Punto 3

Construimos el Polinomio interpolador que pasa Por $(a, f(a)), (x_m, f(x_m)), (b, f(b))$

$$P_2(x) = \frac{f(a)}{2}\left(\frac{(x-x_m)(x-b)}{(a-x_m)(a-b)}\right) + \frac{f(x_m)}{2} \cdot \left(\frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}\right)$$

$$+ \frac{f(b)}{2}\left(\frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}\right)$$

$$\int_a^b P_2(x)\,dx$$

Se simplifica a:

$$\int_a^b P_2(x)\,dx = \frac{b-a}{6}\left[f(a) + 4f(x_m) + f(b)\right]$$

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a) + 4f(x_m + f(b))\right]$$

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$$f(x) = \sum_{n=0}^{N} C_n P_n(x)$$

Son ortogonales en $[-1,1]$ con respecto al peso 1

$$\int_{-1}^{1} P_m(x) P_n(x)\, dx = \begin{cases} \frac{2}{2n+1} & \text{si } m=n \\ 0 & \text{si } m \neq n \end{cases}$$

multiplicamos por $P_m(x)$

$$\int_{-1}^{1} f(x) P_m(x)\, dx = \int_{-1}^{1} \left(\sum_{n=0}^{N} C_n P_n(x) \right) P_m(x)\, dx$$

$$\int_{-1}^{1} f(x) P_m(x)\, dx = \sum_{n=0}^{N} C_n \int_{-1}^{1} P_n(x) P_m(x)\, dx$$

Aplicamos ortogonalidad

$$\int_{-1}^{1} f(x) P_m(x)\, dx = C_m \frac{2}{2m+1}$$

$$C_m = \frac{2m+1}{2} \int_{-1}^{1} f(x) P_m(x)\, dx$$