# CS4495/6495
# Introduction to Computer Vision

8B-L1 *Classification: Generative models*

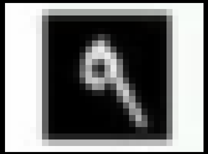
# *Supervised* classification

Given a collection of labeled examples, come up with a function that will predict the labels of new examples.

Training examples

"four"

"nine"



Novel input

?

Kristen Grauman

# Supervised classification

How good is the function we come up with to do the classification? (What does "good" mean?)

Depends on:

- What mistakes does it make
- Cost associated with the mistakes

Kristen Grauman

# Supervised classification

Since we know the desired labels of training data, we want to *minimize the expected misclassification*

# Supervised classification

Two general strategies

- Use the training data to build representative probability model; separately model class-conditional densities and priors (***Generative***)
- Directly construct a good decision boundary, model the posterior (***Discriminative***)

# Supervised classification: Generative

Given labeled training examples, predict labels for new examples

- Notation: $(4 \rightarrow 9)$ - object is a ‘4’ but you call it a ‘9’
- We’ll assume the cost of $(X \rightarrow X)$ is zero.

Kristen Grauman

# Supervised classification: Generative

Consider the two-class (binary) decision problem:

- $L(4 \rightarrow 9)$: Loss of classifying a 4 as a 9
- $L(9 \rightarrow 4)$: Loss of classifying a 9 as a 4

Kristen Grauman

# Supervised classification: Generative

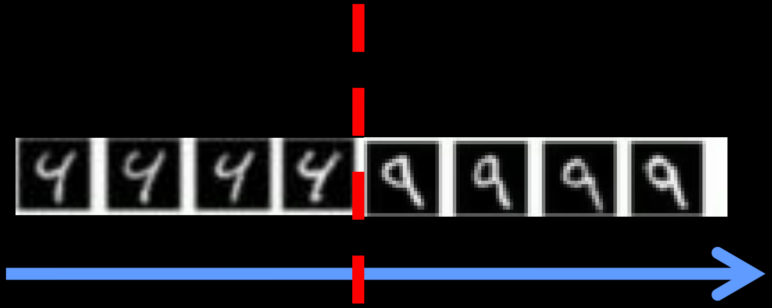
***Risk*** of a classifier strategy ***S*** is expected loss:

$$
\begin{aligned}
R(\mathrm{S}) = \Pr\left(4 \to 9 \mid \text{using S}\right) L\left(4 \to 9\right) \\
+ \Pr\left(9 \to 4 \mid \text{using S}\right) L\left(9 \to 4\right)
\end{aligned}
$$

We want to choose a classifier so as to minimize this total risk

Kristen Grauman

# Supervised classification: minimal risk



Feature value $x$

At best decision boundary, either choice of label yields same expected loss.

If we choose class "four" at boundary, expected loss is:

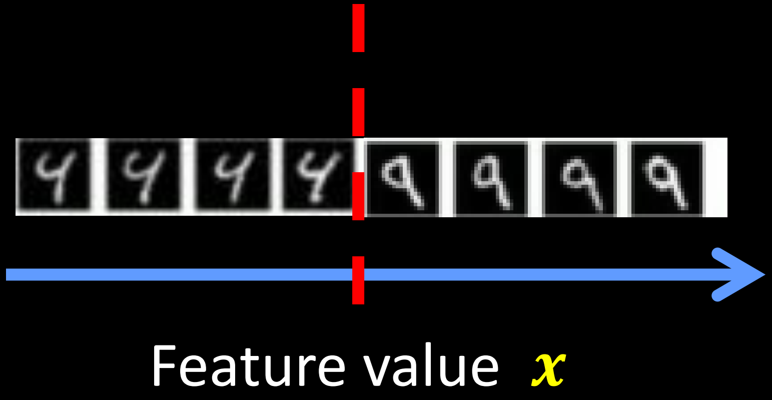
$$= P(\text{class is } 9|\mathbf{x})\ L(9 \to 4) + P(\text{class is } 4\ |\mathbf{x}) L(4 \to 4)$$

If we choose class "nine" at boundary, expected loss is:

$$= P(\text{class is } 4|\mathbf{x})\ L(4 \to 9)$$

Kristen Grauman

# Supervised classification: minimal risk



At best decision boundary, either choice of label yields same expected loss.

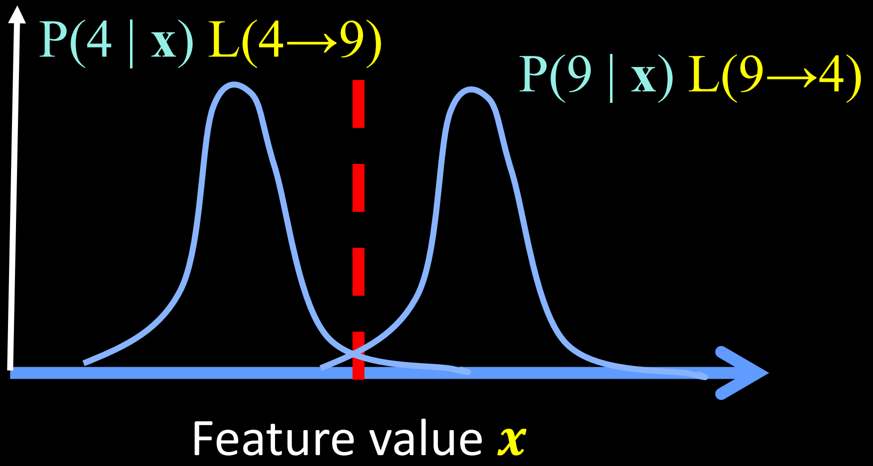
So, best decision boundary is at point **x** where:

$$P(\text{class is } 9|\mathbf{x})\ L(9 \rightarrow 4) = \mathrm{P}(\text{class is } 4|\ \mathbf{x}) L(4 \rightarrow 9)$$

To classify a new point, choose class with lowest expected loss; i.e., choose "four" if: $P(4\,|\,\mathbf{x})L(4 \rightarrow 9) > P(9\,|\,\mathbf{x})L(9 \rightarrow 4)$

Kristen Grauman

# Supervised classification: minimal risk




At best decision boundary, either choice of label yields same expected loss.

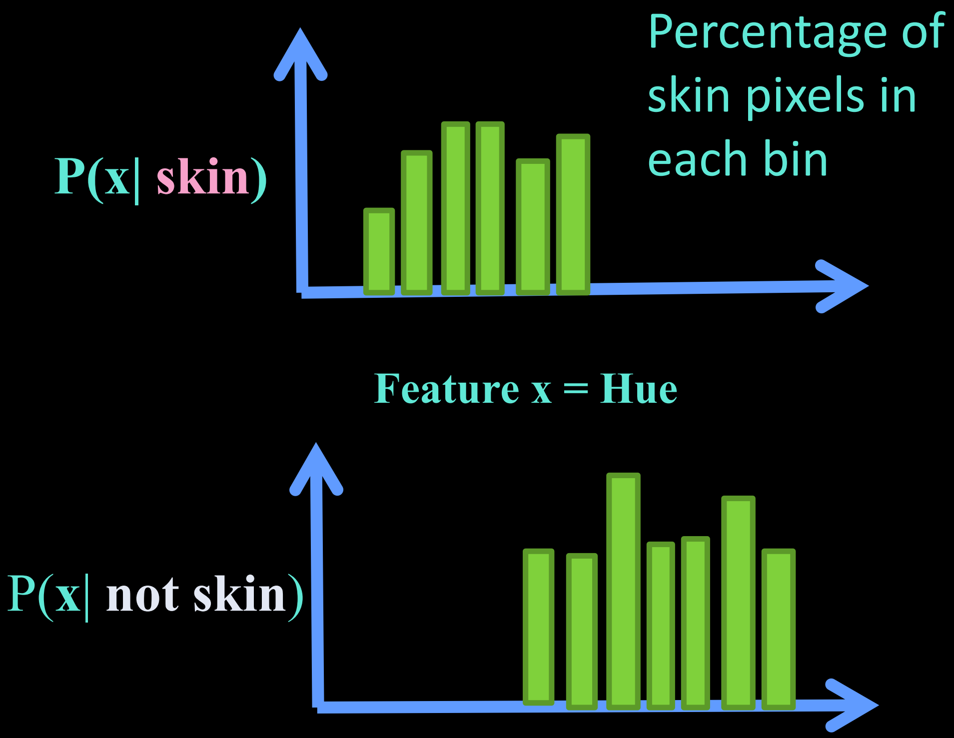
So, best decision boundary is at point **x** where:

$$\boxed{P(\text{class is } 9|\mathbf{x})}\, L(9 \rightarrow 4) = \boxed{P(\text{class is } 4|\, \mathbf{x})} L(4 \rightarrow 9)$$

*How to evaluate these probabilities?*

Kristen Grauman

# Example: learning skin colors




Kristen Grauman

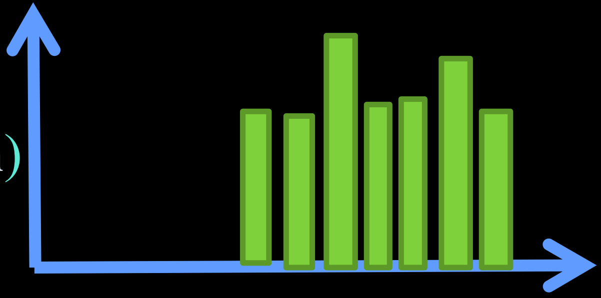
# Example: learning skin colors

P(x| skin)

Percentage of skin pixels in each bin

Feature x = Hue

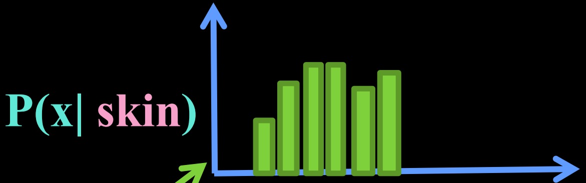
P(x| not skin)



Now we get a new image, and want to label each pixel as skin or non-skin.

Kristen Grauman

# Bayes rule




$$\overbrace{P(\text{skin} \mid x)}^{\text{posterior}} = \frac{\overbrace{P(x \mid \text{skin})}^{\text{likelihood}}\ \overbrace{P(\text{skin})}^{\text{prior}}}{P(x)}$$

$$P(\text{skin} \mid x) \propto P(x \mid \text{skin})\ \boxed{P(\text{skin})}$$

*Where does the prior come from?*

# Bayes rule in (ab)use

Likelihood ratio test (assuming cost of errors is the same):

If $P(skin|\boldsymbol{x}) > P(\sim skin|\boldsymbol{x})$ classify $\mathbf{x}$ as skin

… so ….

If $P(\boldsymbol{x}|skin)P(skin) > P(\boldsymbol{x}|\sim skin)(P(\sim skin)$ classify $\boldsymbol{x}$ as skin (Bayes rule)

*(if the costs are different just re-weight)*

# Bayes rule in (ab)use

… but I don't really know prior $P(skin)$…

… but I can assume it some constant $\Omega$ …

… so with some training data I can ***estimate*** $\Omega$ …

…. and with the same training data I can ***measure*** the ***likelihood densities*** of ***both*** $P(\boldsymbol{x}|skin)$ and $P(\boldsymbol{x}|{\sim}skin)$ …

So…. I can more or less come up with a rule…

Steve Seitz

# Example: classifying skin pixels

Now for every pixel in a new image, we can estimate probability that it is generated by skin:

If $p(skin|x) > \theta$ classify as skin; otherwise not

Brighter pixels are higher probability of being skin

Kristen Grauman

# Example: classifying skin pixels

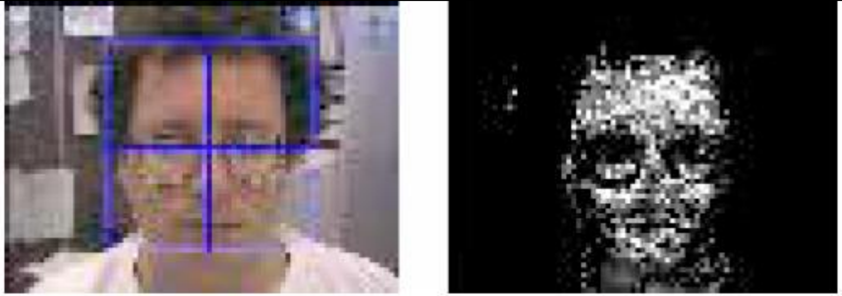


**Figure 6:** A video image and its flesh probability image

**Figure 7:** Orientation of the flesh probability distribution marked on the source video image

Gary Bradski, 1998

# Example: classifying skin pixels

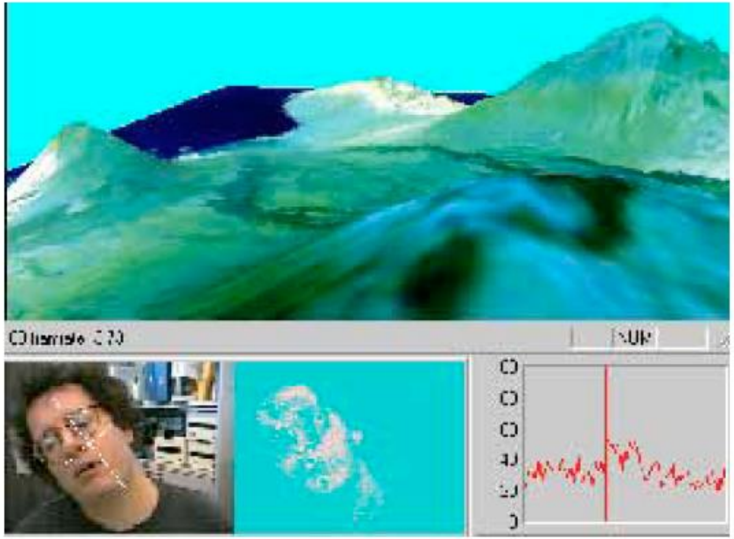


**Figure 13:** CAMSHIFT-based face tracker used to “fly” over a 3D graphic’s model of Hawaii

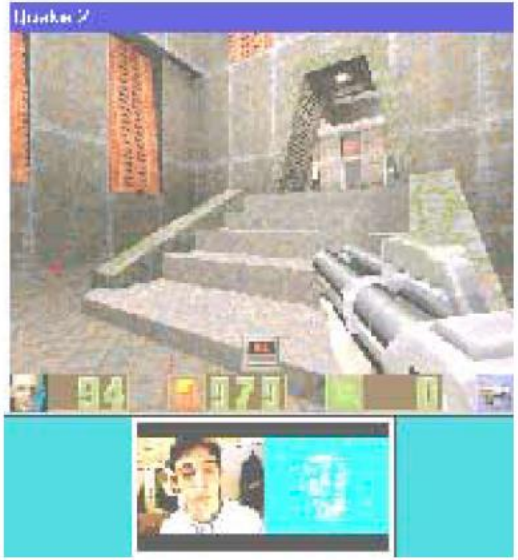


**Figure 12:** CAMSHIFT-based face tracker used to play Quake 2 hands free by inserting control variables into the mouse queue

Gary Bradski, 1998

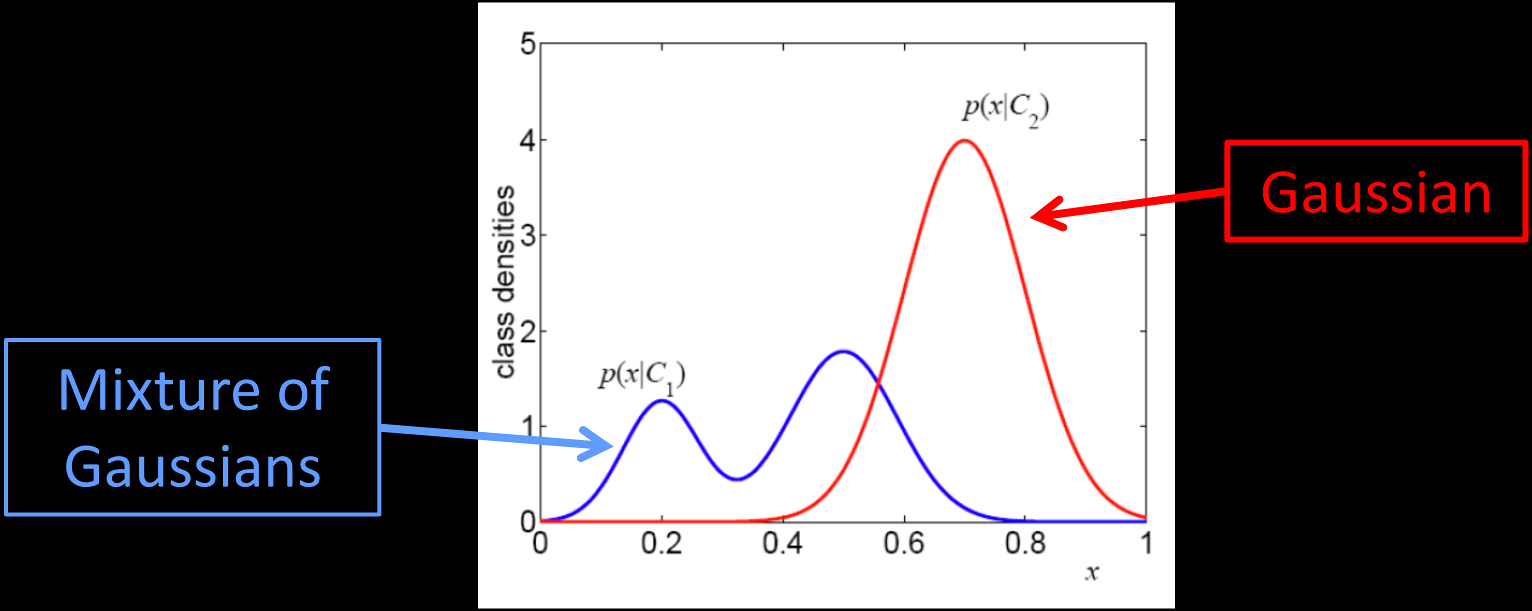
# More general generative models

For a given measurement $\mathbf{x}$ and set of classes $c_i$ choose $c^*$ by:

$$c^* = \arg\max_c p(c \mid \mathbf{x}) = \arg\max_c p(c)\, p(\mathbf{x} \mid c)$$

# Continuous generative models

- If $\mathbf{x}$ is continuous, need *likelihood* density model of $p(\mathbf{x}|c)$
- Typically parametric – Gaussian or mixture of Gaussians



# Continuous generative models

- Why not just some histogram or some KNN (Parzen window) method?
  - You might…
  - But you would need lots and lots of data everywhere you might get a point
  - The whole point of modeling with a parameterized model is not to need lots of data.

# Summary of generative models:

+ Firm probabilistic grounding

+ Allows inclusion of prior knowledge

*+ Parametric modeling of likelihood permits using small number of examples*

*+ New classes do not perturb previous models*

+ Others:

  Can take advantage of unlabelled data

  Can be used to generate samples

# Summary of generative models:

- And just where did you get those priors?
- Why are you modeling those obviously non-C points?
- The example hard cases aren’t special
- If you have lots of data, doesn’t help

# Next…

- A really cool way of building a generative model for face recognition (not detection)
- And then *discriminative* models…