Part 3: Gradient Descent Manual Calculation

Linear Equation $y = mx + b$

initially $m = -1$ Learning Rate $\alpha = 0.1$

Initial $b = 1$ Given points: $(1,3)$ and $(3,6)$

MSE: $J(m,b) = \frac{1}{n} \sum (y_i - \hat{y}_i)^2 x_i$

Gradients $\frac{dJ}{dm} = \frac{-2}{n} \sum (y_i - \hat{y}_i) x_i$

$\frac{dJ}{db} = \frac{-2}{n} \sum (y_i - \hat{y}_i)$

where $\hat{y} = mx + b$

$(x_i, y_i) = (1,3)$ $(x_2, y_2) = (3,6)$

Predictions $(\hat{y}_i)$:

$\hat{y}_1 = (-1)(1) + 1$

$= 0$

$\hat{y}_2 = (-1)(3) + (1)$

$= -2$

② Compute Gradients: $(1,3)$ $(3,6)$

$\frac{dJ}{dm} = \frac{-2}{2} \left[(3-0)(1) + (6-(-2))(3) \right]$

$= -1 \left[3 + 24 \right]$

$= -27$

$m_{new} = m_{old} - \alpha \frac{dJ}{dm}$

$\frac{dJ}{db} = \frac{-2}{2} \left[(3-0) + (6-(-2)) \right]$

$= -1 (3+8)$

$= -11$

$$m_{new} = m_{old} - \alpha \frac{dJ}{dm}$$

$$m_{new} = -1 - 0.1(-27)$$
$$= 1.7$$

$$b_{new} = b_{old} - \alpha \frac{dJ}{db}$$

$$b_{new} = 1 - 0.1(-11)$$
$$= 2.1$$

$$m_{new} = 1.7 \qquad b_{new} = 2.1$$