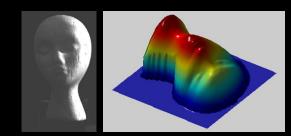
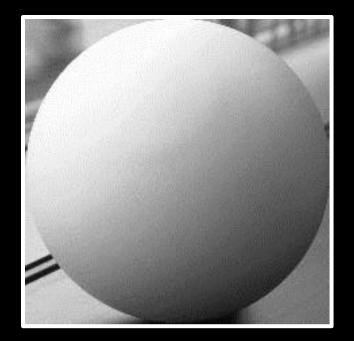
CS4495/6495 Introduction to Computer Vision

5C-L1 Shape from shading



Shape from shading





Shading as a cue for shape reconstruction

Shape from shading/lighting

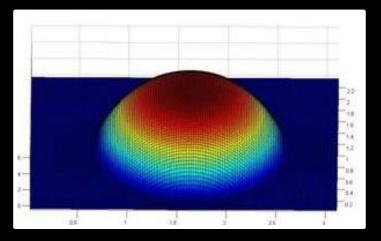
What is the relation between intensity and shape?

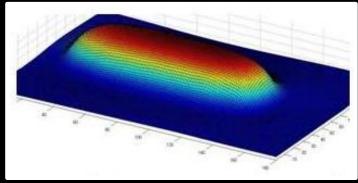
- Need to look at the reflectance function
- Reflectance Map

Surface normals: A bit of math

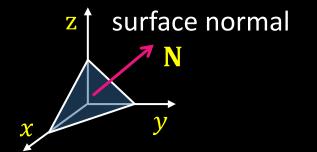
- Let's assume we have a surface z(x, y)
- We can define the following:

$$-\frac{\partial z}{\partial x} = \mathbf{p} \quad -\frac{\partial z}{\partial y} = \mathbf{q}$$





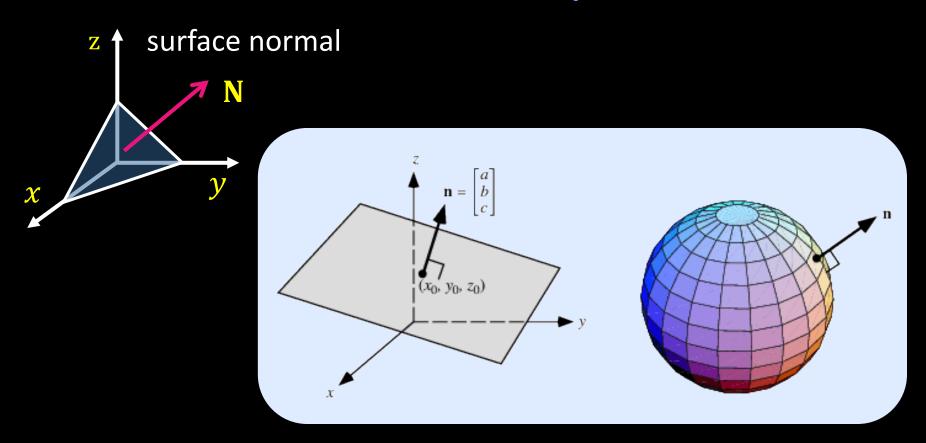
Surface Normal: A bit more math



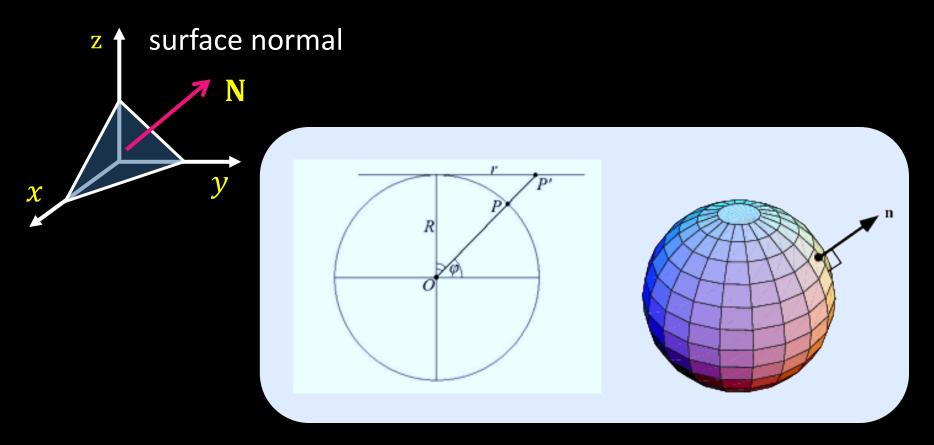
Suppose we have a point on the surface. We can define two tangents: $t_x = (1,0,-p)^T$ and $t_y = (0,1,-q)^T$

$$\mathbf{n} = \frac{N}{\|N\|} = \frac{t_x \times t_y}{\|t_x \times t_y\|} = \frac{1}{\sqrt{p^2 + q^2 + 1}} (p, q, 1)^T$$

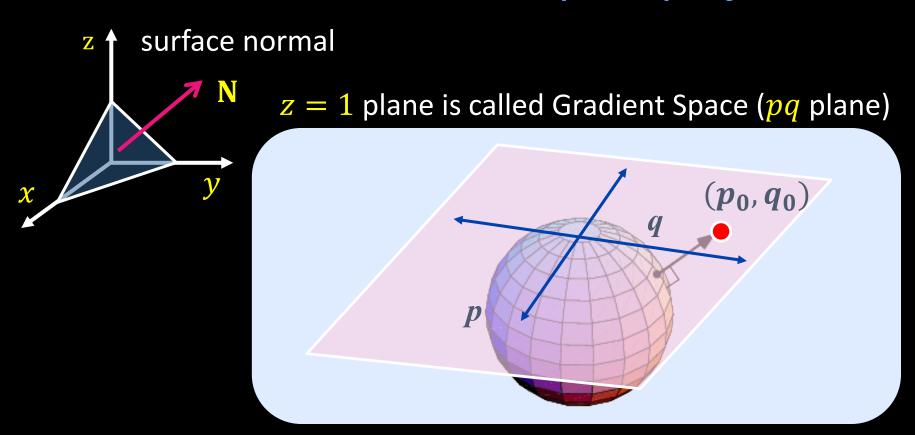
Surface Normal: Gaussian sphere



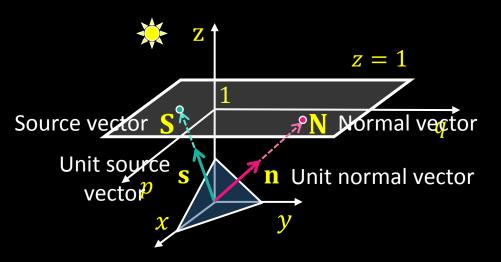
Surface Normal: Gradient space projection



Surface Normal: Gradient space projection



Gradient Space of Source and Normal



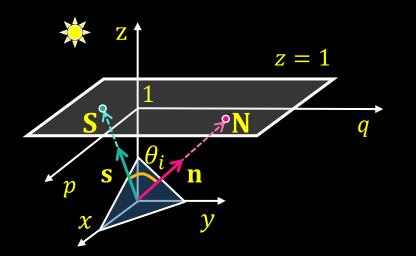
Unit normal vector:

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

Unit source vector:

$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_S, q_S, 1)}{\sqrt{p_S^2 + q_S^2 + 1}}$$

Gradient Space of Source and Normal



Unit normal vector:

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

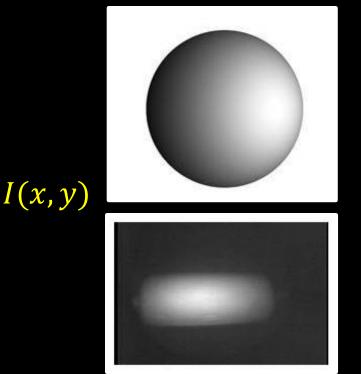
Unit source vector:

$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_S, q_S, 1)}{\sqrt{p_S^2 + q_S^2 + 1}}$$

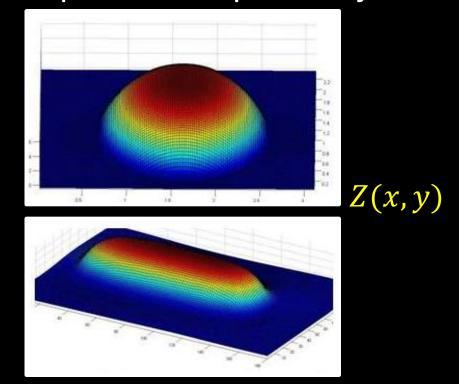
$$\cos \theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_S + qq_S + 1)}{\sqrt{p^2 + q^2 + 1} \sqrt{p_S^2 + q_S^2 + 1}}$$

Shape from shading: Problem definition

Input: 1 or more images



Output: 3D shape of object



Reflectance Map

Relates image brightness I(x, y) to surface orientation (p, q) for given source direction and surface reflectance

Reflectance Map: Lambertian case

Terms:

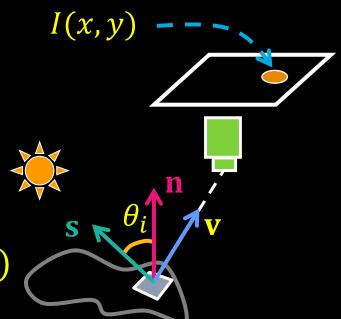
k: source brightness

: surface albedo (reflectance)

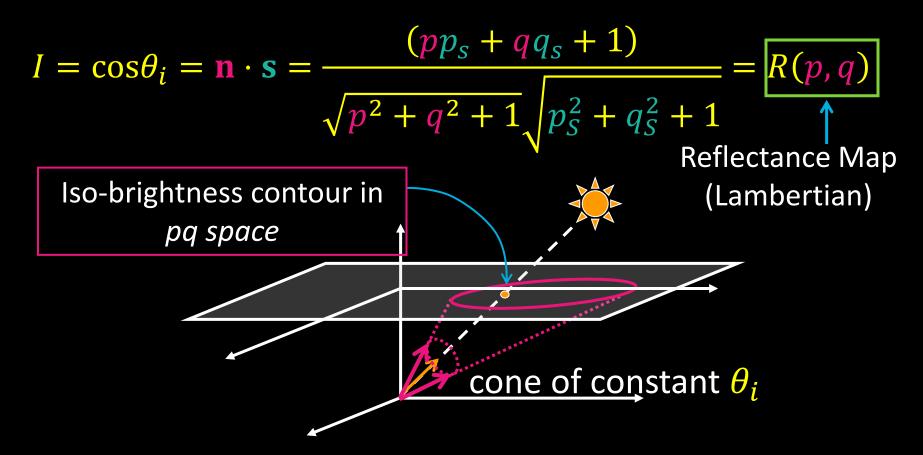
Image brightness:

$$I = \rho \cdot k \cdot \cos \theta_i = \rho \cdot k \; (\mathbf{n} \cdot \mathbf{s})$$

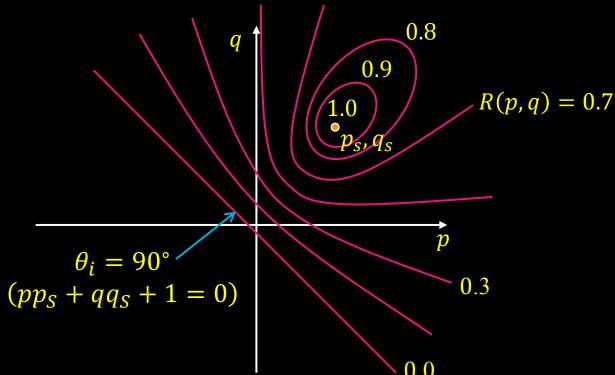
Let $\rho \cdot k = 1$ then $I = \cos \theta_i = \mathbf{n} \cdot \mathbf{s}$



Reflectance Map: Lambertian case



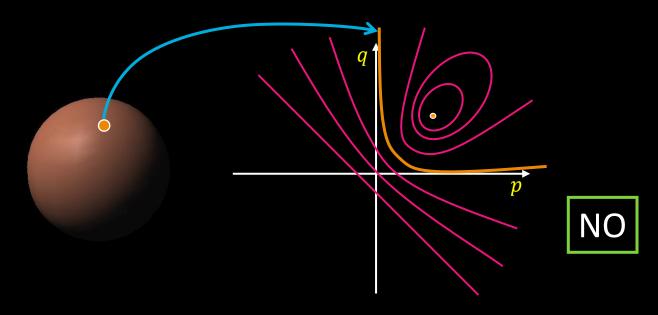
Iso-brightness contours



Note: R(p,q) is maximum when $(p,q) = (p_S, q_S)$

Shape from a single image?

Given R(p,q) ((p_S,q_S)) and surface reflectance) can we determine (p,q) uniquely for each image point?



Shape from Shading

Need more information:

Add more constraints: Shape-from-shading

Take more images: Photometric stereo

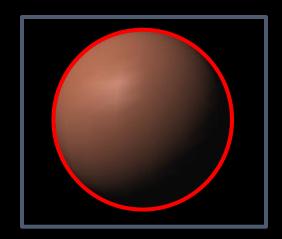
Shape from shading

Given a single image of an object with *known surface* reflectance taken under a known light source, can we recover its shape?

Shape from shading

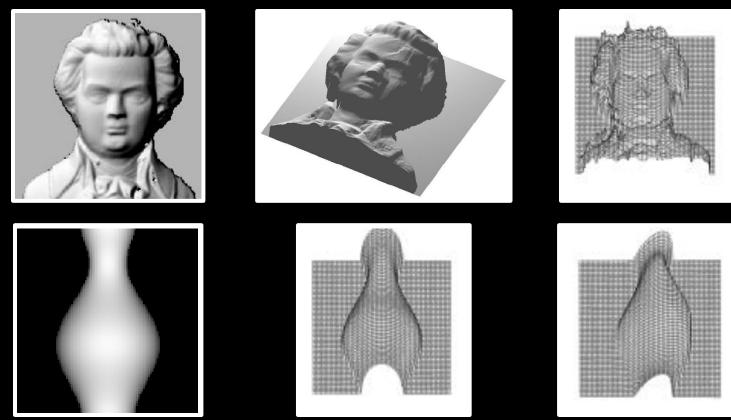
Given R(p,q) ((p_S,q_S)) and surface reflectance) can we determine (p,q) uniquely for each image point?

- Assume shape along the occluding boundary is known
- Constraints on neighboring normals
 - integrability
- Smoothness

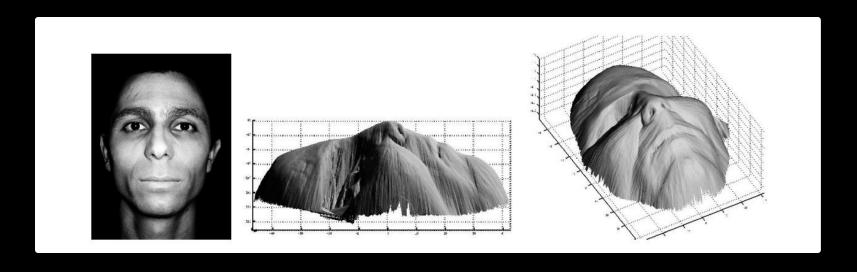


Yes, a slightly ugly optimization

Synthetic results



Shape from Shading: "Real" Results



- These single image methods work poorly in practice
- Why? The assumptions are quite restrictive

Shape from Shading

Need more information:

Add more constraints: Shape-from-shading

• Take more images: *Photometric stereo*

Photometric stereo

Input: Several images

- Same object
- Different lightings
- Same pose

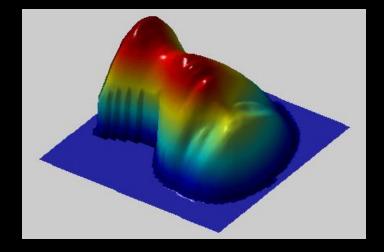




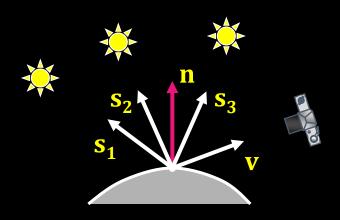


Output:

- 3D shape of object
- Albedo at (x, y)



Photometric stereo



Write this as a matrix equation:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \rho \begin{bmatrix} \mathbf{s_1}^T \\ \mathbf{s_2}^T \\ \mathbf{s_3}^T \end{bmatrix} \mathbf{n}$$

Image brightness:

$$I = \rho \cdot k \cdot \cos \theta_i = \rho \mathbf{n} \cdot \mathbf{s}$$

where $(\mathbf{k} = 1)$

$$I_1 = \rho \mathbf{n} \cdot \mathbf{s_1}$$
$$I_2 = \rho \mathbf{n} \cdot \mathbf{s_2}$$
$$I_3 = \rho \mathbf{n} \cdot \mathbf{s_3}$$

Lambertian!

Solving the equations: Linear

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \mathbf{s_1}^T \\ \mathbf{s_2}^T \\ \mathbf{s_3}^T \end{bmatrix} \rho \mathbf{n}$$

$$\mathbf{I} \qquad \mathbf{S} \qquad \widetilde{\mathbf{n}}$$

$$3 \times 1 \qquad 3 \times 3 \qquad 3 \times 1$$

inverse
$$\widetilde{\mathbf{n}} = \mathbf{S}^{-1} \widetilde{\mathbf{I}}$$

$$\rho = |\widetilde{\mathbf{n}}|$$

$$\mathbf{n} = \frac{\widetilde{\mathbf{n}}}{|\widetilde{\mathbf{n}}|} = \frac{\widetilde{\mathbf{n}}}{\rho}$$

I and S are known

Adding more light sources

Get better results by using more (M) lights:

$$\begin{bmatrix} I_1 \\ \vdots \\ I_M \end{bmatrix} = \begin{bmatrix} \mathbf{s_1}^T \\ \vdots \\ \mathbf{s_M}^T \end{bmatrix} \rho \mathbf{n}$$

Least squares solution:

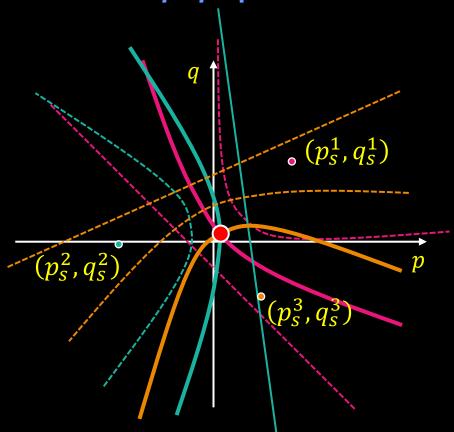
I =
$$S\widetilde{n}$$
 $M \times 1 = (\underline{M} \times 3)(3 \times 1)$

$$S^{T}I = S^{T}S\widetilde{n}$$

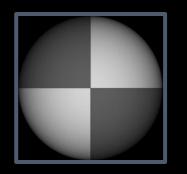
$$\widetilde{n} = (S^{T}S)^{-1}S^{T}I$$

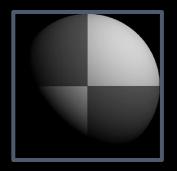
$$\min ||I - S\widetilde{n}||_{2}^{2}$$
Moore-Penrose pseudo inverse
Solve for ρ , n as before

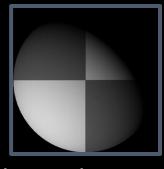
Photometric stereo: pq space

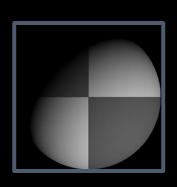


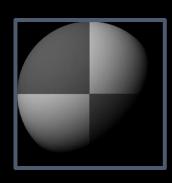
Results: Lambertian sphere





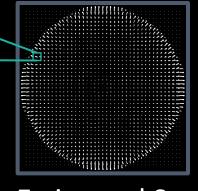




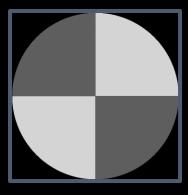


Input Images

Needles are projections of surface normal on image plane







Estimated Albedo

Photometric stereo: Lambertian toy

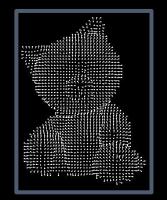








Surface Normals



Input Images



Albedo

Estimated Surface Normals

Estimated Albedo

And from GT: www.trimensional.com



Photometric stereo: Limitations

Big problems

- Doesn't work for shiny things, semi-translucent things
- Shadows, inter-reflections

Smaller problems

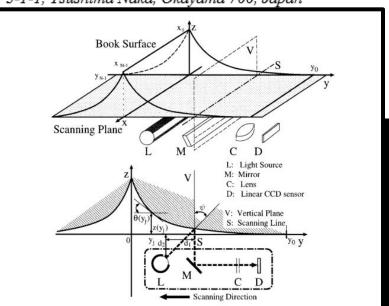
- Camera and lights have to be distant
- Calibration requirements
 - Measure light source directions, intensities
 - Camera response function

A real application that works

Shape from Shading with Interreflections Under a Proximal Light Source: Distortion-Free Copying of an Unfolded Book

TOSHIKAZU WADA[†], HIROYUKI UKIDA[†] AND TAKASHI MATSUYAMA* Department of Information Technology, Faculty of Engineering, Okayama University, 3-1-1, Tsushima Naka, Okayama 700, Japan





A real application that works

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predominant land use in the one cell area (Figure 8.5), or by June 4.5 of land use (Figure 8.5b), as indicated by the number of effect if corpores found. After other and soil conditions are steple feel as recoding into just a handful of categories, each cell is given a security based on the unit condition of some parts. The activities the product of the production of th

the minimum value of the two numbers for a cell is written out is the

The fourth category normalists the creation of accounts, for optital properties, like obvaine or nativeness of regions. It does notice determination of slope and aspect from elsewhere the determination of slope and aspect from elsewhere the determination of slope and aspect from elsewhere the determination and elsewhere the determination and elsewhere the determination and elsewhere the determination with college degrees, that we scalar for restrict and extended neighbourhoods or zones can be examined for establishment of the degree degree that we scalar for examine the scalar properties with increasing distance from a local point, the or area, can be determined by properting notices for defended in defended on elsewhere the determination of the degree of the de

Area and permitter measures for homogeneous brocks of selb-or ordered of contiguous units grouped into ones, perhaps via a special thematic overlay, are obtained, respectively, by cell counts and summing the extreme edges of cells in the rones. Datances oft acid by row and column coordinate differences and applications of the Psthagorian releaseringht transgles, are quite casily obtained adhough, as pointed on Empire in subject to error. Distances for individual vells or a boundary of a case on from a limeter of point feature can be readily as angivined principally by cell counting operations. Supers of blocks or closely one propagally special counting operations. Supers of blocks or closely can be readily as and brock spotal statistics like centroids in coasily determine from row and column propagal statistics like centroids in coasily determine from row and columns interest robusts for section and brock of the coasily controlled in the coasily determined from row and columns.

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8 L2 Spatial modelling with grid-cell data

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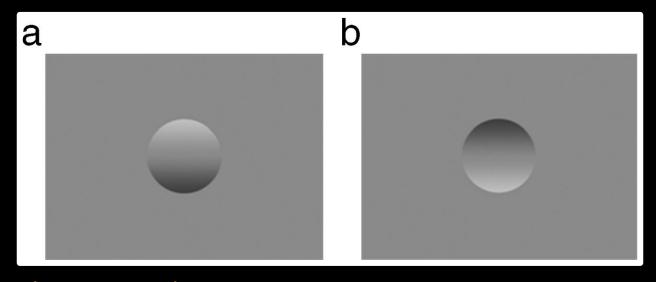
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This procedure has used obtenedate slogical operations in order to apply the termineting. For examile, the flugical test oscial OR has been applied to two attributes, fooking for the esistence of aither D for the first variable, and 23 for the second, and then assigning a weight of this the result of the selection. Gels with the joint anothere of D and 23 are sentified by the logical AND operation. A further stration of selecting the compleximen, of the overlay and he undertaken using the lingual VOR operation. Compiles combinations can be created by satisfied signal statements conducting mentions.

The case of underthing such against processing this conportations is whell data insert from the use of identical spiral data units the gold cells), and of simple source or decord architecture. In a comparison of non-attributes, each of ships, can take on the source experience or absence) denoted by 1 and 0 respectively, the transaction or spectation (logical AND) of the two in the product of the two volues 1. The other drive possibilities each produce 1 acts in product, when mat playing by term. The cause operation produces solutes of 1, 1 or 2 for the days, at OR operation is a diffusion, also dentifying the complement of the growth of the zero volue.

Much and of gradual tensilations for spatial analysis arises from the

Human shape from shading



Thomas R et al. J Vis 2010; 10:6

Also check Ramachandran's work on Shape from Shading by Humans http://psy.ucsd.edu/chip/ramabio.html