Software Requirements Specification for Single Pendulum

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Contents

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

The unit system used throughout is SI (Système International d'Unités). In addition to the basic units, several derived units are also used. For each unit, the Table of Units lists the symbol, a description, and the SI name.

Table 1: Table of Units

Symbol	Description	SI Name
Hz	frequency	hertz
kg	mass	kilogram
m	length	metre
N	force	newton
rad	angle	radian
s	time	second

1.2 Table of Symbols

The symbols used in this document are summarized in the Table of Symbols along with their units. Throughout the document, symbols in bold will represent vectors, and scalars otherwise. The symbols are listed in alphabetical order. For vector quantities, the units shown are for each component of the vector.

Table 2: Table of Symbols

Symbol	Description	Units
a_x	x-component of acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
a_y	y-component of acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
$\mathbf{a}(t)$	Acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
$\mathbf{F}$	Force	N
f	Frequency	Hz
$\mathbf{g}$	Gravitational acceleration	$\frac{\mathrm{m}}{\mathrm{s}^2}$
$\mathbf{I}$	Moment of inertia	kgm^2
$\hat{\mathbf{i}}$	Unit vector	–

Table 2: Table of Symbols (Continued)

Symbol	Description	Units
L_{rod}	Length of the rod	m
m	Mass	kg
p_{x}	x-component of position	m
$p_{\text{x}}{}^{\text{i}}$	x-component of initial position	m
p_{y}	y-component of position	m
$p_{\text{y}}{}^{\text{i}}$	y-component of initial position	m
$\mathbf{p}(t)$	Position	m
T	Period	s
$\mathbf{T}$	Tension	N
t	Time	s
v_{x}	x-component of velocity	$\frac{\text{m}}{\text{s}}$
v_{y}	y-component of velocity	$\frac{\text{m}}{\text{s}}$
$\mathbf{v}(t)$	Velocity	$\frac{\text{m}}{\text{s}}$
α	Angular acceleration	$\frac{\text{rad}}{\text{s}^2}$
θ	Angular displacement	rad
θ_i	Initial pendulum angle	rad
θ_p	Displacement angle of the pendulum	rad
π	Ratio of circumference to diameter for any circle	–
$\boldsymbol{\tau}$	Torque	Nm
Ω	Angular frequency	s
ω	Angular velocity	$\frac{\text{rad}}{\text{s}}$

1.3 Abbreviations and Acronyms

Table 3: Abbreviations and Acronyms

Abbreviation	Full Form
2D	Two-Dimensional
A	Assumption
DD	Data Definition

Table 3: Abbreviations and Acronyms (Continued)

Abbreviation	Full Form
GD	General Definition
GS	Goal Statement
IM	Instance Model
PS	Physical System Description
R	Requirement
RefBy	Referenced by
Refname	Reference Name
SRS	Software Requirements Specification
SglPend	Single Pendulum
TM	Theoretical Model
Uncert.	Typical Uncertainty

2 Introduction

A pendulum consists of mass attached to the end of a rod and its moving curve is highly sensitive to initial conditions. Therefore, it is useful to have a program to simulate the motion of the pendulum to exhibit its chaotic characteristics. The document describes the program called Single Pendulum , which is based on the original, manually created version of Double Pendulum.

The following section provides an overview of the Software Requirements Specification (SRS) for Single Pendulum. This section explains the purpose of this document, the scope of the requirements, the characteristics of the intended reader, and the organization of the document.

2.1 Purpose of Document

The primary purpose of this document is to record the requirements of SglPend. Goals, assumptions, theoretical models, definitions, and other model derivation information are specified, allowing the reader to fully understand and verify the purpose and scientific basis of SglPend. With the exception of system constraints, this SRS will remain abstract, describing what problem is being solved, but not how to solve it.

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documen-

tation and the implementation. Although the SRS fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the waterfall model is not followed, as Parnas and Clements point out [6], the most logical way to present the documentation is still to "fake" a rational design process.

2.2   Scope of Requirements

The scope of the requirements includes the analysis of a two-dimensional (2D) pendulum motion problem with various initial conditions.

2.3   Characteristics of Intended Reader

Reviewers of this documentation should have an understanding of undergraduate level 2 physics, undergraduate level 1 calculus, and ordinary differential equations. The users of SglPend can have a lower level of expertise, as explained in Sec:User Characteristics.

2.4   Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by [5], [8], [9], and [7]. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models and trace back to find any additional information they require.

   The goal statements are refined to the theoretical models and the theoretical models to the instance models.

3   General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics, and lists the system constraints.

3.1   System Context

Fig:sysCtxDiag shows the system context. A circle represents an entity external to the software, the user in this case. A rectangle represents the software system itself (SglPend). Arrows are used to show the data flow between the system and its environment.

Figure 1: System Context

The interaction between the product and the user is through an application programming interface. The responsibilities of the user and the system are as follows:

- User Responsibilities
    - Provide initial conditions of the physical state of the motion and the input data related to the Single Pendulum, ensuring no errors in the data entry.
    - Ensure that consistent units are used for input variables.
    - Ensure required software assumptions are appropriate for any particular problem input to the software.
- SglPend Responsibilities
    - Detect data type mismatch, such as a string of characters input instead of a floating point number.
    - Determine if the inputs satisfy the required physical and software constraints.
    - Calculate the required outputs.
    - Generate the required graphs.

3.2 User Characteristics

The end user of SglPend should have an understanding of high school physics, high school calculus and ordinary differential equations.

3.3 System Constraints

There are no system constraints.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, and definitions that are used.

4.1 Problem Description

A system is needed to predict the motion of a single pendulum.

4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements.

- Gravity: The force that attracts one physical body with mass to another.

- Cartesian coordinate system: A coordinate system that specifies each point uniquely in a plane by a set of numerical coordinates, which are the signed distances to the point from two fixed perpendicular oriented lines, measured in the same unit of length (from [2]).

4.1.2 Physical System Description

The physical system of SglPend, as shown in Fig:sglpend, includes the following elements:

PS1: The rod.

PS2: The mass.

Figure 2: The physical system

4.1.3 Goal Statements

Given the mass and length of the rod, initial angle of the mass and the gravitational constant, the goal statement is:

Motion-of-the-mass: Calculate the motion of the mass.

4.2 Solution Characteristics Specification

The instance models that govern SglPend are presented in the Instance Model Section. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical models by filling in the missing information for the physical system. The assumptions refine the scope by providing more detail.

twoDMotion: The pendulum motion is two-dimensional (2D).

cartSys: A Cartesian coordinate system is used.

cartSysR: The Cartesian coordinate system is right-handed where positive x-axis and y-axis point right up.

yAxisDir: The direction of the y-axis is directed opposite to gravity.

4.2.2 Theoretical Models

This section focuses on the general equations and laws that SglPend is based on.

Refname	TM:acceleration
Label	Acceleration
Equation	$$\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt}$$
Description	$\mathbf{a}(t)$ is the acceleration ($\frac{\text{m}}{\text{s}^2}$) t is the time (s) $\mathbf{v}(t)$ is the velocity ($\frac{\text{m}}{\text{s}}$)
Source	[1]
RefBy	

Refname	TM:velocity
Label	Velocity
Equation	$$\mathbf{v}(t) = \frac{d\mathbf{p}(t)}{dt}$$
Description	$\mathbf{v}(t)$ is the velocity ($\frac{\mathrm{m}}{\mathrm{s}}$) t is the time (s) $\mathbf{p}(t)$ is the position (m)
Source	[3]
RefBy	

Refname	TM:NewtonSecLawMot
Label	Newton's second law of motion
Equation	$$\mathbf{F} = m\,\mathbf{a}(t)$$
Description	$\mathbf{F}$ is the force (N) m is the mass (kg) $\mathbf{a}(t)$ is the acceleration ($\frac{\mathrm{m}}{\mathrm{s}^2}$)
Notes	The net force $\mathbf{F}$ on a body is proportional to the acceleration $\mathbf{a}(t)$ of the body, where m denotes the mass of the body as the constant of proportionality.
Source	–
RefBy	

Refname	TM:NewtonSecLawRotMot
Label	Newton's second law for rotational motion
Equation	

$$\boldsymbol{\tau} = \mathbf{I}\,\alpha$$

Description	$\boldsymbol{\tau}$ is the torque (Nm) $\mathbf{I}$ is the moment of inertia (kgm^2) α is the angular acceleration ($\frac{\text{rad}}{\text{s}^2}$)
Notes	The net torque $\boldsymbol{\tau}$ on a rigid body is proportional to its angular acceleration α, where $\mathbf{I}$ denotes the moment of inertia of the rigid body as the constant of proportionality.
Source	–
RefBy	IM:calOfAngularDisplacement and GD:angFrequencyGD

4.2.3 General Definitions

This section collects the laws and equations that will be used to build the instance models.

Refname	GD:velocityIX
Label	The x-component of velocity of the pendulum
Units	$\frac{\text{m}}{\text{s}}$
Equation	$$v_{\text{x}} = \omega\,L_{\text{rod}}\,\cos\left(\theta_p\right)$$
Description	v_{x} is the x-component of velocity $\left(\frac{\text{m}}{\text{s}}\right)$ ω is the angular velocity $\left(\frac{\text{rad}}{\text{s}}\right)$ L_{rod} is the length of the rod (m) θ_p is the displacement angle of the pendulum (rad)
Source	–
RefBy	

Detailed derivation of the x-component of velocity: At a given point in time, velocity may be defined as

$$\mathbf{v}(t) = \frac{d\mathbf{p}(t)}{dt}$$

We also know the horizontal position

$$p_{\text{x}} = L_{\text{rod}}\,\sin\left(\theta_p\right)$$

Applying this,

$$v_{\text{x}} = \frac{dL_{\text{rod}}\,\sin\left(\theta_p\right)}{dt}$$

L_{rod} is constant with respect to time, so

$$v_{\text{x}} = L_{\text{rod}}\,\frac{d\sin\left(\theta_p\right)}{dt}$$

Therefore, using the chain rule,

$$v_{\text{x}} = \omega\,L_{\text{rod}}\,\cos\left(\theta_p\right)$$

Refname	GD:velocityIY
Label	The y-component of velocity of the pendulum
Units	$\frac{\text{m}}{\text{s}}$
Equation	$$v_\text{y} = \omega\,L_\text{rod}\,\sin\left(\theta_p\right)$$
Description	v_y is the y-component of velocity $\left(\frac{\text{m}}{\text{s}}\right)$ ω is the angular velocity $\left(\frac{\text{rad}}{\text{s}}\right)$ L_rod is the length of the rod (m) θ_p is the displacement angle of the pendulum (rad)
Source	–
RefBy	

Detailed derivation of the y-component of velocity: At a given point in time, velocity may be defined as

$$\mathbf{v}(t) = \frac{d\mathbf{p}(t)}{dt}$$

We also know the vertical position

$$p_\text{y} = -L_\text{rod}\,\cos\left(\theta_p\right)$$

Applying this,

$$v_\text{y} = -\left(\frac{dL_\text{rod}\,\cos(\theta_p)}{dt}\right)$$

L_rod is constant with respect to time, so

$$v_\text{y} = -L_\text{rod}\,\frac{d\cos(\theta_p)}{dt}$$

Therefore, using the chain rule,

$$v_\text{y} = \omega\,L_\text{rod}\,\sin\left(\theta_p\right)$$

Refname	GD:accelerationIX
Label	The x-component of acceleration of the pendulum
Units	$\frac{\text{m}}{\text{s}^2}$
Equation	$$a_\text{x} = -\omega^2\,L_\text{rod}\,\sin\left(\theta_p\right) + \alpha\,L_\text{rod}\,\cos\left(\theta_p\right)$$
Description	a_x is the x-component of acceleration ($\frac{\text{m}}{\text{s}^2}$) ω is the angular velocity ($\frac{\text{rad}}{\text{s}}$) L_rod is the length of the rod (m) θ_p is the displacement angle of the pendulum (rad) α is the angular acceleration ($\frac{\text{rad}}{\text{s}^2}$)
Source	–
RefBy	

Detailed derivation of the x-component of acceleration: Our acceleration is:

$$\mathbf{a}(t) = \tfrac{d\mathbf{v}(t)}{dt}$$

Earlier, we found the horizontal velocity to be

$$v_\text{x} = \omega\,L_\text{rod}\,\cos\left(\theta_p\right)$$

Applying this to our equation for acceleration

$$a_\text{x} = \tfrac{d\omega\,L_\text{rod}\,\cos(\theta_p)}{dt}$$

By the product and chain rules, we find

$$a_\text{x} = \tfrac{d\omega}{dt}\,L_\text{rod}\,\cos\left(\theta_p\right) - \omega\,L_\text{rod}\,\sin\left(\theta_p\right)\,\tfrac{d\theta_p}{dt}$$

Simplifying,

$$a_\text{x} = -\omega^2\,L_\text{rod}\,\sin\left(\theta_p\right) + \alpha\,L_\text{rod}\,\cos\left(\theta_p\right)$$

Refname	GD:accelerationIY
Label	The y-component of acceleration of the pendulum
Units	$\frac{\text{m}}{\text{s}^2}$
Equation	$$a_{\text{y}} = \omega^2\, L_{\text{rod}}\, \cos\left(\theta_p\right) + \alpha\, L_{\text{rod}}\, \sin\left(\theta_p\right)$$
Description	a_{y} is the y-component of acceleration $\left(\frac{\text{m}}{\text{s}^2}\right)$ ω is the angular velocity $\left(\frac{\text{rad}}{\text{s}}\right)$ L_{rod} is the length of the rod (m) θ_p is the displacement angle of the pendulum (rad) α is the angular acceleration $\left(\frac{\text{rad}}{\text{s}^2}\right)$
Source	–
RefBy	

Detailed derivation of the y-component of acceleration: Our acceleration is:

$$\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt}$$

Earlier, we found the vertical velocity to be

$$v_{\text{y}} = \omega\, L_{\text{rod}}\, \sin\left(\theta_p\right)$$

Applying this to our equation for acceleration

$$a_{\text{y}} = \frac{d\omega\, L_{\text{rod}}\, \sin(\theta_p)}{dt}$$

By the product and chain rules, we find

$$a_{\text{y}} = \frac{d\omega}{dt}\, L_{\text{rod}}\, \sin\left(\theta_p\right) + \omega\, L_{\text{rod}}\, \cos\left(\theta_p\right)\, \frac{d\theta_p}{dt}$$

Simplifying,

$$a_{\text{y}} = \omega^2\, L_{\text{rod}}\, \cos\left(\theta_p\right) + \alpha\, L_{\text{rod}}\, \sin\left(\theta_p\right)$$

Refname	GD:hForceOnPendulum
Label	Horizontal force on the pendulum
Units	N
Equation	$$\mathbf{F} = m\,a_{\mathrm{x}} = -\mathbf{T}\,\sin\left(\theta_p\right)$$
Description	$\mathbf{F}$ is the force (N) m is the mass (kg) a_{x} is the x-component of acceleration $\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)$ $\mathbf{T}$ is the tension (N) θ_p is the displacement angle of the pendulum (rad)
Source	–
RefBy	

Detailed derivation of force on the pendulum:

$$\mathbf{F} = m\,a_{\mathrm{x}} = -\mathbf{T}\,\sin\left(\theta_p\right)$$

Refname	GD:vForceOnPendulum
Label	Vertical force on the pendulum
Units	N
Equation	$$\mathbf{F} = m\,a_{\mathrm{y}} = \mathbf{T}\,\cos\left(\theta_p\right) - m\,\mathbf{g}$$
Description	$\mathbf{F}$ is the force (N) m is the mass (kg) a_{y} is the y-component of acceleration $\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)$ $\mathbf{T}$ is the tension (N) θ_p is the displacement angle of the pendulum (rad) $\mathbf{g}$ is the gravitational acceleration $\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)$
Source	–
RefBy	

Detailed derivation of force on the pendulum:

$$\mathbf{F} = m\,a_{\mathrm{y}} = \mathbf{T}\,\cos\left(\theta_p\right) - m\,\mathbf{g}$$

Refname	GD:angFrequencyGD
Label	The angular frequency of the pendulum
Units	s
Equation	$$\Omega = \sqrt{\frac{\mathbf{g}}{L_{\mathrm{rod}}}}$$
Description	Ω is the angular frequency (s) $\mathbf{g}$ is the gravitational acceleration $(\frac{\mathrm{m}}{\mathrm{s}^2})$ L_{rod} is the length of the rod (m)
Notes	The torque is defined in TM:NewtonSecLawRotMot and frequency is f is defined in DD:frequencyDD.
Source	–
RefBy	GD:periodPend and IM:calOfAngularDisplacement

Detailed derivation of the angular frequency of the pendulum: Consider the torque on a pendulum defined in TM:NewtonSecLawRotMot. The force providing the restoring torque is the component of weight of the pendulum bob that acts along the arc length. The torque is the length of the string L_{rod} multiplied by the component of the net force that is perpendicular to the radius of the arc. The minus sign indicates the torque acts in the opposite direction of the angular displacement:

$$\boldsymbol{\tau} = -L_{\mathrm{rod}}\, m\, \mathbf{g}\, \sin\left(\theta_p\right)$$

So then

$$\mathbf{I}\,\alpha = -L_{\mathrm{rod}}\, m\, \mathbf{g}\, \sin\left(\theta_p\right)$$

Therefore,

$$\mathbf{I}\,\frac{d\frac{d\theta_p}{dt}}{dt} = -L_{\mathrm{rod}}\, m\, \mathbf{g}\, \sin\left(\theta_p\right)$$

Substituting for $\mathbf{I}$

$$m \, L_{\mathrm{rod}}^{\,2} \, \frac{d \frac{d\theta_p}{dt}}{dt} = -L_{\mathrm{rod}} \, m \, \mathbf{g} \, \sin\left(\theta_p\right)$$

Crossing out m and L_{rod} we have

$$\frac{d \frac{d\theta_p}{dt}}{dt} = -\left(\frac{\mathbf{g}}{L_{\mathrm{rod}}}\right) \sin\left(\theta_p\right)$$

For small angles, we approximate $\sin \theta_p$ to θ_p

$$\frac{d \frac{d\theta_p}{dt}}{dt} = -\left(\frac{\mathbf{g}}{L_{\mathrm{rod}}}\right) \theta_p$$

Because this equation, has the same form as the equation for simple harmonic motion the solution is easy to find. The angular frequency

$$\Omega = \sqrt{\frac{\mathbf{g}}{L_{\mathrm{rod}}}}$$

Refname	GD:periodPend
Label	The period of the pendulum
Units	s
Equation	$$T = 2\,\pi\,\sqrt{\frac{L_{\mathrm{rod}}}{\mathbf{g}}}$$
Description	T is the period (s) π is the ratio of circumference to diameter for any circle (Unitless) L_{rod} is the length of the rod (m) $\mathbf{g}$ is the gravitational acceleration $\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)$
Notes	The frequency and period are defined in the data definitions for frequency and period respectively
Source	–
RefBy	

Detailed derivation of the period of the pendulum: The period of the pendulum can be defined from the general definition for the equation of angular frequency

$$\Omega = \sqrt{\frac{\mathbf{g}}{L_{\mathrm{rod}}}}$$

Therefore from the data definition of the equation for angular frequency, we have

$$T = 2\,\pi\,\sqrt{\frac{L_{\mathrm{rod}}}{\mathbf{g}}}$$

4.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models.

Refname	DD:positionIX
Label	x-component of initial position
Symbol	$p_{\mathrm{x}}{}^{\mathrm{i}}$
Units	m
Equation	$$p_{\mathrm{x}}{}^{\mathrm{i}} = L_{\mathrm{rod}}\,\sin\left(\theta_i\right)$$
Description	$p_{\mathrm{x}}{}^{\mathrm{i}}$ is the x-component of initial position (m) L_{rod} is the length of the rod (m) θ_i is the initial pendulum angle (rad)
Notes	$p_{\mathrm{x}}{}^{\mathrm{i}}$ is the horizontal position $p_{\mathrm{x}}{}^{\mathrm{i}}$ is shown in Fig:sglpend.
Source	–
RefBy	

Refname	DD:positionIY
Label	y-component of initial position
Symbol	$p_\text{y}{}^\text{i}$
Units	m
Equation	$$p_\text{y}{}^\text{i} = -L_\text{rod}\, \cos\left(\theta_i\right)$$
Description	$p_\text{y}{}^\text{i}$ is the y-component of initial position (m) L_rod is the length of the rod (m) θ_i is the initial pendulum angle (rad)
Notes	$p_\text{y}{}^\text{i}$ is the vertical position $p_\text{y}{}^\text{i}$ is shown in Fig:sglpend.
Source	–
RefBy	

Refname	DD:frequencyDD
Label	Frequency
Symbol	f
Units	Hz
Equation	$$f = \frac{1}{T}$$
Description	f is the frequency (Hz) T is the period (s)
Notes	f is the number of back and forth swings in one second
Source	–
RefBy	GD:periodPend, GD:angFrequencyGD, and DD:periodSHMDD

Refname	DD:angFrequencyDD
Label	Angular frequency
Symbol	Ω
Units	s
Equation	$$\Omega = \frac{2\,\pi}{T}$$
Description	Ω is the angular frequency (s) π is the ratio of circumference to diameter for any circle (Unitless) T is the period (s)
Notes	T is from DD:periodSHMDD
Source	–
RefBy	GD:periodPend

Refname	DD:periodSHMDD
Label	Period
Symbol	T
Units	s
Equation	$$T = \frac{1}{f}$$
Description	T is the period (s) f is the frequency (Hz)
Notes	T is from DD:frequencyDD
Source	–
RefBy	GD:periodPend and DD:angFrequencyDD

4.2.5 Instance Models

This section transforms the problem defined in the problem description into one which is expressed in mathematical terms. It uses concrete symbols defined in the data definitions to replace the abstract symbols in the models identified in theoretical models and general definitions.

Refname	IM:calOfAngularDisplacement
Label	Calculation of angular displacement
Input	L_{rod}, θ_i, $\mathbf{g}$
Output	θ_p
Input Constraints	$$L_{\mathrm{rod}} > 0$$ $$\theta_i > 0$$ $$\mathbf{g} > 0$$
Output Constraints	$$\theta_p > 0$$
Equation	$$\theta_p\,(t) = \theta_i\,\cos\left(\Omega\,t\right)$$
Description	θ_p is the displacement angle of the pendulum (rad) t is the time (s) θ_i is the initial pendulum angle (rad) Ω is the angular frequency (s)
Notes	The constraint $\theta_i > 0$ is required. The angular frequency is defined in GD:angFrequencyGD.
Source	–
RefBy	FR:Output-Values and FR:Calculate-Angular-Position-Of-Mass

Detailed derivation of angular displacement: When the pendulum is displaced to an initial angle and released, the pendulum swings back and forth with periodic motion. By applying Newton's second law for rotational motion, the equation of motion for the pendulum may be obtained:

$$\boldsymbol{\tau} = \mathbf{I}\,\alpha$$

Where $\boldsymbol{\tau}$ denotes the torque, $\mathbf{I}$ denotes the moment of inertia and α denotes the angular acceleration. This implies:

$$-m\,\mathbf{g}\,\sin\left(\theta_p\right)\,L_{\mathrm{rod}} = m\,L_{\mathrm{rod}}{}^2\,\frac{d\frac{d\theta_p}{dt}}{dt}$$

And rearranged as:

$$\frac{d\frac{d\theta_p}{dt}}{dt} + \frac{\mathbf{g}}{L_{\mathrm{rod}}}\,\sin\left(\theta_p\right) = 0$$

If the amplitude of angular displacement is small enough, we can approximate $\sin\left(\theta_p\right) = \theta_p$ for the purpose of a simple pendulum at very small angles. Then the equation of motion reduces to the equation of simple harmonic motion:

$$\frac{d\frac{d\theta_p}{dt}}{dt} + \frac{\mathbf{g}}{L_{\mathrm{rod}}}\,\theta_p = 0$$

Thus the simple harmonic motion is:

$$\theta_p\left(t\right) = \theta_i\,\cos\left(\Omega\,t\right)$$

4.2.6 Data Constraints

The Data Constraints Table shows the data constraints on the input variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise. The constraints are conservative to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Table 4: Input Data Constraints

Var	Physical Constraints	Typical Value	Uncert.
L_{rod}	$L_{\mathrm{rod}} > 0$	44.2 m	10%
θ_i	$\theta_i > 0$	2.1 rad	10%

4.2.7 Properties of a Correct Solution

The Data Constraints Table shows the data constraints on the output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable.

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Table 5: Output Data Constraints

Var	Physical Constraints
α	$\alpha > 0$
θ_p	$\theta_p > 0$

5 Requirements

This section provides the functional requirements, the tasks and behaviours that the software is expected to complete, and the non-functional requirements, the qualities that the software is expected to exhibit.

5.1 Functional Requirements

This section provides the functional requirements, the tasks and behaviours that the software is expected to complete.

Input-Values: Input the values from Tab:ReqInputs.

Verify-Input-Values: Check the entered input values to ensure that they do not exceed the data constraints. If any of the input values are out of bounds, an error message is displayed and the calculations stop.

Calculate-Angular-Position-Of-Mass: Calculate the following values: θ and θ_p (from IM:calOfAngularDisplacement).

Output-Values: Output L_{rod} (from IM:calOfAngularDisplacement).

Table 6: Required Inputs

Symbol	Description	Units
L_{rod}	Length of the rod	m
m	Mass	kg
α	Angular acceleration	$\frac{\text{rad}}{\text{s}^2}$
θ_i	Initial pendulum angle	rad
θ_p	Displacement angle of the pendulum	rad

5.2 Non-Functional Requirements

This section provides the non-functional requirements, the qualities that the software is expected to exhibit.

Correctness: The outputs of the code have the properties of a correct solution.

Portability: The code shall be portable to multiple environments, particularly Windows, Mac OSX, and Linux.

6 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" should be modified as well. Tab:TraceMatAvsA shows the dependencies of the assumptions on each other. Tab:TraceMatAvsAll shows the dependencies of the data definitions, theoretical models, general definitions, instance models, requirements, likely changes, and unlikely changes on the assumptions. Tab:TraceMatRefvsRef shows the dependencies of the data definitions, theoretical models, general definitions, and instance models on each other. Tab:TraceMatAllvsR shows the dependencies of the requirements and goal statements on the data definitions, theoretical models, general definitions, and instance models.

Table 7: Traceability Matrix Showing the Connections Between Assumptions and Other Assumptions

	A:twoDMotion	A:cartSys	A:cartSysR	A:yAxisDir
A:twoDMotion				
A:cartSys				
A:cartSysR				
A:yAxisDir				

Table 8: Traceability Matrix Showing the Connections Between Assumptions and Other Items

	A:twoDMotion	A:cartSys	A:cartSysR	A:yAxisDir
DD:positionIX				
DD:positionIY				
DD:frequencyDD				
DD:angFrequencyDD				
DD:periodSHMDD				
TM:acceleration				

Table 8: Traceability Matrix Showing the Connections Between Assumptions and Other Items (Continued)

	A:twoDMotion	A:cartSys	A:cartSysR	A:yAxisDir
TM:velocity				
TM:NewtonSecLawMot				
TM:NewtonSecLawRotMot				
GD:velocityIX				
GD:velocityIY				
GD:accelerationIX				
GD:accelerationIY				
GD:hForceOnPendulum				
GD:vForceOnPendulum				
GD:angFrequencyGD				
GD:periodPend				
IM:calOfAngularDisplacement				
FR:Input-Values				
FR:Verify-Input-Values				
FR:Calculate-Angular-Position-Of-Mass				
FR:Output-Values				
NFR:Correctness				
NFR:Portability				

	DD:positionIX	DD:positionIY	DD:frequencyDD	DD:angFr
DD:positionIX				
DD:positionIY				
DD:frequencyDD				
DD:angFrequencyDD				
DD:periodSHMDD			X	
TM:acceleration				
TM:velocity				
TM:NewtonSecLawMot				

	DD:positionIX	DD:positionIY	DD:frequencyDD	DD:angFr
TM:NewtonSecLawRotMot				
GD:velocityIX				
GD:velocityIY				
GD:accelerationIX				
GD:accelerationIY				
GD:hForceOnPendulum				
GD:vForceOnPendulum				
GD:angFrequencyGD		X		
GD:periodPend		X	X	
IM:calOfAngularDisplacement				

	DD:positionIX	DD:positionIY	DD:frequencyDD	D
GS:Motion-of-the-mass				
FR:Input-Values				
FR:Verify-Input-Values				
FR:Calculate-Angular-Position-Of-Mass				
FR:Output-Values				
NFR:Correctness				
NFR:Portability				

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent dependencies. The component at the tail of an arrow is depended on by the component at the head of that arrow. Therefore, if a component is changed, the components that it points to should also be changed. Fig:TraceGraphAvsA shows the dependencies of assumptions on each other. Fig:TraceGraphAvsAll shows the dependencies of data definitions, theoretical models, general definitions, instance models, requirements, likely changes, and unlikely changes on the assumptions. Fig:TraceGraphRefvsRef shows the dependencies of data definitions, theoretical models, general definitions, and instance models on each other. Fig:TraceGraphAllvsR shows the dependencies of requirements and goal statements on the data definitions, theoretical models, general definitions, and instance models. Fig:Trace-

GraphAllvsAll shows the dependencies of dependencies of assumptions, models, definitions, requirements, goals, and changes with each other.

Figure 3: TraceGraphAvsA

Figure 4: TraceGraphAvsAll

Figure 5: TraceGraphRefvsRef

Figure 6: TraceGraphAllvsR

Figure 7: TraceGraphAllvsAll

For convenience, the following graphs can be found at the links below:

- TraceGraphAvsA
- TraceGraphAvsAll

- TraceGraphRefvsRef

- TraceGraphAllvsR

- TraceGraphAllvsAll

7 Values of Auxiliary Constants

There are no auxiliary constants.

8 References

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