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2017
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Summary Sheet

Cooperate and Navigate

Summary

In order to analyze the effects of autonomous vehicles on the traffic on highways, our team develop two major models, namely the Highway Network Model, and the Vehicle Model.

For the Highway Network Model, we abstract a hierarchical structure from the real-world highway network, highlighting the modeling of entrances and exits of highways to facilitate further analysis.

For the Vehicle Model, we take a bottom-top approach, which allows us to analyze the behavior of the traffic system as a whole by inspecting individual decisions. The decision-making of autonomous and manual vehicles are studied separately, with an emphasis on their differences. Our models include our creative modifications to classical models, which make them more suitable to the needs of mixed traffic systems.

To evaluate the performance of our traffic model, we develop a set of metrics that differ significantly from existing ones, in that they focus on the welfare of individual drivers, from which the overall performance of the traffic network can be inferred.

The effect of introducing autonomous vehicles is validated by means of computer simulation. The experimental results on the highways of interest provided with the problem indicate a significant improvement in performance as the percentage of autonomous vehicles increases. Specifically, the average welfare increases, while the variance in welfare decreases, resulting in improvements in both efficiency and equity. A percentage of autonomous vehicles of over 25% may lead to the formation of a platoon, and a percentage of over 65% may lead to a rapid increase in the traffic efficiency and a rapid drop of the variance.

Computer simulation also gave us clue on the effect of lanes being dedicated to autonomous vehicles. The introduction of a dedicated lane might be desirable if the percentage of autonomous vehicles exceeds 25%. However, it is meaningless to set a dedicated lane when the percentage exceeds 50%.

Sensitivity analysis shows that our model is proof to minor fluctuations in parameters.

Contents

1 Introduction

Traffic congestion is a worldwide problem. Because of congestion, drivers may experience long delays during peak hours, leading to losses of efficiency. One possible solution without increasing the number of lanes or roads is to introduce autonomous vehicles.

While it is a long-term goal to achieve the full automation, it is likely that autonomous vehicles may appear on highways in the near future. Therefore, a mixed traffic system combining manual and automated vehicles is well worth researching.

In our research, we firstly abstract the road structure of mixed traffic from real-world highway systems, highlighting the models of entrances and exits. Based on this structure, we develop several agent-based models for manual and autonomous vehicles to study the decision-making of individual drivers.

To apply our model, we developed an evaluation metrics for highway systems, and utilized computer simulation to test our theory. Experimental results show significant performance boost brought about by the introduction of autonomous vehicles, and cast new light on how autonomous vehicles should be developed.

This thesis is structured as is shown in fig. 1.

2 Basic Assumptions

- Only two kinds of vehicles are considered, i.e. *autonomous vehicles* (including self-driving vehicles and cooperating vehicles) and *manual vehicles*.

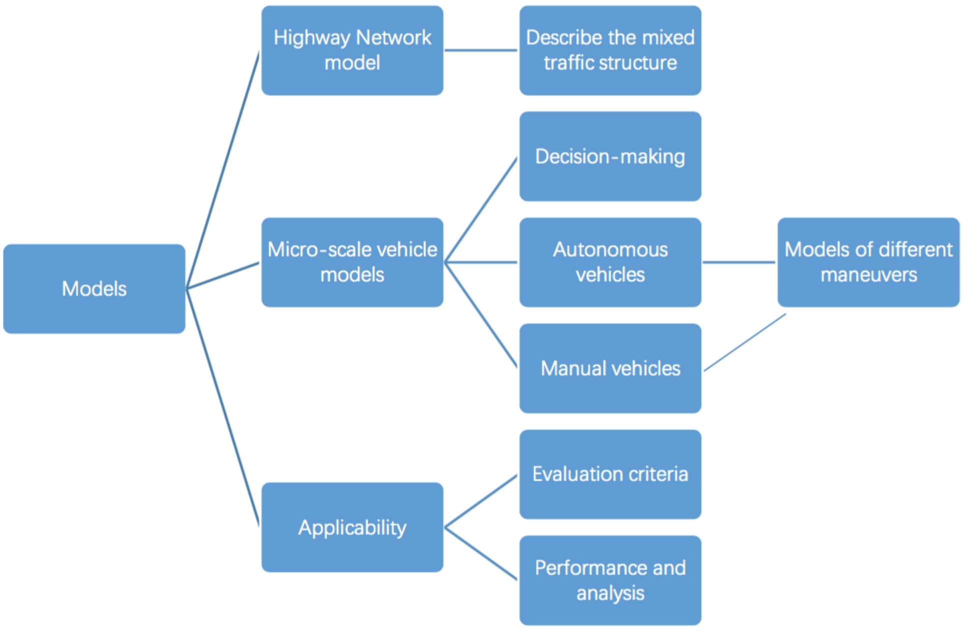


Figure 1: Overview

- An autonomous vehicle is equipped with technologies that enable it to acquire essential information about the surroundings, and to precisely control its motion. The specific kinds of information required will be discussed later.
- A human driver has adequate driving experience. For example, she is familiar with traffic regulations, she can roughly perceive the distance of the preceding vehicle as well as its relative velocity with regard to her vehicle, and she is able to control her vehicle.
- Geographic and climatic features of highways are ignored, i.e. we assume that all highways are straight and flat, and that climate conditions have no effect on the traffic.

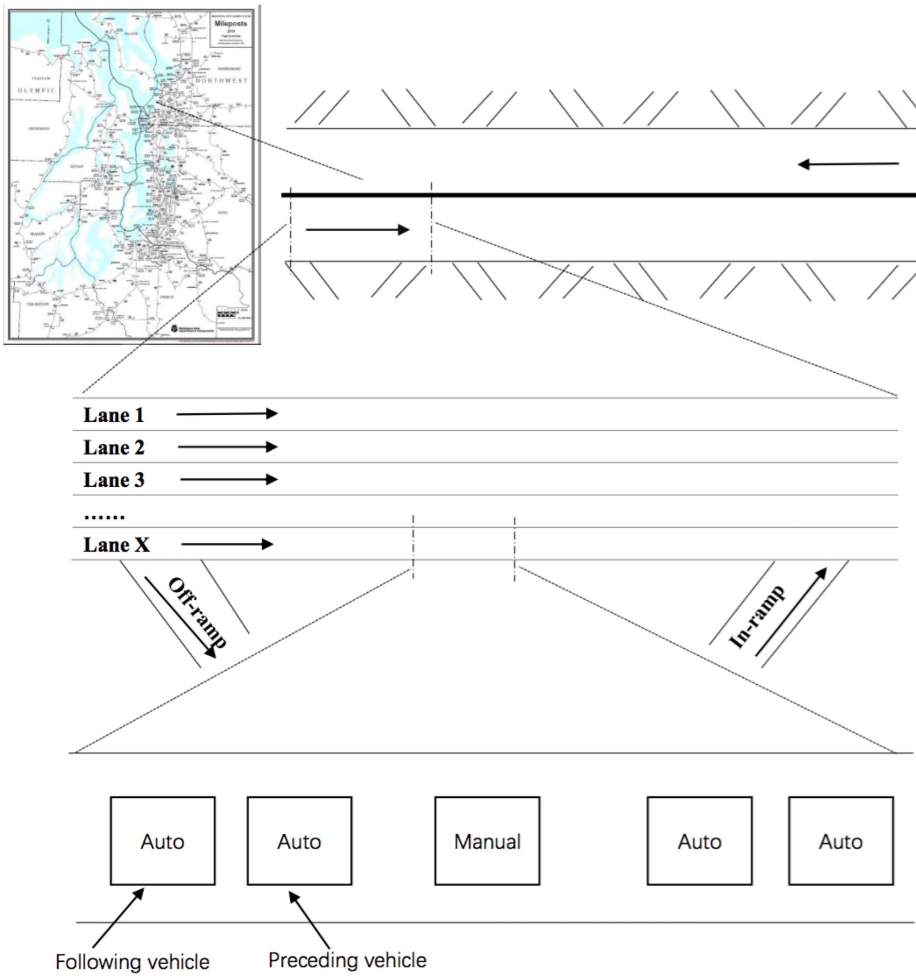


Figure 2: Highway models

3 Highway Network Model

3.1 Hierarchical Structure of the Highway Network

As is shown in fig. 2, we adopt a top-down approach to analyzing the highway network.

The *highway network* comprises a series of *highway*s, among which of particular interest are I-5, I-90, I-405, and SR-520, which we extract from the original map as is shown is fig. 3.

Each highway is bi-directional, with entrances and exits, referred to as *ramp area*s on each direction. Specifically, an *in-ramp* is where vehicles enter a highway, and an *out-ramp* is where vehicles exit a highway. In-ramps and out-ramps always appear in pairs. The properties of ramps will be detailed in sec. 3.2.

A *road section* s is a unidirectional segment of a highway between adjacent ramp areas, comprising n_s *lane*s. The *traffic* of a road section, t_s, is defined as the number of vehicles passing through it within a certain time period.

Two road sections s and $\tilde{s}$, with the same starting and ending mileposts but opposite directions, are *mutually conjugate*. Traffic is assumed to be portioned between mutually conjugate road sections according to the number of lanes, i.e.

$$\frac{t_s}{t_{\tilde{s}}} = \frac{n_s}{n_{\tilde{s}}}.$$

Mixed traffic, comprising both autonomous and manual vehicles, pass through each lane. The two types of vehicles will be modeled respectively in sec. 4.

3.2 Ramp Models

As is specified in sec. 3.1, in-ramps and out-ramps appear in pair. Additionally, an out-ramp always appears before its corresponding in-ramp along the direction of the traffic, so that the possibility of collision between inbound and outbound traffic is eliminated.

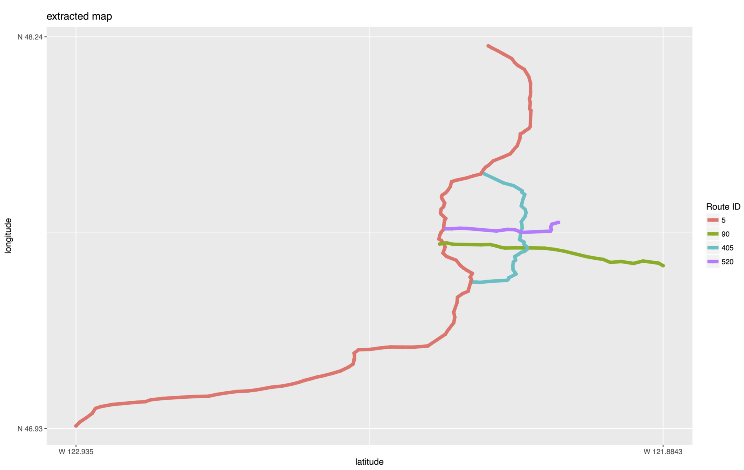


Figure 3: Highways of interest

For the sake of simplicity, and because of the unavailability of detailed inbound/outbound traffic data, we group a pair of in-ramp/out-ramp into a single *net in-ramp*, which increases the traffic on the highway, or a *net out-ramp*, which decreases the traffic on the highway. In other words, a milepost in the data provided is either a net in-ramp or a net out-ramp, which is equivalent to a source of traffic, or a drain of traffic, respectively.

Both the generation and expiration of traffic are modeled as Poisson processes, the intensity of which can be determined by taking differences of the traffic of adjacent road sections in the data provided. Equivalently, the time gap of both the arrival and the departure of vehicles obey exponential distribution, i.e.

$$P((S_n - S_{n-1}) \leq t) = 1 - e^{-\lambda t},$$

where S is the stochastic process denoting the arrival or departure of vehicles, S_n is the arrival or departure time of the n-th vehicle, and λ is the expected number of arriving or departing vehicles per unit time.

The behaviors of vehicles passing through in-ramps and out-ramps are treated differently, the principle of which is to minimize the influence of ramps on the arterial traffic. The details are explained as follows.

3.2.1 In-ramp Model

An in-ramp is modeled as a transfer lane to the side of the highway. An arriving vehicle can enter the highway only when it is eligible to enter the rightmost lane of the highway (see sec. 4.3.3). Arriving vehicles may queue up for entrance when the highway is crowded.

3.2.2 Off-ramp Model

Similar to an in-ramp, an out-ramp is also modeled as a transfer lane to the side of the highway, and we assume that all vehicles with the intention of leaving the highway through an out-ramp have already moved to the rightmost lane of the highway beforehand [1] . The throughput of an out-ramp, however, has no upper bound, i.e. vehicles can leave the highway immediately upon reaching the out-ramp.

3.3 Scenarios Affecting the Highway Network Model: Accidents

We discuss accidents as a part the highway network model, since an accident in effect closes or decelerates one or more lanes, thus affecting the whole highway network.

We classify the accidents by severity. A *major accident* completely blocks vehicles near the ground zero until the accident has been disposed of, while a *minor accident* forces vehicles near the ground zero to decelerate.

[1] In simulation, we adopt the strategy of removing a vehicle from the rightmost lane each time a departure is needed (as required by the stochastic model), so that we do not need any prior knowledge about the destinations of vehicles entering the highway, while retaining the overall behavior.

4 Vehicle Models

4.1 Model Overview

In this section, we discuss the behavior of autonomous and manual vehicles respectively.

Manual and autonomous vehicles share a lot in common in their control strategies. However, autonomous vehicles edge over manual ones in that automatic control systems are by far more precise than human drivers in the perception of the surroundings, in the control strategy free of randomness caused by variance in emotions and driving techniques, and in the actuation of motion. Thus, the introduction of autonomous vehicles promise more desirable traffic conditions.

Both manual and autonomous vehicles adopt different maneuvers according to different situations. Huang et al. divide Autonomous Vehicle Driving System into eight maneuvers, namely AICC, tracking, emergency, change-lane maneuvers, etc. [1]

Upon inspection, we find that some of the maneuvers mentioned above can be merged or simplified, and we divide the control strategy into four maneuvers, i.e.

- *free driving*, in which a vehicle is far enough from the preceding vehicle, so that it can move at its own *target velocity*.
- *car following*, in which a vehicle is within a certain distance of the preceding vehicle, thus its control being affected.
- *jam*, in which a vehicle is too close to the preceding vehicle to perform normal controls, corresponding to a traffic jam from a global perspective.
- *lane change*, in which a vehicle has the intention to move to an adjacent lane, during the entrance into or exit from the highway, or when overtaking.

In each of the maneuver, the acceleration of the vehicle is the controlled variable.

4.2 Notations

Table 1: Declaration of notations

Notation	Description
S	following space
S_n	basic following space
t	current time
T	time scale of control
v_{target}	Target velocity
x_0	position of the vehicle being discussed
v_0	velocity of the vehicle being discussed
a_0	acceleration of the vehicle being discussed
$x_{-1}^{(k)}$	position of the nearest vehicle in front of the vehicle being discussed in lane (k); (k) may be omitted when discussing the current lane

Notation	Description
$v_{-1}^{(k)}$	velocity of the nearest vehicle in front of the vehicle being discussed in lane (k); (k) may be omitted when discussing the current lane
$a_{-1}^{(k)}$	acceleration of the nearest vehicle in front of the vehicle being discussed in lane (k); (k) may be omitted when discussing the current lane
$x_{1}^{(k)}$	position of the nearest vehicle behind the vehicle being discussed in lane (k); (k) may be omitted when discussing the current lane
$v_{1}^{(k)}$	velocity of the nearest vehicle behind the vehicle being discussed in lane (k); (k) may be omitted when discussing the current lane
$a_{1}^{(k)}$	acceleration of the nearest vehicle behind the vehicle being discussed in lane (k); (k) may be omitted when discussing the current lane
L_{ori}	original lane
L_{des}	destination lane
$a_{\max}$	maximum acceleration
a_{mod}	a moderate acceleration
$E_t^{(k)}$	expected time cost in lane (k)
$E_v^{(k)}$	expected velocity in lane (k)
$\Delta_{n,m}x$	distance between the n-th and the m-th vehicle
$\Delta_{n,m}v$	relative velocity between the n-th and the m-th vehicle
$\Delta_1^{(k)}v$	relative velocity between the vehicle being discussed and the one behind it in lane (k); (k) may be omitted when discussing the current lane
$\Delta_1^{(k)}x$	distance between the vehicle being discussed and the one behind it in lane (k); (k) may be omitted when discussing the current lane
$\Delta_{-1}^{(k)}x$	distance between the vehicle being discussed and the one in front of it in lane (k); (k) may be omitted when discussing the current lane
$\Delta_{-1}^{(k)}v$	relative velocity between the vehicle being discussed and the one in front of it in lane (k), (k) may be omitted when discussing the current lane
Δ	penalty time when a vehicle starts from $v = 0$
$t^{(k)}(x)$	the time when the vehicle reaches x in lane (k); (k) may be omitted when discussing the current lane
v_B	threshold of tolerable velocity difference
v_{TH}	the threshold of the deviation of expected speed
S_0	a personalized parameter of a human driver

4.3 Autonomous Vehicle Model

Thanks to the fast-developing technology, we may assume that autonomous vehicles can get precise kinematic parameters of the preceding vehicle and control its own acceleration accurately (either by self-sensing sensors or Cooperative Vehicle Infrastructure System (CVIS)). Our models are based on this assumption.

Before proceeding with specific maneuvers, some parameters of a vehicles need to be defined:

- v_{target}: the *target velocity*. For autonomous vehicles, we define it to be ~80% of the speed limit of the highway, i.e. 50 mph (22 m/s).

- S: the *following space*, the distance a vehicle needs to keep with the preceding distance to avoid collision, as a function of v. The way to determine S will be detailed in sec. 4.3.2.

An overview of the overall control strategy is provided with fig. 4.

4.3.1 Free Driving

When the distance to the preceding vehicle is large enough ($> 4S$), a vehicle can be freely controlled.

When the velocity is low (< 10 mph), we take a large acceleration to accelerate; a medium acceleration is taken while velocity is not so low. After the velocity reaches 50 mph, a random acceleration is adopted. We do not use the normal distribution whose mean is zero because we believe that for a vehicle, i.e.

$$a_0 = \begin{cases} 3, & \text{if } v_0 < 5 \\ 1, & \text{if } 5 < v_0 < 20 \\ X, & \text{if } 26 < v_0 \\ Y, & \text{otherwise} \end{cases},$$

2

where $X \sim N(-1, 0.25^2)$, $Y \sim N(0.5, 0.25^2)$, denoting errors in acceleration, since it is difficult to maintain a constant low acceleration.

4.3.2 Car Following

A classical car following model is set up as

[2] Unless stated otherwise, all numerical quantities are in SystÃĺme international d’unitÃľ (SI).

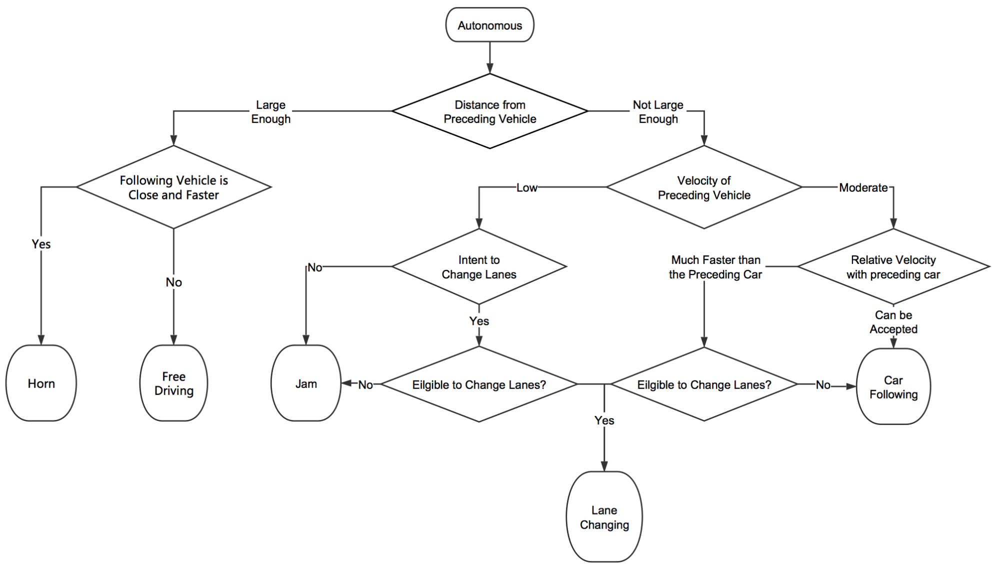


Figure 4: The control strategy of autonomous vehicles

$$a_0(t+T) = a_{-1}(t) + \frac{(\Delta_{-1}v(t))^2}{2(S - \Delta_{-1}x(t))}$$

3

in which $x_n(t), v_n(t), a_n(t)$ stand for the position, velocity and acceleration of the n-th car, respectively. $\Delta_{-1}v(t) \triangleq v_{-1}(t) - v_0(t)$ is the relative velocity,
and $\Delta_{-1}x(t) \triangleq x_{-1}(t) - x_0(t)$ is the distance between the vehicle being discussed and the preceding vehicle.

In practice, however, the performance of the model is unsatisfactory under some circumstances, i.e. when a vehicle being discussed in close to its preceding vehicle, it decelerates even if its current velocity is lower than that of its preceding vehicle, leading to unnecessary extra time cost. To address this issue, we modify the model as follows:

$$a_0(t+T) = \begin{cases} a_{-1}(t) + \frac{(\Delta_{-1}v(t))^2}{2(S-\Delta_{-1}x(t))}, & \text{if } \Delta_{-1}v(t) < 0 \text{ and } \Delta_{-1}x(t) > S \\ a_{-1}(t) - \frac{(\Delta_{-1}v(t))^2}{2(S-\Delta_{-1}x(t))}, & \text{if } \Delta_{-1}v(t) > 0 \text{ and } \Delta_{-1}x(t) > S \\ -a_{\text{mod}}, & \text{if } \Delta_{-1}x(t) < S \\ 0, & \text{if } |\Delta_{-1}x(t) - S| < \varepsilon \text{ and } |\Delta_{-1}v(t)| < \varepsilon \end{cases}, \quad (1)$$

where ε is an minor value.

By applying this model, we find that the distance between adjacent vehicles will fluctuate near S, in essence forming a *platoon*. Experimental results show that our modification to the classical model may result in a sixfold increase in highway throughput under a setting where all vehicles are autonomous.

Some additional notes on this model:

- The threshold distance from the preceding vehicle for the model to function is chosen as a relatively large $4S$, to ensure sufficient reaction time in case the preceding car should stop suddenly, e.g. because of an accident.
- There is an upper limit to the acceleration of a real-world vehicle, so we use a saturation model to get a rectified acceleration a^*:

$$a^* = \begin{cases} a_{\max}, & \text{if } a > a_{\max} \\ -a_{\max}, & \text{if } a < -a_{\max} \\ a, & \text{otherwise} \end{cases}, \quad (2)$$

  where $a_{\max}$ denotes the physical limit to the acceleration of a vehicle. According to empirical observations, we choose $a_{\max} = 5\text{m/s}^2$.

The Determination of Following Space The following space S is roughly the minimum braking distance $S_n = \frac{v_0^2}{2a_{\max}}$. As a modification to this estimation, we consider the type of the preceding vehicle:

[3] A latency T is introduced considering the fact that real-time control on acceleration is not possible in a real-life scenario; this also facilitates computer simulation by discretizing time.

$$S = \begin{cases} \frac{v_0^2}{2a_{\max}} + 20, & \text{if the preceding vehicle is manual} \\ \frac{v_0^2}{2a_{\max}} + 3, & \text{if the preceding vehicle is autonomous} \end{cases}$$

Since the uncertainty of human drivers is higher than that of an autonomous control system, larger following space is chosen.

4.3.3 Lane Change Model

Lane change happens during the entrance and exit of a vehicle, or when the velocity of the preceding vehicle is too low to tolerate.

The Intention of Lane Change To determine whether a driver has the intention [2] to move to another lane, we examine the expected passing time E_t of each lane:

$$E_t^{(k)} \triangleq E(t^{(k)}(X)), \text{ where } X >> \Delta_{-1}^{(k)} x$$

$\Delta_{-1}^{(k)} x$ stands for the distance to the preceding vehicle in lane k, and X stands for some very long distance.

Apparently, lane-changing condition can be expressed as

$$E_t^{(L_{\text{des}})} < E_t^{(L_{\text{ori}})} \tag{3}$$

To faciliate further analysis, we simplify this definition by examining stable velocity. According to sec. 4.3.1 and sec. 4.3.2, if a vehicle were to change its lane, either the free driving model or the car following model would apply after the change. Thus the expected velocity E_v can be defined as:

$$E_v^{(k)} \triangleq \begin{cases} v_{-1}^{(k)}, & \text{if } \Delta_{-1}^{(k)} x < 4S \\ v_{\text{target}}, & \text{otherwise} \end{cases}$$

Then the condition to the lane-changing can be expressed as

$$E_v^{(L_{\text{des}})} > E_v^{(L_{\text{ori}})} \tag{4}$$

It can be proved that the two definitions eq. 3 and eq. 4 are largely equivalent under most circumstances.

Taking into account the threshold and the impact of the target velocity, the lane-changing conditions (to the left, for overtaking) are:

$$\begin{cases} \text{the current lane is not the leftmost lane} \\ E_v^{(L_{\text{des}})} > E_v^{(L_{\text{ori}})} + v_{\text{TH}} \\ E_v^{(L_{\text{ori}})} < v_{\text{target}} - v_{\text{B}} \end{cases}, \tag{5}$$

in which v_{TH} stands for the threshold of the deviation of expected speed, and v_{B} stands for the threshold of the tolerable speed difference. We choose $v_{\mathrm{TH}} = 5\mathrm{m/s}, \mathrm{v_B} = 2\mathrm{m/s}$.

According to our highway network model stated in sec. 3, vehicles may try to move rightward when the lane to the right is vacant. The right-moving condition can be described as:

$$\begin{cases} \text{the current lane is not the rightmost lane} \\ \Delta_1^{(L_{\mathrm{des}})} x < 8S \quad \text{where } L_{\mathrm{des}} \text{ is the right lane} \end{cases} \tag{6}$$

To recapitulate, when condition eq. 5 is met, the vehicle tends to move to the left (the beginning of overtaking); when condition eq. 6 is met, the vehicle tends to move to the right (back to the original lane).

Eligibility of Lane Changing When a vehicle has the intention to change its lane, it checks for the eligibility of this lane changing by verifying there is enough space on the destination lane for it to insert into, i.e.

$$\begin{cases} \Delta_{-1}^{L_{\mathrm{des}}} x > 10 \\ \Delta_1^{L_{\mathrm{des}}} x > 10 \end{cases} \tag{7}$$

Lane Changing Process To depict the effect of a lane change to the traffic, we model it as three steps (referring to the original lane as L_{ori}, and the destination lane as L_{des}):

- **Step 1:** In the first 3 seconds, the vehicle decelerates on L_{ori}.
- **Step 2:** At the end of step 1, an identical vehicle appears at the same position on L_{des}. In the next 2 seconds, the "new" vehicle accelerates on L_{des}, while the "old" vehicle remains on L_{ori}, which describes the situation where the lane-changing process has impact on both lanes.
- **Step 3:** At the end of step 2, the vehicle is remove from L_{ori}, completing the process.

Coordination of Following Vehicles During the lane-changing process, the following vehicle on L_{des} also take action to avoid collision. Condition eq. 7 guarantees that the car following model, as stated in sec. 4.3.2, can depict the behavior of the following vehicle.

Analysis of the In-ramp Process As is stated in sec. 3.2.1, vehicles entering the highway through an in-ramp follow the lane-changing process:

- When the distance to the highway end of an in-ramp is less than $S_0 = 30\mathrm{m}$, a lane-changing intention is generated.
- The eligibility of the intention is checked (see eq. 7).

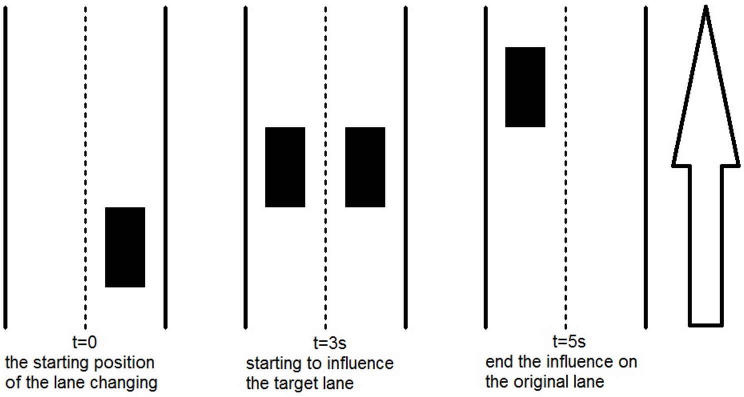


Figure 5: Lane changing process

- If the intention is eligible, the vehicle enters the highway, following a normal lane-changing process.
- If the intention is ineligible, the vehicle decelerates with the target position set to the end of the in-ramp: $a = v^2/2S_0$, waiting for the eligibility for entrance. If v drops to 0 before entrance, a penalty time Δ is incurred when the vehicle starts again.

4.3.4 Jam Model

Traffic jam may occur for various reasons. e.g. accidents, in-ramp traffic flow, etc. The jam model has practical significance.

When a traffic jam occurs, a smaller distance to the preceding car is expected, so the previous car following model is modified accordingly to meet this demand.

Jam model is activated only when the velocity of the preceding vehicle is lower than 5m/s. The model is detailed as follows:

- if the velocity is relatively high, brake with the target position set to 5m behind the preceding vehicle.

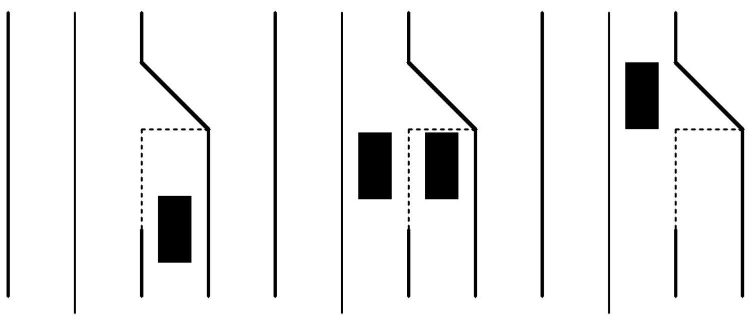


Figure 6: In-ramp process

- if the velocity is low, i.e. no larger than 5m/s, use the same acceleration as the preceding vehicle.
- if v drops to 0 before, a penalty time Δ is incurred when the vehicle starts again.

The jam model is in essence a variant of the car following model under special circumstances. In practice, the jam model translates automatically to the car following model as the traffic jam eliminates.

4.4 Manual Vehicle Model

4.4.1 Aggressiveness

According to [3], the variance in the aggressiveness of drivers lead to different driving behaviors. For instance, a highly aggressive driver tends to drive faster, keep a smaller safe distance to the preceding vehicle, change lanes more frequently, and is more likely to perform a dangerous maneuver than a less aggressive driver.

To depict a driver's aggressiveness, we introduce a non-negative variable obeying logarithmic normal distribution $\alpha \sim LN(1, 0.3)$, which determines a driver's S_0, v_{target}, v_{B} and v_{TH}:

$$v_{\text{target}} = \begin{cases} 16, & \text{if } 16 > 20\alpha \\ 20\alpha, & \text{if } 16 < 20\alpha < 26 \\ 26, & \text{if } 26 < 20\alpha \end{cases} \tag{8}$$

$$S_0 = \begin{cases} -10, & \text{if } 0 > \frac{40}{\alpha} \\ \frac{40}{\alpha} - 10, & \text{if } 0 < \frac{40}{\alpha} < 100 \\ 90, & \text{if } 100 < \frac{40}{\alpha} \end{cases}$$

$$v_{\text{B}} = \begin{cases} 1, & \text{if } 1 > \frac{3}{\alpha} \\ \frac{3}{\alpha}, & \text{if } 1 < \frac{3}{\alpha} < 10 \\ 10, & \text{if } 10 < \frac{3}{\alpha} \end{cases}$$

$$v_{\text{TH}} = \begin{cases} 3, & \text{if } 3 > 5\alpha \\ 5\alpha, & \text{if } 3 < 5\alpha < 10 \\ 10, & \text{if } 10 < 5\alpha \end{cases}$$

The physical meanings of S_0, v_{target}, v_{B} and v_{TH} will be explained in sec. 4.4.3, sec. 4.4.2 and sec. 4.4.4, respectively.

4.4.2 Free Driving Model

The free driving model is virtually identical to the free driving model for autonomous vehicles, as is stated in sec. 4.3.1, except that the personalized parameter v_{target} is determined by the aggressiveness of each driver. ($v_{\text{target}} \propto \alpha$, with saturating limits, as in eq. 8)

4.4.3 Car Following Model

Since a human driver can hardly know acceleration of a preceding vehicle, the control strategy is determined by (rough) distance and relative velocity:

$$a_0 = N + \frac{(\Delta_{-1}v)^2}{2(S - \Delta_{-1}x)} \tag{9}$$

where $N \sim N(0, 1)$, denotes the error term of human control, and S_{man} is the following space of the manual vehicle.

In the absence of knowledge of the acceleration of the preceding vehicle, eq. 9 adopts a more conservative strategy than eq. 1 used by autonomous control systems.

To determine the following space, a similar conservative strategy is adopted:

$$S = \frac{V_0^2}{2a_{\max}} + S_0,$$

in which S_0 is a personalized parameter of a human driver, inversely proportional to her aggressiveness, i.e. an aggressive driver tends to choose smaller headway and following space.

The rectification due to physical limits of vehicles is identical to the one stated in eq. 2.

4.4.4 Lane Change Model

The lane change model is virtually identical to the lane change model for autonomous vehicles, as is stated in sec. 4.3.3, except that personalized parameters V_B, V_{TH} are determined by the aggressiveness of each driver.

4.4.5 Horn Model

We develop a creative model to describe the impact of the following vehicle on the vehicle being discussed. We denominate it has the "horn model", in that the following vehicle may blow the horn to urge the vehicle being discussed to move faster in a real-life scenario.

Horn model is essential to reducing traffic latency and improving its throughput, since slowly moving vehicles may cause blockage on certain road sections and thus affect the whole highway network.

Horn model is effective for human drivers only, because we assume that all autonomous vehicles try to maintain a target velocity close to the speed limit of the highway (see sec. 4.3.1), and there is no need to respond to a "request" to speed up.

The prerequisites to apply horn model include:

- The target velocity of the following vehicle is significantly larger than that of the vehicle currently being discussed.
- The following vehicle is ineligible to change its lane (or it can simply overtake the vehicle being discussed instead of blowing the horn).
- The distance of the following vehicle from the vehicle being discussed is larger than $4S$.

To formally represent the effect of "horn" on the vehicle being discussed:

$$v'_{\text{target}} = \max(22, v_{\text{target}} + u(t - t_0)\lambda t), \tag{10}$$

in which v'_{target} stands for the rectified target velocity due to "horn effect", $u(t) \triangleq \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$ is a step function, λ is a constant coefficient, and t_0 is a personalized tolerable time.

Intuitively, eq. 10 implies that a vehicle tends to move faster if the following vehicle blows horn for a long time, which coincides with empirical observations.

5 Model Stability

Our models of vehicles demonstrate considerable robustness under extreme conditions. To verify this, we design a simulation, where we employ two adjacent vehicles, with the acceleration of the preceding one set to a random value. For instance, given $\Delta x = 15, \Delta v = 10, v_1(0) = 25$, and the acceleration of the preceding vehicle set to a large negative value, we verify that our model is collision-proof through repeated experiments. Such a process is shown in fig. 7, where two vehicles narrowly avoid collision under an extremely adverse initial condition, and quickly converge to a stable car-following pattern.

6 Applicability

6.1 Evaluation Metrics

To test the performance of our models, we develop two metrics to evaluate the overall performance of a traffic system:

- The mean velocity of all vehicles on a certain road section at a certain time section.
- $\tau_{\text{D}} \triangleq \frac{T_{\text{actual}}}{T_{\text{ideal}}}$, i.e. the ratio of the actual time cost of the journey to the ideal time cost (the ideal time cost is defined as the distance of the journey divided by the target velocity). Generally, $\tau_{\text{D}} \geq 1$, due to blockages such as traffic jams and accidents. τ_{D} reflects how satisfactory a journey is from the perspective of an individual driver, the smaller the better. To inspect the overall performance, we take the average of all τ_{D}s: $\tau = \text{mean}(\tau_{\text{D}})$

For a mixed highway system, τ value closer to 1 may indicate smoother traffic.

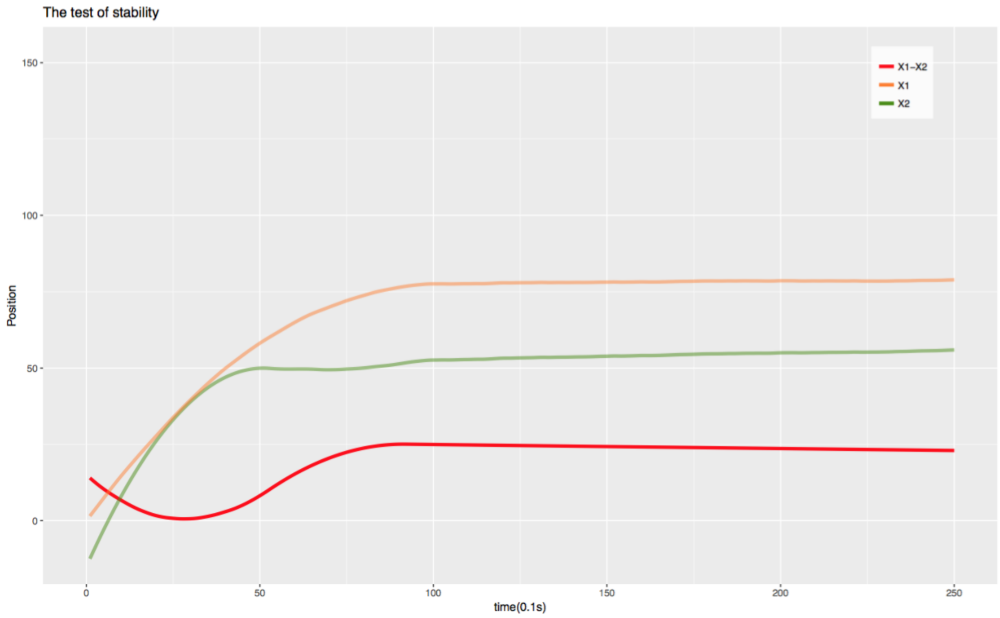


Figure 7: A demonstration of the stability of our model

6.2 Experiment and Analysis

We test our model on the data provided with the problem by running computer simulation on each highway of interest, emulating the behavior of the real-world highway network during a time period of 1800 seconds (30 minutes).

6.2.1 The Effect of Autonomous Vehicles on the Mean Velocity

To test the effect of introducing autonomous vehicles on the mean velocity, we use a set of different configurations for simulation, as is listed in tbl. 2. Picking one parameter from each column each time, 64 configurations can be produced.

Table 2: Simulation configurations

Road No.	Period	Direction	% of autonomous vehicles
5	peak	increasing	0
90	average	decreasing	10
405			50
520			90

Experimental results are shown in fig. 8. In the average velocity heatmap, warmer colors mean lower average velocity on some certain road sections, indicating traffic jams. It can be seen that blockage points are gradually eliminated with the introduction of autonomous vehicles.

6.2.2 The Correlation between the Proportion of Autonomous Vehicles and the Smoothness of Traffic

To obtain more fine-grained knowledge of the proportion of autonomous vehicles on the smoothness of traffic, evaluated by τ_{D} and τ developed in sec. 6.1, we pick out the typical I-5 highway, and experiment with 0%, 10%, 20%, $\cdots$, 90% of autonomous vehicles during peak hours. The distribution of τ_{D} is shown in fig. 9, where red dots stand for autonomous vehicles, and green dots stand for manual vehicles. The trend of mean values and variances of τ_{D}s are shown in fig. 10a and fig. 10b, respectively.

It is apparent that an increase in the proportion of autonomous vehicles results in an increase in the overall efficiency, and eliminates the difference in the efficiency of individual vehicles. It can be inferred that the dominance of autonomous vehicles (>65%) may "regulate" individual behaviors by forming platoons. Notably, the overall efficiency of manual vehicles is also increased with the dominance of autonomous vehicles, which may in essence act as "guides" on the traffic.

6.2.3 The Effect of Dedicated Lanes

As a conclusion to our experiment, we tried allocating dedicated lanes to autonomous vehicles. The results are shown in fig. 11.

It can be seen that highways with lanes dedicated to autonomous start to edge over those without dedicated lanes when roughly a quarter of all vehicles are autonomous, while the advantage fades when the proportion of autonomous vehicles surpasses a half.

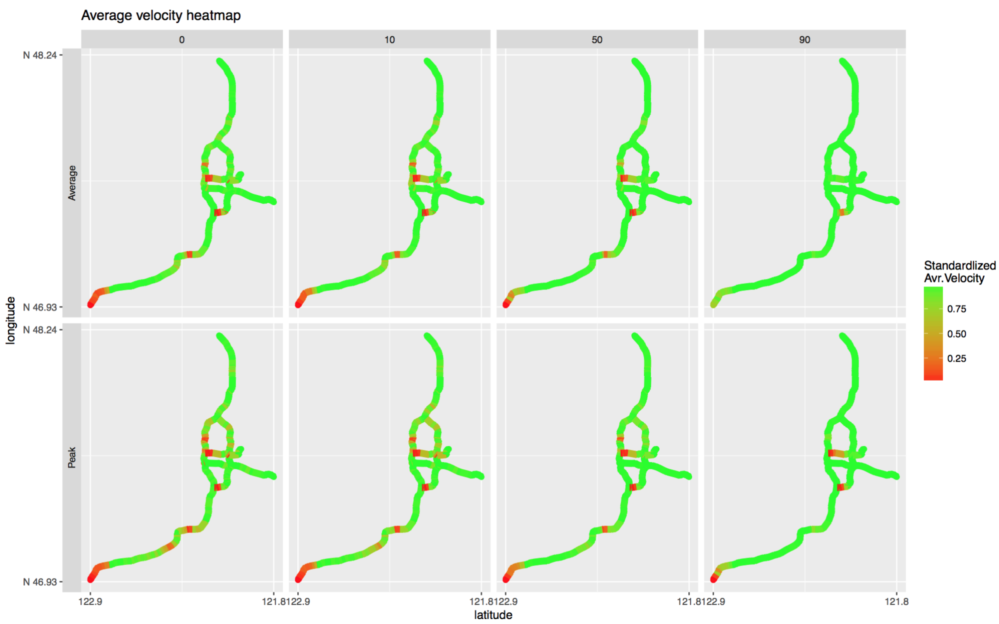


Figure 8: The effect of autonomous vehicles on the mean velocity

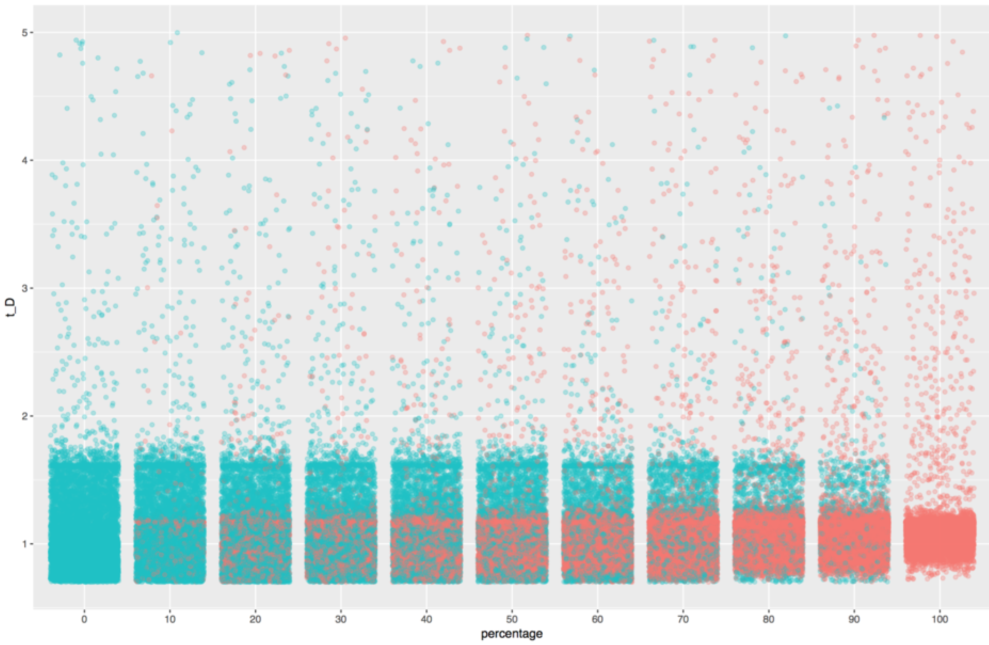
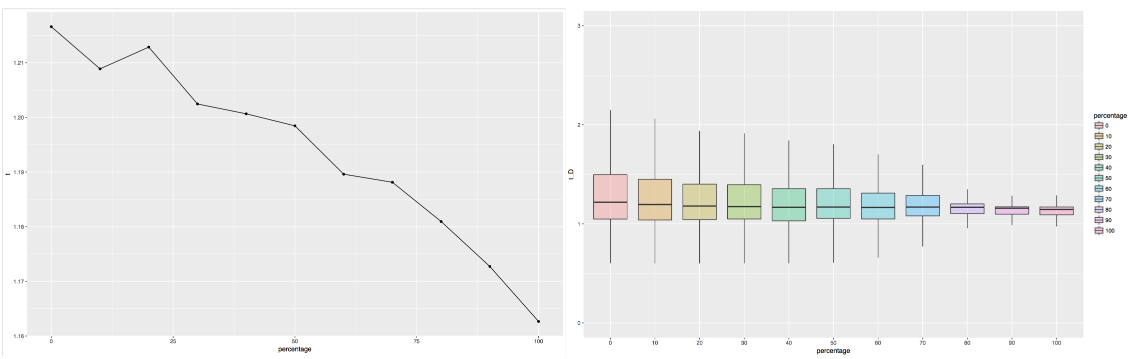


Figure 9: Distribution of τ_{D}



(a) mean value

(b) variance

Figure 10: Statistics of τ_{D}

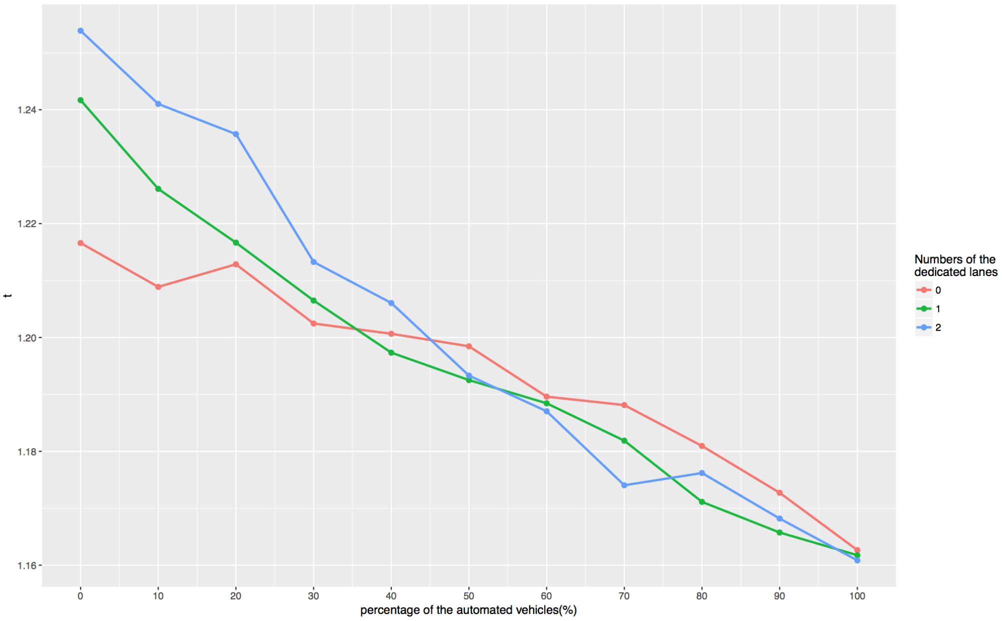


Figure 11: The effect of dedicated lanes

7 Sensitivity Analysis

7.1 Sensitivity of Following Space

In our original model, when the average speed is about 25 m/s, the average following space is about 70m.

By tweaking the following space and re-running the simulation, we get the data shown in tbl. 3.

Table 3: Sensitivity of following space

following space S/m	probability of crash	actual following distance A/m	error space $A - S$ /m
69	1.01%	79.03	10.03
70	**0.92%**	**80.85**	**10.85**
71	0.87%	83.06	12.06

It can be seen that a slight change in S results in a corresponding change in A, keeping an error of about 10m. Notably, the change has little effect on the probability of crashing.

7.2 Sensitivity of Target Velocity

In our original model, we assumed the target velocity of an autonomous vehicle to be about 80% of the speed limit of the highway, or about 50 mph, ignoring the possibility that it might be necessary for the speed limit to change with the actual introduction of autonomous vehicles.

By re-running the simulation with different target velocities, we get the data shown in tbl. 4.

Table 4: Sensitivity of target velocity

target velocity v /mph	probability of crash	τ	average passing time t /s
45	0.83%	1.28	3054.549
50	**0.92%**	**1.25**	**2443.465**
55	1.02%	1.32	1753.091

As is shown in the table, the average passing time keeps a negative correlation with the target velocity, which is intuitively obvious. The probability of crash and the τ of the traffic flow are hardly affected.

8 Extended Model

Besides the models mentioned above, we have some creative ideas that has not been fully validated due to the limited time. Among them is the *traffic light model* for in-ramp vehicles.

According to sec. 3.2.1, vehicles may follow the lane changing model to enter the highway. If the vehicle has stopped before entering the highway, a penalty time cost is incurred. This model is practical, but not efficient enough. In fact, it is empirically a major cause of congestion.

Traffic light model is based on the real-life experience that when congestions occur near entrances, a traffic police tends to regulate the traffic by making vehicles from the in-ramp and those from the arterial highway move in turns, as is shown in fig. 12.

We suggest that in the future, a central system might be setup to regulate the coordination between autonomous vehicles. When congestion happens at an in-ramp, a virtual

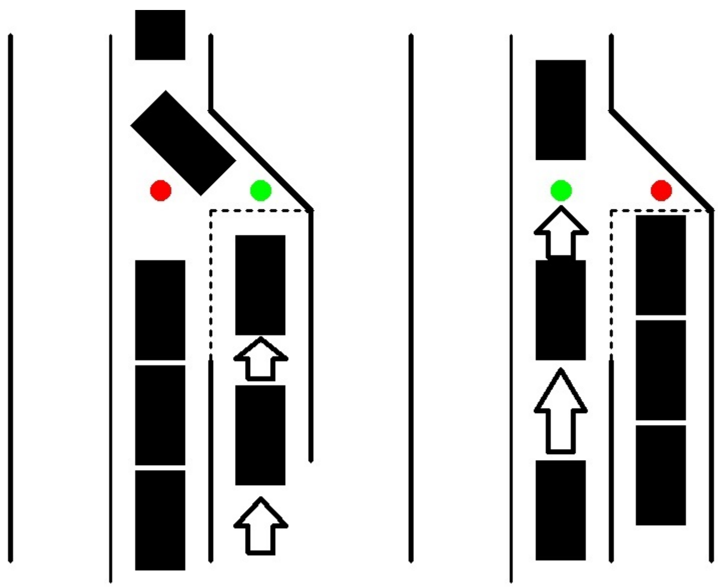


Figure 12: Traffic light model

"traffic light" can be set. Similar to what human police do today, the central system will come up with an optimal plan for the traffic to move and command all autonomous vehicles to obey it.

9 Strengths and Weakness

9.1 Strengths

- **The use of micro-scale models to describe the whole traffic road network:** in order to evaluate a mixed traffic system, we adopt a bottom-top approach by inspecting the behavior of individual vehicles, from which we can successfully describe the big picture of the traffic system.
- **Creative evaluation metrics fit to future traffic systems:** instead of adopting traditional metrics focusing on the number of vehicles on a certain section, we try to measure the welfare of individual drivers, by comparing the actual time cost to the expected time cost, with the belief that future traffic systems would support high throughput while maintaining high velocity.
- **Creative models to provide fine-grained description of certain situations:** For instance, 'horn model' is set up to describe the impact from the following vehicle on the preceding vehicle. 'Traffic light model' is also mentioned to improve the efficiency at the entrance of the highway.

9.2 Weaknesses

- **Horn model requires further modification:** As is found in simulation, some of the manual vehicles may have $\tau_{\mathrm{D}} < 1$. Upon careful inspection, we found horn model responsible for this counter-intuitive behavior. A human driver may be "pushed" by the following vehicle for a long time under heavy traffic. We need more mature metrics to deal with these edge cases.
- **Simulation is computationally expensive:** Our individual-based models require iterating over each vehicle in each time step, incurring large time complexity. In fact, the generation of the data in our thesis took hours on a 400-core server. Further research may require computational optimizations.

10 A Letter to the Governor's Office

Dear officer,

In response to the need to analyze the effects of allowing self-driving and cooperating cars on certain roads in your state, our team has built a complete model considering all the aspects we could imagine. We hope that some of the conclusions we have reached may help improve the highway efficiency in the future.

Our mathematical model is composed of two parts: Highway network model and Vehicle model.

As you can infer from their names, the highway network model extracts real highway to a virtual model. In this model, the entrance and the exit of the highway are fully-described for further analysis.

Vehicle model consists of two parts: autonomous vehicle models and manual vehicle models. We build the vehicle models based on individual actions. For autonomous vehicles, we develop our own driving system model, which clarifies the whole driving process. For manual vehicles, we divide drivers by their aggressiveness, adding personalized factor into our model.

Then we set up our own criteria to evaluate the performance of the highway for the mixed traffic system. We reached some conclusions after applying our model on the highway in your state.

Here are some key questions you may concern about:

Q: what are the prerequisites of your model?

A: We assume that autonomous vehicles can acquire essential information about the surroundings, and can precisely control its motion. We don't care about the realization of the technology.

Q: Will autonomous vehicles improve the traffic condition on the highway?

A: It depends on the percentage of the autonomous vehicles. We found that when the percentage is low (less than 25%), the improvement of the overall traffic condition is slim. During this period, the autonomous vehicles act as pacesetters. They normalize the behaviors of manual vehicles. During the intermediate period (from 25% to 65%), traffic conditions improve faster. When the percentage of the autonomous vehicles reaches a relatively high ratio (65%), platoons are completely formed, the traffic conditions improves significantly at this period. greatly reducing the variance of the whole traffic flow.

Q: Should we add dedicated lanes for the autonomous vehicles? If so, how many lanes are optimal?

A: It also depends on the percentage of the autonomous vehicles. We found that when the percentage is below about 25%, no dedicated lanes is the best choice; when the ratio maintains between 25% and 50%, one dedicated lane is recommended; when the ratio exceeds 50%, actually there is no difference between whether to set dedicated lanes or not. You can make your decision based on the market share of the autonomous vehicle.

Q: Is there any other policy changes suggested by your model?

A: We suggests that the government should require all the autonomous vehicle get a government-regulated device installed. The government should be able to regulate every autonomous vehicle. Then our 'traffic light model' can be applied to the vehicles near the entrance. For further information, you may contact our specialists.

We have great confidence in the reliability of the model. Mixed manual and automated traffic system is an inexorable trend, hopefully this model may somewhat contribute to the planning and decision in the near future.

Yours Sincerely,

Team #60213

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