$$\frac{\dfrac{f(x)-f(x_0)}{x-x_0}-f[x_1,x_0]}{x-x_1} = \frac{f(x)-f(x_0)-(x-x_0)f[x_1,x_0]}{(x-x_0).(x-x_1)}$$

$(\Rightarrow)\quad f(x) = f(x_0) + (x - x_0)\, f[x_1, x_0] + (x - x_0)\,(x - x_1)\, f[x_0, x_1, x]$

$(\Rightarrow)\qquad f(x) = P_1(x) + E_1(x) \qquad (4.2.1\text{-}5)$

$$\boxed{\begin{aligned} P_1(x) &= P_0(x) + (x - x_0)\, f[x_0, x_1] \\ E_1(x) &= (x - x_0)\,(x - x_1)\, f[x_0, x_1, x] \end{aligned}}$$

(4.3.1-7)

VERIFICAÇÃO: $P_1(x)$ interpola $f(x)$ em x_0 e em x_1?

$P_1(x_0) = f(x_0)$

$$P_1(x_1) = f(x_0) + (x_1 - x_0)\left[\frac{f(x_1)-f(x_0)}{x_1-x_0}\right] = f(x_1)$$

Agora, construa $P_2(x)$ o polinômio de grau menor que 2 que interpola $f(x)$ em x_0, x_1, x_2.

$$f[x_0, x_1, x_2, x] = f[x_2, x_1, x_0, x] = \frac{f[x_1,x_0,x]-f[x_2,x_1,x_0]}{x-x_2} \qquad (\Rightarrow)$$

$$\boxed{\begin{aligned} P_2(x) &= P_1(x) + (x - x_0)\,(x - x_1)\, f[x_0, x_1, x_2] \\ E_2(x) &= (x - x_0)\,(x - x_1)\,(x - x_2)\, f[x_0, x_1, x] \end{aligned}}$$

(4.3.1-8)

De um modo geral:

$$P_{n+1}(x) = P_n(x) + (x - x_0)\,(x - x_1) \ldots (x - x_n)\, f[x_0, x_1, \ldots, x_n] \qquad (4.3.1\text{-}9)$$

e o erro cometido

$$E_{n+1}(x) = f(x) + P_{n+1}(x) = (x - x_0)\,(x - x_1) \ldots (x - x_{n+1})\, f[x_0, x_1, \ldots, x_n, x_{n+1}]$$

(4.3.1-10)

Teorema 4.3.1-5 : (Fórmula 4.3.1-10) ou

$$E_n(x) = f(x) - P_n(x) = (x - x_0)\,(x - x_1) \ldots (x - x_n)\ .\ \frac{1}{(n+1)!} f^{(n+1)}(c) \qquad (4.3.1\text{-}11)$$

$c \in (x_0, x_n)$.