# Homework 1

1.1)

$$P(z) = \left(z^2 - 2z + \frac{5}{4}\right)\left(4z^4 - 12z^3 + 5z^2 + 11z - 10\right) \qquad z \in \mathbb{C}$$

- 1) The degree is 6
- 2)

$$z^2 - 2z + \frac{5}{4} = 0$$

$$z = \frac{1 \pm \sqrt{1 - \frac{5}{4}}}{1} \longrightarrow z_0 = 1 \pm \sqrt{-\frac{1}{4}} \quad \begin{cases} z_{01} = 1 + \frac{1}{2}i \\ z_{02} = 1 - \frac{1}{2}i \end{cases}$$

$$P(z) = \left(z - 1 - \frac{1}{2}i\right)\left(z - 1 + \frac{1}{2}i\right)\left(4z^4 - 12z^3 + 5z^2 + 11z - 10\right)$$

$$W(z) = \left(4z^4 - 12z^3 + 5z^2 + 11z - 10\right)$$

$$W(-1) = 4 + 12 + 5 - 11 - 10 \qquad x = -1 \text{ ROOT}$$

$$W(z) = (x+1)\,Q(z)$$

$$b_3 = \boxed{\begin{matrix} a_4 = 4 \\ a_3 = -12 \\ a_2 = 5 \\ a_1 = 11 \\ a_0 = -10 \end{matrix}} \qquad \begin{matrix} b_2 = a_3 - 1b_3 = -16 \\ b_1 = a_2 - 1b_2 = 21 \\ b_0 = a_1 - 1b_1 = -10 \end{matrix}$$

$$Q(z) = 4z^3 - 16z^2 + 21z - 10$$

$$P(z) = \left(z - 1 - \frac{1}{2}i\right)\left(z - 1 + \frac{1}{2}i\right)(z+1)\left(4z^3 - 16z^2 + 21z - 10\right)$$

$$W(z) = \left(4z^3 - 16z^2 + 21z - 10\right)$$

$$W(2) = 32 - 64 + 42 - 10 \qquad x = 2 \text{ ROOT}$$

$$W(z) = (z-2)\,Q(z)$$

$$Q(z) = 4z^2 - 8z + 5$$

$$b_2 = \boxed{\begin{matrix} a_3 = 4 \\ a_2 = -16 \\ a_1 = 21 \\ a_0 = -10 \end{matrix}} \qquad \begin{matrix} b_1 = a_2 + 2b_2 = -8 \\ b_0 = a_1 + 2b_1 = 5 \end{matrix}$$

$$P(z) = \left(z - 1 - \frac{1}{2}i\right)\left(z - 1 + \frac{1}{2}i\right)(z+1)(z-2)\left(4z^2 - 8z + 5\right)$$

$$4z^2 - 8z + 5 = 0$$

$$z = \frac{4 \pm \sqrt{16 - 20}}{4} = \frac{4 \pm 2i}{4} \quad \begin{cases} z_{01} = 1 + \frac{1}{2}i \\ z_{02} = 1 - \frac{1}{2}i \end{cases}$$

$$P(z) = \left(z - 1 - \frac{1}{2}i\right)\left(z - 1 + \frac{1}{2}i\right)(z+1)(z-2)\left(z - 1 - \frac{1}{2}i\right)\left(z - 1 + \frac{1}{2}i\right)$$ FACTORIZED FORM

Roots:

| | | |
|---|---|---|
| $1 + \frac{1}{2}i$ | $2$ | $1 + \frac{1}{2}i$ |
| $1 - \frac{1}{2}i$ | $-1$ | $1 - \frac{1}{2}i$ |

Multiplicities:

All the roots have mulplicity = 1 except $1 + \frac{1}{2}i$ & $1 - \frac{1}{2}i$ with mulp. = 2

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## 1.2)

$z_1 = \left(1, -\frac{7}{4}\pi\right) \quad z_2 = (8, 7\pi) \quad z_3 = \left(2, \frac{7}{6}\pi\right)$

• 1) $z_1$ $\qquad$ $z_2$ $\qquad$ $z_3$

$Arg(z_1) = \frac{\pi}{4}$ $\qquad$ $Arg(z_2) = \pi$ $\qquad$ $Arg(z_3) = -\frac{5}{6}\pi$

Re $r\cos\alpha = \frac{\sqrt{2}}{2}$ $\qquad$ Re $r\cos\alpha = -8$ $\qquad$ Re $r\cos\alpha = -\sqrt{3}$

Im $r\sin\alpha = \frac{\sqrt{2}}{2}$ $\qquad$ Im $r\sin\alpha = 0$ $\qquad$ Im $r\sin\alpha = -1$

$z_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$ $\qquad$ $z_2 = -8$ $\qquad$ $z_3 = -\sqrt{3} - i$

$z_1 = e^{\frac{\pi}{4} i}$ $\qquad$ $z_2 = 8e^{i\pi}$ $\qquad$ $z_3 = 2e^{-\frac{5}{6}\pi i}$

• 2) $A = 2 - 2\sqrt{3}\, i$ $\qquad$ $B = \sqrt{6} + \sqrt{2}\, i$ $\qquad$ $C = \frac{A^6}{B^8}$

$A = \sqrt{4+12}\; e^{i\left(-\frac{\pi}{3}\right)}$ $\qquad$ $B = \sqrt{6+2}\; e^{i\frac{\pi}{6}}$ $\qquad$ $C = \frac{\left(4e^{i\left(-\frac{\pi}{3}\right)}\right)^6}{\left(2\sqrt{2}\, e^{i\frac{\pi}{6}}\right)^8}$

$A = 4e^{i\left(-\frac{\pi}{3}\right)}$ $\qquad$ $B = 2\sqrt{2}\, e^{i\frac{\pi}{6}}$

$\cos\alpha = \frac{1}{2}$ $\qquad$ $\cos\alpha = \frac{\sqrt{3}}{2}$ $\qquad$ $C = \left(\frac{4e^{i\left(-\frac{\pi}{3}\right)}}{2\sqrt{2}\, e^{i\frac{\pi}{6}}}\right)^6 \cdot \frac{1}{\left(2\sqrt{2}\, e^{i\frac{\pi}{6}}\right)^2}$

$\sin\alpha = -\frac{\sqrt{3}}{2}$ $\qquad$ $\sin\alpha = \frac{1}{2}$

$C = \left(\sqrt{2}\right)^6 \left(e^{i\left(-\frac{\pi}{3}+\frac{\pi}{6}\right)}\right)^6 \cdot \frac{1}{8e^{i\pi}}$

$C = \frac{8e^{i(-\pi)}}{8e^{i\pi}} = 1$

## 1.3)

• 1) $z^6 = -729 \qquad z \in \mathbb{C}$

$z = \sqrt[6]{729}\; e^{i\left(\frac{\pi}{6} + p\frac{\pi}{3}\right)}$ $\qquad$ $p = 0, 1, 2 \ldots n-1$ $\qquad$ $a = |z|\cos\alpha \quad b = |z|\sin\alpha$

• $z = 3e^{i\frac{\pi}{6}} = \frac{3\sqrt{3}}{2} + \frac{3}{2} i$

• $z = 3e^{i\frac{\pi}{2}} = 3i$

• $z = 3e^{i\frac{5}{6}\pi} = -\frac{3\sqrt{3}}{2} + \frac{3}{2} i$

• $z = 3e^{i\frac{7}{6}\pi} = -\frac{3\sqrt{3}}{2} - \frac{3}{2} i$

• $z = 3e^{i\frac{3}{2}\pi} = -3i$

• $z = 3e^{i\frac{11}{6}\pi} = \frac{3\sqrt{3}}{2} - \frac{3}{2} i$

• 2) $e^z - ie^{\pi} = 0 \qquad z \in \mathbb{C} \qquad |\text{solution}| < 2\pi$

$e^{a+bi} = ie^{\pi}$

$e^{a+bi} = e^{\pi}\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

$e^a = e^{\pi} \longrightarrow a = \pi$

$\mathrm{Arg}(e^z) = \mathrm{Arg}(ie^{\pi}) \longrightarrow b = \frac{\pi}{2} + 2\pi p$

$z = a + bi \longrightarrow z = \pi + \left(\frac{\pi}{2} + 2\pi p\right)i$

$z = \pi + \frac{\pi}{2}i$

1.4)

$\cosh(v) = \dfrac{e^v + e^{-v}}{2}$ and $\sinh(v) = \dfrac{e^v - e^{-v}}{2} \quad v \in \mathbb{R}$

• 1) $\begin{cases} f'(v) = g(v) \\ g'(v) = f(v) \end{cases} \qquad f(0) = 1$ and $g(0) = 0$

$\begin{cases} f'(v) = g(v) \\ g'(v) = f(v) \end{cases} \quad \begin{cases} f''(v) = g'(v) \\ g'(v) = f \end{cases} \longrightarrow f''(v) = f(v)$

$f''(v) - f(v) = 0 \xrightarrow{\text{CORRESPONDING POLY.}} \lambda^2 - 1 = 0 \longrightarrow \lambda = \pm 1$

$\begin{cases} x = 1 & P_1 \\ x = -1 & P_2 \end{cases}$

$f(x) = c_1 e^{P_1 x} + c_2 e^{P_2 x} \longrightarrow f(x) = c_1 e^x + c_2 e^{-x}$

$f'(x) = c_1 e^x - c_2 e^x$

$\begin{cases} f'(0) = g(0) = 0 \\ f(0) = 1 \end{cases} \quad \begin{cases} c_1 - c_2 = 0 \\ c_1 + c_2 = 1 \end{cases} \qquad c_1 = c_2 = \frac{1}{2}$

$$f(x) = \frac{e^x + e^{-x}}{2} \longrightarrow g(x) = \frac{e^x - e^{-x}}{2}$$

• 2) $x^2 - y^2 = 1$

$-\cosh(v) + i\sinh(v)$ left and $\cosh(v) + i\sinh(v)$ right $v \in \mathbb{R}$

$V = -2$

$-\cosh(-2) + i\sinh(-2) \longrightarrow -\frac{1+e^4}{2e^2} + i\left(\frac{1-e^4}{2e^2}\right)$

cosh is always $> 0$ $\forall \in R$, so with - sing in front is always $< 0$. This means that it's in the II and III quadrant.

$$\left(-\frac{1+e^4}{2e^2}\right)^2 - \left(\frac{1-e^4}{2e^2}\right) = 1$$

$$\frac{1+2e^4+e^8}{4e^4} - \frac{1-2e^4+e^8}{4e^4} = 1$$

$\frac{4e^4}{4e^4} = 1$ ✓ It's and the left branch of $x^2-y^2=1$

$V = -2$

$\cosh(-2) + i\sinh(-2) \longrightarrow \frac{1+e^4}{2e^2} + i\left(\frac{1-e^4}{2e^2}\right)$

cosh is always positive $\forall \in R$. In this case it's always in the I and IV quadrat

$$\left(\frac{1+e^4}{2e^2}\right)^2 + \left(\frac{1-e^4}{2e^2}\right)^2 = 1$$

$\frac{4e^4}{4e^4} = 1$ ✓ It's in the right branch of $x^2-y^2=1$

3) $f(x) = \frac{1}{10}\sinh(x^2-x)$ $x \in R$ $x_0 = 0$

| $n$ | $f^{(n)}(x)$ | $f^{(n)}(0)$ |
|---|---|---|
| 0 | $\frac{1}{10}\sinh(x^2-x)$ | 0 |
| 1 | $\frac{2\cosh(x^2-x)x - \cosh(x^2-x)}{10}$ | $-\frac{1}{10}$ |
| 2 | $2\sinh(x^2-x)\ldots$ | $\frac{1}{5}$ |
| 3 | $8\cosh\ldots$ | $-\frac{1}{10}$ |

$$P_3(x) = 0 + \frac{-\frac{1}{10}}{1!}x + \frac{\frac{1}{5}}{2!}x^2 + \frac{-\frac{1}{10}}{3!}x^3$$

$$P_3(x) = -\frac{1}{10}x + \frac{1}{10}x^2 - \frac{1}{60}x^3$$

$$P_2(x) = -\frac{1}{10}x + \frac{1}{10}x^2$$