Hand-in Exercise: Super Mario Run

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1 Super Mario Run

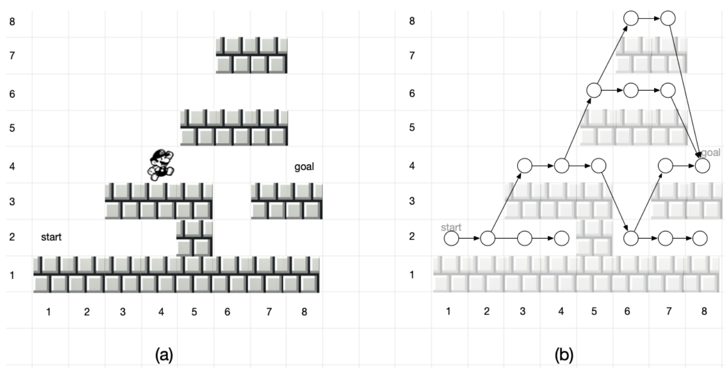

Figure 1: An 8×8 Mario world and corresponding Mario graph.

A *Mario world* M consists of a $k \times k$ grid. Each field in the grid is either *empty* or *brick*. Two empty fields are marked as *start* and *goal* (see Fig. 1(a)). The goal of the game is to move the player, called Mario, from the start field to the goal field. When Mario is in field (x, y) he has the following options:

Forward Mario moves to the field $(x+1, y)$. This move is possible if $(x+1, y)$ is empty and $(x+1, y-1)$ is brick.

Jump Mario moves to the field $(x+1, y+2)$. This move is possible if the fields $(x, y+2)$, $(x, y+1)$, and $(x+1, y+2)$ are empty, and $(x+1, y+1)$ is brick.

Fall Mario moves to the field $(x+1, y')$ where $y' < y$ is the maximal value y' such that $(x+1, y'-1)$ is brick and $(x+1, y), (x+1, y-1), \ldots, (x+1, y')$ are empty.

For instance, Mario in field $(4, 4)$ can move forward to $(5, 4)$ or jump to field $(5, 6)$. A move is *valid* if the move can be done according to the above rules. All fields that can be reached via valid moves from the start field are the *valid fields*. For example, $(4, 4)$ is a valid field, since it can be reached from the start field in $(1, 2)$ doing the sequence of valid moves: forward, forward, jump.

A Mario world M defines a directed graph G with n vertices and m edges (see Fig. 1(b)). All valid fields correspond to a vertex and the valid moves define the edges (there is an edge from field (x, y) to field (x', y') if Mario can move from (x, y) to (x', y') with a valid move). The edges corresponding to forward, jump and fall are denoted by *forward edges*, *jump edges* and *fall edges*.

1.1 Let M be a Mario world defined on a $k \times k$ grid and let G be the corresponding Mario graph. State the maximum number of nodes n and edges m, respectively, that can appear in G as a function of k using asymptotic notation.

1.2 We are now interested in checking if it is possible to go from the start field to the goal field. A Mario graph is *valid* if there exists a path from start to goal. Give an algorithm that given a Mario graph G, decides whether or not there is a path from the start field to the goal field. Analyze the running time of your algorithm in terms of parameters n and m.

1.3 We are now interested in collecting a *mushroom* during the game. A *mushroom graph* is a valid Mario graph where a single node c is marked as a *mushroom node* and there is a path from start to goal that goes through c. Give an algorithm which, given a mushroom graph, finds a shortest path from the start to the goal that goes through c. Analyze the running time of your algorithm in terms of parameters n and m.