Math 415. Practice Exam 1. September 28, 2017

Full Name: ____________________
Net ID: ____________________

- There are 20 problems worth 5 points each.
- You must not communicate with other students.
- No books, notes, calculators, or electronic devices allowed.
- This is a 70 minute exam.
- Do not turn this page until instructed to.
- Fill in the answers on the scantron form provided. Also circle your answers on the exam itself.
- Hand in both the exam and the scantron.
- On the scantron make sure you
- There are several different versions of this exam.

Fill in the following answers on the Scantron form:

On the first page of the scantron, bubble in **your name, your UIN and your NetID**!
On the back of the scantron bubble in the following:

95. D
96. C

1. (5 points) The matrix $A = \begin{bmatrix} 0 & 0 & 2 \\ 3 & 3 & 6 \\ 1 & 1 & 6 \end{bmatrix}$ does not have an $A = LU$ decomposition. However, after *interchanging the first and third rows* of A, we can find a $PA = LU$ decomposition, where P is the permutation matrix that interchanges the first and the third row. What are the appropriate matrices P, L, and U for such a decomposition?

(A) $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & \frac{1}{6} & 1 \end{bmatrix}$, $U = \begin{bmatrix} 1 & 1 & 6 \\ 0 & 0 & -12 \\ 0 & 0 & 0 \end{bmatrix}$

(B) $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{3} & 2 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 3 & 3 & 6 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

(C) $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & -\frac{1}{6} & 1 \end{bmatrix}$, $U = \begin{bmatrix} 1 & 1 & 6 \\ 0 & 0 & -12 \\ 0 & 0 & 0 \end{bmatrix}$

(D) $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{3} & -2 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 3 & 3 & 6 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

2. (5 points) Let $A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ k & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. For which value of k does the system $A\mathbf{x} = b$ have a unique solution?

(A) There is no such value for k.

(B) $k = 0$

(C) $k = -1$

(D) $k = 1$

3. (5 points) Consider the following subsets of $\mathbb{R}^3$:

$$W_1 = \{ \begin{bmatrix} a \\ b \\ 1 \end{bmatrix} : a + b = 0 \}, \; W_2 = \{ \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} : a + b \geq 0 \}.$$

Then:

(A) Both W_1 and W_2 are subspaces of $\mathbb{R}^3$.

(B) Only W_2 is a subspace of $\mathbb{R}^3$.

(C) Neither W_1 nor W_2 is a subspace of $\mathbb{R}^3$.

(D) Only W_1 is a subspace of $\mathbb{R}^3$.

4. (5 points) Which of the following matrices is the inverse of

$$A = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(A) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & -1 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 1 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$

5. (5 points) Let $A = \begin{bmatrix} 1 & 4 \\ 4 & 16 \end{bmatrix}$. Then

(A) $\mathrm{Nul}(A) = \mathbb{R}^2$.

(B) $\mathrm{Nul}(A) = \mathrm{Span}(\begin{bmatrix} 1 \\ 4 \end{bmatrix})$.

(C) $\mathrm{Nul}(A) = \{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\}$.

(D) $\mathrm{Nul}(A) = \mathrm{Span}(\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 16 \end{bmatrix})$.

(E) $\mathrm{Nul}(A) = \mathrm{Span}(\begin{bmatrix} 4 \\ -1 \end{bmatrix})$.

6. (5 points) If $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$, the equation $A\mathbf{x} = \mathbf{b}$ is consistent for

(A) all $\mathbf{b} \in \mathbb{R}^2$.

(B) all $\mathbf{b}$ such that $\mathbf{b} = c \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

(C) all $\mathbf{b}$ such that $\mathbf{b} = c \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

(D) only for $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

(E) no $\mathbf{b} \in \mathbb{R}^2$.

7. (5 points) Find an explicit description of the null space of

$$A = \begin{bmatrix} 1 & 0 & 4 & 2 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

that is, find a minimal set of vectors $\boldsymbol{v_1}, \ldots, \boldsymbol{v_p}$ such that $\mathrm{Nul}(A) = \mathrm{span}\{\boldsymbol{v_1}, \ldots, \boldsymbol{v_p}\}$.

(A) $\begin{bmatrix} -4 \\ 3 \\ 1 \\ 0 \end{bmatrix}$

(B) $\begin{bmatrix} -4 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 \\ 0 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$

8. (5 points) Let A be a 3×2-matrix and $\boldsymbol{b}$ in $\mathbb{R}^3$. Consider the following two statements:

(S1) If $\boldsymbol{b} \neq \mathbf{0}$, then the set of solutions of $A\boldsymbol{x} = \boldsymbol{b}$ can be a plane through the origin.

(S2) If $A\boldsymbol{x} = \boldsymbol{b}$ is consistent, the solution to $A\boldsymbol{x} = \boldsymbol{b}$ is unique if and only if the equation $A\boldsymbol{x} = \mathbf{0}$ has only the trivial solution.

Then

(A) Statement S1 and Statement S2 are correct.

(B) Only Statement S1 is correct.

(C) Only Statement S2 is correct.

(D) Neither Statement S1 nor Statement S2 is correct.

9. (5 points) The matrix $A = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$ is reduced to the echelon matrix $U = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$ using the following row operations (in the given order):

A. $R_2 \to R_2 + 2R_1$,

B. $R_3 \to R_3 - R_1$,

C. $R_3 \to R_3 + R_2$,

D. $R_4 \to R_4 + R_2$.

What is the matrix L in the $A = LU$ decomposition?

(A) $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$

(B) $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(C) $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

(D) $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

10. (5 points) If A and B are $m \times n$-matrices with the same reduced row echelon form U, then there exists a sequence of elementary matrices $E_1, \ldots, E_k$, with each E_i of size $m \times m$, such that

$$E_k \cdots E_1 A = B.$$

(A) Always false

(B) Sometimes true and sometimes false

(C) Always true

11. (5 points) Let A, B be $n \times n$-matrices. Which of the following statements is false?

(A) if A is invertible and its rows are in reverse order in B, then B is invertible.

(B) if A and B are invertible, then BA is invertible.

(C) if A is invertible, A^T is invertible.

(D) if A is invertible, then A can be factored into the product $A = LU$ of a lower triangular matrix L and an upper triangular matrix U.

12. (5 points) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$. Then the inverse of A is

(A) $A^{-1} = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$

(B) $A^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

(C) $A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) $A^{-1} = \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$

(E) A^{-1} does not exist.

13. (5 points)
Let A be an $m \times n$-matrix and let B be an $n \times m$-matrix. Then which of the following statement is not true for all such matrices?

(A) BA is defined

(B) the columns of AB are linear combinations of the columns of B

(C) AB is defined

(D) the columns of AB are linear combinations of the columns of A

14. (5 points) Let $A = \begin{bmatrix} 1 & 7 \\ 2 & 8 \\ 3 & 9 \end{bmatrix}$. Then the *transpose* of A is

(A) $A^T = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix}$

(B) is only defined for $n \times n$-matrices.

(C) $A^T = \begin{bmatrix} 7 & 1 \\ 8 & 2 \\ 9 & 3 \end{bmatrix}$

(D) $A^T = \begin{bmatrix} 7 & 8 & 9 \\ 1 & 2 & 3 \end{bmatrix}$

15. (5 points) Let A be an $\ell \times m$-matrix such that for every $\boldsymbol{b}$ in $\mathbb{R}^\ell$, the equation $A\boldsymbol{x} = \boldsymbol{b}$ has a solution. What does this statement imply about the relative size of ℓ and m?

(A) $\ell \leq m$

(B) $\ell \geq m$

(C) nothing (ℓ and m can be any positive integers)

(D) $\ell = m$

16. (5 points) Consider $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 0 \end{bmatrix}$. Let $\boldsymbol{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Which of the following statements is true?

(A) $\boldsymbol{v}$ is a linear combination of the columns of B but not of the columns of A.

(B) $\boldsymbol{v}$ is a linear combination of the columns of A and of the columns of B.

(C) $\boldsymbol{v}$ is a linear combination of the columns of A but not of the columns of B.

(D) $\boldsymbol{v}$ is neither a linear combination of the columns of A nor of the columns of B.

17. (5 points) Which of the following choices for a makes A invertible, where A is $\begin{bmatrix} 1 & a \\ 1 & a \end{bmatrix}$?

(A) no real number.

(B) Any real number.

(C) any real number except for -1 and 1.

(D) Only for -1 and 1.

18. (5 points) For what values of h is the system

$$\begin{aligned} x_1 + x_2 &= 0 \\ x_2 + x_3 &= h \\ x_1 - x_3 &= 1 \end{aligned}$$

consistent?

(A) It is consistent for $h = 0$.

(B) It is always inconsistent.

(C) It is consistent for $h = 1$.

(D) It is consistent for all values of h.

(E) It is consistent for $h = -1$.

19. (5 points) Consider the following four 3×3-matrices: $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Which matrix is not invertible?

(A) D

(B) C

(C) B

(D) A

20. (5 points) Let A be a 2×3-matrix and b a vector in $\mathbb{R}^2$.

Consider the following two statements:

(P1) A has at most two pivots,

(P2) Assuming $Ax = b$ has a solution, then it has infinitely many solutions.

Then:

(A) Only Statement P2 is correct.

(B) Neither Statement P1 nor Statement P2 is correct.

(C) Only Statement P1 is correct.

(D) Statement P1 and Statement P2 are correct.