PH456 - Lab 1

Random Numbers

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Task 1

Two pseudo-random number generators were selected for study in this lab. The generator used in Python's random module - the Mersenne Twister (MT19937) and the more recently developed 64-bit Permuted Congruential generator (PCG64). Firstly, two strings of 10000 numbers between (0,1) were generated and the processes timed. The results were as follows:

Time taken for PCG64:	0.005555200000003424
Time taken for Mersenne Twister:	0.005260700000000895

The time taken for each pseudo-random number generator (prng) is equal to 3 decimal places with MT19937 the slightly quicker of the two.
In order to test both generators for uniformity the χ^2 test was applied. Using this initial string of 10000 Numbers and setting $M = 9000$ the results were as follows:

PCG64	$\chi^2 = 9409.4$ with $p - value = 0.00127$
MT19937	$\chi^2 = 8853.2$ with $p - value = 0.86164$

This shows that for PCG64, $\chi^2 > M$ meaning PCG64 is not producing a uniform distribution of random numbers. MT19937 has $\chi^2 \leq M$ meaning that according to the χ^2 test, MT19937 does in fact produce numbers with a uniform distribution. If we look at our p-values, PCG64 produces an acceptable p-value but for MT19937 our p-value > 0.05 which is the standard statistically acceptable threshold.

A Sequential correlation test was then executed and the results are shown in figure 1. It can be seen that both prngs performed well with no clumping or uneven distribution present.

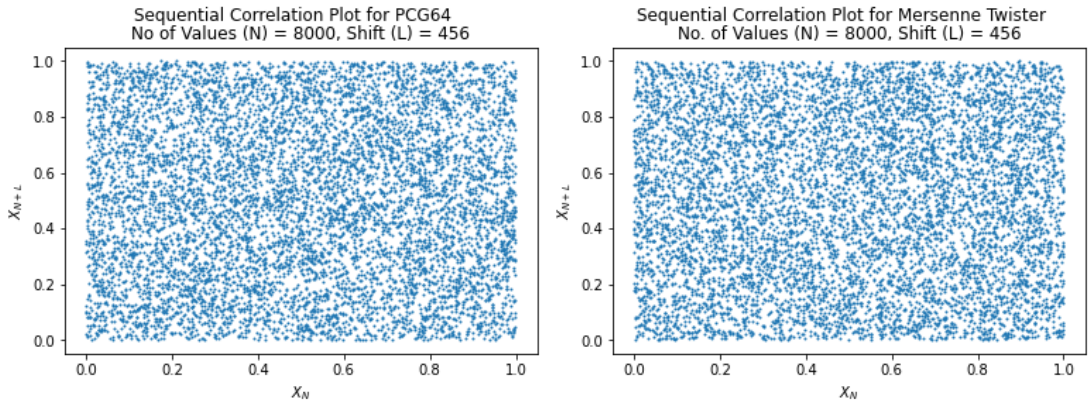

Figure 1: Sequential Correlation Plots for both PCG64 and MT19937. Neither plot shows sequential correlation.

Task 2 & 3

When running the partitioned box simulation, each prng was used in two ways:

- To determine which of the N particles has the potential to pass through the partition per unit time.
- To select a random number between 0 and 1 which is then compared to the probability of a particle crossing through the partition. This was used to determine whether the particle crossed or not.

The results of these simulations are given in Figure 2.

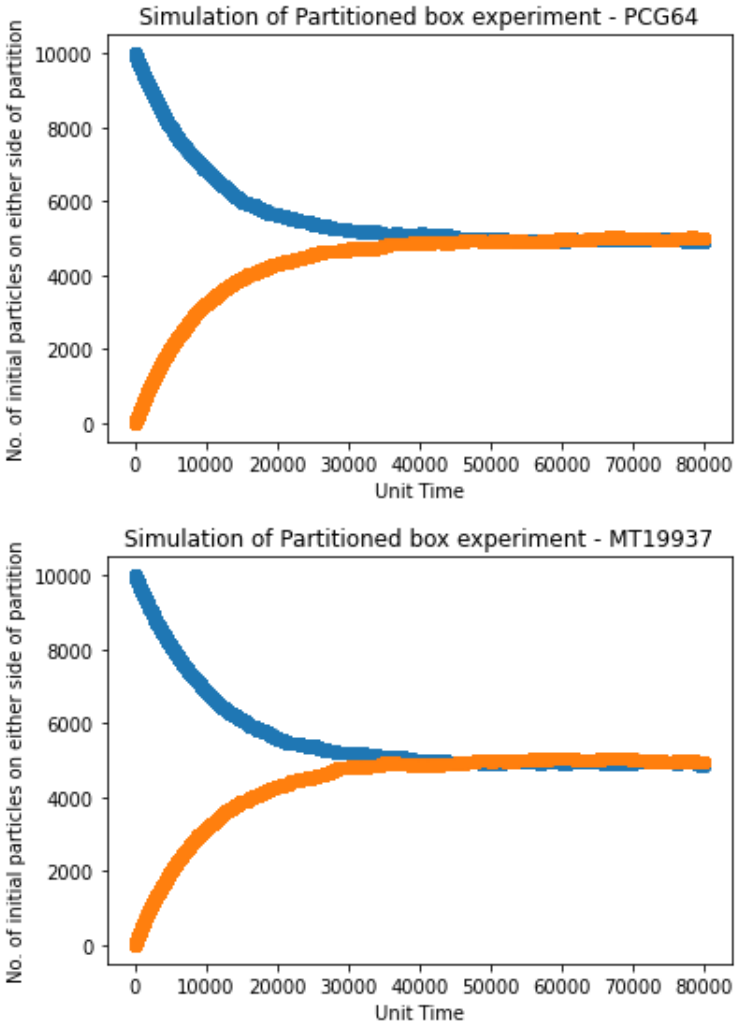

Figure 2: Partitioned Box Simulation for 10000 particles, with equal probability of any 1 particle crossing from one side to the other.

Both Systems reach equilibrium with roughly 5000 particles on either side of the partition after approximately 40000 unit times.

Task 4

The simulation was run again but this time with a 75% probability of a particle crossing from one side to the other and only 25% of a particle crossing the other way. The results are given in Figure 3.

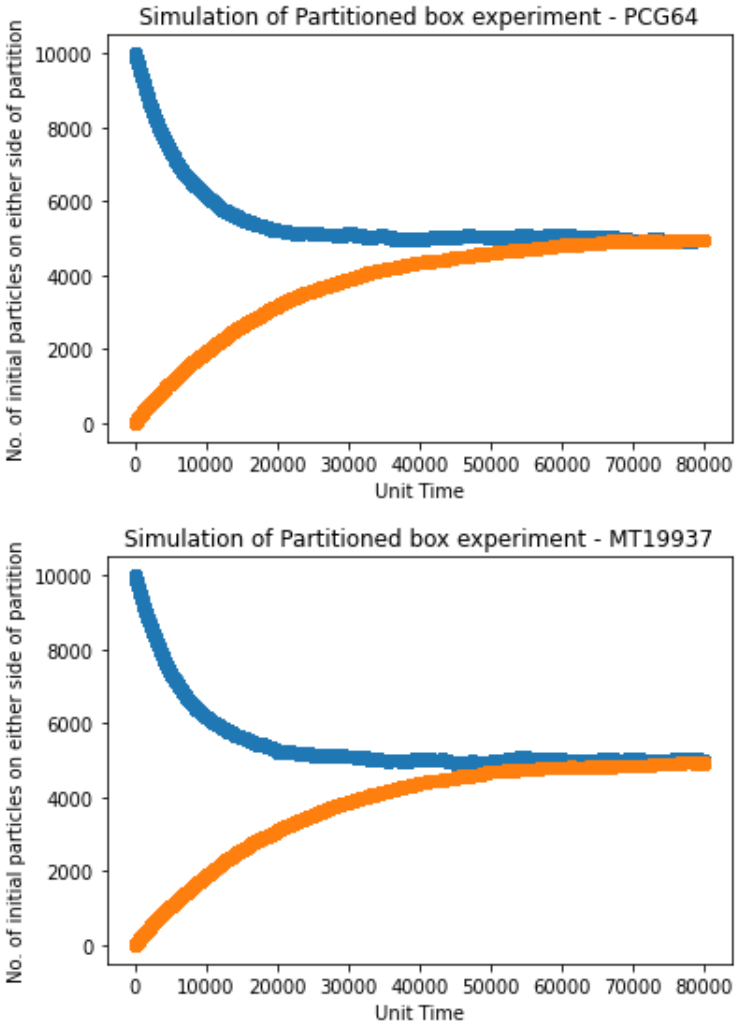

Figure 3: Partitioned Box Simulation for 10000 particles, with 75% probability of a 'blue' particle crossing and 25% probability of an 'orange' particle crossing.

From these plots it can be seen that both systems do still reach a state of equilibrium but this occurs approx. 10000 unit times later than in the 50% probability instance. It can also be noted that with both prngs, the 'blue' particles reach the equilibrium position $\approx$ 5000 particles quicker than in the 50% probability case and the 'orange' particles reach this equilibrium at a slower rate. This is shown by the steeper drop off in the number of blue particles and the more gradual increase in the number of orange particles.