Prisoner’s dilemma

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Background

The prisoner’s dilemma is a fundamental problem in game theory that demonstrates why two people might not cooperate even if it is in both their best interests to do so. A classic example of the prisoner’s dilemma (PD) is presented as follows:
Two suspects are arrested by the police. The police have insufficient evidence for a conviction, and, having separated the prisoners, visit each of them to offer the same deal. If one testifies for the prosecution against the other (defects) and the other remains silent (cooperates), the defector goes free and the silent accomplice receives the full 10-year sentence. If both remain silent, both prisoners are sentenced to only six months in jail for a minor charge. If each betrays the other, each receives a five-year sentence. Each prisoner must choose to betray the other or to remain silent. Each one is assured that the other would not know about the betrayal before the end of the investigation. How should the prisoners act?

Spatial Prisoner’s dilemma

In the iterated prisoner’s dilemma, the game is played repeatedly. Thus each player has an opportunity to punish the other player for previous non-cooperative play. If the number of steps is known by both players in advance, economic theory says that the two players should defect again and again, no matter how many times the game is played. However, this analysis fails to predict the behavior of human players in a real iterated prisoners dilemma situation, and it also fails to predict the optimum algorithm when computer programs play in a tournament. Only when the players play an indefinite or random number of times can cooperation be an equilibrium, technically a subgame perfect equilibrium meaning that both players defecting always remains an equilibrium and there are many other equilibrium outcomes. In this case, the incentive to defect can be overcome by the threat of punishment.
One variation of the two-player game for a mesh of players is to play against all of their neighbours and to subsequently adopt the strategy (Cooperate or Defect) of the neighbouring player with the highest payoff. This is known as the spatial prisoners’ dilemma and it has been studied on square lattices with periodic boundary conditions. Figure shows a prisoner cell surrounded by its four nearest neighbours and its additional four next-nearest neighbours.

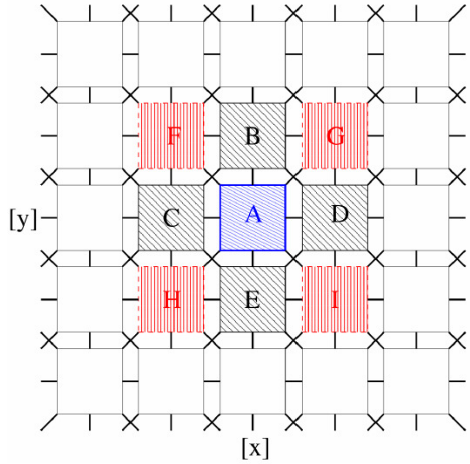

Figure 1: Spatial Prisoner's Dilemma.

Exercises

1. Implement the spatial prisoner's dilemma with the following payoff-matrix

	cooperate	defect
cooperate	7	0
defect	10	0

   on an $N \times N$ grid and run it for n iterations.

   Create two $N \times N \times n$ numpy-arrays S and P and save for each cell and each iteration, the strategy applied by the cell and the received payoff.

2. Plot the strategy as $N \times N$-image for each iteration in a loop using `ion()` and `imshow()` such that you can see how the system evolves in a movie.

3. Plot the number of cooperating players by iteration.

4. Plot histograms for the distributions of

   a) number of consecutive cooperations

   b) total number of cooperations over time

5. Experiment with different strategies to choose defection/cooperation.