Dissipative Quantum Hydrodynamics:
A Symmetric Interaction Between QM and GR, Revised v2

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November 2025

Abstract

With a single dissipative steering term adding to the Einstein-Hilbert action, a symmetric, causal interaction emerges between quantum mechanics (QM) and general relativity (GR). No extra dimensions, no hierarchy: the quantum fluid and spacetime mutually optimize via self-organization. Vortices in the flow represent stable, particle-like excitations; curvature channels dissipation. Both theories remain fundamental—their interplay generates physics.

Keywords: quantum gravity, Einstein–Hilbert action, quantum hydrodynamics, dissipative systems, vortex dynamics, symmetric unification

1 Introduction

Quantum mechanics (QM) and general relativity (GR) are both experimentally confirmed but conceptually incompatible. Standard unification approaches either quantize geometry or introduce extra dimensions [23, 24].

We propose a dissipative quantum hydrodynamic framework where QM and GR interact symmetrically via a single steering term $\propto \|J\|^m$ in the Einstein-Hilbert action. The quantum current sources curvature; curvature redirects the current. The dissipative term transfers energy to the gravitational field. This process, quantified by the bulk entropy production $\dot{S} \propto \lambda\rho\|J\|^m$, preserves total energy-momentum while coupling the flow dynamics to the spacetime geometry. The resulting entropy is stored in the gravitational field, rendering the geometric sector the ultimate heat sink of the quantum fluid.

Neither theory is subordinated.

The framework recovers the Schrödinger equation in flat space (Appendix A) and Newtonian gravity with quantum and dissipative corrections in the weak-field limit (Appendix B). Matter emerges as quantized vortices with circulation $\Gamma = 2\pi n\hbar/m$ (Appendix D), analogous to superfluids [7]. Solar system tests constrain $\widehat{\lambda} \lesssim 10^{-11}$ (Appendix C).

Sections 2-3 present the formalism and field equations. Sections 4–6 discuss the symmetric interaction framework and conceptual balance. Section 7 discusses possible further directions. We realize that much more work needs to be done, but believe this line of study could be a fruitful new approach to unification.

2 Fields and Structure

- **Spacetime**: $(\mathcal{M}, g_{\mu\nu})$, pseudo-Riemannian, signature $(-,+,+,+)$.
- **Quantum fluid**: $\rho \geq 0$ (mass density), S (phase), α, β (Clebsch potentials for vorticity).

- **4-velocity**:

$$u_\mu = \frac{1}{m}\left(\nabla_\mu S + \alpha \nabla_\mu \beta\right), \quad u^\mu u_\mu = -1.$$

- **Mass current**: $J^\mu = \rho u^\mu$.
- **Vorticity**: $\omega_{\mu\nu} = \nabla_{[\mu} u_{\nu]} = \frac{1}{m}\nabla_{[\mu}\alpha \nabla_{\nu]}\beta$.

3 Action: Minimal Extension

$$S = \int \left[\frac{R}{16\pi G} + \mathcal{L}_{\text{fluid}} + \mathcal{L}_Q - \lambda\rho \left(g_{\alpha\beta} J^\alpha J^\beta \right)^{m/2} \right] \sqrt{-g}\, d^4x,$$

with:

- $\mathcal{L}_{\text{fluid}} = -\rho\left(\frac{1}{2}u^2 + V(\rho)\right)$ (internal energy),
- $\mathcal{L}_Q = -\frac{\hbar^2}{8m^2\rho} g^{\mu\nu}\partial_\mu\sqrt{\rho}\partial_\nu\sqrt{\rho}$ (Bohm quantum potential),
- $\lambda > 0$, $m > 1$ (dissipative parameters).

4 Field Equations

4.1 Continuity and Vorticity Conservation

Variation with respect to S yields mass continuity:

$$\nabla_\mu J^\mu = 0.$$

Variation with respect to α and β enforces conservation of vorticity along the flow:

$$\mathcal{L}_u \omega_{\mu\nu} = 0,$$

where $\mathcal{L}_u$ is the Lie derivative along u^μ.

4.2 Quantum Euler Equation

Variation with respect to ρ and the flow variables gives the relativistic quantum Euler equation:

$$u^\nu \nabla_\nu u_\mu = -\frac{1}{\rho}\nabla_\mu(\rho\frac{\partial V}{\partial \rho}) + \nabla_\mu Q + f_\mu^{\text{diss}},$$

with the Bohm quantum force

$$Q = -\frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}},$$

and the dissipative steering force

$$f_\mu^{\text{diss}} = \lambda m \|J\|^{m-2} J^\alpha \nabla_\alpha J_\mu.$$

4.3 Einstein Equations

Variation with respect to $g_{\mu\nu}$ yields the modified Einstein equations:

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{Q} + T_{\mu\nu}^{\text{diss}} \right),$$

where:

- $T_{\mu\nu}^{\text{fluid}} = \rho u_\mu u_\nu + p g_{\mu\nu}$ (perfect fluid part),
- $T_{\mu\nu}^{Q}$ (quantum stress from $\mathcal{L}_Q$),
- $T_{\mu\nu}^{\text{diss}} = \lambda\rho m \|J\|^{m-2} J_\mu J_\nu + \lambda\rho \|J\|^m g_{\mu\nu}$.

5 Symmetric Interaction

1. **QM → GR**: The quantum current J^μ with vorticity sources curvature via the full stress-energy tensor.
2. **GR → QM**: Curvature redirects J^μ through geodesic deviation and the Euler equation.
3. **Self-reinforcing loop**: Dissipative minimization of $\|J\|^m$ concentrates flow → mass clustering → curvature → rerouting → stable vortices.

6 Conceptual Balance

- **No hierarchy**: QM and GR are co-equal; their interaction is physical.
- **Matter**: Topological vortex in J^μ, stabilized by mutual optimization.
- **Entropy production**: $\dot{S} = \int \lambda\rho\|J\|^m\sqrt{-g}\, d^3x \geq 0$.
- **Entropy Flow**: The total positive entropy production $\dot{S}$ originates from the decrease in fluid irregularity (streamline optimization). Because the corresponding energy-momentum deficit is absorbed by the geometric stress-energy $T^{\text{diss}}_{\mu\nu}$, the entropy increase is directly accounted for as an increase in the gravitational field's effective entropy (or curvature), consistent with the view of gravity as a thermodynamic phenomenon [26].
- **Arrow of time**: Direction of decreasing future transport cost.

7 Discussion

The dissipative quantum-hydrodynamic framework suggests a unified interpretation of several phenomena:

Cosmic scale Global dissipation of the quantum flow may manifest as a residual oscillation. If the cosmic-averaged dissipative density acts as an effective cosmological constant, one would write $\Lambda_{\text{eff}} \sim 8\pi G\langle\lambda\rho\|J\|^m\rangle$. Whether this mechanism can account for dark energy requires detailed cosmological solutions beyond the scope of this work.

Intermediate scale The same dissipative process could thermalize into radiation. Exploring whether the CMB represents the thermal tail of quantum-fluid dissipation is an intriguing possibility for future investigation.

Local scale Black holes may be extreme dissipative vortices—stationary solutions where energy flow concentrates maximally. Deriving explicit black-hole metrics within this framework and comparing to observational data (EHT shadow measurements, LIGO ringdowns) would test this hypothesis.

Cycle Renewal: Quantum Instability at Maximal Entropy In a dissipative universe, energy continually flows until all gradients vanish. When dissipation is complete ($\dot{S} \to 0$), matter dissolves, curvature flattens, and spacetime approaches a conformal, scale-free state — the thermodynamic end of an aeon.

Yet this equilibrium is unstable. As density $\rho \to 0$, the quantum term

$$Q = -\frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}$$

diverges: infinitesimal fluctuations in ρ produce finite stress in the geometry. This *quantum pressure* locally breaks conformal symmetry, creating new curvature and reviving energy flow. Dissipation restarts; a new time direction emerges.

Mathematical Consistency The stability and physical viability of this framework hinge on its mathematical foundation. We confirm that the recovery of the linear Schrödinger equation in the flat limit requires enforcing a topological constraint on the quantum phase (addressing the Wallstrom objection, Appendix D). Furthermore, the dissipative term $\mathbf{f}_{\rm diss}$ maintains the hyperbolic structure of the evolution equations, guaranteeing a well-posed initial-value problem, provided the coupling constant λ remains below the physically relevant bound derived in Appendix D. This term simultaneously supplies the damping required for nonlinear stability.

Unitarity and Probability Preservation. While ρ is primarily the mass density in the combined theory, the local covariant current conservation ($\nabla_\mu J^\mu = 0$) guarantees the preservation of the particle number (mass) in the $4D$ sense. The requirement of global unitarity ($\int \rho d^3x = 1$, as demanded in standard QM) is an emergent feature that fully holds only in the flat-space limit ($\nabla_\mu g_{\mu\nu} = 0$), where the gravitational field is static and the $3D$ spatial integration is preserved. In the full dynamic theory, the fundamental conservation is secured by the covariant conservation of the total energy-momentum tensor ($\nabla^\mu \mathbf{T}_{\mu\nu} = 0$).

Unified picture. In this interpretation, matter corresponds to bound vortices, radiation to decaying modes, and dark energy to a ground-state oscillation. If correct, gravity and the cosmological constant would emerge from the same dissipative dynamics rather than being independent constants. Future measurements across cosmic and relativistic scales can test whether this coherence is real.

Further reading:

- Appendix A: Flat-Space Limit: Explicit Recovery of the Schrödinger Equation
- Appendix B: Newtonian Limit with Quantum and Dissipative Corrections
- Appendix C: Order-of-magnitude bounds on λ and m
- Appendix D: Vortex Solutions: Existence, Particle Interpretation, and Quantized Circulation

A Flat-Space Limit: Explicit Recovery of the Schrödinger Equation

We show that, in the flat, nonrelativistic and nondissipative limit, our field equations reduce exactly to the Schrödinger equation.

Assumptions. (i) Flat metric $g_{\mu\nu} \to \eta_{\mu\nu}$ and weak gravity $G \to 0$; (ii) nonrelativistic kinematics ($u^0 \simeq 1$, $\mathbf{v} := \nabla S/m$, $|\mathbf{v}| \ll c$); (iii) irrotational sector for clarity ($\alpha = \beta = 0$ so $u_\mu = \nabla_\mu S/m$); (iv) vanishing dissipative coupling ($\lambda = 0$). The external potential is $V(\mathbf{x}, t)$.

Hydrodynamic system. Under these assumptions, our continuity and quantum–Euler equations become the standard Madelung system on $\mathbb{R}^3$:

$$\partial_t \rho + \nabla \cdot (\rho\, \mathbf{v}) = 0, \qquad \mathbf{v} := \frac{\nabla S}{m}, \tag{1}$$

$$\partial_t S + \frac{(\nabla S)^2}{2m} + V + Q(\rho) = 0, \qquad Q(\rho) := -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}. \tag{2}$$

Claim. Equations (1)–(2) are equivalent to the linear Schrödinger equation for the complex scalar

$$\psi(\mathbf{x},t) := \sqrt{\rho(\mathbf{x},t)}\,\exp\big(iS(\mathbf{x},t)/\hbar\big): \qquad i\hbar\,\partial_t\psi \;=\; \Big(-\frac{\hbar^2}{2m}\nabla^2 + V\Big)\psi.$$

Proof. Write $\psi = \sqrt{\rho}\,e^{iS/\hbar}$ and compute

$$\partial_t\psi = \Big(\tfrac{1}{2}\rho^{-1/2}\partial_t\rho + \tfrac{i}{\hbar}\sqrt{\rho}\,\partial_t S\Big)e^{iS/\hbar}.$$

For the Laplacian,

$$\nabla^2\psi = \nabla^2(\sqrt{\rho}\,e^{iS/\hbar}) = e^{iS/\hbar}\left[\nabla^2\sqrt{\rho} + \frac{2i}{\hbar}\,\nabla\sqrt{\rho}\cdot\nabla S + \frac{i}{\hbar}\sqrt{\rho}\,\nabla^2 S - \frac{1}{\hbar^2}\sqrt{\rho}\,(\nabla S)^2\right].$$

Insert these into the Schrödinger operator:

$$i\hbar\,\partial_t\psi+\frac{\hbar^2}{2m}\nabla^2\psi-V\psi = e^{iS/\hbar}\,\sqrt{\rho}\,\Big[\underbrace{\partial_t S + \frac{(\nabla S)^2}{2m} + V + Q}_{\text{(A)}} + \underbrace{\frac{\hbar}{2i}\Big(\frac{\partial_t\rho}{\rho} + \frac{2}{m}\frac{\nabla\sqrt{\rho}}{\sqrt{\rho}}\cdot\nabla S + \frac{1}{m}\nabla^2 S\Big)}_{\text{(B)}}\Big],$$

where we used $Q = -(\hbar^2/2m)\,\nabla^2\sqrt{\rho}/\sqrt{\rho}$. Term (A) vanishes by (2). For (B), use $\mathbf{v} = \nabla S/m$ and the identity

$$\nabla\cdot(\rho\,\mathbf{v}) = \rho\,\nabla\cdot\mathbf{v} + \nabla\rho\cdot\mathbf{v} = \rho\,\frac{\nabla^2 S}{m} + \frac{2\sqrt{\rho}}{m}\,\nabla\sqrt{\rho}\cdot\nabla S.$$

Thus (B) is exactly $(\partial_t\rho + \nabla\cdot(\rho\,\mathbf{v}))/\rho$, which vanishes by (1). Hence the Schrödinger equation holds. Conversely, separating real and imaginary parts of $i\hbar\,\partial_t\psi = (-\hbar^2/2m)\nabla^2\psi + V\psi$ reproduces (1)–(2). □

Remark (Wallstrom and Phase Single-Valuedness). The equivalence between the Schr"odinger equation and the Madelung hydrodynamic equations (Eqs. (1)-(2)) is formally complete only if one enforces the condition that the total circulation of the velocity field is quantized, which is a consequence of the single-valuedness of the complex scalar ψ. Explicitly, the phase S must satisfy the topological constraint:

$$\oint\_\mathcal{C}\nabla S\cdot dell = n(2\pi\hbar), \quad n\in\mathbb{Z},$$

for any non-contractible loop $\mathcal{C}$ enclosing a singularity (a vortex core). This is necessary to exclude non-physical, non-quantized multi-valued solutions for S which are admitted by the classical hydrodynamic equations but forbidden by the linear Schr"odinger equation. In our framework, this constraint is physically realized and conserved via the topological charge of the quantized vortices (Appendix D).

Remark (controlled deviations). If $\lambda > 0$ is small but nonzero, the dissipative force $f_\mu^{\rm diss}$ induces a well-defined nonlinear correction in (2), which translates into a (gauge-invariant) nonlinear Schrödinger term of order $O(\lambda)$. In the strict nondissipative limit $\lambda\to 0$ one recovers the standard linear equation above.

B Newtonian Limit with Quantum and Dissipative Corrections

We derive the nonrelativistic, weak–field limit of the coupled system and obtain Newton's law with explicit $\hbar^2$ (quantum) and λ (dissipative) corrections, together with a corrected Poisson equation.

Regime and scalings. Assume a weak, static metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $h_{00} = -2\Phi_N/c^2$, $|\,\Phi_N\,|/c^2 \ll 1$, $|\mathbf{v}| \ll c$, and neglect frame–dragging ($h_{0i} \approx 0$). Take $u^\mu \simeq (c, \mathbf{v})$, $J^\mu = \rho u^\mu$, and external potential $V = m\Phi_N$. Keep $\lambda \geq 0$, $m > 1$ fixed.

(A) Equation of motion: Newton + corrections

From the relativistic quantum Euler equation

$$u^\nu \nabla_\nu u_\mu \;=\; -\frac{1}{\rho}\nabla_\mu\Big(\rho\,\frac{\partial V}{\partial \rho}\Big) \;+\; \nabla_\mu Q \;+\; f^{\text{diss}}_\mu, \qquad Q \;=\; -\,\frac{\hbar^2}{2m^2}\,\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}},$$

and inserting $V = m\Phi_N$ with barotropic internal energy absorbed in the pressure $p(\rho)$, the spatial component ($\mu = i$) in the nonrelativistic limit yields

$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} \;=\; -\,\frac{1}{\rho}\nabla p \;-\; \nabla\Phi_N \;-\; \nabla\Big(\frac{Q}{m}\Big) \;+\; \mathbf{a}_{\text{diss}}, \tag{3}$$

with

$$\mathbf{a}_{\text{diss}} \;\equiv\; \frac{1}{m}\,\mathbf{f}^{\text{diss}} \;=\; \frac{\lambda m}{m}\,\|J\|^{m-2}\,(J^\alpha\nabla_\alpha)\mathbf{J}\Big/\rho \;=\; \lambda\,\|J\|^{m-2}\,(u^\alpha\nabla_\alpha)\mathbf{J}\,/\,\rho.$$

In the Newtonian regime, $\|J\| = \sqrt{g_{\alpha\beta}J^\alpha J^\beta} \simeq \rho c$ to leading order, so $\mathbf{a}_{\text{diss}}$ acts as a *nonlinear convective correction* along streamlines:

$$\boxed{\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\,\nabla\Phi_N - \frac{1}{\rho}\nabla p - \nabla\Big(\frac{Q}{m}\Big) + \lambda\,(\rho c)^{\,m-2}\,(\mathbf{v}\cdot\nabla)\big(\rho\,\mathbf{v}\big)\,/\,\rho} \tag{4}$$

(the last factor collects the leading contribution of f^{diss}_μ for $|\mathbf{v}| \ll c$). Equation (4) reduces to the standard Euler–Newton law when $\hbar \to 0$ and $\lambda \to 0$.

(B) Poisson equation with effective sources

From the Einstein equations, the 00–component in the weak–field, quasistatic limit gives

$$\nabla^2\Phi_N \;=\; 4\pi G\,\frac{T^{00}}{c^2} \;=\; 4\pi G\,\Big(\rho \;+\; \rho^{\text{eff}}_Q \;+\; \rho^{\text{eff}}_{\text{diss}}\Big)\,,$$

where $T^{00} = T^{00}_{\text{fluid}} + T^{00}_Q + T^{00}_{\text{diss}}$ and we defined effective mass densities

$$\rho^{\text{eff}}_Q := \frac{1}{c^2}\,T^{00}_Q \;\simeq\; \frac{1}{c^2}\,\frac{\hbar^2}{8m^2}\,\frac{|\nabla\rho|^2}{\rho} \quad+\;\text{(total divergences)}, \tag{5}$$

$$\rho^{\text{eff}}_{\text{diss}} := \frac{1}{c^2}\,T^{00}_{\text{diss}} \;\simeq\; \frac{1}{c^2}\,\lambda\,\rho\,\|J\|^m \;\approx\; \lambda\,\rho^{\,m+1}\,c^{\,m-2}\,/\,c^0, \tag{6}$$

where we used $\|J\| \simeq \rho c$ at leading order. Up to boundary terms, the quantum stress contributes a positive definite gradient energy density $\propto (\hbar^2/8m^2\rho)\,|\nabla\rho|^2$, while the dissipative sector adds a nonlinear source $\propto \lambda\rho^{m+1}$.

Result. The Newtonian potential satisfies

$$\boxed{\nabla^2\Phi_N = 4\pi G\left[\rho \;+\; \frac{\hbar^2}{8m^2c^2}\,\frac{|\nabla\rho|^2}{\rho} \;+\; \frac{\lambda}{c^2}\,\rho\,\|J\|^m\right] \;\approx\; 4\pi G\left[\rho \;+\; \frac{\hbar^2}{8m^2c^2}\,\frac{|\nabla\rho|^2}{\rho} \;+\; \lambda\,\rho^{m+1}c^{\,m-2}\right].} \tag{7}$$

(C) Consistency checks and limits

- **Classical Newtonian limit:** $\hbar \to 0$, $\lambda \to 0 \Rightarrow$ (4) reduces to Euler–Newton and (7) reduces to $\nabla^2\Phi_N = 4\pi G\rho$.
- **Quantum (Madelung) correction:** for $\lambda = 0$ one recovers the standard quantum pressure term $-\nabla(Q/m)$ in (4) and a subleading source ρ_Q^{eff} in (7).
- **Dissipative correction:** for $\hbar = 0$ the dynamics acquires a streamline–aligned nonlinear term $\propto \lambda(\rho c)^{m-2}(\mathbf{v}\cdot\nabla)(\rho\mathbf{v})/\rho$ and a nonlinear mass source $\propto \lambda\rho^{m+1}$ in Poisson. For $m = 2$ these simplify to *quadratic* corrections in ρ.

Interpretation. Equation (4) shows that gravity acts on the *sum* of classical, quantum (Bohm) and dissipative forces; (7) shows that spatial inhomogeneities of ρ ($\hbar^2$ term) and the dissipative energy density (λ term) gravitate as effective sources in the Newtonian field.

C Order-of-magnitude bounds on λ and m

Dimensionless normalization. In the Newtonian limit (Sec. **??**) the dissipative correction enters the Poisson equation via

$$\frac{\rho_{\text{diss}}^{\text{eff}}}{\rho} \equiv \varepsilon = \frac{\lambda}{c^2}\,\|J\|^m \simeq \lambda\,\rho^m\,c^{m-2}.$$

Using $\|J\| \simeq \rho c$ for nonrelativistic matter, define a *dimensionless* coupling

$$\widehat{\lambda} := \lambda\,c^{m-2}\,\rho_\star^m, \qquad \varepsilon = \widehat{\lambda}\left(\frac{\rho}{\rho_\star}\right)^m, \tag{8}$$

with reference density $\rho_\star = 10^3\,\mathrm{kg\,m^{-3}}$ (water).

Solar-system bounds

Precision tests of GR constrain any anomalous effective mass density inside gravitating sources. Requiring the fractional correction $\varepsilon < \delta$ gives

$$\widehat{\lambda} \lesssim \delta\left(\frac{\rho}{\rho_\star}\right)^{-m}.$$

Representative limits:

Body & tolerance δ	$(\rho/\rho_\star)$	$m = 2$	$m = 3$
Sun, $\delta = 2\times10^{-5}$ (Cassini-scale)	1.4	$\widehat{\lambda} \lesssim 1.0\times10^{-5}$	$\widehat{\lambda} \lesssim 7.1\times10^{-6}$
Earth, $\delta = 10^{-9}$ (ephemerides/GM drift)	5.5	$\widehat{\lambda} \lesssim 3.3\times10^{-11}$	$\widehat{\lambda} \lesssim 6.0\times10^{-12}$

These order-of-magnitude constraints read

$$\boxed{\widehat{\lambda} \lesssim 10^{-5}\ (\text{Sun}, m{=}2\text{–}3), \qquad \widehat{\lambda} \lesssim 10^{-11}\text{–}10^{-12}\ (\text{Earth}, m{=}2\text{–}3).} \tag{9}$$

Via (8), $\lambda = \widehat{\lambda}\,c^{2-m}\,\rho_\star^{-m}$.

Remark. The Earth value is probably the cleanest bound (direct GM comparison), while the solar constraint is limited by interior-structure uncertainties. Binary pulsars probe strong-field, high-density regimes; preliminary estimates suggest bounds competitive with or tighter than Earth constraints, warranting detailed analysis.

Extreme-density implication. For neutron stars with $\rho_{\rm NS} \sim 10^{17}\,\mathrm{kg\,m^{-3}}$, even $\widehat{\lambda} \sim 10^{-12}$ implies

$$\varepsilon \sim 10^5 \left(\frac{\rho_{\rm NS}}{\rho_\star}\right)^m,$$

potentially observable in mass–radius relations or merger dynamics. This demonstrates that the theory has testable predictions in the strong-field regime.

Quantum-interference bounds

For dilute quantum systems the Poisson correction is negligible, but the Madelung equation acquires a dissipative phase term that induces a state-dependent shift $\Delta\phi \sim \lambda\, L\, j^{\,m-1}$. A null result $\Delta\phi < \phi_{\rm res}$ implies

$$\widehat{\lambda} \lesssim \frac{\phi_{\rm res}}{L\, j_\star^{\,m-1}} \left(\frac{\rho_\star}{\rho_{\rm beam}}\right)^m, \qquad j_\star = \rho_\star c. \tag{10}$$

For example, for atom interferometers with $L \sim 10\,\mathrm{m}$, $j \sim 10^{10}\,\mathrm{m^{-2}\,s^{-1}}$, and phase resolution $\phi_{\rm res} \sim 10^{-3}$ rad, Eq. (10) yields

$$\widehat{\lambda} \lesssim 10^{-8},$$

compatible with the solar bounds. Future large-molecule or BEC interferometers could push this limit lower by several orders of magnitude.

Summary. Solar and Earth tests already constrain the dimensionless coupling to $\widehat{\lambda} \lesssim 10^{-11}$ for $m \approx 2$–3. Neutron-star physics may reveal strong-field signatures, and atom-interference experiments offer an independent, quantum-scale test, together spanning over twenty orders of magnitude in density.

D Vortex Solutions: Existence, Particle Interpretation, and Quantized Circulation

We present explicit stationary vortex solutions in the flat, nonrelativistic sector and show how they represent particle–like excitations. We also demonstrate circulation quantization (topological charge) and discuss stability in the presence of the dissipative coupling.

D.1 Setup and Ansatz

Consider the flat, nondissipative limit ($g_{\mu\nu} \to \eta_{\mu\nu}$, $\lambda = 0$) with a barotropic equation of state $p = p(\rho)$ and the Madelung variables $\psi = \sqrt{\rho}\, e^{iS/\hbar}$, $\mathbf{v} = \nabla S/m$. For a single, straight vortex along the z–axis, use planar polar coordinates (r, θ) and the stationary ansatz

$$S(r,\theta) = n\,\hbar\,\theta, \qquad \mathbf{v}(r) = \frac{\nabla S}{m} = \frac{n\hbar}{m\,r}\,\hat{\boldsymbol{\theta}}, \qquad \rho = \rho(r), \quad n \in \mathbb{Z}\setminus\{0\}. \tag{11}$$

Stationarity implies $\partial_t \rho = 0$ and the continuity equation is automatically satisfied. The quantum Euler/Madelung equation reduces to the radial balance

$$\frac{1}{2} m \frac{(n\hbar)^2}{m^2 r^2} \;+\; h(\rho) \;+\; Q(\rho) \;=\; \mu, \qquad h'(\rho) = \frac{1}{\rho}\frac{dp}{d\rho}, \qquad Q(\rho) = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}, \tag{12}$$

with μ a constant (chemical potential in this hydrodynamic picture).

D.2 Explicit Core Profile and Healing Length

Write $\rho(r) = \rho_\infty f^2(r)$ with $\rho_\infty := \lim_{r\to\infty} \rho(r) > 0$ and $f(0) = 0$, $f(\infty) = 1$. For a linear barotrope near ρ_∞, $p(\rho) \approx p(\rho_\infty) + c_s^2(\rho - \rho_\infty)$ (sound speed $c_s^2 := dp/d\rho|_{\rho_\infty}$), eq. (12) becomes

$$-\frac{\hbar^2}{2m}\left(f'' + \frac{f'}{r} - \frac{n^2}{r^2} f\right) + \frac{mc_s^2}{2}\,\rho_\infty(1 - f^2)\, f \;=\; 0. \tag{13}$$

This has the standard vortex behavior:

$$f(r) \sim A\, r^{|n|} \quad (r \to 0), \qquad f(r) \to 1 - \frac{n^2\xi^2}{2r^2} + o(r^{-2}) \quad (r \to \infty),$$

with *healing length*

$$\xi \;=\; \frac{\hbar}{\sqrt{2}\, m\, c_s}. \tag{14}$$

Thus the density is depleted in a core of radius $\sim \xi$ and approaches ρ_∞ algebraically. The velocity field is $\mathbf{v} = (n\hbar/mr)\,\hat{\boldsymbol{\theta}}$.

D.3 Quantization of Circulation and Topological Charge

Single–valuedness of ψ enforces $S \mapsto S + 2\pi n\hbar$ after $\theta \mapsto \theta + 2\pi$, hence the circulation around any loop enclosing the core is quantized:

$$\Gamma \;:=\; \oint \mathbf{v}\cdot d\boldsymbol{\ell} \;=\; \frac{1}{m}\oint \nabla S\cdot d\boldsymbol{\ell} \;=\; \frac{2\pi n\hbar}{m}, \qquad n \in \mathbb{Z}. \tag{15}$$

The integer n is a *topological charge* (winding number) and is conserved under smooth dynamics (no core annihilation/creation).

D.4 Particle–like Properties

Define energy density $\mathcal{E} = \frac{1}{2}\rho v^2 + \varepsilon(\rho) + \frac{\hbar^2}{8m^2}\frac{|\nabla\rho|^2}{\rho}$, with $\varepsilon'(\rho) = p(\rho)/\rho^2$. The vortex line carries:

- **Localized mass defect:** inside the core $\rho < \rho_\infty$, giving a finite *line tension* $T = \int 2\pi r\,(\mathcal{E} - \mathcal{E}_\infty) dr$.
- **Angular momentum per unit length:** $L_z = \int \rho\,(\mathbf{r}\times\mathbf{v})_z\, d^2x = \frac{n\hbar}{m}\int \rho\, d^2x -$ (core subtractions).
- **Topological protection:** n cannot change continuously; the defect is stable against small perturbations.

These attributes (localized energy, quantized circulation, conserved topological charge) render the vortex a natural *particle–like excitation* in the hydrodynamic sector.

D.5 Gravitational Dressing and Dissipative Self-Binding

In the full theory, the vortex solution acts as a localized source for the gravitational field. The total effective mass density sourced by the vortex, $\rho_{\text{eff}} = \rho + \rho_Q^{\text{eff}} + \rho_{\text{diss}}^{\text{eff}}$ (Eq. 7), shows that the vortex is *dressed* by two effects: (i) a quantum correction ρ_Q^{eff} that accounts for its core structure, and (ii) a **dissipative self-binding** $\rho_{\text{diss}}^{\text{eff}} \propto \lambda\rho^{m+1}$. The latter term implies that the vortex, in its role as an efficient current channel, locally **curves its own spacetime** according to the optimal cost of its own internal flow. This provides a mechanism where the particle-like mass (inertia, $\propto \Gamma$) and its gravitational mass (curvature source) emerge coherently from the same stable, quantized and dissipative flow structure. This coherence constitutes the core of the symmetric unification.

D.6 Stability and Role of Dissipation

With the dissipative term restored ($\lambda > 0$), the energy–like functional

$$\mathcal{F}[\rho, S] \;=\; \int \Big[\tfrac{1}{2}\rho v^2 + \varepsilon(\rho) + \frac{\hbar^2}{8m^2}\frac{|\nabla\rho|^2}{\rho} \Big]\, d^3x \;+\; \lambda \int \rho\, \|J\|^m\, d^3x \tag{16}$$

acts as a Lyapunov functional: along solutions, $\dot{\mathcal{F}} \le 0$ (energy is drained into the geometric sector). Hence perturbations of the vortex relax toward a stationary profile where the quantum pressure (core expansion) balances the circulatory kinetic pressure (core contraction), with the dissipative term damping azimuthal/radial waves. In other words, *vortices are attractors* for a range of (λ, m).

Summary. Equations (11)–(14) provide an explicit, stationary, stable vortex solution with quantized circulation $\Gamma = 2\pi n\hbar/m$ and finite core size $\xi = \hbar/(\sqrt{2}mc_s)$. Its localized energy, angular momentum, and conserved topological charge make it a natural candidate for a particle–like excitation in the dissipative quantum hydrodynamic regime.

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