Math 231E, 2013. Midterm 2.

- This exam has 30 questions.
- You must not communicate with other students during this test. No books, notes, **calculators**, or electronic devices allowed.
- Please fill out all of the information below. Make sure to fill out your Scantron form as directed in class; fill in name, UIN number, and NetID.
- October is Pumpkin Month. Draw a happy pumpkin somewhere on the test booklet for good luck.

1. Fill in your information:

Full Name: ______________________________

UIN (Student Number): ______________________________

NetID: ______________________________

2. Fill out name, student number (UIN) and NetID on Scantron sheet. Then fill in the following answers on the Scantron form:

Zone 1

1/1. (3 points) Compute the integral

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot(x)dx$$

A. ★ $\frac{\log(2)}{2}$

B. $-\frac{1}{12}$

C. $\frac{\pi}{4}$

D. $\frac{8}{3}$

E. $\frac{\pi^2}{6}$

Solution. We have

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos(x)}{\sin(x)}dx = \log(\sin(x))|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

2/1. (3 points) Compute the area of the region that is above the x-axis, above the line $y = \frac{x}{2}$ and below the curve $y = x^2$.

A. ★ $\dfrac{1}{48}$

B. $\dfrac{5}{12}$

C. $\dfrac{7}{1}2$

D. $-\dfrac{11}{2}4$

E. $\dfrac{1}{24}$

Solution. If we draw a diagram, it is clear that we want to compute the quantity

$$\int_0^{\frac{1}{2}} \frac{x}{2} - x^2 dx = (\frac{x^2}{4} - \frac{x^3}{3})|_0^{\frac{1}{2}} = \frac{1}{48}$$

3/1. (3 points) What is the coefficient of the x^{13} term in the Taylor series for $\dfrac{1}{1+2x}$ at $a = 0$?

A. ★ -2^{13}

B. $(2)^{13}$

C. 1

D. -1

E. 0

Solution.

4/1. (3 points) Compute $f'(x)$, where

$$f(x) = x^2 \cos(2x).$$

A. ★ $2x\cos(2x) - 2x^2\sin(2x)$

B. $2x\cos(2x)$

C. $x^2\ln(2x)$

D. $x^2\sin(2x) + 2\cos(2x)$

E. $-4x\sin(2x)$

Solution. We use the product and chain rules, so

$$\frac{d}{dx}(x^2\cos(2x)) = 2x\cos(2x) + x^2(-2\sin(2x)).$$

4/2. (3 points) Compute $f'(x)$, where

$$f(x) = x^2 \cos(3x).$$

A. ★ $2x\cos(3x) - 3x^2\sin(3x)$

B. $2x\cos(3x)$

C. $x^2\ln(3x)$

D. $x^2\sin(3x) + 3\cos(3x)$

E. $-6x\sin(3x)$

Solution. We use the product and chain rules, so

$$\frac{d}{dx}(x^2\cos(3x)) = 2x\cos(3x) + x^2(-3\sin(3x)).$$

5/1. (3 points) Compute the integral $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

A. ★ $-2\cos(\sqrt{x}) + C$

B. $2\cos(\sqrt{x}) + C$

C. $-\sqrt{x}\sin(\sqrt{x}) + C$

D. $\sqrt{x}\sin(\sqrt{x}) + C$

E. $\sqrt{x}\sin(\sqrt{x}) - \frac{\cos(\sqrt{x})}{\sqrt{x}} + C$

5/2. (3 points) Compute the integral $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

A. ★ $2\sin(\sqrt{x}) + C$

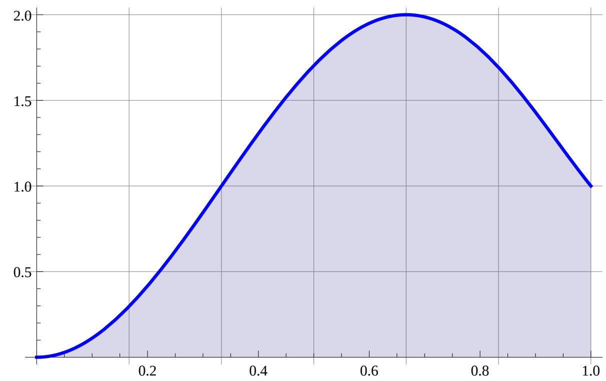
B. $-2\sin(\sqrt{x}) + C$

C. $-\sqrt{x}\cos(\sqrt{x}) + C$

D. $\sqrt{x}\cos(\sqrt{x}) + C$

E. $\sqrt{x}\cos(\sqrt{x}) - \frac{\sin(\sqrt{x})}{\sqrt{x}} + C$

6/1. (3 points) Give your best estimate for the **lower** Riemann sum for the following area between $x = 0$ and $x = 1$ with $n = 3$.



A. ★ $\frac{2}{3}$

B. 2

C. $\frac{5}{3}$

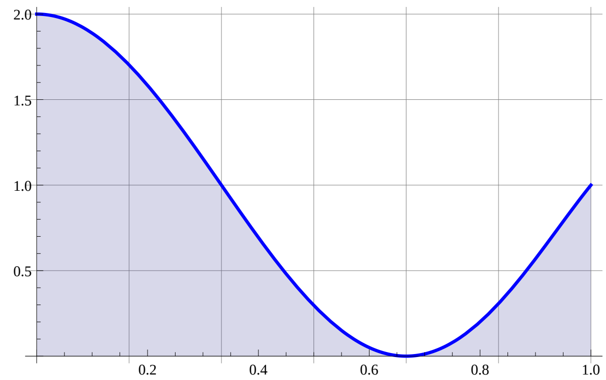
D. $\frac{1}{3}$

E. $\frac{4}{3}$

Solution.

6/2. (3 points) 6/3. (3 points) Give your best estimate for the **lower** Riemann sum for the following

area between $x = 0$ and $x = 1$ with $n = 3$.



A. $\frac{2}{3}$

B. 2

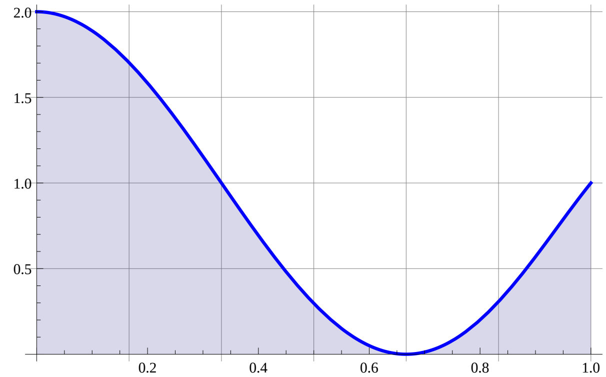
C. $\frac{5}{3}$

D. ★ $\frac{1}{3}$

E. $\frac{4}{3}$

Solution.

7/1. (3 points) Give your best estimate for the **upper** Riemann sum for the following area between $x = 0$ and $x = 1$ with $n = 3$.



A. $\frac{2}{3}$

B. 2

C. ★ $\frac{5}{3}$

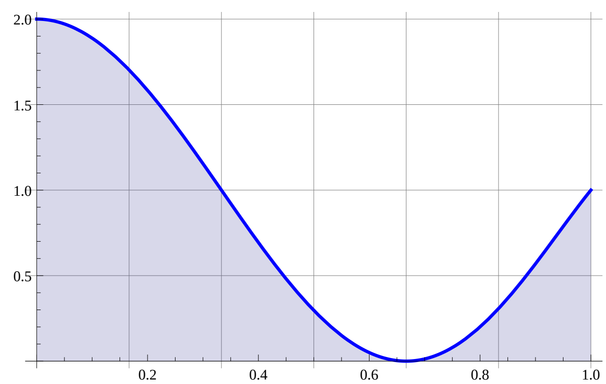
D. $\frac{1}{3}$

E. $\frac{4}{3}$

Solution.

7/2. (3 points) 7/3. (3 points) Give your best estimate for the **upper** Riemann sum for the following

area between $x = 0$ and $x = 1$ with $n = 3$.



A. $\frac{2}{3}$

B. 2

C. $\frac{5}{3}$

D. $\frac{1}{3}$

E. ★ $\frac{4}{3}$

Solution.

8/1. (3 points) Compute the antiderivative

$$\int \sin(2x)\,dx.$$

A. ★ $\frac{1}{2}\cos(2x) + C$

B. $\cos(2x) + C$

C. $2\cos(2x) + C$

D. $\sin(2x) + C$

E. $\frac{1}{2}\sin(2x) + C$

Solution. We make a u-substitution $u = 2x$, which gives $du = 2\,dx$, so we have

$$\int \sin(2x)\,dx = \int \sin(u)\frac{du}{2} = \frac{1}{2}\int \sin(u)\,du = \frac{1}{2}\cos(u) + C = \frac{1}{2}\cos(2x) + C.$$

8/2. (3 points) Compute the antiderivative

$$\int \sin(3x)\,dx.$$

A. ★ $\frac{1}{3}\cos(3x) + C$

B. $\cos(3x) + C$

C. $3\cos(3x) + C$

D. $\sin(3x) + C$

E. $\frac{1}{3}\sin(3x) + C$

Solution. We make a u-substitution $u = 3x$, which gives $du = 3\,dx$, so we have

$$\int \sin(3x)\,dx = \int \sin(u)\frac{du}{3} = \frac{1}{3}\int \sin(u)\,du = \frac{1}{3}\cos(u) + C = \frac{1}{3}\cos(3x) + C.$$

9/1. (3 points) Let $f(x)$ be the function

$$f(x) = \int_{-2}^{x} \sin(2t)\, dt.$$

Compute $f'(x)$.

A. $\bigstar$ $\sin(2x)$

B. $2\sin(2x)$

C. $\frac{1}{2}\cos(x) - \frac{1}{2}\cos(-2)$

D. $\frac{1}{2}\cos(2x) - \frac{1}{2}\cos(-4)$

E. $-2\tan(x+2)$

Solution. The fundamental theorem of calculus tells us that

$$f'(x) = \frac{d}{dx}\int_{-2}^{x} \sin(2t)\, dt = \sin(2x).$$

9/2. (3 points) Let $f(x)$ be the function

$$f(x) = \int_{-2}^{x} \sin(3t)\, dt.$$

Compute $f'(x)$.

A. $\bigstar$ $\sin(3x)$

B. $-2\sin(3x)$

C. $\frac{1}{3}\cos(x) - \frac{1}{3}\cos(-2)$

D. $\frac{\cos}{(}3x) - \frac{1}{3}\cos(-6)$

E. $2\sin(3x)$

Solution. The fundamental theorem of calculus tells us that

$$f'(x) = \frac{d}{dx}\int_{-2}^{x} \sin(3t)\, dt = \sin(3x).$$

Zone 2

10/1. (3 points) What is the best substitution to make in the integral

$$\int \frac{dx}{(x^2+4)^{\frac{3}{2}}}.$$

A. ★ $x = 2\tan(t)$

B. $x = tan(t)$

C. $x = 2\sec(t)$

D. $x = \sec(2t)$

E. $x = 2\cos(t)$

Solution. Making the substitution $x = 2\tan(t)$ and applying the identity $1 + \tan^2(t) = \sec^2(t)$ will eliminate the radical.

10/2. (3 points) What is the best substitution to make in the integral

$$\int \frac{dx}{(x^2+9)^{\frac{5}{2}}}.$$

A. ★ $x = 3\tan(t)$

B. $x = tan(t)$

C. $x = 3\sec(t)$

D. $x = \sec(3t)$

E. $x = 3\sin(t)$

Solution. Making the substitution $x = 3\tan(t)$ and applying the identity $1 + \tan^2(t) = \sec^2(t)$ will eliminate the radical.

11/1. (3 points) Compute the Limit $\lim_{x \to 0^-} \frac{|x|}{\sin(3x)}$

A. ★ $-\frac{1}{3}$

B. $+\frac{1}{3}$

C. 0

D. $+\infty$

E. does not exist

Solution. For $x < 0$ we have $|x| = -x$ and apply l'Hôpital.

11/2. (3 points) Compute the Limit $\lim_{x \to 0^-} \frac{|x|}{\sin(2x)}$

A. ★ $-\dfrac{1}{2}$

B. $+\dfrac{1}{2}$

C. 0

D. $+\infty$

E. does not exist

Solution. For $x < 0$ we have $|x| = -x$ and apply l'Hôpital.

12/1. (3 points) A $8\,\mathrm{m}$ long ladder is propped up against a wall. The ladder begins to slip. At time $t = 3\,\mathrm{s}$, the base of the ladder is $6\,\mathrm{m}$ from the wall and moving away from the wall at 7m/s. How fast is the end of the ladder moving along the wall? (**Note:** We take the convention that negative velocity means downward, positive means upward)

A. ★ $-3\sqrt{7}\,\mathrm{m/s}$

B. $6\sqrt{7}\,\mathrm{m/s}$

C. $-\dfrac{30}{8}$ s

D. $-24\sqrt{5}\,\mathrm{m/s}$

E. $24\sqrt{5}\,\mathrm{m/s}$

Solution. Let us denote the height of the ladder's contact with the wall as h, and the distance from the wall to the base of the ladder as w. Then we know

$$w^2 + h^2 = 64\,\mathrm{m}^2$$

by the Pythagorean theorem. Differentiating gives

$$2w\frac{dw}{dt} + 2h\frac{dh}{dt} = 0.$$

We are given $w = 2\,\mathrm{m}$ and $dw/dt = 6\mathrm{m/s}$. We need h, but we can use the first equation to obtain

$$h = \sqrt{(64\,\mathrm{m})^2 - (2\,\mathrm{m})^2} = \sqrt{16\,\mathrm{m}^2 - 4\,\mathrm{m}^2} = \sqrt{12\,\mathrm{m}^2} = \sqrt{12}\ \mathrm{m} = 2\sqrt{3}\ \mathrm{m}.$$

Thus we plug in, and obtain

$$(2\,\mathrm{m})(3\mathrm{m/s}) + (2\sqrt{3}\ \mathrm{m})\frac{dh}{dt} = 0,$$

or

$$\frac{dh}{dt} = -\frac{6\,\mathrm{m}^2/\mathrm{s}}{2\sqrt{3}\,\mathrm{m}} = \sqrt{3}\,\mathrm{m/s}.$$

12/2. (3 points) A $6\,\mathrm{m}$ long ladder is propped up against a wall. The ladder begins to slip. At time $t = 5\,\mathrm{s}$, the base of the ladder is $3\,\mathrm{m}$ from the wall and moving away from the wall at 1m/s. How fast is the end of the ladder moving down the wall?

A. ★ $-\frac{\sqrt{3}}{3}\,\mathrm{m/s}$

B. $\sqrt{3}\,\mathrm{m/s}$

C. $-\dfrac{13}{3}$ s

D. $-2\sqrt{2}\,\mathrm{m/s}$

E. $2/\sqrt{2}\,\mathrm{m/s}$

Solution. Let us denote the height of the ladder's contact with the wall as h, and the distance from the wall to the base of the ladder as w. Then we know

$$w^2 + h^2 = 36\,\mathrm{m}^2$$

by the Pythagorean theorem. Differentiating gives

$$2w\frac{dw}{dt} + 2h\frac{dh}{dt} = 0.$$

We are given $w = 3\,\mathrm{m}$ and $dw/dt = 2\mathrm{m/s}$. We need h, but we can use the first equation to obtain

$$h = \sqrt{(6\,\mathrm{m})^2 - (3\,\mathrm{m})^2} = \sqrt{36\,\mathrm{m}^2 - 9\,\mathrm{m}^2} = \sqrt{27\,\mathrm{m}^2} = \sqrt{27}\ \mathrm{m} = 3\sqrt{3}\ \mathrm{m}.$$

Thus we plug in, and obtain

$$(3\,\mathrm{m})(2\mathrm{m/s}) + (3\sqrt{3}\ \mathrm{m})\frac{dh}{dt} = 0,$$

or

$$\frac{dh}{dt} = -\frac{6\,\mathrm{m}^2/\mathrm{s}}{3\sqrt{3}\,\mathrm{m}} = 2/\sqrt{3}\,\mathrm{m/s}.$$

13/1. (3 points) A farmer wants to build a rectangular field next to a river. The farmer will use the river as one side of the rectangle, but must build the other three sides of the rectangle with fence. There is a total amount of 120 m of fence available. What is the largest possible area that can be enclosed?

A. ★ $1800\,\mathrm{m}^2$

B. $1600\,\mathrm{m}^2$

C. $900\,\mathrm{m}^2$

D. $14400\,\mathrm{m}^2$

E. $2400\,\mathrm{m}^2$

Solution. Let us denote the length of the side parallel to the river as d_1, and the side perpendicular to the river as d_2. Then we know that $d_1 + 2d_2 = 120\,\mathrm{m}$, or $d_2 = 60\,\mathrm{m} - d_1/2$.

We also know the area is

$$A = d_1 d_2 = d_1(60\,\mathrm{m} - d_1/2) = 60\,\mathrm{m}\ d_1 - \frac{d_1^2}{2}.$$

To find the critical points, we differentiate and set equal to zero, which gives

$$60\,\mathrm{m} - d_1 = 0,$$

or $d_1 = 60\,\mathrm{m}$ (and $d_2 = 30\,\mathrm{m}$). We also have to check the boundaries $d_1 = 0\,\mathrm{m}, 120\,\mathrm{m}$, but these give degenerate rectangles of zero area. Thus the maximum is attained when we choose $d_1 = 60\,\mathrm{m}$, which gives a maximal area of

$$A = (60\,\mathrm{m})(30\,\mathrm{m}) = 1800\,\mathrm{m}^2.$$

13/2. (3 points) A farmer wants to build a rectangular field next to a river. The farmer will use the river as one side of the rectangle, but must build the other three sides of the rectangle with fence. There is a total amount of 180 m of fence available. What is the largest possible area that can be enclosed?

A. ★ $4050\,\mathrm{m}^2$

B. $3600\,\mathrm{m}^2$

C. $2025\,\mathrm{m}^2$

D. $32400\,\mathrm{m}^2$

E. $5400\,\mathrm{m}^2$

Solution. Let us denote the length of the side parallel to the river as d_1, and the side perpendicular to the river as d_2. Then we know that $d_1 + 2d_2 = 180\,\mathrm{m}$, or $d_2 = 90\,\mathrm{m} - d_1/2$.

We also know the area is

$$A = d_1 d_2 = d_1(90\,\mathrm{m} - d_1/2) = 90\,\mathrm{m}\ d_1 - \frac{d_1^2}{2}.$$

To find the critical points, we differentiate and set equal to zero, which gives

$$90\,\mathrm{m} - d_1 = 0,$$

or $d_1 = 90\,\mathrm{m}$ (and $d_2 = 45\,\mathrm{m}$). We also have to check the boundaries $d_1 = 0\,\mathrm{m}, 180\,\mathrm{m}$, but these give degenerate rectangles of zero area. Thus the maximum is attained when we choose $d_1 = 90\,\mathrm{m}$, which gives a maximal area of

$$A = (90\,\mathrm{m})(45\,\mathrm{m}) = 4050\,\mathrm{m}^2.$$

14/1. (3 points) Evaluate the following integral

$$\int x^2 \sin(2x) dx$$

A. ★ $-\frac{x^2}{2}\cos(2x) + \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C$

B. $2x^2\cos(2x) + 8x\sin(2x) + 16\cos(2x) + C$

C. $\frac{x^2}{2}\cos(2x) - \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C$

D. $2x^2\cos(2x) - 8x\sin(2x) + 16\cos(2x) + C$

E. $x^2\cos(2x) - 4x\sin(2x) + 8\cos(2x) + C$

Solution. Keep calm and integrate by parts

14/2. (3 points) 14/3. (3 points) Evaluate the following integral

$$\int x^2 \cos(2x) dx$$

A. ★ $\frac{x^2}{2}\sin(2x) + \frac{x}{2}\cos(2x) - \frac{1}{4}\sin(2x) + C$

B. $2x^2\sin(2x) + 8x\cos(2x) + 16\sin(2x) + C$

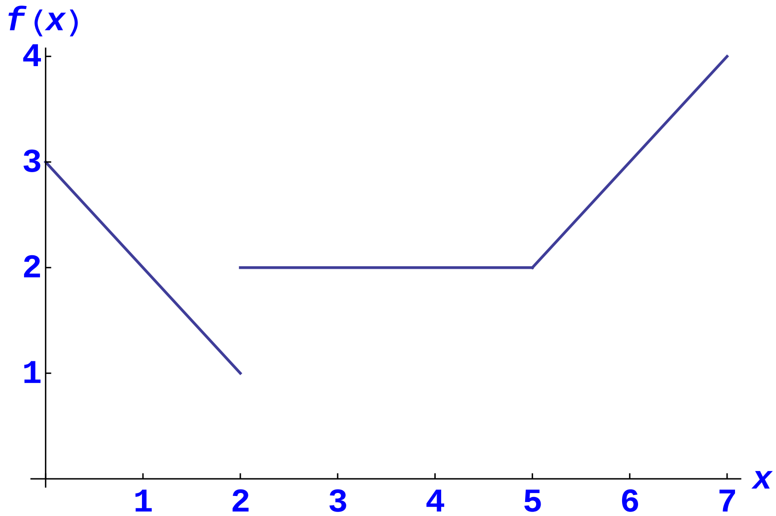
C. $\frac{x^2}{2}\sin(2x) - \frac{x}{2}\cos(2x) + \frac{1}{4}\sin(2x) + C$

D. $2x^2\sin(2x) - 8x\cos(2x) + 16\sin(2x) + C$

E. $x^2\sin(2x) - 4x\cos(2x) + 8\sin(2x) + C$

Solution. Keep calm and integrate by parts

15/1. (3 points) Let $f(x)$ be given by the following graph:



Define $g(x) = \int_0^x f(t)\,dt$ and assume that $g(0) = 1$. Then:

A. ★ $g(2) = 5, \quad g(3) = 7, \quad g(7) = 17.$

B. $g(2) = 4, \quad g(3) = 6, \quad g(7) = 16.$

C. $g(2) = -1, \quad g(3) = 0, \quad g(7) = 2.$

D. $g(2) = 3, \quad g(3) = 4, \quad g(7) = 10.$

E. $g(2) = 5, \quad g(3) = 9, \quad g(7) = 19.$

Solution. We can compute ...

16/1. (3 points) We want to estimate

$$\int_0^1 x^3\,dx,$$

with a Riemann sum of $n = 3$ terms. Let us define L_3 as the Riemann sum if we choose the left endpoints, and R_3 if we choose the right endpoints. Then:

A. ★ $L_3 = \frac{1}{9}, \quad R_3 = \frac{4}{9}$

B. $L_3 = \frac{7}{27}, \quad R_3 = \frac{4}{27}$

C. $L_3 = \frac{15}{81}, \quad R_3 = \frac{41}{81}$

D. $L_3 = \frac{2}{9}, \quad R_3 = \frac{5}{9}$

E. $L_3 = \frac{4}{27}, \quad R_3 = \frac{11}{27}$

Solution. We have $\Delta x = 2/3$. If we choose left-hand endpoints, our x_k^* will be

$$x_1^* = 0, \quad x_2^* = \frac{1}{3}, \quad x_3^* = \frac{2}{3},$$

and so we have

$$\begin{aligned} L_3 &= f(x_1^*)\ \Delta x + f(x_2^*)\ \Delta x + f(x_3^*)\ \Delta x \\ &= 0^2 \cdot \frac{1}{3} + \left(\frac{1}{3}\right)^3 \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^3 \frac{1}{3} \\ &= \frac{1}{27} \cdot \frac{1}{3} + \frac{8}{27} \cdot \frac{1}{3} = \frac{9}{81} = \frac{1}{9}. \end{aligned}$$

If we choose right-hand endpoints, then

$$x_1^* = \frac{1}{3}, \quad x_2^* = \frac{2}{3}, \quad x_3^* = 1,$$

and so we have

$$\begin{aligned} L_3 &= f(x_1^*)\ \Delta x + f(x_2^*)\ \Delta x + f(x_3^*)\ \Delta x \\ &= \left(\frac{1}{3}\right)^3 \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^3 \frac{1}{3} + 1^3 \cdot \frac{1}{3} \\ &= \frac{1}{27} \cdot \frac{1}{3} + \frac{8}{27} \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} = \frac{4}{9}. \end{aligned}$$

17/1. (3 points) If we know that

$$\int_0^2 f(x)\,dx = 3, \quad \int_0^4 f(x)\,dx = 4, \quad \int_{-2}^6 f(x)\,dx = 5, \quad \int_{-2}^0 f(x)\,dx = 7,$$

then compute A, where

$$A = \int_2^6 f(x)\,dx.$$

A. ★ $A = -5$

B. $A = -6$

C. $A = 1$

D. $A = 11$

E. $A = -2$

Solution. We know that

$$\int_2^4 f(x)\,dx = \int_0^4 f(x)\,dx - \int_0^2 f(x)\,dx = 4 - 3 = 1.$$

We also know that

$$\begin{aligned}\int_4^6 f(x)\,dx &= \int_{-2}^6 f(x)\,dx - \int_{-2}^4 f(x)\,dx \\ &= \int_{-2}^6 f(x)\,dx - \left(\int_{-2}^0 f(x)\,dx + \int_0^4 f(x)\,dx\right) \\ &= 5 - (7 + 4) = -6.\end{aligned}$$

Then

$$\int_2^6 f(x)\,dx = \int_2^4 f(x)\,dx + \int_4^6 f(x)\,dx = 1 + (-6) = -5.$$

18/1. (3 points) Compute

$$\int_0^{\pi/2} \sin^3(x)\,dx.$$

A. ★ $\frac{2}{3}$

B. 0

C. 1

D. $\frac{pi}{6}$

E. $\frac{pi}{4}$

Solution. We do a u-substitution $u = \cos(x)$, which gives $du = -\sin(x)\,dx$, and thus

$$\int (u^2 - 1)du.$$

Plugging in $x = \pi/2$, gives $t = 0$, $x = 0$ gives $t = 1$

$$u - \frac{u^3}{3}|_0^1 = \frac{2}{3}$$

19/1. (3 points) Which of these five choices are the same as

$$\int_0^{\pi/2} e^{\cos^2(x)} \sin(x)\, dx.$$

A. ★ $\displaystyle\int_0^1 e^{u^2}\, du$

B. $e^{u^2} + C$

C. $\displaystyle -\int_0^1 e^{u^2}\, du$

D. $\displaystyle\int_0^{\pi/2} e^{u}\, du$

E. $\displaystyle\int_0^{\pi/2} e^{u^2}\, du$

Solution. We use a substitution $u = \cos(x)$, which gives $du = -\sin(x)\, dx$, and thus the integrand is $u\, du$. We also need to change the limits of integration, and we have $x = 0$ gives $u = 1$, and at $x = \pi/2$ gives $u = 0$, so we have

$$\int_0^{\pi/2} e^{\cos^2(x)} \sin(x)\, dx = \int_1^0 e^{u^2}\, (-du) = -\int_1^0 e^{u^2}\, du = \int_0^1 e^{u^2}\, du.$$

Zone 3

20/1. (3 points) Let us define $f(x)$ by

$$f(x) = \int_{\sin(x^2)}^{\cos(x^2)} \frac{1}{1+t^2}\, dt.$$

Compute $f'(x)$.

A. ★ $\frac{-2x\sin(x^2)}{1+\cos^2(x^2)} - \frac{-2x\cos(x^2)}{1+\sin^2(x^2)}$

B. $\frac{-2x\sin(x^2)}{1+\sin^2(x^2)} - \frac{-2x\cos(x^2)}{1+\cos^2(x^2)}$

C. $\frac{2x\sin(x^2)}{1+\sin^2(x^2)} - \frac{-2x\cos(x^2)}{1+\cos^2(x^2)}$

D. $\frac{-2x\sin(x^2)}{1+\cos^2(x^2)} - + \frac{-2x\cos(x^2)}{1+\sin^2(x^2)}$

E. $\frac{2x\sin(x^2)}{1+\cos^2(x^2)} - \frac{-2x\cos(x^2)}{1+\sin^2(x^2)}$

Solution.

21/1. (3 points) A ladder with length L is propped up against a wall. Let us denote by w the distance of the base of the ladder from the wall, and h as the height of the point of contact between ladder and wall. Now imagine that the ladder starts slipping down the wall. Assuming that the top of the ladder has not yet hit the floor, write the rate of change dh/dt in terms of w,L, and dw/dt.

A. ★ $\dfrac{dh}{dt} = \dfrac{-w}{\sqrt{L^2 - w^2}} \cdot \dfrac{dw}{dt}$

B. $\dfrac{dh}{dt} = \dfrac{w}{\sqrt{L^2 - w^2}} \cdot \dfrac{dw}{dt}$

C. $\dfrac{dh}{dt} = w\sqrt{L^2 - \left(\dfrac{dw}{dt}\right)^2}$

D. $\dfrac{dh}{dt} = -\dfrac{w}{L} \cdot \dfrac{dw}{dt}$

E. $\dfrac{dh}{dt} = w\sqrt{L^2 - w^2} \cdot \dfrac{dw}{dt}$

Solution. By the Pythagorean Theorem, we have

$$w^2 + h^2 = L^2.$$

Differentiating gives

$$2w\frac{dw}{dt} + 2h\frac{dh}{dt} = 0,$$

or

$$\frac{dh}{dt} = -\frac{w}{h} \cdot \frac{dw}{dt},$$

and using the first equation to plug in gives

$$\frac{dh}{dt} = -\frac{w}{\sqrt{L^2 - w^2}} \cdot \frac{dw}{dt}.$$

22/1. (3 points) Compute

$$\lim_{x\to 1}\left(\frac{\sin(\pi x)}{\int_1^x t^t\,dt}\right).$$

A. ★ π

B. e

C. 0

D. 1

E. ∞

Solution. FApplying L'Hôpital we get

$$\lim_{x\to 1}\frac{\sin(\pi x)}{\int_1^x t^t\,dt} = \lim_{x\to 1}\frac{\pi\cos(\pi x)}{x^x} = \pi$$

23/1. (3 points) Compute

$$\int_0^{\frac{\sqrt{3}}{2}} \frac{x^3}{\sqrt{1-x^2}} dx$$

A. ★ $\frac{5}{24}$

B. 1

C. $\frac{pi}{4}$

D. $\frac{1}{2}$

E. $\frac{pi}{6}$

Solution.

Zone 4

NEW Compute the following indefinite integral

24/1. (3 points)

$$\int \log(1+x^2)dx$$

A. ★ $x\log(1+x^2) + 2\arctan(x) - 2x + C$

B. $x\log(1+x^2) - 2x + C$

C. $\log(\frac{1+x}{1-x}) + C$

D. $\log(\arctan(x)) + C$

E. $xlog(\arctan(x)) - \frac{1}{1+x^2} + C$

25/1. (3 points) **NEW** Compute the following definite integral

$$\int_0^{\frac{\pi}{4}} \sec(x)\tan^2(x)dx$$

A. ★ $\frac{\sqrt{2}}{2} - \frac{1}{2}\log(1+\sqrt{2})$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. 1

E. $\frac{1}{2}(1+\log(1+\sqrt{2}))$

26/1. (3 points) **NEW** Consider the polynomial $P(x) = x^3 - 6x^2 + 4x + 5$. Let A, B, C, D, E denote the following intervals

- $A = [0, 1]$
- $B = [1, 2]$
- $C = [2, 3]$
- $D = [3, 4]$
- $E = [4, 5]$

In which intervals is the polynomial $P(x)$ guaranteed to have a root

A. ★ Intervals B and E

B. Intervals A and B

C. Intervals A, C and D

D. Intervals B,C,E

E. Intervals A,D

27/1. (3 points) Evaluate the integral

$$\int \frac{\sqrt{x^2-1}}{x}dx$$

A. ★ $\sqrt{x^2-1} + \arctan(\frac{1}{\sqrt{x^2-1}}) + C$

B. $x\sqrt{x^2-1} + \arctan(\frac{1}{\sqrt{x^2-1}}) + C$

C. $\sqrt{x^2-1} + \arctan(\frac{1}{x}) + C$

D. $\sqrt{x^2-1} - \arctan(\frac{1}{\sqrt{x^2-1}}) + C$

E. $x\sqrt{x^2-1} - \arctan(\frac{1}{\sqrt{x^2-1}}) + C$