Accurate Curvature Estimation Along Digital Contours With Maximal Digital Circular Arcs

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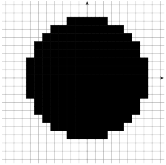
Outline

Plan

Curvature estimation along digital contours

- Input : a digital contour C in the plane = boundary of digital object
- Objective : associate a curvature field to C
- Difficulties : what is the correct curvature estimation ?



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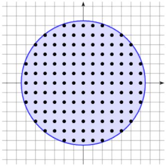

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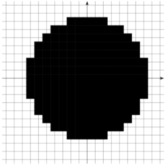

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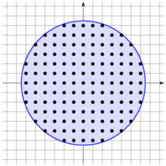

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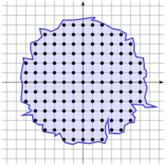

also valid

State of the art

3 families [Worring,Smeulders93], [Vialard94], [Hermann,Klette07].
Curvature approached by :

1. derivative of the tangent orientation

often convolution by a Gaussian derivative kernel in a continuous [Worring,Smeulders93],[Vialard94] or discrete setting [Malgouyres,Brunet,Fourey09], [Fiorio,Mercat,Rieux10]

2. norm of the second derivative of the parameterized contour

local approximation with some polynomial [Marji03], [Hermann,Klette07]

3. inverse of radius of osculating circle

estimation of a local osculating circle [Coeurjolly,Miguet,Tougne01], [Hermann,Klette07], [Fleischmann,Wietzke,Sommer10]

State of the art (II)

Note

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- hard to compare them objectively

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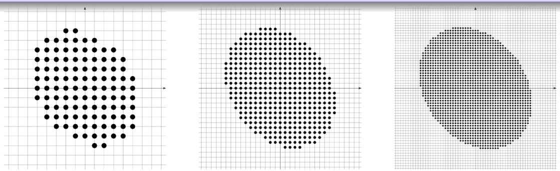
Two parameter-free methods

- Circumscribed circle to the two digital half-tangents [Coeurjolly,Miguet,Tougne01]
  Unstable in practice.
- GMC : Global Min-curvature estimator [Kerautret,Lachaud09]
  Find the shape minimizing its squared-curvature while being digitized as the input contour.
  Stable. Accurate. Can handle noisy data.

What is a good curvature estimator ?

Objective criterion

Asymptotic or Multigrid convergence When $h \to 0$ [Serra 82]



. . .

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Geometric estimator $\hat{\epsilon}$ multigrid convergent for $\mathcal{F}$ to a geom. quantity ϵ
$\forall X \in \mathcal{F}, |\hat{\epsilon}(\mathrm{Dig}_X(h)) - \epsilon(X)| \leq \tau(h)$, with $\lim_{h \to 0} \tau(h) = 0$.

- convergent estimators of area, perimeter, moments, tangents.

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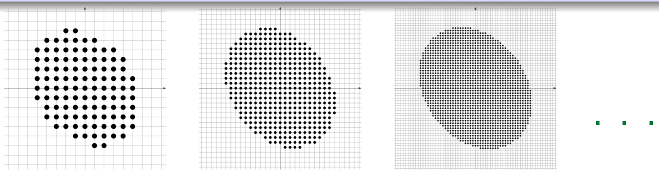

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- convergent estimators of area, perimeter, moments, tangents.
- NO convergent estimators of curvature

Contribution

We present a new curvature estimator to digital contours, based on the osculating circle (3rd family) :

- it is based on maximal digital circular arcs decomposition
- it requires no parameter
- it is multigrid convergent under some conditions
- it outperforms the best known curvature estimators in practice
- it is rather fast to compute (quasi-linear)

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Outline

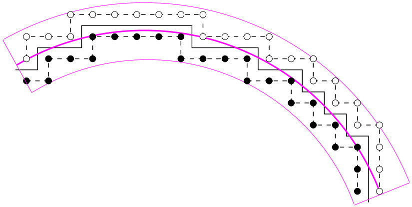
- we present maximal digital circular arcs
- we show why and when it is multigrid convergent
- we illustrates how it outperforms the best known curvature estimators with several experiments

Plan

Digital circular arc

Digital circular arc

A connected part C' of a contour C is a Digital Circular Arc (DCA) iff its interior points and exterior points are circularly separable.



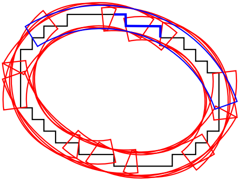
Curvature of a DCA A

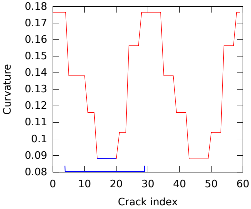
Curvature of A $\kappa(A) = \begin{cases} 0 \text{ if } A \text{ linearly separable,} \\ \text{inverse radius of any separating circle.} \end{cases}$

Curvature estimator based on circular arcs

Maximal Digital circular arc (MDCA)

A DCA A is a MDCA iff all the proper supsets C' of A in the contour C ($A \subset C' \subset C$) are not DCA.





MDCA curvature estimator

Let $p \in C$, h the gridstep. Curvature $\hat{\kappa}^h(p) = \frac{1}{h}\kappa(A)$, where A is the most centered MDCA covering p.

Plan

Definition of multigrid convergence

- Standard definition of multigrid convergence is for global geometric quantities
- Adaptation to local geometric quantities is delicate
- we follow the adaptation to tangent estimation [Lachaud, Vialard, de Vieilleville07]

Multigrid convergence of curvature estimator

The estimator $\hat{\kappa}$ is *multigrid-convergent* for the shapes $\mathbb{X}$ if and only if, for any $X \in \mathbb{X}$, $h > 0$, for any $x \in \partial X$,

$$\forall y \in \partial \mathrm{Dig}_h(X) \text{ with } \|y - x\|_1 \leq h,$$
$$|\hat{\kappa}^h(\mathrm{Dig}_h(X), y) - \kappa(X, x)| \leq \tau_x(h),$$

where $\tau_x : \mathbb{R}^{+*} \to \mathbb{R}^+$ has null limit at 0.

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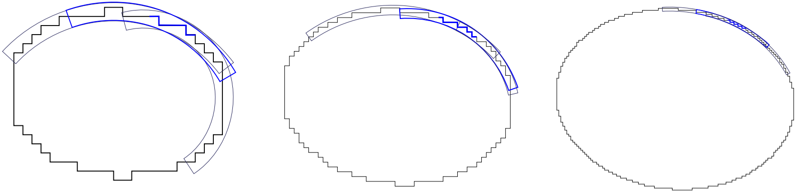
$$\forall y \in \partial \mathrm{Dig}_h(X) \text{ with } \|y - x\|_1 \leq h, \qquad (y \text{ close to } x)$$
$$|\hat{\kappa}^h(\mathrm{Dig}_h(X), y) - \kappa(X, x)| \leq \tau_x(h), \quad (\text{implies } \hat{\kappa} \text{ close to } \kappa)$$

where $\tau_x : \mathbb{R}^{+*} \rightarrow \mathbb{R}^+$ has null limit at 0.

Main theorem

Let $\mathbb{X}$ be the family of compact convex subsets of $\mathbb{R}^2$, whose curvature field is continuous, strictly positive and upper bounded. If the length of MDCA along the digital contour of any $\mathrm{Dig}_h(X)$, $X \in \mathbb{X}$, is lower bounded by $\Omega(h^a)$ and upper bounded by $O(h^b)$, $0 < b \leq a < 1/2$, then the curvature estimator $\hat{\kappa}^h_{MDCA}$ is uniformly multigrid convergent for X, with $\tau = O(h^{\min(1-2a,b)})$.



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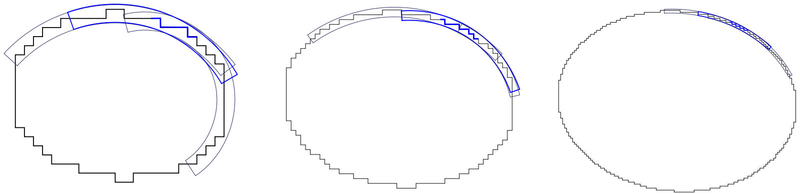


$h = 1$ $h = 0.5$ $h = 0.2$

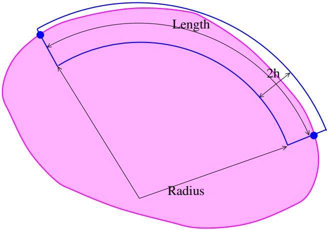
$N = \frac{1}{h}$, $\Omega(\sqrt{N}) <$ Digital length MDCA $< O(N) \Rightarrow$ convergence.

Sketch of the proof

Let X be a convex shape, ∂X its boundary, $\mathrm{Dig}_h(X)$ its digitization.

1. If a piece of ring $\mathcal{R}$ of thickness $2h$ simply covers ∂X, and its Euclidean length is between $\Omega(h^a)$ and $O(h^b)$, then

$$\lim_{h \to 0} \mathrm{radius}(\mathcal{R}) = 1/\kappa(p), \text{for any } p \in \mathcal{R} \cap \partial X$$



Proof uses convex support functions.

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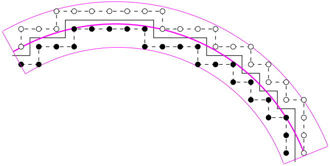
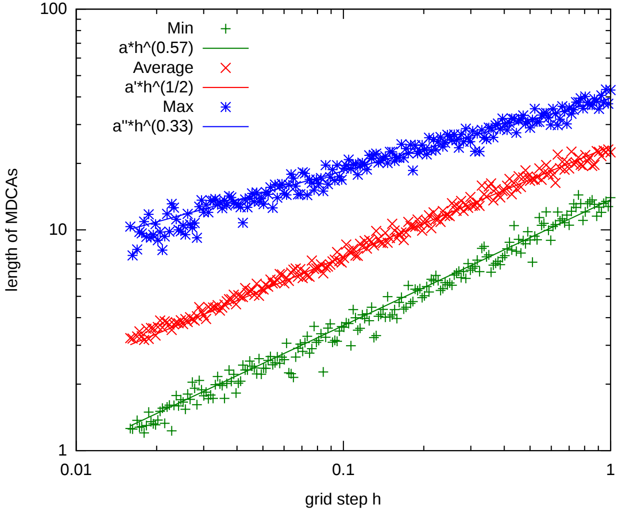
$$\lim_{h \to 0} \mathrm{radius}(\mathcal{R}) = 1/\kappa(p), \text{for any } p \in \mathcal{R} \cap \partial X$$

2. MDCA are pieces of ring simply covering ∂X.



since $\partial\mathrm{Dig}_h(X)$ has same topology as ∂X for small h (par-regularity).

Experimental evaluation of the length of MDCA



Experimental evaluation of the convergence