Comparison of Discrete Curvature Estimators and Application to Corner Detection

Bertrand Kerautret[1] J.-O Lachaud [2] B. Naegel [1]

[1]LORIA/IUT st Dié
Nancy-Universtity
France

[2]LAMA
University of Savoie
France

ISVC - Las Vegas
2, December 2008

. . . under the conditions typical for digital image processing the curvature can rarely be estimated with a precision higher than 50% [Kovalevsky 01].

1. Introduction

Many Applications exploiting curvature estimators :

- Shape analysis.
- Tools for segmentation.
- Concept of “Signature shapes” [Calabi *et al.* 98].
- Corner detection.

1. Introduction

Many Applications exploiting curvature estimators :

- Shape analysis.
- Tools for segmentation.
- Concept of "Signature shapes" [Calabi *et al.* 98].
- Corner detection.

Several criticisms concerning existing curvature estimators :

1. Few precision with low resolution shape.
2. Adapted only for perfect digitization process.

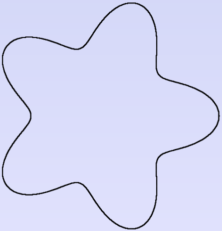
1. Introduction

Many Applications exploiting curvature estimators :

- Shape analysis.
- Tools for segmentation.
- Concept of "Signature shapes" [Calabi *et al.* 98].
- Corner detection.

Several criticisms concerning existing curvature estimators :

1. Few precision with low resolution shape.
2. Adapted only for perfect digitization process.



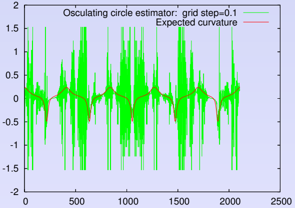
1. Introduction

Many Applications exploiting curvature estimators :

- Shape analysis.
- Tools for segmentation.
- Concept of "Signature shapes" [Calabi *et al.* 98].
- Corner detection.

Several criticisms concerning existing curvature estimators :

1. Few precision with low resolution shape.
2. Adapted only for perfect digitization process.



1. Introduction

Overview of the presentation :

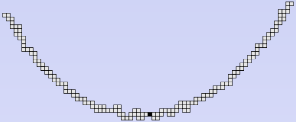
- 2. Comparisons of three recent curvature estimators :
  - 2.1 Method of osculating circles [Coeurjolly *et al.* 99] [Nguyen, Debled 07]
  - 2.2 Estimator based on Global Minimisation [Kerautret, Lachaud 08].
  - 2.3 Estimator based on Binomial Convolutions [Malgouyres *et al.* 08].
- 3. Application to corner detection with noisy shapes.
- 4. Comparison with a recent morphological corner detector.

2.1 CC and NDC curvature estimators

Method of osculating circles [Coeurjolly *et al.* 99] [Nguyen, Debled 07]

Principle :

- Geometric definition of curvature $\kappa = \frac{1}{R}$ with R the radius of the osculating circle.
- Estimation of the circle with discrete tangent segments (CC estimator).
- Integration of blurred segments for noise robustness (NDC estimator).

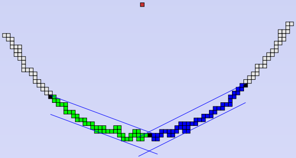


2.1 CC and NDC curvature estimators

Method of osculating circles [Coeurjolly *et al.* 99] [Nguyen, Debled 07]

Principle :

- Geometric definition of curvature $\kappa = \frac{1}{R}$ with R the radius of the osculating circle.
- Estimation of the circle with discrete tangent segments (CC estimator).
- Integration of blurred segments for noise robustness (NDC estimator).

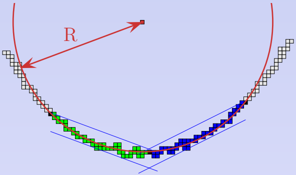


2.1 CC and NDC curvature estimators

Method of osculating circles [Coeurjolly *et al.* 99] [Nguyen, Debled 07]

Principle :

- Geometric definition of curvature $\kappa = \frac{1}{R}$ with R the radius of the osculating circle.
- Estimation of the circle with discrete tangent segments (CC estimator).
- Integration of blurred segments for noise robustness (NDC estimator).

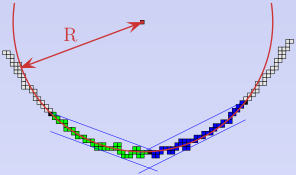


2.1 CC and NDC curvature estimators

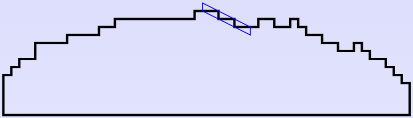
Method of osculating circles [Coeurjolly *et al.* 99] [Nguyen, Debled 07]

Principle :

- Geometric definition of curvature $\kappa = \frac{1}{R}$ with R the radius of the osculating circle.
- Estimation of the circle with discrete tangent segments (CC estimator).
- Integration of blurred segments for noise robustness (NDC estimator).



Parameter ν permits to control the sensibility to noise : (ex $\nu = 1$)

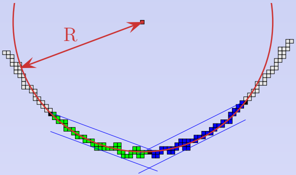


2.1 CC and NDC curvature estimators

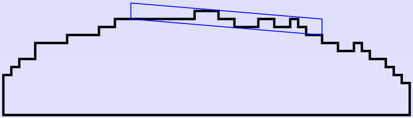
Method of osculating circles [Coeurjolly *et al.* 99] [Nguyen, Debled 07]

Principle :

- Geometric definition of curvature $\kappa = \frac{1}{R}$ with R the radius of the osculating circle.
- Estimation of the circle with discrete tangent segments (CC estimator).
- Integration of blurred segments for noise robustness (NDC estimator).



Parameter ν permits to control the sensibility to noise : (ex $\nu = 2$)

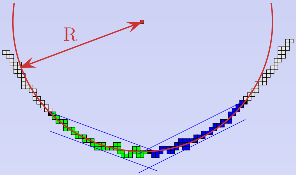


2.1 CC and NDC curvature estimators

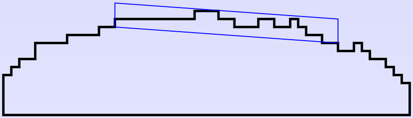
Method of osculating circles [Coeurjolly *et al.* 99] [Nguyen, Debled 07]

Principle :

- Geometric definition of curvature $\kappa = \frac{1}{R}$ with R the radius of the osculating circle.
- Estimation of the circle with discrete tangent segments (CC estimator).
- Integration of blurred segments for noise robustness (NDC estimator).



Parameter ν permits to control the sensibility to noise : (ex $\nu = 3$)

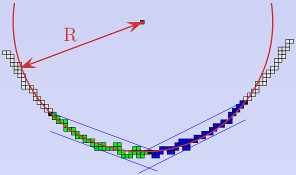


2.1 CC and NDC curvature estimators

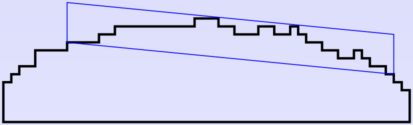
Method of osculating circles [Coeurjolly *et al.* 99] [Nguyen, Debled 07]

Principle :

- Geometric definition of curvature $\kappa = \frac{1}{R}$ with R the radius of the osculating circle.
- Estimation of the circle with discrete tangent segments (CC estimator).
- Integration of blurred segments for noise robustness (NDC estimator).



Parameter ν permits to control the sensibility to noise : (ex $\nu = 5$)

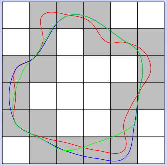


2.2 GMC curvature estimator (1)

Estimator based on Global Minimisation [Kerautret, Lachaud 08]

Main idea :

- Take into account all the real shapes corresponding to the digitized shapes.
  ⇒ Select the more probable real shape.
- Use blurred segments to adapt the estimator to noisy shapes.

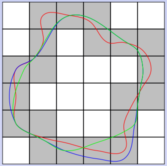


2.2 GMC curvature estimator (1)

Estimator based on Global Minimisation [Kerautret, Lachaud 08]

Main idea :

- Take into account all the real shapes corresponding to the digitized shapes.
  $\Rightarrow$ Select the more probable real shape.
- Use blurred segments to adapt the estimator to noisy shapes.

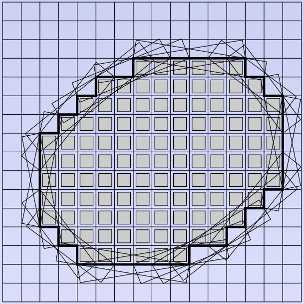


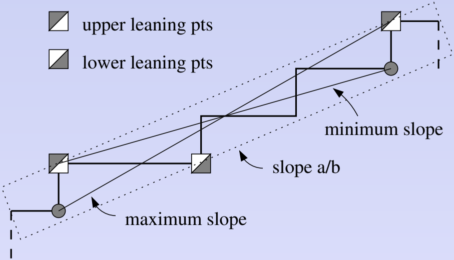
Two main steps of curvature estimation :

- Tangential cover and tangent bound estimation.
- Minimisation of curvature : $\int \kappa^2 ds$

2.2 GMC curvature estimator (2)

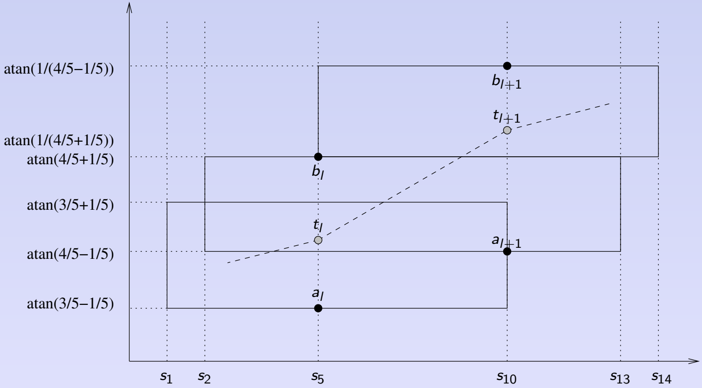
Tangent bound estimation :





2.2 GMC curvature estimator (2)

Minimisation of curvature :



2.3 BCC curvature estimator

Binomial Convolution Curvature Estimator [Malgouyres *et al.* 08]

Main idea :

- Estimate derivative with Binomial Convolution.
- Definition of the operator Ψ_k modifying the function $F : \mathbb{Z} \rightarrow \mathbb{Z}$ with kernel $K : \mathbb{Z} \rightarrow \mathbb{Z}$.

2.3 BCC curvature estimator

Binomial Convolution Curvature Estimator [Malgouyres *et al.* 08]

Main idea :

- Estimate derivative with Binomial Convolution.
- Definition of the operator Ψ_k modifying the function $F : \mathbb{Z} \rightarrow \mathbb{Z}$ with kernel $K : \mathbb{Z} \rightarrow \mathbb{Z}$.

Definition of the derivative kernel :

- Definition of the smoothing kernel :

$$H_n(a) = \begin{cases} \begin{pmatrix} n \\ a + \frac{n}{2} \end{pmatrix} & \text{if } n \text{ is even and } \; a \in \{-\frac{n}{2}, ..., \frac{n}{2}\} \\ \begin{pmatrix} n \\ a + \frac{n+1}{2} \end{pmatrix} & \text{if } n \text{ is odd and } \; a \in \{-\frac{n+1}{2}, ..., \frac{n-1}{2}\} \\ 0 & \textit{otherwise}. \end{cases}$$

- Kernel of backward finite difference : $\delta(a) = \begin{cases} 1 & \text{if } a = 0 \\ -1 & \text{if } a = 1 \\ 0 & \textit{otherwise} \end{cases}$

2.3 BCC curvature estimator (2)

- Derivative kernel D_n defined as : $D_n = \delta * H_n$
- Second order derivative kernel :

$$D_n^2 = \delta * \delta * D_n \tag{1}$$

2.3 BCC curvature estimator (2)

- Derivative kernel D_n defined as : $D_n = \delta * H_n$
- Second order derivative kernel :

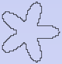
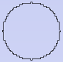
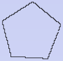
$$D_n^2 = \delta * \delta * D_n \tag{1}$$

The curvature is then deducted as :

$$\frac{D_n^2(x) * D_n(y) - D_n^2(y) * D_n(x)}{D_n(x)^2 + D_n(y)^2} \tag{2}$$

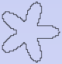
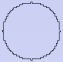
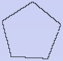
3. Experimental comparison of the three estimators

Mean squared errors on the three estimators :

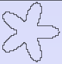
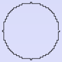
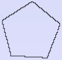
shape	 Flower			 Circle			 Polygon		
h	1	0.1	0.01	1	0.1	0.01	1	0.1	0.01
CC	0.0945	0.0225	0.0079	0.0005	0.0009	0.0013	0.0004	0.0003	8.1e-05
GMC	0.0966	0.0346	0.0049	2.5e-07	3.2e-10	4.3e-08	0.0113	0.3089	3.428
BCC	0.0855	0.0185	0.0081	0.0178	0.0012	0.0001	0.0232	0.0261	0.0510

3. Experimental comparison of the three estimators

Mean squared errors on the three estimators :

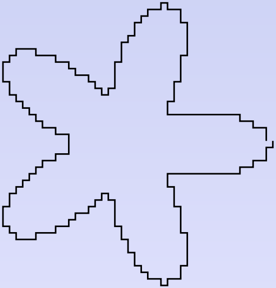
shape	 Flower			 Circle			 Polygon		
h	1	0.1	0.01	1	0.1	0.01	1	0.1	0.01
CC	0.0945	0.0225	0.0079	0.0005	0.0009	0.0013	0.0004	0.0003	8.1e-05
GMC	0.0966	0.0346	0.0049	2.5e-07	3.2e-10	4.3e-08	0.0113	0.3089	3.428
BCC	0.0855	0.0185	0.0081	0.0178	0.0012	0.0001	0.0232	0.0261	0.0510

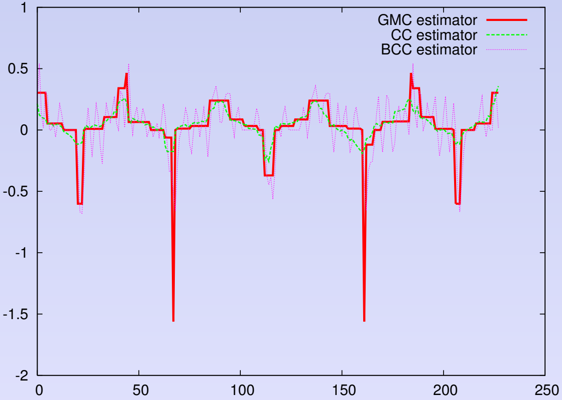
Execution times :

shape	 Flower			 Circle			 Polygon		
h	1	0.1	0.01	1	0.1	0.01	1	0.1	0.01
$\mathrm{CC}_{(\mathrm{ms})}$	6	84	891	6	55	637	8	82	870
$\mathrm{GMC}_{(\mathrm{ms})}$	0	75	2593	2	363	2673	0	4	67
$\mathrm{BCC}_{(\mathrm{ms})}$	0	18	4514	0	14	3275	0	18	4501

3. Experimental comparison of the three estimators

Comparisons on regular shapes : flower

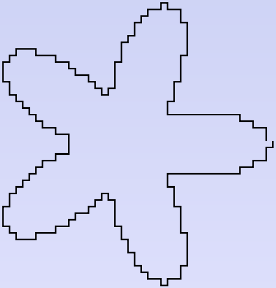


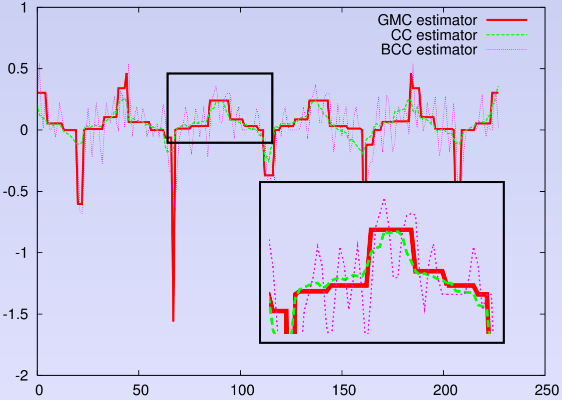


grid size=1

3. Experimental comparison of the three estimators

Comparisons on regular shapes : flower

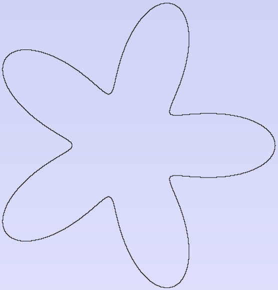


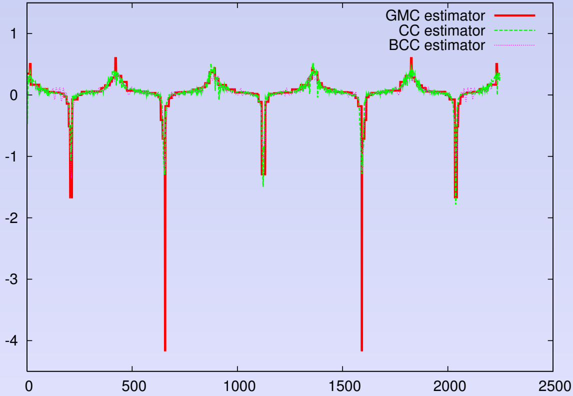


grid size=1

3. Experimental comparison of the three estimators

Comparisons on regular shapes : flower

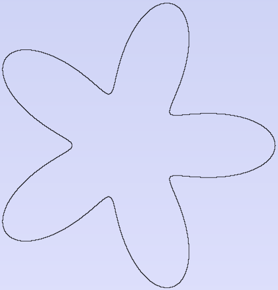


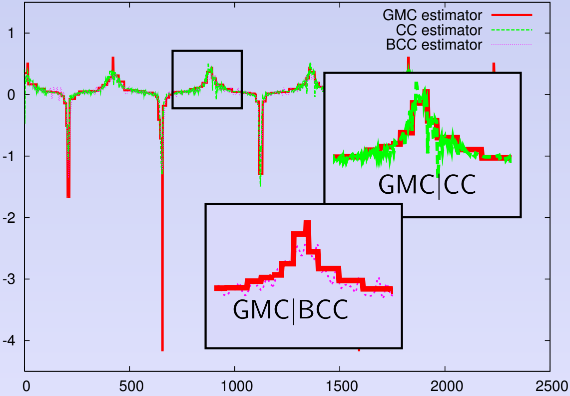


grid size=0.1

3. Experimental comparison of the three estimators

Comparisons on regular shapes : flower

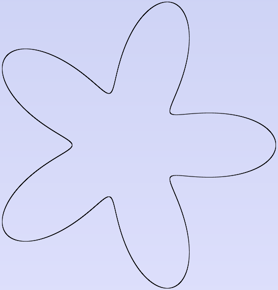


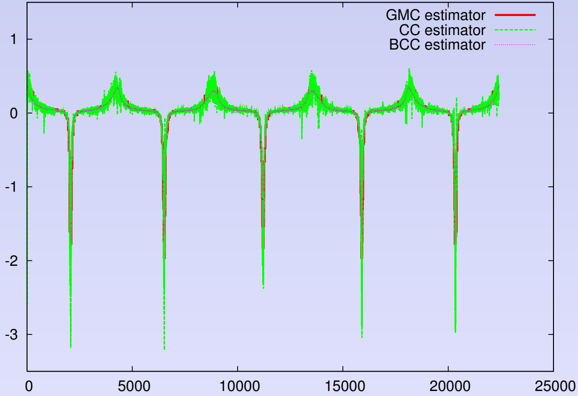


grid size=0.1

3. Experimental comparison of the three estimators

Comparisons on regular shapes : flower

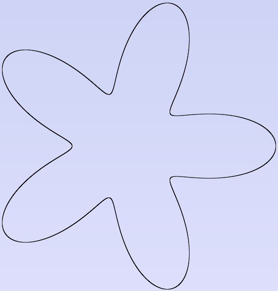


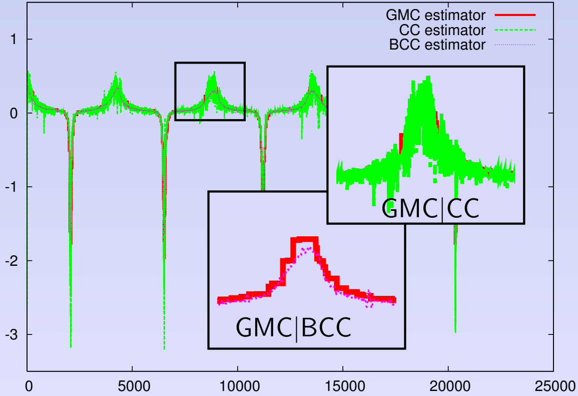


grid size=0.01

3. Experimental comparison of the three estimators

Comparisons on regular shapes : flower

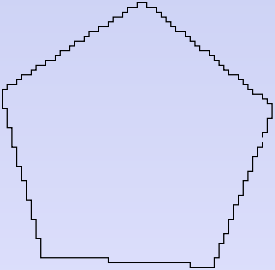


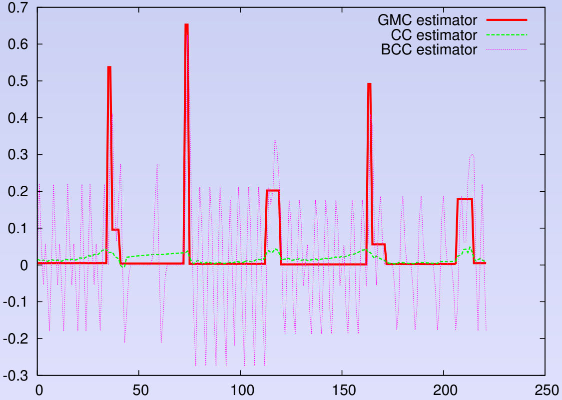


grid size=0.01

3. Experimental comparison of the three estimators

Comparisons on regular shapes : polygon

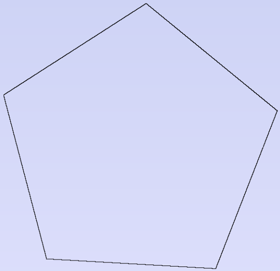


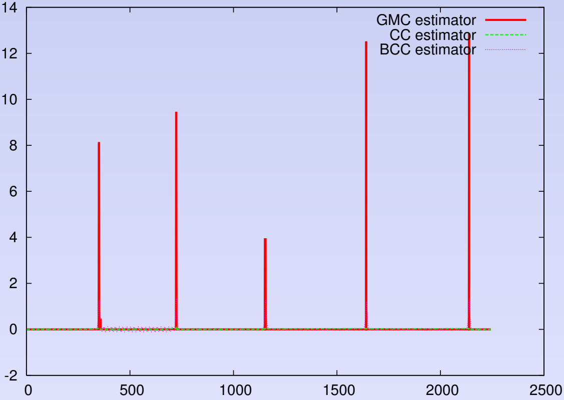


grid size=1

3. Experimental comparison of the three estimators

Comparisons on regular shapes : polygon

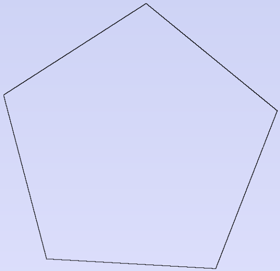


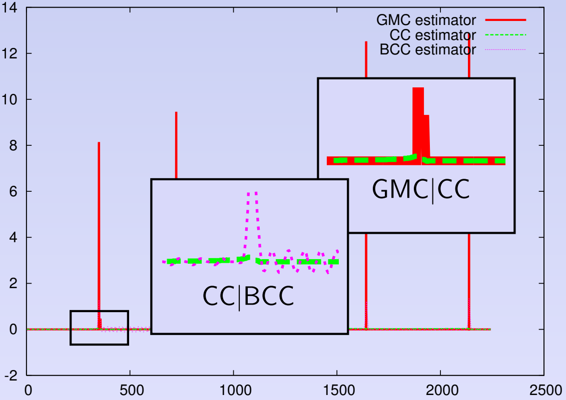


grid size=0.1

3. Experimental comparison of the three estimators

Comparisons on regular shapes : polygon

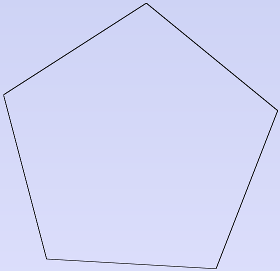


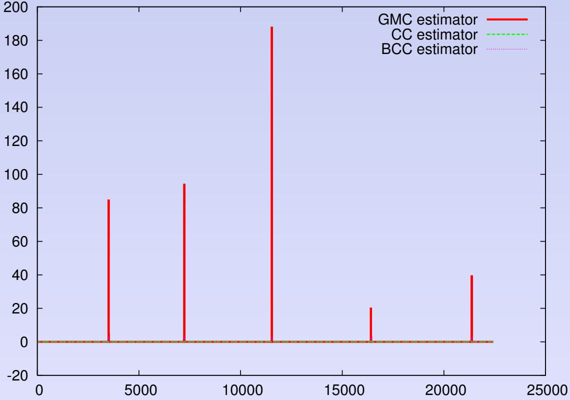


grid size=0.1

3. Experimental comparison of the three estimators

Comparisons on regular shapes : polygon

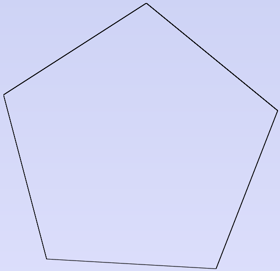


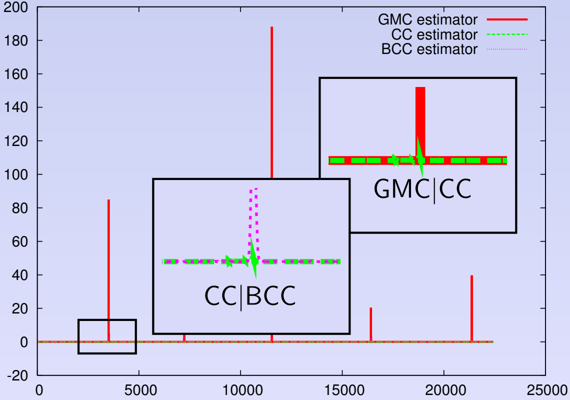


grid size=0.01

3. Experimental comparison of the three estimators

Comparisons on regular shapes : polygon

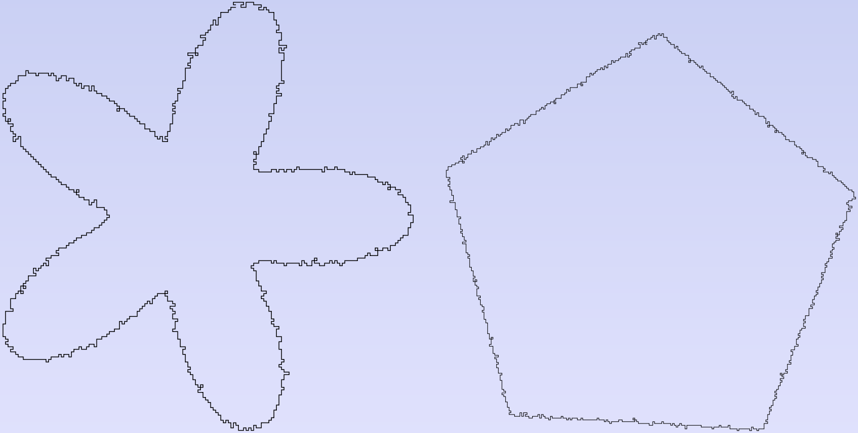




grid size=0.01

3. Experimental comparison of the three estimators

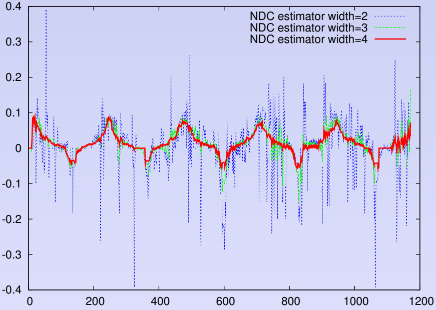
Comparisons on a noisy version of the flower and the polygon :



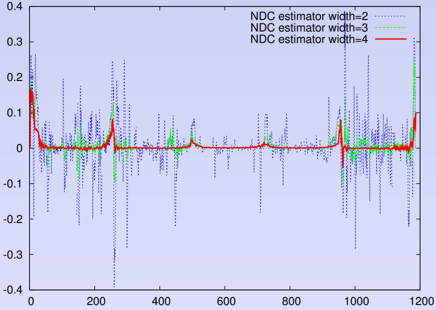
3. Experimental comparison of the three estimators

Comparisons on a noisy version of the flower and the polygon :

NDC on noisy flower



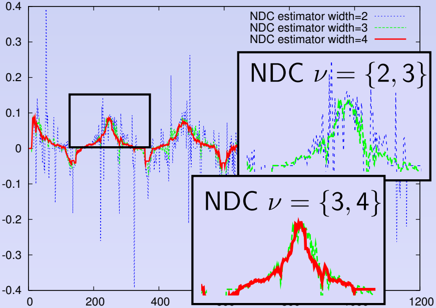
NDC on noisy polygon



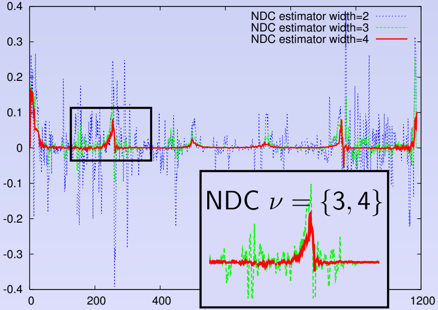
3. Experimental comparison of the three estimators

Comparisons on a noisy version of the flower and the polygon :

NDC on noisy flower

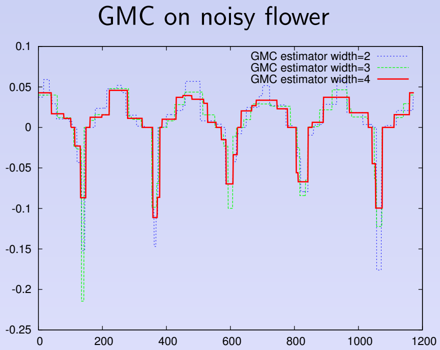


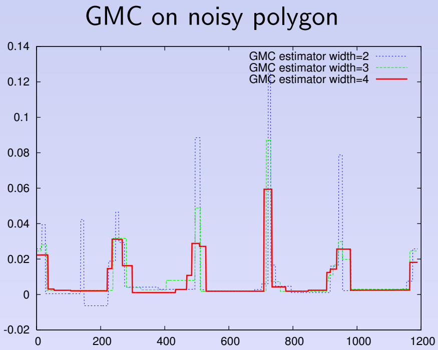
NDC on noisy polygon



3. Experimental comparison of the three estimators

Comparisons on a noisy version of the flower and the polygon :

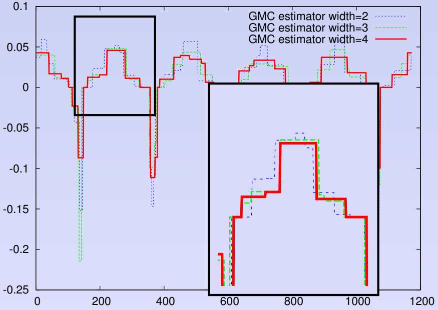




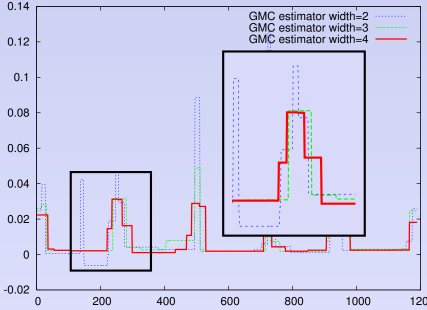
3. Experimental comparison of the three estimators

Comparisons on a noisy version of the flower and the polygon :

GMC on noisy flower



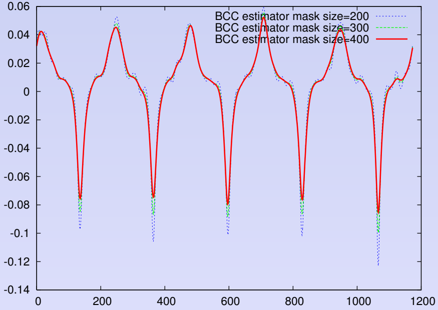
GMC on noisy polygon



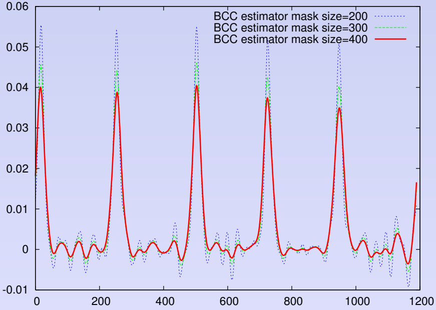
3. Experimental comparison of the three estimators

Comparisons on a noisy version of the flower and the polygon :

BCC on noisy flower



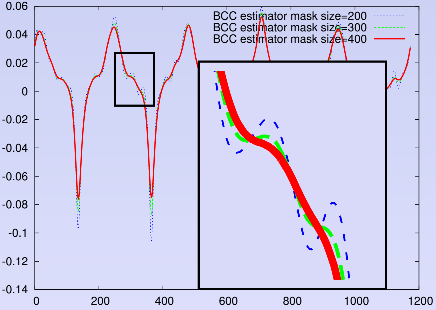
BCC on noisy polygon



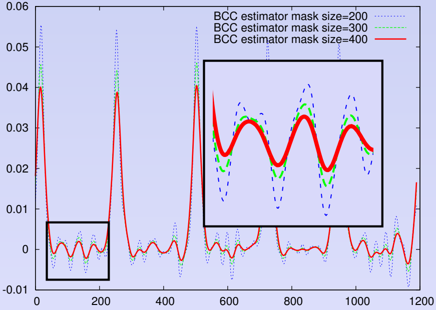
3. Experimental comparison of the three estimators

Comparisons on a noisy version of the flower and the polygon :

BCC on noisy flower



BCC on noisy polygon



4. Application to corner detection

Selection of the GMC estimator :

- Stability, no oscillations in the curvature.
- Easy to analyse.
- Only one parameter associated to the width of the blurred segment.

4. Application to corner detection

Selection of the GMC estimator :

- Stability, no oscillations in the curvature.
- Easy to analyse.
- Only one parameter associated to the width of the blurred segment.

Simple algorithm for corner detection :

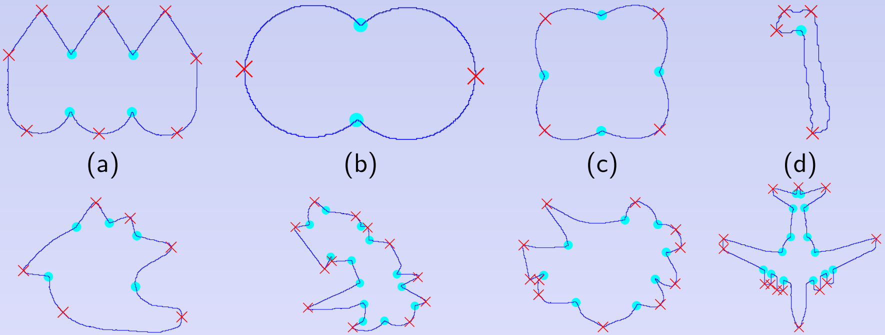
1. Compute $\kappa(p_i)$ for all contour points with GMC and width ν.
2. Detect all the maximal curvature regions defined by sets of consecutive points :

$$R_k = \{(p_i)_{i\in[a,b]} \mid \forall i, (\kappa(p_i) = \kappa(p_a)) \wedge (\kappa(p_{a-1}) < \kappa(p_a)) \wedge (\kappa(p_{b+1}) < \kappa(p_b)) \wedge (\kappa(p_a) > 0)\}$$

3. for each region R_k mark the point $p_{(a+b)/2}$ as a corner.

4. Application to corner detection

Result on known test shapes :



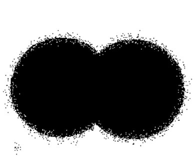
(a) (b) (c) (d)

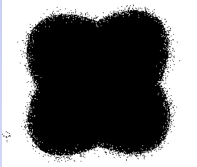
(e) (f) (g) (h)

4. Application to corner detection

Generation of noisy test shapes :


(a)


(b)


(c)


(d)


(e)

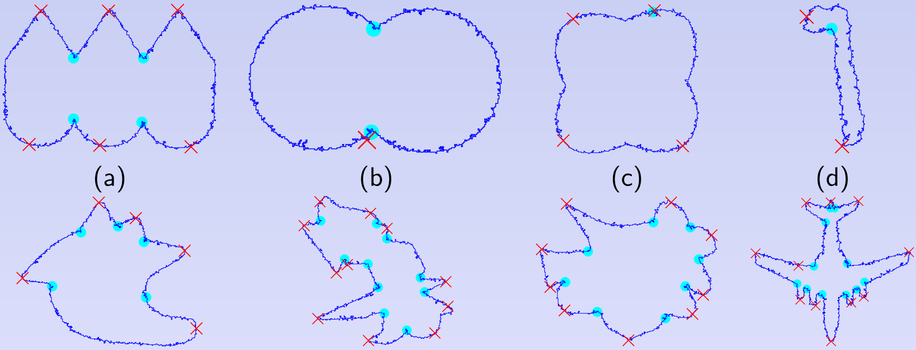

(f)


(g)


(h)

4. Application to corner detection

Result with GMC estimator and width $\nu = 4$:



(a) (b) (c) (d)

(e) (f) (g) (h)

4. Application to corner detection

Comparison with a recent morphological approach [Chang *et al.* 07]

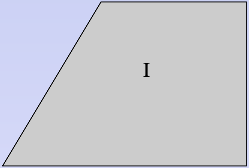
Principle of the BAP detector (Base Angle Point) :

- Application of the "top-hat" operator :

$$C(I) = I \backslash \gamma_{B_\lambda}(I),$$

where $\gamma_B(X) = \cup_i \{B_i \mid B_i \subseteq X\}$.

- Analyse of the residual areas.
- Filter to remove non significant residual areas.



4. Application to corner detection

Comparison with a recent morphological approach [Chang *et al.* 07]

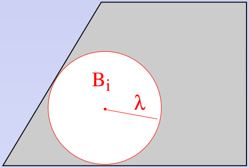
Principle of the BAP detector (Base Angle Point) :

- Application of the “top-hat” operator :

$$C(I) = I \backslash \gamma_{B_\lambda}(I),$$

where $\gamma_B(X) = \cup_i \{B_i \mid B_i \subseteq X\}$.

- Analyse of the residual areas.
- Filter to remove non significant residual areas.



4. Application to corner detection

Comparison with a recent morphological approach [Chang *et al.* 07]

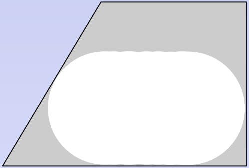
Principle of the BAP detector (Base Angle Point) :

- Application of the "top-hat" operator :

$$C(I) = I \backslash \gamma_{B_\lambda}(I),$$

where $\gamma_B(X) = \cup_i \{B_i \mid B_i \subseteq X\}$.

- Analyse of the residual areas.
- Filter to remove non significant residual areas.



4. Application to corner detection

Comparison with a recent morphological approach [Chang *et al.* 07]

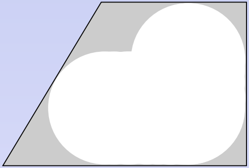
Principle of the BAP detector (Base Angle Point) :

- Application of the "top-hat" operator :

$$C(I) = I \backslash \gamma_{B_\lambda}(I),$$

where $\gamma_B(X) = \cup_i \{B_i \mid B_i \subseteq X\}$.

- Analyse of the residual areas.
- Filter to remove non significant residual areas.



4. Application to corner detection

Comparison with a recent morphological approach [Chang *et al.* 07]

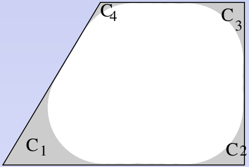
Principle of the BAP detector (Base Angle Point) :

- Application of the "top-hat" operator :

$$C(I) = I \backslash \gamma_{B_\lambda}(I),$$

where $\gamma_B(X) = \cup_i \{B_i \mid B_i \subseteq X\}$.

- Analyse of the residual areas.
- Filter to remove non significant residual areas.



4. Application to corner detection

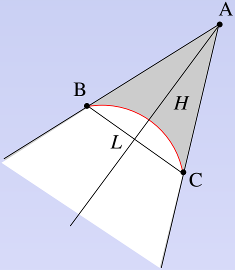
Comparison with a recent morphological approach [Chang *et al.* 07]

Principle of the BAP detector (Base Angle Point) :

- Application of the "top-hat" operator :

$$C(I) = I \backslash \gamma_{B_\lambda}(I),$$

  where $\gamma_B(X) = \cup_i \{B_i \mid B_i \subseteq X\}$.
- Analyse of the residual areas.
- Filter to remove non significant residual areas.



4. Application to corner detection

Comparison with a recent morphological approach [Chang *et al.* 07]

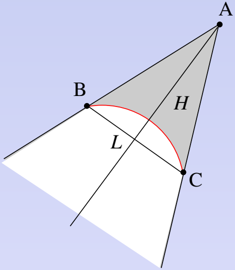
Principle of the BAP detector (Base Angle Point) :

- Application of the "top-hat" operator :

$$C(I) = I \backslash \gamma_{B_\lambda}(I),$$

where $\gamma_B(X) = \cup_i \{B_i \mid B_i \subseteq X\}$.

- Analyse of the residual areas.
- Filter to remove non significant residual areas.



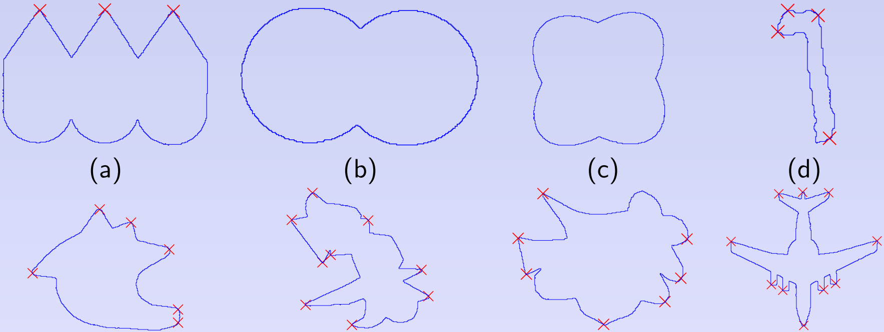
Main drawbacks :

- Noise sensibility.
- Three parameters (λ, L, H) not easy to define for noisy shapes.

4. Application to corner detection

Comparison with a recent morphological approach [Chang *et al.* 07]

Result obtained on normal test images with parameters : $(\lambda, L, H) = (12, 2, 1)$



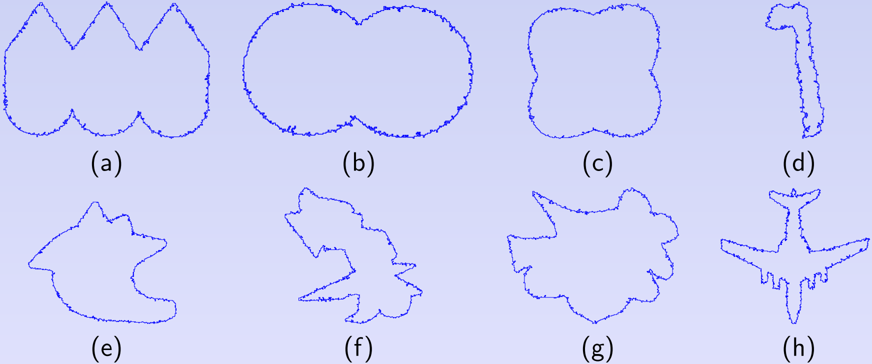
(a) (b) (c) (d)

(e) (f) (g) (h)

4. Application to corner detection

Comparison with a recent morphological approach [Chang *et al.* 07]

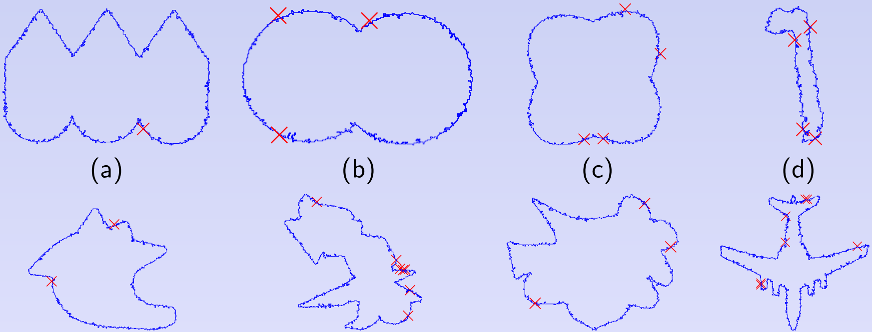
Result obtained on noisy test images with parameters : $(\lambda, L, H) = (12, 2, 1)$



4. Application to corner detection

Comparison with a recent morphological approach [Chang *et al.* 07]

Result obtained on noisy test images with parameters : $(\lambda, L, H) = (3, 1, 1)$

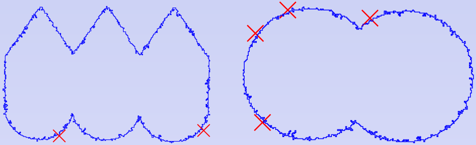


(a) (b) (c) (d)
(e) (f) (g) (h)

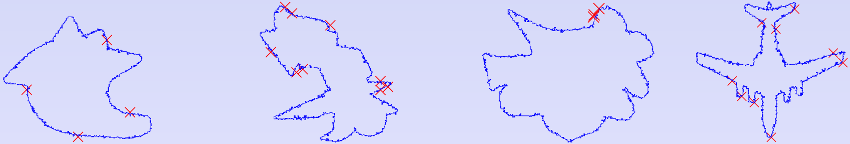
4. Application to corner detection

Comparison with a recent morphological approach [Chang *et al.* 07]

Result obtained on noisy test images with parameters : $(\lambda, L, H) = (2, 2, 1)$



(a) (b) (c) (d)



(e) (f) (g) (h)

5. Conclusion and future work

Conclusion

- Comparisons of tree new estimators adapted to noisy contours.
- The GMC estimator is the more stable.
- Application to a simple corner detector which resists to noise.

5. Conclusion and future work

Conclusion

- Comparisons of tree new estimators adapted to noisy contours.
- The GMC estimator is the more stable.
- Application to a simple corner detector which resists to noise.

Future work

- Comparison with another recent approach [Liu *et al.* 08].
- Application to “curvature based” polygonisation.
- Method for automatically determine the width of blurred segments.

Thanks you for your attention.

- [Kovalevsky 01] Kovalevsky, V.
  International Journal of Pattern Recognition and Artificial Intelligence 2001, 15, 1183-1200
- [Coeurjolly *et al.* 99] Coeurjolly, D., Miguet, S., Tougne, L.
  Discrete curvature based on osculating circle estimation.
  In proc. Int. workshop Visual Form, Volume 2059 of LNCS (Springer), 303–302 2001.
- [Nguyen, Debled *et al.* 07] Nguyen, T., Debled-Rennesson, I.
  Curvature estimation in noisy curves.
  In proc. of Int Conf CAIP, Volume 4673 of LNCS (Springer), 474-481 2007.
- [Kerautret, Lachaud 08] Kerautret, B., Lachaud, J.0-
  Robust estimation of curvature along digital contours with global optimization.
  In : Proc. of Int Conf DGCI, Volume 4992 of LNCS (Springer), 334-345 2008.
- [Malgouyres *et al.* 08] Malgouyres, R., Brunet, F., Fourey, S.
  Binomial convolutions and derivatives estimations from noisy discretizations.
  In : Proc. of Int Conf DGCI, Volume 4992 of LNCS (Springer), 370-379 2008.
- [Chang *et al.* 07] Chang, X., Gao, L., Li, Y.
  Corner detection based on morphological disk element.
  In : Proceedings of the 2007 American Control Conference, IEEE (2007) 1994-1999
- [Liu *et al.* 08] Liu, H., Latecki, L.J., Liu, W.
  A Unified Curvature Definition for Regular, Polygonal, and Digital Planar Curves
  Int J Comput Vis (2008) 80 : 104-124
- [Calabi *et al.* 98] Calabi, E., Olver, P. J., Shakiban, C., Tannenbaum, A., Haker, S.
  Differential and numerically invariant signature curves applied to object recognition.
  International Journal of Computer Vision, 26, 107-135