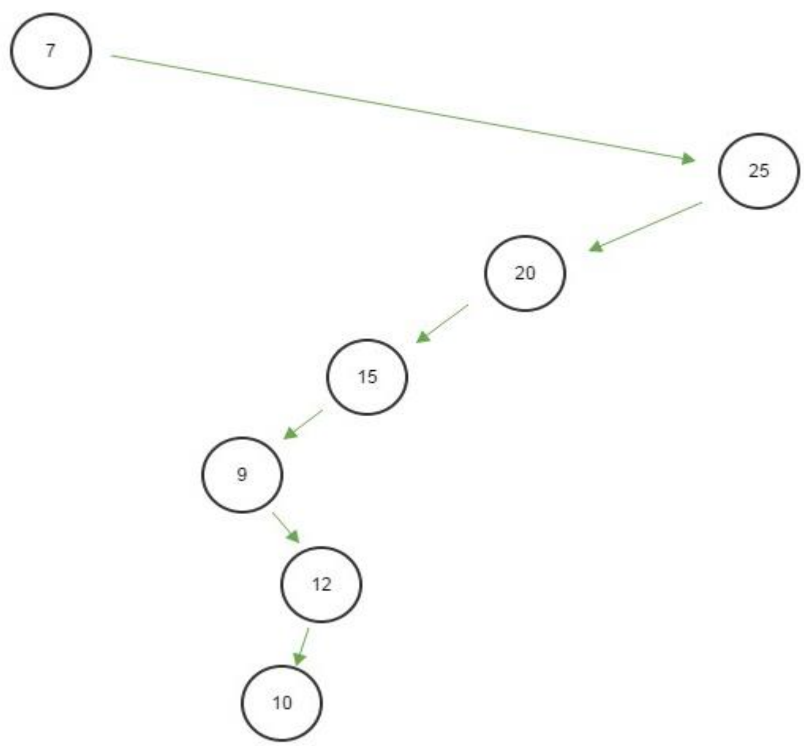
Gray Houston
ghousto
0026483532

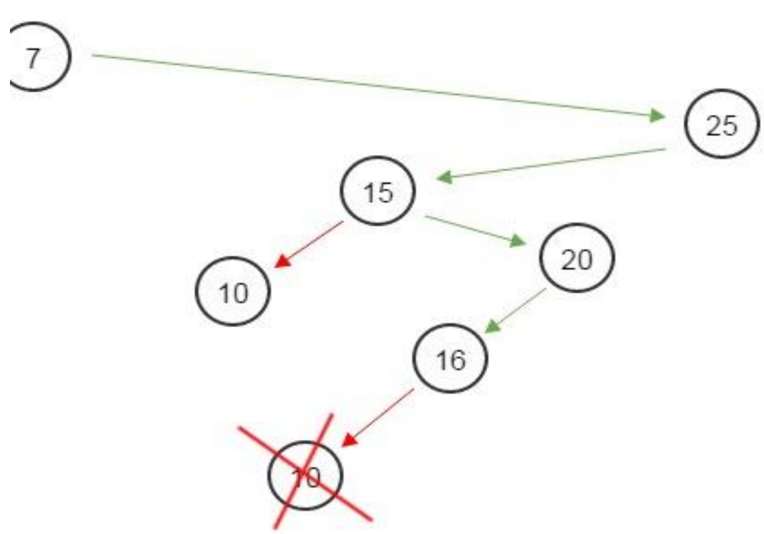
Problem Set 1

1.
   a. 7 → 25 → 20 → 15 → 9 → 12 → 10

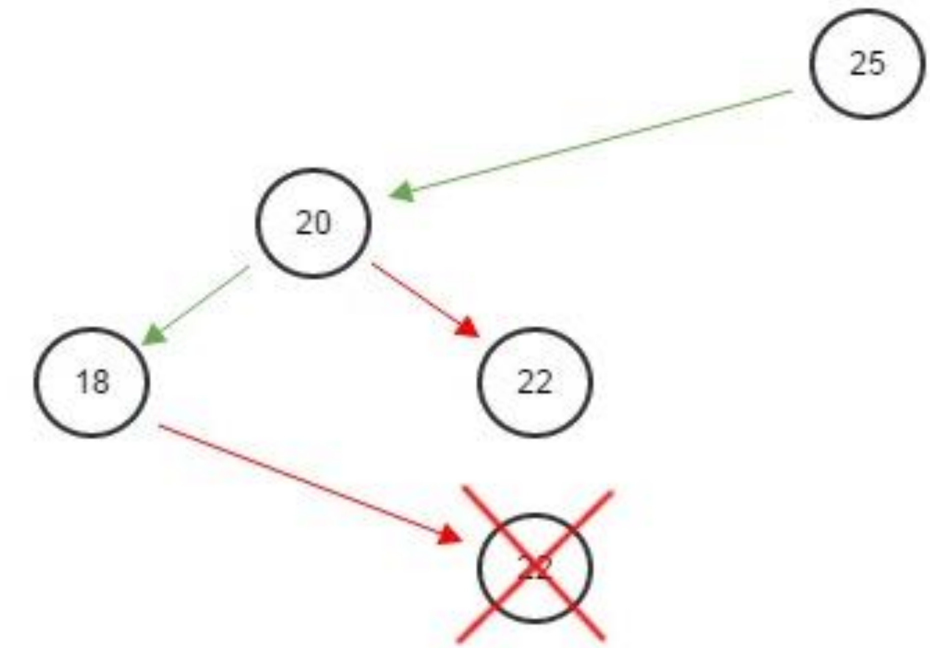


Tree exists

   b. 7 → 25 → 15 → 20 → 16 → 10 Not a valid tree



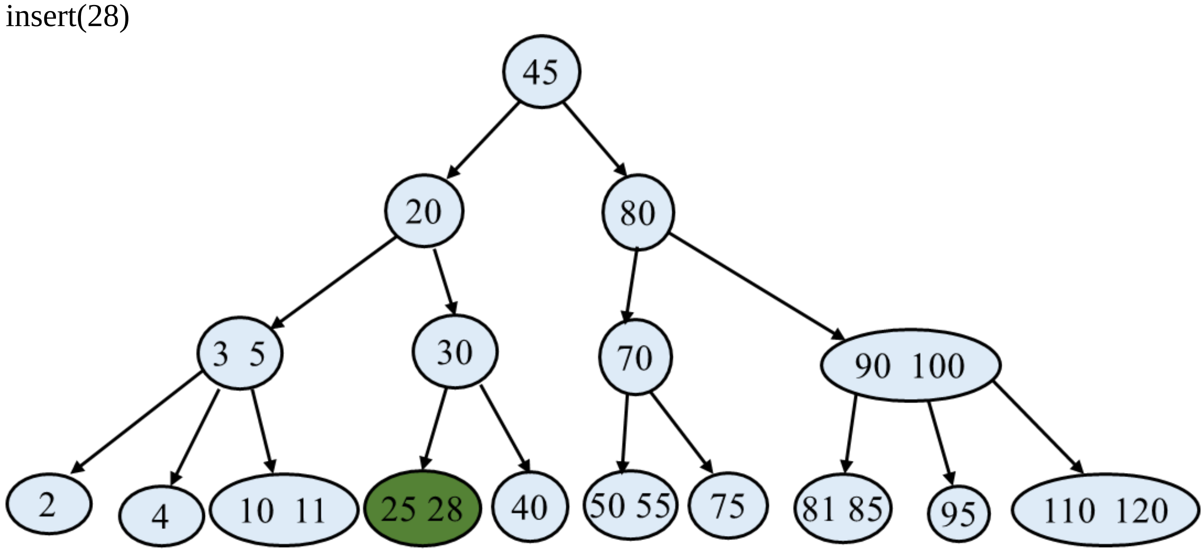
~~c.~~ 25 → 20 → 18 → ~~22 → 16 → 10~~



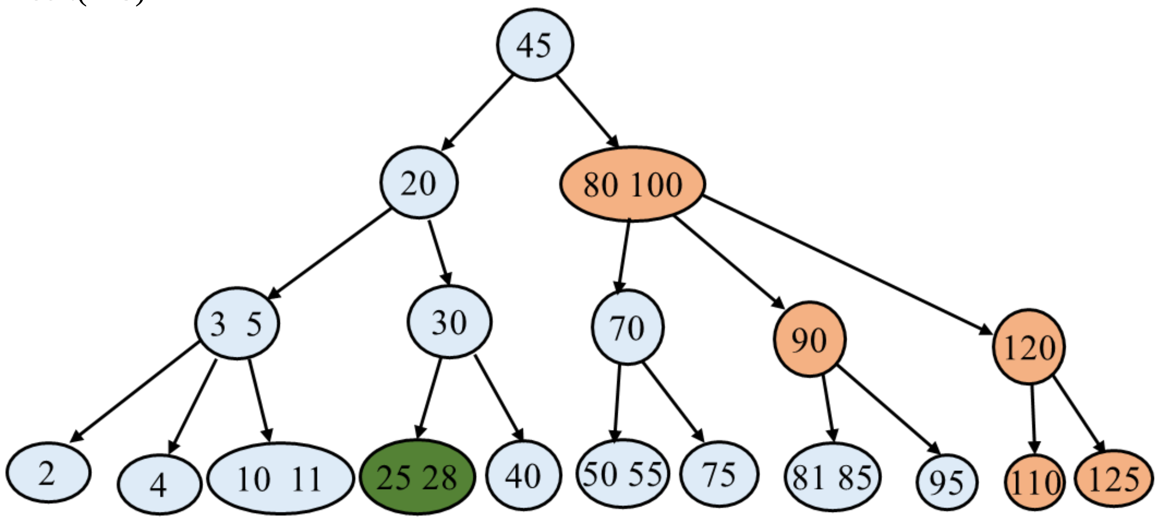
Therefore, this is not a valid tree.

2.

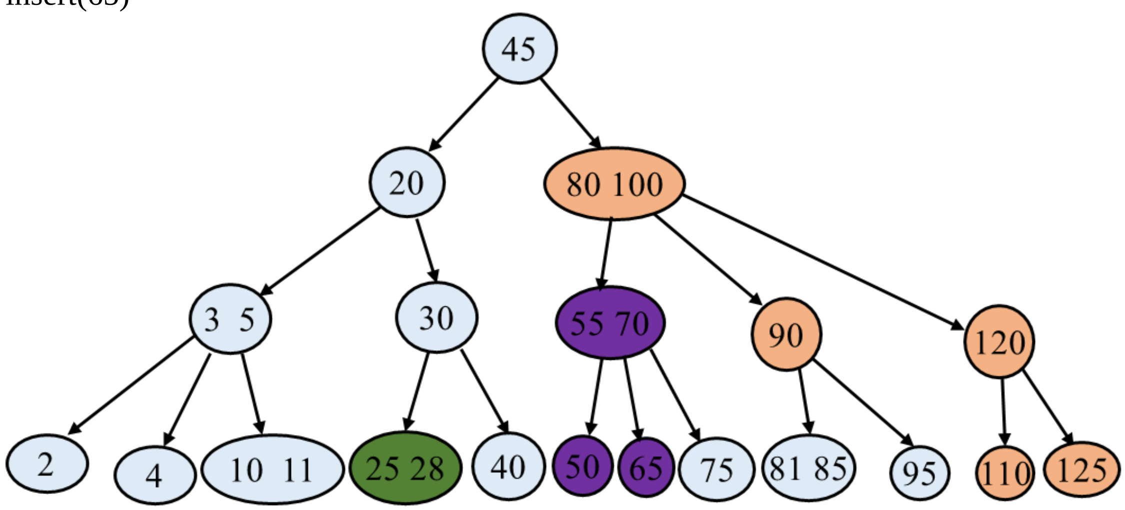
a. insert(28)



b. insert(125)

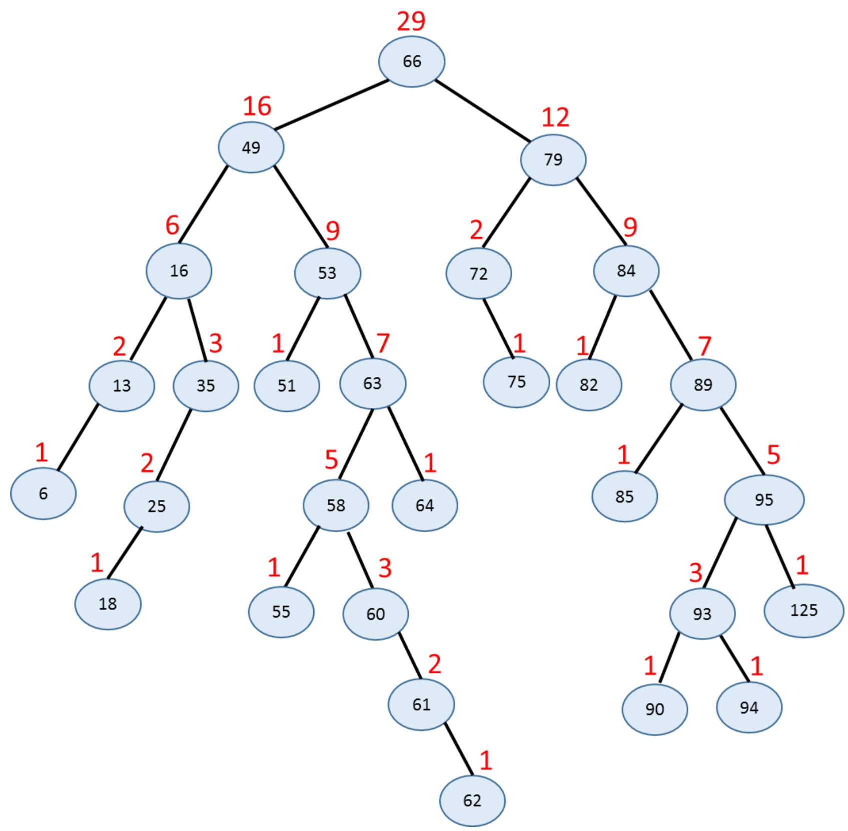


c. insert(65)

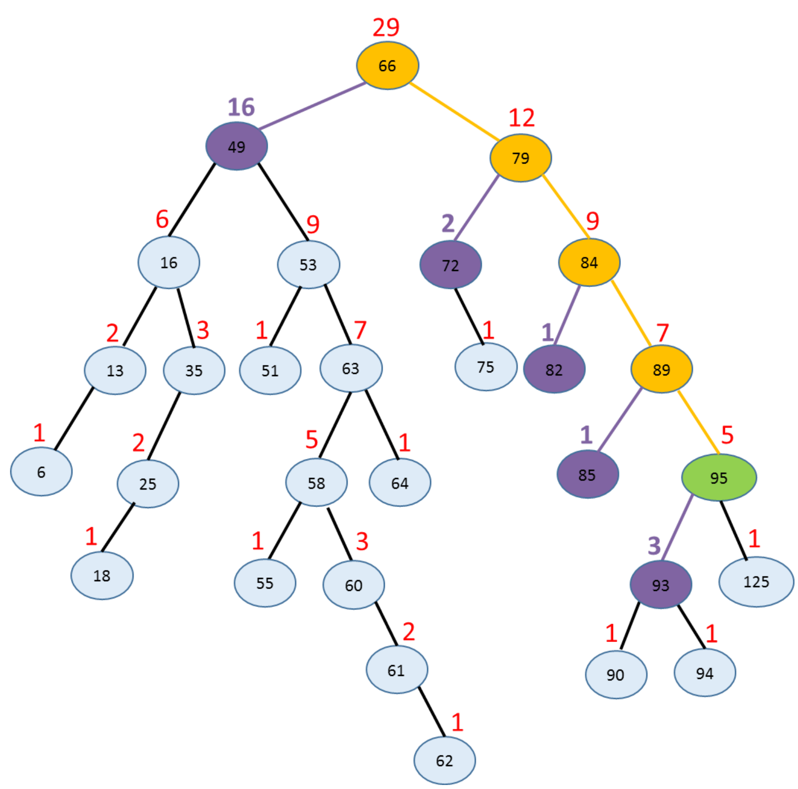


3.

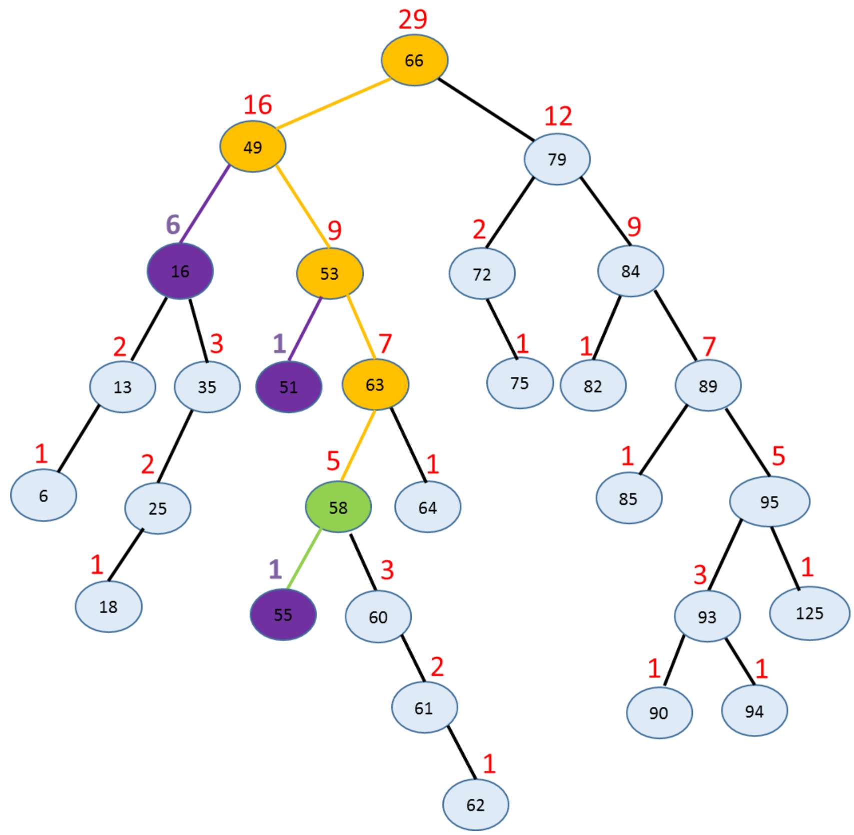
a.



b.



Rank(95) = (16+1) + (2+1) + (1+1) + (1+1) + 3 = 27 c.



Rank(58) = (6+1) + (1+1) + 1 = 10

4. For the smallest case, a 2-3 tree with the minimum number of nodes (assuming a complete tree), each node only contains 1 key. If each node only has 1 key, this structure is a balanced binary search tree (BBST). A complete BBST of height h will contain (at most) $2^0 + 2^1 + \ldots + 2^h = 2^{h+1} - 1$. Therefore, because a 2-3 tree with minimum nodes is a BBST, this minimal 2-3 tree will have $n = 2^{h+1} - 1$ nodes.

   In the case of a 2-3 tree with the maximum number of nodes, the number of nodes at each height h is equal to $3^0 + 3^1 + 3^2 + \ldots + 3^h$. This sums to $3^{h+1} - 1$. Therefore, for a 2-3 tree of height h with the maximum possible number of nodes, the number of nodes $n = 3^{h+1} - 1$.