POLITECNICO DI MILANO

AUTONOMOUS VEHICLES

S. Arrigoni

POLITECNICO
MILANO 1863

Motion Planning

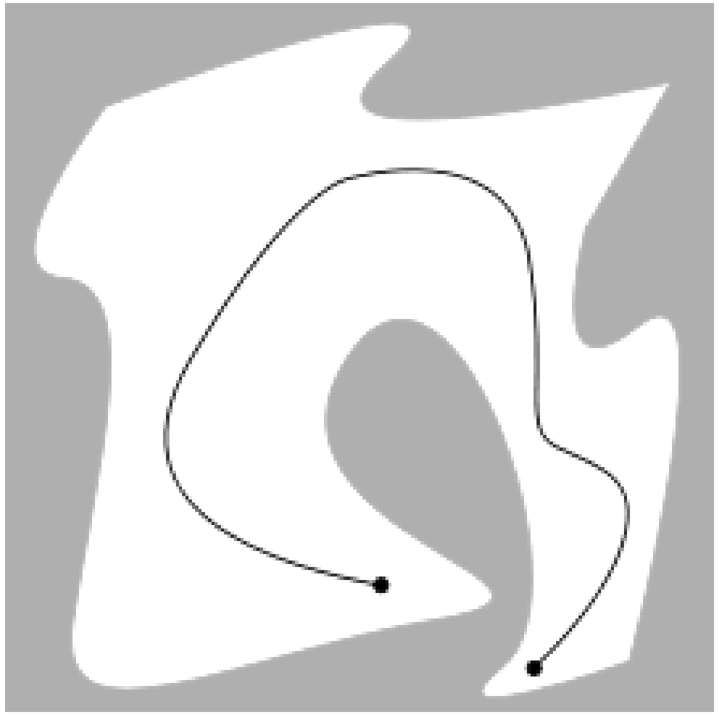
in-depth study

Introduction

What is motion planning?

Define a sequence of actions to take a robot from one known state (**initial**) to another known state (**goal**), while:

- avoiding obstacles;
- respecting motion constraints;
- optimizing a cost function (*hopefully*)



Introduction

Assumptions

- Ideal knowledge of the robot and the world
- A priori knowledge of actual and goal positions
- motion rules of our robot are fixed

Introduction

Popular approaches:

- Potential fields [*Rimon, Koditschek, '92*]: create forces on the robot that pull it toward the goal and push it away from obstacles
- Grid-based planning [*Stentz, '94*]: discretizes problem into grid and runs a graph-search algorithm (Dijkstra, A*, …)
- Combinatorial planning [*LaValle, '06*]: constructs structures in the configuration (C-) space that completely capture all information needed for planning
- Sampling-based planning [*Kavraki et al, '96; LaValle, Kuffner, '06, etc.*]: uses collision detection algorithms to probe and incrementally search the C-space for a solution, rather than completely characterizing all of the C_{free} structure

Introduction

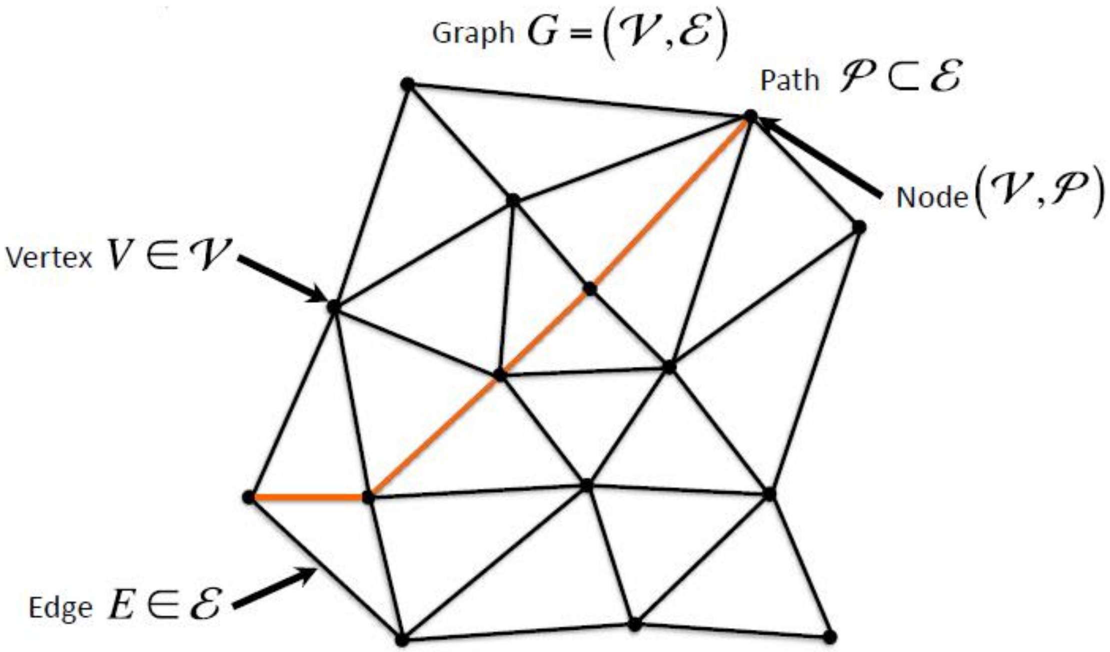
Discrete graph representation

It is common to convert the planning problem into a (<u>discrete</u>) ***graph*** representation

→ use one of the existing search algorithms on the graph

Where ***edges*** can:

- Have a direction (*directed graph*)
- Have a cost (*weighted graph*)

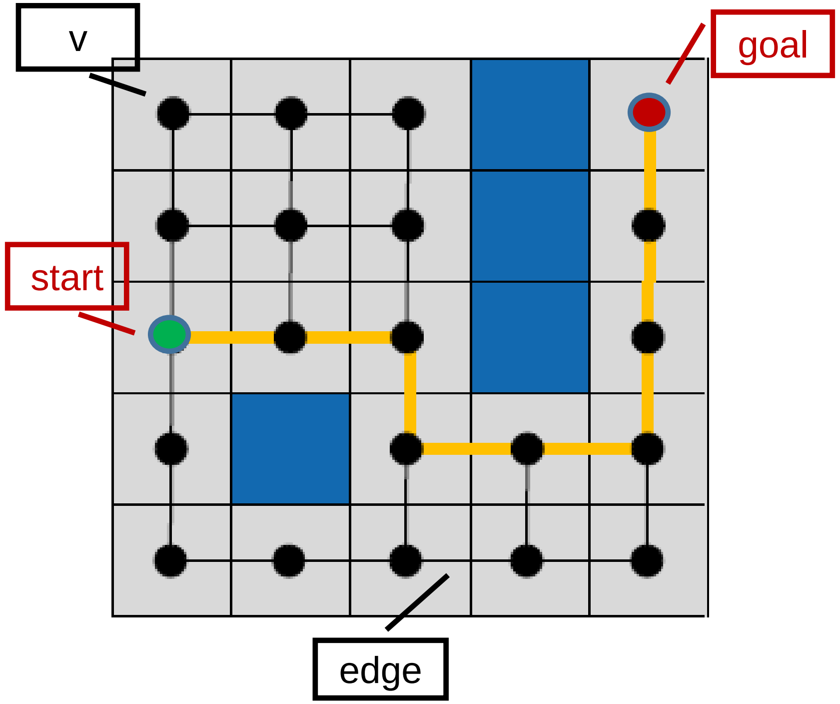


Introduction

Grid based approach

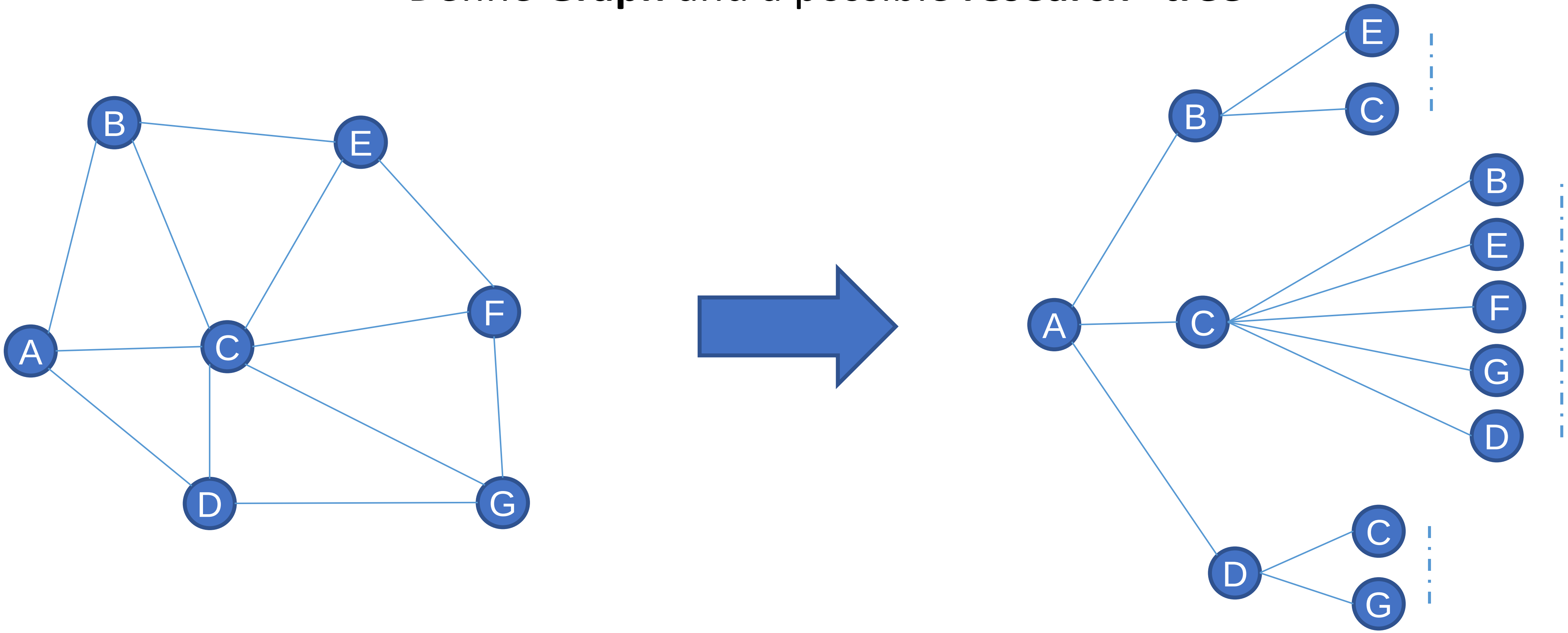
Discretize the continuous world **into a grid**:
- Each cell is either free or forbidden
- Robot moves between adjacent free cells
- Goal: find sequence of free cells from start

Mathematically, this corresponds to pathfinding in a discrete graph $G = (V, E)$
- Each vertex $v \in V$ represents a free cell
- Edges $(v, u) \in E$ connect adjacent grid cells



Introduction

Define **Graph** and a possible **research "tree"**



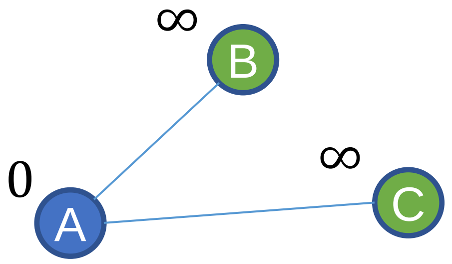
Introduction

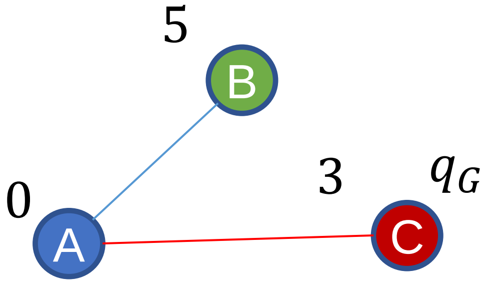
Graph search algorithms

Label correcting algorithms:

best path optimizing *cost-of-arrival* $C(q)$

q_I to q



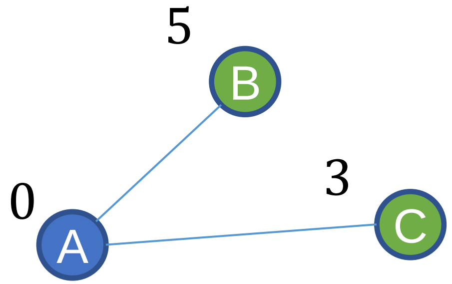


Define neighbours of **q**:

Evaluate and **update C(q)**:

Define new $\boldsymbol{q}$

Repeat until $\boldsymbol{q = q_G}$



Introduction

Label correcting algorithms:

best path optimizing *cost-of-arrival* $C(q)$

q_I to q

Initialize C(nodes) =∞:

Remove q from queue and **find** $\mathbf{q'}$:

Evaluate and **update C(q')**:

if lower set **q** as **q'** <u>parent</u>

if $\boldsymbol{q' \mathrel{!=} q_G}$

Add q' in queue

Get next (q)

terminate

Introduction

How to get next node?

Breadth-First-Search (BFS, Bellman-Ford): Maintain Q as a **list** – First in/first out

Q: frontier/open/alive set of nodes
= next nodes to be visited

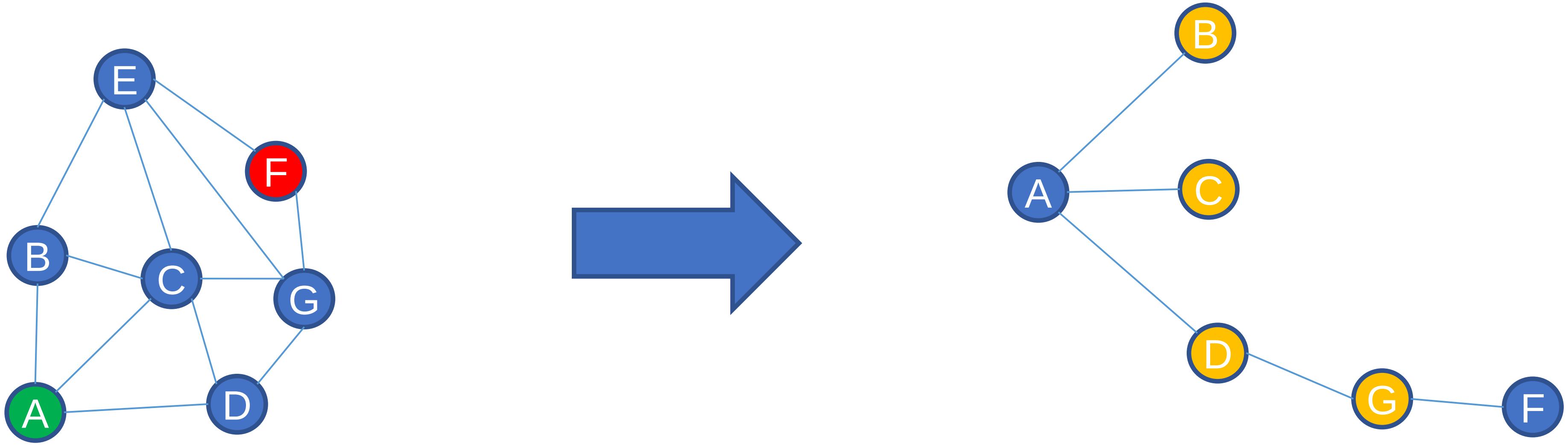
Update cost for **all edges up to current depth** before proceeding to greater depth

- Complete (will find the solution if it exists)
- Guaranteed to find the shortest (number of edges) path
- First solution found is the optimal path
- Can deal with negative edge (transition) costs

Introduction

How to get next node?

Breadth-First-Search (BFS, Bellman-Ford): Maintain Q as a **list** – First in/first out



Introduction

How to get next node?

Depth-First-Search (DFS): Maintain Q as a **stack** – Last in/first out

- Lower memory requirement (only need to store part of graph)

Q: frontier/open/alive set of nodes
= next nodes to be visited

Starts at the root node and explores as far as possible
along each branch before backtracking

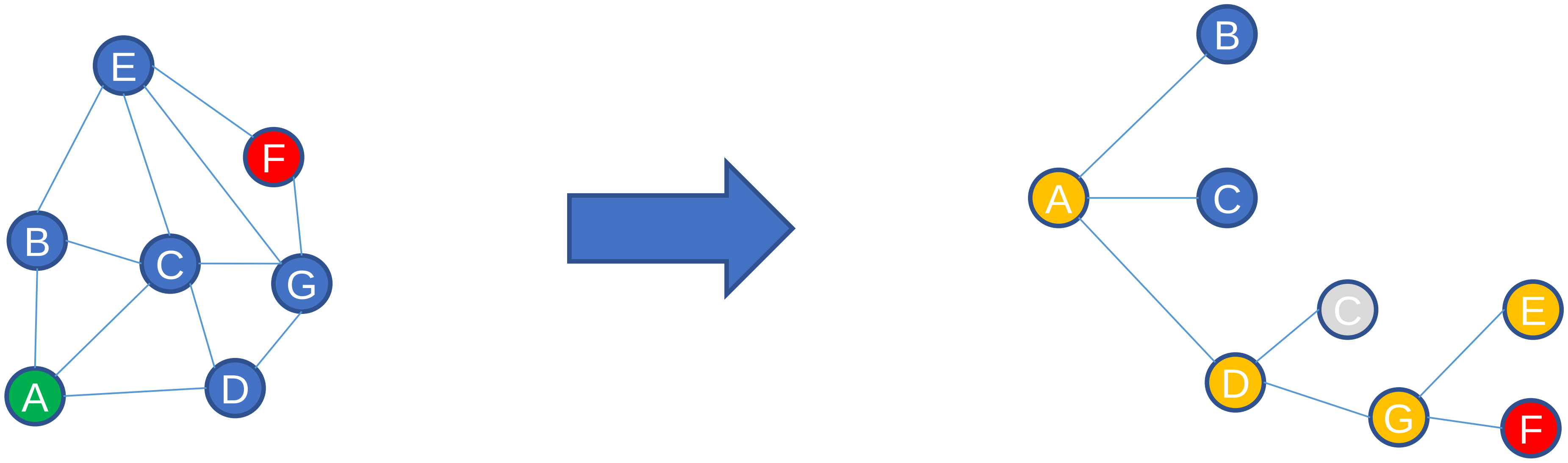
- Lower memory footprint than BFS with high-branching
- Both BFS and DFS are simple to implement (but generally inefficient).
- not complete for infinite trees

Introduction

How to get next node?

Depth-First-Search (DFS): Maintain Q as a **stack** – Last in/first out

- Lower memory requirement (only need to store part of graph)



Dijkstra

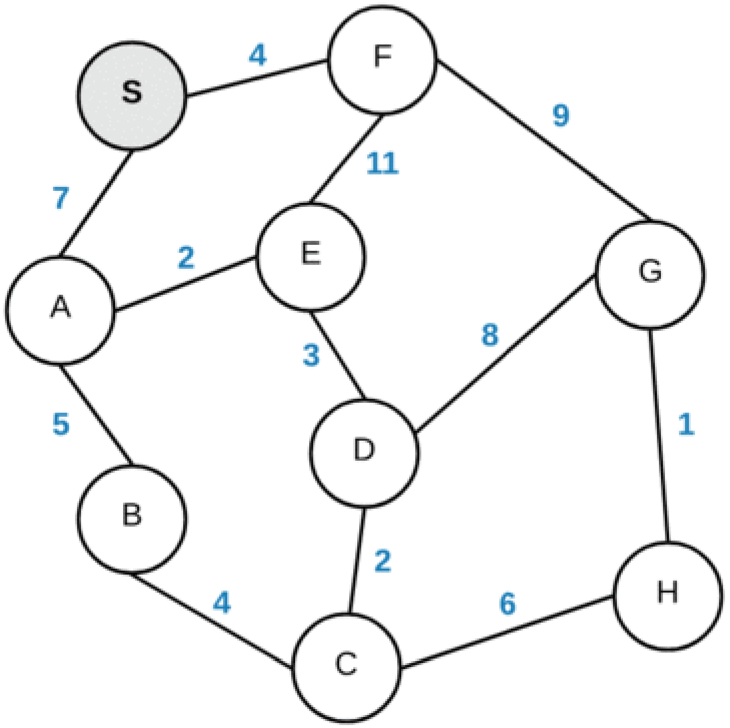
Dijkstra

Best-First (BF, Dijkstra): next q: $\rightarrow q = argmin_{q \in Q} C(q)$

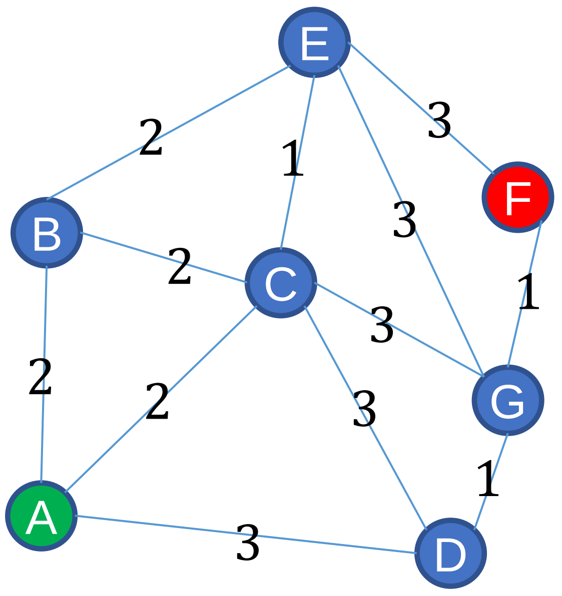
- Node will enter Q at most ***<u>once</u>***
- <u>Requires costs to be non-negative</u>

Expanding **from closest** to start (BFS with edge costs)

Q is ordered according to <u>currently known</u> best cost to arrive



Dijkstra

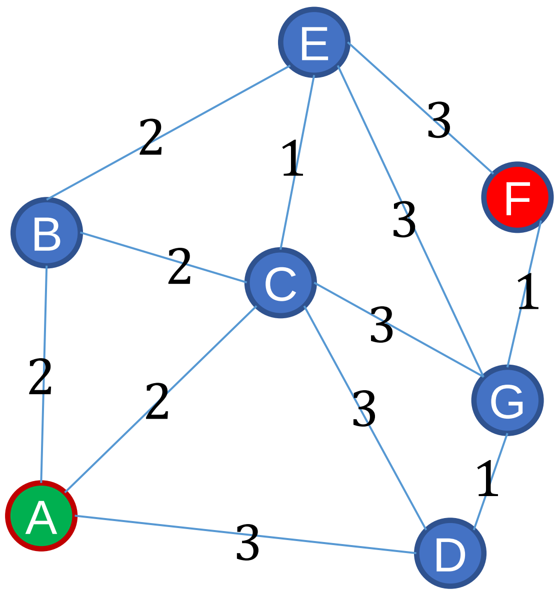


How it works

Q	Checked
A(0)	

Add in Q ordered by cost

Dijkstra

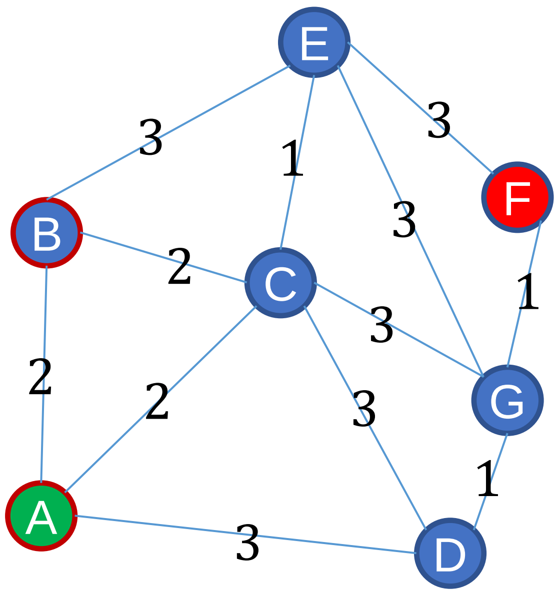


How it works

Q	Checked
B(2),C(2), D(3)	A(0)

Move in checked;
place connected in Q ranked by cost

Dijkstra

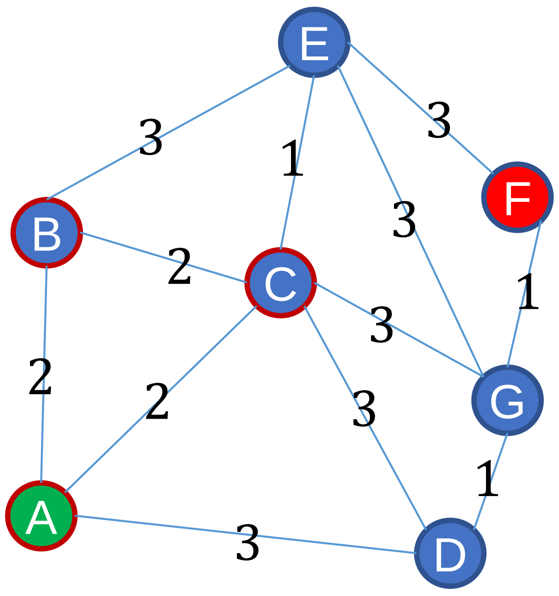


How it works

Q	Checked
C(2), D(3),E(3)	A(0), B(2)

Move in checked;
place connected in Q ranked by cost
(update if the same is lower)

Dijkstra

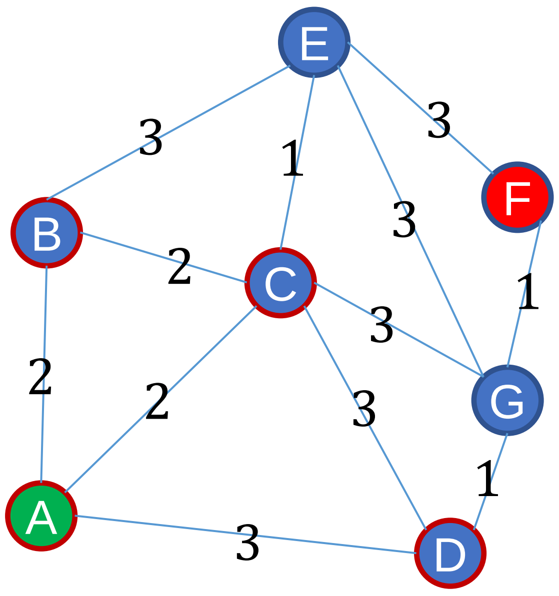


How it works

Q	Checked
D(3),E(3), G(5)	A(0), B(2),C(2)

Move in checked;
place connected in Q ranked by cost
(update if the same is lower)

Dijkstra

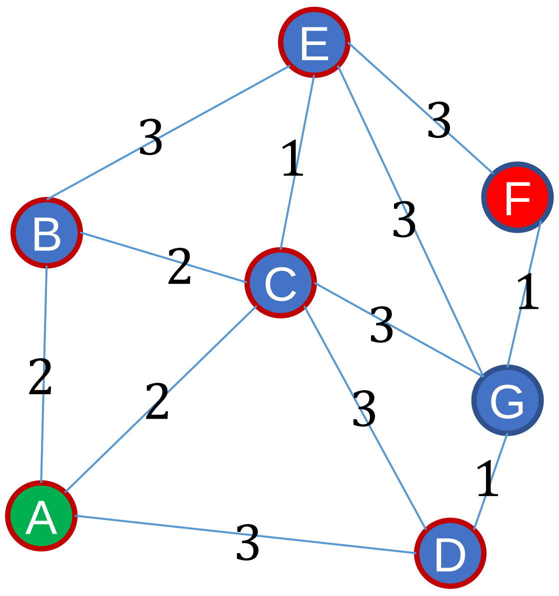


How it works

Q	Checked
E(3),G(4)	A(0), B(2),C(2), D(3)

Move in checked;
place connected in Q ranked by cost
(update if the same is lower)

Dijkstra

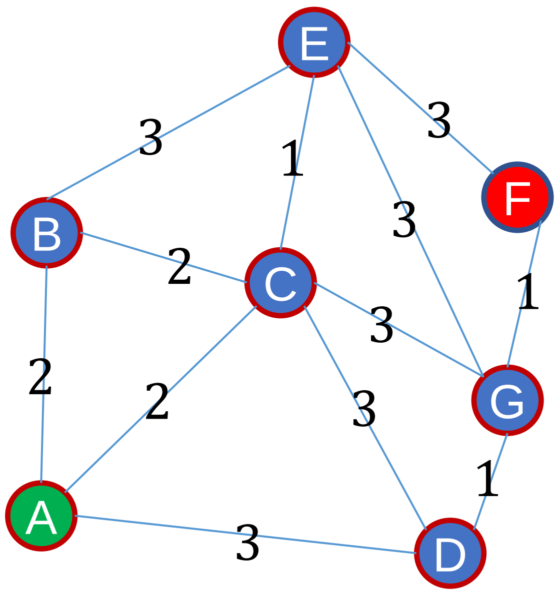


How it works

Q	Checked
G(4),F(6)	A(0), B(2),C(2), D(3), E(3)

Move in checked;
place connected in Q ranked by cost
(update if the same is lower)

Dijkstra



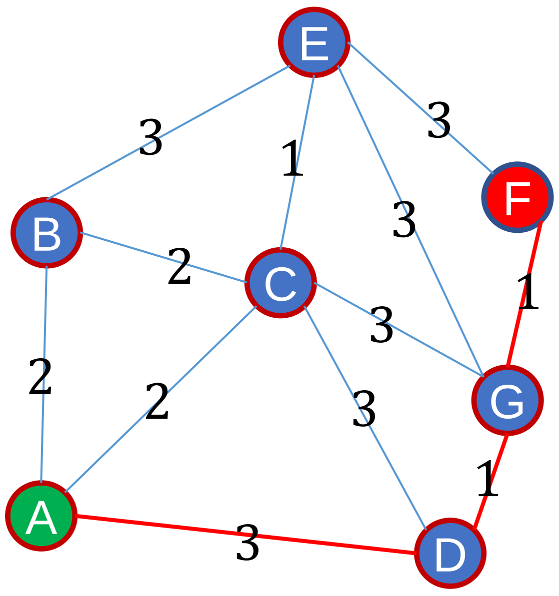
How it works

Q	Checked
F(5)	A(0), B(2),C(2), D(3), E(3), G(4)

Move in checked;
place connected in Q ranked by cost
(update if the same is lower)

Dijkstra

How it works



Q	Checked
[ ]	A(0), B(2),C(2), D(3), E(3), G(4), F(5)

Goal reached

Consider parenting: $F \leftarrow G \leftarrow A$

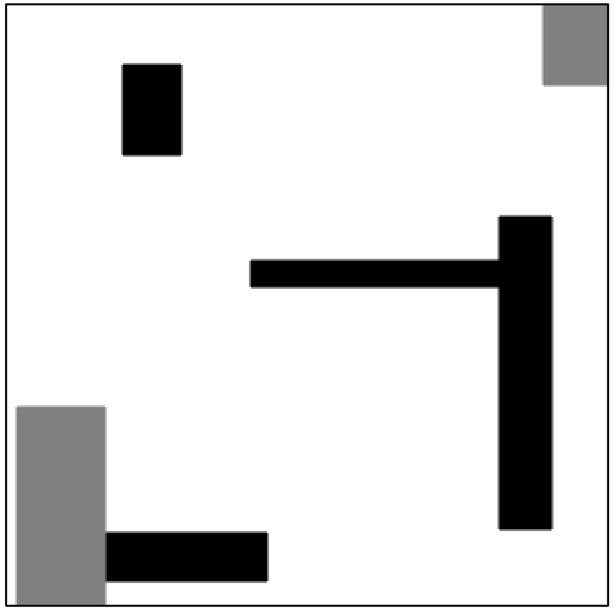
Software requirements

MATLAB®

Implementation: an example

Create Image

- Dimension: ~= 30x30 pixels;
- black walls; grey unknown;
- Save as *.png (24 bit);



- ✓ Win: Paint
- ✓ Ubuntu: KolourPaint

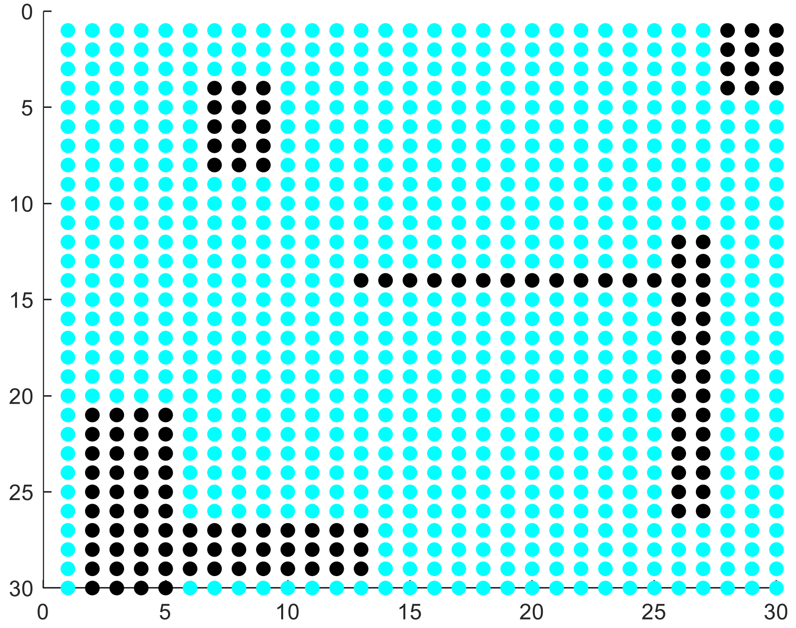
Implementation: an example

Image-Map

```
map_rgb = imread('mappa_test_red.png');
BW = im2bw(map_rgb,0.7);
```

RGB to bool Matrix
(try different threshold values..)

```
plot_map(BW)
```



Implementation: an example

Initialize variables & edge - matrix

```
% initialize variables

G=-1*ones(size(BW,1)*size(BW,2));

dist=inf*ones(size(BW,1)*size(BW,2),1);

prec=inf*ones(size(BW,1)*size(BW,2),1);

nodelist=-1*ones(size(BW,1)*size(BW,2),1);
```

Edge- matrix (tree dependencies)

C(q)

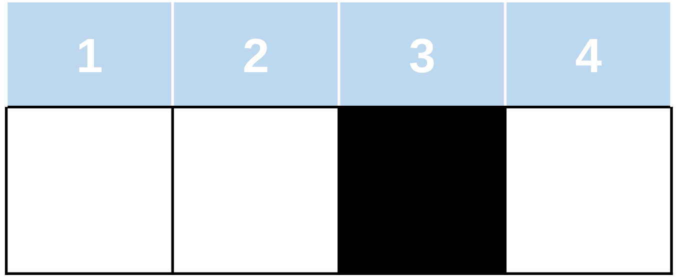
parenting

Q (open queue)

Implementation: an example

Edge - matrix

Example: 4 nodes in a row



Is it connected to that node?

Node considered		1	2	3	4
	1	0	1	-1	-1
	2	1	0	-1	-1
	3	-1	1	-1	1
	4	-1	-1	-1	0

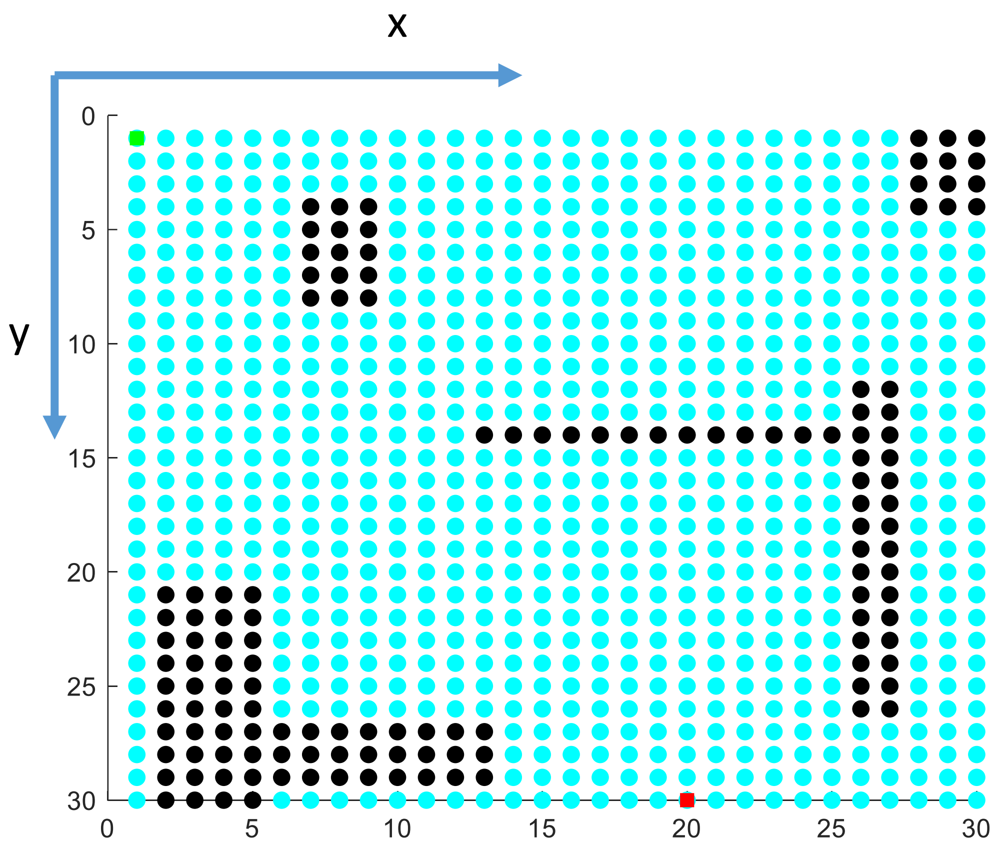
Implementation: an example

Start- goal

```
start_pos = [1,1];
goal_pos = [20,30];

start= (start_pos(2)-1)*size(BW,1)+start_pos(1);
goal= (goal_pos(2)-1)*size(BW,1)+goal_pos(1);

plot(start_pos(1),start_pos(2),'sg','MarkerFaceColor','g')
plot(goal_pos(1),goal_pos(2),'sr','MarkerFaceColor','r')
```



Implementation: an example

Dijkstra: initialization

```
dist(start)=0;
act_node=start;


[~,con_nodes]=find(G(act_node,:)>0);
nodelist(con_nodes,1) = 1;
nodelist(act_node,1)=0;
```

C(q) = 0

actual q

Find connected & add in Q

remove q from Q

Implementation: an example

Dijkstra: steps 1/2

```
while any(nodelist(:,1)==1) && act_node~=goal
        i_con=length(con_nodes);
        while i_con>0
                if dist(con_nodes(i_con))>dist(act_node)+1
                        dist(con_nodes(i_con))= dist(act_node)+1;
                        prec(con_nodes(i_con))=act_node;
                end
                i_con=i_con-1;
        end
```

Q ~= 0 & q ~= goal

evaluate C(q')

update parenting

Implementation: an example

Dijkstra: steps 2/2

```
        [min_val,~]=min(dist(nodelist(:,1)==1));
        new_nodes=find(dist==min_val);
        tmp_i=1;
        while nodelist(new_nodes(tmp_i),1)~=1
                tmp_i=tmp_i+1;
        end
        act_node=new_nodes(tmp_i);
        nodelist(act_node,1)=0;
        [~,con_nodes]=find(G(act_node,:)>0);
        i_con=length(con_nodes);
        while i_con>0
                if nodelist(con_nodes(i_con),1) ~= 0
                                nodelist(con_nodes(i_con),1) = 1;
                end
                i_con=i_con-1;
        end
end
```

select q’ in Q according to C(q’)

remove q from Q

Find connected & add in Q

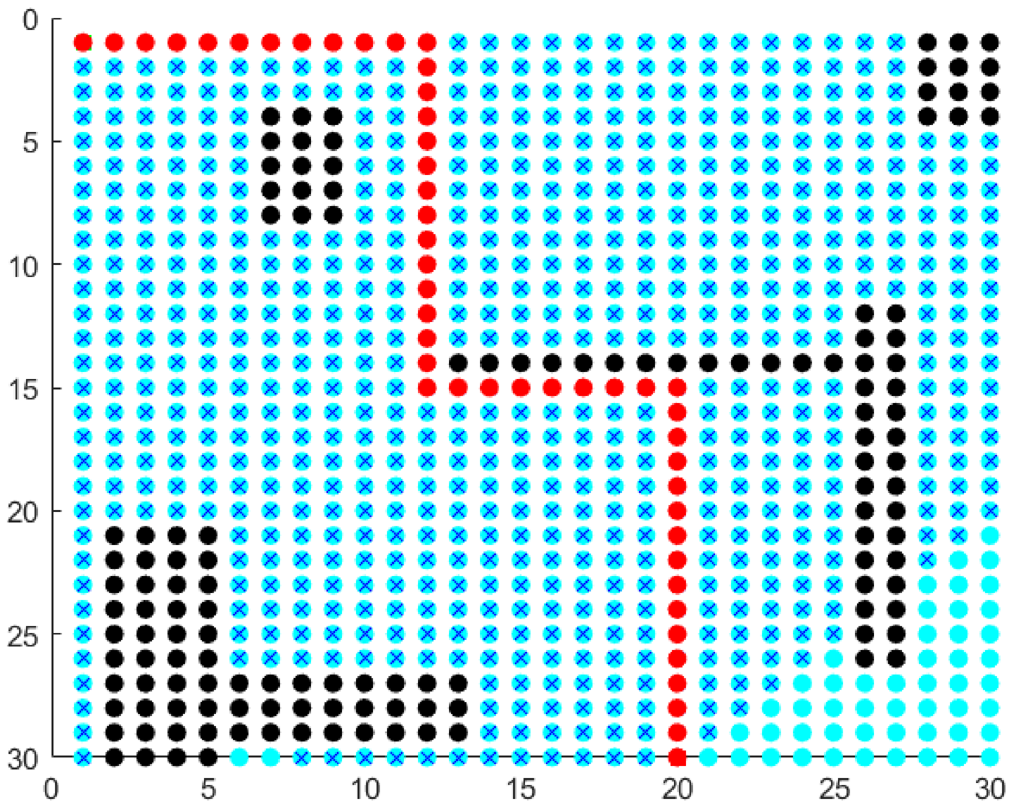
Implementation: an example

Dijkstra: shortest path

```
if dist(goal)<inf
        sol_id=goal;
        path=[];
        while(sol_id~=start)
                path=[sol_id path];
                sol_id=prec(sol_id);
        end
        path=[sol_id path];
```

If a solution is available

recorsively define the sequence



ROS

Assignment IV

What we expect from you

Starting from initial Dijkstra code (v-I) provided:

1. Modify the code to consider diagonals movements as allowed (v-II);
2. Modify the code to implement A* algorithm (v-III);
3. Implement both (v-IV).

Apply the algorithms (all) to the maps created from images (estimate also start&goal) provided and compare the results.

Hints: diagonals & A*

Diagonals:

- Adjacent nodes dist = 1;
- Diagonal nodes dist = $\sqrt{2}$;

A*:

Dijkstra orders by optimal “*cost-to-arrival*”
A* orders by “*cost-to-arrival*”+ *(approximate)* “*cost-to-go*”

$$C(q') = C(q) + C(q, q') + h(q')$$

where $h(q')$ is an underestimate of “cost-to-go

Results

- What is mandatory for the report?

    - Plot of shortest path;
    - Evaluate total number of nodes analyzed;
    - Plot the sequence of «actual nodes» computed
    - (compare & comment results)

That’s it for today…

See you next time!

S. Arrigoni

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