

Punto 26

$$a) \Delta x = \frac{(b-a)}{N}$$

$$b=2$$

$$a=0$$

$$\Delta x = \frac{2-0}{N} = \frac{2}{N}$$

$$b) x_i = a + i \Delta x$$

$$a=0$$

$$\Delta x = \frac{2}{N}$$

$$x_i = 0 + i \cdot \frac{2}{N} = \frac{2i}{N} \rightarrow \text{PARA } i=0,1,2 \text{ hasta } N-1(29)$$

$$c) f(x_i) = x_i^3$$

REEMPLAZAMOS

$$f(x_i) = \left(\frac{2i}{N}\right)^3 = \left(\frac{8i^3}{N^3}\right)$$

$$\text{PARA } i=0,1,2 \text{ hasta } N-1(29)$$

2

D) La Suma de Riemann es

$$I \approx \sum_{i=0}^{N-1} f(x_i) \Delta x$$

Como $f(x_i) = \frac{8i^3}{N^3}$ y $\Delta x = \frac{2}{N}$

$$I \approx \sum_{i=0}^{N-1} \frac{8i^3}{N^3} \cdot \frac{2}{N}$$

Simplificamos

$$I \approx \frac{16}{N^4} \sum_{i=0}^{N-1} i^3$$

Sabiendo que

Sustituimos

$$\sum_{i=0}^{N-1} i^3 = \frac{(N(N-1))^2}{4} \rightarrow I \approx \frac{16}{N^4} \cdot \frac{N^2(N-1)^2}{4}$$

Simplificando y expandiendo

$$I \approx 4 \cdot \frac{(N-1)^2}{N^2} = 4 \cdot \left(1 - \frac{2}{N} + \frac{1}{N^2}\right)$$

llegamos

$$I \approx 4 \left(1 - \frac{2}{N} + \frac{1}{N^2}\right)$$