Technical University of Denmark

Written examination - Test Exam, May, 2023.

Course name: Algorithms and Data Structures 1

Course number: 02105

Aids: all aids, no internet access.

Duration: 4 hours.

Weight: written part (exercise 1) - 50%, multiple choice part 40%, mandatory exercise - 10%. The weight is approximate. The grade is based on an overall assessment of the multiple-choice part, the written part of the exam, and the mandatory exercise.

Instructions for the exercise

Write your answers to the exercises and submit them using the digital exam system. Your answer should be in the form of a `pdf`-file containing exactly 1 page in the a4-format. The top of the page should clearly list your your study id. The rest of page should contain your answers to the each of the exercises. Number each of your answers with the number of the corresponding exercise. Use a clearly readable font of size 10pt or more and margins of 2cm or more.

Asymptotic bounds should be as tight as possible. Unless otherwise specified, the base of all logarithms is 2 and $\log^k n$ means $(\log n)^k$.

"Give an algorithm" means that you should describe your solution in a short, precise, and unambiguous manner and analyze the complexity of your solution. Unless specified otherwise, the description should be in natural language and not pseudocode. The analysis should explain how you derived the complexity bound.

"Argue correctness of your algorithm" means that you should provide a short argument for correctness of your algorithm.

"Analyze the running time of your algorithm in the relevant parameters (parameters x, y, ..) of the problem" means that you should analyze the running time using the explicitly stated parameters of the problem (parameters x, y, ..).

1 Zombies

A *zombie outbreak* consists of a set of persons and a set of infections. Each infection is a pair of persons (p_1, p_2) and we say that p_1 has *infected* p_2. Note that multiple persons may infect the same person. For instance, $\{A, B, C, D, E\}$ and $\{(A, B), (B, C), (C, D), (A, E), (E, C)\}$ is a zombie outbreak with 5 persons and 5 infections. Throughout the exercise, let P denote the number of persons and I the number of infections.

1.1 Briefly describe how to model a zombie outbreak as a graph.

1.2 We are interested in investigating the origin of the outbreak. A person p is a *patient-zero* if p did not get infected by any other person. Give an algorithm, that given a zombie outbreak, prints out all patient-zero persons. Argue correctness of your algorithm. Analyze the running time of your algorithm in terms of parameters P and/or I.

1.3 We are now interested in reconstructing the details of the zombie outbreak and want to ensure that the data is correct. An *infection chain* is a sequence of persons $p_0, \ldots, p_{k-1}$ such that p_i has infected p_{i+1} for $i = 0, \ldots, k-2$. A zombie outbreak is *inconsistent* if there exists an infection chain that starts and ends with the same person. If a zombie outbreak is not inconsistent it is *consistent*. Give an algorithm, that given a zombie outbreak, determines if the zombie outbreak is inconsistent or consistent. Argue correctness of your algorithm. Analyze the running time of your algorithm in terms of parameters P and/or I.

1.4 We now associate to every infection i an *infection speed*, *speed*(i). We define the infection speed of an infection chain to be the sum of the infection speeds of each infection in the chain. We are interested in computing the fastest infection chain between two persons in a consistent zombie outbreak. Given an algorithm, that given a consistent zombie outbreak and two persons p_1 and p_2 computes a fastest infection chain from p_1 to p_2. Argue correctness of your algorithm. Analyze the running time of your algorithm in terms of parameters P and/or I.