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Theories and Data Structures

—Draft—

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Abstract

Showing how some simple mathematical theories naturally give rise to some common data-structures

—Source: https://github.com/JacquesCarette/TheoriesAndDataStructures—

Contents

1 README JC & WK

- ◇ `C-c C-e` then `l o` to produce the PDF from the org file.

  Or simply press `f7` if you've allowed the local variables in this file.

- ◇ `\edcomm{Person}{Comment}` to make first-class literate comments: **Comment: MA** Please read this document;
  thanks **End Comment**

  Since `remark` is a macro that takes 2 arguments, separated by commas, so not use any `,` in the your editorial comment.

- ◇ "From monoids to near-semirings: the essence of MonadPlus and Alternative", https://usuarios.fceia.unr.edu.ar/~mauro/pubs/FromMonoidstoNearsemirings.pdf.

  Very good paper indeed.

2 We want to be systematic about

Exploring Magma-based theories see https://en.wikipedia.org/wiki/Magma_(algebra) where we want to at least explore all the properties that are affine. These are interesting things said at https://en.wikipedia.org/wiki/Category_of_magmas which should be better understood.

Pointed theories There is not much to be said here. Although I guess 'contractible' can be defined already here.

Pointed Magma theories Interestingly, non-associative pointed Magma theories don't show up in the nice summary above. Of course, this is where Monoid belongs. But it is worth exploring all of the combinations too.

unary theories wikipedia sure doesn't spend much time on these (see https://en.wikipedia.org/wiki/Algebraic_structure) but there are some interesting ones, because if the unary operation is 'f' things like forall x. f (f x) = x is **linear**, because x is used exactly once on each side. The non-linearity of 'f' doesn't count (else associativity wouldn't work either, as $\_*\_$ is used funnily there too). So "iter 17 f x = x" is a fine axiom here too. [iter is definable in the ground theory]

This is actually where things started, as 'involution' belongs here.

And is the first weird one.

Pointed unary theories E.g., the natural numbers

Pointer binary theories need to figure out which are expressible

more semiring, near-ring, etc. Need a sampling. But quasigroup (with 3 operations!) would be neat to look at.

Also, I think we want to explore

- ◇ Free Theories
- ◇ Initial Objects
- ◇ Cofree Theories (when they exist)

Then the potential 'future work' is huge. But that can be left for later. We want to have all the above rock solid first.

3 Relationship with 700 modules

To make it a POPL paper, as well as related to your module work, it is also going to be worthwhile to notice and abstract the patterns. Such as generating induction principles and recursors.

A slow-paced introduction to reflection in Agda: https://github.com/alhassy/gentle-intro-to-reflect

4 Timeline

Regarding POPL: https://popl20.sigplan.org/track/POPL-2020-Research-Papers#POPL-2020-Call-for There is no explicit Pearl category, nor any mention of that style. Nevertheless, I think it's worth a shot, as I think by being systematic, we'll "grab" in a lot of things that are not usually considered part of one's basic toolkit.

However, to have a chance, the technical content of the paper should be done by June 17th, and the rest of the time should be spent on the presentation of the material. The bar is very high at POPL.