02105 Algorithms and Data Structures 1 - MC Part Test Exam 2023

Der anvendes en scoringsalgoritme, som er baseret på "One best answer"

Dette betyder følgende:
Der er altid netop ét svar som er mere rigtigt end de andre
Studerende kan kun vælge ét svar per spørgsmål

The following approach to scoring responses is implemented and is based on "One best answer"
There is always only one correct answer – a response that is more correct than the rest
Students are only able to select one answer per question

For each question below, mark whether or not the statement is correct.

Select the correct answers

	Yes	No
$3n^4 + 2n^3 = O(n^3)$	○	⊗
$4n + 45 \log n = \Theta(n)$	⊗	○
$\frac{1}{3}n^5 = \Omega(n^4)$	⊗	○
$2^{n+2} = \Theta(2^n)$	⊗	○
$(\frac{1}{3}n^5 + n^2 + n)n = O(n^6)$	⊗	○

Mark the running time in O-notation of each of the following algorithms as a function n. Mark the best bound.

ALG1(n)
$c = 0$
$j = n - 4$
while $j \leq n$ **do**
  $j = j + 1$
  for $i = 1$ **to** n **do**
    $c = j + i$
  end for
end while

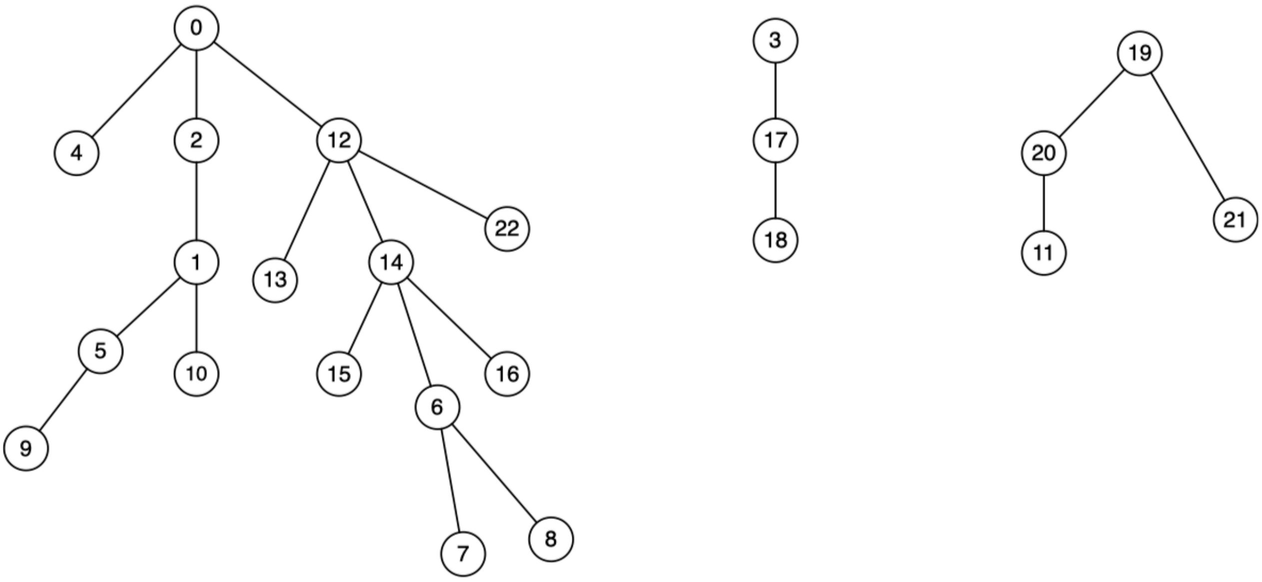
ALG2(n)
if $n \leq 1$ **then**
  return 1
else
  return $1 +$ ALG2(1) $+$ ALG2(1)
end if

ALG3(n)
$c = 0$
for $i = 1$ **to** $\lceil \log n \rceil$ **do**
  for $j = i$ **to** n **do**
    $c = i + j$
  end for
end for

Select the correct answers	$O(1)$	$O(\log n)$	$O(\sqrt{n})$	$O(n)$	$O(n \log n)$	$O(n^2)$	$O(n^2 \log n)$	$O(n^3)$	$O(2^n)$
Alg1	○	○	○	○	○	☒	○	○	○
Alg2	☒	○	○	○	○	○	○	○	○
Alg3	○	○	○	○	☒	○	○	○	○

Union Find

Consider the following forest of trees representing a family of sets in a union find data structure constructed using the quick union algorithm.



Mark the result of the following Find(.) operations.

Select the correct answers	0	2	3	8	11	14	18	19	20	21
Find(2)	X									
Find(18)			X							
Find(0)	X									
Find(8)	X									
Find(11)								X		
Find(14)	X									

Suppose we now use path compression on the forest above. Construct the forest of trees F after a Find(6) operation and answer the following questions.

Select the correct answers	1	2	3	4	5	8	10	12	14	16
The total number of leaves in all trees in F is										
The maximum depth of a tree in F is										
The minimum depth of a tree in F is										
The maximum number of children of a root of a tree in F is										
The minimum number of children of a root of a tree in F is										

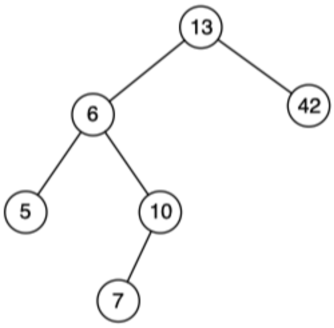
Consider the following data structures for representing a set of integers. We are interested in supporting the operation Member(x), that given an integer x should return yes if x is in the set and no otherwise. For instance, if a data structure stores the set {32, 6, 18, 7, 2} then Member(6) should return yes while Member(4) should return no. For each data structure state the runtime of Member(x) operation in O-notation as a function of n, where n is the number of elements in the data structure.

Select the correct answers

	$O(1)$	$O(\log n)$	$O(\sqrt{n})$	$O(n)$	$O(n \log n)$	$O(n^2)$	$O(2^n)$
Sorted array	○	X	○	○	○	○	○
Max-heap	○	○	○	X	○	○	○
Min-heap	○	○	○	X	○	○	○
Array (not sorted)	○	○	○	X	○	○	○
Binary search tree	○	○	○	X	○	○	○

Binary Search Trees

Consider the following binary search tree.



Construct the preorder, inorder, and postorder traversal of the tree and answer the following questions.

Select the correct answers

	5	6	7	10	13	42
The key of the first node is the preorder traversal is					X	
The key of the first node in the inorder traversal is	X					
The key of the first node in the postorder traversal is	X					
The key of the last node in the preorder traversal is						X
The key of the last node in the inorder traversal is						X
The key of the last node in the postorder traversal is					X	
The key of the 4th node in the preorder traversal is				X		
The key of the 4th node in the inorder traversal is				X		
The key of the 4th node in the ~~inorder~~ postorder traversal is		X				

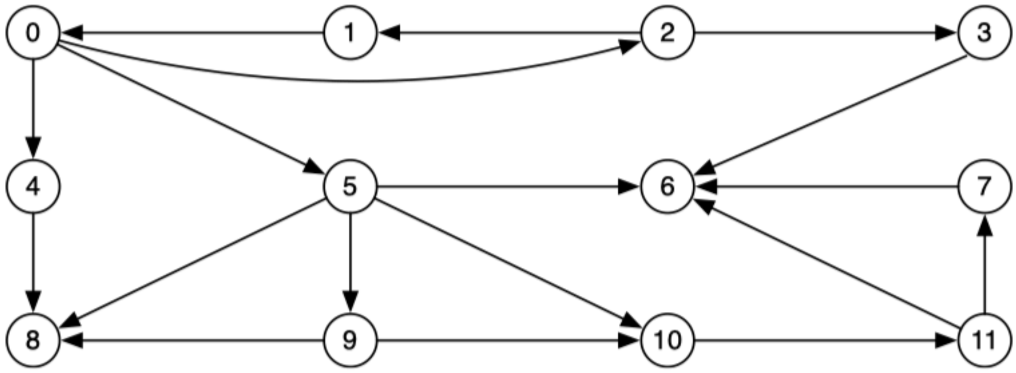
Insert a new node with key 9 into the above tree and let T be the resulting binary search tree. Answer the following questions.

Select the correct answers

	0	1	2	3	4	5
The height of T is	○	○	○	○	⊗	○
The number of nodes with 2 children in T is	○	○	⊗	○	○	○
The number of nodes with 1 child in T is	○	○	⊗	○	○	○

Graph Search

Consider the following graph G.



Construct the DFS tree T_1 for G when starting in vertex 0. Assume that the adjacency lists are sorted in increasing order and T_1 is rooted in vertex 0. Answer the following questions.

Select the correct answers

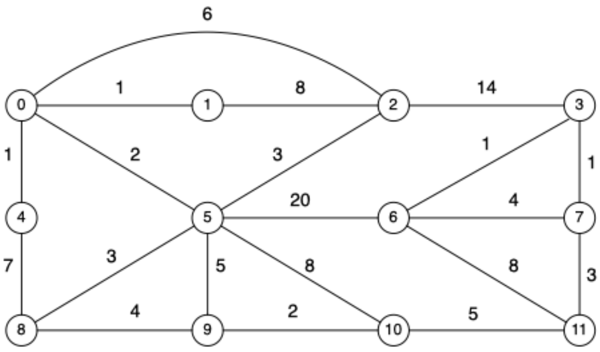
	1	2	3	4	5	6	7
The depth of the tree is	○	○	○	○	☒	○	○
The number of leaves of the tree is	○	○	○	☒	○	○	○
The maximum number of children of a node in the tree is	○	○	☒	○	○	○	○

Construct the BFS tree T_2 for G when starting in node 0. Assume that the adjacency lists are sorted in increasing order and T_2 is rooted in vertex 0. Answer the following questions.

Select the correct answers

	1	2	3	4	5	6	7
The depth of the tree is	○	○	○	☒	○	○	○
The number of leaves of the tree is	○	○	○	○	○	☒	○
The maximum number of children of a node in the tree is	○	○	☒	○	○	○	○

Minimum Spanning Trees
Consider the following graph.



Construct a minimum spanning tree T for the graph. Assume that T is rooted in vertex 0 and answer the following questions.

Answer the following questions.

Select the correct answers

	2	3	4	5	6	7	8	9
The depth of T is	○	○	○	○	○	○	✗	○
The number of leaves of T is	○	○	✗	○	○	○	○	○
The maximum of children of a node in T is	○	✗	○	○	○	○	○	○

State the total weight of T. 26

Signe Tudent s101010

Exercise 1.1

We construct a directed graph where the vertices are the zombies and the edges are the infections. An infection (p_1, p_2) is directed from p_1 to p_2.

Exercise 1.2

Algorithm We do the following steps.

1. Construct the graph G of the zombie outbreak.
2. Compute the in-degree of each vertex in G. To do so initialize an array A of length P with 0 in all entries. Then, process all edges. For each edge (p_1, p_2) we add 1 to $A[p_2]$.
3. Finally, report all vertices with in-degree 0 in a single traversal of A.

Correctness A patient 0 is by definition a vertex with in-degree 0. Hence, the algorithm is correct.

Analysis Step 1 uses $O(P+I)$ time, step 2 uses $O(I)$ time, and step 3 uses $O(P)$ time. Hence, in total the algorithm uses $O(P+I)$ time.

Exercise 1.3

Algorithm Construct the graph G and run a topological sort on G. Output "consistent" if and only if G is a DAG.

Correctness By definition, an infection chain is a path in G. Thus, the zombie outbreak is inconsistent if and only it contains a cycle. Hence, the algorithm returns consistent if and only if G does not contain a cycle and is thus correct.

Analysis Constructing the graph and running topological sort uses $O(P+I)$ time. Hence, in total the algorithm uses $O(P+I)$ time.

Exercise 1.4

Algorithm Construct the graph G with edge weights, where an edge is weighted by the speed of that edge. Then, compute the shortest path from p_1 to p_2 using the shortest path algorithm for DAGs.

Correctness By definition the graph G is a weighted DAG and the fastest infection chain between two persons is the shortest path between them. Hence, the algorithm is correct.

Analysis Both steps uses $O(P+I)$ time. Hence, in total the algorithm uses $O(P+I)$ time.