

Types for references and memory

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Adapted from “Types and Programming Languages” by Benjamin C. Pierce

Sums

$$\begin{array}{l} \langle t \rangle ::= \dots \\ \quad | \text{ inl } t \\ \quad | \text{ inr } t \\ \quad | \text{ case } t \text{ of inl } x \Rightarrow t \mid \text{ inr } x \Rightarrow t \end{array} \qquad \begin{array}{l} \langle v \rangle ::= \text{ inl } v \\ \quad | \text{ inr } v \\ \langle T \rangle ::= \dots \\ \quad | T + T \end{array}$$

Evaluation rules

$$\frac{\text{case } (\text{inl } v_0) \text{ of inl } x_1 \Rightarrow t_1 \mid \text{ inr } x_2 \Rightarrow t_2}{[x_1 \mapsto v_0] t_1} \quad (\text{E-CaseInl})$$

$$\frac{\text{case } (\text{inr } v_0) \text{ of inl } x_1 \Rightarrow t_1 \mid \text{ inr } x_2 \Rightarrow t_2}{[x_2 \mapsto v_0] t_2} \quad (\text{E-CaseInr})$$

$$\frac{t_0 \rightarrow t'_0}{\text{case } t_0 \text{ of inl } x_1 \Rightarrow t_1 \mid \text{ inr } x_2 \Rightarrow t_2 \rightarrow \text{case } t'_0 \text{ of inl } x_1 \Rightarrow t_1 \mid \text{ inr } x_2 \Rightarrow t_2} \quad (\text{E-Case})$$

$$\frac{t_1 \rightarrow t'_1}{\text{inl } t_1 \rightarrow t'_1} \quad (\text{E-Inl})$$

$$\frac{t_1 \rightarrow t'_1}{\text{inr } t_1 \rightarrow t'_1} \quad (\text{E-Inr})$$

Typing rules

$$\frac{\Gamma \vdash t_1 : T_1}{\Gamma \vdash \text{inl } t_1 : T_1 + T_2} \quad (\text{T-Inl})$$

$$\frac{\Gamma \vdash t_1 : T_2}{\Gamma \vdash \text{inr } t_1 : T_1 + T_2} \quad (\text{T-Inr})$$

$$\frac{\Gamma \vdash t_0 : T_1 + T_2 \quad \Gamma, x_1 : t_1 \vdash t_1 : T \quad \Gamma, x_2 : t_2 \vdash t_2 : T}{\Gamma \vdash \text{case } t_0 \text{ of } \text{inl } x_1 \Rightarrow t_1 \mid \text{inr } x_2 \Rightarrow t_2 : T} \quad (\text{T-Case})$$