

$$1. (a) \sqrt{24.5} - \sqrt{12.5} = \sqrt{\frac{49}{2}} - \sqrt{\frac{25}{2}} = \frac{\sqrt{49}}{\sqrt{2}} - \frac{\sqrt{25}}{\sqrt{2}} = \frac{7}{\sqrt{2}} - \frac{5}{\sqrt{2}} \\ = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$(b) \frac{\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} = \frac{\sqrt{2}-2}{1-\sqrt{2}+\sqrt{2}-2} = \frac{\sqrt{2}-2}{-1} = 2-\sqrt{2}$$

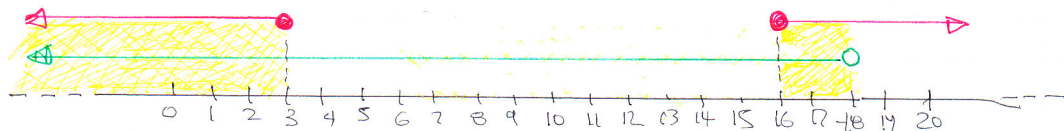
$$2. (a) 8+3x > 4(x-3)+2 \\ \Rightarrow 8+3x > 4x-12+2 \\ \Rightarrow 8+3x > 4x-10 \\ \Rightarrow -x > -18 \\ \Rightarrow x < 18$$

$$(b) (x-10)(x-4) \geq 5(x-1)-3 \\ \Rightarrow x^2-4x-10x+40 \geq 5x-5-3 \\ \Rightarrow x^2-14x+40 \geq 5x-8 \\ \Rightarrow x^2-19x+48 \geq 0 \\ \Rightarrow (x-16)(x-3) \geq 0 \\ \Rightarrow C.V = \begin{matrix} 16 \\ 3 \end{matrix}$$



$$\therefore x \leq 3 \text{ OR } x \geq 16$$

(c)



$$\therefore x \leq 3 \text{ OR } 16 \leq x < 18$$

3.

$$\text{EQUAL ROOTS} \Rightarrow b^2-4ac=0 \\ \Rightarrow (2m)^2-4 \times 1 \times (3m+4)=0 \\ \Rightarrow 4m^2-4(3m+4)=0 \\ \Rightarrow 4m^2-12m-16=0 \\ \Rightarrow m^2-3m-4=0 \\ \Rightarrow (m-4)(m+1)=0$$

$$m = \begin{matrix} 4 \\ -1 \end{matrix}$$

4. $\begin{cases} x+y=9 \\ x^2-3xy+2y^2=0 \end{cases} \Rightarrow \boxed{x=9-y} \Rightarrow (2y-9)(y-3)=0$

SUBSTITUTE INTO THE QUADRATIC

$$\Rightarrow (9-y)^2 - 3(9-y)y + 2y^2 = 0$$

$$\Rightarrow 81 - 18y + y^2 - 3y(9-y) + 2y^2 = 0$$

$$\Rightarrow 81 - 18y + y^2 - 27y + 3y^2 + 2y^2 = 0$$

$$\Rightarrow 6y^2 - 45y + 81 = 0$$

$$\Rightarrow 2y^2 - 15y + 27 = 0$$

$y = \begin{cases} 3 \\ \frac{9}{2} \end{cases}$

$x = \begin{cases} 9-3 = 6 \\ 9-\frac{9}{2} = \frac{9}{2} \end{cases}$

$\therefore (6, 3) \text{ and } (\frac{9}{2}, \frac{9}{2})$

5. (a) $f(x) = 0$

$$9x^2 + 18x - 7 = 0$$

$$(3x-1)(3x+7) = 0$$

$x = \begin{cases} \frac{1}{3} \\ -\frac{7}{3} \end{cases}$

(b) $f(x) = 9x^2 + 18x - 7$

$$= 9 \left[x^2 + 2x - \frac{7}{9} \right]$$

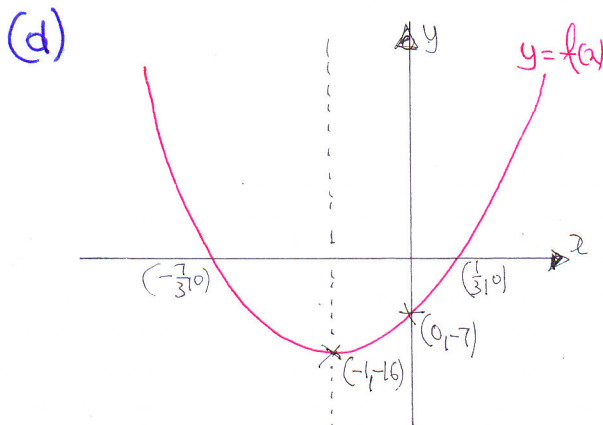
$$= 9 \left[(x+1)^2 - 1 - \frac{7}{9} \right]$$

$$= 9(x+1)^2 - 9 - 7$$

$$= 9(x+1)^2 - 16$$

$A=1$
 $B=-16$

(c) MINIMUM VALUE IS -16 (IT OCCURS WHEN $x = -1$)



- $+9x^2$ \cup
- $x=0, y=-7$ $(0, -7)$
- $y=0, x = \begin{cases} \frac{1}{3} \\ -\frac{7}{3} \end{cases}$ $(\frac{1}{3}, 0)$ $(-\frac{7}{3}, 0)$
- MIN AT $(-1, -16)$

8. (a) $S_n = \frac{n}{2} [2a + (n-1)d]$

$$\Rightarrow 15360 = \frac{12}{2} [2 \times 1500 + 11d]$$

$$\Rightarrow 15360 = 6 [3000 + 11d]$$

$$\Rightarrow \frac{15360}{6} = 3000 + 11d$$

$$\Rightarrow \frac{15000 + 360}{6} = 3000 + 11d$$

$$\Rightarrow 2500 + 60 = 3000 + 11d$$

$$\Rightarrow 2560 - 3000 = 11d$$

$$\Rightarrow 11d = -440$$

$$\Rightarrow \boxed{d = -40}$$

$$\therefore a = 40$$

(b) $a = 1500$
 $d = -40$

$$T_n = \frac{n}{2} [2 \times 1500 + (n-1)(-40)]$$

$$T_n = \frac{n}{2} [3000 + 40 - 40n]$$

$$T_n = n [1500 + 20 - 20n]$$

$$T_n = n [1520 - 20n]$$

$$T_n = 20n [76 - n] \quad \text{As required}$$

(c) $T_n = 26000$

$$\Rightarrow 20n(76 - n) = 26000$$

$$\Rightarrow n(76 - n) = 1300$$

BY INSPECTION

| | |
|--------|-----------------------|
| $n=10$ | $10 \times 66 = 660$ |
| $n=20$ | $20 \times 56 = 1120$ |
| $n=30$ | $30 \times 46 = 1380$ |
| $n=26$ | $26 \times 50 = 1300$ |

Thus

$$\Rightarrow 76n - n^2 = 1300$$

$$\Rightarrow 0 = n^2 - 76n + 1300$$

$$\Rightarrow 0 = (n-26)(n-50)$$

$$\therefore n = \begin{cases} 26 \\ 50 \end{cases}$$

(d) $U_n = a + (n-1)d$

● IF $n=26$

$$U_{26} = 1500 + 25(-40)$$

$$U_{26} = 1500 - 1000$$

$$U_{26} = 500$$

● IF $n=50$

$$U_{50} = 1500 + 49(-40)$$

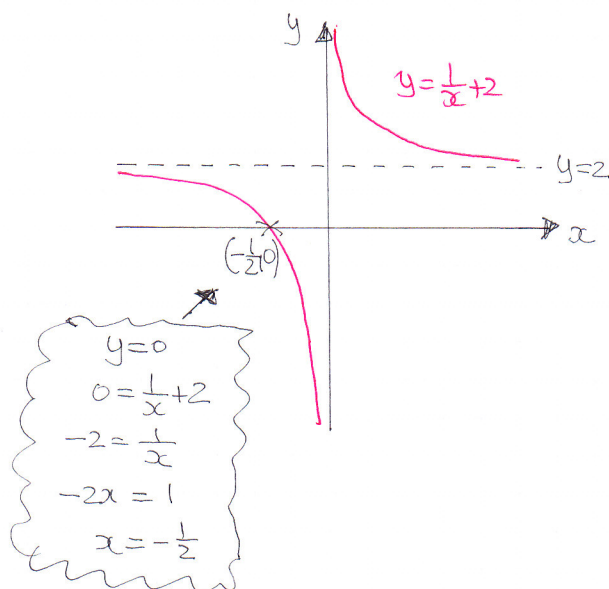
$$U_{50} = 1500 - 1960$$

$$U_{50} = -460$$

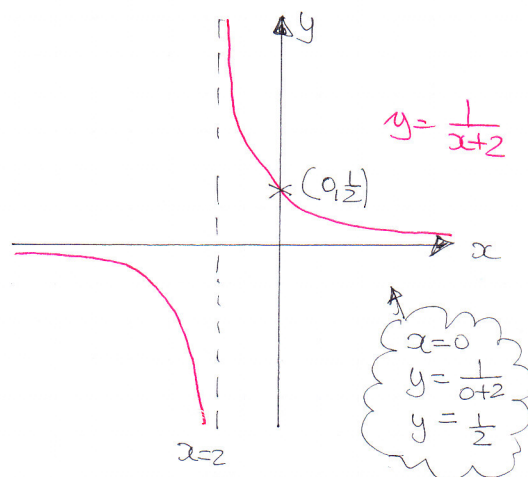
IMPOSSIBLE

$$\therefore n = 26$$

6. $y = \frac{1}{x} + 2$ IS A TRANSLATION OF $y = \frac{1}{x}$, UPWARDS BY 2 UNITS



- $y = \frac{1}{x+2}$ IS A TRANSLATION OF $y = \frac{1}{x}$, 2 UNITS TO THE LEFT



7. (a) AB: $5x + 4y = 7$
 $4y = -5x + 7$
 $y = -\frac{5}{4}x + \frac{7}{4}$
 GRAD OF AB

"CD" IS PARALLEL & PASSES THROUGH (4, 7)

$$y - y_0 = m(x - x_0)$$

$$y - 7 = -\frac{5}{4}(x - 4)$$

$$4y - 28 = -5x + 20$$

$$4y + 5x = 48$$

AS REQUIRED

- (b) GRADIENT AD IS $\frac{4}{5}$ (PERPENDICULAR)

$$y - 7 = \frac{4}{5}(x - 4)$$

$$5y - 35 = 4x - 16$$

$$5y = 4x + 19$$

(c) [CD]: $5x + 4y = 48$

[BC]: $x + 9y + 15 = 0$

$$\downarrow$$

$$x = -9y - 15$$

BY SUBSTITUTION

$$5(-9y - 15) + 4y = 48$$

$$-45y - 75 + 4y = 48$$

$$-123 = 41y$$

$$y = -3$$

$$\therefore x = -9y - 15$$

$$x = -9(-3) - 15$$

$$x = 27 - 15$$

$$x = 12$$

$$\therefore C(12, -3)$$

AS REQUIRED

9. (a) $y = x^3 - 10x + 2$
 $\frac{dy}{dx} = 3x^2 - 10$
 $\left. \frac{dy}{dx} \right|_{x=2} = 3 \times 2^2 - 10 = 12 - 10 = 2$

• TANGENT l_1
 $y - y_0 = m(x - x_0)$
 $y + 10 = 2(x - 2)$
 $y + 10 = 2x - 4$
 $y = 2x - 14$

• NORMAL l_2
 $y - y_0 = m(x - x_0)$
 $y + 10 = -\frac{1}{2}(x - 2)$
 $2y + 20 = -x + 2$
 $x + 2y + 18 = 0$

(b) • WHEN $y = 0$

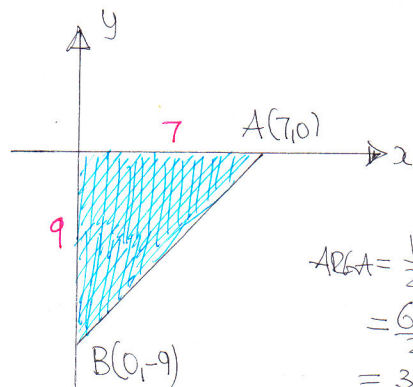
$l_1: 0 = 2x - 14$
 $14 = 2x$
 $x = 7$

$\therefore A(7, 0)$

• WHEN $x = 0$

$l_2: 0 + 2y + 18 = 0$
 $2y = -18$
 $y = -9$

$\therefore B(0, -9)$



$AREA = \frac{1}{2} \times 7 \times 9$
 $= \frac{63}{2}$
 $= 31.5$

10. (a) $f'(x) = \frac{12x-1}{\sqrt{x}} = \frac{12x-1}{x^{\frac{1}{2}}} = \frac{12x}{x^{\frac{1}{2}}} - \frac{1}{x^{\frac{1}{2}}} = 12x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

$\therefore f(x) = \int 12x^{\frac{1}{2}} - x^{-\frac{1}{2}} dx$

$f(x) = \left(\frac{12}{\frac{3}{2}} \right) x^{\frac{3}{2}} - \left(\frac{1}{-\frac{1}{2}} \right) x^{\frac{1}{2}} + C$

$f(x) = 8x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + C$

when $x = 0$ $y = 0$ (PASSES THROUGH 0)

$0 = 8 \times 0^{\frac{3}{2}} - 2 \times 0^{\frac{1}{2}} + C$

$0 = C$

$\therefore f(x) = 8x^{\frac{3}{2}} - 2x^{\frac{1}{2}}$

(b) $f(x) = 8x^{\frac{3}{2}} - 2x^{\frac{1}{2}}$

$$0 = 8x^{\frac{3}{2}} - 2x^{\frac{1}{2}}$$

$$0 = 2x^{\frac{1}{2}}(4x - 1)$$

$$\bullet x^{\frac{1}{2}} = 0$$

$$x = 0$$

↑
ORIGIN

$$\bullet 4x - 1$$

$$x = \frac{1}{4}$$

$$\therefore P\left(\frac{1}{4}, 0\right) //$$