Classical Symbolic Retrodictive Execution of Quantum Circuits

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1 Main

Retrodictive quantum theory [4], retrocausality [2], and the time-symmetry of physical laws [13] suggest that partial knowledge about the future can be exploited to understand the present. We demonstrate the even stronger proposition that, in concert with the computational concepts of *demand-driven lazy evaluation* [9] and *symbolic partial evaluation* [8], retrodictive reasoning can be used as a computational resource to de-quantize some quantum algorithms, i.e., to provide efficient classical algorithms inspired by their quantum counterparts.

Symbolic Execution of Classical Programs Applied to Quantum Oracles. A well-established technique to simultaneously explore multiple paths that a classical program could take under different inputs is *symbolic execution* [3, 5, 7, 10, 11]. In this execution scheme, concrete values are replaced by symbols which are initially unconstrained. As the execution proceeds, the symbols interact with program constructs and this typically introduces constraints on the possible values that the symbols represent. At the end of the execution, these constraints can be solved to infer properties of the program under consideration. The idea is also applicable to quantum circuits as the following example illustrates.

Let $[\mathbf{n}]$ denote the finite set $\{0, 1, \ldots, (n-1)\}$. In Simon's problem, we are given a 2-1 (classical) function $f : [\mathbf{2^n}] \to [\mathbf{2^n}]$ with the property that there exists an a such $f(x) = f(x \oplus a)$ for all x; the goal is to determine a. The circuit in Fig. 1 implements the quantum algorithm when $n = 2$ and $a = 3$. In the circuit, the gates between barriers (1) and (2) implement a quantum oracle $U_f(x, 0) = (x, f(x))$ that encapsulates the function f of interest. A direct classical simulation of the quantum cir-

Figure 1: Circuit for Simon's Algorithm $n = 2$ and $a = 3$

cuit would need to execute the U_f block four times, once for each possible value $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ for the top two wires. Instead, let us introduce two symbols x_0 representing the top wire and x_1 representing the wire below it, and let's proceed with the execution symbolically. The state at barrier (1) is initially $|x_0 x_1 00\rangle$. At the first CX-gate, we symbolically calculate the result of the target wire as $x_0 \oplus 0 = x_0$ evolving the state to $|x_0 x_1 x_0 0\rangle$. Going through the next three CX-gates, the state evolves as $|x_0 x_1 x_0 x_0\rangle$, $|x_0 x_1 (x_0 \oplus x_1) x_0\rangle$, and $|x_0 x_1 (x_0 \oplus x_1)(x_0 \oplus x_1)\rangle$ at barrier (2). At that point, we have established that the bottom two wires are equal; the result of their measurement can only be 00 or 11. Since the function is promised to be 2-1 for all inputs, it is sufficient to analyze one case, say when the measurement at barrier (3) produces 00. This measurement collapses the top wires to $|x_0 x_1\rangle$ subject to the constraint that $x_0 \oplus x_1 = 0$ or equivalently that $x_0 = x_1$. We have thus inferred that both $x_0 = x_1 = 0$ and $x_0 = x_1 = 1$ produce the same measurement result at barrier (3) and hence that $f(00) = f(11) = f(00 \oplus 11)$ which reveals that a is 11 in binary notation.
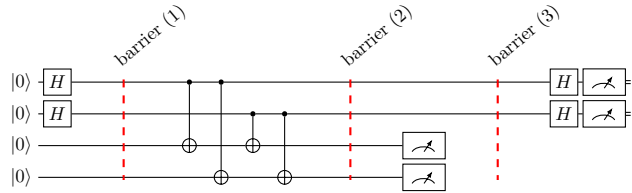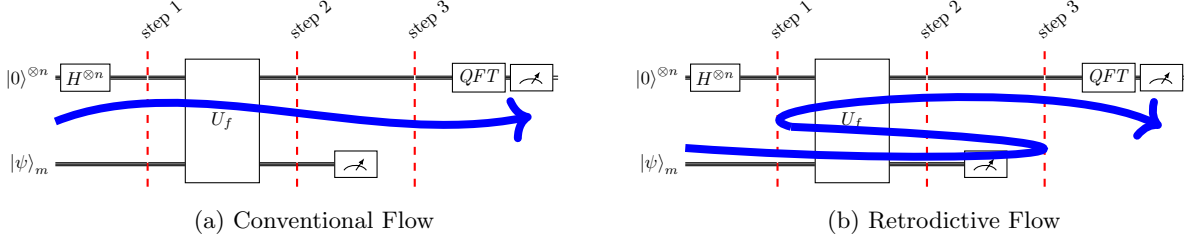
(a) Conventional Flow (b) Retrodictive Flow

Figure 2: Template quantum circuit

Since the quantum circuit between barriers (1) and (2) is reversible, we can perform the analysis above in a mixed predictive and symbolic retrodictive execution to make the flow of information conceptually clearer. We start a forward classical simulation with one arbitrary state at barrier (1), say $|0100\rangle$. This state evolves to $|0100\rangle$, then $|0100\rangle$ again, then $|0110\rangle$, and finally $|0111\rangle$. In this case, the result of measuring the bottom two wires is 11. Having produced a possible measurement at barrier (3), we start a retrodictive execution to find out what other input states might be compatible with this future measurement. To that end, we execute the circuit backwards with the symbolic state $|x_0 x_1 11\rangle$; that execution evolves to $|x_0 x_1 1(1 \oplus x_1)\rangle$, then $|x_0 x_1 (1 \oplus x_1)(1 \oplus x_1)\rangle$, then $|x_0 x_1 (1 \oplus x_1)(1 \oplus x_0 \oplus x_1)\rangle$, and finally $|x_0 x_1 (1 \oplus x_0 \oplus x_1)(1 \oplus x_0 \oplus x_1)\rangle$. Having reached the initial conditions on the bottom two wires, we reconcile them with the collected constraints to conclude that $1 \oplus x_0 \oplus x_1 = 0$ or equivalently that $x_0 \neq x_1$. The measurement of 11 at barrier (3) is consistent with not just the state $|01\rangle$ we started with but also with the state $|10\rangle$. In other words, we have $f(01) = f(10) = f(01 \oplus 11)$ and the hidden value of a is revealed to be 11.

Representing Wavefunctions Symbolically. A symbolic variable represents a boolean value that can be 0 or 1; this is similar to a qubit in a superposition $(1/\sqrt{2})(|0\rangle \pm |1\rangle)$. Thus, it appears that $H|0\rangle$ could be represented by a symbol x to denote the uncertainty. Surprisingly, this idea scales to even represent maximally entangled states. Fig. 3(left) shows a circuit to generate the Bell state $(1/\sqrt{2})(|00\rangle + |11\rangle)$. By using the symbol x for $H|0\rangle$, the input to the CX-gate is $|x0\rangle$ which evolves to $|xx\rangle$. By sharing the same symbol in two positions, the symbolic state accurately represents the entangled Bell state. Similarly, for the circuit in Fig. 3(right), the state after the Hadamard gate is $|x00\rangle$ which evolves to $|xx0\rangle$ and then to $|xxx\rangle$ again accurately capturing the entanglement correlations.
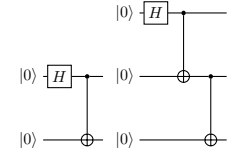

Figure 3: Bell and GHZ States

This insight allows us to symbolically execute the many quantum algorithms that match the template in Fig. 2 (including Deutsch, Deutsch-Jozsa, Bernstein-Vazirani, Simon, Grover, and Shor's algorithms). Specifically, in all these algorithms, the top collection of wires (which we will call the computational register) is prepared in a uniform superposition which can be represented using symbolic variables. Below, we report on the results of such symbolic executions. In each case, instead of the conventional execution flow depicted in Fig. 2(a), we find a possible measurement outcome w at barrier (3) and perform a retrodictive execution with a state $|xw\rangle$ going backwards to collect the constraints on x that enable us to solve the problem in question.

Deutsch. The quantum circuit in Fig. 4 determines if the function $[2] \rightarrow [2]$ encapsulated in the quantum oracle U_f is constant or balanced. Since 0 is always a possible measurement of the ancilla register, we start a retrodictive execution of the U_f block with state $|x0\rangle$. This execution terminates with a state $|xr\rangle$ where r is a formula expressing the dependencies of the ancilla on x. Running the experiment with different choices for f, the resulting formula always perfectly describes f. Specifically when f is the constant function that returns 0, we have $r = 0$; when f is the constant function that returns 1, we have $r = 0$; when f is the balanced function that
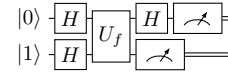

Figure 4: Deutsch

78 returns its input, we have $r = x$; and when f is the balanced function that returns the negation of its input,
79 we have $r = 1 \oplus x$.

80 **Deutsch-Jozsa.** The problem is a generalization of the previous one. We are given a function $[\mathbf{2^n}] \to [\mathbf{2}]$
81 that is promised to be constant or balanced and we need to decide distinguish the two cases. The quantum
82 circuit generalizes the one in Fig. 4 to use n-wires for the computation register. Similarly to before, we
83 perform a retrodictive execution of the U_f block with the state $|x_{n-1} \cdots x_1 x_0 0\rangle$ and observe the resulting
84 formula r. Like before, when the function is constant, the formula r is the corresponding constant and when
85 the function is balanced, the formula r completely describes how the result is computed from the symbols
86 $x_{n-1}, \cdots, x_1, x_0$. For example, for $n = 6$, the resulting formulae for three balanced functions were: x_0,
87 $x_0 \oplus x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5$, and $x_0 x_1 x_2 \oplus x_0 x_1 x_2 x_3 x_4 \oplus x_0 x_1 x_2 x_3 x_5 \oplus x_0 x_1 x_2 x_4 \oplus x_0 x_1 x_2 x_4 x_5 \oplus x_0 x_1 x_3 x_4 \oplus$
88 $x_0 x_1 x_3 x_5 \oplus x_0 x_1 x_4 \oplus x_0 x_1 x_4 x_5 \oplus x_0 x_2 \oplus x_0 x_2 x_3 x_5 \oplus x_0 x_2 x_4 x_5 \oplus x_0 x_3 \oplus x_0 x_3 x_4 x_5 \oplus x_0 x_3 x_5 \oplus x_1 x_2 x_3 x_5 \oplus$
89 $x_1 x_2 x_4 x_5 \oplus x_1 x_3 x_4 x_5 \oplus x_1 x_3 x_5 \oplus x_1 x_5 \oplus x_2 x_3 x_4 x_5 \oplus x_2 x_3 x_5 \oplus x_2 x_4 \oplus x_3 x_4 x_5 \oplus x_3 x_5$. In the first case, the
90 function is balanced because its output depends on just one variable (which is 0 in half the possible inputs);
91 in the second case the output of the function is the exclusive-or of all the input variables which is an easy
92 instance of a balanced function. The last case is a cryptographically strong balanced function whose output
93 pattern is, by design, difficult to discern [6]. An important insight in the case of the Deutsch-Jozsa problem
94 is that, since we are promised the function is either constant or balanced, then any formula that refers to at
95 least one variable must indicate a balanced function. In other words, the outcome of the algorithm can be
96 immediately decided if the formula is anything other than 0 or 1. We confirmed this observation by running
97 the experiment on all 12870 balanced functions from $[\mathbf{2^4}] \to [\mathbf{2}]$ and correctly identifying them as such. This is
98 significant as some of these functions produce complicated entangled patterns during quantum evolution and
99 could not be de-quantized using previous approaches [1]. The catch is that symbolic retrodictive execution
100 is not consistent with "query complexity" as it operates in time proportional to the depth of the quantum
101 oracle and the size of the formula.

102 **Bernstein-Vazirani.** We are given a function $f : [\mathbf{2^n}] \to$
103 $[\mathbf{2}]$ that hides a secret number $s \in [\mathbf{2^n}]$. We are promised the
104 function is defined using the binary representations $\sum_i^{n-1} x_i$
105 and $\sum_i^{n-1} s_i$ of x and s respectively as $f(x) = \sum_{i=0}^{n-1} s_i x_i$
106 mod 2. The goal is to determine the secret number s. The
107 circuit in Fig. 5 solves the problem for $n = 8$ and a hidden
108 number 92 ($= 00111010$ in binary notation with the right-
109 most bit at index 0). Retrodictive execution starting with
110 the state $|x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 0\rangle$ terminates with the formula
111 $x_1 \oplus x_3 \oplus x_4 \oplus x_5$. The secret string can be immediately read
112 from the formula as the indices $\{1, 3, 4, 5\}$ of the symbols are
113 exactly the positions at which the secret string has a 1.
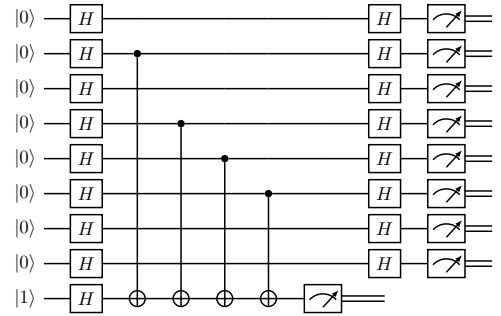
Figure 5: Circuit for Bernstein-Vazirani Algo-
rithm ($n = 8$, $s = 92$, least significant bit is
the top wire)

114 **Grover.** We are given a function $f : [\mathbf{2^n}] \to [\mathbf{2}]$ with the
115 property that there exists only one input u such $f(u) = 1$.
116 The goal is to find u. The conventional presentation of the
117 quantum algorithm does not fit the template of Fig. 2. But
118 it is still possible to construct a quantum oracle U_f from the
119 given f and perform retrodictive execution starting from an
120 ancilla measurement of 1 corresponding to the input pattern
121 we are interested in. The resulting equations for $n = 4$ and u in the range $\{0..15\}$ are in Fig. 6. In some cases
122 (e.g. $u = 15$) the equations immediately reveal u; in others, retrodictive executive provides no advantage
123 since solving arbitrary equations over boolean variables is, in general, an NP-complete problem.

3

$$
\begin{aligned}
u = 0 \quad & 1 \oplus x_0 \oplus x_0 x_1 \oplus x_0 x_1 x_2 \oplus x_0 x_1 x_2 x_3 \oplus x_0 x_1 x_3 \oplus x_0 x_2 \oplus x_0 x_2 x_3 \oplus x_0 x_3 \oplus x_1 \oplus x_1 x_2 \oplus \\
& x_1 x_2 x_3 \oplus x_1 x_3 \oplus x_2 \oplus x_2 x_3 \oplus x_3 \\
u = 1 \quad & x_0 \oplus x_0 x_1 \oplus x_0 x_1 x_2 \oplus x_0 x_1 x_2 x_3 \oplus x_0 x_1 x_3 \oplus x_0 x_2 \oplus x_0 x_2 x_3 \oplus x_0 x_3 \\
u = 2 \quad & x_0 x_1 \oplus x_0 x_1 x_2 \oplus x_0 x_1 x_2 x_3 \oplus x_0 x_1 x_3 \oplus x_1 \oplus x_1 x_2 \oplus x_1 x_2 x_3 \oplus x_1 x_3 \\
u = 3 \quad & x_0 x_1 \oplus x_0 x_1 x_2 \oplus x_0 x_1 x_2 x_3 \oplus x_0 x_1 x_3 \\
u = 4 \quad & x_0 x_1 x_2 \oplus x_0 x_1 x_2 x_3 \oplus x_0 x_2 \oplus x_0 x_2 x_3 \oplus x_1 x_2 \oplus x_1 x_2 x_3 \oplus x_2 \oplus x_2 x_3 \\
u = 5 \quad & x_0 x_1 x_2 \oplus x_0 x_1 x_2 x_3 \oplus x_0 x_2 \oplus x_0 x_2 x_3 \\
u = 6 \quad & x_0 x_1 x_2 \oplus x_0 x_1 x_2 x_3 \oplus x_1 x_2 \oplus x_1 x_2 x_3 \\
u = 7 \quad & x_0 x_1 x_2 \oplus x_0 x_1 x_2 x_3 \\
u = 8 \quad & x_0 x_1 x_2 x_3 \oplus x_0 x_1 x_3 \oplus x_0 x_2 x_3 \oplus x_0 x_3 \oplus x_1 x_2 x_3 \oplus x_1 x_3 \oplus x_2 x_3 \oplus x_3 \\
u = 9 \quad & x_0 x_1 x_2 x_3 \oplus x_0 x_1 x_3 \oplus x_0 x_2 x_3 \oplus x_0 x_3 \\
u = 10 \quad & x_0 x_1 x_2 x_3 \oplus x_0 x_1 x_3 \oplus x_1 x_2 x_3 \oplus x_1 x_3 \\
u = 11 \quad & x_0 x_1 x_2 x_3 \oplus x_0 x_1 x_3 \\
u = 12 \quad & x_0 x_1 x_2 x_3 \oplus x_0 x_2 x_3 \oplus x_1 x_2 x_3 \oplus x_2 x_3 \\
u = 13 \quad & x_0 x_1 x_2 x_3 \oplus x_0 x_2 x_3 \\
u = 14 \quad & x_0 x_1 x_2 x_3 \oplus x_1 x_2 x_3 \\
u = 15 \quad & x_0 x_1 x_2 x_3
\end{aligned}
$$

Figure 6: Result of retrodictive execution for the Grover oracle ($n = 4$, w in the range $\{0..15\}$).

Shor 15. The circuit in Fig. 7 uses a hand-optimized implementation of the modular exponentiation $4^x \bmod 15$ to factor 15 using Shor's algorithm. In a conventional forward execution, the state before the QFT block is:

$$
\frac{1}{2\sqrt{2}} \left((|0\rangle + |2\rangle + |4\rangle + |6\rangle) |1\rangle + (|1\rangle + |3\rangle + |5\rangle + |7\rangle) |4\rangle \right)
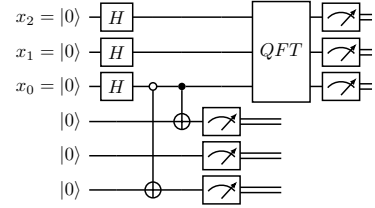$$

Figure 7: Finding the period of 4^x mod 15

At this point, the ancilla register is measured to either $|1\rangle$ or $|4\rangle$. In either case, the computational register snaps to a state of the form $\sum_{r=0}^{3} |a + 2r\rangle$ whose QFT has peaks at $|0\rangle$ or $|4\rangle$ making them the most likely outcomes of measurements of the computational register. If we measure $|0\rangle$, we repeat the experiment; otherwise we infer that the period is 2.

In the retrodictive execution, we can start with the state $|x_2 x_1 x_0 001\rangle$ since 1 is guaranteed to be a possible ancilla measurement. The first CX-gate changes the state to $|x_2 x_1 x_0 x_0 01\rangle$ and the second CX-gate produces $|x_2 x_1 x_0 x_0 0 x_0\rangle$. At that point, we reconcile the retrodictive result of the ancilla register $|x_0 0 x_0\rangle$ with the initial condition $|000\rangle$ to conclude that $x_0 = 0$. In other words, in order to observe the ancilla at 001, the computational register must be initialized to a superposition of the form $|??0\rangle$ where the least significant bit must be 0 and the other two bits are unconstrained. Expanding the possibilities, the first register needs to be in a superposition of the states $|000\rangle, |010\rangle, |100\rangle$ or $|110\rangle$ and we have just inferred using purely classical but retrodictive reasoning that the period is 2. Significantly, this approach is robust and does not require small hand-optimized circuits. Indeed, following the methods for producing quantum circuits for arithmetic operations from first principles using adders and multipliers [12], our implementation for a general circuit for $a^x \bmod 15$ has 56538 generalized Toffoli gates over 9 qubits, and yet the equations resulting from the retrodictive execution in Fig. 8 are trivial and immediately solvable as they only involve either the least significant bit x_0 (when $a \in \{4, 11, 14\}$) or the least significant two bits x_0 and x_1 (when $a \in \{2, 7, 8, 13\}$). When the solution is $x_0 = 0$, the period is 2. When the solution is $x_0 = 0, x_1 = 0$, the period is 4.

Shor 21. The examples presented so far demonstrate that some instances of quantum algorithms can be solved via classical symbolic retrodictive execution. But as was already apparent in some examples (e.g.

$a = 11$	$x_0 = 0$			$\mid x_0 = 0$
$a = 4, 14$	$1 \oplus x_0 = 1$	$x_0 = 0$		$\mid x_0 = 0$
$a = 7, 13$	$1 \oplus x_0 x_1 \oplus x_1 = 1$	$x_0 x_1 = 0$	$x_0 \oplus x_0 x_1 \oplus x_1 = 0$ $\quad x_0 \oplus x_0 x_1 = 0$	$\mid x_0 = 0, x_1 = 0$
$a = 2, 8$	$1 \oplus x_0 \oplus x_0 x_1 \oplus x_1 = 1$	$x_0 x_1 = 0$	$x_0 x_1 \oplus x_1 = 0$ $\quad\quad x_0 \oplus x_0 x_1 = 0$	$\mid x_0 = 0, x_1 = 0$

Figure 8: Equations generated by retrodictive execution of $a^x \mod 15$ starting from observed result 1 and unknown $x_8 x_7 x_6 x_5 x_4 x_3 x_2 x_1 x_0$. The solution for the unknown variables is given in the last column.

Grover), running retrodictive execution may produce large residual equations that are difficult to solve. To appreciate how large these equations may be, we include the full set of equations produced for a retrodictive execution of Shor's algorithm for factoring 21. Unlike the number 15 which corresponds to a rare occurrence of products of Fermat primes producing a period that is a power of 2 and hence trivial to represent by equations of binary numbers, the period of 21 is not easily representable as a system of equations over binary numbers. The equations which span about five pages in Sec. 2 glaringly show the limitations of the basic retrodictive execution approach and the need for additional insights.

Retrodictive Executions and Function Pre-images. Given finite sets A and B, a function $f : A \to B$ and an element $y \in B$, we define $\{\cdot \xleftarrow{f} y\}$, the pre-image of y under f, as the set $\{x \in A \mid f(x) = y\}$. For example, let $A = B = [\mathbf{2^4}]$ and let $f(x) = 7^x \mod 15$, then the collection of values that f maps to 4, $\{\cdot \xleftarrow{f} 4\}$, is the set $\{2, 6, 10, 14\}$ as shown in Fig. 9. Symbolic retrodictive execution can be seen as a method to generate boolean formulae that describe the pre-image of the function f under study. For the example in Fig. 9, retrodictive execution might generate the formulae $x_1 = 1$ and $x_0 = 0$. The (trivial in this case) solution for the formulae is indeed the set $\{2, 6, 10, 14\}$. The critical points to note, however, are that: (i) solving the equations describing the pre-image is in general an intractable (even for quantum computers) NP-complete problem, and (ii) solving the equations is not needed for the quantum algorithms in the previous section. *Only some global properties of the pre-image are needed!* Indeed, we have already seen that for solving the Deutsch-Jozsa problem, the only thing needed was whether the formula contains some variables. Also for the Bernstein-Vazirani problem, the only thing needed was the indices of the variables occurring in the formula. For Grover's algorithm, we only need to extract the singleton element in the pre-image and for Shor's algorithm we only need to extract the periodicity of the elements in the pre-image but retrodictive execution as presented so far is only able to de-quantize some rare instances of algorithms.
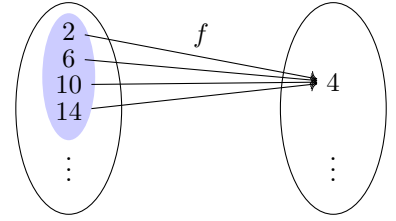

Figure 9: The pre-image of 4 under $f(x) = 7^x \mod 15$.

do communication protocols too ?

extensional vs intensional reasoning about functions

graph state: H,H,CZ 00 00 01 01 10 10 11 -11

check if H commutes with x and cx and ccx so we only need H at beginning and end

insight: QFT insensitive to 0+2+4... vs 1+3+5... so insensitive to where lsb is 0/1 so we only need to know if a variable is constant or varying fourier transform classical efficient in some cases

Ewin Tang papers

Kochen-Specker; interactive QM; observer free will; choice backtracks

universe uses lazy evaluation?

algebra of Toffoli and Hadamard ZX calculus

values going at different speeds; intervals ideas; path types

https://quantumalgorithmzoo.org

$|-\rangle$; two classes of vars; +vars and -vars; -vars infect +vars in control gates; We have two operations +red (add red) -red (remove red) Remember cx(+,-) = (-,-) Some interactions (Toffoli) want to create more refined operations +/-(1/2)(red) +/-(red) The more you do these operations the more precise it wants to be +/-(1/4)(red) +/-(1/2) red +/-(red) taint analysis with increasing precisions; truncate at desired precision (more and more colors) The taint analysis groups variables in "waves" (superpositions) of things that have the same color so the values we propagate are "red: phase=p; frequency=f; involved variables=x1,x2,..."

Possibility that collapse of wave function is information flow back from measured future to present unknown initial conditions and then back to rest of wave that was not measured

We need to explain ideas about time-reversal, prediction and retrodiction in physics. The laws of computation and the laws of physics are intimately related. When does knowing something about the future help us unveil the structure or symmetries of the past? It is like a detective story, but one with ramifications in complexity and/or efficiency. Problems involving questions where answers demand a Many(past)-to-one(future) map are at the root of our proposal

Possibility that collapse of wave function is information flow back from measured future to present unknown initial conditions and then back to rest of wave that was not measured transactional interpretation?

instead of generating one formula; generating many formulas with different weights or with various patterns of negative weights... and sum them to get the patterns we need

- Symbolic (retrodictive) evaluation as a broader perspective to classical computation

- Symbolic execution allows you to express/discover interference via shared variables

- When interference pattern is simple symbolic execution reveals solutions faster (and completely classically)

- Symbolic execution as a "classical waves" computing paradigm

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2   Methods

> You can't connect the dots looking forward; you can only connect them looking backwards. So you have to trust that the dots will somehow connect in your future. *Steve Jobs*

Lazy Evaluation.   Consider a program that searches for three different numbers x, y, and z each in the range $[1..n]$ and that sum to s. A well-established design principle for solving such problems is the *generate-and-test* computational paradigm. Following this principle, a simple program to solve this problem in the programming language Haskell is:

```
generate :: Int -> [(Int,Int,Int)]
generate n =  [(x,y,z) | x <- [1..n], y <- [1..n], z <- [1..n]]

test :: Int -> [(Int,Int,Int)] -> [(Int,Int,Int)]
test s nums = [(x,y,z) | (x,y,z) <- nums, x /= y, x /= z, y /= z, x+y+z == s]

find :: Int -> Int -> (Int,Int,Int)
find s =  head . test s . generate
```

The program consists of three functions: `generate` that produces all triples `(x,y,z)` from `(1,1,1)` to `(n,n,n)`; `test` that checks that the numbers are different and that their sum is equal to `s`; and `find` that

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composes the two functions: generating all triples, testing the ones that satisfy the condition, and returning the first solution. Running this program to find numbers in the range $[1..6]$ that sum to 15 immediately produces $(4, 5, 6)$ as expected.

But what if the range of interest was $[1..10000000]$? A naïve execution of the generate-and-test method would be prohibitively expensive as it would spend all its time generating an enormous number of triples that are un-needed. Lazy demand-driven evaluation as implemented in Haskell succeeds in a few seconds with the result $(1, 2, 12)$, however. The idea is simple: instead of eagerly generating all the triples, generate a process that, when queried, produces one triple at a time on demand. Conceptually the execution starts from the observer site which is asking for the first element of a list; this demand is propagated to the function `test` which itself propagates the demand to the function `generate`. As each triple is generated, it is tested until one triple passes the test. This triple is immediately returned without having to generate any additional values.

Partial Evaluation. Below is a Haskell program that computes a^n by repeated squaring:

```
power :: Int -> Int -> Int
power a n
    | n == 0     = 1
    | n == 1     = a
    | even n     = let r = power a (n 'div' 2) in r * r
    | otherwise  = a * power a (n-1)
```

When both inputs are known, e.g., `a = 3` and `n = 5`, the program evaluates as follows:

```
    power 3 5
=   3 * power 3 4
=   3 * (let r1 = power 3 2 in r1 * r1)
=   3 * (let r1 = (let r2 = power 3 1 in r2 * r2) in r1 * r1)
=   3 * (let r1 = (let r2 = 3 in r2 * r2) in r1 * r1)
=   3 * (let r1 = 9 in r1 * r1)
=   243
```

Partial evaluation is used when we only have partial information about the inputs. Say we only know $n = 5$. A partial evaluator then attempts to evaluate `power` with symbolic input `a` and actual input `n=5`. This evaluation proceeds as follows:

```
    power a 5
=   a * power a 4
=   a * (let r1 = power a 2 in r1 * r1)
=   a * (let r1 = (let r2 = power a 1 in r2 * r2) in r1 * r1)
=   a * (let r1 = (let r2 = a in r2 * r2) in r1 * r1)
=   a * (let r1 = a * a in r1 * r1)
=   let r1 = a * a in a * r1 * r1
```

All of this evaluation, simplification, and specialization happens without knowledge of `a`. Just knowing `n` was enough to produce a residual program that is much simpler.

The evolution of a quantum system is typically understood as proceeding forwards in time — from the present to the future. As shown in Fig. 2(a),

Since the conventional execution starts with complete ignorance about the future, the initial state is prepared as a superposition that includes every possibility. In a well-designed algorithm, , by the time the computation reaches the measurement stages, the relative phases and probability amplitudes in that enormous superposition have become biased towards states of interest which are projected to produce the final answer.

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Algebraic Normal Form (ANF).

> circuits have generalized toffoli gates: semantics (and of controls; xor with target); ANF uses exactly those two primitives; explain
> The resulting expressions are in algebraic normal form [5] where + denotes exclusive-or.
> instances with no 'and' easy to solve
> if only x and cx then symbolic execution is efficient; no need for last batch of H
> can solve problem classically
> connect with Gottsman-Knill

Function Pre-Images and NP-Complete Problems. To appreciate the difficulty of computing pre-images in general, note that finding the pre-image of a function subsumes several challenging computational problems such as pre-image attacks on hash functions [4], predicting environmental conditions that allow certain reactions to take place in computational biology [1, 2], and finding the pre-image of feature vectors in the space induced by a kernel in neural networks [3]. More to the point, the boolean satisfiability problem SAT is expressible as a boolean function over the input variables and solving a SAT problem is asking for the pre-image of true. Indeed, based on the conjectured existence of one-way functions which itself implies $P \neq NP$, all these pre-images calculations are believed to be computationally intractable in their most general setting.

Complexity Analysis.

> one pass over circuit BUT size of circuit may be exponential and complexity of normalizing to ANF not trivial

Discussion.

> observer 1 measures wires a,b; obs2 measures wires b,c; not commuting; each obs gives partial solution to equations; but partial solutions cannot lead to a global solution
> KS suggests that equations do not have unique solutions; only materialize when you measure; can associate a probability with each variable in a equation: look at all solutions and see the contribution of each variable to these solutions.

Data Availability. All execution results will be made available and can be replicated by executing the associated software.

Code Availability. The computer programs used to generate the circuits and symbolically execute the quantum algorithms retrodictively will be made publicly available.

Author Contributions. The idea of symbolic evaluation is due to A.S. The connection to retrodictive quantum mechanics is due to G.O. The connection to partial evaluation is due to J.C. Both A.S. and J.C. contributed to the software code to run the experiments. Both A.S. and G.O. contributed to the analysis of the quantum algorithms and their de-quantization. All authors contributed to the writing of the document.

Competing Interests. No competing interests.

Materials & Correspondence. The corresponding author is Gerardo Ortiz.

Supplementary Information. Equations generated by retrodictive execution of 4^x mod 21 starting from observed result 1 and unknown x. The circuit consists of 9 qubits, 36400 cx-gates, 38200 ccx-gates, and 4000 cccx-gates. There are only three equations but each equation is exponentially large.

$1 \oplus x_0 \oplus x_0 x_1 x_2 \oplus x_0 x_1 x_2 x_3 x_4 \oplus x_0 x_1 x_2 x_3 x_4 x_5 x_6 \oplus x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 \oplus x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_9 \oplus x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_8 \oplus$
$x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_8 x_9 \oplus x_0 x_1 x_2 x_3 x_4 x_5 x_7 \oplus x_0 x_1 x_2 x_3 x_4 x_5 x_7 x_8 x_9 \oplus x_0 x_1 x_2 x_3 x_4 x_5 x_7 x_9 \oplus x_0 x_1 x_2 x_3 x_4 x_5 x_8 \oplus$

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$299 \quad x_0x_1x_2x_3x_4x_5x_9 \oplus x_0x_1x_2x_3x_4x_6 \oplus x_0x_1x_2x_3x_4x_6x_7 \oplus x_0x_1x_2x_3x_4x_6x_7x_8 \oplus x_0x_1x_2x_3x_4x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_6x_8x_9 \oplus$

$300 \quad x_0x_1x_2x_3x_4x_6x_9 \oplus x_0x_1x_2x_3x_4x_7x_8 \oplus x_0x_1x_2x_3x_4x_7x_9 \oplus x_0x_1x_2x_3x_4x_8 \oplus x_0x_1x_2x_3x_4x_8x_9 \oplus x_0x_1x_2x_3x_5 \oplus$

$301 \quad x_0x_1x_2x_3x_5x_6x_7 \oplus x_0x_1x_2x_3x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_5x_6x_7x_9 \oplus x_0x_1x_2x_3x_5x_6x_8 \oplus x_0x_1x_2x_3x_5x_6x_9 \oplus x_0x_1x_2x_3x_5x_7 \oplus$

$302 \quad x_0x_1x_2x_3x_5x_7x_8 \oplus x_0x_1x_2x_3x_5x_7x_8x_9 \oplus x_0x_1x_2x_3x_5x_8x_9 \oplus x_0x_1x_2x_3x_5x_9 \oplus x_0x_1x_2x_3x_6 \oplus x_0x_1x_2x_3x_6x_7x_8 \oplus$

$303 \quad x_0x_1x_2x_3x_6x_7x_9 \oplus x_0x_1x_2x_3x_6x_8 \oplus x_0x_1x_2x_3x_6x_8x_9 \oplus x_0x_1x_2x_3x_7 \oplus x_0x_1x_2x_3x_7x_8x_9 \oplus x_0x_1x_2x_3x_7x_9 \oplus$

$304 \quad x_0x_1x_2x_3x_8 \oplus x_0x_1x_2x_3x_9 \oplus x_0x_1x_2x_4 \oplus x_0x_1x_2x_4x_5 \oplus x_0x_1x_2x_4x_5x_6 \oplus x_0x_1x_2x_4x_5x_6x_7 \oplus x_0x_1x_2x_4x_5x_6x_7x_8 \oplus$

$305 \quad x_0x_1x_2x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_4x_5x_6x_8x_9 \oplus x_0x_1x_2x_4x_5x_6x_9 \oplus x_0x_1x_2x_4x_5x_7x_8 \oplus x_0x_1x_2x_4x_5x_7x_9 \oplus x_0x_1x_2x_4x_5x_8 \oplus$

$306 \quad x_0x_1x_2x_4x_5x_8x_9 \oplus x_0x_1x_2x_4x_6x_7 \oplus x_0x_1x_2x_4x_6x_7x_8x_9 \oplus x_0x_1x_2x_4x_6x_7x_9 \oplus x_0x_1x_2x_4x_6x_8 \oplus x_0x_1x_2x_4x_6x_9 \oplus$

$307 \quad x_0x_1x_2x_4x_7 \oplus x_0x_1x_2x_4x_7x_8 \oplus x_0x_1x_2x_4x_7x_8x_9 \oplus x_0x_1x_2x_4x_8x_9 \oplus x_0x_1x_2x_4x_9 \oplus x_0x_1x_2x_5x_6 \oplus x_0x_1x_2x_5x_6x_7x_8 \oplus$

$308 \quad x_0x_1x_2x_5x_6x_7x_9 \oplus x_0x_1x_2x_5x_6x_8 \oplus x_0x_1x_2x_5x_6x_8x_9 \oplus x_0x_1x_2x_5x_7 \oplus x_0x_1x_2x_5x_7x_8x_9 \oplus x_0x_1x_2x_5x_7x_9 \oplus$

$309 \quad x_0x_1x_2x_5x_8 \oplus x_0x_1x_2x_5x_9 \oplus x_0x_1x_2x_6 \oplus x_0x_1x_2x_6x_7 \oplus x_0x_1x_2x_6x_7x_8 \oplus x_0x_1x_2x_6x_7x_8x_9 \oplus x_0x_1x_2x_6x_8x_9 \oplus$

$310 \quad x_0x_1x_2x_6x_9 \oplus x_0x_1x_2x_7x_8 \oplus x_0x_1x_2x_7x_9 \oplus x_0x_1x_2x_8 \oplus x_0x_1x_2x_8x_9 \oplus x_0x_1x_3 \oplus x_0x_1x_3x_4x_5 \oplus x_0x_1x_3x_4x_5x_6x_7 \oplus$

$311 \quad x_0x_1x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_3x_4x_5x_6x_7x_9 \oplus x_0x_1x_3x_4x_5x_6x_8 \oplus x_0x_1x_3x_4x_5x_6x_9 \oplus x_0x_1x_3x_4x_5x_7 \oplus x_0x_1x_3x_4x_5x_7x_8 \oplus$

$312 \quad x_0x_1x_3x_4x_5x_7x_8x_9 \oplus x_0x_1x_3x_4x_5x_8x_9 \oplus x_0x_1x_3x_4x_5x_9 \oplus x_0x_1x_3x_4x_6 \oplus x_0x_1x_3x_4x_6x_7x_8 \oplus x_0x_1x_3x_4x_6x_7x_9 \oplus$

$313 \quad x_0x_1x_3x_4x_6x_8 \oplus x_0x_1x_3x_4x_6x_8x_9 \oplus x_0x_1x_3x_4x_7 \oplus x_0x_1x_3x_4x_7x_8x_9 \oplus x_0x_1x_3x_4x_7x_9 \oplus x_0x_1x_3x_4x_8 \oplus x_0x_1x_3x_4x_9 \oplus$

$314 \quad x_0x_1x_3x_5 \oplus x_0x_1x_3x_5x_6 \oplus x_0x_1x_3x_5x_6x_7 \oplus x_0x_1x_3x_5x_6x_7x_8 \oplus x_0x_1x_3x_5x_6x_7x_8x_9 \oplus x_0x_1x_3x_5x_6x_8x_9 \oplus x_0x_1x_3x_5x_6x_9 \oplus$

$315 \quad x_0x_1x_3x_5x_7x_8 \oplus x_0x_1x_3x_5x_7x_9 \oplus x_0x_1x_3x_5x_8 \oplus x_0x_1x_3x_5x_8x_9 \oplus x_0x_1x_3x_6x_7 \oplus x_0x_1x_3x_6x_7x_8x_9 \oplus x_0x_1x_3x_6x_7x_9 \oplus$

$316 \quad x_0x_1x_3x_6x_8 \oplus x_0x_1x_3x_6x_9 \oplus x_0x_1x_3x_7 \oplus x_0x_1x_3x_7x_8 \oplus x_0x_1x_3x_7x_8x_9 \oplus x_0x_1x_3x_8x_9 \oplus x_0x_1x_3x_9 \oplus x_0x_1x_4 \oplus$

$317 \quad x_0x_1x_4x_5x_6 \oplus x_0x_1x_4x_5x_6x_7x_8 \oplus x_0x_1x_4x_5x_6x_7x_9 \oplus x_0x_1x_4x_5x_6x_8 \oplus x_0x_1x_4x_5x_6x_8x_9 \oplus x_0x_1x_4x_5x_7 \oplus x_0x_1x_4x_5x_7x_8x_9 \oplus$

$318 \quad x_0x_1x_4x_5x_7x_9 \oplus x_0x_1x_4x_5x_8 \oplus x_0x_1x_4x_5x_9 \oplus x_0x_1x_4x_6 \oplus x_0x_1x_4x_6x_7 \oplus x_0x_1x_4x_6x_7x_8 \oplus x_0x_1x_4x_6x_7x_8x_9 \oplus$

$319 \quad x_0x_1x_4x_6x_8x_9 \oplus x_0x_1x_4x_6x_9 \oplus x_0x_1x_4x_7x_8 \oplus x_0x_1x_4x_7x_9 \oplus x_0x_1x_4x_8 \oplus x_0x_1x_4x_8x_9 \oplus x_0x_1x_5 \oplus x_0x_1x_5x_6x_7 \oplus$

$320 \quad x_0x_1x_5x_6x_7x_8x_9 \oplus x_0x_1x_5x_6x_7x_9 \oplus x_0x_1x_5x_6x_8 \oplus x_0x_1x_5x_6x_9 \oplus x_0x_1x_5x_7 \oplus x_0x_1x_5x_7x_8 \oplus x_0x_1x_5x_7x_8x_9 \oplus$

$321 \quad x_0x_1x_5x_8x_9 \oplus x_0x_1x_5x_9 \oplus x_0x_1x_6 \oplus x_0x_1x_6x_7x_8 \oplus x_0x_1x_6x_7x_9 \oplus x_0x_1x_6x_8 \oplus x_0x_1x_6x_8x_9 \oplus x_0x_1x_7 \oplus x_0x_1x_7x_8x_9 \oplus$

$322 \quad x_0x_1x_7x_9 \oplus x_0x_1x_8 \oplus x_0x_1x_9 \oplus x_0x_2 \oplus x_0x_2x_3 \oplus x_0x_2x_3x_4 \oplus x_0x_2x_3x_4x_5 \oplus x_0x_2x_3x_4x_5x_6 \oplus x_0x_2x_3x_4x_5x_6x_7 \oplus$

$323 \quad x_0x_2x_3x_4x_5x_6x_7x_8 \oplus x_0x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_2x_3x_4x_5x_6x_8x_9 \oplus x_0x_2x_3x_4x_5x_6x_9 \oplus x_0x_2x_3x_4x_5x_7x_8 \oplus x_0x_2x_3x_4x_5x_7x_9 \oplus$

$324 \quad x_0x_2x_3x_4x_5x_8 \oplus x_0x_2x_3x_4x_5x_8x_9 \oplus x_0x_2x_3x_4x_6x_7 \oplus x_0x_2x_3x_4x_6x_7x_8 \oplus x_0x_2x_3x_4x_6x_7x_9 \oplus x_0x_2x_3x_4x_6x_8 \oplus$

$325 \quad x_0x_2x_3x_4x_6x_9 \oplus x_0x_2x_3x_4x_7 \oplus x_0x_2x_3x_4x_7x_8 \oplus x_0x_2x_3x_4x_7x_8x_9 \oplus x_0x_2x_3x_4x_8x_9 \oplus x_0x_2x_3x_4x_9 \oplus x_0x_2x_3x_5x_6 \oplus$

$326 \quad x_0x_2x_3x_5x_6x_7x_8 \oplus x_0x_2x_3x_5x_6x_7x_9 \oplus x_0x_2x_3x_5x_6x_8 \oplus x_0x_2x_3x_5x_6x_8x_9 \oplus x_0x_2x_3x_5x_7 \oplus x_0x_2x_3x_5x_7x_8x_9 \oplus$

$327 \quad x_0x_2x_3x_5x_7x_9 \oplus x_0x_2x_3x_5x_8 \oplus x_0x_2x_3x_5x_9 \oplus x_0x_2x_3x_6 \oplus x_0x_2x_3x_6x_7 \oplus x_0x_2x_3x_6x_7x_8 \oplus x_0x_2x_3x_6x_7x_8x_9 \oplus$

$328 \quad x_0x_2x_3x_6x_8x_9 \oplus x_0x_2x_3x_6x_9 \oplus x_0x_2x_3x_7x_8 \oplus x_0x_2x_3x_7x_9 \oplus x_0x_2x_3x_8 \oplus x_0x_2x_3x_8x_9 \oplus x_0x_2x_4x_5 \oplus x_0x_2x_4x_5x_6x_7 \oplus$

$329 \quad x_0x_2x_4x_5x_6x_7x_8x_9 \oplus x_0x_2x_4x_5x_6x_7x_9 \oplus x_0x_2x_4x_5x_6x_8 \oplus x_0x_2x_4x_5x_6x_9 \oplus x_0x_2x_4x_5x_7 \oplus x_0x_2x_4x_5x_7x_8 \oplus$

$330 \quad x_0x_2x_4x_5x_7x_8x_9 \oplus x_0x_2x_4x_5x_8x_9 \oplus x_0x_2x_4x_5x_9 \oplus x_0x_2x_4x_6 \oplus x_0x_2x_4x_6x_7x_8 \oplus x_0x_2x_4x_6x_7x_9 \oplus x_0x_2x_4x_6x_8 \oplus$

$331 \quad x_0x_2x_4x_6x_8x_9 \oplus x_0x_2x_4x_7 \oplus x_0x_2x_4x_7x_8x_9 \oplus x_0x_2x_4x_7x_9 \oplus x_0x_2x_4x_8 \oplus x_0x_2x_4x_9 \oplus x_0x_2x_5 \oplus x_0x_2x_5x_6 \oplus$

$332 \quad x_0x_2x_5x_6x_7 \oplus x_0x_2x_5x_6x_7x_8 \oplus x_0x_2x_5x_6x_7x_8x_9 \oplus x_0x_2x_5x_6x_8x_9 \oplus x_0x_2x_5x_6x_9 \oplus x_0x_2x_5x_7x_8 \oplus x_0x_2x_5x_7x_9 \oplus$

$333 \quad x_0x_2x_5x_8 \oplus x_0x_2x_5x_8x_9 \oplus x_0x_2x_6x_7 \oplus x_0x_2x_6x_7x_8x_9 \oplus x_0x_2x_6x_7x_9 \oplus x_0x_2x_6x_8 \oplus x_0x_2x_6x_9 \oplus x_0x_2x_7 \oplus$

$334 \quad x_0x_2x_7x_8 \oplus x_0x_2x_7x_8x_9 \oplus x_0x_2x_8x_9 \oplus x_0x_2x_9 \oplus x_0x_3x_4 \oplus x_0x_3x_4x_5x_6 \oplus x_0x_3x_4x_5x_6x_7x_8 \oplus x_0x_3x_4x_5x_6x_7x_9 \oplus$

$335 \quad x_0x_3x_4x_5x_6x_8 \oplus x_0x_3x_4x_5x_6x_8x_9 \oplus x_0x_3x_4x_5x_7 \oplus x_0x_3x_4x_5x_7x_8x_9 \oplus x_0x_3x_4x_5x_7x_9 \oplus x_0x_3x_4x_5x_8 \oplus x_0x_3x_4x_5x_9 \oplus$

$336 \quad x_0x_3x_4x_6 \oplus x_0x_3x_4x_6x_7 \oplus x_0x_3x_4x_6x_7x_8 \oplus x_0x_3x_4x_6x_7x_8x_9 \oplus x_0x_3x_4x_6x_8x_9 \oplus x_0x_3x_4x_6x_9 \oplus x_0x_3x_4x_7x_8 \oplus$

$337 \quad x_0x_3x_4x_7x_9 \oplus x_0x_3x_4x_8 \oplus x_0x_3x_4x_8x_9 \oplus x_0x_3x_5 \oplus x_0x_3x_5x_6x_7 \oplus x_0x_3x_5x_6x_7x_8x_9 \oplus x_0x_3x_5x_6x_7x_9 \oplus x_0x_3x_5x_6x_8 \oplus$

$338 \quad x_0x_3x_5x_6x_9 \oplus x_0x_3x_5x_7 \oplus x_0x_3x_5x_7x_8 \oplus x_0x_3x_5x_7x_8x_9 \oplus x_0x_3x_5x_8x_9 \oplus x_0x_3x_5x_9 \oplus x_0x_3x_6 \oplus x_0x_3x_6x_7x_8 \oplus$

$339 \quad x_0x_3x_6x_7x_9 \oplus x_0x_3x_6x_8 \oplus x_0x_3x_6x_8x_9 \oplus x_0x_3x_7 \oplus x_0x_3x_7x_8x_9 \oplus x_0x_3x_7x_9 \oplus x_0x_3x_8 \oplus x_0x_3x_9 \oplus x_0x_4 \oplus x_0x_4x_5 \oplus$

$340 \quad x_0x_4x_5x_6 \oplus x_0x_4x_5x_6x_7 \oplus x_0x_4x_5x_6x_7x_8 \oplus x_0x_4x_5x_6x_7x_8x_9 \oplus x_0x_4x_5x_6x_8x_9 \oplus x_0x_4x_5x_6x_9 \oplus x_0x_4x_5x_7x_8 \oplus$

$341 \quad x_0x_4x_5x_7x_9 \oplus x_0x_4x_5x_8 \oplus x_0x_4x_5x_8x_9 \oplus x_0x_4x_6x_7 \oplus x_0x_4x_6x_7x_8x_9 \oplus x_0x_4x_6x_7x_9 \oplus x_0x_4x_6x_8 \oplus x_0x_4x_6x_9 \oplus$

$342 \quad x_0x_4x_7 \oplus x_0x_4x_7x_8 \oplus x_0x_4x_7x_8x_9 \oplus x_0x_4x_8x_9 \oplus x_0x_4x_9 \oplus x_0x_5x_6 \oplus x_0x_5x_6x_7x_8 \oplus x_0x_5x_6x_7x_9 \oplus x_0x_5x_6x_8 \oplus$

$343 \quad x_0x_5x_6x_8x_9 \oplus x_0x_5x_7 \oplus x_0x_5x_7x_8x_9 \oplus x_0x_5x_7x_9 \oplus x_0x_5x_8 \oplus x_0x_5x_9 \oplus x_0x_6 \oplus x_0x_6x_7 \oplus x_0x_6x_7x_8 \oplus x_0x_6x_7x_8x_9 \oplus$

$344 \quad x_0x_6x_8x_9 \oplus x_0x_6x_9 \oplus x_0x_7x_8 \oplus x_0x_7x_9 \oplus x_0x_8 \oplus x_0x_8x_9 \oplus x_1 \oplus x_1x_2x_3 \oplus x_1x_2x_3x_4x_5 \oplus x_1x_2x_3x_4x_5x_6x_7 \oplus$

$345 \quad x_1x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_1x_2x_3x_4x_5x_6x_7x_9 \oplus x_1x_2x_3x_4x_5x_6x_8 \oplus x_1x_2x_3x_4x_5x_6x_9 \oplus x_1x_2x_3x_4x_5x_7 \oplus x_1x_2x_3x_4x_5x_7x_8 \oplus$

$346 \quad x_1x_2x_3x_4x_5x_7x_8x_9 \oplus x_1x_2x_3x_4x_5x_8x_9 \oplus x_1x_2x_3x_4x_5x_9 \oplus x_1x_2x_3x_4x_6 \oplus x_1x_2x_3x_4x_6x_7x_8 \oplus x_1x_2x_3x_4x_6x_7x_9 \oplus$

$347 \quad x_1x_2x_3x_4x_6x_8 \oplus x_1x_2x_3x_4x_6x_8x_9 \oplus x_1x_2x_3x_4x_7 \oplus x_1x_2x_3x_4x_7x_8x_9 \oplus x_1x_2x_3x_4x_7x_9 \oplus x_1x_2x_3x_4x_8 \oplus x_1x_2x_3x_4x_9 \oplus$

$348 \quad x_1x_2x_3x_5 \oplus x_1x_2x_3x_5x_6 \oplus x_1x_2x_3x_5x_6x_7 \oplus x_1x_2x_3x_5x_6x_7x_8 \oplus x_1x_2x_3x_5x_6x_7x_8x_9 \oplus x_1x_2x_3x_5x_6x_8x_9 \oplus x_1x_2x_3x_5x_6x_9 \oplus$

$349 \quad x_1x_2x_3x_5x_7x_8 \oplus x_1x_2x_3x_5x_7x_9 \oplus x_1x_2x_3x_5x_8 \oplus x_1x_2x_3x_5x_8x_9 \oplus x_1x_2x_3x_6x_7 \oplus x_1x_2x_3x_6x_7x_8x_9 \oplus x_1x_2x_3x_6x_7x_9 \oplus$

$x_1x_2x_3x_6x_8 \oplus x_1x_2x_3x_6x_9 \oplus x_1x_2x_3x_7 \oplus x_1x_2x_3x_7x_8 \oplus x_1x_2x_3x_7x_8x_9 \oplus x_1x_2x_3x_8x_9 \oplus x_1x_2x_3x_9 \oplus x_1x_2x_4 \oplus$

$x_1x_2x_4x_5x_6 \oplus x_1x_2x_4x_5x_6x_7x_8 \oplus x_1x_2x_4x_5x_6x_7x_9 \oplus x_1x_2x_4x_5x_6x_8 \oplus x_1x_2x_4x_5x_6x_8x_9 \oplus x_1x_2x_4x_5x_7 \oplus x_1x_2x_4x_5x_7x_8x_9 \oplus$

$x_1x_2x_4x_5x_7x_9 \oplus x_1x_2x_4x_5x_8 \oplus x_1x_2x_4x_5x_9 \oplus x_1x_2x_4x_6 \oplus x_1x_2x_4x_6x_7 \oplus x_1x_2x_4x_6x_7x_8 \oplus x_1x_2x_4x_6x_7x_8x_9 \oplus$

$x_1x_2x_4x_6x_8x_9 \oplus x_1x_2x_4x_6x_9 \oplus x_1x_2x_4x_7x_8 \oplus x_1x_2x_4x_7x_9 \oplus x_1x_2x_4x_8 \oplus x_1x_2x_4x_8x_9 \oplus x_1x_2x_5 \oplus x_1x_2x_5x_6x_7 \oplus$

$x_1x_2x_5x_6x_7x_8x_9 \oplus x_1x_2x_5x_6x_7x_9 \oplus x_1x_2x_5x_6x_8 \oplus x_1x_2x_5x_6x_9 \oplus x_1x_2x_5x_7 \oplus x_1x_2x_5x_7x_8 \oplus x_1x_2x_5x_7x_8x_9 \oplus$

$x_1x_2x_5x_8x_9 \oplus x_1x_2x_5x_9 \oplus x_1x_2x_6 \oplus x_1x_2x_6x_7x_8 \oplus x_1x_2x_6x_7x_9 \oplus x_1x_2x_6x_8 \oplus x_1x_2x_6x_8x_9 \oplus x_1x_2x_7 \oplus x_1x_2x_7x_8x_9 \oplus$

$x_1x_2x_7x_9 \oplus x_1x_2x_8 \oplus x_1x_2x_9 \oplus x_1x_3 \oplus x_1x_3x_4 \oplus x_1x_3x_4x_5 \oplus x_1x_3x_4x_5x_6 \oplus x_1x_3x_4x_5x_6x_7 \oplus x_1x_3x_4x_5x_6x_7x_8 \oplus$

$x_1x_3x_4x_5x_6x_7x_8x_9 \oplus x_1x_3x_4x_5x_6x_8x_9 \oplus x_1x_3x_4x_5x_6x_9 \oplus x_1x_3x_4x_5x_7x_8 \oplus x_1x_3x_4x_5x_7x_9 \oplus x_1x_3x_4x_5x_8 \oplus$

$x_1x_3x_4x_5x_8x_9 \oplus x_1x_3x_4x_6x_7 \oplus x_1x_3x_4x_6x_7x_8x_9 \oplus x_1x_3x_4x_6x_7x_9 \oplus x_1x_3x_4x_6x_8 \oplus x_1x_3x_4x_6x_9 \oplus x_1x_3x_4x_7 \oplus$

$x_1x_3x_4x_7x_8 \oplus x_1x_3x_4x_7x_8x_9 \oplus x_1x_3x_4x_8x_9 \oplus x_1x_3x_4x_9 \oplus x_1x_3x_5x_6 \oplus x_1x_3x_5x_6x_7x_8 \oplus x_1x_3x_5x_6x_7x_9 \oplus x_1x_3x_5x_6x_8 \oplus$

$x_1x_3x_5x_6x_8x_9 \oplus x_1x_3x_5x_7 \oplus x_1x_3x_5x_7x_8x_9 \oplus x_1x_3x_5x_7x_9 \oplus x_1x_3x_5x_8 \oplus x_1x_3x_5x_9 \oplus x_1x_3x_6 \oplus x_1x_3x_6x_7 \oplus$

$x_1x_3x_6x_7x_8 \oplus x_1x_3x_6x_7x_8x_9 \oplus x_1x_3x_6x_8x_9 \oplus x_1x_3x_6x_9 \oplus x_1x_3x_7x_8 \oplus x_1x_3x_7x_9 \oplus x_1x_3x_8 \oplus x_1x_3x_8x_9 \oplus x_1x_4x_5 \oplus$

$x_1x_4x_5x_6x_7 \oplus x_1x_4x_5x_6x_7x_8x_9 \oplus x_1x_4x_5x_6x_7x_9 \oplus x_1x_4x_5x_6x_8 \oplus x_1x_4x_5x_6x_9 \oplus x_1x_4x_5x_7 \oplus x_1x_4x_5x_7x_8 \oplus$

$x_1x_4x_5x_7x_8x_9 \oplus x_1x_4x_5x_8x_9 \oplus x_1x_4x_5x_9 \oplus x_1x_4x_6 \oplus x_1x_4x_6x_7x_8 \oplus x_1x_4x_6x_7x_9 \oplus x_1x_4x_6x_8 \oplus x_1x_4x_6x_8x_9 \oplus$

$x_1x_4x_7 \oplus x_1x_4x_7x_8x_9 \oplus x_1x_4x_7x_9 \oplus x_1x_4x_8 \oplus x_1x_4x_9 \oplus x_1x_5 \oplus x_1x_5x_6 \oplus x_1x_5x_6x_7 \oplus x_1x_5x_6x_7x_8 \oplus x_1x_5x_6x_7x_8x_9 \oplus$

$x_1x_5x_6x_8x_9 \oplus x_1x_5x_6x_9 \oplus x_1x_5x_7x_8 \oplus x_1x_5x_7x_9 \oplus x_1x_5x_8 \oplus x_1x_5x_8x_9 \oplus x_1x_6x_7 \oplus x_1x_6x_7x_8x_9 \oplus x_1x_6x_7x_9 \oplus$

$x_1x_6x_8 \oplus x_1x_6x_9 \oplus x_1x_7 \oplus x_1x_7x_8 \oplus x_1x_7x_8x_9 \oplus x_1x_8x_9 \oplus x_1x_9 \oplus x_2 \oplus x_2x_3x_4 \oplus x_2x_3x_4x_5x_6 \oplus x_2x_3x_4x_5x_6x_7x_8 \oplus$

$x_2x_3x_4x_5x_6x_7x_9 \oplus x_2x_3x_4x_5x_6x_8 \oplus x_2x_3x_4x_5x_6x_8x_9 \oplus x_2x_3x_4x_5x_7 \oplus x_2x_3x_4x_5x_7x_8x_9 \oplus x_2x_3x_4x_5x_7x_9 \oplus$

$x_2x_3x_4x_5x_8 \oplus x_2x_3x_4x_5x_9 \oplus x_2x_3x_4x_6 \oplus x_2x_3x_4x_6x_7 \oplus x_2x_3x_4x_6x_7x_8 \oplus x_2x_3x_4x_6x_7x_8x_9 \oplus x_2x_3x_4x_6x_8x_9 \oplus$

$x_2x_3x_4x_6x_9 \oplus x_2x_3x_4x_7x_8 \oplus x_2x_3x_4x_7x_9 \oplus x_2x_3x_4x_8 \oplus x_2x_3x_4x_8x_9 \oplus x_2x_3x_5 \oplus x_2x_3x_5x_6x_7 \oplus x_2x_3x_5x_6x_7x_8x_9 \oplus$

$x_2x_3x_5x_6x_7x_9 \oplus x_2x_3x_5x_6x_8 \oplus x_2x_3x_5x_6x_9 \oplus x_2x_3x_5x_7 \oplus x_2x_3x_5x_7x_8 \oplus x_2x_3x_5x_7x_8x_9 \oplus x_2x_3x_5x_8x_9 \oplus x_2x_3x_5x_9 \oplus$

$x_2x_3x_6 \oplus x_2x_3x_6x_7x_8 \oplus x_2x_3x_6x_7x_9 \oplus x_2x_3x_6x_8 \oplus x_2x_3x_6x_8x_9 \oplus x_2x_3x_7 \oplus x_2x_3x_7x_8x_9 \oplus x_2x_3x_7x_9 \oplus x_2x_3x_8 \oplus$

$x_2x_3x_9 \oplus x_2x_4 \oplus x_2x_4x_5 \oplus x_2x_4x_5x_6 \oplus x_2x_4x_5x_6x_7 \oplus x_2x_4x_5x_6x_7x_8 \oplus x_2x_4x_5x_6x_7x_8x_9 \oplus x_2x_4x_5x_6x_8x_9 \oplus$

$x_2x_4x_5x_6x_9 \oplus x_2x_4x_5x_7x_8 \oplus x_2x_4x_5x_7x_9 \oplus x_2x_4x_5x_8 \oplus x_2x_4x_5x_8x_9 \oplus x_2x_4x_6x_7 \oplus x_2x_4x_6x_7x_8x_9 \oplus x_2x_4x_6x_7x_9 \oplus$

$x_2x_4x_6x_8 \oplus x_2x_4x_6x_9 \oplus x_2x_4x_7 \oplus x_2x_4x_7x_8 \oplus x_2x_4x_7x_8x_9 \oplus x_2x_4x_8x_9 \oplus x_2x_4x_9 \oplus x_2x_5x_6 \oplus x_2x_5x_6x_7x_8 \oplus$

$x_2x_5x_6x_7x_9 \oplus x_2x_5x_6x_8 \oplus x_2x_5x_6x_8x_9 \oplus x_2x_5x_7 \oplus x_2x_5x_7x_8x_9 \oplus x_2x_5x_7x_9 \oplus x_2x_5x_8 \oplus x_2x_5x_9 \oplus x_2x_6 \oplus x_2x_6x_7 \oplus$

$x_2x_6x_7x_8 \oplus x_2x_6x_7x_8x_9 \oplus x_2x_6x_8x_9 \oplus x_2x_6x_9 \oplus x_2x_7x_8 \oplus x_2x_7x_9 \oplus x_2x_8 \oplus x_2x_8x_9 \oplus x_3 \oplus x_3x_4x_5 \oplus x_3x_4x_5x_6x_7 \oplus$

$x_3x_4x_5x_6x_7x_8x_9 \oplus x_3x_4x_5x_6x_7x_9 \oplus x_3x_4x_5x_6x_8 \oplus x_3x_4x_5x_6x_9 \oplus x_3x_4x_5x_7 \oplus x_3x_4x_5x_7x_8 \oplus x_3x_4x_5x_7x_8x_9 \oplus$

$x_3x_4x_5x_8x_9 \oplus x_3x_4x_5x_9 \oplus x_3x_4x_6 \oplus x_3x_4x_6x_7x_8 \oplus x_3x_4x_6x_7x_9 \oplus x_3x_4x_6x_8 \oplus x_3x_4x_6x_8x_9 \oplus x_3x_4x_7 \oplus x_3x_4x_7x_8x_9 \oplus$

$x_3x_4x_7x_9 \oplus x_3x_4x_8 \oplus x_3x_4x_9 \oplus x_3x_5 \oplus x_3x_5x_6 \oplus x_3x_5x_6x_7 \oplus x_3x_5x_6x_7x_8 \oplus x_3x_5x_6x_7x_8x_9 \oplus x_3x_5x_6x_8x_9 \oplus$

$x_3x_5x_6x_9 \oplus x_3x_5x_7x_8 \oplus x_3x_5x_7x_9 \oplus x_3x_5x_8 \oplus x_3x_5x_8x_9 \oplus x_3x_6x_7 \oplus x_3x_6x_7x_8x_9 \oplus x_3x_6x_7x_9 \oplus x_3x_6x_8 \oplus x_3x_6x_9 \oplus$

$x_3x_7 \oplus x_3x_7x_8 \oplus x_3x_7x_8x_9 \oplus x_3x_8x_9 \oplus x_3x_9 \oplus x_4 \oplus x_4x_5x_6 \oplus x_4x_5x_6x_7x_8 \oplus x_4x_5x_6x_7x_9 \oplus x_4x_5x_6x_8 \oplus x_4x_5x_6x_8x_9 \oplus$

$x_4x_5x_7 \oplus x_4x_5x_7x_8x_9 \oplus x_4x_5x_7x_9 \oplus x_4x_5x_8 \oplus x_4x_5x_9 \oplus x_4x_6 \oplus x_4x_6x_7 \oplus x_4x_6x_7x_8 \oplus x_4x_6x_7x_8x_9 \oplus x_4x_6x_8x_9 \oplus$

$x_4x_6x_9 \oplus x_4x_7x_8 \oplus x_4x_7x_9 \oplus x_4x_8 \oplus x_4x_8x_9 \oplus x_5 \oplus x_5x_6x_7 \oplus x_5x_6x_7x_8x_9 \oplus x_5x_6x_7x_9 \oplus x_5x_6x_8 \oplus x_5x_6x_9 \oplus x_5x_7 \oplus$

$x_5x_7x_8 \oplus x_5x_7x_8x_9 \oplus x_5x_8x_9 \oplus x_5x_9 \oplus x_6 \oplus x_6x_7x_8 \oplus x_6x_7x_9 \oplus x_6x_8 \oplus x_6x_8x_9 \oplus x_7 \oplus x_7x_8x_9 \oplus x_7x_9 \oplus x_8 \oplus x_9 = 1$

$x_0x_1 \oplus x_0x_1x_2x_3 \oplus x_0x_1x_2x_3x_4x_5 \oplus x_0x_1x_2x_3x_4x_5x_6x_7 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_9 \oplus$

$x_0x_1x_2x_3x_4x_5x_6x_8 \oplus x_0x_1x_2x_3x_4x_5x_6x_9 \oplus x_0x_1x_2x_3x_4x_5x_7 \oplus x_0x_1x_2x_3x_4x_5x_7x_8 \oplus x_0x_1x_2x_3x_4x_5x_7x_8x_9 \oplus$

$x_0x_1x_2x_3x_4x_5x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_9 \oplus x_0x_1x_2x_3x_4x_6 \oplus x_0x_1x_2x_3x_4x_6x_7x_8 \oplus x_0x_1x_2x_3x_4x_6x_7x_9 \oplus x_0x_1x_2x_3x_4x_6x_8 \oplus$

$x_0x_1x_2x_3x_4x_6x_8x_9 \oplus x_0x_1x_2x_3x_4x_7 \oplus x_0x_1x_2x_3x_4x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_7x_9 \oplus x_0x_1x_2x_3x_4x_8 \oplus x_0x_1x_2x_3x_4x_9 \oplus$

$x_0x_1x_2x_3x_5 \oplus x_0x_1x_2x_3x_5x_6 \oplus x_0x_1x_2x_3x_5x_6x_7 \oplus x_0x_1x_2x_3x_5x_6x_7x_8 \oplus x_0x_1x_2x_3x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_5x_6x_8x_9 \oplus$

$x_0x_1x_2x_3x_5x_6x_9 \oplus x_0x_1x_2x_3x_5x_7x_8 \oplus x_0x_1x_2x_3x_5x_7x_9 \oplus x_0x_1x_2x_3x_5x_8 \oplus x_0x_1x_2x_3x_5x_8x_9 \oplus x_0x_1x_2x_3x_6x_7 \oplus$

$x_0x_1x_2x_3x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_6x_7x_9 \oplus x_0x_1x_2x_3x_6x_8 \oplus x_0x_1x_2x_3x_6x_9 \oplus x_0x_1x_2x_3x_7 \oplus x_0x_1x_2x_3x_7x_8 \oplus$

$x_0x_1x_2x_3x_7x_8x_9 \oplus x_0x_1x_2x_3x_8x_9 \oplus x_0x_1x_2x_3x_9 \oplus x_0x_1x_2x_4 \oplus x_0x_1x_2x_4x_5x_6 \oplus x_0x_1x_2x_4x_5x_6x_7x_8 \oplus x_0x_1x_2x_4x_5x_6x_7x_9 \oplus$

$x_0x_1x_2x_4x_5x_6x_8 \oplus x_0x_1x_2x_4x_5x_6x_8x_9 \oplus x_0x_1x_2x_4x_5x_7 \oplus x_0x_1x_2x_4x_5x_7x_8x_9 \oplus x_0x_1x_2x_4x_5x_7x_9 \oplus x_0x_1x_2x_4x_5x_8 \oplus$

$x_0x_1x_2x_4x_5x_9 \oplus x_0x_1x_2x_4x_6 \oplus x_0x_1x_2x_4x_6x_7 \oplus x_0x_1x_2x_4x_6x_7x_8 \oplus x_0x_1x_2x_4x_6x_7x_8x_9 \oplus x_0x_1x_2x_4x_6x_8x_9 \oplus$

$x_0x_1x_2x_4x_6x_9 \oplus x_0x_1x_2x_4x_7x_8 \oplus x_0x_1x_2x_4x_7x_9 \oplus x_0x_1x_2x_4x_8 \oplus x_0x_1x_2x_4x_8x_9 \oplus x_0x_1x_2x_5 \oplus x_0x_1x_2x_5x_6x_7 \oplus$

$x_0x_1x_2x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_5x_6x_7x_9 \oplus x_0x_1x_2x_5x_6x_8 \oplus x_0x_1x_2x_5x_6x_9 \oplus x_0x_1x_2x_5x_7 \oplus x_0x_1x_2x_5x_7x_8 \oplus$

$x_0x_1x_2x_5x_7x_8x_9 \oplus x_0x_1x_2x_5x_8x_9 \oplus x_0x_1x_2x_5x_9 \oplus x_0x_1x_2x_6 \oplus x_0x_1x_2x_6x_7x_8 \oplus x_0x_1x_2x_6x_7x_9 \oplus x_0x_1x_2x_6x_8 \oplus$

$x_0x_1x_2x_6x_8x_9 \oplus x_0x_1x_2x_7 \oplus x_0x_1x_2x_7x_8x_9 \oplus x_0x_1x_2x_7x_9 \oplus x_0x_1x_2x_8 \oplus x_0x_1x_2x_9 \oplus x_0x_1x_3 \oplus x_0x_1x_3x_4 \oplus$

$x_0x_1x_3x_4x_5 \oplus x_0x_1x_3x_4x_5x_6 \oplus x_0x_1x_3x_4x_5x_6x_7 \oplus x_0x_1x_3x_4x_5x_6x_7x_8 \oplus x_0x_1x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_3x_4x_5x_6x_8x_9 \oplus$

$x_0x_1x_3x_4x_5x_6x_9 \oplus x_0x_1x_3x_4x_5x_7x_8 \oplus x_0x_1x_3x_4x_5x_7x_9 \oplus x_0x_1x_3x_4x_5x_8 \oplus x_0x_1x_3x_4x_5x_8x_9 \oplus x_0x_1x_3x_4x_6x_7 \oplus$

401   $x_0x_1x_3x_4x_6x_7x_8x_9 \oplus x_0x_1x_3x_4x_6x_7x_9 \oplus x_0x_1x_3x_4x_6x_8 \oplus x_0x_1x_3x_4x_6x_9 \oplus x_0x_1x_3x_4x_7 \oplus x_0x_1x_3x_4x_7x_8 \oplus$

402   $x_0x_1x_3x_4x_7x_8x_9 \oplus x_0x_1x_3x_4x_8x_9 \oplus x_0x_1x_3x_4x_9 \oplus x_0x_1x_3x_5x_6 \oplus x_0x_1x_3x_5x_6x_7x_8 \oplus x_0x_1x_3x_5x_6x_7x_9 \oplus x_0x_1x_3x_5x_6x_8 \oplus$

403   $x_0x_1x_3x_5x_6x_8x_9 \oplus x_0x_1x_3x_5x_7 \oplus x_0x_1x_3x_5x_7x_8x_9 \oplus x_0x_1x_3x_5x_7x_9 \oplus x_0x_1x_3x_5x_8 \oplus x_0x_1x_3x_5x_9 \oplus x_0x_1x_3x_6 \oplus$

404   $x_0x_1x_3x_6x_7 \oplus x_0x_1x_3x_6x_7x_8 \oplus x_0x_1x_3x_6x_7x_8x_9 \oplus x_0x_1x_3x_6x_8x_9 \oplus x_0x_1x_3x_6x_9 \oplus x_0x_1x_3x_7x_8 \oplus x_0x_1x_3x_7x_9 \oplus$

405   $x_0x_1x_3x_8 \oplus x_0x_1x_3x_8x_9 \oplus x_0x_1x_4x_5 \oplus x_0x_1x_4x_5x_6x_7 \oplus x_0x_1x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_4x_5x_6x_7x_9 \oplus x_0x_1x_4x_5x_6x_8 \oplus$

406   $x_0x_1x_4x_5x_6x_9 \oplus x_0x_1x_4x_5x_7 \oplus x_0x_1x_4x_5x_7x_8 \oplus x_0x_1x_4x_5x_7x_8x_9 \oplus x_0x_1x_4x_5x_8x_9 \oplus x_0x_1x_4x_5x_9 \oplus x_0x_1x_4x_6 \oplus$

407   $x_0x_1x_4x_6x_7x_8 \oplus x_0x_1x_4x_6x_7x_9 \oplus x_0x_1x_4x_6x_8 \oplus x_0x_1x_4x_6x_8x_9 \oplus x_0x_1x_4x_7 \oplus x_0x_1x_4x_7x_8x_9 \oplus x_0x_1x_4x_7x_9 \oplus$

408   $x_0x_1x_4x_8 \oplus x_0x_1x_4x_9 \oplus x_0x_1x_5 \oplus x_0x_1x_5x_6 \oplus x_0x_1x_5x_6x_7 \oplus x_0x_1x_5x_6x_7x_8 \oplus x_0x_1x_5x_6x_7x_8x_9 \oplus x_0x_1x_5x_6x_8x_9 \oplus$

409   $x_0x_1x_5x_6x_9 \oplus x_0x_1x_5x_7x_8 \oplus x_0x_1x_5x_7x_9 \oplus x_0x_1x_5x_8 \oplus x_0x_1x_5x_8x_9 \oplus x_0x_1x_6x_7 \oplus x_0x_1x_6x_7x_8x_9 \oplus x_0x_1x_6x_7x_9 \oplus$

410   $x_0x_1x_6x_8 \oplus x_0x_1x_6x_9 \oplus x_0x_1x_7 \oplus x_0x_1x_7x_8 \oplus x_0x_1x_7x_8x_9 \oplus x_0x_1x_8x_9 \oplus x_0x_1x_9 \oplus x_0x_2 \oplus x_0x_2x_3x_4 \oplus x_0x_2x_3x_4x_5x_6 \oplus$

411   $x_0x_2x_3x_4x_5x_6x_7x_8 \oplus x_0x_2x_3x_4x_5x_6x_7x_9 \oplus x_0x_2x_3x_4x_5x_6x_8 \oplus x_0x_2x_3x_4x_5x_6x_8x_9 \oplus x_0x_2x_3x_4x_5x_7 \oplus x_0x_2x_3x_4x_5x_7x_8x_9 \oplus$

412   $x_0x_2x_3x_4x_5x_7x_9 \oplus x_0x_2x_3x_4x_5x_8 \oplus x_0x_2x_3x_4x_5x_9 \oplus x_0x_2x_3x_4x_6 \oplus x_0x_2x_3x_4x_6x_7 \oplus x_0x_2x_3x_4x_6x_7x_8 \oplus x_0x_2x_3x_4x_6x_7x_8x_9 \oplus$

413   $x_0x_2x_3x_4x_6x_8x_9 \oplus x_0x_2x_3x_4x_6x_9 \oplus x_0x_2x_3x_4x_7x_8 \oplus x_0x_2x_3x_4x_7x_9 \oplus x_0x_2x_3x_4x_8 \oplus x_0x_2x_3x_4x_8x_9 \oplus x_0x_2x_3x_5 \oplus$

414   $x_0x_2x_3x_5x_6x_7 \oplus x_0x_2x_3x_5x_6x_7x_8x_9 \oplus x_0x_2x_3x_5x_6x_7x_9 \oplus x_0x_2x_3x_5x_6x_8 \oplus x_0x_2x_3x_5x_6x_9 \oplus x_0x_2x_3x_5x_7 \oplus$

415   $x_0x_2x_3x_5x_7x_8 \oplus x_0x_2x_3x_5x_7x_8x_9 \oplus x_0x_2x_3x_5x_8x_9 \oplus x_0x_2x_3x_5x_9 \oplus x_0x_2x_3x_6 \oplus x_0x_2x_3x_6x_7x_8 \oplus x_0x_2x_3x_6x_7x_9 \oplus$

416   $x_0x_2x_3x_6x_8 \oplus x_0x_2x_3x_6x_8x_9 \oplus x_0x_2x_3x_7 \oplus x_0x_2x_3x_7x_8x_9 \oplus x_0x_2x_3x_7x_9 \oplus x_0x_2x_3x_8 \oplus x_0x_2x_3x_9 \oplus x_0x_2x_4 \oplus$

417   $x_0x_2x_4x_5 \oplus x_0x_2x_4x_5x_6 \oplus x_0x_2x_4x_5x_6x_7 \oplus x_0x_2x_4x_5x_6x_7x_8 \oplus x_0x_2x_4x_5x_6x_7x_8x_9 \oplus x_0x_2x_4x_5x_6x_8x_9 \oplus x_0x_2x_4x_5x_6x_9 \oplus$

418   $x_0x_2x_4x_5x_7x_8 \oplus x_0x_2x_4x_5x_7x_9 \oplus x_0x_2x_4x_5x_8 \oplus x_0x_2x_4x_5x_8x_9 \oplus x_0x_2x_4x_6x_7 \oplus x_0x_2x_4x_6x_7x_8x_9 \oplus x_0x_2x_4x_6x_7x_9 \oplus$

419   $x_0x_2x_4x_6x_8 \oplus x_0x_2x_4x_6x_9 \oplus x_0x_2x_4x_7 \oplus x_0x_2x_4x_7x_8 \oplus x_0x_2x_4x_7x_8x_9 \oplus x_0x_2x_4x_8x_9 \oplus x_0x_2x_4x_9 \oplus x_0x_2x_5x_6 \oplus$

420   $x_0x_2x_5x_6x_7x_8 \oplus x_0x_2x_5x_6x_7x_9 \oplus x_0x_2x_5x_6x_8 \oplus x_0x_2x_5x_6x_8x_9 \oplus x_0x_2x_5x_7 \oplus x_0x_2x_5x_7x_8x_9 \oplus x_0x_2x_5x_7x_9 \oplus$

421   $x_0x_2x_5x_8 \oplus x_0x_2x_5x_9 \oplus x_0x_2x_6 \oplus x_0x_2x_6x_7 \oplus x_0x_2x_6x_7x_8 \oplus x_0x_2x_6x_7x_8x_9 \oplus x_0x_2x_6x_8x_9 \oplus x_0x_2x_6x_9 \oplus x_0x_2x_7x_8 \oplus$

422   $x_0x_2x_7x_9 \oplus x_0x_2x_8 \oplus x_0x_2x_8x_9 \oplus x_0x_3 \oplus x_0x_3x_4x_5 \oplus x_0x_3x_4x_5x_6x_7 \oplus x_0x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_3x_4x_5x_6x_7x_9 \oplus$

423   $x_0x_3x_4x_5x_6x_8 \oplus x_0x_3x_4x_5x_6x_9 \oplus x_0x_3x_4x_5x_7 \oplus x_0x_3x_4x_5x_7x_8 \oplus x_0x_3x_4x_5x_7x_8x_9 \oplus x_0x_3x_4x_5x_8x_9 \oplus x_0x_3x_4x_5x_9 \oplus$

424   $x_0x_3x_4x_6 \oplus x_0x_3x_4x_6x_7x_8 \oplus x_0x_3x_4x_6x_7x_9 \oplus x_0x_3x_4x_6x_8 \oplus x_0x_3x_4x_6x_8x_9 \oplus x_0x_3x_4x_7 \oplus x_0x_3x_4x_7x_8x_9 \oplus$

425   $x_0x_3x_4x_7x_9 \oplus x_0x_3x_4x_8 \oplus x_0x_3x_4x_9 \oplus x_0x_3x_5 \oplus x_0x_3x_5x_6 \oplus x_0x_3x_5x_6x_7 \oplus x_0x_3x_5x_6x_7x_8 \oplus x_0x_3x_5x_6x_7x_8x_9 \oplus$

426   $x_0x_3x_5x_6x_8x_9 \oplus x_0x_3x_5x_6x_9 \oplus x_0x_3x_5x_7x_8 \oplus x_0x_3x_5x_7x_9 \oplus x_0x_3x_5x_8 \oplus x_0x_3x_5x_8x_9 \oplus x_0x_3x_6x_7 \oplus x_0x_3x_6x_7x_8x_9 \oplus$

427   $x_0x_3x_6x_7x_9 \oplus x_0x_3x_6x_8 \oplus x_0x_3x_6x_9 \oplus x_0x_3x_7 \oplus x_0x_3x_7x_8 \oplus x_0x_3x_7x_8x_9 \oplus x_0x_3x_8x_9 \oplus x_0x_3x_9 \oplus x_0x_4 \oplus x_0x_4x_5x_6 \oplus$

428   $x_0x_4x_5x_6x_7x_8 \oplus x_0x_4x_5x_6x_7x_9 \oplus x_0x_4x_5x_6x_8 \oplus x_0x_4x_5x_6x_8x_9 \oplus x_0x_4x_5x_7 \oplus x_0x_4x_5x_7x_8x_9 \oplus x_0x_4x_5x_7x_9 \oplus$

429   $x_0x_4x_5x_8 \oplus x_0x_4x_5x_9 \oplus x_0x_4x_6 \oplus x_0x_4x_6x_7 \oplus x_0x_4x_6x_7x_8 \oplus x_0x_4x_6x_7x_8x_9 \oplus x_0x_4x_6x_8x_9 \oplus x_0x_4x_6x_9 \oplus x_0x_4x_7x_8 \oplus$

430   $x_0x_4x_7x_9 \oplus x_0x_4x_8 \oplus x_0x_4x_8x_9 \oplus x_0x_5 \oplus x_0x_5x_6x_7 \oplus x_0x_5x_6x_7x_8x_9 \oplus x_0x_5x_6x_7x_9 \oplus x_0x_5x_6x_8 \oplus x_0x_5x_6x_9 \oplus$

431   $x_0x_5x_7 \oplus x_0x_5x_7x_8 \oplus x_0x_5x_7x_8x_9 \oplus x_0x_5x_8x_9 \oplus x_0x_5x_9 \oplus x_0x_6 \oplus x_0x_6x_7x_8 \oplus x_0x_6x_7x_9 \oplus x_0x_6x_8 \oplus x_0x_6x_8x_9 \oplus$

432   $x_0x_7 \oplus x_0x_7x_8x_9 \oplus x_0x_7x_9 \oplus x_0x_8 \oplus x_0x_9 \oplus x_1 \oplus x_1x_2 \oplus x_1x_2x_3 \oplus x_1x_2x_3x_4 \oplus x_1x_2x_3x_4x_5 \oplus x_1x_2x_3x_4x_5x_6 \oplus$

433   $x_1x_2x_3x_4x_5x_6x_7 \oplus x_1x_2x_3x_4x_5x_6x_7x_8 \oplus x_1x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_1x_2x_3x_4x_5x_6x_8x_9 \oplus x_1x_2x_3x_4x_5x_6x_9 \oplus x_1x_2x_3x_4x_5x_7x_8 \oplus$

434   $x_1x_2x_3x_4x_5x_7x_9 \oplus x_1x_2x_3x_4x_5x_8 \oplus x_1x_2x_3x_4x_5x_8x_9 \oplus x_1x_2x_3x_4x_6x_7 \oplus x_1x_2x_3x_4x_6x_7x_8x_9 \oplus x_1x_2x_3x_4x_6x_7x_9 \oplus$

435   $x_1x_2x_3x_4x_6x_8 \oplus x_1x_2x_3x_4x_6x_9 \oplus x_1x_2x_3x_4x_7 \oplus x_1x_2x_3x_4x_7x_8 \oplus x_1x_2x_3x_4x_7x_8x_9 \oplus x_1x_2x_3x_4x_8x_9 \oplus x_1x_2x_3x_4x_9 \oplus$

436   $x_1x_2x_3x_5x_6 \oplus x_1x_2x_3x_5x_6x_7x_8 \oplus x_1x_2x_3x_5x_6x_7x_9 \oplus x_1x_2x_3x_5x_6x_8 \oplus x_1x_2x_3x_5x_6x_8x_9 \oplus x_1x_2x_3x_5x_7 \oplus x_1x_2x_3x_5x_7x_8x_9 \oplus$

437   $x_1x_2x_3x_5x_7x_9 \oplus x_1x_2x_3x_5x_8 \oplus x_1x_2x_3x_5x_9 \oplus x_1x_2x_3x_6 \oplus x_1x_2x_3x_6x_7 \oplus x_1x_2x_3x_6x_7x_8 \oplus x_1x_2x_3x_6x_7x_8x_9 \oplus$

438   $x_1x_2x_3x_6x_8x_9 \oplus x_1x_2x_3x_6x_9 \oplus x_1x_2x_3x_7x_8 \oplus x_1x_2x_3x_7x_9 \oplus x_1x_2x_3x_8 \oplus x_1x_2x_3x_8x_9 \oplus x_1x_2x_4x_5 \oplus x_1x_2x_4x_5x_6x_7 \oplus$

439   $x_1x_2x_4x_5x_6x_7x_8x_9 \oplus x_1x_2x_4x_5x_6x_7x_9 \oplus x_1x_2x_4x_5x_6x_8 \oplus x_1x_2x_4x_5x_6x_9 \oplus x_1x_2x_4x_5x_7 \oplus x_1x_2x_4x_5x_7x_8 \oplus$

440   $x_1x_2x_4x_5x_7x_8x_9 \oplus x_1x_2x_4x_5x_8x_9 \oplus x_1x_2x_4x_5x_9 \oplus x_1x_2x_4x_6 \oplus x_1x_2x_4x_6x_7x_8 \oplus x_1x_2x_4x_6x_7x_9 \oplus x_1x_2x_4x_6x_8 \oplus$

441   $x_1x_2x_4x_6x_8x_9 \oplus x_1x_2x_4x_7 \oplus x_1x_2x_4x_7x_8x_9 \oplus x_1x_2x_4x_7x_9 \oplus x_1x_2x_4x_8 \oplus x_1x_2x_4x_9 \oplus x_1x_2x_5 \oplus x_1x_2x_5x_6 \oplus$

442   $x_1x_2x_5x_6x_7 \oplus x_1x_2x_5x_6x_7x_8 \oplus x_1x_2x_5x_6x_7x_8x_9 \oplus x_1x_2x_5x_6x_8x_9 \oplus x_1x_2x_5x_6x_9 \oplus x_1x_2x_5x_7x_8 \oplus x_1x_2x_5x_7x_9 \oplus$

443   $x_1x_2x_5x_8 \oplus x_1x_2x_5x_8x_9 \oplus x_1x_2x_6x_7 \oplus x_1x_2x_6x_7x_8x_9 \oplus x_1x_2x_6x_7x_9 \oplus x_1x_2x_6x_8 \oplus x_1x_2x_6x_9 \oplus x_1x_2x_7 \oplus$

444   $x_1x_2x_7x_8 \oplus x_1x_2x_7x_8x_9 \oplus x_1x_2x_8x_9 \oplus x_1x_2x_9 \oplus x_1x_3x_4 \oplus x_1x_3x_4x_5x_6 \oplus x_1x_3x_4x_5x_6x_7x_8 \oplus x_1x_3x_4x_5x_6x_7x_9 \oplus$

445   $x_1x_3x_4x_5x_6x_8 \oplus x_1x_3x_4x_5x_6x_8x_9 \oplus x_1x_3x_4x_5x_7 \oplus x_1x_3x_4x_5x_7x_8x_9 \oplus x_1x_3x_4x_5x_7x_9 \oplus x_1x_3x_4x_5x_8 \oplus x_1x_3x_4x_5x_9 \oplus$

446   $x_1x_3x_4x_6 \oplus x_1x_3x_4x_6x_7 \oplus x_1x_3x_4x_6x_7x_8 \oplus x_1x_3x_4x_6x_7x_8x_9 \oplus x_1x_3x_4x_6x_8x_9 \oplus x_1x_3x_4x_6x_9 \oplus x_1x_3x_4x_7x_8 \oplus$

447   $x_1x_3x_4x_7x_9 \oplus x_1x_3x_4x_8 \oplus x_1x_3x_4x_8x_9 \oplus x_1x_3x_5 \oplus x_1x_3x_5x_6x_7 \oplus x_1x_3x_5x_6x_7x_8x_9 \oplus x_1x_3x_5x_6x_7x_9 \oplus x_1x_3x_5x_6x_8 \oplus$

448   $x_1x_3x_5x_6x_9 \oplus x_1x_3x_5x_7 \oplus x_1x_3x_5x_7x_8 \oplus x_1x_3x_5x_7x_8x_9 \oplus x_1x_3x_5x_8x_9 \oplus x_1x_3x_5x_9 \oplus x_1x_3x_6 \oplus x_1x_3x_6x_7x_8 \oplus$

449   $x_1x_3x_6x_7x_9 \oplus x_1x_3x_6x_8 \oplus x_1x_3x_6x_8x_9 \oplus x_1x_3x_7 \oplus x_1x_3x_7x_8x_9 \oplus x_1x_3x_7x_9 \oplus x_1x_3x_8 \oplus x_1x_3x_9 \oplus x_1x_4 \oplus x_1x_4x_5 \oplus$

450   $x_1x_4x_5x_6 \oplus x_1x_4x_5x_6x_7 \oplus x_1x_4x_5x_6x_7x_8 \oplus x_1x_4x_5x_6x_7x_8x_9 \oplus x_1x_4x_5x_6x_8x_9 \oplus x_1x_4x_5x_6x_9 \oplus x_1x_4x_5x_7x_8 \oplus$

451   $x_1x_4x_5x_7x_9 \oplus x_1x_4x_5x_8 \oplus x_1x_4x_5x_8x_9 \oplus x_1x_4x_6x_7 \oplus x_1x_4x_6x_7x_8x_9 \oplus x_1x_4x_6x_7x_9 \oplus x_1x_4x_6x_8 \oplus x_1x_4x_6x_9 \oplus$

$$x_1x_4x_7 \oplus x_1x_4x_7x_8 \oplus x_1x_4x_7x_8x_9 \oplus x_1x_4x_8x_9 \oplus x_1x_4x_9 \oplus x_1x_5x_6 \oplus x_1x_5x_6x_7x_8 \oplus x_1x_5x_6x_7x_9 \oplus x_1x_5x_6x_8 \oplus$$

$$x_1x_5x_6x_8x_9 \oplus x_1x_5x_7 \oplus x_1x_5x_7x_8x_9 \oplus x_1x_5x_7x_9 \oplus x_1x_5x_8 \oplus x_1x_5x_9 \oplus x_1x_6 \oplus x_1x_6x_7 \oplus x_1x_6x_7x_8 \oplus x_1x_6x_7x_8x_9 \oplus$$

$$x_1x_6x_8x_9 \oplus x_1x_6x_9 \oplus x_1x_7x_8 \oplus x_1x_7x_9 \oplus x_1x_8 \oplus x_1x_8x_9 \oplus x_2x_3 \oplus x_2x_3x_4x_5 \oplus x_2x_3x_4x_5x_6x_7 \oplus x_2x_3x_4x_5x_6x_7x_8x_9 \oplus$$

$$x_2x_3x_4x_5x_6x_7x_9 \oplus x_2x_3x_4x_5x_6x_8 \oplus x_2x_3x_4x_5x_6x_9 \oplus x_2x_3x_4x_5x_7 \oplus x_2x_3x_4x_5x_7x_8 \oplus x_2x_3x_4x_5x_7x_8x_9 \oplus x_2x_3x_4x_5x_8x_9 \oplus$$

$$x_2x_3x_4x_5x_9 \oplus x_2x_3x_4x_6 \oplus x_2x_3x_4x_6x_7x_8 \oplus x_2x_3x_4x_6x_7x_9 \oplus x_2x_3x_4x_6x_8 \oplus x_2x_3x_4x_6x_8x_9 \oplus x_2x_3x_4x_7 \oplus x_2x_3x_4x_7x_8x_9 \oplus$$

$$x_2x_3x_4x_7x_9 \oplus x_2x_3x_4x_8 \oplus x_2x_3x_4x_9 \oplus x_2x_3x_5 \oplus x_2x_3x_5x_6 \oplus x_2x_3x_5x_6x_7 \oplus x_2x_3x_5x_6x_7x_8 \oplus x_2x_3x_5x_6x_7x_8x_9 \oplus$$

$$x_2x_3x_5x_6x_8x_9 \oplus x_2x_3x_5x_6x_9 \oplus x_2x_3x_5x_7x_8 \oplus x_2x_3x_5x_7x_9 \oplus x_2x_3x_5x_8 \oplus x_2x_3x_5x_8x_9 \oplus x_2x_3x_6x_7 \oplus x_2x_3x_6x_7x_8x_9 \oplus$$

$$x_2x_3x_6x_7x_9 \oplus x_2x_3x_6x_8 \oplus x_2x_3x_6x_9 \oplus x_2x_3x_7 \oplus x_2x_3x_7x_8 \oplus x_2x_3x_7x_8x_9 \oplus x_2x_3x_8x_9 \oplus x_2x_3x_9 \oplus x_2x_4 \oplus x_2x_4x_5x_6 \oplus$$

$$x_2x_4x_5x_6x_7x_8 \oplus x_2x_4x_5x_6x_7x_9 \oplus x_2x_4x_5x_6x_8 \oplus x_2x_4x_5x_6x_8x_9 \oplus x_2x_4x_5x_7 \oplus x_2x_4x_5x_7x_8x_9 \oplus x_2x_4x_5x_7x_9 \oplus$$

$$x_2x_4x_5x_8 \oplus x_2x_4x_5x_9 \oplus x_2x_4x_6 \oplus x_2x_4x_6x_7 \oplus x_2x_4x_6x_7x_8 \oplus x_2x_4x_6x_7x_8x_9 \oplus x_2x_4x_6x_8x_9 \oplus x_2x_4x_6x_9 \oplus x_2x_4x_7x_8 \oplus$$

$$x_2x_4x_7x_9 \oplus x_2x_4x_8 \oplus x_2x_4x_8x_9 \oplus x_2x_5 \oplus x_2x_5x_6x_7 \oplus x_2x_5x_6x_7x_8x_9 \oplus x_2x_5x_6x_7x_9 \oplus x_2x_5x_6x_8 \oplus x_2x_5x_6x_9 \oplus$$

$$x_2x_5x_7 \oplus x_2x_5x_7x_8 \oplus x_2x_5x_7x_8x_9 \oplus x_2x_5x_8x_9 \oplus x_2x_5x_9 \oplus x_2x_6 \oplus x_2x_6x_7x_8 \oplus x_2x_6x_7x_9 \oplus x_2x_6x_8 \oplus x_2x_6x_8x_9 \oplus$$

$$x_2x_7 \oplus x_2x_7x_8x_9 \oplus x_2x_7x_9 \oplus x_2x_8 \oplus x_2x_9 \oplus x_3 \oplus x_3x_4 \oplus x_3x_4x_5 \oplus x_3x_4x_5x_6 \oplus x_3x_4x_5x_6x_7 \oplus x_3x_4x_5x_6x_7x_8 \oplus$$

$$x_3x_4x_5x_6x_7x_8x_9 \oplus x_3x_4x_5x_6x_8x_9 \oplus x_3x_4x_5x_6x_9 \oplus x_3x_4x_5x_7x_8 \oplus x_3x_4x_5x_7x_9 \oplus x_3x_4x_5x_8 \oplus x_3x_4x_5x_8x_9 \oplus$$

$$x_3x_4x_6x_7 \oplus x_3x_4x_6x_7x_8x_9 \oplus x_3x_4x_6x_7x_9 \oplus x_3x_4x_6x_8 \oplus x_3x_4x_6x_9 \oplus x_3x_4x_7 \oplus x_3x_4x_7x_8 \oplus x_3x_4x_7x_8x_9 \oplus x_3x_4x_8x_9 \oplus$$

$$x_3x_4x_9 \oplus x_3x_5x_6 \oplus x_3x_5x_6x_7x_8 \oplus x_3x_5x_6x_7x_9 \oplus x_3x_5x_6x_8 \oplus x_3x_5x_6x_8x_9 \oplus x_3x_5x_7 \oplus x_3x_5x_7x_8x_9 \oplus x_3x_5x_7x_9 \oplus$$

$$x_3x_5x_8 \oplus x_3x_5x_9 \oplus x_3x_6 \oplus x_3x_6x_7 \oplus x_3x_6x_7x_8 \oplus x_3x_6x_7x_8x_9 \oplus x_3x_6x_8x_9 \oplus x_3x_6x_9 \oplus x_3x_7x_8 \oplus x_3x_7x_9 \oplus x_3x_8 \oplus$$

$$x_3x_8x_9 \oplus x_4x_5 \oplus x_4x_5x_6x_7 \oplus x_4x_5x_6x_7x_8x_9 \oplus x_4x_5x_6x_7x_9 \oplus x_4x_5x_6x_8 \oplus x_4x_5x_6x_9 \oplus x_4x_5x_7 \oplus x_4x_5x_7x_8 \oplus$$

$$x_4x_5x_7x_8x_9 \oplus x_4x_5x_8x_9 \oplus x_4x_5x_9 \oplus x_4x_6 \oplus x_4x_6x_7x_8 \oplus x_4x_6x_7x_9 \oplus x_4x_6x_8 \oplus x_4x_6x_8x_9 \oplus x_4x_7 \oplus x_4x_7x_8x_9 \oplus$$

$$x_4x_7x_9 \oplus x_4x_8 \oplus x_4x_9 \oplus x_5 \oplus x_5x_6 \oplus x_5x_6x_7 \oplus x_5x_6x_7x_8 \oplus x_5x_6x_7x_8x_9 \oplus x_5x_6x_8x_9 \oplus x_5x_6x_9 \oplus x_5x_7x_8 \oplus$$

$$x_5x_7x_9 \oplus x_5x_8 \oplus x_5x_8x_9 \oplus x_6x_7 \oplus x_6x_7x_8x_9 \oplus x_6x_7x_9 \oplus x_6x_8 \oplus x_6x_9 \oplus x_7 \oplus x_7x_8 \oplus x_7x_8x_9 \oplus x_8x_9 \oplus x_9 = 0$$

$$x_0 \oplus x_0x_1 \oplus x_0x_1x_2 \oplus x_0x_1x_2x_3 \oplus x_0x_1x_2x_3x_4 \oplus x_0x_1x_2x_3x_4x_5 \oplus x_0x_1x_2x_3x_4x_5x_6 \oplus x_0x_1x_2x_3x_4x_5x_6x_7 \oplus$$

$$x_0x_1x_2x_3x_4x_5x_6x_7x_8 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_6x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_6x_9 \oplus x_0x_1x_2x_3x_4x_5x_7x_8 \oplus$$

$$x_0x_1x_2x_3x_4x_5x_7x_9 \oplus x_0x_1x_2x_3x_4x_5x_8 \oplus x_0x_1x_2x_3x_4x_5x_8x_9 \oplus x_0x_1x_2x_3x_4x_6x_7 \oplus x_0x_1x_2x_3x_4x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_6x_7x_9 \oplus$$

$$x_0x_1x_2x_3x_4x_6x_8 \oplus x_0x_1x_2x_3x_4x_6x_9 \oplus x_0x_1x_2x_3x_4x_7 \oplus x_0x_1x_2x_3x_4x_7x_8 \oplus x_0x_1x_2x_3x_4x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_8x_9 \oplus$$

$$x_0x_1x_2x_3x_4x_9 \oplus x_0x_1x_2x_3x_5x_6 \oplus x_0x_1x_2x_3x_5x_6x_7x_8 \oplus x_0x_1x_2x_3x_5x_6x_7x_9 \oplus x_0x_1x_2x_3x_5x_6x_8 \oplus x_0x_1x_2x_3x_5x_6x_8x_9 \oplus$$

$$x_0x_1x_2x_3x_5x_7 \oplus x_0x_1x_2x_3x_5x_7x_8x_9 \oplus x_0x_1x_2x_3x_5x_7x_9 \oplus x_0x_1x_2x_3x_5x_8 \oplus x_0x_1x_2x_3x_5x_9 \oplus x_0x_1x_2x_3x_6 \oplus$$

$$x_0x_1x_2x_3x_6x_7 \oplus x_0x_1x_2x_3x_6x_7x_8 \oplus x_0x_1x_2x_3x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_6x_8x_9 \oplus x_0x_1x_2x_3x_6x_9 \oplus x_0x_1x_2x_3x_7x_8 \oplus$$

$$x_0x_1x_2x_3x_7x_9 \oplus x_0x_1x_2x_3x_8 \oplus x_0x_1x_2x_3x_8x_9 \oplus x_0x_1x_2x_4x_5 \oplus x_0x_1x_2x_4x_5x_6x_7 \oplus x_0x_1x_2x_4x_5x_6x_7x_8x_9 \oplus$$

$$x_0x_1x_2x_4x_5x_6x_7x_9 \oplus x_0x_1x_2x_4x_5x_6x_8 \oplus x_0x_1x_2x_4x_5x_6x_9 \oplus x_0x_1x_2x_4x_5x_7 \oplus x_0x_1x_2x_4x_5x_7x_8 \oplus x_0x_1x_2x_4x_5x_7x_8x_9 \oplus$$

$$x_0x_1x_2x_4x_5x_8x_9 \oplus x_0x_1x_2x_4x_5x_9 \oplus x_0x_1x_2x_4x_6 \oplus x_0x_1x_2x_4x_6x_7x_8 \oplus x_0x_1x_2x_4x_6x_7x_9 \oplus x_0x_1x_2x_4x_6x_8 \oplus$$

$$x_0x_1x_2x_4x_6x_8x_9 \oplus x_0x_1x_2x_4x_7 \oplus x_0x_1x_2x_4x_7x_8x_9 \oplus x_0x_1x_2x_4x_7x_9 \oplus x_0x_1x_2x_4x_8 \oplus x_0x_1x_2x_4x_9 \oplus x_0x_1x_2x_5 \oplus$$

$$x_0x_1x_2x_5x_6 \oplus x_0x_1x_2x_5x_6x_7 \oplus x_0x_1x_2x_5x_6x_7x_8 \oplus x_0x_1x_2x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_5x_6x_8x_9 \oplus x_0x_1x_2x_5x_6x_9 \oplus$$

$$x_0x_1x_2x_5x_7x_8 \oplus x_0x_1x_2x_5x_7x_9 \oplus x_0x_1x_2x_5x_8 \oplus x_0x_1x_2x_5x_8x_9 \oplus x_0x_1x_2x_6x_7 \oplus x_0x_1x_2x_6x_7x_8x_9 \oplus x_0x_1x_2x_6x_7x_9 \oplus$$

$$x_0x_1x_2x_6x_8 \oplus x_0x_1x_2x_6x_9 \oplus x_0x_1x_2x_7 \oplus x_0x_1x_2x_7x_8 \oplus x_0x_1x_2x_7x_8x_9 \oplus x_0x_1x_2x_8x_9 \oplus x_0x_1x_2x_9 \oplus x_0x_1x_3x_4 \oplus$$

$$x_0x_1x_3x_4x_5x_6 \oplus x_0x_1x_3x_4x_5x_6x_7x_8 \oplus x_0x_1x_3x_4x_5x_6x_7x_9 \oplus x_0x_1x_3x_4x_5x_6x_8 \oplus x_0x_1x_3x_4x_5x_6x_8x_9 \oplus x_0x_1x_3x_4x_5x_7 \oplus$$

$$x_0x_1x_3x_4x_5x_7x_8x_9 \oplus x_0x_1x_3x_4x_5x_7x_9 \oplus x_0x_1x_3x_4x_5x_8 \oplus x_0x_1x_3x_4x_5x_9 \oplus x_0x_1x_3x_4x_6 \oplus x_0x_1x_3x_4x_6x_7 \oplus$$

$$x_0x_1x_3x_4x_6x_7x_8 \oplus x_0x_1x_3x_4x_6x_7x_8x_9 \oplus x_0x_1x_3x_4x_6x_8x_9 \oplus x_0x_1x_3x_4x_6x_9 \oplus x_0x_1x_3x_4x_7x_8 \oplus x_0x_1x_3x_4x_7x_9 \oplus$$

$$x_0x_1x_3x_4x_8 \oplus x_0x_1x_3x_4x_8x_9 \oplus x_0x_1x_3x_5 \oplus x_0x_1x_3x_5x_6x_7 \oplus x_0x_1x_3x_5x_6x_7x_8x_9 \oplus x_0x_1x_3x_5x_6x_7x_9 \oplus x_0x_1x_3x_5x_6x_8 \oplus$$

$$x_0x_1x_3x_5x_6x_9 \oplus x_0x_1x_3x_5x_7 \oplus x_0x_1x_3x_5x_7x_8 \oplus x_0x_1x_3x_5x_7x_8x_9 \oplus x_0x_1x_3x_5x_8x_9 \oplus x_0x_1x_3x_5x_9 \oplus x_0x_1x_3x_6 \oplus$$

$$x_0x_1x_3x_6x_7x_8 \oplus x_0x_1x_3x_6x_7x_9 \oplus x_0x_1x_3x_6x_8 \oplus x_0x_1x_3x_6x_8x_9 \oplus x_0x_1x_3x_7 \oplus x_0x_1x_3x_7x_8x_9 \oplus x_0x_1x_3x_7x_9 \oplus$$

$$x_0x_1x_3x_8 \oplus x_0x_1x_3x_9 \oplus x_0x_1x_4 \oplus x_0x_1x_4x_5 \oplus x_0x_1x_4x_5x_6 \oplus x_0x_1x_4x_5x_6x_7 \oplus x_0x_1x_4x_5x_6x_7x_8 \oplus x_0x_1x_4x_5x_6x_7x_8x_9 \oplus$$

$$x_0x_1x_4x_5x_6x_8x_9 \oplus x_0x_1x_4x_5x_6x_9 \oplus x_0x_1x_4x_5x_7x_8 \oplus x_0x_1x_4x_5x_7x_9 \oplus x_0x_1x_4x_5x_8 \oplus x_0x_1x_4x_5x_8x_9 \oplus x_0x_1x_4x_6x_7 \oplus$$

$$x_0x_1x_4x_6x_7x_8 \oplus x_0x_1x_4x_6x_7x_9 \oplus x_0x_1x_4x_6x_8 \oplus x_0x_1x_4x_6x_9 \oplus x_0x_1x_4x_7 \oplus x_0x_1x_4x_7x_8 \oplus x_0x_1x_4x_7x_8x_9 \oplus$$

$$x_0x_1x_4x_8x_9 \oplus x_0x_1x_4x_9 \oplus x_0x_1x_5x_6 \oplus x_0x_1x_5x_6x_7x_8 \oplus x_0x_1x_5x_6x_7x_9 \oplus x_0x_1x_5x_6x_8 \oplus x_0x_1x_5x_6x_8x_9 \oplus x_0x_1x_5x_7 \oplus$$

$$x_0x_1x_5x_7x_8 \oplus x_0x_1x_5x_7x_9 \oplus x_0x_1x_5x_8 \oplus x_0x_1x_5x_9 \oplus x_0x_1x_6 \oplus x_0x_1x_6x_7 \oplus x_0x_1x_6x_7x_8 \oplus x_0x_1x_6x_7x_8x_9 \oplus$$

$$x_0x_1x_6x_8x_9 \oplus x_0x_1x_6x_9 \oplus x_0x_1x_7x_8 \oplus x_0x_1x_7x_9 \oplus x_0x_1x_8 \oplus x_0x_1x_8x_9 \oplus x_0x_2x_3 \oplus x_0x_2x_3x_4x_5 \oplus x_0x_2x_3x_4x_5x_6x_7 \oplus$$

$$x_0x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_2x_3x_4x_5x_6x_7x_9 \oplus x_0x_2x_3x_4x_5x_6x_8 \oplus x_0x_2x_3x_4x_5x_6x_9 \oplus x_0x_2x_3x_4x_5x_7 \oplus x_0x_2x_3x_4x_5x_7x_8 \oplus$$

$$x_0x_2x_3x_4x_5x_7x_8x_9 \oplus x_0x_2x_3x_4x_5x_8x_9 \oplus x_0x_2x_3x_4x_5x_9 \oplus x_0x_2x_3x_4x_6 \oplus x_0x_2x_3x_4x_6x_7x_8 \oplus x_0x_2x_3x_4x_6x_7x_9 \oplus$$

$$x_0x_2x_3x_4x_6x_8 \oplus x_0x_2x_3x_4x_6x_8x_9 \oplus x_0x_2x_3x_4x_7 \oplus x_0x_2x_3x_4x_7x_8x_9 \oplus x_0x_2x_3x_4x_7x_9 \oplus x_0x_2x_3x_4x_8 \oplus x_0x_2x_3x_4x_9 \oplus$$

$$x_0x_2x_3x_5 \oplus x_0x_2x_3x_5x_6 \oplus x_0x_2x_3x_5x_6x_7 \oplus x_0x_2x_3x_5x_6x_7x_8 \oplus x_0x_2x_3x_5x_6x_7x_8x_9 \oplus x_0x_2x_3x_5x_6x_8x_9 \oplus x_0x_2x_3x_5x_6x_9 \oplus$$

$x_0x_2x_3x_5x_7x_8 \oplus x_0x_2x_3x_5x_7x_9 \oplus x_0x_2x_3x_5x_8 \oplus x_0x_2x_3x_5x_8x_9 \oplus x_0x_2x_3x_6x_7 \oplus x_0x_2x_3x_6x_7x_8x_9 \oplus x_0x_2x_3x_6x_7x_9 \oplus$

$x_0x_2x_3x_6x_8 \oplus x_0x_2x_3x_6x_9 \oplus x_0x_2x_3x_7 \oplus x_0x_2x_3x_7x_8 \oplus x_0x_2x_3x_7x_8x_9 \oplus x_0x_2x_3x_8x_9 \oplus x_0x_2x_3x_9 \oplus x_0x_2x_4 \oplus$

$x_0x_2x_4x_5x_6 \oplus x_0x_2x_4x_5x_6x_7x_8 \oplus x_0x_2x_4x_5x_6x_7x_9 \oplus x_0x_2x_4x_5x_6x_8 \oplus x_0x_2x_4x_5x_6x_8x_9 \oplus x_0x_2x_4x_5x_7 \oplus x_0x_2x_4x_5x_7x_8x_9 \oplus$

$x_0x_2x_4x_5x_7x_9 \oplus x_0x_2x_4x_5x_8 \oplus x_0x_2x_4x_5x_9 \oplus x_0x_2x_4x_6 \oplus x_0x_2x_4x_6x_7 \oplus x_0x_2x_4x_6x_7x_8 \oplus x_0x_2x_4x_6x_7x_8x_9 \oplus$

$x_0x_2x_4x_6x_8x_9 \oplus x_0x_2x_4x_6x_9 \oplus x_0x_2x_4x_7x_8 \oplus x_0x_2x_4x_7x_9 \oplus x_0x_2x_4x_8 \oplus x_0x_2x_4x_8x_9 \oplus x_0x_2x_5 \oplus x_0x_2x_5x_6x_7 \oplus$

$x_0x_2x_5x_6x_7x_8x_9 \oplus x_0x_2x_5x_6x_7x_9 \oplus x_0x_2x_5x_6x_8 \oplus x_0x_2x_5x_6x_9 \oplus x_0x_2x_5x_7 \oplus x_0x_2x_5x_7x_8 \oplus x_0x_2x_5x_7x_8x_9 \oplus$

$x_0x_2x_5x_8x_9 \oplus x_0x_2x_5x_9 \oplus x_0x_2x_6 \oplus x_0x_2x_6x_7x_8 \oplus x_0x_2x_6x_7x_9 \oplus x_0x_2x_6x_8 \oplus x_0x_2x_6x_8x_9 \oplus x_0x_2x_7 \oplus x_0x_2x_7x_8x_9 \oplus$

$x_0x_2x_7x_9 \oplus x_0x_2x_8 \oplus x_0x_2x_9 \oplus x_0x_3 \oplus x_0x_3x_4 \oplus x_0x_3x_4x_5 \oplus x_0x_3x_4x_5x_6 \oplus x_0x_3x_4x_5x_6x_7 \oplus x_0x_3x_4x_5x_6x_7x_8 \oplus$

$x_0x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_3x_4x_5x_6x_8x_9 \oplus x_0x_3x_4x_5x_6x_9 \oplus x_0x_3x_4x_5x_7x_8 \oplus x_0x_3x_4x_5x_7x_9 \oplus x_0x_3x_4x_5x_8 \oplus$

$x_0x_3x_4x_5x_8x_9 \oplus x_0x_3x_4x_6x_7 \oplus x_0x_3x_4x_6x_7x_8x_9 \oplus x_0x_3x_4x_6x_7x_9 \oplus x_0x_3x_4x_6x_8 \oplus x_0x_3x_4x_6x_9 \oplus x_0x_3x_4x_7 \oplus$

$x_0x_3x_4x_7x_8 \oplus x_0x_3x_4x_7x_8x_9 \oplus x_0x_3x_4x_8x_9 \oplus x_0x_3x_4x_9 \oplus x_0x_3x_5x_6 \oplus x_0x_3x_5x_6x_7x_8 \oplus x_0x_3x_5x_6x_7x_9 \oplus x_0x_3x_5x_6x_8 \oplus$

$x_0x_3x_5x_6x_8x_9 \oplus x_0x_3x_5x_7 \oplus x_0x_3x_5x_7x_8x_9 \oplus x_0x_3x_5x_7x_9 \oplus x_0x_3x_5x_8 \oplus x_0x_3x_5x_9 \oplus x_0x_3x_6 \oplus x_0x_3x_6x_7 \oplus$

$x_0x_3x_6x_7x_8 \oplus x_0x_3x_6x_7x_8x_9 \oplus x_0x_3x_6x_8x_9 \oplus x_0x_3x_6x_9 \oplus x_0x_3x_7x_8 \oplus x_0x_3x_7x_9 \oplus x_0x_3x_8 \oplus x_0x_3x_8x_9 \oplus x_0x_4x_5 \oplus$

$x_0x_4x_5x_6x_7 \oplus x_0x_4x_5x_6x_7x_8x_9 \oplus x_0x_4x_5x_6x_7x_9 \oplus x_0x_4x_5x_6x_8 \oplus x_0x_4x_5x_6x_9 \oplus x_0x_4x_5x_7 \oplus x_0x_4x_5x_7x_8 \oplus$

$x_0x_4x_5x_7x_8x_9 \oplus x_0x_4x_5x_8x_9 \oplus x_0x_4x_5x_9 \oplus x_0x_4x_6 \oplus x_0x_4x_6x_7x_8 \oplus x_0x_4x_6x_7x_9 \oplus x_0x_4x_6x_8 \oplus x_0x_4x_6x_8x_9 \oplus$

$x_0x_4x_7 \oplus x_0x_4x_7x_8x_9 \oplus x_0x_4x_7x_9 \oplus x_0x_4x_8 \oplus x_0x_4x_9 \oplus x_0x_5 \oplus x_0x_5x_6 \oplus x_0x_5x_6x_7 \oplus x_0x_5x_6x_7x_8 \oplus x_0x_5x_6x_7x_8x_9 \oplus$

$x_0x_5x_6x_8x_9 \oplus x_0x_5x_6x_9 \oplus x_0x_5x_7x_8 \oplus x_0x_5x_7x_9 \oplus x_0x_5x_8 \oplus x_0x_5x_8x_9 \oplus x_0x_6x_7 \oplus x_0x_6x_7x_8x_9 \oplus x_0x_6x_7x_9 \oplus$

$x_0x_6x_8 \oplus x_0x_6x_9 \oplus x_0x_7 \oplus x_0x_7x_8 \oplus x_0x_7x_8x_9 \oplus x_0x_8x_9 \oplus x_0x_9 \oplus x_1x_2 \oplus x_1x_2x_3x_4 \oplus x_1x_2x_3x_4x_5x_6 \oplus$

$x_1x_2x_3x_4x_5x_6x_7x_8 \oplus x_1x_2x_3x_4x_5x_6x_7x_9 \oplus x_1x_2x_3x_4x_5x_6x_8 \oplus x_1x_2x_3x_4x_5x_6x_8x_9 \oplus x_1x_2x_3x_4x_5x_7 \oplus x_1x_2x_3x_4x_5x_7x_8x_9 \oplus$

$x_1x_2x_3x_4x_5x_7x_9 \oplus x_1x_2x_3x_4x_5x_8 \oplus x_1x_2x_3x_4x_5x_9 \oplus x_1x_2x_3x_4x_6 \oplus x_1x_2x_3x_4x_6x_7 \oplus x_1x_2x_3x_4x_6x_7x_8 \oplus x_1x_2x_3x_4x_6x_7x_8x_9 \oplus$

$x_1x_2x_3x_4x_6x_8x_9 \oplus x_1x_2x_3x_4x_6x_9 \oplus x_1x_2x_3x_4x_7x_8 \oplus x_1x_2x_3x_4x_7x_9 \oplus x_1x_2x_3x_4x_8 \oplus x_1x_2x_3x_4x_8x_9 \oplus x_1x_2x_3x_5 \oplus$

$x_1x_2x_3x_5x_6x_7 \oplus x_1x_2x_3x_5x_6x_7x_8x_9 \oplus x_1x_2x_3x_5x_6x_7x_9 \oplus x_1x_2x_3x_5x_6x_8 \oplus x_1x_2x_3x_5x_6x_9 \oplus x_1x_2x_3x_5x_7 \oplus$

$x_1x_2x_3x_5x_7x_8 \oplus x_1x_2x_3x_5x_7x_8x_9 \oplus x_1x_2x_3x_5x_8x_9 \oplus x_1x_2x_3x_5x_9 \oplus x_1x_2x_3x_6 \oplus x_1x_2x_3x_6x_7x_8 \oplus x_1x_2x_3x_6x_7x_9 \oplus$

$x_1x_2x_3x_6x_8 \oplus x_1x_2x_3x_6x_8x_9 \oplus x_1x_2x_3x_7 \oplus x_1x_2x_3x_7x_8x_9 \oplus x_1x_2x_3x_7x_9 \oplus x_1x_2x_3x_8 \oplus x_1x_2x_3x_9 \oplus x_1x_2x_4 \oplus$

$x_1x_2x_4x_5 \oplus x_1x_2x_4x_5x_6 \oplus x_1x_2x_4x_5x_6x_7 \oplus x_1x_2x_4x_5x_6x_7x_8 \oplus x_1x_2x_4x_5x_6x_7x_8x_9 \oplus x_1x_2x_4x_5x_6x_8x_9 \oplus x_1x_2x_4x_5x_6x_9 \oplus$

$x_1x_2x_4x_5x_7x_8 \oplus x_1x_2x_4x_5x_7x_9 \oplus x_1x_2x_4x_5x_8 \oplus x_1x_2x_4x_5x_8x_9 \oplus x_1x_2x_4x_6x_7 \oplus x_1x_2x_4x_6x_7x_8x_9 \oplus x_1x_2x_4x_6x_7x_9 \oplus$

$x_1x_2x_4x_6x_8 \oplus x_1x_2x_4x_6x_9 \oplus x_1x_2x_4x_7 \oplus x_1x_2x_4x_7x_8 \oplus x_1x_2x_4x_7x_8x_9 \oplus x_1x_2x_4x_8x_9 \oplus x_1x_2x_4x_9 \oplus x_1x_2x_5x_6 \oplus$

$x_1x_2x_5x_6x_7x_8 \oplus x_1x_2x_5x_6x_7x_9 \oplus x_1x_2x_5x_6x_8 \oplus x_1x_2x_5x_6x_8x_9 \oplus x_1x_2x_5x_7 \oplus x_1x_2x_5x_7x_8x_9 \oplus x_1x_2x_5x_7x_9 \oplus$

$x_1x_2x_5x_8 \oplus x_1x_2x_5x_9 \oplus x_1x_2x_6 \oplus x_1x_2x_6x_7 \oplus x_1x_2x_6x_7x_8 \oplus x_1x_2x_6x_7x_8x_9 \oplus x_1x_2x_6x_8x_9 \oplus x_1x_2x_6x_9 \oplus x_1x_2x_7x_8 \oplus$

$x_1x_2x_7x_9 \oplus x_1x_2x_8 \oplus x_1x_2x_8x_9 \oplus x_1x_3 \oplus x_1x_3x_4x_5 \oplus x_1x_3x_4x_5x_6x_7 \oplus x_1x_3x_4x_5x_6x_7x_8x_9 \oplus x_1x_3x_4x_5x_6x_7x_9 \oplus$

$x_1x_3x_4x_5x_6x_8 \oplus x_1x_3x_4x_5x_6x_9 \oplus x_1x_3x_4x_5x_7 \oplus x_1x_3x_4x_5x_7x_8 \oplus x_1x_3x_4x_5x_7x_8x_9 \oplus x_1x_3x_4x_5x_8x_9 \oplus x_1x_3x_4x_5x_9 \oplus$

$x_1x_3x_4x_6 \oplus x_1x_3x_4x_6x_7x_8 \oplus x_1x_3x_4x_6x_7x_9 \oplus x_1x_3x_4x_6x_8 \oplus x_1x_3x_4x_6x_8x_9 \oplus x_1x_3x_4x_7 \oplus x_1x_3x_4x_7x_8x_9 \oplus$

$x_1x_3x_4x_7x_9 \oplus x_1x_3x_4x_8 \oplus x_1x_3x_4x_9 \oplus x_1x_3x_5 \oplus x_1x_3x_5x_6 \oplus x_1x_3x_5x_6x_7 \oplus x_1x_3x_5x_6x_7x_8 \oplus x_1x_3x_5x_6x_7x_8x_9 \oplus$

$x_1x_3x_5x_6x_8x_9 \oplus x_1x_3x_5x_6x_9 \oplus x_1x_3x_5x_7x_8 \oplus x_1x_3x_5x_7x_9 \oplus x_1x_3x_5x_8 \oplus x_1x_3x_5x_8x_9 \oplus x_1x_3x_6x_7 \oplus x_1x_3x_6x_7x_8x_9 \oplus$

$x_1x_3x_6x_7x_9 \oplus x_1x_3x_6x_8 \oplus x_1x_3x_6x_9 \oplus x_1x_3x_7 \oplus x_1x_3x_7x_8 \oplus x_1x_3x_7x_8x_9 \oplus x_1x_3x_8x_9 \oplus x_1x_3x_9 \oplus x_1x_4 \oplus$

$x_1x_4x_5x_6 \oplus x_1x_4x_5x_6x_7x_8 \oplus x_1x_4x_5x_6x_7x_9 \oplus x_1x_4x_5x_6x_8 \oplus x_1x_4x_5x_6x_8x_9 \oplus x_1x_4x_5x_7 \oplus x_1x_4x_5x_7x_8x_9 \oplus$

$x_1x_4x_5x_7x_9 \oplus x_1x_4x_5x_8 \oplus x_1x_4x_5x_9 \oplus x_1x_4x_6 \oplus x_1x_4x_6x_7 \oplus x_1x_4x_6x_7x_8 \oplus x_1x_4x_6x_7x_8x_9 \oplus x_1x_4x_6x_8x_9 \oplus$

$x_1x_4x_6x_9 \oplus x_1x_4x_7x_8 \oplus x_1x_4x_7x_9 \oplus x_1x_4x_8 \oplus x_1x_4x_8x_9 \oplus x_1x_5 \oplus x_1x_5x_6x_7 \oplus x_1x_5x_6x_7x_8x_9 \oplus x_1x_5x_6x_7x_9 \oplus$

$x_1x_5x_6x_8 \oplus x_1x_5x_6x_9 \oplus x_1x_5x_7 \oplus x_1x_5x_7x_8 \oplus x_1x_5x_7x_8x_9 \oplus x_1x_5x_8x_9 \oplus x_1x_5x_9 \oplus x_1x_6 \oplus x_1x_6x_7x_8 \oplus x_1x_6x_7x_9 \oplus$

$x_1x_6x_8 \oplus x_1x_6x_8x_9 \oplus x_1x_7 \oplus x_1x_7x_8x_9 \oplus x_1x_7x_9 \oplus x_1x_8 \oplus x_1x_9 \oplus x_2 \oplus x_2x_3 \oplus x_2x_3x_4 \oplus x_2x_3x_4x_5 \oplus x_2x_3x_4x_5x_6 \oplus$

$x_2x_3x_4x_5x_6x_7 \oplus x_2x_3x_4x_5x_6x_7x_8 \oplus x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_2x_3x_4x_5x_6x_8x_9 \oplus x_2x_3x_4x_5x_6x_9 \oplus x_2x_3x_4x_5x_7x_8 \oplus$

$x_2x_3x_4x_5x_7x_9 \oplus x_2x_3x_4x_5x_8 \oplus x_2x_3x_4x_5x_8x_9 \oplus x_2x_3x_4x_6x_7 \oplus x_2x_3x_4x_6x_7x_8x_9 \oplus x_2x_3x_4x_6x_7x_9 \oplus x_2x_3x_4x_6x_8 \oplus$

$x_2x_3x_4x_6x_9 \oplus x_2x_3x_4x_7 \oplus x_2x_3x_4x_7x_8 \oplus x_2x_3x_4x_7x_8x_9 \oplus x_2x_3x_4x_8x_9 \oplus x_2x_3x_4x_9 \oplus x_2x_3x_5x_6 \oplus x_2x_3x_5x_6x_7x_8 \oplus$

$x_2x_3x_5x_6x_7x_9 \oplus x_2x_3x_5x_6x_8 \oplus x_2x_3x_5x_6x_8x_9 \oplus x_2x_3x_5x_7 \oplus x_2x_3x_5x_7x_8x_9 \oplus x_2x_3x_5x_7x_9 \oplus x_2x_3x_5x_8 \oplus x_2x_3x_5x_9 \oplus$

$x_2x_3x_6 \oplus x_2x_3x_6x_7 \oplus x_2x_3x_6x_7x_8 \oplus x_2x_3x_6x_7x_8x_9 \oplus x_2x_3x_6x_8x_9 \oplus x_2x_3x_6x_9 \oplus x_2x_3x_7x_8 \oplus x_2x_3x_7x_9 \oplus$

$x_2x_3x_8 \oplus x_2x_3x_8x_9 \oplus x_2x_4x_5 \oplus x_2x_4x_5x_6x_7 \oplus x_2x_4x_5x_6x_7x_8x_9 \oplus x_2x_4x_5x_6x_7x_9 \oplus x_2x_4x_5x_6x_8 \oplus x_2x_4x_5x_6x_9 \oplus$

$x_2x_4x_5x_7 \oplus x_2x_4x_5x_7x_8 \oplus x_2x_4x_5x_7x_8x_9 \oplus x_2x_4x_5x_8x_9 \oplus x_2x_4x_5x_9 \oplus x_2x_4x_6 \oplus x_2x_4x_6x_7x_8 \oplus x_2x_4x_6x_7x_9 \oplus$

$x_2x_4x_6x_8 \oplus x_2x_4x_6x_8x_9 \oplus x_2x_4x_7 \oplus x_2x_4x_7x_8x_9 \oplus x_2x_4x_7x_9 \oplus x_2x_4x_8 \oplus x_2x_4x_9 \oplus x_2x_5 \oplus x_2x_5x_6 \oplus x_2x_5x_6x_7 \oplus$

$x_2x_5x_6x_7x_8 \oplus x_2x_5x_6x_7x_8x_9 \oplus x_2x_5x_6x_8x_9 \oplus x_2x_5x_6x_9 \oplus x_2x_5x_7x_8 \oplus x_2x_5x_7x_9 \oplus x_2x_5x_8 \oplus x_2x_5x_8x_9 \oplus x_2x_6x_7 \oplus$

$x_2x_6x_7x_8x_9 \oplus x_2x_6x_7x_9 \oplus x_2x_6x_8 \oplus x_2x_6x_9 \oplus x_2x_7 \oplus x_2x_7x_8 \oplus x_2x_7x_8x_9 \oplus x_2x_8x_9 \oplus x_2x_9 \oplus x_3x_4 \oplus x_3x_4x_5x_6 \oplus$

$x_3x_4x_5x_6x_7x_8 \oplus x_3x_4x_5x_6x_7x_9 \oplus x_3x_4x_5x_6x_8 \oplus x_3x_4x_5x_6x_8x_9 \oplus x_3x_4x_5x_7 \oplus x_3x_4x_5x_7x_8x_9 \oplus x_3x_4x_5x_7x_9 \oplus$

$x_3x_4x_5x_8 \oplus x_3x_4x_5x_9 \oplus x_3x_4x_6 \oplus x_3x_4x_6x_7 \oplus x_3x_4x_6x_7x_8 \oplus x_3x_4x_6x_7x_8x_9 \oplus x_3x_4x_6x_8x_9 \oplus x_3x_4x_6x_9 \oplus x_3x_4x_7x_8 \oplus$

$x_3x_4x_7x_9 \oplus x_3x_4x_8 \oplus x_3x_4x_8x_9 \oplus x_3x_5 \oplus x_3x_5x_6x_7 \oplus x_3x_5x_6x_7x_8x_9 \oplus x_3x_5x_6x_7x_9 \oplus x_3x_5x_6x_8 \oplus x_3x_5x_6x_9 \oplus$

$x_3x_5x_7 \oplus x_3x_5x_7x_8 \oplus x_3x_5x_7x_8x_9 \oplus x_3x_5x_8x_9 \oplus x_3x_5x_9 \oplus x_3x_6 \oplus x_3x_6x_7x_8 \oplus x_3x_6x_7x_9 \oplus x_3x_6x_8 \oplus x_3x_6x_8x_9 \oplus$

$x_3x_7 \oplus x_3x_7x_8x_9 \oplus x_3x_7x_9 \oplus x_3x_8 \oplus x_3x_9 \oplus x_4 \oplus x_4x_5 \oplus x_4x_5x_6 \oplus x_4x_5x_6x_7 \oplus x_4x_5x_6x_7x_8 \oplus x_4x_5x_6x_7x_8x_9 \oplus$

$x_4x_5x_6x_8x_9 \oplus x_4x_5x_6x_9 \oplus x_4x_5x_7x_8 \oplus x_4x_5x_7x_9 \oplus x_4x_5x_8 \oplus x_4x_5x_8x_9 \oplus x_4x_6x_7 \oplus x_4x_6x_7x_8x_9 \oplus x_4x_6x_7x_9 \oplus$

$x_4x_6x_8 \oplus x_4x_6x_9 \oplus x_4x_7 \oplus x_4x_7x_8 \oplus x_4x_7x_8x_9 \oplus x_4x_8x_9 \oplus x_4x_9 \oplus x_5x_6 \oplus x_5x_6x_7x_8 \oplus x_5x_6x_7x_9 \oplus x_5x_6x_8 \oplus$

$x_5x_6x_8x_9 \oplus x_5x_7 \oplus x_5x_7x_8x_9 \oplus x_5x_7x_9 \oplus x_5x_8 \oplus x_5x_9 \oplus x_6 \oplus x_6x_7 \oplus x_6x_7x_8 \oplus x_6x_7x_8x_9 \oplus x_6x_8x_9 \oplus x_6x_9 \oplus$

$x_7x_8 \oplus x_7x_9 \oplus x_8 \oplus x_8x_9 = 0$

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