Home assignment 4

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Exercise 4.1

a) Show using induction that

$$10^n \equiv 1^n \pmod 9 \tag{1}$$

holds for $n \in \mathbb{N}$.

For the next part of the exercise, we need to introduce some notation to talk about the decimal number system. The number 152 is made up of the three digits 1, 5 and 2. Let us denote the right most-digit by d_0, the digit to its left as d_1, and in general the digit to the left of d_i as d_{i+1}. Thus in the case of 152, we have $d_0 = 2$, $d_1 = 5$ and $d_2 = 1$ that respectively denote the total quantity of 1's, 10's and 100's in the number. As another example, the number 3754 would have $d_0 = 4$, $d_1 = 5$, $d_2 = 7$ and $d_3 = 3$.

We will now define the *digit sum*. To calculate the digit sum from a given number, we sum all digits of that number. For example, the digit sum of 152 is $2 + 5 + 1 = 8$ while the digit sum for 3754 is $4 + 5 + 7 + 3 = 19$. In general the digit sum for a number $d_{n-1}d_{n-2}\ldots d_0$ is equal to $d_0 + d_1 + d_2 + \cdots + d_{n-1}$.

b) Explain briefly why the following holds: If $k \in \mathbb{N}$ has the digit sum α, then

$$k \equiv \alpha \pmod 9.$$

Hint: We have for example $152 = 1 \cdot 10^2 + 5 \cdot 10^1 + 2 \cdot 10^0$. Think about how you could combine what you showed in part a) with Lemma 5.4 from the notes.

Exercise 4.2

a) A vending machine accepts two types of coins: 2-krone coins and 5-krone coins. A 2-krone coin weighs 59 dg while a 5-krone coin weighs 92 dg (dg stands for decigram but the unit is not really relevant to the exercise). During some day, the vending machine receives coins with a total weight of 7229 dg. Let x be the number of received 2-krone coins and let y be the number of received 5-krone coins. Explain why the following congruence relation holds:

$$59x \equiv 7229 \pmod{92}$$

Hint: $92y \equiv 0 \pmod{92}$.

b) Show that the above congruence relation can be transformed to the following:

$$59x \equiv 53 \pmod{92}$$

c) Solve this congruence relation for x. (There are a priori infinitely many solutions.)

Once you know x, it is easy to solve for y.

d) Solve for y and use the fact that both $x \geq 0$ og $y \geq 0$ to limit the number of solutions.

Hint: In this exercise there is only one possible solution for (x, y).

Exercise 4.3

a) Find the set of solutions for x in the following system of congruence relations:

$$x \equiv 250 \pmod{439} \tag{2}$$
$$118x \equiv 590 \pmod{1121} \tag{3}$$

Hint: Rewrite the first congruence (3) and use the Chinese remainder theorem.