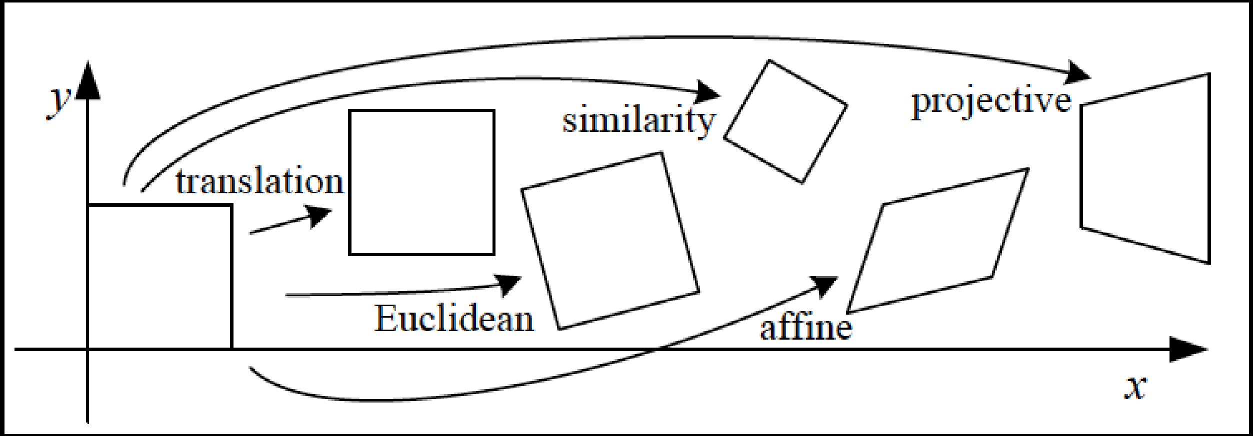
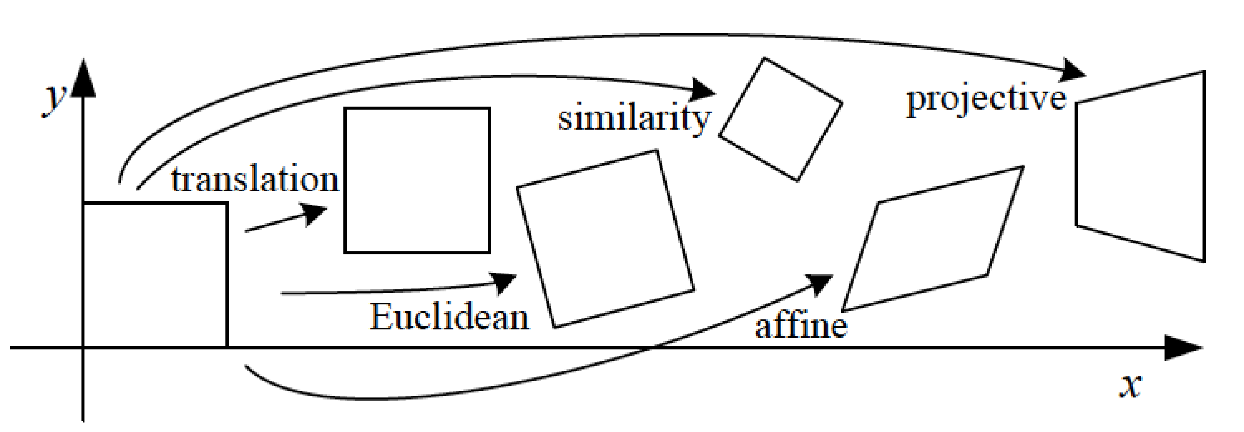
CS4495/6495
Introduction to Computer Vision

3D-L1 *Image to image projections*

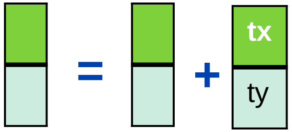
2D Transformations

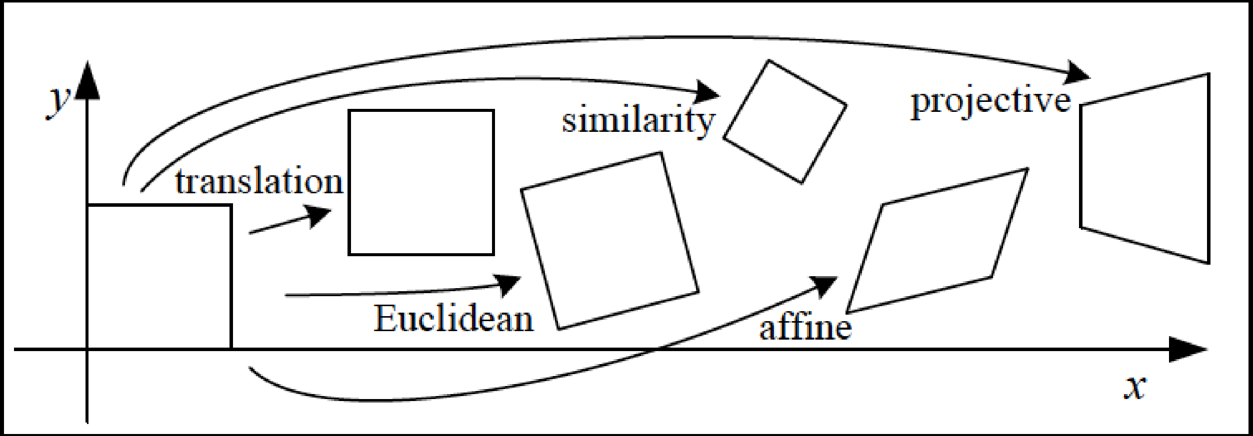




Example: translation

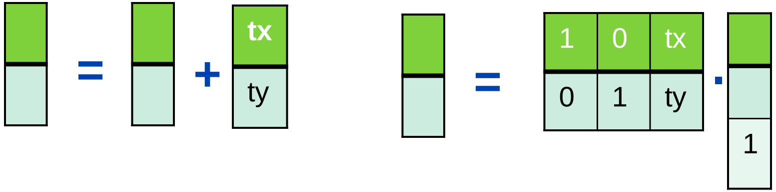
$$x' = x + t$$

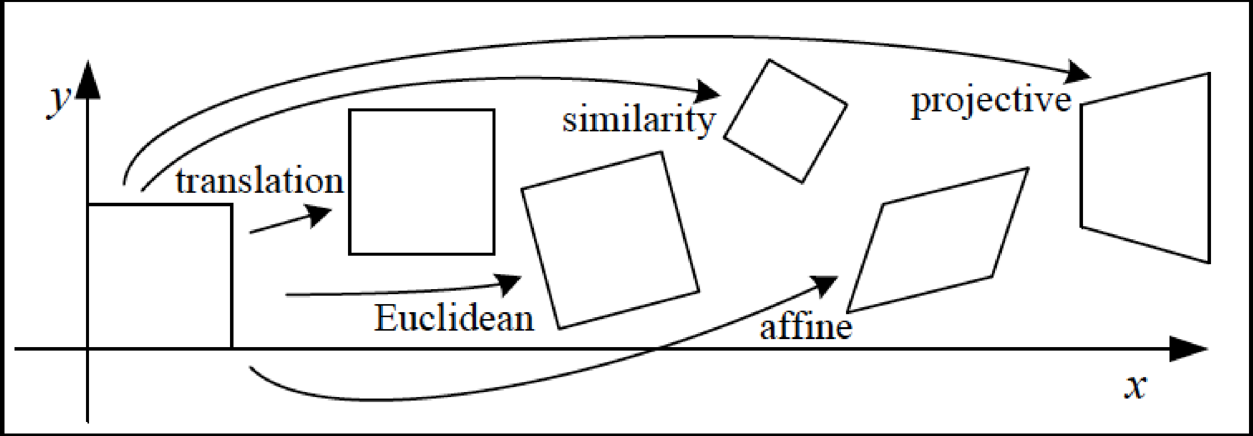




Example: translation

$$x' = x + t \qquad x' = \begin{bmatrix} I & \vec{t} \end{bmatrix} \bar{x}$$





Homogenous vector

Example: translation

$$x' = x + t \qquad x' = \begin{bmatrix} I & \vec{t} \end{bmatrix} \bar{x} \qquad \bar{x}' = \begin{bmatrix} I & \vec{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \bar{x}$$

$$\begin{bmatrix} \; \\ \; \end{bmatrix} = \begin{bmatrix} \; \\ \; \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix} \qquad \begin{bmatrix} \; \\ \; \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \end{bmatrix} \cdot \begin{bmatrix} \; \\ \; \\ 1 \end{bmatrix} \qquad \begin{bmatrix} \; \\ \; \\ \; \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \; \\ \; \\ \; \end{bmatrix}$$

[BTW: Now we can chain transformations]

Projective Transformations

- *Projective* transformations: for 2D images it's a 3x3 matrix applied to homogenous coordinates

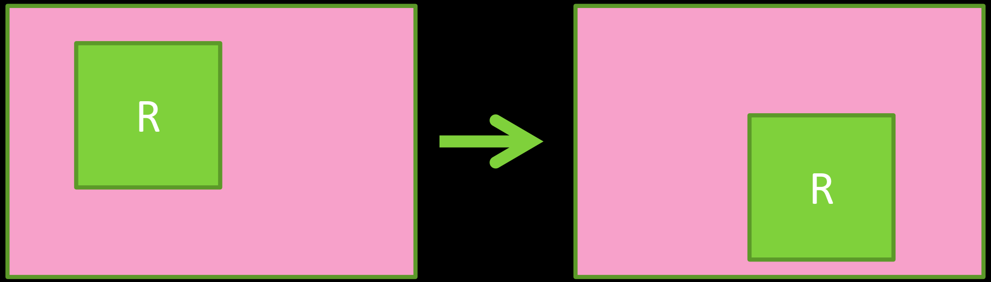
$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Special Projective Transformations

- Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Preserves:
  - Lengths/Areas
  - Angles
  - Orientation
  - Lines

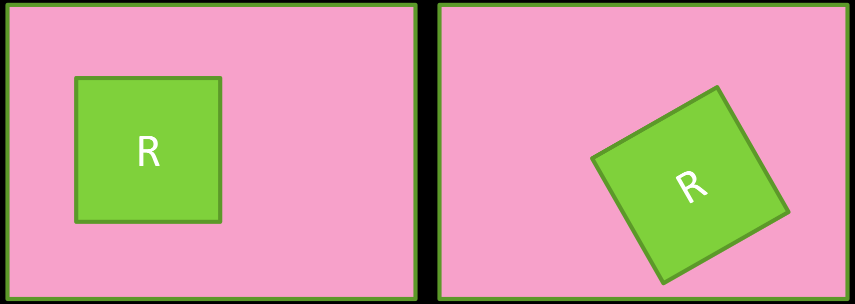


Special Projective Transformations

- Euclidean (Rigid body)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & t_x \\ \sin(\theta) & \cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Preserves:
  - Lengths/Areas
  - Angles
  - Lines

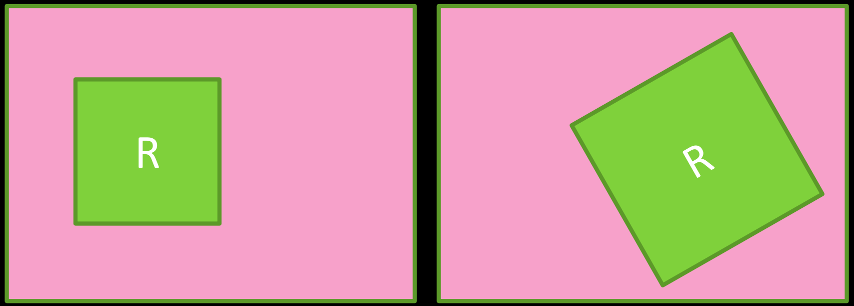


Special Projective Transformations

- Similarity (trans, rot, scale) transform

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a\cos(\theta) & -a\sin(\theta) & t_x \\ a\sin(\theta) & a\cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Preserves:
  - Ratios of Areas
  - Angles
  - Lines

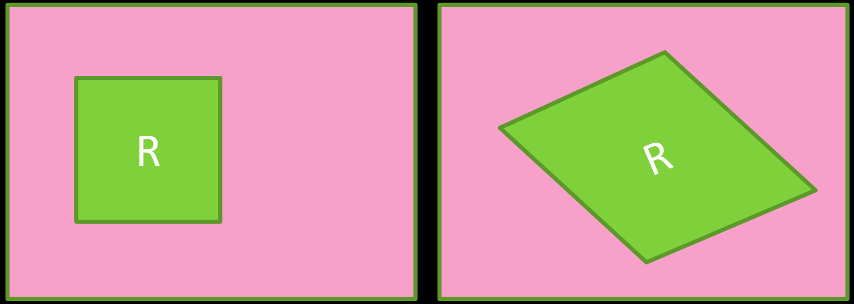


Special Projective Transformations

- Affine transform

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Preserves:
  - Parallel lines
  - Ratio of Areas
  - Lines



Projective Transformations

- Remember, these are homogeneous coordinates

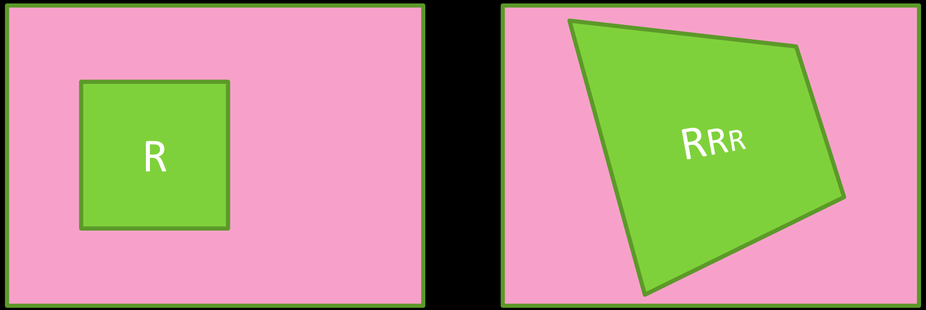
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \cong \begin{bmatrix} sx' \\ sy' \\ s \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Projective Transformations

- General projective transform (or Homography)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \cong \begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Preserves:
  - Lines
  - Also cross ratios (maybe later)



Projective Transformations

- General projective transform (or Homography)

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- Preserves:
  - Lines
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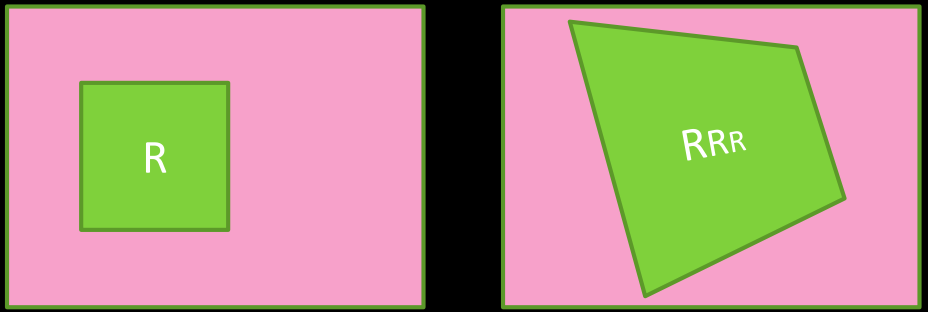


Quiz 1

Suppose I told you the transform from image A to image B is a ***translation***. How many pairs of corresponding points would you need to know to compute the transformation?

a) 3

b) 1

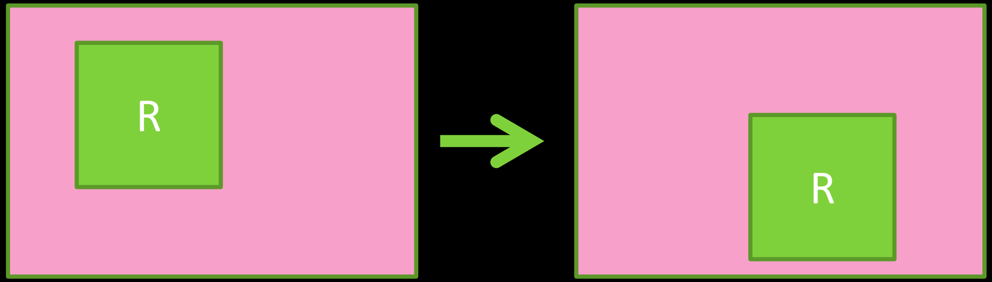
c) 2

d) 4

Quiz 1 – answer

- Translation: a 1 point transformation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



Quiz 2

Suppose I told you the transform from image A to image B is ***affine***. How many pairs of corresponding points would you need to know to compute the transformation?

a) 3

b) 1

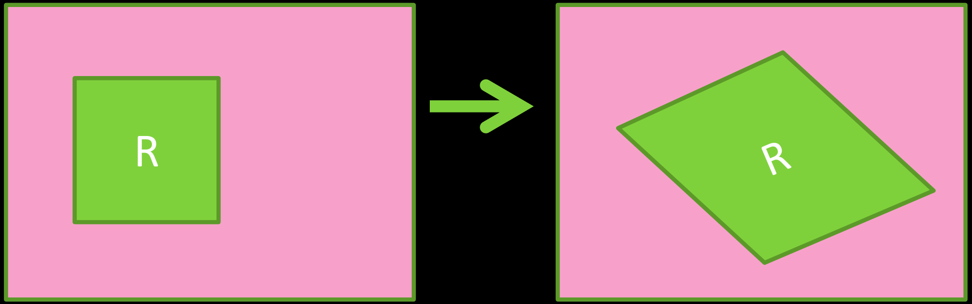
c) 2

d) 4

Quiz 2 – answer

- Affine transform: a 3 point transformation
  - 6 unknowns – each point pair gives two equations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



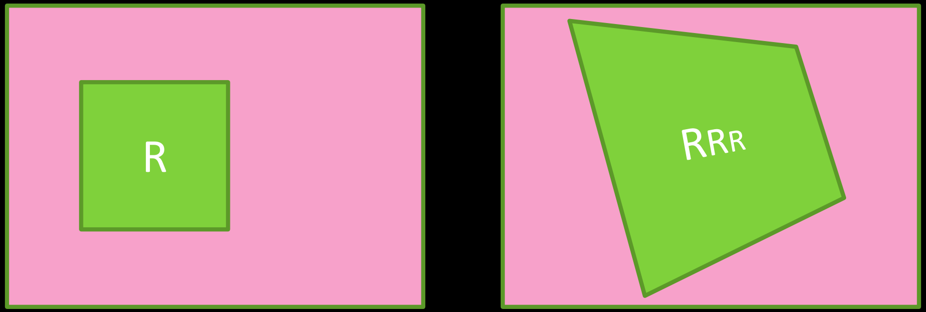
Quiz 3

Suppose I told you the transform from image A to image B is a ***homography***. How many pairs of corresponding points would you need to know to compute the transformation?

a) 3
b) 1
c) 2
d) 4

Quiz 3 – answer

- Homography:
  4 points



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \simeq \begin{bmatrix} w'x' \\ w'y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$