Math 231E, 2013. Midterm 1.

- This exam has 25 questions, each worth 4 points.
- If you mark in the letter C in problem 42 on your scantron, you will receive two bonus points.
- You must not communicate with other students during this test. No books, notes, **calculators**, or electronic devices allowed.
- Please fill out all of the information below. Make sure to fill out your Scantron form as directed in class; fill in name, UIN number, and NetID.

1. Fill in your information:

Full Name: ______________________________

UIN (Student Number): ______________________________

NetID: ______________________________

2. Fill out name, student number (UIN) and NetID on Scantron sheet. Then fill in the following answers on the Scantron form:

89. A
90. A
91. A
92. A
93. A
94. D
95. C
96. E

1. (4 points) What is the third-order Taylor polynomial for $\sin(x)$ at $a = 0$?

(A) $1 + \sin(x) + \frac{\sin(x)^2}{2} + \frac{\sin(x)^3}{6}$

(B) $1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

(C) ★ $x - \frac{x^3}{6}$

(D) $1 - \frac{x^2}{2}$

(E) $1 + x + x^2 + x^3$

Solution. The definition of the third-order polynomial of $f(x)$ at $a = 0$ is

$$T_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3.$$

Noting that $f(x) = -f''(x) = \sin(x)$, and $f'(x) = -f'''(x) = \cos(x)$ we have

$$f'(0) = -f'''(0) = 1, \qquad f(0) = f''(0) = 0$$

so

$$T_3(x) = x - \frac{x^3}{6}.$$

---

2. (4 points) Compute the limit

$$\lim_{x \to 3} e^{4x}.$$

(A) ★ e^{12}

(B) e^3

(C) $4e^{12}$

(D) does not exist

(E) $4e^3$

Solution. This is a continuous function, so we can just plug in. Thus

$$\lim_{x \to 3} e^{4x} = e^{4 \cdot 3} = e^{12}.$$

---

3. (4 points) Evaluate $\lim\limits_{x\to\infty} \dfrac{\sin(2x)}{x}$.

(A) ★ 0

(B) $+\infty$

(C) 2

(D) 3

(E) does not exist

Solution. The definition of the limit at ∞ means

$$\lim_{x\to\infty} \frac{\sin(2x)}{x} = \lim_{x\to 0+} \frac{\sin(2/x)}{1/x} = \lim_{x\to 0+} x\sin(2/x).$$

We can use the Squeeze Theorem as such:

$$\begin{aligned} -1 &< \sin(2/x) < 1, \\ -x &< x\sin(2/x) < x, \end{aligned}$$

and

$$\lim_{x\to 0} x = \lim_{x\to 0}(-x) = 0,$$

so the limit is 0.

---

4. (4 points) Compute the limit

$$\lim_{x\to 2} \frac{x^2-3}{2x}$$

(A) 1

(B) $-\dfrac{3}{2}$

(C) 2

(D) does not exist

(E) ★ $\dfrac{1}{4}$

Solution. We can use the limit laws. We have

$$\lim_{x\to 2} x^2 - 3 = 1, \quad \lim_{x\to 2} 2x = 4,$$

and thus

$$\lim_{x\to 2} \frac{x^2-3}{2x} = \frac{1}{4}.$$

---

5. (4 points) Express the following complex number

$$z = \frac{1}{4+i}.$$

in the form $z = a + ib$.

(A) $z = 4 - i$

(B) ★ $z = \frac{4}{17} - \frac{i}{17}$

(C) $z = \frac{1}{3} - \frac{i}{5}$

(D) $z = \cos(4) + i\sin(1)$

(E) $z = \frac{1}{4} + \frac{i}{4}$

Solution. We use the formula

$$\frac{1}{z} = \frac{\overline{z}}{|z|^2} = \frac{4-i}{4^2+1^2} = \frac{4-i}{17}.$$

---

6. (4 points) What is the second-order Taylor series for $\dfrac{1}{1+x}$ at $a = 0$?

(A) ★ $1 - x + x^2 + O(x^3)$

(B) $1 - \dfrac{x^2}{2} + O(x^3)$

(C) $1 + \dfrac{1}{x} + \dfrac{1}{x^2} + O\left(\dfrac{1}{x^3}\right)$

(D) $1 + x + \dfrac{x^2}{2} + O(x^3)$

(E) $1 + x + x^2 + O(x^3)$

Solution. The definition of the second-order series of $f(x)$ at $a = 0$ is

$$T_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + O(x^3)$$

Noting that $f(x) = \dfrac{1}{1+x}$, $f'(x) = -\dfrac{1}{(1+x)^2}$, $f''(x) = \dfrac{2}{(1+x)^3}$, we have

$$1 - x + x^2 + O(x^3).$$

---

7. (4 points) $\lim_{x\to\infty} x^2 = \infty$. Suppose $M = 400$. How large must x be chosen to guarantee that $x^2 > M$

(A) $x < -4$

(B) $x > 10$

(C) ★ $x > 20$

(D) $x > 4$

(E) $x < 0$

Solution.

$$\begin{aligned} x^2 &> M = 400 \\ x &> \sqrt{M} = 20 \end{aligned}$$

---

8. (4 points) Compute $\dfrac{d}{dx}\sin(e^{x^2/2})$.

(A) $-e^{x^2/2}\cos(e^{x^2/2})$

(B) ★ $xe^{x^2/2}\cos(e^{x^2/2})$

(C) $-\cos(e^{x^2/2})$

(D) $e^{x^2/2}\cos(e^{x^2/2})$

(E) $\cos(e^{x^2/2})$

Solution. Chain Rule time! We have

$$\frac{d}{dx}\sin(e^{x^2/2}) = \cos(e^{x^2/2})\cdot\frac{d}{dx}e^{x^2/2} = \cos(e^{x^2/2})\cdot e^{x^2/2}\cdot\frac{d}{dx}\frac{x^2}{2} = x\cos(e^{x^2/2})e^{x^2/2}.$$

---

9. (4 points) Suppose that the function $f(x)$ has a Taylor series at $a = 0$ that begins $f(x) = 1 + 2x - \frac{5}{2}x^2 + \frac{11}{3}x^3 + O(x^4)$. What is the Taylor series for $f'(x)$ at $a = 0$?

(A) $f'(x) = 2 - 5x + 22x^2 + O(x^3)$

(B) $f'(x) = 3 + 6x + \frac{15}{2}x^2 + O(x^3)$

(C) ★ $f'(x) = 2 - 5x + 11x^2 + O(x^3)$

(D) $f'(x) = x + x^2 - \frac{5}{6}x^3 + O(x^4)$

(E) $f'(x) = 1 + x + x^2 - \frac{5}{6}x^3 + O(x^4)$

Solution. We can obtain this one of two ways: we can first differentiate the original series, and obtain

$$f'(x) = \frac{d}{dx}\left(1 + 2x - \frac{5}{2}x^2 + \frac{11}{3}x^3 + O(x^4)\right) = 2 - 5x + 11x^2 + O(x^3),$$

noting that any term with an x^3 in it will, after differentiation, have an x^2 in it.

Another approach is to reason indirectly. The fact that $f(x)$ has this Taylor series is equivalent to saying that

$$f(0) = 1, \quad f'(0) = 2, \quad \frac{f''(0)}{2} = -\frac{5}{2}, \quad \frac{f'''(0)}{6} = \frac{11}{3},$$

so that $f''(0) = -5$ and $f'''(0) = 22$. If we write $g(x) = f'(x)$, then

$$g(0) = f'(0) = 2, \quad g'(0) = f''(0) = -5, \quad g''(0) = f'''(0) = 22,$$

so

$$g(x) = g(0) + g'(0)x + \frac{g''(0)}{2}x^2 + O(x^3) = 2 - 5x + 11x^2 + O(x^3).$$

---

10. (4 points) Compute $\dfrac{d}{dx}(\arctan(x))$. (Recall that $\arctan(x)$ is also sometimes written $\tan^{-1}(x)$!)

(A) $\dfrac{x}{\sqrt{1-x^2}}$

(B) $\dfrac{x}{1+x^2}$

(C) $\dfrac{1}{1-x^2}$

(D) $\dfrac{1}{\sqrt{1-x^2}}$

(E) ★ $\dfrac{1}{1+x^2}$

Solution. If $y = \arctan(x)$, then $x = \tan(y)$. Differentiating both sides of this equation gives

$$1 = \sec^2(y)\frac{dy}{dx},$$

or

$$\frac{dy}{dx} = \cos^2(y) = \cos^2(\tan^{-1}(x)).$$

We need to draw a right triangle with an interior angle θ such that $\tan(\theta) = x$, and we do so by choosing the opposite side to have length x and the near side to have length 1. Then, by Pythagoras, the hypothenuse has length $\sqrt{1+x^2}$, so

$$\cos(\tan^{-1}(x)) = \sqrt{1+x^2}, \quad \cos^2(\tan^{-1}(x)) = 1 + x^2.$$

---

11. (4 points) Compute $f'(x)$, where
$$f(x) = x^2 \cos(2x).$$

(A) $x^2 \ln(2x)$

(B) $2x \cos(2x)$

(C) ★ $2x \cos(2x) - 2x^2 \sin(2x)$

(D) $-4x \sin(2x)$

(E) $x^2 \sin(2x) + 2\cos(2x)$

Solution. We use the product and chain rules, so
$$\frac{d}{dx}(x^2 \cos(2x)) = 2x \cos(2x) + x^2(-2\sin(2x)).$$

---

12. (4 points) What is the third-order Taylor polynomial for e^x at $a = 7$?

(A) ★ $e^7 + e^7(x-7) + \frac{e^7}{2}(x-7)^2 + \frac{e^7}{6}(x-7)^3$

(B) $3 + 3(x-7) + \frac{3}{2}(x-7)^2 + \frac{1}{2}(x-7)^3$

(C) $1 + e(x-7) + e^2(x-7)^2 + e^3(x-7)^3$

(D) $e^3 + e^3(x-3) + \frac{e^3}{2}(x-3)^2 + \frac{e^3}{6}(x-3)^3$

(E) $1 + (x-7) + \frac{1}{2}(x-7)^2 + \frac{1}{6}(x-7)^3$

Solution. The definition of the third-order polynomial of $f(x)$ at $a = 7$ is

$$T_3(x) = f(7) + f'(7)(x-7) + \frac{f''(7)}{2}(x-7)^2 + \frac{f'''(7)}{6}(x-7)^3.$$

Noting that $f(x) = f'(x) = f''(x) = f'''(x) = e^x$, we have

$$f(7) = f'(7) = f''(7) = f'''(7) = e^7,$$

so

$$T_3(x) = e^7 + e^7(x-7) + \frac{e^7}{2}(x-7)^2 + \frac{e^7}{6}(x-7)^3.$$

---

13. (4 points) Compute the limit $\lim_{x\to\infty} \frac{\sin(x)+x}{x^2}$

(A) 1

(B) ∞

(C) ★ 0

(D) $-\frac{1}{6}$

(E) does not exist

Solution. First write

$$\lim_{x\to\infty} \frac{\sin(x)+x}{x^2} = \lim_{x\to\infty} \frac{\sin(x)}{x^2} + \lim_{x\to\infty} \frac{1}{x}.$$

The first piece can be handled by a Squeeze Theorem, namely:

$$\lim_{x\to\infty} \frac{\sin(x)}{x^2} = \lim_{x\to 0+} x^2 \sin(1/x),$$

and

$$-1 < \sin(1/x) < 1,$$
$$-x^2 < x^2 \sin(1/x) < x^2,$$

and since $\lim_{x\to 0+} x^2 = 0$, the first limit is zero. The second piece is easier, since

$$\lim_{x\to\infty} \frac{1}{x} = \lim_{x\to 0+} x = 0.$$

Therefore

$$\lim_{x\to\infty} \frac{\sin(x)+x}{x^2} = \lim_{x\to\infty} \frac{\sin(x)}{x^2} + \lim_{x\to\infty} \frac{1}{x} = 0+0=0.$$

---

14. (4 points) Compute the limit

$$\lim_{x\to\infty} \frac{11x^{2013} + 57x^{1967} + 128x^{1492}}{22x^{2013} + 88x^{2001} + 195x^{1066}}.$$

(A) $\frac{128}{195}$

(B) does not exist

(C) $+\infty$

(D) 0

(E) ★ $\frac{1}{2}$

Solution. Divide both top and bottom by x^{2013}, and we obtain

$$\lim_{x\to\infty} \frac{11x^{2013} + 57x^{1967} + 128x^{1492}}{22x^{2013} + 88x^{2001} + 195x^{1066}} = \lim_{x\to\infty} \frac{11 + 57x^{-46} + 128x^{-521}}{22 + 88x^{-12} + 195x^{-947}} = \frac{11}{22} = \frac{1}{2}.$$

---

15. (4 points) Find L, where

$$\lim_{x\to 0}\frac{\sin(\alpha x)}{x} = L.$$

(A) ★ α

(B) $-\dfrac{a^3}{6}$

(C) 1

(D) 0

(E) L does not exist

Solution. We see that if we plug in, we get $\dfrac{0}{0}$, which is indeterminant; moreover, we cannot use a Limit Law at this stage. There are several ways to proceed.

One is l'Hôpital's Rule: Noting that

$$\lim_{x\to 0}\frac{\sin(\alpha x)}{x} = \lim_{x\to 0}\frac{\frac{d}{dx}\sin(\alpha x)}{\frac{d}{dx}x} = \lim_{x\to 0}\frac{\alpha\cos(\alpha x)}{1},$$

and this is one we can plug into, so we obtain a.

Alternately, we can use Taylor Series at 0, writing

$$\sin(x) = x - \frac{x^3}{6} + O(x^5),$$
$$\sin(\alpha x) = \alpha x - \frac{(\alpha x)^3}{6} + O((\alpha x)^5),$$

so

$$\frac{\sin(\alpha x)}{x} = \frac{\alpha x - \frac{(\alpha x)^3}{6} + O((\alpha x)^5)}{x} = \alpha - \frac{\alpha^3 x^2}{6} + O(x^4),$$

so when $x \to 0$ we obtain α.

---

16. (4 points) Compute the third-order Taylor series for $f(x) = e^x \cos(x)$ at $a = 0$.

(A) ★ $1 + x - \frac{x^3}{3} + O(x^4)$

(B) $1 + 2x + x^2 - \frac{x^3}{3} + O(x^4)$

(C) $2 + x - \frac{3}{2}x^2 + \frac{x^3}{6} + O(x^4)$

(D) $1 + x - x^2 + x^3 + O(x^4)$

(E) $2 + x + \frac{x^3}{6} + O(x^4)$

Solution. We expand out each piece of the product then multiply together. Since we want a final answer that involves the third-order term, we must expand each term to third order. So:

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + O(x^4), \\ \cos(x) &= 1 - \frac{x^2}{2} + O(x^4). \end{aligned}$$

Multiplying the two polynomials gives

$$\begin{aligned} \left((1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)\left(1 - \frac{x^2}{2}\right) &= \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) - \frac{x^2}{2}\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) \\ &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^2}{2} - \frac{x^3}{2} + O(x^4) \\ &= 1 + x - \frac{x^3}{3} + O(x^4). \end{aligned}$$

---

17. (4 points) Compute the Taylor series for

$$f(x) = \frac{1 - \cos(x)}{x^2}$$

to second order.

(A) $f(x) = \frac{1}{2} + \frac{x}{2} + \frac{x^2}{6} + O(x^3)$

(B) ★ $f(x) = \frac{1}{2} - \frac{x^2}{24} + O(x^4)$

(C) $f(x) = \frac{1}{6} - \frac{x^2}{120} + O(x^4)$

(D) $f(x) = \frac{1}{2} + \frac{x^2}{24} + O(x^4)$

(E) $f(x) = \frac{1}{6} + \frac{x^2}{120} + O(x^4)$

Solution. Use the known series for $\cos(x)$ and algebraic manipulations.

$$\begin{aligned}
\cos(x) &= 1 - \frac{x^2}{2} + \frac{x^4}{24} + O(x^6) \\
1 - \cos(x) &= \frac{x^2}{2} - \frac{x^4}{24} + O(x^6) \\
\frac{1 - \cos(x)}{x^2} &= \frac{1}{2} - \frac{x^2}{24} + O(x^4)
\end{aligned}$$

---

18. (4 points) Compute the third-order Taylor series for

$$f(x) = \frac{\sin(x) + 1 - \cos(x)}{x}.$$

(A) ★ $1 + \frac{x}{2} - \frac{x^2}{6} - \frac{x^3}{24} + O(x^4)$

(B) $x - \frac{x^3}{6} + O(x^4)$

(C) $1 + x + \frac{x^2}{2} + O(x^4)$

(D) $1 + x + x^2 + x^3 + O(x^4)$

(E) $1 - \frac{x^2}{6} + O(x^4)$

Solution. Using the standard series

$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} + O(x^7),$$
$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} + O(x^6),$$

we have

$$\sin(x) + 1 - \cos(x) = x - \frac{x^3}{6} + \frac{x^5}{120} + \frac{x^2}{2} - \frac{x^4}{24} + O(x^6),$$

and thus

$$\frac{\sin(x) + 1 - \cos(x)}{x} = 1 + \frac{x}{2} - \frac{x^2}{6} - \frac{x^3}{24} + O(x^4).$$

---

19. (4 points) Compute the Limit $\lim_{x\to 0^-} \frac{|x|}{\sin(2x)}$

(A) 0

(B) does not exist

(C) $+\infty$

(D) $+\frac{1}{2}$

(E) ★ $-\frac{1}{2}$

Solution. For $x < 0$ we have $|x| = -x$ and apply l'Hôpital.

---

20. (4 points) Recall the notation that $\exp(A) = e^A$ for any expression A. Then find L, where

$$\lim_{x \to 0} \frac{\sin(x^4) - x^4}{3\exp(x^{12}) - 2} = L.$$

(A) $\frac{1}{6}$

(B) does not exist

(C) 1

(D) ★ $-\frac{1}{18}$

(E) $-\frac{1}{3}$

Solution. Plugging in gives $\frac{0}{0}$, so we need some technique. l'Hôpital's Rule is applicable, but will get us nowhere. So we use Taylor Series. Expanding the numerator gives

$$\begin{aligned} \sin(x) &= x - \frac{x^3}{6} + O(x^5), \\ \sin(x^4) &= x^4 - \frac{x^{12}}{6} + O(x^{20}), \\ \sin(x^4) - x^4 &= -\frac{x^{12}}{6} + O(x^{20}). \end{aligned}$$

Expanding the denominator gives

$$\begin{aligned} 3e^x &= 3 + 3x + \frac{3}{2}x^2 + O(x^3), \\ 3e^{x^{12}} &= 3 + 3x^{12} + \frac{3}{2}x^{24} + O(x^{36}), \\ 3e^{x^{12}} - 3 &= 3x^{12} + \frac{3}{2}x^{24} + O(x^{36}), \end{aligned}$$

so

$$\frac{\sin(x^4) - x^4}{3\exp(x^{12}) - 3} = \frac{-\frac{x^{12}}{6} + O(x^{20})}{3x^{12} + \frac{3}{2}x^{24} + O(x^{36})} = \frac{-\frac{1}{6} + O(x^{14})}{3 + \frac{3}{2}x^{21} + O(x^{42})},$$

and as $x \to 0$ this limit is clearly $-1/18$.

---

21. (4 points) Compute the limit $\lim_{x\to\infty} x^{16}e^{-x}$.

(A) $-\infty$

(B) ★ 0

(C) $\frac{1}{12!}$

(D) 1

(E) ∞

Solution. There are many ways to approach this problem. One of them is to use the result proved in class, namely that $\lim_{x\to\infty} x^n e^{-x} = 0$ for any integer n.

Another way is to use the ideas behind that result, and notice that

$$e^x > \frac{x^{17}}{17!},$$

since that is just one term in the Taylor series, and so we have

$$\frac{x^{16}}{e^x} < \frac{17!x^{16}}{x^{17}} = \frac{17!}{x}.$$

Since $\lim_{x\to\infty} \frac{17!}{x} = 0$, and since $x^{16}e^{-x} > 0$, this means by the Squeeze Theorem that our limit is 0 as well.

Finally, one could apply l'Hôpital's Rule. However, it would take 16 applications to do this, but it would again give the correct answer.

---

22. (4 points) Let $y = f(x)$ be defined implicitly by

$$y^3 - xy = 8.$$

Find $y' = f'(x)$ in terms of y and x.

(A) $y' = \dfrac{-y}{3y^2 - x}$

(B) $y' = \dfrac{8}{3y^2 - x}$

(C) ★ $y' = \dfrac{y}{3y^2 - x}$

(D) $y' = \arctan(x/y)$

(E) $y' = \dfrac{8 + xy}{3y^2}$

Solution. The derivative is given by implicit differentiation

$$\begin{aligned} y^3 - xy &= 8 \\ 3y^2\frac{dy}{dx} - x\frac{dy}{dx} - y &= 0 \\ \frac{dy}{dx} = \frac{y}{3y^2 - x} \end{aligned}$$

---

23. (4 points) Compute L, where

$$L = \lim_{x \to 0} \frac{e^x \cos(x) - 1 - x}{x^3}$$

(A) -1

(B) $1/2$

(C) ★ $-1/3$

(D) does not exist

(E) 1

Solution. Let us first compute a Taylor series of the numerator; of course, we have already done this problem earlier.

We expand out each piece of the product then multiply together. Since we want a final answer that involves the third-order term, we must expand each term to third order. So:

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + O(x^4), \\ \cos(x) &= 1 - \frac{x^2}{2} + O(x^4). \end{aligned}$$

Multiplying the two polynomials gives

$$\begin{aligned} \left((1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)\left(1 - \frac{x^2}{2}\right) &= \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) - \frac{x^2}{2}\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) \\ &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^2}{2} - \frac{x^3}{2} + O(x^4) \\ &= 1 + x - \frac{x^3}{3} + O(x^4). \end{aligned}$$

Then we have

$$e^x \cos(x) - 1 - x = -\frac{x^3}{3} + O(x^4),$$

so

$$\frac{e^x \cos(x) - 1 - x}{x^3} = -\frac{1}{3} + O(x),$$

and clearly as $x \to 0$, this limit is $-1/3$.

---

24. (4 points) Let $y = f(x)$ be defined implicitly by

$$y^3 - xy = 27.$$

and suppose that $y(0) = 3$. Find the first two terms of the Taylor series for y about the point 0.

(A) $y = 1 + \frac{x}{81} + O(x^2)$

(B) ★ $y = 3 + \frac{x}{9} + O(x^2)$

(C) $y = 3 - \frac{x}{12} + O(x^2)$

(D) $y = 1 + x + 2x^2 + O(x^3)$

(E) $y = 3 - \frac{2x}{9} + O(x^2)$

Solution. The derivative is given implicitly by

$$\begin{aligned} y^3 - xy &= 27 \\ 3y^2\frac{dy}{dx} - x\frac{dy}{dx} - y &= 0 \\ \frac{dy}{dx} &= \frac{y}{3y^2 - x} \end{aligned}$$

Plugging in $y = 3, x = 0$ gives

$$\left.\frac{dy}{dx}\right|_{y=3,x=0} = \frac{3}{3 \cdot 3^2 - 0} = \frac{3}{27} = \frac{1}{9}.$$

Thus

$$y(x) = y(0) + x\frac{dy}{dx}(0) + O(x^2) = 3 + \frac{x}{9} + O(x^2)$$

---

25. (4 points) Compute

$$L = \lim_{x \to 0} \cos\left(\frac{\pi \sin(7x)}{7x}\right).$$

(A) $L = 1$

(B) L does not exist

(C) ★ $L = -1$

(D) $L = 0$

(E) $L = \infty$

Solution. Let us first look at the "inside" part of the function, namely $\sin(7x)/7x$. We compute:

$$\lim_{x \to 0} \frac{\sin(7x)}{7x} = \lim_{x \to 0} \frac{7\cos(7x)}{7} = \frac{7}{7} = 1.$$

Since $\sin(x)$ is continuous,

$$\lim_{x \to 0} \cos\left(\frac{\pi \sin(7x)}{7x}\right) = \cos\left(\lim_{x \to 0} \pi \frac{\sin(7x)}{7x}\right) = \cos(\pi) = -1.$$
