

Software Requirements Specification for Solar Water Heating Systems Incorporating PCM

Thulasi Jegatheesan, Brooks MacLachlan, and W. Spencer Smith

May 1, 2019

Contents

1	Reference Material	2
1.1	Table of Units	2
1.2	Table of Symbols	2
1.3	Abbreviations and Acronyms	4
2	Introduction	5
2.1	Purpose of Document	5
2.2	Scope of Requirements	6
2.3	Characteristics of Intended Reader	6
2.4	Organization of Document	6
3	General System Description	6
3.1	System Context	7
3.2	User Characteristics	7
3.3	System Constraints	7
4	Specific System Description	7
4.1	Problem Description	8
4.1.1	Terminology and Definitions	8
4.1.2	Physical System Description	8
4.1.3	Goal Statements	8
4.2	Solution Characteristics Specification	10
4.2.1	Assumptions	10
4.2.2	Theoretical Models	11
4.2.3	General Definitions	14
4.2.4	Data Definitions	17
4.2.5	Instance Models	22
4.2.6	Data Constraints	30
4.2.7	Properties of a Correct Solution	31

5	Requirements	31
5.1	Functional Requirements	31
5.2	Non-Functional Requirements	33
6	Likely Changes	33
7	Unlikely Changes	34
8	Traceability Matrices and Graphs	34
9	Values of Auxiliary Constants	37
10	References	38

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

The unit system used throughout is SI (Système International d'Unités). In addition to the basic units, several derived units are also used. For each unit, the table lists the symbol, a description and the SI name.

Symbol	Description
°C	temperature (centigrade)
J	energy (joule)
kg	mass (kilogram)
m	length (metre)
s	time (second)
W	power (watt)

1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the heat transfer literature and with existing documentation for solar water heating systems. The symbols are listed in alphabetical order.

Symbol	Description	Units
A_C	Heating coil surface area	m^2
A_{in}	Surface area over which heat is transferred in	m^2
A_{out}	Surface area over which heat is transferred out	m^2
A_P	Phase change material surface area	m^2
C	Specific heat capacity	$\frac{J}{(kg^\circ C)}$
C^L	Specific heat capacity of a liquid	$\frac{J}{(kg^\circ C)}$
C^S	Specific heat capacity of a solid	$\frac{J}{(kg^\circ C)}$
C^V	Specific heat capacity of a vapour	$\frac{J}{(kg^\circ C)}$
C_W	Specific heat capacity of water	$\frac{J}{(kg^\circ C)}$
C_P^L	Specific heat capacity of PCM as a liquid	$\frac{J}{(kg^\circ C)}$

Symbol	Description	Units
C_P^S	Specific heat capacity of PCM as a solid	$\frac{\text{J}}{(\text{kg}^\circ\text{C})}$
D	Diameter of tank	m
E	Sensible heat	J
E_P	Change in heat energy in the PCM	J
E_W	Change in heat energy in the water	J
$E_{P_{melt}}^{init}$	Change in heat energy in the PCM at the instant when melting begins	J
g	Volumetric heat generation per unit volume	$\frac{\text{W}}{\text{m}^3}$
h	Convective heat transfer coefficient	$\frac{\text{W}}{(\text{m}^2^\circ\text{C})}$
h_C	Convective heat transfer coefficient between coil and water	$\frac{\text{W}}{(\text{m}^2^\circ\text{C})}$
H_f	Specific latent heat of fusion	$\frac{\text{J}}{\text{kg}}$
h_{min}	Minimum thickness of a sheet of PCM	m
h_P	Convective heat transfer coefficient between PCM and water	$\frac{\text{W}}{(\text{m}^2^\circ\text{C})}$
L	Length of tank	m
m	Mass	kg
m_P	Mass of phase change material	kg
m_W	Mass of water	kg
\mathbf{n}	Unit outward normal vector for a surface	—
Q	Latent heat	J
q	Heat flux	$\frac{\text{W}}{\text{m}^2}$
q_C	Heat flux into the water from the coil	$\frac{\text{W}}{\text{m}^2}$
q_{in}	Heat flux input	$\frac{\text{W}}{\text{m}^2}$
q_{out}	Heat flux output	$\frac{\text{W}}{\text{m}^2}$
q_P	Heat flux into the PCM from water	$\frac{\text{W}}{\text{m}^2}$
Q_P	Latent heat energy added to PCM	J
\mathbf{q}	Thermal flux vector	$\frac{\text{W}}{\text{m}^2}$
S	Surface	m^2
T	Temperature	$^\circ\text{C}$
t	Time	s
ΔT	Change in temperature	$^\circ\text{C}$
T_{boil}	Boiling point temperature	$^\circ\text{C}$
T_C	Temperature of the heating coil	$^\circ\text{C}$

Symbol	Description	Units
T_{env}	Temperature of the environment	$^{\circ}\text{C}$
t_{final}	Final time	s
T_{init}	Initial temperature	$^{\circ}\text{C}$
T_{melt}	Melting point temperature	$^{\circ}\text{C}$
T_P	Temperature of the phase change material	$^{\circ}\text{C}$
T_W	Temperature of the water	$^{\circ}\text{C}$
t_{melt}^{final}	Time at which melting of PCM ends	s
t_{melt}^{init}	Time at which melting of PCM begins	s
T_{melt}^P	Melting point temperature for PCM	$^{\circ}\text{C}$
V	Volume	m^3
V_P	Volume of PCM	m^3
V_{tank}	Volume of the cylindrical tank	m^3
V_W	Volume of water	m^3
η	ODE parameter	—
ρ	Density	$\frac{\text{kg}}{\text{m}^3}$
ρ_P	Density of PCM	$\frac{\text{kg}}{\text{m}^3}$
ρ_W	Density of water	$\frac{\text{kg}}{\text{m}^3}$
τ	Dummy variable for integration over time	s
τ_W	ODE parameter for water	s
τ_P^L	ODE parameter for liquid PCM	s
τ_P^S	ODE parameter for solid PCM	s
ϕ	Melt fraction	—
∇	Gradient	—

1.3 Abbreviations and Acronyms

Abbreviation	Full Form
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model

Abbreviation	Full Form
LC	Likely Change
ODE	Ordinary Differential Equation
PCM	Phase Change Material
PS	Physical System Description
R	Requirement
RHS	Right Hand Side
SRS	Software Requirements Specification
SWHS	Solar Water Heating System
T	Theoretical Model
UC	Unlikely Change
Uncert.	Typical Uncertainty

2 Introduction

Due to increasing cost, diminishing availability, and negative environmental impact of fossil fuels, there is a higher demand for renewable energy sources and energy storage technology. Solar water heating systems incorporating phase change material (PCM) use renewable energy sources and provide a novel way of storing energy. Solar water heating systems incorporating PCM improve over the traditional solar water heating systems because of their smaller size. The smaller size is possible because of the ability of PCM to store thermal energy as latent heat, which allows higher thermal energy storage capacity per unit weight. The following section provides an overview of the Software Requirements Specification (SRS) for solar water heating systems incorporating PCM. The developed program will be referred to as Solar Water Heating System (SWHS). This section explains the purpose of this document, the scope of the system, the characteristics of the intended reader, and the organization of the document.

2.1 Purpose of Document

The main purpose of this document is to describe the modelling of solar water heating systems incorporating PCM. The goal statements and theoretical models used in the SWHS code are provided, with an emphasis on explicitly identifying assumptions and unambiguous definitions. This document is intended to be used as a reference to provide ad hoc access to all information necessary to understand and verify the model. The SRS is abstract because the contents say what problem is being solved, but do not say how to solve it. This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation.

Although the SRS fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the waterfall model is not followed, as Parnas and Clements point out [**parnasClements1986**], the most logical way to present the documentation is still to “fake” a rational design process.

2.2 Scope of Requirements

The scope of the requirements includes thermal analysis of a single solar water heating tank incorporating PCM. Given the appropriate inputs, SWHS predicts the temperature and thermal energy histories for the water and the PCM. This entire document is written assuming that the substances inside the solar water heating tank are water and PCM.

2.3 Characteristics of Intended Reader

Reviewers of this documentation should have an understanding of heat transfer theory from level 3 or 4 mechanical engineering and differential equations from level 1 and 2 calculus. The users of SWHS can have a lower level of expertise, as explained in [Section: User Characteristics](#).

2.4 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by [**dParnas1972**] and [**parnasClements1984**]. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models in [Section: Instance Models](#) and trace back to find any additional information they require. The goal statements ([Section: Goal Statements](#)) are refined to the theoretical models, and the theoretical models ([Section: Theoretical Models](#)) to the instance models ([Section: Instance Models](#)). The instance models to be solved are referred to as [IM: eBalanceOnWtr](#), [IM: eBalanceOnPCM](#), [IM: heatEInWtr](#), and [IM: heatEInPCM](#). The instance models provide the Ordinary Differential Equation (ODEs) and algebraic equations that model the solar water heating systems incorporating PCM. SWHS solves these ODEs.

3 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics, and lists the system constraints.



Figure 1: **Fig:SysCon**: System Context

3.1 System Context

Fig:SysCon shows the system context. A circle represents an external entity outside the software, the user in this case. A rectangle represents the software system itself (SWHS). Arrows are used to show the data flow between the system and its environment. SWHS is mostly self-contained. The only external interaction is through the user interface. The responsibilities of the user and the system are as follows:

- User Responsibilities:
 - Provide the input data to the system, ensuring no errors in the data entry
 - Take care that consistent units are used for input variables
- SWHS Responsibilities:
 - Detect data type mismatch, such as a string of characters instead of a floating point number
 - Determine if the inputs satisfy the required physical and software constraints
 - Calculate the required outputs

3.2 User Characteristics

The end user of SWHS should have an understanding of undergraduate Level 1 Calculus and Physics.

3.3 System Constraints

There are no system constraints.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, and definitions that are used.

4.1 Problem Description

SWHS is a computer program developed to investigate the effect of employing PCM within a solar water heating tank.

4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements.

- Heat flux: The rate of thermal energy transfer through a given surface per unit time.
- Phase change material: A substance that uses phase changes (such as melting) to absorb or release large amounts of heat at a constant temperature.
- Specific heat capacity: The amount of energy required to raise the temperature of the unit mass of a given substance by a given amount.
- Thermal conduction: The transfer of heat energy through a substance.
- Transient: Changing with time.

4.1.2 Physical System Description

The physical system of SWHS, as shown in [Fig:Tank](#), includes the following elements:

PS1: Tank containing water.

PS2: Heating coil at bottom of tank. (q_C represents the heat flux into the water from the coil.)

PS3: PCM suspended in tank. (q_P represents the heat flux into the PCM from water.)

4.1.3 Goal Statements

Given the temperature of the heating coil, initial conditions for the temperature of the water and the temperature of the phase change material, and material properties, the goal statements are:

GS1: Predict the temperature of the water over time.

GS2: Predict the temperature of the phase change material over time.

GS3: Predict the change in heat energy in the water over time.

GS4: Predict the change in heat energy in the PCM over time.



Figure 2: Solar water heating tank, with heat flux into the water from the coil of q_C and heat flux into the PCM from water of q_P

4.2 Solution Characteristics Specification

The instance models that govern SWHS are presented in [Section: Instance Models](#). The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the Theoretical Models [Section: Theoretical Models](#), General Definitions [Section: General Definitions](#), Data Definitions [Section: Data Definitions](#), Instance Models [Section: Instance Models](#), Likely Changes [Section: Likely Changes](#), or Unlikely Changes [Section: Unlikely Changes](#), in which the respective assumption is used.

- Thermal-Energy-Only: The only form of energy that is relevant for this problem is thermal energy. All other forms of energy, such as mechanical energy, are assumed to be negligible. [TM: cons-ThermE](#).
- Heat-Coeffs-Constant: All heat transfer coefficients are constant over time. [GD: nwtmCooling](#).
- Temp-Across-Tank: The water in the tank is fully mixed, so the temperature of the water is the same throughout the entire tank. [GD: rocTempSimp](#) [DD: ht_flux_P](#).
- Temp-Across-Volume: The temperature of the phase change material is the same throughout the volume of PCM. [GD: rocTempSimp](#) [LC: Uniform-Temperature-PCM](#) [DD: ht_flux_P](#).
- Density-over-Volume: The density of water and density of PCM have no spatial variation; that is, they are each constant over their entire volume. [GD: rocTempSimp](#).
- Specific-heat-over-Volume: The specific heat capacity of water, specific heat capacity of PCM as a solid, and specific heat capacity of PCM as a liquid have no spatial variation; that is, they are each constant over their entire volume. [GD: rocTempSimp](#).
- Cooling-Coil-Water: Newton's law of convective cooling applies between the heating coil and the water. [DD: ht_flux_C](#).
- Temperature-over-Time: The temperature of the heating coil is constant over time. [LC: Temperature-Coil-Variable-Over-Day](#) [DD: ht_flux_C](#).
- Temperature-over-Length: The temperature of the heating coil does not vary along its length. [LC: Temperature-Coil-Variable-Over-Length](#) [DD: ht_flux_C](#).
- Cooling-Water-PCM: Newton's law of convective cooling applies between the water and the PCM. [DD: ht_flux_P](#).

o-Temp-Discharge: The model only accounts for charging of the tank, not discharging. The temperature of the water and temperature of the phase change material can only increase, or remain constant; they do not decrease. This implies that the initial temperature **A: Same-Initial-Temp-Water-PCM** is less than (or equal) to the temperature of the heating coil. **LC: Discharging-Tank IM: eBalanceOnWtr.**

Temp-Water-PCM: The initial temperature of the water and the PCM is the same. **LC: Different-Initial-Temps-PCM-Water IM: eBalanceOnPCM A: Charging-Tank-No-Temp-Discharge.**

CM-Initially-Solid: The simulation will start with the PCM in a solid state. **IM: heatEInPCM IM: eBalanceOnPCM.**

Water-Always-Liquid: The operating temperature range of the system is such that the water is always in liquid state. That is, the temperature will not drop below the melting point temperature of water, or rise above its boiling point temperature. **UC: Water-PCM-Fixed-States IM: heatEInWtr IM: eBalanceOnWtr.**

Perfect-Insulation-Tank: The tank is perfectly insulated so that there is no heat loss from the tank. **LC: Tank-Lose-Heat IM: eBalanceOnWtr.**

Non-By-Water-PCM: No internal heat is generated by either the water or the PCM ; therefore, the volumetric heat generation per unit volume is zero. **UC: No-Internal-Heat-Generation IM: eBalanceOnWtr IM: eBalanceOnPCM.**

Volume-Change-PCM-Negligible: The volume change of the PCM due to melting is negligible. **IM: eBalanceOnPCM.**

Gaseous-State-PCM: The PCM is either in a liquid state or a solid state but not a gaseous state. **UC: Water-PCM-Fixed-States UC: No-Gaseous-State IM: heatEInPCM IM: eBalanceOnPCM.**

Atmospheric-Pressure-Tank: The pressure in the tank is atmospheric, so the melting point temperature and boiling point temperature are 0°C and 100°C, respectively. **IM: heatEInWtr IM: eBalanceOnWtr.**

Heating-Coil-Negligible: When considering the volume of water in the tank, the volume of the heating coil is assumed to be negligible. **FR: Input-Initial-Quantities.**

4.2.2 Theoretical Models

This section focuses on the general equations and laws that SWHS is based on.

Refname	TM:consThermE
Label	Conservation of thermal energy
Equation	$-\nabla \cdot \mathbf{q} + g = \rho C \frac{\partial T}{\partial t}$
Description	<p> ∇ is the gradient (Unitless) \mathbf{q} is the thermal flux vector ($\frac{\text{W}}{\text{m}^2}$) g is the volumetric heat generation per unit volume ($\frac{\text{W}}{\text{m}^3}$) ρ is the density ($\frac{\text{kg}}{\text{m}^3}$) C is the specific heat capacity ($\frac{\text{J}}{(\text{kg}^\circ\text{C})}$) t is the time (s) T is the temperature ($^\circ\text{C}$) </p>
Notes	<p> The above equation gives the law of conservation of energy for transient heat transfer in a material of specific heat capacity C ($\frac{\text{J}}{(\text{kg}^\circ\text{C})}$) and density, ρ ($\frac{\text{kg}}{\text{m}^3}$), where \mathbf{q} is the thermal flux vector ($\frac{\text{W}}{\text{m}^2}$), g is the volumetric heat generation per unit volume ($\frac{\text{W}}{\text{m}^3}$), T is the temperature ($^\circ\text{C}$), t is time (s), and ∇ is the degree of steepness of a graph at any point. For this equation to apply, other forms of energy, such as mechanical energy, are assumed to be negligible in the system (A: Thermal-Energy-Only). </p>
Source	Fourier Law of Heat Conduction and Heat Equation
RefBy	GD: rocTempSimp.

Refname	TM:sensHtE
Label	Sensible heat energy
Equation	$E = \begin{cases} C^S m \Delta T, & T < T_{melt} \\ C^L m \Delta T, & T_{melt} < T < T_{boil} \\ C^V m \Delta T, & T_{boil} < T \end{cases}$
Description	<p>E is the sensible heat (J) C^S is the specific heat capacity of a solid ($\frac{\text{J}}{(\text{kg}^\circ\text{C})}$) m is the mass (kg) ΔT is the change in temperature ($^\circ\text{C}$) C^L is the specific heat capacity of a liquid ($\frac{\text{J}}{(\text{kg}^\circ\text{C})}$) C^V is the specific heat capacity of a vapour ($\frac{\text{J}}{(\text{kg}^\circ\text{C})}$) T is the temperature ($^\circ\text{C}$) T_{melt} is the melting point temperature ($^\circ\text{C}$) T_{boil} is the boiling point temperature ($^\circ\text{C}$)</p>
Notes	<p>E is the change in sensible heat energy (J). C^S, C^L, C^V are the specific heat capacity of a solid, specific heat capacity of a liquid, and specific heat capacity of a vapour, respectively ($\frac{\text{J}}{(\text{kg}^\circ\text{C})}$). m is the mass (kg). T is the temperature ($^\circ\text{C}$), and ΔT is the change in temperature ($^\circ\text{C}$). T_{melt} and T_{boil} are the melting point temperature and boiling point temperature, respectively ($^\circ\text{C}$). Sensible heating occurs as long as the material does not reach a temperature where a phase change occurs. A phase change occurs if $T=T_{boil}$ or $T=T_{melt}$. If this is the case, refer to TM: latentHtE, Latent heat energy.</p>
Source	Definition of Sensible Heat
RefBy	IM: heatEInWtr IM: heatEInPCM IM: heatEInPCM IM: heatEInPCM.

Refname	TM:latentHtE
Label	Latent heat energy
Equation	$Q(t) = \int_0^t \frac{dQ(\tau)}{d\tau} d\tau$
Description	<p>Q is the latent heat (J) t is the time (s) τ is the dummy variable for integration over time (s)</p>
Notes	<p>Q is the change in thermal energy (J), latent heat energy. $Q(t) = \int_0^t \frac{dQ(\tau)}{d\tau} d\tau$ is the rate of change of Q with respect to time τ (s). t is the time (s) elapsed, as long as the phase change is not complete. The status of the phase change depends on the melt fraction, DD: htFusion. T_{melt} and T_{boil} are the melting point temperature and boiling point temperature, respectively (°C). Latent heating stops when all material has changed to the new phase.</p>
Source	Definition of Latent Heat
RefBy	TM: sensHtE IM: heatEInPCM.

4.2.3 General Definitions

This section collects the laws and equations that will be used to build the instance models.

Refname	GD:nwtnCooling
Label	Newton's law of cooling
Units	$\frac{W}{m^2}$
Equation	$q(t) = h\Delta T(t)$
Description	<p>q is the heat flux ($\frac{W}{m^2}$)</p> <p>t is the time (s)</p> <p>h is the convective heat transfer coefficient ($\frac{W}{m^2\text{ }^\circ\text{C}}$)</p> <p>$\Delta T$ is the change in temperature ($^\circ\text{C}$)</p>
Notes	<p>Newton's law of cooling describes convective cooling from a surface. The law is stated as: the rate of heat loss from a body is proportional to the difference in temperatures between the body and its surroundings. $\mathbf{q}(t)$ is the thermal flux ($\frac{W}{m^2}$). h is the heat transfer coefficient, assumed independant of T (A: Heat-Transfer-Coeffs-Constant) ($\frac{W}{m^2\text{ }^\circ\text{C}}$). $\Delta T(t) = T(t) - T_{env}(t)$ is the time-dependant thermal gradient between the environment and the object ($^\circ\text{C}$).</p>
Source	[incroperaEtAl2007]
RefBy	

Refname	GD:rocTempSimp
Label	Simplified rate of change of temperature
Equation	$mC\frac{dT}{dt} = q_{in}A_{in} - q_{out}A_{out} + gV$
Description	<p> m is the mass (kg) C is the specific heat capacity ($\frac{J}{(kg^{\circ}C)}$) t is the time (s) T is the temperature ($^{\circ}C$) q_{in} is the heat flux input ($\frac{W}{m^2}$) A_{in} is the surface area over which heat is transferred in (m^2) q_{out} is the heat flux output ($\frac{W}{m^2}$) A_{out} is the surface area over which heat is transferred out (m^2) g is the volumetric heat generation per unit volume ($\frac{W}{m^3}$) V is the volume (m^3) </p>
Notes	<p>The basic equation governing the rate of change of temperature, for a given volume V, with time. m is the mass (kg). C is the specific heat capacity ($\frac{J}{(kg^{\circ}C)}$). T is the temperature ($^{\circ}C$) and t is the time (s). q_{in} and q_{out} are the in and out heat transfer rates, respectively ($\frac{W}{m^2}$). A_{in} and A_{out} are the surface areas over which the heat is being transferred in and out, respectively (m^2). g is the volumetric heat generated ($\frac{W}{m^3}$). V is the volume (m^3).</p>
Source	
RefBy	GD: rocTempSimp IM: eBalanceOnWtr IM: eBalanceOnPCM.
Detailed derivation of simplified rate of change of temperature : Integrating TM: cons-ThermE over a volume (V), we have:	

$$-\int_V \nabla \cdot \mathbf{q} dV + \int_V g dV = \int_V \rho C \frac{\partial T}{\partial t} dV$$

Applying Gauss's Divergence Theorem to the first term over the surface S of the volume, with \mathbf{q} as the thermal flux vector for the surface and \mathbf{n} as a unit outward normal vector for

a surface:

$$-\int_S \mathbf{q} \cdot \mathbf{n} dS + \int_V g dV = \int_V \rho C \frac{\partial T}{\partial t} dV$$

We consider an arbitrary volume. The volumetric heat generation per unit volume is assumed constant. Then (1) can be written as:

$$q_{in}A_{in} - q_{out}A_{out} + gV = \int_V \rho C \frac{\partial T}{\partial t} dV$$

Where q_{in} , q_{out} , A_{in} , and A_{out} are explained in [GD: rocTempSimp](#). Assuming ρ , C and T are constant over the volume, which is true in our case by [A: Constant-Water-Temp-Across-Tank](#), [A: Temp-PCM-Constant-Across-Volume](#), [A: Density-Water-PCM-Constant-over-Volume](#), and [A: Specific-Heat-Energy-Constant-over-Volume](#), we have:

$$\rho CV \frac{dT}{dt} = q_{in}A_{in} - q_{out}A_{out} + gV$$

Using the fact that $\rho=m/V$, (2) can be written as:

$$mC \frac{dT}{dt} = q_{in}A_{in} - q_{out}A_{out} + gV$$

4.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models.

Refname	DD:ht.flux.C
Label	Heat flux into the water from the coil
Symbol	q_C
Units	$\frac{\text{W}}{\text{m}^2}$
Equation	$q_C = h_C (T_C - T_W(t))$
Description	<p>q_C is the heat flux into the water from the coil ($\frac{\text{W}}{\text{m}^2}$)</p> <p>$h_C$ is the convective heat transfer coefficient between coil and water ($\frac{\text{W}}{\text{m}^2\text{°C}}$)</p> <p>$T_C$ is the temperature of the heating coil (°C)</p> <p>T_W is the temperature of the water (°C)</p> <p>t is the time (s)</p>
Notes	A: Newton-Law-Convective-Cooling-Coil-Water A: Temp-Heating-Coil-Constant-over-Time A: Temp-Heating-Coil-Constant-over-Length
Source	[koothoor2013]
RefBy	IM: eBalanceOnWtr.

Refname	DD:ht.flux.P
Label	Heat flux into the PCM from water
Symbol	q_P
Units	$\frac{\text{W}}{\text{m}^2}$
Equation	$q_P = h_P(T_W(t) - T_P(t))$
Description	<p>q_P is the heat flux into the PCM from water ($\frac{\text{W}}{\text{m}^2}$)</p> <p>$h_P$ is the convective heat transfer coefficient between PCM and water ($\frac{\text{W}}{\text{m}^2\text{°C}}$)</p> <p>$T_W$ is the temperature of the water (°C)</p> <p>t is the time (s)</p> <p>T_P is the temperature of the phase change material (°C)</p>
Notes	A: Constant-Water-Temp-Across-Tank A: Temp-PCM-Constant-Across-Volume A: Law-Convective-Cooling-Water-PCM
Source	[koothoor2013]
RefBy	IM: eBalanceOnWtr IM: eBalanceOnPCM.

Refname	DD:BalanceSolidPCM
Label	ODE parameter for solid PCM
Symbol	τ_P^S
Units	s
Equation	$\tau_P^S = \frac{m_P C_P^S}{h_P A_P}$
Description	<p> τ_P^S is an ODE parameter for solid PCM (s) m_P is the mass of the PCM (kg) C_P^S is the specific heat capacity of the solid PCM ($\frac{\text{J}}{(\text{kg}^\circ\text{C})}$) h_P is the heat transfer coefficient of the PCM ($\frac{\text{W}}{(\text{m}^2^\circ\text{C})}$) A_P is the surface area of the PCM (m^2) </p>
Source	[Lightstone2012]
RefBy	IM: eBalanceOnPCM

Refname	DD:htFusion
Label	Specific latent heat of fusion
Symbol	H_f
Units	$\frac{\text{J}}{\text{kg}}$
Equation	$H_f = \frac{Q}{m}$
Description	<p>H_f is the specific latent heat of fusion ($\frac{\text{J}}{\text{kg}}$)</p> <p>$Q$ is the latent heat (J)</p> <p>m is the mass (kg)</p>
Source	[bueche1986]
RefBy	DD: melt_frac TM: latentHtE IM: heatEInPCM IM: eBalanceOnWtr IM: eBalanceOnWtr.

Refname	DD:melt.frac
Label	Melt fraction
Symbol	ϕ
Units	Unitless
Equation	$\phi = \frac{Q_P}{H_f m_P}$
Description	ϕ is the melt fraction (Unitless) Q_P is the latent heat energy added to PCM (J) H_f is the specific latent heat of fusion ($\frac{\text{J}}{\text{kg}}$) m_P is the mass of phase change material (kg)
Notes	DD: htFusion
Source	[koothoor2013]
RefBy	IM: eBalanceOnWtr IM: eBalanceOnPCM.

4.2.5 Instance Models

This section transforms the problem defined in [Section: Problem Description](#) into one which is expressed in mathematical terms. It uses concrete symbols defined in [Section: Data Definitions](#) to replace the abstract symbols in the models identified in [Section: Theoretical Models](#) and [Section: General Definitions](#).

Refname	IM:eBalanceOnWtr		
Label	Energy balance on water to find the temperature of the water		
Input	$m_W, C_W, h_C, A_P, h_P, A_C, T_P, t_{final}, T_C, T_{init}$		
Output	T_W		
Input Constraints	$T_{init} < T_C$		
Output constraints	Con-	$0 < t < t_{final}$	
Equation	$\frac{dT_W}{dt} = \frac{1}{\tau_W} (T_C - T_W(t) + \eta(T_P(t) - T_W(t)))$		
Description	<p>t is the time (s)</p> <p>T_W is the temperature of the water (°C)</p> <p>τ_W is the ODE parameter for water (s)</p> <p>T_C is the temperature of the heating coil (°C)</p> <p>η is the ODE parameter (Unitless)</p> <p>T_P is the temperature of the phase change material (°C)</p>		
Notes	<p>$\tau_W, t_{final}, T_C, T_P$ from (IM: eBalanceOnPCM). The input is constrained so that $T_{init} \leq T_C$. (A: Charging-Tank-No-Temp-Discharge) T_W is the temperature of the water (°C). T_P is the temperature of the phase change material (°C). T_C is the temperature of the heating coil (°C). $\tau_W = \frac{m_W C_W}{h_C A_C}$ is a constant (DD: htFusion) (s). $\eta = \frac{h_P A_P}{h_C A_C}$ is a constant (dimensionless). The above equation applies as long as the water is in liquid form, $0 < T_W < 100$ (°C) where 0 (°C) and 100 (°C) are the melting and boiling point temperatures of water, respectively (A: Water-Always-Liquid, A: Atmospheric-Pressure-Tank).</p>		
Source	[koothoor2013]		

Derivation of the energy balance on water: To find the rate of change of T_W , we look at the energy balance on water. The volume being considered is the volume of water V_W , which has mass of water m_W and specific heat capacity of water, C_W . q_C represents the heat transfer into the water from the heating coil and q_P represents the heat transfer into the PCM from water, over heating coil surface area and phase change material surface area of A_C and A_P , respectively. No heat transfer occurs to the outside of the tank, since it has been assumed to be perfectly insulated (**A: Perfect-Insulation-Tank**). Assuming no volumetric heat generation per unit volume (**A: No-Internal-Heat-Generation-By-Water-PCM**), $g = 0$. Therefore, the equation for **GD: rocTempSimp** can be written as:

$$m_W C_W \frac{dT_W}{dt} = q_C A_C - q_P A_P$$

Using **DD: ht_flux_C** and **DD: ht_flux_P** for q_C and q_P respectively, this can be written as:

$$m_W C_W \frac{dT_W}{dt} = h_C A_C (T_C - T_W) - h_P A_P (T_W - T_P)$$

Dividing (3) by $m_W C_W$, we obtain:

$$\frac{dT_W}{dt} = \frac{h_C A_C}{m_W C_W} (T_C - T_W) - \frac{h_P A_P}{m_W C_W} (T_W - T_P)$$

Factoring the negative sign out of the second term of the RHS of Equation (4) and multiplying it by $h_C A_C / h_C A_C$ yields:

$$\frac{dT_W}{dt} = \frac{h_C A_C}{m_W C_W} (T_C - T_W) + \frac{h_C A_C}{h_C A_C} \frac{h_P A_P}{m_W C_W} (T_P - T_W)$$

Which simplifies to:

$$\frac{dT_W}{dt} = \frac{h_C A_C}{m_W C_W} (T_C - T_W) + \frac{h_P A_P}{h_C A_C} \frac{h_C A_C}{m_W C_W} (T_P - T_W)$$

Setting $\tau_W = \frac{m_W C_W}{h_C A_C}$ (**DD: htFusion**) and $\eta = \frac{h_P A_P}{h_C A_C}$ (**DD: melt_frac**), Equation (5) can be written as:

$$\frac{dT_W}{dt} = \frac{1}{\tau_W} (T_C - T_W) + \frac{\eta}{\tau_W} (T_P - T_W)$$

Finally, factoring out $\frac{1}{\tau_W}$, we are left with the governing ODE for (**IM: eBalanceOnWtr**):

$$\frac{dT_W}{dt} = \frac{1}{\tau_W} (T_C - T_W + \eta (T_P - T_W))$$

Refname	IM:eBalanceOnPCM		
Label	Energy Balance on PCM to Find T_p		
Input	$T_{melt}^P, t_{final}, T_{init}, A_P, h_P, m_P, C_P^S, C_P^L$		
Output	T_P		
Input Constraints	$T_{init} < T_{melt}^P$		
Output constraints	Con-	$0 < t < t_{final}$	
Equation	$\frac{dT_P}{dt} = \begin{cases} \frac{1}{\tau_P^S} (T_W(t) - T_P(t)), & T_P < T_{melt}^P \\ \frac{1}{\tau_P^L} (T_W(t) - T_P(t)), & T_P > T_{melt}^P \\ 0, & T_P = T_{melt}^P \\ 0, & 0 < \phi < 1 \end{cases}$		
Description	<p>t is the time (s)</p> <p>T_P is the temperature of the phase change material (°C)</p> <p>τ_P^S is the ODE parameter for solid PCM (s)</p> <p>T_W is the temperature of the water (°C)</p> <p>τ_P^L is the ODE parameter for liquid PCM (s)</p> <p>T_{melt}^P is the melting point temperature for PCM (°C)</p> <p>ϕ is the melt fraction (Unitless)</p>		
Notes	<p>$T_{melt}^P, t_{final}, T_{init}, h_P, m_P, C_P^S, C_P^L$ form (IM: eBalanceOnWtr). The input is constrained so that $T_{init} < T_{melt}^P$ (A: PCM-Initially-Solid) $T_P, 0 < t < t_{final}$, with initial conditions, $T_W = T_P = T_{init}$, $\text{FIXME } t_w(0) = t_p(0)$, A: Same-Initial-Temp-Water-PCM, and T_W from IM1, such that the following governing ODE is satisfied. The temperature remains constant at T_{melt}^P, even with the heating (or cool-ing), until the phase change has occurred for all of the material; that is as long as $0 < \phi < 1$ (from DD: melt_frac) is determined as part of the heat energy in the PCM, as given in (IM: heatEInPCM).</p>		

Detailed derivation of the energy balance on the PCM during sensible heating phase: To find the rate of change of T_P , we look at the energy balance on the PCM. The volume being considered is the volume of PCM, V_P . The derivation that follows is initially for the solid PCM. The mass of phase change material is m_P and the specific heat capacity of PCM as a solid is C_P^S . The heat flux into the PCM from water is q_P over phase change material surface area A_P . There is no heat flux output. Assuming no volumetric heat generation per unit volume (**A: No-Internal-Heat-Generation-By-Water-PCM**), $g = 0$, the equation for **GD: rocTempSimp** can be written as:

$$m_P C_P^S \frac{dT_P}{dt} = q_P A_P$$

Using **DD: ht_flux_P** for q_P , this equation can be written as:

$$m_P C_P^S \frac{dT_P}{dt} = h_P A_P (T_W - T_P)$$

Dividing by $m_P C_P^S$ we obtain:

$$\frac{dT_P}{dt} = \frac{h_P A_P}{m_P C_P^S} (T_W - T_P)$$

Setting $\tau_P^S = m_P C_P^S / h_P A_P$, this can be written as:

$$\frac{dT_P}{dt} = \frac{1}{\tau_P^S} (T_W - T_P)$$

Equation (6) applies for the solid PCM. In the case where all of the PCM is melted, the same derivation applies, except that C_P^S is replaced by C_P^L , and thus τ_P^S is replaced by τ_P^L . Although a small change in surface area would be expected with melting, this is not included, since the volume change of the PCM with melting is assumed to be negligible (**A: Volume-Change-Melting-PCM-Negligible**). In the case where $T_P = T_{melt}^P$ and not all of the PCM is melted, the temperature of the phase change material does not change. Therefore, in this case $d T_P / d t = 0$. This derivation does not consider the boiling of the PCM, as the PCM is assumed to either be in a solid state or a liquid state (**A: No-Gaseous-State-PCM**).

Refname	IM:heatEInWtr		
Label	Heat energy in the water		
Input	T_{init}, m_W, C_W, m_W		
Output	E_W		
Input Constraints			
Output constraints	Con-	$0 < t < t_{final}$	
Equation			
$E_W(t) = C_W m_W (T_W(t) - T_{init})$			
Description			
E_W is the change in heat energy in the water (J) t is the time (s) C_W is the specific heat capacity of water ($\frac{\text{J}}{(\text{kg}^\circ\text{C})}$) m_W is the mass of water (kg) T_W is the temperature of the water ($^\circ\text{C}$) T_{init} is the initial temperature ($^\circ\text{C}$)			
Notes			
The above equation is derived using TM: sensHtE . E_W is the change in thermal energy of the liquid water relative to the energy at the initial temperature (T_{init}) (J). C_W is the specific heat capacity of liquid water ($\frac{\text{J}}{(\text{kg}^\circ\text{C})}$) and m_W is the mass of the water (kg). The change in temperature is the difference between the temperature at time t (s), T_W and the initial temperature, T_{init} ($^\circ\text{C}$). This equation applies as long as $0 < T_W < 100^\circ\text{C}$ (A: Water-Always-Liquid, A: Atmospheric-Pressure-Tank).			
Source	[koothoor2013]		
RefBy	FR: Find-Mass FR: Calculate-Change-Heat_Energy-Water-Over-Time.		

Refname	IM:heatEInPCM		
Label	Heat energy in the PCM		
Input	$T_{melt}^P, t_{final}, T_{init}, A_P, h_P, m_P, C_P^S, C_P^L, T_P, H_f, t_{melt}^{init}$		
Output	E_P		
Input Constraints	$T_{init} < T_{melt}^P$		
Output constraints	Con-	$0 < t < t_{final}$	
Equation	$E_P = \begin{cases} C_P^S m_P (T_P(t) - T_{init}), & T_P < T_{melt}^P \\ E_{P_{melt}^{init}} + H_f m_P + C_P^L m_P (T_P(t) - T_{melt}^P), & T_P > T_{melt}^P \\ E_{P_{melt}^{init}} + Q_P(t), & T_P = T_{melt}^P \\ E_{P_{melt}^{init}} + Q_P(t), & 0 < \phi < 1 \end{cases}$		
Description	<p> E_P is the change in heat energy in the PCM (J) C_P^S is the specific heat capacity of PCM as a solid ($\frac{J}{(kg^{\circ}C)}$) m_P is the mass of phase change material (kg) T_P is the temperature of the phase change material ($^{\circ}C$) t is the time (s) T_{init} is the initial temperature ($^{\circ}C$) $E_{P_{melt}^{init}}$ is the change in heat energy in the PCM at the instant when melting begins (J) H_f is the specific latent heat of fusion ($\frac{J}{kg}$) C_P^L is the specific heat capacity of PCM as a liquid ($\frac{J}{(kg^{\circ}C)}$) T_{melt}^P is the melting point temperature for PCM ($^{\circ}C$) Q_P is the latent heat energy added to PCM (J) ϕ is the melt fraction (Unitless) </p>		

Notes	<p>The above equation is derived using TM: sensHtE and TM: laten-tHtE. E_P is the change in thermal energy of the PCM relative to the energy at the initial temperature (T_{init}) J. E_P for the solid PCM is found using TM: sensHtE for sensible heat ing, with the specific heat capacity of the solid PCM, C_P^S ($\frac{J}{(kg^{\circ}C)}$), and the change in</p>
-------	--

4.2.6 Data Constraints

[Table:InDataConstraints](#) and [Table:OutDataConstraints](#) show the data constraints on the input and output variables, respectively. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario. The column for software constraints restricts the range of inputs to reasonable values. (*) These quantities cannot be equal to zero, or there will be a divide by zero in the model. (+) These quantities cannot be zero, or there would be freezing ([A: PCM-Initially-Solid](#)). (++) The constraints on the surface area are calculated by considering the surface area to volume ratio. The assumption is that the lowest ratio is 1 and the highest possible is $\frac{2}{h_{min}}$, where h_{min} is the thickness of a “sheet” of PCM. A thin sheet has the greatest surface area to volume ratio. (**) The constraint on the maximum time at the end of the simulation is the total number of seconds in one day.

Var	Physical Constraints	Software Constraints	Typical Value	Uncert.
A_C	$A_C > 0$	$A_C \leq A_C^{max}$	$120.0 \cdot 10^{-3} \text{ m}^2$	10.0%
A_P	$A_P > 0$	$V_P \leq A_P \leq \frac{2}{h_{min}} V_{tank}$	1.2 m^2	10.0%
C_W	$C_W > 0$	$C_W^{min} < C_W < C_W^{max}$	$4.186 \cdot 10^3 \frac{\text{J}}{(\text{kg}^\circ\text{C})}$	10.0%
C_P^L	$C_P^L > 0$	$C_P^{Lmin} < C_P^L < C_P^{Lmax}$	$2.27 \cdot 10^3 \frac{\text{J}}{(\text{kg}^\circ\text{C})}$	10.0%
C_P^S	$C_P^S > 0$	$C_P^{Smin} < C_P^S < C_P^{Smax}$	$1.76 \cdot 10^3 \frac{\text{J}}{(\text{kg}^\circ\text{C})}$	10.0%
D	$D > 0$	–	$412.0 \cdot 10^{-3} \text{ m}$	10.0%
h_C	$h_C > 0$	$h_C^{min} \leq h_C \leq h_C^{max}$	$1.0 \cdot 10^3 \frac{\text{W}}{(\text{m}^2\text{C})}$	10.0%
H_f	$H_f > 0$	$H_{fmin} < H_f < H_{fmax}$	$211.6 \cdot 10^3 \frac{\text{J}}{\text{kg}}$	10.0%
h_P	$h_P > 0$	$h_P^{min} \leq h_P \leq h_P^{max}$	$1.0 \cdot 10^3 \frac{\text{W}}{(\text{m}^2\text{C})}$	10.0%
L	$L > 0$	$L_{min} \leq L \leq L_{max}$	1.5 m	10.0%
T_C	$0 < T_C < 100$	–	$50.0 \text{ }^\circ\text{C}$	10.0%
t_{final}	$t_{final} > 0$	$t_{final} < t_{final}^{max}$	$50.0 \cdot 10^3 \text{ s}$	10.0%
T_{init}	$0 < T_{init} < T_{melt}$	–	$40.0 \text{ }^\circ\text{C}$	10.0%
T_{melt}^P	$0 < T_{melt}^P < T_C$	–	$44.2 \text{ }^\circ\text{C}$	10.0%
V_P	$0 < V_P < V_{tank}$	$V_P \geq MINFRACT V_{tank}$	$500.0 \cdot 10^{-3} \text{ m}^3$	10.0%
ρ_P	$\rho_P^{min} < \rho_P < \rho_P^{max}$	–	$1.007 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$	10.0%
ρ_W	$\rho_W > 0$	$\rho_W^{min} < \rho_W \leq \rho_W^{max}$	$1.0 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$	10.0%

Table 2: Input Data Constraints

Var	Physical Constraints
T_W	$T_{init} \leq T_W \leq T_C$
T_P	$T_{init} \leq T_P \leq T_C$
E_W	$E_W \geq 0$
E_P	$E_P \geq 0$

Table 3: Output Data Constraints

4.2.7 Properties of a Correct Solution

A correct solution must exhibit the law of conservation of energy. This means that the change in heat energy in the water should equal the difference between the total energy input from the heating coil and the energy output to the PCM. This can be shown as an equation by taking **DD: ht_flux_C** and **DD: ht_flux_P**, multiplying each by their respective surface area of heat transfer, and integrating each over the simulation time, as follows:

$$E_W = \int_0^t h_C A_C (T_C - T_W(t)) dt - \int_0^t h_P A_P (T_W(t) - T_P(t)) dt$$

In addition, the change in heat energy in the PCM should equal the energy input to the PCM from the water. This can be expressed as

$$E_P = \int_0^t h_P A_P (T_W(t) - T_P(t)) dt$$

Equations (FIXME: Equation 7) and (FIXME: Equation 8) can be used as “sanity” checks to gain confidence in any solution computed by SWHS. The relative error between the results computed by SWHS and the results calculated from the RHS of these equations should be less than 0.001

5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the non-functional requirements, the qualities that the software is expected to exhibit.

5.1 Functional Requirements

-Initial-Quantities: Input the following quantities described in **Table:Input-Variable-Requirements**, which define the tank parameters, material properties and initial conditions. **A: Volume-Coil-Negligible**.

- Find-Mass: Use the inputs in **FR: Input-Initial-Quantities** to find the mass needed for **IM: eBalanceOnWtr**, **IM: eBalanceOnPCM**, **IM: heatEInWtr**, and **IM: heatEInPCM**, using $m_W = V_W \rho_W = (V_{tank} - V_P) \rho_W = (\frac{D}{2}L - V_P) \rho_W$ and $m_P = V_P \rho_P$, where V_W is the volume of water and V_{tank} is the volume of the cylindrical tank.
- Physical_Constraints: Verify that the inputs satisfy the required physical constraints.
- Derived-Quantities: Output the input quantities and derived quantities in the following list: the quantities from **FR: Input-Initial-Quantities**, the masses from **FR: Find-Mass**, τ_W (from **IM: eBalanceOnWtr**), η (from **IM: eBalanceOnWtr**), τ_P^S (from **IM: eBalanceOnPCM**) and τ_P^L (from **IM: eBalanceOnPCM**).
- Water-Over-Time: Calculate and output the temperature of the water ($T_W(t)$) over the simulation time (from **IM: eBalanceOnWtr**).
- PCM-Over-Time: Calculate and output the temperature of the phase change material ($T_P(t)$) over the simulation time (from **IM: eBalanceOnPCM**).
- Water-Over-Time: Calculate and output the change in heat energy in the water ($E_W(t)$) over the simulation time (from **IM: heatEInWtr**).
- PCM-Over-Time: Calculate and output the change in heat energy in the PCM ($E_P(t)$) over the simulation time (from **IM: heatEInPCM**).
- Conservation-of-Energy: Verify that the energy outputs ($E_W(t)$ and $E_P(t)$) follow the law of conservation of energy with relative error no greater than 0.001
- Melt-Begin-Time: Calculate and output the time at which the PCM begins to melt t_{melt}^{init} (from **IM: eBalanceOnPCM**).
- Melt-End-Time: Calculate and output the time at which the PCM stops melting t_{melt}^{final} (from **IM: eBalanceOnPCM**).

Symbol	Unit	Description
L	m	length of tank
D	m	diameter of tank
V_P	m ³	volume of PCM
A_P	m ²	phase change material surface area
ρ_P	$\frac{\text{kg}}{\text{m}^3}$	density of PCM
T_{melt}^P	°C	melting point temperature for PCM
C_P^S	$\frac{\text{J}}{(\text{kg}^\circ\text{C})}$	specific heat capacity of PCM as a solid
C_P^L	$\frac{\text{J}}{(\text{kg}^\circ\text{C})}$	specific heat capacity of PCM as a liquid

Symbol	Unit	Description
H_f	$\frac{\text{J}}{\text{kg}}$	specific latent heat of fusion
A_C	m^2	heating coil surface area
T_C	$^{\circ}\text{C}$	temperature of the heating coil
ρ_W	$\frac{\text{kg}}{\text{m}^3}$	density of water
C_W	$\frac{\text{J}}{(\text{kg}^{\circ}\text{C})}$	specific heat capacity of water
h_C	$\frac{\text{W}}{(\text{m}^2^{\circ}\text{C})}$	convective heat transfer coefficient between coil and water
h_P	$\frac{\text{W}}{(\text{m}^2^{\circ}\text{C})}$	convective heat transfer coefficient between PCM and water
T_{init}	$^{\circ}\text{C}$	initial temperature
t_{final}	s	final time

Table 4: Input Variable Requirements

5.2 Non-Functional Requirements

This problem is small in size and relatively simple, so performance is not a priority. Any reasonable implementation will be very quick and use minimal storage. Rather than performance, the non-functional requirement priorities are correctness, verifiability, understandability, reusability, and maintainability.

6 Likely Changes

- Temperature-PCM: **A: Temp-PCM-Constant-Across-Volume** - PCM is actually a poor thermal conductor, so the assumption of uniform temperature of the phase change material is not likely.
- Variable-Over-Day: **A: Temp-Heating-Coil-Constant-over-Time** - The temperature of the heating coil will change over the course of the day, depending on the energy received from the sun.
- Variable-Over-Length: **A: Temp-Heating-Coil-Constant-over-Length** - The temperature of the heating coil will actually change along its length as the water within it cools.
- Discharging-Tank: **A: Charging-Tank-No-Temp-Discharge** - The model currently only accounts for charging of the tank. A more complete model would also account for discharging of the tank.
- Temps-PCM-Water: **A: Same-Initial-Temp-Water-PCM** - To add more flexibility to the simulation, the initial temperature of the water and the PCM could be allowed to have different values.
- Tank-Lose-Heat: **A: Perfect-Insulation-Tank** - Any real tank cannot be perfectly insulated and will lose heat.

7 Unlikely Changes

PCM-Fixed-States: **A: Water-Always-Liquid** , **A: No-Gaseous-State-PCM** - It is unlikely for the change of water from liquid to a solid or the state change of the phase change material from a liquid to a gas to be considered.

l-Heat-Generation: **A: No-Internal-Heat-Generation-By-Water-PCM** - Is used for the derivations of **IM: eBalanceOnWtr** and **IM: eBalanceOnPCM**.

No-Gaseous-State: **A: No-Gaseous-State-PCM** - Is used for the derivation of **IM: eBalanceOnPCM** and for the equation given by **IM: heatEInPCM** to be valid.

8 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the row of that component that are marked with an “X” should be modified as well. **Table:Tracey** shows the dependencies of theoretical models, general definitions, data definitions, and instance models with each other. **Table:Tracey2** shows the dependencies of instance models, requirements, and data constraints on each other. **Table:Tracey1** shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.

	FR: Input-Initial-Quantities	IM: heatEInWtr	IM: eBalanceOnPCM
Table:Input-Variable-Requirements	X		
A: Atmospheric-Pressure-Tank		X	X
A: Charging-Tank-No-Temp-Discharge			X
A: Constant-Water-Temp-Across-Tank			
A: Density-Water-PCM-Constant-over-Volume			
A: Heat-Transfer-Coeffs-Constant			
A: Newton-Law-Convective-Cooling-Coil-Water			
A: Law-Convective-Cooling-Water-PCM			
A: No-Gaseous-State-PCM			
A: No-Internal-Heat-Generation-By-Water-PCM			X
A: PCM-Initially-Solid			
A: Perfect-Insulation-Tank			X
A: Specific-Heat-Energy-Constant-over-Volume			
A: Same-Initial-Temp-Water-PCM			
A: Thermal-Energy-Only			
A: Temp-Heating-Coil-Constant-over-Length			
A: Temp-Heating-Coil-Constant-over-Time			
A: Temp-PCM-Constant-Across-Volume			
A: Volume-Change-Melting-PCM-Negligible			

	FR: Input-Initial-Quantities	IM: heatEInWtr	IM: eBalanceOnPCM
A: Volume-Coil-Negligible	X		
A: Water-Always-Liquid		X	X
TM: consThermE			
IM: eBalanceOnPCM			X
IM: eBalanceOnWtr			X
FR: Find-Mass			
IM: heatEInPCM			
IM: heatEInWtr			
DD: htFusion			X
DD: ht_flux_C			X
DD: ht_flux_P			X
FR: Input-Initial-Quantities			
TM: latentHtE			
DD: melt_frac			X
GD: rocTempSimp			X
TM: sensHtE		X	

	T1 (TM: consThermE)	T2 (TM: sensHtE)	T3 (TM: latentHtE)	GD1 (GD: nwtNCooling)
T1 (TM: consThermE)				
T2 (TM: sensHtE)			X	
T3 (TM: latentHtE)				
GD1 (GD: nwtNCooling)				
GD2 (GD: rocTempSimp)	X			
DD1 (DD: ht_flux_C)				X
DD2 (DD: ht_flux_P)				X
DD3 (DD: htFusion)				
DD4 (DD: melt_frac)				
IM1 (IM: eBalanceOnWtr)				
IM2 (IM: eBalanceOnPCM)				
IM3 (IM: heatEInWtr)		X		
IM4 (IM: heatEInPCM)		X	X	

	IM1 (IM: eBalanceOnWtr)	IM2 (IM: eBalanceOnPCM)
IM1 (IM: eBalanceOnWtr)		X
IM2 (IM: eBalanceOnPCM)	X	

	IM1 (IM: eBalanceOnWtr)	IM2 (IM: eBalanceOnPCM)
IM3 (IM: heatEInWtr)		
IM4 (IM: heatEInPCM)		X
R1 (FR: Input-Initial-Quantities)		
R2 (FR: Find-Mass)		
R3 (FR: Check-Input-with-Physical_Constraints)		
R4 (FR: Output-Input-Derived-Quantities)	X	X
R5 (FR: Calculate-Temperature-Water-Over-Time)	X	
R6 (FR: Calculate-Temperature-PCM-Over-Time)		X
R7 (FR: Calculate-Change-Heat_Energy-Water-Over-Time)		
R8 (FR: Calculate-Change-Heat_Energy-PCM-Over-Time)		
R9 (FR: Verify-Energy-Output-Follow-Conservation-of-Energy)		
R10 (FR: Calculate-PCM-Melt-Begin-Time)		X
R11 (FR: Calculate-PCM-Melt-End-Time)		X

	A1 (A: Thermal-Energy-Only)	A2 (A: Heat-Transfer-Only)
T1 (TM: consThermE)	X	
T2 (TM: sensHtE)		
T3 (TM: latentHtE)		
GD1 (GD: nwtN_Cooling)		X
GD2 (GD: rocTempSimp)		
DD1 (DD: ht_flux_C)		
DD2 (DD: ht_flux_P)		
DD3 (DD: htFusion)		
DD4 (DD: melt_frac)		
IM1 (IM: eBalanceOnWtr)		
IM2 (IM: eBalanceOnPCM)		
IM3 (IM: heatEInWtr)		
IM4 (IM: heatEInPCM)		
LC1 (LC: Uniform-Temperature-PCM)		
LC2 (LC: Temperature-Coil-Variable-Over-Day)		
LC3 (LC: Temperature-Coil-Variable-Over-Length)		
LC4 (LC: Discharging-Tank)		
LC5 (LC: Different-Initial-Temps-PCM-Water)		
LC6 (LC: Tank-Lose-Heat)		

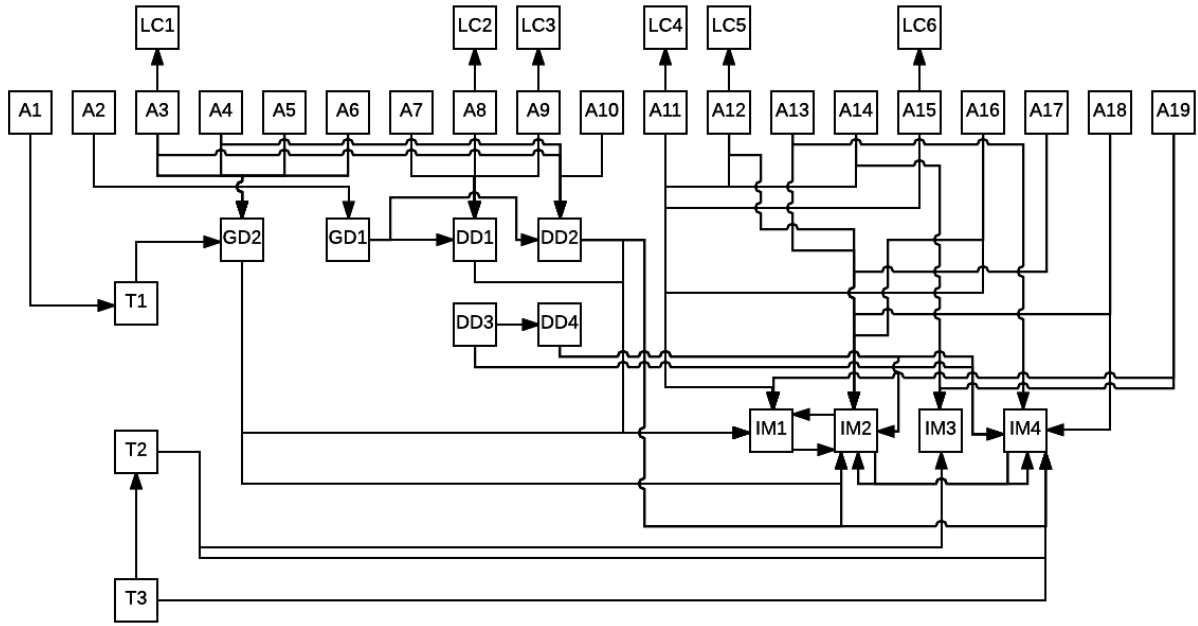


Figure 3: Traceability Graph Showing the Connections Between Items of Different Sections

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent dependencies. The component at the tail of an arrow is depended on by the component at the head of that arrow. Therefore, if a component is changed, the components that it points to should also be changed. **Fig:TraceyA** shows the dependencies of theoretical models, general definitions, data definitions, instance models, likely changes, and assumptions on each other. **Fig:TraceyR** shows the dependencies of instance models, requirements, and data constraints on each other.

9 Values of Auxiliary Constants

This section contains the standard values that are used for calculations in SWHS.

Symbol	Description	Value	Unit
C_W^{max}	maximum specific heat capacity of water	4210	$\frac{J}{(kg^{\circ}C)}$
C_W^{min}	minimum specific heat capacity of water	4170	$\frac{J}{(kg^{\circ}C)}$
$C_P^L_{max}$	maximum specific heat capacity of PCM as a liquid	5000	$\frac{J}{(kg^{\circ}C)}$

Symbol	Description	Value	Unit
$C_P^L{}_{min}$	minimum specific heat capacity of PCM as a liquid	100	$\frac{\text{J}}{(\text{kg}^\circ\text{C})}$
$C_P^S{}_{max}$	maximum specific heat capacity of PCM as a solid	4000	$\frac{\text{J}}{(\text{kg}^\circ\text{C})}$
$C_P^S{}_{min}$	minimum specific heat capacity of PCM as a solid	100	$\frac{\text{J}}{(\text{kg}^\circ\text{C})}$
h_C^{max}	maximum convective heat transfer coefficient between coil and water	10000	$\frac{\text{W}}{(\text{m}^2\text{C})}$
h_C^{min}	minimum convective heat transfer coefficient between coil and water	10	$\frac{\text{W}}{(\text{m}^2\text{C})}$
h_P^{max}	maximum convective heat transfer coefficient between PCM and water	10000	$\frac{\text{W}}{(\text{m}^2\text{C})}$
h_P^{min}	minimum convective heat transfer coefficient between PCM and water	10	$\frac{\text{W}}{(\text{m}^2\text{C})}$
L_{max}	maximum length of tank	50	m
L_{min}	minimum length of tank	$100.0 \cdot 10^{-3}$	m
$MINFRACT$	minimum fraction of the tank volume taken up by the PCM	$1.0 \cdot 10^{-6}$	—
t_{final}^{max}	maximum final time	86400	s
ρ_P^{max}	maximum density of PCM	20000	$\frac{\text{kg}}{\text{m}^3}$
ρ_P^{min}	minimum density of PCM	500	$\frac{\text{kg}}{\text{m}^3}$
ρ_W^{max}	maximum density of water	1000	$\frac{\text{kg}}{\text{m}^3}$
ρ_W^{min}	minimum density of water	950	$\frac{\text{kg}}{\text{m}^3}$

Table 9: Auxiliary Constants

10 References

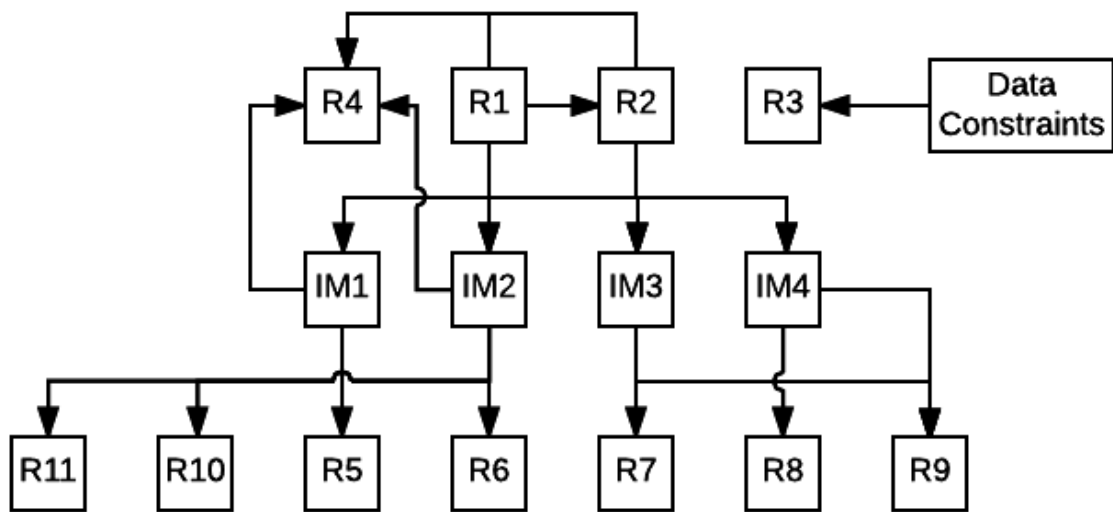


Figure 4: Traceability Graph Showing the Connections Between Instance Models, Requirements, and Data Constraints