Name: _____ **Date:**_____

Class: 4th (D-E) **Duration:** 50 minutes

Math Exam

5% of your grade is allocated for presentation and proper use of notation.

I- Circle the best answer. **Show your work**. **(35 points)**

1) Which of the below represents a quadratic expression?

a) x^3+2x^2-3x+1

b) $x+2$

c) $5x-x^2$ highest degree = 2

d) $\dfrac{x^2+2x+6}{x^4}$

2) Consider the expression $A=x^2+bx+c$. If the equation $A=0$ has two roots -1 and 3, then A can be written in factorized form as:

a) $(x+1)(x-3)$

b) $(x-1)(x+3)$

c) $(x-1)(2x+6)$

d) $(2x+2)(x-3)$

3) Which of the below has no solution?

a) $(2x+5)^2-6=-2$ $(2x+5)^2=4$ ✓

b) $3x^2+x+1=0$ $\Delta = 1-4(3) = 1-12 = -11 < 0$

c) $4x^2+20x+25=0$ $(2x+5)^2=0$ ✓

d) $x^2+3x+2=0$ $\Delta = 9-4(1)(2) = 1$ ✓

4) If -2 is a solution for the equation $mx^2 + mx - 4 = 0$, then $m =$

(a) 2

b) $\frac{1}{2}$

c) 0

d) cannot be determined

$m(-2)^2 + m(-2) - 4 = 0$

$4m - 2m - 4 = 0$

$2m = 4$

$m = 2$.

5) Let $ax^2 + bx + c = 3(x+5)(x-1)$, then $a + b + c =$

(a) 0

b) 1

c) -1

d) 2

$ax^2 + bx + c = 3(x^2 - x + 5x - 5)$

$ax^2 + bx + c = 3(x^2 + 4x - 5)$

$ax^2 + bx + c = 3x^2 + 12x - 15$

$a = 3$

$b = 12$

$c = -15$

$a + b + c = 0$

6) The product of two consecutive positive integers is 5 more than their sum. The greatest of these two integers is:

(a) 4

b) 3

c) 6

d) 5

$x, x+1$

$x(x+1) = 5 + (x + x + 1)$

$x^2 + x = 5 + 2x + 1$

$x^2 + x = 6 + 2x$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ or $x = -2$ ✓ rejected

So the 2 integers are 3 and 4

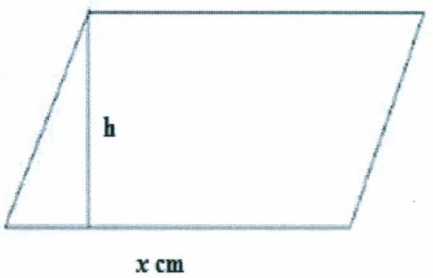
7) The area of a parallelogram is given by the formula A = base x height. If the height of a parallelogram is 11 cm less than twice its base of length x cm, and the area of the parallelogram is 21 cm^2, then the measure of the height is:

a) 7

(b) 3

c) 11

d) 14



Area = $x \cdot h$

$21 = x(2x - 11)$

$21 = 2x^2 - 11x$

$2x^2 - 11x - 21 = 0$

Product = -42, Sum = -11 } -14 and 3

$2x^2 - 14x + 3x - 21 = 0$

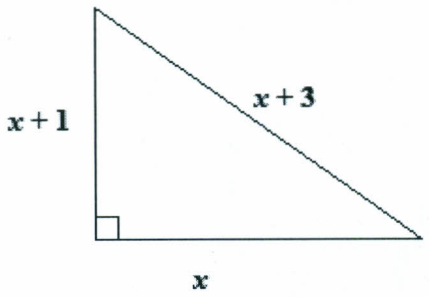
$2x(x-7) + 3(x-7) = 0$

$(x-7)(2x+3) = 0$

$x = 7$ or $x = -3/2$. rejected

so $h = 2(7) - 11 = 3$

9) The value of x in the figure below is:



a) $2+2\sqrt{3}$

b) $2\pm2\sqrt{3}$

c) $4+\sqrt{48}$

d) $2-2\sqrt{3}$

$(x+3)^2 = x^2 + (x+1)^2$

$x^2+6x+9 = x^2 + x^2+2x+1$

$x^2-4x-8=0$

$\Delta = 16-4(1)(-8)$

$= 16+32$

$= 48$

$x = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$

$x = 2 \pm 2\sqrt{3}$

but $2-2\sqrt{3}$ is $<0 \Rightarrow$ rejected

10) A solution of the equation $a^2 - b^2 + 10b - 25 = 0$ is:

a) $a = b-5$

b) $b = \frac{5}{2}$

c) $a = b$

d) $a = -b$

$a^2 - (b^2 - 10b + 25) = 0$

$a^2 - (b-5)^2 = 0$

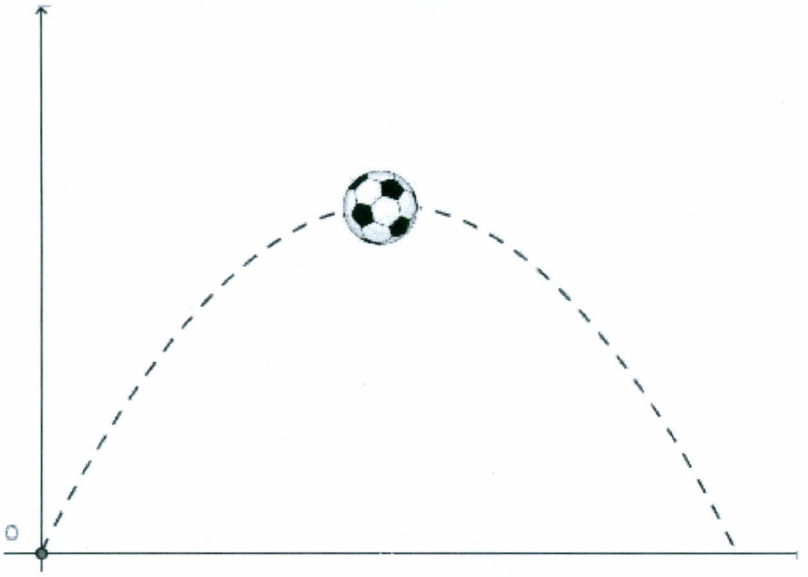
$(a+b-5)(a-b+5) = 0$

$a = b-5$ or $a = 5-b$

II- A football is kicked from point O(0,0) towards a goal to the right of O. The ball follows a path that can be modeled by the equation $h = \dfrac{-x^2 + 8x}{4}$

where h represents the height of the ball above the ground in meters (m) and x represents the horizontal distance of the ball from O also measured in meters (m).

(15 points)



1) Find the height of the ball above the ground when the ball is at 2m away from O.

$h = \dfrac{-2^2+8(2)}{4} = \dfrac{-4+16}{4} = 12/4 = 3$ m

2) When will the ball hit the ground for the first time?

$h = 0$ (hit the ground)

$\Rightarrow \dfrac{-x^2+8x}{4} = 0 \Rightarrow 8x - x^2 = 0$

$x(8-x) = 0 \quad x = 0$ or $\boxed{x = 8 \text{ m}}$ ✓

3) Using "completing the square" method, find the horizontal distance for which the ball reaches a height of $\dfrac{5}{2}$ m above the ground.

$h = \dfrac{5}{2}$

$\dfrac{5}{2} = \dfrac{-x^2+8x}{4} \Rightarrow -2x^2 + 16x = 20$

$\Rightarrow x^2 - 8x = -10$

$x^2 - 8x + 10 = 0$

$x^2 - 8x + 16 + 10 = 16$

$(x-4)^2 = 6$

$x - 4 = \pm\sqrt{6}$

$x = 4 \pm \sqrt{6}$ m (both accepted).

III- A factory manufactures plastic toys and sells them in the market at a selling price S. The selling price is given by the equation $S = -2x^2 + 9x - 10$ where x represents the number of toys in hundreds of pieces (i.e 100 toys correspond to $x = 1$, 200 toys correspond to $x = 2$, etc….) The cost of producing a toy is expressed by the equation $C = (2-x)^2 - 2x + 4$ **(12 points)**

1) Factorize each of S and C.

$S = -2x^2 + 9x - 10 \qquad P = 20 \qquad S = 9$

$= -2x^2 + 4x + 5x - 10$

$= -2x(x-2) + 5(x-2)$

$= (x-2)(-2x+5)$

$C = (2-x)^2 - 2(x-2)$

$= (x-2)^2 - 2(x-2)$

$= (x-2)[(x-2)-2] = (x-2)(x-4)$

2) Knowing that the profit is the difference between the selling price and cost (profit = selling price – cost), show that the profit P is expressed as $P = -3(x-2)(x-3)$.

$P = S - C$

$= (x-2)(-2x+5) - (x-2)(x-4)$

$= (x-2)[-2x+5 - x+4]$

$= (x-2)(-3x+9)$

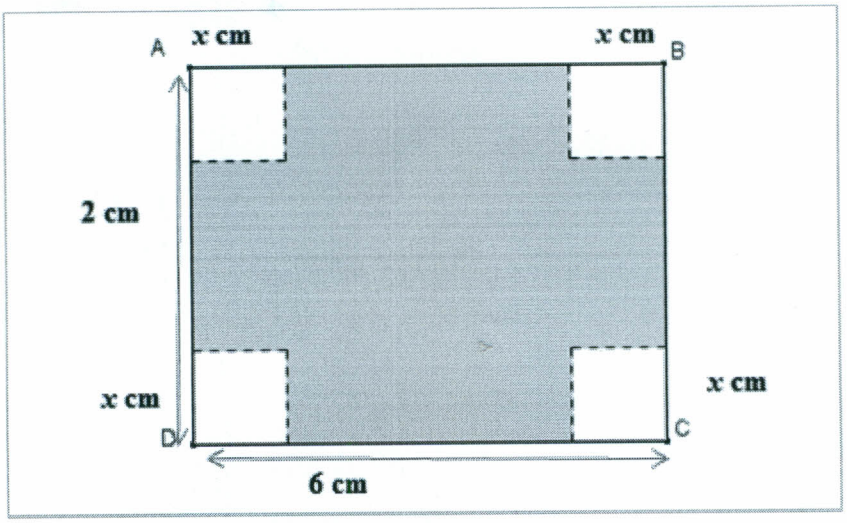
$= -3(x-2)(x-3)$

3) Find the profit made when 200 toys are sold. Comment on your result.

$P(2) = -3(2-2)(2-3) = 0$

No profit is made.

IV- In the figure below, ABCD is a rectangular piece with AD = 2 cm and DC= 6cm. Squares of sides x cm are cut from each corner. **(5 points)**



(figure not drawn to scale)

Express the area of the shaded region in fully factorized form.

Area of ABCD = $6 \times 2 = 12$

Area of small white square = x^2

Area of shaded region = $12 - 4x^2$

$= 4(3 - x^2)$

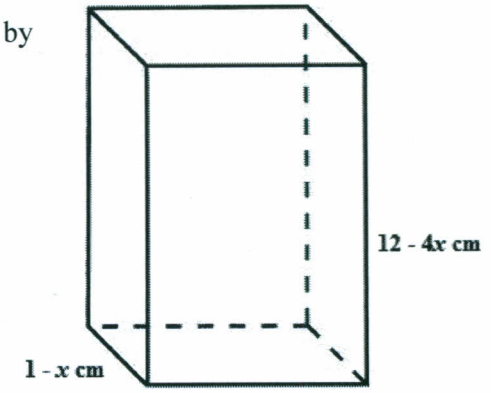
$= 4(\sqrt{3} - x)(\sqrt{3} + x)$ cm^2

V- Solid ***S*** shown in the figure below is a square based prism having the dimensions $l =$ $w = 1- x$ cm and $h = 12- 4x$ cm, where l represents the length, w the width, and h the height. **(12 points)**

1) Knowing that the volume of a cuboid is given by the formula $V = l \times w \times h$, write the volume of cuboid ***S*** in factorized form.

$V = (1-x)(1-x)(12-4x)$

$= 4(1-x)^2(3-x)$



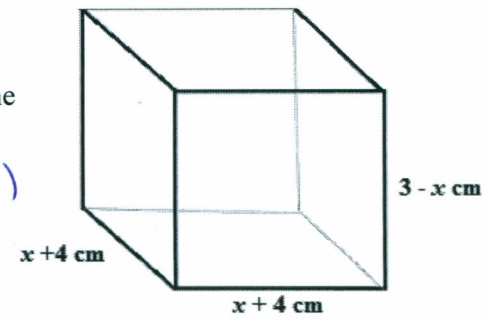
The cuboid contains 9 small cuboid blocks each of dimensions $l = w = x + 4$ cm and $h = 3- x$ cm.

2) Show that the volume of each small block is $(x+4)^2(3-x)$ cm^3, and then deduce the volume of the 9 small blocks.

$V = (x+4)(x+4)(3-x) = (x+4)^2(3-x)$

$\Rightarrow$ 9 small blocks have volume

$V = 9(x+4)^2(3-x)$



3) Show that the volume of empty space in ***S*** is $-5(3-x)(x+14)(x+2)$ cm^3.

Volume of empty space $= 4(1-x)^2(3-x) - 9(x+4)^2(3-x)$

$= (3-x)\left[4(1-x)^2 - 9(x+4)^2\right]$

$= (3-x)\left[(2-2x)^2 - (3x+12)^2\right]$

$= (3-x)(2-2x+3x+12)(2-2x-3x-12)$

$= (3-x)(x+14)(-5x-10)$

$= -5(3-x)(x+14)(x+2)$

4) Deduce the volume of empty space when $x =$ -3.

$V(3) = -5(3-(-3))(-3+14)(-3+2)$

$= -5(6)(11)(-1)$

$= 30 \times 11 = 330$ cm^3.