Classical Symbolic Retrodictive Execution of Quantum Circuits

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Retrodictive quantum theory [4], retrocausality [1], and the time-symmetry of physical laws [16] suggest that partial knowledge about the future can be exploited to understand the present. We demonstrate the even stronger proposition that, in concert with the computational concepts of *demand-driven lazy evaluation* [9] and *symbolic partial evaluation* [8], retrodictive reasoning can be used as a computational resource to de-quantize some quantum algorithms, i.e., to provide efficient classical algorithms inspired by their quantum counterparts.

Symbolic Execution of Classical Programs Applied to Quantum Oracles. A well-established technique to simultaneously explore multiple paths that a classical program could take under different inputs is *symbolic execution* [3, 5, 7, 10, 11]. In this execution scheme, concrete values are replaced by symbols which are initially unconstrained. As the execution proceeds, the symbols interact with program constructs and this typically introduces constraints on the possible values that the symbols represent. At the end of the execution, these constraints can be solved to infer properties of the program under consideration. The idea is also applicable to quantum circuits as the following example illustrates.

Let $[\mathbf{n}]$ denote the finite set $\{0, 1, \ldots, (n-1)\}$. In Simon's problem, we are given a 2-1 (classical) function $f : [\mathbf{2^n}] \to [\mathbf{2^n}]$ with the property that there exists an a such $f(x) = f(x \oplus a)$ for all x; the goal is to determine a. The circuit in Fig. 1 implements the quantum algorithm when $n = 2$ and $a = 3$. In the circuit, the gates between barrier (1) and barrier (2) implement a quantum oracle $U_f(x, 0) = (x, f(x))$ that encapsulates the function f of interest. A direct classical simulation of the quantum circuit would need to execute the U_f block four times, once for each possible value $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ for the top two wires. Instead, let us introduce two symbols x_0 representing the top wire and x_1 representing the wire below it, and let's proceed with the execution symbolically. The state at barrier (1) is initially $|x_0x_100\rangle$. At the first CX-gate, we symbolically calculate the result of the target wire as $x_0 \oplus 0 = x_0$ evolving the state to $|x_0x_1x_00\rangle$. Going through the next three CX-gates, the state evolves as $|x_0x_1x_0x_0\rangle$, $|x_0x_1(x_0 \oplus x_1)x_0\rangle$, and $|x_0x_1(x_0 \oplus x_1)(x_0 \oplus x_1)\rangle$ at barrier (2). At that point, we have established that the bottom two wires are equal; the result of their measurement can only be 00 or 11. Since the function is promised to be 2-1 for all inputs, it is sufficient to analyze one case, say when the measurement at barrier (3) produces 00. This measurement collapses the top wires to $|x_0x_1\rangle$ subject to the constraint that $x_0 \oplus x_1 = 0$ or equivalently that $x_0 = x_1$. We have thus inferred that both $x_0 = x_1 = 0$ and $x_0 = x_1 = 1$ produce the same measurement result at barrier (3) and hence that $f(00) = f(11) = f(00 \oplus 11)$ which reveals that a is 11 in binary notation.

Since the quantum circuit between barriers (1) and (2) is reversible, we can perform the analysis above in a mixed predictive and retrodictive symbolic execution to make the flow of information conceptually clearer. We start a forward classical simulation with one arbitrary state at barrier (1), say $|0100\rangle$. This state evolves to $|0100\rangle$, then $|0100\rangle$ again, then $|0110\rangle$, and finally $|0111\rangle$. In this case, the result of measuring the bottom two wires is 11. Having produced a possible measurement at barrier (3), we start a retrodictive execution to find out what other input states might be compatible with this future measurement. To that end, we execute the circuit backwards with the symbolic state $|x_0x_111\rangle$; that execution evolves to $|x_0x_11(1 \oplus x_1)\rangle$, then $|x_0x_1(1 \oplus x_1)(1 \oplus x_1)\rangle$, then $|x_0x_1(1 \oplus x_1)(1 \oplus x_0 \oplus x_1)\rangle$, and finally $|x_0x_1(1 \oplus x_0 \oplus x_1)(1 \oplus x_0 \oplus x_1)\rangle$. Having reached the initial conditions on the bottom two wires, we reconcile them with the collected constraints to conclude that $1 \oplus x_0 \oplus x_1 = 0$ or equivalently that $x_0 \neq x_1$. The measurement of 11 at barrier

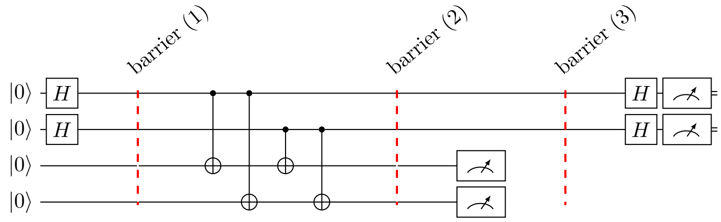
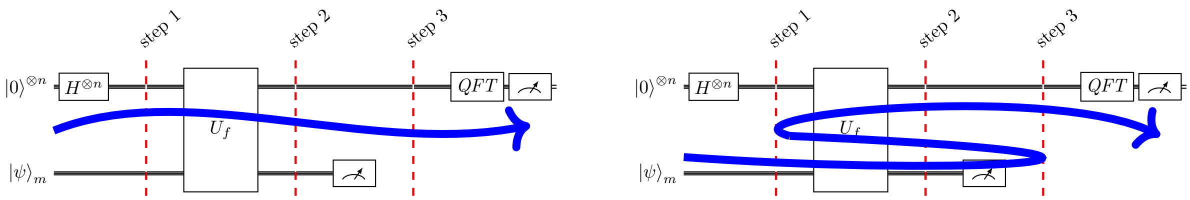


Figure 1: Circuit for Simon's Algorithm $n = 2$ and $a = 3$



(a) Conventional Flow

(b) Retrodictive Flow

Figure 2: Template quantum circuit

(3) is consistent with not just the state $|01\rangle$ we started with but also with the state $|10\rangle$. In other words, we have $f(01) = f(10) = f(01 \oplus 11)$ and the hidden value of a is revealed to be 11.

Representing Wavefunctions Symbolically. A symbolic variable represents an boolean value that can be 0 or 1; this is similar to a qubit in a superposition $(1/\sqrt{2})(|0\rangle \pm |1\rangle)$. Thus, it appear possible in any circuit to represent $H(|0\rangle)$ by a symbol x. Surprisingly, this idea scales to even represent maximally entangled states. Fig. 3(left) shows a circuit to generate the Bell state $(1/\sqrt{2})(|00\rangle + |11\rangle)$. By using the symbol x for $H(|0\rangle)$, the input to the CX-gate is $|x0\rangle$ which evolves to $|xx\rangle$. By sharing the same symbol in two positions, the symbolic state accurately represents the entangled Bell state. Similarly, for the circuit in Fig. 3(right), the state after the Hadamard gate is $|x00\rangle$ which evolves to $|xx0\rangle$ and then to $|xxx\rangle$ again accurately capturing the entanglement correlations.

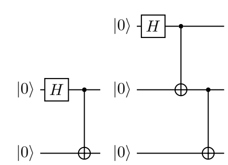


Figure 3: Bell and GHZ States

This insight allows us to symbolically execute the many quantum algorithms that match the template in Fig. 2 (including Deutsch, Deutsch-Jozsa, Bernstein-Vazirani, Simon, Grover, and Shor's algorithms). Specifically, in all these algorithms, the top collection of wires (which we will call the computational register) is prepared in a uniform superposition which can be represented using symbolic variables. Below, we report on the results of such symbolic executions. In each case, instead of the conventional execution flow depicted in Fig. 2(a), we find a possible measurement outcome w at barrier (3) and perform a retrodictive execution with a state $|x\rangle\,|w\rangle$ going backwards to collect the constraints on x that enable us to solve the problem in question.

A First Collection of Quantum Algorithms. In order to assess whether this idea works for a broad class of situations including different algorithms and different circuit sizes, we implemented a collection of software tools that perform retrodictive symbolic evaluation for circuits matching the template shown in Fig. 2. Each circuit consists of three stages: preparation, unitary evolution, and measurement in the Hadamard / Fourier basis. Our execution replaces the conventional flow of information in Fig. 2(a) with the novel flow in Fig. 2(b). In the latter model, a forward classical execution is performed to determine a possible measurement result for the bottom register; using this information, a retrodictive classical execution is performed to determine the

$$
\begin{array}{lllll|l}
a = 11 & x_0 = 0 & & & & x_0 = 0 \\
a = 4, 14 & 1 \oplus x_0 = 1 & x_0 = 0 & & & x_0 = 0 \\
a = 7, 13 & 1 \oplus x_0x_1 \oplus x_1 = 1 & x_0x_1 = 0 & x_0 \oplus x_0x_1 \oplus x_1 = 0 & x_0 \oplus x_0x_1 = 0 & x_0 = 0, x_1 = 0 \\
a = 2, 8 & 1 \oplus x_0 \oplus x_0x_1 \oplus x_1 = 1 & x_0x_1 = 0 & x_0x_1 \oplus x_1 = 0 & x_0 \oplus x_0x_1 = 0 & x_0 = 0, x_1 = 0
\end{array}
$$

Figure 5: Equations generated by retrodictive execution of $a^x \mod 15$ starting from observed result 1 and unknown $x_8x_7x_6x_5x_4x_3x_2x_1x_0$. The solution for the unknown variables is given in the last column.

initial states of the first register that are consistent with this measurement. These states are then analyzed depending on the algorithm in question. As we demonstrate below, retrodictive symbolic evaluation provides additional *classical* computational resources that are powerful enough to solve instances of Deutsch-Jozsa, Bernstein-Vazirani, and Simon problems, as well as some instances of Grover's and Shor's algorithms.

Shor 15. The circuit in Fig. 4 uses a hand-optimized implementation of the modular exponentiation $4^x \mod 15$ to factor 15 using Shor's algorithm. In a conventional forward execution, the state at step (3) is:

$$\frac{1}{2\sqrt{2}}((|0\rangle + |2\rangle + |4\rangle + |6\rangle)\,|1\rangle + (|1\rangle + |3\rangle + |5\rangle + |7\rangle)\,|4\rangle)$$

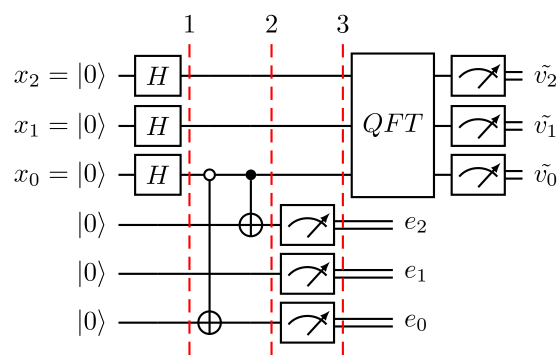


Figure 4: Finding the period of $4^x \mod 15$

At this point, the bottom register is measured. The result of the measurement can be either $|1\rangle$ or $|4\rangle$. In either case, the top register snaps to a state of the form $\sum_{r=0}^{3} |a + 2r\rangle$ whose QFT has peaks at $|0\rangle$ or $|4\rangle$. If we measure $|0\rangle$ for the top register, we repeat the experiment; otherwise we infer that the period is 2. Instead of this forward execution, we can reason as follows. Since $x^0 = 1$ for all x, we know that $|1\rangle$ is a possible measurement of the second register. We can therefore proceed in a retrodictive fashion with the state $|x_2x_1x_0\rangle\,|001\rangle$ at step (2) and compute backwards. The first CX-gate changes the state to $|x_2x_1x_0\rangle\,|x_001\rangle$ and the second CX-gate produces $|x_2x_1x_0\rangle\,|x_00x_0\rangle$. At that point, we reconcile the retrodictive result of the second register $|x_00x_0\rangle$ with the initial condition $|000\rangle$ to conclude that $x_0 = 0$. In other words, in order to observe $e_2e_1e_0 = 001$, the first register must be initialized to a superposition of the form $|??0\rangle$ where the least significant bit must be 0 and the other two bits are unconstrained. Expanding the possibilities, the first register needs to be in a superposition of the states $|000\rangle, |010\rangle, |100\rangle$ or $|110\rangle$ and we have just inferred using purely classical but retrodictive reasoning that the period is 2. Significantly, this approach is robust and does not require small hand-optimized circuits. Indeed, following the methods for producing quantum circuits for arithmetic operations from first principles using adders and multipliers [15], our implementation for $a^x \mod 15$ has 56538 generalized Toffoli gates over 9 qubits, and yet the equations resulting from the retrodictive execution in Fig. 5 are trivial and immediately solvable as they only involve either the least significant bit x_0 (when $a \in \{4, 11, 14\}$) or the least significant two bits x_0 and x_1 (when $a \in \{2, 7, 8, 13\}$). When the solution is $x_0 = 0$, the period is 2. When the solution is $x_0 = 0, x_1 = 0$, the period is 4.

Deutsch. The problem is to determine if a given function $[\mathbf{2}] \to [\mathbf{2}]$ is constant or balanced. It is assumed that the function is embedded in a quantum circuit U_f, typically composed of X and CX gate, and the goal is to use U_f just once. The textbook quantum algorithm prepares a quantum superposition that propagates through the quantum oracle U_f in the forward direction and then performs a measurement that deterministically solves the problem. Instead, we fix the ancilla output to a possible boundary condition,

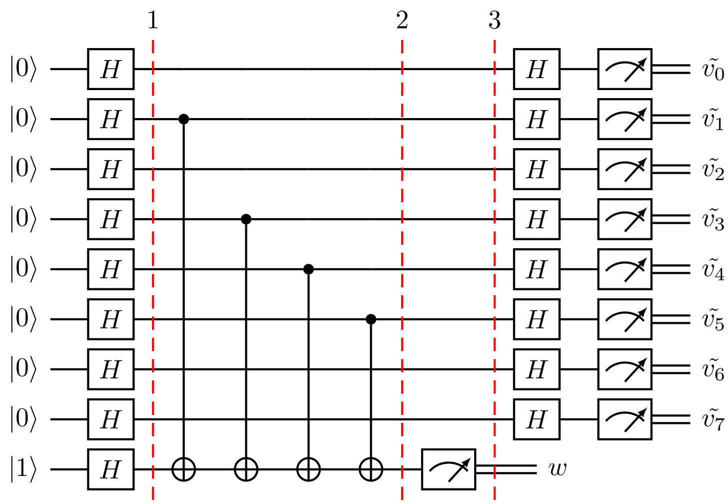


Figure 6: Circuit for Bernstein-Vazirani Algorithm ($n = 8$, $s = 92$, least significant bit is the top wire)

say $|0\rangle$, provide a symbolic state $|x\rangle$ for the top register, and perform a retrodictive execution of the quantum oracle. The execution starts from the output side with the state $|x\rangle\,|0\rangle$ and terminates on the input side with a state $|x\rangle\,|y\rangle$ where y is a symbolic expression that captures the necessary initial conditions to produce the partial observation $|0\rangle$ on the ancilla register. Running the experiment, we get one of the following four symbolic expressions 0, 1, x, or $1 \oplus x$ depending on the function f. In the first two cases, the observation of the ancilla is independent of x, i.e. the function is constant. In the last two cases, the ancilla depends on x (or its negation), and the function must be balanced.

Deutsch-Jozsa. The problem is a generalization of the previous one: we are given a function $[\mathbf{n}] \to [\mathbf{2}]$ that is promised to be constant or balanced and we need to decide distinguish the two cases. Again, we fix the ancillary output to a possible boundary condition, say $|0\rangle$, and perform a retrodictive execution of the circuit to calculate a symbolic expression. Running the experiment for the two constant functions, the result is 0 or 1 indicating no dependency of the ancilla on the input. As small examples of balanced functions with $n = 6$, the resulting expression was x_0 in one case, $x_0 \oplus x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5$ in another, and $x_0x_1x_2 \oplus x_0x_1x_2x_3x_4 \oplus x_0x_1x_2x_3x_5 \oplus x_0x_1x_2x_4 \oplus x_0x_1x_2x_4x_5 \oplus x_0x_1x_3x_4 \oplus x_0x_1x_3x_5 \oplus x_0x_1x_4 \oplus x_0x_1x_4x_5 \oplus x_0x_2 \oplus x_0x_2x_3x_5 \oplus x_0x_2x_4x_5 \oplus x_0x_3 \oplus x_0x_3x_4x_5 \oplus x_0x_3x_5 \oplus x_1x_2x_3x_5 \oplus x_1x_2x_4x_5 \oplus x_1x_3x_4x_5 \oplus x_1x_3x_5 \oplus x_1x_5 \oplus x_2x_3x_4x_5 \oplus x_2x_3x_5 \oplus x_2x_4 \oplus x_3x_4x_5 \oplus x_3x_5$ in the last. In the first case, the function is balanced because its output depends on just one variable (which is 0 half the time); in the second case the output of the function is the exclusive-or of all the input variables which is an easy instance of a balanced function. The last case is a cryptographically strong balanced function whose output pattern is, by design, difficult to discern [6]. Since we are promised the function is either constant or balanced, then any output that depends on at least one symbolic variable is incompatible with a constant function; the details of the dependency are not relevant. We confirmed this observation by running the experiment on all 12870 balanced function from $[\mathbf{2^4}] \to [\mathbf{2}]$.

Bernstein-Vazirani. We are given a function $f : [\mathbf{2^n}] \to [\mathbf{2}]$ that hides a secret number $s \in [\mathbf{2^n}]$. We are promised the function is defined using the binary representations $\sum_i^{n-1} x_i$ and $\sum_i^{n-1} s_i$ of x and s respectively as $f(x) = \sum_{i=0}^{n-1} s_i x_i \mod 2$. The goal is to determine the secret number s. The circuit in Fig. 6 solves the problem for $n = 8$ and a hidden number 92 (= 00111010 in binary notation with the rightmost bit at index 0). The gates between slice (1) and slice (2) collect the sum of the x_i at positions that match the occurrences of 1 in the secret string. The retrodictive execution proceeds from slice (2) backwards with the

$$
\begin{array}{ll}
w = 0 & 1 \oplus x_0 \oplus x_0x_1 \oplus x_0x_1x_2 \oplus x_0x_1x_2x_3 \oplus x_0x_1x_3 \oplus x_0x_2 \oplus x_0x_2x_3 \oplus x_0x_3 \oplus x_1 \oplus x_1x_2 \oplus \\
 & \quad x_1x_2x_3 \oplus x_1x_3 \oplus x_2 \oplus x_2x_3 \oplus x_3 \\
w = 1 & x_0 \oplus x_0x_1 \oplus x_0x_1x_2 \oplus x_0x_1x_2x_3 \oplus x_0x_1x_3 \oplus x_0x_2 \oplus x_0x_2x_3 \oplus x_0x_3 \\
w = 2 & x_0x_1 \oplus x_0x_1x_2 \oplus x_0x_1x_2x_3 \oplus x_0x_1x_3 \oplus x_1 \oplus x_1x_2 \oplus x_1x_2x_3 \oplus x_1x_3 \\
w = 3 & x_0x_1 \oplus x_0x_1x_2 \oplus x_0x_1x_2x_3 \oplus x_0x_1x_3 \\
w = 4 & x_0x_1x_2 \oplus x_0x_1x_2x_3 \oplus x_0x_2 \oplus x_0x_2x_3 \oplus x_1x_2 \oplus x_1x_2x_3 \oplus x_2 \oplus x_2x_3 \\
w = 5 & x_0x_1x_2 \oplus x_0x_1x_2x_3 \oplus x_0x_2 \oplus x_0x_2x_3 \\
w = 6 & x_0x_1x_2 \oplus x_0x_1x_2x_3 \oplus x_1x_2 \oplus x_1x_2x_3 \\
w = 7 & x_0x_1x_2 \oplus x_0x_1x_2x_3 \\
w = 8 & x_0x_1x_2x_3 \oplus x_0x_1x_3 \oplus x_0x_2x_3 \oplus x_0x_3 \oplus x_1x_2x_3 \oplus x_1x_3 \oplus x_2x_3 \oplus x_3 \\
w = 9 & x_0x_1x_2x_3 \oplus x_0x_1x_3 \oplus x_0x_2x_3 \oplus x_0x_3 \\
w = 10 & x_0x_1x_2x_3 \oplus x_0x_1x_3 \oplus x_1x_2x_3 \oplus x_1x_3 \\
w = 11 & x_0x_1x_2x_3 \oplus x_0x_1x_3 \\
w = 12 & x_0x_1x_2x_3 \oplus x_0x_2x_3 \oplus x_1x_2x_3 \oplus x_2x_3 \\
w = 13 & x_0x_1x_2x_3 \oplus x_0x_2x_3 \\
w = 14 & x_0x_1x_2x_3 \oplus x_1x_2x_3 \\
w = 15 & x_0x_1x_2x_3
\end{array}
$$

Figure 7: Result of retrodictive execution for the Grover oracle ($n = 4$, w in the range $\{0..15\}$).

state $|x_0x_1x_2x_3x_4x_5x_6x_70\rangle$; upon termination the last qubit has the symbolic value $x_1 \oplus x_3 \oplus x_4 \oplus x_5$. The indices $\{1, 3, 4, 5\}$ are exactly the positions in which the secret string has a 1.

Grover. We are given a function $f; [\mathbf{2^n}] \to [\mathbf{2}]$ with the property that there exists only one input w such $f(wx) = 1$. The goal is to find w. The conventional presentation of the quantum algorithm does not exactly fit the template of Fig. 2. But it is possible to construct a quantum oracle U_f from the given f and perform retrodictive execution. The resulting equations for $n = 4$ and w in the range $\{0..15\}$ are in Fig. 7. In some cases (e.g. $w = 15$) the equations immediately reveal w; in others non-trivial steps would be needed to solve the equations.

Shor 21. The sample examples presented so far demonstrate that some instances of quantum algorithms can be solved via classical symbolic retrodictive execution. We now show an instance that glaringly shows the limitations of the basic retrodictive execution, do a theoretical analysis, and show how to tune the basic idea to solve more and more instances of quantum algorithms. As is already apparent in some examples, running retrodictive execution may produce large equations. To appreciate how large these equations may be, we include the full set of equations producing for a retrodictive execution of Shor's algorithm for factoring 21. Unlike the number 15 and the rare occurrences of products of Fermat primes which result in a period that is a power of 2 and hence trivial to represent by equations of binary numbers, the period of 21 is not easily representable as a system of equations over binary numbers. See Sec. 1.

Retrodictive Executions, Function Pre-images, and NP-Complete Problems. We now express the computational problems above uniformly as queries over function pre-images. Given finite sets A and B, a function $f : A \to B$ and an element $y \in B$, we define $\{\cdot \xleftarrow{f} y\}$, the pre-image of y under f, as the set $\{x \in A \mid f(x) = y\}$. For example, let $A = B = \{0, 1, \ldots, 15\}$ and let $f(x) = 7^x \mod 15$, then the collection of values that f maps to 4, $\{\cdot \xleftarrow{f} 4\}$, is the set $\{2, 6, 10, 14\}$.

Referring back to Fig. 2, we observe that the quantum algorithm can be decomposed into: (a) the computation up to step (3) which just computes the pre-image of the ancilla measurement under f, and (b) a module performing Hadamard of QFT to analyze this pre-image. For example, the pre-image of 4 under $f(x) = 7^x \mod 15$ displayed in Fig. 8 would be represented as the superposition $|\psi\rangle = 1/2(|2\rangle + |6\rangle + |10\rangle + |14\rangle)$ at step (3) of Shor's algorithm. What is crucial is that although the quantum state $|\psi\rangle$ is not directly

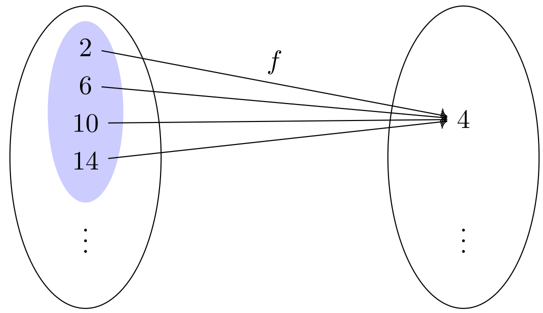


Figure 8: The pre-image of 4 under $f(x) = 7^x \mod 15$.

observable, this is of no concern. Shor's algorithm does not actually care about the full description of the pre-image, only about a global property of the pre-image: its period. Indeed, in the quantum algorithms we discussed, the full calculation of pre-image is never needed: each algorithm computes a particular global property of the corresponding pre-image. The Deutsch and Deutsch-Jozsa algorithms only need to distinguish whether the pre-image of either 1 or 0 is empty, contains half the elements, or the entire set. The Bernstein-Vazirani algorithm only needs n queries over the pre-image of either 1 or 0: query i asks whether 2^i is a member of the pre-image and the answer determines bit i of the secret s. Indeed, by definition, $f(2^i) = s_i$ and hence s_i is 1 iff 2^i is a member of the pre-image of 1. In the case of the Simon problem, we calculate $f(x) = w$ for some x and query the pre-image of w to get the other value in the pre-image.

To summarize, quantum algorithms compute "simple queries" over pre-images, and in fact, unless $P = NP$, such simple queries are the only possibility since the full calculation of a pre-image is an NP-complete problem, and it is believed that even full fledged quantum computers cannot solve NP-complete problems. To appreciate the difficulty of computing pre-images in general, note that finding the pre-image of a function is subsumes several challenging computational problems such as pre-image attacks on hash functions [14], predicting environmental conditions that allow certain reactions to take place in computational biology [2, 12], and finding the pre-image of feature vectors in the space induced by a kernel in neural networks [13]. More to the point, the boolean satisfiability problem SAT is expressible as a boolean function over the input variables and solving a SAT problem is asking for the pre-image of `true`. Indeed, based on the conjectured existence of one-way functions which itself implies $P \neq NP$, all these pre-images calculations are believed to be computationally intractable in their most general setting.

we have enough to analyze many quantum algorithms; say we have an implementation and show result of running on many circuits

do communication protocols too ??

then get to $|-\rangle$

graph state: H,H,CZ

00 00 01 01 10 10 11 -11

H control +/- distinction not important so use one class of vars H target +/- distinction important; use two classes of vars

just run forward symbolically retrodictive is not fundamental here

symbolic exec H introduces uncertainty; forget +/- distinction for now; use variable safe is H wires are used as control wires but not targets run symbolically; can represent entanglement; e.g. bell state xx check if H commutes with x and cx so we only need H at beginning and end introduce vars at beginning and run symbolically if only x and cx then symbolic execution is efficient; no need for last batch of H can solve problem classically connect with Gottsman-Knill

What is have ccx sometimes fine; shor 15 example; still fine sometimes we get very complicated representation of wavefunction but if we are following up with QFT; QFT insensitive to offset, don't care variables no need to keep track of values of vars; only need to know if they are constant or not

what is H wires are used as targets; need two flavors of variables; +vars and -vars; -vars infect +vars in control gates; taint analysis with increasing precisions (more and more colors)

retrodictive? Kochen-Specker; interactive QM; observer free will; choice backtracks

values going at different speeds; intervals ideas; path types

The quantum circuit model consists of two classes of gates: (i) quantum counterparts to classical reversible gates (e.g., Toffoli gates), and (ii) genuine quantum gates with no classical counterpart (e.g., Hadamard and phase gates). We make the remarkable observation, that, for a number of well-established quantum algorithms, judicious reasoning about the classical components, ignoring all the quantum gates, is sufficient. Put differently, in those cases, the quantum gates serve no fundamental purpose and are actually distracting from an underlying efficient classical algorithm. The result relies on the ability to symbolically execute circuits, especially in a retrodictive fashion, i.e., by making partial observations at the output site and proceeding backwards to infer the implied initial conditions.

> You can't connect the dots looking forward; you can only connect them looking backwards. So you have to trust that the dots will somehow connect in your future. *Steve Jobs*

extract vars; all we need for some algos

The obvious question to ask now is whether the retrodictive execution can be tuned to only produce the required statistics instead producing the full description of pre-images.

insight 1: qft does not care about 0+2+4.... vs 1+3+5....

0 0 ? 0 1 ? 1 0 ? 1 1 ?

equiv no matter what ? is ? is used in the computation (don't care about value) others not used so we just need to keep track of which vars are used

run experiments with PEX and PEY

2. Hadamard basis: Toffoli + Hadamard is universal so we "just" need to understand how to run in X basis.

Get rid of all quantum gates and run just the reversible classical part but with different taint analyses

Essentially we have two colors and we do taint analysis

Blue and Red; when blue interacts with red it gets tainted

We have two operations +red (add red) -red (remove red)

Remember cx(+,-) = (-,-)

Some interactions (Toffoli) want to create more refined operations +/-(1/2)(red) +/-(red) The more you do these operations the more precise it wants to be +/-(1/4)(red) +/-(1/2) red +/-(red)

And so on

You can truncate at the desired level of accuracy

The taint analysis groups variables in "waves" (superpositions) of things that have the same color so the values we

propagate are "red: phase=p; frequency=f; involved variables=x1,x2,. . . "

Seems that naive taint analysis is just keep track of which variable is used

run again; refined pe; var used; if used twice then disappears

go back to that stupid paper about logic programming and xor

The equations turn out to be trivial when the period is a power of 2. This occurs when the number to factor is a product of Fermat primes: 3, 5, 17, 257, 65537, ... The equations generated for some of these cases are in . . .

need stats only PEX , PEY ...

core of many quantum algos is quantum oracle uf two inputs; two outputs system; ancilla; normal eval; control ancilla; system unknown; so throw in complete superposition and eval forward

Retrodictive QFT. only need number of vars !!!!

solve other problems with just knowing which vars are involved

Discussion. Normal quantum evolution: from present to future

Now what if I had partial knowledge about the future; what can you say about the present? (And then about the rest of the unknown future)

Can this help flow of information, complexity, etc?

In some cases, partial knowledge about the future is enough to predict the present accurately enough

to then predict everything about the future; in some cases it is not enough

Possibility that collapse of wave function is information flow back from measured future to present unknown initial conditions and then back to rest of wave that was not measured

Provide a general introduction to the topic and a brief non-technical summary of your main results and their implication.

200 words ??

main text 2000-2500 words 3-4 figures 30-50 references

Methods section 3000 words more references ok

Author contributions

Code available

https://quantumalgorithmzoo.org

every quantum circuit can be written using Toffoli and Hadamard retro just go through Toffoli; ignore Had; but of course we are using symbolic eval

can H be moved past Toffoli?

universe uses lazy evaluation?

algebra of Toffoli and Hadamard ZX calculus

fourier transform classical efficient in some cases

Ewin Tang papers

kochen specker ??

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1 Methods

Lazy Evaluation. Consider a program that searches for three different numbers x, y, and z each in the range $[1..n]$ and that sum to s. A well-established design principle for solving such problems is the *generate-and-test* computational paradigm. Following this principle, a simple program to solve this problem in the programming language Haskell is:

```
generate :: Int -> [(Int,Int,Int)]
generate n =  [(x,y,z) | x <- [1..n], y <- [1..n], z <- [1..n]]

test :: Int -> [(Int,Int,Int)] -> [(Int,Int,Int)]
test s nums = [(x,y,z) | (x,y,z) <- nums, x /= y, x /= z, y /= z, x+y+z == s]
```

```
find :: Int -> Int -> (Int,Int,Int)
find s =  head . test s . generate
```

The program consists of three functions: `generate` that produces all triples `(x,y,z)` from `(1,1,1)` to `(n,n,n)`; `test` that checks that the numbers are different and that their sum is equal to `s`; and `find` that composes the two functions: generating all triples, testing the ones that satisfy the condition, and returning the first solution. Running this program to find numbers in the range [1..6] that sum to 15 immediately produces $(4, 5, 6)$ as expected.

But what if the range of interest was [1..10000000] ? A naïve execution of the generate-and-test method would be prohibitively expensive as it would spend all its time generating an enormous number of triples that are un-needed. Lazy demand-driven evaluation as implemented in Haskell succeeds in a few seconds with the result $(1, 2, 12)$, however. The idea is simple: instead of eagerly generating all the triples, generate a process that, when queried, produces one triple at a time on demand. Conceptually the execution starts from the observer site which is asking for the first element of a list; this demand is propagated to the function `test` which itself propagates the demand to the function `generate`. As each triple is generated, it is tested until one triple passes the test. This triple is immediately returned without having to generate any additional values.

Partial Evaluation. Below is a Haskell program that computes a^n by repeated squaring:

```
power :: Int -> Int -> Int
power a n
  | n == 0     = 1
  | n == 1     = a
  | even n     = let r = power a (n `div` 2) in r * r
  | otherwise  = a * power a (n-1)
```

When both inputs are known, e.g., `a = 3` and `n = 5`, the program evaluates as follows:

```
   power 3 5
=  3 * power 3 4
=  3 * (let r1 = power 3 2 in r1 * r1)
=  3 * (let r1 = (let r2 = power 3 1 in r2 * r2) in r1 * r1)
=  3 * (let r1 = (let r2 = 3 in r2 * r2) in r1 * r1)
=  3 * (let r1 = 9 in r1 * r1)
=  243
```

Partial evaluation is used when we only have partial information about the inputs. Say we only know $n = 5$. A partial evaluator then attempts to evaluate `power` with symbolic input `a` and actual input `n=5`. This evaluation proceeds as follows:

```
   power a 5
=  a * power a 4
=  a * (let r1 = power a 2 in r1 * r1)
=  a * (let r1 = (let r2 = power a 1 in r2 * r2) in r1 * r1)
=  a * (let r1 = (let r2 = a in r2 * r2) in r1 * r1)
=  a * (let r1 = a * a in r1 * r1)
=  let r1 = a * a in a * r1 * r1
```

All of this evaluation, simplification, and specialization happens without knowledge of `a`. Just knowing `n` was enough to produce a residual program that is much simpler.

The evolution of a quantum system is typically understood as proceeding forwards in time — from the present to the future. As shown in Fig. 2(a), Since the conventional execution starts with complete ignorance about the future, the initial state is prepared as a superposition that includes every possibility. In a well-designed algorithm, , by the time the computation reaches the measurement stages, the relative phases and probability amplitudes in that enormous superposition have become biased towards states of interest which are projected to produce the final answer.

Data Availability. available

Discussion. Possibility that collapse of wave function is information flow back from measured future to present unknown initial conditions and then back to rest of wave that was not measured

transactional interpretation?

Luckily, the problems of concern to us are quite special: (i) the functions are not arbitrary but have additional structure that can be exploited, and (ii) we never need access to all the elements in the pre-image; we just need to answer aggregate queries about the pre-images. Quantum algorithms somehow exploit these properties along with some physical principles to solve these problems efficiently. To understand the precise way in which this is happening, we start with the template of the quantum circuit used for solving all the problems above in Fig. 2.

The core of the circuit is the U_f block which can be assumed to be implemented using only generalized Toffoli gates. The block implements the unitary transformation: $U_f(|x\rangle\,|y\rangle) = |x\rangle\,|f(x) \oplus y\rangle$ where $\oplus$ is the (bitwise) exclusive-or operation; it defines the function of interest whose pre-image properties are to be calculated. The inputs of the U_f block are grouped in two registers: the top register contains an equal superposition of all possible inputs to f; the second register is prepared in initial states that depend on the specific algorithm. Thus, the state at slice (1) in the figure is:

$$\frac{1}{\sqrt{2^n}\sqrt{2^m}} \sum_{x=0}^{2^n-1} \sum_{y=0}^{2^m-1} |x\rangle\,|y\rangle$$

This is transformed by U_f to:

$$\frac{1}{\sqrt{2^n}\sqrt{2^m}} \sum_{x=0}^{2^n-1} \sum_{y=0}^{2^m-1} |x\rangle\,|f(x) \oplus y\rangle$$

So far, nothing too interesting is happening: we have just produced a superposition of states where each state is a possible input to f, say x, tensored with $f(x) \oplus y$, the result of applying f to this particular input adjusted by the second register y. At slice (3), something remarkable occurs; the result w of measuring the second register "kicks back" information to the first register whose state becomes a superposition of those values x that are consistent with the measurement, i.e., *the pre-image of w under f!* That pre-image representation is then analyzed using the Quantum Fourier Transform (QFT) to produce the final result.

Quantum algorithms typically operate on a *black box* holding a classical function whose properties need to be computed. The general structure of these algorithms is to (i) create a superposition of values to be passed as inputs to the black box, (ii) apply the operation inside the black box, and (iii) post-process the output of the black box. We observe that, in quite a few cases, steps (i) and (iii) are actually unnecessary and that the entire "quantum" algorithm can be executed by forward or backward, full or partial, efficient classical *symbolic execution* of the black box.

typical use: superposition, Uf, measure second register; we only care about which x has f(x) = r

By default all functions are reversible.

To make them irreversible you fix h and delete g. If you delete too much the function becomes very expensive to reverse. So one way functions emerge

simplify function has polynomial realization and we want statistics about the kernel (not necessarily compute it exactly)

collect assumptions:

important that no matter what measurement we do on w, properly we want is the same since we say that algos related to pre-images lets do naive thing and eval backwards

assumptions we have a rev circuit efficient forward two inputs: first is full superposition; second whatever first output same as first input; but that is only at point 2; at point 3 explain kick back; misleading to think it is the same after 3 second output is result of function; measure; have element of range; go back with that elem if we knew first output as well as w then eval backwards same complexity but we only know w and we don't know first output; because we are starting at 3 not 2

we have no use for H block; it was only there for the forward exec to express our complete ignorance o the future; prepared with every x but if we have knowledge about future (w measured) we go back to find the values of x in the present that would be consistent with w so general circuit reduces to :

...

fix pics to have amplitudes with y (most general)

To what extent are the quantum algorithms above taking advantage of non-classical features. We posit that pre-image computation can be, at least for some of the some of the algorithms, be performed classically. The main insight needed for that is to perform the execution *symbolically*. We illustrate the idea with two examples.

We need to explain ideas about time-reversal, prediction and retrodiction in physics. The laws of computation and the laws of physics are intimately related. When does knowing something about the future help us unveil the structure or symmetries of the past? It is like a detective story, but one with ramifications in complexity and/or efficiency. Problems involving questions where answers demand a Many(past)-to-one(future) map are at the root of our proposal.... Difference between exploiting or not entanglement in the unitary evolution.

As we demonstrate, the family of quantum algorithms initiated by Deutsch's algorithm and culminating with Shor's algorithm (i) solves variants of the pre-image problem efficiently, and, in that context, (ii) answering queries about pre-images is closely related to *retrodictive quantum theory* [2], retrocausality [1], and the time-symmetry of physical laws [4].

- Retrodictive execution more efficient in some cases. What cases?
- Here are three examples: Deutsch-Jozsa, Simon, Shor when period is close to a power of 2
- Symbolic (retrodictive) evaluation as a broader perspective to classical computation
- Symbolic execution allows you to express/discover interference via shared variables
- When interference pattern is simple symbolic execution reveals solutions faster (and completely classically)
- Symbolic execution as a "classical waves" computing paradigm

to represent unequal superpositions do multiple runs with vars the first has x1 x2 etc the second has y1 2y2 etc or y2/2 etc, or with various patterns of negative weights.... And then the punchline would be to interpret the negative backwards. So instead of all forward or all retro we have some values going forward and then backwards

Start with the story about function many to one etc why superpositions because we don't know which values so we try all easy to represent by unknown vars so we can represent superpositions as vars and equations between them but at the end we want stats about superpositions slow way is to generate all equations and solve faster way is generate many sets of equations with different weights and sum to get your stats

Partial Symbolic Evaluation with Algebraic Normal Form (ANF). The resulting expressions are in algebraic normal form [3] where + denotes exclusive-or.

We should use two prototypical examples to illustrate main ideas before going to the complex ones. The examples I have in mind are: Deutsch-Jozsa and Simon (precursor of Shor's). There are prior works on dequantization of the first problem and should make contact with their resolution. Perhaps we can show that they are as efficient classically? That would justify retrodiction alone. The more complex (and important) case of factorization should be the natural follow up.

The idea of symbolic execution is not tied to forward or backward execution. We should introduce it in a way that is independent of the direction of execution. What the idea depends on however is that the wave function, at least in the cases we are considering, can be represented as equations over booleans.

Wave Functions as Equations over Booleans

in the typical scenario for using quantum oracles, we can represent wave function as equations over booleans; equations represent the wave function but the solution is unobservable just like the components of the superposition in the wave function are not observable; just like we don't directly get access to the components of the wave function; we don't directly get access to the solution of the equations; need to "observe" the equations

we can go backwards with an equation (representing a wave function sigma x where f(x) = r and go back towards the present to calculate the wave function (represented as equations again)

Musing: how to explain complementarity when wave function is represented as an equation? Kochen specker;

or contextuality

observer 1 measures wires a,b; obs2 measures wires b,c; not commuting; each obs gives partial solution to equations; but partial solutions cannot lead to a global solution

KS suggests that equations do not have unique solutions; only materialize when you measure;

can associate a probability with each variable in a equation: look at all solutions and see the contribution of each variable to these solutions.

Complexity Analysis. one pass over circuit BUT complexity of normalizing to ANF not trivial; be careful

Supplementary Information. Equations generated by retrodictive execution of $4^x \mod 21$ starting from observed result 1 and unknown x. The circuit consists of 9 qubits, 36400 CX-gates, 38200 CCX-gates, and 4000 CCCX-gates. There are only three equations but each equation is exponentially large.

$1 \oplus x_0 \oplus x_0x_1x_2 \oplus x_0x_1x_2x_3x_4 \oplus x_0x_1x_2x_3x_4x_5x_6 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_8 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_9 \oplus x_0x_1x_2x_3x_4x_5x_6x_8 \oplus$
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$x_0x_1x_5x_8x_9 \oplus x_0x_1x_5x_9 \oplus x_0x_1x_6 \oplus x_0x_1x_6x_7x_8 \oplus x_0x_1x_6x_7x_9 \oplus x_0x_1x_6x_8 \oplus x_0x_1x_6x_8x_9 \oplus x_0x_1x_7 \oplus x_0x_1x_7x_8x_9 \oplus$
$x_0x_1x_7x_9 \oplus x_0x_1x_8 \oplus x_0x_1x_9 \oplus x_0x_2 \oplus x_0x_2x_3 \oplus x_0x_2x_3x_4 \oplus x_0x_2x_3x_4x_5 \oplus x_0x_2x_3x_4x_5x_6 \oplus x_0x_2x_3x_4x_5x_6x_7 \oplus$
$x_0x_2x_3x_4x_5x_6x_7x_8 \oplus x_0x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_2x_3x_4x_5x_6x_8x_9 \oplus x_0x_2x_3x_4x_5x_6x_9 \oplus x_0x_2x_3x_4x_5x_7x_8 \oplus x_0x_2x_3x_4x_5x_7x_9 \oplus$
$x_0x_2x_3x_4x_5x_8 \oplus x_0x_2x_3x_4x_5x_8x_9 \oplus x_0x_2x_3x_4x_6x_7 \oplus x_0x_2x_3x_4x_6x_7x_8x_9 \oplus x_0x_2x_3x_4x_6x_7x_9 \oplus x_0x_2x_3x_4x_6x_8 \oplus$
$x_0x_2x_3x_4x_6x_9 \oplus x_0x_2x_3x_4x_7 \oplus x_0x_2x_3x_4x_7x_8 \oplus x_0x_2x_3x_4x_7x_8x_9 \oplus x_0x_2x_3x_4x_8x_9 \oplus x_0x_2x_3x_4x_9 \oplus x_0x_2x_3x_5x_6 \oplus$
$x_0x_2x_3x_5x_6x_7x_8 \oplus x_0x_2x_3x_5x_6x_7x_9 \oplus x_0x_2x_3x_5x_6x_8 \oplus x_0x_2x_3x_5x_6x_8x_9 \oplus x_0x_2x_3x_5x_7 \oplus x_0x_2x_3x_5x_7x_8x_9 \oplus$
$x_0x_2x_3x_5x_7x_9 \oplus x_0x_2x_3x_5x_8 \oplus x_0x_2x_3x_5x_9 \oplus x_0x_2x_3x_6 \oplus x_0x_2x_3x_6x_7 \oplus x_0x_2x_3x_6x_7x_8 \oplus x_0x_2x_3x_6x_7x_8x_9 \oplus$
$x_0x_2x_3x_6x_8x_9 \oplus x_0x_2x_3x_6x_9 \oplus x_0x_2x_3x_7x_8 \oplus x_0x_2x_3x_7x_9 \oplus x_0x_2x_3x_8 \oplus x_0x_2x_3x_8x_9 \oplus x_0x_2x_4x_5 \oplus x_0x_2x_4x_5x_6x_7 \oplus$
$x_0x_2x_4x_5x_6x_7x_8x_9 \oplus x_0x_2x_4x_5x_6x_7x_9 \oplus x_0x_2x_4x_5x_6x_8 \oplus x_0x_2x_4x_5x_6x_9 \oplus x_0x_2x_4x_5x_7 \oplus x_0x_2x_4x_5x_7x_8 \oplus$
$x_0x_2x_4x_5x_7x_8x_9 \oplus x_0x_2x_4x_5x_8x_9 \oplus x_0x_2x_4x_5x_9 \oplus x_0x_2x_4x_6 \oplus x_0x_2x_4x_6x_7x_8 \oplus x_0x_2x_4x_6x_7x_9 \oplus x_0x_2x_4x_6x_8 \oplus$
$x_0x_2x_4x_6x_8x_9 \oplus x_0x_2x_4x_7 \oplus x_0x_2x_4x_7x_8x_9 \oplus x_0x_2x_4x_7x_9 \oplus x_0x_2x_4x_8 \oplus x_0x_2x_4x_9 \oplus x_0x_2x_5 \oplus x_0x_2x_5x_6 \oplus$
$x_0x_2x_5x_6x_7 \oplus x_0x_2x_5x_6x_7x_8 \oplus x_0x_2x_5x_6x_7x_8x_9 \oplus x_0x_2x_5x_6x_8x_9 \oplus x_0x_2x_5x_6x_9 \oplus x_0x_2x_5x_7x_8 \oplus x_0x_2x_5x_7x_9 \oplus$
$x_0x_2x_5x_8 \oplus x_0x_2x_5x_8x_9 \oplus x_0x_2x_6x_7 \oplus x_0x_2x_6x_7x_8x_9 \oplus x_0x_2x_6x_7x_9 \oplus x_0x_2x_6x_8 \oplus x_0x_2x_6x_9 \oplus x_0x_2x_7 \oplus$
$x_0x_2x_7x_8 \oplus x_0x_2x_7x_8x_9 \oplus x_0x_2x_8x_9 \oplus x_0x_2x_9 \oplus x_0x_3x_4 \oplus x_0x_3x_4x_5x_6 \oplus x_0x_3x_4x_5x_6x_7x_8 \oplus x_0x_3x_4x_5x_6x_7x_9 \oplus$
$x_0x_3x_4x_5x_6x_8 \oplus x_0x_3x_4x_5x_6x_8x_9 \oplus x_0x_3x_4x_5x_7 \oplus x_0x_3x_4x_5x_7x_8x_9 \oplus x_0x_3x_4x_5x_7x_9 \oplus x_0x_3x_4x_5x_8 \oplus x_0x_3x_4x_5x_9 \oplus$
$x_0x_3x_4x_6 \oplus x_0x_3x_4x_6x_7 \oplus x_0x_3x_4x_6x_7x_8 \oplus x_0x_3x_4x_6x_7x_8x_9 \oplus x_0x_3x_4x_6x_8x_9 \oplus x_0x_3x_4x_6x_9 \oplus x_0x_3x_4x_7x_8 \oplus$
$x_0x_3x_4x_7x_9 \oplus x_0x_3x_4x_8 \oplus x_0x_3x_4x_8x_9 \oplus x_0x_3x_5 \oplus x_0x_3x_5x_6x_7 \oplus x_0x_3x_5x_6x_7x_8x_9 \oplus x_0x_3x_5x_6x_7x_9 \oplus x_0x_3x_5x_6x_8 \oplus$
$x_0x_3x_5x_6x_9 \oplus x_0x_3x_5x_7 \oplus x_0x_3x_5x_7x_8 \oplus x_0x_3x_5x_7x_8x_9 \oplus x_0x_3x_5x_8x_9 \oplus x_0x_3x_5x_9 \oplus x_0x_3x_6 \oplus x_0x_3x_6x_7x_8 \oplus$
$x_0x_3x_6x_7x_9 \oplus x_0x_3x_6x_8 \oplus x_0x_3x_6x_8x_9 \oplus x_0x_3x_7 \oplus x_0x_3x_7x_8x_9 \oplus x_0x_3x_7x_9 \oplus x_0x_3x_8 \oplus x_0x_3x_9 \oplus x_0x_4 \oplus x_0x_4x_5 \oplus$
$x_0x_4x_5x_6 \oplus x_0x_4x_5x_6x_7 \oplus x_0x_4x_5x_6x_7x_8 \oplus x_0x_4x_5x_6x_7x_8x_9 \oplus x_0x_4x_5x_6x_8x_9 \oplus x_0x_4x_5x_6x_9 \oplus x_0x_4x_5x_7x_8 \oplus$
$x_0x_4x_5x_7x_9 \oplus x_0x_4x_5x_8 \oplus x_0x_4x_5x_8x_9 \oplus x_0x_4x_6x_7 \oplus x_0x_4x_6x_7x_8x_9 \oplus x_0x_4x_6x_7x_9 \oplus x_0x_4x_6x_8 \oplus x_0x_4x_6x_9 \oplus$
$x_0x_4x_7 \oplus x_0x_4x_7x_8 \oplus x_0x_4x_7x_8x_9 \oplus x_0x_4x_8x_9 \oplus x_0x_4x_9 \oplus x_0x_5x_6 \oplus x_0x_5x_6x_7x_8 \oplus x_0x_5x_6x_7x_9 \oplus x_0x_5x_6x_8 \oplus$
$x_0x_5x_6x_8x_9 \oplus x_0x_5x_7 \oplus x_0x_5x_7x_8x_9 \oplus x_0x_5x_7x_9 \oplus x_0x_5x_8 \oplus x_0x_5x_9 \oplus x_0x_6 \oplus x_0x_6x_7 \oplus x_0x_6x_7x_8 \oplus x_0x_6x_7x_8x_9 \oplus$
$x_0x_6x_8x_9 \oplus x_0x_6x_9 \oplus x_0x_7x_8 \oplus x_0x_7x_9 \oplus x_0x_8 \oplus x_0x_8x_9 \oplus x_1 \oplus x_1x_2x_3 \oplus x_1x_2x_3x_4x_5 \oplus x_1x_2x_3x_4x_5x_6x_7 \oplus$
$x_1x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_1x_2x_3x_4x_5x_6x_7x_9 \oplus x_1x_2x_3x_4x_5x_6x_8 \oplus x_1x_2x_3x_4x_5x_6x_9 \oplus x_1x_2x_3x_4x_5x_7 \oplus x_1x_2x_3x_4x_5x_7x_8 \oplus$
$x_1x_2x_3x_4x_5x_7x_8x_9 \oplus x_1x_2x_3x_4x_5x_8x_9 \oplus x_1x_2x_3x_4x_5x_9 \oplus x_1x_2x_3x_4x_6 \oplus x_1x_2x_3x_4x_6x_7x_8 \oplus x_1x_2x_3x_4x_6x_7x_9 \oplus$
$x_1x_2x_3x_4x_6x_8 \oplus x_1x_2x_3x_4x_6x_8x_9 \oplus x_1x_2x_3x_4x_7 \oplus x_1x_2x_3x_4x_7x_8x_9 \oplus x_1x_2x_3x_4x_7x_9 \oplus x_1x_2x_3x_4x_8 \oplus x_1x_2x_3x_4x_9 \oplus$
$x_1x_2x_3x_5 \oplus x_1x_2x_3x_5x_6 \oplus x_1x_2x_3x_5x_6x_7 \oplus x_1x_2x_3x_5x_6x_7x_8 \oplus x_1x_2x_3x_5x_6x_7x_8x_9 \oplus x_1x_2x_3x_5x_6x_8x_9 \oplus x_1x_2x_3x_5x_6x_9 \oplus$
$x_1x_2x_3x_5x_7x_8 \oplus x_1x_2x_3x_5x_7x_9 \oplus x_1x_2x_3x_5x_8 \oplus x_1x_2x_3x_5x_8x_9 \oplus x_1x_2x_3x_6x_7 \oplus x_1x_2x_3x_6x_7x_8x_9 \oplus x_1x_2x_3x_6x_7x_9 \oplus$
$x_1x_2x_3x_6x_8 \oplus x_1x_2x_3x_6x_9 \oplus x_1x_2x_3x_7 \oplus x_1x_2x_3x_7x_8 \oplus x_1x_2x_3x_7x_8x_9 \oplus x_1x_2x_3x_8x_9 \oplus x_1x_2x_3x_9 \oplus x_1x_2x_4 \oplus$
$x_1x_2x_4x_5x_6 \oplus x_1x_2x_4x_5x_6x_7x_8 \oplus x_1x_2x_4x_5x_6x_7x_9 \oplus x_1x_2x_4x_5x_6x_8 \oplus x_1x_2x_4x_5x_6x_8x_9 \oplus x_1x_2x_4x_5x_7 \oplus x_1x_2x_4x_5x_7x_8x_9 \oplus$
$x_1x_2x_4x_5x_7x_9 \oplus x_1x_2x_4x_5x_8 \oplus x_1x_2x_4x_5x_9 \oplus x_1x_2x_4x_6 \oplus x_1x_2x_4x_6x_7 \oplus x_1x_2x_4x_6x_7x_8 \oplus x_1x_2x_4x_6x_7x_8x_9 \oplus$
$x_1x_2x_4x_6x_8x_9 \oplus x_1x_2x_4x_6x_9 \oplus x_1x_2x_4x_7x_8 \oplus x_1x_2x_4x_7x_9 \oplus x_1x_2x_4x_8 \oplus x_1x_2x_4x_8x_9 \oplus x_1x_2x_5 \oplus x_1x_2x_5x_6x_7 \oplus$
$x_1x_2x_5x_6x_7x_8x_9 \oplus x_1x_2x_5x_6x_7x_9 \oplus x_1x_2x_5x_6x_8 \oplus x_1x_2x_5x_6x_9 \oplus x_1x_2x_5x_7 \oplus x_1x_2x_5x_7x_8 \oplus x_1x_2x_5x_7x_8x_9 \oplus$
$x_1x_2x_5x_8x_9 \oplus x_1x_2x_5x_9 \oplus x_1x_2x_6 \oplus x_1x_2x_6x_7x_8 \oplus x_1x_2x_6x_7x_9 \oplus x_1x_2x_6x_8 \oplus x_1x_2x_6x_8x_9 \oplus x_1x_2x_7 \oplus x_1x_2x_7x_8x_9 \oplus$
$x_1x_2x_7x_9 \oplus x_1x_2x_8 \oplus x_1x_2x_9 \oplus x_1x_3 \oplus x_1x_3x_4 \oplus x_1x_3x_4x_5 \oplus x_1x_3x_4x_5x_6 \oplus x_1x_3x_4x_5x_6x_7 \oplus x_1x_3x_4x_5x_6x_7x_8 \oplus$
$x_1x_3x_4x_5x_6x_7x_8x_9 \oplus x_1x_3x_4x_5x_6x_8x_9 \oplus x_1x_3x_4x_5x_6x_9 \oplus x_1x_3x_4x_5x_7x_8 \oplus x_1x_3x_4x_5x_7x_9 \oplus x_1x_3x_4x_5x_8 \oplus$
$x_1x_3x_4x_5x_8x_9 \oplus x_1x_3x_4x_6x_7 \oplus x_1x_3x_4x_6x_7x_8x_9 \oplus x_1x_3x_4x_6x_7x_9 \oplus x_1x_3x_4x_6x_8 \oplus x_1x_3x_4x_6x_9 \oplus x_1x_3x_4x_7 \oplus$
$x_1x_3x_4x_7x_8 \oplus x_1x_3x_4x_7x_8x_9 \oplus x_1x_3x_4x_8x_9 \oplus x_1x_3x_4x_9 \oplus x_1x_3x_5x_6 \oplus x_1x_3x_5x_6x_7x_8 \oplus x_1x_3x_5x_6x_7x_9 \oplus x_1x_3x_5x_6x_8 \oplus$
$x_1x_3x_5x_6x_8x_9 \oplus x_1x_3x_5x_7 \oplus x_1x_3x_5x_7x_8x_9 \oplus x_1x_3x_5x_7x_9 \oplus x_1x_3x_5x_8 \oplus x_1x_3x_5x_9 \oplus x_1x_3x_6 \oplus x_1x_3x_6x_7 \oplus$
$x_1x_3x_6x_7x_8 \oplus x_1x_3x_6x_7x_8x_9 \oplus x_1x_3x_6x_8x_9 \oplus x_1x_3x_6x_9 \oplus x_1x_3x_7x_8 \oplus x_1x_3x_7x_9 \oplus x_1x_3x_8 \oplus x_1x_3x_8x_9 \oplus x_1x_4x_5 \oplus$
$x_1x_4x_5x_6x_7 \oplus x_1x_4x_5x_6x_7x_8x_9 \oplus x_1x_4x_5x_6x_7x_9 \oplus x_1x_4x_5x_6x_8 \oplus x_1x_4x_5x_6x_9 \oplus x_1x_4x_5x_7 \oplus x_1x_4x_5x_7x_8 \oplus$
$x_1x_4x_5x_7x_8x_9 \oplus x_1x_4x_5x_8x_9 \oplus x_1x_4x_5x_9 \oplus x_1x_4x_6 \oplus x_1x_4x_6x_7x_8 \oplus x_1x_4x_6x_7x_9 \oplus x_1x_4x_6x_8 \oplus x_1x_4x_6x_8x_9 \oplus$
$x_1x_4x_7 \oplus x_1x_4x_7x_8x_9 \oplus x_1x_4x_7x_9 \oplus x_1x_4x_8 \oplus x_1x_4x_9 \oplus x_1x_5 \oplus x_1x_5x_6 \oplus x_1x_5x_6x_7 \oplus x_1x_5x_6x_7x_8 \oplus x_1x_5x_6x_7x_8x_9 \oplus$
$x_1x_5x_6x_8x_9 \oplus x_1x_5x_6x_9 \oplus x_1x_5x_7x_8 \oplus x_1x_5x_7x_9 \oplus x_1x_5x_8 \oplus x_1x_5x_8x_9 \oplus x_1x_6x_7 \oplus x_1x_6x_7x_8x_9 \oplus x_1x_6x_7x_9 \oplus$
$x_1x_6x_8 \oplus x_1x_6x_9 \oplus x_1x_7 \oplus x_1x_7x_8 \oplus x_1x_7x_8x_9 \oplus x_1x_8x_9 \oplus x_1x_9 \oplus x_2 \oplus x_2x_3x_4 \oplus x_2x_3x_4x_5x_6 \oplus x_2x_3x_4x_5x_6x_7x_8 \oplus$
$x_2x_3x_4x_5x_6x_7x_9 \oplus x_2x_3x_4x_5x_6x_8 \oplus x_2x_3x_4x_5x_6x_8x_9 \oplus x_2x_3x_4x_5x_7 \oplus x_2x_3x_4x_5x_7x_8x_9 \oplus x_2x_3x_4x_5x_7x_9 \oplus$
$x_2x_3x_4x_5x_8 \oplus x_2x_3x_4x_5x_9 \oplus x_2x_3x_4x_6 \oplus x_2x_3x_4x_6x_7 \oplus x_2x_3x_4x_6x_7x_8 \oplus x_2x_3x_4x_6x_7x_8x_9 \oplus x_2x_3x_4x_6x_8x_9 \oplus$
$x_2x_3x_4x_6x_9 \oplus x_2x_3x_4x_7x_8 \oplus x_2x_3x_4x_7x_9 \oplus x_2x_3x_4x_8 \oplus x_2x_3x_4x_8x_9 \oplus x_2x_3x_5 \oplus x_2x_3x_5x_6x_7 \oplus x_2x_3x_5x_6x_7x_8x_9 \oplus$
$x_2x_3x_5x_6x_7x_9 \oplus x_2x_3x_5x_6x_8 \oplus x_2x_3x_5x_6x_9 \oplus x_2x_3x_5x_7 \oplus x_2x_3x_5x_7x_8 \oplus x_2x_3x_5x_7x_8x_9 \oplus x_2x_3x_5x_8x_9 \oplus x_2x_3x_5x_9 \oplus$
$x_2x_3x_6 \oplus x_2x_3x_6x_7x_8 \oplus x_2x_3x_6x_7x_9 \oplus x_2x_3x_6x_8 \oplus x_2x_3x_6x_8x_9 \oplus x_2x_3x_7 \oplus x_2x_3x_7x_8x_9 \oplus x_2x_3x_7x_9 \oplus x_2x_3x_8 \oplus$

$x_2x_3x_9 \oplus x_2x_4 \oplus x_2x_4x_5 \oplus x_2x_4x_5x_6 \oplus x_2x_4x_5x_6x_7 \oplus x_2x_4x_5x_6x_7x_8 \oplus x_2x_4x_5x_6x_7x_8x_9 \oplus x_2x_4x_5x_6x_8x_9 \oplus$
$x_2x_4x_5x_6x_9 \oplus x_2x_4x_5x_7x_8 \oplus x_2x_4x_5x_7x_9 \oplus x_2x_4x_5x_8 \oplus x_2x_4x_5x_8x_9 \oplus x_2x_4x_6x_7 \oplus x_2x_4x_6x_7x_8x_9 \oplus x_2x_4x_6x_7x_9 \oplus$
$x_2x_4x_6x_8 \oplus x_2x_4x_6x_9 \oplus x_2x_4x_7 \oplus x_2x_4x_7x_8 \oplus x_2x_4x_7x_8x_9 \oplus x_2x_4x_8x_9 \oplus x_2x_4x_9 \oplus x_2x_5x_6 \oplus x_2x_5x_6x_7x_8 \oplus$
$x_2x_5x_6x_7x_9 \oplus x_2x_5x_6x_8 \oplus x_2x_5x_6x_8x_9 \oplus x_2x_5x_7 \oplus x_2x_5x_7x_8x_9 \oplus x_2x_5x_7x_9 \oplus x_2x_5x_8 \oplus x_2x_5x_9 \oplus x_2x_6 \oplus x_2x_6x_7 \oplus$
$x_2x_6x_7x_8 \oplus x_2x_6x_7x_8x_9 \oplus x_2x_6x_8x_9 \oplus x_2x_6x_9 \oplus x_2x_7x_8 \oplus x_2x_7x_9 \oplus x_2x_8 \oplus x_2x_8x_9 \oplus x_3 \oplus x_3x_4x_5 \oplus x_3x_4x_5x_6x_7 \oplus$
$x_3x_4x_5x_6x_7x_8x_9 \oplus x_3x_4x_5x_6x_7x_9 \oplus x_3x_4x_5x_6x_8 \oplus x_3x_4x_5x_6x_9 \oplus x_3x_4x_5x_7 \oplus x_3x_4x_5x_7x_8 \oplus x_3x_4x_5x_7x_8x_9 \oplus$
$x_3x_4x_5x_8x_9 \oplus x_3x_4x_5x_9 \oplus x_3x_4x_6 \oplus x_3x_4x_6x_7x_8 \oplus x_3x_4x_6x_7x_9 \oplus x_3x_4x_6x_8 \oplus x_3x_4x_6x_8x_9 \oplus x_3x_4x_7 \oplus x_3x_4x_7x_8x_9 \oplus$
$x_3x_4x_7x_9 \oplus x_3x_4x_8 \oplus x_3x_4x_9 \oplus x_3x_5 \oplus x_3x_5x_6 \oplus x_3x_5x_6x_7 \oplus x_3x_5x_6x_7x_8 \oplus x_3x_5x_6x_7x_8x_9 \oplus x_3x_5x_6x_8x_9 \oplus$
$x_3x_5x_6x_9 \oplus x_3x_5x_7x_8 \oplus x_3x_5x_7x_9 \oplus x_3x_5x_8 \oplus x_3x_5x_8x_9 \oplus x_3x_6x_7 \oplus x_3x_6x_7x_8x_9 \oplus x_3x_6x_7x_9 \oplus x_3x_6x_8 \oplus x_3x_6x_9 \oplus$
$x_3x_7 \oplus x_3x_7x_8 \oplus x_3x_7x_8x_9 \oplus x_3x_8x_9 \oplus x_3x_9 \oplus x_4 \oplus x_4x_5x_6 \oplus x_4x_5x_6x_7x_8 \oplus x_4x_5x_6x_7x_9 \oplus x_4x_5x_6x_8 \oplus x_4x_5x_6x_8x_9 \oplus$
$x_4x_5x_7 \oplus x_4x_5x_7x_8x_9 \oplus x_4x_5x_7x_9 \oplus x_4x_5x_8 \oplus x_4x_5x_9 \oplus x_4x_6 \oplus x_4x_6x_7 \oplus x_4x_6x_7x_8 \oplus x_4x_6x_7x_8x_9 \oplus x_4x_6x_8x_9 \oplus$
$x_4x_6x_9 \oplus x_4x_7x_8 \oplus x_4x_7x_9 \oplus x_4x_8 \oplus x_4x_8x_9 \oplus x_5 \oplus x_5x_6x_7 \oplus x_5x_6x_7x_8x_9 \oplus x_5x_6x_7x_9 \oplus x_5x_6x_8 \oplus x_5x_6x_9 \oplus x_5x_7 \oplus$
$x_5x_7x_8 \oplus x_5x_7x_8x_9 \oplus x_5x_8x_9 \oplus x_5x_9 \oplus x_6 \oplus x_6x_7x_8 \oplus x_6x_7x_9 \oplus x_6x_8 \oplus x_6x_8x_9 \oplus x_7 \oplus x_7x_8x_9 \oplus x_7x_9 \oplus x_8 \oplus x_9 = 1$

$x_0x_1 \oplus x_0x_1x_2x_3 \oplus x_0x_1x_2x_3x_4x_5 \oplus x_0x_1x_2x_3x_4x_5x_6x_7 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_9 \oplus$
$x_0x_1x_2x_3x_4x_5x_6x_8 \oplus x_0x_1x_2x_3x_4x_5x_6x_9 \oplus x_0x_1x_2x_3x_4x_5x_7 \oplus x_0x_1x_2x_3x_4x_5x_7x_8 \oplus x_0x_1x_2x_3x_4x_5x_7x_8x_9 \oplus$
$x_0x_1x_2x_3x_4x_5x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_9 \oplus x_0x_1x_2x_3x_4x_6 \oplus x_0x_1x_2x_3x_4x_6x_7x_8 \oplus x_0x_1x_2x_3x_4x_6x_7x_9 \oplus x_0x_1x_2x_3x_4x_6x_8 \oplus$
$x_0x_1x_2x_3x_4x_6x_8x_9 \oplus x_0x_1x_2x_3x_4x_7 \oplus x_0x_1x_2x_3x_4x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_7x_9 \oplus x_0x_1x_2x_3x_4x_8 \oplus x_0x_1x_2x_3x_4x_9 \oplus$
$x_0x_1x_2x_3x_5 \oplus x_0x_1x_2x_3x_5x_6 \oplus x_0x_1x_2x_3x_5x_6x_7 \oplus x_0x_1x_2x_3x_5x_6x_7x_8 \oplus x_0x_1x_2x_3x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_5x_6x_8x_9 \oplus$
$x_0x_1x_2x_3x_5x_6x_9 \oplus x_0x_1x_2x_3x_5x_7x_8 \oplus x_0x_1x_2x_3x_5x_7x_9 \oplus x_0x_1x_2x_3x_5x_8 \oplus x_0x_1x_2x_3x_5x_8x_9 \oplus x_0x_1x_2x_3x_6x_7 \oplus$
$x_0x_1x_2x_3x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_6x_7x_9 \oplus x_0x_1x_2x_3x_6x_8 \oplus x_0x_1x_2x_3x_6x_9 \oplus x_0x_1x_2x_3x_7 \oplus x_0x_1x_2x_3x_7x_8 \oplus$
$x_0x_1x_2x_3x_7x_8x_9 \oplus x_0x_1x_2x_3x_8x_9 \oplus x_0x_1x_2x_3x_9 \oplus x_0x_1x_2x_4 \oplus x_0x_1x_2x_4x_5x_6 \oplus x_0x_1x_2x_4x_5x_6x_7x_8 \oplus x_0x_1x_2x_4x_5x_6x_7x_9 \oplus$
$x_0x_1x_2x_4x_5x_6x_8 \oplus x_0x_1x_2x_4x_5x_6x_8x_9 \oplus x_0x_1x_2x_4x_5x_7 \oplus x_0x_1x_2x_4x_5x_7x_8x_9 \oplus x_0x_1x_2x_4x_5x_7x_9 \oplus x_0x_1x_2x_4x_5x_8 \oplus$
$x_0x_1x_2x_4x_5x_9 \oplus x_0x_1x_2x_4x_6 \oplus x_0x_1x_2x_4x_6x_7 \oplus x_0x_1x_2x_4x_6x_7x_8 \oplus x_0x_1x_2x_4x_6x_7x_8x_9 \oplus x_0x_1x_2x_4x_6x_8x_9 \oplus$
$x_0x_1x_2x_4x_6x_9 \oplus x_0x_1x_2x_4x_7x_8 \oplus x_0x_1x_2x_4x_7x_9 \oplus x_0x_1x_2x_4x_8 \oplus x_0x_1x_2x_4x_8x_9 \oplus x_0x_1x_2x_5 \oplus x_0x_1x_2x_5x_6x_7 \oplus$
$x_0x_1x_2x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_5x_6x_7x_9 \oplus x_0x_1x_2x_5x_6x_8 \oplus x_0x_1x_2x_5x_6x_9 \oplus x_0x_1x_2x_5x_7 \oplus x_0x_1x_2x_5x_7x_8 \oplus$
$x_0x_1x_2x_5x_7x_8x_9 \oplus x_0x_1x_2x_5x_8x_9 \oplus x_0x_1x_2x_5x_9 \oplus x_0x_1x_2x_6 \oplus x_0x_1x_2x_6x_7x_8 \oplus x_0x_1x_2x_6x_7x_9 \oplus x_0x_1x_2x_6x_8 \oplus$
$x_0x_1x_2x_6x_8x_9 \oplus x_0x_1x_2x_7 \oplus x_0x_1x_2x_7x_8x_9 \oplus x_0x_1x_2x_7x_9 \oplus x_0x_1x_2x_8 \oplus x_0x_1x_2x_9 \oplus x_0x_1x_3 \oplus x_0x_1x_3x_4 \oplus$
$x_0x_1x_3x_4x_5 \oplus x_0x_1x_3x_4x_5x_6 \oplus x_0x_1x_3x_4x_5x_6x_7 \oplus x_0x_1x_3x_4x_5x_6x_7x_8 \oplus x_0x_1x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_3x_4x_5x_6x_8x_9 \oplus$
$x_0x_1x_3x_4x_5x_6x_9 \oplus x_0x_1x_3x_4x_5x_7x_8 \oplus x_0x_1x_3x_4x_5x_7x_9 \oplus x_0x_1x_3x_4x_5x_8 \oplus x_0x_1x_3x_4x_5x_8x_9 \oplus x_0x_1x_3x_4x_6x_7 \oplus$
$x_0x_1x_3x_4x_6x_7x_8x_9 \oplus x_0x_1x_3x_4x_6x_7x_9 \oplus x_0x_1x_3x_4x_6x_8 \oplus x_0x_1x_3x_4x_6x_9 \oplus x_0x_1x_3x_4x_7 \oplus x_0x_1x_3x_4x_7x_8 \oplus$
$x_0x_1x_3x_4x_7x_8x_9 \oplus x_0x_1x_3x_4x_8x_9 \oplus x_0x_1x_3x_4x_9 \oplus x_0x_1x_3x_5x_6 \oplus x_0x_1x_3x_5x_6x_7x_8 \oplus x_0x_1x_3x_5x_6x_7x_9 \oplus x_0x_1x_3x_5x_6x_8 \oplus$
$x_0x_1x_3x_5x_6x_8x_9 \oplus x_0x_1x_3x_5x_7 \oplus x_0x_1x_3x_5x_7x_8x_9 \oplus x_0x_1x_3x_5x_7x_9 \oplus x_0x_1x_3x_5x_8 \oplus x_0x_1x_3x_5x_9 \oplus x_0x_1x_3x_6 \oplus$
$x_0x_1x_3x_6x_7 \oplus x_0x_1x_3x_6x_7x_8 \oplus x_0x_1x_3x_6x_7x_8x_9 \oplus x_0x_1x_3x_6x_8x_9 \oplus x_0x_1x_3x_6x_9 \oplus x_0x_1x_3x_7x_8 \oplus x_0x_1x_3x_7x_9 \oplus$
$x_0x_1x_3x_8 \oplus x_0x_1x_3x_8x_9 \oplus x_0x_1x_4x_5 \oplus x_0x_1x_4x_5x_6x_7 \oplus x_0x_1x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_4x_5x_6x_7x_9 \oplus x_0x_1x_4x_5x_6x_8 \oplus$
$x_0x_1x_4x_5x_6x_9 \oplus x_0x_1x_4x_5x_7 \oplus x_0x_1x_4x_5x_7x_8 \oplus x_0x_1x_4x_5x_7x_8x_9 \oplus x_0x_1x_4x_5x_8x_9 \oplus x_0x_1x_4x_5x_9 \oplus x_0x_1x_4x_6 \oplus$
$x_0x_1x_4x_6x_7x_8 \oplus x_0x_1x_4x_6x_7x_9 \oplus x_0x_1x_4x_6x_8 \oplus x_0x_1x_4x_6x_8x_9 \oplus x_0x_1x_4x_7 \oplus x_0x_1x_4x_7x_8x_9 \oplus x_0x_1x_4x_7x_9 \oplus$
$x_0x_1x_4x_8 \oplus x_0x_1x_4x_9 \oplus x_0x_1x_5 \oplus x_0x_1x_5x_6 \oplus x_0x_1x_5x_6x_7 \oplus x_0x_1x_5x_6x_7x_8 \oplus x_0x_1x_5x_6x_7x_8x_9 \oplus x_0x_1x_5x_6x_8x_9 \oplus$
$x_0x_1x_5x_6x_9 \oplus x_0x_1x_5x_7x_8 \oplus x_0x_1x_5x_7x_9 \oplus x_0x_1x_5x_8 \oplus x_0x_1x_5x_8x_9 \oplus x_0x_1x_6x_7 \oplus x_0x_1x_6x_7x_8x_9 \oplus x_0x_1x_6x_7x_9 \oplus$
$x_0x_1x_6x_8 \oplus x_0x_1x_6x_9 \oplus x_0x_1x_7 \oplus x_0x_1x_7x_8 \oplus x_0x_1x_7x_8x_9 \oplus x_0x_1x_8x_9 \oplus x_0x_1x_9 \oplus x_0x_2 \oplus x_0x_2x_3x_4 \oplus x_0x_2x_3x_4x_5x_6 \oplus$
$x_0x_2x_3x_4x_5x_6x_7x_8 \oplus x_0x_2x_3x_4x_5x_6x_7x_9 \oplus x_0x_2x_3x_4x_5x_6x_8 \oplus x_0x_2x_3x_4x_5x_6x_8x_9 \oplus x_0x_2x_3x_4x_5x_7 \oplus x_0x_2x_3x_4x_5x_7x_8x_9 \oplus$
$x_0x_2x_3x_4x_5x_7x_9 \oplus x_0x_2x_3x_4x_5x_8 \oplus x_0x_2x_3x_4x_5x_9 \oplus x_0x_2x_3x_4x_6 \oplus x_0x_2x_3x_4x_6x_7 \oplus x_0x_2x_3x_4x_6x_7x_8 \oplus x_0x_2x_3x_4x_6x_7x_8x_9 \oplus$
$x_0x_2x_3x_4x_6x_8x_9 \oplus x_0x_2x_3x_4x_6x_9 \oplus x_0x_2x_3x_4x_7x_8 \oplus x_0x_2x_3x_4x_7x_9 \oplus x_0x_2x_3x_4x_8 \oplus x_0x_2x_3x_4x_8x_9 \oplus x_0x_2x_3x_5 \oplus$
$x_0x_2x_3x_5x_6x_7 \oplus x_0x_2x_3x_5x_6x_7x_8x_9 \oplus x_0x_2x_3x_5x_6x_7x_9 \oplus x_0x_2x_3x_5x_6x_8 \oplus x_0x_2x_3x_5x_6x_9 \oplus x_0x_2x_3x_5x_7 \oplus$
$x_0x_2x_3x_5x_7x_8 \oplus x_0x_2x_3x_5x_7x_8x_9 \oplus x_0x_2x_3x_5x_8x_9 \oplus x_0x_2x_3x_5x_9 \oplus x_0x_2x_3x_6 \oplus x_0x_2x_3x_6x_7x_8 \oplus x_0x_2x_3x_6x_7x_9 \oplus$
$x_0x_2x_3x_6x_8 \oplus x_0x_2x_3x_6x_8x_9 \oplus x_0x_2x_3x_7 \oplus x_0x_2x_3x_7x_8x_9 \oplus x_0x_2x_3x_7x_9 \oplus x_0x_2x_3x_8 \oplus x_0x_2x_3x_9 \oplus x_0x_2x_4 \oplus$
$x_0x_2x_4x_5 \oplus x_0x_2x_4x_5x_6 \oplus x_0x_2x_4x_5x_6x_7 \oplus x_0x_2x_4x_5x_6x_7x_8 \oplus x_0x_2x_4x_5x_6x_7x_8x_9 \oplus x_0x_2x_4x_5x_6x_8x_9 \oplus x_0x_2x_4x_5x_6x_9 \oplus$
$x_0x_2x_4x_5x_7x_8 \oplus x_0x_2x_4x_5x_7x_9 \oplus x_0x_2x_4x_5x_8 \oplus x_0x_2x_4x_5x_8x_9 \oplus x_0x_2x_4x_6x_7 \oplus x_0x_2x_4x_6x_7x_8x_9 \oplus x_0x_2x_4x_6x_7x_9 \oplus$
$x_0x_2x_4x_6x_8 \oplus x_0x_2x_4x_6x_9 \oplus x_0x_2x_4x_7 \oplus x_0x_2x_4x_7x_8 \oplus x_0x_2x_4x_7x_8x_9 \oplus x_0x_2x_4x_8x_9 \oplus x_0x_2x_4x_9 \oplus x_0x_2x_5x_6 \oplus$
$x_0x_2x_5x_6x_7x_8 \oplus x_0x_2x_5x_6x_7x_9 \oplus x_0x_2x_5x_6x_8 \oplus x_0x_2x_5x_6x_8x_9 \oplus x_0x_2x_5x_7 \oplus x_0x_2x_5x_7x_8x_9 \oplus x_0x_2x_5x_7x_9 \oplus$
$x_0x_2x_5x_8 \oplus x_0x_2x_5x_9 \oplus x_0x_2x_6 \oplus x_0x_2x_6x_7 \oplus x_0x_2x_6x_7x_8 \oplus x_0x_2x_6x_7x_8x_9 \oplus x_0x_2x_6x_8x_9 \oplus x_0x_2x_6x_9 \oplus x_0x_2x_7x_8 \oplus$
$x_0x_2x_7x_9 \oplus x_0x_2x_8 \oplus x_0x_2x_8x_9 \oplus x_0x_3 \oplus x_0x_3x_4x_5 \oplus x_0x_3x_4x_5x_6x_7 \oplus x_0x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_3x_4x_5x_6x_7x_9 \oplus$

$x_0x_3x_4x_5x_6x_8 \oplus x_0x_3x_4x_5x_6x_9 \oplus x_0x_3x_4x_5x_7 \oplus x_0x_3x_4x_5x_7x_8 \oplus x_0x_3x_4x_5x_7x_8x_9 \oplus x_0x_3x_4x_5x_8x_9 \oplus x_0x_3x_4x_5x_9 \oplus$
$x_0x_3x_4x_6 \oplus x_0x_3x_4x_6x_7x_8 \oplus x_0x_3x_4x_6x_7x_9 \oplus x_0x_3x_4x_6x_8 \oplus x_0x_3x_4x_6x_8x_9 \oplus x_0x_3x_4x_7 \oplus x_0x_3x_4x_7x_8x_9 \oplus$
$x_0x_3x_4x_7x_9 \oplus x_0x_3x_4x_8 \oplus x_0x_3x_4x_9 \oplus x_0x_3x_5 \oplus x_0x_3x_5x_6 \oplus x_0x_3x_5x_6x_7 \oplus x_0x_3x_5x_6x_7x_8 \oplus x_0x_3x_5x_6x_7x_8x_9 \oplus$
$x_0x_3x_5x_6x_8x_9 \oplus x_0x_3x_5x_6x_9 \oplus x_0x_3x_5x_7x_8 \oplus x_0x_3x_5x_7x_9 \oplus x_0x_3x_5x_8 \oplus x_0x_3x_5x_8x_9 \oplus x_0x_3x_6x_7 \oplus x_0x_3x_6x_7x_8x_9 \oplus$
$x_0x_3x_6x_7x_9 \oplus x_0x_3x_6x_8 \oplus x_0x_3x_6x_9 \oplus x_0x_3x_7 \oplus x_0x_3x_7x_8 \oplus x_0x_3x_7x_8x_9 \oplus x_0x_3x_8x_9 \oplus x_0x_3x_9 \oplus x_0x_4 \oplus x_0x_4x_5x_6 \oplus$
$x_0x_4x_5x_6x_7x_8 \oplus x_0x_4x_5x_6x_7x_9 \oplus x_0x_4x_5x_6x_8 \oplus x_0x_4x_5x_6x_8x_9 \oplus x_0x_4x_5x_7 \oplus x_0x_4x_5x_7x_8x_9 \oplus x_0x_4x_5x_7x_9 \oplus$
$x_0x_4x_5x_8 \oplus x_0x_4x_5x_9 \oplus x_0x_4x_6 \oplus x_0x_4x_6x_7 \oplus x_0x_4x_6x_7x_8 \oplus x_0x_4x_6x_7x_8x_9 \oplus x_0x_4x_6x_8x_9 \oplus x_0x_4x_6x_9 \oplus x_0x_4x_7x_8 \oplus$
$x_0x_4x_7x_9 \oplus x_0x_4x_8 \oplus x_0x_4x_8x_9 \oplus x_0x_5 \oplus x_0x_5x_6x_7 \oplus x_0x_5x_6x_7x_8x_9 \oplus x_0x_5x_6x_7x_9 \oplus x_0x_5x_6x_8 \oplus x_0x_5x_6x_9 \oplus$
$x_0x_5x_7 \oplus x_0x_5x_7x_8 \oplus x_0x_5x_7x_8x_9 \oplus x_0x_5x_8x_9 \oplus x_0x_5x_9 \oplus x_0x_6 \oplus x_0x_6x_7x_8 \oplus x_0x_6x_7x_9 \oplus x_0x_6x_8 \oplus x_0x_6x_8x_9 \oplus$
$x_0x_7 \oplus x_0x_7x_8x_9 \oplus x_0x_7x_9 \oplus x_0x_8 \oplus x_0x_9 \oplus x_1 \oplus x_1x_2 \oplus x_1x_2x_3 \oplus x_1x_2x_3x_4 \oplus x_1x_2x_3x_4x_5 \oplus x_1x_2x_3x_4x_5x_6 \oplus$
$x_1x_2x_3x_4x_5x_6x_7 \oplus x_1x_2x_3x_4x_5x_6x_7x_8 \oplus x_1x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_1x_2x_3x_4x_5x_6x_8x_9 \oplus x_1x_2x_3x_4x_5x_6x_9 \oplus x_1x_2x_3x_4x_5x_7x_8 \oplus$
$x_1x_2x_3x_4x_5x_7x_9 \oplus x_1x_2x_3x_4x_5x_8 \oplus x_1x_2x_3x_4x_5x_8x_9 \oplus x_1x_2x_3x_4x_6x_7 \oplus x_1x_2x_3x_4x_6x_7x_8x_9 \oplus x_1x_2x_3x_4x_6x_7x_9 \oplus$
$x_1x_2x_3x_4x_6x_8 \oplus x_1x_2x_3x_4x_6x_9 \oplus x_1x_2x_3x_4x_7 \oplus x_1x_2x_3x_4x_7x_8 \oplus x_1x_2x_3x_4x_7x_8x_9 \oplus x_1x_2x_3x_4x_8x_9 \oplus x_1x_2x_3x_4x_9 \oplus$
$x_1x_2x_3x_5x_6 \oplus x_1x_2x_3x_5x_6x_7x_8 \oplus x_1x_2x_3x_5x_6x_7x_9 \oplus x_1x_2x_3x_5x_6x_8 \oplus x_1x_2x_3x_5x_6x_8x_9 \oplus x_1x_2x_3x_5x_7 \oplus x_1x_2x_3x_5x_7x_8x_9 \oplus$
$x_1x_2x_3x_5x_7x_9 \oplus x_1x_2x_3x_5x_8 \oplus x_1x_2x_3x_5x_9 \oplus x_1x_2x_3x_6 \oplus x_1x_2x_3x_6x_7 \oplus x_1x_2x_3x_6x_7x_8 \oplus x_1x_2x_3x_6x_7x_8x_9 \oplus$
$x_1x_2x_3x_6x_8x_9 \oplus x_1x_2x_3x_6x_9 \oplus x_1x_2x_3x_7x_8 \oplus x_1x_2x_3x_7x_9 \oplus x_1x_2x_3x_8 \oplus x_1x_2x_3x_8x_9 \oplus x_1x_2x_4x_5 \oplus x_1x_2x_4x_5x_6x_7 \oplus$
$x_1x_2x_4x_5x_6x_7x_8x_9 \oplus x_1x_2x_4x_5x_6x_7x_9 \oplus x_1x_2x_4x_5x_6x_8 \oplus x_1x_2x_4x_5x_6x_9 \oplus x_1x_2x_4x_5x_7 \oplus x_1x_2x_4x_5x_7x_8 \oplus$
$x_1x_2x_4x_5x_7x_8x_9 \oplus x_1x_2x_4x_5x_8x_9 \oplus x_1x_2x_4x_5x_9 \oplus x_1x_2x_4x_6 \oplus x_1x_2x_4x_6x_7x_8 \oplus x_1x_2x_4x_6x_7x_9 \oplus x_1x_2x_4x_6x_8 \oplus$
$x_1x_2x_4x_6x_8x_9 \oplus x_1x_2x_4x_7 \oplus x_1x_2x_4x_7x_8x_9 \oplus x_1x_2x_4x_7x_9 \oplus x_1x_2x_4x_8 \oplus x_1x_2x_4x_9 \oplus x_1x_2x_5 \oplus x_1x_2x_5x_6 \oplus$
$x_1x_2x_5x_6x_7 \oplus x_1x_2x_5x_6x_7x_8 \oplus x_1x_2x_5x_6x_7x_8x_9 \oplus x_1x_2x_5x_6x_8x_9 \oplus x_1x_2x_5x_6x_9 \oplus x_1x_2x_5x_7x_8 \oplus x_1x_2x_5x_7x_9 \oplus$
$x_1x_2x_5x_8 \oplus x_1x_2x_5x_8x_9 \oplus x_1x_2x_6x_7 \oplus x_1x_2x_6x_7x_8x_9 \oplus x_1x_2x_6x_7x_9 \oplus x_1x_2x_6x_8 \oplus x_1x_2x_6x_9 \oplus x_1x_2x_7 \oplus$
$x_1x_2x_7x_8 \oplus x_1x_2x_7x_8x_9 \oplus x_1x_2x_8x_9 \oplus x_1x_2x_9 \oplus x_1x_3x_4 \oplus x_1x_3x_4x_5x_6 \oplus x_1x_3x_4x_5x_6x_7x_8 \oplus x_1x_3x_4x_5x_6x_7x_9 \oplus$
$x_1x_3x_4x_5x_6x_8 \oplus x_1x_3x_4x_5x_6x_8x_9 \oplus x_1x_3x_4x_5x_7 \oplus x_1x_3x_4x_5x_7x_8x_9 \oplus x_1x_3x_4x_5x_7x_9 \oplus x_1x_3x_4x_5x_8 \oplus x_1x_3x_4x_5x_9 \oplus$
$x_1x_3x_4x_6 \oplus x_1x_3x_4x_6x_7 \oplus x_1x_3x_4x_6x_7x_8 \oplus x_1x_3x_4x_6x_7x_8x_9 \oplus x_1x_3x_4x_6x_8x_9 \oplus x_1x_3x_4x_6x_9 \oplus x_1x_3x_4x_7x_8 \oplus$
$x_1x_3x_4x_7x_9 \oplus x_1x_3x_4x_8 \oplus x_1x_3x_4x_8x_9 \oplus x_1x_3x_5 \oplus x_1x_3x_5x_6x_7 \oplus x_1x_3x_5x_6x_7x_8x_9 \oplus x_1x_3x_5x_6x_7x_9 \oplus x_1x_3x_5x_6x_8 \oplus$
$x_1x_3x_5x_6x_9 \oplus x_1x_3x_5x_7 \oplus x_1x_3x_5x_7x_8 \oplus x_1x_3x_5x_7x_8x_9 \oplus x_1x_3x_5x_8x_9 \oplus x_1x_3x_5x_9 \oplus x_1x_3x_6 \oplus x_1x_3x_6x_7x_8 \oplus$
$x_1x_3x_6x_7x_9 \oplus x_1x_3x_6x_8 \oplus x_1x_3x_6x_8x_9 \oplus x_1x_3x_7 \oplus x_1x_3x_7x_8x_9 \oplus x_1x_3x_7x_9 \oplus x_1x_3x_8 \oplus x_1x_3x_9 \oplus x_1x_4 \oplus x_1x_4x_5 \oplus$
$x_1x_4x_5x_6 \oplus x_1x_4x_5x_6x_7 \oplus x_1x_4x_5x_6x_7x_8 \oplus x_1x_4x_5x_6x_7x_8x_9 \oplus x_1x_4x_5x_6x_8x_9 \oplus x_1x_4x_5x_6x_9 \oplus x_1x_4x_5x_7x_8 \oplus$
$x_1x_4x_5x_7x_9 \oplus x_1x_4x_5x_8 \oplus x_1x_4x_5x_8x_9 \oplus x_1x_4x_6x_7 \oplus x_1x_4x_6x_7x_8x_9 \oplus x_1x_4x_6x_7x_9 \oplus x_1x_4x_6x_8 \oplus x_1x_4x_6x_9 \oplus$
$x_1x_4x_7 \oplus x_1x_4x_7x_8 \oplus x_1x_4x_7x_8x_9 \oplus x_1x_4x_8x_9 \oplus x_1x_4x_9 \oplus x_1x_5x_6 \oplus x_1x_5x_6x_7x_8 \oplus x_1x_5x_6x_7x_9 \oplus x_1x_5x_6x_8 \oplus$
$x_1x_5x_6x_8x_9 \oplus x_1x_5x_7 \oplus x_1x_5x_7x_8x_9 \oplus x_1x_5x_7x_9 \oplus x_1x_5x_8 \oplus x_1x_5x_9 \oplus x_1x_6 \oplus x_1x_6x_7 \oplus x_1x_6x_7x_8 \oplus x_1x_6x_7x_8x_9 \oplus$
$x_1x_6x_8x_9 \oplus x_1x_6x_9 \oplus x_1x_7x_8 \oplus x_1x_7x_9 \oplus x_1x_8 \oplus x_1x_8x_9 \oplus x_2x_3 \oplus x_2x_3x_4x_5 \oplus x_2x_3x_4x_5x_6x_7 \oplus x_2x_3x_4x_5x_6x_7x_8x_9 \oplus$
$x_2x_3x_4x_5x_6x_7x_9 \oplus x_2x_3x_4x_5x_6x_8 \oplus x_2x_3x_4x_5x_6x_9 \oplus x_2x_3x_4x_5x_7 \oplus x_2x_3x_4x_5x_7x_8 \oplus x_2x_3x_4x_5x_7x_8x_9 \oplus x_2x_3x_4x_5x_8x_9 \oplus$
$x_2x_3x_4x_5x_9 \oplus x_2x_3x_4x_6 \oplus x_2x_3x_4x_6x_7x_8 \oplus x_2x_3x_4x_6x_7x_9 \oplus x_2x_3x_4x_6x_8 \oplus x_2x_3x_4x_6x_8x_9 \oplus x_2x_3x_4x_7 \oplus x_2x_3x_4x_7x_8x_9 \oplus$
$x_2x_3x_4x_7x_9 \oplus x_2x_3x_4x_8 \oplus x_2x_3x_4x_9 \oplus x_2x_3x_5 \oplus x_2x_3x_5x_6 \oplus x_2x_3x_5x_6x_7 \oplus x_2x_3x_5x_6x_7x_8 \oplus x_2x_3x_5x_6x_7x_8x_9 \oplus$
$x_2x_3x_5x_6x_8x_9 \oplus x_2x_3x_5x_6x_9 \oplus x_2x_3x_5x_7x_8 \oplus x_2x_3x_5x_7x_9 \oplus x_2x_3x_5x_8 \oplus x_2x_3x_5x_8x_9 \oplus x_2x_3x_6x_7 \oplus x_2x_3x_6x_7x_8x_9 \oplus$
$x_2x_3x_6x_7x_9 \oplus x_2x_3x_6x_8 \oplus x_2x_3x_6x_9 \oplus x_2x_3x_7 \oplus x_2x_3x_7x_8 \oplus x_2x_3x_7x_8x_9 \oplus x_2x_3x_8x_9 \oplus x_2x_3x_9 \oplus x_2x_4 \oplus x_2x_4x_5x_6 \oplus$
$x_2x_4x_5x_6x_7x_8 \oplus x_2x_4x_5x_6x_7x_9 \oplus x_2x_4x_5x_6x_8 \oplus x_2x_4x_5x_6x_8x_9 \oplus x_2x_4x_5x_7 \oplus x_2x_4x_5x_7x_8x_9 \oplus x_2x_4x_5x_7x_9 \oplus$
$x_2x_4x_5x_8 \oplus x_2x_4x_5x_9 \oplus x_2x_4x_6 \oplus x_2x_4x_6x_7 \oplus x_2x_4x_6x_7x_8 \oplus x_2x_4x_6x_7x_8x_9 \oplus x_2x_4x_6x_8x_9 \oplus x_2x_4x_6x_9 \oplus x_2x_4x_7x_8 \oplus$
$x_2x_4x_7x_9 \oplus x_2x_4x_8 \oplus x_2x_4x_8x_9 \oplus x_2x_5 \oplus x_2x_5x_6x_7 \oplus x_2x_5x_6x_7x_8x_9 \oplus x_2x_5x_6x_7x_9 \oplus x_2x_5x_6x_8 \oplus x_2x_5x_6x_9 \oplus$
$x_2x_5x_7 \oplus x_2x_5x_7x_8 \oplus x_2x_5x_7x_8x_9 \oplus x_2x_5x_8x_9 \oplus x_2x_5x_9 \oplus x_2x_6 \oplus x_2x_6x_7x_8 \oplus x_2x_6x_7x_9 \oplus x_2x_6x_8 \oplus x_2x_6x_8x_9 \oplus$
$x_2x_7 \oplus x_2x_7x_8x_9 \oplus x_2x_7x_9 \oplus x_2x_8 \oplus x_2x_9 \oplus x_3 \oplus x_3x_4 \oplus x_3x_4x_5 \oplus x_3x_4x_5x_6 \oplus x_3x_4x_5x_6x_7 \oplus x_3x_4x_5x_6x_7x_8 \oplus$
$x_3x_4x_5x_6x_7x_8x_9 \oplus x_3x_4x_5x_6x_8x_9 \oplus x_3x_4x_5x_6x_9 \oplus x_3x_4x_5x_7x_8 \oplus x_3x_4x_5x_7x_9 \oplus x_3x_4x_5x_8 \oplus x_3x_4x_5x_8x_9 \oplus$
$x_3x_4x_6x_7 \oplus x_3x_4x_6x_7x_8x_9 \oplus x_3x_4x_6x_7x_9 \oplus x_3x_4x_6x_8 \oplus x_3x_4x_6x_9 \oplus x_3x_4x_7 \oplus x_3x_4x_7x_8 \oplus x_3x_4x_7x_8x_9 \oplus x_3x_4x_8x_9 \oplus$
$x_3x_4x_9 \oplus x_3x_5x_6 \oplus x_3x_5x_6x_7x_8 \oplus x_3x_5x_6x_7x_9 \oplus x_3x_5x_6x_8 \oplus x_3x_5x_6x_8x_9 \oplus x_3x_5x_7 \oplus x_3x_5x_7x_8x_9 \oplus x_3x_5x_7x_9 \oplus$
$x_3x_5x_8 \oplus x_3x_5x_9 \oplus x_3x_6 \oplus x_3x_6x_7 \oplus x_3x_6x_7x_8 \oplus x_3x_6x_7x_8x_9 \oplus x_3x_6x_8x_9 \oplus x_3x_6x_9 \oplus x_3x_7x_8 \oplus x_3x_7x_9 \oplus x_3x_8 \oplus$
$x_3x_8x_9 \oplus x_4x_5 \oplus x_4x_5x_6x_7 \oplus x_4x_5x_6x_7x_8x_9 \oplus x_4x_5x_6x_7x_9 \oplus x_4x_5x_6x_8 \oplus x_4x_5x_6x_9 \oplus x_4x_5x_7 \oplus x_4x_5x_7x_8 \oplus$
$x_4x_5x_7x_8x_9 \oplus x_4x_5x_8x_9 \oplus x_4x_5x_9 \oplus x_4x_6 \oplus x_4x_6x_7x_8 \oplus x_4x_6x_7x_9 \oplus x_4x_6x_8 \oplus x_4x_6x_8x_9 \oplus x_4x_7 \oplus x_4x_7x_8x_9 \oplus$
$x_4x_7x_9 \oplus x_4x_8 \oplus x_4x_9 \oplus x_5 \oplus x_5x_6 \oplus x_5x_6x_7 \oplus x_5x_6x_7x_8 \oplus x_5x_6x_7x_8x_9 \oplus x_5x_6x_8x_9 \oplus x_5x_6x_9 \oplus x_5x_7x_8 \oplus$
$x_5x_7x_9 \oplus x_5x_8 \oplus x_5x_8x_9 \oplus x_6x_7 \oplus x_6x_7x_8x_9 \oplus x_6x_7x_9 \oplus x_6x_8 \oplus x_6x_9 \oplus x_7 \oplus x_7x_8 \oplus x_7x_8x_9 \oplus x_8x_9 \oplus x_9 = 0$

$x_0 \oplus x_0x_1 \oplus x_0x_1x_2 \oplus x_0x_1x_2x_3 \oplus x_0x_1x_2x_3x_4 \oplus x_0x_1x_2x_3x_4x_5 \oplus x_0x_1x_2x_3x_4x_5x_6 \oplus x_0x_1x_2x_3x_4x_5x_6x_7 \oplus$

$x_0x_1x_2x_3x_4x_5x_6x_7x_8 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_6x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_6x_9 \oplus x_0x_1x_2x_3x_4x_5x_7x_8 \oplus$
$x_0x_1x_2x_3x_4x_5x_7x_9 \oplus x_0x_1x_2x_3x_4x_5x_8 \oplus x_0x_1x_2x_3x_4x_5x_8x_9 \oplus x_0x_1x_2x_3x_4x_6x_7 \oplus x_0x_1x_2x_3x_4x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_6x_7x_9 \oplus$
$x_0x_1x_2x_3x_4x_6x_8 \oplus x_0x_1x_2x_3x_4x_6x_9 \oplus x_0x_1x_2x_3x_4x_7 \oplus x_0x_1x_2x_3x_4x_7x_8 \oplus x_0x_1x_2x_3x_4x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_8x_9 \oplus$
$x_0x_1x_2x_3x_4x_9 \oplus x_0x_1x_2x_3x_5x_6 \oplus x_0x_1x_2x_3x_5x_6x_7x_8 \oplus x_0x_1x_2x_3x_5x_6x_7x_9 \oplus x_0x_1x_2x_3x_5x_6x_8 \oplus x_0x_1x_2x_3x_5x_6x_8x_9 \oplus$
$x_0x_1x_2x_3x_5x_7 \oplus x_0x_1x_2x_3x_5x_7x_8x_9 \oplus x_0x_1x_2x_3x_5x_7x_9 \oplus x_0x_1x_2x_3x_5x_8 \oplus x_0x_1x_2x_3x_5x_9 \oplus x_0x_1x_2x_3x_6 \oplus$
$x_0x_1x_2x_3x_6x_7 \oplus x_0x_1x_2x_3x_6x_7x_8 \oplus x_0x_1x_2x_3x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_6x_8x_9 \oplus x_0x_1x_2x_3x_6x_9 \oplus x_0x_1x_2x_3x_7x_8 \oplus$
$x_0x_1x_2x_3x_7x_9 \oplus x_0x_1x_2x_3x_8 \oplus x_0x_1x_2x_3x_8x_9 \oplus x_0x_1x_2x_4x_5 \oplus x_0x_1x_2x_4x_5x_6x_7 \oplus x_0x_1x_2x_4x_5x_6x_7x_8x_9 \oplus$
$x_0x_1x_2x_4x_5x_6x_7x_9 \oplus x_0x_1x_2x_4x_5x_6x_8 \oplus x_0x_1x_2x_4x_5x_6x_9 \oplus x_0x_1x_2x_4x_5x_7 \oplus x_0x_1x_2x_4x_5x_7x_8 \oplus x_0x_1x_2x_4x_5x_7x_8x_9 \oplus$
$x_0x_1x_2x_4x_5x_8x_9 \oplus x_0x_1x_2x_4x_5x_9 \oplus x_0x_1x_2x_4x_6 \oplus x_0x_1x_2x_4x_6x_7x_8 \oplus x_0x_1x_2x_4x_6x_7x_9 \oplus x_0x_1x_2x_4x_6x_8 \oplus$
$x_0x_1x_2x_4x_6x_8x_9 \oplus x_0x_1x_2x_4x_7 \oplus x_0x_1x_2x_4x_7x_8x_9 \oplus x_0x_1x_2x_4x_7x_9 \oplus x_0x_1x_2x_4x_8 \oplus x_0x_1x_2x_4x_9 \oplus x_0x_1x_2x_5 \oplus$
$x_0x_1x_2x_5x_6 \oplus x_0x_1x_2x_5x_6x_7 \oplus x_0x_1x_2x_5x_6x_7x_8 \oplus x_0x_1x_2x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_5x_6x_8x_9 \oplus x_0x_1x_2x_5x_6x_9 \oplus$
$x_0x_1x_2x_5x_7x_8 \oplus x_0x_1x_2x_5x_7x_9 \oplus x_0x_1x_2x_5x_8 \oplus x_0x_1x_2x_5x_8x_9 \oplus x_0x_1x_2x_6x_7 \oplus x_0x_1x_2x_6x_7x_8x_9 \oplus x_0x_1x_2x_6x_7x_9 \oplus$
$x_0x_1x_2x_6x_8 \oplus x_0x_1x_2x_6x_9 \oplus x_0x_1x_2x_7 \oplus x_0x_1x_2x_7x_8 \oplus x_0x_1x_2x_7x_8x_9 \oplus x_0x_1x_2x_8x_9 \oplus x_0x_1x_2x_9 \oplus x_0x_1x_3x_4 \oplus$
$x_0x_1x_3x_4x_5x_6 \oplus x_0x_1x_3x_4x_5x_6x_7x_8 \oplus x_0x_1x_3x_4x_5x_6x_7x_9 \oplus x_0x_1x_3x_4x_5x_6x_8 \oplus x_0x_1x_3x_4x_5x_6x_8x_9 \oplus x_0x_1x_3x_4x_5x_7 \oplus$
$x_0x_1x_3x_4x_5x_7x_8x_9 \oplus x_0x_1x_3x_4x_5x_7x_9 \oplus x_0x_1x_3x_4x_5x_8 \oplus x_0x_1x_3x_4x_5x_9 \oplus x_0x_1x_3x_4x_6 \oplus x_0x_1x_3x_4x_6x_7 \oplus$
$x_0x_1x_3x_4x_6x_7x_8 \oplus x_0x_1x_3x_4x_6x_7x_8x_9 \oplus x_0x_1x_3x_4x_6x_8x_9 \oplus x_0x_1x_3x_4x_6x_9 \oplus x_0x_1x_3x_4x_7x_8 \oplus x_0x_1x_3x_4x_7x_9 \oplus$
$x_0x_1x_3x_4x_8 \oplus x_0x_1x_3x_4x_8x_9 \oplus x_0x_1x_3x_5 \oplus x_0x_1x_3x_5x_6x_7 \oplus x_0x_1x_3x_5x_6x_7x_8x_9 \oplus x_0x_1x_3x_5x_6x_7x_9 \oplus x_0x_1x_3x_5x_6x_8 \oplus$
$x_0x_1x_3x_5x_6x_9 \oplus x_0x_1x_3x_5x_7 \oplus x_0x_1x_3x_5x_7x_8 \oplus x_0x_1x_3x_5x_7x_8x_9 \oplus x_0x_1x_3x_5x_8x_9 \oplus x_0x_1x_3x_5x_9 \oplus x_0x_1x_3x_6 \oplus$
$x_0x_1x_3x_6x_7x_8 \oplus x_0x_1x_3x_6x_7x_9 \oplus x_0x_1x_3x_6x_8 \oplus x_0x_1x_3x_6x_8x_9 \oplus x_0x_1x_3x_7 \oplus x_0x_1x_3x_7x_8x_9 \oplus x_0x_1x_3x_7x_9 \oplus$
$x_0x_1x_3x_8 \oplus x_0x_1x_3x_9 \oplus x_0x_1x_4 \oplus x_0x_1x_4x_5 \oplus x_0x_1x_4x_5x_6 \oplus x_0x_1x_4x_5x_6x_7 \oplus x_0x_1x_4x_5x_6x_7x_8 \oplus x_0x_1x_4x_5x_6x_7x_8x_9 \oplus$
$x_0x_1x_4x_5x_6x_8x_9 \oplus x_0x_1x_4x_5x_6x_9 \oplus x_0x_1x_4x_5x_7x_8 \oplus x_0x_1x_4x_5x_7x_9 \oplus x_0x_1x_4x_5x_8 \oplus x_0x_1x_4x_5x_8x_9 \oplus x_0x_1x_4x_6x_7 \oplus$
$x_0x_1x_4x_6x_7x_8x_9 \oplus x_0x_1x_4x_6x_7x_9 \oplus x_0x_1x_4x_6x_8 \oplus x_0x_1x_4x_6x_9 \oplus x_0x_1x_4x_7 \oplus x_0x_1x_4x_7x_8 \oplus x_0x_1x_4x_7x_8x_9 \oplus$
$x_0x_1x_4x_8x_9 \oplus x_0x_1x_4x_9 \oplus x_0x_1x_5x_6 \oplus x_0x_1x_5x_6x_7x_8 \oplus x_0x_1x_5x_6x_7x_9 \oplus x_0x_1x_5x_6x_8 \oplus x_0x_1x_5x_6x_8x_9 \oplus x_0x_1x_5x_7 \oplus$
$x_0x_1x_5x_7x_8x_9 \oplus x_0x_1x_5x_7x_9 \oplus x_0x_1x_5x_8 \oplus x_0x_1x_5x_9 \oplus x_0x_1x_6 \oplus x_0x_1x_6x_7 \oplus x_0x_1x_6x_7x_8 \oplus x_0x_1x_6x_7x_8x_9 \oplus$
$x_0x_1x_6x_8x_9 \oplus x_0x_1x_6x_9 \oplus x_0x_1x_7x_8 \oplus x_0x_1x_7x_9 \oplus x_0x_1x_8 \oplus x_0x_1x_8x_9 \oplus x_0x_2x_3 \oplus x_0x_2x_3x_4x_5 \oplus x_0x_2x_3x_4x_5x_6x_7 \oplus$
$x_0x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_2x_3x_4x_5x_6x_7x_9 \oplus x_0x_2x_3x_4x_5x_6x_8 \oplus x_0x_2x_3x_4x_5x_6x_9 \oplus x_0x_2x_3x_4x_5x_7 \oplus x_0x_2x_3x_4x_5x_7x_8 \oplus$
$x_0x_2x_3x_4x_5x_7x_8x_9 \oplus x_0x_2x_3x_4x_5x_8x_9 \oplus x_0x_2x_3x_4x_5x_9 \oplus x_0x_2x_3x_4x_6 \oplus x_0x_2x_3x_4x_6x_7x_8 \oplus x_0x_2x_3x_4x_6x_7x_9 \oplus$
$x_0x_2x_3x_4x_6x_8 \oplus x_0x_2x_3x_4x_6x_8x_9 \oplus x_0x_2x_3x_4x_7 \oplus x_0x_2x_3x_4x_7x_8x_9 \oplus x_0x_2x_3x_4x_7x_9 \oplus x_0x_2x_3x_4x_8 \oplus x_0x_2x_3x_4x_9 \oplus$
$x_0x_2x_3x_5 \oplus x_0x_2x_3x_5x_6 \oplus x_0x_2x_3x_5x_6x_7 \oplus x_0x_2x_3x_5x_6x_7x_8 \oplus x_0x_2x_3x_5x_6x_7x_8x_9 \oplus x_0x_2x_3x_5x_6x_8x_9 \oplus x_0x_2x_3x_5x_6x_9 \oplus$
$x_0x_2x_3x_5x_7x_8 \oplus x_0x_2x_3x_5x_7x_9 \oplus x_0x_2x_3x_5x_8 \oplus x_0x_2x_3x_5x_8x_9 \oplus x_0x_2x_3x_6x_7 \oplus x_0x_2x_3x_6x_7x_8x_9 \oplus x_0x_2x_3x_6x_7x_9 \oplus$
$x_0x_2x_3x_6x_8 \oplus x_0x_2x_3x_6x_9 \oplus x_0x_2x_3x_7 \oplus x_0x_2x_3x_7x_8 \oplus x_0x_2x_3x_7x_8x_9 \oplus x_0x_2x_3x_8x_9 \oplus x_0x_2x_3x_9 \oplus x_0x_2x_4 \oplus$
$x_0x_2x_4x_5x_6 \oplus x_0x_2x_4x_5x_6x_7x_8 \oplus x_0x_2x_4x_5x_6x_7x_9 \oplus x_0x_2x_4x_5x_6x_8 \oplus x_0x_2x_4x_5x_6x_8x_9 \oplus x_0x_2x_4x_5x_7 \oplus x_0x_2x_4x_5x_7x_8x_9 \oplus$
$x_0x_2x_4x_5x_7x_9 \oplus x_0x_2x_4x_5x_8 \oplus x_0x_2x_4x_5x_9 \oplus x_0x_2x_4x_6 \oplus x_0x_2x_4x_6x_7 \oplus x_0x_2x_4x_6x_7x_8 \oplus x_0x_2x_4x_6x_7x_8x_9 \oplus$
$x_0x_2x_4x_6x_8x_9 \oplus x_0x_2x_4x_6x_9 \oplus x_0x_2x_4x_7x_8 \oplus x_0x_2x_4x_7x_9 \oplus x_0x_2x_4x_8 \oplus x_0x_2x_4x_8x_9 \oplus x_0x_2x_5 \oplus x_0x_2x_5x_6x_7 \oplus$
$x_0x_2x_5x_6x_7x_8x_9 \oplus x_0x_2x_5x_6x_7x_9 \oplus x_0x_2x_5x_6x_8 \oplus x_0x_2x_5x_6x_9 \oplus x_0x_2x_5x_7 \oplus x_0x_2x_5x_7x_8 \oplus x_0x_2x_5x_7x_8x_9 \oplus$
$x_0x_2x_5x_8x_9 \oplus x_0x_2x_5x_9 \oplus x_0x_2x_6 \oplus x_0x_2x_6x_7x_8 \oplus x_0x_2x_6x_7x_9 \oplus x_0x_2x_6x_8 \oplus x_0x_2x_6x_8x_9 \oplus x_0x_2x_7 \oplus x_0x_2x_7x_8x_9 \oplus$
$x_0x_2x_7x_9 \oplus x_0x_2x_8 \oplus x_0x_2x_9 \oplus x_0x_3 \oplus x_0x_3x_4 \oplus x_0x_3x_4x_5 \oplus x_0x_3x_4x_5x_6 \oplus x_0x_3x_4x_5x_6x_7 \oplus x_0x_3x_4x_5x_6x_7x_8 \oplus$
$x_0x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_3x_4x_5x_6x_8x_9 \oplus x_0x_3x_4x_5x_6x_9 \oplus x_0x_3x_4x_5x_7x_8 \oplus x_0x_3x_4x_5x_7x_9 \oplus x_0x_3x_4x_5x_8 \oplus$
$x_0x_3x_4x_5x_8x_9 \oplus x_0x_3x_4x_6x_7 \oplus x_0x_3x_4x_6x_7x_8x_9 \oplus x_0x_3x_4x_6x_7x_9 \oplus x_0x_3x_4x_6x_8 \oplus x_0x_3x_4x_6x_9 \oplus x_0x_3x_4x_7 \oplus$
$x_0x_3x_4x_7x_8 \oplus x_0x_3x_4x_7x_8x_9 \oplus x_0x_3x_4x_8x_9 \oplus x_0x_3x_4x_9 \oplus x_0x_3x_5x_6 \oplus x_0x_3x_5x_6x_7x_8 \oplus x_0x_3x_5x_6x_7x_9 \oplus x_0x_3x_5x_6x_8 \oplus$
$x_0x_3x_5x_6x_8x_9 \oplus x_0x_3x_5x_7 \oplus x_0x_3x_5x_7x_8x_9 \oplus x_0x_3x_5x_7x_9 \oplus x_0x_3x_5x_8 \oplus x_0x_3x_5x_9 \oplus x_0x_3x_6 \oplus x_0x_3x_6x_7 \oplus$
$x_0x_3x_6x_7x_8 \oplus x_0x_3x_6x_7x_8x_9 \oplus x_0x_3x_6x_8x_9 \oplus x_0x_3x_6x_9 \oplus x_0x_3x_7x_8 \oplus x_0x_3x_7x_9 \oplus x_0x_3x_8 \oplus x_0x_3x_8x_9 \oplus x_0x_4x_5 \oplus$
$x_0x_4x_5x_6x_7 \oplus x_0x_4x_5x_6x_7x_8x_9 \oplus x_0x_4x_5x_6x_7x_9 \oplus x_0x_4x_5x_6x_8 \oplus x_0x_4x_5x_6x_9 \oplus x_0x_4x_5x_7 \oplus x_0x_4x_5x_7x_8 \oplus$
$x_0x_4x_5x_7x_8x_9 \oplus x_0x_4x_5x_8x_9 \oplus x_0x_4x_5x_9 \oplus x_0x_4x_6 \oplus x_0x_4x_6x_7x_8 \oplus x_0x_4x_6x_7x_9 \oplus x_0x_4x_6x_8 \oplus x_0x_4x_6x_8x_9 \oplus$
$x_0x_4x_7 \oplus x_0x_4x_7x_8x_9 \oplus x_0x_4x_7x_9 \oplus x_0x_4x_8 \oplus x_0x_4x_9 \oplus x_0x_5 \oplus x_0x_5x_6 \oplus x_0x_5x_6x_7 \oplus x_0x_5x_6x_7x_8 \oplus x_0x_5x_6x_7x_8x_9 \oplus$
$x_0x_5x_6x_8x_9 \oplus x_0x_5x_6x_9 \oplus x_0x_5x_7x_8 \oplus x_0x_5x_7x_9 \oplus x_0x_5x_8 \oplus x_0x_5x_8x_9 \oplus x_0x_6x_7 \oplus x_0x_6x_7x_8x_9 \oplus x_0x_6x_7x_9 \oplus$
$x_0x_6x_8 \oplus x_0x_6x_9 \oplus x_0x_7 \oplus x_0x_7x_8 \oplus x_0x_7x_8x_9 \oplus x_0x_8x_9 \oplus x_0x_9 \oplus x_1x_2 \oplus x_1x_2x_3x_4 \oplus x_1x_2x_3x_4x_5x_6 \oplus$
$x_1x_2x_3x_4x_5x_6x_7x_8 \oplus x_1x_2x_3x_4x_5x_6x_7x_9 \oplus x_1x_2x_3x_4x_5x_6x_8 \oplus x_1x_2x_3x_4x_5x_6x_8x_9 \oplus x_1x_2x_3x_4x_5x_7 \oplus x_1x_2x_3x_4x_5x_7x_8x_9 \oplus$
$x_1x_2x_3x_4x_5x_7x_9 \oplus x_1x_2x_3x_4x_5x_8 \oplus x_1x_2x_3x_4x_5x_9 \oplus x_1x_2x_3x_4x_6 \oplus x_1x_2x_3x_4x_6x_7 \oplus x_1x_2x_3x_4x_6x_7x_8 \oplus x_1x_2x_3x_4x_6x_7x_8x_9 \oplus$
$x_1x_2x_3x_4x_6x_8x_9 \oplus x_1x_2x_3x_4x_6x_9 \oplus x_1x_2x_3x_4x_7x_8 \oplus x_1x_2x_3x_4x_7x_9 \oplus x_1x_2x_3x_4x_8 \oplus x_1x_2x_3x_4x_8x_9 \oplus x_1x_2x_3x_5 \oplus$
$x_1x_2x_3x_5x_6x_7 \oplus x_1x_2x_3x_5x_6x_7x_8x_9 \oplus x_1x_2x_3x_5x_6x_7x_9 \oplus x_1x_2x_3x_5x_6x_8 \oplus x_1x_2x_3x_5x_6x_9 \oplus x_1x_2x_3x_5x_7 \oplus$

$$
\begin{aligned}
&x_1x_2x_3x_5x_7x_8 \oplus x_1x_2x_3x_5x_7x_8x_9 \oplus x_1x_2x_3x_5x_8x_9 \oplus x_1x_2x_3x_5x_9 \oplus x_1x_2x_3x_6 \oplus x_1x_2x_3x_6x_7x_8 \oplus x_1x_2x_3x_6x_7x_9 \oplus \\
&x_1x_2x_3x_6x_8 \oplus x_1x_2x_3x_6x_8x_9 \oplus x_1x_2x_3x_7 \oplus x_1x_2x_3x_7x_8x_9 \oplus x_1x_2x_3x_7x_9 \oplus x_1x_2x_3x_8 \oplus x_1x_2x_3x_9 \oplus x_1x_2x_4 \oplus \\
&x_1x_2x_4x_5 \oplus x_1x_2x_4x_5x_6 \oplus x_1x_2x_4x_5x_6x_7 \oplus x_1x_2x_4x_5x_6x_7x_8 \oplus x_1x_2x_4x_5x_6x_7x_8x_9 \oplus x_1x_2x_4x_5x_6x_8x_9 \oplus x_1x_2x_4x_5x_6x_9 \oplus \\
&x_1x_2x_4x_5x_7x_8 \oplus x_1x_2x_4x_5x_7x_9 \oplus x_1x_2x_4x_5x_8 \oplus x_1x_2x_4x_5x_8x_9 \oplus x_1x_2x_4x_6x_7 \oplus x_1x_2x_4x_6x_7x_8x_9 \oplus x_1x_2x_4x_6x_7x_9 \oplus \\
&x_1x_2x_4x_6x_8 \oplus x_1x_2x_4x_6x_9 \oplus x_1x_2x_4x_7 \oplus x_1x_2x_4x_7x_8 \oplus x_1x_2x_4x_7x_8x_9 \oplus x_1x_2x_4x_8x_9 \oplus x_1x_2x_4x_9 \oplus x_1x_2x_5x_6 \oplus \\
&x_1x_2x_5x_6x_7x_8 \oplus x_1x_2x_5x_6x_7x_9 \oplus x_1x_2x_5x_6x_8 \oplus x_1x_2x_5x_6x_8x_9 \oplus x_1x_2x_5x_7 \oplus x_1x_2x_5x_7x_8x_9 \oplus x_1x_2x_5x_7x_9 \oplus \\
&x_1x_2x_5x_8 \oplus x_1x_2x_5x_9 \oplus x_1x_2x_6 \oplus x_1x_2x_6x_7 \oplus x_1x_2x_6x_7x_8 \oplus x_1x_2x_6x_7x_8x_9 \oplus x_1x_2x_6x_8x_9 \oplus x_1x_2x_6x_9 \oplus x_1x_2x_7x_8 \oplus \\
&x_1x_2x_7x_9 \oplus x_1x_2x_8 \oplus x_1x_2x_8x_9 \oplus x_1x_3 \oplus x_1x_3x_4x_5 \oplus x_1x_3x_4x_5x_6x_7 \oplus x_1x_3x_4x_5x_6x_7x_8x_9 \oplus x_1x_3x_4x_5x_6x_7x_9 \oplus \\
&x_1x_3x_4x_5x_6x_8 \oplus x_1x_3x_4x_5x_6x_9 \oplus x_1x_3x_4x_5x_7 \oplus x_1x_3x_4x_5x_7x_8 \oplus x_1x_3x_4x_5x_7x_8x_9 \oplus x_1x_3x_4x_5x_8x_9 \oplus x_1x_3x_4x_5x_9 \oplus \\
&x_1x_3x_4x_6 \oplus x_1x_3x_4x_6x_7x_8 \oplus x_1x_3x_4x_6x_7x_9 \oplus x_1x_3x_4x_6x_8 \oplus x_1x_3x_4x_6x_8x_9 \oplus x_1x_3x_4x_7 \oplus x_1x_3x_4x_7x_8x_9 \oplus \\
&x_1x_3x_4x_7x_9 \oplus x_1x_3x_4x_8 \oplus x_1x_3x_4x_9 \oplus x_1x_3x_5 \oplus x_1x_3x_5x_6 \oplus x_1x_3x_5x_6x_7 \oplus x_1x_3x_5x_6x_7x_8 \oplus x_1x_3x_5x_6x_7x_8x_9 \oplus \\
&x_1x_3x_5x_6x_8x_9 \oplus x_1x_3x_5x_6x_9 \oplus x_1x_3x_5x_7x_8 \oplus x_1x_3x_5x_7x_9 \oplus x_1x_3x_5x_8 \oplus x_1x_3x_5x_8x_9 \oplus x_1x_3x_6x_7 \oplus x_1x_3x_6x_7x_8x_9 \oplus \\
&x_1x_3x_6x_7x_9 \oplus x_1x_3x_6x_8 \oplus x_1x_3x_6x_9 \oplus x_1x_3x_7 \oplus x_1x_3x_7x_8 \oplus x_1x_3x_7x_8x_9 \oplus x_1x_3x_8x_9 \oplus x_1x_3x_9 \oplus x_1x_4 \oplus \\
&x_1x_4x_5x_6 \oplus x_1x_4x_5x_6x_7x_8 \oplus x_1x_4x_5x_6x_7x_9 \oplus x_1x_4x_5x_6x_8 \oplus x_1x_4x_5x_6x_8x_9 \oplus x_1x_4x_5x_7 \oplus x_1x_4x_5x_7x_8x_9 \oplus \\
&x_1x_4x_5x_7x_9 \oplus x_1x_4x_5x_8 \oplus x_1x_4x_5x_9 \oplus x_1x_4x_6 \oplus x_1x_4x_6x_7 \oplus x_1x_4x_6x_7x_8 \oplus x_1x_4x_6x_7x_8x_9 \oplus x_1x_4x_6x_8x_9 \oplus \\
&x_1x_4x_6x_9 \oplus x_1x_4x_7x_8 \oplus x_1x_4x_7x_9 \oplus x_1x_4x_8 \oplus x_1x_4x_8x_9 \oplus x_1x_5 \oplus x_1x_5x_6x_7 \oplus x_1x_5x_6x_7x_8x_9 \oplus x_1x_5x_6x_7x_9 \oplus \\
&x_1x_5x_6x_8 \oplus x_1x_5x_6x_9 \oplus x_1x_5x_7 \oplus x_1x_5x_7x_8 \oplus x_1x_5x_7x_8x_9 \oplus x_1x_5x_8x_9 \oplus x_1x_5x_9 \oplus x_1x_6 \oplus x_1x_6x_7x_8 \oplus x_1x_6x_7x_9 \oplus \\
&x_1x_6x_8 \oplus x_1x_6x_8x_9 \oplus x_1x_7 \oplus x_1x_7x_8x_9 \oplus x_1x_7x_9 \oplus x_1x_8 \oplus x_1x_9 \oplus x_2 \oplus x_2x_3 \oplus x_2x_3x_4 \oplus x_2x_3x_4x_5 \oplus x_2x_3x_4x_5x_6 \oplus \\
&x_2x_3x_4x_5x_6x_7 \oplus x_2x_3x_4x_5x_6x_7x_8 \oplus x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_2x_3x_4x_5x_6x_8x_9 \oplus x_2x_3x_4x_5x_6x_9 \oplus x_2x_3x_4x_5x_7x_8 \oplus \\
&x_2x_3x_4x_5x_7x_9 \oplus x_2x_3x_4x_5x_8 \oplus x_2x_3x_4x_5x_8x_9 \oplus x_2x_3x_4x_6x_7 \oplus x_2x_3x_4x_6x_7x_8x_9 \oplus x_2x_3x_4x_6x_7x_9 \oplus x_2x_3x_4x_6x_8 \oplus \\
&x_2x_3x_4x_6x_9 \oplus x_2x_3x_4x_7 \oplus x_2x_3x_4x_7x_8 \oplus x_2x_3x_4x_7x_8x_9 \oplus x_2x_3x_4x_8x_9 \oplus x_2x_3x_4x_9 \oplus x_2x_3x_5x_6 \oplus x_2x_3x_5x_6x_7x_8 \oplus \\
&x_2x_3x_5x_6x_7x_9 \oplus x_2x_3x_5x_6x_8 \oplus x_2x_3x_5x_6x_8x_9 \oplus x_2x_3x_5x_7 \oplus x_2x_3x_5x_7x_8x_9 \oplus x_2x_3x_5x_7x_9 \oplus x_2x_3x_5x_8 \oplus x_2x_3x_5x_9 \oplus \\
&x_2x_3x_6 \oplus x_2x_3x_6x_7 \oplus x_2x_3x_6x_7x_8 \oplus x_2x_3x_6x_7x_8x_9 \oplus x_2x_3x_6x_8x_9 \oplus x_2x_3x_6x_9 \oplus x_2x_3x_7x_8 \oplus x_2x_3x_7x_9 \oplus \\
&x_2x_3x_8 \oplus x_2x_3x_8x_9 \oplus x_2x_4x_5 \oplus x_2x_4x_5x_6x_7 \oplus x_2x_4x_5x_6x_7x_8x_9 \oplus x_2x_4x_5x_6x_7x_9 \oplus x_2x_4x_5x_6x_8 \oplus x_2x_4x_5x_6x_9 \oplus \\
&x_2x_4x_5x_7 \oplus x_2x_4x_5x_7x_8 \oplus x_2x_4x_5x_7x_8x_9 \oplus x_2x_4x_5x_8x_9 \oplus x_2x_4x_5x_9 \oplus x_2x_4x_6 \oplus x_2x_4x_6x_7x_8 \oplus x_2x_4x_6x_7x_9 \oplus \\
&x_2x_4x_6x_8 \oplus x_2x_4x_6x_8x_9 \oplus x_2x_4x_7 \oplus x_2x_4x_7x_8x_9 \oplus x_2x_4x_7x_9 \oplus x_2x_4x_8 \oplus x_2x_4x_9 \oplus x_2x_5 \oplus x_2x_5x_6 \oplus x_2x_5x_6x_7 \oplus \\
&x_2x_5x_6x_7x_8 \oplus x_2x_5x_6x_7x_8x_9 \oplus x_2x_5x_6x_8x_9 \oplus x_2x_5x_6x_9 \oplus x_2x_5x_7x_8 \oplus x_2x_5x_7x_9 \oplus x_2x_5x_8 \oplus x_2x_5x_8x_9 \oplus x_2x_6x_7 \oplus \\
&x_2x_6x_7x_8x_9 \oplus x_2x_6x_7x_9 \oplus x_2x_6x_8 \oplus x_2x_6x_9 \oplus x_2x_7 \oplus x_2x_7x_8 \oplus x_2x_7x_8x_9 \oplus x_2x_8x_9 \oplus x_2x_9 \oplus x_3x_4 \oplus x_3x_4x_5x_6 \oplus \\
&x_3x_4x_5x_6x_7x_8 \oplus x_3x_4x_5x_6x_7x_9 \oplus x_3x_4x_5x_6x_8 \oplus x_3x_4x_5x_6x_8x_9 \oplus x_3x_4x_5x_7 \oplus x_3x_4x_5x_7x_8x_9 \oplus x_3x_4x_5x_7x_9 \oplus \\
&x_3x_4x_5x_8 \oplus x_3x_4x_5x_9 \oplus x_3x_4x_6 \oplus x_3x_4x_6x_7 \oplus x_3x_4x_6x_7x_8 \oplus x_3x_4x_6x_7x_8x_9 \oplus x_3x_4x_6x_8x_9 \oplus x_3x_4x_6x_9 \oplus x_3x_4x_7x_8 \oplus \\
&x_3x_4x_7x_9 \oplus x_3x_4x_8 \oplus x_3x_4x_8x_9 \oplus x_3x_5 \oplus x_3x_5x_6x_7 \oplus x_3x_5x_6x_7x_8x_9 \oplus x_3x_5x_6x_7x_9 \oplus x_3x_5x_6x_8 \oplus x_3x_5x_6x_9 \oplus \\
&x_3x_5x_7 \oplus x_3x_5x_7x_8 \oplus x_3x_5x_7x_8x_9 \oplus x_3x_5x_8x_9 \oplus x_3x_5x_9 \oplus x_3x_6 \oplus x_3x_6x_7x_8 \oplus x_3x_6x_7x_9 \oplus x_3x_6x_8 \oplus x_3x_6x_8x_9 \oplus \\
&x_3x_7 \oplus x_3x_7x_8x_9 \oplus x_3x_7x_9 \oplus x_3x_8 \oplus x_3x_9 \oplus x_4 \oplus x_4x_5 \oplus x_4x_5x_6 \oplus x_4x_5x_6x_7 \oplus x_4x_5x_6x_7x_8 \oplus x_4x_5x_6x_7x_8x_9 \oplus \\
&x_4x_5x_6x_8x_9 \oplus x_4x_5x_6x_9 \oplus x_4x_5x_7x_8 \oplus x_4x_5x_7x_9 \oplus x_4x_5x_8 \oplus x_4x_5x_8x_9 \oplus x_4x_6x_7 \oplus x_4x_6x_7x_8x_9 \oplus x_4x_6x_7x_9 \oplus \\
&x_4x_6x_8 \oplus x_4x_6x_9 \oplus x_4x_7 \oplus x_4x_7x_8 \oplus x_4x_7x_8x_9 \oplus x_4x_8x_9 \oplus x_4x_9 \oplus x_5x_6 \oplus x_5x_6x_7x_8 \oplus x_5x_6x_7x_9 \oplus x_5x_6x_8 \oplus \\
&x_5x_6x_8x_9 \oplus x_5x_7 \oplus x_5x_7x_8x_9 \oplus x_5x_7x_9 \oplus x_5x_8 \oplus x_5x_9 \oplus x_6 \oplus x_6x_7 \oplus x_6x_7x_8 \oplus x_6x_7x_8x_9 \oplus x_6x_8x_9 \oplus x_6x_9 \oplus \\
&x_7x_8 \oplus x_7x_9 \oplus x_8 \oplus x_8x_9 = 0
\end{aligned}
$$

Author Contributions.

Competing Interests.

Materials & Correspondence.

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