Math 231E, 2013. Midterm 1.

- This exam has 25 questions, each worth 4 points.
- If you mark in the letter C in problem 42 on your scantron, you will receive two bonus points.
- You must not communicate with other students during this test. No books, notes, **calculators**, or electronic devices allowed.
- Please fill out all of the information below. Make sure to fill out your Scantron form as directed in class; fill in name, UIN number, and NetID.

1. Fill in your information:

Full Name: _____

UIN (Student Number): _____

NetID: _____

2. Fill out name, student number (UIN) and NetID on Scantron sheet. Then fill in the following answers on the Scantron form:

89. A
90. A
91. A
92. A
93. A
94. D
95. C
96. E

1. (4 points) What is the third-order Taylor polynomial for $\sin(x)$ at $a = 0$?

(A) $1 + \sin(x) + \dfrac{\sin(x)^2}{2} + \dfrac{\sin(x)^3}{6}$

(B) $1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{6}$

(C) $x - \dfrac{x^3}{6}$

(D) $1 - \dfrac{x^2}{2}$

(E) $1 + x + x^2 + x^3$

2. (4 points) Compute the limit

$$\lim_{x \to 3} e^{4x}.$$

(A) e^{12}

(B) e^3

(C) $4e^{12}$

(D) does not exist

(E) $4e^3$

3. (4 points) Evaluate $\lim_{x\to\infty} \dfrac{\sin(2x)}{x}$.

(A) 0

(B) $+\infty$

(C) 2

(D) 3

(E) does not exist

4. (4 points) Compute the limit

$$\lim_{x\to 2} \frac{x^2-3}{2x}$$

(A) 1

(B) $-\dfrac{3}{2}$

(C) 2

(D) does not exist

(E) $\dfrac{1}{4}$

5. (4 points) Express the following complex number

$$z = \frac{1}{4+i}.$$

in the form $z = a + ib$.

(A) $z = 4 - i$

(B) $z = \dfrac{4}{17} - \dfrac{i}{17}$

(C) $z = \dfrac{1}{3} - \dfrac{i}{5}$

(D) $z = \cos(4) + i\sin(1)$

(E) $z = \dfrac{1}{4} + \dfrac{i}{4}$

6. (4 points) What is the second-order Taylor series for $\dfrac{1}{1+x}$ at $a = 0$?

(A) $1 - x + x^2 + O(x^3)$

(B) $1 - \dfrac{x^2}{2} + O(x^3)$

(C) $1 + \dfrac{1}{x} + \dfrac{1}{x^2} + O\left(\dfrac{1}{x^3}\right)$

(D) $1 + x + \dfrac{x^2}{2} + O(x^3)$

(E) $1 + x + x^2 + O(x^3)$

7. (4 points) $\lim_{x\to\infty} x^2 = \infty$. Suppose $M = 400$. How large must x be chosen to guarantee that $x^2 > M$

(A) $x < -4$

(B) $x > 10$

(C) $x > 20$

(D) $x > 4$

(E) $x < 0$

8. (4 points) Compute $\dfrac{d}{dx}\sin(e^{x^2/2})$.

(A) $-e^{x^2/2}\cos(e^{x^2/2})$

(B) $xe^{x^2/2}\cos(e^{x^2/2})$

(C) $-\cos(e^{x^2/2})$

(D) $e^{x^2/2}\cos(e^{x^2/2})$

(E) $\cos(e^{x^2/2})$

9. (4 points) Suppose that the function $f(x)$ has a Taylor series at $a = 0$ that begins $f(x) = 1 + 2x - \frac{5}{2}x^2 + \frac{11}{3}x^3 + O(x^4)$. What is the Taylor series for $f'(x)$ at $a = 0$?

(A) $f'(x) = 2 - 5x + 22x^2 + O(x^3)$

(B) $f'(x) = 3 + 6x + \frac{15}{2}x^2 + O(x^3)$

(C) $f'(x) = 2 - 5x + 11x^2 + O(x^3)$

(D) $f'(x) = x + x^2 - \frac{5}{6}x^3 + O(x^4)$

(E) $f'(x) = 1 + x + x^2 - \frac{5}{6}x^3 + O(x^4)$

10. (4 points) Compute $\frac{d}{dx}(\arctan(x))$. (Recall that $\arctan(x)$ is also sometimes written $\tan^{-1}(x)$!)

(A) $\frac{x}{\sqrt{1-x^2}}$

(B) $\frac{x}{1+x^2}$

(C) $\frac{1}{1-x^2}$

(D) $\frac{1}{\sqrt{1-x^2}}$

(E) $\frac{1}{1+x^2}$

11. (4 points) Compute $f'(x)$, where

$$f(x) = x^2 \cos(2x).$$

(A) $x^2 \ln(2x)$

(B) $2x \cos(2x)$

(C) $2x \cos(2x) - 2x^2 \sin(2x)$

(D) $-4x \sin(2x)$

(E) $x^2 \sin(2x) + 2 \cos(2x)$

12. (4 points) What is the third-order Taylor polynomial for e^x at $a = 7$?

(A) $e^7 + e^7(x-7) + \dfrac{e^7}{2}(x-7)^2 + \dfrac{e^7}{6}(x-7)^3$

(B) $3 + 3(x-7) + \dfrac{3}{2}(x-7)^2 + \dfrac{1}{2}(x-7)^3$

(C) $1 + e(x-7) + e^2(x-7)^2 + e^3(x-7)^3$

(D) $e^3 + e^3(x-3) + \dfrac{e^3}{2}(x-3)^2 + \dfrac{e^3}{6}(x-3)^3$

(E) $1 + (x-7) + \dfrac{1}{2}(x-7)^2 + \dfrac{1}{6}(x-7)^3$

13. (4 points) Compute the limit $\lim_{x\to\infty} \dfrac{\sin(x)+x}{x^2}$

(A) 1

(B) ∞

(C) 0

(D) $-\dfrac{1}{6}$

(E) does not exist

14. (4 points) Compute the limit

$$\lim_{x\to\infty} \frac{11x^{2013}+57x^{1967}+128x^{1492}}{22x^{2013}+88x^{2001}+195x^{1066}}.$$

(A) $\dfrac{128}{195}$

(B) does not exist

(C) $+\infty$

(D) 0

(E) $\dfrac{1}{2}$

15. (4 points) Find L, where

$$\lim_{x \to 0} \frac{\sin(\alpha x)}{x} = L.$$

(A) α

(B) $-\dfrac{a^3}{6}$

(C) 1

(D) 0

(E) L does not exist

16. (4 points) Compute the third-order Taylor series for $f(x) = e^x \cos(x)$ at $a = 0$.

(A) $1 + x - \dfrac{x^3}{3} + O(x^4)$

(B) $1 + 2x + x^2 - \dfrac{x^3}{3} + O(x^4)$

(C) $2 + x - \dfrac{3}{2}x^2 + \dfrac{x^3}{6} + O(x^4)$

(D) $1 + x - x^2 + x^3 + O(x^4)$

(E) $2 + x + \dfrac{x^3}{6} + O(x^4)$

17. (4 points) Compute the Taylor series for

$$f(x) = \frac{1 - \cos(x)}{x^2}$$

to second order.

(A) $f(x) = \frac{1}{2} + \frac{x}{2} + \frac{x^2}{6} + O(x^3)$

(B) $f(x) = \frac{1}{2} - \frac{x^2}{24} + O(x^4)$

(C) $f(x) = \frac{1}{6} - \frac{x^2}{120} + O(x^4)$

(D) $f(x) = \frac{1}{2} + \frac{x^2}{24} + O(x^4)$

(E) $f(x) = \frac{1}{6} + \frac{x^2}{120} + O(x^4)$

18. (4 points) Compute the third-order Taylor series for

$$f(x) = \frac{\sin(x) + 1 - \cos(x)}{x}.$$

(A) $1 + \frac{x}{2} - \frac{x^2}{6} - \frac{x^3}{24} + O(x^4)$

(B) $x - \frac{x^3}{6} + O(x^4)$

(C) $1 + x + \frac{x^2}{2} + O(x^4)$

(D) $1 + x + x^2 + x^3 + O(x^4)$

(E) $1 - \frac{x^2}{6} + O(x^4)$

19. (4 points) Compute the Limit $\lim_{x \to 0^-} \dfrac{|x|}{\sin(2x)}$

(A) 0

(B) does not exist

(C) $+\infty$

(D) $+\dfrac{1}{2}$

(E) $-\dfrac{1}{2}$

20. (4 points) Recall the notation that $\exp(A) = e^A$ for any expression A. Then find L, where

$$\lim_{x \to 0} \frac{\sin(x^4) - x^4}{3\exp(x^{12}) - 2} = L.$$

(A) $\dfrac{1}{6}$

(B) does not exist

(C) 1

(D) $-\dfrac{1}{18}$

(E) $-\dfrac{1}{3}$

21. (4 points) Compute the limit $\lim_{x\to\infty} x^{16}e^{-x}$.

(A) $-\infty$

(B) 0

(C) $\dfrac{1}{12!}$

(D) 1

(E) ∞

22. (4 points) Let $y = f(x)$ be defined implicitly by

$$y^3 - xy = 8.$$

Find $y' = f'(x)$ in terms of y and x.

(A) $y' = \dfrac{-y}{3y^2 - x}$

(B) $y' = \dfrac{8}{3y^2 - x}$

(C) $y' = \dfrac{y}{3y^2 - x}$

(D) $y' = \arctan(x/y)$

(E) $y' = \dfrac{8 + xy}{3y^2}$

23. (4 points) Compute L, where

$$L = \lim_{x \to 0} \frac{e^x \cos(x) - 1 - x}{x^3}$$

(A) -1

(B) $1/2$

(C) $-1/3$

(D) does not exist

(E) 1

24. (4 points) Let $y = f(x)$ be defined implicitly by

$$y^3 - xy = 27.$$

and suppose that $y(0) = 3$. Find the first two terms of the Taylor series for y about the point 0.

(A) $y = 1 + \dfrac{x}{81} + O(x^2)$

(B) $y = 3 + \dfrac{x}{9} + O(x^2)$

(C) $y = 3 - \dfrac{x}{12} + O(x^2)$

(D) $y = 1 + x + 2x^2 + O(x^3)$

(E) $y = 3 - \dfrac{2x}{9} + O(x^2)$

25. (4 points) Compute

$$L = \lim_{x \to 0} \cos\left(\frac{\pi \sin(7x)}{7x}\right).$$

(A) $L = 1$

(B) L does not exist

(C) $L = -1$

(D) $L = 0$

(E) $L = \infty$