Equivalence between n-surfaces and regular n-G-maps

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Outline

- Background and motivations
- Models description
- Main ideas underlying the proof
- Future work

Background

- Topological representation of space subdivisions
  - Geometric modeling, Computational geometry, Image analysis
  - Dedicated structures :
    incidence graphs, combinatorial maps, generalized maps, cell-tuples, simplicial complexes, simplicial sets, orders. . .
  - Specific tools and algorithms :
    construction operators, topological operators. . .

Motivations

- Transfer tools and notions from one model to another

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  - n-surface (Image analysis) :
    marching-cube like algorithms, homotopic thinning. . .
  - n-G-map (Topological modeling) :
    efficient data structures, construction operators. . .

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- Transfer tools and notions from one model to another
- Design a general framework to represent the topology of subdivisions
- Use several models in a single processing sequence
  - Obtain an n-surface from an image
  - Transform it into an n-G-map
  - Handle it with n-G-maps operators

Motivations

- Transfer tools and notions from one model to another
- Design a general framework to represent the topology of subdivisions
- Use several models in a single processing sequence

$\Rightarrow$ Compare these structures

$\Rightarrow$ Highlight their similarities and specificities

Previous work

- quad-edge, facet-edge, cell-tuples, n-dimensional map (generalized or not) (Brisson 89, Lienhardt 91)
- dual graphs, combinatorial maps (Brun and Kropatsch 01)
- subclass of orders, cell complexes (Alayrangues and Lachaud 02)

n-surfaces and n-G-maps

- n-surfaces (subclass of orders)
  - *Image analysis*
  - subclass of pseudo-manifolds without boundary
  - Recursive definition
- Generalized maps
  - *Geometric and topological modeling*
  - Quasi-manifolds with or without boundary, oriented or not

n-surfaces and n-G-maps

- n-surfaces (subclass of orders)
  - subclass of pseudo-manifolds without boundary
- Generalized maps
  - Quasi-manifolds $\subset$ pseudo-manifolds

n-surfaces $\longleftarrow ? \longrightarrow$ subclass of n-G-maps

Orders and n-G-maps

- Order $|X| = (X, \alpha)$
  - X set of elements equipped with the order relation α

Orders and n-G-maps

- C Order $|X| = (X, \alpha)$
  - X set of elements equipped with the order relation α
  - X Countable

Orders and n-G-maps

- CF- Order $|X| = (X, \alpha)$
  - X set of elements equipped with the order relation α
  - X Countable and locally Finite

Orders and n-G-maps

- CF- Order $|X| = (X, \alpha)$
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$\Rightarrow$ may be represented by a DAG

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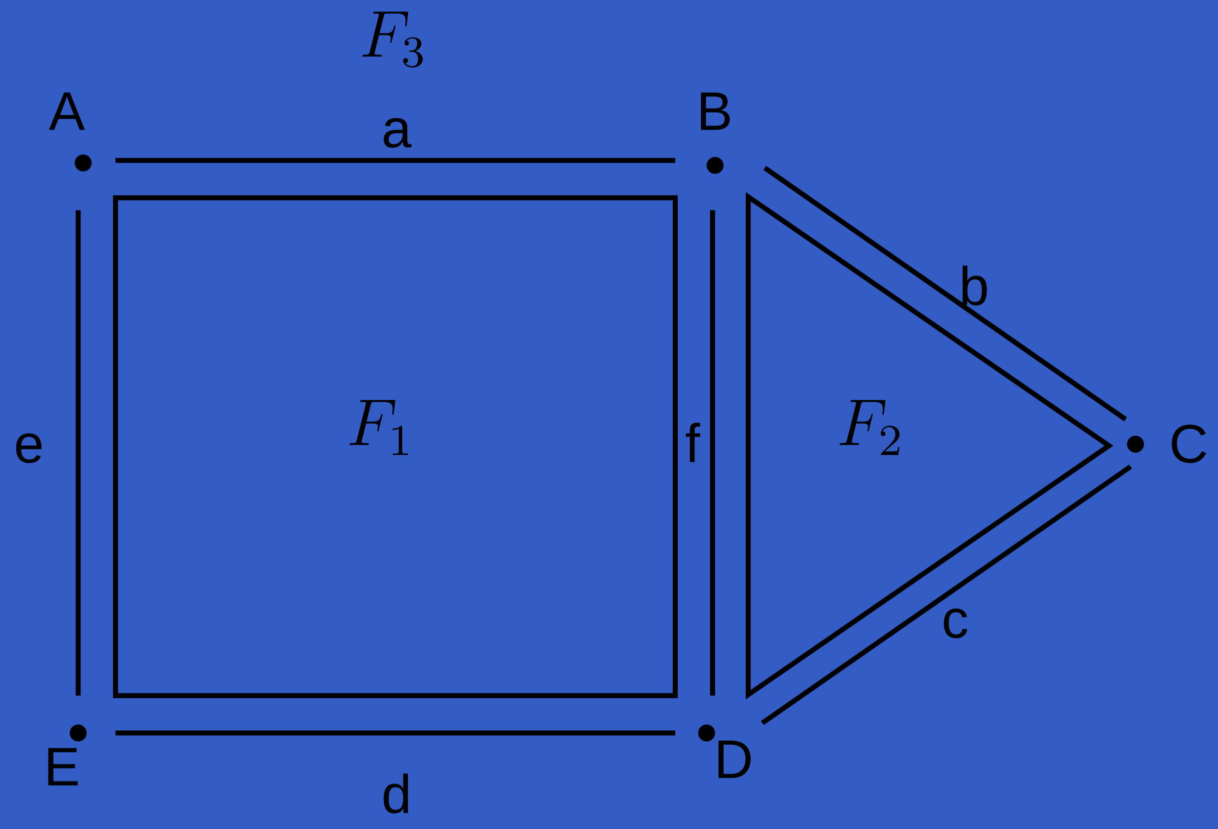
  $\Rightarrow$ may be represented by a DAG
- n-G-map $G = (D, \alpha_0, \cdots, \alpha_n)$
  - D set of darts,
  - α_i, $i \in \{0, \cdots, n\}$, involutions
  - $\alpha_i\alpha_j$ involution, $i \leq j - 2$

First comparison difficulty



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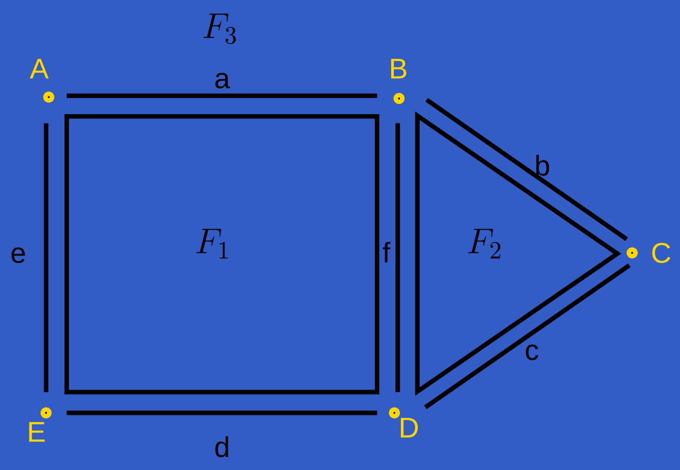

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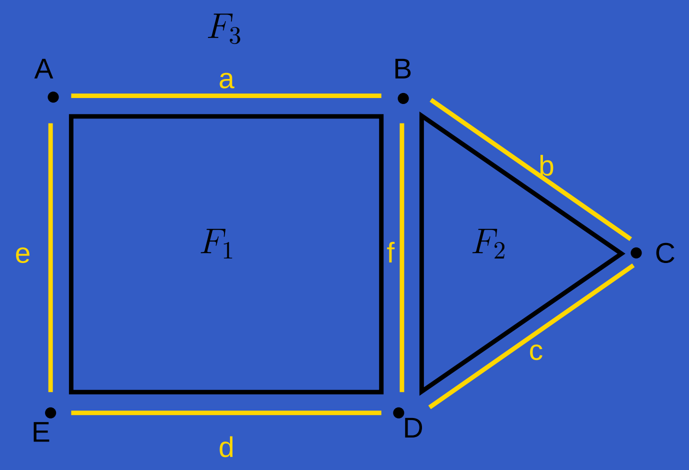

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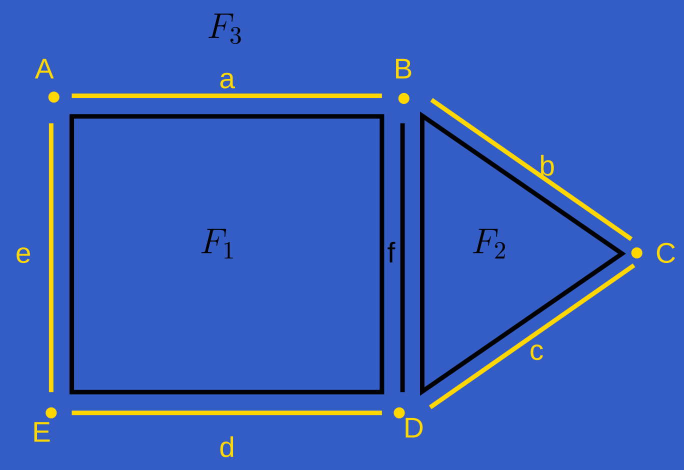

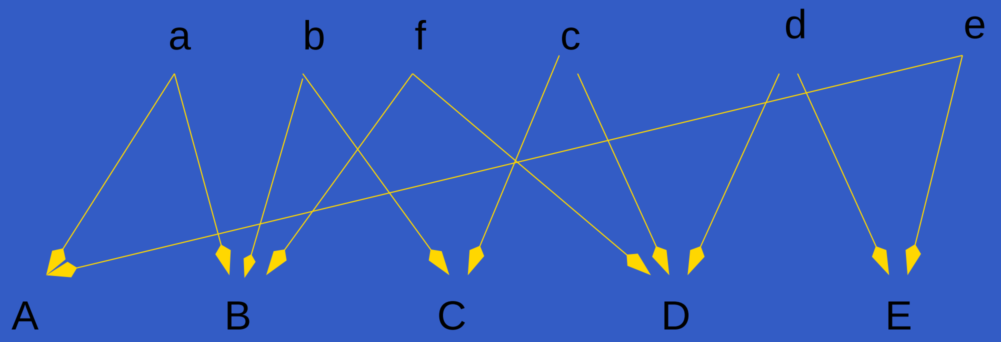

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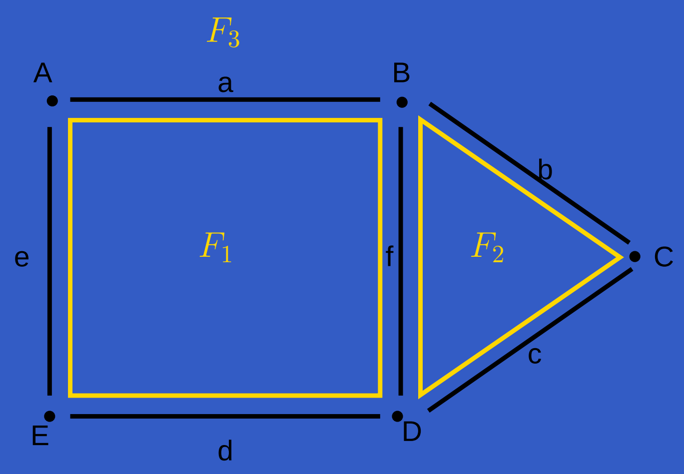

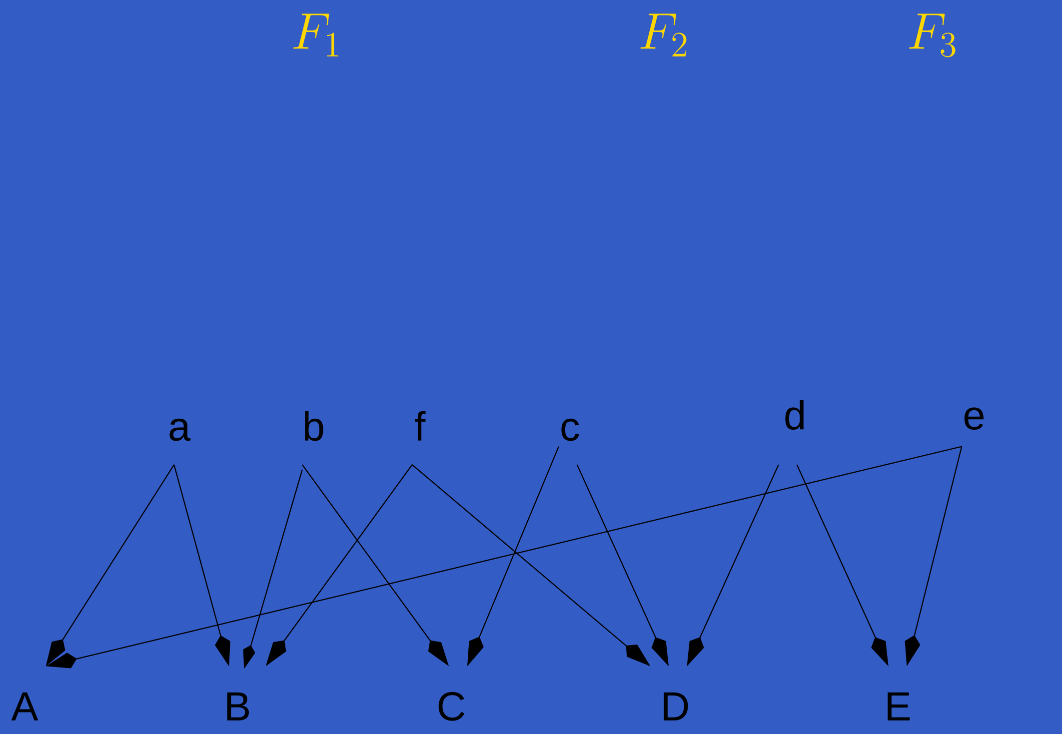

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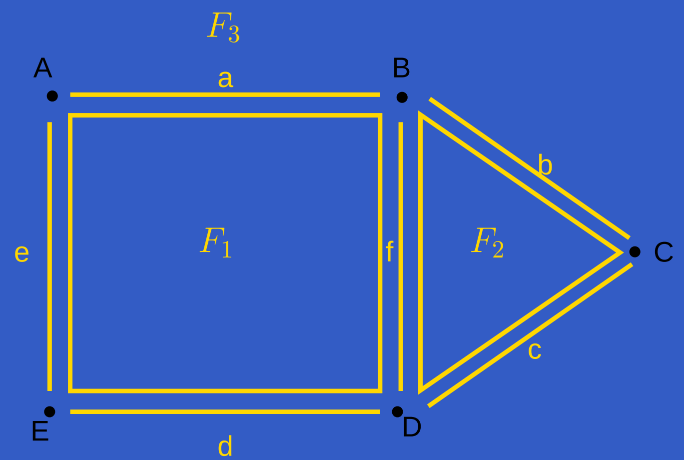

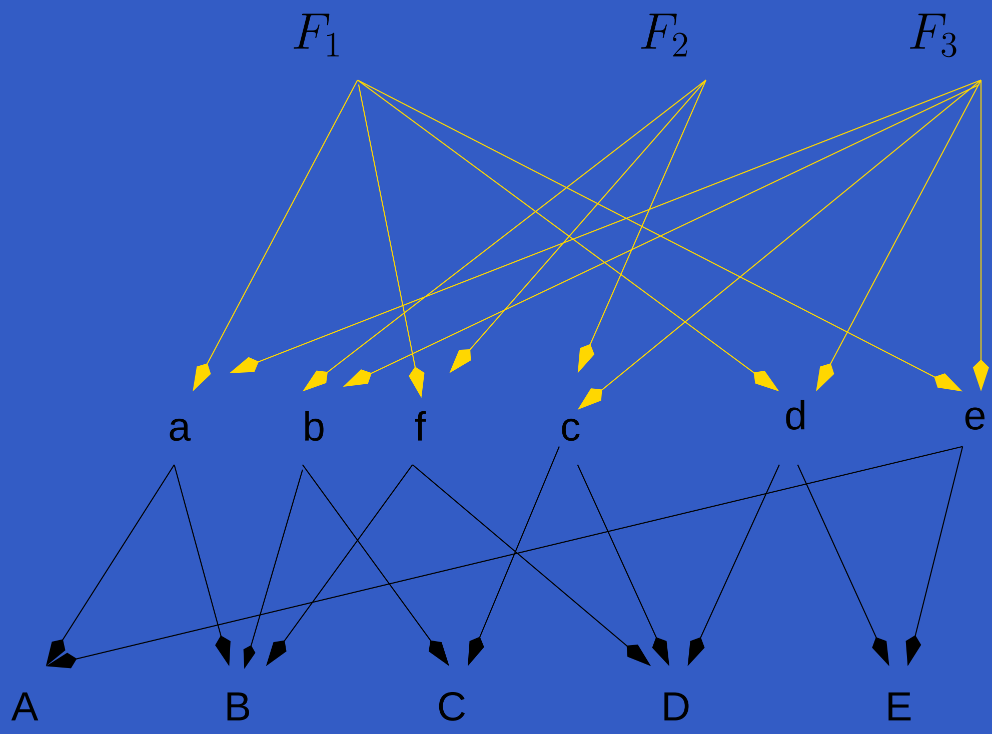

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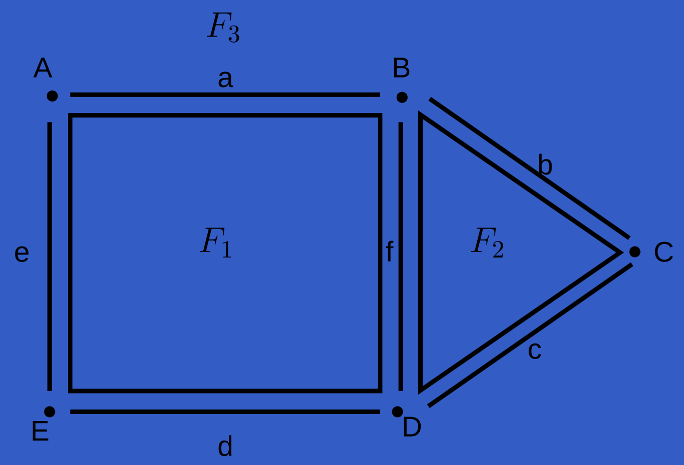

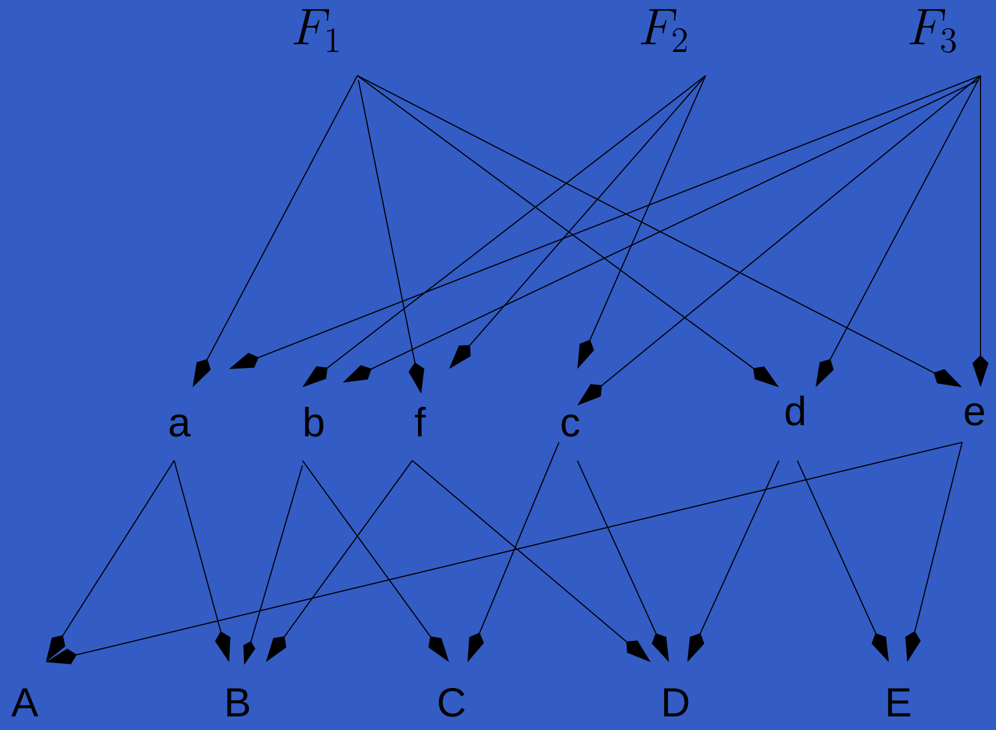

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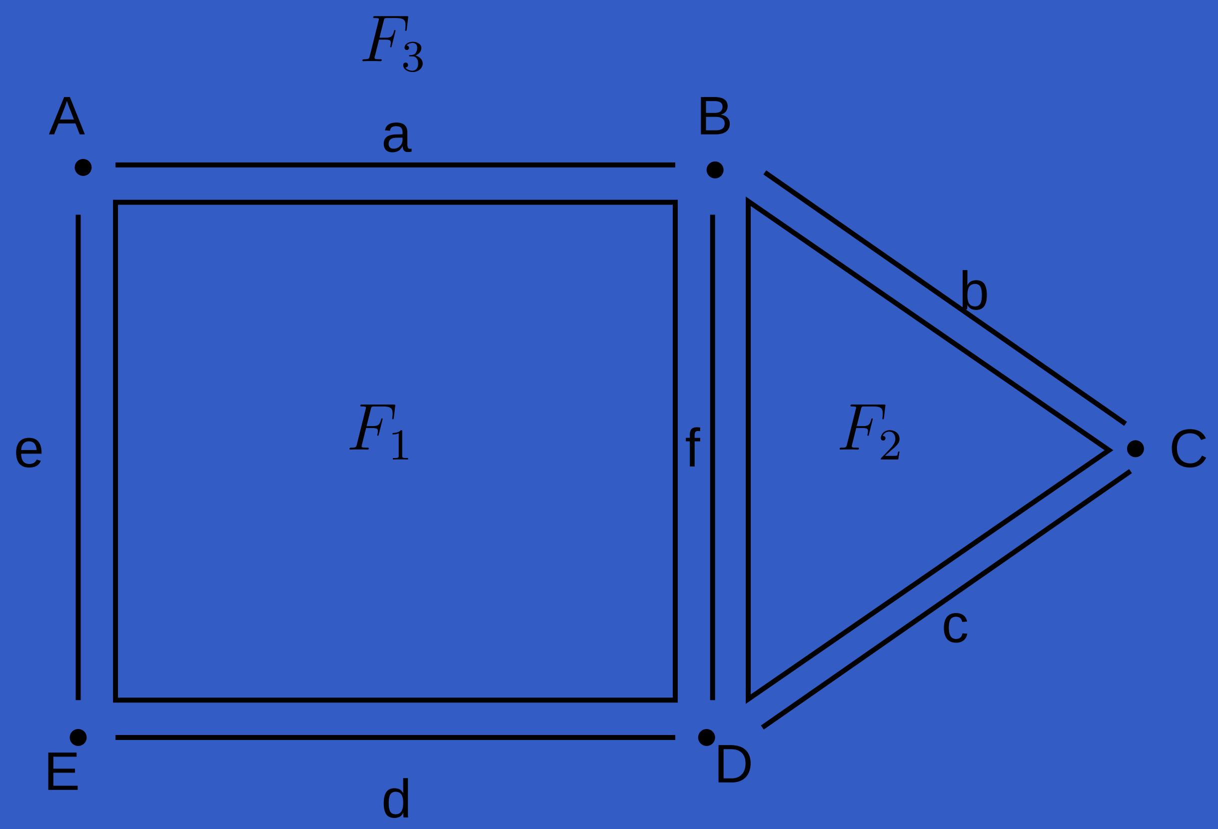

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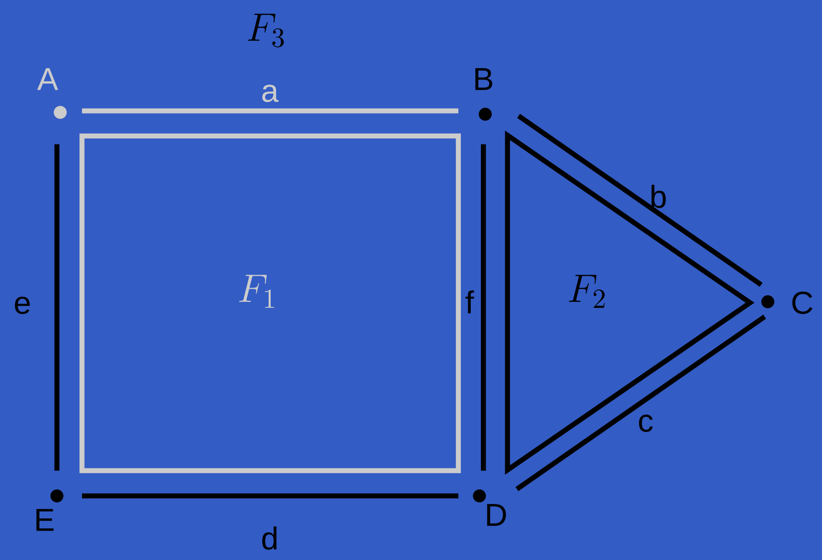

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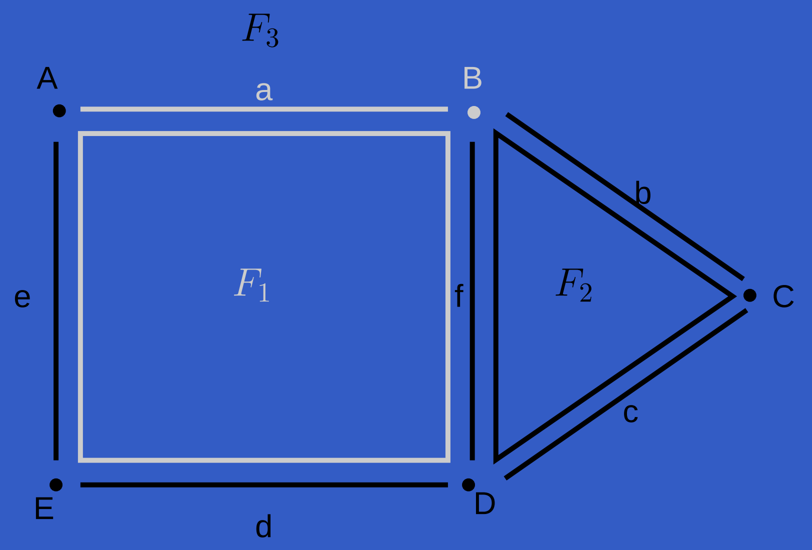

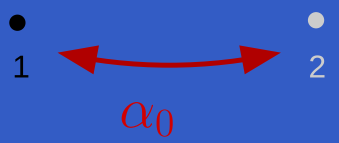

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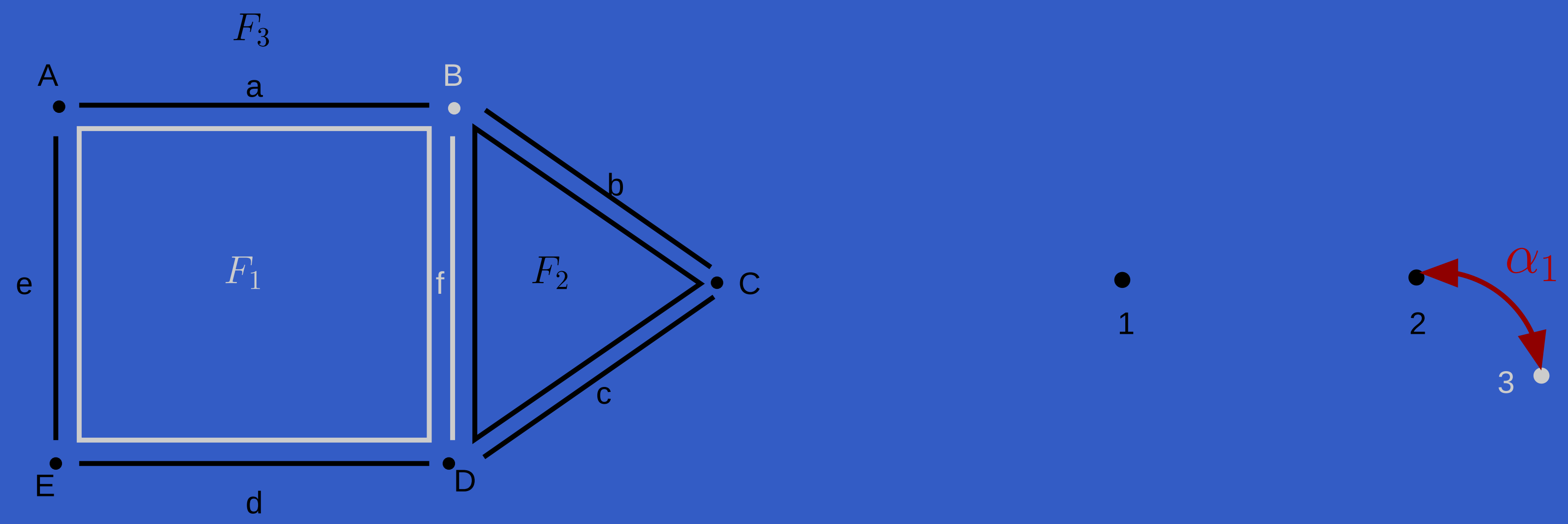

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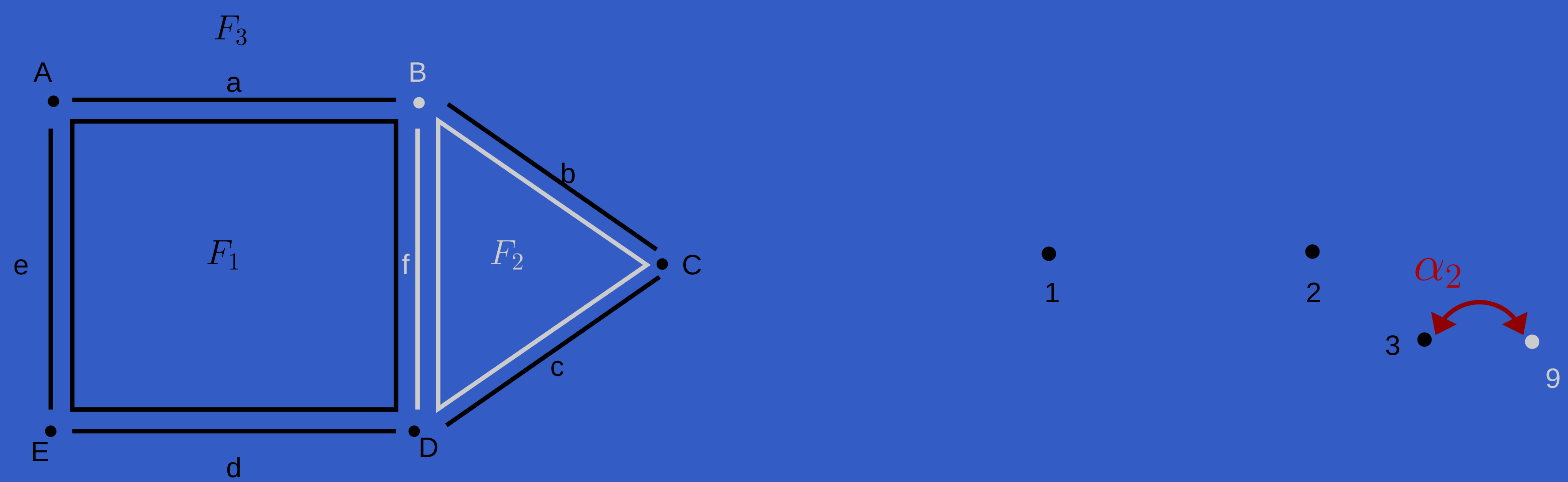

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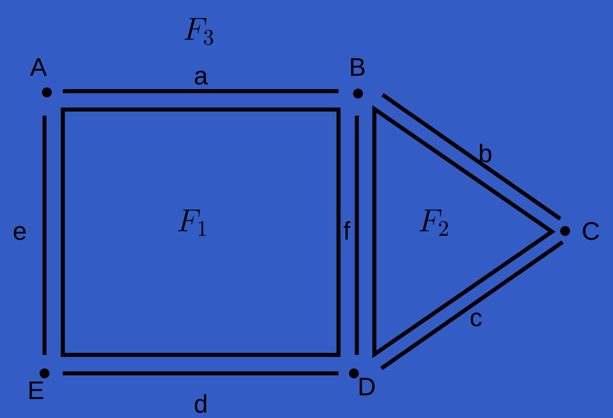
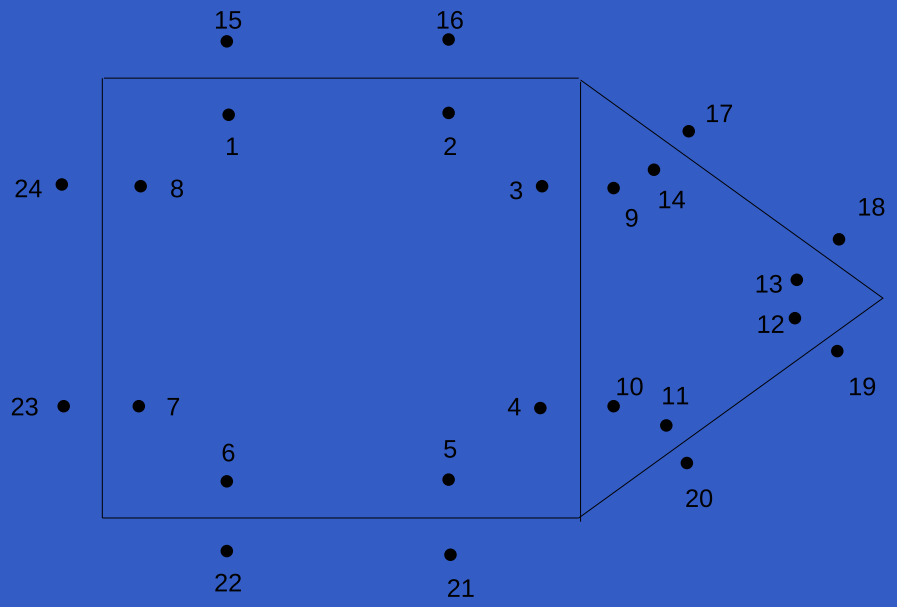




Second difficulty

- n-surface : subclass of connected orders

  $\Rightarrow$ locally everywhere an $(n-1)$-surface

  - 0-surface : 2 elements θ-disconnected
  - n-surface, $n > 0$, $\theta(x) \backslash \{x\}$ $(n-1)$-surface

  $\Rightarrow$ Recursive definition

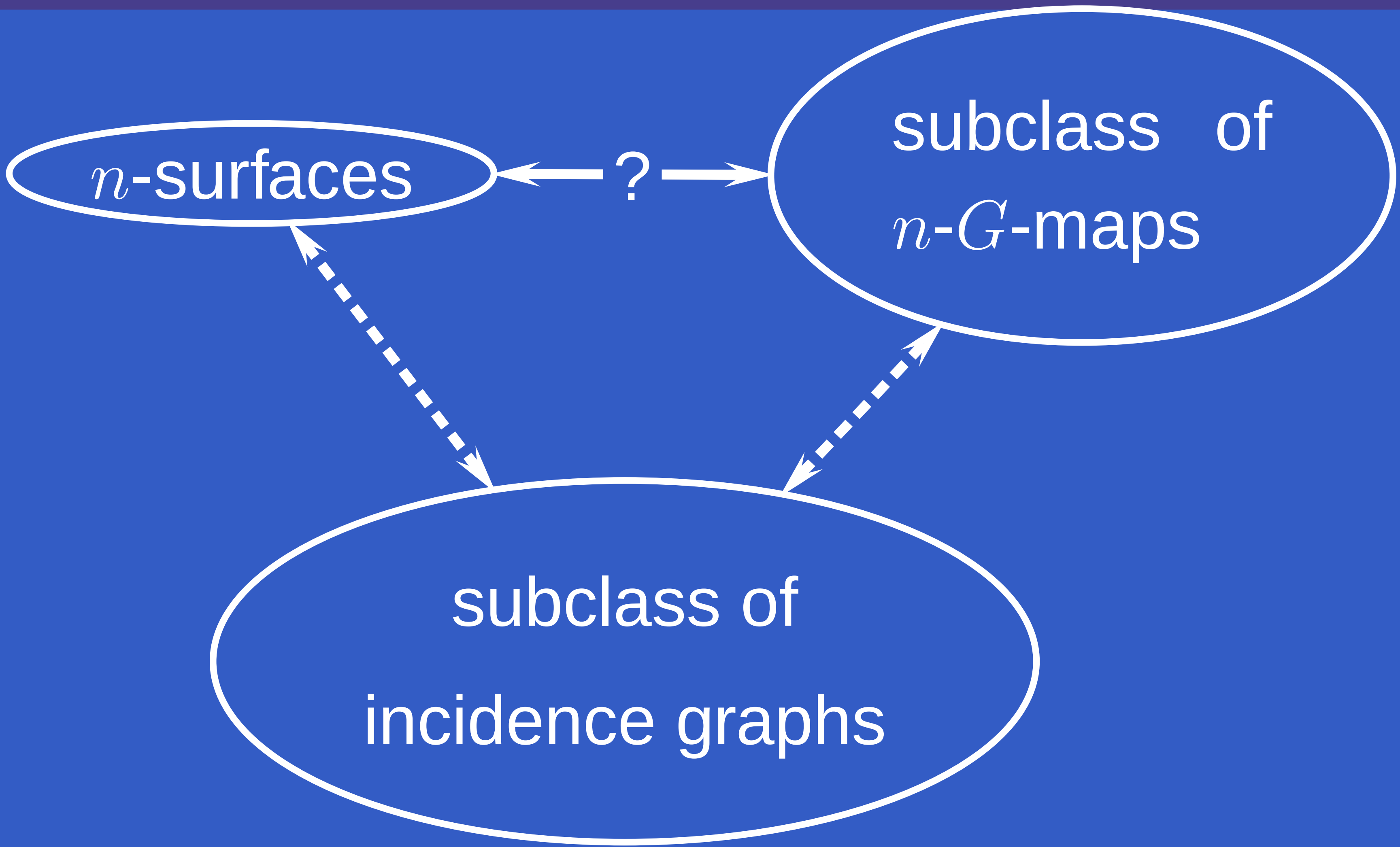
- n-G-maps
  - Constructive definition

$\Rightarrow$ How to characterize a subclass of n-G-maps equivalent to n-surfaces ?

Methodology

n-surfaces $\longleftarrow$? $\longrightarrow$ subclass of n-G-maps

Methodology



Augmented Incidence Graphs (AIG)

- Incidence graphs of subdivided d-manifolds (Brisson 89)
  - subdivided d-manifolds $\subset$ quasi-manifold
  - each cell belongs to at least one maximal chain
  - local property called `switch` property :

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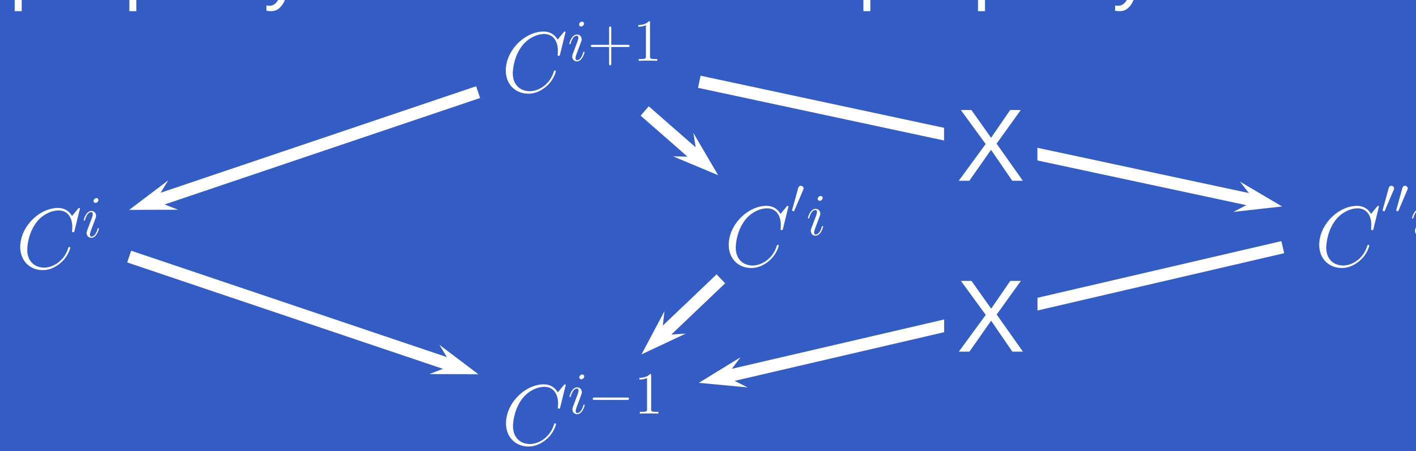

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$\Rightarrow$ But no complete characterization of such graphs

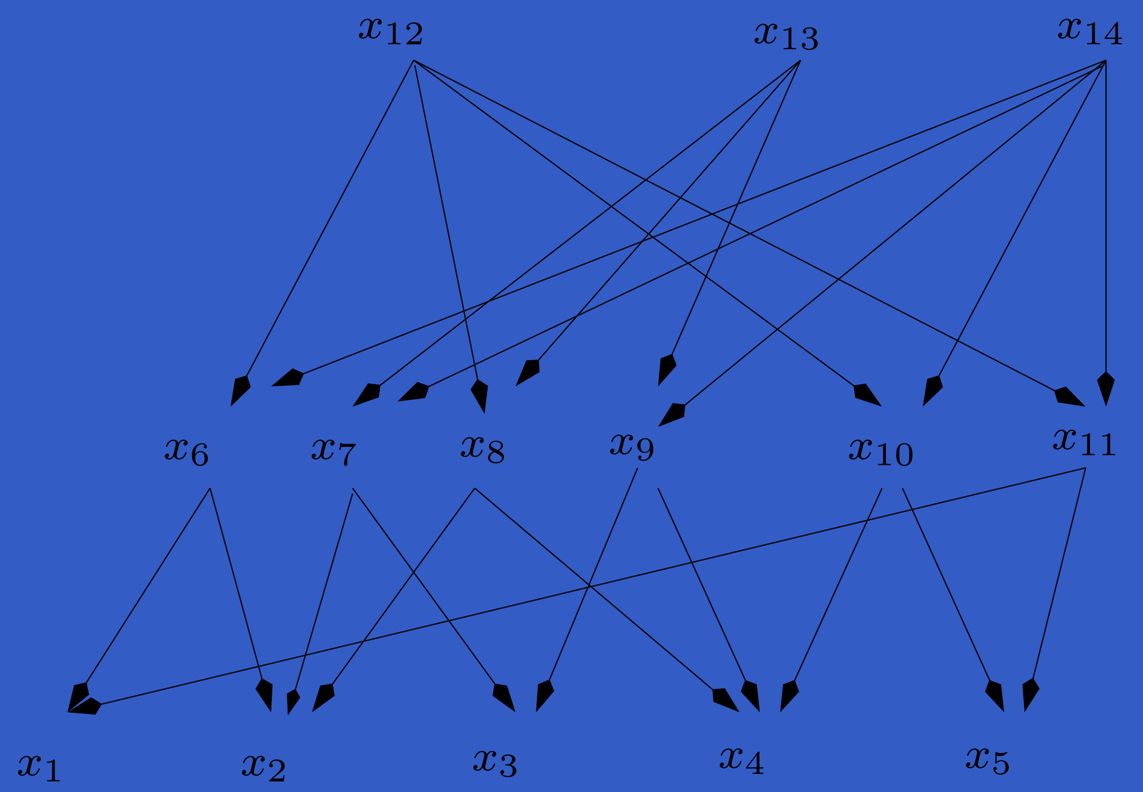
AIG and n-surfaces

- Recursive characterization of AIG
  - an incidence graph which is everywhere an AIG also is an AIG
  - an AIG is locally everywhere an AIG
- an AIG of dimension 0 is isomorphic to a 0-surface

  $\boxed{\Rightarrow \text{Equivalence between } AIG \text{ and } n\text{-surface}}$
- Note : proof not fully completed in the paper

n-surface

Consequence : `switch` property on n-surfaces



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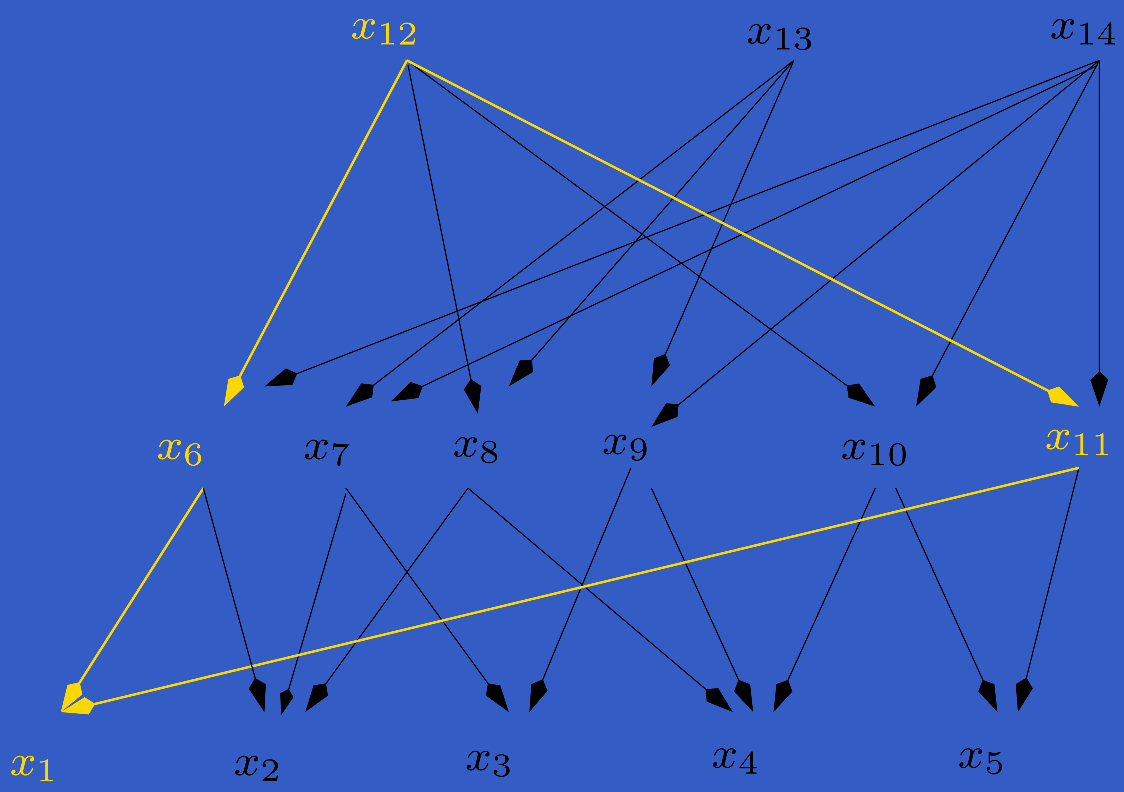

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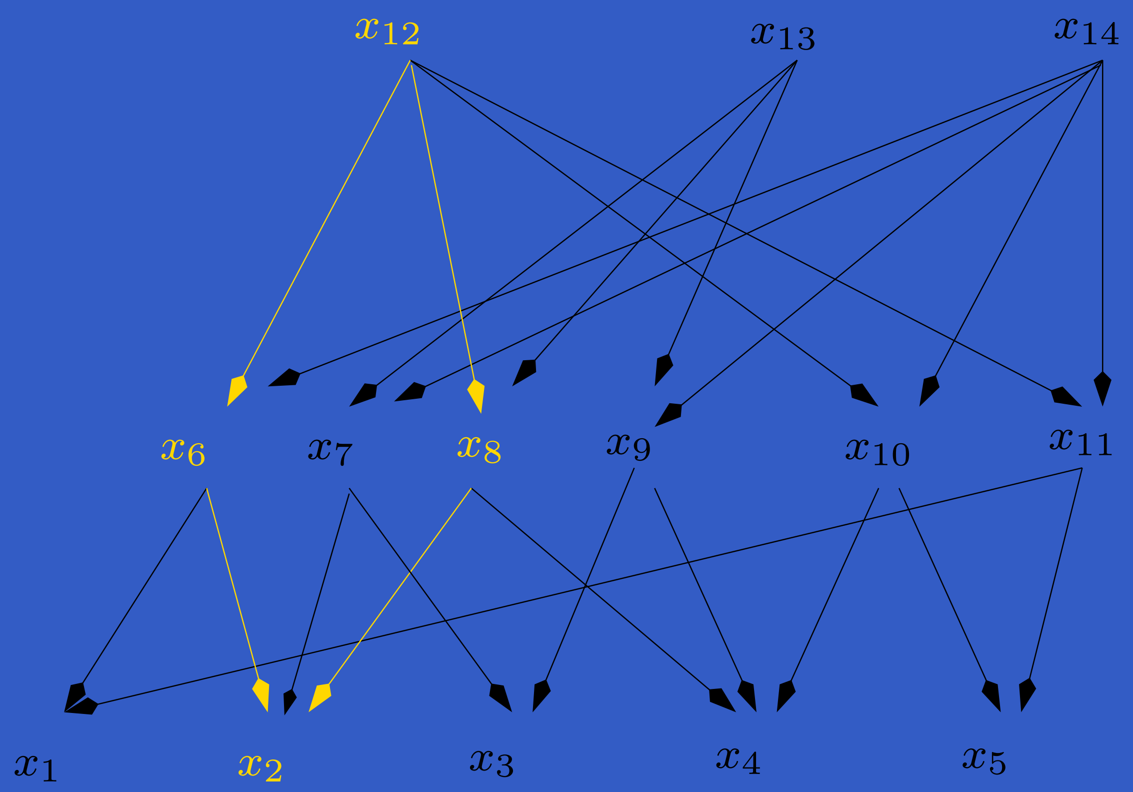

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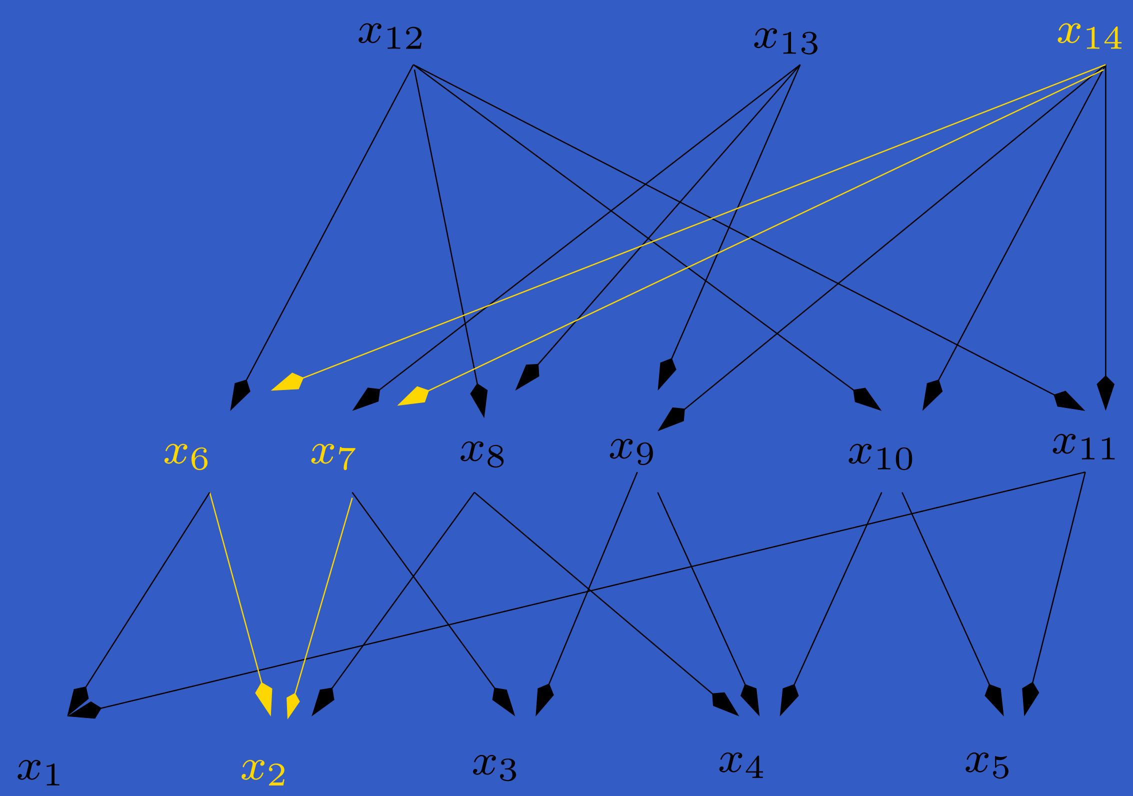

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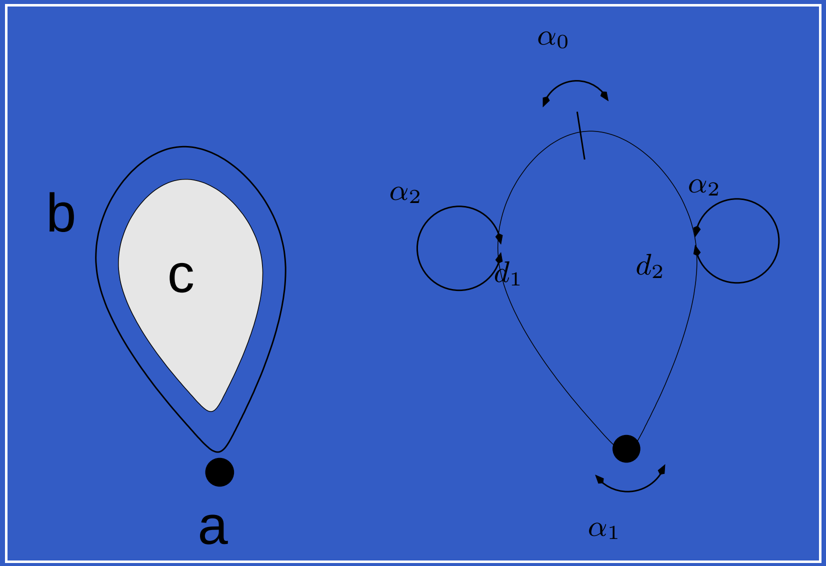

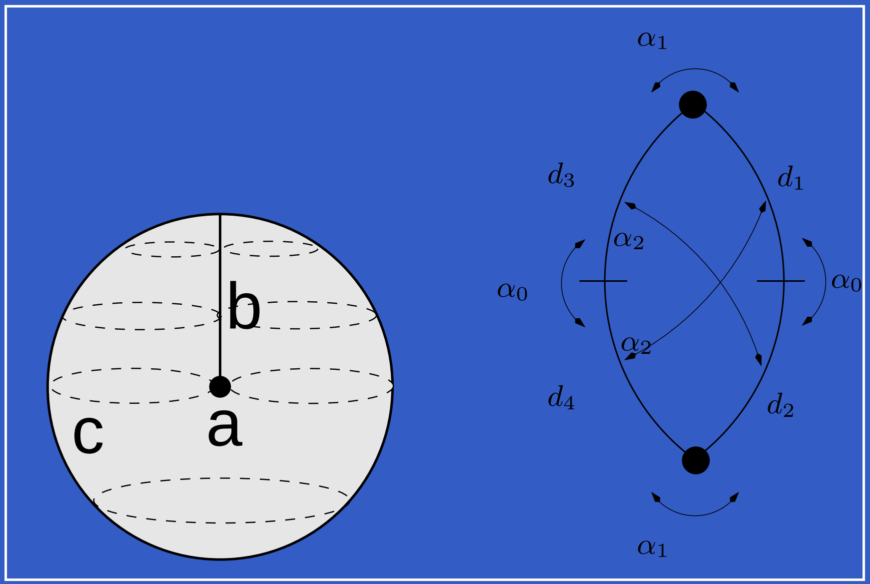

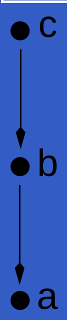

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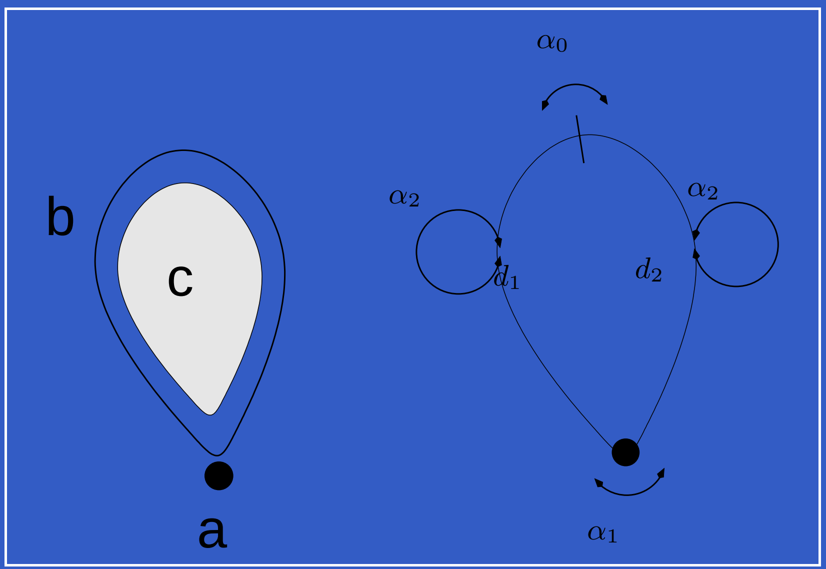
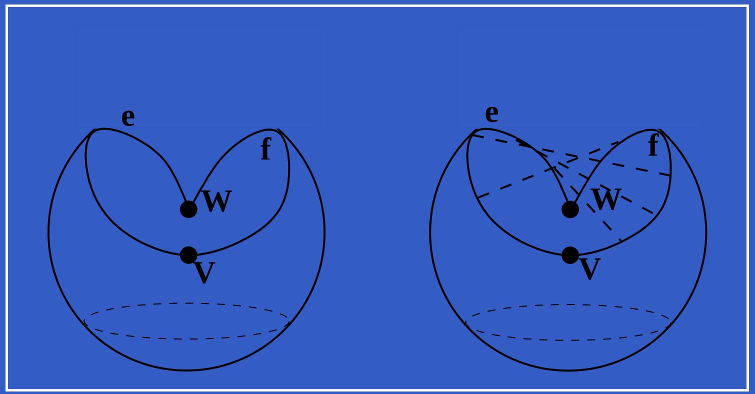
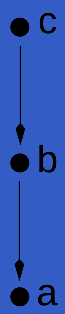
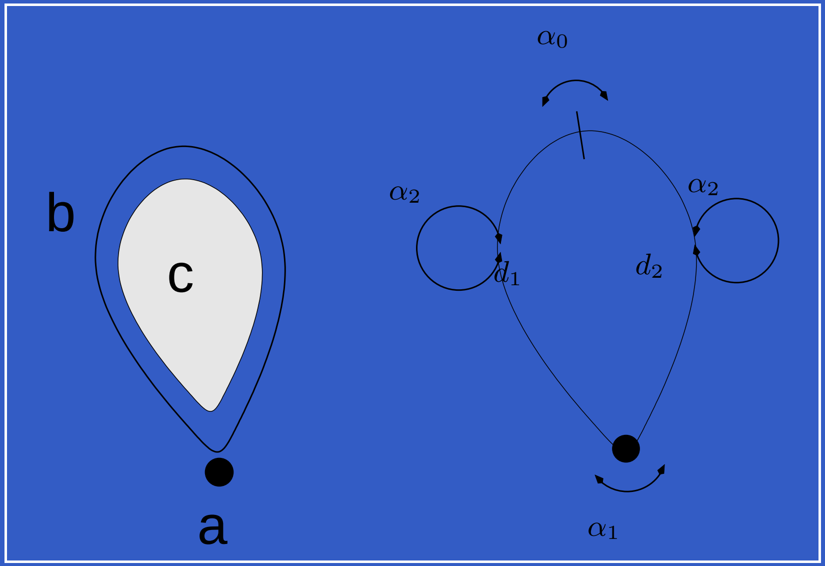
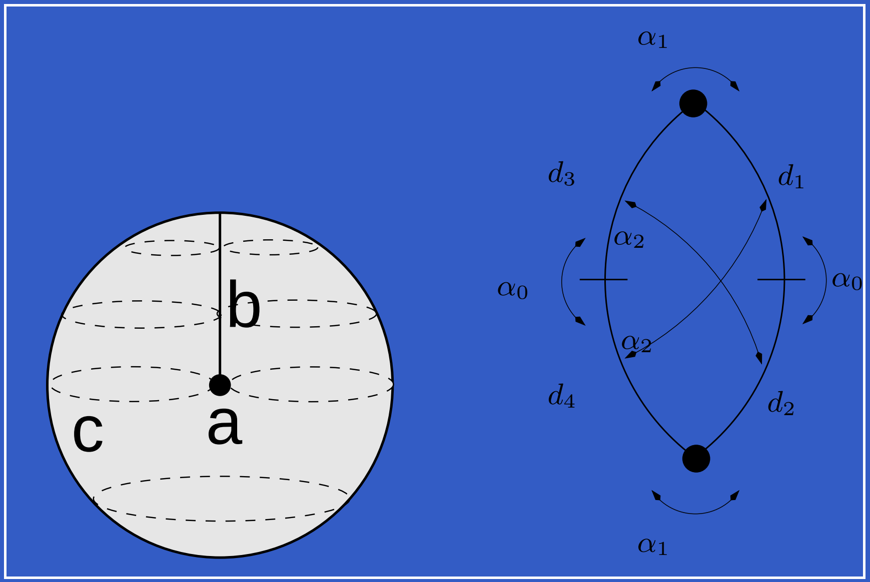


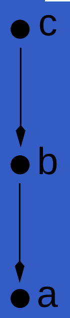




AIG and n-G-maps







Conclusion and Future work

- Achievement :

  $\Rightarrow$ Characterization of a subclass of n-G-maps equivalent to n-surfaces

- Future work :
  - effectively use this equivalence
  - study n-G-maps with boundary, oriented or not

    $\Rightarrow$ define such notions on orders
  - focus on a wider range of objects

    $\Rightarrow$ chains of maps (Elter, Lienhardt)