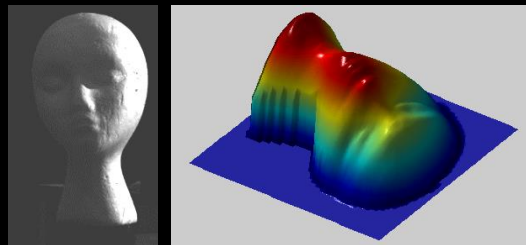


CS4495/6495

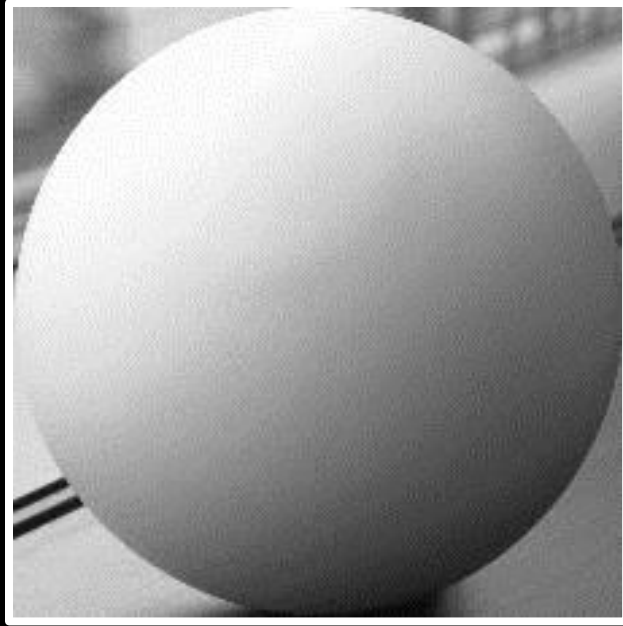
Introduction to Computer Vision

5C-L1 *Shape from shading*



Thanks to Srinivasa Narasimhan, Shree Nayar, David Kriegman, Marc Pollefeys

Shape from shading



Shading as a cue for shape reconstruction

Shape from shading/lighting

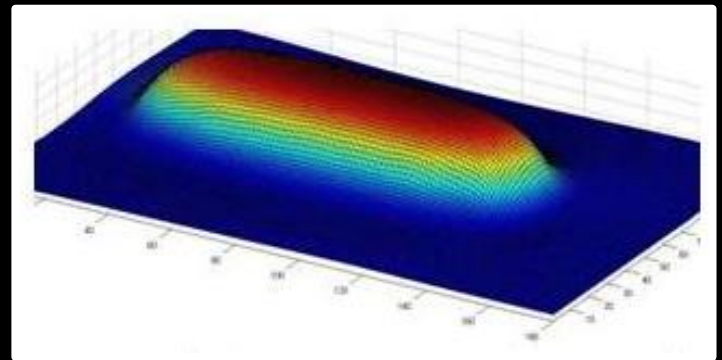
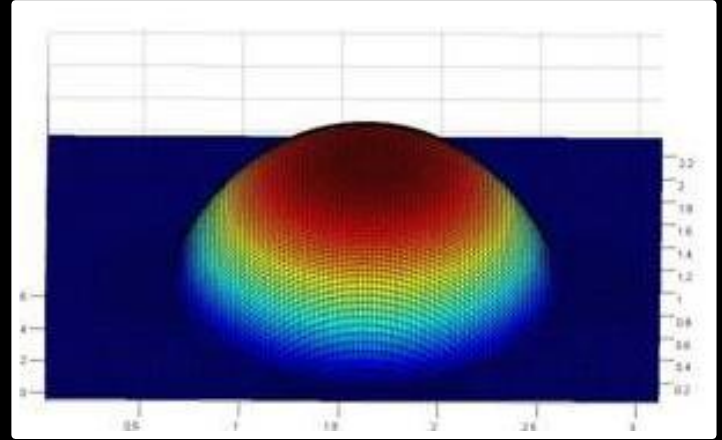
What is the relation between intensity and shape?

- Need to look at the reflectance function
- *Reflectance Map*

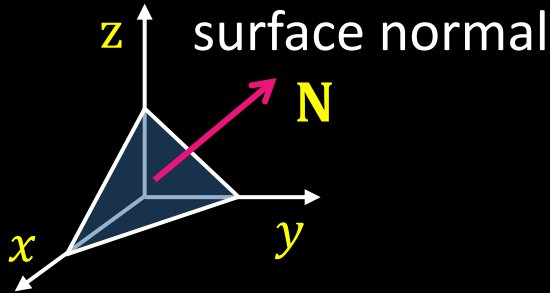
Surface normals: A bit of math

- Let's assume we have a surface $z(x, y)$
- We can define the following:

$$-\frac{\partial z}{\partial x} = p \quad -\frac{\partial z}{\partial y} = q$$



Surface Normal: A bit more math

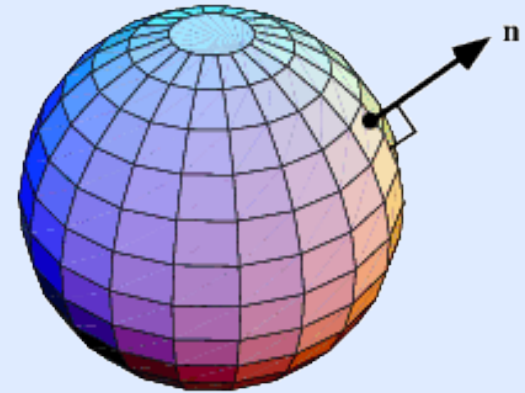
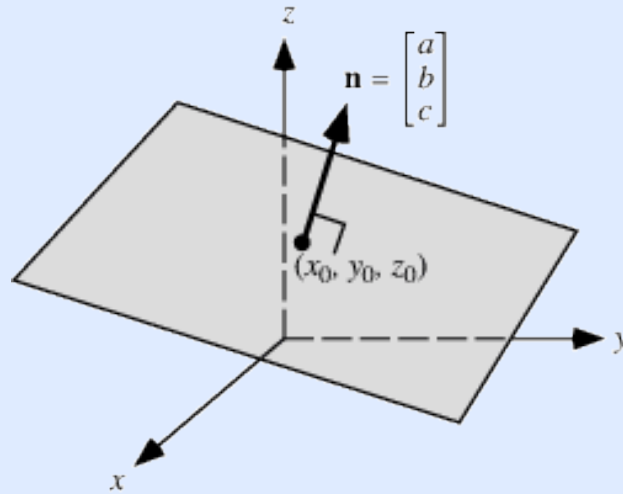
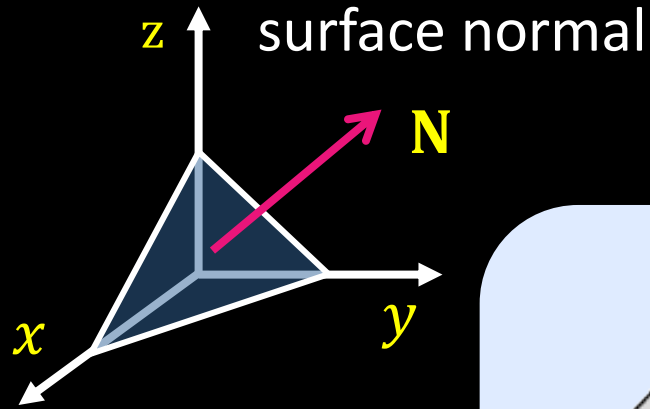


Suppose we have a point on the surface. We can define two tangents:

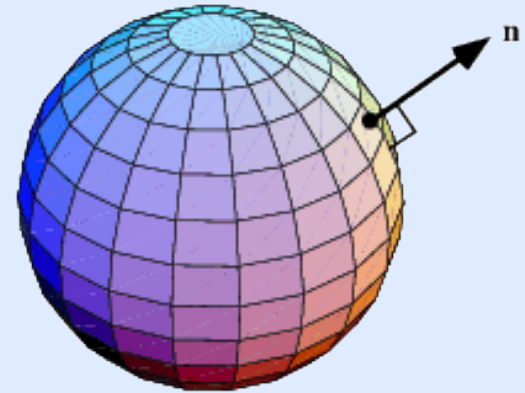
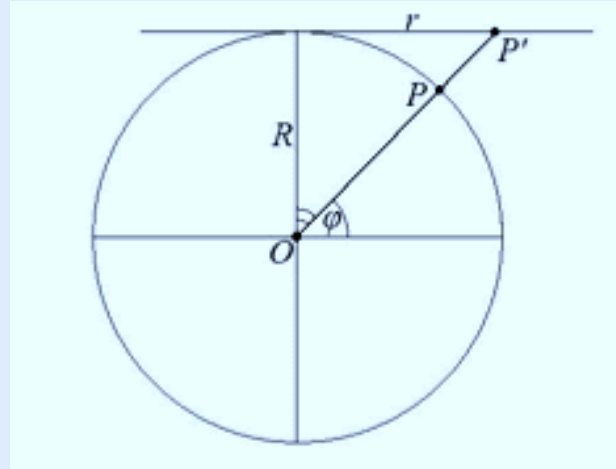
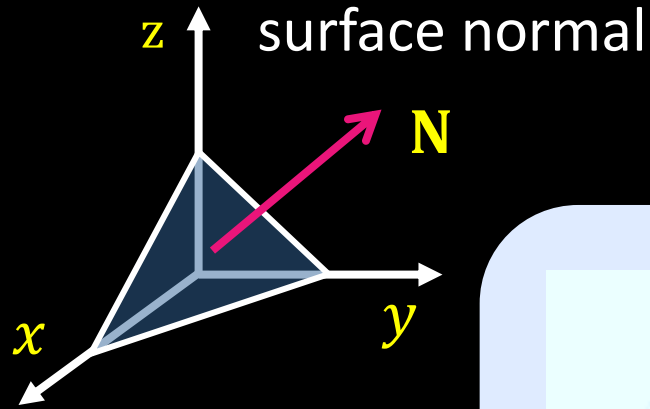
$$t_x = (1, 0, -p)^T \text{ and } t_y = (0, 1, -q)^T$$

$$\mathbf{n} = \frac{N}{\|N\|} = \frac{t_x \times t_y}{\|t_x \times t_y\|} = \frac{1}{\sqrt{p^2 + q^2 + 1}} (p, q, 1)^T$$

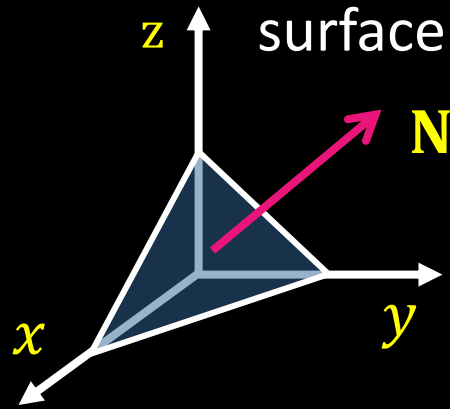
Surface Normal: Gaussian sphere



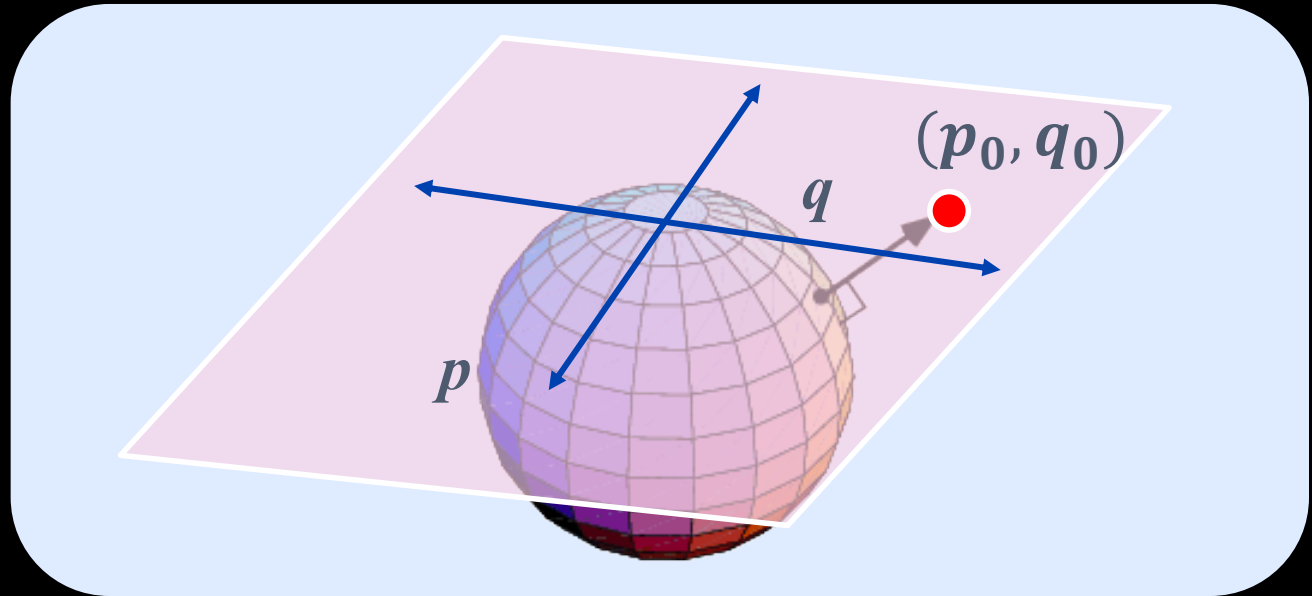
Surface Normal: Gradient space projection



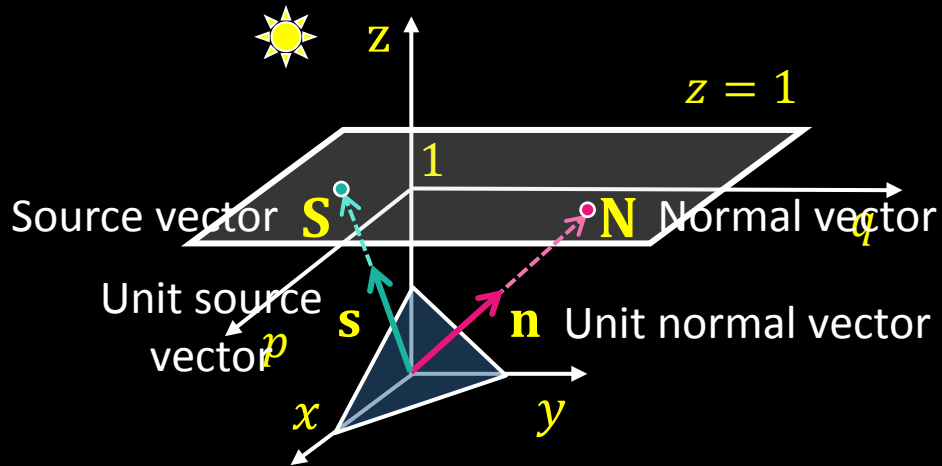
Surface Normal: Gradient space projection



$z = 1$ plane is called Gradient Space (pq plane)



Gradient Space of Source and Normal



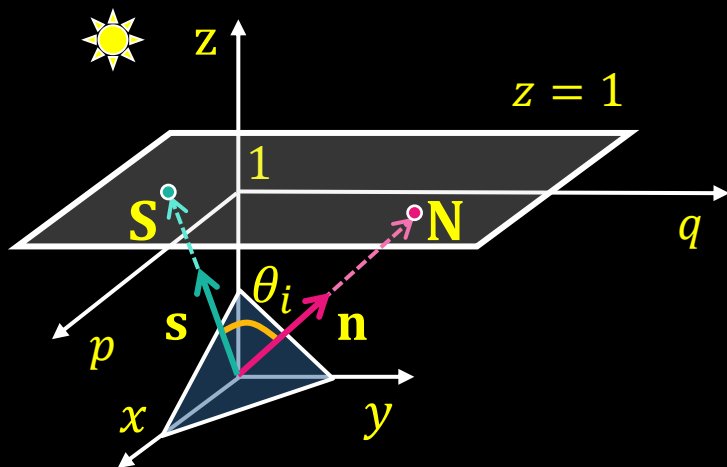
Unit normal vector:

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

Unit source vector:

$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_s, q_s, 1)}{\sqrt{p_s^2 + q_s^2 + 1}}$$

Gradient Space of Source and Normal



Unit normal vector:

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

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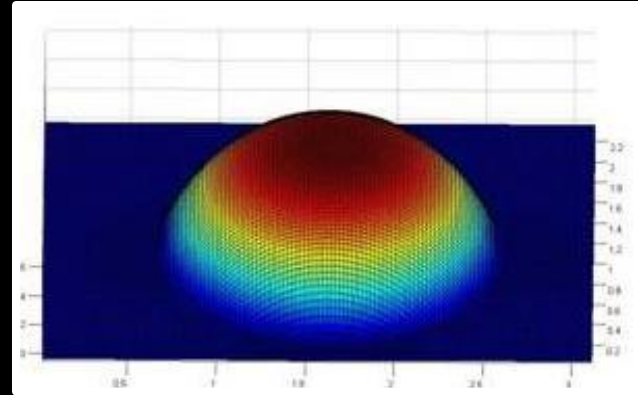
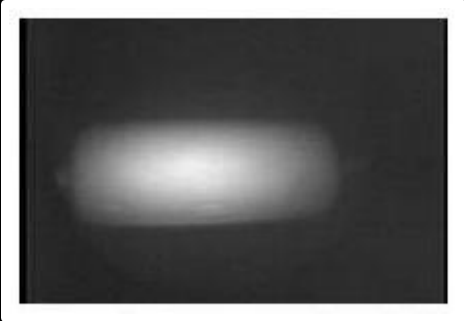
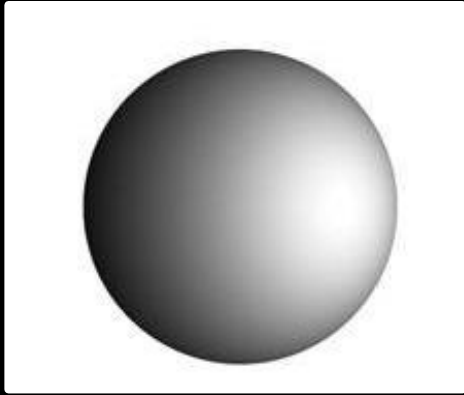
$$\cos\theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2 + q^2 + 1} \sqrt{p_s^2 + q_s^2 + 1}}$$

Shape from shading: Problem definition

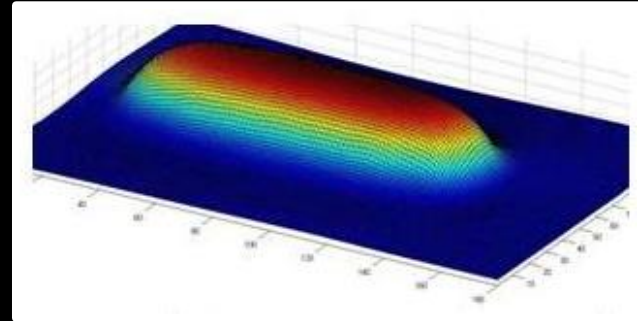
Input: 1 or more images

Output: 3D shape of object

$I(x, y)$



$Z(x, y)$



Reflectance Map

Relates image brightness $I(x, y)$ to surface orientation (p, q) for *given* source direction and *surface reflectance*

Reflectance Map: Lambertian case

Terms:

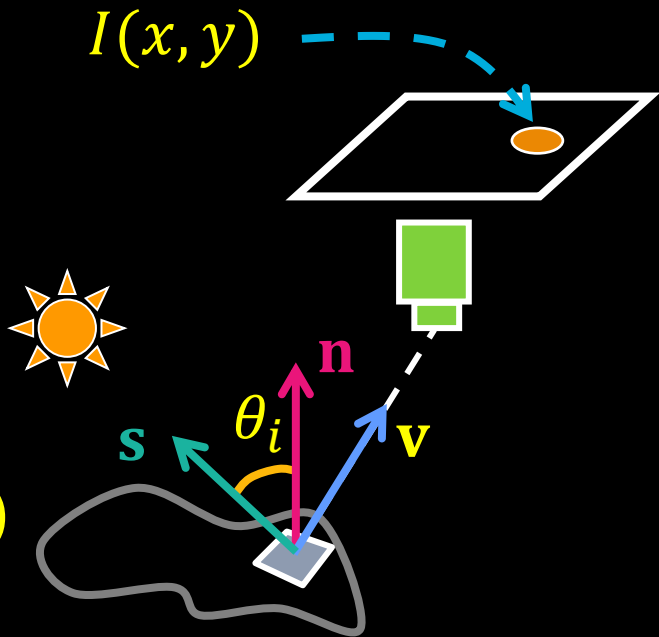
k : source brightness

ρ : surface albedo (reflectance)

Image brightness:

$$I = \rho \cdot k \cdot \cos\theta_i = \rho \cdot k (\mathbf{n} \cdot \mathbf{s})$$

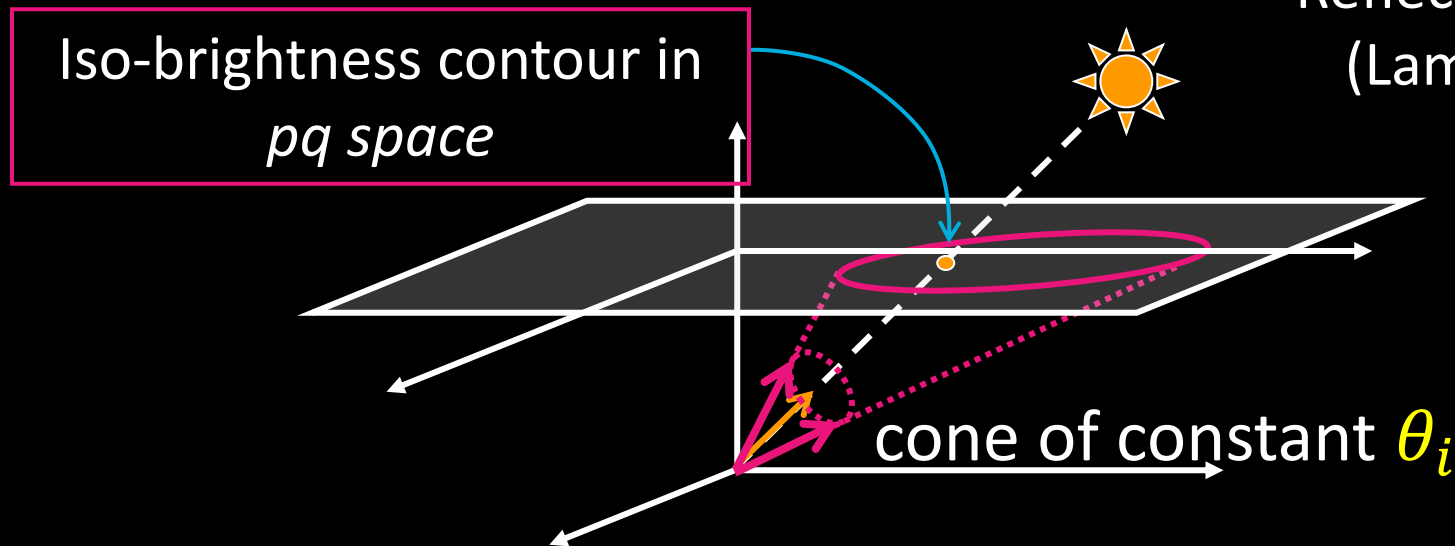
Let $\rho \cdot k = 1$ then $I = \cos\theta_i = \mathbf{n} \cdot \mathbf{s}$



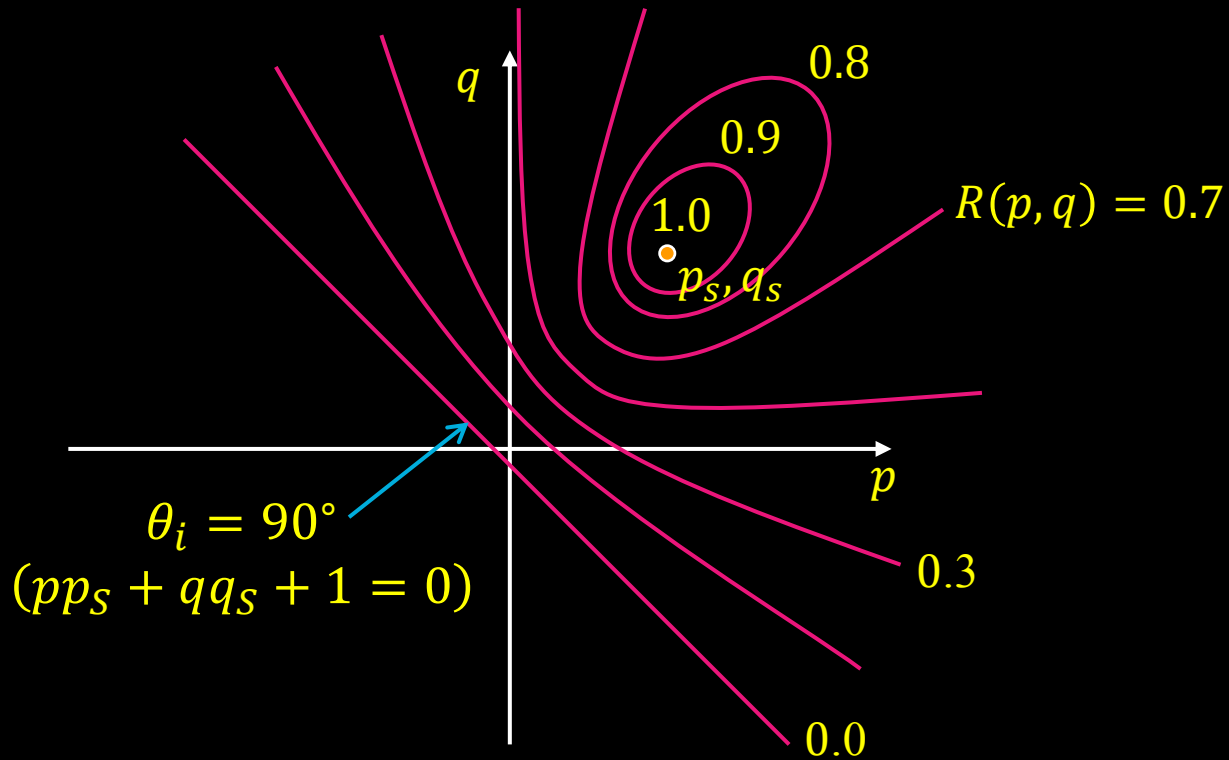
Reflectance Map: Lambertian case

$$I = \cos\theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(p p_s + q q_s + 1)}{\sqrt{p^2 + q^2 + 1} \sqrt{p_s^2 + q_s^2 + 1}} = \boxed{R(p, q)}$$

Reflectance Map (Lambertian)



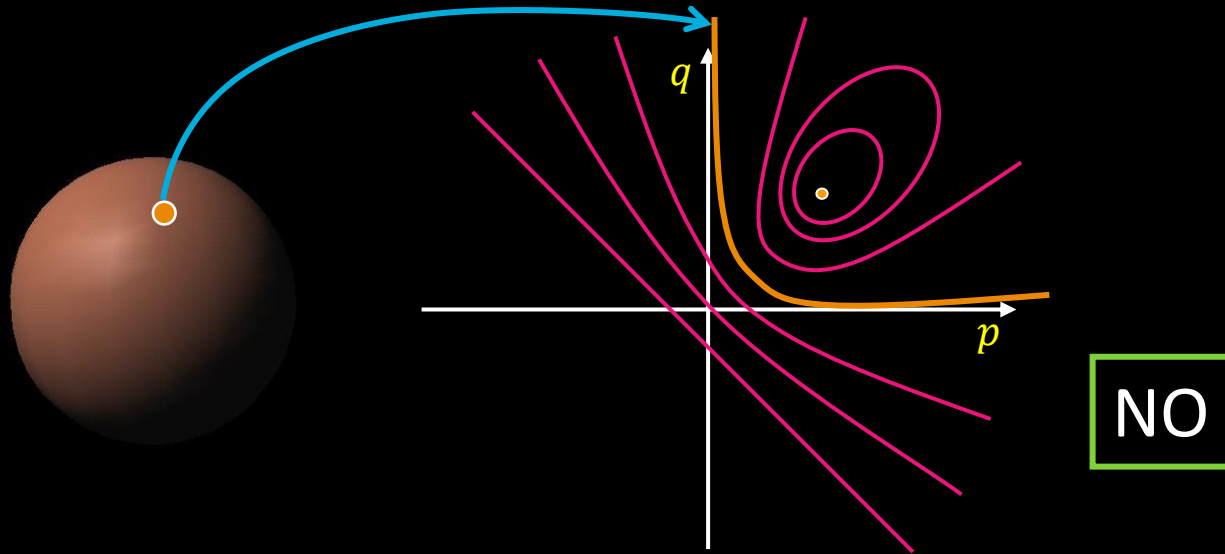
Iso-brightness contours



Note: $R(p, q)$ is maximum when $(p, q) = (p_s, q_s)$

Shape from a *single* image?

Given $R(p, q)$ ((p_s, q_s) and surface reflectance) can we determine (p, q) uniquely for each image point?



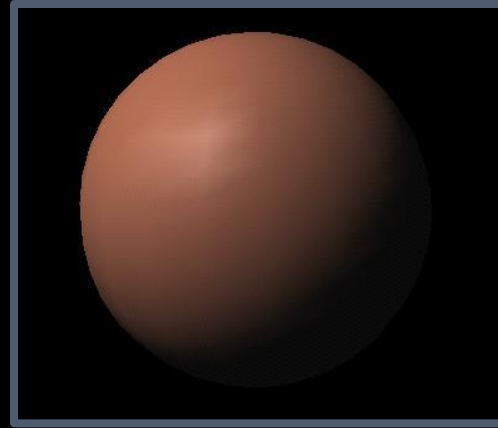
Shape from Shading

Need more information:

- Add more constraints: *Shape-from-shading*
- Take more images: *Photometric stereo*

Shape from shading

Given a single image of an object with *known surface reflectance* taken under *a known light source*, can we recover its shape?



Shape from shading

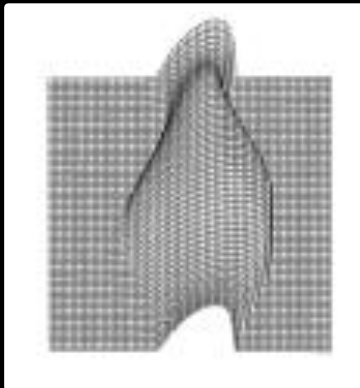
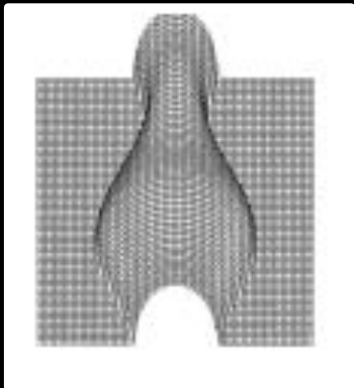
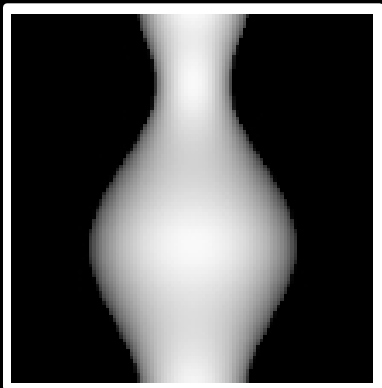
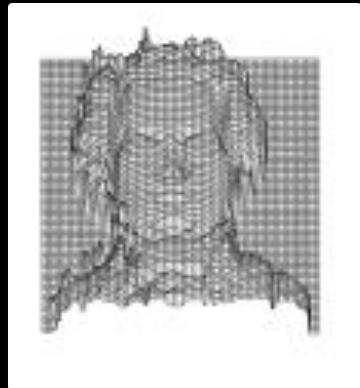
Given $R(p, q)$ ((p_s, q_s) and surface reflectance) can we determine (p, q) uniquely for each image point?

- Assume shape along the occluding boundary is known
- Constraints on neighboring normals
 - *integrability*
- Smoothness

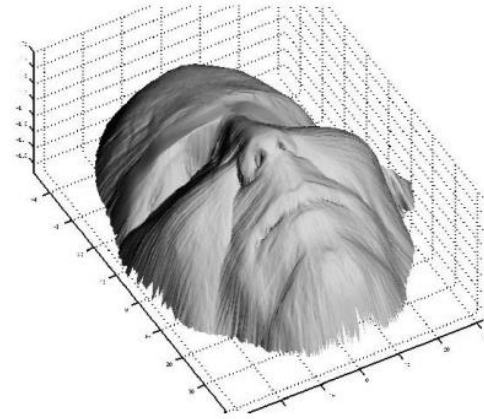
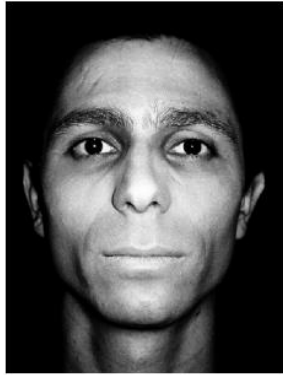


Yes, a slightly ugly optimization

Synthetic results



Shape from Shading: “Real” Results



- These *single* image methods work poorly in practice
- Why? The assumptions are quite restrictive

Shape from Shading

Need more information:

- Add more constraints: *Shape-from-shading*
- Take more images: *Photometric stereo*

Photometric stereo

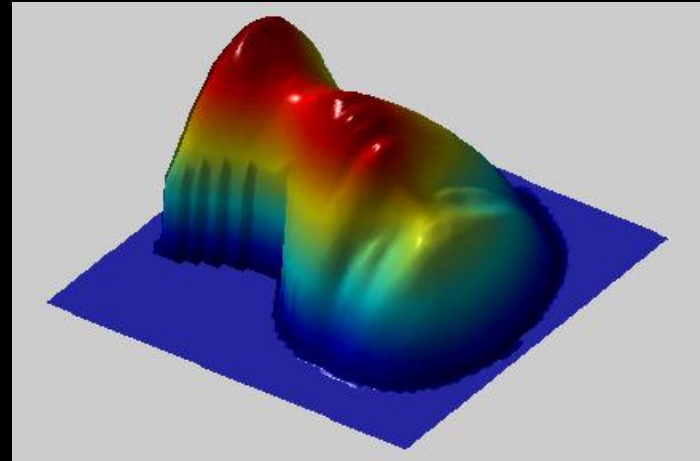
Input: Several images

- Same object
- Different lightings
- Same pose



Output:

- 3D shape of object
- Albedo at (x, y)



Photometric stereo

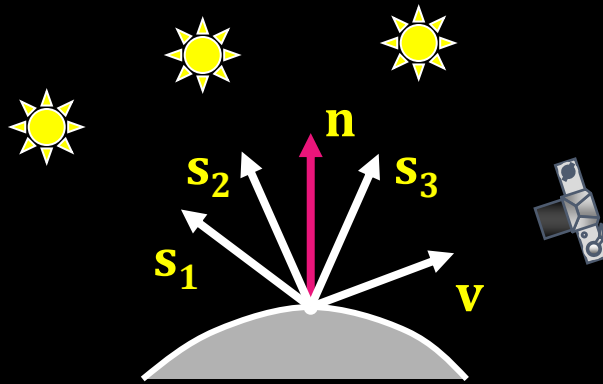


Image brightness:

$$I = \rho \cdot k \cdot \cos\theta_i = \rho \mathbf{n} \cdot \mathbf{s}$$

where ($k = 1$)

$$I_1 = \rho \mathbf{n} \cdot \mathbf{s}_1$$

$$I_2 = \rho \mathbf{n} \cdot \mathbf{s}_2$$

$$I_3 = \rho \mathbf{n} \cdot \mathbf{s}_3$$

Write this as a matrix equation:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \rho \begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \mathbf{s}_3^T \end{bmatrix} \mathbf{n}$$

Lambertian!

Solving the equations: Linear

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{\mathbf{I}} = \underbrace{\begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \mathbf{s}_3^T \end{bmatrix}}_{\mathbf{S}} \underbrace{\rho \mathbf{n}}_{\tilde{\mathbf{n}}}$$

$3 \times 1 \quad 3 \times 3 \quad 3 \times 1$

I and **S** are known

$$\tilde{\mathbf{n}} = \mathbf{S}^{-1} \mathbf{I}$$

inverse

$$\rho = |\tilde{\mathbf{n}}|$$

$$\mathbf{n} = \frac{\tilde{\mathbf{n}}}{|\tilde{\mathbf{n}}|} = \frac{\tilde{\mathbf{n}}}{\rho}$$

Adding more light sources

Get better results by using more (M) lights:

$$\begin{bmatrix} I_1 \\ \vdots \\ I_M \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1^T \\ \vdots \\ \mathbf{s}_M^T \end{bmatrix} \rho \mathbf{n}$$

Least squares solution:

$$\mathbf{I} = \mathbf{S} \tilde{\mathbf{n}} \quad \leftarrow M \times 1 = (\underline{M \times 3})(3 \times 1)$$

$$\mathbf{S}^T \mathbf{I} = \mathbf{S}^T \mathbf{S} \tilde{\mathbf{n}}$$

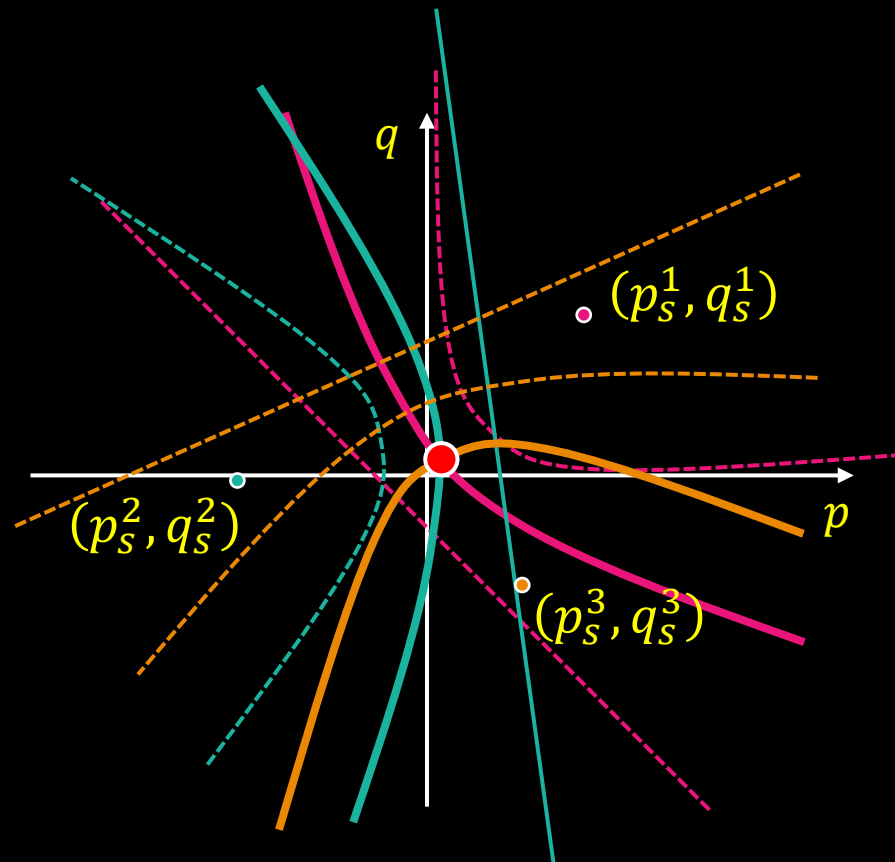
$$\tilde{\mathbf{n}} = (\mathbf{S}^T \mathbf{S})^{-1} \mathbf{S}^T \mathbf{I}$$

$$\min \|\mathbf{I} - \mathbf{S} \tilde{\mathbf{n}}\|_2^2$$

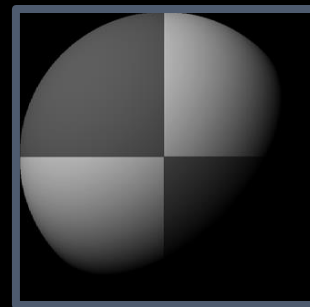
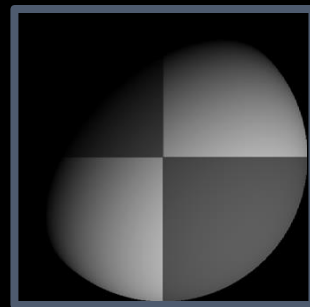
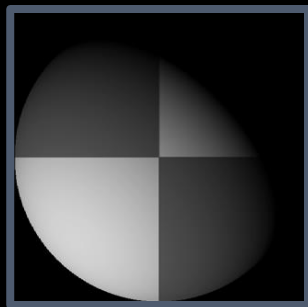
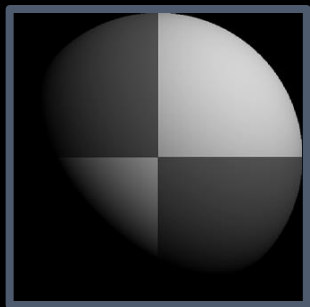
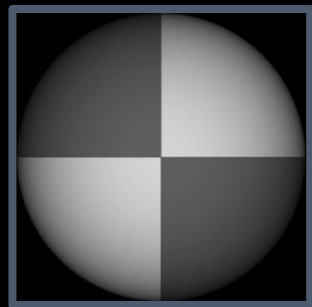
Moore-Penrose pseudo inverse

Solve for ρ , \mathbf{n} as before

Photometric stereo: pq space



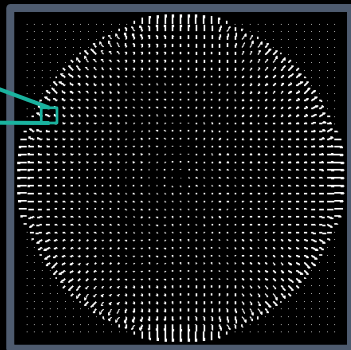
Results: Lambertian sphere



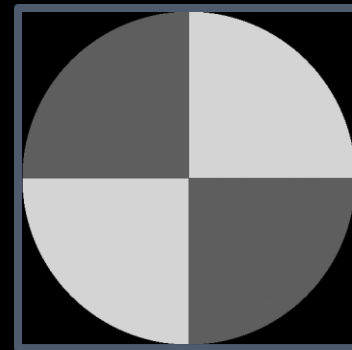
Input Images



Needles are
projections of
surface normal on
image plane

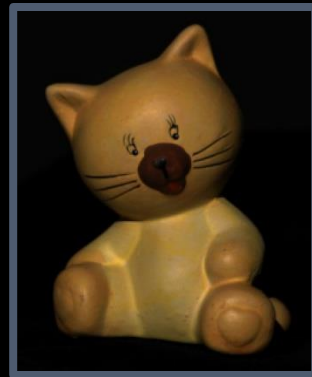
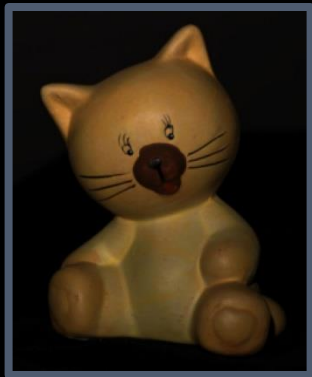


Estimated Surface Normals



Estimated Albedo

Photometric stereo: Lambertian toy



Input Images

Surface
Normals



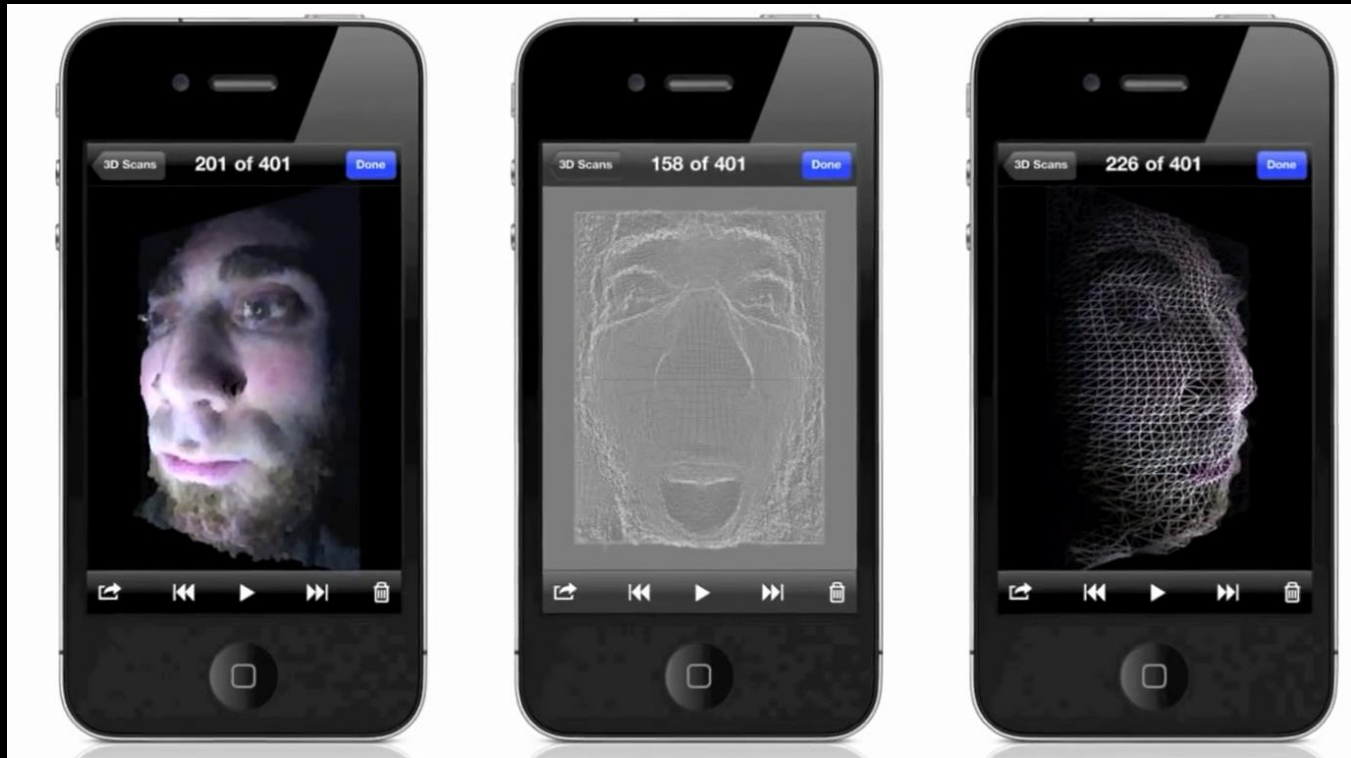
Estimated Surface Normals



Albedo

Estimated Albedo

And from GT: www.trimensional.com



Photometric stereo: Limitations

Big problems

- Doesn't work for shiny things, semi-translucent things
- Shadows, inter-reflections

Smaller problems

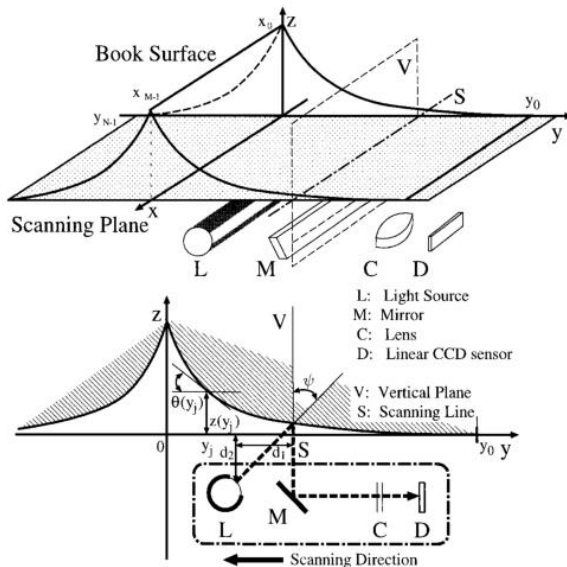
- Camera and lights have to be distant
- Calibration requirements
 - Measure light source directions, intensities
 - Camera response function

A real application that works

Shape from Shading with Interreflections Under a Proximal Light Source: Distortion-Free Copying of an Unfolded Book

TOSHIKAZU WADA[†], HIROYUKI UKIDA[†] AND TAKASHI MATSUYAMA^{*}

*Department of Information Technology, Faculty of Engineering, Okayama University,
3-1-1, Tsushima Naka, Okayama 700, Japan*



A real application that works

preliminary land use in the nine cell area (Figure 8.4b) of land use (Figure 8.4b) as indicated by the numbers of categories found. After slope and soil conditions are resolved into just a handful of categories, each cell is again based on the joint condition of slope and soil, and the minimum value of the two numbers for a cell is the result.

The fourth category includes the creation of measures of properties, like distance or narrowness of regions. It also includes determination of slope and aspect from elevation data by looking at the difference between a cell's value and that of its neighbours. Gradients may also be computed for other variables, like percentage population with college degrees, that are extended neighbourhoods or zones can be examined. For spatial properties like length or area of objects, or gradients, accuracies of properties with increasing distance from a focal point can be determined by spreading outwards in distance increments and counting the numerical values for an attribute for the cells falling in the distance zones (Figure 8.4c).

Area and perimeter measures for homogeneous blocks of cells or other sets of contiguous units grouped into zones, perhaps by a special thematic overlay, are obtained, respectively, by adding up the exterior edges of cells in the zones. Distances between columns (horizontal differences) and application of the Pythagorean rule to right triangles are quite easily obtained arithmetically. Chapter 6, subject to error. Distances for individual cells or a zone or from a linear or point feature can be computed by cell counting operations. A third situation of selecting the complement of the overlap can be undertaken using the logical XOR operation. Complex combinations can be created by variable logical statements combining more than two attributes.

The case of undertaking such logical processing (based on set operations) with cell data arises from the use of identical spatial data units (the grid cells), and of simple binary or decimal arithmetic. In a comparison of two attributes, each of which can take on two states (presence or absence) denoted by 1 and 0 respectively, the intersection set operation (logical AND) of the two is the product of the two values (1 × 1 = 1, 0 × 1 = 0, 1 × 0 = 0, 0 × 0 = 0). The union operation produces values of 0, 1 or 2 for the two values. The logical OR operation is identical, also identifying the complement of the union by the zero value.

8.1.2 Spatial modelling with grid-cell data

The first of data encoding facilitates map analysis involving map overlay processing steps. Figure 8.4a serves to illustrate some of the procedures commonly encountered in working with cell data for a map overlay modelling task. Preliminary planning has produced a forecast of operations required to produce a single scale of numbers representing the potential for residential development in areas not yet built on. The data processing consists of a mixture of operations drawn from the four categories noted above.

Topography and highways is represented by incremental distance values inserted to indicate declining desirability with distance. The distance distance is in the case of the highways. Original cell values have been assigned code numbers suggesting that there are no constraints, especially the soil system. Slopes have been computed from elevation data. The variety of soil types in cells, computed by scanning the neighbourhood, is a measure of the likelihood of finding homogeneous conditions. For the relative likelihood of slope and soil types is combined by logical values that is cross-classification operation. New cell values are assigned ways to obtain the composite final scale for residential development, as shown.

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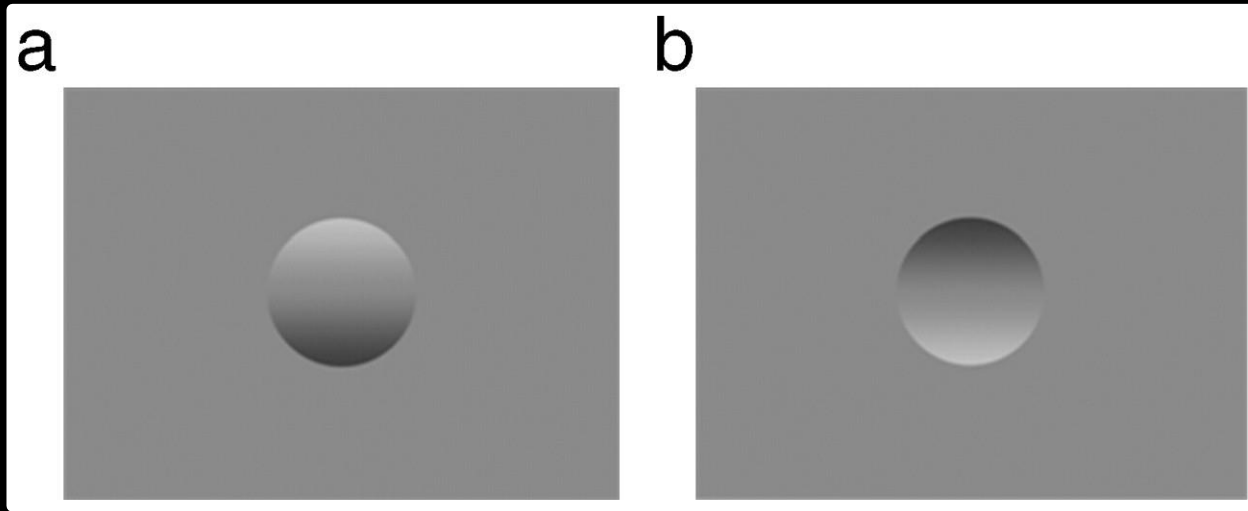
Topography and highways is represented by incremental distance values inserted to indicate declining desirability with distance or based on a minimum distance as in the case of the highways. Original cell values have been assigned code numbers suggesting that there are no constraints, especially the soil system. Slopes have been computed from elevation data. The variety of soil types in cells, computed by scanning the neighbourhood, is a measure of the likelihood of finding homogeneous conditions. For the relative influence of slope and soil types is combined by logical values that is cross-classification operation. New cell values are computed in various ways to obtain the composite final scale for residential development, as shown.

This procedure has used elementary logical operations in order to apply the reasoning. For example, the logical test logical OR has been applied to two attributes, looking for the existence of either D for the first variable, and 23 for the second, and then assigning a weight of 10 to the result of the selection. Cells with the joint condition of D and 23 are identified by the logical AND operation. A third situation of selecting the complement of the overlap can be undertaken using the logical XOR operation. Complex combinations can be created by variable logical statements combining more than two attributes.

The case of undertaking such logical processing (based on set operations) with cell data arises from the use of identical spatial data units (the grid cells), and of simple binary or decimal arithmetic. In a comparison of two attributes, each of which can take on two states (presence or absence) denoted by 1 and 0 respectively, the intersection set operation (logical AND) of the two is the product of the two values (1 × 1 = 1, 0 × 1 = 0, 1 × 0 = 0, 0 × 0 = 0). The union operation produces values of 0, 1 or 2 for the two values. The logical OR operation is identical, also identifying the complement of the union by the zero value.

Area and perimeter measures for homogeneous blocks of cells or other sets of contiguous units grouped into zones, perhaps by a special thematic overlay, are obtained, respectively, by adding up the exterior edges of cells in the zones. Distances between columns (horizontal differences) and application of the Pythagorean rule to right triangles are quite easily obtained arithmetically. Chapter 6, subject to error. Distances for individual cells or a zone or from a linear or point feature can be computed by cell counting operations. A third situation of selecting the complement of the overlap can be undertaken using the logical XOR operation. Complex combinations can be created by variable logical statements combining more than two attributes.

Human shape from shading



Thomas R et al. J Vis 2010; 10:6

Also check Ramachandran's work on Shape from Shading by Humans

<http://psy.ucsd.edu/chip/ramabio.html>