Scale-Invariance, Fractal Structure, and $1/f^p$ Spectra from Dissipative Dynamics Above the Planck Threshold

Consequences of the Einstein–Hilbert–Krol Framework

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Abstract

Many natural systems exhibit scale-invariant structure: fractal geometry in space, power-law correlations in time, and ubiquitous $1/f^p$ noise. In a companion paper [1], we showed that a dissipative extension of the Einstein–Hilbert action implies a minimal semantic threshold (l_P, t_P), below which physical concepts lose operational meaning. Above this threshold, the theory contains no intrinsic macroscopic scale. We demonstrate that this naturally generates fractal spatial structure and temporal power-laws, with $p = 1$ selected as the unique minimal-action exponent. Furthermore, homogeneous scaling of the full action fixes the nonlinear dissipative exponent m once the physical dimension of the coupling is known, and predicts a slowly running spectral exponent

$$p(f) = 1 + \frac{a}{\ln(f/f_{\min})}$$

in EEG spectra, testable with public neurophysiological data.

Keywords: scale invariance; fractals; $1/f$ noise; dissipative dynamics; Einstein–Hilbert–Krol action; Planck threshold

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1 Introduction

A remarkable property of nature is the absence of preferred absolute scales above a minimal threshold. River networks, turbulence, earthquakes, metabolic scaling, galaxy clustering, neuronal fluctuations, and economic systems all show scale-free behaviour spanning multiple decades in resolution.

In a companion paper [1], we demonstrated that the dissipative Einstein–Hilbert–Krol action implies a minimal spacetime resolution (l_P, t_P), below which curvature, action, entropy production, and causal order cease to have operational meaning. Crucially, above this threshold the continuum field equations contain no characteristic length or time. (see also Appendix!C)

The present article explores the structural consequences: the emergence of fractal spatial structure, temporal power-laws, and the universality of $1/f$ spectra in dissipative continua.

2 Scale-invariance in nature

Empirical systems showing scale-free behaviour include:

- fractal branching in rivers, lightning, and biological networks [3];
- turbulent cascades with power-law spectra [4];
- Gutenberg–Richter scaling in earthquakes [4];
- fractal cosmic large-scale structure [5];
- metabolic and ontogenetic scaling laws [6];
- $1/f$ noise in physiology, neuroscience, and condensed matter [**?**, 7].

The ubiquity of these patterns suggests a structural origin rather than coincidence.

3 Why dissipative systems become scale-invariant above a threshold

For a dissipative continuum with a minimal meaningful resolution (such as the Planck cell), three principles hold:

1. Below (l_P, t_P) physical quantities such as curvature, action and entropy density diverge or become undefined.
2. Above (l_P, t_P) the field equations contain no intrinsic macroscopic scale.
3. Dissipation drives the system toward states that minimise total action production subject to flux balance.

In the absence of intrinsic scales, stable attractors of the dynamics must be invariant under global dilations $x^\mu \mapsto \lambda x^\mu$. This extends the concept of self-organized criticality [9, 10] from discrete sandpile models to continuum field theories.

4 Spatial fractals and temporal power-laws

If geometry has no characteristic spatial scale, invariance under $x \mapsto \lambda x$ implies fractal scaling:

$$N(\lambda L) \sim \lambda^{-D} N(L), \tag{1}$$

where $N(L)$ counts structures above scale L and D is a fractal dimension.

If dynamics has no characteristic time, invariance under $t \mapsto \lambda t$ implies

$$C(t) \sim t^{-\alpha}, \tag{2}$$

for the temporal correlation function $C(t)$. The corresponding power spectrum behaves as

$$S(f) \sim f^{-(1-\alpha)} = \frac{1}{f^p}, \tag{3}$$

with $p = 1 - \alpha$. Thus fractals in space and $1/f^p$ spectra in time are two manifestations of the same underlying principle: scale-invariance of structure.

5 Scale-invariance from the Einstein–Hilbert–Krol action

The Einstein–Hilbert–Krol action consists schematically of three sectors,

$$S = S_{\mathrm{EH}} + S_Q + S_{\mathrm{diss}}, \tag{4}$$

with:

- S_{EH} the Einstein–Hilbert term,
- S_Q a Bohm-type quantum term,
- S_{diss} a dissipative term depending on the fluid current J^μ and density ρ.

Consider a global dilation of coordinates

$$x^\mu \mapsto x'^\mu = \lambda x^\mu. \tag{5}$$

Under this transformation

$$\partial_\mu \mapsto \lambda^{-1}\partial_\mu, \qquad d^4x \mapsto \lambda^4 d^4x, \tag{6}$$

and we treat the metric $g_{\mu\nu}$ as dimensionless under this rescaling. The curvature scalar then scales as $R \mapsto \lambda^{-2}R$, so the Einstein–Hilbert action

$$S_{\mathrm{EH}} = \int \frac{R}{16\pi G}\sqrt{-g}\, d^4x \tag{7}$$

transforms as

$$S_{\mathrm{EH}} \mapsto \lambda^2 S_{\mathrm{EH}}. \tag{8}$$

The Bohm term S_Q has Lagrangian density

$$\mathcal{L}_Q = -\frac{\hbar^2}{8m^2\rho}\, g^{\mu\nu}\partial_\mu\sqrt{\rho}\,\partial_\nu\sqrt{\rho}. \tag{9}$$

With $\rho \mapsto \lambda^{-3}\rho$, we have $\sqrt{\rho} \mapsto \lambda^{-3/2}\sqrt{\rho}$ and $\partial_\mu\sqrt{\rho} \mapsto \lambda^{-5/2}\partial_\mu\sqrt{\rho}$, so

$$g^{\mu\nu}\partial_\mu\sqrt{\rho}\,\partial_\nu\sqrt{\rho} \mapsto \lambda^{-5}\, g^{\mu\nu}\partial_\mu\sqrt{\rho}\,\partial_\nu\sqrt{\rho}, \tag{10}$$

while $1/\rho \mapsto \lambda^3/\rho$. Hence

$$\mathcal{L}_Q \mapsto \lambda^{-2}\mathcal{L}_Q, \tag{11}$$

and the quantum action scales as

$$S_Q = \int \mathcal{L}_Q\sqrt{-g}\, d^4x \mapsto \lambda^2 S_Q. \tag{12}$$

For the dissipative sector we write

$$S_{\mathrm{diss}} = -\int \lambda_{\mathrm{diss}}\,\rho\,\|J\|^m\sqrt{-g}\, d^4x, \tag{13}$$

where λ_{diss} is a coupling and $m > 1$ a nonlinearity exponent. With

$$\rho \mapsto \lambda^{-3}\rho, \qquad J^\mu \mapsto \lambda^{-3}J^\mu, \qquad \|J\|^2 = g_{\mu\nu}J^\mu J^\nu \mapsto \lambda^{-6}\|J\|^2, \tag{14}$$

we obtain

$$\rho\,\|J\|^m \mapsto \lambda^{-3}\lambda^{-3m}\,\rho\,\|J\|^m = \lambda^{-3(m+1)}\,\rho\,\|J\|^m. \tag{15}$$

If we allow the dissipative coupling to carry a scaling dimension

$$\lambda_{\mathrm{diss}} \mapsto \lambda^\alpha\lambda_{\mathrm{diss}}, \tag{16}$$

then the Lagrangian density transforms as

$$\mathcal{L}_{\text{diss}} \mapsto \lambda^{\alpha-3(m+1)}\mathcal{L}_{\text{diss}}, \tag{17}$$

and the action as

$$S_{\text{diss}} \mapsto \lambda^{\alpha+4-3(m+1)}S_{\text{diss}}. \tag{18}$$

Requiring *homogeneous* scaling of all sectors,

$$S_{\text{EH}},\ S_Q,\ S_{\text{diss}} \ \mapsto\ \lambda^2(S_{\text{EH}}, S_Q, S_{\text{diss}}), \tag{19}$$

imposes the constraint

$$\alpha + 4 - 3(m+1) = 2 \quad\Longrightarrow\quad \alpha = 3m - 2. \tag{20}$$

Thus, once the physical (engineering) dimension of λ_{diss} is fixed experimentally, the exponent m is *no longer a free parameter*: it is uniquely determined by the requirement that the full Einstein–Hilbert–Krol action has no preferred macroscopic scale above (l_P, t_P). The scaling derivation is given in detail in Appendix A.

This structural scale-freedom allows dissipative continua to organise into fractal spatial patterns and power-law temporal statistics.

Example: viscous fluids and the determination of m

The relation

$$\alpha = 3m - 2$$

does not introduce a new free parameter. The exponent α is fixed once the physical (engineering) dimension of the dissipative coupling λ_{diss} is known. Hence m is fully determined by the microscopic form of the dissipation.

To illustrate this, consider ordinary viscous fluids. The dissipative power density has the schematic form

$$\mathcal{P}_{\text{visc}} \sim \eta\,(\partial v)^2,$$

with shear viscosity $[\eta] = M\,L^{-1}T^{-1}$. For a fluid with mass density $[\rho] = ML^{-3}$ and mass current $[J^\mu] = ML^{-2}T^{-1}$, a dissipative action term of the form $\lambda_{\text{diss}}\,\rho\,\|J\|^2$ has dimension

$$[\rho\,\|J\|^2] = M^3L^{-7}T^{-2}.$$

Matching this to a viscous dissipative power density determines the dimension of λ_{diss} uniquely:

$$[\lambda_{\text{diss}}] = M^{-2}L^6T^{-1}.$$

This dimensionality corresponds to a definite scaling exponent α in the dilation law $\lambda_{\text{diss}} \mapsto \lambda^\alpha\lambda_{\text{diss}}$. Inserting this fixed physical value of α into

$$m = \frac{\alpha+2}{3}$$

yields a unique value of m. For viscous fluids this gives

$$\boxed{m = 7/3}.$$

Thus, once the physical scaling of the dissipative coupling is known, the value of m is fully determined. The pair $(\lambda_{\text{diss}}, m)$ is therefore not phenomenological, but constrained by the requirement of homogeneous scaling of the full Einstein–Hilbert–Krol action.

The scaling condition derived in Section A,

$$\alpha = 3m - 2,$$

does not make λ_{diss} a universal constant. On the contrary: the physical (engineering) dimensions of λ_{diss} are determined by the microscopic properties of the medium in which the Einstein–Hilbert–Krol continuum description is applied. This is analogous to viscosity, thermal conductivity, or diffusivity in ordinary hydrodynamics: the *form* of the dissipative term is universal, but the *magnitude* of the coupling is system-dependent.

Once the physical dimension of λ_{diss} is known, the scaling relation fixes the nonlinearity exponent m uniquely,

$$m = \frac{\alpha + 2}{3}.$$

Thus $(\lambda_{\text{diss}}, m)$ are not free phenomenological parameters: their combination is constrained by the requirement that all sectors of the action scale homogeneously above the Planck threshold (l_P, t_P). The universality lies in the structure of the dissipative term, not in the numerical value of its coupling.

6 Why $1/f$ is the minimal-dissipation spectrum

We now deepen the analysis of why dissipative, scale-free systems select the spectrum

$$S(f) \sim f^{-p} \tag{21}$$

with exponent $p = 1$ as the unique stable, minimal-action solution.

6.1 The action integral constraint

A signal with power spectrum $S(f) = f^{-p}$ contributes to the total action as

$$S_{\text{act}} \propto \int_{f_{\min}}^{f_{\max}} f^{-p}\, df, \tag{22}$$

where $f_{\min}$ is set by the macroscopic system size and $f_{\max}$ is bounded by the microscopic cutoff (ultimately t_P^{-1}). A physically admissible spectrum must yield a finite action as $f_{\min} \to 0$ (infrared) and $f_{\max} \to t_P^{-1}$ (ultraviolet).

6.2 Why $p < 1$ fails (IR divergence)

For $p < 1$ we have

$$\int_{0}^{f_{\max}} f^{-p}\, df = \frac{f_{\max}^{1-p}}{1-p}, \tag{23}$$

which diverges as $f_{\min} \to 0$. Physically this corresponds to unbounded long-term correlations and infinite memory: an accumulation of energy at very low frequencies that cannot be dissipated away. Such behaviour is incompatible with stable dissipative continua, which cannot support arbitrarily large IR energy.

6.3 Why $p > 1$ fails (UV divergence)

For $p > 1$,

$$\int_{f_{\min}}^{\infty} f^{-p}\, df = \frac{f_{\min}^{1-p}}{p-1}, \tag{24}$$

which diverges as $f_{\max} \to t_P^{-1}$. This implies unbounded energy at very high frequencies, violating the Planck-scale cutoff derived in [1]. Dissipation near the UV threshold cannot support arbitrarily large short-timescale fluctuations.

6.4 $p = 1$ as the unique marginal case

This marginality condition was first noted in sandpile models [Bak1987] and RG analyses of diffusion-noise systems. Here we derive it ab initio from action scaling in a relativistic dissipative field theory.

For $p = 1$ the action integral becomes logarithmic:

$$\int_{f_{\min}}^{f_{\max}} \frac{df}{f} = \ln\left(\frac{f_{\max}}{f_{\min}}\right), \tag{25}$$

which diverges only logarithmically and remains under control for physically reasonable cutoffs. This spectrum is *marginal*: it avoids both IR and UV dominance and is the only exponent compatible with:

- finite action between the macroscopic and Planck scales,
- scale-invariance of the spectrum,
- dissipative flux balance.

Thus $p = 1$ identifies $1/f$ noise as the natural fixed point of dissipative continuum dynamics above the Planck threshold.

6.5 Logarithmic corrections and observational tests

Real systems rarely realise a perfect $1/f$ law over all frequencies. Instead, one often observes spectra of the form

$$S(f) = \frac{1}{f}\left(1 + a\ln f + bf^{\epsilon} + \dots\right), \tag{26}$$

with small parameters a, b and ϵ. Such corrections arise from finite-size effects, weak external forcing, or slow renormalization-group flow away from the strict fixed point.

Observable signatures include:

- a running effective exponent

$$p(f) = 1 + \frac{a}{\ln f} + O\left(\frac{1}{(\ln f)^2}\right), \tag{27}$$

- IR roll-off below $f_{\min}$ set by system size,
- UV cutoff or steepening near microscopic dissipative scales,
- universality: very different systems share the same asymptotic exponent $p \approx 1$.

These features are consistent with observations in turbulence, seismicity, financial time series, neurophysiological signals, and condensed matter noise [4, 7, 9].

Empirical prediction: running exponent in EEG spectra

The logarithmic corrections discussed above imply that real systems should not exhibit a perfectly constant $p = 1$ over many decades, but rather a very slowly running exponent

$$S(f) \sim f^{-p(f)}, \qquad p(f) = 1 + \frac{a}{\ln(f/f_0)},$$

with a a small dimensionless constant and f_0 a reference frequency of order the infrared cutoff $f_{\min}$ set by the effective system size.

EEG signals are a natural testing ground for this prediction. Human resting-state EEG typically displays a broadband approximate $1/f^p$ background with superposed alpha-band oscillations around 8–12 Hz. In the Einstein–Hilbert–Krol picture, the infrared cutoff $f_{\min}$ is associated with the largest coherent integration time of the cortical network. For large-scale human EEG this is of order

$$T_{\max} \sim 10\,\mathrm{s} \quad \Rightarrow \quad f_{\min} \sim 0.1\,\mathrm{Hz},$$

suggesting $f_0 \approx f_{\min}$.

The concrete prediction is then

$$p(f) \;=\; 1 + \frac{a}{\ln\!\big(f/f_{\min}\big)}, \qquad f_{\min} \sim 0.1\,\mathrm{Hz},$$

in the empirical band $f \in [0.5, 50]\,\mathrm{Hz}$. In this range $\ln(f/f_{\min})$ varies only moderately, so that $p(f)$ deviates from unity by a small, slowly varying amount. We hypothesize that fits will yield of human alpha-band EEG spectra (8–12 Hz) over the range 0.5–50 Hz yield values

$$a \approx 0.3\text{–}0.5,$$

corresponding to effective exponents $p(f)$ in the range 1.05–1.2, consistent with an infrared cutoff set by a system size of order $T_{\max} \sim 10\,\mathrm{s}$.

A systematic test of the theory would consist in estimating the running exponent $p(f)$ from high-resolution EEG or MEG data by local slopes in log-log spectra, and comparing the frequency dependence to the parameter-free functional form $1+a/\ln(f/f_{\min})$, with $f_{\min}$ constrained by independent estimates of the largest correlation time in the underlying neural network.

7 Conclusion

The Einstein–Hilbert–Krol framework implies a minimal semantic threshold (l_P, t_P) below which physical quantities lose operational meaning. Above this threshold the action contains no intrinsic macroscopic scale: curvature, Bohm pressure, and dissipation scale homogeneously under global dilations. Requiring homogeneous scaling of all sectors fixes the relation between the dissipative coupling λ_{diss} and the nonlinearity exponent m, so that m is determined once the physical scaling of λ_{diss} is known.

As a consequence, dissipative continua naturally evolve toward scale-invariant states with fractal spatial structure and power-law temporal correlations. Among all power-law spectra, $1/f$ emerges as the unique marginal, minimal-action solution consistent with both IR and UV constraints. The ubiquity of $1/f$ noise across physics, biology, and economics thus finds a structural explanation in the combination of a Planck-scale semantic threshold and dissipative, scale-free dynamics.

A Scaling of the Einstein–Hilbert–Krol action

In this appendix we collect the scaling relations used in the main text and show explicitly how the homogeneous scaling constraint fixes the relation between λ_{diss} and m.

A.1 Geometric sector

Under the global dilation

$$x^\mu \mapsto \lambda x^\mu, \tag{28}$$

derivatives and measure transform as

$$\partial_\mu \mapsto \lambda^{-1}\partial_\mu, \qquad d^4x \mapsto \lambda^4 d^4x, \tag{29}$$

and we treat $g_{\mu\nu}$ as dimensionless. The curvature scalar scales as $R \mapsto \lambda^{-2} R$, hence

$$S_{\text{EH}} = \int \frac{R}{16\pi G} \sqrt{-g}\, d^4x \mapsto \lambda^2 S_{\text{EH}}. \tag{30}$$

A.2 Quantum Bohm sector

The Bohm term has Lagrangian density

$$\mathcal{L}_Q = -\frac{\hbar^2}{8m^2\rho}\, g^{\mu\nu} \partial_\mu \sqrt{\rho}\, \partial_\nu \sqrt{\rho}. \tag{31}$$

With density scaling

$$\rho \mapsto \lambda^{-3} \rho, \tag{32}$$

we have

$$\sqrt{\rho} \mapsto \lambda^{-3/2} \sqrt{\rho}, \qquad \partial_\mu \sqrt{\rho} \mapsto \lambda^{-1} \lambda^{-3/2} \partial_\mu \sqrt{\rho} = \lambda^{-5/2} \partial_\mu \sqrt{\rho}. \tag{33}$$

Thus

$$g^{\mu\nu} \partial_\mu \sqrt{\rho}\, \partial_\nu \sqrt{\rho} \mapsto \lambda^{-5} g^{\mu\nu} \partial_\mu \sqrt{\rho}\, \partial_\nu \sqrt{\rho}, \tag{34}$$

and with $1/\rho \mapsto \lambda^3/\rho$ we find

$$\mathcal{L}_Q \mapsto \lambda^{-2} \mathcal{L}_Q. \tag{35}$$

The action transforms as

$$S_Q = \int \mathcal{L}_Q \sqrt{-g}\, d^4x \mapsto \lambda^2 S_Q, \tag{36}$$

matching the Einstein–Hilbert scaling.

A.3 Dissipative sector and constraint on m

For the dissipative term

$$S_{\text{diss}} = -\int \lambda_{\text{diss}}\, \rho\, \|J\|^m \sqrt{-g}\, d^4x, \tag{37}$$

we take

$$\rho \mapsto \lambda^{-3} \rho, \qquad u^\mu \mapsto u^\mu, \qquad J^\mu = \rho u^\mu \mapsto \lambda^{-3} J^\mu, \tag{38}$$

so

$$\|J\|^2 = g_{\mu\nu} J^\mu J^\nu \mapsto \lambda^{-6} \|J\|^2, \qquad \|J\|^m \mapsto \lambda^{-3m} \|J\|^m. \tag{39}$$

Thus

$$\rho\, \|J\|^m \mapsto \lambda^{-3(m+1)} \rho\, \|J\|^m. \tag{40}$$

If the coupling carries a scaling dimension

$$\lambda_{\text{diss}} \mapsto \lambda^\alpha \lambda_{\text{diss}}, \tag{41}$$

then the Lagrangian density scales as

$$\mathcal{L}_{\text{diss}} \mapsto \lambda^{\alpha - 3(m+1)} \mathcal{L}_{\text{diss}}, \tag{42}$$

and the action as

$$S_{\text{diss}} = \int \mathcal{L}_{\text{diss}} \sqrt{-g}\, d^4x \mapsto \lambda^{\alpha + 4 - 3(m+1)} S_{\text{diss}}. \tag{43}$$

Requiring homogeneous scaling of all sectors,

$$S_{\text{EH}},\ S_Q,\ S_{\text{diss}} \mapsto \lambda^2 (S_{\text{EH}}, S_Q, S_{\text{diss}}), \tag{44}$$

imposes

$$\alpha + 4 - 3(m+1) = 2 \quad \Longrightarrow \quad \alpha = 3m - 2. \tag{45}$$

Therefore, once the physical scaling (engineering dimension) of λ_{diss} is fixed by experiment or microscopic theory, the exponent m is *determined* by this constraint: the pair $(\lambda_{\text{diss}}, m)$ cannot be chosen arbitrarily if the Einstein–Hilbert–Krol theory is to remain scale-free above (l_P, t_P).

Coordinate scaling versus physical scaling

In standard general relativity, coordinates x^μ are labels without intrinsic physical meaning; physical distances are encoded in the line element $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. One may assign dimensions either to the coordinates or to the metric. In this work we adopt the convention $[x^\mu] = L$, $[ds] = L$, so that the metric components $g_{\mu\nu}$ are dimensionless. This is a choice of units, not a dynamical statement.

A global transformation $x^\mu \mapsto \lambda x^\mu$ can be interpreted in two different ways. As a passive coordinate change it is simply a relabeling of the same physical geometry (a diffeomorphism), under which the proper length ds and any physical cutoff remain unchanged. In the scaling analysis of this paper, however, we use $x^\mu \mapsto \lambda x^\mu$ as an *active* dilation of the fields, in the spirit of a renormalization group transformation: we ask how each sector of the action scales under a change of resolution, while keeping the Planck threshold (l_P, t_P) as an invariant semantic bound on ds.

With this interpretation, treating $g_{\mu\nu}$ as dimensionless is consistent even in the presence of a physical UV cutoff: the cutoff is a bound on the invariant line element ds, not on the coordinates x^μ themselves. The homogeneous scaling relations derived in Section 5 and Appendix A should therefore be understood as statements about the relative scaling of the action sectors above (l_P, t_P), rather than as coordinate transformations of a fixed background geometry.

B Viscous dissipation: tensorial form and dimensional analysis

For completeness we record the full continuum–mechanical expression for viscous dissipation in a Newtonian fluid, and show why the schematic form used in the main text is sufficient for dimensional and scaling arguments.

A. Full tensorial form

The viscous power density (power dissipated per unit volume) is

$$\mathcal{P}_{\text{visc}} = \tau_{ij}\, \partial_i v_j,$$

where τ_{ij} is the viscous stress tensor

$$\tau_{ij} = \eta \left(\partial_i v_j + \partial_j v_i - \tfrac{2}{3} \delta_{ij}\, \nabla \cdot v \right).$$

In compact notation this may be written as

$$\mathcal{P}_{\text{visc}} = \eta : \nabla v \otimes \nabla v.$$

All three terms in τ_{ij} carry the same engineering dimension and transform identically under a global dilation $x^\mu \mapsto \lambda x^\mu$.

B. Schematic form for scaling

For the purposes of dimensional analysis it is therefore sufficient to retain only the schematic dependence

$$\mathcal{P}_{\text{visc}} \sim \eta\, (\partial v)^2.$$

Indeed,

$$[\eta] = M L^{-1} T^{-1}, \qquad [\partial v] = T^{-1},$$

so that

$$[\mathcal{P}_{\text{visc}}] = M L^{-1} T^{-3},$$

independent of whether one keeps the symmetrised derivatives, the divergence term, or tensor contractions.

Moreover, under a dilation $x^\mu \to \lambda x^\mu$,

$$\partial_i v_j \mapsto \lambda^{-1}\,\partial_i v_j, \qquad (\partial v)^2 \mapsto \lambda^{-2}\,(\partial v)^2,$$

and each contribution in the full tensorial expression for $\mathcal{P}_{\text{visc}}$ scales in exactly the same way. Hence the schematic form $\eta(\partial v)^2$ captures the correct scaling behaviour of the dissipative sector.

This justifies the use of the simplified expression in the main text for comparing the dissipative coupling λ_{diss} in the Einstein–Hilbert–Krol framework with familiar dissipative terms in continuum hydrodynamics.

C Emergence of the Planck Threshold from the Dissipative Einstein–Hilbert–Krol Action

The existence of a minimal spacetime scale (l_P, t_P) is not an assumption but a necessary consequence of the combined structure of the Einstein–Hilbert term, the Bohm quantum potential, and the dissipative current term in the Einstein–Hilbert–Krol (EHK) action. Below we summarise the key points; full details are provided in the companion paper
citeKrol2025DQH.

The total action reads

$$S = \int \left[\frac{R}{16\pi G} - \rho\left(\tfrac{1}{2}u^2 + V(\rho)\right) - \frac{\hbar^2}{8m^2\rho} g^{\mu\nu}\partial_\mu\sqrt{\rho}\,\partial_\nu\sqrt{\rho} - \lambda_{\text{diss}}\,\rho\,\|J\|^m \right] \sqrt{-g}\,d^4x. \tag{46}$$

We now show that this structure enforces a lower bound on resolvable lengths and times.

1. Quantum term divergence

The Bohm quantum potential contains second derivatives of $\sqrt{\rho}$ and scales as

$$|Q| \sim \frac{\hbar^2}{m^2}\frac{1}{L^2} \tag{47}$$

for variations over length scale L. As $L \to 0$, the contribution diverges as $1/L^2$, implying that any attempt to resolve features below a certain length injects arbitrarily large stress-energy into the geometry.

2. Dissipative divergence

The dissipative contribution scales as

$$\rho\,\|J\|^m \sim L^{-3}\,L^{-m} = L^{-(m+3)}, \tag{48}$$

hence becomes dominant for small L. For $m > 1$ (required by the homogeneity constraints of the full action), the stress-energy contribution diverges faster than the quantum term.

3. Geometric inconsistency below a threshold

The Einstein tensor remains finite for any smooth metric. Therefore, if the stress-energy entering the Einstein equations diverges as $L \to 0$, the field equations cannot be satisfied. Hence spacetime cannot support coherent structures below a certain minimal scale, which we identify as l_P.

4. Determination of the scale

Balancing the smallest consistent quantum and geometric contributions yields

$$l_P^2 = \frac{\hbar G}{c^3}, \qquad t_P^2 = \frac{\hbar G}{c^5}, \tag{49}$$

the standard Planck scales. Thus (l_P, t_P) arise necessarily from the EHK action, not by assumption.

A Detailed Derivation of the Planck Threshold

We give a compact derivation of the minimal spacetime scale arising from the dissipative EHK action.

A. Scaling of the quantum sector

Let a fluctuation occur on spatial scale L. Then

$$\partial_\mu \sqrt{\rho} \sim L^{-1}, \qquad Q \sim \frac{\hbar^2}{m^2} L^{-2}. \tag{50}$$

The stress-energy contribution of Q scales as L^{-2}.

B. Scaling of the dissipative sector

With $\rho \sim L^{-3}$ and $J \sim L^{-2}$,

$$T_{\mu\nu}^{\text{diss}} \sim \rho \, \|J\|^m \sim L^{-3-m}. \tag{51}$$

The full action scaling conditions require $m > 1$, so this term diverges at least as fast as L^{-4}.

C. Consistency requirement of Einstein equations

The Einstein tensor contains at most two derivatives of the metric and remains finite provided the metric is smooth. Hence the field equations

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu} \tag{52}$$

cannot hold if $T_{\mu\nu} \to \infty$ as $L \to 0$. Therefore a minimal length scale must exist.

D. Identifying the minimal scale

Balancing geometric curvature $R \sim L^{-2}$ with the quantum contribution $Q \sim L^{-2}$ yields a scale L such that

$$L^2 \sim \frac{\hbar G}{c^3} = l_P^2, \tag{53}$$

with a corresponding minimal time t_P. Any attempt to probe below this scale leads to divergent stress-energy and loss of operational meaning of physical quantities.

This establishes (l_P, t_P) as intrinsic semantic thresholds of the Einstein–Hilbert–Krol framework.

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