

# FINN 6216 Quantitative Risk Management

## Trading Volatility

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March 25, 2021

*The slides and codes can be accessed on my website: [@JacquesYL](#)*

# Volatility

- Volatility is a measure of market risk

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (1)$$

- Volatility is **Unobservable**.
- Thus we need to estimate it.
  - Sample moment: use the historical data (method of moments)
  - Statistical approach: ARCH, GARCH, EGARCH
  - Forward approach: Can we use options to backout volatility?

# Realized Volatility & Implied Volatility

- Realized Volatility

$$RV_t = \sqrt{\frac{1}{T-1} \sum_{k=0}^{T-1} R_{t-k}^2}, \quad \text{where} \quad R_t = \log(S_t) - \log(S_{t-1}) \quad (2)$$

- Implied Volatility:
  - Inferred by the market price of options traded in a liquid option market
  - A forward looking perspective
  - Offers crucial information about the market's expectation of future volatility
- The difference between the above two is sometimes called “Volatility Risk Premium”

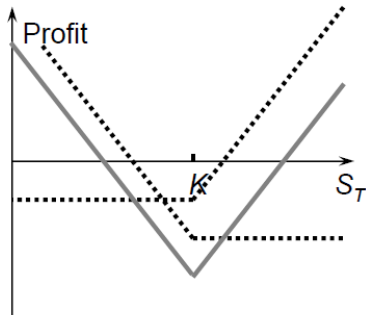
# Trading Volatility

- Volatility is increasingly becoming an asset class in itself, for indices as well as for single stocks.
- Construct Equity/Index options strategy to bet against future volatility
- Trade derivatives (futures or options) on volatility, either in Exchange or OTC

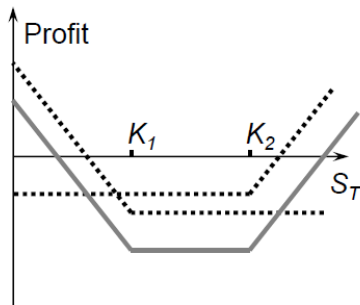
## Straddle & Strangle

- A straddle is constructed by a long position in a call and a put of the same strike and maturity.
- This strategy pays off only if the stock moves far enough away from its current value, no matter in which direction, **hence a bet on the volatility**
- A straddle may be quite costly
- A strangle is less costly. The difference in a strangle is that we employ two strike prices,  $K_1$  and  $K_2$

## Straddle & Strangle (Gupta, 2013)

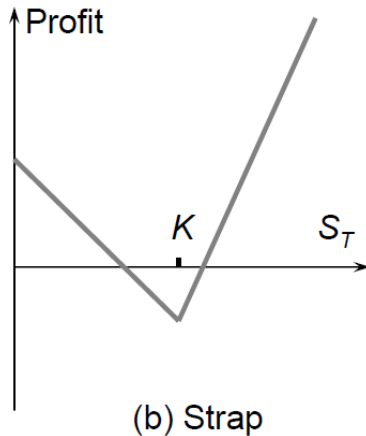
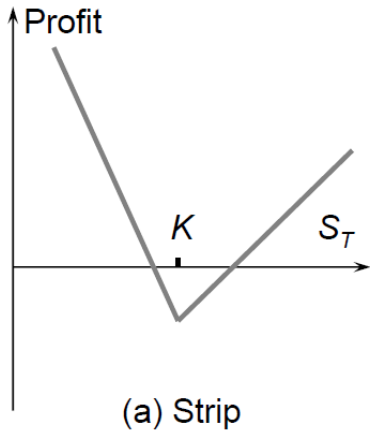


(a) A Straddle



(b) A Strangle

## Strip & Strap (Gupta, 2013)

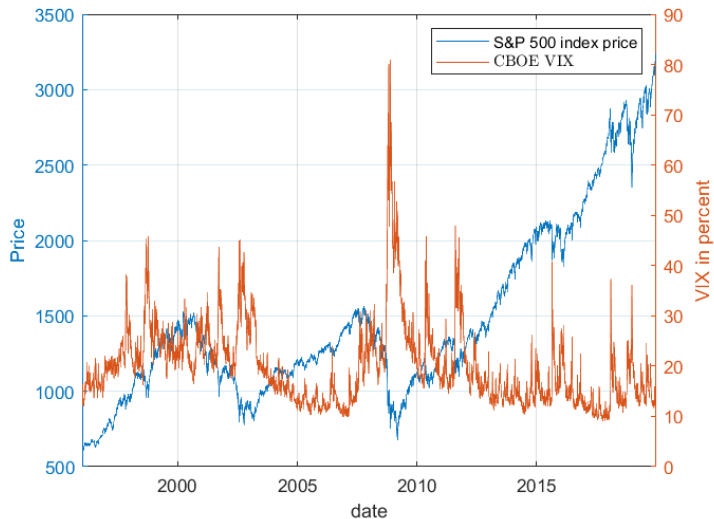


# Strip & Strap

- The investors may have different reaction to bad or good news
- A strip has a bias to a higher likelihood of a downward move
- A strap has a bias to a higher likelihood for the stock to move up
- Question: How to construct a strip?



# VIX Index



# VIX Index

- VIX was introduced initially in 1993 to track the BSM implied volatility of options on S&P 100 with near-the-money strikes.
- On September 22, 2003, the CBOE uses the more actively traded S&P 500 index options to compute VIX
- The old index is kept and was renamed as VXO
- A “fear” index? — High VIX or Low VIX, when should be you worried about?

## VIX Index

- Let  $R_{f,t \rightarrow t+T}$  denote the gross risk-free return over  $[t, t+T]$
- VIX index can be computed from ( $T = 30$  days)

$$VIX_{t \rightarrow t+T}^2 = \frac{2R_{f,t \rightarrow t+T}}{T} \left\{ \int_0^{F_{t \rightarrow t+T}} \frac{1}{K^2} Put_{t,t+T}(K) dK + \int_{F_{t \rightarrow t+T}}^{\infty} \frac{1}{K^2} Call_{t,t+T}(K) dK \right\} \quad (3)$$

- VIX is a weighted sum of market prices for a range of options on the S&P 500 stock index.

## Technical Issues Behind VIX

- Given a  $C^2$  function  $\phi(x)$  and  $x_0$  is one in the defined region, then

$$\phi(x) = \phi(x_0) + (x - x_0)\phi'(x_0) + \int_{x_0}^{\infty} \phi''(K)(x - K)^+ dK + \int_0^{x_0} \phi''(K)(K - x)^+ dK. \quad (4)$$

- Replace  $x$  by  $S_{t+T}$ , the stock price at time  $t + T$ ,
- Replace  $x_0$  by  $F_{t \rightarrow t+T}$ , the futures price that matures at time  $t + T$ ,

# Carr-Madan Formula

$$\begin{aligned}\phi(S_{t+T}) &= \phi(F_{t \rightarrow t+T}) \text{ ( a bond position)} \\ &+ \phi'(F_{t \rightarrow t+T})(S_{t+T} - F_{t \rightarrow t+T}) \text{ ( a stock position)} \\ &+ \int_{F_{t \rightarrow t+T}}^{\infty} \phi''(K)(S_{t+T} - K)^+ dK \text{ ( a sequence of call options )} \\ &+ \int_0^{F_{t \rightarrow t+T}} \phi''(K)(K - S_{t+T})^+ dK \text{ ( a sequence of put options)}\end{aligned}\tag{5}$$

# Carr-Madan Formula

- The formula allows you to replicate any European option with payoff function  $\phi(\cdot)$  using the plain-vanilla Call and Put options for all strikes
  - For instance, this formula is used in the valuation of a variance swap
  - Also, an approximation for VIX construction

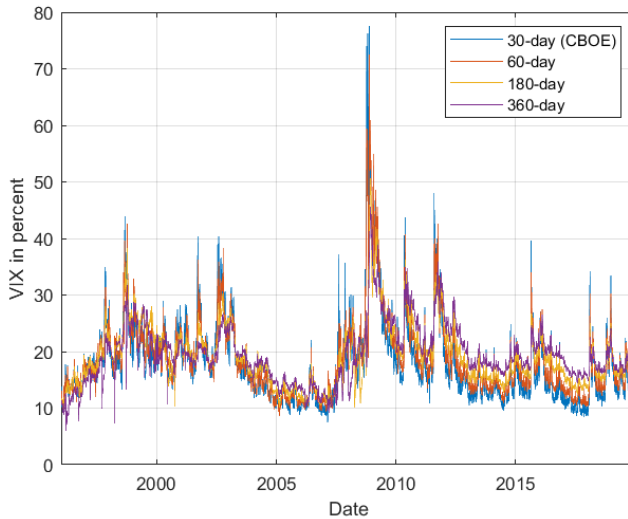
## VIX Term Structure

- VIX index can be computed from

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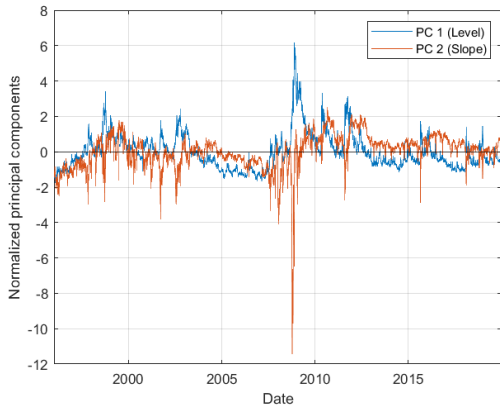
- We can choose different  $T$  and compute VIX over the future 30, 60, 90, 120, 180, 360 days
- This gives us a term structure of VIX index.

# VIX Term Structure

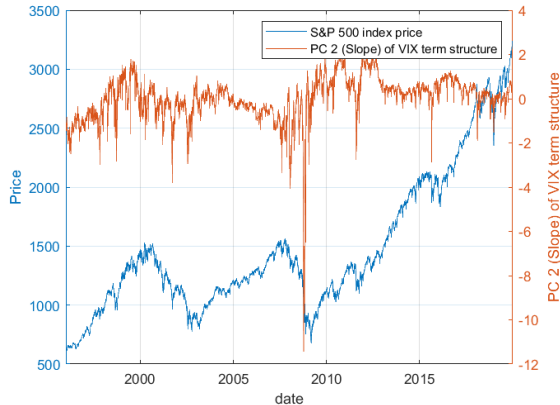




# VIX Index, Term Structure, and S&P 500 Index



(a) 1st & 2nd Principal components

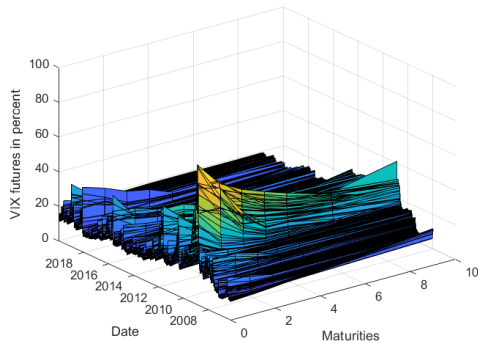


(b) VIX PC 2 & S&P 500 index

# VIX Futures

- VIX futures contract were first introduced on March 26, 2004 by CBOE Futures Exchange (CFE).
- In the early days, 4 futures contracts were listed every day. After October 2006, 9 contracts are listed every day.
- Initially, VIX futures prices were quoted as the VIX times ten and the contract multiplier was \$100.
- Starting on March 26, 2007, CFE modified the contract specification by dividing futures prices by ten and increasing the contract multiplier to \$1000.

# VIX Futures



- In normal times, VIX futures prices tend to have an upward-sloping term structure, suggesting a premium paid by long-time investors.
- The upward term structure of VIX futures prices to maturity is known as “contango trap”. Thus, rolling a futures contract is associated with substantial losses
- During market turbulence, i.e., the financial crisis in 2008/09, the futures curve tends to become inverted or hump-shaped.

# VIX Options

- CBOE launched European options on the VIX index on February 24, 2006.
- Like VIX futures, they are cash settled according to the difference between the value of the VIX at expiration and their strike price
- SPX Index Options: European options on S&P 500 index

# VIX Options & SPX Index Options

- Question: which market is larger for SPX options: Call or Put?

# VIX Options & SPX Index Options

- Question: which market is larger for SPX options: Call or Put?
  - Put options: hedging
  - Buying OTM put options is “equivalent” to short selling stocks

# VIX Options & SPX Index Options

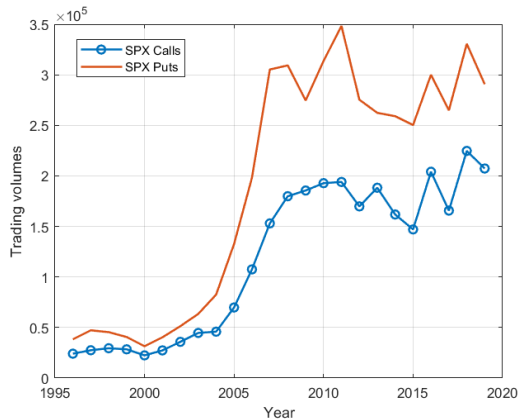
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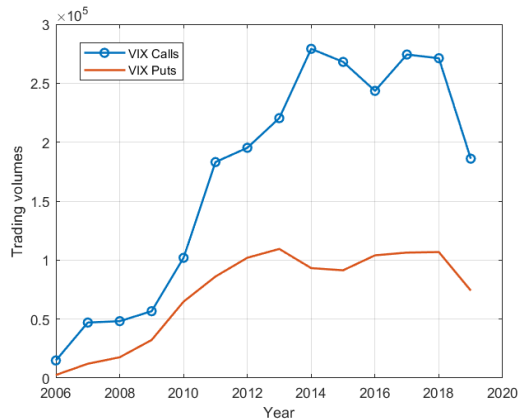
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- Question: which market is larger for VIX options: Call or Put?
  - The VIX call option market is much larger than the VIX put option market in terms of volume, open interest, and the number of quotes.



# Option Volumes (Jacobs and Mai, 2020)

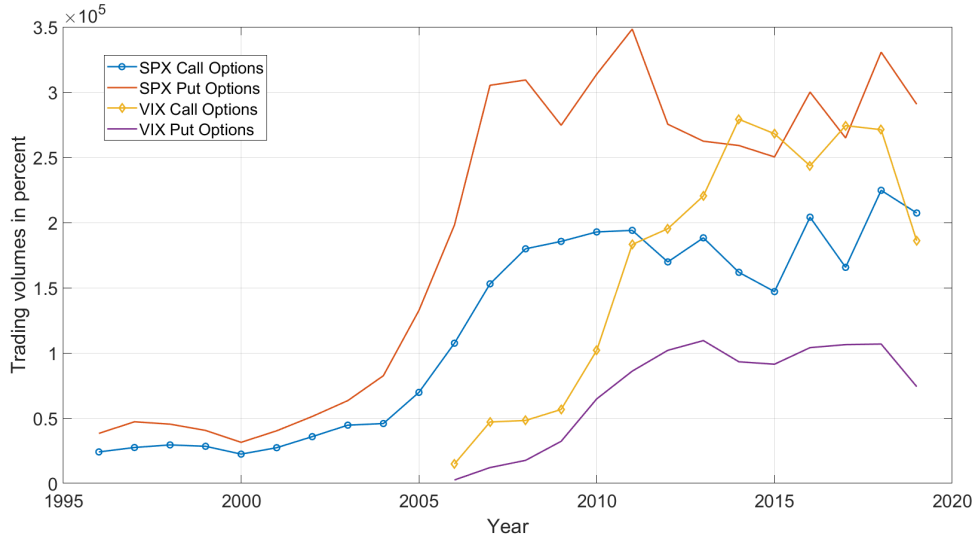


(c) SPX Index Options



(d) VIX Options

# Option Volumes (Jacobs and Mai, 2020)



# VIX Options & SPX Index Options

- VIX derivative positions can be used to hedge the risks of investments in the S&P 500 index.
  - By holding VIX derivatives, investors can achieve exposure to S&P 500 volatility without having to delta-hedge the option positions with index itself.
  - It is **cheaper** to buy OTM VIX call options than to buy OTM index put options.
  - VIX options are the **ONLY** asset in which open interest is **highest** for OTM call strikes

# Volatility Derivatives Pricing

- Volatility derivatives is different from equity (stock) derivatives
  1. Volatility is not “tradable”.
  2. Volatility is generally believed to be both stochastic and mean-reverting
- Researchers have proposed many advanced techniques in pricing options on derivatives.
- Grünbichler and Longstaff (1996) presents a simple closed-form solution for volatility option prices that is similar to BSM equity option pricing formula
- You can find the paper [here](#)

## Underlying Process

- Let  $V$  denote the current value of the standard deviation of a stock index return
- The dynamics of  $V$  is assumed to follow a Feller process (“CIR” process)

$$dV_t = (\alpha - \kappa V_t)dt + \sigma \sqrt{V_t}dZ, \quad (6)$$

- where  $\alpha$ ,  $\kappa$ , and  $\sigma$  are constants.
- $\alpha/\kappa$  is thus the long-term mean level of volatility
- A standard CIR process specifies that,

$$dr_t = a(b - r_t)dt + \sigma \sqrt{r_t}dW.$$

## Risk Neutral Process

- Consider a GBM for stock index dynamism in real-world and risk-neutral measures

$$dS_t = \alpha S_t dt + \sigma S_t dZ_t \quad \implies \quad dS_t = r S_t dt + \sigma S_t dZ_t \quad (7)$$

- In comparison, the risk-neutral process for Feller process in Equation (6)

$$dV_t = (\alpha - \beta V_t)dt + \sigma \sqrt{V_t} dZ_t, \quad \text{where} \quad \beta = \kappa + \xi \quad (8)$$

- The underlying in the volatility case is not tradable.
- We have to adjust for the market price of risk.

## Volatility Futures

- Let  $F(V, T)$  denote the futures price for a futures contract on  $V$  with maturity  $T$ .

$$F(V, T) = \mathbb{E}^Q [V_T] \quad (9)$$

- Evaluating this expectation gives the following expression for the volatility futures price

$$F(V, T) = \frac{\alpha}{\beta} [1 - \exp(-\beta T)] + \exp(-\beta T) V_0 \quad (10)$$

where  $\beta = \gamma + \xi$ , is from the risk-adjusted (risk-neutral) process

- The futures prices are exponentially weighted averages of the current value of  $V_0$  and the long-run mean  $\frac{\alpha}{\beta}$  of the risk-neutral process

## Volatility Options

- Let  $C(V, K, T)$  denote the current value of a call option on  $V$ , where  $K$  is the strike price of the option and  $T$  is the time until expiration.

$$C(V, K, T) = D(T) \mathbb{E}^Q [\max(0, V_T - K)] \quad (11)$$

where  $D(T)$  is the discounting process.

- Evaluating this expectation gives the following closed-form expression

$$\begin{aligned} C(V, K, T) &= D(T) \exp(-\beta T) V_0 Q(\gamma \kappa | \nu + 4, \lambda) \\ &+ D(T) \frac{\alpha}{\beta} [1 - \exp(-\beta T)] Q(\gamma \kappa | \nu + 2, \lambda) - D(T) K Q(\gamma \kappa | \nu, \lambda) \end{aligned}$$

where  $Q(\gamma \kappa | \nu, \lambda) = 1 - N(d)$ .



# Monte Carlo Simulation

- Use Monte Carlo simulation to evaluate the  $\mathbb{E}^Q [\max(0, V_T - K)]$  or  $\mathbb{E}^Q [V_T]$

$$dV_t = (\alpha - \beta V_t)dt + \sigma \sqrt{V_t} dZ_t,$$

- Consider the S&P 500 index, use VIX historical data to calibrate the above process.
- One numerical issue is that the discretization of Feller process may lead to negative value of  $V$  during simulation
  - Full truncation: replace negative value with zero
  - Reflection: replace negative value with the absolute value.

# Questions?

- Email: [ylu28@uncc.edu](mailto:ylu28@uncc.edu)
- Webpage: [JacquesYL.github.io](https://JacquesYL.github.io)

# References I

Andreas Grünbichler and Francis A Longstaff. Valuing futures and options on volatility. *Journal of Banking & Finance*, 20 (6):985–1001, 1996.

Aparna Gupta. *Risk management and simulation*. CRC Press, 2013.

Kris Jacobs and Anh Thu Mai. The role of intermediaries in derivatives markets: Evidence from vix options. *Available at SSRN 3635087*, 2020.