

01019 Discrete Mathematics E22

Home assignment 1

To be handed in no later than Wednesday September 28 at
11:59pm

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Important regarding the home assignments: Remember to respect the policy on collaborated work stated in the welcome text on the Campusnet page! Please note that the home assignments must be handed in in pairs of two students.

Exercise A (= exercise 2, exercises 3rd week)

Use the tableau method to determine which of the following formulas are valid. For the non-valid formulas you must state a truth assignment that makes the formula false (read from an open, saturated tableau branch).

Try to apply the decomposition rules in an order that minimizes the size of the tableaux. Also remember that you do not have to continue the tableau constriction if you have found just a single open, saturated branch.

(a) $((p \rightarrow r) \vee (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$

(b) $p \rightarrow ((q \rightarrow r) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)))$

Exercise B (= exercise 5, exercises 3rd week)

Consider the following inference:

If Per moves his bishop, René will break the chess board.

René will move his rook if and only if Per moves his queen.

It is not the case that Per will move his queen and not move his bishop.

If Per moves his queen, René will move his rook and break the chess board.

Show that the inference is logically correct.

Exercise C (= exercise 3, exercises 4th week)

State all the elements of the following sets:

- a) $A = \{x \in \mathbb{R} \mid x^2 - 3x = 4\}$.
- b) $B = \{x \in \mathbb{Z} \mid -3 \leq x < 3\}$.
- c) $C = \{x \in \mathbb{Z} \mid -3 \leq x < 3 \wedge x^2 - 3x = 4\}$.
- d) $D = \{x \in \mathbb{Z} \mid -3 \leq x < 3 \wedge x^2 - 3x \neq 4\}$.

Then show that the sets C and D can be described from the sets A and B using the set operations (\cup , \cap and $-$). Explain your result. *Hint:* If e.g. we had $E = \{x \in \mathbb{R} \mid x > 2\}$, $F = \{x \in \mathbb{R} \mid x < 3\}$ and $G = \{x \in \mathbb{R} \mid 2 < x < 3\}$ we would describe G from E and F by the expression $G = E \cap F$.

Exercise D (= exercise 5, exercises 4th week)

Prove that the following equalities hold for all sets A , B and C . It is up to you whether you use a linguistic proof or show it by first translating to propositional logic.

- a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
- b) $A - (A \cap B) = A - B$.
- c) $A \cap (A \cup B) = A$.

Exercise E

In the following exercise it is very important to explain your solution and argue for its correctness. The task is not trivial, so start thinking about it in due time!

On a remote island there are two types of people:

- Truth sayers, that always tell the truth (everything they say is true).
- Liars, that always lie (everything they say is false).

It is not possible to distinguish between them by the way they look, but only by the truth or falsehood of the things they say.

1. Once a foreigner visited the island. There, he met two of the island's inhabitants, Peter and Signe. He asked them: "Are any of you liars?". "At least one of us is a liar", Peter replied. What are Peter and Signe (truth sayers or liars)?

Hint: Use a propositional variable p to denote the proposition "Peter is a truth sayers" and a propositional variable s to denote "Signe is a truth sayers". We can then e.g. express the proposition "Peter and Signe are both truth sayers" by the formula $p \wedge s$. How do you then express "at least one of them is a liar"? Now find a formula with propositional variables p and s , that must be true for the described situation. By looking at which truth values of p and s that make this formula true (e.g. by truth table), you are able to determine what Peter and Signe are.

2. While the the foreigner wandered around the island, he met two other inhabitants, Anne and Bob. He asked Anne: “Are any of you truth sayers?”. “If Bob is a liar, then I am also a liar”, Anne replied. What are Anne and Bob?

Hint: Use the same strategy as above. Let a denote “Anne is a truth sayer” and b denote “Bob is a truth sayer”. How do you then express “If Bob is a liar, then Anne is also a liar”? Now find a formula with propositional variables a and b , that must be true for the described situation, and use this formula to determine the truth values of a and b .

3. The foreigner kept wandering. At night he started searching for shelter for the night, but was very careful, as he knew some of the island’s inhabitants to be cannibals, and it was not possible to distinguish between them by their looks. He then met three of the island’s inhabitants, Carsten, Diana and Erika. He asked Carsten: “How many of you are truth sayers?”. “Flam flim”, Carsten replied in his own language. “What did he say?”, the stranger asked Dianna. “He said ‘Only one’ ”, Diana replied. “Don’t trust Diana, she is a liar. Come with me, I am not a cannibal”, Erika said. “No, come with me, I am not a cannibal”, Diana said. What should the stranger do?

Hint: Like in the previous questions we can let c denote that Carsten is a truth sayer, d for Dianna being a truth sayer and e for Erika being a truth sayer. Then start by writing the formula that expresses “only one of us is a truth sayer”, where “us” is Carsten, Diana and Erika. Carsten’s proposition is “flim flam”. Diana claims that “flim flam” means “only one of us is a truth sayer”. So if Diana tells the truth (is a truth sayer), then Carsten’s claim is that “only one of us is a truth sayer”, and this claim must then be true exactly when Carsten is a truth sayer. Try to write down a formula that captures all of this, and determine which truth assignments that satisfy this formula.

Exercise F: Only for 3-person groups

Prove the following lemma.

Lemma 1. *The formulas A and B are logically equivalent if and only if the formula $A \leftrightarrow B$ is valid.*

Hint: You can look for inspiration in the solution to Exercise 7 from the week 4 Exercises.