Problem §1.1: 8(a,d,f,h): Let p and q be the propositions

p: I bought a lottery ticket this week.
q: I won the million dollar jackpot.

Express each of these propositions as an English sentence.

(a) $\neg p$

(d) $p \wedge q$

(f) $(\neg p) \implies (\neg q)$

(h) $(\neg p) \vee (p \wedge q)$

Solution. Below are the propositions in plain English

(a) I didn't buy a lottery ticket this week.

(d) I bought a lottery ticket this week and I won the million dollar jackpot.

(f) If I didn't buy a lottery ticket this week, then I didn't win the million dollar jackpot.

(h) I either didn't buy a lottery ticket this week or I bought one and won the million dollar jackpot.

□

Problem §1.2: 6: Use a truth table to verify the first De Morgan law $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

Solution.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

□

Problem §1.4: 14(a,d): Express each of these quantifications in English, if the domain consists of all real numbers. Then, determine the truth value of the statement

(a) $\exists x(x^3 = -1)$

(d) $\forall x(2x > x)$

Solution. Here are the quantifications in English

(a) There exits an x; and the statement is True

(d) For all x; and the statement is False

□

Problem §2.1: 10(a,c,e,g): Determine whether the following statements are true or false.

(a) $\emptyset \in \{\emptyset\}$

(c) $\{\emptyset\} \in \{\emptyset\}$

(e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$

(g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

Solution. Is an element of and is a subset of.

(a) True

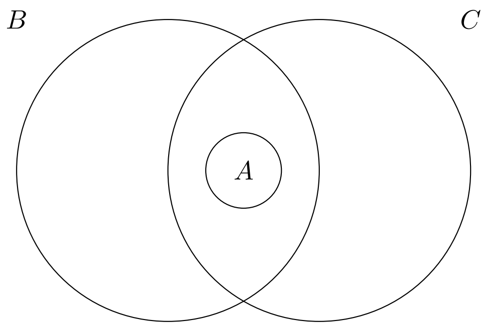
(c) False

(e) True

(g) True

□

Problem §2.1: 16: Use a Venn diagram to illustrate the relationships $A \subset B$ and $A \subset C$.



Solution.

□

Problem §2.1: 20: What is the cardinality of each of the following sets?

(a) $\emptyset$

(b) $\{\emptyset\}$

(c) $\{\emptyset, \{\emptyset\}\}$

(d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

Solution. Following are the cardinality of each set

(a) zero

(b) one

(c) two

(d) three

□

Problem §2.1: 26: Show that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$.

Proof. Let $a \in A$ and $b \in B$. By definition of cartesian product, $A \times B = \{(a, b)\}$. It is given that $A \subseteq C$ and $B \subseteq D$, then $a \in C$ and $b \in D$. So $\forall a, b, (a, b) \in C \times D$. Hence $A \times B \subseteq D \times D$ □

Problem §2.1: 32(a,c): Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find the following Cartesian products.

(a) $A \times B \times C$

(c) $C \times A \times B$

Solution.

$$\begin{aligned} A \times B \times C = \{&(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), \\ &(b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1), \\ &(c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\} \end{aligned}$$

$$\begin{aligned} C \times A \times B = \{&(0, a, x), (0, a, y), (0, b, x), (0, b, y), \\ &(0, c, x), (0, c, y), (1, a, x), (1, a, y), \\ &(1, b, x), (1, b, y), (1, c, x), (1, c, y)\} \end{aligned}$$

□

Problem §2.2: 4: Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find:

(a) $A \cup B$.

(b) $A \cap B$.

(c) $A - B$.

(d) $B - A$.

Solution.

(a) $A \cup B = \{a, b, c, d, e, f, g, h\}$.

(b) $A \cap B = \{a, b, c, d, e\}$.

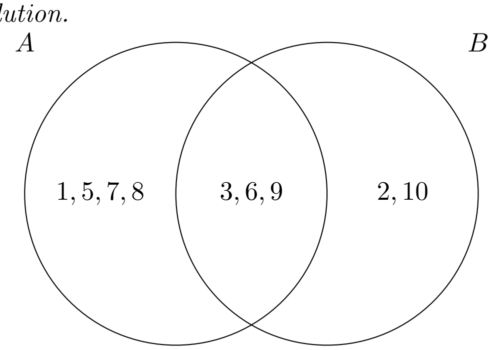
(c) $A - B = \emptyset$.

(d) $B - A = \{f, g, h\}$.

□

Problem §2.2: 14: Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

Solution.



As presented by the graph, $A = \{1, 5, 7, 8, 3, 6, 9\}, B = \{3, 6, 9, 2, 10\}$

□

Problem §2.2: 15: Prove the second De Morgan law in Table 1 by showing that if A and B are sets, then $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (a) showing each side is a subset of the other side and (b) by using a membership table.

Solution. Proof. Method 1:

For the first method we need to show that $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$ and $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$.

Starting with $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$:

Let $x \in \overline{A \cup B}$, then $x \notin A$ and $x \notin B$. Then $x \in \overline{A} \cap \overline{B}$, which means $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$.

Then to prove $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$:

Let $x \in \overline{A} \cap \overline{B}$, then $x \in \overline{A}$ and $x \in \overline{B}$. Then $x \notin A$ and $x \notin B$. So $x \notin A \cup B$ which means $x \in \overline{A \cup B}$. Hence $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$.

Because $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$ and $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$, $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

□

Proof. Method 2:

In this method, we list the membership table of both sets and if the columns of both sets are identical, both sets are equivalent.

A	B	$A \cup B$	$\overline{A \cup B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cap \overline{B}$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

From the table we can tell that $\overline{A \cup B}$ and $\overline{A} \cap \overline{B}$ are identical in columns. Hence $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

□

□

Problem §2.2: 24: Let A, B, and C be sets. Show that $(A - B) - C = (A - C) - (B - C)$.

Solution. We are checking the membership table of both left side and right side of the equation we need proof. If coloms of $(A - B) - C$ is the same as $(A - C) - (B - C)$

A	B	C	$A - B$	$(A - B) - C$	$A - C$	$B - C$	$(A - C) - (B - C)$
1	1	1	0	0	0	0	0
1	1	0	0	0	1	1	0
1	0	1	1	0	0	0	0
1	0	0	1	1	1	0	1
0	1	1	0	0	0	0	0
0	1	0	0	0	0	1	0
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0

As the columns of both sides of equation are equivalent, $(A - B) - C = (A - C) - (B - C)$.

□

Problem §2.2: 26: Draw the Venn diagrams for each of the following combinations of the sets A, B, and C.

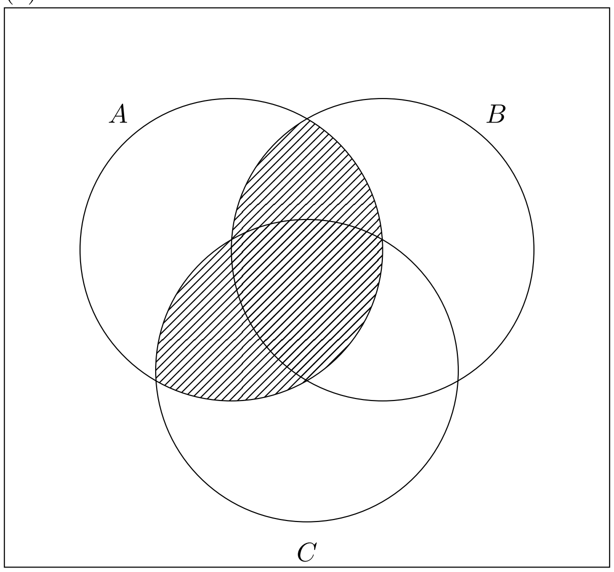
(a) $A \cap (B \cup C)$

(b) $\overline{A} \cap \overline{B} \cap \overline{C}$

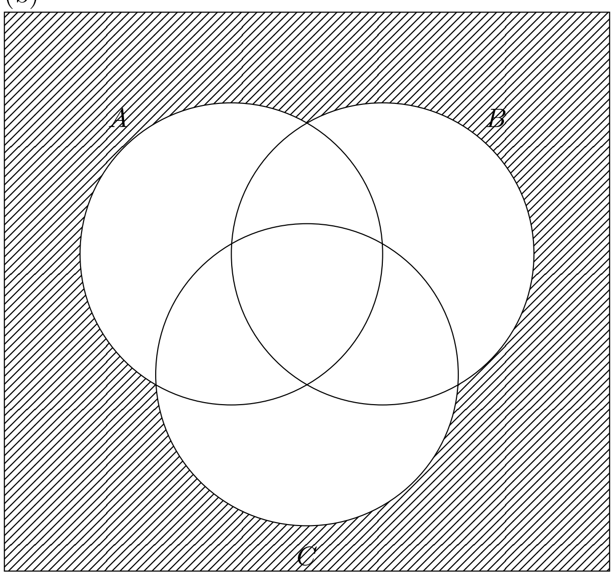
(c) $(A - B) \cup (A - C) \cup (B - C)$

Solution.

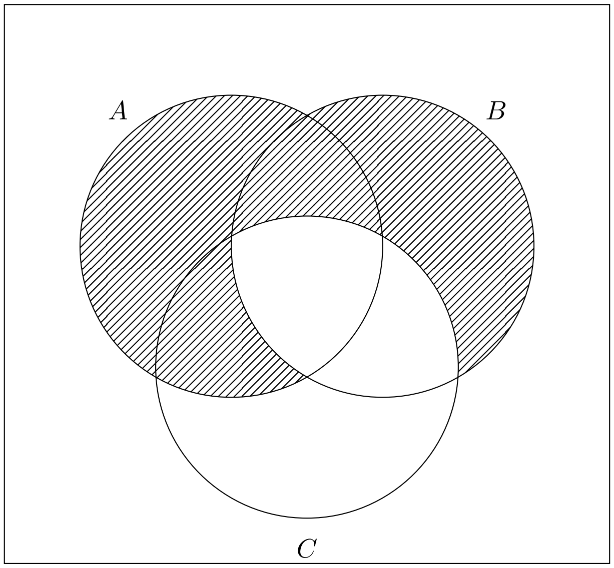
(a)



(b)



(c)


A
B
C

□