Exam 3 Review Session
Math 231E

Please join the queue for attendance!

Outline

1. Please join the queue
2. Mini review of some topics covered
3. Practice! → CARE Worksheet, Practice Exams
    a. Please raise hands for questions rather than put them in the queue

Need extra help? → 4th Floor Grainger Library

Subject	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Math 231 (E)	4pm-10pm	1pm-5pm 8pm-10pm		1pm-5pm 8pm-10pm	6pm-8pm		2pm-4pm

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Improper Integrals

- Improper Integrals: FTC does not hold since functions are **not continuous along the interval of integration.**
- Type I: Infinite Interval

$$\int_a^\infty f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$$

$$\int_{-\infty}^b f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$$

- Type II: Discontinuous Interval

$$\int_a^b f(x)dx = \lim_{t\to b-} \int_a^t f(x)dx$$

$$\int_a^b f(x)dx = \lim_{t\to a+} \int_t^b f(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Improper Integrals

- Comparison Principal: If given two functions g(x) and h(x) and we want to take the integral to infinity, **and we know that g(x) is always smaller than h(x),** then:
  - If g(x) diverges, then h(x) must as well.
  - If h(x) converges, then g(x) must as well.

Applications

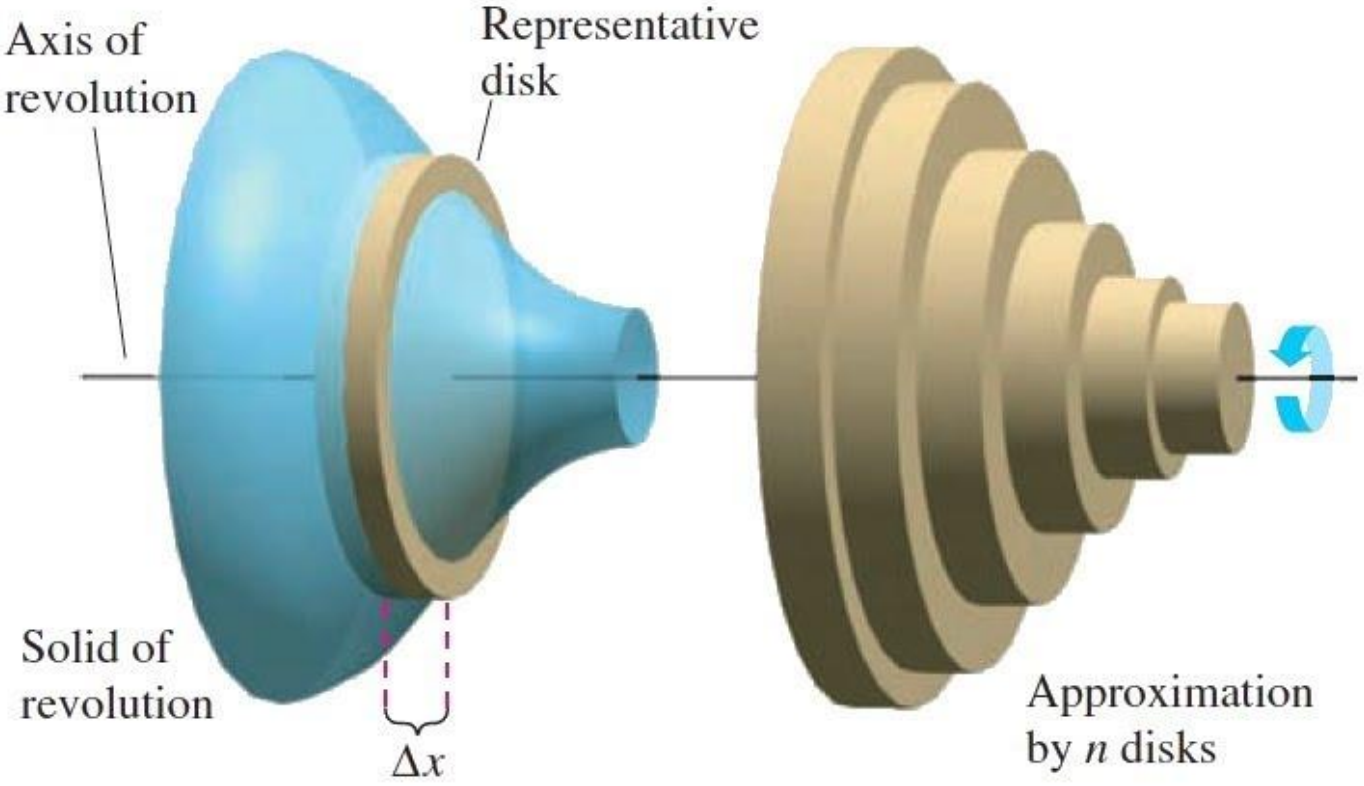
- Volume

Disk Method

- Using the formula for area of a circle
    - Putting it in the integral adds each circle in the bounds

$$V = \int_a^b \pi r^2 dx$$

Where the $r = f(x)$



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Applications

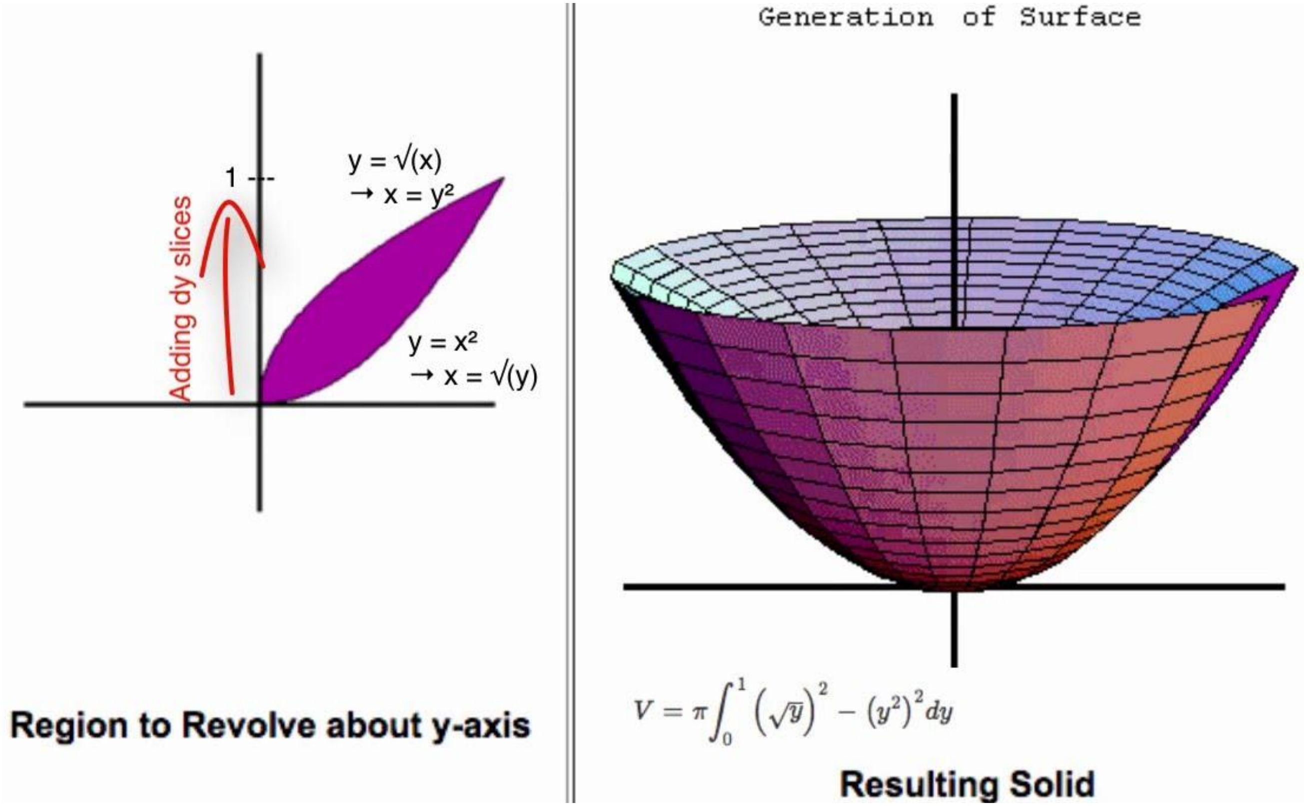
- Volume

Washer Method

- Must subtract big function minus small function to find the in between region
  $(Top - Bottom)$
  $(Right - Left)$

$$V = \int_a^b \pi(R^2 - r^2)dx$$

Where $\mathrm{R} = f(x)$
$r = g\ (x)$
$\mathrm{f(x)} > g(x)$



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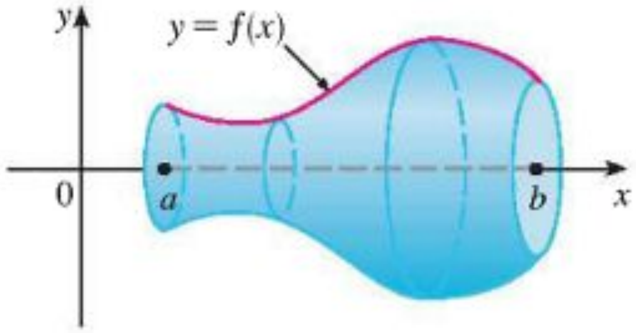
Applications

- Arclength ("height"):

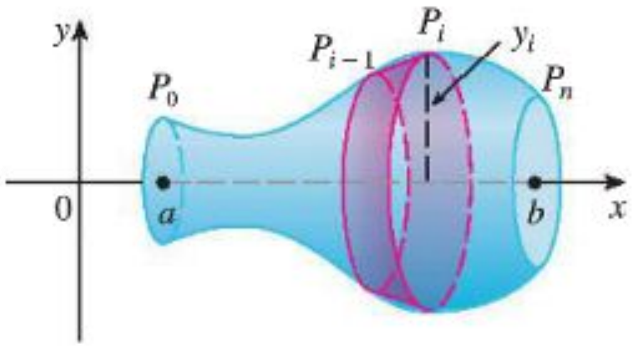
$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \text{ if } y = f(x)$$
$$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \text{ if x} = h(y)$$

- Surface of Revolution:

$$SA = \int 2\pi y ds \quad \text{rotation about x-axis}$$
$$SA = \int 2\pi x ds \text{ rotation about y-axis}$$



(a) Surface of revolution



(b) Approximating band

Applications Example

- How to set up the surface area equation when rotating $y = \sqrt{9 - x^2}$ about the y-axis?
  - Because about y-axis, using $SA = \int 2\pi x ds$
    - We need x and ds

To get x, rearrange the given equation

$$y = \sqrt{9 - x^2}$$
$$y^2 = 9 - x^2$$
$$x^2 = 9 - y^2$$
$$x = \sqrt{9 - y^2}$$

Substituting…

$$SA = \int 2\pi\sqrt{9 - y^2}ds$$

To get ds, use the equation for ds that gives us a dy (since x is in terms of y)

$$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \text{ if } x = h(y)$$

Derivative of x…

$$\frac{dx}{dy} = \frac{-y}{\sqrt{9-y^2}}$$

Plug back in…

Work

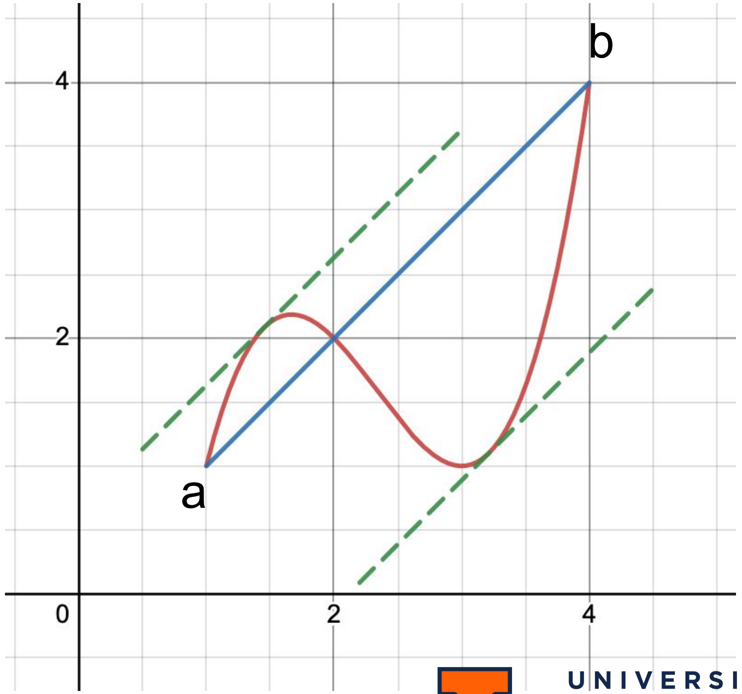
- Work: Force over a distance

$$W = \int F(x)dx$$

  - If the force is not constant.

- Average Value of a function over an interval

$$y_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$$



Series / Sequences

- **Sequence:** Just the list of the numbers
  - Limits of sequences
    - Treat it like a function
  - Convergence
    - Treat it like a function
    - Derivative can tell you if it is always increasing or decreasing
- **Series**: The sum of a sequence
  - If a series converges, then the sequence must converge as well.
  - **However:** If sequence converges, then the series may or may not converge.
  - Σa_n converges if the limit of the series converges.

Integral Test

- Let an = f(n):
  - $\int f(x)dx$ (from k to infinity) converges if the series converges (Σa_k).

    Must be:
    - Continuous
    - Positive
    - Decreasing

1. If $\displaystyle\int_k^\infty f(x)\,dx$ is convergent so is $\displaystyle\sum_{n=k}^{\infty} a_n$.

2. If $\displaystyle\int_k^\infty f(x)\,dx$ is divergent so is $\displaystyle\sum_{n=k}^{\infty} a_n$.

- P-test:
  - The series $\Sigma\frac{1}{n^p}$ converges if p > 1 and diverges if p ≤ 1.

Comparison/Limit Tests

- **Direct Comparison Test:**
  - Let $0 \leq an \leq bn$.
    - If the series of bn converges, then the series of an does as well.
    - If the series of an diverges, then the series of bn does as well.
- **Limit Comparison Test:**
  - Let $0 \leq an, bn$
    - If the limit of an/bn = C, and C is a nonzero, finite number (ie. not zero or infinity)
    - Then one of two things:
      - Both an and bn converge.
      - Both an and bn diverge.

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Alternating Series Test

- **What is an Alternating Series?**
  - The series is changing signs with each subsequent term
  - $\Sigma a_n(-1)^{n+1}$
- **Alternating Series Test**
  - With series Σa_n, $a_n = (-1)^n b_n$ OR $a_n = (-1)^{n+1} b_n$
    - If $\lim_{n\to\infty} b_n = 0$

      AND
    - b_n is a decreasing sequence
  - **The series Σa_n is convergent**

Absolute Convergence

- **Absolute Convergence**:
  - If the absolute value of a series, then the series is absolutely convergent.
- **Conditional Convergence**:
  - If a series if convergent, but the absolute value of the series diverges, then the series is conditionally convergent.
- **Negative signs can only help convergence!**

Strategies

1. **Check divergence with limit**
2. Look for easy P-Test/Geometric
3. Inspection

TEST	SERIES	CONVERGES IF...	DIVERGES IF...	COMMENTS
*n*th Term Test for Divergence	$\sum_{n=1}^{\infty} a_n$	n/a	$\lim_{n\to\infty} \neq 0$	should be first test used. Inconclusive if limit = 0.
Geometric Series Test	$\sum_{n=1}^{\infty} a_n\, r^{n-1}$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	use if there is a "common ratio" $S_n = \frac{a}{1-r}$
P-Series Test	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	harmonic series when p=1. Useful for comparison tests.
Integral Test	$\sum_{n=1}^{\infty} a_n$, $a_n = f(x)$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	$f(x)$ must be continuous, positive, and decreasing
Direct Comparison Test	$\sum_{n=1}^{\infty} a_n$	$0 \leq a_n \leq b_n$, $\sum_{n=1}^{\infty} b_n$ converges	$0 \leq b_n \leq a_n$, $\sum_{n=1}^{\infty} b_n$ diverges	to show convergence, find a larger series. to show divergence, find a smaller series.
Limit Comparison Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$, $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$, $\sum_{n=1}^{\infty} b_n$ diverges	apply l'hospital's rule if necessary; inconclusive if limit equals 0 or ∞
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$	$a_{n+1} \leq a_n$, $\lim_{n\to\infty} a_n = 0$	$\lim_{n\to\infty} a_n \neq 0$	must prove that the limit equals 0 and that terms are decreasing

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