Emergence of Planck Scales from a Dissipative Extension of the Einstein–Hilbert Action

A Unified Framework for l_P, t_P, E_P, $\hbar$, c, G and k_B

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Draft — November 2025

Abstract

We consider a dissipative extension of the Einstein–Hilbert action, in which a quantum fluid with Bohm-type quantum potential and a nonlinear current term evolves in curved spacetime. In this framework, time and space emerge from a universal minimal update rate and a minimal correlation length, respectively. We show that the structure of the modified action forces the existence of a smallest spacetime cell, and that dimensional consistency then uniquely selects the Planck length l_P and Planck time t_P. Once these two scales are identified, the constants c, $\hbar$, G, and k_B can be interpreted as derived quantities that characterize the granularity of dissipative updates in spacetime.

1 Introduction

In conventional physics, the constants c, G, $\hbar$ and k_B are taken as independent primitives. Time is introduced as an external parameter, and space as a geometric stage on which fields live. In a deeply dissipative universe, however, it is natural to expect that both space and time emerge from the structure of the fundamental action itself.

In earlier work [1], now published on Figshare with DOI[1], a model of dissipative quantum hydrodynamics in curved spacetime was proposed, based on a modification of the Einstein–Hilbert action by a nonlinear current term. In the present article we ask a specific question:

Given this dissipative extension of the Einstein–Hilbert action, does the model enforce the existence of a minimal length and a minimal time, and can these be identified with the Planck scales l_P and t_P?

We show that the answer is yes, in the following precise sense:

- the combination of curvature, quantum gradients and dissipation makes it impossible to push the theory to arbitrarily small spacetime scales without contradiction;
- the requirement of a smallest consistent spacetime cell introduces a fundamental pair (L, T);
- since the action already contains G, $\hbar$ and the Lorentzian causal structure (via c), dimensional analysis then uniquely identifies L and T with the Planck length and Planck time.

Once l_P en t_P are fixed, it is natural to reinterpret

$$c = \frac{l_P}{t_P}, \qquad \hbar \sim E_P t_P, \qquad G \sim \frac{l_P^3}{\hbar t_P},$$

and to view k_B as the entropy quantum per fundamental dissipative update.

[1]DOI: `10.6084/m9.figshare.30615254`.

2 Dissipative Extension of the Einstein–Hilbert Action

The starting point is the total action introduced in [1],

$$S[g,\rho,S,\alpha,\beta] = \int \left[\frac{R}{16\pi G} + \mathcal{L}_{\text{fluid}} + \mathcal{L}_Q - \lambda\rho\left(g_{\alpha\beta}J^\alpha J^\beta\right)^{m/2}\right]\sqrt{-g}\,d^4x, \tag{1}$$

with

$$\mathcal{L}_{\text{fluid}} = -\rho\left(\frac{1}{2}u^2 + V(\rho)\right), \qquad \text{(internal energy)}, \tag{2}$$

$$\mathcal{L}_Q = -\frac{\hbar^2}{8m^2\rho}\,g^{\mu\nu}\partial_\mu\sqrt{\rho}\,\partial_\nu\sqrt{\rho}, \qquad \text{(Bohm quantum potential)}, \tag{3}$$

$$J^\mu = \rho u^\mu, \qquad \lambda > 0,\ m > 1. \tag{4}$$

The explicitly dissipative contribution is

$$S_{\text{diss}}[g,\rho,u] = -\lambda\int \rho\left(g_{\alpha\beta}J^\alpha J^\beta\right)^{m/2}\sqrt{-g}\,d^4x, \tag{5}$$

which encodes bulk entropy production and defines a preferred causal direction via the norm

$$\|J\| := \sqrt{-g_{\alpha\beta}J^\alpha J^\beta}. \tag{6}$$

Variation of (1) with respect to the fields yields [1]:

- covariant mass conservation,
$$\nabla_\mu J^\mu = 0, \tag{7}$$
- a quantum–Euler equation with dissipative steering force,
$$f^\mu_{\text{diss}} = \lambda m\|J\|^{m-2}J^\alpha\nabla_\alpha J^\mu, \tag{8}$$
- and modified Einstein equations
$$G_{\mu\nu} = 8\pi G\left(T^{\text{fluid}}_{\mu\nu} + T^Q_{\mu\nu} + T^{\text{diss}}_{\mu\nu}\right), \tag{9}$$
  with the dissipative stress–energy tensor
$$T^{\text{diss}}_{\mu\nu} = \lambda\rho m\|J\|^{m-2}J_\mu J_\nu + \lambda\rho\|J\|^m g_{\mu\nu}. \tag{10}$$

In the present article we take these equations as given, and we focus on what the structure of the action (1) implies for the *fundamental scales* of spacetime itself.

3 Derivation of l_P and t_P from the Dissipative Action

To show explicitly that the Planck length and Planck time follow from the model, we analyse the interplay of the three key ingredients present in the action (1):

1. Curvature via the Einstein–Hilbert term $R/16\pi G$;
2. Quantum gradients via the Bohm term $\mathcal{L}_Q$ containing $\hbar$;
3. Dissipation via the nonlinear current term $\lambda\rho(g_{\alpha\beta}J^\alpha J^\beta)^{m/2}$.

These three structures carry the constants G and $\hbar$, and the Lorentzian causal structure determined by c in the metric. We now show that their coexistence forces the existence of a minimal spacetime cell, which must be of Planck size.

3.1 Step 1: Relevant dimensions at small scales

We work with a characteristic spatial scale L and temporal scale T associated with local variations of the fields. At small scales, the following behaviours are relevant:

- The curvature scalar scales as
$$[R] \sim L^{-2}. \tag{11}$$
- The Bohm quantum term contains gradients of $\sqrt{\rho}$ and the factor $\hbar^2$, giving an effective scaling
$$[\mathcal{L}_Q] \sim \frac{\hbar^2}{m^2} L^{-2}, \tag{12}$$
i.e. a quantum pressure opposing arbitrarily small L.
- The dissipative term involves
$$\Phi := \rho \, \|J\|^m, \tag{13}$$
and the coupling λ. For a fluid with typical velocity scale $u \sim L/T$, one has $J^\mu \sim \rho u^\mu$, so that
$$\|J\| \sim \rho \frac{L}{T}. \tag{14}$$
The combination $\lambda\Phi$ can be viewed as an effective entropy production density with dimension
$$[\lambda\Phi] \sim T^{-1} L^{-3}. \tag{15}$$

Thus, curvature and quantum gradients introduce L^{-2} behaviour, while dissipation introduces a T^{-1} factor and thereby a time scale.

3.2 Step 2: Existence of a minimal (L, T)

At arbitrarily small scales, three runaway behaviours would occur if no new scale intervenes:

1. The curvature R can grow without bound as L^{-2};
2. The quantum gradient energy in $\mathcal{L}_Q$ diverges as L^{-2};
3. For sufficiently large $\|J\|$, the dissipative entropy production rate $\lambda\Phi$ diverges as well.

The model is only self-consistent if there is a smallest scale (L, T) at which these three tendencies balance. Physically, this is the smallest spacetime cell in which the geometry, quantum pressure and dissipation can coexist in a stationary configuration. We therefore impose that at this scale the contributions are of the same order:

$$R \sim \frac{1}{L^2} \quad \text{and} \quad \lambda\Phi \sim \frac{1}{T}\frac{1}{L^3}, \quad \mathcal{L}_Q \sim \frac{\hbar^2}{m^2}\frac{1}{L^2}. \tag{16}$$

The existence of such a balance defines a fundamental pair (L, T). The exact numerical factors and the dependence on ρ and m are model-dependent, but the crucial point is that the model itself forbids $L \to 0$ and $T \to 0$.

3.3 Step 3: Identification with Planck scales

The action (1) already contains the constants

$$G, \qquad \hbar, \qquad c,$$

and no additional fundamental constants are introduced in the construction of the model. Therefore, any fundamental spacetime scale (L, T) that arises from the interplay of curvature, quantum gradients and dissipation must be expressible solely in terms of G, $\hbar$ and c.

Dimensional analysis then leaves only one possible length and one possible time:

$$L = \alpha\, l_P, \qquad T = \beta\, t_P, \tag{17}$$

with

$$l_P = \sqrt{\frac{\hbar G}{c^3}}, \qquad t_P = \sqrt{\frac{\hbar G}{c^5}}, \tag{18}$$

and α, β dimensionless numbers of order unity.

Because the dissipative model is supposed to be *fundamental* rather than effective, there is no deeper scale below (L, T) that could regularize the theory. It is therefore natural to set

$$L = l_P, \qquad T = t_P. \tag{19}$$

In this sense, the Planck scales are not put in by hand; they are the unique minimal scales compatible with the constants and structures already present in the action.

4 Dimensional Argument for Fundamental Spatiotemporal Quanta

The previous section showed that the modified action enforces the existence of a smallest spacetime cell, and that dimensional analysis singles out the Planck scales as the natural candidates. Here we summarize the argument in a compact form.

- The Einstein–Hilbert term introduces G and c and contributes a curvature energy density $\sim (16\pi G)^{-1} R$ with $[R] \sim L^{-2}$.
- The Bohm term introduces $\hbar$ and a quantum pressure $\sim \hbar^2 L^{-2}$.
- The dissipative term introduces a time scale via $\lambda \rho \|J\|^m$ with effective dimension $T^{-1} L^{-3}$.

If there were no smallest spacetime cell, the theory would suffer from simultaneous UV divergences in curvature, quantum pressure and entropy production. The existence of a minimal consistent configuration implies a fundamental pair (L, T). Since the only constants in the model with dimensions of length, time and action are G, $\hbar$ and c, the unique choice is

$$L = l_P, \qquad T = t_P.$$

Thus, the Planck length and Planck time emerge as the minimal spacetime resolution compatible with the dissipative quantum hydrodynamic model and the Einstein–Hilbert geometry.

5 Emergence of c, $\hbar$, G, and k_B

Once l_P and t_P are treated as fundamental, the remaining constants can be reinterpreted as derived parameters that characterize the granularity of dissipative updates.

5.1 Speed of Light

If l_P and t_P are the minimal spatial and temporal update quanta, the maximal signal speed is simply

$$c = \frac{l_P}{t_P}. \tag{20}$$

The speed of light is thus the universal ratio of the fundamental spacetime quanta.

5.2 Planck's Constant as Minimal Action per Update

Define the Planck energy

$$E_P = \sqrt{\frac{\hbar c^5}{G}}. \tag{21}$$

The minimal action associated with a single fundamental update of duration t_P at energy scale E_P is

$$\Delta S_{\text{diss}} = E_P t_P. \tag{22}$$

Identifying this with Planck's constant,

$$\hbar = E_P t_P, \tag{23}$$

we interpret $\hbar$ as the minimal action transferred in a single dissipative spacetime update.

5.3 Newton's Constant as Emergent Coupling

From the definition of the Planck length,

$$l_P^2 = \frac{\hbar G}{c^3}, \tag{24}$$

we can solve for G:

$$G = \frac{l_P^2 c^3}{\hbar}. \tag{25}$$

Newton's constant thus encodes how the discretized dissipative structure of spacetime manifests as curvature in the Einstein equations.

5.4 Boltzmann's Constant as Entropy Quantum

If each fundamental dissipative update produces a fixed minimal entropy increment ΔS, it is natural to identify

$$k_B \sim \Delta S. \tag{26}$$

In this view, Boltzmann's constant reflects the granularity of entropy production per elementary dissipative event. A more detailed microscopic model would be required to fix the proportionality constant by counting the accessible microstates per update.

6 Conclusion

We have shown that a specific dissipative modification of the Einstein–Hilbert action, realized through a quantum fluid with Bohm-type quantum potential and a nonlinear current-dependent term, naturally leads to the existence of a minimal spacetime cell. The interplay of curvature, quantum gradients, and dissipation forbids arbitrarily small scales and forces the introduction of a fundamental pair (L, T).

Because the model contains only G, $\hbar$ and the Lorentzian causal structure (with speed c), dimensional analysis leaves the Planck length l_P and Planck time t_P as the unique candidates for these minimal scales. From this perspective, c, $\hbar$, G, and k_B can be reinterpreted as derived quantities that describe, respectively, the ratio of spacetime quanta, the minimal action per update, the coupling between energy and curvature, and the entropy increment per dissipative event.

This suggests that a genuine unification of quantum mechanics and general relativity may not lie in quantizing a pre-existing geometry, but in understanding geometry and quantum behaviour as two emergent aspects of a single dissipative action principle. The dissipative quantum hydrodynamic model of [1] provides a concrete realization of this idea and a natural route toward a unified description of microscopic dynamics and spacetime structure.

References

[1] J. W. Krol, *Dissipative Quantum Hydrodynamics in Curved Spacetime*, Figshare (2025), DOI: 10.6084/m9.figshare.30615254.