FINN 6216 Quantitative Risk Management

Historical Simulation & Stress Testing

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The slide and MATLAB code can be accessed on my website: @JacquesYL

Outlines

- Review of Value-at-Risk (VaR)
- Analytic Variance-Covariance Approach (Review Lecture 1 - 3)
- Historical Simulation Approach
- Monte Carlo Simulation Approach
- Back Testing
- Stress Testing

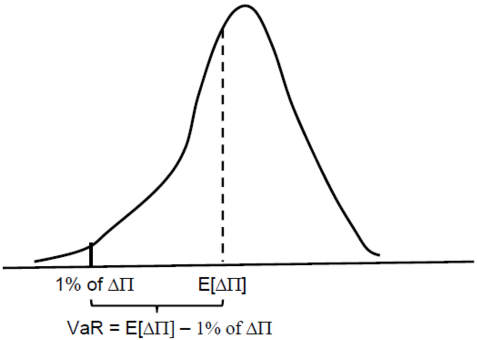
Value-at-Risk

- VaR is powerful for measuring overall market risk
  - over short horizon (10-day period)
  - under ‘normal’ market conditions (not normal distribution)
  - sudden jumps and extreme-events are not incorporated
- VaR is NOT telling you the worst-case scenario or how much losses may excedd the VaR figure. It just states how likely or unlikely it is for the VaR to be exceeded.
- For long time-periods and for abnormal market conditions, supplemental methodologies are necessary
  - stress Testing & worst-case scenario

Value-at-Risk

- Example: A position has a daily VaR of $10 million at the 99 percent confidence level
- Interpretation 1: You are 99% confident that, on average, the maximum realized loss in one day will not exceed %10 million
- Interpretation 2: Realized daily losses from the position will on average be higher than $10 million on only ONE out of HUNDRED trading days.
  - 100 trading days $\times$ (1 - 99%) = 1 trading day

Two Terminologies in Capital Regulatory



- The change in the value of the portfolio is the key distribution

$$\Delta\Pi = \Pi_{\Delta T} - \Pi_0$$

- Absolute VaR = worst-case loss at (1 - α)% confidence level
    - Also known as "Conservative VaR" or "VaR without the mean"
- VaR = Expected profit/loss – Absolute VaR
    - Also known as "Relative VaR"

Value-at-Risk

- Two steps to calculate VaR
  - Derive the forward distribution of the portfolio, or returns on the portfolio at a chosen horizon (one day).
  - Identify the required percentile of the distribution so the particular Loss Number can be read off.
- Question: how to derive the forward distribution?
  - Analytic variance-covariance approach
  - Historical simulation (Non-parametric)
  - Monte Carlo analysis

Analytic Variance-Covariance (VC) Approach

Analytic Variance-Covariance Approach

- Also called the ”Delta Normal” Approach
- It assumes that risk factors and the portfolio values are lognormally distributed, or equivalently, log returns are normally distributed
  - For instance, Geometric Brownian Motion & Black-Scholes-Merton formula
- Normal distributions are completely characterized by their first two moments - mean and variance!
  - An analytical formula that links the central risk measure, standard deviation, with the tail risk measure, VaR.

Analytical Formula

- In a Normal distribution, VaR corresponds to a factor of the Standard Deviation (Sigma) of the position.

- Analytical Formula for loss distribution of 1 dollar (we ignore the mean)

$$
\begin{aligned}
\text{1-day VaR} &= \sigma_{\text{1-day}} * \Phi^{-1}(\alpha) && (1)\\
\text{N-day VaR} &= \sigma_{\text{N-day}} * \Phi^{-1}(\alpha) && (2)\\
&= \sqrt{N} \times \text{1-day VaR} && (3)
\end{aligned}
$$

- For instance, VaR (99%) = 2.33 Sigma, VaR (95%) = 1.65 Sigma
  - Here, VaR and Sigma are measured over the same time horizon

- If corr = 1, VaR = sum of individual VaRs

Verification

- Suppose the future return of 1-dollar follow the normal distribution $\hat{r} \sim N(\mu, \sigma)$
- Given a confidence level, (1 - α)%, the worst-case threshold is

$$r^* = \left(\mu - \sigma * \Phi^{-1}(\alpha)\right) \tag{4}$$

- VaR = Expected future value - worst expected future value = 33

$$\begin{aligned} \text{VaR}(\alpha) &= \underbrace{[1 * (1 + \mu)]}_{\text{mean level}} - \underbrace{[1 * (1 + r^*)]}_{\text{worst-case}} = \mu - r^* \\ &= \sigma * \Phi^{-1}(\alpha) \end{aligned} \tag{5}$$

Example

- Initial value/wealth: $w0 = \$100$
- Expected return and volatility: $\mu = 10\%$, and $\sigma = 20\%$
- Compute the VaR(95%) assuming a normal distribution of future return

Example

- The threshold return of 95% confidence level under normal distribution is

$$r^* = (\mu - 1.65 * \sigma) = -23\% \tag{6}$$

- Use the definition

$$\begin{aligned}
\text{mean level} &= w0 * (1 + \mu) = 100 * (1 + 10\%) = 110, && (7)\\
\text{worst-case} &= w0 * (1 + r^*) = 100 * (1 - 23\%) = 77, && (8)\\
\text{VaR} &= 110 - 77 = 33 && (9)
\end{aligned}$$

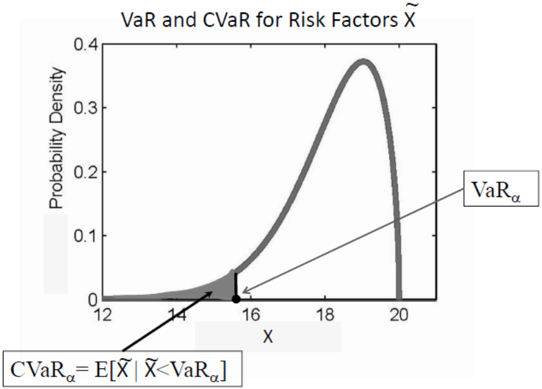
- Use the formula

$$\text{VaR} = 1.65 * \sigma * w0 = 33 \tag{10}$$

Is Normal Distribution Valid?

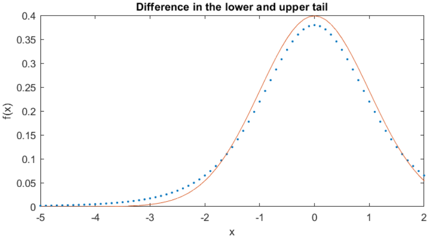
- A well-diversified portofio: may exhibit (near) normal dsitribution due to
  - **Central Limit Theorem:** independent RVs of well-behaved distribution will have a mean that converges, in large samples, to a normal distribution
- Caution: Fat tails, lumpy portfolio, highly correlated risk factors should send a warning signal to managers who seek comfort in VaR numbers for gaining control over risk levels.

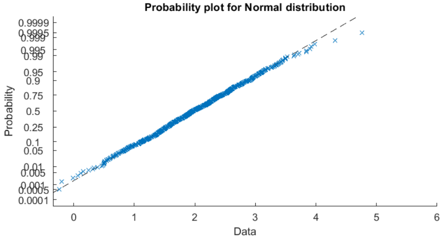
Something to Notice

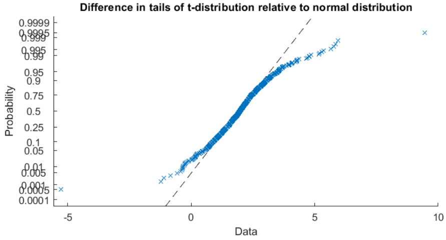


- We use Value-at-Risk as a measure for tail risk
- But in calculating VaR, if we use analytic variance-covariance approach, we make an explicit assumption that the distribution is normal.
- If everything is normal, no "tail-risk".
- Also, VaR is not a coherent risk measure. Instead, Conditional VaR, or known as Expected Shortfall, is a coherent risk measure.

Tail Risk Characteristics







Historical Simulation (Non-parametric) Approach

Steps to compute 99% VaR

1. Identify the market variables affecting the portfolios. For example, stock return, equity price, interest rate, exchange rates etc.

2. Portfolio under examination is revalued, using changes in risk factors derived from historical data, to create distribution of the portfolio returns from which the VaR of the portfolio is derived.

    - Collect data on the movements in these market variables over the most recent 500 days.

3. Each simulated change in the value of the portfolio becomes an observation in the distribution. Sum these changes across all positions, keeping days synchronized.

4. Construct a histogram of portfolio values and identify the VaR that isolates the first percentile of the distribution in the left end tail.

Attractions & Drawbacks

- Major attraction: completely non-parametric; no assumptions about distributions of the risk factors! No problems with fat tails.
- No variances and correlations to estimate, historical volatilities and correlations are already reflected in the data set.
- Drawbacks
  - Completely depends on the particular set of historical data and its idiosyncrasies.
  - Past is perfectly reliable representation of the future - this may be flawed, since there have been crashes, or unusual low volatilities, which may not be repeated.
  - What if there are structural changes in the market - such as - introduction of Euro in the FX market at beginning of 1999.
  - Data availability could be the other challenge. 1 year of data = 250 trading days = 250 scenarios. These could be quite little to accurately estimate the VaR.

Example

- See the excel sheet of VaR_Calculations_Worksheet.xlsx in the course page
- Use both analytic variance-covariance approach and historical simulation approach to compute 10-day VaR at 99% level.

Monte Carlo Simulation Approach

Monte Carlo Simulation Approach

- This involves repeatedly simulating the random processes that govern the risk factors.

- Each scenario corresponds to possible value of the portfolio at the target horizon – 1-day or 10-days.

- Three steps:

  1. Specify all relevant risk factors. We also need to specify the dynamics of each of the risk factors, i.e. stochastic processes, parameters estimated – volatilities, correlations, mean-reversion for interest rates, etc.
  2. Construct price paths using random number generators and the stochastic processes, incorporating correlations and multivariate distributions where necessary.
  3. Value of the portfolio for each scenario is determined. 10,000 of such scenarios create the distribution of the portfolio, from which the 1st percentile and mean is read off to compute the VaR.

Monte Carlo Simulation Approach

- Monte Carlo simulation allows computing Confidence Intervals on the VaR.
- It allows performing sensitivity analysis by changing market parameters.
- Drawback:
  - **Model risk:** Have to estimate parameters in the models – means, variances, covariances.
  - **Computational difficulty:** Computer resources needed are quite high, especially for large and complex portfolios.

Example: Question 6 of Assignment 1

- A bank has written a call option on one stock and a put option on another stock. For the first option the stock price is 50, the strike price is 40, the volatility is 25% per annum, and the time to maturity is 9 months. For the second option the stock price is 20, the strike price is 20, the volatility is 20% per annum, and the time to maturity is 1 year. Neither stock pays a dividend, the risk-free rate is 5% per annum, and the correlation between stock price return is 0.4. Calculate a 10-day 99% VaR using delta. Then try your best to calculate the VaR using both delta and gamma.

- We may use Monte Carlo simulation when it comes to both delta and gamma, but it is still the analytic variance-covariance approach, not the Monte Carlo simulation approach.

Example: Question 6 of Assignment 1

- The key is to find the the Standard Deviation of the position.

$$\Delta P = \left[\delta_1 S_1(\Delta x_1) + \frac{1}{2}\gamma_1 S_1^2(\Delta x_1)^2\right] + \left[\delta_2 S_2(\Delta x_2) + \frac{1}{2}\gamma_2 S_2^2(\Delta x_2)^2\right] \tag{11}$$

  where $\Delta x_i = \frac{\Delta S_i}{S_i}$ is the stock return.

- Use Monte Carlo simulation to find the 1-day $\sigma_{\Delta P}$, and then plug into the formula

$$\text{N-day VaR} = \quad 2.33 \times \sqrt{10} \times \sigma_{\Delta P} \tag{12}$$

- See the MATLAB code here: @JacquesYL
  - DeltaGammaVAR_JacquesYL.m

Back Testing

Back Testing

- Back testing gives the criteria of checking whether a risk (VaR) measure is good enough.

- Suppose that we have developed a procedure to compute a 1-day 99% VaR

- We look at how often the loss in ONE day exceeds the 1-day 99% VaR calculated using the procedure for that day

- Days when the actual change exceeds VaR are referred to as **exceptions**.
  - If exceptions happen on about 1% of the days, we are comfortable with the VaR model
  - If exceptions happen about 10% of the days, we underestimate the VaR
  - If exceptions happen about 0.01% of the days, we overestimate the VaR

Back Testing

- Suppose that the time horizon is one day and the confidence level limit is X (for instance, X = 0.95; 0.99; 0.9999). If the VaR mode is accurate, the probability of the VaR being exceed on any given day is $p = 1 - X$.

- Consider a total of n days, the probability of the VaR limit being exceeded on exactly m days is

$$\binom{n}{m} p^m (1-p)^{n-m} = \frac{n!}{m!(n-m)!} p^m (1-p)^{n-m} \tag{13}$$

- The probability of the VaR limit being exceeds on m or more days is

$$\sum_{k=m}^{n} \binom{n}{k} p^k (1-p)^{n-k} = \sum_{k=m}^{n} \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} \tag{14}$$

Binomial Distribution & Normal Distribution

- Let ξ be the number of exceptions in n observations, and it follows a binomial distribution: mean is pn and variance is $p(1-p)n$.
- By the central limit theorem, when n goes to infinity, then

$$\frac{\xi - pn}{\sqrt{p(1-p)n}} \rightarrow N(0,1) \tag{15}$$

- Binomial tree (Lattice) pricing model & BSM option pricing model (See John Hull's book for derivation

Statistical Inference: One-way Tailed Test

- **Null Hypothesis:** The probability of an exception on any given day is p.
- An often-used confidence level in statistical test is 5% (Regulator might want 1% to be consistent with the VaR confidence level).
- If the probability of the VaR limit being exceeded on m or more days is less than $p = 0.05$, we reject the Null Hypothesis
- Similar to the statistical significance in regression using p-value

Back Testing Example

- *Suppose that we back test a VaR model using 600 days of data. The VaR confidence level is 99%, so the expected number of exceptions is 6. If we observe 9 exceptions, should we reject the VaR model?*
- Recall, the probability of the VaR being exceed on any given day is $p = 1 - 99\% = 0.01$.
- We compute The probability of 9 or more exceptions

$$Pr = \sum_{k=9}^{600} \frac{600!}{k!(600-k)!} 0.01^k (1 - 0.01)^{600-k} = 0.1517 > 0.05 \tag{16}$$

- Thus, we should NOT reject the null
- The number of 9 is not statistically different from 6, in the context of 600 days of data.

Stress Testing

Stress Testing

- VaR measure extent of risk exposure under ‘normal market risk variations’
- Sudden jumps and extreme-events are not incorporated
- Jumps and extreme events correspond to periods of market crises characterized by large price changes, high volatility, and breakdowns in the correlations among risk factors.
- These extreme events have very small probability, but substantial impact.
- Regulators see Stress Testing and Scenario Analysis are necessary complement to VaR models

What Makes a Good Stress Test?

- The goal of stress testing is to uncover potential concentrations and make risks more transparent

- Good stress tests should
  - be relevant to current positions
  - consider changes in all relevant market rates
  - examine potential regime shifts
  - spur discussion
  - consider market illiquidity, and
  - consider the interplay of market and credit risk.

Three Basic Steps for Stress Testing

- **Step 1**: Generate scenarios
  - The most challenging aspect of stress testing is generating credible worst-case scenarios that are relevant to portfolio positions.
  - Scenarios should address both the magnitude of movement of individual market variables and the interrelationship of variables (i.e., correlation or causality).
- **Step 2**: Revalue portfolio
  - Revaluing a portfolio involves marking-to-market all financial instruments under new worst-case market rates. Stress test results are generally changes in present value, not VaR.
- **Step 3**: Summarize result
  - A summary of results should show expected levels of mark-to-market loss (or gain) for each stress scenario and in which business areas the losses would be concentrated.

Stresses Banks for Derivative Exposures

- Parallel yield-curve shift of +/- 100 bp
- Yield-curve twist of +/- 25 bp.
- Equity index values change of +/- 10%
- Currency changes of +/- 6%
- Volatility changes of +/- 20%
- All relevant mixed effects
- Scenarios for liquidity, credit and operational risk are also included.

Stress Testing Envelopes

- Stress-envelopes methodology combines categories of worst possible stress shocks across all possible markets for every business.

- Stress envelope is the change in the market value of a business position in a certain currency/market due to a particular stress shock

- Scenario is then created by combining several stress shocks and their envelopes.

- Scenarios can also be constructed matching **historical extreme events**

Historical Extreme Event: Crash of October 1987

- Equity markets globally fall by 20% on average
  - Some even more (Hong Kong by 30%)
- Implied volatility shoots from 20% to 50%
- USD rallies against other currencies
  - Asian currencies lose up to 10% again USD
- Interest rates fall in the Western markets
  - HK long-term interest rates rise by 40bp, and short-term by 100bp
- Commodity prices drop in fear of recession
  - Copper and Oil prices decline by 5%.

Readings on Stress Testing

- Dodd-Frank Act stress testing is a forward-looking exercise that assesses the impact on capital levels that would result from immediate financial shocks and nine quarters of adverse economic conditions.
    - Stress Test Scenarios: (Link)
    - Dodd-Frank Act Stress Test 2019: (Link)
- Articles from Global Association of Risk Professionals (GARP)