COMP 5711: Advanced Algorithm
Assignment 4

SKI (30 pts) The following problem is known as the *ski rental problem*. Suppose you are going skiing for an unknown number of days (e.g., you don't know when you might lose interest, encounter bad weather, or break your legs). Assume that renting skis costs 1 dollar per day and buying skis costs n dollars. Of course, you no longer need to rent after you have bought it. Every day you have to decide whether to continue renting skis for one more day or buy one.

(a) Design a deterministic strategy that achieves a competitive ratio of at most 2. [Hint: Rent until day n.]

(b) Show that no deterministic strategy can achieve a competitive ratio better than $2 - 1/n$ in the worst case, using an adversarial argument.

(c) Design a randomized strategy that has a better competitive ratio in expectation. You may assume that n is an even number. [Hint: Rent until day $n/2$. Then with some probability you buy; otherwise you continue to rent until day n.]

MG (30 pts) We showed that the error of the MG algorithm in estimating the frequency of any item is at most $N/(k+1)$ when using k counters. Below you are asked to get tighter error bounds:

(a) Let M be the sum of the k counters. Show that the error is at most $(N - M)/(k+1)$.

(b) Let $N^{\mathrm{res}(t)} = N - \sum_{i=1}^{t} f_i$ be the *residual count* of the stream, where f_i denotes the frequency of the i-th most frequent item. Show that the error is at most $N^{\mathrm{res}(t)}/(k+1-t)$ for any $0 \le t \le k$. Note that this means the MG algorithms offers better error guarantees for skewed distributions. [Hint: Use the result above.]

Parallel (20 pts) Design a parallel algorithm for the following problem: Given an array A of n bits, find the location (i.e., the index of A) of the first 1. Your algorithm should have $O(\log n)$ time and $O(n)$ work.