

## 一、导数基本公式表

$$C' = 0$$

$$(x^a)' = a \cdot x^{a-1}$$

$$(a^x)' = a^x \ln a \quad (a > 0 \text{ 且 } a \neq 1) \rightarrow (e^x)' = e^x$$

$$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0 \text{ 且 } a \neq 1) \rightarrow (\ln x)' = \frac{1}{x}$$

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$$(\sin x)' = \cos x$$

$$(\sec x)' = \sec x \cdot \tan x$$

$$(\cos x)' = -\sin x$$

$$(\csc x)' = -\csc x \cdot \cot x$$

$$(\tan x)' = \sec^2 x = \frac{1}{\cos^2 x}$$

$$(\cot x)' = -\csc^2 x = -\frac{1}{\sin^2 x}$$

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$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

## 二、求导法则

### 1. 导数的四则运算

$$(u \pm v)' = u' \pm v'$$

$$(u \cdot v)' = u'v + uv'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

## 2. 反函数的导数

$$f'(x) = \frac{1}{\varphi'(y)} \quad (\text{即 } \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}})$$

## 3. 复合函数的求导法则 (链式法则)

$$[f(g(x))]' = f'(g(x)) \cdot g'(x) \quad \text{即 } \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx}$$

## 4. 对数求导法

## 5. 隐函数求导

偏导数

## 6. 参数方程求导

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

## 三. 高阶导数 $\frac{d^ny}{dx^n}$

求法 =

1. 找规律 (注意代数变形)

2. 记住常见的几个  $n$  阶导数

$$(x^{\alpha})^{(n)} = \frac{\alpha!}{(\alpha-n)!} x^{\alpha-n}$$

$$(x^n)^{(n)} = n!$$

$$(x^n)^{(n+k)} = 0$$

$$\left(\frac{1}{x}\right)^{(n)} = (-1)^n \cdot \frac{n!}{x^{n+1}}$$

$$(e^{ax})^{(n)} = a^n \cdot e^{ax}$$

3. 莱布尼兹公式:

$$(uv)^{(n)} = \sum_{k=0}^n \binom{n}{k} u^{(n-k)} v^{(k)}$$