j3drsk

§ Lecture 2

Last time: linear equation, linear system, fundamental questions, elem row ops

row reduction

Example:

$$\begin{cases} x_2 - x_3 = 0 \\ x_1 + 2x_2 + 3x_3 = 1 \\ x_1 + x_2 + 4x_3 = 1 \end{cases} \xrightarrow{R_1 \leftrightarrow R_2} \begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ x_2 - x_3 = 0 \\ \boxed{x_1} + x_2 + 4x_3 = 1 \end{cases} \xrightarrow[\text{new } R_3]{-R_1+R_3}$$

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ x_2 - x_3 = 0 \\ \boxed{-x_2} + x_3 = 0 \end{cases} \xrightarrow[\text{new } R_3]{R_2+R_3} \begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ x_2 - x_3 = 0 \\ 0 = 0 \end{cases}$$

let $x_3 = r \in \mathbb{R}$ free variable

$x_2 = x_3 = r$

$x_1 = 1 - 2x_2 - 3x_3$

$= 1 - 2r - 3r = 1 - 5r$

solutions $\begin{cases} x_1 = 1-5r \\ x_2 = r \\ x_3 = r \end{cases}$ $r \in \mathbb{R}$

Existence: ✓

Uniqueness: ✗

augmented matrix

$$\left(\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 4 & 1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & -1 & 0 \\ \boxed{1} & 1 & 4 & 1 \end{array}\right) \xrightarrow[\text{new } R_3]{-R_1+R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & \boxed{-1} & 1 & 0 \end{array}\right) \xrightarrow[\text{new } R_3]{R_2+R_3}$$

pivots

$$\left(\begin{array}{ccc|c} \overset{x_1}{1} & \overset{x_2}{2} & \overset{x_3}{3} & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

row-echelon form (REF)

x_1, x_2 pivot variables

x_3 free variable

Example:

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

Obstruction to existence

x_2 free variable

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right)$$

Existence:	✗	✓	✓
Uniqueness:	—	✗	✓

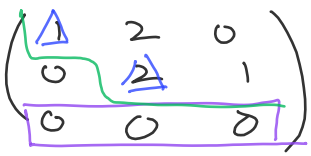
Moral: Fundamental questions are "easy" for systems in REF

- existence: look for obstruction
- uniqueness: look for free variables

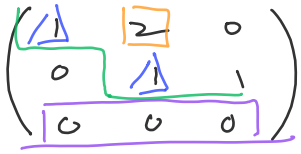
Special REF called <u>reduced row-echelon form</u> (RREF)

REF + { ① all pivots = 1
② every entry above each pivot = 0

Example:



REF not RREF



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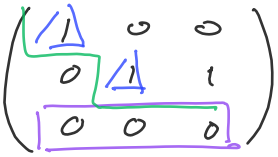

REF also RREF