{r} &= (9+18t+27t^2)\mathbf{i} + (2t^2)\mathbf{j} \\ \mathbf{r} &= (9+26+27)\mathbf{i} + (2)\mathbf{j} \\ \mathbf{r} &= 47\mathbf{i} + 4\mathbf{j}\end{aligned}$$

etc etc