



**POLITECNICO
DI TORINO**

Exam Time Table Model

Group 3 [MA-ZZ]

Jacopo Maggio
Stefano Munna
Jacopo Nasi
Andrea Santu
Marco Torlaschi

20 novembre 2017

1 Mathematical Model

This document contains the mathematical model designed for the assignment of Optimization Methods and Algorithms (2017/2018) for the GROUP 3 of the second part of the course [MA-ZZ].

1.1 Variables

We have starting defining 2 main boolean variables. The first is related to the exam in time slots:

$$x_{e,t} = \begin{cases} 1 \rightarrow \text{If the exam } e \text{ is assigned to timeslot } t \\ 0 \rightarrow \text{Otherwise} \end{cases} \quad (1)$$

The second one is used for student and exam enrollment:

$$y_{s,e} = \begin{cases} 1 \rightarrow \text{If the student } s \text{ is enrolled in exam } e \\ 0 \rightarrow \text{Otherwise} \end{cases} \quad (2)$$

We have also defined the number of students enrolled in both conflicting exams e and e' with the variable:

$$\begin{aligned} n_{e,e'} &= \sum_{s=1}^S y_{s,e} \cdot y_{s,e'} \\ \forall e, e' &\in \{1, \dots, E\} \wedge (e \neq e') \\ s &\in \{1, \dots, S\} \end{aligned} \quad (3)$$

1.2 Objective Function

Our model is designed to MINIMIZE the following expression:

$$\sum_{e=1}^{E-1} \sum_{e'=e+1}^E \sum_{t=1}^{t_{max}-5} \sum_{t'=t}^{t+5} \left(\frac{2^{5-i}}{|S|} \cdot n_{e,e'} \cdot x_{e,t} \cdot x_{e',t'} \right) \quad (4)$$

the only problem of this equation is that is not a linear function (due to the product of two decision variables). We need to linearize it:

$$K_{e,t,e',t'} = x_{e,t} \cdot x_{e',t'} \quad (5)$$

this is not enough, we need to define also other constraints in the equation number 8.

1.3 Constrains

There are two main constraints that need to be defined for generating feasible solution, the first is the one related to the possibility to define only once an exam during the examination period:

$$\sum_{t=1}^{t_{max}} x_{e,t} = 1 \quad (6)$$

$$\forall e, e' \in \{1, \dots, E\}$$

The second one is to avoid that two conflict exams can be held in the same time slots:

$$\begin{aligned} n_{e,e'} &\leq |S| \cdot w_{e,e'} \\ K_{e,t,e',t'} &\leq 1 - w_{e,e'} \\ w_{e,e'} &\in \{0, 1\} \end{aligned} \quad (7)$$

The following constraints are defined due to **linearize the objective function**:

$$\begin{aligned} K_{e,t,e',t'} &\leq x_{e,t} \\ K_{e,t,e',t'} &\leq x_{e',t'} \\ K_{e,t,e',t'} &\geq x_{e,t} + x_{e',t'} - 1 \end{aligned} \quad (8)$$

1.4 Final Objective Function

The final version of the objective function is obtained merging the equation 4 and 5 with the following result:

$$\sum_{e=1}^{E-1} \sum_{e'=e+1}^E \sum_{t=1}^{t_{max}-5} \sum_{t'=t}^{t+5} \left(\frac{2^{5-i}}{|S|} \cdot n_{e,e'} \cdot K_{e,t,e',t'} \right) \quad (9)$$