CS4495/6495
Introduction to Computer Vision

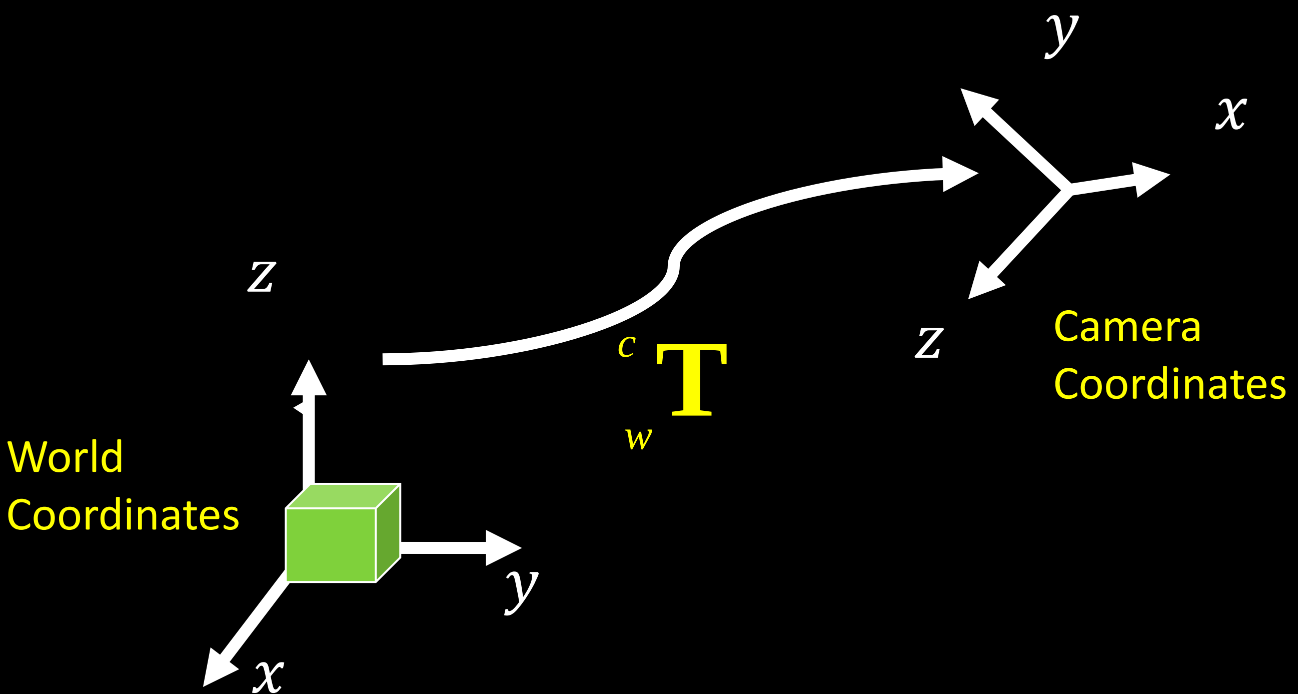
3C-L2 *Intrinsic camera calibration*

Geometric Camera calibration

Composed of 2 transformations:

- From some (arbitrary) world coordinate system to the camera's 3D coordinate system. *Extrinisic parameters (or camera pose)*

Camera Pose



From World to Camera

$$\begin{pmatrix} {}^{C}\vec{p} \\ \\ \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} - & - & - \\ - & {}^{C}_{W}R & - \\ - & - & - \end{matrix}} & \boxed{\begin{matrix} | \\ {}^{C}_{W}\vec{t} \\ | \end{matrix}} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{pmatrix} \begin{pmatrix} {}^{W}\vec{p} \\ \\ \end{pmatrix}$$

Homogeneous coordinates

From world to camera is the ***extrinsic*** *parameter matrix (4x4)*

(sometimes 3x4 if using for next step in projection – not worrying about inversion)

Geometric Camera calibration

Composed of 2 transformations:

- From some (arbitrary) world coordinate system to the camera’s 3D coordinate system. *Extrinisic parameters (or camera pose)*

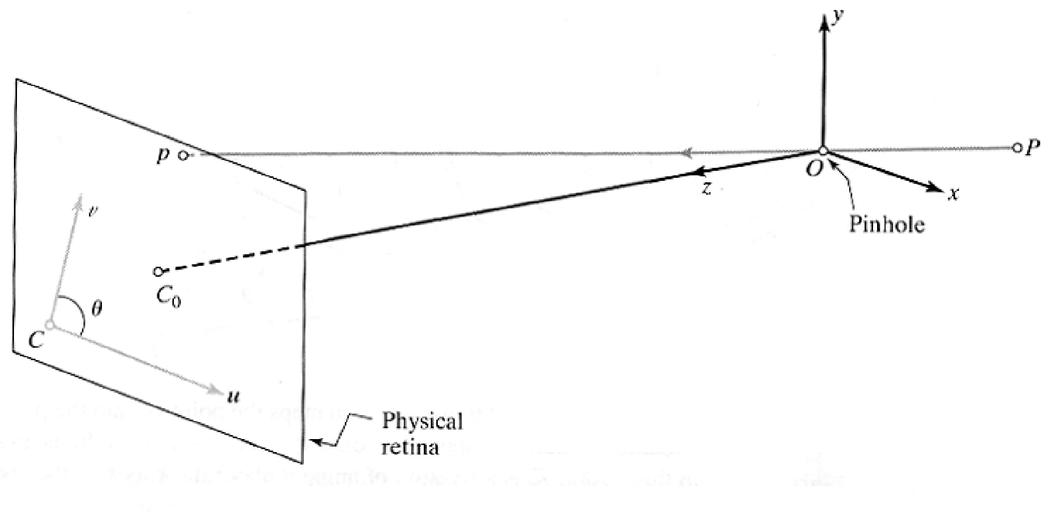
- From the 3D coordinates in the camera frame to the 2D image plane via projection.
  Intrinisic parameters

Ideal intrinsic parameters

Ideal Perspective projection:

$$u = f\,\frac{x}{z}$$

$$v = f\,\frac{y}{z}$$

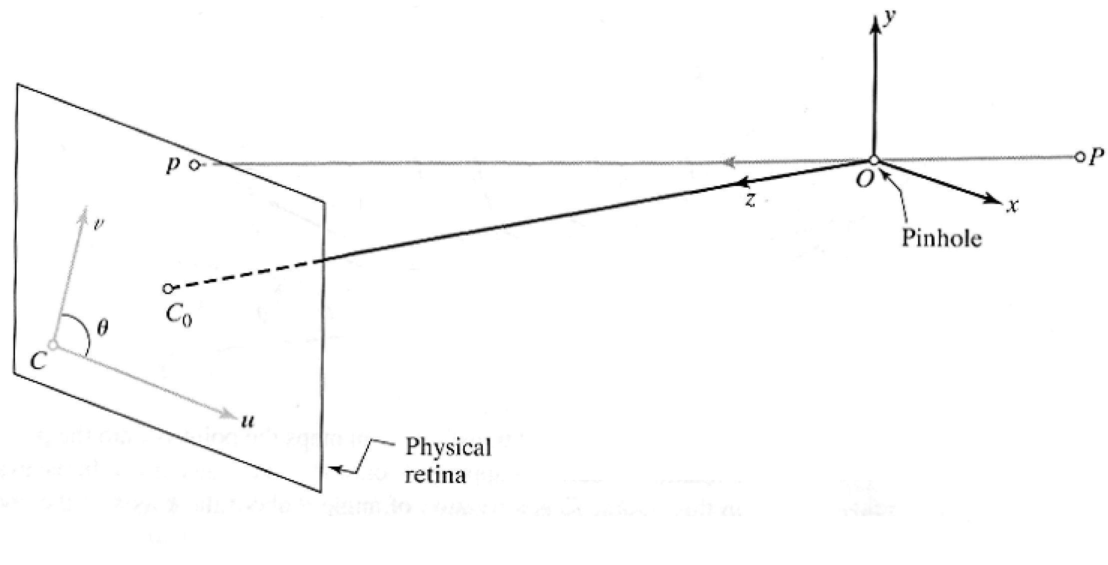


Real intrinsic parameters (1)

But "pixels" are in some arbitrary spatial units

$$u = \alpha \frac{x}{z}$$

$$v = \alpha \frac{y}{z}$$

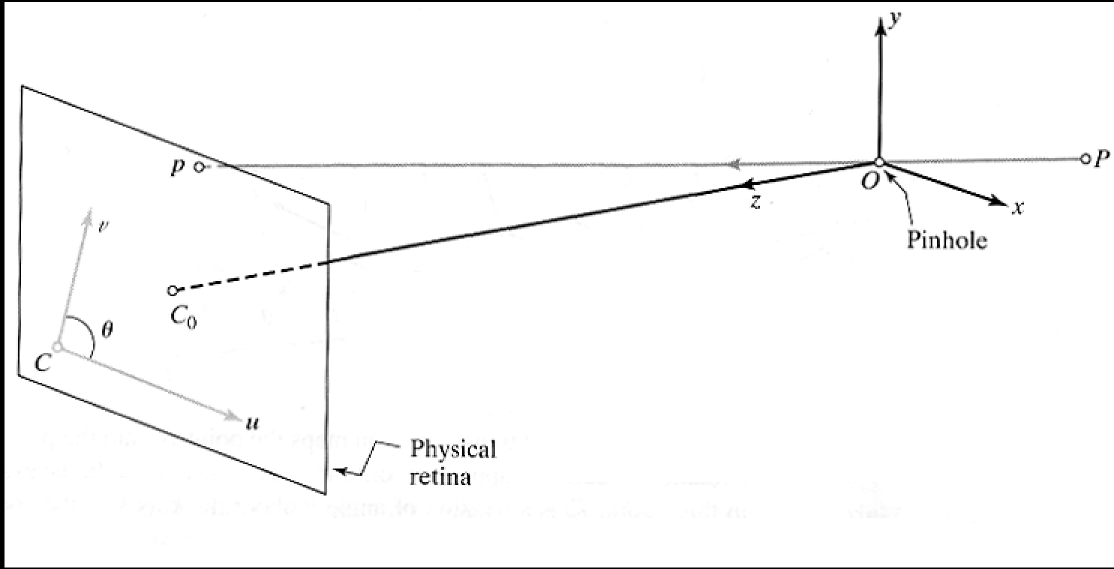


Real intrinsic parameters (2)

Maybe pixels are not square

$$u = \alpha \frac{x}{z}$$

$$v = \beta \frac{y}{z}$$

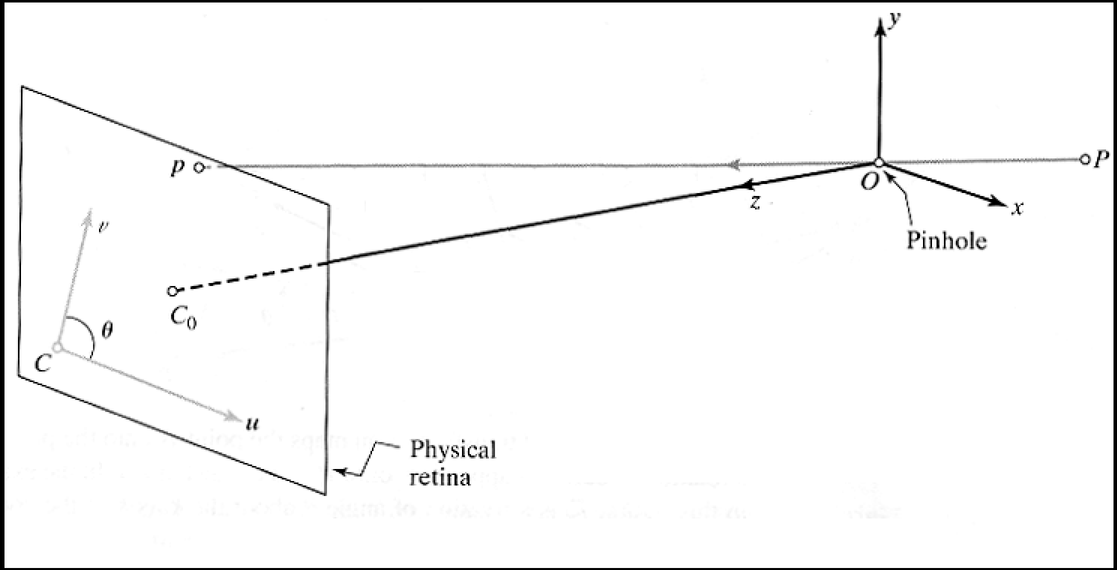


Real intrinsic parameters (3)

We don't know the origin of our camera pixel coordinates

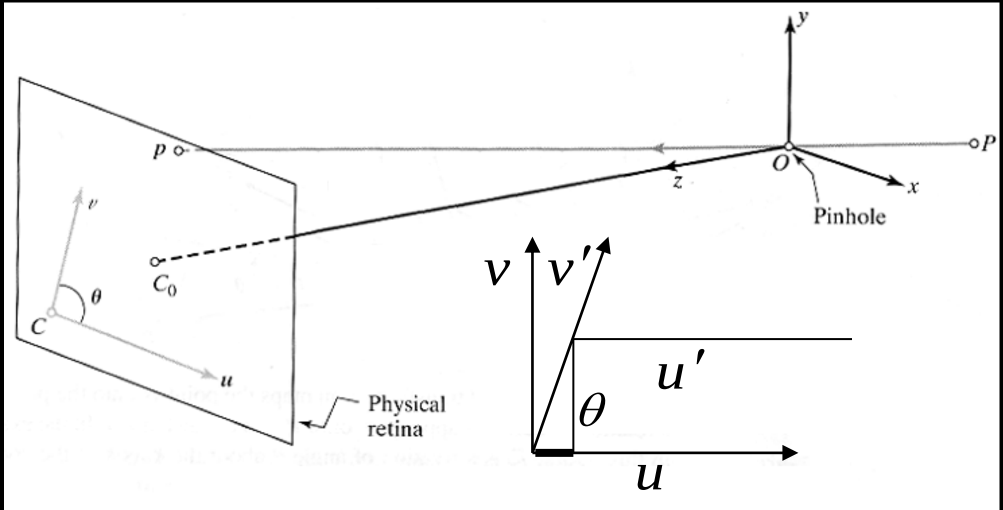
$$u = \alpha \frac{x}{z} + u_0$$

$$v = \beta \frac{y}{z} + v_0$$



Really ugly intrinsic parameters (4)

May be skew between camera pixel axes



$$v'\sin(\theta) = v$$

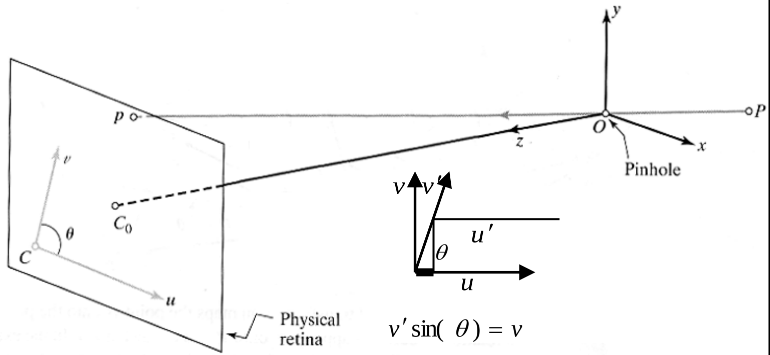
$$u' = u - \cos(\theta)v' = u - \cot(\theta)v$$

Really ugly intrinsic parameters (4)

May be skew between camera pixel axes

$$u = \alpha \frac{x}{z} - \alpha \cot(\theta)\frac{y}{z} + u_0$$

$$v = \frac{\beta}{\sin(\theta)}\frac{y}{z} + v_0$$



$$v'\sin(\theta) = v$$

$$u' = u - \cos(\theta)v' = u - \cot(\theta)v$$

Intrinsic parameters, non-homogeneous coords

Notice division by z

$$u = \alpha \frac{x}{z} - \alpha \cot(\theta) \frac{y}{z} + u_0$$

$$v = \frac{\beta}{\sin(\theta)} \frac{y}{z} + v_0$$

Intrinsic parameters, *homogeneous* coords

$$\begin{pmatrix} z*u \\ z*v \\ z \end{pmatrix} = \begin{pmatrix} \alpha & -\alpha\cot(\theta) & u_0 & 0 \\ 0 & \dfrac{\beta}{\sin(\theta)} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\vec{\mathrm{p}}' = \mathrm{K} \; {}^{C}\vec{p}$$

In homogeneous pixels

Intrinsic matrix

In camera-based 3D coords

Kinder, gentler intrinsics

- Can use simpler notation for intrinsics – remove last column which is zero:

$$K = \begin{bmatrix} f & s & c_x \\ 0 & af & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

f –focal length

s – skew

a – aspect ratio

c_x, c_y - offset

(5 DOF)

Kinder, gentler intrinsics

- If square pixels, no skew, and optical center is in the center (assume origin in the middle):

$$K = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In this case only one DOF, focal length f

Kinder, gentler intrinsics

- Can use simpler notation for intrinsics – remove last column which is zero:

$$K = \begin{bmatrix} f & s & c_x \\ 0 & af & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

f –focal length

s – skew

a – aspect ratio

c_x, c_y - offset

(5 DOF)

Quiz

The intrinsics have the following: a focal length, a pixel x size, a pixel y size, two offsets and a skew. That's 6. But we've said there are only 5 DOFS. What happened:

a) Because *f* always multiplies the pixel sizes, those 3 numbers are really only 2 DOFs.

b) In modern cameras, the skew is always zero so we don't count it.

c) In CCDs or CMOS cameras, the aspect is carefully controlled to be 1.0, so it is no longer modeled.

Combining extrinsic and intrinsic calibration parameters

Pixels

$$\vec{p}\,' = K\ {}^{C}\vec{p}$$

Intrinsic

World 3D coordinates

Camera 3D coordinates

$$\begin{pmatrix} {}^{C}\vec{p} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} - & - & - & | \\ - & {}^{C}_{W}R & - & {}^{C}_{W}\vec{t} \\ - & - & - & | \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} {}^{W}\vec{p} \\ \\ \\ \end{pmatrix}$$

Extrinsic

Combining extrinsic and intrinsic calibration parameters

$$\vec{p}\,' = \underset{\boxed{\substack{\mathrm{K} \\ 3\mathrm{x}3}}}{K} \underbrace{\left({}^{C}_{W}R \quad {}^{C}_{W}\vec{t} \right)}_{\boxed{3\mathrm{x}4}} {}^{W}\vec{p}$$

$$\vec{p}\,' = M \; {}^{W}\vec{p}$$

Other ways to write the same equation

pixel coordinates

world coordinates

$$\vec{p}\,' = M \; {}^{W}\vec{p}$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \simeq \begin{pmatrix} s*u \\ s*v \\ s \end{pmatrix} = \begin{pmatrix} . & m_1^T & . & . \\ . & m_2^T & . & . \\ . & m_3^T & . & . \end{pmatrix} \begin{pmatrix} {}^{W}p_x \\ {}^{W}p_y \\ {}^{W}p_z \\ 1 \end{pmatrix}$$

projectively similar

Conversion back from homogeneous coordinates

$$u = \frac{\vec{m}_1 \cdot \vec{P}}{\vec{m}_3 \cdot \vec{P}}$$

$$v = \frac{\vec{m}_2 \cdot \vec{P}}{\vec{m}_3 \cdot \vec{P}}$$

Finally: Camera parameters

- A camera (and its matrix) M (or Π) is described by several parameters
  - Translation **T** of the optical center from the origin of world coordinates
  - Rotation **R** of the camera system
  - focal length and aspect **(f, a)** [or pixel size **(s_x, s_y)**] , principle point **(x'_c, y'_c)**, and skew **(s)**
  - **blue** parameters are called "**extrinsics**," red are "intrinsics"

Finally: Camera parameters

- Projection equation – the cumulative effect of all parameters:

$$\mathbf{x} \simeq \begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{M}\,\mathbf{X}$$

Finally: Camera parameters

- Projection equation – the cumulative effect of all parameters:

$$\underset{(3x4)}{\mathbf{M}} = \begin{bmatrix} f & s & x'_c \\ 0 & af & y'_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{T}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}$$

intrinsics projection rotation translation

$$\boxed{\textit{DoFs: } 5+0+3+3 = 11}$$