Agent-Based Modeling:
Modeling Asset Price Processes in Financial Markets

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1 Introduction

Time series of financial assets typically exhibit notable phenomena such as volatility clustering, jumps in asset prices, and leptokurtic distributions of the probability density of log price returns. Volatility clustering means that large price movements are followed by large price movements and small price movements are followed by small price movements and are most evident in plots of the log-returns of the asset over time. Jumps can be observed in the price paths as time progresses where the asset price seemingly moves between two very different values connected by a single straight line.

As financial markets are complex systems made up of interacting traders, the volatility clustering and leptokurtic log return distributions can be interpreted as emergent phenomena. Therefore, given an appropriately constructed agent-based model (ABM), it is expected that these phenomena can be replicated within the ABM. Examples of trading and price process ABMs include the Genoa model by Raberto et al., the work by Inoua, and the work by Shi et al.. By implementing various agent trading strategies and degrees of rationality, each of these works was able to reproduce volatility clustering and leptokurtic distribution of log-returns as well as the cubic power law of the tail of the returns distribution.

Unlike ABM, stochastic differential equation (SDE) models do not exhibit emergent phenomena. The celebrated Black-Scholes model [4] models asset prices as an SDE in the form of a geometric Brownian motion where the change in the asset price depends on a deterministic drift related to the change in time and a stochastic increment related to the product of the asset volatility and the Brownian motion process value. A major drawback of this model is that the volatility of the asset is assumed to be constant and, as a result, the occurrence of volatility clustering and price jumps must be modelled explicitly. Examples include the Heston model [5] where the volatility is modelled as a stochastic process itself with a mean reversion and the Merton jump-diffusion model [6] where jumps in the asset price are modelled as a Poisson process. The stochastic volatility with jumps (SVJ) model by Bates combines the Heston and Merton models.

In this work, we simulate the price evolution of an asset in a financial market using ABM. Within the scope of this work, we compare the dynamics obtained using the Black-Scholes model, the SVJ model, the Genoa model, and our own implemented asset trading ABM, which incorporates elements of evolutionary computing, to the historical prices of the S&P 500 index. To this end, we use Monte Carlo simulation to generate price data under the Black-Scholes and SVJ models and we implement our ABM in the Mesa framework [8]. We are mainly interested in seeing whether volatility clustering and leptokurtic log return distributions occur in our ABM as well as which, if any, emergent phenomena arise. We consider the SDE models only for visual comparisons and therefore we do not calibrate the model parameters thoroughly; conversely, we give a detailed description of our ABM according to the overview, design concepts, details, and decisions (ODD + D) protocol [9] and we perform a global sensitivity analysis.

1.1 ABM: Genoa Model

The original Genoa artificial market model [1] is an ABM that simulates a financial market. Heterogeneous agents are initialized with a finite amount of cash and assets. There is no cash generation process so that the total amount of cash in the system remains constant in time. At each time step, traders decide whether to buy or sell assets according to a probability which is set to 0.5 for random traders. Buy and sell order limit prices of the ith trader at time h are respectively computed as

$$b = p(h)\mathcal{N}(\mu, \sigma_i), \quad s = p(h)/\mathcal{N}(\mu, \sigma_i)$$

where $\mathcal{N}$ is a normal distribution with $\mu = 1.01$ and $\sigma_i = k\sigma(T_i)$ with $k = 3.5$ and T_i is a time window of 20 days. The corresponding buy and sell order quantities are computed as $a_i^b = [r_i C_i(h)/b_i]$ and $a_i^s = [r_i A_i(h)]$, respectively, where r_i is a random draw from the uniform distribution on $[0, 1]$ and $C_i(h)$ and $A_i(h)$ are the total numbers of cash and assets of the ith trader at time h.

The price-formation process of the market is initiated by constructing demand and supply curves for the asset using the buy and sell order quantities and limit prices. In a system with random strategies only, the resulting price process has a Gaussian distribution of log-returns and mean-reverting behaviour but no volatility clustering or fat tails [1].

To reproduce volatility clustering and fat tails, Raberto et al. introduced agent aggregation as clustering in a random graph: at each time step, random pairs of traders are selected and a cluster of these traders is formed with some probability. This process leads to the clustering of traders in separate groups which are then activated or deactivated according to a random draw. If activated, each cluster changes its buy probability from 0.5 to either 1 or 0 meaning that all traders within the cluster adopt the same strategy of buying or selling at that particular time point.

The Genoa model was extended to accommodate the possible imbalance of supply and demand by having a market maker inject any additionally needed cash or assets [10]. Additionally, more sophisticated trading strategies were analyzed [11]. *Momentum traders* speculate that if prices are rising they will keep rising and vice versa for falling prices: using a time window of past prices T_i they compute the current trend $\mathcal{D}(h+1, T_i)$ of the prices as the difference between the previous price $p(h)$ and the price at the start of the time window $p(h - T_i)$, divided by the size of the time window. For $\mathcal{D}(h+1, T_i) > 0$ and $\mathcal{D}(h+1, T_i) < 0$ the trader issues a buy or sell order, respectively, with limit price equal to $p(h) + \mathcal{D}(h+1, T_i)$. *Contrarian traders* speculate that if prices are rising, they will stop rising soon and start decreasing as the stock value will exceed the total cash value of the market. These traders take time windows of 10 to 50 days for which they compute the trends and, being contrarian, they will sell if $\mathcal{D}(h+1, T_i) > 0$ and buy if $\mathcal{D}(h+1, T_i) < 0$. Finally, *fundamentalist traders* do not speculate on short-term price changes but instead, believe that the asset has a fundamental price due to factors external to the market. These traders try to make steady gains in the long run and do so by buying the asset if the current price is lower than their assumed fundamental price and selling the asset if it is higher.

1.2 SDE Models: Black-Scholes and SVJ

The Black-Scholes (BS) model is an option pricing model that was introduced in 1973 and is still widely used for the pricing of European-style option contracts. A European option gives the holder the right but not the obligation to buy (call option) or sell (put option) an underlying asset at the end of the contract against a predetermined strike price. In the BS model, the evolution of the underlying asset under the real-world measure $\mathbb{P}$ is assumed to be a geometric Brownian motion as given by Equation (1)

$$S_t = S_0 \text{exp}(\mu t + \sigma W_t) \tag{1}$$

where S_0 is the asset value at time 0, μ is the drift of the asset, dt is the time increment, σ is the volatility of the asset, and W_t is the increment of a Brownian motion. A Brownian motion is a martingale, i.e., a random process with expected value 0, expected variance t, and the notable property that its expected value at any time t equals its value at the immediately preceding time s: $\mathbb{E}[W_t|W_s] = W_s$. This way, the asset prices follow a lognormal distribution. Using Itô calculus, the process S_t can be shown to solve the stochastic differential equation (SDE) as given by Equation (2) [12]

$$dS_t = (\mu - \frac{\sigma^2}{2})S_t dt + \sigma S_t dW_t \tag{2}$$

One limitation of the standard BS model is that the volatility is assumed to be constant. A more realistic approach is to implement stochastic volatility, such as in the Heston model where the dynamics of the the volatility process v_t and the asset process S_t under $\mathbb{P}$ are given by Equations (3) and (4)

$$dv_t = \kappa(\theta - v_t)dt + \xi\sqrt{v_t}dW_t^v \tag{3}$$

$$dS_t = \mu S_t dt + \sqrt{v_t}S_t dW_t^S \tag{4}$$

where v_t is the stochastic volatility. The Brownian motions W_t^S and W_t^v are correlated to some degree given by ρ so that $dW_t^S dW_t^v = \rho dt$. The stochastic volatility process is a mean-reverting square-root process with reversion to the value θ at a rate κ and its own volatility parameter ξ.

Another limitation in both the standard BS model and the Heston model is the lack of jumps. Jumps can be modeled by adding to the asset price SDE a term dJ_t with [13]

$$J_t = \sum_{j=1}^{N(t)} (Y_j - 1)$$

where $Y_1, Y_2, ...$ are random variables and N(t) is a counting process that counts the number of random arrivals in $[0,t]$. Thus for the random arrival times $0 < \tau_1 < \tau_2 < ...$, the counting process is given by $N(t) = \sup(n : \tau_n \leq t)$. This method of adding jumps was first introduced by Merton who interpreted the jumps as shocks affecting an individual company but not the entire market as a whole.

Bates combined the Heston model with the Merton jumps under the stochastic volatility jump (SVJ) model. By combining the two models, the following dynamics are obtained.

$$dv_t = \kappa(\theta - v_t)dt + \xi\sqrt{v_t}dW_t^v \tag{5}$$

$$dS_t = (\mu - \lambda J)S_t dt + \sqrt{v_t}S_t dW_t^S + JS_t dN_t \tag{6}$$

Simulation of asset price processes with these SDE models can be done with Monte Carlo simulation where a large number of asset paths are simulated using a discretization of the asset price process. The mean of the prices at each time point is the estimate of the true asset price [13]. The pricing of financial derivatives using these models requires a change of measure to the risk-neutral measure $\mathbb{Q}$ and the corresponding change of time-based drift in the dynamics of the asset path which we will not consider in this work as we focus on the realized real-world asset paths.

2 Methods

We take the S&P 500 daily price data from the period of January 1, 2016, to December 13, 2018. This time frame was selected in order to avoid any extraordinary stock effects that may have been caused by the COVID-19 pandemic. We discretize our SDE and ABM models such that their simulation times correspond to the number of business days within this time frame which is a total of 754 business days.

2.1 SDE Models: Black-Scholes and SVJ

As we are interested in the evolution of asset prices and not the pricing of derivatives, we use the BS and SVJ models under the real-world probability $\mathbb{P}$. We implement these models in our Monte Carlo simulations using Euler discretization. Therefore we discretize the real-world asset price of the BS model given by Equation (1) using Equation (7).

$$S_{t+1} = S_t \exp(\mu\Delta t + \sigma\tilde{W}_t) \tag{7}$$

We discretize the solutions to the real-world volatility and asset price processes of the SVJ model given by Equations (5) and (6) by applying Euler discretization as in Equations (8) and (9), respectively

$$v_{t+1} = v_t + \kappa(\theta - v_t^+)\Delta t + \xi\sqrt{v_t^+}\Delta W_{t+1}^v \tag{8}$$

$$S_{t+1} = S_t \left\{ \exp\left((\mu - \lambda J - \frac{v_t^+}{2})\Delta t + \sqrt{v_t^+}\sqrt{\Delta t}\Delta W_{t+1}^S \right) + JdN_t \right\} \tag{9}$$

where we apply the full truncation mechanism in order to prevent negative volatilities using $v_t^+ = \max(v_t, 0)$. The correlation between the Brownian motions is given by

$$W_t^S = \rho W_t^v + \sqrt{1-\rho^2}W_t^i \tag{10}$$

with W_t^i a third, independent Brownian motion. J and the Brownian motions are sampled from the standard normal distribution and dN_t is sampled from a Poisson distribution with intensity $\lambda \cdot \Delta t$.

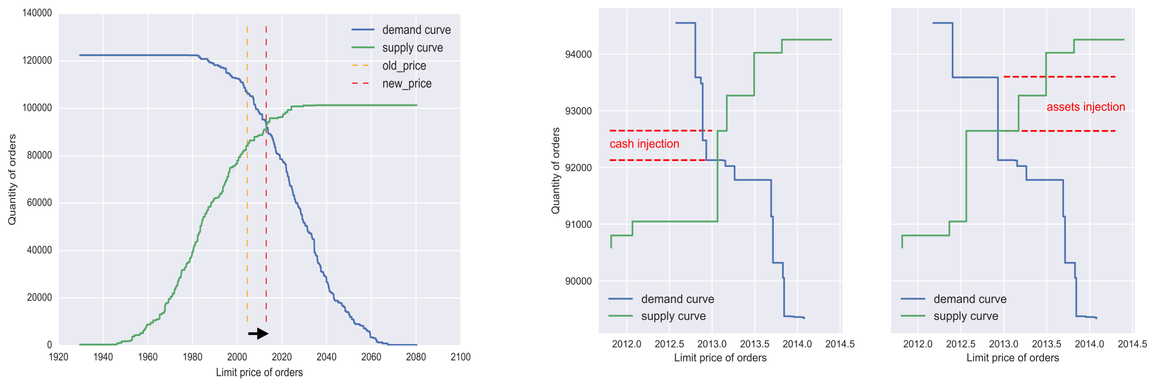
To obtain statistically relevant estimates of the asset prices, we perform Monte Carlo simulations with 100,000 runs each. The BS and SVJ parameters as used in our simulations are displayed in Table 1. As previously stated, our main focus in this work is our ABM. Therefore, the model parameters were not thoroughly calibrated to the market data.

Parameter	Value
S_0	2012.66
μ	0
σ	0.2247
T	2.066
κ	2
ξ	0.2247
ρ	0.25
λ	1
$N_{\text{time points}}$	754

Table 1: Parameters as used in the Monte Carlo simulations of the BS and SVJ models.

2.2 Agent-Based Model

The base of our ABM is inspired by the way the Genoa model puts in trade orders and determines the market value of the asset. As in the Genoa model, our model has traders put in their trade orders (or decide not to trade) after which the cumulative order quantities for buying and selling are computed as functions of the order limit prices. As shown in Figure 1 (a), The intersection of these demand and supply curves yield the market price of the asset as the corresponding value of the x-axis. Orders with compatible limit prices (buy orders with a maximum price lower than or equal to the new price and sell orders with a minimum price higher than or equal to the new price) are satisfied, while others are discarded.



(a) (b)

Figure 1: Price formation process (a) and Injections of cash or assets(b)

If the size of compatible sell orders is larger than the size of compatible buy orders, as shown in Fig 1 (b), the market adds cash to the trader and subtracts assets from it. On the contrary, if the size of sell orders is smaller than the size of buy orders, the market maker adds assets to the trader and subtracts cash from it.

Two variables are critical in this system: the probability of putting in a trade order and the probability of that trade order being a buy order. To extend this base model, we implement an evolutionary computation (EC) component where the trade and buy probabilities evolve in time. The objective fitness for agents is the total wealth owned (i.e. cash plus assets times current market price). The evolutionary agents actively copy and mutate strategies represented by two probabilities mentioned above from top performers in the market.

We use 100 runs in each of our ABM simulations as a compromise between program runtime and the statistical significance of our results. We further describe our ABM according to the ODD + D protocol in the sections below.

2.2.1 Overview

Purpose This work introduces a framework in which evolving trading strategies can be analyzed. The financial stock market is dependent on individual decision making, however since there are a limited amount of strategies available for each individual this leads to clusters of decision-makers. The purpose of this work is to analyze the evolution strategy of the top-performing traders in the financial stock market, this could lead to new insights for practitioners in the real stock market.

Entities, State Variables and Scales Multiple entities are present in our model. Firstly, the market maker represents the stock market or the main exchange. The market maker keeps track of the asset price, the number of traders, handles the orders and finds the market price after each iteration of buying and selling. Secondly, the agents each have a unique ID. They keep track of the number of assets and cash they have at their disposal. Each has to decide based on a probability whether they will even trade at all at each timestep. Hereafter each agent can either buy and sell per timestep depending on whether they have cash or assets. The probabilities of trading are initialized with a random distribution between 0 and 1. Buying and selling are initialized with a probability of 0.5, and they all evolve as time progresses.

Process Overview and Scheduling In each time step, each trader is prompted to submit buying or selling orders. The market maker receives orders from all traders and decides the new asset price by matching the overall amount of demand and supply. At the end of each step, the market maker executes orders with acceptable limit prices and updates each trader's current cash and assets. The scheduling of requesting and executing orders is asynchronous. Thus each trader is processed one after another.

2.2.2 Design Concepts

Theoretical and Empirical Background Our basic model is originated from Genoa artificial market model and characterized by heterogeneous traders exhibiting fully random behaviour. Therefore the price path should be a random walk and no emergent behaviour is expected. As mentioned above, we incorporate evolutionary agents as an extension to see whether volatility clustering or any emergent phenomena exists which is the case in the real world financial asset returns.

Individual Decision Making, Learning, Sensing, Prediction The agents make random choices on placing orders and whether buying or selling for each of these orders is based on two parameters, in other words, the probability of putting in a trade order and the probability of that trade order being a buy order. In the basic model, this random process repeats in each time step. There is no individual learning, sensing and prediction in this case. On the other hand, for evolutionary agents, the new generation of agents inherit and mutate two critical probabilities from parents with the high wealth given by fitness function. These evolving steps could be recognized as individuals adapting to the market and learning from others. Since all agents trade in this virtual market, space is not taken into consideration. No local information needed to be sensed and no prediction exists either.

Interaction There is no direct interaction between agents. However, since the market executes orders based on matching total demand and supply, agents can be seen as trading with each other.

Collectives We don't cluster the agents explicitly as Raberto et al. paired agents in a random graph. Similar aggregations can be found in the context that evolutionary agents tend to take a similar approach for better fitness.

Heterogeneity For both the basic model and the evolutionary model, agents exhibit heterogeneity due to random decisions on orders and their different transaction probabilities and buying probabilities.

Stochasticity The process of placing orders and generating bonded limit prices are randomly decided. And the evolutionary algorithm itself is a stochastic approach for each trader to search for an optimal strategy.

Observation We are mainly focusing on whether the volatility clustering and leptokurtic daily log return distributions occur in our ABM and if any emergent phenomena arise.

2.2.3 Details

Implementation Details Our ABM is implemented in the Python framework for agent-based modelling called Mesa. Traders correspond to the agents in the module. And all market behaviours are built in the class definition of the market which inherits from the base class model in Mesa. The scheduler module controls the activation of agents one after another. The "Datacollector" module enables us to collect the model-level and agent-level data during each step.

Initialisation All traders are initialized with a certain amount of cash and assets. The quotient of the initial cash and initial assets is the initial stock price. Each agent has 10 opportunities to place an order with 0.1 as the initial probability to submit an order. And the probability for buying each order is set to 0.5, thus selling and buying have the same chance. In terms of evolutionary agents, the mutation rate is an add-on parameter.

Input Data To compare with the real S&P 500 data and SDE models, the market price is initialized as 2012.66 which means each trader is endowed with 2012660 cash and 1000 share of assets.

3 Results

3.1 SDE Models: Black-Scholes and SVJ

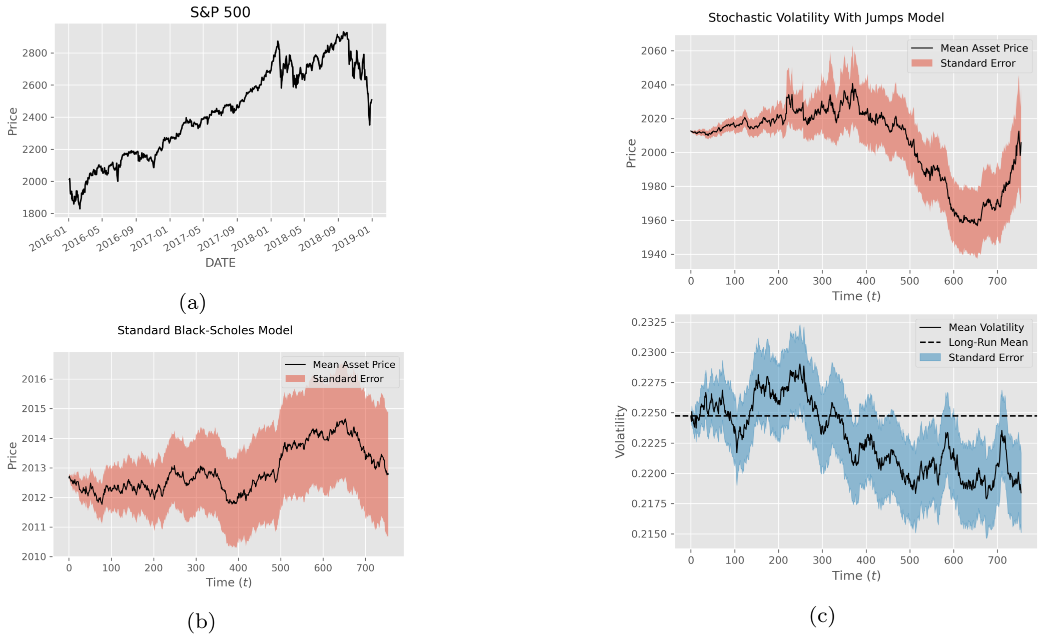
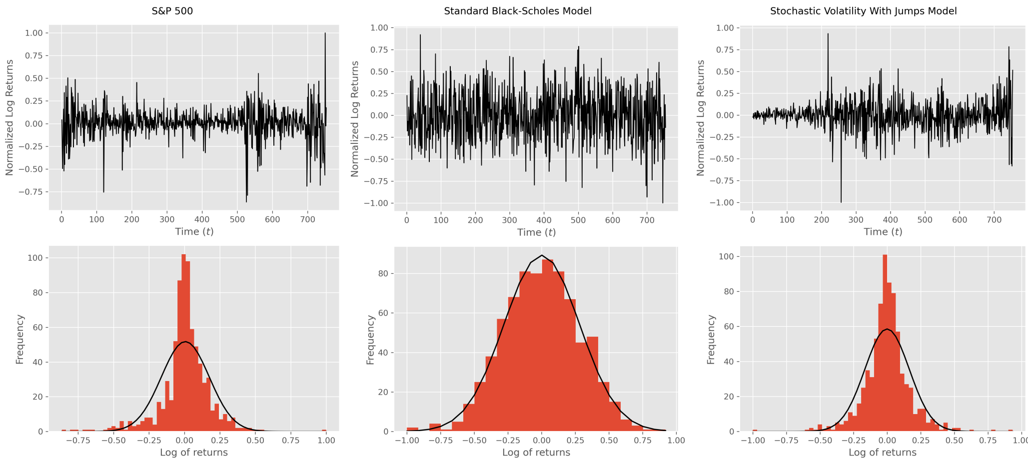


Figure 2: Evolution of asset prices over time for the S&P 500 (a), the BS model (b), and the SVJ model (c). Also depicted in (c) is the evolution of the stochastic volatility. The highlighted areas indicate the standard errors around the means.

Figure 2 shows the evolution over time of the asset prices of the S&P 500 for the period from January 1st 2016, to December 31st 2018. The figure also depicts the evolution of the asset paths in the BS and SVJ models for a corresponding number of time points. The BS model shows a fluctuating, stochastic mean realization of price paths that is superficially similar to the price path of the S&P 500. However, there is a notable lack of grouped spikes in the price value and

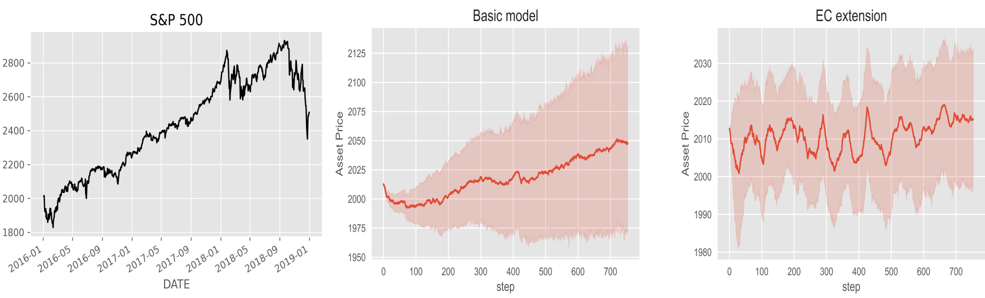
Figure 3 shows the normalized daily log returns over time and their corresponding histograms for the S&P 500 (for the period from January 1st 2016, to December 31st 2018) and the BS and SVJ models. The S&P 500 data clearly shows volatility clustering in the plot of the log-returns: there is a relatively low level of fluctuating returns with intermittent spikes of large returns. The histogram shows that the daily log returns exhibit a notable degree of leptokurtosis. As expected, the BS model log-returns resemble Gaussian noise and the histogram fits closely within the probability density function of a normal distribution. The SVJ model results are much more in line with the S&P 500 data: the log-returns show similar spikes against a lower level of fluctuating returns and the histogram has a narrower and higher peak with wider tails than the normal distribution.



(a) (b) (c)

Figure 3: Normalized daily log returns over time and histograms of the S&P 500 (a), the BS model (b), and the SVJ model (c). Overlaid on the histograms are the probability density functions of normal distributions with means and standard deviations corresponding to the daily log return data.

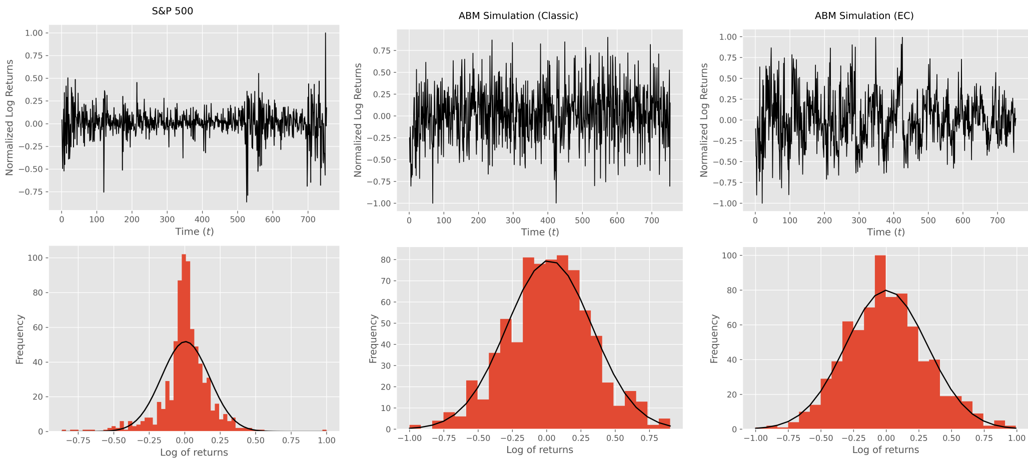
3.2 Agent-Based Model



(a) (b) (c)

Figure 4: Evolution of asset prices over time for the S&P 500 (a) our ABM model without EC (b) and with EC (c) over 100 simulations. The highlighted areas indicate the standard errors around the means.

Figure 4 shows that the price movement in both the EC extended Genoa model we implemented mimics price movement behaviour similar to the S&P 500 price movement. We see clear spikes and large volatility compared to for example the BS model in Figure 2, although these price movements appear to be highly cyclical. In our basic model, the price process dynamics are very similar to the BS model.



(a) (b) (c)

Figure 5: Normalized daily log returns over time and histograms of the S&P 500 (a) our ABM model without EC (b) and with EC (c). Overlaid on the histograms are the probability density functions of normal distributions with means and standard deviations corresponding to the daily log return data.

Looking at Figure 5 we see that a clear distinction can be made between the daily log return of both models. In (b) we see the log-returns of the normal Genoa model as we have implemented it. Here we observe no volatility clustering with the daily log returns plot and the histogram is very similar to those of the BS model in Figure 3. Moving on to (c) the EC extended Genoa Model, on the other hand, we do observe possible clustering albeit not as clear as in the S&P 500 in (a). The distribution of the daily log returns in both of our implementations mimics the normal distribution, which is not identical to the S&P 500.

3.2.1 Global Sensitivity Analysis

For the sensitivity analysis, we analyze the attributes and ranges shown in Table 2.

Attribute	Range
asset price	[1, 5000]
initial cash	[5000, 50000]
initial assets	[100, 10000]
std	[0, 0.1]
mutation	[0, 1]

Table 2: Attributes and range of values for OFAT and SOBOL

We observe the following patterns.

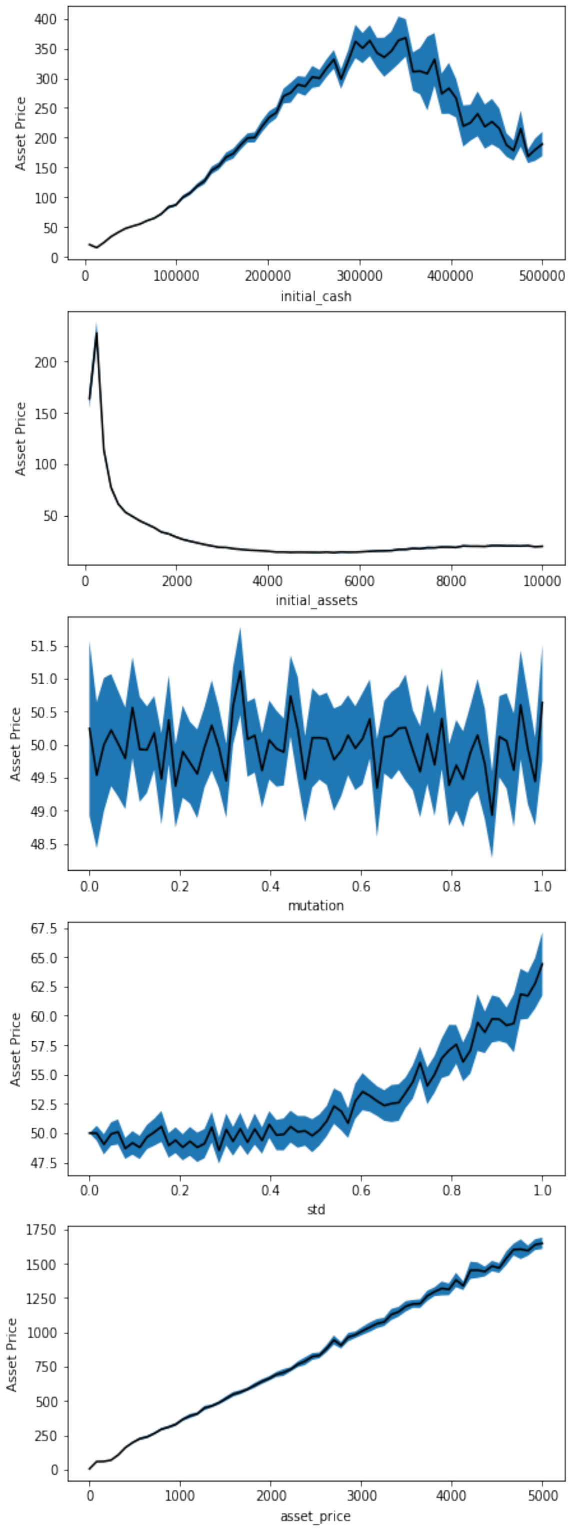


Figure 6: Sensitivity Analysis: OFAT

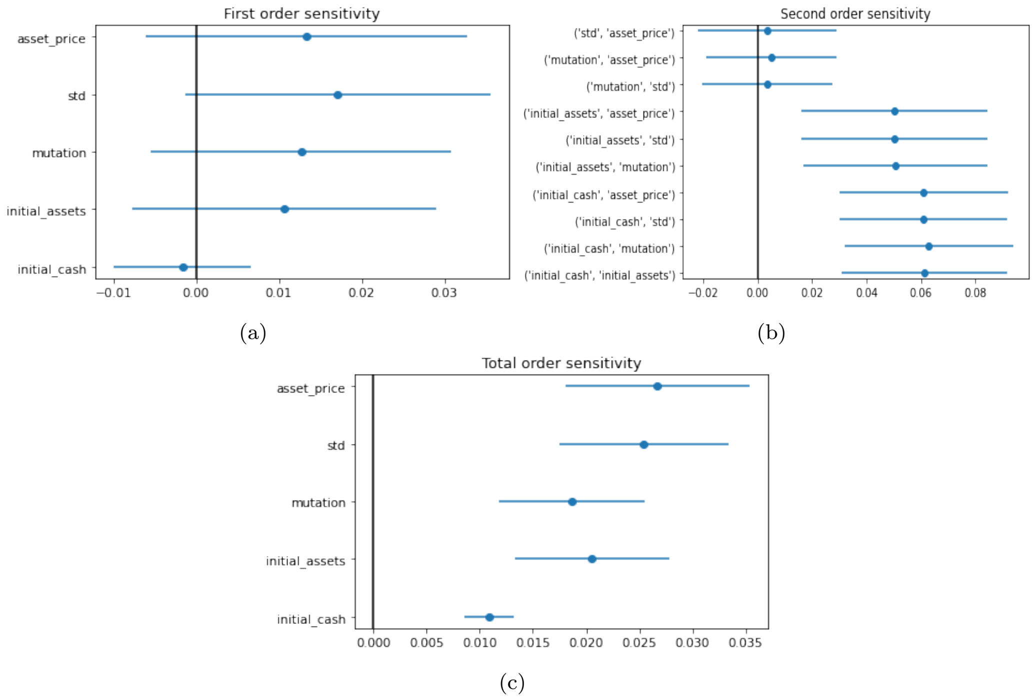


Figure 7: Sensitivity Analysis: SOBOL

The results of the sensitivity analysis gives us insight into individual contribution of every variable in our model.

The initial OFAT analysis gives us some preliminary results 6, while SOBOL, predictably, gives us a much better insight into the relationship between the variables.

In general, no variable seems to make the model truly unstable. “Initial asset price”, according to OFAT, has a linear relationship with final asset price, this is reflected in SOBOL, which consistently scores high deviation and high influence for first, second and total order sensitivity. “Mutation” and “STD” show similar behaviours on OFAT, high-variance of results and making a small difference in overall price long-term. SOBOL shows a different story, with STD and mutation having a big impact on total order sensitivity. “Initial cash” and “initial assets”, which in OFAT seem like important players, show little variance and impact on SOBOL, particularly “initial cash” in total order sensitivity, with a very unremarkable standard deviation.

4 Discussion & Conclusion

Of the SDE models, the Black-Scholes model did not yield results in line with realistic asset price processes. The SVJ model was shown to be effective in the modelling of price processes with volatility clustering and leptokurtic daily log return distributions. The Agent-Based Model was found to successfully reproduce the artificial trading market as the original Genoa did and demonstrate various emergent behaviours among agents. However, the volatility clustering was not as distinctive and the distributions of daily log return remained normal. This could be the result of the insignificant aggregation derived by our evolutionary strategies. As Raberto et al. managed to obtain a definite leptokurtic distribution by adding direct strong links among traders and thus constructing networks of similar trading strategies, further investigation measuring how our Evolutionary Algorithm infects the herd behaviour should be carried on.

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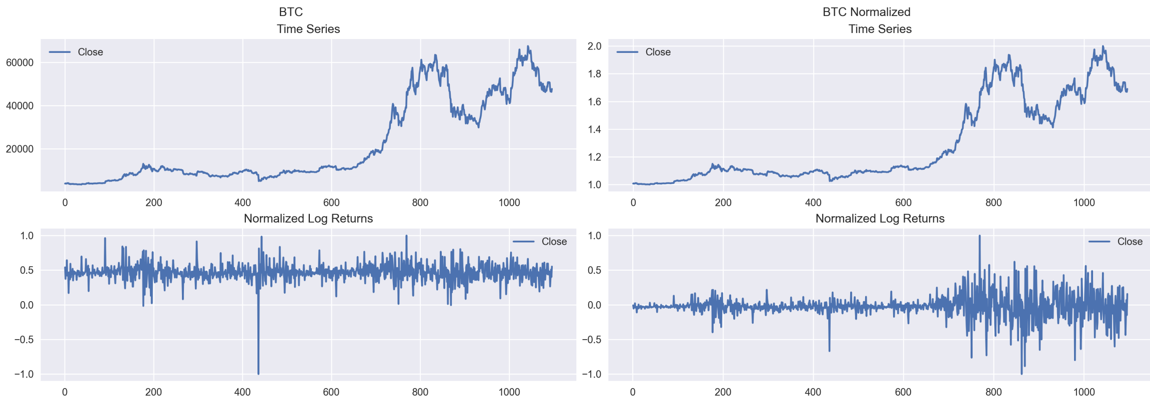
5 Appendix A: Normalization of Price Path

In early experimentation, an attempt to normalize the asset price in a range between 0 and 1 was made. This was done with the intent to ensure that the price between different models could be equally compared. As can be seen from the results in 8, this doesn't always produce

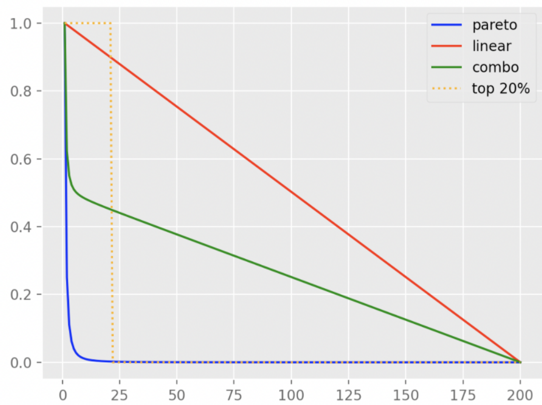


(a) (b)

Figure 8

6 Appendix B: Fitness Function

To select the top-performing individuals in the evolutionary model, a fitness function was used. We experimented with different types of fitness functions, describing the weights of selection of the top-performing individuals. We implemented the selections procedures seen in 9, the final model uses "Combo".



(a)

Figure 9