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An Introduction to JuMP.jl

JuDO Meeting Presentation

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What is JuMP?

- JuMP is a modelling language for mathematical optimization in Julia. It allows us to easily formulate optimization problems into structures that solvers can actually parse.
- JuMP is **NOT** a solver. It interfaces with different solvers through MathOptInterface.jl to solve optimization problems.

Official JuMP Tutorials

`https://jump.dev/JuMP.jl/stable/tutorials/getting_started/introduction/`

Solver Examples

Solver packages that will be used in these examples:

- **IPOPT**: Widely used interior point optimizer for continuous nonlinear systems.
- **HiGHS**: Sparse solver package for continuous and mixed integer linear programs, as well as continuous quadratic programs. Uses revised simplex, interior point, and active set solvers for different problems.

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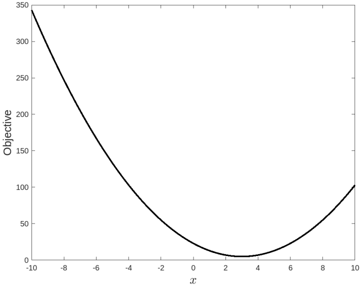
Example 1: 1D Minimization

Consider a simple 1-dimensional optimization problem:

$$\min_{x} \; 2(x-3)^2 + 5$$



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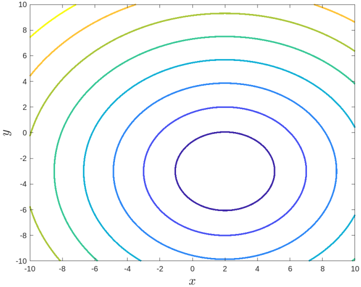
Example 2: Multivariable Problem

We can define optimization problems in more than one variable:

$$\min_{x,y} \, (x-2)^2 + (y+3)^2 + 3$$



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Example 3: Constrained Optimization

Now add constraints to the problem of Example 2:

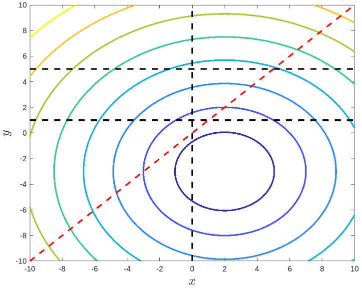
$$\min_{x,y} \; (x-2)^2 + (y+3)^2 + 3$$

s.t.

$$x \geq 0$$

$$1 \leq y \leq 5$$

$$-x + y \geq 0$$



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Example 4: Nonlinear Problem

Next, consider a problem with a nonlinear objective and constraints:

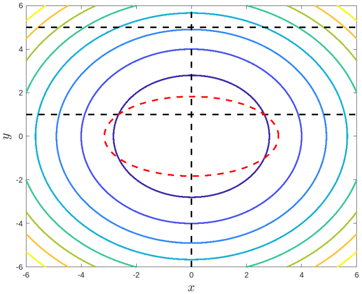
$$\min_{x,y} \; e^{(x^2+y^2)} + 5(\text{atan}(x-3)y^2)^2$$

s.t.

$$x \geq 0$$

$$1 \leq y \leq 5$$

$$x^2 + 3y^2 \geq 10$$



Modify solver properties

ex: `set_optimizer_attribute(ex4,"max_iter",5)`

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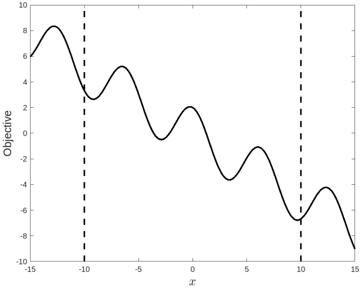
Example 5: Non-Convex Problem

Take care when solving problems which are non-convex over the domain:

$$\min_{x} \; 2\cos(x) - 0.5x$$

s.t.

$$-10 \le x \le 10$$



Set the initial guess for a variable

ex: `set_start_value(ex5[:x],5)`

Example 6: Optimal Control Problem

Consider the system with dynamics

$$\dot{x} = v \qquad \dot{v} = u$$

$$\text{s.t} \quad 0 \leq x \leq 10 \qquad 0 \leq y \leq 1 \qquad -0.3 \leq u \leq 0.5$$

Starting from $x = 0$ and $v = 0$, find the control input trajectory $\mathbf{u}$ that reaches the state $x = 10$, $v = 0$ in minimum time.

The solution presented transcribes the problem using a uniform mesh, piecewise constant inputs, and dynamics enforced through collocation at the mesh nodes using a midpoint rule.