FINN 6216 Quantitative Risk Management

Trading Volatility

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Volatility

- Volatility is a measure of market risk

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{1}$$

- Volatility is Unobservable.
- Thus we need to estimate it.
  - Sample moment: use the historical data (method of moments)
  - Statistical approach: ARCH, GARCH, EGARCH
  - Forward approach: Can we use options to backout volatility?

Realized Volatility & Implied Volatility

- Realized Volatility

$$RV_t = \sqrt{\frac{1}{T-1}\sum_{k=0}^{T-1} R_{t-k}^2}, \quad \text{where} \quad R_t = \log(S_t) - \log(S_{t-1}) \tag{2}$$

- Implied Volatility:
    - Inferred by the market price of options traded in a liquid option market
    - A forward looking perspective
    - Offers crucial information about the market's expectation of future volatility
- The difference between the above two is sometimes called "Volatility Risk Premium"

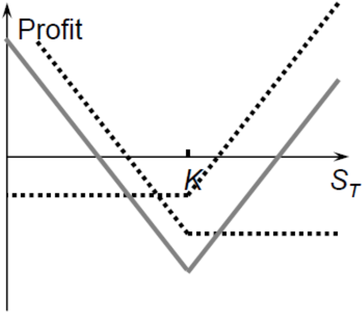
Trading Volatility

- Volatility is increasingly becoming an asset class in itself, for indices as well as for single stocks.
- Construct Equity/Index options strategy to bet against future volatility
- Trade derivatives (futures or options) on volatility, either in Exchange or OTC

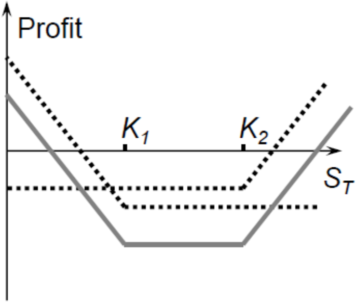
Straddle & Strangle

- A straddle is constructed by a long position in a call and a put of the same strike and maturity.
- This strategy pays off only if the stock moves far enough away from its current value, no matter in which direction, hence a bet on the volatility
- A straddle may be quite costly
- A strangle is less costly. The difference in a strangle is that we employ two strike prices, K1 and K2

Straddle & Strangle



(a) A Straddle

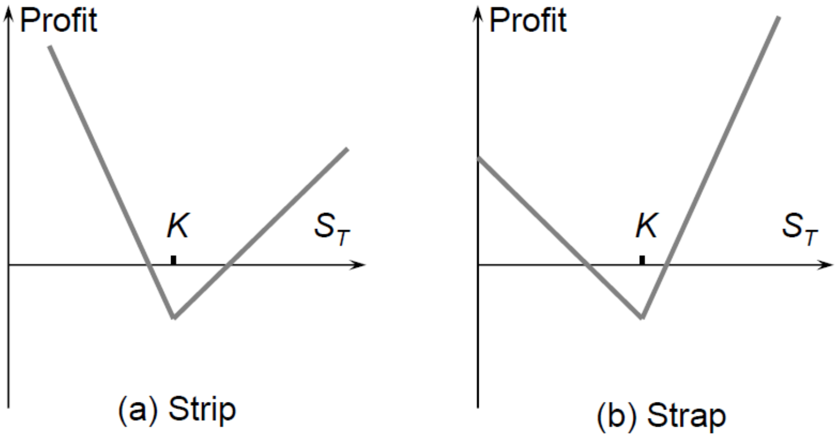


(b) A Strangle

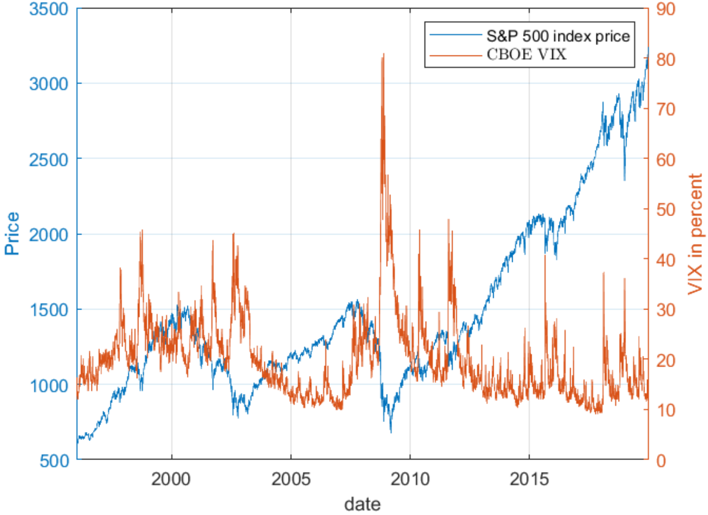
Strip & Strap

- The investors may have different reaction to bad or good news
- A strip has a bias to a higher likelihood of a downward move
- A strap has a bias to a higher likelihood for the stock to move up

Strip & Strap



VIX Index



S&P 500 index price
CBOE VIX
Price
VIX in percent
date
3500
3000
2500
2000
1500
1000
500
90
80
70
60
50
40
30
20
10
0
2000
2005
2010
2015

VIX Index

- VIX was introduced initially in 1993 to track the BSM implied volatility of options on S&P 100 with near-the-money strikes.
- On September 22, 2003, the CBOE uses the more actively traded S&P 500 index options to compute VIX
- The old index is kept and was renamed as VXO
- A "fear" index? – High VIX or Low VIX, when should be you worried about?

VIX Index

- Let $R_{f,t\to t+T}$ denote the gross risk-free return over $[t, t+T]$
- VIX index can be computed from ($T = 30$ days)

$$VIX^2_{t\to t+T} = \frac{2R_{f,t\to t+T}}{T}\left\{\int_0^{F_{t\to t+T}} \frac{1}{K^2} Put_{t,t+T}(K)dK + \int_{F_{t\to t+T}}^{\infty} \frac{1}{K^2} Call_{t,t+T}(K)dK\right\} \quad (3)$$

- VIX is a weighted sum of market prices for a range of options on the S&P 500 stock index.

Technical Issues Behind VIX

- Given a C^2 function $\phi(x)$ and x_0 is one in the defined region, then

$$\phi(x) = \phi(x_0) + (x - x_0)\phi'(x_0) + \int_{x_0}^{\infty} \phi''(K)(x - K)^+ dK + \int_0^{x_0} \phi''(K)(K - x)^+ dK. \tag{4}$$

- Replace x by S_{t+T}, the stock price at time $t + T$,
- Replace x_0 by $F_{t \to t+T}$, the futures price that matures at time $t + T$,

Carr-Madan Formula

$$\begin{aligned}
\phi(S_{t+T}) &= \phi(F_{t\to t+T}) \text{ (a bond position)} \\
&+ \phi'(F_{t\to t+T})(S_{t+T} - F_{t\to t+T}) \text{ (a stock position)} \\
&+ \int_{F_{t\to t+T}}^{\infty} \phi''(K)(S_{t+T} - K)^{+} dK \text{ (a sequence of call options)} \\
&+ \int_{0}^{F_{t\to t+T}} \phi''(K)(K - S_{t+T})^{+} dK \text{ (a sequence of put options)}
\end{aligned} \tag{5}$$

Carr-Madan Formula

- The formula allows you to replicate any European option with payoff function $\phi(.)$ using the plain-vanilla Call and Put options for all strikes
  - For instance, this formula is used in the valuation of a variance swap
  - Also, an approximation for VIX construction

VIX Term Structure

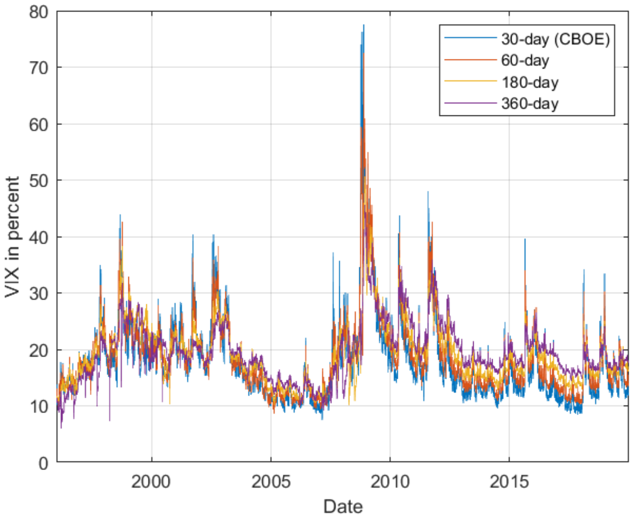
- VIX index can be computed from

$$VIX^2_{t\to t+T} = \frac{2R_{f,t\to t+T}}{T}\left\{\int_0^{F_{t\to t+T}} \frac{1}{K^2} Put_{t,t+T}(K)dK + \int_{F_{t\to t+T}}^{\infty} \frac{1}{K^2} Call_{t,t+T}(K)dK\right\}$$

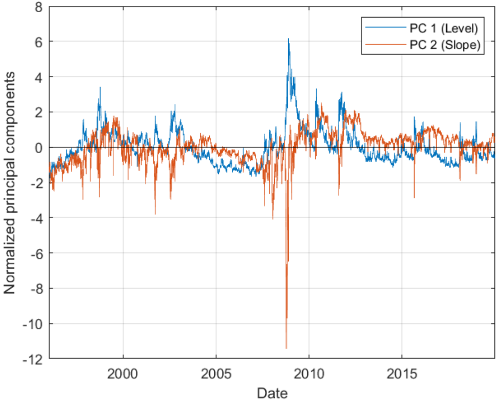
- We can choose different T and compute VIX over the future 30, 60, 90, 120, 180, 360 days

- This gives us a term structure of VIX index.

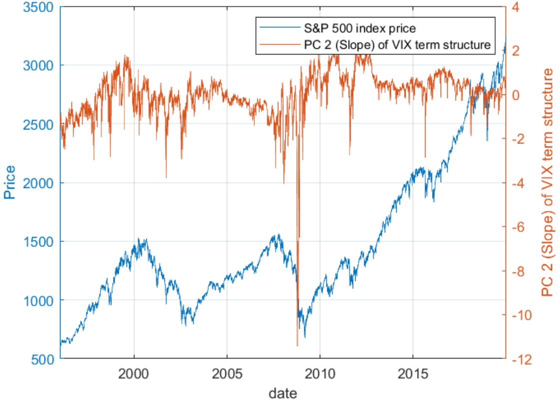
VIX Term Structure



VIX Index, Term Structure, and S&P 500 Index



(a) 1st & 2nd Principal components

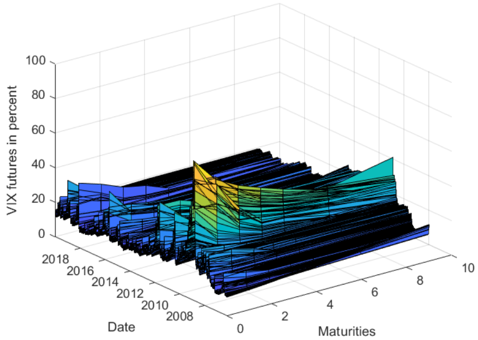


(b) VIX PC 2 & S&P 500 index

VIX Futures

- VIX futures contract were first introduced on March 26, 2004 by CBOE Futures Exchange (CFE).
- In the early days, 4 futures contracts were listed every day. After October 2006, 9 contracts are listed every day.
- Initially, VIX futures prices were quoted as the VIX times ten and the contract multiplier was $100.
- Starting on March 26, 2007, CFE modified the contract specification by dividing futures prices by ten and increasing the contract multiplier to $1000.

VIX Futures

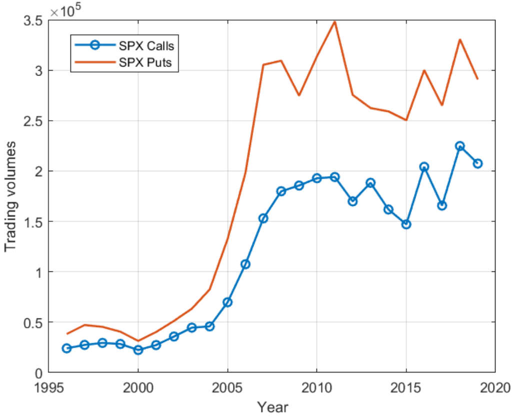


- In normal times, VIX futures prices tend to have an upward-sloping term structure, suggesting a premium paid by long-time investors.
- The upward term structure of VIX futures prices to maturity is known as "contango trap". Thus, rolling a futures contract is associated with substantial losses
- During market turbulence, i.e., the financial crisis in 2008/09, the futures curve tends to become inverted or hump-shaped.

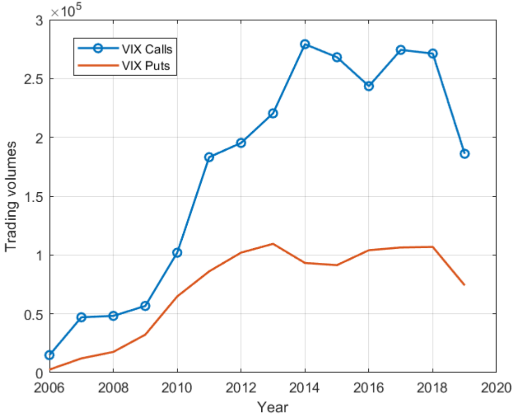
VIX Options

- CBOE launched European options on the VIX index on February 24, 2006.
- Like VIX futures,they are cash settled according to the difference between the value of the VIX at expiration and their strike price
- VIX derivative positions can be used to hedge the risks of investments in the S&P 500 index.
  - By holding VIX derivatives, investors can achieve exposure to S&P 500 volatility without having to delta-hedge the option positions with index itself.
  - It is cheaper to buy OTM VIX call options than to buy OTM index put options.
  - VIX options are the ONLY asset in which open interest is highest for OTM call strikes
- The VIX call option market is much larger than the VIX put option market in terms of volume, open interest, and the number of quotes.

Option Volumes (Jacobs and Mai, 2020)

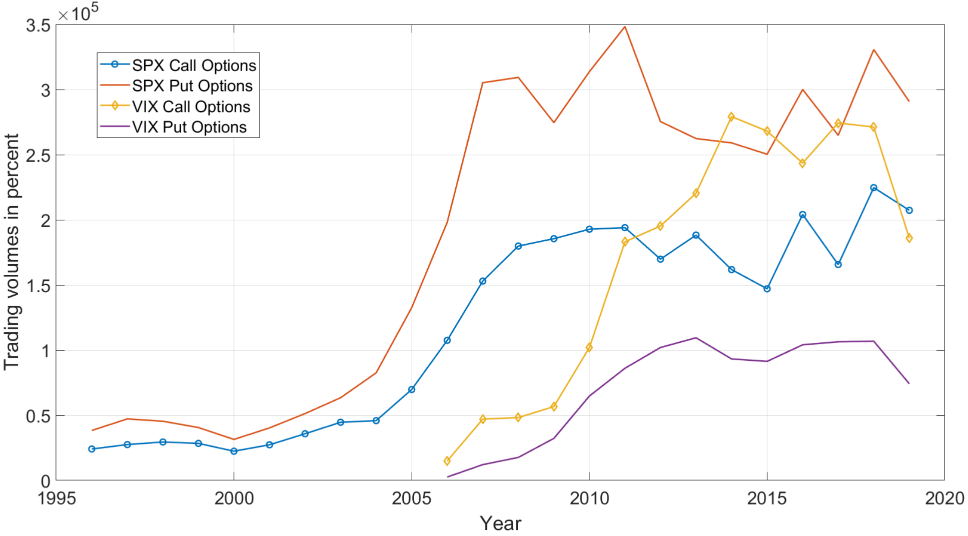


(c) SPX Index Options



(d) VIX Options

Option Volumes (Jacobs and Mai, 2020)



Volatility Derivatives Pricing

- Volatility derivatives is different from equity (stock) derivatives
    1. Volatility is not "tradable".
    2. Volatility is generally believed to be both stochastic and mean-reverting
- Researchers have proposed many advanced techniques in pricing options on derivatives.
- Grünbichler and Longstaff (1996) presents a simple closed-form solution for volatility option prices that is similar to BSM equity option pricing formula
- You can find the paper here

Underlying Process

- Let V denote the current value of the standard deviation of a stock index return
- The dynamics of V is assumed to follow a Feller process ("CIR" process)

$$dV_t = (\alpha - \kappa V_t)dt + \sigma\sqrt{V_t}dZ, \tag{6}$$

  - where α, κ, and σ are constants.
  - α/κ is thus the long-term mean level of volatility
- A standard CIR process specifies that,

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}dW.$$

Risk Neutral Process

- Consider a GBM for stock index dynamism in real-world and risk-neutral measures

$$dS_t = \alpha S_t dt + \sigma S_t dZ_t \quad \Longrightarrow \quad dS_t = r S_t dt + \sigma S_t dZ_t \tag{7}$$

- In comparison, the risk-neutral process for Feller process in Equation (6)

$$dV_t = (\alpha - \beta V_t)dt + \sigma\sqrt{V_t}dZ_t, \quad \text{where} \quad \beta = \kappa + \xi \tag{8}$$

- The underlying in the volatility case is not tradable.

- We have to adjust for the market price of risk.

Volatility Futures

- Let $F(V,T)$ denote the futures price for a futures contract on V with maturity T.

$$F(V,T) = \mathbb{E}^Q[V_T] \tag{9}$$

- Evaluating this expectation gives the following expression for the volatility futures price

$$F(V,T) = \frac{\alpha}{\beta}\left[1 - exp(-\beta T)\right] + exp(-\beta T)V_0 \tag{10}$$

  where $\beta = \gamma + \xi$, is from the risk-adjusted (risk-neutral) process

- The futures prices are exponentially weighted averages of the current value of V_0 and the long-run mean $\frac{\alpha}{\beta}$ of the risk-neutral process

Volatility Options

- Let $C(V, K, T)$ denote the current value of a call option on V, where K is the strike price of the option and T is the time until expiration.

$$C(V, K, T) = D(T)\mathbb{E}^{Q}\left[\max(0, V_T - K)\right] \tag{11}$$

where $D(T)$ is the discounting process.

- Evaluating this expectation gives the following closed-form expression

$$\begin{aligned} C(V, K, T) &= D(T)exp(-\beta T)V_0 Q(\gamma\kappa|\nu + 4, \lambda) \\ &+ D(T)\frac{\alpha}{\beta}\left[1 - exp(-\beta T)\right] Q(\gamma\kappa|\nu + 2, \lambda) - D(T)KQ(\gamma\kappa|\nu, \lambda) \end{aligned}$$

where $Q(\gamma\kappa|\nu, \lambda) = 1 - N(d)$.

Monte Carlo Simulation

- Use Monte Carlo simulation to evaluate the $\mathbb{E}^Q\left[\max(0, V_T - K)\right]$ or $\mathbb{E}^Q\left[V_T\right]$

$$dV_t = (\alpha - \beta V_t)dt + \sigma\sqrt{V_t}dZ_t,$$

- Consider the S&P 500 index, use VIX historical data to calibrate the above process.
- One numerical issue is that the discretization of Feller process may lead to negative value of V during simulation
  - Full truncation: replace negative value with zero
  - Reflection: replace negative value with the absolute value.

Questions?

- Email: ylu28@uncc.edu
- Webpage: JacquesYL.github.io

References I

Andreas Grünbichler and Francis A Longstaff. Valuing futures and options on volatility. *Journal of Banking & Finance*, 20 (6):985–1001, 1996.

Kris Jacobs and Anh Thu Mai. The role of intermediaries in derivatives markets: Evidence from vix options. *Available at SSRN 3635087*, 2020.