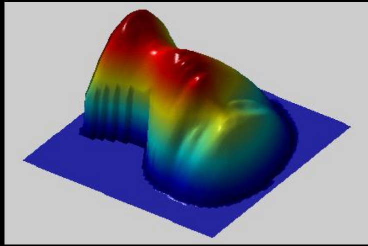
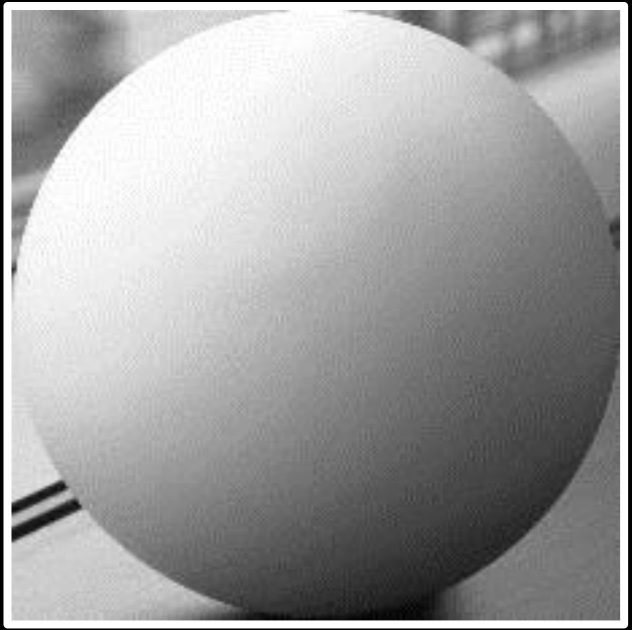
CS4495/6495
Introduction to Computer Vision

5C-L1 *Shape from shading*



Thanks to Srinivasa Narasimhan, Shree Nayar, David Kreigman, Marc Pollefeys

Shape from shading



Shading as a cue for shape reconstruction

Shape from shading/lighting

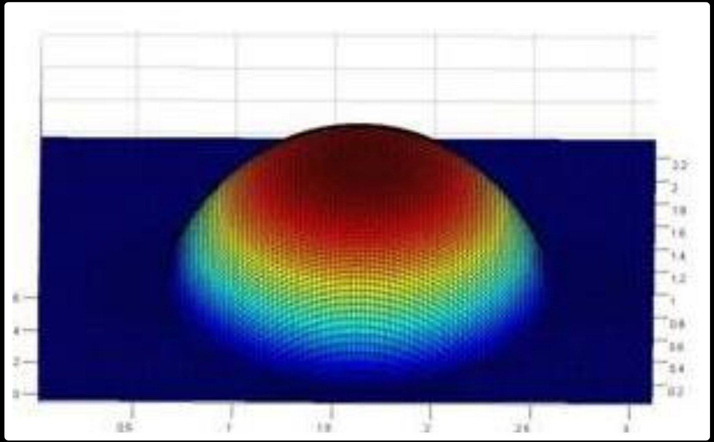
What is the relation between intensity and shape?

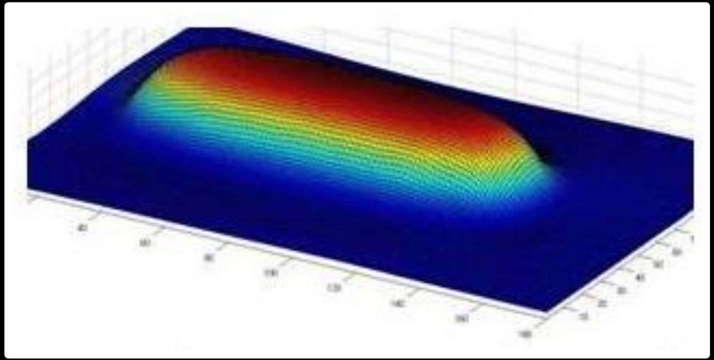
- Need to look at the reflectance function
- *Reflectance Map*

Surface normals: A bit of math

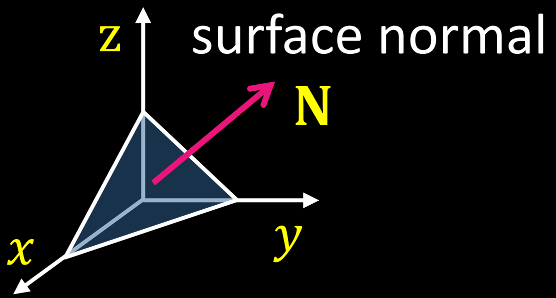
- Let's assume we have a surface $z(x, y)$
- We can define the following:

$$-\frac{\partial z}{\partial x} = p \qquad -\frac{\partial z}{\partial y} = q$$





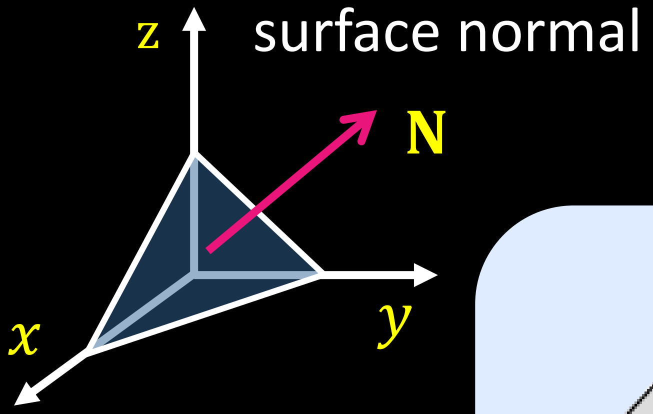
Surface Normal: A bit more math

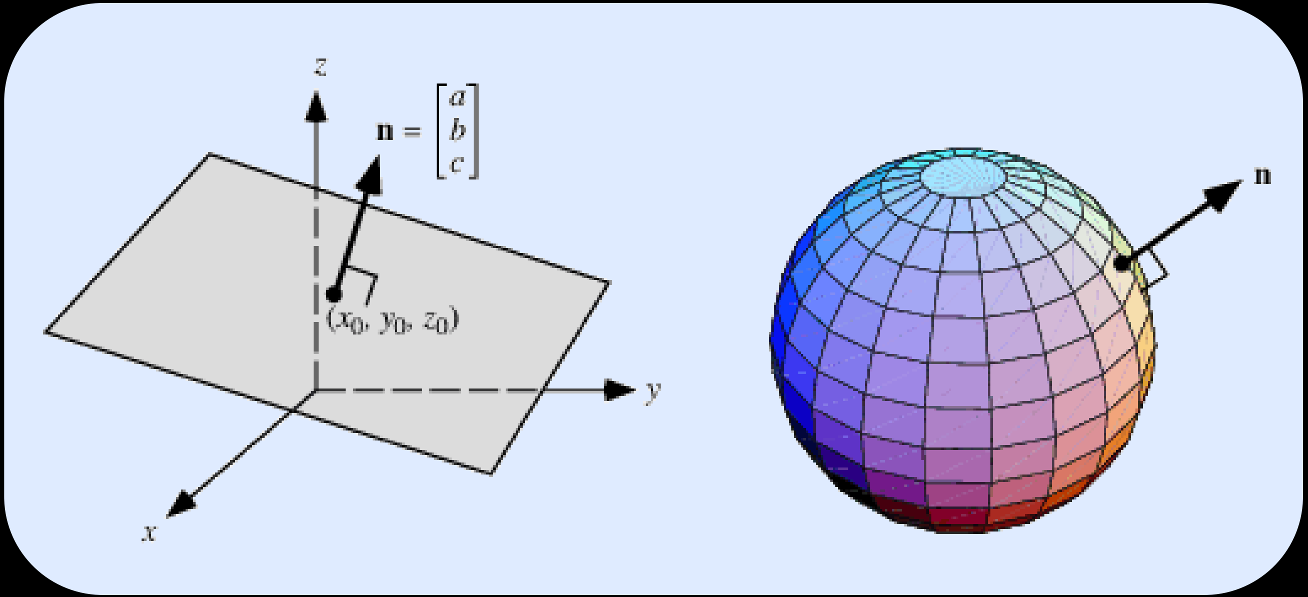


Suppose we have a point on the surface. We can define two tangents: $t_x = (1,0,-p)^T$ and $t_y = (0,1,-q)^T$

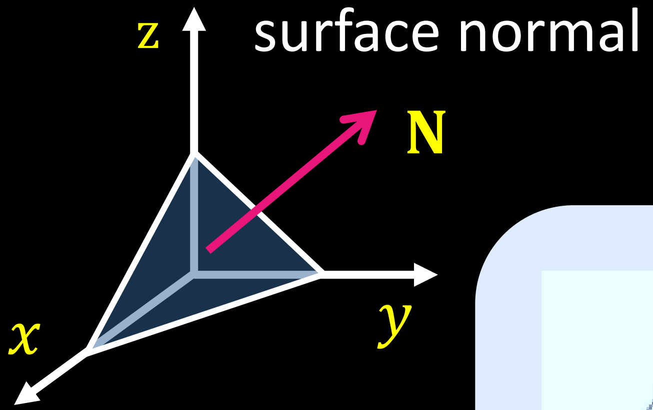
$$\mathbf{n} = \frac{N}{\|N\|} = \frac{t_x \times t_y}{\| t_x \times t_y \|} = \frac{1}{\sqrt{p^2 + q^2 + 1}} (p, q, 1)^T$$

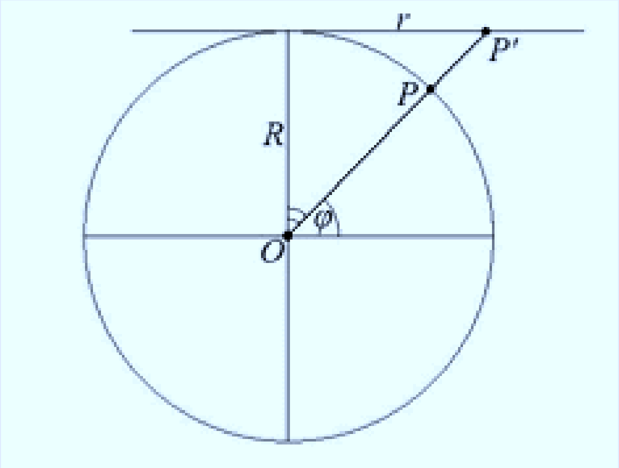
Surface Normal: Gaussian sphere

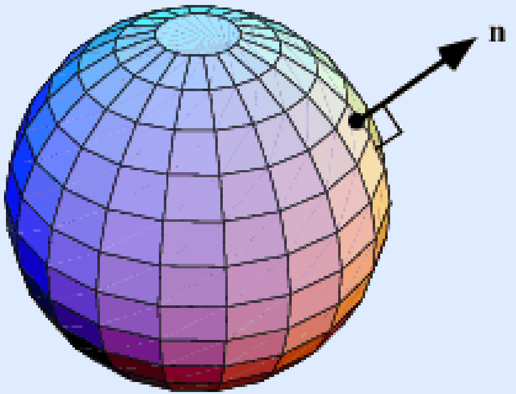




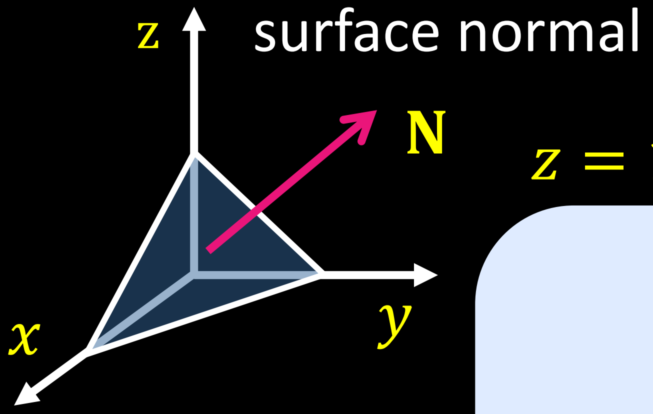
Surface Normal: Gradient space projection



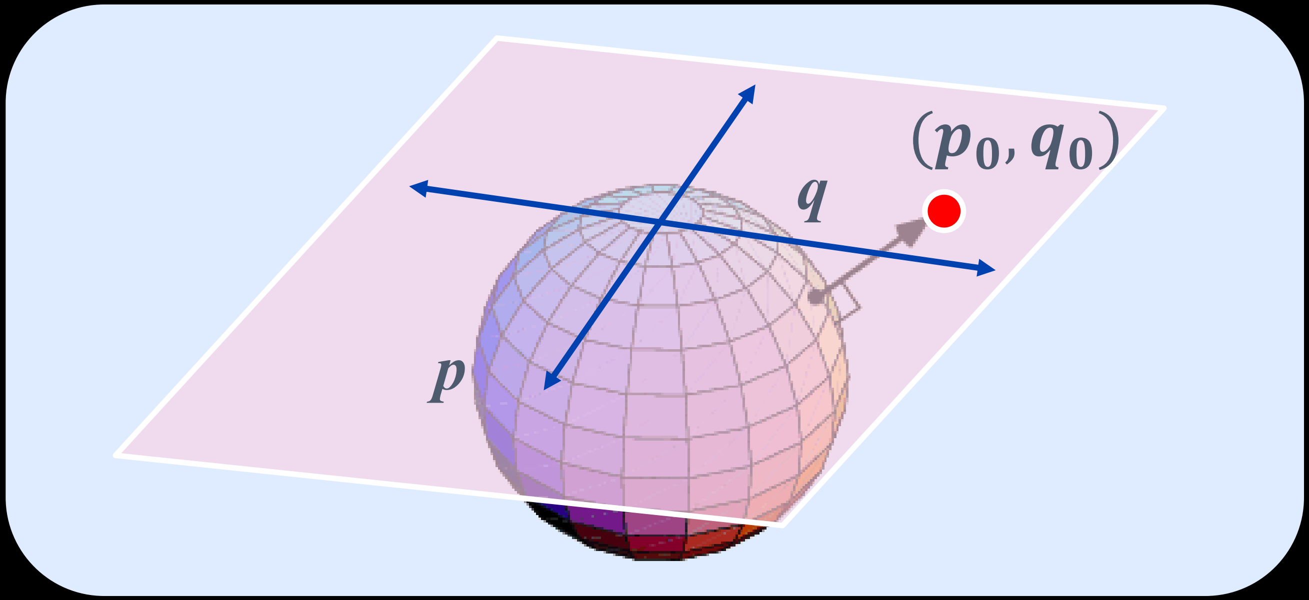




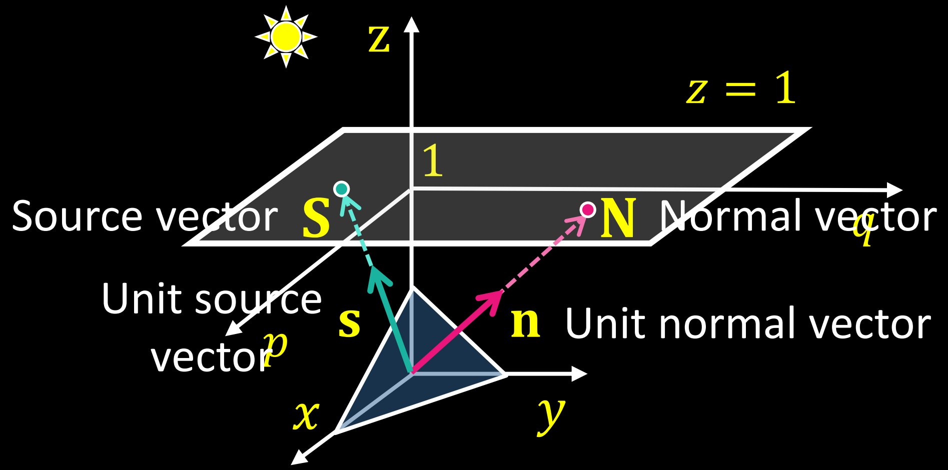
Surface Normal: Gradient space projection



$z = 1$ plane is called Gradient Space (pq plane)



Gradient Space of Source and Normal



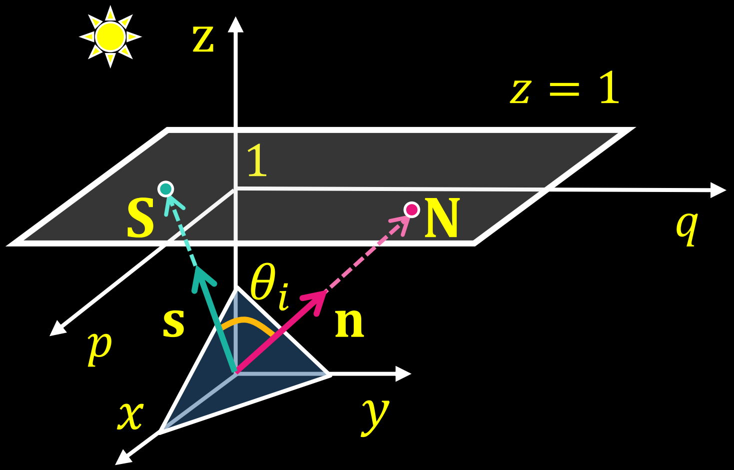
Unit normal vector:

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

Unit source vector:

$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_S, q_S, 1)}{\sqrt{p_S^2 + q_S^2 + 1}}$$

Gradient Space of Source and Normal



Unit normal vector:

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

Unit source vector:

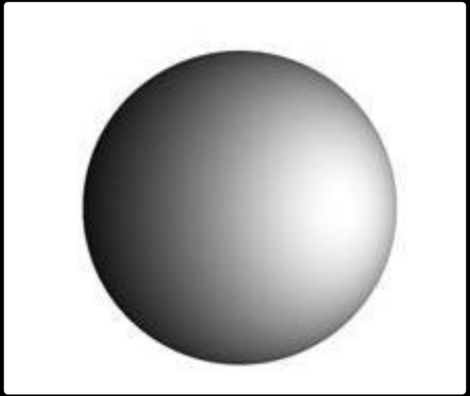
$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_S, q_S, 1)}{\sqrt{p_S^2 + q_S^2 + 1}}$$

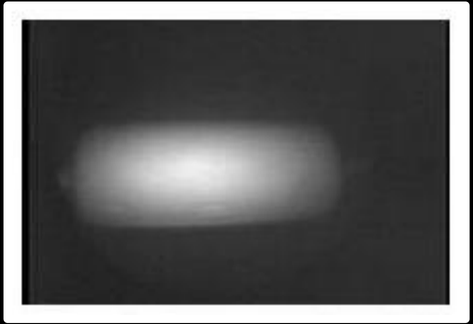
$$\cos\theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_S + qq_S + 1)}{\sqrt{p^2 + q^2 + 1}\sqrt{p_S^2 + q_S^2 + 1}}$$

Shape from shading: Problem definition

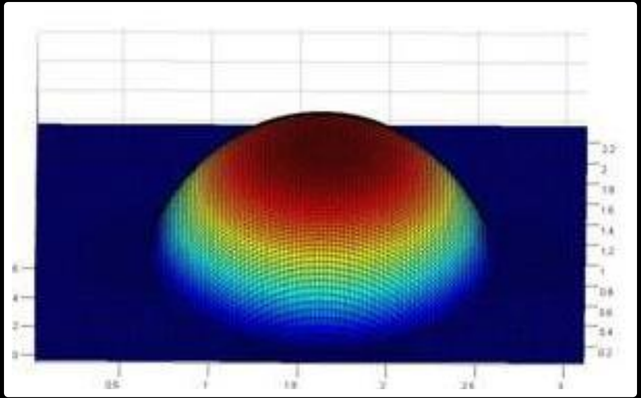
Input: 1 or more images

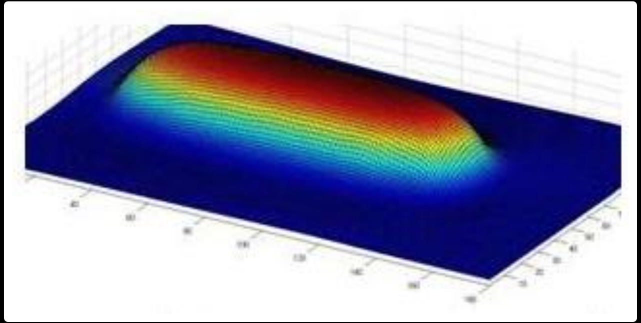
$I(x, y)$





Output: 3D shape of object





$Z(x, y)$

Reflectance Map

Relates image brightness $I(x, y)$ to surface orientation (p, q) for ***given*** *source direction* and *surface reflectance*

Reflectance Map: Lambertian case

Terms:

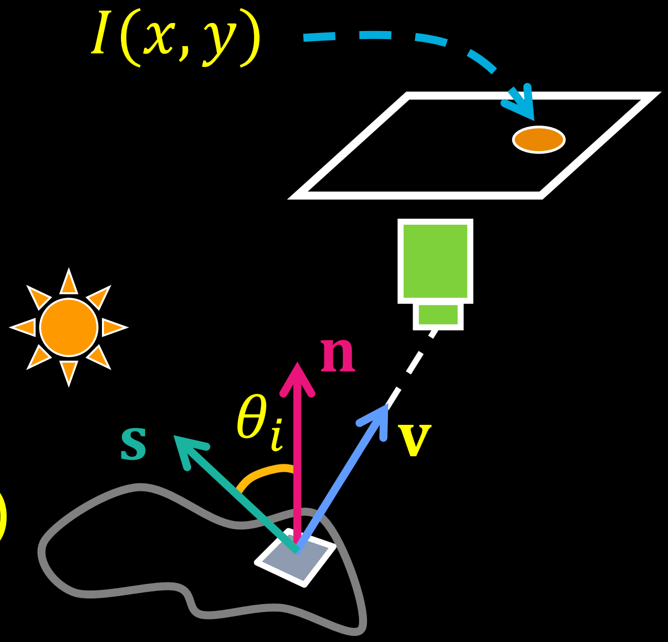
k: source brightness

ρ: surface albedo (reflectance)

Image brightness:

$$I = \rho \cdot k \cdot \cos\theta_i = \rho \cdot k\,(\mathbf{n} \cdot \mathbf{s})$$

Let $\rho \cdot k = 1$ then $I = \cos\theta_i = \mathbf{n} \cdot \mathbf{s}$

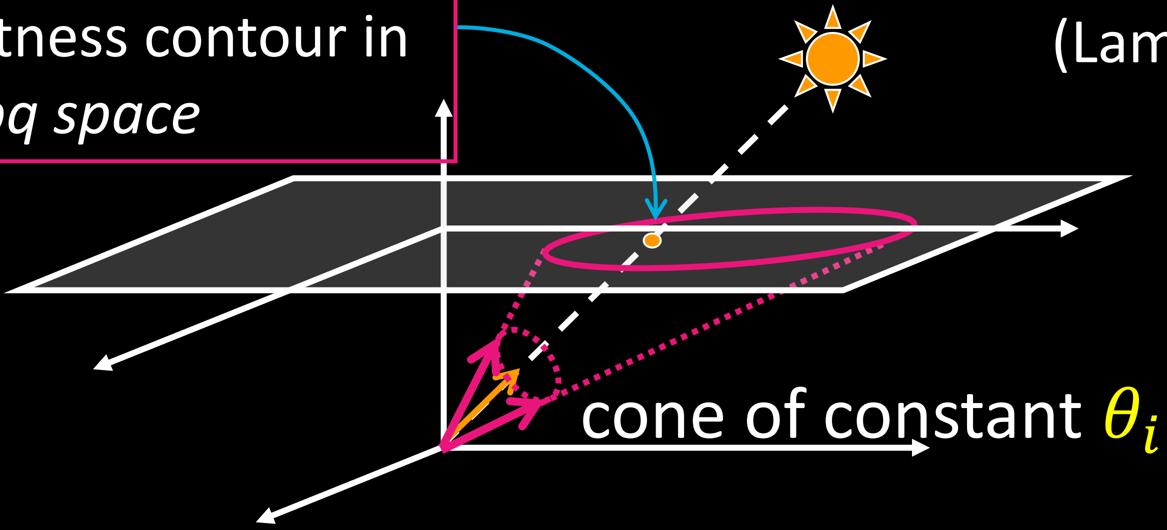


Reflectance Map: Lambertian case

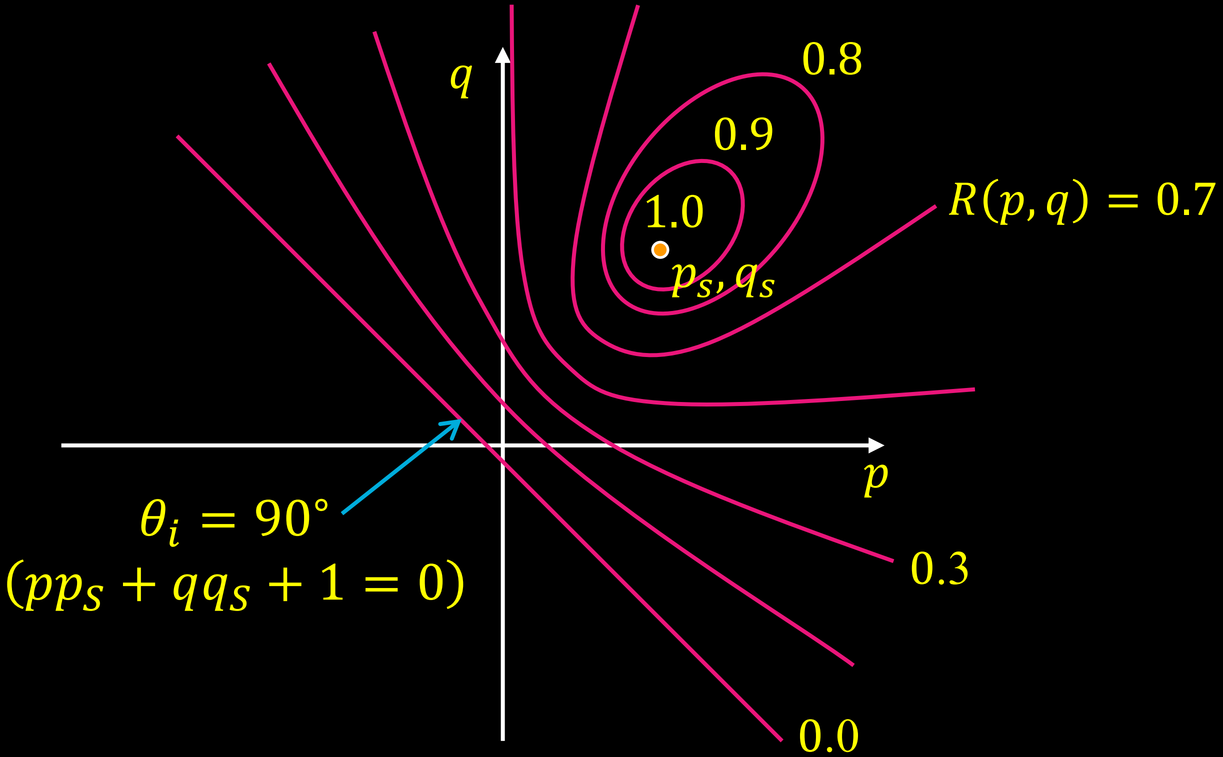
$$I = \cos\theta_i = \mathbf{n}\cdot\mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2+q^2+1}\sqrt{p_s^2+q_s^2+1}} = \boxed{R(p,q)}$$

Reflectance Map
(Lambertian)

Iso-brightness contour in
pq space



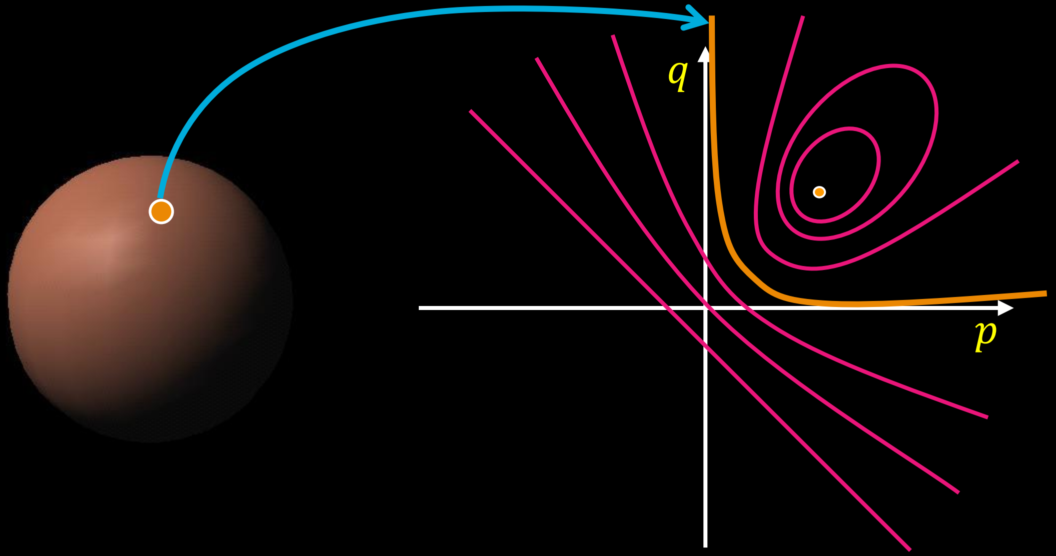
Iso-brightness contours



Note: $R(p, q)$ is maximum when $(p, q) = (p_s, q_s)$

Shape from a *single* image?

Given $R(p,q)$ ((p_S,q_S) and surface reflectance) can we determine (p,q) uniquely for each image point?



NO

Shape from Shading

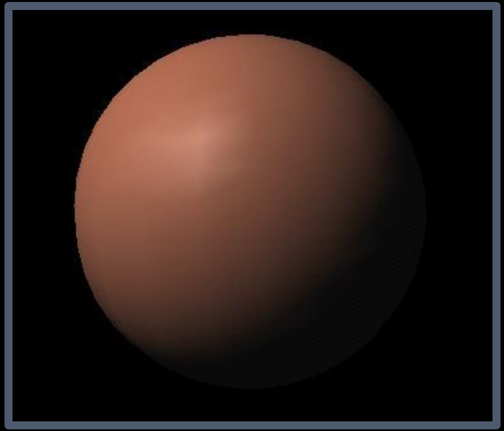
Need more information:

- Add more constraints: *Shape-from-shading*

- Take more images: *Photometric stereo*

Shape from shading

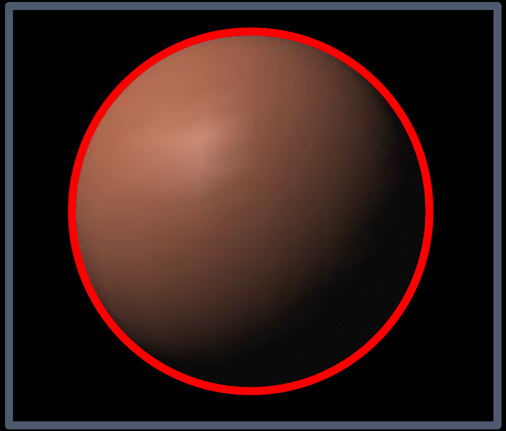
Given a single image of an object with *known surface reflectance* taken under *a known light source*, can we recover its shape?



Shape from shading

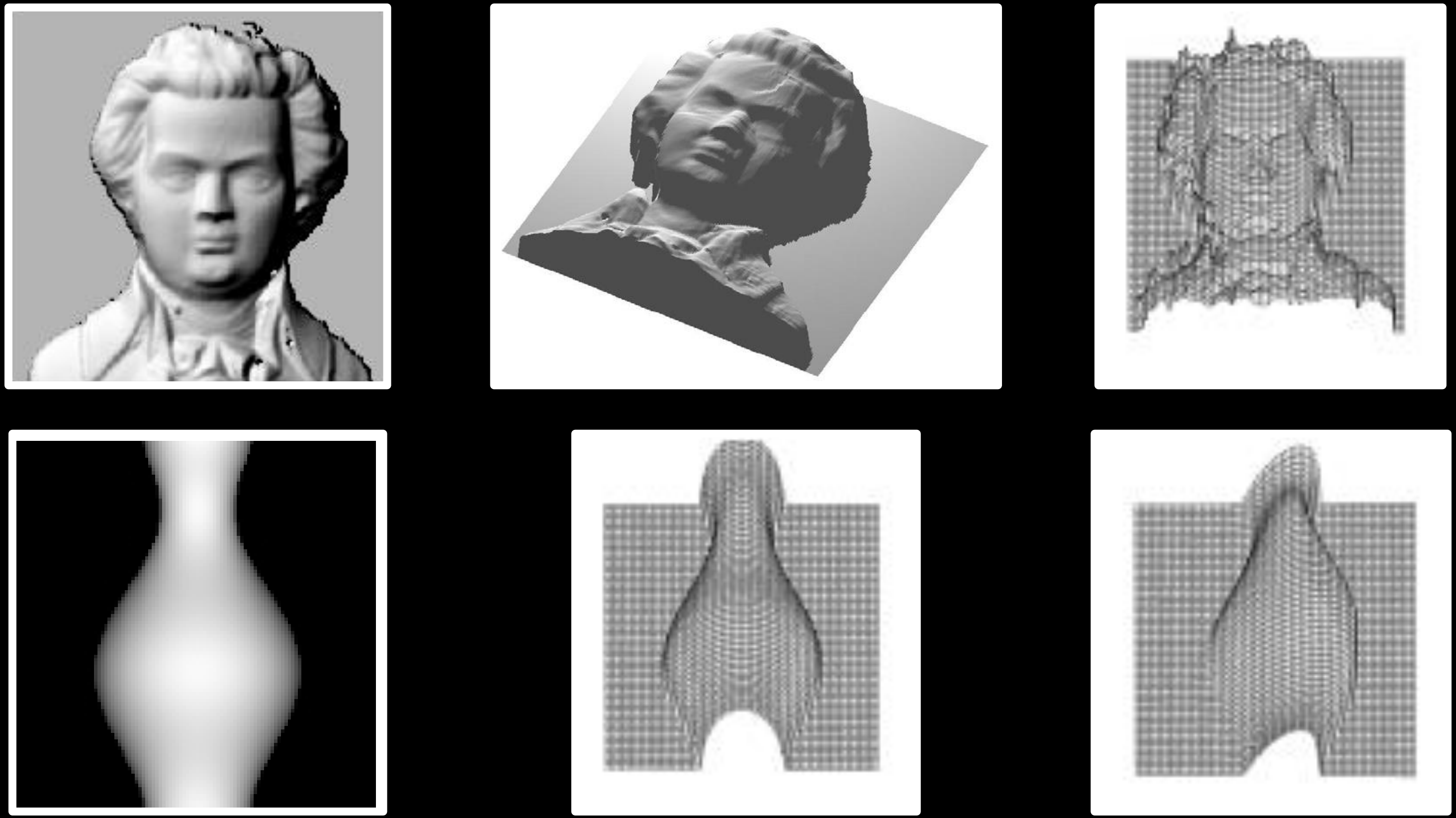
Given $R(p,q)$ ((p_S, q_S) and surface reflectance) can we determine (p,q) uniquely for each image point?

- Assume shape along the occluding boundary is known
- Constraints on neighboring normals
  — *integrability*
- Smoothness

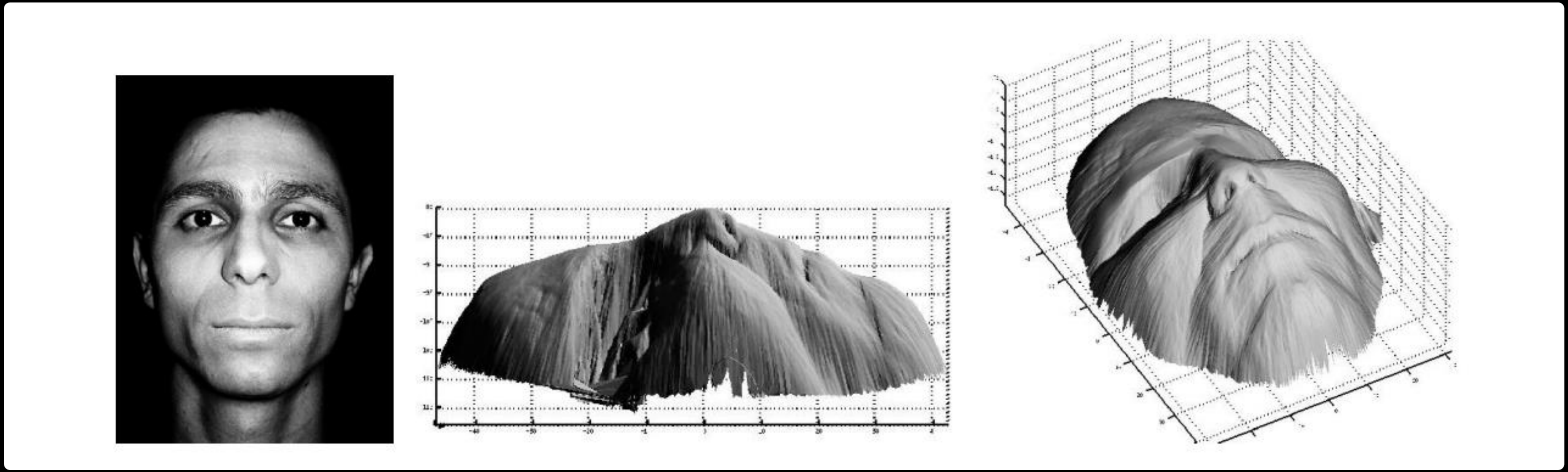


Yes, a slightly ugly optimization

Synthetic results



Shape from Shading: “Real” Results



- These *single* image methods work poorly in practice
- Why? The assumptions are quite restrictive

Shape from Shading

Need more information:

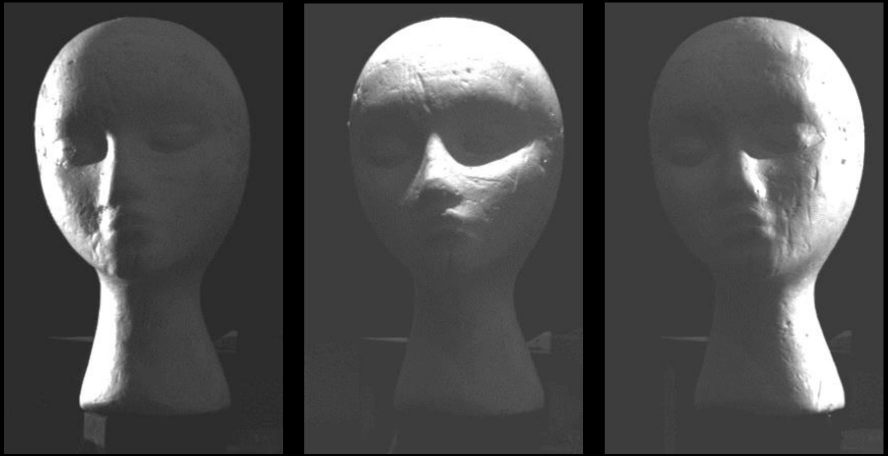
- Add more constraints: *Shape-from-shading*

- Take more images: *Photometric stereo*

Photometric stereo

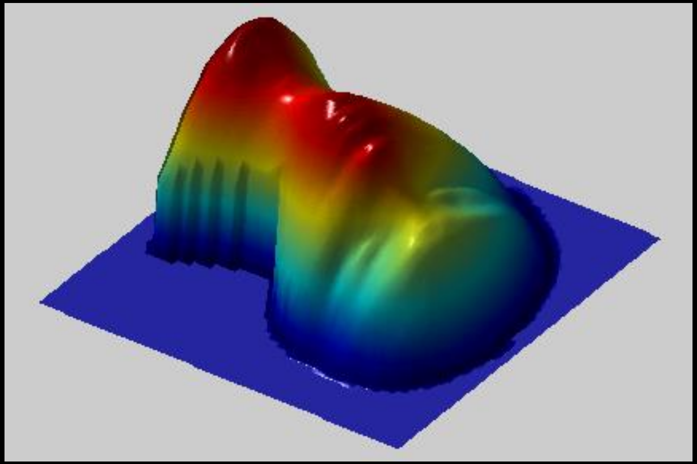
Input: Several images

- Same object
- Different lightings
- Same pose



Output:

- 3D shape of object
- Albedo at (x, y)



Photometric stereo

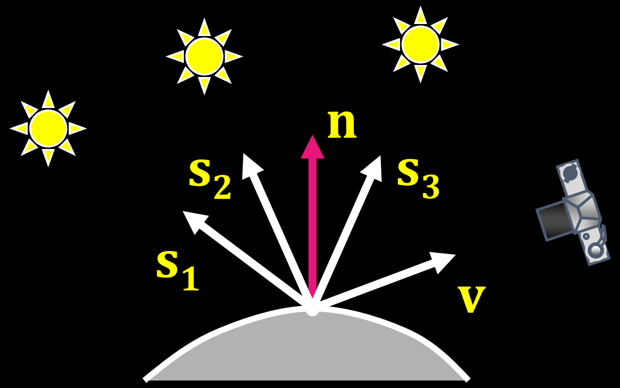


Image brightness:

$$I = \rho \cdot k \cdot \cos\theta_i = \rho \mathbf{n} \cdot \mathbf{s}$$

where $(\mathrm{k} = 1)$

$$I_1 = \rho \mathbf{n} \cdot \mathbf{s_1}$$

$$I_2 = \rho \mathbf{n} \cdot \mathbf{s_2}$$

$$I_3 = \rho \mathbf{n} \cdot \mathbf{s_3}$$

Write this as a matrix equation:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \rho \begin{bmatrix} \mathbf{s_1}^T \\ \mathbf{s_2}^T \\ \mathbf{s_3}^T \end{bmatrix} \mathbf{n}$$

Lambertian!

Solving the equations: Linear

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{\substack{\mathbf{I} \\ 3\times 1}} = \underbrace{\begin{bmatrix} \mathbf{s_1}^T \\ \mathbf{s_2}^T \\ \mathbf{s_3}^T \end{bmatrix}}_{\substack{\mathbf{S} \\ 3\times 3}} \underbrace{\rho\mathbf{n}}_{\substack{\tilde{\mathbf{n}} \\ 3\times 1}}$$

inverse

$$\tilde{\mathbf{n}} = \mathbf{S}^{-1}\mathbf{I}$$

$$\rho = |\tilde{\mathbf{n}}|$$

$$\mathbf{n} = \frac{\tilde{\mathbf{n}}}{|\tilde{\mathbf{n}}|} = \frac{\tilde{\mathbf{n}}}{\rho}$$

$\mathbf{I}$ and $\mathbf{S}$ are known

Adding more light sources

Get better results by using more (M) lights:

$$\begin{bmatrix} I_1 \\ \vdots \\ I_M \end{bmatrix} = \begin{bmatrix} \mathbf{s_1}^T \\ \vdots \\ \mathbf{s}_M{}^T \end{bmatrix} \rho \mathbf{n}$$

Least squares solution:

$$\mathbf{I} = \mathbf{S}\tilde{\mathbf{n}}$$

$$M \times 1 = \underline{(M \times 3)}(3 \times 1)$$

$$\mathbf{S}^T\mathbf{I} = \mathbf{S}^T\mathbf{S}\tilde{\mathbf{n}}$$

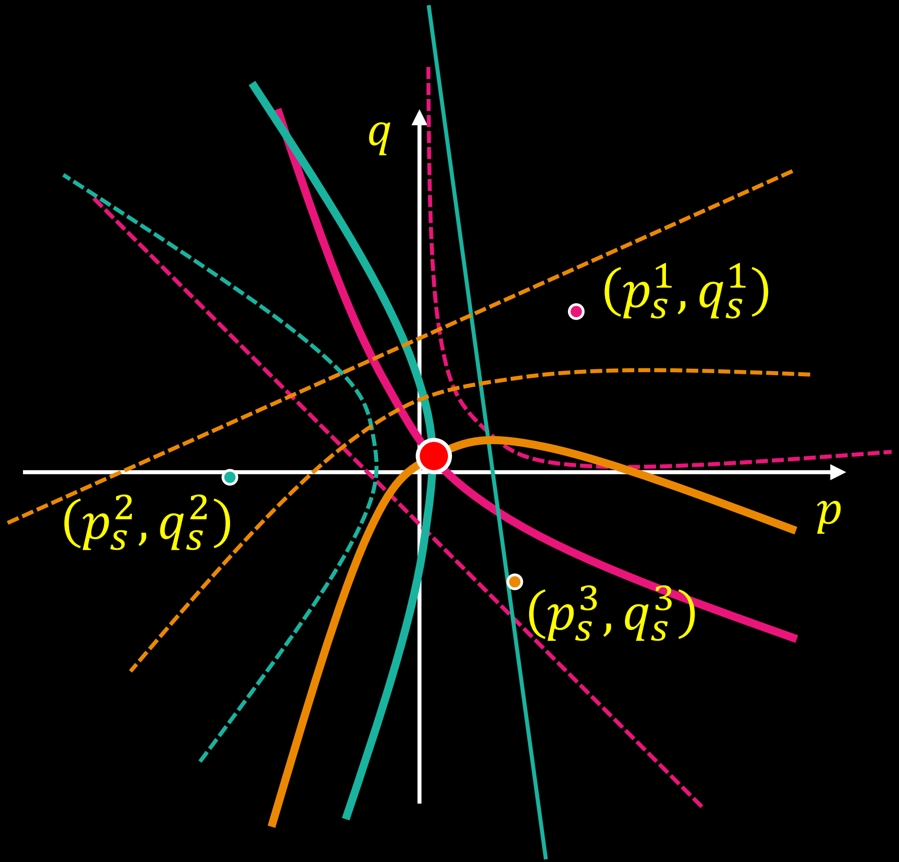
$$\tilde{\mathbf{n}} = \boxed{(\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T}\mathbf{I}$$

$$\min \|\mathbf{I} - \mathbf{S}\tilde{\mathbf{n}}\|_2^2$$

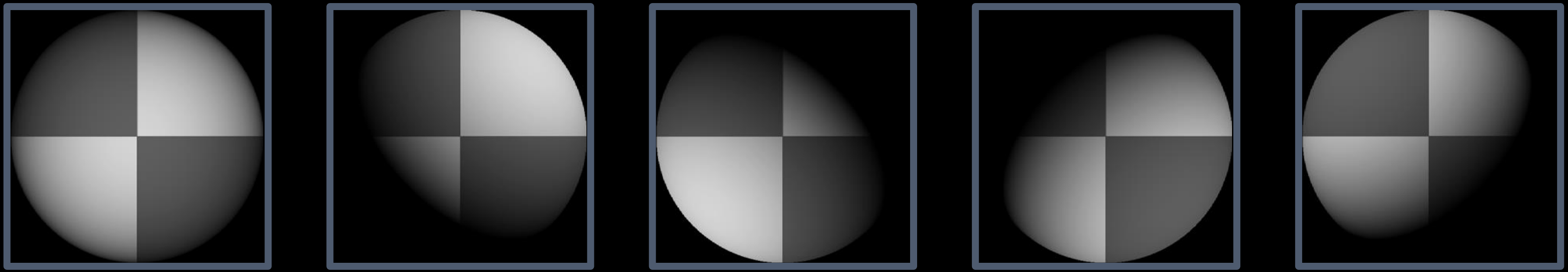
Moore-Penrose pseudo inverse

Solve for ρ, $\mathbf{n}$ as before

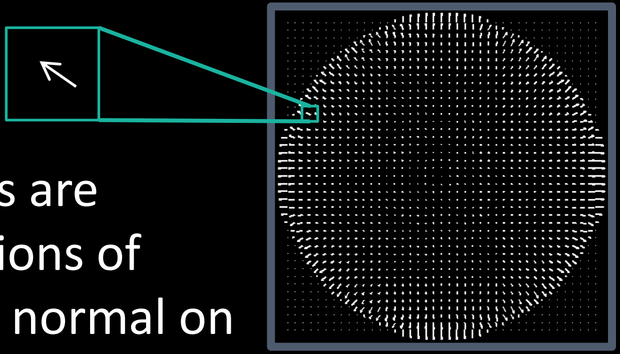
Photometric stereo: *pq* space



Results: Lambertian sphere

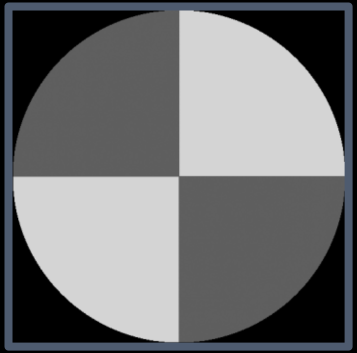


Input Images



Needles are projections of surface normal on image plane

Estimated Surface Normals



Estimated Albedo

Photometric stereo: Lambertian toy

Input Images

Surface Normals

Estimated Surface Normals

Albedo

Estimated Albedo

And from GT: www.trimensional.com

Photometric stereo: Limitations

Big problems

- Doesn’t work for shiny things, semi-translucent things
- Shadows, inter-reflections

Smaller problems

- Camera and lights have to be distant
- Calibration requirements
  - Measure light source directions, intensities
  - Camera response function

A real application that works

Shape from Shading with Interreflections Under a Proximal Light Source: Distortion-Free Copying of an Unfolded Book

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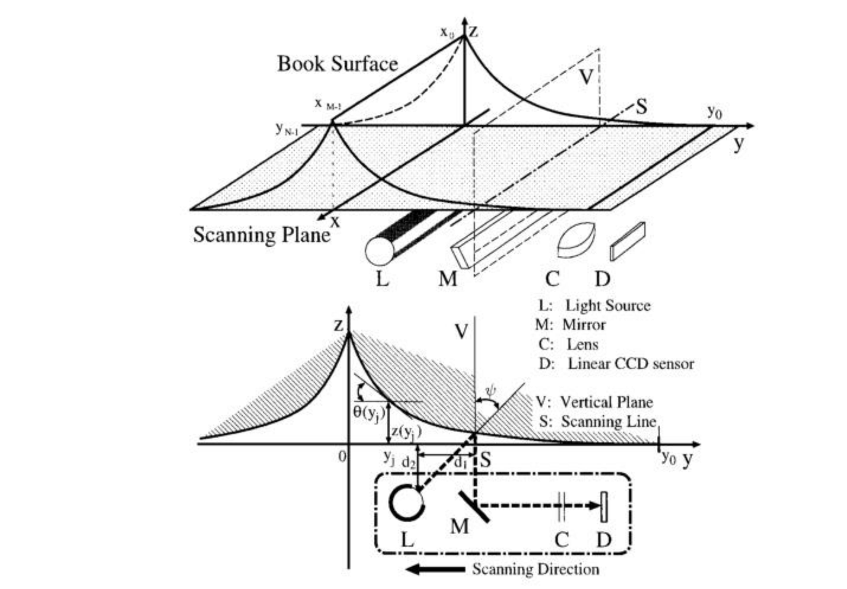
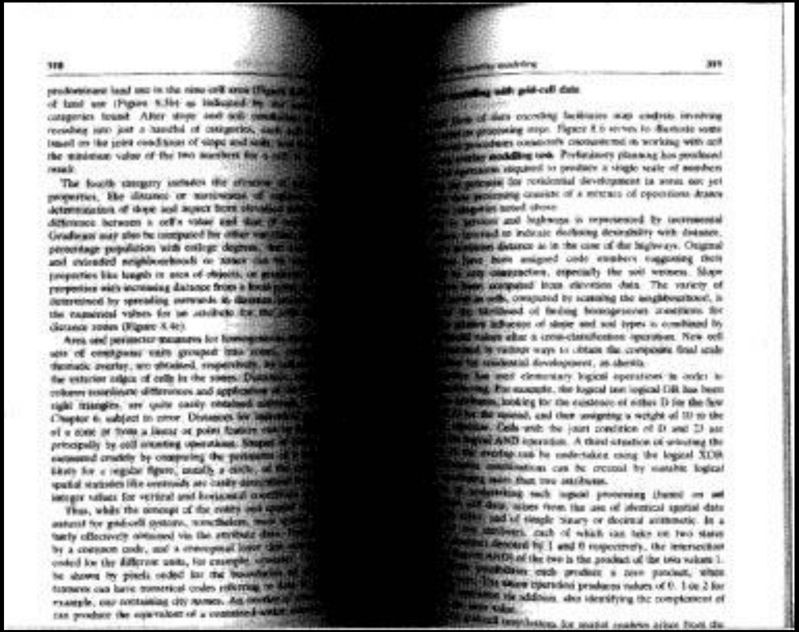
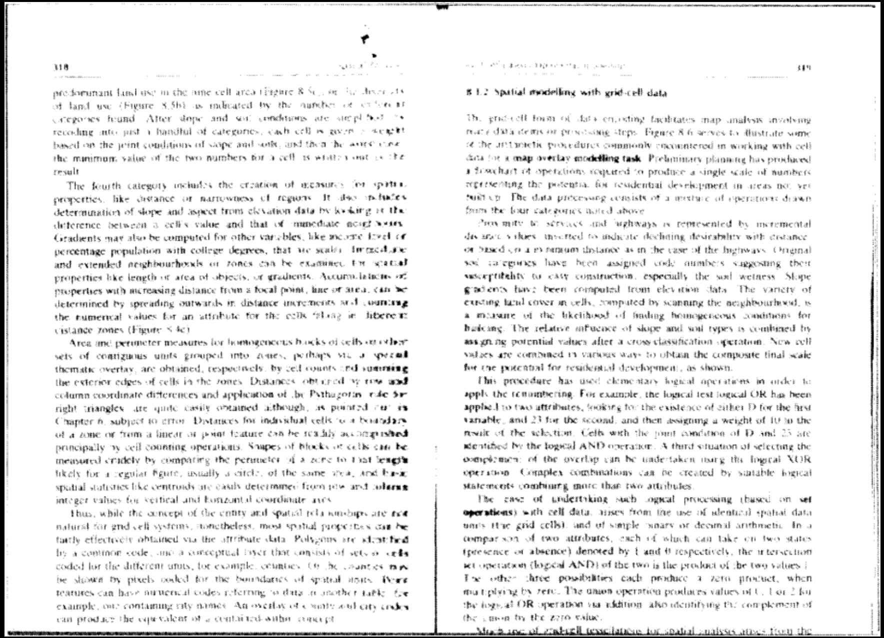
Department of Information Technology, Faculty of Engineering, Okayama University, 3-1-1, Tsushima Naka, Okayama 700, Japan

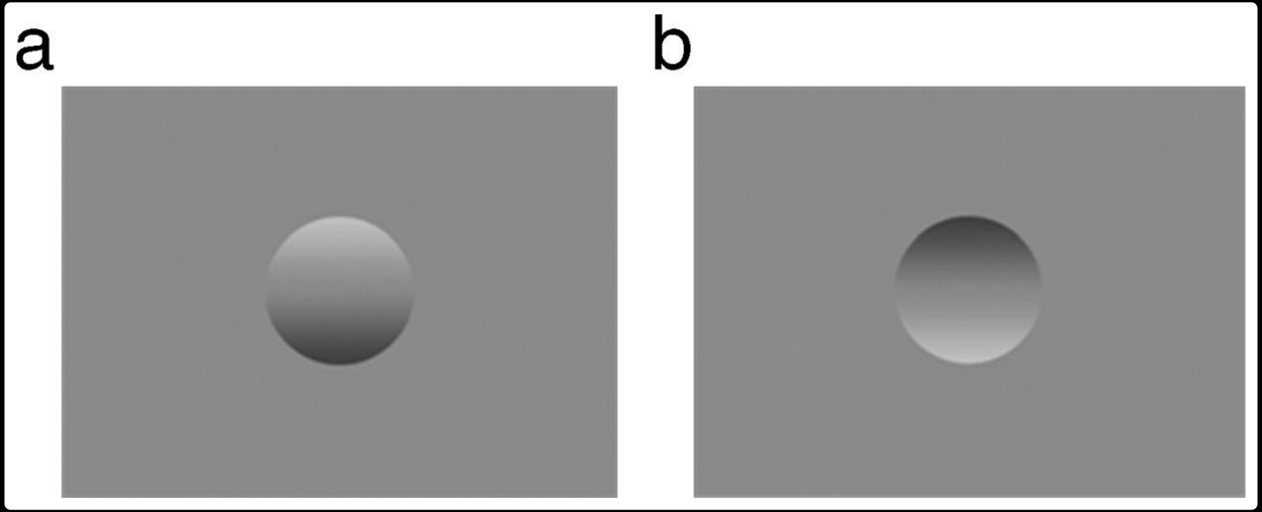


A real application that works





Human shape from shading



Thomas R et al. J Vis 2010; 10:6

Also check Ramachandran's work on Shape from Shading by Humans
http://psy.ucsd.edu/chip/ramabio.html