Math 231E, 2013. Midterm 2.

- This exam has 30 questions.
- You must not communicate with other students during this test. No books, notes, **calculators**, or electronic devices allowed.
- Please fill out all of the information below. Make sure to fill out your Scantron form as directed in class; fill in name, UIN number, and NetID.
- October is Pumpkin Month. Draw a happy pumpkin somewhere on the test booklet for good luck.

1. Fill in your information:

Full Name: ________________________________

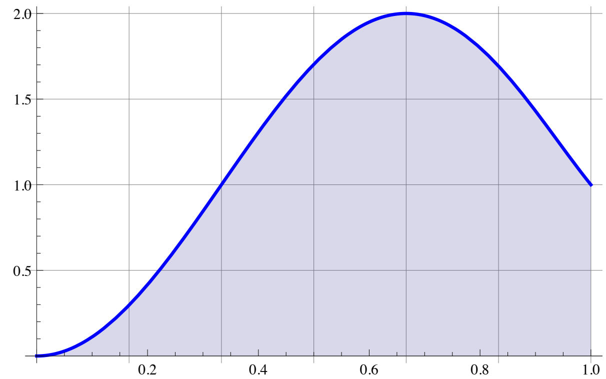
UIN (Student Number): ________________________________

NetID: ________________________________

2. Fill out name, student number (UIN) and NetID on Scantron sheet. Then fill in the following answers on the Scantron form:

89. E
90. E
91. C
92. A
93. B
94. E
95. B
96. B

1. (3 points) Give your best estimate for the **lower** Riemann sum for the following area between $x = 0$ and $x = 1$ with $n = 3$.



(A) $\frac{4}{3}$

(B) $\frac{5}{3}$

(C) $\frac{2}{3}$

(D) $\frac{1}{3}$

(E) 2

2. (3 points) Compute the integral $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

(A) $2\cos(\sqrt{x}) + C$

(B) $-2\cos(\sqrt{x}) + C$

(C) $-\sqrt{x}\sin(\sqrt{x}) + C$

(D) $\sqrt{x}\sin(\sqrt{x}) + C$

(E) $\sqrt{x}\sin(\sqrt{x}) - \frac{\cos(\sqrt{x})}{\sqrt{x}} + C$

3. (3 points) What is the coefficient of the x^{13} term in the Taylor series for $\dfrac{1}{1+2x}$ at $a=0$?

(A) 0

(B) -2^{13}

(C) $(2)^{13}$

(D) -1

(E) 1

4. (3 points) Compute the area of the region that is above the x-axis, below the line $y=\frac{x}{2}$ and above the curve $y=x^2$.

(A) $\dfrac{1}{48}$

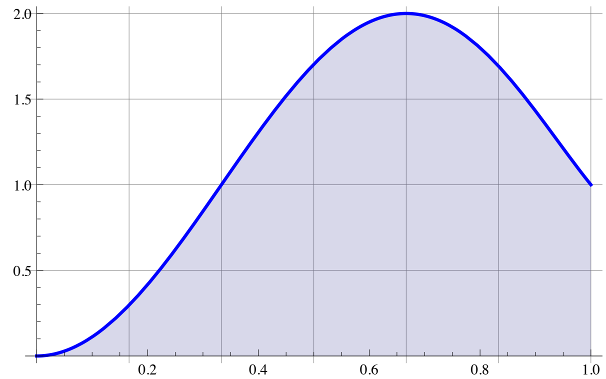
(B) $\dfrac{1}{24}$

(C) $\dfrac{7}{12}$

(D) $-\dfrac{11}{24}$

(E) $\dfrac{5}{12}$

5. (3 points) Give your best estimate for the **upper** Riemann sum for the following area between $x = 0$ and $x = 1$ with $n = 3$.



(A) $\frac{1}{3}$

(B) $\frac{4}{3}$

(C) $\frac{2}{3}$

(D) 2

(E) $\frac{5}{3}$

6. (3 points) Let $f(x)$ be the function

$$f(x) = \int_{-2}^{x} \sin(2t)\, dt.$$

Compute $f'(x)$.

(A) $2\sin(2x)$

(B) $\dfrac{1}{2}\cos(x) - \dfrac{1}{2}\cos(-2)$

(C) $\sin(2x)$

(D) $\dfrac{1}{2}\cos(2x) - \dfrac{1}{2}\cos(-4)$

(E) $2\cos(2x)$

7. (4 points) Compute the antiderivative

$$\int x\cos(x^2)\, dx.$$

(A) $\sin(x^2) + C$

(B) $\cos(x^2) + C$

(C) $\frac{1}{2}\sin(x^2) + C$

(D) $x\cos(x^2) + C$

(E) $\frac{1}{2}\cos(x^2) + C$

8. (3 points) Suppose that the function $f(x)$ is continuous on $[0,1]$ and differentiable on $(0,1)$, and that $f(0)=1$ and $f(1)=0$. Which of the following statements are guaranteed to be true.

1 $f(x)$ is monotone decreasing.

2 There is a point $b \in (0,1)$ such that $f(b)=1/2$

3 There is a point $c \in (0,1)$ such that $f'(c)=-1$

4 There is a point $d \in (0,1)$ such that $f'(d)=1/2$

5 There is a point $e \in (0,1)$ such that $f(e)=-1$

(A) Statements 1,3 and 5 must be true.

(B) Statements 2 and 3 must be true.

(C) Statements 4 and 5 must be true.

(D) Statements 1,2, and 4 must be true.

(E) None of these statements are guaranteed to be true.

9. (3 points) Compute the integral

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot(x)dx$$

(A) $\frac{\pi}{4}$

(B) $\frac{8}{3}$

(C) $\frac{\pi^2}{6}$

(D) $\frac{\ln(2)}{2}$

(E) $-\frac{1}{12}$

10. (4 points) Evaluate the following integral

$$\int x^2e^{-3x}dx$$

(A) $\frac{x^3}{3} - \frac{e^{-3x}}{3} + C$

(B) $-\frac{e^{-3x}}{27}(2x^2 - 9x + 6) + C$

(C) $\frac{x^3}{3}\frac{e^{-3x}}{-3} + C$

(D) $-\frac{e^{-3x}}{27}(9x^2 + 6x + 2) + C$

(E) $\frac{e^{-3x}}{27}(9x^2 - 6x + 2) + C$

11. (4 points) If we know that

$$\int_0^2 f(x)\,dx = 3, \quad \int_0^4 f(x)\,dx = 4, \quad \int_{-2}^6 f(x)\,dx = 5, \quad \int_{-2}^0 f(x)\,dx = 7,$$

then compute A, where

$$A = \int_2^6 f(x)\,dx.$$

(A) $A = -5$

(B) $A = 1$

(C) $A = 11$

(D) $A = -6$

(E) $A = -2$

12. (3 points) What is the smallest surface area possible for a cylinder with a circular base that has volume equal to $16\pi\, cm^3$? (The surface area should include the area of the top and bottom of the cylinder.)

(A) $27\pi\, cm^2$

(B) $16\pi\, cm^2$

(C) $2\, cm^2$

(D) $24\pi\, cm^2$

(E) $54\pi\, cm^2$

13. (3 points) What is the correct partial fractions form to simplify the integral

$$\int \frac{2x+1}{x^3(x^2+1)} dx$$

(A) $\frac{2x+1}{x^3(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+1} + \frac{E}{x-1}$

(B) $\frac{2x+1}{x^3(x^2+1)} = A + Bx + \frac{C}{x^3} + \frac{Dx+E}{x^2+1}$

(C) $\frac{2x+1}{x^3(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+1}$

(D) $\frac{2x+1}{x^3(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^2+1}$

(E) $\frac{2x+1}{x^3(x^2+1)} = \frac{A}{x^3} + \frac{Bx+C}{x^2+1}$

14. (3 points) Which of these five choices are the same as

$$\int_0^{\pi/2} e^{\cos^2(x)} \sin(x)\, dx.$$

(A) $e^{u^2} + C$

(B) $-\int_0^1 e^{u^2}\, du$

(C) $\int_0^{\pi/2} e^{u^2}\, du$

(D) $\int_0^{\pi/2} e^{u}\, du$

(E) $\int_0^1 e^{u^2}\, du$

15. (4 points) Compute

$$L = \lim_{x \to 1} \frac{\sin(x^2 - 1)}{x - 1}$$

(A) $L = 2$

(B) $L = 1$

(C) $L = 1/2$

(D) L does not exist

(E) $L = \infty$

16. (4 points) A 8 m long ladder is propped up against a wall. The ladder begins to slip. At time $t = 3\,\mathrm{s}$, the base of the ladder is 6 m from the wall and moving away from the wall at 7m/s. How fast is the end of the ladder moving along the wall?

(A) $-24\sqrt{5}\,\mathrm{m/s}$

(B) $-\dfrac{30}{8}\ \mathrm{s}$

(C) $24\sqrt{5}\,\mathrm{m/s}$

(D) $3\sqrt{7}\,\mathrm{m/s}$

(E) $-3\sqrt{7}\,\mathrm{m/s}$

17. (4 points) A farmer wants to build a rectangular field next to a river. The farmer will use the river as one side of the rectangle, but must build the other three sides of the rectangle with fence. There is a total amount of 180 m of fence available. What is the largest possible area that can be enclosed?

(A) $2025\,\mathrm{m}^2$

(B) $3600\,\mathrm{m}^2$

(C) $4050\,\mathrm{m}^2$

(D) $32400\,\mathrm{m}^2$

(E) $5400\,\mathrm{m}^2$

18. (4 points) We want to estimate

$$\int_0^1 x^3\,dx,$$

with a Riemann sum of $n = 3$ terms. Let us define L_3 as the Riemann sum if we choose the left endpoints, and R_3 if we choose the right endpoints. Then:

(A) $L_3 = \frac{15}{81}, \quad R_3 = \frac{41}{81}$

(B) $L_3 = \frac{2}{9}, \quad R_3 = \frac{5}{9}$

(C) $L_3 = \frac{4}{27}, \quad R_3 = \frac{11}{27}$

(D) $L_3 = \frac{7}{27}, \quad R_3 = \frac{4}{27}$

(E) $L_3 = \frac{1}{9}, \quad R_3 = \frac{4}{9}$

19. (4 points) What is the best substitution to make in the integral

$$\int \frac{dx}{(x^2+9)^{\frac{5}{2}}}.$$

(A) $x = 3\sec(t)$

(B) $x = 3\sin(t)$

(C) $x = 3\tan(t)$

(D) $x = \tan(t)$

(E) $x = \sec(3t)$

20. (3 points) Compute

$$\int_0^{\pi/2} \sin^3(x)\cos^2(x)\,dx.$$

(A) $\frac{1}{3} - \frac{1}{5}$

(B) 0

(C) 1

(D) $\frac{1}{3}$

(E) $\frac{\pi}{6}$

21. (4 points) Find

$$\int \frac{1}{x^2\sqrt{x^2+9}}\,dx$$

(A) $\frac{9x}{\sqrt{x^2+9}} + C$

(B) $-\frac{9x}{\sqrt{x^2+9}} + C$

(C) $\frac{9x}{x^2+9} + C$

(D) $\frac{\sqrt{x^2+9}}{9x} + C$

(E) $-\frac{\sqrt{x^2+9}}{9x} + C$

22. (3 points) Consider the polynomial $P(x) = x^3 - 6x^2 + 4x + 6$. Let A, B, C, D, E denote the following intervals

- $A = [0, 1]$
- $B = [1, 2]$
- $C = [2, 3]$
- $D = [3, 4]$
- $E = [4, 5]$

In which intervals is the polynomial $P(x)$ guaranteed to have a root

(A) Intervals A, C and D must each contain a root.

(B) Intervals B,C and E must each contain a root.

(C) Intervals A,D must each contain a root.

(D) Intervals B and E must each contain a root.

(E) Intervals A and B must each contain a root.

23. (3 points) Let us define $f(x)$ by

$$f(x) = \int_{\sin(x^2)}^{\cos(x^2)} e^t \, dt.$$

Compute $f'(x)$.

(A) $2x(e^{\cos(x^2)} - e^{\sin(x^2)})$

(B) $e^{\cos(x^2)} - e^{\sin(x^2)}$

(C) $-2x(e^{\cos(x^2)} \cos(x^2) + e^{\sin(x^2)} \sin(x^2))$

(D) $-2x(e^{\cos(x^2)} \cos(x^2) - e^{\sin(x^2)} \sin(x^2))$

(E) $-2x(e^{\cos(x^2)} \sin(x^2) + e^{\sin(x^2)} \cos(x^2))$

24. (3 points) Compute

$$\lim_{x \to 1} \left(\frac{\sin(\frac{\pi}{4}x) - 2^{-1/2}}{\int_1^x \sin(\frac{\pi}{4}t) \, dt} \right).$$

(A) π

(B) $\pi/4$

(C) 0

(D) $\pi/2$

(E) ∞

25. (3 points) Compute the following definite integral

$$\int_0^{\frac{\pi}{4}} \sec(x)\tan^2(x)dx$$

(A) $\frac{1}{2}(1+\ln(1+\sqrt{2}))$

(B) 1

(C) $\frac{\sqrt{2}}{2} - \frac{1}{2}\ln(1+\sqrt{2})$

(D) $\frac{\pi}{3}$

(E) $\frac{\pi}{2}$

26. (3 points) Compute the following indefinite integral

$$\int \ln(1+x^2)dx$$

(A) $x\ln(1+x^2) + 2\arctan(x) - 2x + C$

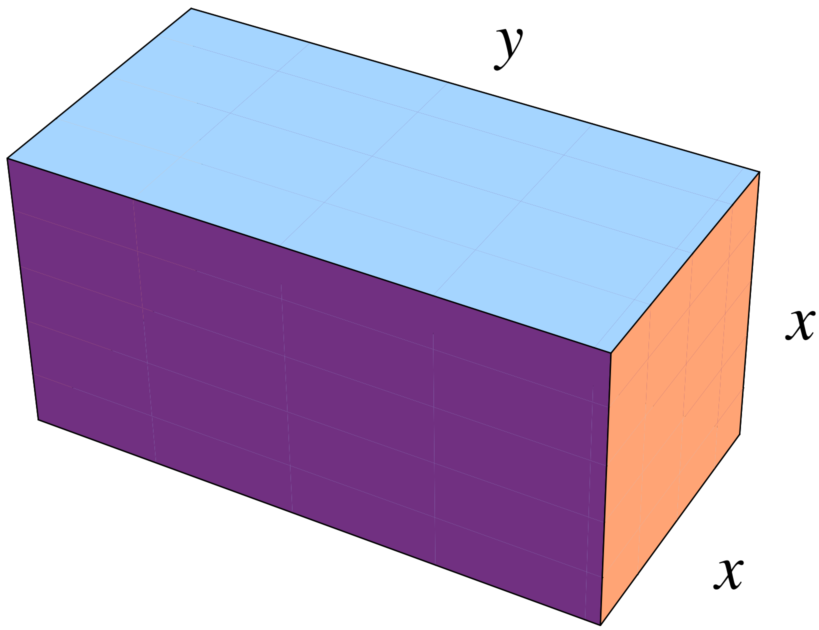
(B) $\ln(\arctan(x)) + C$

(C) $\ln(\frac{1+x}{1-x}) + C$

(D) $x\ln(\arctan(x)) - \frac{1}{1+x^2} + C$

(E) $x\ln(1+x^2) - 2x + C$

27. (3 points) The length of a rectangular box is defined to be the length of the longest side, and the girth is defined to be the circumference in the other two directions. For instance a $1\,\mathrm{m} \times 2\,\mathrm{m} \times 3\,\mathrm{m}$ rectangular box has length $3\,\mathrm{m}$ and girth$1 + 2 + 1 + 2 = 6\,\mathrm{m}$.
Suppose that a box has a square cross section, as shown in the illustration, and that the sum of the girth and the length is exactly 1 m. What is the largest possible volume of the box?



(A) $\frac{3}{144}\,\mathrm{m}^3$

(B) $\frac{\sqrt{3}}{17}\,\mathrm{m}^3$

(C) $\frac{1}{81}\,\mathrm{m}^3$

(D) $\frac{1}{64}\,\mathrm{m}^3$

(E) $\frac{1}{108}\,\mathrm{m}^3$

28. (3 points) Compute

$$\int_0^{\frac{\sqrt{3}}{2}} \frac{x^3}{\sqrt{1-x^2}} dx$$

(A) $\frac{\pi}{6}$

(B) $\frac{1}{2}$

(C) $\frac{5}{24}$

(D) 1

(E) $\frac{\pi}{4}$

29. (3 points) Evaluate the integral

$$\int \frac{\sqrt{x^2-1}}{x} dx$$

(A) $\sqrt{x^2-1} - \operatorname{arcsec}(x) + C$

(B) $x\sqrt{x^2-1} + \operatorname{arcsec}(x) + C$

(C) $x\sqrt{x^2-1} - \operatorname{arcsec}(\sqrt{x^2-1}) + C$

(D) $\sqrt{x^2-1} - \operatorname{arcsec}(\frac{1}{x}) + C$

(E) $\sqrt{x^2-1} + \operatorname{arcsec}(\frac{1}{\sqrt{x^2-1}}) + C$

30. (4 points) Compute the following definite integral

$$\int_0^1 \frac{dx}{(x+4)(x+6)}$$

(A) $\ln(\sqrt{\frac{21}{20}})$

(B) $\ln(\frac{5}{4})$

(C) $\ln(\frac{15}{14})$

(D) $\ln(\frac{21}{20})$

(E) $\ln(\sqrt{\frac{15}{14}})$

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