

# Software Requirements Specification for GlassBR

Nikitha Krithnan and W. Spencer Smith

September 26, 2025

## Contents

<b>1</b>	<b>Reference Material</b>	<b>3</b>
1.1	Table of Units . . . . .	3
1.2	Table of Symbols . . . . .	3
1.3	Abbreviations and Acronyms . . . . .	5
<b>2</b>	<b>Introduction</b>	<b>6</b>
2.1	Purpose of Document . . . . .	6
2.2	Scope of Requirements . . . . .	7
2.3	Characteristics of Intended Reader . . . . .	7
2.4	Organization of Document . . . . .	7
<b>3</b>	<b>Stakeholders</b>	<b>7</b>
3.1	The Client . . . . .	7
3.2	The Customer . . . . .	7
<b>4</b>	<b>General System Description</b>	<b>8</b>
4.1	System Context . . . . .	8
4.2	User Characteristics . . . . .	8
4.3	System Constraints . . . . .	9
<b>5</b>	<b>Specific System Description</b>	<b>9</b>
5.1	Problem Description . . . . .	9
5.1.1	Terminology and Definitions . . . . .	9
5.1.2	Physical System Description . . . . .	11
5.1.3	Goal Statements . . . . .	11
5.2	Solution Characteristics Specification . . . . .	11
5.2.1	Assumptions . . . . .	12
5.2.2	Theoretical Models . . . . .	12
5.2.3	General Definitions . . . . .	13
5.2.4	Data Definitions . . . . .	14

5.2.5	Instance Models . . . . .	20
5.2.6	Data Constraints . . . . .	30
5.2.7	Properties of a Correct Solution . . . . .	31
<b>6</b>	<b>Requirements</b>	<b>31</b>
6.1	Functional Requirements . . . . .	31
6.2	Non-Functional Requirements . . . . .	34
<b>7</b>	<b>Likely Changes</b>	<b>34</b>
<b>8</b>	<b>Unlikely Changes</b>	<b>35</b>
<b>9</b>	<b>Traceability Matrices and Graphs</b>	<b>35</b>
<b>10</b>	<b>Values of Auxiliary Constants</b>	<b>41</b>
<b>11</b>	<b>References</b>	<b>41</b>
<b>12</b>	<b>Appendix</b>	<b>42</b>

# 1 Reference Material

This section records information for easy reference.

## 1.1 Table of Units

The unit system used throughout is SI (Système International d’Unités). In addition to the basic units, several derived units are also used. For each unit, the [Table of Units](#) lists the symbol, a description, and the SI name.

Table 1: Table of Units

Symbol	Description	SI Name
kg	mass	kilogram
m	length	metre
N	force	newton
Pa	pressure	pascal
s	time	second

## 1.2 Table of Symbols

The symbols used in this document are summarized in the [Table of Symbols](#) along with their units. The symbols are listed in alphabetical order.

Table 2: Table of Symbols

Symbol	Description	Units
$a$	Plate length (long dimension)	m
$AR$	Aspect ratio	—
$AR_{\max}$	Maximum aspect ratio	—
$B$	Risk of failure	—
$b$	Plate width (short dimension)	m
$capacity$	Capacity or load resistance	Pa
$d_{\max}$	Maximum value for one of the dimensions of the glass plate	m
$d_{\min}$	Minimum value for one of the dimensions of the glass plate	m
$E$	Modulus of elasticity of glass	Pa
$g$	Glass type	—

Continued on next page

Table 2: Table of Symbols (Continued)

Symbol	Description	Units
$GTF$	Glass type factor	—
$h$	Minimum thickness	m
$interpY$	InterpY	—
$interpZ$	InterpZ	—
$isSafeLoad$	Load resistance safety requirement	—
$isSafeLR$	3 second load equivalent resistance safety requirement	—
$isSafePb$	Probability of glass breakage safety requirement	—
$isSafeProb$	Probability of failure safety requirement	—
$J$	Stress distribution factor (Function)	—
$J_{\max}$	Maximum value for the stress distribution factor	—
$J_{\min}$	Minimum value for the stress distribution factor	—
$J_{\text{tol}}$	Tolerable stress distribution factor	—
$k$	Surface flaw parameter	$\frac{\text{m}^{12}}{\text{N}^7}$
$LDF$	Load duration factor	—
$Load$	Applied load (demand) or pressure	Pa
$LR$	Load resistance	Pa
$LSF$	Load share factor	—
$m$	Surface flaw parameter	$\frac{\text{m}^{12}}{\text{N}^7}$
$NFL$	Non-factored load	Pa
$P_b$	Probability of breakage	—
$P_{\text{btol}}$	Tolerable probability of breakage	—
$P_f$	Probability of failure	—
$P_{\text{ftol}}$	Tolerable probability of failure	—
$q$	Applied load (demand)	Pa
$\hat{q}$	Dimensionless load	—
$\hat{q}_{\text{tol}}$	Tolerable load	—
$SD$	Stand off distance	m
$SD_{\max}$	Maximum stand off distance permissible for input	m
$SD_{\min}$	Minimum stand off distance permissible for input	m
$SD_x$	Stand off distance ( $x$ -component)	m

Continued on next page

Table 2: Table of Symbols (Continued)

Symbol	Description	Units
$SD_y$	Stand off distance ( $y$ -component)	m
$SD_z$	Stand off distance ( $z$ -component)	m
$t$	Nominal thickness $t$ is in {2.5, 2.7, 3.0, 4.0, 5.0, 6.0, 8.0, 10.0, 12.0, 16.0, 19.0, 22.0}	mm
$t_d$	Duration of load	s
$TNT$	TNT equivalent factor	—
$w$	Charge weight	kg
$w_{\max}$	Maximum permissible input charge weight	kg
$w_{\min}$	Minimum permissible input charge weight	kg
$w_{TNT}$	Equivalent TNT charge mass	kg

### 1.3 Abbreviations and Acronyms

Table 3: Abbreviations and Acronyms

Abbreviation	Full Form
A	Assumption
AN	Annealed
AR	Aspect Ratio
DD	Data Definition
FT	Fully Tempered
GS	Goal Statement
GTF	Glass Type Factor
HS	Heat Strengthened
IG	Insulating Glass
IM	Instance Model
LC	Likely Change
LDF	Load Duration Factor
LG	Laminated Glass
LR	Load Resistance
LSF	Load Share Factor

Continued on next page

Table 3: Abbreviations and Acronyms (Continued)

Abbreviation	Full Form
NFL	Non-Factored Load
PS	Physical System Description
R	Requirement
RefBy	Referenced by
Refname	Reference Name
SD	Stand Off Distance
SRS	Software Requirements Specification
TM	Theoretical Model
UC	Unlikely Change
Uncert.	Typical Uncertainty

## 2 Introduction

Explosions in downtown areas are dangerous from the blast itself and also potentially from the secondary effect of falling glass. Therefore, software is needed to predict whether a glass slab can withstand a blast under given conditions. For example, we might wish to know whether a pane of glass fails from a gas main explosion or from a small fertilizer truck bomb. The document describes the program called GlassBR , which is based on the original, manually created version of [GlassBR](#).

The following section provides an overview of the Software Requirements Specification (SRS) for GlassBR. This section explains the purpose of this document, the scope of the requirements, the characteristics of the intended reader, and the organization of the document.

### 2.1 Purpose of Document

The primary purpose of this document is to record the requirements of GlassBR. Goals, assumptions, theoretical models, definitions, and other model derivation information are specified, allowing the reader to fully understand and verify the purpose and scientific basis of GlassBR. With the exception of **system constraints**, this SRS will remain abstract, describing what problem is being solved, but not how to solve it.

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation. Although the SRS fits in a series of documents that follow

the so-called waterfall model, the actual development process is not constrained in any way. Even when the waterfall model is not followed, as Parnas and Clements point out [7], the most logical way to present the documentation is still to “fake” a rational design process.

## 2.2 Scope of Requirements

The scope of the requirements includes determining the safety of a glass slab under a blast loading following the ASTM standard ([1]).

## 2.3 Characteristics of Intended Reader

Reviewers of this documentation should have an understanding of second year calculus, structural mechanics, glass breakage, blast risk, computer applications in civil engineering, and applicable standards for constructions using glass from [1], [3], and [2] in references. The users of GlassBR can have a lower level of expertise, as explained in [Sec:User Characteristics](#).

## 2.4 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by [6], [9], [10], and [8]. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the [data definitions](#) and trace back to find any additional information they require.

The [goal statements](#) are refined to the theoretical models and the [theoretical models](#) to the [instance models](#). The data definitions are used to support the definitions of the different models.

# 3 Stakeholders

This section describes the stakeholders: the people who have an interest in the product.

## 3.1 The Client

The client for GlassBR is a company named Entuitive. It is developed by Dr. Manuel Campidelli. The client has the final say on acceptance of the product.

## 3.2 The Customer

The customers are the end user of GlassBR.

## 4 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics, and lists the system constraints.

### 4.1 System Context

**Fig:sysCtxDiag** shows the system context. A circle represents an external entity outside the software, the user in this case. A rectangle represents the software system itself (GlassBR). Arrows are used to show the data flow between the system and its environment.



Figure 1: System Context

The interaction between the product and the user is through a user interface. The responsibilities of the user and the system are as follows:

- User Responsibilities
  - Provide the input data related to the glass slab and blast type, ensuring no errors in the data entry.
  - Ensure that consistent units are used for input variables.
  - Ensure required **software assumptions** are appropriate for any particular problem input to the software.
- GlassBR Responsibilities
  - Detect data type mismatch, such as a string of characters input instead of a floating point number.
  - Determine if the inputs satisfy the required physical and software constraints.
  - Predict whether the glass slab is safe or not.

### 4.2 User Characteristics

- The end user of GlassBR is expected to have completed at least the equivalent of the second year of an undergraduate degree in civil engineering or structural engineering.



- The end user is expected to have an understanding of theory behind glass breakage and blast risk.
- The end user is expected to have basic computer literacy to handle the software.

### 4.3 System Constraints

There are no system constraints.

## 5 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, and definitions that are used.

### 5.1 Problem Description

A system is needed to predict whether a glass slab can withstand a blast under given conditions.

#### 5.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements. All of the terms are extracted from [1].

1. Glass breakage - The fracture or breakage of any lite or ply in monolithic, laminated, or insulating glass.
2. Lateral - Perpendicular to the glass surface.
3. Lite - Pieces of glass that are cut, prepared, and used to create the window or door.
4. Specifying authority - The design professional responsible for interpreting applicable regulations of authorities having jurisdiction and considering appropriate site specific factors to determine the appropriate values used to calculate the specified design load, and furnishing other information required to perform this practice.
5. Blast resistant glazing - Glazing that provides protection against air blast pressure generated by explosions.
6. Equivalent TNT charge mass - Mass of TNT placed on the ground in a hemisphere that represents the design explosive threat.
7. Glass Type:

- Annealed (AN) - A flat, monolithic, glass lite which has uniform thickness where the residual surface stresses are almost zero, as defined in [2].
  - Fully tempered (FT) - A flat, monolithic, glass lite of uniform thickness that has been subjected to a special heat treatment process where the residual surface compression is not less than 69 MPa (10 000 psi) or the edge compression not less than 67 MPa (9700 psi), as defined in [3].
  - Heat strengthened (HS) - A flat, monolithic, glass lite of uniform thickness that has been subjected to a special heat treatment process where the residual surface compression is not less than 24 MPa (3500psi) or greater than 52 MPa (7500 psi), as defined in [3].
8. Applied load (demand) or pressure - A uniformly distributed lateral pressure.
    - Load resistance (LR) - The uniform lateral load that a glass construction can sustain based upon a given probability of breakage and load duration as defined in [1, (pp. 1 and 53)].
    - Non-factored load (NFL) - Three second duration uniform load associated with a probability of breakage less than or equal to 8 lites per 1000 for monolithic AN glass.
    - Glass weight load - The dead load component of the glass weight.
    - Short duration load - Any load lasting 3 seconds or less.
    - Specified design load - The magnitude in Pa (psf), type (for example, wind or snow) and duration of the load given by the specifying authority.
    - Long duration load - Any load lasting approximately 30 days.
  9. Stand off distance (SD) - The distance from the glazing surface to the centroid of a hemispherical high explosive charge.
  10. Load share factor (LSF) - A multiplying factor derived from the load sharing between the double glazing, of equal or different thicknesses and types (including the layered behaviour of LG under long duration loads), in a sealed IG unit.
  11. Glass type factor (GTF) - A multiplying factor for adjusting the LR of different glass type, that is, AN, FT, or HS, in monolithic glass, LG (Laminated Glass), or IG (Insulating Glass) constructions.
  12. Aspect ratio (AR) - The ratio of the long dimension of the glass to the short dimension of the glass. For glass supported on four sides, the aspect ratio is always equal to or greater than 1.0. For glass supported on three sides, the ratio of the length of one of the supported edges perpendicular to the free edge, to the length of the free edge, is equal to or greater than 0.5.

13. Probability of breakage ( $P_b$ ) - The fraction of glass lites or plies that would break at the first occurrence of a specified load and duration, typically expressed in lites per 1000 ([2]).

### 5.1.2 Physical System Description

The physical system of GlassBR, as shown in [Fig:physSystImage](#), includes the following elements:

PS1: The glass slab.

PS2: The point of explosion. Where the bomb, or any kind of man-made explosion, is located. The stand off distance is the distance between the point of explosion and the glass.



Figure 2: The physical system

### 5.1.3 Goal Statements

Given the dimensions of the glass plane, the glass type, the characteristics of the explosion, and the tolerable probability of breakage, the goal statement is:

Predict-Glass-Withstands-Explosion: Analyze and predict whether the glass slab under consideration will be able to withstand the explosion of a certain degree which is calculated based on user input.

## 5.2 Solution Characteristics Specification

The instance models that govern GlassBR are presented in the [Instance Model Section](#). The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

### 5.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical models by filling in the missing information for the physical system. The assumptions refine the scope by providing more detail.

glassType: The standard E1300-09a for calculation applies only to monolithic, laminated, or insulating glass constructions of rectangular shape with continuous lateral support along one, two, three, or four edges. This practice assumes that: (1) the supported glass edges for two, three and four-sided support conditions are simply supported and free to slip in plane; (2) glass supported on two sides acts as a simply supported beam; and (3) glass supported on one side acts as a cantilever.

glassCondition: Following [1, (pg. 1)], this practice does not apply to any form of wired, patterned, etched, sandblasted, drilled, notched, or grooved glass with surface and edge treatments that alter the glass strength. (RefBy: [UC:Accommodate-Altered-Glass](#).)

explainScenario: This system only considers the external explosion scenario for its calculations. (RefBy: [LC:Calculate-Internal-Blast-Risk](#).)

standardValues: The values provided in [Sec:Values of Auxiliary Constants](#) are assumed for the duration of load ( $t_d$ ), and the material properties of  $m$ ,  $k$ , and  $E$ . (RefBy: [IM:sdfTol](#), [IM:nFL](#), [IM:dimlessLoad](#), [LC:Variable-Values-of-m,k,E](#), [A:ldfConstant](#), and [DD:loadDurFactor](#).)

glassLite: Glass under consideration is assumed to be a single lite; hence, the value of LSF is equal to 1 for all calculations in GlassBR. (RefBy: [LC:Accomodate-More-than-Single-Lite](#).)

boundaryConditions: Boundary conditions for the glass slab are assumed to be 4-sided support for calculations. (RefBy: [LC:Accomodate-More-Boundary-Conditions](#).)

responseType: The response type considered in GlassBR is flexural. (RefBy: [LC:Consider-More-than-Flexure-Glass](#).)

ldfConstant: With reference to [A:standardValues](#), the value of load duration factor ( $LDF$ ) is a constant in GlassBR. (RefBy: [LC:Variable-Values-of-m,k,E](#) and [DD:loadDurFactor](#).)

### 5.2.2 Theoretical Models

This section focuses on the general equations and laws that GlassBR is based on.

<b>Refname</b>	<b>TM:isSafeProb</b>
Label	Safety Probability
Equation	$isSafeProb = P_f < P_{ftol}$
Description	<p><i>isSafeProb</i> is the probability of failure safety requirement (Unitless)</p> <p><math>P_f</math> is the probability of failure (Unitless)</p> <p><math>P_{ftol}</math> is the tolerable probability of failure (Unitless)</p>
Notes	If <i>isSafeProb</i> , the structure is considered safe.
Source	[1]
RefBy	
<b>Refname</b>	<b>TM:isSafeLoad</b>
Label	Safety Load
Equation	$isSafeLoad = capacity > Load$
Description	<p><i>isSafeLoad</i> is the load resistance safety requirement (Unitless)</p> <p><i>capacity</i> is the capacity or load resistance (Pa)</p> <p><i>Load</i> is the applied load (demand) or pressure (Pa)</p>
Notes	If <i>isSafeLoad</i> , the structure is considered safe.
Source	[1]
RefBy	

### 5.2.3 General Definitions

There are no general definitions.

### 5.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models.

Refname	DD:minThick
Label	Minimum thickness
Symbol	$h$
Units	m
Equation	$h = \frac{1}{1000} \begin{cases} 2.16, & t = 2.5 \\ 2.59, & t = 2.7 \\ 2.92, & t = 3.0 \\ 3.78, & t = 4.0 \\ 4.57, & t = 5.0 \\ 5.56, & t = 6.0 \\ 7.42, & t = 8.0 \\ 9.02, & t = 10.0 \\ 11.91, & t = 12.0 \\ 15.09, & t = 16.0 \\ 18.26, & t = 19.0 \\ 21.44, & t = 22.0 \end{cases}$
Description	<p><math>h</math> is the minimum thickness (m)</p> <p><math>t</math> is the nominal thickness <math>t</math> is in</p> <p><math>\{2.5, 2.7, 3.0, 4.0, 5.0, 6.0, 8.0, 10.0, 12.0, 16.0, 19.0, 22.0\}</math> (mm)</p>
Notes	$t$ is a function that maps from the nominal thickness ( $h$ ) to the minimum thickness.
Source	[1]
RefBy	IM:sdfTol, IM:riskFun, IM:nFL, and IM:dimlessLoad

Refname	DD:loadDurFactor
Label	Load duration factor
Symbol	$LDF$
Units	Unitless
Equation	$LDF = \left(\frac{t_d}{60}\right)^{\frac{m}{16}}$
Description	<p><math>LDF</math> is the load duration factor (Unitless)</p> <p><math>t_d</math> is the duration of load (s)</p> <p><math>m</math> is the surface flaw parameter (<math>\frac{m^{12}}{N^7}</math>)</p>
Notes	<p><math>t_d</math> and <math>m</math> come from <a href="#">A:standardValues</a>.</p> <p><math>LDF</math> is assumed to be constant (from <a href="#">A:ldfConstant</a>).</p>
Source	<a href="#">[1]</a>
RefBy	<a href="#">IM:sdfTol</a> and <a href="#">IM:riskFun</a>

Refname	DD:gTF
Label	Glass type factor
Symbol	$GTF$
Units	Unitless
Equation	$GTF = \begin{cases} 1, & g = \text{“AN”} \\ 4, & g = \text{“FT”} \\ 2, & g = \text{“HS”} \end{cases}$
Description	<p><math>GTF</math> is the glass type factor (Unitless)</p> <p><math>g</math> is the glass type (Unitless)</p>
Notes	<p>AN is annealed glass.</p> <p>FT is fully tempered glass.</p> <p>HS is heat strengthened glass.</p>
Source	<a href="#">[1]</a>
RefBy	IM:calofCapacity and IM:dimlessLoad



Refname	DD:standOffDist
Label	Stand off distance
Symbol	$SD$
Units	m
Equation	$SD = \sqrt{SD_x^2 + SD_y^2 + SD_z^2}$
Description	<p><math>SD</math> is the stand off distance (m)</p> <p><math>SD_x</math> is the stand off distance (<math>x</math>-component) (m)</p> <p><math>SD_y</math> is the stand off distance (<math>y</math>-component) (m)</p> <p><math>SD_z</math> is the stand off distance (<math>z</math>-component) (m)</p>
Source	<a href="#">[1]</a>
RefBy	DD:calofDemand

Refname	DD:aspectRatio
Label	Aspect ratio
Symbol	$AR$
Units	Unitless
Equation	$AR = \frac{a}{b}$
Description	<p><math>AR</math> is the aspect ratio (Unitless)</p> <p><math>a</math> is the plate length (long dimension) (m)</p> <p><math>b</math> is the plate width (short dimension) (m)</p>
Notes	$a$ and $b$ are the dimensions of the plate, where ( $a \geq b$ ).
Source	<a href="#">[1]</a>
RefBy	IM:tolLoad and IM:stressDistFac

Refname	DD:eqTNTW
Label	Equivalent TNT charge mass
Symbol	$w_{TNT}$
Units	kg
Equation	$w_{TNT} = w \ TNT$
Description	<p><math>w_{TNT}</math> is the equivalent TNT charge mass (kg)</p> <p><math>w</math> is the charge weight (kg)</p> <p><math>TNT</math> is the TNT equivalent factor (Unitless)</p>
Source	<a href="#">[1]</a>
RefBy	DD:calofDemand

Refname	DD:calofDemand
Label	Applied load (demand)
Symbol	$q$
Units	Pa
Equation	$q = \text{interpY}(\text{"TSD.txt"}, SD, w_{TNT})$
Description	$q$ is the applied load (demand) (Pa) $\text{interpY}$ is the interpY (Unitless) $SD$ is the stand off distance (m) $w_{TNT}$ is the equivalent TNT charge mass (kg)
Notes	$q$ , or applied load (demand), is the 3 second duration equivalent pressure obtained from Fig:demandVSsod by interpolation using stand off distance ( $SD$ ) and $w_{TNT}$ as parameters. $w_{TNT}$ is defined in DD:eqTNTW. $SD$ is the stand off distance as defined in DD:standOffDist.
Source	[1]
RefBy	IM:isSafeLR and IM:dimlessLoad

### 5.2.5 Instance Models

This section transforms the problem defined in the [problem description](#) into one which is expressed in mathematical terms. It uses concrete symbols defined in the [data definitions](#) to replace the abstract symbols in the models identified in [theoretical models](#) and [general definitions](#).

The goal [GS:Predict-Glass-Withstands-Explosion](#) is met by [IM:isSafePb](#), [IM:isSafeLR](#).

Refname	IM:riskFun
Label	Risk of failure
Input	$E, LDF, J, k, m, h, a, b$
Output	$B$
Input Constraints	$a > 0$ $0 < b \leq a$
Output Constraints	
Equation	$B = \frac{k}{(a b)^{m-1}} (E h^2)^m LDF e^J$
Description	<p><math>B</math> is the risk of failure (Unitless)</p> <p><math>k</math> is the surface flaw parameter (<math>\frac{\text{m}^{12}}{\text{N}^7}</math>)</p> <p><math>a</math> is the plate length (long dimension) (m)</p> <p><math>b</math> is the plate width (short dimension) (m)</p> <p><math>m</math> is the surface flaw parameter (<math>\frac{\text{m}^{12}}{\text{N}^7}</math>)</p> <p><math>E</math> is the modulus of elasticity of glass (Pa)</p> <p><math>h</math> is the minimum thickness (m)</p> <p><math>LDF</math> is the load duration factor (Unitless)</p> <p><math>J</math> is the stress distribution factor (Function) (Unitless)</p>
Notes	<p><math>a</math> and <math>b</math> are the dimensions of the plate, where (<math>a \geq b</math>).</p> <p><math>h</math> is defined in <a href="#">DD:minThick</a> and is based on the nominal thicknesses.</p> <p><math>LDF</math> is defined in <a href="#">DD:loadDurFactor</a>.</p> <p><math>J</math> is defined in <a href="#">IM:stressDistFac</a>.</p>
Source	[1], [4, (Eqs. 4-5)], and [5, (Eq. 14)]
RefBy	<a href="#">IM:probOfBreak</a>

Refname	IM:stressDistFac
Label	Stress distribution factor (Function)
Input	$AR, \hat{q}$
Output	$J$
Input Constraints	$AR \geq 1$
Output Constraints	$J_{\min} \leq J \leq J_{\max}$
Equation	$J = \text{interpZ}(\text{"SDF.txt"}, AR, \hat{q})$
Description	<p><math>J</math> is the stress distribution factor (Function) (Unitless)</p> <p><math>\text{interpZ}</math> is the interpZ (Unitless)</p> <p><math>AR</math> is the aspect ratio (Unitless)</p> <p><math>\hat{q}</math> is the dimensionless load (Unitless)</p>
Notes	<p><math>J</math> is obtained by interpolating from data shown in <a href="#">Fig:dimlessloadVSaspect</a>.</p> <p><math>AR</math> is defined in <a href="#">DD:aspectRatio</a>.</p> <p><math>\hat{q}</math> is defined in <a href="#">IM:dimlessLoad</a>.</p>
Source	<a href="#">[1]</a>
RefBy	<a href="#">IM:riskFun</a>

Refname	IM:nFL
Label	Non-factored load
Input	$\hat{q}_{\text{tol}}, E, h, a, b$
Output	$NFL$
Input Constraints	$a > 0$ $0 < b \leq a$
Output Constraints	
Equation	$NFL = \frac{\hat{q}_{\text{tol}} E h^4}{(a b)^2}$
Description	<p><math>NFL</math> is the non-factored load (Pa)  <math>\hat{q}_{\text{tol}}</math> is the tolerable load (Unitless)  <math>E</math> is the modulus of elasticity of glass (Pa)  <math>h</math> is the minimum thickness (m)  <math>a</math> is the plate length (long dimension) (m)  <math>b</math> is the plate width (short dimension) (m)</p>
Notes	<p><math>\hat{q}_{\text{tol}}</math> is defined in <a href="#">IM:tolLoad</a>.  <math>E</math> comes from <a href="#">A:standardValues</a>.  <math>h</math> is defined in <a href="#">DD:minThick</a> and is based on the nominal thicknesses.  <math>a</math> and <math>b</math> are the dimensions of the plate, where (<math>a \geq b</math>).</p>
Source	<a href="#">[1]</a>
RefBy	<a href="#">IM:calofCapacity</a>

Refname	IM:dimlessLoad
Label	Dimensionless load
Input	$q, E, h, GTF, a, b$
Output	$\hat{q}$
Input Constraints	$a > 0$ $0 < b \leq a$
Output Constraints	
Equation	$\hat{q} = \frac{q(a b)^2}{E h^4 GTF}$
Description	<p> <math>\hat{q}</math> is the dimensionless load (Unitless)  <math>q</math> is the applied load (demand) (Pa)  <math>a</math> is the plate length (long dimension) (m)  <math>b</math> is the plate width (short dimension) (m)  <math>E</math> is the modulus of elasticity of glass (Pa)  <math>h</math> is the minimum thickness (m)  <math>GTF</math> is the glass type factor (Unitless) </p>
Notes	<p> <math>q</math> is the 3 second duration equivalent pressure, as given in <a href="#">DD:calofDemand</a>.  <math>a</math> and <math>b</math> are the dimensions of the plate, where <math>(a \geq b)</math>.  <math>E</math> comes from <a href="#">A:standardValues</a>.  <math>h</math> is defined in <a href="#">DD:minThick</a> and is based on the nominal thicknesses.  <math>GTF</math> is defined in <a href="#">DD:gTF</a>. </p>
Source	[1] and [5, (Eq. 7)]
RefBy	IM:stressDistFac



Refname	IM:tolLoad
Label	Tolerable load
Input	$AR, J_{\text{tol}}$
Output	$\hat{q}_{\text{tol}}$
Input Constraints	$AR \geq 1$
Output Constraints	
Equation	$\hat{q}_{\text{tol}} = \text{interpY}(\text{"SDF.txt"}, AR, J_{\text{tol}})$
Description	$\hat{q}_{\text{tol}}$ is the tolerable load (Unitless) $\text{interpY}$ is the interpY (Unitless) $AR$ is the aspect ratio (Unitless) $J_{\text{tol}}$ is the tolerable stress distribution factor (Unitless)
Notes	$\hat{q}_{\text{tol}}$ is obtained by interpolating from data shown in <b>Fig:dimlessloadVSaspect</b> . $AR$ is defined in <b>DD:aspectRatio</b> . $J_{\text{tol}}$ is defined in <b>IM:sdfTol</b> .
Source	<a href="#">[1]</a>
RefBy	<b>IM:nFL</b>

Refname	IM:sdfTol
Label	Tolerable stress distribution factor
Input	$LDF, P_{\text{btol}}, E, a, b, m, k, h$
Output	$J_{\text{tol}}$
Input Constraints	$0 \leq P_{\text{btol}} \leq 1$ $a > 0$ $0 < b \leq a$
Output Constraints	
Equation	$J_{\text{tol}} = \ln \left( \ln \left( \frac{1}{1-P_{\text{btol}}} \right) \frac{(a b)^{m-1}}{k (E h^2)^m LDF} \right)$
Description	<p> <math>J_{\text{tol}}</math> is the tolerable stress distribution factor (Unitless)  <math>P_{\text{btol}}</math> is the tolerable probability of breakage (Unitless)  <math>a</math> is the plate length (long dimension) (m)  <math>b</math> is the plate width (short dimension) (m)  <math>m</math> is the surface flaw parameter (<math>\frac{\text{m}^{12}}{\text{N}^7}</math>)  <math>k</math> is the surface flaw parameter (<math>\frac{\text{m}^{12}}{\text{N}^7}</math>)  <math>E</math> is the modulus of elasticity of glass (Pa)  <math>h</math> is the minimum thickness (m)  <math>LDF</math> is the load duration factor (Unitless) </p>
Notes	<p> <math>P_{\text{btol}}</math> is entered by the user.  <math>a</math> and <math>b</math> are the dimensions of the plate, where (<math>a \geq b</math>).  <math>m</math>, <math>k</math>, and <math>E</math> come from <a href="#">A:standardValues</a>.  <math>h</math> is defined in <a href="#">DD:minThick</a> and is based on the nominal thicknesses.  <math>LDF</math> is defined in <a href="#">DD:loadDurFactor</a>. </p>
Source	<a href="#">[1]</a>
RefBy	<a href="#">IM:tolLoad</a>

Refname	IM:probOfBreak	
Label	Probability of breakage	
Input	$B$	
Output	$P_b$	
Input Constraints		
Output Constraints	$0 \leq P_b \leq 1$	
Equation	$P_b = 1 - e^{-B}$	
Description	$P_b$ is the probability of breakage (Unitless) $B$ is the risk of failure (Unitless)	
Notes	$B$ is defined in <a href="#">IM:riskFun</a> .	
Source	<a href="#">[1]</a> and <a href="#">[4]</a>	
RefBy	<a href="#">IM:isSafePb</a>	

Refname	IM:calofCapacity
Label	Load resistance
Input	$NFL$ , $GTF$ , $LSF$
Output	$LR$
Input Constraints	
Output Constraints	
Equation	$LR = NFL \ GTF \ LSF$
Description	<p><math>LR</math> is the load resistance (Pa)</p> <p><math>NFL</math> is the non-factored load (Pa)</p> <p><math>GTF</math> is the glass type factor (Unitless)</p> <p><math>LSF</math> is the load share factor (Unitless)</p>
Notes	<p><math>LR</math> is also called capacity.</p> <p><math>NFL</math> is defined in <b>IM:nFL</b>.</p> <p><math>GTF</math> is defined in <b>DD:gTF</b>.</p>
Source	[1]
RefBy	<b>IM:isSafeLR</b>

Refname	IM:isSafePb
Label	Safety Req-Pb
Input	$P_b, P_{\text{btol}}$
Output	<i>isSafePb</i>
Input Constraints	$0 \leq P_b \leq 1$ $0 \leq P_{\text{btol}} \leq 1$
Output Constraints	
Equation	$isSafePb = P_b < P_{\text{btol}}$
Description	<p><i>isSafePb</i> is the probability of glass breakage safety requirement (Unitless)</p> <p><math>P_b</math> is the probability of breakage (Unitless)</p> <p><math>P_{\text{btol}}</math> is the tolerable probability of breakage (Unitless)</p>
Notes	<p>If <i>isSafePb</i>, the glass is considered safe. <i>isSafePb</i> and <i>isSafeLR</i> (from <b>IM:isSafeLR</b>) are either both True or both False.</p> <p><math>P_b</math> is defined in <b>IM:probOfBreak</b>.</p> <p><math>P_{\text{btol}}</math> is entered by the user.</p>
Source	[1]
RefBy	<b>IM:isSafeLR</b> and <b>FR:Check-Glass-Safety</b>

Refname	IM:isSafeLR
Label	Safety Req-LR
Input	$LR, q$
Output	$isSafeLR$
Input Constraints	$LR > 0$ $q > 0$
Output Constraints	
Equation	$isSafeLR = LR > q$
Description	<p><math>isSafeLR</math> is the 3 second load equivalent resistance safety requirement (Unitless)</p> <p><math>LR</math> is the load resistance (Pa)</p> <p><math>q</math> is the applied load (demand) (Pa)</p>
Notes	<p>If <math>isSafeLR</math>, the glass is considered safe. <math>isSafePb</math> (from IM:isSafePb) and <math>isSafeLR</math> are either both True or both False.</p> <p><math>LR</math> is defined in IM:calofCapacity and is also called capacity.</p> <p><math>q</math> is the 3 second duration equivalent pressure, as given in DD:calofDemand.</p>
Source	[1]
RefBy	IM:isSafePb and FR:Check-Glass-Safety

### 5.2.6 Data Constraints

The **Data Constraints Table** shows the data constraints on the input variables. The column for physical constraints gives the physical limitations on the range of values that can be taken

by the variable. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise. The constraints are conservative to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario. The **auxiliary constants** give the values of the specification parameters used in the **Data Constraints Table**.

Table 4: Input Data Constraints

Var	Physical Constraints	Software Constraints	Typical Value	Uncert.
$a$	$a > 0 \wedge a \geq b$	$d_{\min} \leq a \leq d_{\max}$	1.5 m	10%
$AR$	$AR \geq 1$	$AR \leq AR_{\max}$	1.5	10%
$b$	$0 < b \leq a$	$d_{\min} \leq b \leq d_{\max}$	1.2 m	10%
$P_{\text{btol}}$	$0 \leq P_{\text{btol}} \leq 1$	–	0.008	0.1%
$SD$	$SD > 0$	$SD_{\min} \leq SD \leq SD_{\max}$	45 m	10%
$TNT$	$TNT > 0$	–	1	10%
$w$	$w > 0$	$w_{\min} \leq w \leq w_{\max}$	42 kg	10%

### 5.2.7 Properties of a Correct Solution

The **Data Constraints Table** shows the data constraints on the output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable.

Table 5: Output Data Constraints

Var	Physical Constraints
$P_b$	$0 \leq P_b \leq 1$
$J$	$J_{\min} \leq J \leq J_{\max}$

## 6 Requirements

This section provides the functional requirements, the tasks and behaviours that the software is expected to complete, and the non-functional requirements, the qualities that the software is expected to exhibit.

### 6.1 Functional Requirements

This section provides the functional requirements, the tasks and behaviours that the software is expected to complete.

Input-Values: Input the values from **Tab:ReqInputs**, which define the glass dimensions, type of glass, tolerable probability of failure, and the characteristics of the blast.

System-Set-Values-Following-Assumptions: The system shall set the known values as described in the table for **Required Assignments**.

Check-Input-with-Data\_Constraints: The system shall check the entered input values to ensure that they do not exceed the **data constraints**. If any of the input values are out of bounds, an error message is displayed and the calculations stop.

Output-Values-and-Known-Values: Output the input values from **FR:Input-Values** and the known values from **FR:System-Set-Values-Following-Assumptions**.

Check-Glass-Safety: If  $isSafePb \wedge isSafeLR$  (from **IM:isSafePb** and **IM:isSafeLR**), output the message “For the given input parameters, the glass is considered safe.” If the condition is false, then output the message “For the given input parameters, the glass is NOT considered safe.”

Output-Values: Output the values from the table for **Required Outputs**.

Table 6: Required Inputs

Symbol	Description	Units
$a$	Plate length (long dimension)	m
$b$	Plate width (short dimension)	m
$g$	Glass type	—
$P_{\text{btol}}$	Tolerable probability of breakage	—
$SD_x$	Stand off distance ( $x$ -component)	m
$SD_y$	Stand off distance ( $y$ -component)	m
$SD_z$	Stand off distance ( $z$ -component)	m
$t$	Nominal thickness $t$ is in {2.5, 2.7, 3.0, 4.0, 5.0, 6.0, 8.0, 10.0, 12.0, 16.0, 19.0, 22.0}	mm
$TNT$	TNT equivalent factor	—
$w$	Charge weight	kg



Table 7: Required Assignments

Symbol	Description	Source	Units
$AR$	Aspect ratio	DD:aspectRatio	–
$E$	Modulus of elasticity of glass	A:standardValues	Pa
$GTF$	Glass type factor	DD:gTF	–
$h$	Minimum thickness	DD:minThick	m
$k$	Surface flaw parameter	A:standardValues	$\frac{m^{12}}{N^7}$
$LDF$	Load duration factor	DD:loadDurFactor	–
$LSF$	Load share factor	A:glassLite	–
$m$	Surface flaw parameter	A:standardValues	$\frac{m^{12}}{N^7}$
$SD$	Stand off distance	DD:standOffDist	m
$t_d$	Duration of load	A:standardValues	s

Table 8: Required Outputs

Symbol	Description	Source	Units
$AR$	Aspect ratio	DD:aspectRatio	–
$B$	Risk of failure	IM:riskFun	–
$GTF$	Glass type factor	DD:gTF	–
$h$	Minimum thickness	DD:minThick	m
$isSafeLR$	3 second load equivalent resistance safety requirement	IM:isSafeLR	–
$isSafePb$	Probability of glass breakage safety requirement	IM:isSafePb	–
$J$	Stress distribution factor (Function)	IM:stressDistFac	–
$J_{tol}$	Tolerable stress distribution factor	IM:sdfTol	–
$LR$	Load resistance	IM:calofCapacity	Pa
$NFL$	Non-factored load	IM:nFL	Pa
$P_b$	Probability of breakage	IM:probOfBreak	–
$\hat{q}$	Dimensionless load	IM:dimlessLoad	–
$\hat{q}_{tol}$	Tolerable load	IM:tolLoad	–

## 6.2 Non-Functional Requirements

This section provides the non-functional requirements, the qualities that the software is expected to exhibit.

Correctness: The outputs of the code have the **properties of a correct solution**.

Verifiability: The code is tested with complete verification and validation plan.

Understandability: The code is modularized with complete module guide and module interface specification.

Reusability: The code is modularized.

Maintainability: If a likely change is made to the finished software, it will take at most 10% of the original development time, assuming the same development resources are available.

Portability: The code shall be portable to multiple environments, particularly Windows, Mac OSX, and Linux.

## 7 Likely Changes

This section lists the likely changes to be made to the software.

Calculate-Internal-Blast-Risk: **A:explainScenario** - The system currently only calculates for external blast risk. In the future, calculations can be added for the internal blast risk.

Variable-Values-of-m,k,E: **A:standardValues**, **A:ldfConstant** - Currently, the values for  $m$ ,  $k$ , and  $E$  are assumed to be the same for all glass. In the future, these values can be changed to variable inputs.

Accomodate-More-than-Single-Lite: **A:glassLite** - The software may be changed to accomodate more than a single lite.

Accomodate-More-Boundary-Conditions: **A:boundaryConditions** - The software may be changed to accommodate more boundary conditions than 4-sided support.

Consider-More-than-Flexure-Glass: **A:responseType** - The software may be changed to consider more than just flexure of the glass.

## 8 Unlikely Changes

This section lists the unlikely changes to be made to the software.

**Predict-Withstanding-of-Certain-Degree:** The goal of the system is to predict whether the glass slab under consideration can withstand an explosion of a certain degree.

**Accommodate-Altered-Glass:** **A:glassCondition** requires that the glass is not altered in any way. Therefore, this cannot be used on altered glass.

## 9 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an “X” should be modified as well. **Tab:TraceMatAvsA** shows the dependencies of the assumptions on each other. **Tab:TraceMatAvsAll** shows the dependencies of the data definitions, theoretical models, general definitions, instance models, requirements, likely changes, and unlikely changes on the assumptions. **Tab:TraceMatRefvsRef** shows the dependencies of the data definitions, theoretical models, general definitions, and instance models on each other. **Tab:TraceMatAllvsR** shows the dependencies of the requirements and goal statements on the data definitions, theoretical models, general definitions, and instance models.

Table 9: Traceability Matrix Showing the Connections Between

	<b>A:glassType</b>	<b>A:glassCondition</b>	<b>A:explainScenario</b>	<b>A:standardValues</b>
<b>A:glassType</b>				
<b>A:glassCondition</b>				
<b>A:explainScenario</b>				
<b>A:standardValues</b>				
<b>A:glassLite</b>				
<b>A:boundaryConditions</b>				
<b>A:responseType</b>				
<b>A:ldfConstant</b>				X

Table 10: Traceability Matrix Showing the Co

	A:glassType	A:glassCondition	A:explainScenar
DD:minThick			
DD:loadDurFactor			
DD:gTF			
DD:standOffDist			
DD:aspectRatio			
DD:eqTNTW			
DD:calofDemand			
TM:isSafeProb			
TM:isSafeLoad			
IM:riskFun			
IM:stressDistFac			
IM:nFL			
IM:dimlessLoad			
IM:tolLoad			
IM:sdfTol			
IM:probOfBreak			
IM:calofCapacity			
IM:isSafePb			
IM:isSafeLR			
NFR:Correctness			
NFR:Verifiability			
NFR:Understandability			
NFR:Reusability			
NFR:Maintainability			
NFR:Portability			
FR:Input-Values			
FR:System-Set-Values-Following-Assumptions			
FR:Check-Input-with-Data_Constraints			
FR:Output-Values-and-Known-Values			
FR:Check-Glass-Safety			

Table 10: Traceability Matrix Showing the Connecti

	A:glassType	A:glassCondition	A:explainScenar
FR:Output-Values			
LC:Calculate-Internal-Blast-Risk			X
LC:Variable-Values-of-m,k,E			
LC:Accomodate-More-than-Single-Lite			
LC:Accomodate-More-Boundary-Conditions			
LC:Consider-More-than-Flexure-Glass			
UC:Predict-Withstanding-of-Certain-Degree			
UC:Accommodate-Altered-Glass		X	
	</		

	DD:minThick	DD:loadDurFactor	DD:gTF	DD:standOffDist	DD:asp
IM:isSafeLR					
GS:Predict-Glass-Withstands-Explosion					
NFR:Correctness					
NFR:Verifiability					
NFR:Understandability					
NFR:Reusability					
NFR:Maintainability					
NFR:Portability					
FR:Input-Values					
FR:System-Set-Values-Following-Assumptions					
FR:Check-Input-with-Data_Constraints					
FR:Output-Values-and-Known-Values					
FR:Check-Glass-Safety					
FR:Output-Values					

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent dependencies. The component at the tail of an arrow is depended on by the component at the head of that arrow. Therefore, if a component is changed, the components that it points to should also be changed. [Fig:TraceGraphAvsA](#) shows the dependencies of assumptions on each other. [Fig:TraceGraphAvsAll](#) shows the dependencies of data definitions, theoretical models, general definitions, instance models, requirements, likely changes, and unlikely changes on the assumptions. [Fig:TraceGraphRefvsRef](#) shows the dependencies of data definitions, theoretical models, general definitions, and instance models on each other. [Fig:TraceGraphAllvsR](#) shows the dependencies of requirements and goal statements on the data definitions, theoretical models, general definitions, and instance models. [Fig:TraceGraphAllvsAll](#) shows the dependencies of dependencies of assumptions, models, definitions, requirements, goals, and changes with each other.



Figure 3: TraceGraphAvsA



Figure 4: TraceGraphAvsAll



Figure 5: TraceGraphRefvsRef





- [TraceGraphAllvsR](#)
- [TraceGraphAllvsAll](#)

## 10 Values of Auxiliary Constants

This section contains the standard values that are used for calculations in GlassBR.

Table 13: Auxiliary Constants

Symbol	Description	Value	Unit
$AR_{\max}$	maximum aspect ratio	5	–
$d_{\max}$	maximum value for one of the dimensions of the glass plate	5	m
$d_{\min}$	minimum value for one of the dimensions of the glass plate	0.1	m
$E$	modulus of elasticity of glass	$71.7 \cdot 10^9$	Pa
$J_{\max}$	maximum value for the stress distribution factor	32	–
$J_{\min}$	minimum value for the stress distribution factor	1	–
$k$	surface flaw parameter	$28.6 \cdot 10^{-54}$	$\frac{\text{m}^{12}}{\text{N}^7}$
$LSF$	load share factor	1	–
$m$	surface flaw parameter	7	$\frac{\text{m}^{12}}{\text{N}^7}$
$SD_{\max}$	maximum stand off distance permissible for input	130	m
$SD_{\min}$	minimum stand off distance permissible for input	6	m
$t_d$	duration of load	3	s
$w_{\max}$	maximum permissible input charge weight	910	kg
$w_{\min}$	minimum permissible input charge weight	4.5	kg

## 11 References

- [1] ASTM. *Standard Practice for Determining Load Resistance of Glass in Buildings*. <https://www.astm.org>. 2009.
- [2] ASTM. *Standard specification for Flat Glass*. <https://doi.org/10.1520/C1036-16>. West Conshohocken, PA, 2016.
- [3] ASTM. *Standard Specification for Heat-Strengthened and Fully Tempered Flat Glass*. <https://doi.org/10.1520/C1048-12E01>. West Conshohocken, PA, 2012.

- [4] W. Lynn Beason, Terry L. Kohutek, and Joseph M. Bracci. *Basis for ASTM E 1300 Annealed Glass Thickness Selection Charts*. [https://doi.org/10.1061/\(ASCE\)0733-9445\(1998\)124:2\(215\)](https://doi.org/10.1061/(ASCE)0733-9445(1998)124:2(215)). Feb. 1998.
- [5] Manuel Campidelli. *Glass-BR Software for the design and risk assessment of glass facades subjected to blast loading*.
- [6] Nirmitha Koothoor. “A Document Driven Approach to Certifying Scientific Computing Software”. MA thesis. Hamilton, ON, Canada: McMaster University, 2013.
- [7] David L. Parnas and P. C. Clements. “A rational design process: How and why to fake it”. In: *IEEE Transactions on Software Engineering* 12.2 (Feb. 1986), pp. 251–257.
- [8] W. Spencer Smith and Nirmitha Koothoor. “A Document-Driven Method for Certifying Scientific Computing Software for Use in Nuclear Safety Analysis”. In: *Nuclear Engineering and Technology* 48.2 (Apr. 2016), pp. 404–418.
- [9] W. Spencer Smith and Lei Lai. “A new requirements template for scientific computing”. In: *Proceedings of the First International Workshop on Situational Requirements Engineering Processes - Methods, Techniques and Tools to Support Situation-Specific Requirements Engineering Processes, SREP’05*. Ed. by PJ Agerfalk, N. Kraiem, and J. Ralyte. In conjunction with 13th IEEE International Requirements Engineering Conference, Paris, France, 2005, pp. 107–121.
- [10] W. Spencer Smith, Lei Lai, and Ridha Khedri. “Requirements Analysis for Engineering Computation: A Systematic Approach for Improving Software Reliability”. In: *Reliable Computing, Special Issue on Reliable Engineering Computation* 13.1 (Feb. 2007), pp. 83–107.

## 12 Appendix

This appendix holds the graphs ([Fig:demandVSsod](#) and [Fig:dimlessloadVSaspect](#)) used for interpolating values needed in the models.



Figure 8: 3 second duration equivalent pressure ( $q$ ) versus Stand off distance (SD) versus Charge weight ( $w$ )

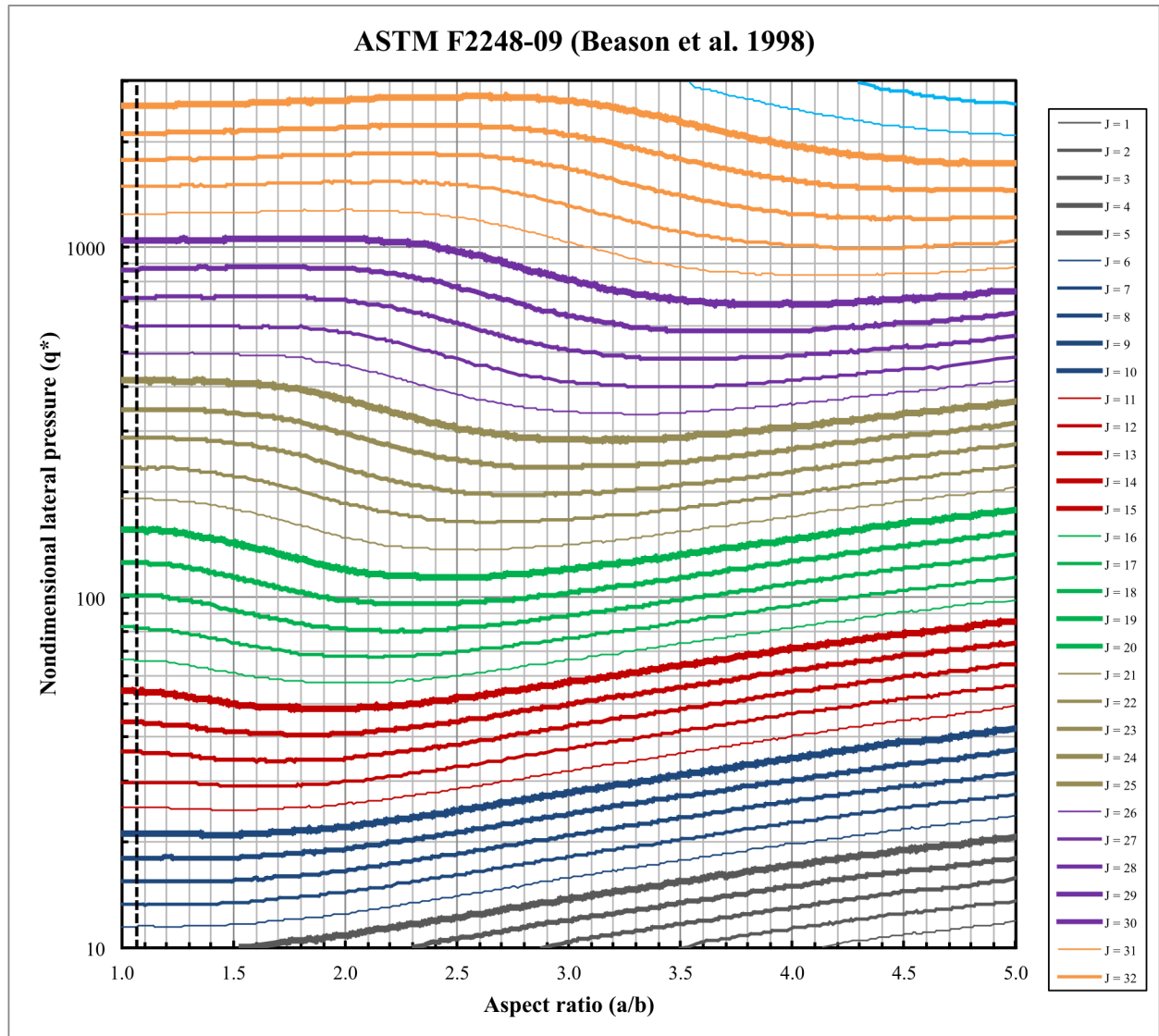


Figure 9: Non dimensional lateral applied load (demand) or pressure ( $\hat{q}$ ) versus Aspect Ratio (AR) versus Stress distribution factor (Function) ( $J$ )