Essay Homework 5

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A function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ is given by:

$$f(x, y) = \frac{5}{4}x^2 y - \frac{1}{4}x^4 - y^2$$

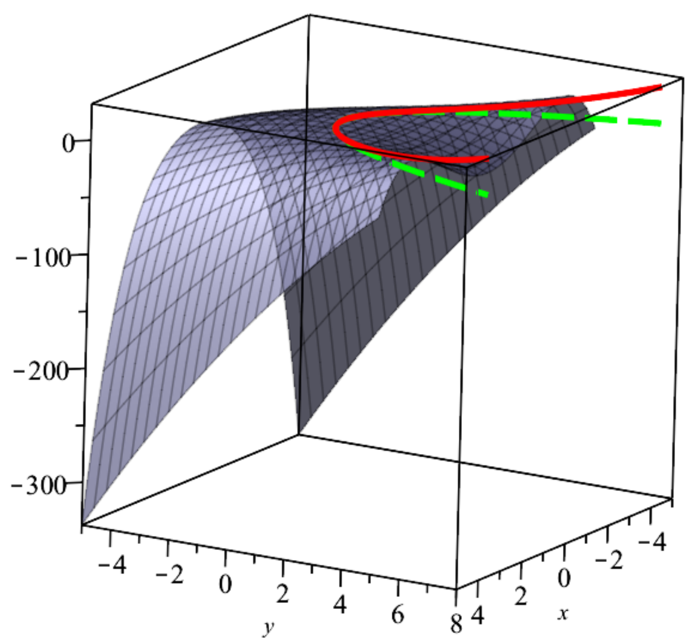
We are also given a curve K in the parametric representation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{r}(u) = \left(u, \frac{1}{2}u^2\right), \quad u \in R$$

A)

$h(u) = f(\mathbf{r}(u)), u \in \mathbb{R}$

The plot of the function $h(u)$ (red) over the surface of the curve $f(x, y)$ and the function K (green dashed) is :



To find the values of u for which $h(u) = 1$ and $h'(u) = 0$ I used the command "solve" and "diff" to find the first derivative respect to u.

```
solve(h(u)=1, u)
```

$$2^{3/4}, I\,2^{3/4}, -2^{3/4}, -I\,2^{3/4}$$

```
hu:=unapply( diff(h(u),u), u):
'hu(u)'=hu(u)
```

$$hu(u) = \frac{u^3}{2}$$

```
solve(hu(u)=0, u)
```

$$0, 0, 0$$

Form here we can get that $u = 2^{\frac{3}{4}}$ and $u = -2^{\frac{3}{4}}$ are the real values for which $h(u) = 1$ and $u = 0$ $(triple)$ are the vaules for which $h'(u) = 0$.

$$h'(u) = \frac{u^3}{2}$$

The function has the following intervals:
Positive:]-∞;0[
Negative:]0;+∞[
And is equal to 0 when $u = 0$

B)
Now we consider the points: $A = (0, -1)$ and $B = (0, 0)$.

To find the Hessian matrix for f in A we first need to find the $f'(x, y)$ respect to x and y and after that find all the $f''(x, y)$ respect to: x, y and the mixed one (that is the same according to theorem 19.34 "If all 4 double derivatives of a given function $f(x, y)$ are continuous on an open set then, on the whole they are the same").

```
fx:=diff(f(x,y),x):
fy:=diff(f(x,y),y):
fxx:=diff(fx,x);
fyy:=diff(fy,y);
fxy:=diff(fx,y);
fyx:=diff(fy,x)
```

$$fxx := \frac{5y}{2} - 3x^2$$
$$fyy := -2$$
$$fxy := \frac{5x}{2}$$
$$fyx := \frac{5x}{2}$$

```
H:=unapply( <fxx,fxy|fyx,fyy>, x,y):
H(x,y)
```

$$\begin{bmatrix} \frac{5y}{2} - 3x^2 & \frac{5x}{2} \\ \frac{5x}{2} & -2 \end{bmatrix}$$

Now plug in the values of the point A to get:

```
H(A)
```

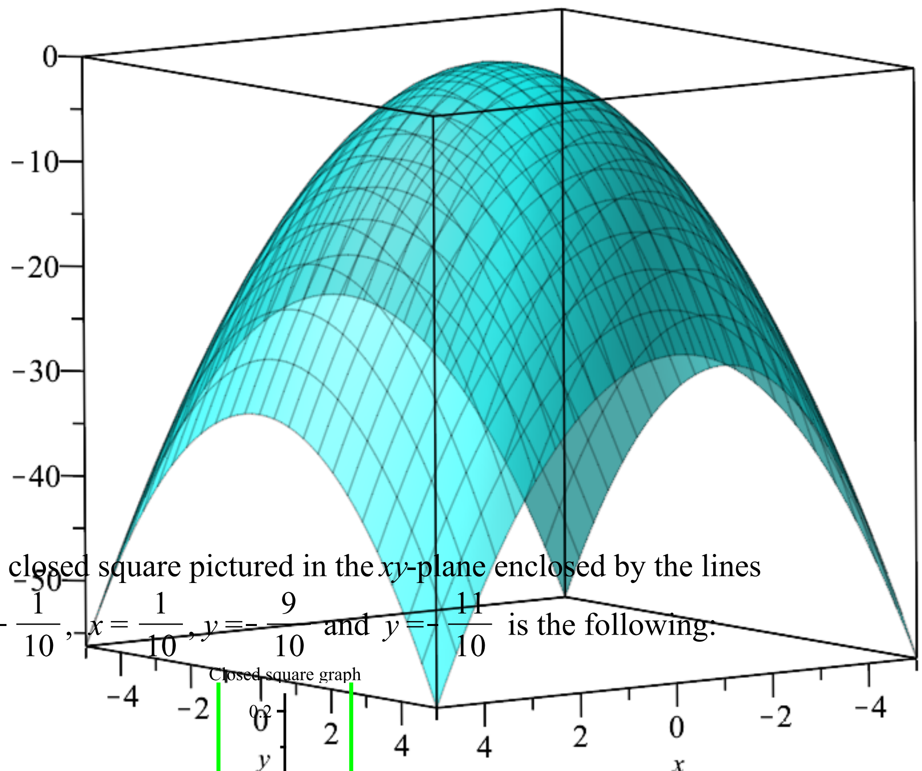
$$\begin{bmatrix} -\frac{5}{2} & 0 \\ 0 & -2 \end{bmatrix}$$

The second degree polynomial P_2 of f, with point of expansion A is found by using the Taylor approximation formula for functions in two variables and it can be easly done using the command:

```
P2:=unapply( mtaylor(f(x,y),[x=0,y=-1],3),x,y)
```

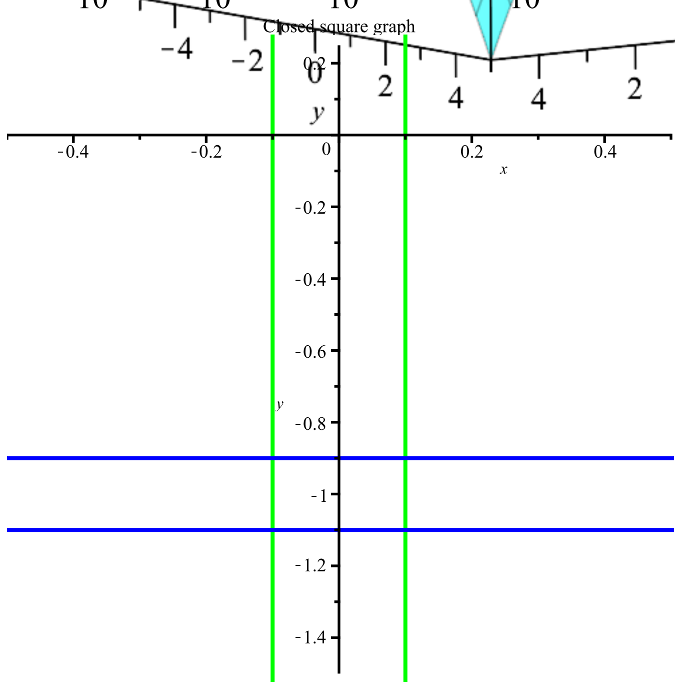
$$P2 := (x, y) \mapsto 1 + 2 \cdot y - \frac{5 \cdot x^2}{4} - (y + 1)^2$$

Its geometric representation is an elliptic paraboloid that is shown below:



The closed square pictured in the xy-plane enclosed by the lines

$x=-\frac{1}{10}$, $x=\frac{1}{10}$, $y=-\frac{9}{10}$ and $y=-\frac{11}{10}$ is the following:



To determine the error that occurs using P_2 instead of f we first generate the distance function

$d(x, y) = (f(x, y) - P2)^2$

and we elevate it to the power of two (this provides a positive value at the end). NB: could have also done this by taking the absolute value of the function $d(x, y)$.

```
f(x,y)-P2(x,y)
```

$$\frac{5x^2 y}{4} - \frac{x^4}{4} - y^2 - 1 - 2y + \frac{5x^2}{4} + (y+1)^2$$

```
simplify(%^2)
```

$$\frac{x^4 \left(x^2 - 5y - 5\right)^2}{16}$$

```
d:=unapply(%,x,y):
'd(x,y)'=d(x,y)
```

$$d(x, y) = \frac{x^4 \left(x^2 - 5y - 5\right)^2}{16}$$

Now we need to procede in the global extrema investigation for this function; the first thing is finding all the stationary points setting the gradiant for $d(x, y)$ equal to zero.

```
dx:=diff(d(x,y),x);
dy:=diff(d(x,y),y)
```

$$dx := \frac{x^3 \left(x^2 - 5y - 5\right)^2}{4} + \frac{x^5 \left(x^2 - 5y - 5\right)}{4}$$

$$dy := -\frac{5x^4 \left(x^2 - 5y - 5\right)}{8}$$

```
solve({dx=0,dy=0})
```

$$\{x = 0, y = y\}, \left\{x = x, y = \frac{x^2}{5} - 1\right\}$$

```
C1:=(0,u):
C2:=(u,u^2/5-1):
```

Notice that the possible candidates are represented by the parametrized line $(0, u)$ $u \in \left[-\frac{9}{10}, -\frac{11}{10}\right]$ and the segment of the parabola parametrized as $\left(u, \frac{u^2}{5} - 1\right)$ $u \in \left[-\frac{1}{10}, \frac{1}{10}\right]$

For the investigation in the boundary points I proceeded parametrizing the edges and finding the values of the parametrization in the function d:

```
l1:=unapply( <u,-9/10> ,u):
'l1(u)'=l1(u)
```

$$l1(u) = \begin{bmatrix} u \\ -\frac{9}{10} \end{bmatrix}$$

```
d(u,-9/10)
```

$$\frac{u^4 \left(u^2 - \frac{1}{2}\right)^2}{16}$$

```
diff(d(u,-9/10),u)
```

$$\frac{u^3 \left(u^2 - \frac{1}{2}\right)^2}{4} + \frac{u^5 \left(u^2 - \frac{1}{2}\right)}{4}$$

```
solve(%=0)
```

$$0, 0, 0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, \frac{1}{2}, -\frac{1}{2}$$

l1 in those points:

```
l1(sqrt(2)/2)
```

$$\begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{9}{10} \end{bmatrix}$$

```
l1(-sqrt(2)/2)
```

$$\begin{bmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{9}{10} \end{bmatrix}$$

```
l1(1/2)
```

$$\begin{bmatrix} \frac{1}{2} \\ -\frac{9}{10} \end{bmatrix}$$

```
l1(-1/2)
```

$$\begin{bmatrix} -\frac{1}{2} \\ -\frac{9}{10} \end{bmatrix}$$

None of them is in the closed square, no other candidates for now.

```
l2:=unapply( <u,-11/10> ,u):
'l2(u)'=l2(u)
```

$$l2(u) = \begin{bmatrix} u \\ -\frac{11}{10} \end{bmatrix}$$

```
d(u,-11/10)
```

$$\frac{u^4 \left(u^2 + \frac{1}{2}\right)^2}{16}$$

```
diff(d(u,-11/10),u)
```

$$\frac{u^3 \left(u^2 + \frac{1}{2}\right)^2}{4} + \frac{u^5 \left(u^2 + \frac{1}{2}\right)}{4}$$

```
solve(%=0)
```

$$0, 0, 0, \frac{I}{2}\sqrt{2}, -\frac{I}{2}\sqrt{2}, \frac{I}{2}, -\frac{I}{2}$$

Also for l2 there are no other possible candidates because the values found above are not in $\mathbb{R}$.

```
l3:=unapply( <-1/10,u> ,u):
'l3(u)'=l3(u)
```

$$l3(u) = \begin{bmatrix} -\frac{1}{10} \\ u \end{bmatrix}$$

```
d(-1/10,u)
```

$$\frac{\left(-\frac{499}{100} - 5u\right)^2}{160000}$$

```
diff(d(-1/10,u),u)
```

$$\frac{499}{1600000} + \frac{u}{3200}$$

```
solve(%=0)
```

$$-\frac{499}{500}$$

```
l3(-499/500)
```

$$\begin{bmatrix} -\frac{1}{10} \\ -\frac{499}{500} \end{bmatrix}$$

```
C3:=(-1/10,-499/500):
```

In l3 we have a possible candidate C3 that we will analyze at the end.

```
l4:=unapply( <1/10,u> ,u):
'l4(u)'=l4(u)
```

$$l4(u) = \begin{bmatrix} \frac{1}{10} \\ u \end{bmatrix}$$

```
d(1/10,u)
```

$$\frac{\left(-\frac{499}{100} - 5\,u\right)^2}{160000}$$

```
diff(d(1/10,u),u)
```

$$\frac{499}{1600000} + \frac{u}{3200}$$

```
solve(%=0)
```

$$-\frac{499}{500}$$

```
l4(-499/500)
```

$$\begin{bmatrix} \frac{1}{10} \\ -\frac{499}{500} \end{bmatrix}$$

```
C4:=(1/10,-499/500):
```

In l4 we have a possible candidate aswell, called C4. Remember to also take as possible candidates all the corners:

```
C5:=(-1/10,-9/10);
C6:=(-1/10,-11/10);
C7:=(1/10,-9/10);
C8:=(1/10,-11/10)
```

$$C5 := -\frac{1}{10}, -\frac{9}{10}$$

$$C6 := -\frac{1}{10}, -\frac{11}{10}$$

$$C7 := \frac{1}{10}, -\frac{9}{10}$$

$$C8 := \frac{1}{10}, -\frac{11}{10}$$

And now let's evaluate the function d in all of these possible points to find the global maximum in the closed square set:

```
d(C1);
d(C2);
d(C3);
d(C4);
evalf(d(C5));
evalf(d(C6));
evalf(d(C7));
evalf(d(C8))
```

$$0$$
$$0$$
$$0$$
$$0$$
$$1.500625000 \times 10^{6}$$
$$1.625625000 \times 10^{6}$$
$$1.500625000 \times 10^{6}$$
$$1.625625000 \times 10^{6}$$

The maximum points in the set are C6 and C8. From here we can evaluate the maximum delta "error" that we have in the set taking the square root of the value (because we previously elevated the function to the power of two).

```
delta:=sqrt(d(C8))
```

$$\delta := \frac{51}{40000}$$

```
evalf(%)
```

$$0.001275000000$$

The error is: $1.275 \cdot 10^{-3}$

C)

First let's verify that B is a stationary point for f. To do so we set the first partial derivatives respect to x and y equal to zero:

```
solve({fx=0,fy=0})
```

$$\{x = 0, y = 0\}$$

The point found solving the system is exactly our point B = $(0, 0)$ so it can be said that B is a sationary point. Now we should use the Hessian matrix precedently found to evaluate whether B is a local extrrma. Finding the Eigenvalues to this matrix will also tell us the type of local extrema.

```
H(B)
```

$$\begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix}$$

```
Eigenvalues(%)
```

$$\begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

Unfortunately the Eigenvalues for the matrix are $\lambda_1 = 0$ and $\lambda_2 = -2$. This does not give us enugh informations to determine if B is a local extremum.

D)

First thing is finding all the set of straight lines that pass through the point B. This is done by substituting in the formula the coordinates of point B:

$$y - y_0 = k\left(x - x_0\right)$$

$$y - 0 = k(x - 0)$$

$$y = kx$$

is then the set of these lines

To find the type of extremum that is present in B we need to apply f to the set of all lines that pass through B

```
'f(x)'=f(x,k*x)
```

$$f(x) = \frac{5}{4}x^3 k - \frac{1}{4}x^4 - k^2 x^2$$

This is basically a function of one variable where k is cosidered as a "parameter" (which describes the angular coefficient of the lines through B). Now we study the function and in particular its extrema. Let's find the first derivative and set it to zero:

```
g:=unapply( diff(%,x), x):
'g(x)'=g(x)
```

$$g(x) = \frac{15}{4}x^2 k - x^3 - 2k^2 x$$

```
solve(g(x)=0,x)
```

$$0, \left(\frac{15}{8} + \frac{\sqrt{97}}{8}\right)k, \left(\frac{15}{8} - \frac{\sqrt{97}}{8}\right)k$$

From here we get the three roots 0, $k\left(\frac{15}{8} + \frac{\sqrt{97}}{8}\right)$, $k\left(\frac{15}{8} - \frac{\sqrt{97}}{8}\right)$

Evaluaing the following inequality we can "study the sign" of the first derivative and find the type of extrema in those points. Unfortunately Maple gives me an error whenever I try to do so:

```
> solve(g(x)>0 ,x)
Warning, solutions may have been lost
```

So I proceeded doing it by hand:

$g(x) > 0$

$\frac{15}{4}x^2k - x^3 - 2k^2x > 0$

$x\left(-x^2 + \frac{15}{4}xk - 2k^2\right) > 0$

$x > 0$

$\left(\frac{15-\sqrt{97}}{8}\right)k < x < \left(\frac{15+\sqrt{97}}{8}\right)k$

roots:

$0, \left(\frac{15-\sqrt{97}}{8}\right)k, \left(\frac{15+\sqrt{97}}{8}\right)k$

	0		$\frac{15-\sqrt{97}}{8}k$		$\frac{15+\sqrt{97}}{8}k$	
$-$	|	$+$	|	$+$	|	$+$
$-$	|	$-$	|	$+$	|	$-$
$+$	|	$-$	|	$+$	|	$-$

increase decrease

We can see that the function increases before the root 0 and decreases right after. This means the in point B = (0, 0) the function f has a local maximum.

Appendix

```
> restart:
> with(plots):
> with(LinearAlgebra):
```

Part A

```
> f:=unapply( 5/4*x^2*y-1/4*x^4-y^2 , x,y):
  'f(x,y)'=f(x,y)
```

$$f(x, y) = \frac{5}{4}\, x^2\, y - \frac{1}{4}\, x^4 - y^2 \tag{1.1}$$

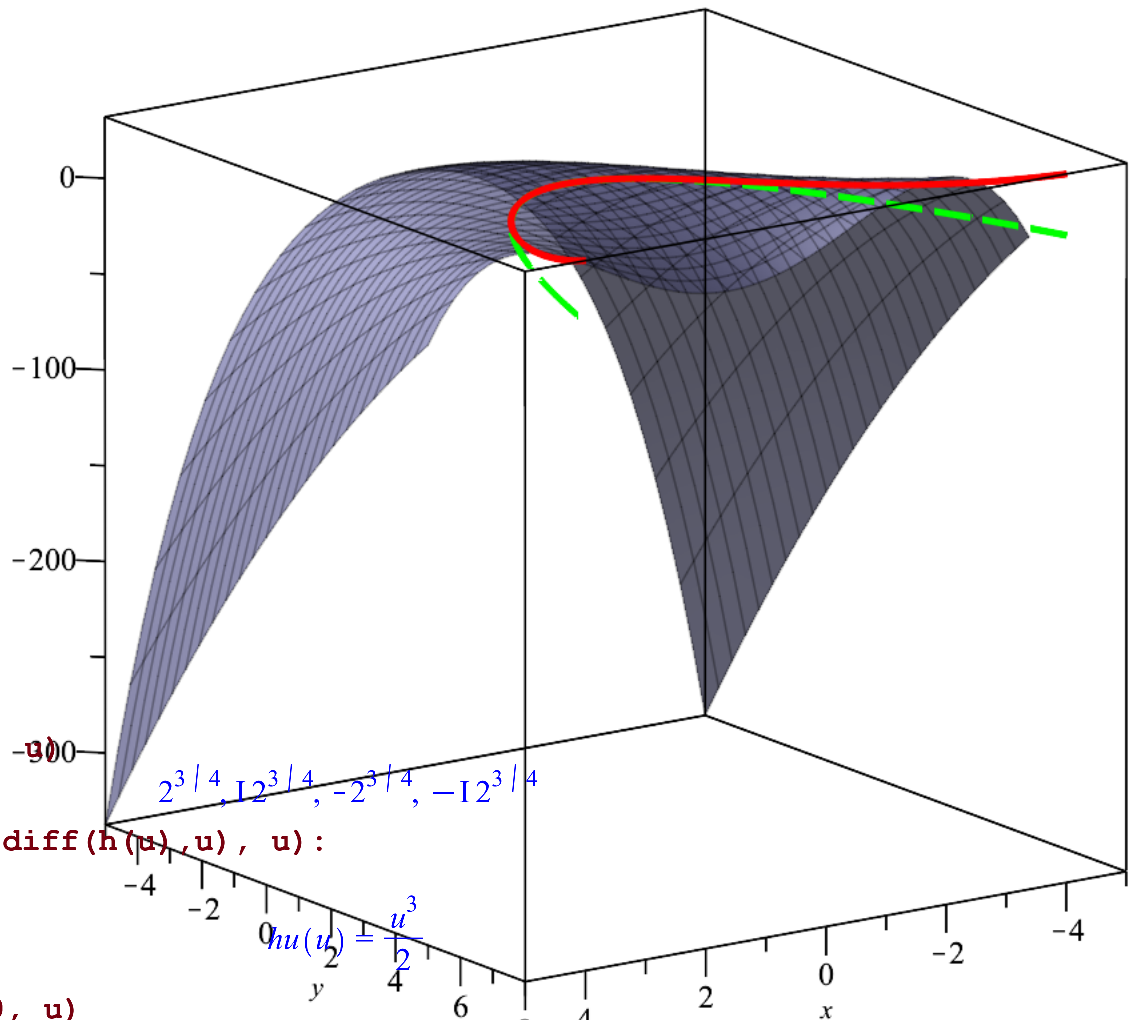
```
> R:=(u,1/2*u^2):
  r:=unapply( [u,1/2*u^2], u):
  'r(u)'=r(u)
```

$$r(u) = \left[u,\ \frac{u^2}{2}\right] \tag{1.2}$$

```
> h:=unapply(f(R),u):
  'h(u)'=h(u)
```

$$h(u) = \frac{u^4}{8} \tag{1.3}$$

```
> surface:=plot3d(f(x,y),x=-5..5,y=-5..5,color="Nautical
  GrayViolet",transparency=0.2):
  h_curve:=spacecurve(<u,u^2/2,u^4/8>,u=-4..4,thickness=5,color=
  red):
> k_curve:=spacecurve(<u,u^2/2,0>,u=-4..4,thickness=5,color=green,
  linestyle=dash):
> display(surface,k_curve,h_curve)
```




```
> solve(h(u)=1, u)
```

$$2^{3/4}, I\,2^{3/4}, -2^{3/4}, -I\,2^{3/4}$$

```
> hu:=unapply( diff(h(u),u), u):
  'hu(u)'=hu(u)
```

$$hu(u) = \frac{u^3}{2}$$

```
> solve(hu(u)=0, u)
```

$$0, 0, 0$$

Part B

```
> A:=(0,-1);
  B:=(0,0)
```

$$A := 0, -1$$

$$B := 0, 0 \qquad \textbf{(1.7)}$$

```
> fx:=diff(f(x,y),x):
  fy:=diff(f(x,y),y):
> fxx:=diff(fx,x);
  fyy:=diff(fy,y);
  fxy:=diff(fx,y);
  fyx:=diff(fy,x)
```

$$fxx := \frac{5\,y}{2} - 3\,x^2$$

$$fyy := -2$$

$$fxy := \frac{5\,x}{2}$$

$$fyx := \frac{5\,x}{2} \tag{1.8}$$

```
> H:=unapply( <fxx,fxy|fyx,fyy>, x,y):
  H(x,y)
```

$$\begin{bmatrix} \frac{5\,y}{2} - 3\,x^2 & \frac{5\,x}{2} \\ \frac{5\,x}{2} & -2 \end{bmatrix} \tag{1.9}$$

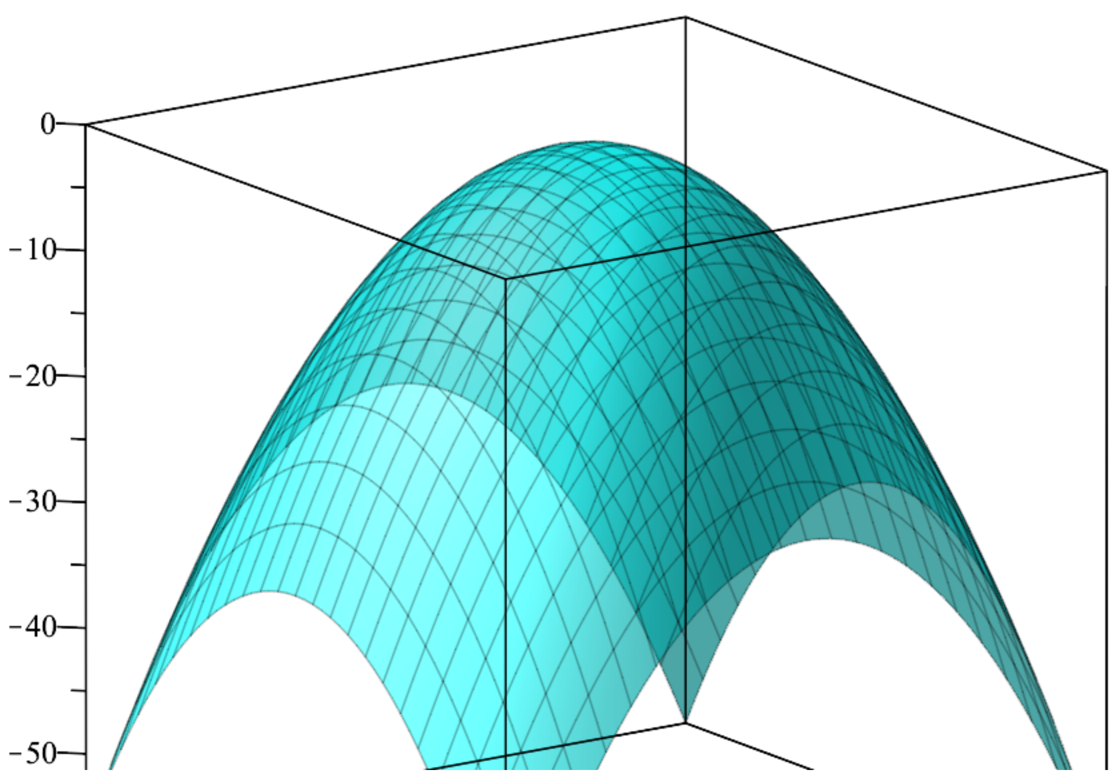
```
> H(A)
```

$$\begin{bmatrix} -\frac{5}{2} & 0 \\ 0 & -2 \end{bmatrix} \tag{1.10}$$

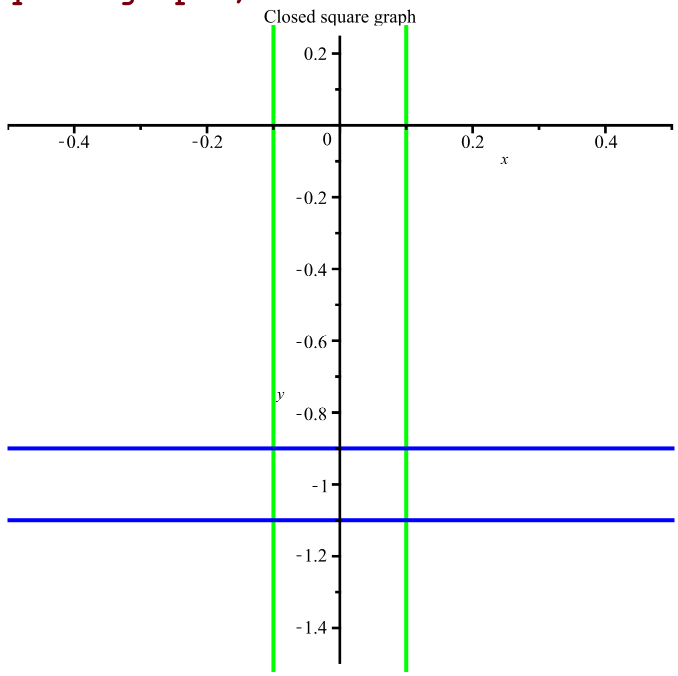
```
> P2:=unapply( mtaylor(f(x,y),[x=0,y=-1],3),x,y)
```

$$P2 := (x, y) \mapsto 1 + 2 \cdot y - \frac{5 \cdot x^2}{4} - (y+1)^2 \tag{1.11}$$

```
> plot3d(P2(x,y),x=-5..5,y=-5..5,axes=box,color=cyan,transparency=
  0.3)
```



```
> l1:=plot([-1/10,t,t=-2..1/2],color=green):
  l2:=plot([1/10,t,t=-2..1/2],color=green):
  l3:=plot(-9/10,color=blue):
  l4:=plot(-11/10,color=blue):
  display(l1,l2,l3,l4,view=[-1/2..1/2,-3/2..1/4],labels=[x,y],
  title="Closed square graph")
```




```
> f(x,y)-P2(x,y)
```

$$\frac{5x^2 y}{4} - \frac{x^4}{4} - y^2 - 1 - 2y + \frac{5x^2}{4} + (y+1)^2 \quad \textbf{(1.12)}$$

```
> simplify(%^2)
```

$$\frac{x^4 (x^2 - 5y - 5)^2}{16} \quad \textbf{(1.13)}$$

```
> d:=unapply(%,x,y):
  'd(x,y)'=d(x,y)
```

$$d(x,y) = \frac{x^4 (x^2 - 5y - 5)^2}{16} \quad \textbf{(1.14)}$$

```
> dx:=diff(d(x,y),x);
  dy:=diff(d(x,y),y)
```

$$dx := \frac{x^3 (x^2 - 5y - 5)^2}{4} + \frac{x^5 (x^2 - 5y - 5)}{4}$$

$$dy := -\frac{5x^4 (x^2 - 5y - 5)}{8} \quad \textbf{(1.15)}$$

```
> solve({dx=0,dy=0})
```

$$\{x = 0, y = y\}, \left\{x = x, y = \frac{x^2}{5} - 1\right\} \quad \textbf{(1.16)}$$

```
> C1:=(0,u):
> C2:=(u,u^2/5-1):
```

```
> l1:=unapply( <u,-9/10> ,u):
  'l1(u)'=l1(u)
```

$$l1(u) = \begin{bmatrix} u \\ -\frac{9}{10} \end{bmatrix} \tag{1.17}$$

```
> d(u,-9/10)
```

$$\frac{u^4 \left(u^2 - \frac{1}{2}\right)^2}{16} \tag{1.18}$$

```
> diff(d(u,-9/10),u)
```

$$\frac{u^3 \left(u^2 - \frac{1}{2}\right)^2}{4} + \frac{u^5 \left(u^2 - \frac{1}{2}\right)}{4} \tag{1.19}$$

```
> solve(%=0)
```

$$0, 0, 0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, \frac{1}{2}, -\frac{1}{2} \tag{1.20}$$

```
> l1(sqrt(2)/2)
```

$$\begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{9}{10} \end{bmatrix} \tag{1.21}$$

```
> l1(-sqrt(2)/2)
```

$$\begin{bmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{9}{10} \end{bmatrix} \tag{1.22}$$

```
> l1(1/2)
```

$$\begin{bmatrix} \frac{1}{2} \\ -\frac{9}{10} \end{bmatrix} \tag{1.23}$$

```
> l1(-1/2)
```

$$\begin{bmatrix} -\frac{1}{2} \\ -\frac{9}{10} \end{bmatrix} \tag{1.24}$$

```
> l2:=unapply( <u,-11/10> ,u):
  'l2(u)'=l2(u)
```

$$l2(u) = \begin{bmatrix} u \\ -\frac{11}{10} \end{bmatrix} \quad (1.25)$$

```
> d(u,-11/10)
```

$$\frac{u^4 \left(u^2 + \frac{1}{2}\right)^2}{16} \quad (1.26)$$

```
> diff(d(u,-11/10),u)
```

$$\frac{u^3 \left(u^2 + \frac{1}{2}\right)^2}{4} + \frac{u^5 \left(u^2 + \frac{1}{2}\right)}{4} \quad (1.27)$$

```
> solve(%=0)
```

$$0, 0, 0, \frac{I}{2}\sqrt{2}, -\frac{I}{2}\sqrt{2}, \frac{I}{2}, -\frac{I}{2} \quad (1.28)$$

```
> l3:=unapply( <-1/10,u> ,u):
  'l3(u)'=l3(u)
```

$$l3(u) = \begin{bmatrix} -\frac{1}{10} \\ u \end{bmatrix} \quad (1.29)$$

```
> d(-1/10,u)
```

$$\frac{\left(-\frac{499}{100} - 5u\right)^2}{160000} \quad (1.30)$$

```
> diff(d(-1/10,u),u)
```

$$\frac{499}{1600000} + \frac{u}{3200} \quad (1.31)$$

```
> solve(%=0)
```

$$-\frac{499}{500} \quad (1.32)$$

```
> l3(-499/500)
```

$$\begin{bmatrix} -\frac{1}{10} \\ -\frac{499}{500} \end{bmatrix} \quad (1.33)$$

```
> C3:=(-1/10,-499/500):
> l4:=unapply( <1/10,u> ,u):
  'l4(u)'=l4(u)
```

(1.34)

$$l4(u) = \begin{bmatrix} \frac{1}{10} \\ u \end{bmatrix} \quad (1.34)$$

```
> d(1/10,u)
```

$$\frac{\left(-\frac{499}{100} - 5\,u\right)^2}{160000} \quad (1.35)$$

```
> diff(d(1/10,u),u)
```

$$\frac{499}{1600000} + \frac{u}{3200} \quad (1.36)$$

```
> solve(%=0)
```

$$-\frac{499}{500} \quad (1.37)$$

```
> l4(-499/500)
```

$$\begin{bmatrix} \frac{1}{10} \\ -\frac{499}{500} \end{bmatrix} \quad (1.38)$$

```
> C4:=(1/10,-499/500):
> C5:=(-1/10,-9/10);
  C6:=(-1/10,-11/10);
  C7:=(1/10,-9/10);
  C8:=(1/10,-11/10)
```

$$C5 := -\frac{1}{10}, -\frac{9}{10}$$

$$C6 := -\frac{1}{10}, -\frac{11}{10}$$

$$C7 := \frac{1}{10}, -\frac{9}{10}$$

$$C8 := \frac{1}{10}, -\frac{11}{10} \quad (1.39)$$

```
> d(C1);
  d(C2);
  d(C3);
  d(C4);
  evalf(d(C5));
  evalf(d(C6));
  evalf(d(C7));
  evalf(d(C8))
```

$$0$$

$$0$$

$$0$$

$$0$$
$$1.500625000 \times 10^{-6}$$
$$1.625625000 \times 10^{-6}$$
$$1.500625000 \times 10^{-6}$$
$$1.625625000 \times 10^{-6} \quad (1.40)$$

```
> delta:=sqrt(d(C8))
```

$$\delta := \frac{51}{40000} \quad (1.41)$$

```
> evalf(%)
```

$$0.001275000000 \quad (1.42)$$

Part C

```
> solve({fx=0,fy=0})
```

$$\{x=0, y=0\} \quad (1.43)$$

```
> H(B)
```

$$\begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \quad (1.44)$$

```
> Eigenvalues(%)
```

$$\begin{bmatrix} 0 \\ -2 \end{bmatrix} \quad (1.45)$$

Pard D

```
> y-0=k(x-0)
```

$$y=k(x) \quad (1.46)$$

```
> 'f(x)'=f(x,k*x)
```

$$f(x) = \frac{5}{4}\,x^3\,k - \frac{1}{4}\,x^4 - k^2\,x^2 \quad (1.47)$$

```
> g:=unapply( diff(5/4*x^3*k-1/4*x^4-k^2*x^2 ,x), x):
  'g(x)'=g(x)
```

$$g(x) = \frac{15}{4}\,x^2\,k - x^3 - 2\,k^2\,x \quad (1.48)$$

```
> solve(g(x)=0,x)
```

$$0, \left(\frac{15}{8} + \frac{\sqrt{97}}{8}\right)k, \left(\frac{15}{8} - \frac{\sqrt{97}}{8}\right)k \quad (1.49)$$

```
> solve(g(x)>0 ,x)
```

Warning, solutions may have been lost