

# Retrodictive Execution of Quantum Circuits

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## Abstract

## 1 Introduction

*Retrodictive quantum theory* [3], retrocausality [1], and the time-symmetry of physical laws [9] suggest that partial knowledge about the future can be exploited to understand the present. We demonstrate the even stronger proposition that, in concert with the computational concepts of *demand-driven lazy evaluation* [5] and *symbolic partial evaluation* [4], retrodictive reasoning can be used to de-quantize some quantum algorithms, i.e., to provide efficient classical algorithms inspired by their quantum counterparts.

We begin by introducing the principles of demand-driven lazy evaluation, symbolic partial evaluation, and then apply them to the quantum circuit model to de-quantize quantum algorithms.

**Lazy Evaluation.** Consider a program that searches for three different numbers  $x$ ,  $y$ , and  $z$  each in the range  $[1..n]$  and that sum to  $s$ . A well-established design principle for solving such problems is the *generate-and-test* computational paradigm. Following this principle, a simple program to solve this problem in the programming language Haskell is:

```
generate :: Int -> [(Int,Int,Int)]
generate n = [(x,y,z) | x <- [1..n], y <- [1..n], z <- [1..n]]

test :: Int -> [(Int,Int,Int)] -> [(Int,Int,Int)]
test s nums = [(x,y,z) | (x,y,z) <- nums, x /= y, x /= z, y /= z, x+y+z == s]

find :: Int -> Int -> (Int,Int,Int)
find s = head . test s . generate
```

The program consists of three functions: **generate** that produces all triples  $(x,y,z)$  from  $(1,1,1)$  to  $(n,n,n)$ ; **test** that checks that the numbers are different and that their sum is equal to  $s$ ; and **find** that composes the two functions: generating all triples, testing the ones that satisfy the condition, and returning the first solution. Running this program to find numbers in the range  $[1..6]$  that sum to 15 immediately produces  $(4,5,6)$  as expected. But what if the range of interest was  $[1..10000000]$ ? A naïve execution of the generate-and-test method would be prohibitively expensive as it would spend all its time generating an enormous number of triples that are un-needed.

Lazy demand-driven evaluation as implemented in Haskell succeeds in a few seconds with the result  $(1, 2, 12)$ , however. The idea is simple: instead of eagerly generating all the triples, generate a process that, when queried, produces one triple at a time on demand. Conceptually the execution starts from the observer side which is asking for the first element of a list; this demand is propagated to the function **test** which itself propagates the demand to the function **generate**. As each triple is generated, it is tested until one triple passes the test. This triple is immediately returned without having to generate any additional values.

**Partial Evaluation.** Below is a Haskell program that computes  $a^n$  by repeated squaring:

```
power :: Int -> Int -> Int
power a n
  | n == 0    = 1
  | n == 1    = a
  | even n    = let r = power a (n `div` 2) in r * r
  | otherwise = a * power a (n-1)
```

When both inputs are known, e.g.,  $a = 3$  and  $n = 5$ , the program evaluates as follows:

```
power 3 5
= 3 * power 3 4
= 3 * (let r1 = power 3 2 in r1 * r1)
= 3 * (let r1 = (let r2 = power 3 1 in r2 * r2) in r1 * r1)
= 3 * (let r1 = (let r2 = 3 in r2 * r2) in r1 * r1)
= 3 * (let r1 = 9 in r1 * r1)
= 243
```

Partial evaluation is used when we only have partial information about the inputs. Say we only know  $n = 5$ . A partial evaluator then attempts to evaluate `power` with symbolic input `a` and actual input `n=5`. This evaluation proceeds as follows:

```
power a 5
= a * power a 4
= a * (let r1 = power a 2 in r1 * r1)
= a * (let r1 = (let r2 = power a 1 in r2 * r2) in r1 * r1)
= a * (let r1 = (let r2 = a in r2 * r2) in r1 * r1)
= a * (let r1 = a * a in r1 * r1)
= let r1 = a * a in a * r1 * r1
```

All of this evaluation, simplification, and specialization happens without knowledge of `a`. Just knowing `n` was enough to produce a residual program that is much simpler.

**Quantum Circuit Model.** Many quantum algorithms can be expressed using circuits consisting of three stages: preparation, unitary evolution, and measurement. For a large number of quantum algorithms, the unitary evolution is a quantum oracle  $U_f$  that encapsulates a classical function  $f$  to be analyzed as shown in Fig. 1.

In the conventional execution model of quantum circuits, which is the conventional way to use quantum mechanics as a predictive theory, the  $U_f$  block receives both inputs and evolves in the forwards direction to produce the outputs. The preceding discussion about lazy evaluation and partial evaluation suggests, however, more creative ways to execute the  $U_f$  block as shown in Fig. 2. In this scenario, the second register has both a known initial and final observed result. Using this knowledge, it is possible to perform a demand-driven partial evaluation with a symbolic placeholder for  $x$ . This execution produces information about the necessary initial conditions on  $x$  that lead to the observed final result.

The circuit in Fig. 3 explains the idea in a simple but realistic scenario. The circuit uses a hand-optimized  $U_f$  that implements the modular exponentiation  $4^x \bmod 15$  in order to factor 15 using Shor's algorithm. In a conventional setting, the execution proceeds as follows. At step (1), we

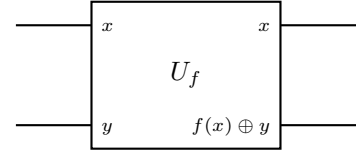


Figure 1: Quantum oracle

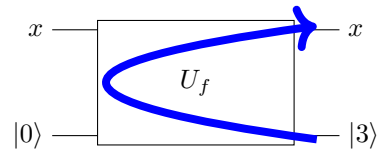


Figure 2: Retrodictive execution flow

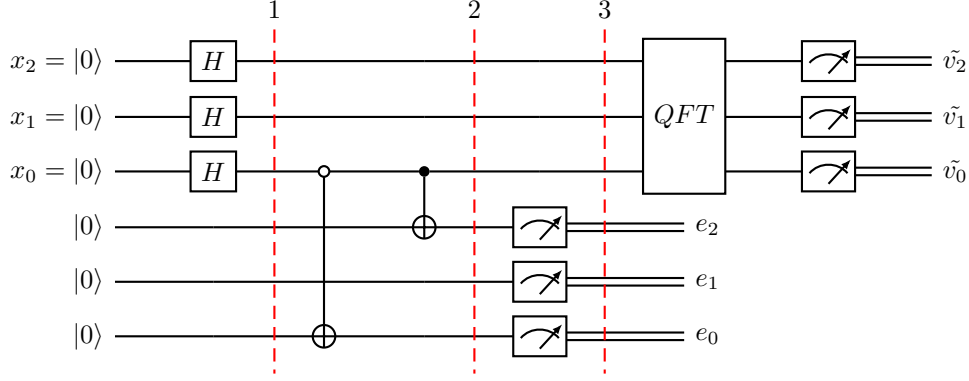


Figure 3: Finding the period of  $4^x \bmod 15$

have the initial state  $(1/2\sqrt{2}) \sum_{i=0}^7 |i\rangle |0\rangle$ . The state evolves through the  $U_f$  block between (1) and (2) to become:

$$\frac{1}{2\sqrt{2}} ((|0\rangle + |2\rangle + |4\rangle + |6\rangle) |1\rangle + (|1\rangle + |3\rangle + |5\rangle + |7\rangle) |4\rangle)$$

At this point, the second register is measured. The result of the measurement can be either  $|1\rangle$  or  $|4\rangle$ . In either case, the top register snaps to a state of the form  $\sum_{r=0}^3 |a + 2r\rangle$  whose QFT has peaks at  $|0\rangle$  or  $|4\rangle$ . If we measure  $|0\rangle$  we repeat the experiment; otherwise we infer that the period is 2.

Instead of this forward execution, we can reason as follows. Since  $x^0 = 1$  for all  $x$ , we know that  $|1\rangle$  is a possible measurement of the second register. We can therefore proceed in a retrodictive fashion with the state  $|x_2 x_1 x_0\rangle |001\rangle$  at step 2 and computing backwards. In this symbolic execution, the first CX-gate changes the state to  $|x_2 x_1 x_0\rangle |x_0 01\rangle$  and the second CX-gate produces  $|x_2 x_1 x_0\rangle |x_0 0 x_0\rangle$ . At that point, we reconcile the retrodictive result of the second register  $|x_0 0 x_0\rangle$  with the initial condition  $|000\rangle$  to conclude that  $x_0 = 0$ . In other the first register must be in a superposition of basis states of the form  $|??0\rangle$  where the least significant bit must be 0 and the other two bits are unconstrained. Expanding the possibilities, the first register need to be in a superposition of the states  $|000\rangle, |010\rangle, |100\rangle$  or  $|110\rangle$ . We have just inferred, using purely classical but retrodictive reasoning, that the period is 2.

In order to assess whether this idea works for a broader class of situations including different algorithms and different circuit sizes, we implemented the demand-driven symbolic partial evaluator and ran it on a variety of circuits. The first experiment generalizes the simple example above by using more qubits and circuits that are constructed automatically without any manual optimization. In particular, we generated 8 qubit modular exponentiations circuits to compute  $a^x \bmod 15$  for  $a \in \{2, 4, 7, 8, 11, 13, 14\}$ . Each of the circuits, automatically constructed from first principles using adders and multipliers, has 26244 CX(controlled not) gates, 27378 CCX(Toffoli) gates, and 2916 CCCX (generalized Toffoli) gates. Running the retrodictive partial evaluator with an observed value of 1, produces the equations in Fig. 4.

Perhaps surprisingly, even though there are 8 qubits in the circuit and thousands of controlled gates, the equations are trivial and immediately solvable as they only involve either the least significant bit  $x_0$  (when  $a \in \{4, 11, 14\}$ ) or the least significant two bits  $x_0$  and  $x_1$  (when  $a \in \{2, 7, 8, 13\}$ ). When the solution is  $x_0 = 0$ , the period is 2. When the solution is  $x_0 = 0, x_1 = 0$ , the period is 4.

The equations turn out to be trivial when the period is a power of 2. This occurs when the number to factor is a product of Fermat primes: 3, 5, 17, 257, 65537, .... The equations generated for some of these cases are in Fig. ??.

$a = 11$	$x_0 = 0$				$x_0 = 0$
$a = 4, 14$	$1 \oplus x_0 = 1$	$x_0 = 0$			$x_0 = 0$
$a = 7, 13$	$1 \oplus x_0x_1 \oplus x_1 = 1$	$x_0x_1 = 0$	$x_0 \oplus x_0x_1 \oplus x_1 = 0$	$x_0 \oplus x_0x_1 = 0$	$x_0 = 0, x_1 = 0$
$a = 2, 8$	$1 \oplus x_0 \oplus x_0x_1 \oplus x_1 = 1$	$x_0x_1 = 0$	$x_0x_1 \oplus x_1 = 0$	$x_0 \oplus x_0x_1 = 0$	$x_0 = 0, x_1 = 0$

Figure 4: Equations generated by retrodictive execution of  $a^x \bmod 15$  starting from observed result 1 and unknown  $x_7x_6x_5x_4x_3x_2x_1x_0$ . The solution for the unknown variables is given in the last column.

**Equations generated by retrodictive execution of  $4^x \bmod 21$  starting from observed result 1 and unknown  $x$ .** The circuit consists of 9 qubits, 36400 CX-gates, 38200 CCX-gates, and 4000 CCCX-gates. There are only three equations but each equation is exponentially large.

$$\begin{aligned}
& 1 \oplus x_0 \oplus x_0x_1x_2 \oplus x_0x_1x_2x_3x_4 \oplus x_0x_1x_2x_3x_4x_5x_6 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_8 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_9 \oplus x_0x_1x_2x_3x_4x_5x_6x_8 \oplus \\
& x_0x_1x_2x_3x_4x_5x_6x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_7 \oplus x_0x_1x_2x_3x_4x_5x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_7x_9 \oplus x_0x_1x_2x_3x_4x_5x_8 \oplus \\
& x_0x_1x_2x_3x_4x_5x_9 \oplus x_0x_1x_2x_3x_4x_6 \oplus x_0x_1x_2x_3x_4x_6x_7 \oplus x_0x_1x_2x_3x_4x_6x_7x_8 \oplus x_0x_1x_2x_3x_4x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_6x_8x_9 \oplus \\
& x_0x_1x_2x_3x_4x_6x_9 \oplus x_0x_1x_2x_3x_4x_7x_8 \oplus x_0x_1x_2x_3x_4x_7x_9 \oplus x_0x_1x_2x_3x_4x_8 \oplus x_0x_1x_2x_3x_4x_8x_9 \oplus x_0x_1x_2x_3x_5 \oplus \\
& x_0x_1x_2x_3x_5x_6x_7 \oplus x_0x_1x_2x_3x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_5x_6x_7x_9 \oplus x_0x_1x_2x_3x_5x_6x_8 \oplus x_0x_1x_2x_3x_5x_6x_9 \oplus x_0x_1x_2x_3x_5x_7 \oplus \\
& x_0x_1x_2x_3x_5x_7x_8 \oplus x_0x_1x_2x_3x_5x_7x_8x_9 \oplus x_0x_1x_2x_3x_5x_8x_9 \oplus x_0x_1x_2x_3x_5x_9 \oplus x_0x_1x_2x_3x_6 \oplus x_0x_1x_2x_3x_6x_7x_8 \oplus \\
& x_0x_1x_2x_3x_6x_7x_9 \oplus x_0x_1x_2x_3x_6x_8 \oplus x_0x_1x_2x_3x_6x_8x_9 \oplus x_0x_1x_2x_3x_6x_9 \oplus x_0x_1x_2x_3x_7 \oplus x_0x_1x_2x_3x_7x_8x_9 \oplus x_0x_1x_2x_3x_7x_9 \oplus \\
& x_0x_1x_2x_3x_8 \oplus x_0x_1x_2x_3x_9 \oplus x_0x_1x_2x_4 \oplus x_0x_1x_2x_4x_5 \oplus x_0x_1x_2x_4x_5x_6 \oplus x_0x_1x_2x_4x_5x_6x_7 \oplus x_0x_1x_2x_4x_5x_6x_7x_8 \oplus \\
& x_0x_1x_2x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_4x_5x_6x_8x_9 \oplus x_0x_1x_2x_4x_5x_6x_9 \oplus x_0x_1x_2x_4x_5x_7x_8 \oplus x_0x_1x_2x_4x_5x_7x_9 \oplus x_0x_1x_2x_4x_5x_8 \oplus \\
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& x_0x_1x_2x_4x_7 \oplus x_0x_1x_2x_4x_7x_8 \oplus x_0x_1x_2x_4x_7x_8x_9 \oplus x_0x_1x_2x_4x_8x_9 \oplus x_0x_1x_2x_4x_9 \oplus x_0x_1x_2x_5x_6 \oplus x_0x_1x_2x_5x_6x_7x_8 \oplus \\
& x_0x_1x_2x_5x_6x_7x_9 \oplus x_0x_1x_2x_5x_6x_8 \oplus x_0x_1x_2x_5x_6x_8x_9 \oplus x_0x_1x_2x_5x_7 \oplus x_0x_1x_2x_5x_7x_8x_9 \oplus x_0x_1x_2x_5x_7x_9 \oplus \\
& x_0x_1x_2x_5x_8 \oplus x_0x_1x_2x_5x_9 \oplus x_0x_1x_2x_6 \oplus x_0x_1x_2x_6x_7 \oplus x_0x_1x_2x_6x_7x_8 \oplus x_0x_1x_2x_6x_7x_8x_9 \oplus x_0x_1x_2x_6x_8x_9 \oplus x_0x_1x_2x_6x_9 \oplus \\
& x_0x_1x_2x_6x_9 \oplus x_0x_1x_2x_7x_8 \oplus x_0x_1x_2x_7x_9 \oplus x_0x_1x_2x_8 \oplus x_0x_1x_2x_8x_9 \oplus x_0x_1x_3 \oplus x_0x_1x_3x_4x_5 \oplus x_0x_1x_3x_4x_5x_6x_7 \oplus \\
& x_0x_1x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_3x_4x_5x_6x_7x_9 \oplus x_0x_1x_3x_4x_5x_6x_8 \oplus x_0x_1x_3x_4x_5x_6x_9 \oplus x_0x_1x_3x_4x_5x_7 \oplus x_0x_1x_3x_4x_5x_7x_8 \oplus \\
& x_0x_1x_3x_4x_5x_7x_8x_9 \oplus x_0x_1x_3x_4x_5x_8x_9 \oplus x_0x_1x_3x_4x_5x_9 \oplus x_0x_1x_3x_4x_6 \oplus x_0x_1x_3x_4x_6x_7x_8 \oplus x_0x_1x_3x_4x_6x_7x_9 \oplus \\
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& x_0x_1x_3x_5 \oplus x_0x_1x_3x_5x_6 \oplus x_0x_1x_3x_5x_6x_7 \oplus x_0x_1x_3x_5x_6x_7x_8 \oplus x_0x_1x_3x_5x_6x_7x_8x_9 \oplus x_0x_1x_3x_5x_6x_8x_9 \oplus x_0x_1x_3x_5x_6x_9 \oplus \\
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& x_0x_1x_4x_5x_7x_9 \oplus x_0x_1x_4x_5x_8 \oplus x_0x_1x_4x_5x_9 \oplus x_0x_1x_4x_6 \oplus x_0x_1x_4x_6x_7 \oplus x_0x_1x_4x_6x_7x_8 \oplus x_0x_1x_4x_6x_7x_8x_9 \oplus \\
& x_0x_1x_4x_6x_8x_9 \oplus x_0x_1x_4x_6x_9 \oplus x_0x_1x_4x_7x_8 \oplus x_0x_1x_4x_7x_9 \oplus x_0x_1x_4x_8 \oplus x_0x_1x_4x_8x_9 \oplus x_0x_1x_5 \oplus x_0x_1x_5x_6x_7 \oplus \\
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& x_0x_1x_7x_9 \oplus x_0x_1x_8 \oplus x_0x_1x_9 \oplus x_0x_2 \oplus x_0x_2x_3 \oplus x_0x_2x_3x_4 \oplus x_0x_2x_3x_4x_5 \oplus x_0x_2x_3x_4x_5x_6 \oplus x_0x_2x_3x_4x_5x_6x_7 \oplus \\
& x_0x_2x_3x_4x_5x_6x_7x_8 \oplus x_0x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_2x_3x_4x_5x_6x_8x_9 \oplus x_0x_2x_3x_4x_5x_6x_9 \oplus x_0x_2x_3x_4x_5x_7x_8 \oplus x_0x_2x_3x_4x_5x_7x_9 \oplus \\
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& x_0x_2x_3x_4x_6x_9 \oplus x_0x_2x_3x_4x_7 \oplus x_0x_2x_3x_4x_7x_8 \oplus x_0x_2x_3x_4x_7x_8x_9 \oplus x_0x_2x_3x_4x_8x_9 \oplus x_0x_2x_3x_4x_9 \oplus x_0x_2x_3x_5x_6 \oplus \\
& x_0x_2x_3x_5x_6x_7x_8 \oplus x_0x_2x_3x_5x_6x_7x_9 \oplus x_0x_2x_3x_5x_6x_8 \oplus x_0x_2x_3x_5x_6x_8x_9 \oplus x_0x_2x_3x_5x_7 \oplus x_0x_2x_3x_5x_7x_8x_9 \oplus \\
& x_0x_2x_3x_5x_7x_9 \oplus x_0x_2x_3x_5x_8 \oplus x_0x_2x_3x_5x_9 \oplus x_0x_2x_3x_6 \oplus x_0x_2x_3x_6x_7 \oplus x_0x_2x_3x_6x_7x_8 \oplus x_0x_2x_3x_6x_7x_8x_9 \oplus \\
& x_0x_2x_3x_6x_8x_9 \oplus x_0x_2x_3x_6x_9 \oplus x_0x_2x_3x_7x_8 \oplus x_0x_2x_3x_7x_9 \oplus x_0x_2x_3x_8 \oplus x_0x_2x_3x_8x_9 \oplus x_0x_2x_4x_5 \oplus x_0x_2x_4x_5x_6x_7 \oplus \\
& x_0x_2x_4x_5x_6x_7x_8x_9 \oplus x_0x_2x_4x_5x_6x_7x_9 \oplus x_0x_2x_4x_5x_6x_8 \oplus x_0x_2x_4x_5x_6x_9 \oplus x_0x_2x_4x_5x_7 \oplus x_0x_2x_4x_5x_7x_8 \oplus \\
& x_0x_2x_4x_5x_7x_8x_9 \oplus x_0x_2x_4x_5x_8x_9 \oplus x_0x_2x_4x_5x_9 \oplus x_0x_2x_4x_6 \oplus x_0x_2x_4x_6x_7x_8 \oplus x_0x_2x_4x_6x_7x_9 \oplus x_0x_2x_4x_6x_8 \oplus \\
& x_0x_2x_4x_6x_8x_9 \oplus x_0x_2x_4x_7 \oplus x_0x_2x_4x_7x_8x_9 \oplus x_0x_2x_4x_7x_9 \oplus x_0x_2x_4x_8 \oplus x_0x_2x_4x_9 \oplus x_0x_2x_5 \oplus x_0x_2x_5x_6 \oplus \\
& x_0x_2x_5x_6x_7 \oplus x_0x_2x_5x_6x_7x_8 \oplus x_0x_2x_5x_6x_7x_8x_9 \oplus x_0x_2x_5x_6x_8x_9 \oplus x_0x_2x_5x_6x_9 \oplus x_0x_2x_5x_7x_8 \oplus x_0x_2x_5x_7x_9 \oplus \\
& x_0x_2x_5x_8 \oplus x_0x_2x_5x_8x_9 \oplus x_0x_2x_6x_7 \oplus x_0x_2x_6x_7x_8x_9 \oplus x_0x_2x_6x_7x_9 \oplus x_0x_2x_6x_8 \oplus x_0x_2x_6x_9 \oplus x_0x_2x_7 \oplus \\
& x_0x_2x_7x_8 \oplus x_0x_2x_7x_8x_9 \oplus x_0x_2x_8x_9 \oplus x_0x_2x_9 \oplus x_0x_3x_4 \oplus x_0x_3x_4x_5x_6 \oplus x_0x_3x_4x_5x_6x_7x_8 \oplus x_0x_3x_4x_5x_6x_7x_9 \oplus
\end{aligned}$$

$$\begin{aligned}
& x_0x_3x_4x_5x_6x_8 \oplus x_0x_3x_4x_5x_6x_8x_9 \oplus x_0x_3x_4x_5x_7 \oplus x_0x_3x_4x_5x_7x_8x_9 \oplus x_0x_3x_4x_5x_7x_9 \oplus x_0x_3x_4x_5x_8 \oplus x_0x_3x_4x_5x_9 \oplus \\
& x_0x_3x_4x_6 \oplus x_0x_3x_4x_6x_7 \oplus x_0x_3x_4x_6x_7x_8 \oplus x_0x_3x_4x_6x_7x_8x_9 \oplus x_0x_3x_4x_6x_8x_9 \oplus x_0x_3x_4x_6x_9 \oplus x_0x_3x_4x_7x_8 \oplus \\
& x_0x_3x_4x_7x_9 \oplus x_0x_3x_4x_8 \oplus x_0x_3x_4x_8x_9 \oplus x_0x_3x_5 \oplus x_0x_3x_5x_6x_7 \oplus x_0x_3x_5x_6x_7x_8x_9 \oplus x_0x_3x_5x_6x_7x_9 \oplus x_0x_3x_5x_6x_8 \oplus \\
& x_0x_3x_5x_6x_9 \oplus x_0x_3x_5x_7 \oplus x_0x_3x_5x_7x_8 \oplus x_0x_3x_5x_7x_8x_9 \oplus x_0x_3x_5x_8x_9 \oplus x_0x_3x_5x_9 \oplus x_0x_3x_6 \oplus x_0x_3x_6x_7x_8 \oplus \\
& x_0x_3x_6x_7x_9 \oplus x_0x_3x_6x_8 \oplus x_0x_3x_6x_8x_9 \oplus x_0x_3x_7 \oplus x_0x_3x_7x_8x_9 \oplus x_0x_3x_7x_9 \oplus x_0x_3x_8 \oplus x_0x_3x_9 \oplus x_0x_4 \oplus x_0x_4x_5 \oplus \\
& x_0x_4x_5x_6 \oplus x_0x_4x_5x_6x_7 \oplus x_0x_4x_5x_6x_7x_8 \oplus x_0x_4x_5x_6x_7x_8x_9 \oplus x_0x_4x_5x_6x_8x_9 \oplus x_0x_4x_5x_6x_9 \oplus x_0x_4x_5x_7x_8 \oplus \\
& x_0x_4x_5x_7x_9 \oplus x_0x_4x_5x_8 \oplus x_0x_4x_5x_8x_9 \oplus x_0x_4x_6x_7 \oplus x_0x_4x_6x_7x_8x_9 \oplus x_0x_4x_6x_7x_9 \oplus x_0x_4x_6x_8 \oplus x_0x_4x_6x_9 \oplus \\
& x_0x_4x_7 \oplus x_0x_4x_7x_8 \oplus x_0x_4x_7x_8x_9 \oplus x_0x_4x_8x_9 \oplus x_0x_4x_9 \oplus x_0x_5x_6 \oplus x_0x_5x_6x_7x_8 \oplus x_0x_5x_6x_7x_9 \oplus x_0x_5x_6x_8 \oplus \\
& x_0x_5x_6x_8x_9 \oplus x_0x_5x_7 \oplus x_0x_5x_7x_8x_9 \oplus x_0x_5x_7x_9 \oplus x_0x_5x_8 \oplus x_0x_5x_9 \oplus x_0x_6 \oplus x_0x_6x_7 \oplus x_0x_6x_7x_8 \oplus x_0x_6x_7x_8x_9 \oplus \\
& x_0x_6x_8x_9 \oplus x_0x_6x_9 \oplus x_0x_7x_8 \oplus x_0x_7x_9 \oplus x_0x_8 \oplus x_0x_8x_9 \oplus x_1 \oplus x_1x_2x_3 \oplus x_1x_2x_3x_4x_5 \oplus x_1x_2x_3x_4x_5x_6x_7 \oplus \\
& x_1x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_1x_2x_3x_4x_5x_6x_7x_9 \oplus x_1x_2x_3x_4x_5x_6x_8 \oplus x_1x_2x_3x_4x_5x_6x_9 \oplus x_1x_2x_3x_4x_5x_7 \oplus x_1x_2x_3x_4x_5x_7x_8 \oplus \\
& x_1x_2x_3x_4x_5x_7x_8x_9 \oplus x_1x_2x_3x_4x_5x_8x_9 \oplus x_1x_2x_3x_4x_5x_9 \oplus x_1x_2x_3x_4x_6 \oplus x_1x_2x_3x_4x_6x_7x_8 \oplus x_1x_2x_3x_4x_6x_7x_9 \oplus \\
& x_1x_2x_3x_4x_6x_8 \oplus x_1x_2x_3x_4x_6x_8x_9 \oplus x_1x_2x_3x_4x_7 \oplus x_1x_2x_3x_4x_7x_8x_9 \oplus x_1x_2x_3x_4x_7x_9 \oplus x_1x_2x_3x_4x_8 \oplus x_1x_2x_3x_4x_9 \oplus \\
& x_1x_2x_3x_5 \oplus x_1x_2x_3x_5x_6 \oplus x_1x_2x_3x_5x_6x_7 \oplus x_1x_2x_3x_5x_6x_7x_8 \oplus x_1x_2x_3x_5x_6x_7x_8x_9 \oplus x_1x_2x_3x_5x_6x_8x_9 \oplus x_1x_2x_3x_5x_6x_9 \oplus \\
& x_1x_2x_3x_5x_7x_8 \oplus x_1x_2x_3x_5x_7x_9 \oplus x_1x_2x_3x_5x_8 \oplus x_1x_2x_3x_5x_8x_9 \oplus x_1x_2x_3x_6x_7 \oplus x_1x_2x_3x_6x_7x_8x_9 \oplus x_1x_2x_3x_6x_7x_9 \oplus \\
& x_1x_2x_3x_6x_8 \oplus x_1x_2x_3x_6x_9 \oplus x_1x_2x_3x_7 \oplus x_1x_2x_3x_7x_8 \oplus x_1x_2x_3x_7x_8x_9 \oplus x_1x_2x_3x_8x_9 \oplus x_1x_2x_3x_9 \oplus x_1x_2x_4 \oplus \\
& x_1x_2x_4x_5x_6 \oplus x_1x_2x_4x_5x_6x_7x_8 \oplus x_1x_2x_4x_5x_6x_7x_9 \oplus x_1x_2x_4x_5x_6x_8 \oplus x_1x_2x_4x_5x_6x_8x_9 \oplus x_1x_2x_4x_5x_7 \oplus x_1x_2x_4x_5x_7x_8x_9 \oplus \\
& x_1x_2x_4x_5x_7x_9 \oplus x_1x_2x_4x_5x_8 \oplus x_1x_2x_4x_5x_9 \oplus x_1x_2x_4x_6 \oplus x_1x_2x_4x_6x_7 \oplus x_1x_2x_4x_6x_7x_8 \oplus x_1x_2x_4x_6x_7x_8x_9 \oplus \\
& x_1x_2x_4x_6x_8x_9 \oplus x_1x_2x_4x_6x_9 \oplus x_1x_2x_4x_7x_8 \oplus x_1x_2x_4x_7x_9 \oplus x_1x_2x_4x_8 \oplus x_1x_2x_4x_8x_9 \oplus x_1x_2x_5 \oplus x_1x_2x_5x_6x_7 \oplus \\
& x_1x_2x_5x_6x_7x_8x_9 \oplus x_1x_2x_5x_6x_7x_9 \oplus x_1x_2x_5x_6x_8 \oplus x_1x_2x_5x_6x_9 \oplus x_1x_2x_5x_7 \oplus x_1x_2x_5x_7x_8 \oplus x_1x_2x_5x_7x_8x_9 \oplus \\
& x_1x_2x_5x_8x_9 \oplus x_1x_2x_5x_9 \oplus x_1x_2x_6 \oplus x_1x_2x_6x_7x_8 \oplus x_1x_2x_6x_7x_9 \oplus x_1x_2x_6x_8 \oplus x_1x_2x_6x_8x_9 \oplus x_1x_2x_7 \oplus x_1x_2x_7x_8x_9 \oplus \\
& x_1x_2x_7x_9 \oplus x_1x_2x_8 \oplus x_1x_2x_9 \oplus x_1x_3 \oplus x_1x_3x_4 \oplus x_1x_3x_4x_5 \oplus x_1x_3x_4x_5x_6 \oplus x_1x_3x_4x_5x_6x_7 \oplus x_1x_3x_4x_5x_6x_7x_8 \oplus \\
& x_1x_3x_4x_5x_6x_7x_8x_9 \oplus x_1x_3x_4x_5x_6x_8x_9 \oplus x_1x_3x_4x_5x_6x_9 \oplus x_1x_3x_4x_5x_7x_8 \oplus x_1x_3x_4x_5x_7x_9 \oplus x_1x_3x_4x_5x_8 \oplus \\
& x_1x_3x_4x_5x_8x_9 \oplus x_1x_3x_4x_6x_7 \oplus x_1x_3x_4x_6x_7x_8x_9 \oplus x_1x_3x_4x_6x_7x_9 \oplus x_1x_3x_4x_6x_8 \oplus x_1x_3x_4x_6x_9 \oplus x_1x_3x_4x_7 \oplus \\
& x_1x_3x_4x_7x_8 \oplus x_1x_3x_4x_7x_8x_9 \oplus x_1x_3x_4x_8x_9 \oplus x_1x_3x_4x_9 \oplus x_1x_3x_5x_6 \oplus x_1x_3x_5x_6x_7x_8 \oplus x_1x_3x_5x_6x_7x_9 \oplus x_1x_3x_5x_6x_8 \oplus \\
& x_1x_3x_5x_6x_8x_9 \oplus x_1x_3x_5x_7 \oplus x_1x_3x_5x_7x_8x_9 \oplus x_1x_3x_5x_7x_9 \oplus x_1x_3x_5x_8 \oplus x_1x_3x_5x_9 \oplus x_1x_3x_6 \oplus x_1x_3x_6x_7 \oplus \\
& x_1x_3x_6x_7x_8 \oplus x_1x_3x_6x_7x_8x_9 \oplus x_1x_3x_6x_8x_9 \oplus x_1x_3x_6x_9 \oplus x_1x_3x_7x_8 \oplus x_1x_3x_7x_9 \oplus x_1x_3x_8 \oplus x_1x_3x_8x_9 \oplus x_1x_4x_5 \oplus \\
& x_1x_4x_5x_6x_7 \oplus x_1x_4x_5x_6x_7x_8x_9 \oplus x_1x_4x_5x_6x_7x_9 \oplus x_1x_4x_5x_6x_8 \oplus x_1x_4x_5x_6x_9 \oplus x_1x_4x_5x_7 \oplus x_1x_4x_5x_7x_8 \oplus \\
& x_1x_4x_5x_7x_8x_9 \oplus x_1x_4x_5x_8x_9 \oplus x_1x_4x_5x_9 \oplus x_1x_4x_6 \oplus x_1x_4x_6x_7x_8 \oplus x_1x_4x_6x_7x_9 \oplus x_1x_4x_6x_8 \oplus x_1x_4x_6x_8x_9 \oplus \\
& x_1x_4x_7 \oplus x_1x_4x_7x_8x_9 \oplus x_1x_4x_7x_9 \oplus x_1x_4x_8 \oplus x_1x_4x_9 \oplus x_1x_5 \oplus x_1x_5x_6 \oplus x_1x_5x_6x_7 \oplus x_1x_5x_6x_7x_8 \oplus x_1x_5x_6x_7x_8x_9 \oplus \\
& x_1x_5x_6x_8x_9 \oplus x_1x_5x_6x_9 \oplus x_1x_5x_7x_8 \oplus x_1x_5x_7x_9 \oplus x_1x_5x_8 \oplus x_1x_5x_8x_9 \oplus x_1x_6x_7 \oplus x_1x_6x_7x_8x_9 \oplus x_1x_6x_7x_9 \oplus \\
& x_1x_6x_8 \oplus x_1x_6x_9 \oplus x_1x_7 \oplus x_1x_7x_8 \oplus x_1x_7x_8x_9 \oplus x_1x_8x_9 \oplus x_1x_9 \oplus x_2 \oplus x_2x_3x_4 \oplus x_2x_3x_4x_5x_6 \oplus x_2x_3x_4x_5x_6x_7x_8 \oplus \\
& x_2x_3x_4x_5x_6x_7x_9 \oplus x_2x_3x_4x_5x_6x_8 \oplus x_2x_3x_4x_5x_6x_8x_9 \oplus x_2x_3x_4x_5x_7 \oplus x_2x_3x_4x_5x_7x_8x_9 \oplus x_2x_3x_4x_5x_7x_9 \oplus \\
& x_2x_3x_4x_5x_8 \oplus x_2x_3x_4x_5x_9 \oplus x_2x_3x_4x_6 \oplus x_2x_3x_4x_6x_7 \oplus x_2x_3x_4x_6x_7x_8 \oplus x_2x_3x_4x_6x_7x_8x_9 \oplus x_2x_3x_4x_6x_8x_9 \oplus \\
& x_2x_3x_4x_6x_9 \oplus x_2x_3x_4x_7x_8 \oplus x_2x_3x_4x_7x_9 \oplus x_2x_3x_4x_8 \oplus x_2x_3x_4x_8x_9 \oplus x_2x_3x_5 \oplus x_2x_3x_5x_6x_7 \oplus x_2x_3x_5x_6x_7x_8x_9 \oplus \\
& x_2x_3x_5x_6x_7x_9 \oplus x_2x_3x_5x_6x_8 \oplus x_2x_3x_5x_6x_9 \oplus x_2x_3x_5x_7 \oplus x_2x_3x_5x_7x_8 \oplus x_2x_3x_5x_7x_8x_9 \oplus x_2x_3x_5x_8x_9 \oplus x_2x_3x_5x_9 \oplus \\
& x_2x_3x_6 \oplus x_2x_3x_6x_7x_8 \oplus x_2x_3x_6x_7x_9 \oplus x_2x_3x_6x_8 \oplus x_2x_3x_6x_8x_9 \oplus x_2x_3x_7 \oplus x_2x_3x_7x_8x_9 \oplus x_2x_3x_7x_9 \oplus x_2x_3x_8 \oplus \\
& x_2x_3x_9 \oplus x_2x_4 \oplus x_2x_4x_5 \oplus x_2x_4x_5x_6 \oplus x_2x_4x_5x_6x_7 \oplus x_2x_4x_5x_6x_7x_8 \oplus x_2x_4x_5x_6x_7x_8x_9 \oplus x_2x_4x_5x_6x_8x_9 \oplus \\
& x_2x_4x_5x_6x_9 \oplus x_2x_4x_5x_7x_8 \oplus x_2x_4x_5x_7x_9 \oplus x_2x_4x_5x_8 \oplus x_2x_4x_5x_8x_9 \oplus x_2x_4x_6x_7 \oplus x_2x_4x_6x_7x_8x_9 \oplus x_2x_4x_6x_7x_9 \oplus \\
& x_2x_4x_6x_8 \oplus x_2x_4x_6x_9 \oplus x_2x_4x_7 \oplus x_2x_4x_7x_8 \oplus x_2x_4x_7x_8x_9 \oplus x_2x_4x_8x_9 \oplus x_2x_4x_9 \oplus x_2x_5x_6 \oplus x_2x_5x_6x_7x_8 \oplus \\
& x_2x_5x_6x_7x_9 \oplus x_2x_5x_6x_8 \oplus x_2x_5x_6x_8x_9 \oplus x_2x_5x_7 \oplus x_2x_5x_7x_8x_9 \oplus x_2x_5x_7x_9 \oplus x_2x_5x_8 \oplus x_2x_5x_9 \oplus x_2x_6 \oplus x_2x_6x_7 \oplus \\
& x_2x_6x_7x_8 \oplus x_2x_6x_7x_8x_9 \oplus x_2x_6x_8x_9 \oplus x_2x_6x_9 \oplus x_2x_7x_8 \oplus x_2x_7x_9 \oplus x_2x_8 \oplus x_2x_8x_9 \oplus x_3 \oplus x_3x_4x_5 \oplus x_3x_4x_5x_6x_7 \oplus \\
& x_3x_4x_5x_6x_7x_8x_9 \oplus x_3x_4x_5x_6x_7x_9 \oplus x_3x_4x_5x_6x_8 \oplus x_3x_4x_5x_6x_9 \oplus x_3x_4x_5x_7 \oplus x_3x_4x_5x_7x_8 \oplus x_3x_4x_5x_7x_8x_9 \oplus \\
& x_3x_4x_5x_8x_9 \oplus x_3x_4x_5x_9 \oplus x_3x_4x_6 \oplus x_3x_4x_6x_7x_8 \oplus x_3x_4x_6x_7x_9 \oplus x_3x_4x_6x_8 \oplus x_3x_4x_6x_8x_9 \oplus x_3x_4x_7 \oplus x_3x_4x_7x_8x_9 \oplus \\
& x_3x_4x_7x_9 \oplus x_3x_4x_8 \oplus x_3x_4x_9 \oplus x_3x_5 \oplus x_3x_5x_6 \oplus x_3x_5x_6x_7 \oplus x_3x_5x_6x_7x_8 \oplus x_3x_5x_6x_7x_8x_9 \oplus x_3x_5x_6x_8x_9 \oplus \\
& x_3x_5x_6x_9 \oplus x_3x_5x_7x_8 \oplus x_3x_5x_7x_9 \oplus x_3x_5x_8 \oplus x_3x_5x_8x_9 \oplus x_3x_6x_7 \oplus x_3x_6x_7x_8x_9 \oplus x_3x_6x_7x_9 \oplus x_3x_6x_8 \oplus x_3x_6x_9 \oplus \\
& x_3x_7 \oplus x_3x_7x_8 \oplus x_3x_7x_8x_9 \oplus x_3x_8x_9 \oplus x_3x_9 \oplus x_4 \oplus x_4x_5x_6 \oplus x_4x_5x_6x_7x_8 \oplus x_4x_5x_6x_7x_9 \oplus x_4x_5x_6x_8 \oplus x_4x_5x_6x_8x_9 \oplus \\
& x_4x_5x_7 \oplus x_4x_5x_7x_8x_9 \oplus x_4x_5x_7x_9 \oplus x_4x_5x_8 \oplus x_4x_5x_9 \oplus x_4x_6 \oplus x_4x_6x_7 \oplus x_4x_6x_7x_8 \oplus x_4x_6x_7x_8x_9 \oplus x_4x_6x_8x_9 \oplus \\
& x_4x_6x_9 \oplus x_4x_7x_8 \oplus x_4x_7x_9 \oplus x_4x_8 \oplus x_4x_8x_9 \oplus x_5 \oplus x_5x_6x_7 \oplus x_5x_6x_7x_8x_9 \oplus x_5x_6x_7x_9 \oplus x_5x_6x_8 \oplus x_5x_6x_9 \oplus x_5x_7 \oplus \\
& x_5x_7x_8 \oplus x_5x_7x_8x_9 \oplus x_5x_8x_9 \oplus x_5x_9 \oplus x_6 \oplus x_6x_7x_8 \oplus x_6x_7x_9 \oplus x_6x_8 \oplus x_6x_8x_9 \oplus x_7 \oplus x_7x_8x_9 \oplus x_7x_9 \oplus x_8 \oplus x_9 = 1 \\
& x_0x_1 \oplus x_0x_1x_2x_3 \oplus x_0x_1x_2x_3x_4x_5 \oplus x_0x_1x_2x_3x_4x_5x_6x_7 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_9 \oplus
\end{aligned}$$

200	$x_0x_1x_2x_3x_4x_5x_6x_8 \oplus x_0x_1x_2x_3x_4x_5x_6x_9 \oplus x_0x_1x_2x_3x_4x_5x_7 \oplus x_0x_1x_2x_3x_4x_5x_7x_8 \oplus x_0x_1x_2x_3x_4x_5x_7x_8x_9 \oplus$
201	$x_0x_1x_2x_3x_4x_5x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_9 \oplus x_0x_1x_2x_3x_4x_6 \oplus x_0x_1x_2x_3x_4x_6x_7x_8 \oplus x_0x_1x_2x_3x_4x_6x_7x_9 \oplus x_0x_1x_2x_3x_4x_6x_8 \oplus$
202	$x_0x_1x_2x_3x_4x_6x_8x_9 \oplus x_0x_1x_2x_3x_4x_7 \oplus x_0x_1x_2x_3x_4x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_7x_9 \oplus x_0x_1x_2x_3x_4x_8 \oplus x_0x_1x_2x_3x_4x_9 \oplus$
203	$x_0x_1x_2x_3x_5 \oplus x_0x_1x_2x_3x_5x_6 \oplus x_0x_1x_2x_3x_5x_6x_7 \oplus x_0x_1x_2x_3x_5x_6x_7x_8 \oplus x_0x_1x_2x_3x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_5x_6x_8x_9 \oplus$
204	$x_0x_1x_2x_3x_5x_6x_9 \oplus x_0x_1x_2x_3x_5x_7x_8 \oplus x_0x_1x_2x_3x_5x_7x_9 \oplus x_0x_1x_2x_3x_5x_8 \oplus x_0x_1x_2x_3x_5x_8x_9 \oplus x_0x_1x_2x_3x_6x_7 \oplus$
205	$x_0x_1x_2x_3x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_6x_7x_9 \oplus x_0x_1x_2x_3x_6x_8 \oplus x_0x_1x_2x_3x_6x_9 \oplus x_0x_1x_2x_3x_7 \oplus x_0x_1x_2x_3x_7x_8 \oplus$
206	$x_0x_1x_2x_3x_7x_8x_9 \oplus x_0x_1x_2x_3x_8x_9 \oplus x_0x_1x_2x_3x_9 \oplus x_0x_1x_2x_4 \oplus x_0x_1x_2x_4x_5x_6 \oplus x_0x_1x_2x_4x_5x_6x_7x_8 \oplus x_0x_1x_2x_4x_5x_6x_7x_9 \oplus$
207	$x_0x_1x_2x_4x_5x_6x_8 \oplus x_0x_1x_2x_4x_5x_6x_8x_9 \oplus x_0x_1x_2x_4x_5x_7 \oplus x_0x_1x_2x_4x_5x_7x_8x_9 \oplus x_0x_1x_2x_4x_5x_7x_9 \oplus x_0x_1x_2x_4x_5x_8 \oplus$
208	$x_0x_1x_2x_4x_5x_9 \oplus x_0x_1x_2x_4x_6 \oplus x_0x_1x_2x_4x_6x_7 \oplus x_0x_1x_2x_4x_6x_7x_8 \oplus x_0x_1x_2x_4x_6x_7x_8x_9 \oplus x_0x_1x_2x_4x_6x_8x_9 \oplus$
209	$x_0x_1x_2x_4x_6x_9 \oplus x_0x_1x_2x_4x_7x_8 \oplus x_0x_1x_2x_4x_7x_9 \oplus x_0x_1x_2x_4x_8 \oplus x_0x_1x_2x_4x_8x_9 \oplus x_0x_1x_2x_5 \oplus x_0x_1x_2x_5x_6x_7 \oplus$
210	$x_0x_1x_2x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_5x_6x_7x_9 \oplus x_0x_1x_2x_5x_6x_8 \oplus x_0x_1x_2x_5x_6x_9 \oplus x_0x_1x_2x_5x_7 \oplus x_0x_1x_2x_5x_7x_8 \oplus$
211	$x_0x_1x_2x_5x_7x_8x_9 \oplus x_0x_1x_2x_5x_8x_9 \oplus x_0x_1x_2x_5x_9 \oplus x_0x_1x_2x_6 \oplus x_0x_1x_2x_6x_7x_8 \oplus x_0x_1x_2x_6x_7x_9 \oplus x_0x_1x_2x_6x_8 \oplus$
212	$x_0x_1x_2x_6x_8x_9 \oplus x_0x_1x_2x_7 \oplus x_0x_1x_2x_7x_8x_9 \oplus x_0x_1x_2x_7x_9 \oplus x_0x_1x_2x_8 \oplus x_0x_1x_2x_9 \oplus x_0x_1x_3 \oplus x_0x_1x_3x_4 \oplus$
213	$x_0x_1x_3x_4x_5 \oplus x_0x_1x_3x_4x_5x_6 \oplus x_0x_1x_3x_4x_5x_6x_7 \oplus x_0x_1x_3x_4x_5x_6x_7x_8 \oplus x_0x_1x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_3x_4x_5x_6x_8x_9 \oplus$
214	$x_0x_1x_3x_4x_5x_6x_9 \oplus x_0x_1x_3x_4x_5x_7x_8 \oplus x_0x_1x_3x_4x_5x_7x_9 \oplus x_0x_1x_3x_4x_5x_8 \oplus x_0x_1x_3x_4x_5x_8x_9 \oplus x_0x_1x_3x_4x_6x_7 \oplus$
215	$x_0x_1x_3x_4x_6x_7x_8x_9 \oplus x_0x_1x_3x_4x_6x_7x_9 \oplus x_0x_1x_3x_4x_6x_8 \oplus x_0x_1x_3x_4x_6x_9 \oplus x_0x_1x_3x_4x_7 \oplus x_0x_1x_3x_4x_7x_8 \oplus$
216	$x_0x_1x_3x_4x_7x_8x_9 \oplus x_0x_1x_3x_4x_8x_9 \oplus x_0x_1x_3x_4x_9 \oplus x_0x_1x_3x_5x_6 \oplus x_0x_1x_3x_5x_6x_7x_8 \oplus x_0x_1x_3x_5x_6x_7x_9 \oplus x_0x_1x_3x_5x_6x_8 \oplus$
217	$x_0x_1x_3x_5x_6x_8x_9 \oplus x_0x_1x_3x_5x_7 \oplus x_0x_1x_3x_5x_7x_8x_9 \oplus x_0x_1x_3x_5x_7x_9 \oplus x_0x_1x_3x_5x_8 \oplus x_0x_1x_3x_5x_9 \oplus x_0x_1x_3x_6 \oplus$
218	$x_0x_1x_3x_6x_7 \oplus x_0x_1x_3x_6x_7x_8 \oplus x_0x_1x_3x_6x_7x_8x_9 \oplus x_0x_1x_3x_6x_8x_9 \oplus x_0x_1x_3x_6x_9 \oplus x_0x_1x_3x_7x_8 \oplus x_0x_1x_3x_7x_9 \oplus$
219	$x_0x_1x_3x_8 \oplus x_0x_1x_3x_8x_9 \oplus x_0x_1x_4x_5 \oplus x_0x_1x_4x_5x_6x_7 \oplus x_0x_1x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_4x_5x_6x_7x_9 \oplus x_0x_1x_4x_5x_6x_8 \oplus$
220	$x_0x_1x_4x_5x_6x_9 \oplus x_0x_1x_4x_5x_7 \oplus x_0x_1x_4x_5x_7x_8 \oplus x_0x_1x_4x_5x_7x_8x_9 \oplus x_0x_1x_4x_5x_8x_9 \oplus x_0x_1x_4x_5x_9 \oplus x_0x_1x_4x_6 \oplus$
221	$x_0x_1x_4x_6x_7x_8 \oplus x_0x_1x_4x_6x_7x_9 \oplus x_0x_1x_4x_6x_8 \oplus x_0x_1x_4x_6x_8x_9 \oplus x_0x_1x_4x_7 \oplus x_0x_1x_4x_7x_8x_9 \oplus x_0x_1x_4x_7x_9 \oplus$
222	$x_0x_1x_4x_8 \oplus x_0x_1x_4x_9 \oplus x_0x_1x_5 \oplus x_0x_1x_5x_6 \oplus x_0x_1x_5x_6x_7 \oplus x_0x_1x_5x_6x_7x_8 \oplus x_0x_1x_5x_6x_7x_8x_9 \oplus x_0x_1x_5x_6x_8x_9 \oplus$
223	$x_0x_1x_5x_6x_9 \oplus x_0x_1x_5x_7x_8 \oplus x_0x_1x_5x_7x_9 \oplus x_0x_1x_5x_8 \oplus x_0x_1x_5x_8x_9 \oplus x_0x_1x_6x_7 \oplus x_0x_1x_6x_7x_8x_9 \oplus x_0x_1x_6x_7x_9 \oplus$
224	$x_0x_1x_6x_8 \oplus x_0x_1x_6x_9 \oplus x_0x_1x_7 \oplus x_0x_1x_7x_8 \oplus x_0x_1x_7x_8x_9 \oplus x_0x_1x_8x_9 \oplus x_0x_1x_9 \oplus x_0x_2 \oplus x_0x_2x_3x_4 \oplus x_0x_2x_3x_4x_5x_6 \oplus$
225	$x_0x_2x_3x_4x_5x_6x_7x_8 \oplus x_0x_2x_3x_4x_5x_6x_7x_9 \oplus x_0x_2x_3x_4x_5x_6x_8 \oplus x_$

$$\begin{aligned}
& x_1x_2x_3x_5x_7x_9 \oplus x_1x_2x_3x_5x_8 \oplus x_1x_2x_3x_5x_9 \oplus x_1x_2x_3x_6 \oplus x_1x_2x_3x_6x_7 \oplus x_1x_2x_3x_6x_7x_8 \oplus x_1x_2x_3x_6x_7x_8x_9 \oplus \\
& x_1x_2x_3x_6x_8x_9 \oplus x_1x_2x_3x_6x_9 \oplus x_1x_2x_3x_7x_8 \oplus x_1x_2x_3x_7x_9 \oplus x_1x_2x_3x_8 \oplus x_1x_2x_3x_8x_9 \oplus x_1x_2x_4x_5 \oplus x_1x_2x_4x_5x_6x_7 \oplus \\
& x_1x_2x_4x_5x_6x_7x_8x_9 \oplus x_1x_2x_4x_5x_6x_7x_9 \oplus x_1x_2x_4x_5x_6x_8 \oplus x_1x_2x_4x_5x_6x_9 \oplus x_1x_2x_4x_5x_7 \oplus x_1x_2x_4x_5x_7x_8 \oplus \\
& x_1x_2x_4x_5x_7x_8x_9 \oplus x_1x_2x_4x_5x_8x_9 \oplus x_1x_2x_4x_5x_9 \oplus x_1x_2x_4x_6 \oplus x_1x_2x_4x_6x_7x_8 \oplus x_1x_2x_4x_6x_7x_9 \oplus x_1x_2x_4x_6x_8 \oplus \\
& x_1x_2x_4x_6x_8x_9 \oplus x_1x_2x_4x_7 \oplus x_1x_2x_4x_7x_8x_9 \oplus x_1x_2x_4x_7x_9 \oplus x_1x_2x_4x_8 \oplus x_1x_2x_4x_9 \oplus x_1x_2x_5 \oplus x_1x_2x_5x_6 \oplus \\
& x_1x_2x_5x_6x_7 \oplus x_1x_2x_5x_6x_7x_8 \oplus x_1x_2x_5x_6x_7x_8x_9 \oplus x_1x_2x_5x_6x_8x_9 \oplus x_1x_2x_5x_6x_9 \oplus x_1x_2x_5x_7x_8 \oplus x_1x_2x_5x_7x_9 \oplus \\
& x_1x_2x_5x_8 \oplus x_1x_2x_5x_8x_9 \oplus x_1x_2x_6x_7 \oplus x_1x_2x_6x_7x_8x_9 \oplus x_1x_2x_6x_7x_9 \oplus x_1x_2x_6x_8 \oplus x_1x_2x_6x_9 \oplus x_1x_2x_7 \oplus \\
& x_1x_2x_7x_8 \oplus x_1x_2x_7x_8x_9 \oplus x_1x_2x_8x_9 \oplus x_1x_2x_9 \oplus x_1x_3x_4 \oplus x_1x_3x_4x_5x_6 \oplus x_1x_3x_4x_5x_6x_7x_8 \oplus x_1x_3x_4x_5x_6x_7x_9 \oplus \\
& x_1x_3x_4x_5x_6x_8 \oplus x_1x_3x_4x_5x_6x_8x_9 \oplus x_1x_3x_4x_5x_7 \oplus x_1x_3x_4x_5x_7x_8x_9 \oplus x_1x_3x_4x_5x_7x_9 \oplus x_1x_3x_4x_5x_8 \oplus x_1x_3x_4x_5x_9 \oplus \\
& x_1x_3x_4x_6 \oplus x_1x_3x_4x_6x_7 \oplus x_1x_3x_4x_6x_7x_8 \oplus x_1x_3x_4x_6x_7x_8x_9 \oplus x_1x_3x_4x_6x_8x_9 \oplus x_1x_3x_4x_6x_9 \oplus x_1x_3x_4x_7x_8 \oplus \\
& x_1x_3x_4x_7x_9 \oplus x_1x_3x_4x_8 \oplus x_1x_3x_4x_8x_9 \oplus x_1x_3x_5 \oplus x_1x_3x_5x_6x_7 \oplus x_1x_3x_5x_6x_7x_8x_9 \oplus x_1x_3x_5x_6x_7x_9 \oplus x_1x_3x_5x_6x_8 \oplus \\
& x_1x_3x_5x_6x_9 \oplus x_1x_3x_5x_7 \oplus x_1x_3x_5x_7x_8 \oplus x_1x_3x_5x_7x_8x_9 \oplus x_1x_3x_5x_8x_9 \oplus x_1x_3x_5x_9 \oplus x_1x_3x_6 \oplus x_1x_3x_6x_7x_8 \oplus \\
& x_1x_3x_6x_7x_9 \oplus x_1x_3x_6x_8 \oplus x_1x_3x_6x_8x_9 \oplus x_1x_3x_7 \oplus x_1x_3x_7x_8x_9 \oplus x_1x_3x_7x_9 \oplus x_1x_3x_8 \oplus x_1x_3x_9 \oplus x_1x_4 \oplus x_1x_4x_5 \oplus \\
& x_1x_4x_5x_6 \oplus x_1x_4x_5x_6x_7 \oplus x_1x_4x_5x_6x_7x_8 \oplus x_1x_4x_5x_6x_7x_8x_9 \oplus x_1x_4x_5x_6x_8x_9 \oplus x_1x_4x_5x_6x_9 \oplus x_1x_4x_5x_7x_8 \oplus \\
& x_1x_4x_5x_7x_9 \oplus x_1x_4x_5x_8 \oplus x_1x_4x_5x_8x_9 \oplus x_1x_4x_6x_7 \oplus x_1x_4x_6x_7x_8x_9 \oplus x_1x_4x_6x_7x_9 \oplus x_1x_4x_6x_8 \oplus x_1x_4x_6x_9 \oplus \\
& x_1x_4x_7 \oplus x_1x_4x_7x_8 \oplus x_1x_4x_7x_8x_9 \oplus x_1x_4x_8x_9 \oplus x_1x_4x_9 \oplus x_1x_5x_6 \oplus x_1x_5x_6x_7x_8 \oplus x_1x_5x_6x_7x_9 \oplus x_1x_5x_6x_8 \oplus \\
& x_1x_5x_6x_8x_9 \oplus x_1x_5x_7 \oplus x_1x_5x_7x_8x_9 \oplus x_1x_5x_7x_9 \oplus x_1x_5x_8 \oplus x_1x_5x_9 \oplus x_1x_6 \oplus x_1x_6x_7 \oplus x_1x_6x_7x_8 \oplus x_1x_6x_7x_8x_9 \oplus \\
& x_1x_6x_8x_9 \oplus x_1x_6x_9 \oplus x_1x_7x_8 \oplus x_1x_7x_9 \oplus x_1x_8 \oplus x_1x_8x_9 \oplus x_2x_3 \oplus x_2x_3x_4x_5 \oplus x_2x_3x_4x_5x_6x_7 \oplus x_2x_3x_4x_5x_6x_7x_8x_9 \oplus \\
& x_2x_3x_4x_5x_6x_7x_9 \oplus x_2x_3x_4x_5x_6x_8 \oplus x_2x_3x_4x_5x_6x_9 \oplus x_2x_3x_4x_5x_7 \oplus x_2x_3x_4x_5x_7x_8 \oplus x_2x_3x_4x_5x_7x_8x_9 \oplus x_2x_3x_4x_5x_8x_9 \oplus \\
& x_2x_3x_4x_5x_9 \oplus x_2x_3x_4x_6 \oplus x_2x_3x_4x_6x_7x_8 \oplus x_2x_3x_4x_6x_7x_9 \oplus x_2x_3x_4x_6x_8 \oplus x_2x_3x_4x_6x_8x_9 \oplus x_2x_3x_4x_7 \oplus x_2x_3x_4x_7x_8x_9 \oplus \\
& x_2x_3x_4x_7x_9 \oplus x_2x_3x_4x_8 \oplus x_2x_3x_4x_9 \oplus x_2x_3x_5 \oplus x_2x_3x_5x_6 \oplus x_2x_3x_5x_6x_7 \oplus x_2x_3x_5x_6x_7x_8 \oplus x_2x_3x_5x_6x_7x_8x_9 \oplus \\
& x_2x_3x_5x_6x_8x_9 \oplus x_2x_3x_5x_6x_9 \oplus x_2x_3x_5x_7x_8 \oplus x_2x_3x_5x_7x_9 \oplus x_2x_3x_5x_8 \oplus x_2x_3x_5x_8x_9 \oplus x_2x_3x_6x_7 \oplus x_2x_3x_6x_7x_8x_9 \oplus \\
& x_2x_3x_6x_7x_9 \oplus x_2x_3x_6x_8 \oplus x_2x_3x_6x_9 \oplus x_2x_3x_7 \oplus x_2x_3x_7x_8 \oplus x_2x_3x_7x_8x_9 \oplus x_2x_3x_8x_9 \oplus x_2x_3x_9 \oplus x_2x_4 \oplus x_2x_4x_5x_6 \oplus \\
& x_2x_4x_5x_6x_7x_8 \oplus x_2x_4x_5x_6x_7x_9 \oplus x_2x_4x_5x_6x_8 \oplus x_2x_4x_5x_6x_8x_9 \oplus x_2x_4x_5x_7 \oplus x_2x_4x_5x_7x_8x_9 \oplus x_2x_4x_5x_7x_9 \oplus \\
& x_2x_4x_5x_8 \oplus x_2x_4x_5x_9 \oplus x_2x_4x_6 \oplus x_2x_4x_6x_7 \oplus x_2x_4x_6x_7x_8 \oplus x_2x_4x_6x_7x_8x_9 \oplus x_2x_4x_6x_8x_9 \oplus x_2x_4x_6x_9 \oplus x_2x_4x_7x_8 \oplus \\
& x_2x_4x_7x_9 \oplus x_2x_4x_8 \oplus x_2x_4x_8x_9 \oplus x_2x_5 \oplus x_2x_5x_6x_7 \oplus x_2x_5x_6x_7x_8x_9 \oplus x_2x_5x_6x_7x_9 \oplus x_2x_5x_6x_8 \oplus x_2x_5x_6x_9 \oplus \\
& x_2x_5x_7 \oplus x_2x_5x_7x_8 \oplus x_2x_5x_7x_8x_9 \oplus x_2x_5x_8x_9 \oplus x_2x_5x_9 \oplus x_2x_6 \oplus x_2x_6x_7x_8 \oplus x_2x_6x_7x_9 \oplus x_2x_6x_8 \oplus x_2x_6x_8x_9 \oplus \\
& x_2x_7 \oplus x_2x_7x_8x_9 \oplus x_2x_7x_9 \oplus x_2x_8 \oplus x_2x_9 \oplus x_3 \oplus x_3x_4 \oplus x_3x_4x_5 \oplus x_3x_4x_5x_6 \oplus x_3x_4x_5x_6x_7 \oplus x_3x_4x_5x_6x_7x_8 \oplus \\
& x_3x_4x_5x_6x_7x_8x_9 \oplus x_3x_4x_5x_6x_8x_9 \oplus x_3x_4x_5x_6x_9 \oplus x_3x_4x_5x_7x_8 \oplus x_3x_4x_5x_7x_9 \oplus x_3x_4x_5x_8 \oplus x_3x_4x_5x_8x_9 \oplus \\
& x_3x_4x_6x_7 \oplus x_3x_4x_6x_7x_8x_9 \oplus x_3x_4x_6x_7x_9 \oplus x_3x_4x_6x_8 \oplus x_3x_4x_6x_9 \oplus x_3x_4x_7 \oplus x_3x_4x_7x_8 \oplus x_3x_4x_7x_8x_9 \oplus x_3x_4x_8x_9 \oplus \\
& x_3x_4x_9 \oplus x_3x_5x_6 \oplus x_3x_5x_6x_7x_8 \oplus x_3x_5x_6x_7x_9 \oplus x_3x_5x_6x_8 \oplus x_3x_5x_6x_8x_9 \oplus x_3x_5x_7 \oplus x_3x_5x_7x_8x_9 \oplus x_3x_5x_7x_9 \oplus \\
& x_3x_5x_8 \oplus x_3x_5x_9 \oplus x_3x_6 \oplus x_3x_6x_7 \oplus x_3x_6x_7x_8 \oplus x_3x_6x_7x_8x_9 \oplus x_3x_6x_8x_9 \oplus x_3x_6x_9 \oplus x_3x_7x_8 \oplus x_3x_7x_9 \oplus x_3x_8 \oplus \\
& x_3x_8x_9 \oplus x_4x_5 \oplus x_4x_5x_6x_7 \oplus x_4x_5x_6x_7x_8x_9 \oplus x_4x_5x_6x_7x_9 \oplus x_4x_5x_6x_8 \oplus x_4x_5x_6x_9 \oplus x_4x_5x_7 \oplus x_4x_5x_7x_8 \oplus \\
& x_4x_5x_7x_8x_9 \oplus x_4x_5x_8x_9 \oplus x_4x_5x_9 \oplus x_4x_6 \oplus x_4x_6x_7x_8 \oplus x_4x_6x_7x_9 \oplus x_4x_6x_8 \oplus x_4x_6x_8x_9 \oplus x_4x_7 \oplus x_4x_7x_8x_9 \oplus \\
& x_4x_7x_9 \oplus x_4x_8 \oplus x_4x_9 \oplus x_5 \oplus x_5x_6 \oplus x_5x_6x_7 \oplus x_5x_6x_7x_8 \oplus x_5x_6x_7x_8x_9 \oplus x_5x_6x_8x_9 \oplus x_5x_6x_9 \oplus x_5x_7x_8 \oplus \\
& x_5x_7x_9 \oplus x_5x_8 \oplus x_5x_8x_9 \oplus x_6x_7 \oplus x_6x_7x_8x_9 \oplus x_6x_7x_9 \oplus x_6x_8 \oplus x_6x_9 \oplus x_7 \oplus x_7x_8 \oplus x_7x_8x_9 \oplus x_8x_9 \oplus x_9 = 0 \\
& x_0 \oplus x_0x_1 \oplus x_0x_1x_2 \oplus x_0x_1x_2x_3 \oplus x_0x_1x_2x_3x_4 \oplus x_0x_1x_2x_3x_4x_5 \oplus x_0x_1x_2x_3x_4x_5x_6 \oplus x_0x_1x_2x_3x_4x_5x_6x_7 \oplus \\
& x_0x_1x_2x_3x_4x_5x_6x_7x_8 \oplus x_0x_1x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_6x_8x_9 \oplus x_0x_1x_2x_3x_4x_5x_6x_9 \oplus x_0x_1x_2x_3x_4x_5x_7x_8 \oplus \\
& x_0x_1x_2x_3x_4x_5x_7x_9 \oplus x_0x_1x_2x_3x_4x_5x_8 \oplus x_0x_1x_2x_3x_4x_5x_8x_9 \oplus x_0x_1x_2x_3x_4x_6x_7 \oplus x_0x_1x_2x_3x_4x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_6x_7x_9 \oplus \\
& x_0x_1x_2x_3x_4x_6x_8 \oplus x_0x_1x_2x_3x_4x_6x_9 \oplus x_0x_1x_2x_3x_4x_7 \oplus x_0x_1x_2x_3x_4x_7x_8 \oplus x_0x_1x_2x_3x_4x_7x_8x_9 \oplus x_0x_1x_2x_3x_4x_8x_9 \oplus \\
& x_0x_1x_2x_3x_4x_9 \oplus x_0x_1x_2x_3x_5x_6 \oplus x_0x_1x_2x_3x_5x_6x_7x_8 \oplus x_0x_1x_2x_3x_5x_6x_7x_9 \oplus x_0x_1x_2x_3x_5x_6x_8 \oplus x_0x_1x_2x_3x_5x_6x_8x_9 \oplus \\
& x_0x_1x_2x_3x_5x_7 \oplus x_0x_1x_2x_3x_5x_7x_8x_9 \oplus x_0x_1x_2x_3x_5x_7x_9 \oplus x_0x_1x_2x_3x_5x_8 \oplus x_0x_1x_2x_3x_5x_9 \oplus x_0x_1x_2x_3x_6 \oplus \\
& x_0x_1x_2x_3x_6x_7 \oplus x_0x_1x_2x_3x_6x_7x_8 \oplus x_0x_1x_2x_3x_6x_7x_8x_9 \oplus x_0x_1x_2x_3x_6x_8x_9 \oplus x_0x_1x_2x_3x_6x_9 \oplus x_0x_1x_2x_3x_7x_8 \oplus \\
& x_0x_1x_2x_3x_7x_9 \oplus x_0x_1x_2x_3x_8 \oplus x_0x_1x_2x_3x_8x_9 \oplus x_0x_1x_2x_4x_5 \oplus x_0x_1x_2x_4x_5x_6x_7 \oplus x_0x_1x_2x_4x_5x_6x_7x_8x_9 \oplus \\
& x_0x_1x_2x_4x_5x_6x_7x_9 \oplus x_0x_1x_2x_4x_5x_6x_8 \oplus x_0x_1x_2x_4x_5x_6x_9 \oplus x_0x_1x_2x_4x_5x_7 \oplus x_0x_1x_2x_4x_5x_7x_8 \oplus x_0x_1x_2x_4x_5x_7x_8x_9 \oplus \\
& x_0x_1x_2x_4x_5x_8x_9 \oplus x_0x_1x_2x_4x_5x_9 \oplus x_0x_1x_2x_4x_6 \oplus x_0x_1x_2x_4x_6x_7x_8 \oplus x_0x_1x_2x_4x_6x_7x_9 \oplus x_0x_1x_2x_4x_6x_8 \oplus \\
& x_0x_1x_2x_4x_6x_8x_9 \oplus x_0x_1x_2x_4x_7 \oplus x_0x_1x_2x_4x_7x_8x_9 \oplus x_0x_1x_2x_4x_7x_9 \oplus x_0x_1x_2x_4x_8 \oplus x_0x_1x_2x_4x_9 \oplus x_0x_1x_2x_5 \oplus \\
& x_0x_1x_2x_5x_6 \oplus x_0x_1x_2x_5x_6x_7 \oplus x_0x_1x_2x_5x_6x_7x_8 \oplus x_0x_1x_2x_5x_6x_7x_8x_9 \oplus x_0x_1x_2x_5x_6x_8x_9 \oplus x_0x_1x_2x_5x_6x_9 \oplus \\
& x_0x_1x_2x_5x_7x_8 \oplus x_0x_1x_2x_5x_7x_9 \oplus x_0x_1x_2x_5x_8 \oplus x_0x_1x_2x_5x_8x_9 \oplus x_0x_1x_2x_6x_7 \oplus x_0x_1x_2x_6x_7x_8x_9 \oplus x_0x_1x_2x_6x_7x_9 \oplus \\
& x_0x_1x_2x_6x_8 \oplus x_0x_1x_2x_6x_9 \oplus x_0x_1x_2x_7 \oplus x_0x_1x_2x_7x_8 \oplus x_0x_1x_2x_7x_8x_9 \oplus x_0x_1x_2x_8x_9 \oplus x_0x_1x_2x_9 \oplus x_0x_1x_3x_4 \oplus \\
& x_0x_1x_3x_4x_5x_6 \oplus x_0x_1x_3x_4x_5x_6x_7x_8 \oplus x_0x_1x_3x_4x_5x_6x_7x_9 \oplus x_0x_1x_3x_4x_5x_6x_8 \oplus x_0x_1x_3x_4x_5x_6x_8x_9 \oplus x_0x_1x_3x_4x_5x_7 \oplus
\end{aligned}$$





$x_1x_4x_5x_7x_9 \oplus x_1x_4x_5x_8 \oplus x_1x_4x_5x_9 \oplus x_1x_4x_6 \oplus x_1x_4x_6x_7 \oplus x_1x_4x_6x_7x_8 \oplus x_1x_4x_6x_7x_8x_9 \oplus x_1x_4x_6x_8x_9 \oplus$   
 $x_1x_4x_6x_9 \oplus x_1x_4x_7x_8 \oplus x_1x_4x_7x_9 \oplus x_1x_4x_8 \oplus x_1x_4x_8x_9 \oplus x_1x_5 \oplus x_1x_5x_6x_7 \oplus x_1x_5x_6x_7x_8x_9 \oplus x_1x_5x_6x_7x_9 \oplus$   
 $x_1x_5x_6x_8 \oplus x_1x_5x_6x_9 \oplus x_1x_5x_7 \oplus x_1x_5x_7x_8 \oplus x_1x_5x_7x_8x_9 \oplus x_1x_5x_8x_9 \oplus x_1x_5x_9 \oplus x_1x_6 \oplus x_1x_6x_7x_8 \oplus x_1x_6x_7x_9 \oplus$   
 $x_1x_6x_8 \oplus x_1x_6x_8x_9 \oplus x_1x_7 \oplus x_1x_7x_8x_9 \oplus x_1x_7x_9 \oplus x_1x_8 \oplus x_1x_9 \oplus x_2 \oplus x_2x_3 \oplus x_2x_3x_4 \oplus x_2x_3x_4x_5 \oplus x_2x_3x_4x_5x_6 \oplus$   
 $x_2x_3x_4x_5x_6x_7 \oplus x_2x_3x_4x_5x_6x_7x_8 \oplus x_2x_3x_4x_5x_6x_7x_8x_9 \oplus x_2x_3x_4x_5x_6x_8x_9 \oplus x_2x_3x_4x_5x_6x_9 \oplus x_2x_3x_4x_5x_7x_8 \oplus$   
 $x_2x_3x_4x_5x_7x_9 \oplus x_2x_3x_4x_5x_8 \oplus x_2x_3x_4x_5x_8x_9 \oplus x_2x_3x_4x_6x_7 \oplus x_2x_3x_4x_6x_7x_8x_9 \oplus x_2x_3x_4x_6x_7x_9 \oplus x_2x_3x_4x_6x_8 \oplus$   
 $x_2x_3x_4x_6x_9 \oplus x_2x_3x_4x_7 \oplus x_2x_3x_4x_7x_8 \oplus x_2x_3x_4x_7x_8x_9 \oplus x_2x_3x_4x_8x_9 \oplus x_2x_3x_4x_9 \oplus x_2x_3x_5x_6 \oplus x_2x_3x_5x_6x_7x_8 \oplus$   
 $x_2x_3x_5x_6x_7x_9 \oplus x_2x_3x_5x_6x_8 \oplus x_2x_3x_5x_6x_8x_9 \oplus x_2x_3x_5x_7 \oplus x_2x_3x_5x_7x_8x_9 \oplus x_2x_3x_5x_7x_9 \oplus x_2x_3x_5x_8 \oplus x_2x_3x_5x_9 \oplus$   
 $x_2x_3x_6 \oplus x_2x_3x_6x_7 \oplus x_2x_3x_6x_7x_8 \oplus x_2x_3x_6x_7x_8x_9 \oplus x_2x_3x_6x_8x_9 \oplus x_2x_3x_6x_9 \oplus x_2x_3x_7x_8 \oplus x_2x_3x_7x_9 \oplus$   
 $x_2x_3x_8 \oplus x_2x_3x_8x_9 \oplus x_2x_4x_5 \oplus x_2x_4x_5x_6x_7 \oplus x_2x_4x_5x_6x_7x_8x_9 \oplus x_2x_4x_5x_6x_7x_9 \oplus x_2x_4x_5x_6x_8 \oplus x_2x_4x_5x_6x_9 \oplus$   
 $x_2x_4x_5x_7 \oplus x_2x_4x_5x_7x_8 \oplus x_2x_4x_5x_7x_8x_9 \oplus x_2x_4x_5x_8x_9 \oplus x_2x_4x_5x_9 \oplus x_2x_4x_6 \oplus x_2x_4x_6x_7x_8 \oplus x_2x_4x_6x_7x_9 \oplus$   
 $x_2x_4x_6x_8 \oplus x_2x_4x_6x_8x_9 \oplus x_2x_4x_7 \oplus x_2x_4x_7x_8x_9 \oplus x_2x_4x_7x_9 \oplus x_2x_4x_8 \oplus x_2x_4x_9 \oplus x_2x_5 \oplus x_2x_5x_6 \oplus x_2x_5x_6x_7 \oplus$   
 $x_2x_5x_6x_7x_8 \oplus x_2x_5x_6x_7x_8x_9 \oplus x_2x_5x_6x_8x_9 \oplus x_2x_5x_6x_9 \oplus x_2x_5x_7x_8 \oplus x_2x_5x_7x_9 \oplus x_2x_5x_8 \oplus x_2x_5x_8x_9 \oplus x_2x_6x_7 \oplus$   
 $x_2x_6x_7x_8x_9 \oplus x_2x_6x_7x_9 \oplus x_2x_6x_8 \oplus x_2x_6x_9 \oplus x_2x_7 \oplus x_2x_7x_8 \oplus x_2x_7x_8x_9 \oplus x_2x_8x_9 \oplus x_2x_9 \oplus x_3x_4 \oplus x_3x_4x_5x_6 \oplus$   
 $x_3x_4x_5x_6x_7x_8 \oplus x_3x_4x_5x_6x_7x_9 \oplus x_3x_4x_5x_6x_8 \oplus x_3x_4x_5x_6x_8x_9 \oplus x_3x_4x_5x_7 \oplus x_3x_4x_5x_7x_8x_9 \oplus x_3x_4x_5x_7x_9 \oplus$   
 $x_3x_4x_5x_8 \oplus x_3x_4x_5x_9 \oplus x_3x_4x_6 \oplus x_3x_4x_6x_7 \oplus x_3x_4x_6x_7x_8 \oplus x_3x_4x_6x_7x_8x_9 \oplus x_3x_4x_6x_8x_9 \oplus x_3x_4x_6x_9 \oplus x_3x_4x_7x_8 \oplus$   
 $x_3x_4x_7x_9 \oplus x_3x_4x_8 \oplus x_3x_4x_8x_9 \oplus x_3x_5 \oplus x_3x_5x_6x_7 \oplus x_3x_5x_6x_7x_8x_9 \oplus x_3x_5x_6x_7x_9 \oplus x_3x_5x_6x_8 \oplus x_3x_5x_6x_9 \oplus$   
 $x_3x_5x_7 \oplus x_3x_5x_7x_8 \oplus x_3x_5x_7x_8x_9 \oplus x_3x_5x_8x_9 \oplus x_3x_5x_9 \oplus x_3x_6 \oplus x_3x_6x_7x_8 \oplus x_3x_6x_7x_9 \oplus x_3x_6x_8 \oplus x_3x_6x_8x_9 \oplus$   
 $x_3x_7 \oplus x_3x_7x_8x_9 \oplus x_3x_7x_9 \oplus x_3x_8 \oplus x_3x_9 \oplus x_4 \oplus x_4x_5 \oplus x_4x_5x_6 \oplus x_4x_5x_6x_7 \oplus x_4x_5x_6x_7x_8 \oplus x_4x_5x_6x_7x_8x_9 \oplus$   
 $x_4x_5x_6x_8x_9 \oplus x_4x_5x_6x_9 \oplus x_4x_5x_7x_8 \oplus x_4x_5x_7x_9 \oplus x_4x_5x_8 \oplus x_4x_5x_8x_9 \oplus x_4x_6x_7 \oplus x_4x_6x_7x_8x_9 \oplus x_4x_6x_7x_9 \oplus$   
 $x_4x_6x_8 \oplus x_4x_6x_9 \oplus x_4x_7 \oplus x_4x_7x_8 \oplus x_4x_7x_8x_9 \oplus x_4x_8x_9 \oplus x_4x_9 \oplus x_5x_6 \oplus x_5x_6x_7x_8 \oplus x_5x_6x_7x_9 \oplus x_5x_6x_8 \oplus$   
 $x_5x_6x_8x_9 \oplus x_5x_7 \oplus x_5x_7x_8x_9 \oplus x_5x_7x_9 \oplus x_5x_8 \oplus x_5x_9 \oplus x_6 \oplus x_6x_7 \oplus x_6x_7x_8 \oplus x_6x_7x_8x_9 \oplus x_6x_8x_9 \oplus x_6x_9 \oplus$   
 $x_7x_8 \oplus x_7x_9 \oplus x_8 \oplus x_8x_9 = 0$

## 2 Retrodictive QFT

only need number of vars !!!!

solve other problems with just knowing which vars are involved

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