

Given:

linear equation: $y = mx + b$

$$m = 1.7 \quad b = 2.1 \quad \alpha = 0.1$$

$$(1, 3) \text{ and } (3, 6)$$

$$p = 2$$

gradient formula:

$$\frac{\partial J}{\partial m} = -\frac{2}{n} \sum (y_i - \hat{y}_i) x_i$$

$$\frac{\partial J}{\partial b} = -\frac{2}{n} \sum (y_i - \hat{y}_i)$$

$$m_{\text{new}} = m_{\text{old}} - \alpha \left(\frac{\partial J}{\partial m} \right)$$

$$b_{\text{new}} = b_{\text{old}} - \alpha \left(\frac{\partial J}{\partial b} \right)$$

1) predictions (\hat{y}_i), $(1, 3)$ $(3, 6)$
 $y = mx + b$

$$\hat{y}_1 = 1.7(1) + 2.1, \quad \hat{y}_1 = 3.8$$

$$\hat{y}_2 = 1.7(3) + 2.1, \quad \hat{y}_2 = 7.2$$

2. Gradients

$$\frac{\partial J}{\partial m} = \frac{-2}{n} \sum (y_i - \hat{y}_i) x_i \quad \text{where } n=2$$

$$= \frac{-2}{2} [(3 - 3.8)(1) + (6 - 7.2)(3)]$$

$$= -1 [(-0.8) + (-3.6)]$$

$$= -1(-4.4) = 4.4$$

$$\frac{\partial J}{\partial b} = \frac{-2}{n} \sum (y_i - \hat{y}_i)$$

$$= -\frac{2}{2} [(3 - 3.8) + (6 - 7.2)]$$

$$= -1 [(-0.8) + (-1.2)]$$

$$= -1(-2) = 2$$

$$y_1 = 3$$

$$y_2 = 6$$

$$\hat{y}_1 = 3$$

$$\hat{y}_2 = 7$$

$$x_1 = 1$$

$$x_2 = 3$$

3. Updating parameters, where $m_{old} = 1.7$

$$b_{old} = 2.1$$

$$m_{new} = m_{old} - \alpha \frac{\partial J}{\partial m}$$

$$\alpha = 0.1$$

$$b_{new} = b_{old} - \alpha \frac{\partial J}{\partial b}$$

$$\frac{\partial J}{\partial m} = 4.4$$

$$\frac{\partial J}{\partial b} = 2$$

$$\rightarrow m_{new} = 1.7 - 0.1 (4.4) \\ = 1.26$$

$$b_{new} = 2.1 - 0.1 (2) \\ = 1.9$$

$$m_{new} = 1.26, b_{new} = 1.9$$