

# Unsupervised, Fast and Precise Recognition of Digital Arcs in Noisy Images

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# Outline

- 1 Introduction
- 2 Arc segmentation
- 3 Unsupervised Noise Detection
- 4 A framework for arc recognition along noisy curves
- 5 Experimentations
- 6 Conclusions and futur work

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# Introduction

## Motivation

- Arc and circle are basic objects in discrete geometry.
- ⇒ The study of these primitives are important.
- Arc and circle appear often also in images.
- Due to the effect of acquisition phase, there is often **noise** in images
- ⇒ **The detection, recognition of these primitives in noisy condition are interesting topic in pattern recognition.**

Reel image

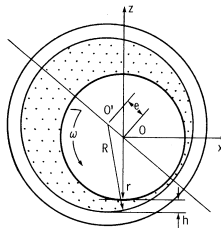


# Introduction

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## Document graphic



# Discrete circle

## Discrete circle

- Basic object in discrete geometry.
- Based on the discretization of a reel circle.

## Existing definitions

- Kim's definition
- Nakamura's definition
- Andres' definition



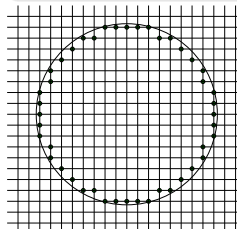
C. E. Kim.

Digital disks.

*Pattern Analysis and Machine Intelligence, IEEE Transactions on, PAMI-6(3) :372–374, May 1984.*

## Définition

A discrete circle ([Kim84]) is constructed from digital points that are the most nearest and interior in a discrete circle.



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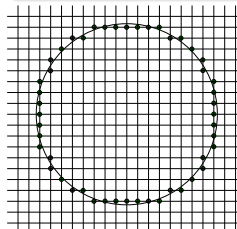
A. Nakamura and K. Aizawa.

Digital circles.

*Computer Vision, Graphics, and Image Processing*, 26(2) :242–255, 1984.

## Définition

A discrete circle ([Nakamura84]) is a sequenque of digital points that are nearest a discrete circle.



# Discrete circle

## Discrete circle

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E. Andres.

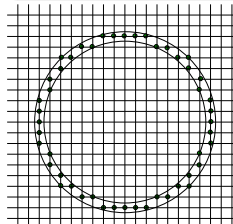
Discrete circles, rings and spheres.

*Computers & Graphics*, 18(5) :695–706, 1994.

## Définition

A digital circle ([Andres95]) is a sequence of digital points that verifies :

$$(R - \frac{w}{2})^2 \leq (x - x_0)^2 + (y - y_0)^2 < (R + \frac{w}{2})^2 \}$$





# Outline

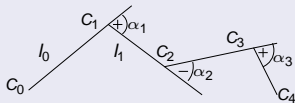
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## Tangent space representation [Arkin91], [Latecki00]

### Input

$C = \{C_i\}_{i=0}^n$  is a polygonal curve with

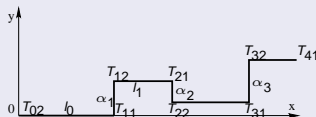
- $\alpha_i = \angle(\overrightarrow{C_{i-1}C_i}, \overrightarrow{C_iC_{i+1}})$
- $l_i$  is the length of segment  $C_iC_{i+1}$ .
- $\alpha_i > 0$  if  $C_{i+1}$  is at the right side of  $\overrightarrow{C_{i-1}C_i}$ ,  $\alpha_i < 0$  otherwise.



### Output

We consider the transform that associates polygon  $C$  of  $\mathbb{Z}^2$  to a polygon of  $\mathbb{R}^2$  constituted by the segments  $T_{i2}T_{(i+1)1}$ ,  $T_{(i+1)1}T_{(i+1)2}$  for  $i$  from 0 to  $n-1$  with

- $T_{02} = (0, 0)$
- $T_{i1} = (T_{(i-1)2} \cdot x + l_{i-1}, T_{(i-1)2} \cdot y)$ , pour  $i$  de 1  $\tilde{\sim}$   $n$ ,
- $T_{i2} = (T_{i1} \cdot x, T_{i1} \cdot y + \alpha_i)$ , pour  $i$  de 1  $\tilde{\sim}$   $n-1$ .



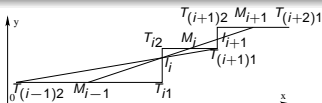
## Property of an arc in tangent space

### Principal result

Suppose that

- $C = \{C_i\}_{i=0}^n$  is a polygon with  $\alpha_i = \angle(\overrightarrow{C_{i-1}C_i}, \overrightarrow{C_iC_{i+1}})$  such that  $\sin \alpha_i \simeq \alpha_i$  for  $i \in \{1, \dots, n-1\}$
- $T(C)$  its representation in the tangent space, constituted by segments  $T_{i2}T_{(i+1)1}, T_{(i+1)1}T_{(i+1)2}$  for  $i$  from 0 to  $n-1$
- $\{M_i\}_{i=0}^{n-1}$  is a set of central point of  $\{T_{i2}T_{(i+1)1}\}_{i=1}^{n-1}$ .

Therefore,  $C$  is a polygon that approximates an arc of circle if and only if  $\{M_i\}_{i=0}^{n-1}$  is a set of collinear points.



## Consequence

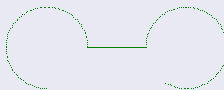
### Interest of this result

Reconnaissance  
de cercle (arc)

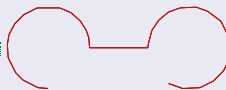


Reconnaissance  
de droite

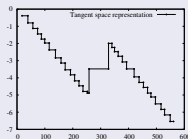
### Example



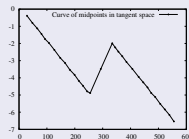
(a) Entry curve



(b) Approximated polygon



(c) Tangent space representation



(d) Central curve in the tangent space

# Arc recognition and segmentation of a digital curve into arcs

## Recognition of digital arc

- 1 Polygonalize the input curve
- 2 Transform this polygon to tangent space
- 3 Construct the middle curve in this tangent space
- 4 Verify the collinearity of points in this curve
  - A parameter to control the approximation error

## Complexity

- Use [Debled et al. 06] for pour accomplishing steps 1 and 4 in linear time
  - Step 2 and 3 are done in linear time
- ⇒ The proposed method is linear



Thanh Phuong Nguyen et Isabelle Debled-Rennesson :  
Segmentation en arcs discrets en temps linéaire.  
*In RFA, 2010.*

# Arc recognition and segmentation of a digital curve into arcs

## Segmentation of a curve into arcs

- 1 Polygonalize the input curve
- 2 Transform this polygon to tangent space
- 3 Construct the middle curve in this tangent space
- 4 Polygonalize the middle curve in the tangent space
  - Utilize parameter  $\alpha$  to verify detected arcs

## Complexity

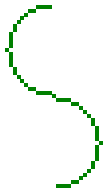
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# Experimentation on arc detection



## Experimentation

- Input curve
- Polygonalization
- Representation in tangent space
- Middle curve in the tangent space
- Detect arcs by using blurred segment to verify the collinearity of the middle curve

# Experimentation on arc detection

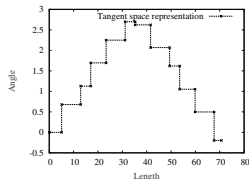


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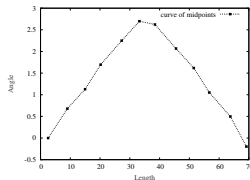
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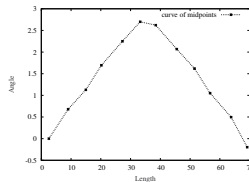
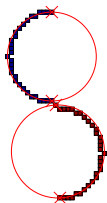
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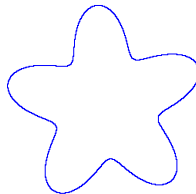
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## Automatic detection of significant scales [KerLach09]

### Principal idea

- 1 Exploit the asymptotic properties of perfect shape discretization.
- 2 Estimate these properties from multiscale representation.
- 3 Compare them to determine significant scale.

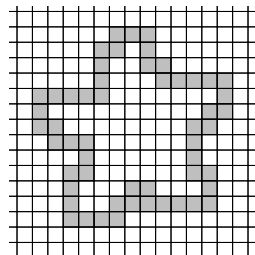


X

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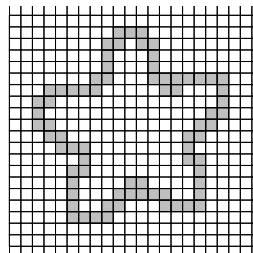


$Dig_{20}(X)$

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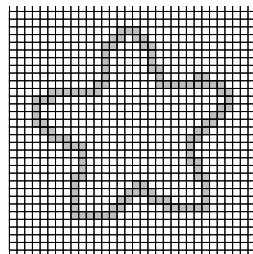


$Dig_{15}(X)$

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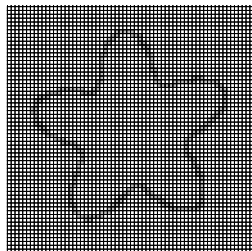
$Dig_{10}(X)$



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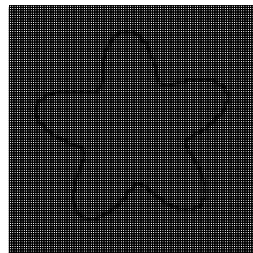


$Dig_5(X)$

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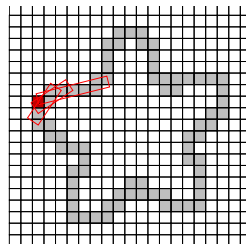


$Dig_3(X)$

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- 3 Compare them to determine significant scale.



$Dig_{15}(X)$

### Asymptotic properties of the length of **maximal segments** :

- Standard discrete line (discretizations 4-connexe)
- Segment of discrete line (SDL), a part of connected discrete line.
- Maximal segment of a contour  $C$  : SDL of  $C$  inextended neither to right side nor to left side.

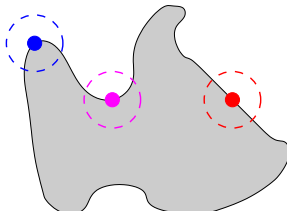
## Asymtotic results of maximal segments

### Theorem [Lachaud 06] : asymtotic behavior of length of maximal segments

- $X$  simple connected shape in  $R^2$  with the boundary  $\partial X$  with a piecewise boundary  $C^3$ ,
- $U$  an open connected neighborhood of  $p \in \partial X$ ,
- $(L_j^h)$  the digital lengths of the maximal segments covering  $p$  along the boundary of  $Dig_h(X)$ ,

$$\text{if } U \text{ is strictly convex or concave, then } \Omega(1/h^{1/3}) \leq L_j^h \leq O(1/h^{1/2}) \quad (1)$$

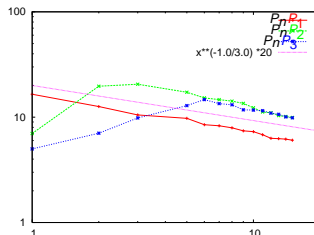
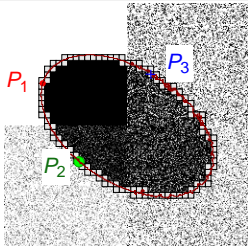
$$\text{if } U \text{ has null curvature everywhere, then } \Omega(1/h) \leq L_j^h \leq O(1/h) \quad (2)$$



## Multiscale profile

### Multiscale profile of a point $P$ on a discrete contour

- Multiscale profile** :  $P_n P = \text{sequence } (\log i, \log(E(L^{h_i})))_{i=1..n}$ ,  
 with  $E$  mean operator,  $L^{h_i}$  are the digital lengths of of the maximal segments  
 covering  $P$  sont les longueurs discrètes des segments for all of subsampling  $i \times i$ .



## Meaningful scales and noise detection

### Meaningful scales

A **meaningful scale** of a multiscale profile  $(X_i, Y_i)_{1 \leq i \leq n}$  is then a pair  $(i_1, i_2)$   $1 \leq i_1 \leq i_2 \leq n$ , such that for all  $i$ ,  $i_1 \leq i < i_2$ ,

$$\frac{Y_{i+1} - Y_i}{X_{i+1} - X_i} \leq t_m,$$

and not true for  $i_1 - 1$  et  $i_2$ .

Parameter  $t_m$  = threshold of noise level for separate noisy/non-noisy zones.

## Meaningful scales and noise detection

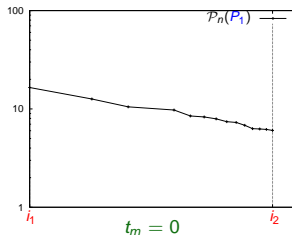
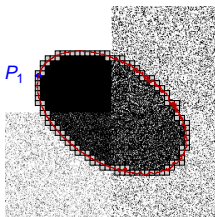
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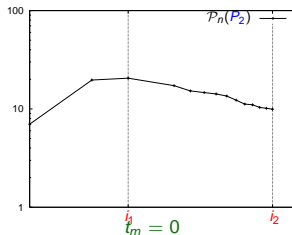
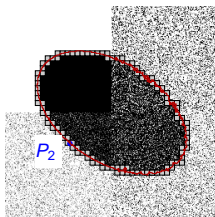
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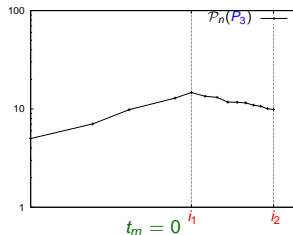
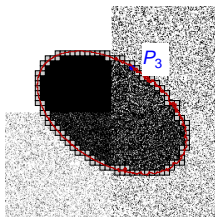
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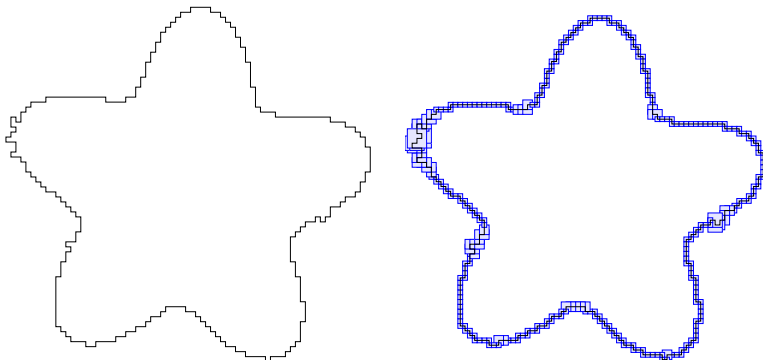
Parameter  $t_m$  = threshold of noise level for separate noisy/non-noisy zones.

### Noise level at a point $P$

If  $(i_1, i_2)$  is the first meaningful scale of point  $P$ , the **noise level** at  $P$  is  $i_1 - 1$ .

## Experimentations on noise detection

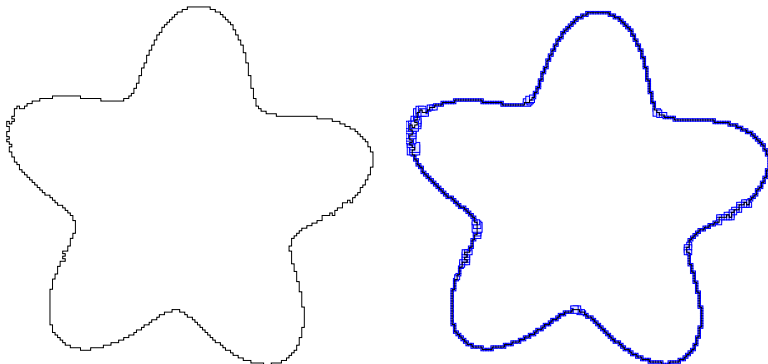
Flower with local noise insertion



Local noise at résolution  $R_0$

## Experimentations on noise detection

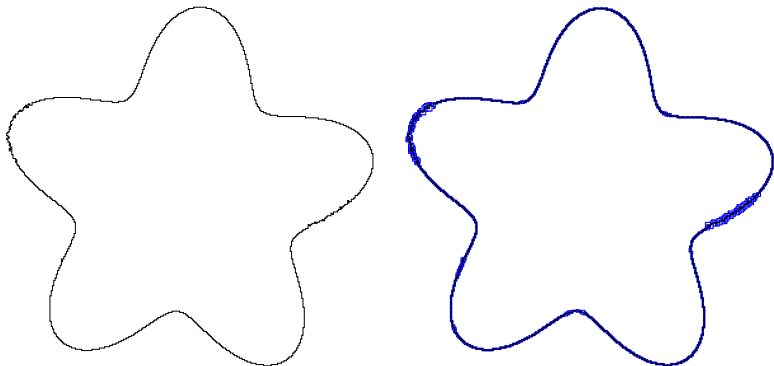
Flower with local noise insertion



Local noise at résolution  $R1$

## Experimentations on noise detection

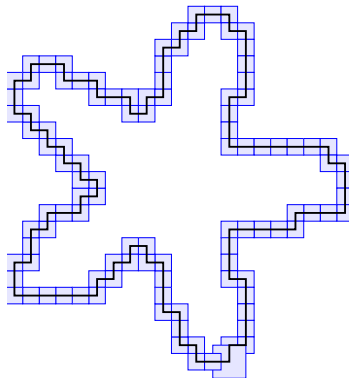
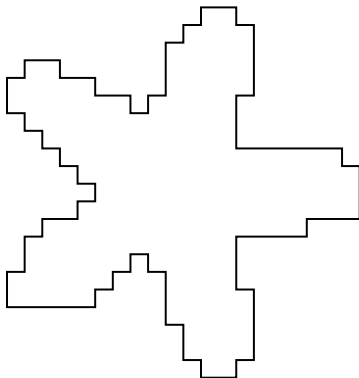
Flower with local noise insertion



Local noise at résolution  $R2$

## Experimentations on noise detection

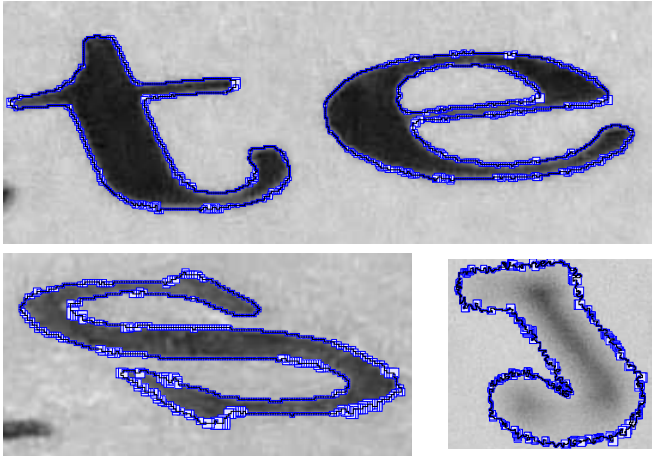
Flower at low resolution without noise



## Noise detection on real images



## Noise detection on real images





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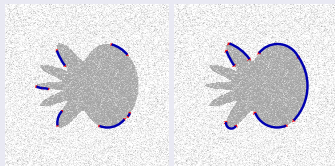
## Problem of arc detection on noisy curves

### Remarks concerning the arc detection algorithm

- A parameter  $\nu_1$  to take into account the amount of noise in the polygonalization step
  - This parameter is adjusted manually
- ⇒ For each noisy curve, how can we choose the value of  $\nu_1$  to obtain the best result ?

### Our proposed solution

- Use [KerautretLauchaud09] to determine noise level of the noisy curve
- Construct approximated polygon based on this noise information



(e) Width=2

(f) Width=3

# Polygonalization adapted to noisy curves

## Proposed solution

- Two solutions for taking into account the noise of discrete contour
- The first one considers the hypothesis of uniform distribution
- The second one considers the hypothesis of non-uniform distribution

**Algorithm 1** : Polygonalization based on unsupervised noise detection.

**Data** :  $C = \{C_i\}_{i=0}^n$  digital curve,  $\nu = \{\nu_i\}_{i=0}^n$  noise information, *uniformNoise*- *true* if uniform noise distribution, *false* otherwise

**Result** : P-approximated polygon

**begin**

$b \leftarrow 0$ ; Add  $C_b$  to  $P$ ;

**if** *!uniformNoise* **then**

**while**  $b < n$  **do**

      Use [DEB05] to recognize  $\{C_b, \dots, C_e\}$  as blurred segment of width  $\nu_b$ ;

      Add  $C_b$  to  $P$ ;  $b \leftarrow e$ ;

**else**

$\bar{\nu} \leftarrow$  mean value of  $\nu = \{\nu_i\}_{i=0}^n$ ;

**while**  $b < n$  **do**

      Use [DEB05] to recognize  $\{C_b, \dots, C_e\}$  as blurred segment of width  $\bar{\nu}$ ;

      Add  $C_b$  to  $P$ ;  $b \leftarrow e$ ;

**end**

## Arc recognition along noisy curves

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### Algorithm 2 : Arc segmentation along a noisy digital curve

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**Data :**  $C = \{C_i\}_{i=0}^n$  noisy digital curve

**Result :** ARC- sequence of extracted arcs

**begin**

$N \leftarrow \{N_i\}_{i=0}^n$  noise information determined by [1] (see Section ??);

$ARC \leftarrow \emptyset$ ; Use Algorithm 1 to polygonalize  $C$  in  $P = \{P\}_{i=0}^m$ ;

Represent  $P$  in the tangent space by  $T(P)$  (see Section ??);

Determine the midpoint set  $MpC = \{M_i\}_{i=1}^n$  (see Section ??);

Use [DEB05] to polygonalize  $MpC$  into a sequence  $S = \{S\}_{i=0}^k$  of blurred segments of width 0.25;

**for**  $i$  **from** 0 **to**  $k - 1$  **do**

$\{M_j\}_{j=b}^e$  : sequence of points of  $MpC$  that corresponds to  $S_i S_{i+1}$ ;

$C'$  : part of  $C$  that corresponds to  $S_i S_{i+1}$ ;

$isArc \leftarrow true$ ;

**for**  $i$  **from**  $b$  **to**  $e - 1$  **do**

**if**  $M_{i+1} \cdot y - M_i \cdot y > \frac{\pi}{4}$  **then**  $isArc \leftarrow false$

**if**  $isArc$  **then** Add  $C'$  to  $ARC$

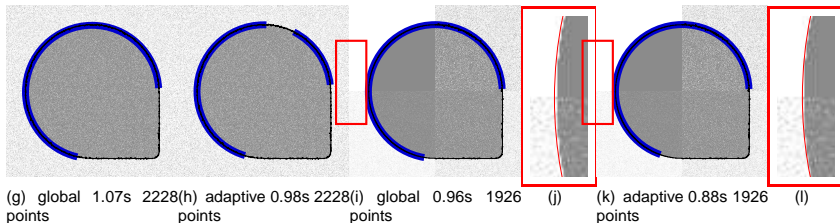
**end**

---

# Outline

- 1 Introduction
- 2 Arc segmentation
- 3 Unsupervised Noise Detection
- 4 A framework for arc recognition along noisy curves
- 5 Experimentations**
- 6 Conclusions and futur work

## Experimentation



**FIGURE:** Arcs detection from the global noise based approach (a,d) and the adaptive approach (b,e) (image size 512x512). (d,f) close-up view of (c,e).

## Experimentation



(a) source image                      (b) contours                      (c) result (uniform noise)  
**FIGURE:** Arc detection with our method on an image of a car (size 4000x2672 pixels).

## Experimentation



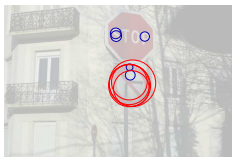
(a) Source



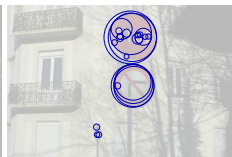
(b) Global : 6.57s, 12126 pts



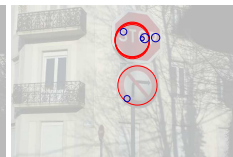
(c) Adaptif : 6s, 12126 pts



(d) CHT  
(1024x684)



(Kimme1975) (e) MHT (Rad2003) (1024x684)



(f) FHT (Dav1984) (4000x2672)

**FIGURE:** Application of our method on a real picture (size 4000x2672 pixels) with the possible values for *uniformNoise* (a-c), and comparison with three methods based on Hough transform (d-f).  
The corresponding parametres : (d) -  $\mu_C = (70, 2, 25)$  1m19s  $\mu_C = (5, 1, 20)$  1m0s, (e)-  
 $\mu_M = (10, 190)$  2.0s, (f)-  $\mu_F = (200, 330, 100)$  1m27s  $\mu_F = (170, 50, 100)$  4m26s



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# Conclusions

## Conclusions

- A new approach for arc segmentation of digital curves in noisy images
  - Combination between arc detection method and an unsupervised noise detector
- ⇒ an efficient arc detector in images. Our method
- is better than methods based on the Hough transform which require both large memory and execution time
  - is not dependent to the need to set a specific parameter

## Futur work

We plan to integrate the detection of curved zone in noisy curves of [Kerautret-Lachaud09] as a preprocessing step to enhance the robustness of the arc detector.

## References

-  [Kimme1975 ] Carolyn Kimme and Dana Ballard and Jack Sklansky  
Finding Circles by an Array of Accumulators  
Short Communications Graphics and Image Processing **18** (1975) 120–122
-  [Rad03 ] A. A. Rad and K. Faez and N. Qaragozlou  
Fast circle detection using gradient pair vectors  
Digital Image Comp. : Techniques and Applications (2003), 879–887
-  [E.R Davies84] E.R Davies  
A modified Hough scheme for general circle location  
Pattern Recognition Letters **(7)** (1984), 37–43
-  [Debled06 ] Debled-Rennesson, I. ; Feschet, F. ; Rouyer-Degli  
Optimal Blurred Segments Decomposition of Noisy Shapes in Linear Times  
Comp. & Graphics **30** (2006) 30–36
-  [NguyenDebled10] T. P. Nguyen et I. Debled-Rennesson  
A linear method for segmentation of digital arcs  
Technical report. 2010.  
[http://www.loria.fr/~nguyentp/pubs/techreport\\_arcsegmentation.pdf](http://www.loria.fr/~nguyentp/pubs/techreport_arcsegmentation.pdf)
-  [KerLach09] Kerautret, B. ; Lachaud, J.-O.  
Multi-scale Analysis of Discrete Contours for Unsupervised Noise Detection.