Tangent estimation along 3D digital curves

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Digital Geometry

Digital shapes arise naturally in several contexts e.g. image analysis, approximation, word combinatorics, tilings, cellular automata, computational geometry, biomedical imaging ...

Digital shape analysis requires a sound digital geometry which is a geometry in $\mathbb{Z}^n$

Geometrical Properties

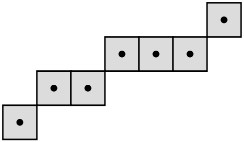
The classical problem in the digital geometry is to estimate geometrical properties of the digitalized shapes without any knowledge of the underlying continuous shape.

- length
- area
- perimeter
- convexity/concavity
- tangents
- curvature
- torsion
- ...

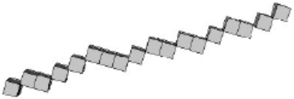
Discrete Curves

Many vision, image analysis and pattern recognition applications relay on the estimation of the geometry of the discrete curves.

$\mathbb{Z}^2$



$\mathbb{Z}^3$

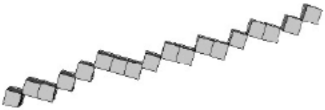


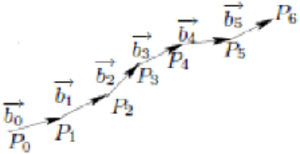
The digital curves can be, for example, result of

- discretization
- segmentation
- skeletonization
- boundary tracking

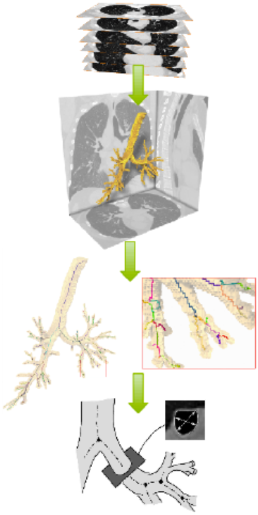
Discrete Tangent Estimator

The discrete tangent estimator evaluate tangent direction along all points of the discrete curve.





Discrete Tangent Estimator Application



Methods

In the framework of digital geometry, there exist few studies on 3D discrete curves yet while there are numerous methods performed on 2D.

- Approximation techniques in the continuous Euclidean space.

(+) very good accuracy

(-) require to set parameters
(-) can be costly
(-) poor behavior on sharp corners

- Methods which are work in discrete space directly.

(+) good accuracy
(+) no need to set any parameters
(+) simple and fast

(-) poor behavior on corrupted curves

Computational window

The size of the computational window is fixed globally and is not adopted to the local curve geometry.

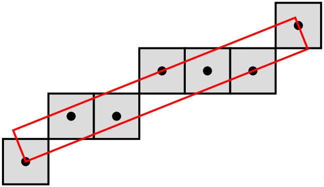
Computational window

The size of the computational window can be adopted to the local curve geometry thanks to notion of Maximal Digital Straight Segments.

2D Digital Straight Segments

Definition

Given a discrete curve C, a set of its consecutive points $C_{i,j}$ where $1 \leq i \leq j \leq |C|$ is said to be a digital straight segment (or $S(i,j)$) iff there exists a digital line $\mathcal{D}$ containing all the points of $C_{i,j}$.



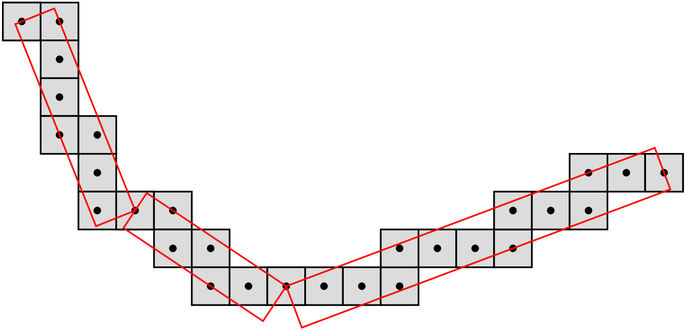
$\mathcal{D}(a, b, \mu, e)$ is defined as the set of points $(x, y) \in \mathbb{Z}^2$ which satisfy the diophantine inequality:

$$\mu \leq ax - by < \mu + e,$$

Maximal Segments

Definition

Any subset $C_{i,j}$ of C is called a maximal segment iff $S(i,j)$ and $\neg S(i,j+1)$ and $\neg S(i-1,j)$.



The λ-MST Estimator (Lachaud et al., 2007)

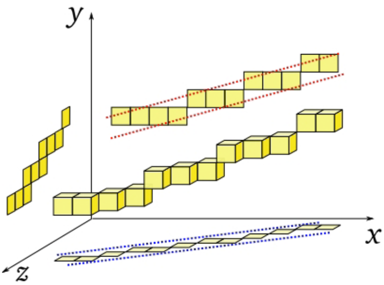
The λ-MST, was originally designed for estimating tangents on 2D digital contours. It is a simple parameter-free method based on maximal straight segments recognition along digital contour

- linear computation complexity
- accurate results
- multigrid convergence

3D Digital Straight Segments

Property

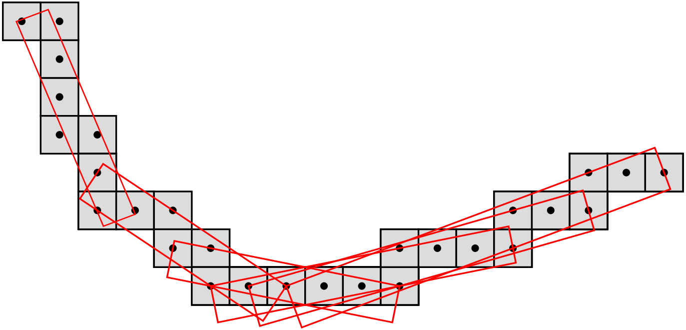
In 3D case, $S(i,j)$ is verified iff two of the three projections of $C_{i,j}$ on the basic planes O_{XY}, O_{XZ} and O_{YZ} are 2D digital straight segments.



Tangential Cover

Property

For any discrete curve C, there is a unique set $\mathcal{M}$ of its maximal segments, called the tangential cover.

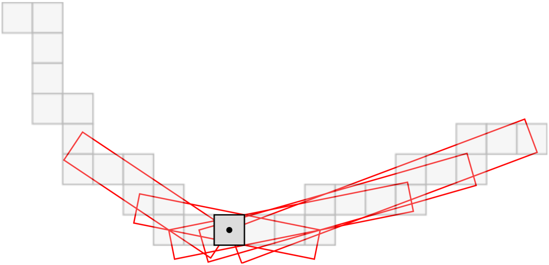


Pencil of Maximal Segments

Definition

The set of all maximal segments going through a point $x \in C$ is called the pencil of maximal segments around x and defined by

$$P(x) = \{M_i \in \mathcal{M} \mid x \in M_i\}$$

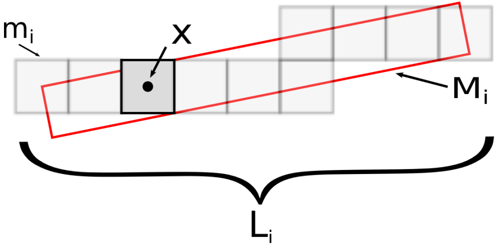


Eccentricity

Definition

The eccentricity $e_i(x)$ of a point x with respect to a maximal segment M_i is its relative position between the extremities of M_i such that

$$e_i(x) = \begin{cases} \frac{\|x - m_i\|_1}{L_i} & \text{if } M_i \in P(x), \\ 0 & \text{otherwise.} \end{cases}$$



The 3D λ-MST

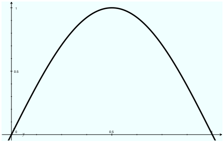
Definition

The 3D λ-MST direction $\mathbf{t}(x)$ at point x of a curve C is defined as a weighted combination of the vectors $\mathbf{t_i}$ of the covering maximal segments M_i such that

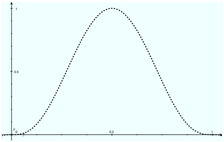
$$\mathbf{t}(x) = \frac{\sum_{M_i \in P(x)} \lambda(e_i(x)) \frac{\mathbf{t_i}}{|\mathbf{t_i}|}}{\sum_{M_i \in P(x)} \lambda(e_i(x))}.$$

The Function λ

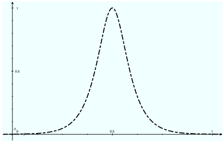
The function λ maps from $[0, 1]$ to $\mathbb{R}_+$ with $\lambda(0) = \lambda(1) = 0$ and $\lambda > 0$ elsewhere and need to satisfy convexity/concavity property.



$sin(\pi x)$

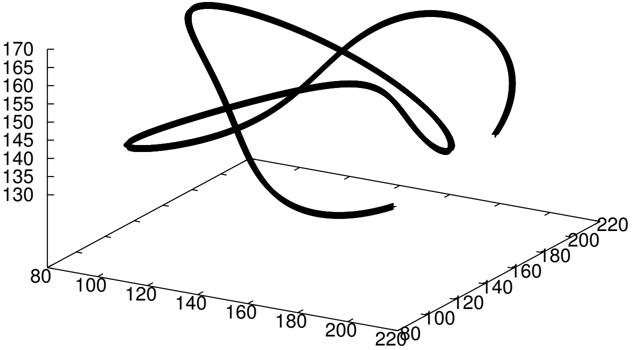


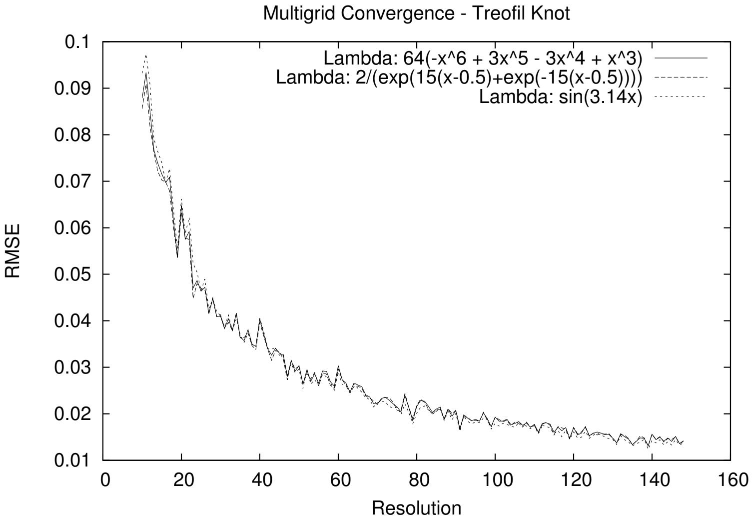
$64\left(-x^6 + 3x^5 - 3x^4 + x^3\right)$

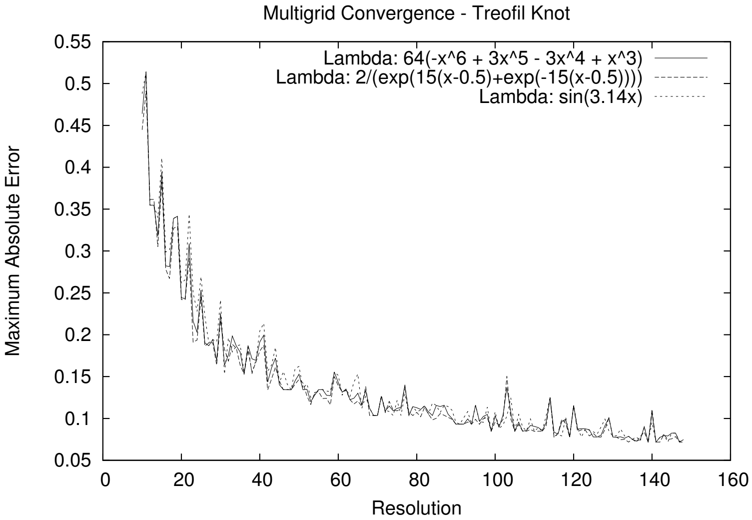


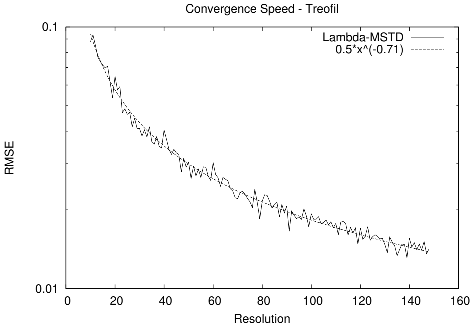
$$\frac{2}{e^{15(x-0.5)} + e^{-15(x-0.5)}}$$

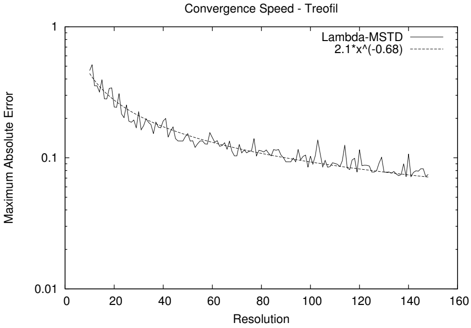
Treofil Knot < cos(2t)*(3+cos(3t)), sin(2t)*(3+cos(3t)), sin(3t) >

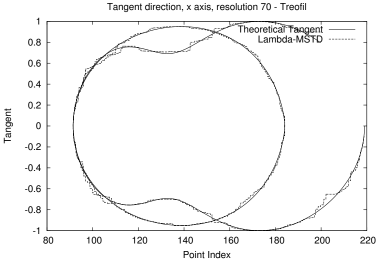


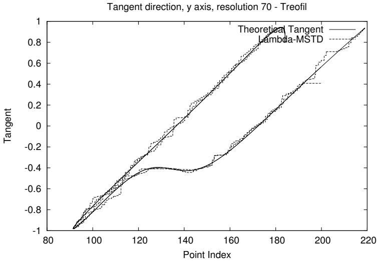

Multigrid Convergence - Treofil Knot
Lambda: 64(-x^6 + 3x^5 - 3x^4 + x^3)
Lambda: 2/(exp(15(x-0.5)+exp(-15(x-0.5))))
Lambda: sin(3.14x)
RMSE
Resolution
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
20
40
60
80
100
120
140
160

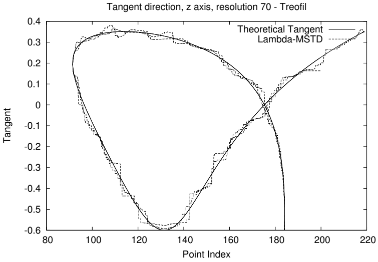

Multigrid Convergence - Treofil Knot
Lambda: 64(-x^6 + 3x^5 - 3x^4 + x^3)
Lambda: 2/(exp(15(x-0.5)+exp(-15(x-0.5))))
Lambda: sin(3.14x)
Maximum Absolute Error
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
20
40
60
80
100
120
140
160
Resolution

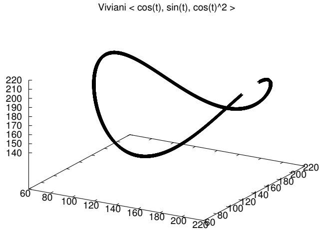

Convergence Speed - Treofil
Lambda-MSTD
0.5*x^(-0.71)
RMSE
0.1
0.01
0
20
40
60
80
100
120
140
160
Resolution

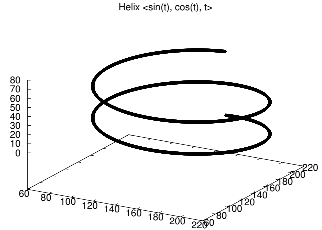

Convergence Speed - Treofil
Lambda-MSTD
2.1*x^(-0.68)
Maximum Absolute Error
1
0.1
0.01
0
20
40
60
80
100
120
140
160
Resolution

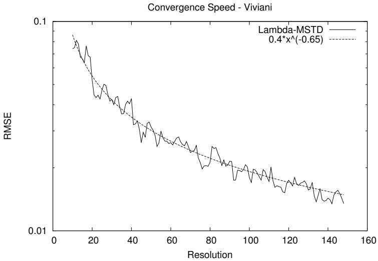

Tangent direction, x axis, resolution 70 - Treofil
Theoretical Tangent
Lambda-MSTD
Tangent
Point Index

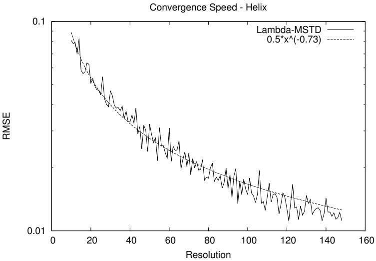

Tangent direction, y axis, resolution 70 - Treofil
Theoretical Tangent
Lambda-MSTD
Tangent
Point Index


Tangent direction, z axis, resolution 70 - Treofil
Theoretical Tangent
Lambda-MSTD
Tangent
Point Index


Viviani < cos(t), sin(t), cos(t)^2 >


Helix <sin(t), cos(t), t>


Convergence Speed - Viviani
Lambda-MSTD
0.4*x^(-0.65)
RMSE
Resolution


Convergence Speed - Helix
Lambda-MSTD
0.5*x^(-0.73)
RMSE
Resolution

Conclusions

We have proposed a new tangent estimator for 3D digital curves which is an extension of the 2D λ-MST estimator.

- We keep the same time complexity and accuracy as the original algorithm
- Asymptotic behavior evaluated experimentally on several space parametric curves is promising

Thank You for your attention!

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