Exam Time Table Model

Group 3 [MA-ZZ]

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1 Mathematical Model

This document contains the mathematical model designed for the assignment of Optimization Methods and Algorithms (2017/2018) for the GROUP 3 of the second part of the course [MA-ZZ].

1.1 Variables

We have defined two main boolean variables. The first is related to the exam in time slots:

$$x_{e,t} = \begin{cases} 1 \rightarrow \textit{If the exam } \boldsymbol{e} \textit{ is assigned to timeslot } \boldsymbol{t} \\ 0 \rightarrow \textit{Otherwise} \end{cases} \tag{1}$$

The second one is used for student and exam enrollment:

$$y_{s,e} = \begin{cases} 1 \rightarrow \textit{If the student } \boldsymbol{s} \textit{ is enrolled in exam } \boldsymbol{e} \\ 0 \rightarrow \textit{Otherwise} \end{cases} \tag{2}$$

We have also defined the number of students enrolled in both conflicting exams e and e' with the variable:

$$\begin{gathered} n_{e,e'} = \sum_{s=1}^{S} y_{s,e} \cdot y_{s,e'} \\ \forall e, e' \in \{1, ..., E\} \wedge (e \neq e') \\ s \in \{1, ..., S\} \end{gathered} \tag{3}$$

There are also other standard variables defined by the assignement:

- S: Number of student.
- t_{max}: Number of time slots.
- e: Number of exam.

1.2 Objective Function

The pourpose of the model is to MINIMIZE the following expression:

$$\begin{gathered} \sum_{e=1}^{E-1} \sum_{e'=e+1}^{E} \sum_{t=1}^{t_{max}-5} \sum_{t'=t}^{t+5} (\frac{2^{5-i}}{|S|} \cdot n_{e,e'} \cdot x_{e,t} \cdot x_{e',t'}) \\ i = t' - t \end{gathered} \tag{4}$$

the problem of this equation is that is not a linear function (due to the product of two decision variables). We need linearize it:

$$K_{e,t,e',t'} = x_{e,t} \cdot x_{e',t'} \tag{5}$$

this is not enough, we also need to define other constraints for forcing the linearization. They can be found in the block of equation 8.

1.3 Constraints

There are two main constraints to define to generating feasible solution, the first is the one related to the possibility to schedule only once an exam during the period:

$$\begin{gathered}\sum_{t=1}^{t_{max}} x_{e,t} = 1 \\ \forall e, e' \in \{1, ..., E\}\end{gathered} \tag{6}$$

The second one is to avoid that two conflict exams can be helded in the same time slots:

$$\begin{gathered}n_{e,e'} \leq |S| \cdot w_{e,e'} \\ K_{e,t,e',t'} \leq 1 - w_{e,e'} \\ w_{e,e'} \in \{0, 1\}\end{gathered} \tag{7}$$

The following constraints are defined due to **linearize the objective function**:

$$\begin{gathered}K_{e,t,e',t'} \leq x_{e,t} \\ K_{e,t,e',t'} \leq x_{e',t'} \\ K_{e,t,e',t'} \geq x_{e,t} + x_{e',t'} - 1\end{gathered} \tag{8}$$

1.4 Final Objective Function

The final version of the objective function is obtained merging the equation 4 and 5 with the following result:

$$\sum_{e=1}^{E-1} \sum_{e'=e+1}^{E} \sum_{t=1}^{t_{max}-5} \sum_{t'=t}^{t+5} \left(\frac{2^{5-i}}{|S|} \cdot n_{e,e'} \cdot K_{e,t,e',t'}\right) \tag{9}$$