Introduction

1. Context

- Energy minimization methods for image segmentation.
  - Classical formulation:
    - object boundary = curve in the image space,
    - curve energy = image contour matching + curve smoothness,
    - limited due to their fixed topology.
  - Highly deformable models:
    - adaptive topology but heavy computational costs.
- Acquisition devices → higher and higher image resolutions.

Goal: Segmentation algorithms more independent from image resolution.

2. Our approach

Deformable model

- Explicit shape description:
  - object boundary = sets of vertices and edges,
  - special procedures to maintain topological consistency.

Main idea

- Time complexity directly dependent on the number of vertices.
- High accuracy over the whole image = waste of time and memory.

⇒ Adapting the model resolution according to its position in the image.

How

- Changing the Euclidean metric with a deformed metric to geometrically expand the "interesting parts" of the image.
- Keeping the vertex density on the mesh regular with that new metric.

⇒ Number of vertices locally adapted to the required accuracy.

Highly Deformable Model

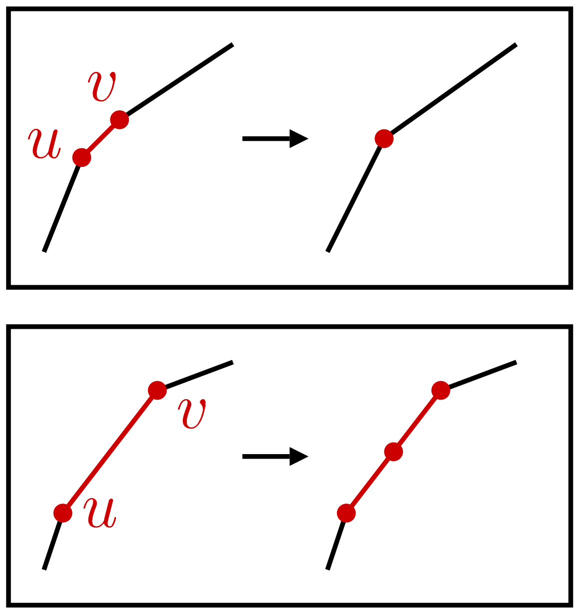
1. Mesh Regularity

Distance constraints between neighbour vertices:

$$\delta \leq d(u,v) \leq \zeta\delta$$

To maintain constraints:

- too short edge $\Rightarrow$ merge
- too long edge $\Rightarrow$ split

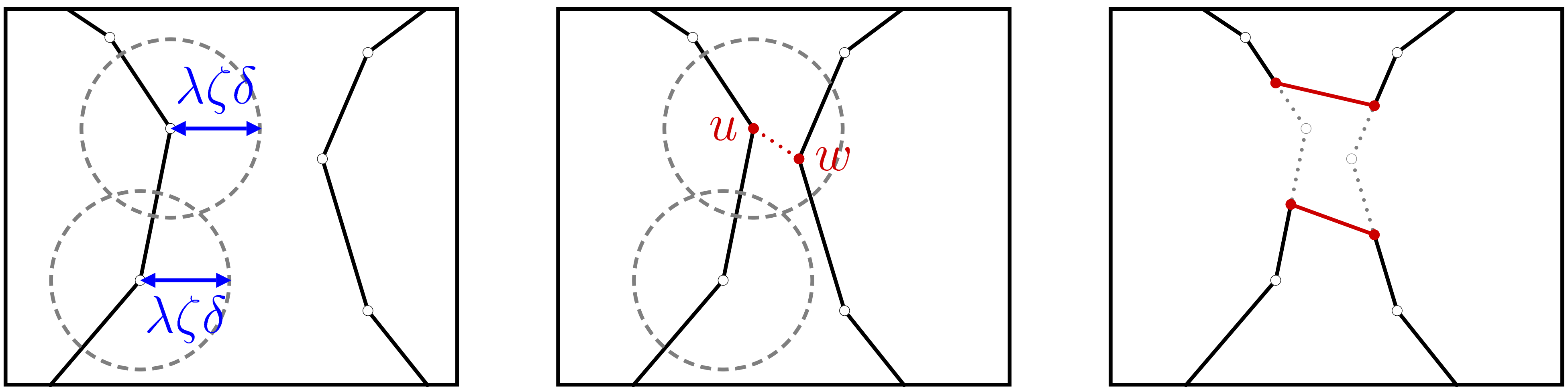


2. Topology Adaptation

Distance constraint between non-neighbour vertices:

$$\lambda\zeta\delta \leq d(u,w)$$

To maintain the constraint: topological operators.



3. Changing Distance Estimations

overestimated distances $\Rightarrow$ too long edges $\Rightarrow$ vertex insertion
underestimated distances $\Rightarrow$ too short edges $\Rightarrow$ edge contraction

Locally changing distance estimations = Locally adapting vertex density.

Changing Metrics

Riemannian Metric:
A metric is a $\mathcal{C}^1$ mapping that associates each point (x, y) with a symmetric positive-definite matrix $G_{(x,y)}$ ($i.e.$ with a dot product).

Length of a vector:

$$\|\overrightarrow{ds}\|_R^2 = {}^t\overrightarrow{ds} \times G_{(x,y)} \times \overrightarrow{ds}$$

At a given point, G has a spectral decomposition $\{(\overrightarrow{v_1}, \mu_1), (\overrightarrow{v_2}, \mu_2)\}$. $(\overrightarrow{v_1}, \overrightarrow{v_2})$ is an orthonormal base, $0 < \mu_1$, and $0 < \mu_2$.

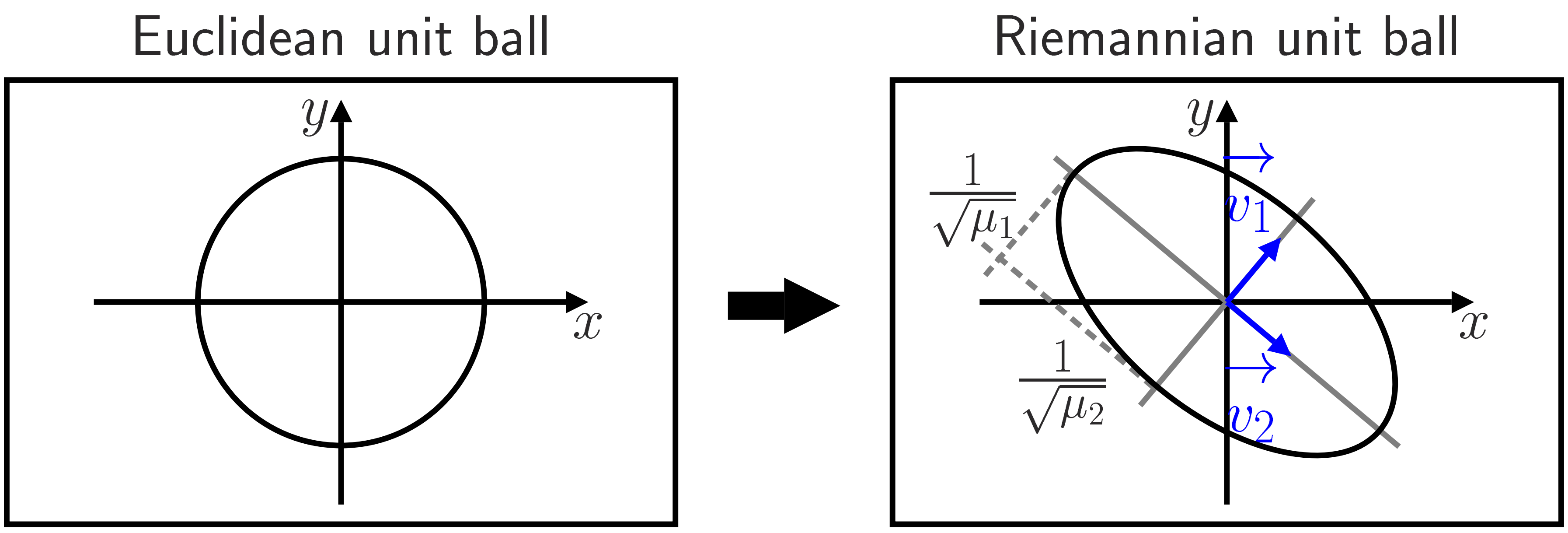
$$\overrightarrow{ds} = x_1\overrightarrow{v_1} + x_2\overrightarrow{v_2} \quad \Rightarrow \quad \|\overrightarrow{ds}\|_R^2 = \mu_1 x_1^2 + \mu_2 x_2^2$$

$\Rightarrow$ The length of a vector depends on both its origin and its direction.

Length of a path:

$$L_R(\gamma) = \int_0^1 \left({}^t\overrightarrow{\gamma'(u)} \times G_{\gamma(u)} \times \overrightarrow{\gamma'(u)}\right)^{\frac{1}{2}} du$$

Local behaviour:



Changing the metric $=$ Locally contracting/dilating the space along the local eigendirections of the metric.

Model Dynamics

1. Physical interpretation of the problem

Continuous formulation:

- continuous curve: $c : [0,1] \to \mathbb{R}^2$
- energy functional: $E(c)$

Discretized formulation:

- polygonal curve: $(c_0, \dots c_{n-1})$
- discretized energy: $E(c_0, \dots c_n)$

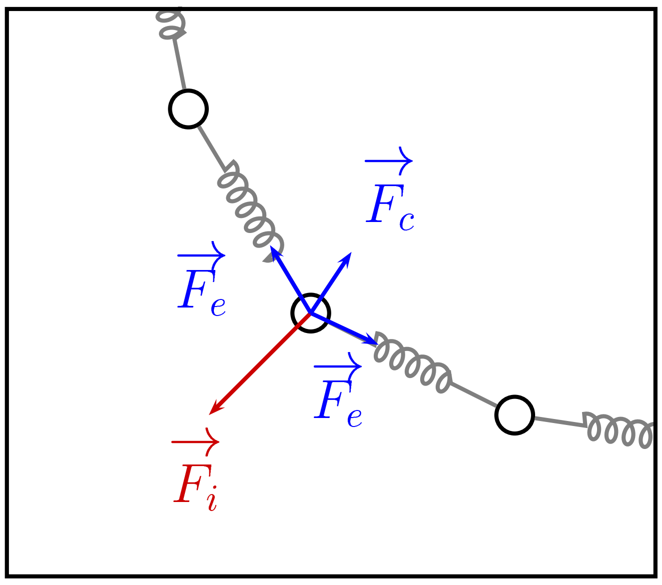
Optimization Method

Steepest descent methods: each vertex c_i moves along the line of steepest descent of E as a function of c_0, c_1, $\dots c_n$.

Physical Interpretation

Mass-spring system under the action of forces deriving from E:

- image force $\overrightarrow{F_i}$,
- elastic and curvature force $\overrightarrow{F_e}$ and $\overrightarrow{F_c}$,
- additional forces that improve convergence and/or segmentation.



2. Motion Equations

Lagrangian Formulation

Any variation $\overrightarrow{\delta x}$ of the trajectory x of a particle is such that:

$$\int_{t_1}^{t_2} \left(\delta T + \overrightarrow{F} \cdot \overrightarrow{\delta x} \right) dt = 0, \quad \text{with } T = \tfrac{1}{2}\, m\, \overrightarrow{\dot{x}} \cdot \overrightarrow{\dot{x}}$$

The dotproduct "·" takes account of the metric.

$$\boxed{\text{Euler-Lagrange equations lead to: } m\ddot{x}_k + \sum_{i,j=1}^{2} \Gamma^k_{ij} \dot{x}_i \dot{x}_j = F_k}$$

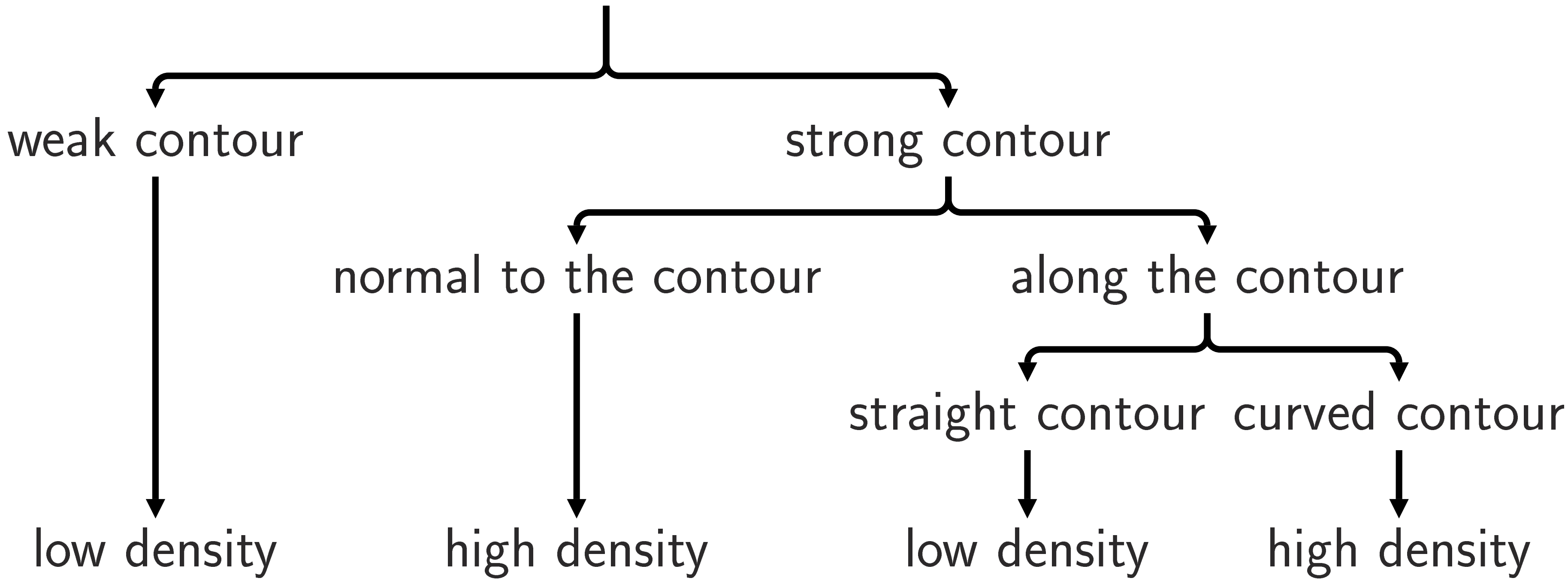
Christoffel's symbols: $\Gamma^k_{ij} = \frac{1}{2} \sum_{l=1}^{2} g^{kl} \left(\frac{\partial g_{il}}{\partial x_j} + \frac{\partial g_{lj}}{\partial x_i} - \frac{\partial g_{ij}}{\partial x_l} \right)$

Defining Metrics

1. Expected Behaviour

Vertex density should

- reflect the reliability of contour information,
- optimize the shape description quality.



2. Effective Metric Choice

For each point: two orthogonal directions and two eigenvalues.

Places with no contour information

- no prefered direction and low accuracy $\Rightarrow \mu_1 = \mu_2 = 1$,
- $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ have no influence $\Rightarrow \overrightarrow{v_1} = \overrightarrow{i}$ and $\overrightarrow{v_2} = \overrightarrow{j}$

Place with significant contour information

Eigendirections: $\overrightarrow{v_1} = \frac{\overrightarrow{\nabla I}}{\|\nabla I\|}$ and $\overrightarrow{v_2} = \frac{\overrightarrow{\nabla I^{\perp}}}{\|\nabla I\|}$.

Eigenvalues:

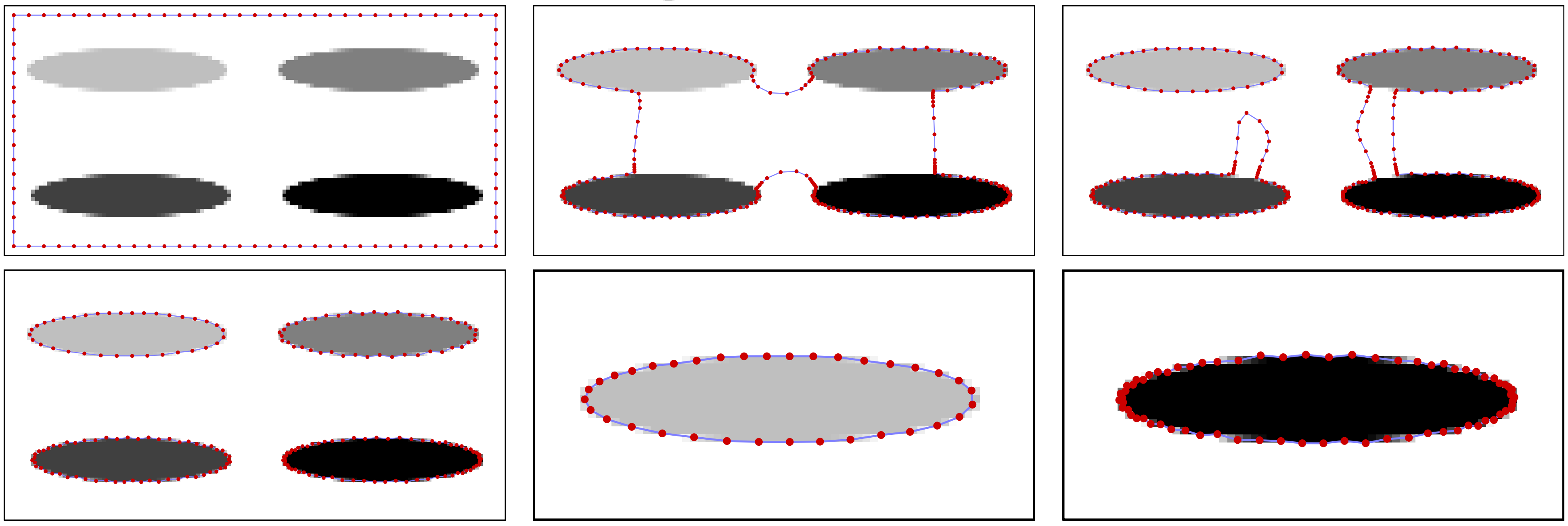
$\mu_1 = 1 + \|\overrightarrow{\nabla I}\|$, used when the mesh is orthogonal to the contour,

$\mu_2 = 1 + \kappa$, where κ is an estimation of the contour curvature.

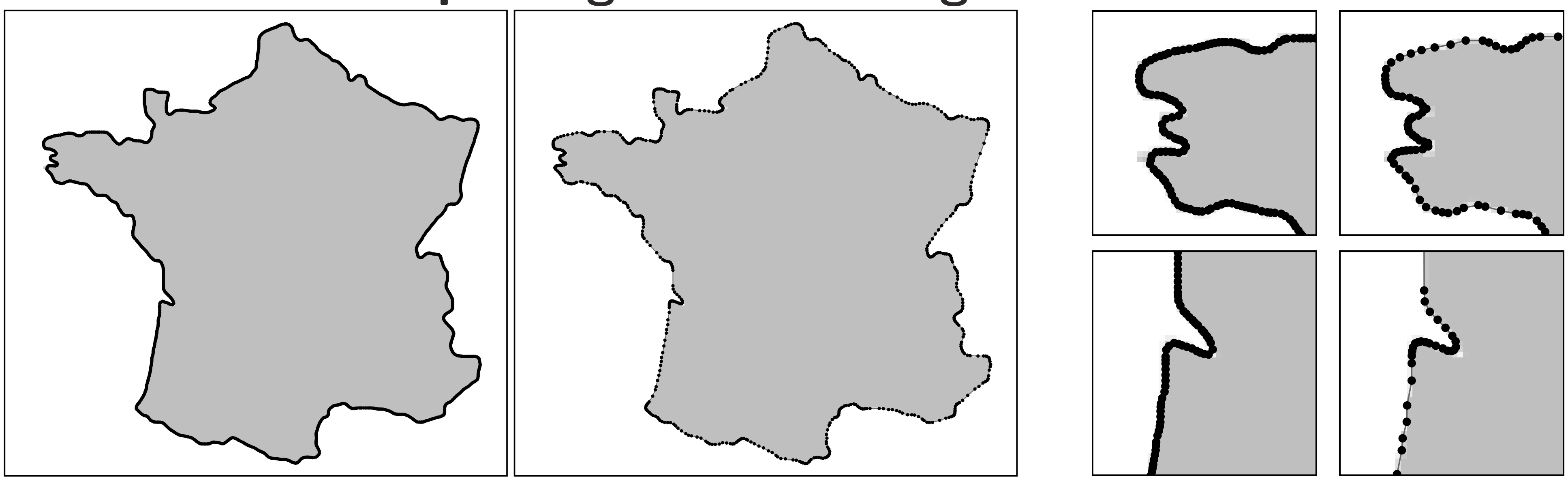
(contour curvature is estimated using the properties of the structure tensor)

Results - Perspectives

Influence of Contour Strength and Curvature on Vertex Density

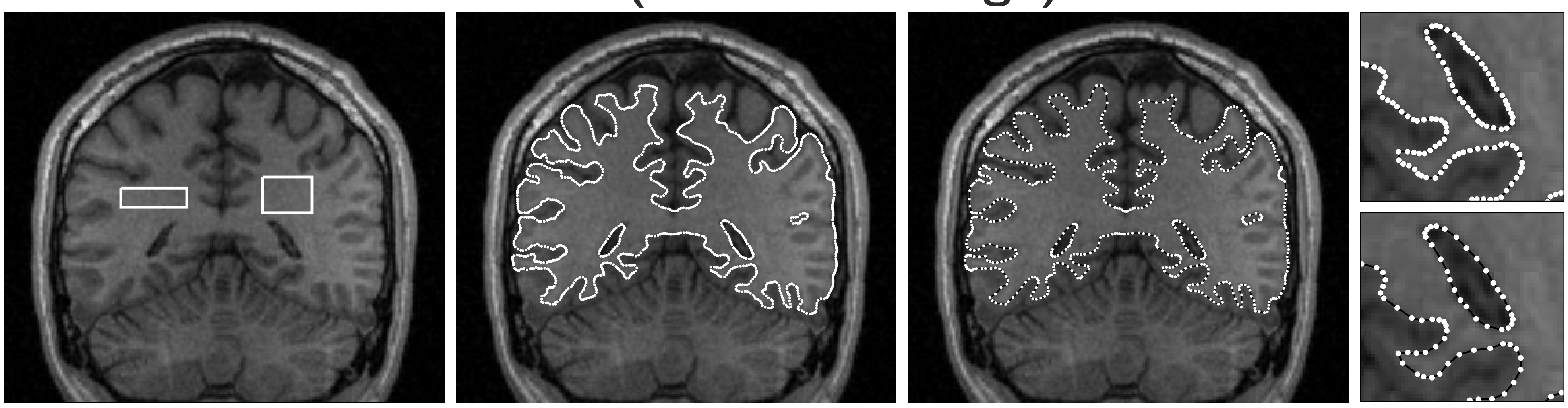


Results on a Computer-generated Image



1314 vertices 627 vertices

Results on Medical Data (brain MR image)



initial model 1262 vertices 574 vertices

Perspectives

- Development of a 3D prototype,
- Optimization of metrics computations.