CS4495/6495
Introduction to Computer Vision

6B-L1 *Dense flow: Brightness constraint*

Motion estimation techniques

Feature-based methods

Direct, dense methods

Motion estimation techniques

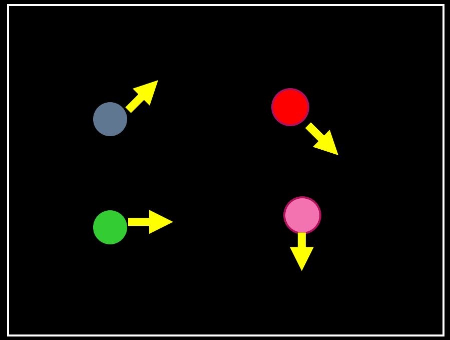
Direct, dense methods

- Directly recover image motion at each pixel from spatio-temporal image brightness variations
- Dense motion fields, but sensitive to appearance variations
- Suitable for video and when image motion is small

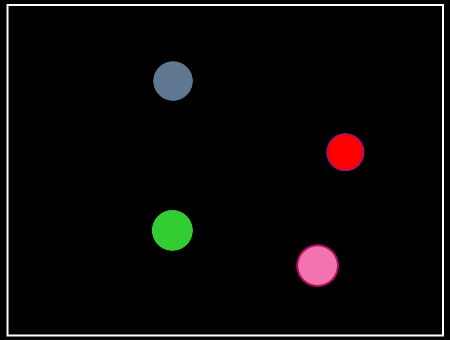
Motion estimation: Optic flow

Optic flow is the **apparent** motion of objects or surfaces

Problem definition: Optic flow

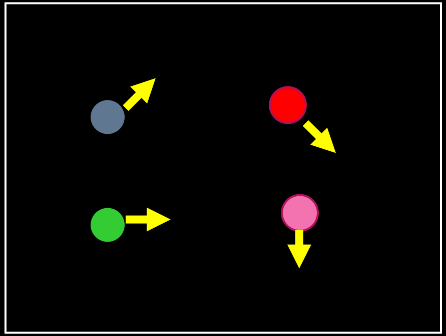


$I(x, y, t)$

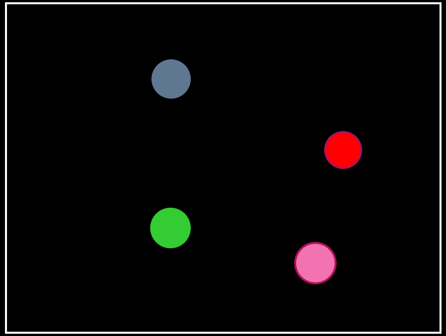


$I(x, y, t+1)$

How to estimate pixel motion
from image $I(x, y, t)$ to $I(x, y, t+1)$?



$I(x, y, t)$



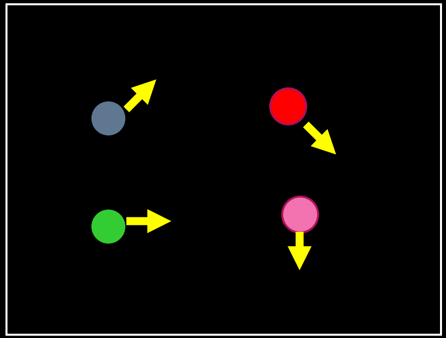
$I(x, y, t+1)$

How to estimate pixel motion from image $I(x, y, t)$ to $I(x, y, t+1)$?

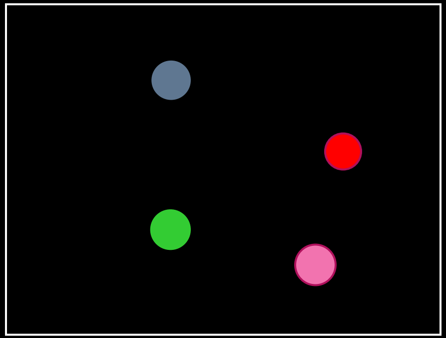
=> Solve pixel correspondence problem

- Given a pixel in $I(x, y, t)$, look for nearby pixels of the same color in $I(x, y, t + 1)$

This is the optic flow problem.



$I(x, y, t)$



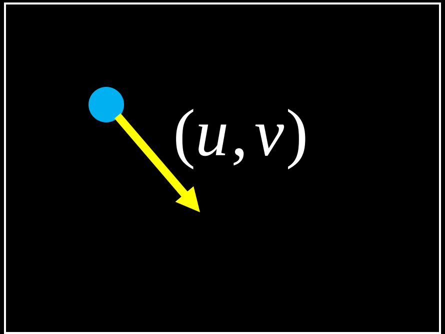
$I(x, y, t+1)$

How to estimate pixel motion from image $I(x, y, t)$ to $I(x, y, t+1)$?

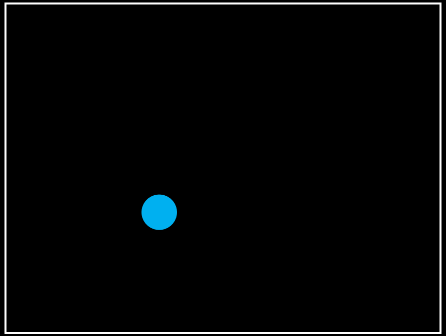
Key assumptions

- **color constancy**: a point in $I(x, y, t)$ looks the same in $I(x', y', t+1)$
  - For grayscale images, this is ***brightness constancy***
- **small motion**: points do not move very far

Optic flow constraints (grayscale images)



$I(x, y, t)$

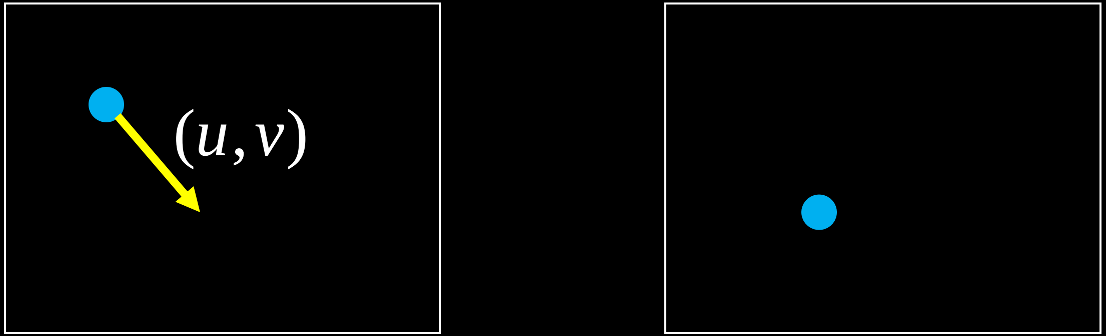


$I(x, y, t+1)$

1) Brightness constancy constraint (equation)

$$I(x, y, t) = I(x+u, y+v, t+1)$$

Optic flow constraints (grayscale images)

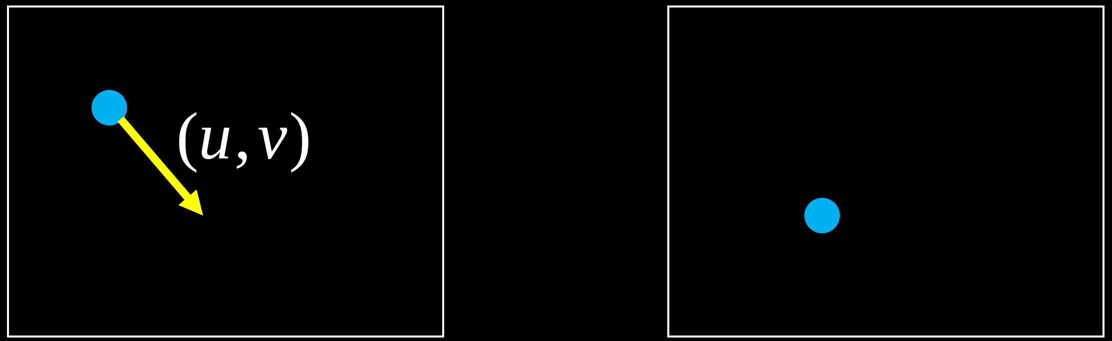


$I(x,y,t)$ $\qquad$ $I(x,y,t+1)$

1) Brightness constancy constraint (equation)

$$0 = I(x+u, y+v, t+1) - I(x,y,t)$$

Optic flow constraints (grayscale images)



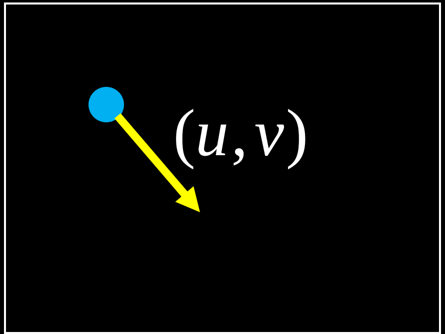
$I(x,y,t)$ $\qquad$ $I(x,y,t+1)$

2) Small motion: (u and v are less than 1 pixel, or smooth)

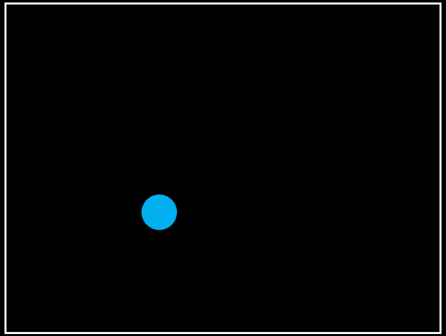
Taylor series expansion of I:

$$I(x+u,y+v) = I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + [\text{higher order terms}]$$

Optic flow constraints (grayscale images)



$I(x, y, t)$



$I(x, y, t+1)$

2) Small motion: (u and v are less than 1 pixel, or smooth)

Taylor series expansion of I:

$$I(x+u, y+v) \approx I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v$$

Combining these two equations:

$$
\begin{aligned}
0 &= I(x+u, y+v, t+1) - I(x, y, t) \\
&\approx I(x, y, t+1) + I_x u + I_y v - I(x, y, t)
\end{aligned}
$$

(Short hand: $I_x = \frac{\partial I}{\partial x}$
for *t* **or** *t+1)*

Combining these two equations:

$$
\begin{aligned}
0 &= I(x+u, y+v, t+1) - I(x, y, t) \\
&\approx I(x, y, t+1) + I_x u + I_y v - I(x, y, t) \\
&\approx [I(x, y, t+1) - I(x, y, t)] + I_x u + I_y v \\
&\approx I_t + I_x u + I_y v
\end{aligned}
$$

(Short hand: $I_x = \frac{\partial I}{\partial x}$
for *t* **or** *t+1*)

Combining these two equations:

$$
\begin{aligned}
0 &= I(x+u, y+v, t+1) - I(x, y, t) \\
&\approx I(x, y, t+1) + I_x u + I_y v - I(x, y, t) \\
&\approx [I(x, y, t+1) - I(x, y, t)] + I_x u + I_y v \\
&\approx I_t + I_x u + I_y v \\
&\approx I_t + \nabla I \cdot \langle u, v \rangle
\end{aligned}
$$

(Short hand: $I_x = \frac{\partial I}{\partial x}$

for *t* **or** *t+1*)

Combining these two equations:

In the limit as u and v approaches zero, this becomes exact:

$$0 = I_t + \nabla I \cdot < u, v >$$

$$0 \approx I_t + \nabla I \cdot < u, v >$$

Combining these two equations:

$$0 \approx I_t + \nabla I \cdot < u, v >$$

In the limit as u and v approaches zero, this becomes exact:

$$0 = I_t + \nabla I \cdot < u, v >$$

Brightness constancy constraint equation

$$\boxed{I_x u + I_y v + I_t = 0}$$

Gradient component of flow

$$0 = I_t + \nabla I \cdot < u, v > \quad \text{or} \quad I_x u + I_y v + I_t = 0$$

Q: How many unknowns and equations per pixel?

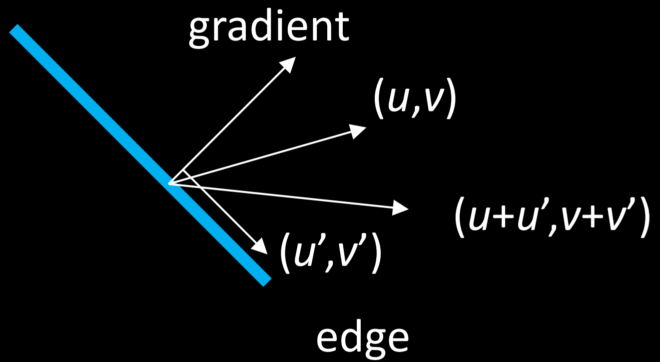
2 unknowns (u,v) but 1 equation!

Gradient component of flow

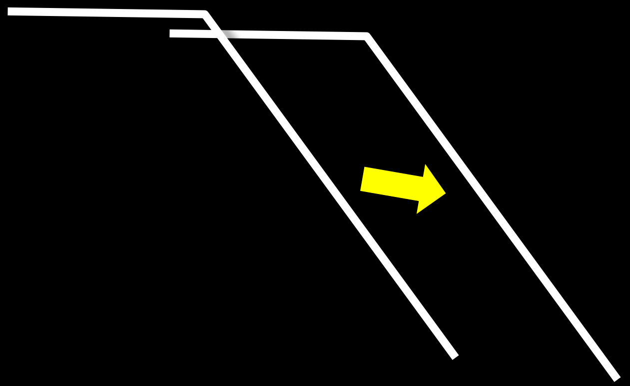
$$0 = I_t + \nabla I \cdot \langle u, v \rangle \quad \text{or} \quad I_x u + I_y v + I_t = 0$$

Intuitively, what does this constraint mean?

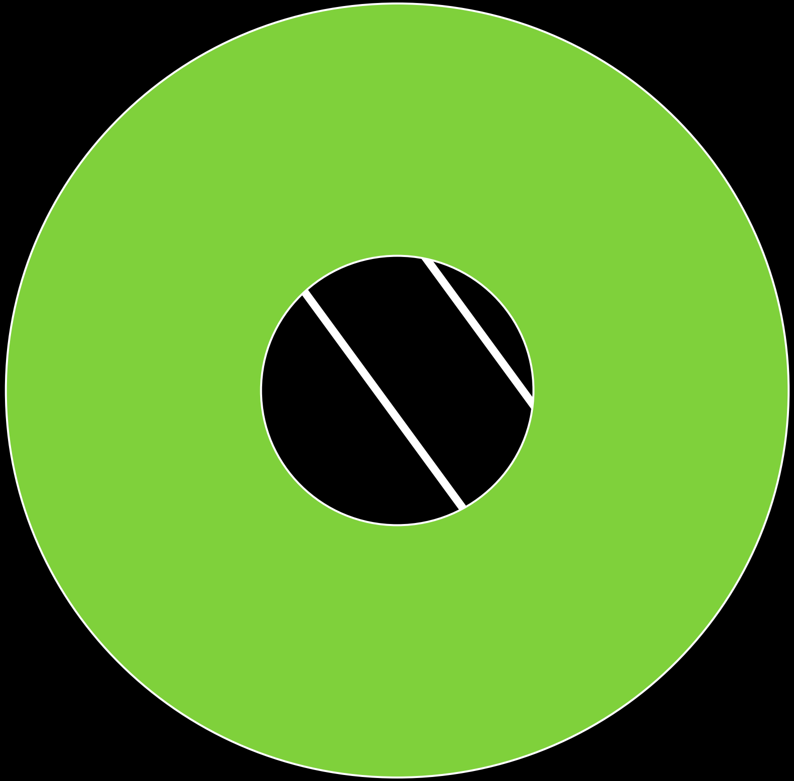
- The component of the flow in the gradient direction is determined
- The component of the flow parallel to an edge is unknown



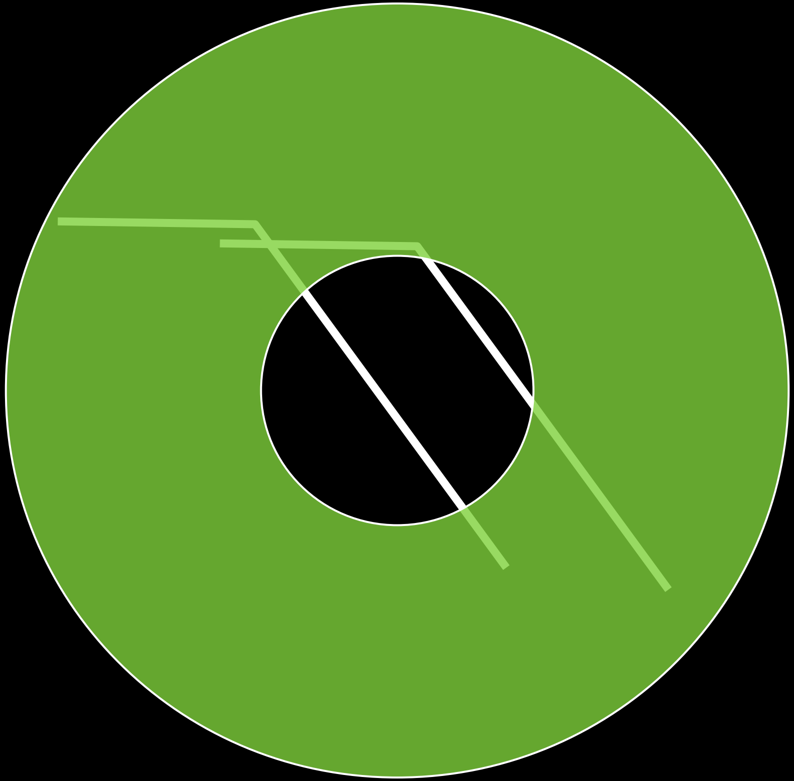
Aperture problem



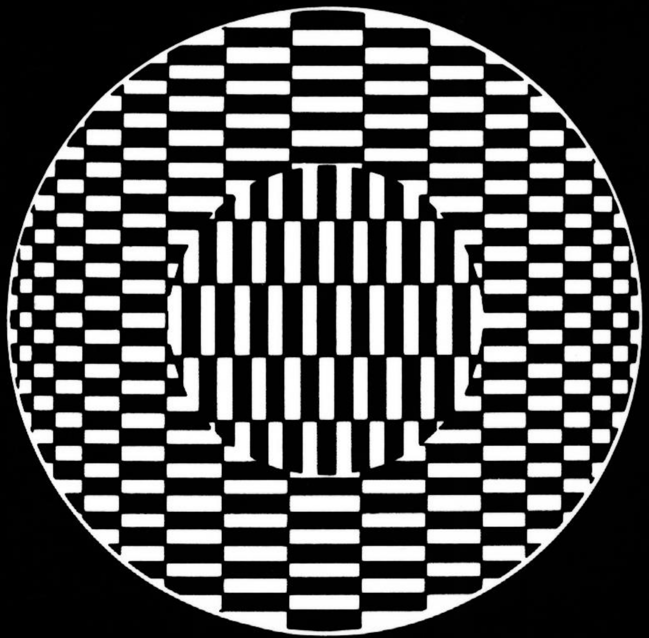
Aperture problem



Aperture problem



Apparently an aperture problem



See: http://www.cfar.umd.edu/~fer/optical/movement2.html

Gradient component of flow

Some folks say: “This explains the Barber Pole illusion”

http://www.sandlotscience.com/Ambiguous/Barberpole_Illusion.htm

http://www.liv.ac.uk/~marcob/Trieste/barberpole.html

Not quite… where do the vectors point?

(See Hildreth, a long time ago…)

No. of unknowns vs equations (pixels)

So if the brightness constraint equation gives us more unknowns than pixels, how do we recover motion?

Smooth Optical Flow (Horn & Schunk, 1980)

- Formulate Error in Optical Flow constraint:

$$e_c = \iint_{image} (I_x u + I_y v + I_t)^2 \, dx \, dy$$

- We need additional constraints (pardon the integrals)

Smooth Optical Flow (Horn & Schunk, 1980)

- Smoothness constraint: Motion field tends to vary smoothly over the image

$$e_s = \iint_{image} (u_x^2 + u_y^2) + (v_x^2 + v_y^2) \; dx\, dy$$

- Penalized for changes in u and v over image

Smooth Optical Flow (Horn & Schunk, 1980)

Given both terms:

$$e_c = \iint_{image} (I_x u + I_y v + I_t)^2 \, dx \, dy$$

$$e_s = \iint_{image} (u_x^2 + u_y^2) + (v_x^2 + v_y^2) \;\; dx \, dy$$

Smooth Optical Flow (Horn & Schunk, 1980)

Find (u, v) at each image point that minimizes:

$$e = e_s + \lambda e_c$$

weighting factor

Dense Flow: Summary

- Impose a constraint on the flow field in general to make the problem solvable
- Strength: Allows you to bias your solution with a prior (if you have one)
- But there are better ways to increase the number of equations…