MGCCC PRESENTS

USA COLLEGE OF ENGINEERING AND COMPUTING

Come learn about the programs offered at the University of South Alabama in Engineering and Computer Science!

JANUARY 28, 2020

12:30 PM TO 1:20 PM

ROOM F-5

Objective #2: Solve quadratic equations by using the Square Root Property.

Circle each quadratic equation below. Then, draw a small star next to the quadratic equations on which the Zero-Factor Property could be easily used to solve for x.

1. $4\sqrt{x-3}=0$ 2. $2x^2-5=0$ 3. $x^2+3x+2=0$ 4. $\frac{x^2-4}{x+1}=0$ 5. $(6x+3)^2=4$ 6. $|6x+3|=0$

KEY IDEAS

We will study four ways to solve a quadratic equation:

1. Zero-Factor Property (sometimes works)
2. Square Root Theorem (sometimes works)
3. Complete the Square
4. Quadratic Formula

Process:

1. $\boxed{\text{Isolate}}$ the squared stuff; rewrite the equation in the form $(variable\ expression)^2 = k$
2. Take the square root of both sides of the equation. Don't forget the "$\pm$" symbol.
3. Isolate the variable, if needed.
4. Simplify, if needed.

Square Root Property

If $x^2=k$, then $x=\sqrt{k}$ or $x=-\sqrt{k}$.

The solution set of $x^2=k$ is $\{\sqrt{k},-\sqrt{k}\}$, which may be abbreviated as $\{\pm\sqrt{k}\}$.

k	Solutions
$k>0$	Both solutions are real
$k<0$	Both solutions are pure imaginary

Solve each equation by using the Square Root Property.

quad. eqn.

isolate squared stuff.

7. $2x^2-32=0$
$+32\ \ +32$
$\frac{2x^2}{2}=\frac{32}{2}$
$x^2=16$
$\sqrt{x^2}=\pm\sqrt{16}$
$x=\pm4$

8. $x^2-20=0$
$+20\ \ +20$
$x^2=20$
$\sqrt{x^2}=\pm\sqrt{20}$
$x=\pm2\sqrt{5}$

9. $\frac{1}{2}x^2+\cancel{1}=10$
$\cancel{-1}\ \ -1$
$\cancel{2}\cdot\cancel{\frac{1}{2}}x^2=9\cdot2$
$x^2=18$
$\sqrt{x^2}=\pm\sqrt{18}$
$x=\pm3\sqrt{2}$

10. $(3x-3)^2-7=11$
$+7\ \ +7$
$\sqrt{(3x-3)^2}=\pm\sqrt{18}$
$3x-\cancel{3}=\pm3\sqrt{2}$
$\cancel{+3}\ \ +3$
$\frac{\cancel{3}x}{\cancel{3}}=\frac{3}{3}\pm\frac{3\sqrt{2}}{3}$
$x=1\pm\sqrt{2}$

11. $\frac{2(2x+1)^2}{2}=\frac{-6}{2}$
$\sqrt{(2x+1)^2}=\pm\sqrt{-3}$
$2x+1=\pm i\sqrt{3}$
$-1\ \ -1$
$\frac{2x}{2}=\frac{-1\pm i\sqrt{3}}{2}$
$x=\frac{-1\pm i\sqrt{3}}{2}$

12. $(3x-3)^2+7=7$
$-7\ \ -7$
$(3x-3)^2=0$
$\sqrt{(3x-3)^2}=\pm\sqrt{0}$
$3x-3=0$
$+3\ \ +3$
$\frac{3x}{3}=\frac{3}{3}$
$x=1$

quad. eqn. isolate sq. stuff.

Memorize

Objective #3: Solve quadratic equations by using Completing the Square.

Explore Perfect Square Trinomials.

Starting Expression ($with\ a = 1$)	Evaluate $\left(\frac{1}{2}\cdot b\right)^2$	Create Perfect Square Trinomial	Factored Form
1. $x^2 + 8x$ $b=8$	$(\frac{1}{2}\cdot 8)^2 = (4)^2 = 16$	$x^2+8x+16$	$(x+4)^2$
2. $x^2 + 12x$ $b=12$	$(\frac{1}{2}\cdot 12)^2 = (6)^2 = 36$	$x^2+12x+36$	$(x+6)^2$
3. $x^2 - 20x$ $b=-20$	$(\frac{1}{2}\cdot -20)^2 = (-10)^2 = 100$	$x^2-20x+100$	$(x-10)^2$
4. $x^2 - 5x$ $b=-5$	$(\frac{1}{2}\cdot -5)^2 = (-5/2)^2 = 25/4$	$x^2-5x+25/4$	$(x-5/2)^2$
5. $x^2 + 7x$ $b=7$			

KEY IDEAS

We will study four ways
to solve a quadratic equation:

1. Zero-Factor Property (sometimes works)
2. Square Root Theorem (sometimes works)
3. Complete the Square (always works)
4. Quadratic Formula

Process:

To solve $ax^2 + bx + c = 0$, where $a \neq 0$, by completing the square:

1. If $a \neq 1$, divide both sides of the equation by a.
2. Rewrite the equation so that the constant term is alone on one side of the equality symbol.
3. Evaluate $\left(\frac{1}{2}\cdot b\right)^2$ and add it to both sides of the equation.
4. Factor the resulting perfect square trinomial and combine like terms on the other side.
5. Use the square root property to complete the solution.

Solve each equation by using Completing the Square.

6. $x^2 + 10x + 21 = 0$

$a=1 \quad b=10$

$(\frac{1}{2}\cdot b)^2 = (\frac{1}{2}\cdot 10)^2 = (5)^2 = 25$

$-21 \quad -21$

$x^2 + 10x + 25 = -21 + 25$

$\sqrt{(x+5)^2} = \pm\sqrt{4}$ Sq. Rt. Prop.

$x+5 = \pm 2$

$-5 \quad -5$

$x = -5 \pm 2$

$x = \pm 2 - 5$

$2-5 \quad -2-5$

$-5+2 \quad -5-2$

$-3 \quad -7$

7. $x^2 + 8x + 7 = 0$

$a=1 \quad b=8$

$(\frac{1}{2}\cdot b)^2 = (\frac{1}{2}\cdot 8)^2 = (4)^2 = 16$

$-7 \quad -7$

$x^2 + 8x + 16 = -7 + 16$

$\sqrt{(x+4)^2} = \pm\sqrt{9}$

$x+4 = \pm 3$

$-4 \quad -4$

$x = -4 \pm 3$

$-4+3 \quad -4-3$

$-1 \quad -7$

8. $(x-1)(x+4) = 9$

$a=1 \quad b=3$

$(\frac{1}{2}\cdot b)^2 = (\frac{1}{2}\cdot 3)^2 = (\frac{3}{2})^2 = \frac{9}{4}$

$x^2 + 4x - 1x - 4 = 9$

$+4 \quad +4$

$x^2 + 3x + \frac{9}{4} = 13 + \frac{9}{4}$

$\sqrt{(x+\frac{3}{2})^2} = \pm\sqrt{\frac{61}{4}}$

$x + \frac{3}{2} = \pm\frac{\sqrt{61}}{2}$

$-\frac{3}{2} \quad -\frac{3}{2}$

$x = -\frac{3}{2} \pm \frac{\sqrt{61}}{2} = \frac{-3 \pm \sqrt{61}}{2}$

Objective #3: Solve quadratic equations by using Completing the Square.

Solve each equation by using Completing the Square.

9. $(x+5)(x+3) = 24$

foil

$x^2+3x+5x+15 = 24$

$x^2+8x+15 = 24$

$-15 \quad -15$

$x^2+8x+16 = 9+16$

$a=1 \quad b=8$

$\left(\frac{1}{2}\cdot 8\right)^2 = (4)^2 = 16$

$(x+4)^2 = 25$

$\sqrt{(x+4)^2} = \pm\sqrt{25}$

$x+4 = \pm 5$

$-4 \quad -4$

$x = -4 \pm 5$

$-4+5$ → $+1$

$-4-5$ → -9

10. $4x^2 - 24x - 1 = 0$

$+1 \quad +1$

$\frac{4x^2}{4} - \frac{24x}{4} = \frac{1}{4}$

$a=1 \quad b=-6$

$\left(\frac{1}{2}\cdot b\right)^2 = \left(\frac{1}{2}\cdot -6\right)^2 = (-3)^2 = 9$

$x^2 - 6x + 9 = 1/4 + 9$

$(x-3)^2 = \frac{37}{4}$

$\sqrt{(x-3)^2} = \pm\sqrt{\frac{37}{4}} \Rightarrow \pm\frac{\sqrt{37}}{\sqrt{4}}$

$x-3 = \pm\frac{\sqrt{37}}{2}$

$+3 \quad +3$

$x = 3 \pm \frac{\sqrt{37}}{2}$

get common denominator!

$x = \frac{3\cdot 2}{1\cdot 2} \pm \frac{\sqrt{37}}{2}$

$= \frac{6}{2} \pm \frac{\sqrt{37}}{2}$

$= \frac{6\pm\sqrt{37}}{2}$

11. $2x^2 - 5x - 12 = 0$

$+12 \quad +12$

$\frac{2x^2}{2} - \frac{5x}{2} = \frac{12}{2}$

$a=1 \quad b=-\frac{5}{2}$

$\left(\frac{1}{2}\cdot b\right)^2 = \left(\frac{1}{2}\cdot -\frac{5}{2}\right)^2$

$= \left(-\frac{5}{4}\right)^2 = \frac{25}{16}$

$x^2 - \frac{5}{2}x + \frac{25}{16} = 6 + \frac{25}{16}$

$\sqrt{\left(x - \frac{5}{4}\right)^2} = \pm\sqrt{\frac{121}{16}} \quad \frac{\sqrt{121}}{\sqrt{16}}$

$x - \cancel{\frac{5}{4}} = \pm\frac{11}{4}$

$\cancel{+\frac{5}{4}} \quad +\frac{5}{4}$

$x = \frac{5}{4} \pm \frac{11}{4}$

$\frac{5\pm 11}{4}$

$\frac{5}{4} + \frac{11}{4}$

$\frac{16}{4}$

4

$\frac{5}{4} - \frac{11}{4}$

$-\frac{6}{4}$

$-\frac{3}{2}$

12. $3x^2 - 4x - 2 = 0$

$+2 \quad +2$

$\frac{3x^2}{3} - \frac{4x}{3} = \frac{2}{3}$

$a=1 \quad b=-\frac{4}{3}$

$\left(\frac{1}{2}\cdot -\frac{4}{3}\right)^2 = \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$

$x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{2}{3} + \frac{4}{9}$

$\left(x - \frac{2}{3}\right)^2 = \frac{10}{9}$

$\sqrt{(x-2/3)^2} = \pm\sqrt{\frac{10}{9}} = \pm\frac{\sqrt{10}}{\sqrt{9}}$

$x - \frac{2}{3} = \pm\frac{\sqrt{10}}{3}$

$+2/3 \quad +2/3$

$x = \frac{2}{3} \pm \frac{\sqrt{10}}{3} = \frac{2\pm\sqrt{10}}{3}$

KEY IDEAS

We will study four ways to solve a quadratic equation:

1. Zero-Factor Property (sometimes works)
2. Square Root Theorem (sometimes works)
3. Complete the Square (always works)
4. Quadratic Formula (always works)

Process:

To solve $ax^2 + bx + c = 0$, where $a \neq 0$, by the quadratic formula:

1. Write the equation in standard form and identify the values of $a, b,$ and c.
2. Substitute the values of $a, b,$ and c into the quadratic formula, $\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$.
3. Simplify the expression found in step 2.

▶ Caution ***Remember to extend the fraction bar in the quadratic formula extends under the –b term in the numerator.***

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve each equation by using the Quadratic Formula.

1. $2x^2 + 11x + 15 = 0$

✓quad. eqn.
✓ =0
✓desc. ord

$a=2 \quad b=11 \quad c=15$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-11 \pm \sqrt{(11)^2-4(2)(15)}}{2(2)}$$

$$= \frac{-11 \pm \sqrt{121-120}}{4} = \frac{-11 \pm \sqrt{1}}{4} = \frac{-11 \pm 1}{4}$$

$$\frac{-11+1}{4} = \frac{-10}{4} = \frac{-5}{2} \qquad \frac{-11-1}{4} = \frac{-12}{4} = -3$$

2. $4x(x-3) + 9 = 0$

✓=0
✓desc.ord.

$4x^2 - 12x + 9 = 0$

$a=4 \quad b=-12 \quad c=9$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{12 \pm \sqrt{(-12)^2-4(4)(9)}}{2(4)}$$

$$= \frac{12 \pm \sqrt{144-144}}{8} = \frac{12 \pm \sqrt{0}}{8} = \frac{12 \pm 0}{8} = \frac{12}{8}$$

$$= \frac{3}{2}$$

3. $4x^2 + 1 = 8x$

$-8x \quad -8x$

$4x^2 - 8x + 1 = 0$

✓quad eqn.
✓ =0
✓desc.ord.

$a=4 \quad b=-8 \quad c=1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{8 \pm \sqrt{(-8)^2-4(4)(1)}}{2(4)}$$

$$= \frac{8 \pm \sqrt{64-16}}{8} = \frac{8 \pm \sqrt{48}}{8} = \frac{8 \pm 4\sqrt{3}}{8}$$

$\div 4 \quad \div 4 \quad \div 4$

$$= \frac{2 \pm \sqrt{3}}{2}$$

4. $2\left(\frac{1}{2}x^2\right) + 2\left(\frac{1}{2}x\right) + 2(4) = (0)2$

common denom=2

$x^2 + x + 8 = 0$

✓quad.eqn
✓=0
✓desc.ord.

$a=1 \quad b=1 \quad c=8$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{(1)^2-4(1)(8)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1-32}}{2} = \frac{-1 \pm \sqrt{-31}}{2} = \frac{-1 \pm i\sqrt{31}}{2}$$