SAS assignment

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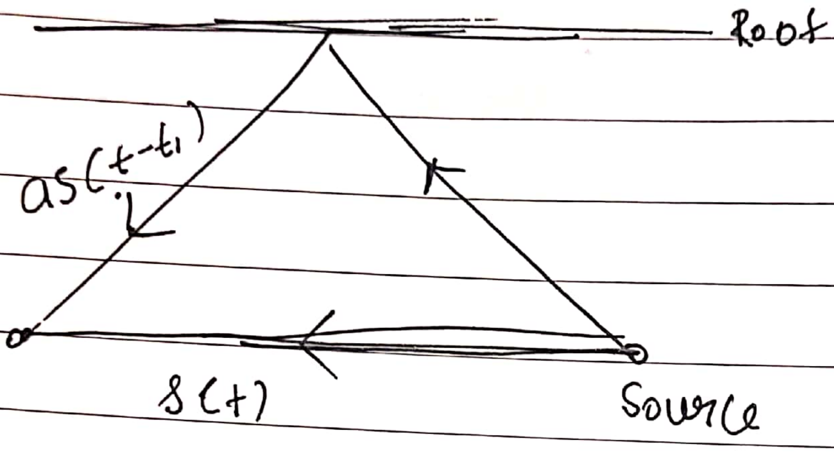
Q1



$0 < a < 1 \rightarrow$ Attentoation

(i) $z(t)$

signal on reciver side = signal from source + signal reflected from roof

$$\boxed{z(t) = s(t) + as(t-t_1)}$$

(ii) Let

$$s(t) \xleftrightarrow{\mathcal{L}} S(s)$$

$$\Rightarrow \quad s(t-t_1) \xleftrightarrow{\mathcal{L}} e^{-st_1} S(s)$$

Now, $z(t) = s(t) + as(t-t_1)$

$$\Rightarrow \quad Z(s) = S(s) + ae^{-st_1} S(s)$$

$$\Rightarrow Z(s) = S(s) \left[1 + ae^{-st_1}\right]$$

$$= S(s) \left[1 + ae^{-st_1}\right]$$

$$= S(s)\, H(s)$$

Transfer function

$$H(s) = 1 + ae^{-st_1}$$

(ii) ~~[illegible]~~ To cascade a system $G(s)$ to recover the original signal $S(s)$

$$S(s) \longrightarrow \boxed{H(s)} \xrightarrow{Z(s)} \boxed{G(s)} \xrightarrow{S(s)}$$

$$S(s) \cdot H(s) \cdot G(s) = S(s)$$

$$G(s)\, H(s) = 1$$

$$\boxed{G(s) = \frac{1}{1 + a e^{-st_1}}}$$

To check stability

Let $s = \sigma + j\omega$

Poles: $1 + a e^{-st_1} = 0$

$$e^{-\sigma t_1}\, e^{-j\omega t_1} = -1/a$$

$$e^{-\sigma t_1} = \frac{1}{a} \Rightarrow \sigma = \frac{1}{t} \ln a < 0$$

$(a < 1)$

$\therefore$ ~~Poles lie in~~

Pole lies in LHP $\Rightarrow$ System is stable

Q2

$x(t) = 2\cos(100\pi t + \pi/2) + \cos(300\pi t)$

$x(t) = -2\sin(100\pi t) + \cos(300\pi t)$

$\omega_1 = 100\pi$ rad/s
$\Rightarrow f_1 = 50$ Hz

$\omega_2 = 300\pi$ rad/s
$f_2 = 150$ Hz

(i) Output is same as input
∴ No loss in frequency
Nyquist criterion stay valid

$f_s \geq 2 f_{max}$

$f_{max} = 150$ Hz

$$\boxed{f_s \geq 300 \text{ Hz}}$$

Sampling freq $\geq$ 300 samples / second.

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ii) Now,

$f_s = 250$ samples / sec

$= 250$ Hz

$\because 250 < 300$

Aliasing occurs

$$x[n] = x(nT_s) = x\left[\frac{n}{250}\right]$$

$$= 2\cos\left[\frac{100\pi}{250} + \frac{\pi}{2}\right] + \cos\left[\frac{300}{250}n\pi\right]$$

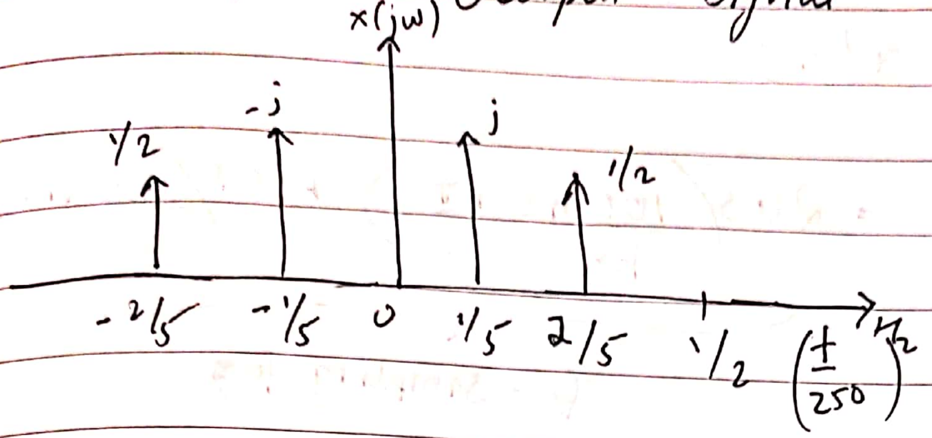
$$= 2\cos\left[\frac{2n\pi}{5} + \frac{\pi}{2}\right] + \cos\left[\frac{6n\pi}{5}\right]$$

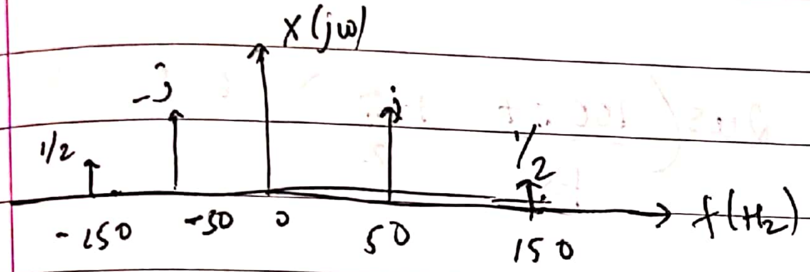
$$= 2\cos\left[\frac{2n\pi}{5} + \frac{\pi}{2}\right] + \cos\left[\frac{6n\pi}{5} - 2n\pi\right]$$

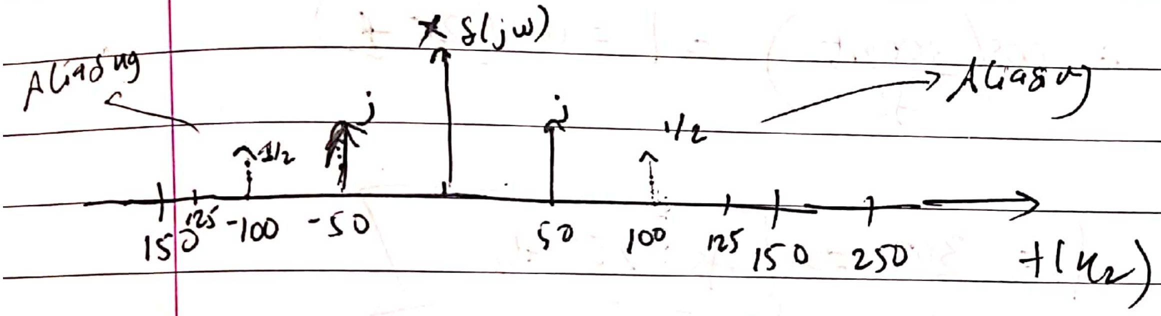
$$\boxed{x[n] = 2\cos\left(\frac{2\pi n}{5} + \frac{\pi}{2}\right) + \cos\left(\frac{4n\pi}{5}\right)}$$

Scanned with CamScanner

(iii) One period: Output Signal Spectrum







(iv) $x[f] = x[n]$

$$= 2\cos\left(\frac{100}{fs}\pi t + \frac{\pi}{2}\right) + \cos\left(\frac{300}{fs}\pi t\right)$$

fs = sampling freq.

Given $y(t) = 2\cos\left(\frac{100\pi t}{fs} + \frac{\pi}{2}\right) + 1$

$$\therefore \cos\left(\frac{300\pi t}{fs}\right) = 1 = \cos(2\pi t)$$

$$fs = \frac{300}{2} \Rightarrow \boxed{fs = 150\ Hz}$$