Sketch of SRS for Projectile

① Theoretical Models

acceleration
velocity

Acceleration $\quad \underline{a} = \dfrac{d\underline{v}}{dt}$ (TM1) ($\underline{a}$ and $\underline{v}$ are general, abstract vectors. We have not yet stated a basis for them.)

position

Velocity $\quad \underline{v} = \dfrac{d\underline{u}}{dt}$ (TM2) ($\underline{v}$ and $\underline{u}$ are general, abstract vectors. We have not yet stated a basis for them.)

② Assumptions (There are relationships b/w assumptions, but we cannot currently capture those.)

this should be part of the terminology (so should target)

A1 – 2D

A2 – Cartesian coordinate system

A3 – the origin is located coincident with the launcher

A4 – up is positive (we'll label this direction y)

A5 – to the right is positive (we'll label this direction x)

A6 – the acceleration is constant

A7 – the acceleration in the x-direction is zero

A8 – the acceleration in the y-direction is the acceleration due to gravity downward (−g)

A9 – air drag is neglected (which is part of why we can say the acceleration is constant)

A10 – the mass of the projectile is constant

A11 – the distance is small enough that the curvature of the Earth can be neglected

A12 – the size and shape of the projectile are negligible, so that it can be modelled as a point mass

A13 – time starts at ∅

3 General Definition

GD1 Rectilinear velocity as a function of time for constant acceleration

~~By A2 we have a Cartesian coordinate system, which means that the motion component of motion in each coordinate direction is rectilinear, or in a straight line.~~

~~For~~ Assume we have rectilinear motion of a particle (of negligible size and shape A12), that is motion in a straight line. The velocity is v and the acceleration is a.

The motion in TM1 is now one-dimensional with a constant acceleration, represented by a^c. The initial velocity (at $t=0$) is represented by v^i.

From TM1, using the above symbols we have:

$$a^c = \frac{dv}{dt}$$

Rearranging and integrating, we have:

$$\int_{v^i}^{v} dv = \int_0^t a^c \, dt$$

~~Integrating we have~~ Performing the integration, we have:

$$v = v^i + a^c t$$

GD2 Rectilinear Position as a Function of Time for Constant Acceleration

- Constant acceleration a^c
- Initial velocity v^i
- Initial position u^i

From TM2 $\quad v = \dfrac{du}{dt}$ (*)

~~From TM GD1 $v = v^i + a^c t$~~

Rearrange and Integrate equation *

$$\int_{u^i}^{u} du = \int_0^t v\,dt = \cancel{\int_0^t (v^i + a^c t)\,dt}$$

From GD1 we can replace v:

$$\int_{u^i}^{u} du = \int_0^t (v^i + a^c t)\,dt$$

$$\therefore \quad u = u^i + v^i t + \frac{1}{2} a^c t^2$$

GD3 Velocity Vector as a Function of Time

~~By A1~~ For a 2D Cartesian Coordinate System (A1 & A2), we can represent the velocity vector as $\underline{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$ and the acceleration vector by $\underline{a} = \begin{bmatrix} a_x \\ a_y \end{bmatrix}$. The acceleration is assumed constant (A6). Let us represent the constant acceleration as a^c_x as $[a^c_x, a^c_y]^T$. The initial velocity (at $t=0$) is represented by $\underline{v}^i = [v^i_x \; v^i_y]$. ~~From the relat~~ Since we have a Cartesian coordinate system, GD1 can be applied to each coordinate direction, to yield

$$\underline{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} v^i_x + a^c_x t \\ v^i_y + a^c_y t \end{bmatrix}$$

GD4 Position Vector as a Function of Time

For a 2D Cartesian Coordinate System (A1 & A2), we can represent the position vector as $\underline{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix}$, the velocity vector as $\underline{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$ and the acceleration by $\underline{a} = \begin{bmatrix} a_x \\ a_y \end{bmatrix}$. The acceleration is assumed constant (A6). The constant acceleration is represented by $\underline{a}^c = \begin{bmatrix} a_x^c \\ a_y^c \end{bmatrix}$. The initial velocity (at $t=0$) is rep. by $\underline{v}^i = \begin{bmatrix} v_x^i \\ v_y^i \end{bmatrix}$.

Since we have a Cartesian coordinate system, GD1 can be applied component-wise in each direction, to yield:

$$\underline{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} u_x^i + v_x^i t + \frac{1}{2} a_x^c t^2 \\ u_y^i + v_y^i t + \frac{1}{2} a_y^c t^2 \end{bmatrix}$$

DD1 Initial Velocity Vector X Components from Magnitude and Direction

Symbol v_x^i

units m/s

Equation $v_x^i = |v| \sin\theta$

Description v_x^i is the x-component of the initial velocity

$|v|$ is the magnitude of the initial velocity vector

θ is the angle between the +ve x-direction and the velocity vector (could reference the diagram of the problem here)

DD2 Initial Velocity Vector y-component from Magnitude and Direction

(similar to DD1) $v_y^i = |v| \cos\theta$

IM1 Calculation of Landing Position and Time

Input: $|v|, \theta$

Output: d', t'

Input Constraints: $|v| > 0$, $0 < \theta < \frac{\pi}{2}$

Output Constraints: $d' > 0$, $t' > 0$

Equation: $t' = \frac{2v_y^i}{g}$

$$d' = \frac{v_x^i \, 2v_y^i}{g}$$

Description

t' is the time when the projectile lands

v_y^i is the ~~initial~~ ... given by DD2

$|v|$ ...

θ ...

d' ...

g ...

Derivation of Landing Position and Time

We know $a_x^c = 0$ (A7) and $a_y^c = -g$ (A8). We also know $u_x^i = u_y^i = 0$ (A3). Substituting these values into GD3, we have

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} v_x^i \\ v_y^i - gt \end{bmatrix}$$

Substituting a_x^c, a_y^c, u_x^i and u_y^i into GD4, we have:

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} v_x^i t \\ v_y^i t - \frac{1}{2} g t^2 \end{bmatrix}$$

To find the time that the projectile lands, we want to find the t value (t') where $u_y = 0$ (since $u_y = 0$ corresponds to the ground). Using the equation for u_y, we have

$$v_y^i t' - \frac{1}{2} g t'^2 = 0$$

Divide by t', to get:

$$v_y^i - \frac{1}{2} g t' = 0$$

Solve for $t' = \frac{2 v_y^i}{g}$ ← the time when the projectile lands

To find the vertical distance travelled, the equation for position in the x-direction d' is used with this time:

$$d' = v_x^i t = \frac{2 v_x^i v_y^i}{g}$$

Requirements

R1 Inputs $\vec{v}$, θ, d

R2 Check Input Validity

R3 Calculate t' and d' by IM1

R4 Output t' and d'

R5 Output $\left[\left(\frac{d-d'}{d} \right) \leq \varepsilon \Rightarrow \text{"Hit target"} \;\middle|\; \text{True} \Rightarrow \text{"Missed"} \right]$

We need to define this constant somewhere. ~~If~~ In previous examples we made the values of constants an assumption.

Thoughts

- The refinement from theory to instance model is meaningful in this problem, but for a different problem, TM1 and TM2 might be data definitions
- There are ~~different kinds of assum~~ relationships between assumptions. For instance A6 says the accel is constant, where A7 and A8 give specific values. The accel is constant because of A9, A10
- Ideally, every assumption should be used somewhere, which might mean one assumption ~~uses~~ referencing another assumption
- the data definitions don't reference anything ~~but then~~. If the properties of vectors were used, then the DDs would become GDs.
- The GDs were written as generally as possible for reuse
- The IM put everything together for this specific problem ~~Still~~
- The requirements bring in the ideas of whether the target was hit or not